Abstract

The ordered alternatives in a one-way layout with \( k \) ordered treatment levels are appropriate for many applications, especially in psychology and medicine. There is an extensive literature in this area, and many parametric and nonparametric approaches have been introduced. This study uses rank based estimators to robustify the Abelson-Tukey method. The approach extends to the two-way layout with \( h \) blocks and \( k \) treatments. One of the two statistics having maximum asymptotic local power and the greatest efficiency when the alternative hypothesis consists of a specific pattern, and extended research has been completed on detecting ordered alternatives with unknown peaks.

Introduction

The null hypothesis of interest is that there are no differences in locations. The alternatives considered that there is a trend (increasing, decreasing or umbrella) in the locations. For example, consider a randomized group experiment with five levels of the independent variables, where the treatments are doses of a drug (say, 10, 20, 30, 40 and 50). With increasing doses level, the performance of the treatment improves. This is the alternative of interest.

Suppose that \( Y_{ij} \) is distributed with continuous cdf \( F(y - \theta_j) \) for \( j = 1, \ldots, k \) and \( i = 1, \ldots, \nu_i \) and the observations are mutually independent. The null and alternative hypotheses are

\[
H_0: \theta_1 = \cdots = \theta_k \quad \text{versus} \quad H_1: \theta_1 = \cdots = \theta_{\nu_0} < \theta_j, (j > 0), \quad j = 1, \ldots, k,
\]

where \( \nu_1, \cdots, \nu_k \) is a given set of constants that specifies the pattern of the alternative.

Initial empirical studies on existing methods show that the Abelson-Tukey (AT) method, though, is not robust and not more powerful than the Jonckheere-Terpstra (JT), the Spearman (SP) and the Hettmansperger-Norton (HN) nonparametric tests at normal errors for moderate sample sizes, and is even worse with heavy-tailed errors. But the AT test, is easily extended to general linear and mixed models.

Method I: RAT

A general linear model can be written as

\[
Y = a + X\beta + \epsilon,
\]

where \( Y \) is the \( n \times 1 \) vector of responses, \( X \) is the \( n \times (k - 1) \) design matrix, and \( \epsilon \) is the \( n \times 1 \) vector of error terms. Based on the dispersion function of Jaeckel (1972), a rank-based estimator of \( \beta \) is consistent and asymptotically normal (Hettmansperger and McKean, 2011), and can be summarized as

\[
\hat{\beta} = \frac{1}{N} \sum_{j=1}^{k} (Y_j - \bar{Y}) (X_j - \bar{X})
\]

where \( \bar{Y} \) and \( \bar{X} \) are the sample means of the response and the explanatory variables, respectively.

The null hypothesis of interest is that there are no differences in locations. The alternative hypothesis is given by

\[
H_1: \theta_1 = \cdots = \theta_{\nu_0} < \theta_j, (j > 0), \quad j = 1, \ldots, k
\]

where \( \theta_1, \cdots, \theta_k \) are the unknown parameters. The Pitman efficacy of the test, which is the greatest efficiency when the alternative hypothesis consists of a specific pattern, and extended research has been completed on detecting ordered alternatives with unknown peaks.

Method II: RSM

The Mann-Whitney-Wilcoxon estimates of shifts \( \Delta_i \) are estimators that are used in RSM.

\[
\Delta_i = \text{med}(Y_i - Y_j) - \text{med}(Y_j - Y_i)
\]

where \( \text{med}(\cdot) \) is the sample median.

Under \( H_1: \theta_1 = \cdots = \theta_{\nu_0} < \theta_j, (j > 0), \quad j = 1, \ldots, k, \) and \( X \) has a density function with \( f(X|\theta, \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \),

\[
\Delta_i = \frac{1}{\sigma \sqrt{2\pi}} \int_{\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx
\]

The asymptotic power of the test based on RSM is given by

\[
P(\Delta_i \geq z_{1-\alpha}) = 1 - \Phi(z_{1-\alpha} - \Delta_i)
\]

where \( \Phi \) is the standard normal cdf.

References