Middle Terminal Cell Models for Efficient Over-the-Cell Routing

Siddharth Bhingarde

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MIDDLE TERMINAL CELL MODELS FOR EFFICIENT OVER-THE-CELL ROUTING

by

Siddharth Bhingarde

A Thesis
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Faculty of The Graduate College
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MIDDLE TERMINAL CELL MODELS FOR EFFICIENT OVER-THE-CELL ROUTING

Siddharth Bhingarde, M.S.
Western Michigan University, 1993

In this thesis, we introduce a new class of cell models called Middle Terminal Models (MTM) which combines the advantages of existing cell models: BTM and CTM. This class contains the models 2MTM, 3MTM-V, and 3MTM+V depending on the number of metal layers and the permissibility of vias in over-the-cell areas. In MTM, two rows of terminals are located in the middle of the cell. This partitions over-the-cell area into three regions and allows allocation of more nets to over-the-cell area in congested channels. We prove that when vias are allowed over-the-cell, in “almost all” cases the MTM based layouts have smaller overall height as compared to the layouts based on the existing cell models.

We have implemented two MTM routers: MTM+V and MTM-V. MTM-V router, which does not allow vias in over-the-cell areas, is based on two key algorithms. First, we develop an approximation algorithm to select a planar set of nets for routing between two terminal rows of a cell row. Second, we develop an optimal algorithm for planar routing between the terminal row and the cell boundary. MTM+V router, which allows vias in over-the-cell areas, is based on two key algorithms: an optimal algorithm for terminal row selection and a greedy routing algorithm for over-the-cell and channel routing.

Experimentally, MTM based layouts are significantly better than the layouts based on existing cell models, irrespective of permissibility of vias over-the-cell.
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I would like to thank my colleague Anand Panyam. The results presented in this thesis are based on our collaboration. I would also like to thank the members of “Nite Group” for their cooperation and friendship.

Siddharth Bhingarde
To My Parents
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CHAPTER I

INTRODUCTION

The goal of a routing algorithm in standard cell environment is to minimize the area used in routing and thus reduce the die size of the chip. In standard cell design, traditional approaches to detailed routing have considered routing of nets within the channel and several algorithms have been proposed which obtain near optimal layout (See [13], Chapter 7). Recently, a new approach to standard cell routing, called over-the-cell routing, has been proposed [5, 6, 7, 8, 9]. The basic idea of over-the-cell routing is to partition the given net set into two sets. The first set of nets is routed in over-the-cell areas, while the second set is routed in the channel. The main objective of this approach is to route as many nets as possible over-the-cell in order to reduce the channel height.

Over-the-cell routing problem is clearly NP-Hard, since it is a generalization of a known intractable problem, the channel routing problem [11]. In view of NP-Hardness of the over-the-cell routing problem, several heuristic approaches have been proposed for the two and three metal layer process [5, 6, 7, 8, 9]. For example, the routers presented in [6, 9] produce a 20% reduction in the channel height as compared to a conventional two-layer channel router. The router presented in [8] produces a 35% reduction in channel height as compared to a two-layer channel router. Recently, a three layer over-the-cell router has been reported, which produces 74% better results than a conventional two layer router [8].

The effectiveness of over-the-cell routing depends on the cell model as it determines the amount of over-the-cell area and its utilization [6, 7, 8, 15]. The cell model specifies the location of terminals, the location of \( VDD \) and \( GND \) lines, number of metal layers and permissibility of vias in over-the-cell areas. The
first two specifications are related to the cell layout, whereas, the last two are related to the fabrication process used. Based on the location of terminals, two classes of cell models have been proposed: Boundary Terminal Models (BTM) and Center Terminal Models (CTM) [6, 7, 8, 15]. These two classes contain several cell models (2BTM, 3BTM-V, 3BTM+V, 2CTM, 3CTM-V, 3CTM+V) based on the number of metal layers and permissibility of vias in over-the-cell areas. When vias are not allowed over-the-cell, BTM models are suitable for the layout of the channels with large set of pairwise independent nets, whereas, CTM based layouts are suitable for channels with long vertical constraint chains. However, in practice the channel height may depend on both the size of pairwise independent nets and length of longest vertical constraint chain. As a result, neither BTM nor CTM based layouts are suitable for practical examples. In addition, neither cell model is amiable to high performance designs and may lead to longer wire lengths for critical nets. As a result, it is necessary to develop new cell models. In addition, it would be necessary to develop routers which take advantage of new cell models for high performance circuits.

In this thesis, we introduce a new class of cell models called Middle Terminal Models (MTM) which combines the advantages of BTM and CTM. This class contains the models 2MTM, 3MTM-V, and 3MTM+V depending on the number of metal layers and the permissibility of vias in over-the-cell areas. In MTM, two rows of terminals are located in the middle of the cell. This partitions over-the-cell area into three regions and allows allocation of more over-the-cell area to nets in congested channels. We prove that when vias are allowed over-the-cell, in "almost all" cases the MTM based layouts have smaller overall height as compared to the layouts based on the existing cell models.

Experimentally, MTM based layouts are significantly better than the layouts based on existing cell models, irrespective of permissibility of vias over-the-cell. We have implemented two MTM routers: MTM+V and MTM-V. The first
router allows vias over-the-cell, whereas, the second does not. MTM-V router is based on two key algorithms. First, we develop a $\frac{1}{2}\min\{1, \frac{k_2}{d}\}$ approximation algorithm to select a planar set of nets for routing between two terminal rows of a cell row, where $k_2$ is the number of tracks between two terminal rows of a cell row, and $d$ is the optimal number of tracks required for routing an intermediate set of nets in a planar fashion, without use of doglegs. Second, we develop an optimal algorithm for planar routing between the terminal row and the cell boundary. Similarly, MTM+V router is based on two key algorithms: an optimal algorithm for terminal row selection and a greedy routing algorithm for over-the-cell and channel routing. The time complexity of the terminal row selection algorithm is $\Theta(K)$, where $K$ is total number of cell rows in the layout. MTM routers improves performance in two ways. Firstly, they allow minimum length routing of critical nets. Secondly, they produce minimum height layouts which improves performance even further.

We have implemented proposed routers in C on a SUN SPARCstation 1+ and have tested these routers on several benchmarks including PRIMARY I and PRIMARY II from MCNC. MTM-V router completes the layout of entire PRIMARY I in 205 tracks using 2-layer middle terminal model, which is 4.20% and 31.20% better than the best results [7, 15] for 2BTM and 2CTM respectively. In three layer environment, MTM-V router produces a 63 track layout of PRIMARY I using 3MTM-V, which is 11.26% and 39.42% better than the results for 3BTM-V and 3CTM-V respectively, using the best available routers [8, 15]. MTM+V router produces a channel-less layout for PRIMARY I using 3MTM+V.

The outline of the remaining thesis is as follows. In Chapter II, the cell design parameters and existing cell models are discussed. A new cell model and the motivation behind it is presented in Chapter III. Chapter IV, and V contain the MTM+V router, and 3MTM-V router respectively. Chapter VI contains the experimental results. Whereas, Chapter VII contains the conclusions.
CHAPTER II

STANDARD CELLS: DESIGN PARAMETERS AND EXISTING MODELS

In this chapter, we give a brief overview of standard cell design parameters and the existing cell models. We also discuss the utilization of the standard cell routing models based on these parameters and the over-the-cell routers associated with these models.

2.1 Standard Cell Design Parameters

In this section, we discuss the layout and process related parameters, and their effect on utilization of over-the-cell area.

1. Layout Related Parameters: The layout related parameters include the locations of terminals, $VDD$, and $GND$ lines as discussed below:

(a) Location of Terminals: Location of terminals is one of the important parameters that determines the effectiveness of utilization of the over-the-cell metal layers. Terminals may or may not be aligned. Most of the existing models assume that the terminals are aligned to ease the routing process. Even though, misaligned terminals may ease the cell design to some degree, they do complicate the routing task. Standard cell routing problem with arbitrarily located terminals in cells, may necessitate use of less developed area routing techniques, as compared to well understood and well developed channel routing techniques which can be used in case of aligned terminals. When the terminals are aligned, there may be one or more rows of terminals. Historically speaking, one terminal row was located at each cell boundary to enable channel routing since no routing layers were available for over-the-cell
routing [6, 7]. Whereas, some other cells have just one row of terminals in the center [15].

(b) Location of VDD and GND Lines: Power and ground lines normally run along the width of the cells. As feedthroughs are perpendicular to VDD and GND lines, they cannot be routed on the same layer. If the terminals are located on the cell boundaries and if the feedthroughs are routed in M1, then VDD and VSS lines have to be in M2 layer. Furthermore, when vias are not allowed in over-the-cell areas, the M2 layer between VDD and VSS lines is not available for routing. Thus, VDD and VSS lines need be very close to each other. Therefore, in this case, the most suitable location of VDD and VSS lines would be in M2 layer, at the center of the cell as it allows equal over-the-cell area for the nets in the upper as well as lower channel (See Figure 1(a)).

If the terminals are located in the center, the nets have to be brought to boundaries using M2 layer. In this case, routing VDD and VSS lines in M2 obstructs the routes of nets from terminals to the cell boundaries. Also, the
M1 layer in the middle of cells is used for intra-cell routing. Thus, \( VDD \) and \( VSS \) lines may be routed in M1 along the cell boundaries (See Figure 1(b)).

2. Process Related Parameters: The fabrication technology used in processing of chips dictate several parameters as discussed below:

(a) Number of Layers: Currently two and three metal layers are allowed for routing in standard cell layouts and cell libraries are available for both technologies. Even though, three layer technology is expensive when compared with the two layer technology, the third metal layer provides more routing space resulting in layouts with smaller area and shorter wire lengths. Therefore, three metal layers are used when performance is the most important criteria for design.

(b) Permissibility of Vias Over-The-Cell: In some existing process technologies, vias are not allowed between the metal layers over the active diffusion areas. One of the reasons for this restriction is that, the active surfaces are not planar, but have hills and valleys (See Figure 2). If a via is introduced in one of the valleys over the active elements, the oxide layer, which tends to be thinner at the edges breaks, thus causing a short circuit between the M1 and active element. Also, it may introduce an open-circuit in M2 at the point of
via. Recent technologies allow vias over the cell as they use the planarization techniques to eliminate hills and valleys.

Depending upon the process technology used for fabricating the chip, vias may or may not be allowed in over-the-cell areas. When vias are not allowed, over-the-cell metal layers can only be used for planar routing.

2.2 Existing Cell Models and Their Associated Routers

Based on the locations of the terminals there are two major classes of cell models in use: Boundary Terminal Models (BTM), and the Center Terminal Models (CTM). Each of these classes contain several cell models based on the variations in other routing parameters, i.e., the number of metal layers and permissibility of vias in over-the-cell areas. BTM contains, 2BTM (2 layer process), 3BTM-V (3 layer process when vias are not allowed in over-the-cell areas), and 3BTM+V (3 layer process when vias are allowed in over-the-cell areas). This section contains discussion of these models and their associated routers.

1. Boundary Terminal Model (BTM): This is the traditional cell model. This was introduced when only two metal layers were available for routing. In the following we discuss the layout style of BTM and briefly review the existing routers for BTM.

(a) Layout Style: In BTM, there are two parallel horizontal diffusion rows, one for the P-type transistors and the other for N-type transistors. The first metal layer (M1) is used to complete connections which are internal to the cells. The power and ground rails are in M2 layer, adjacent to each other, in the center of the cell row. Terminal rows are available in all layers and are located on the boundaries of the cells [7]. This leaves a rectangular, over-the-cell routing area for each terminal row of the standard cells. The number of tracks available for over-the-cell routing is determined by the height of these
rectangular areas and may vary depending on the cell library used. The entire over-the-cell area may be used for routing in the third metal (M3) layer. This model is used by most existing over-the-cell routers [6, 7, 8]. This class of cell models is referred to as BTM or class of Boundary Terminal Models (See Figure 1(a)).

(b) Routers for BTM: Several routers for 2BTM have been presented [6, 7]. The key idea used in [5] is to route two maximum sets of pairwise independent nets in the over-the-cell area. The first set is routed in the upper half and second set is routed in the lower half. This reduces the number of nets to be routed in the channel area resulting in reduction of the channel height. Over-the-cell routing is significantly improved by the use of vacant terminals and abutments [7]. It was shown that entire PRIMARY I can be routed using 214 tracks.

A router for 3BTM-V has been presented in [8]. The key idea in this router is similar to that used in 2-layer router. It routes four pairwise independent sets in over-the-cell area. Two sets are routed in the M2 and M3 layers in the upper half and other two are routed in the M2 and M3 layers in the lower half. It uses HVH reserved layer routing in the channel area. This router completes entire PRIMARY I routing in 71 tracks.

It is easy to see that over-the-cell routing is equivalent to HV reserved layer routing problem for 3BTM+V. In [12], it has been reported that PRIMARY I can be routed in 19 tracks. HV routing style is used in OTC area and HVH routing style is used in the channels.

2. Center Terminal Model(CTM): As more layers became available, it became necessary to develop a new class of cell model. A class of standard cell models that has terminals in the center of the cell has been proposed in [15]. The three layer model from this class is popular for the process which allows vias in
over-the-cell areas. The layout style of CTM and brief review of the routers for CTM are discussed below.

(a) Physical Layout: This class of cell models is quite different than BTM in terms of terminal location. In CTM, the terminals are located in M2, in the middle of the cell. The power and ground rails are in M1 near the top and bottom cell boundaries respectively. Connections within the cell are completed in M1. Thus, M2 is only blocked by terminals, and M3 is completely unblocked (See Figure 1(b)). Over-the-cell routers may use two rectangular regions (about thirteen tracks wide) in M2 and M3.

(b) Routers for CTM: A router for 2CTM that uses M2 layer in the over-the-cell area to river route the nets from the terminal row to the cell boundaries has been developed [15]. River routing is done in order to reduce the vertical constraints between the nets to be routed in the channel area. It uses HV reserved layer routing in the channel area. Entire PRIMARY I can be routed in 298 tracks, using this router.

A router for 3CTM-V uses M2 layer in the same manner as is used in 2CTM [15]. It uses M3 layer to route two sets of pairwise independent nets. Using this router, entire PRIMARY I can be routed in 162 tracks.

Over-the-cell routing problem in 3CTM+V can be viewed as HV reserved layer channel routing in the over-the-cell area and HVH reserved layer routing in the channel area [15]. With this approach, a channelless layout can be obtained for entire PRIMARY I.

BTM models are suitable for the layout of the channels with large set of pairwise independent nets [7, 8], whereas, CTM based layouts are suitable for channels with long vertical constraint chains [15].
CHAPTER III

NEW CELL MODEL

In practice, the channel height may depend on both the size of pairwise independent nets and length of longest vertical constraint chain. As a result, neither BTM nor CTM based layouts are suitable for majority of practical examples and it is necessary to develop new cell models. In this chapter, we present the new cell model suitable for most of the practical examples.

3.1 Motivation for a New Cell Model

Over-the-cell routing is used to eliminate as many nets as possible from the channel so as to reduce the channel height. In this section, we discuss the factors such as vertical/horizontal constraints, and intra-row/inter-row connections that decide the channel height and the effectiveness of over-the-cell area utilization.

1. High Performance Design: The cell design should allow minimum length routing of critical nets. The intra-row critical nets should be routable without vias while inter-row critical nets should be routable with minimum length and minimum vias.

2. Vertical and Horizontal Constraints: In order to remove vertical constraints, we need to offset terminal locations using river routing in M2 [15]. Thus, it is advantageous to place the terminals as far away from the boundary as possible to allow more flexibility in river routing. The center terminal model is ideal for this purpose since thirteen tracks are available for river routing. However, experimental results show that much less tracks are sufficient for this purpose. In fact, five to six tracks would be sufficient for most practical channels.
The horizontal constraints are indirectly removed by selecting a subset of nets and routing them in planar fashion. Boundary terminal models are suitable for this purpose as in BTM M2 layer in over-the-cell area is available for planer routing, in addition to M3. The planarity condition is only necessary when vias are not allowed in over the cell areas. Experimental evidence shows that density of independent sets rarely goes above five to seven [7].

Thus, in order to remove vertical and horizontal constraints, terminals must be placed at least five to seven tracks away from the cell boundary.

3. Intra-Row and Inter-Row Connections: A channel routing problem consists of two type of nets. The first type of nets connect terminals in the same row, and are called Intra-Row Nets, while the nets, which connect terminals on different rows are referred to as Inter-Row Nets. The routing of inter-row nets is somewhat rigid, in the sense, that they must pass through the channel. However, intra-row nets do not have to pass through the channel. In some sense, these nets are ideally suited for OTC routing. It has been shown that routing the intra-row connections in over-the-cell regions leads to an improvement of 15-20% [6]. However, due to planarity condition, not all intra-row nets could be removed from the channel. It is clear that if all intra-row nets are removed from the channel, the resulting channel can be routed with minimal height. The over-the-cell area available for intra-row routing can be maximized if the terminals are towards the cell boundary.

These factors indicate that a favourable location of terminals is neither at the boundary, nor at the center. In fact, terminals placed at some middle location would be ideal. Since the area between the terminals and boundary can be used to remove vertical constraints by river routing. The minimization of horizontal constraints is achieved by routing an independent set of nets in M3. In addition,
the area between the two terminal rows can be used to route a large portion of intra-row nets, thus removing their effect on channel density.

### 3.2 Middle Terminal Cell Model

Middle Terminal Model (MTM) differs from the BTM and CTM in terms of terminal locations. In MTM, the terminals are located in two rows, one row is located $k_1$ tracks below the upper cell boundary and another is located $k_3$ tracks above the lower cell boundary. As in CTM, in MTM, terminals are available only in M2 and the power and ground rails are in M1 near the top and bottom cell boundaries respectively (See Figure 3). Both terminals in a column of a cell are equi-potential. Intra-cell routing is completed in poly and M1, and does not block M2. As opposed to two rectangular regions in CTM, over-the-cell routers for MTM may use three rectangular regions in M2 and M3 as discussed below:

1. $T$ area: $k_1$ track wide area between the upper cell boundary and the upper terminal row,
2. C area: $k_2$ track wide area between the lower terminal row and the upper terminal row,

3. B area: $k_3$ track wide area between the lower terminal row and the lower cell boundary.

MTM improves performance in two ways. Firstly, it allows minimum length routing of critical nets. Secondly, it results in minimum height layouts which improves performance even further. The values of $k_1$, $k_2$ and $k_3$ should be selected in order to minimize the average overall channel height over several multi-channel examples. In addition, we consider the factors such as terminal alignment, cell rotation while selecting these values. It is known that the alignment of terminals makes the routing easier and several channel routers have been developed that assume alligned terminals (See [13], Chapter 7). Also, sometimes the standard cells are used in reflected fashion and it is necessary that the terminals be alligned even after the rotation. This is possible only if $k_1 = k_3$. Now for a given value of $k_1$ the value of $k_2$ is given as $k_2 = k_0 - 2k_1$, where $k_0$ is the total number of tracks available over-the-cell. For a cell with 150A height, $k_0 = 24$. The computation shows that the most suitable values of $k_1$, $k_2$, and $k_3$ are 6, 12, and 6 respectively, for $k_0 = 24$.

3.3 Single Cell Comparison

In order to evaluate the effect of cell model on area, delay and electrical properties of individual cell, various standard cells were laid out in all of the three cell models (See Figure 4). Layouts were done using MAGIC Ver. 6.3, on a Sun Sparc 1+ workstation. These layouts were extracted and then simulated using SPICE to study their electrical characteristics and also their feasability. It was found that the MTM based cells have similar delay characteristics as those based on BTM and CTM. Let's use a three input NAND gate for our discussion. (The
results are similar for all comparable cells.) A transient analysis was performed for NAND gates based on these models, with $V_{DD}$ (input power supply voltage) of 5V. The switching speed of a CMOS gate is dependent on the time taken to charge and discharge the load capacitance $C_L$. A change in the input logic level results in an output transition that either charges $C_L$ towards $V_{DD}$ or discharges $C_L$ towards $V_{SS}$ (input ground level). The load capacitance $C_L$ is determined by adding the capacitances at the inputs of the next gate, routing capacitance and the output capacitance of the gate under consideration.

There are several factors that contribute to the delay of gate, as explained below:

1. The rise time ($t_r$) is the time required for the signal to rise from 10 % to 90 % of $V_{DD}$.

2. The fall time ($t_r$) is the time required for the signal to fall from 90 % to 10 % of $V_{DD}$.
3. The delay time is the time difference between $t_1$ and $t_2$, where, $t_1(t_2)$ is the time taken by input(output) to change from the initial level to the 50% of the final level. Delay time is qualitatively defined as the time taken for a logic transition from input to output. Figure 5 shows delay characteristics of a typical CMOS gate.

The delay of a single gate is dominated by the output rise and fall times, the delay is approximately given by $t_{dr} = \frac{t_f}{2}$ and $t_{df} = \frac{t_f}{2}$ [14], the average gate delay ($t_{av}$) for rising and falling transitions is then calculated as $t_{av} = \frac{t_{dr} + t_{df}}{2} = \frac{t_f + t_f}{4}$. Table 1 shows the circuit delays for cells based on different models, for a load capacitance of 1pF. The cell based on MTM has better performance when compared with the cells based on CTM and BTM. Figure 6 shows the comparative delay performance of a NAND gate cell in three models for different load capacitances. The delay characteristics of cells based on all the three models are similar, with MTM based cell being slightly superior.

The three models, when compared at the cell level, have identical physical and electrical characteristics. Hence, the performance of each cell has be analysed.
Table 1
Single Cell Model Comparisons

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<tr>
<th>Cell Model</th>
<th>Total Area</th>
<th>OTC Routable Area</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>BTM</td>
<td>$150\lambda \times 48\lambda$</td>
<td>24 tracks</td>
<td>2.00ns</td>
</tr>
<tr>
<td>CTM</td>
<td>$150\lambda \times 48\lambda$</td>
<td>24 tracks</td>
<td>2.04ns</td>
</tr>
<tr>
<td>MTM</td>
<td>$150\lambda \times 51\lambda$</td>
<td>24 tracks</td>
<td>1.86ns</td>
</tr>
</tbody>
</table>

Figure 6. Delay Performance of Different Cell Models.
at large circuit levels. A large circuit layout comprises of the area occupied by the cells and the channel area. Since the cell areas are similar for cells in all the three models, the cell model that uses minimum channel height, generates layouts with minimum areas. Minimum channel heights are obtained by optimal use of over-the-cell area for routing. Therefore, to obtain minimum area layouts in standard cell designs, the cell model that allows the best utilization of over-the-cell area for routing, has to be selected. Our experimental results in Chapter VI show that layouts in MTM have smaller routing area than those in BTM and CTM.
CHAPTER IV

ROUTING STRATEGY FOR 3MTM+V

A 3MTM+V cell contains two rows of terminals such that two terminals in a column of a cell are equi-potential. Selecting some terminals from each terminal rows of a cell row may create routing regions with irregular boundaries, thus complicating the task of router. However, regular boundaries can be obtained by selecting all terminals from only one of the two rows of terminals in each cell row. Following is the overview of two major steps in the MTM+V router.

1. Terminal Row Selection: Channel height can be reduced by allocating more over-the-cell area to congested channels. Such allocation of over-the-cell area can be achieved by selecting an appropriate terminal row from two terminal rows available in each cell row. The objective of this step is to select a terminal row for each cell row such that the overall layout height is minimized.

2. Over-the-Cell and Channel Routing: After the terminal row selection, the nets are routed using a composite router that assumes availability of two and three layers in the over-the-cell and channel areas, respectively.

In sections 4.1 and 4.2, we describe these steps in detail.

4.1 Terminal Row Selection

Presence of two terminal rows in each cell row gives rise to an important problem of terminal row selection in order to minimize the overall channel height. In this section, we discuss the terminal row selection problem for 3MTM+V, in detail. Figure 7(a) is an example showing the channel heights obtained by using a 2-layer conventional channel router. An overall channel height of 168 tracks was
Figure 7. An Example of Terminal Row Assignment.
obtained for this example. Figure 7(b) shows that an overall height of 8 tracks is obtained when terminal row selection is done in a sequential manner from top to bottom, for an example in Figure 7(a). Employing 'high density first' strategy for terminal row selection, to the same example, results in an overall channel height of 10, as shown in Figure 7(c). However, Figure 7(d) shows an optimal terminal row assignment giving an overall channel height of 6. In Figure 7(a), (b), (c) and (d), the numbers between the two adjacent cell rows indicate the channel height in number of tracks, whereas, the number at the bottom indicate the overall channel height.

The remaining part of this section addresses the terminal row selection problem for 3MTM+V, in detail. First the required terminology is developed, the terminal row selection problem is formally defined and then the problem is transformed to a shortest-path problem related to weighted directed multi-stage graphs.

For 3MTM+V, the best routing can be accomplished by using only one terminal row (u or l) in each cell row. This transforms the MTM cell layout into a CTM layout with an offset in the terminal row location. Let \( \mathcal{R} = \{R_1, R_2, \ldots, R_K\} \) be a set of cell rows, such that \( R_i \) and \( R_{i+1} \) are adjacent to each other. Let \( f: \mathcal{R} \rightarrow \{u, l\} \) be a function determining the assignment of terminal row to the cell rows in \( \mathcal{R} \). For two adjacent cell rows \( R_i \) and \( R_{i+1} \) in \( \mathcal{R} \), there are four choices for selection of the terminal rows. These are (1) \( f(R_i) = f(R_{i+1}) = u \), (2) \( f(R_i) = f(R_{i+1}) = l \), (3) \( f(R_i) = l, f(R_{i+1}) = u \) and (4) \( f(R_i) = u, f(R_{i+1}) = l \).

Let \( g: \{1, 2, \ldots, K - 1\} \rightarrow \mathbb{Z}^+ \) be a function that gives the number of tracks available in over-the-cell area between the selected terminal rows of \( R_i \) and \( R_{i+1} \). Function \( g \) is defined below and illustrated in Figure 8.
Figure 8. Choices of Terminal Row Selection for Two Adjacent Cell Rows.

\[ g(i) = \begin{cases} 
  k_2 + k_3 + k_1 & \text{if } f(R_i) = f(R_{i+1}) = u \\
  k_2 + k_3 + k_1 & \text{if } f(R_i) = f(R_{i+1}) = l \\
  k_3 + k_1 & \text{if } f(R_i) = l \text{ and } f(R_{i+1}) = u \\
  k_3 + 2k_2 + k_1 & \text{if } f(R_i) = u \text{ and } f(R_{i+1}) = l 
\end{cases} \]

Let the net density between the cell rows \( R_i \) and \( R_{i+1} \) using conventional 2-layer channel routing be given by \( d_i \). We assume the following:

1. If \( g(i) \geq d_i \), then channel is not required between \( R_i \) and \( R_{i+1} \).

2. If \( g(i) < d_i \), then in addition to the tracks available in the over-the-cell area, a channel of height \( \lceil \frac{g(i) - d_i}{2} \rceil \) tracks is required for completing the routing.

These assumptions are valid for the channels that do not have vertical constraints. They are also reasonable for the channels having vertical constraints as several routers have been developed that route such channels producing channel heights very close to their lower bounds (See [13], Chapter 7).

Let \( t : \{1, 2, \ldots, K - 1\} \times \{u, l\} \times \{u, l\} \rightarrow \mathbb{Z}^+ \) be a function, such that \( t(i, p, q) \) gives the height of \( i^{th} \) channel when a terminal row \( p \in \{l, u\} \) is selected for \( R_i \) and \( q \in \{l, u\} \) for \( R_{i+1} \).

The objective of this step is to find if there is any terminal row assignment to the cell rows such that a channel-less layout can be obtained. The formal statement of this problem (TRSP-1) is as follows:
INSTANCE (TRSP-1): Given,

1. $\mathcal{R} = \{R_1, R_2, \ldots, R_K\}$, a set of cell rows such that $R_i$ is adjacent to $R_{i+1}$ for all $i = 1, 2, \ldots, K - 1$ and it is also adjacent to $R_{i-1}$ for all $i = 2, 3, \ldots, K$. A cell row $R_i \in \mathcal{R}$ can have one of its two possible terminal rows $l$ or $u$,

2. $D = \{d_1, d_2, \ldots, d_{K-1}\}$, a set of densities, where $d_i$ indicates the density of nets between $R_i$ and $R_{i+1}$ using conventional 2-layer channel router.

QUESTION: Does there exist a function $f : \mathcal{R} \rightarrow \{l, u\}$, such that,

$$\sum_{i=1}^{K-1} t(i, f(R_i), f(R_{i+1})) = 0.$$  

In order to answer this question we model the instance of TSRP-1 using a weighted, directed, multi-stage graph $G = (V, E)$, as follows:

Corresponding to two terminal row positions ($l$ and $u$) in each $R_i$, $G$ has two vertices, $v_{il}$ and $v_{iu}$, respectively. In addition, it has a source vertex $s$ and a destination vertex $t$, i.e.,

$$V = \{v_{ij} | 1 \leq i \leq K, j \in \{l, u\}\} \cup \{s, t\}$$

$G$ has directed edges between the vertices corresponding to the adjacent cell rows. In addition, $G$ has the edges $(s, v_{il})$, $(s, v_{iu})$, $(v_{Ki}, t)$, and $(v_{Ku}, t)$. The direction of an edge between $v_{ij}$ and $v_{(i+1)j}$ is from $v_{ij}$ to $v_{(i+1)j}$. Edges $(s, v_{il})$ and $(s, v_{iu})$ are directed away from $s$, whereas, $(v_{Ki}, t)$, and $(v_{Ku}, t)$ are directed towards $t$.

$$E = \{(v_{ij}, v_{(i+1)j}) | 1 \leq i \leq K - 1, j \in \{l, u\}, k \in \{l, u\}\} \cup $$

$$\{(s, v_{il}), (s, v_{iu}), (v_{Ki}, t), (v_{Ku}, t)\}$$

The weight of an edge $(v_{ij}, v_{(i+1)j})$ indicates the number of tracks introduced in $i^{th}$ channel (channel between $R_i$ and $R_{i+1}$), when terminal rows $j \in \{l, u\}$ and $k \in \{l, u\}$ are selected on $R_i$ and $R_{i+1}$ respectively.
Based on the assumption stated above, the weight of an edge \((u, v) \in E\) is given by the following function:

\[
    w(u, v) = \begin{cases} 
        0 & \text{if } u = s \text{ or } v = t \\
        \max([d|_{k_2+k_3} - (k_2+k_3)], 0) & \text{if } u = v_{i+1} \text{ and } v = v_{i+1} \\
        \max([d|_{k_2+k_3}], 0) & \text{if } u = v_{i+1} \text{ and } v = v_{i+1} \\
        \max([d|_{k_2+k_3} - (k_2+k_3)], 0) & \text{if } u = v_{i+1} \text{ and } v = v_{i+1} \\
    \end{cases}
\]

Figure 9 illustrates construction of such a graph for the example in Figure 7(a).

A \((v_i, v_j)\)-path is defined as a path from vertex \(v_i\) to vertex \(v_j\). Cost of a \((v_i, v_j)\)-path is defined as the sum of the weights of the edges along \((v_i, v_j)\)-path.

Let there be a zero-cost \((s, t)\)-path in the weighted, directed, multi-stage graph \(G\) obtained as explained above. Let \(S\) be the set of vertices in the shortest path from \(s\) to \(t\) and let \(S' = S - \{s, t\}\). Note that \(S'\) contains exactly one vertex corresponding to each cell row. If \(u_{ij} \in S'\) then for cell row \(R_i\), terminal row at position \(j\) can be selected, i.e., \(f(R_i) = j\). A channel-less layout can be obtained with the terminal row assignment given by function \(f\).

Similarly, it can be proved that if there exists a channel-less layout for an instance of TRSP-1 then there exists a zero-cost \((s, t)\)-path in the corresponding \(G\).
Based on the above discussion, we state the following theorem.

**Theorem 1** A channel-less layout can be obtained for an instance of TRSP-1 if and only if there exists a zero-cost $(s,t)$-path in the corresponding $G$.

When a channel-less layout is not possible, the objective of this step should be to minimize the overall height of the channel, i.e., $\sum_{i=1}^{K-1} t(i, f(R_i), f(R_{i+1}))$ is minimized. This problem is referred to as TRSP-2.

**Theorem 2** TRSP-2 can be optimally solved in $O(K)$ time.

**Proof:** TRSP-2 also can be solved by modeling its instance using a weighted, directed, multi-stage graph as described above and finding a minimum cost $(s,t)$-path in it. Let $S$ be the set of vertices in the shortest path from $s$ to $t$. Let $S' = S - \{s,t\}$. Note that $S'$ contains exactly one vertex corresponding to each cell row. If $v_{ij} \in S'$ then for cell row $R_i$, terminal at position $j \in \{l,u\}$ can be selected, i.e., $f(R_i) = j$.

Time complexity of finding minimum cost $(s,t)$-path in a weighted, directed, multi-stage graph is $O(n + e)$, where $n$ and $e$ indicate the number of vertices and edges in the graph. As the graph $G$ has $2K + 2$ vertices and $4K$ edges respectively, the complexity of solving TRSP-2 is $O(K)$. It is to be noted that the time complexity of solving TRSP-2 can not be reduced further as any other algorithm to solve TRSP-2 would have $K$ assignments, one for each cell row. Hence, we solve the TRSP-2 in $O(K)$ time complexity. □

In Figure 9, the minimum cost $(s,t)$-path is shown by thick lines. Figure 7(d) shows the optimal terminal row selection for the example in Figure 7(a). Note that the optimal terminal selection corresponds to the vertices on the minimum cost $(s,t)$-path. Using TRSP-2, optimal overall channel heights can be obtained for the examples in which channels do not have vertical constraints. Experimental results have shown that using the terminal row selection method.
explained above gives close to optimal channel heights for the examples having vertical constraints.

Let the total number of tracks introduced in the channels of an over-the-cell routing problem $P$, using the cell models $M$ be given by $height(P, M)$. The following theorem proves that for an example $P$, a smaller or same layout height can be obtained by using the MTM+V model than that obtained by using CTM+V model, under same assumptions as those in TRSP-2.

**Theorem 3** Given an over-the-cell routing problem $P$, $height(P, 3MTM+V) \leq height(P, 3CTM+V)$.

**Proof:** Let the number of tracks in the over-the-cell area between two terminal rows using $3CTM+V$ model be $k$ then by selecting the upper terminal row in all the cell rows the same number of tracks can be obtained between the any two adjacent cell rows in $3MTM+V$. If the channel height between two adjacent cell rows using $3CTM+V$ is $p$ then the same channel can be routed by using a channel height of $k + 2p$ tracks using a 2-layer conventional channel router. The same example can, therefore, be routed by using $3MTM+V$ model with a channel height of $p$ tracks. Thus, in this case $height(P, 3MTM+V) = height(P, 3CTM+V)$.

MTM models give flexibility of selecting one of the two terminal rows. This flexibility can be utilized for reducing the channel height. Figure 10(a) shows an example routed by using a 2-layer conventional channel router. An overall channel height of 54 tracks is obtained for this example. Figure 10(b) shows that an overall height of 3 tracks is obtained using $3CTM+V$ for the example in Figure 10(a). However, Figure 10(c) shows that by using $3MTM+V$ with an optimal terminal row selection, an overall height of 1 tracks is obtained for the same example in Figure 10(a). Thus, $height(P, 3MTM + V) \leq height(P, 3CTM + V)$. □
Figure 10. 3MTM+V Gives Less Overall Channel Height When Compared With 3CTM+V.

4.2 Over-the-Cell and Channel Routing

After the terminal row selection, the nets between each adjacent pair of cell rows are routed using a composite router. This router is obtained by modifying the track assignment step of original greedy channel router (See [13], Chapter 7), such that (1) it does HV and HVH reserved layer routing in the over-the-cell and channel areas, respectively, and (2) it uses the tracks in over-the-cell areas before introducing any track in the channel area. The remaining three steps of the greedy router viz; join split nets as much as possible, bring split nets close together by jogging, and bring nets closer to the boundary of their closest terminal, are not changed. We have optimized the composite router to route the critical nets in minimal length, as straight as possible, with minimum jogs, and with minimum vias.

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CHAPTER V

ROUTING STRATEGY FOR 3MTM-V

In MTM-V, the routing in over-the-cell areas must be planar. Furthermore, the terminals cannot be 'brought-up' to M3, as that would require a via. Thus wires from all the terminals must be routed on M2, till they 'reach' cell boundary, where vias may be used to complete the connection. The basic steps of the MTM-V router for 3MTM-V model are given below:

Step 1: Net Classification, Decomposition and Weighting: In this step, all the nets are decomposed into two terminal nets and classified into two types. A TYPE I net has terminals on the same cell row. A TYPE II net has terminals on different cell rows (One on the top terminal row and one on the bottom terminal row). After net decomposition and classification, all the nets are assigned weights. We use two weighting functions in the MTM-V router. The first weight function is based on criticality of nets. The weight of a net is directly proportional to its criticality. This enables the router to select most of the critical nets for routing in M2 layer of C area. The routes of nets in C area are shorter and have no vias. As a result, these nets have small delay. After the net selection in C area, nets are reweighted using a second weighting function which is based on their contribution to the channel congestion. This weighting function is designed such that the router selects the nets in congested areas of channels to route in the M3 layer of the over-the-cell area. As a consequence, the channel height is reduced. Several such weight functions are discussed in [15]. The critical nets are routed with minimal length in the channel. Let the weight of a net $n_i$ be represented as $w(n_i)$. 

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Step 2: Maximum Planar Equi-potential Set Selection for M2 Layer in C area: In this step, a subset of intra-row nets is selected for routing in C area of a cell row $R_i$. Based on the weights assigned to the nets, we find a maximum planar independent subset of the intra-row nets, which can be routed in $k_2$ tracks available in the C area using one of the four different paths, as shown in Figure 11. We prove that this problem is NP-hard and as a result present an approximation algorithm to select such a subset. The nets selected for routing in C area of $R_i$ are routed in planar fashion. This step is executed for all the cell rows.

The TYPE I nets not routed in C area of the cell row $R_i$ are considered for routing in the channel between $R_i$ and $R_{i+1}$, and in the M3 layer of over-the-cell area of $R_i$ and $R_{i+1}$.

Step 3: Boundary Terminal Assignment and M2 River Routing: The cell boundary is essentially considered a row of possible terminal locations. In this step, each net terminal is assigned a terminal location on the boundary considering river routing constraints, while minimizing the channel density. A river router is used to complete the connections as specified by the terminal assignment, in a planar fashion in M2 layer of T and B areas.

Step 4: Routing in M3 Layer of OTC Areas, and in Channel Areas: After step 3, the layout is transformed to an instance for 3BTM-V routing, with only one metal layer (M3) available over-the-cell. Thus we use the steps presented in [8] to complete the routing.

For most of the examples, channelless routing can be obtained, as the two layers in over-the-cell area are sufficient to complete the routing.

The steps for over-the-cell routing for 2MTM are similar to that for 3MTM-V except that the Step 4 is performed by a reserved layer (HV) conventional channel router.
In the following sections we explain step 2 and step 3 of MTM-V router, in detail.

5.1 Maximum Planar Equi-potential Set Selection for C(M2) Area

In this step, a set of nets is selected for routing in the M2 layer in C area of the cell row \( R_i \). As vias are not allowed over-the-cell, the nets in this set must be planar. Let \( N \) be the set of TYPE I nets for a given cell row. The terminal positions in the upper(lower) terminal row are numbered from \( t_{iu}(t_{il}) \) to \( t_{lu}(t_{ll}) \). Let \( n_i \in N \) be represented as \( n_i = (t_j, t_k) \), such that \( \text{COL}(t_j) < \text{COL}(t_k) \). As terminals \( t_{ii} \) and \( t_{iu} \) are equi-potential, four different routing choices are available for routing \( n_i \) in C area: \( (t_{ju}, t_{ku}) \) top routing choice (t), \( (t_{ji}, t_{ki}) \) bottom routing choice (b), \( (t_{ju}, t_{kl}) \) right crossing routing choice (r), and \( (t_{ji}, t_{ku}) \) left crossing routing choice (l) (See Figure 11).

Given, a positive integer \( \alpha \), a set \( \beta \subseteq \{t, b, r, l\} \), and a set \( (N) \) of TYPE I nets, we say a set \( S \) is an \( EPS(\alpha, \beta, N) \), if \( S \subseteq N \), and all nets in \( S \) are routable using one of the routing choices in \( \beta \), in a planar fashion in \( \alpha \) tracks. The maximum

![Figure 11. Four Routing Choices of a TYPE I Net.](image-url)
weighted \( EPS(\alpha, \beta, N) \) is referred to as \( MEPS(\alpha, \beta, N) \). For short we denote \( MEPS(\alpha, \beta, N) \) as \( S(\alpha, \beta, N) \). The weight of a track in the routing of a netlist \( N \) is defined as the summation of the weights of the nets routed on that track. Let \( S^*(\alpha, \beta, N) \) be a set of nets in the \( k_2 \) highest weighted tracks of the optimal no-dogleg planar routing of \( S(\infty, \beta, N) \).

Let the sets \( S(k_2, \{ t, b, r, l \}, N) \), \( S(\infty, \{ t, b, r, l \}, N) \), \( S(\infty, \{ b, r, l \}, N) \), and \( S^*(k_2, \{ b, r, l \}, N) \) be denoted by \( S_1 \), \( S_2 \), \( S_3 \), and \( S_4 \) respectively.

The objective of this step is to find \( S(k_2, \{ t, b, r, l \}, N) \). We call the problem of finding \( S(k_2, \{ t, b, r, l \}, N) \) as the MEPSP-1. We show that MEPSP-1 is computationally hard problem.

**Theorem 4** MEPSP-1 is NP-Hard.

**Proof:** To prove that MEPSP-1 is NP-Hard, we reduce the instance of the problem of finding a maximum weighted bi-partite set (MWBPSP) in a circle graph to an instance of MEPSP-1. MWBPSP is known to be NP-Hard (See [4] for references).

Let \( I = \{ C \} \) be an instance of MWBPSP, such that, \( C \) is a set of \( k \) chords of a circle. Let weight of a chord \( c_i \in C \) be given as \( w(c_i) \). Let the end points of these chords be numbered from 1 to \( 2k \). We construct the instance \( I' \) of MEPSP-1 as follows: For each chord \( c_i = (j, k) \in C \), we add a net \( n_i = (t_j, t_k) \) to \( N \) and assign a weight \( w(c_i) \) to \( n_i \). In addition, we add a net \( n_0 = (t_0, t_{2k+1}) \) to \( N \) and assign a weight \( \sum_{i=1}^{k} w(c_i) + 1 \) to \( n_0 \). Figure 12(b) shows an instance of MEPSP-1 constructed from an instance of MWBPSP shown in Figure 12(a).

Let there be an algorithm \( A \) that optimally solves MEPSP-1. Let \( \eta \) be the optimal solution to the instance \( I' \) of MEPSP-1. \( \eta \) contains \( n_0 \), as \( n_0 \) has more weight than the summation of weights of all other nets in \( N \). Selecting \( n_0 \) does not allow the left and the right routing choices of the nets to be selected for the nets in \( N - \{ n_0 \} \). Thus, a nets in \( \eta - \{ n_0 \} \) has either top routing choice or a
Figure 12. Transformation of an Instance From WMBS Into WMPES.

bottom routing choice. Let $X(Y)$ be the subset of $\eta - \{n_0\}$ such that each net in $X(Y)$ has top(bottom) routing choice.

We construct the solution to the instance $I$ of MWBPSP as follows: Let $C_1 = \{c_i | n_i \in X\}$ and $C_2 = \{c_i | n_i \in Y\}$ then $C_1 \cup C_2$ is maximum weighted bipartite subset of $C$ and the two partites are $C_1$ and $C_2$ respectively. Figure 12(c) shows solution to the instance of MEPSP-1 in Figure 12(b). Figure 12(d) shows solution to the instance of WMBPSP shown in Figure 12(a), constructed from the solution of corresponding instance of MEPSP-1.

However, the MWBPSP is known to be NP-Hard, thus MEPSP-1 is NP-Hard. □

In view of NP-hardness of MEPSP-1, we develop a $\frac{1}{2} \min\{1, \frac{k+1}{n}\}$ approximation algorithm.

Let the problem of finding $S_2$ be called as MEPSP-2. In order to find $S_2$, we represent the terminal rows, and the $b$, $l$, and $r$ routing choices of the nets in $N$ using a circle diagram. The terminals $t_{1u}$ through $t_{Lu}$ are represented as
Figure 13. (a) Chords Representing the $b$, $l$, and $r$ Routing Choices of a Net, (b) Circle-chord Representation, (c) Maximum Independent Set of Chords.

points $p_{1u}$ through $p_{Lu}$ on the circumference of a circle, in the clockwise direction. Similarly, terminals $t_{1l}$ through $t_{Li}$ are represented as points $p_{1l}$ through $p_{Li}$ on the circumference of the circle, in the anti-clockwise direction. For each point $p_{il}$, let $p_{il}' = p_{il} + \epsilon$ and $p_{il}'' = p_{il} - \epsilon$ be new points on the right and the left of $p_{il}$, respectively. Let $C$ represent the set of chords of the circle. For each net $n_{ij} = (t_j, t_k) \in N$, $C$ contains three chords $c_{ib} = (p_{jt}, p_{kt})$, $c_{ir} = (p_{jt}, p_{kl})$, and $c_{il} = (p_{jt}, p_{ku})$ representing the top, right, and left routing choices of net $n_{ij}$, respectively (See Figure 12(a)). Each chord $c_{iz} \in C$ ($z \in \{t, r, l\}$) has a weight $w(n_{ij})$ associated with it. Let $C^*$ be the set of maximum weighted independent chords in $C$. Note that for a net $n_{ij}$, $C^*$ may contain at most one chord among $c_{ib}$, $c_{ir}$, and $c_{il}$ as each pair of chords in $\{c_{ib}, c_{ir}, c_{il}\}$ intersect each other. $S_2$ is represented by $\{n_{ij}|c_{iz} \in C^* \& z \in \{b, r, l\}\}$.

Based on the observation that $|C| = 3|N|$, and that $C^*$ can be found from $C$ in $O(|C|^2)$ time complexity, we state the following theorem.

**Theorem 5** **MEPS-2** can be optimally solved in $O(|N|^2)$ time complexity.
Figure 14. A Set of TYPE-I Nets.

Figure 13(b) shows the circle diagram of the nets in Figure 14. Figure 13(c) shows a maximum weighted independent set of chords for the example in Figure 13(b).

Let $S_2$ be partitioned into two sets $S_5$ and $S_6$ such that $S_5 = \{ n_i | n_i \in S_2$ & $n_i$ has top routing choice $\}$, and $S_6 = S_2 - S_5$. Let $w(S)$ denote the summation of weights of nets in a set $S$.

The performance ratio of an approximation algorithm is defined as the ratio of solution produced by the approximation to the optimal solution, i.e.,

$$\rho = \frac{w(S_4)}{w(S_1)},$$

where $S_4$ is the solution produced by the approximation algorithm and $S_1$ is the optimal solution.

**Theorem 6** The performance ratio of the algorithm for MEPSP-1 is

$$\rho \geq \frac{1}{2}(\min\{1, \frac{k_2}{d}\}),$$

where $d$ is the optimal number of tracks required for routing $S_3$ in a planar fashion, without use of doglegs.

**Proof:** First we prove that $w(S_3) \geq w(S_5)$, by contradiction. Let's assume $w(S_3) < w(S_5)$. Let $S_7$ be the set of all the nets in $S_5$, except that, each of them
is assigned bottom routing choice. Since changing the routing choice of a net does not affect its weight, it is easy to note that \( w(S_7) = w(S_5) \). Hence \( w(S_3) < w(S_7) \), which is a contradiction as \( S_3 \) is \( S(\infty, \{b, r, l\}, N) \). Thus,

\[
 w(S_3) \geq w(S_5). \tag{5.1}
\]

Similarly, it is easy to note that,

\[
 w(S_6) \geq w(S_7). \tag{5.2}
\]

Next we prove that \( w(S_3) \geq \frac{w(S_1)}{2} \). From equations 5.1, 5.2,

\[
 2 \times w(S_3) \geq w(S_5) + w(S_6)
\]

Since, \( w(S_2) = w(S_5) + w(S_6) \)

\[
 w(S_3) \geq \frac{1}{2} w(S_2)
\]

Since, \( w(S_2) \geq w(S_1) \),

\[
 w(S_3) \geq \frac{w(S_1)}{2}
\]

Therefore, if \( k_2 \geq d \), then \( w(S_4) = w(S_3) \). If \( k_2 < d \), then the average track weight of the \( k_2 \) highest weighted tracks is more than or equal to the average track weight of all \( d \) tracks, i.e.,

\[
 \frac{w(S_4)}{k_2} \geq \frac{w(S_1)}{d}
\]

\[
 w(S_4) \geq \frac{w(S_1)}{2} \min\{1, \frac{k_2}{d}\}
\]

\[
 \rho \geq \frac{1}{2} \min\{1, \frac{k_2}{d}\}
\]

Which concludes our proof. \( \square \)

Our experimental results have shown that \( w(S_4) \) is 70-75\% of \( w(S_1) \) for most of the practical examples. Figure 15 shows the routing of \( S_4 \) for the set of nets shown in Figure 14. The utilization of \( C \) area can be improved by routing a maximum weighted planar subset of \( N_2 \cup S_4 \) in \( C \) area, where \( N_2 = \{n_i | n_i \in N \)
Figure 15. Net in $S_4$ Routed in the M2 Layer of C Area ($k_2 = 3$).

& $n_i \not\in S_4$ & $n_i$ is assigned top routing choice } Such a set is represented by the maximum independent subset of the chords in the circle-chord representation of $N_2 \cup S_4$. If this planar subset is not routable in $k_2$ tracks, then the nets in $k_2$ highest weighted tracks are selected for routing in C area.

5.2 Boundary Terminal Assignment

The cell boundary is essentially considered a row of possible terminal locations. In this step, each net terminal is assigned a terminal location on the boundary considering river routing constraints, while minimizing the channel density. From Figure 16, it can be seen that the simple approach of bringing straight wires to boundary does not minimize that channel height. A river router is used to complete the connections as specified by the terminal assignment, in a planar fashion in M2 layer of T and B areas.

The Boundary Terminal Assignment Problem (BTAP) can be stated as follows:

INSTANCE (BTAP): Given, (1) $L$, total number of terminals available in a row, (2) $TOP = \{t_1, t_2, \ldots, t_p\}, p \leq L$, is the set of terminals in the lower terminal row of top cell row, (3) $BOT = \{b_1, b_2, \ldots, b_q\}, q \leq L$, is the set of terminals in the
upper terminal row of bottom cell row, (4) $T_{OB.BO}U = \{u_1, u_2, \ldots, u_p\}$, an ordered set of positions on the lower boundary of the top cell row, (5) $B_{OT.BO}U = \{t_1, t_2, \ldots, t_L\}$, an ordered set of positions on the upper boundary of the bottom cell row, (6) $k_3$, number of tracks in the B area of top cell row, (7) $k_1$, number of tracks in the T area of bottom cell row, (8) $N = \{n_1, n_2, \ldots, n_{N_{max}}\}$ is a net-list which is a partition of $T_{OP}\cup B_{OT}$.

**PROBLEM:** To find two injective functions $f$ and $g$ such that, (1) $f(t_k) \in T_{OP}.B_{OU}$, (2) $g(b_k) \in B_{OT}.B_{OU}$, (3) if $U = \{a_1, a_2, \ldots, a_p\}$ is a set of nets, where $a_i = \{t_i, f(t_i)\}$, then, $U$ is river routable in $k_3$ tracks, (4) if $L = \{c_1, c_2, \ldots, c_q\}$ is a set of nets, where $c_i = \{b_i, g(b_i)\}$, then, $L$ is river routable in $k_1$ tracks, (5) if $N' = \{n'_1, n'_2, \ldots, n'_{N_{max}}\}$ is another set of nets, where $n'_i = \{f(t) | t \in n_i \} \cup \{g(t) | t \in n_i \}$ then, the density of nets in $N'$ is minimized.

**Theorem 7** *Problem BTAP can be optimally solved in $O(pqL)$ time complexity.*

**Proof:** Nets in set $U$ have to be planar, this imposes linear order constraints on the function $f$, i.e. in the set $T_{OP}$ sorted on column position of terminals, if terminals $t_i$ and $t_j$ are adjacent to each other such that $\text{COL}(t_i) < \text{COL}(t_j)$,
then \( \text{COL}(f(t_i)) < \text{COL}(f(t_j)) \). Note that there are only \( O(p) \) order constraints on \( f \). Nets in set \( U \) have to be river routable in \( k_3 \) tracks, this imposes position constraints \( T_i \) on \( f(t_i) \), for each \( t_i \in \text{TOP} \). The position constraints corresponding to \( t_i \) are dependent on the value of \( k_3 \) and the number of nets starting from the positions on the left and the right side of the \( t_i \). Let \( t_i \in \text{TOP} \) be the \( k^{th} \) terminal on the left side of \( t_i \), and \( t_r \in \text{TOP} \) be the the \( k^{th} \) terminal on the right side of \( t_i \). Let the function call \( \text{COL}(t) \) return the column range of a terminal \( t \in \text{TOP} \cup \text{BOU} \). Let the function call \( \text{NUM}(c_1, c_2) \) return the number of terminals from set \( \text{TOP} \) that have columns position in \([c_1, c_2]\). We define \( T_i = \{y_l, \ldots, y_r\} \subseteq \text{TOP} \cup \text{BOU} \), such that \( y_l = u_p \) and \( y_r = u_q \), where

\[
P = \begin{cases} 
\text{COL}(t_l) + k_3 & \text{if } t_l \text{ exists} \\
1 + \text{NUM}(1, \text{COL}(t_l) - 1) & \text{if } t_l \text{ does not exist}
\end{cases}
\]

\[
q = \begin{cases} 
\text{COL}(t_r) - k_3 & \text{if } t_r \text{ exists} \\
L - \text{NUM}(\text{COL}(t_r) + 1, L) & \text{if } t_r \text{ does not exist}
\end{cases}
\]

Note that the total number of position constraints on \( f \) is \( O(p) \). Figures 17(a) and (c) show valid river routings. In Figure 17(a), \( t_i \) exists for \( t_2 \), and \( y_{2l} = u_{11} \). Figure 17(b) shows that the net starting at \( t_5 \) is unroutable, if the net starting at \( t_2 \) is routed on the left of \( u_{11} \). In Figure 17(a), \( t_i \) does not exist for \( t_6 \), and \( y_{6l} = u_3 \). Figure 17(b) shows that the net starting at \( t_4 \) is unroutable, if the net starting at \( t_6 \) is routed on the left of \( u_3 \).
Similarly, the river routability constraints on $L$ impose order and position constraints on $g$. With the transformation of the river routability constraints on $U$ and $L$ into the order and position constraints on $f$ and $g$, BTAP is transforms to CPA [16]. Generating order and position constraints on $f$ requires sorting $TOP$ on the column positions of terminals, i.e., it requires $O(p \log p)$ time. Similarly, to generate position constraints on $g$ requires $O(q \log q)$ time. Thus, $BTA$ can be transformed into $CPA$ in $O(p \log p + q \log q)$ time. Since CPA can be optimally solved in $O(pqL)$ time, the theorem follows. □

In [15], a faster algorithm with the primary objective of elimination of vertical constraints has been proposed for boundary terminal assignment. However, this algorithm requires the presence of a large number of vacant terminals. Also, it may not produce optimal channel heights.

Figure 18 shows the routing after the Step 4.
CHAPTER VI

EXPERIMENTAL RESULTS

In this chapter, we compare several models based on the area required for routing a benchmark example, PRIMARY I. PRIMARY I benchmark from MCNC has 17 cell rows. The BTM, CTM, and MTM cell models were adapted for PRIMARY I and a comparison is made between these cell models for various parameters like Track and Area Utilizations. Table 2 gives the channel height utilization by the PRIMARY I with different cell models. Table 3 compares the overall height of the PRIMARY I laid out with different cell models. Table 4 gives the channel height compared with that of 2-layer conventional channel routing, for PRIMARY I laid out with different cell models. Table 5 gives the overall channel height compared with overall layout height for PRIMARY I. The tabulation is done for both two layered and three layered processes, and when vias are allowed in over-the-cell area and also when vias are not allowed in over-the-cell area. This comparison shows that MTM gives layouts with smallest overall heights for

<table>
<thead>
<tr>
<th>Cell Model</th>
<th>Two Layer Process</th>
<th>Three Layer Process</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>with vias over-the-cell</td>
<td>without vias over-the-cell</td>
</tr>
<tr>
<td>MTM</td>
<td>205</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2

Channel Height (in Tracks) Comparison - PRIMARY I

39
Table 3
Overall Layout Height (in $\lambda$) Comparison - PRIMARY I

<table>
<thead>
<tr>
<th>Cell Model</th>
<th>Two Layer Process</th>
<th>Three Layer Process</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>with vias over-the-cell</td>
</tr>
<tr>
<td>BTM</td>
<td>3852</td>
<td>2682</td>
</tr>
<tr>
<td>CTM</td>
<td>4356</td>
<td>2568</td>
</tr>
<tr>
<td>MTM</td>
<td>3798</td>
<td>2568</td>
</tr>
</tbody>
</table>

Table 4
% Overall Height Comparison With 2-layer Conventional Channel Routing - PRIMARY I

<table>
<thead>
<tr>
<th>Cell Model</th>
<th>Two Layer Process</th>
<th>Three Layer Process</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>with vias over-the-cell</td>
</tr>
<tr>
<td>BTM</td>
<td>88.42</td>
<td>61.57</td>
</tr>
<tr>
<td>CTM</td>
<td>100</td>
<td>58.95</td>
</tr>
<tr>
<td>MTM</td>
<td>87.19</td>
<td>58.95</td>
</tr>
</tbody>
</table>
Table 5

% Overall Channel Height Compared With Overall Layout Height - PRIMARY I

<table>
<thead>
<tr>
<th>Cell Model</th>
<th>Two Layer Process</th>
<th>Three Layer Process</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>with vias over-the-cell</td>
<td>without vias over-the-cell</td>
</tr>
<tr>
<td>BTM</td>
<td>33.80</td>
<td>04.92</td>
</tr>
<tr>
<td>CTM</td>
<td>41.46</td>
<td>00.00</td>
</tr>
<tr>
<td>MTM</td>
<td>32.85</td>
<td>00.00</td>
</tr>
</tbody>
</table>

PRIMARY I, regardless of number of layers available and permissibility of vias over-the-cell.

Table 6 gives height of each channel in PRIMARY I in the layouts produced using 3CTM-V, 3BTM-V, and 3MTM-V cells, respectively. Figure 19 shows the information in Table 6 in the graphical form. The entire PRIMARY I was routed in 2 minutes by MTM-V router. MTM routers allow minimum length routing of critical nets. In addition, they produce minimum height layouts which improves performance even further.
Table 6
Channel Height - PRIMARY I

<table>
<thead>
<tr>
<th>Channel #</th>
<th># of tracks produced</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3CTM-V</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
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<td>12</td>
<td>5</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>104</td>
</tr>
</tbody>
</table>
Figure 19. Channel Height - PRIMARY I.
CHAPTER VII

CONCLUSIONS

In this thesis, we have investigated the effect of cell model on over-the-cell routing. Based on this investigation, we have developed a new class of cell models called Middle Terminal Models (MTM) that combines the advantages of both existing cell models. This class contains the models 2MTM, 3MTM-V and 3MTM+V depending on the number of metal layers and permissibility of vias over-the-cell. We have implemented two MTM routers; MTM+V and MTM-V. The first router allows vias over-the-cell, whereas, the second does not. These routers have been implemented in C on a SUN SPARCstation 1+ and have been tested on several benchmarks including PRIMARY I and PRIMARY II from MCNC. MTM based layouts produced by our routers are significantly better than the layouts based on existing cell models, irrespective of permissibility of vias over-the-cell. MTM cell models and routers are suitable for high performance circuits. They allow minimum length routing of critical nets, and they produce minimum height layouts which improves performance even further.

The results reported in this thesis are very close to optimal in terms of total layout height. This indicates that further improvement in layout by use of over-the-cell routing is only possible by improving the placement algorithms. This is due to the fact that the layouts is so compact that actual cell model can no longer be ignored in placement process. We are currently developing a placement algorithm for standard cell designs, which takes into account the cell model and optimizes, not only channel routing areas, but also over-the-cell routing areas. Our preliminary results indicate that large gains in chip layout area are possible for industrial benchmarks by cell model specific placement algorithms.
REFERENCES


