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ANALYSIS OF SANDWICH PLATES SUBJECTED TO BLAST LOADING

by

Vijay Prasad Bulla

A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
requirements for the
Degree of Master of Science
Department of Mechanical and
Aeronautical Engineering

Western Michigan University
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ANALYSIS OF SANDWICH PLATES SUBJECTED TO BLAST LOADING

Vijay Prasad Bulla, M.S.

Western Michigan University, 1993

A dynamic analysis is presented for the bending response of square sandwich plates with isotropic core and facings under blast type pressure. The maximum central deflections of simply supported plates under static and dynamic loadings are compared for various thicknesses and elastic moduli. The deviation of the thick sandwich plate results from the pure-bending theory results is presented for various core properties. Small deflection dynamic iso-response plots are shown for different core rigidities. To study the limits of small deformation linearity for various sandwich plates, non-linear results for deformations under high pressure loads are compared with the linear results. The results show that the maximum occurs at a frequency after the resonant frequency.

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ACKNOWLEDGEMENTS

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Next, heartfelt thanks to my family members who supported and encouraged me in the pursuance of my master's. My thanks are to my mother who has sacrificed a lot for my education.

I dedicate this thesis in memory of my father, Dr. Somasekar without whose support I would have never made this far.

Vijay Prasad Bulla

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LIST OF FIGURES

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LIST OF SYMBOLS

a	-	Plate Size [m]
D	-	Plate Bending Stiffness [N-m]
E_c	-	Elastic Modulus of Core [GPa]
E_f	-	Elastic Modulus of Facing [GPa]
f	-	Fundamental Natural Frequency of the Structure [Hz]
h	-	Total Thickness of the Panel [m]
h_c	-	Core Thickness [m]
h_f	-	Facing Thickness [m]
I	-	Impulse due to pressure p [Pa-sec]
I_m	-	Impulse due to pressure p_m [Pa-sec], Eqn.(7)
I_{imp}	-	Minimum impulse required to cause deflection [Pa-sec]
N	-	Load duration/Half Natural period
p_m	-	Peak pressure applied [Pa]
p_{st}	-	Static Pressure [Pa]
T	-	First Natural Period of the Structure [sec]
T_1	-	Time for linear pressure increase [sec]
T_2	-	Time taken for pressure pulse to decay to zero [sec]
T_3	-	Total time of the pulse including free vibration [sec]
w_m	-	Peak Dynamic Deflection at the Center of the Plate [m]

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List of Symbols - continued

w_{st}	-	Maximum Static Deflection at the Center [m]
ρ_c	-	Density of the Core [Kg/m^3]
ρ_f	-	Density of the Facing [Kg/m^3]
ρ_{av}	-	Average Density of the Panel [Kg/m^3]
ν_c	-	Core Poisson ratio
ν_f	-	Facing Poisson ratio

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CHAPTER I

INTRODUCTION

Composite sandwich structures have become important in the fields of aerospace and marine structures for the design of light-weight high-stiffness structures. Many structures are replaced by light composite sandwich structures. A lot of work has been done on the bending and vibration characteristics of composite sandwich structures. However, considerably less work has been done on the transient response characteristics of sandwich plates. Watanabe et al. [7], developed a general finite element method for the bending and modal analysis of sandwich plates with general anisotropic composite laminates. Frostig and Baruch [1], have analyzed the bending behavior of foam cored sandwich beams. Kanematsu and Hirano [2] have presented a linear analysis for stiffness and vibration of sandwich plates for unbalanced facings with an orthotropic core. Rao [4] studied the buckling of anisotropic sandwich plates. Ibrahim et al. [3] have presented formulations for analyzing sandwich plates with unequal facings eliminating the coupling of membrane and stiffness actions.

The goal of the present study was to investigate the lateral response of sandwich structures subjected to blast-type pressure.

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CHAPTER II

LOADING CHARACTERISTICS

General

A typical pressure wave from an air-blast rises sharply to its peak and then gradually decreases to zero (see Figure 1). The history of loading p(t) may be described as the loading phase and then free vibration phase. In the loading phase the pressure may be approximated as a linear increase from zero to its peak p_m at time T_1 and then an exponential decay to zero. The loading is then described by

$$p(t)=p_m\cdot\frac{t}{T_1},\ T_1\geq t\geq 0 \tag{1}$$

$$p(t)=p_m\cdot\exp(-\lambda\cdot(t-T_1)),\ t\geq T_1 \tag{2}$$

where,

T_1 = The time taken for linear pressure increase from zero to peak.

T_2 = The time when the pressure pulse decays to zero.

The parameter λ describes the rate of pressure decay.

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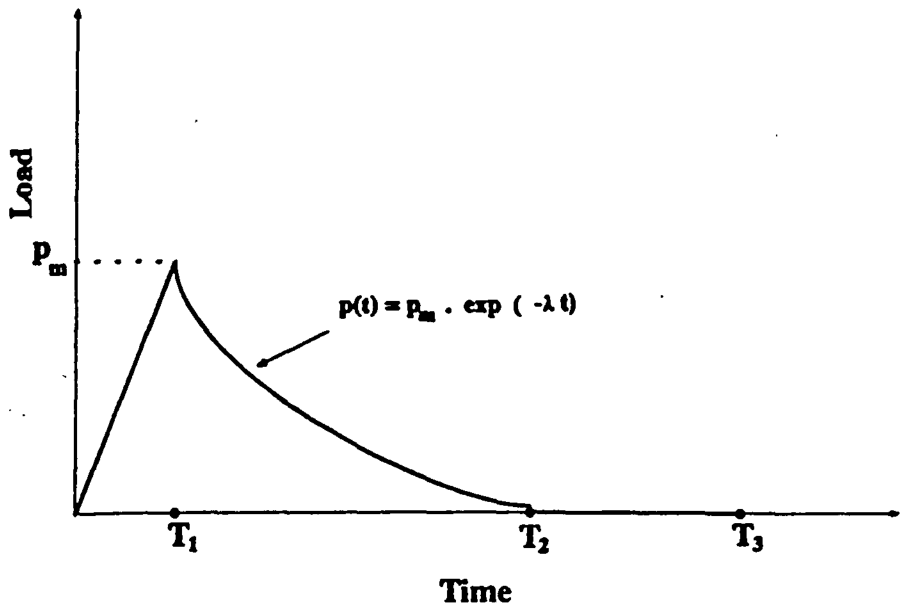

Figure 1. Load vs Time.

If T_3 is the total time of the pulse, including the free vibration, no load is applied from T_2 to T_3. The effect of a blast is the gradual decrease of the load in an exponential manner which goes to zero in infinite time. In the present study the load is assumed to decay to 1/1000 of its peak pressure at time T_2.

$$p(t=T_2) = p_m \cdot \exp[-\lambda \cdot (T_2 - T_1)] \tag{3}$$

$$p(t=T_2) = \frac{p_m}{1000} \tag{4}$$

$$\lambda = \frac{\ln(0.001)}{(T_2 - T_1)} \tag{5}$$

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For dynamic loading the period of loading is approximately equal to half of the fundamental natural period of the structure. Since the blast load is impulsive in nature, the load is applied only for half of the natural period of the structure. This is done so that the pressure loading is applied only in the direction of the natural vibration of the motion structure. The response is studied for different E_c/E_f and h_c/h_f. Effects of impulsive and quasi-static loadings have also been shown. The total impulse for a pressure p is

$$I=\int_0^\infty (p \cdot dt) \tag{6}$$

The impulse for the loading represented in Equations 1 and 2 for time period 0 to T_2 is given by

$$I_m=\int_0^{T_2} p \cdot dt \tag{7}$$

$$I_m=p_m \cdot [(\frac{T_1}{2})+\frac{1}{\lambda}\exp(-\lambda(T_2-T_1))] \tag{8}$$

Duration Ranges

The structural response to the dynamic loading depends also on the duration of loading. This can be explained best by a plot of w_m/p_m versus N shown in Figure

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2 where N is the ratio of the load duration (T_2) to the half of the natural period of the sandwich plate (T/2). The response of the sandwich plate is studied for N ranging from 0.1333 to 133.3. In Figure 2, for N < 4 the loading is impulsive as the deflection of the sandwich plate is smaller than the static response for the applied load. For N > 18 the response is approximately the same as the static response. This is the quasi-static range. For 4 > N >18 the deflection is larger than the static response, which is the dynamic realm, where a dynamic amplification of the deflection occurs.

When N < 4 the response is impulsive in nature. For N > 18 it is quasi-static. In the intermediate range the behavior is dynamic. In the dynamic realm less pressure is needed to cause the peak deflection w_m. In the impulsive realm a minimum impulse has to be applied to obtain the same response w_m. For the sandwich plate (see Figure 3), at least an impulse of 46822.3 Pa-s/m has to be applied to cause the peak deflection w_m. In the quasi-static realm, the impulse is very large but the load is applied slowly. Since the response is dominated by the maximum pressure, the deflection w_m is observed only if the required pressure is applied. For the plate of Figure 3, the ratio of static pressure to the deflection of the sandwich plate due to a static pressure is 0.5055e9 Pa/m. In the impulsive realm a very high pressure is applied for a very short period. Even when the applied load is large a minimum impulse is required for the response of the sandwich structure. In both the impulsive and quasi-static loadings, the loading history is not a dominant factor, but the impulse and peak pressure, respectively, are the only important characteristics. Based on the p_m/w_m vs N plot it can be determined when the history of loading is insignificant and

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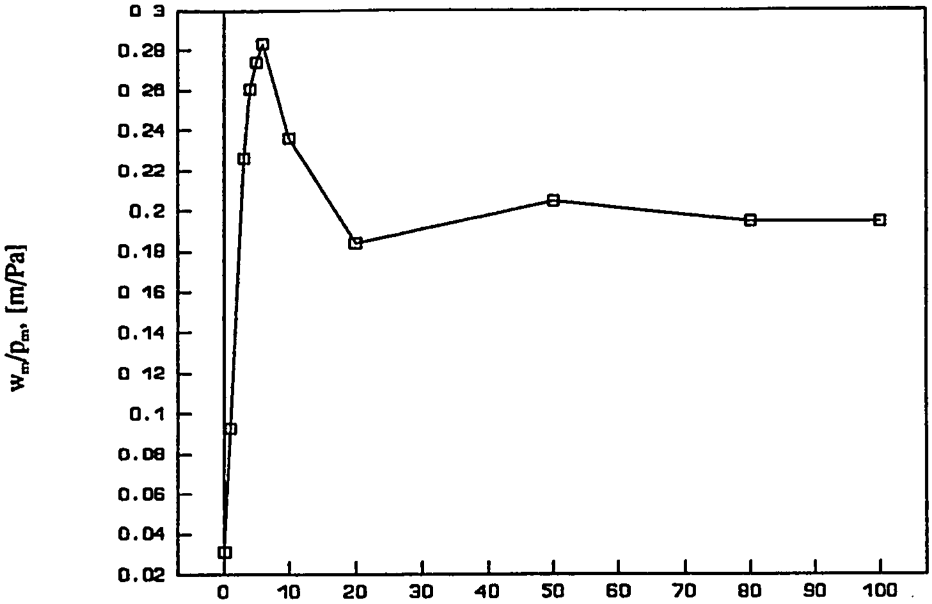

Figure 2. w_m/p_m vs N for $E_c/E_f = 1$.

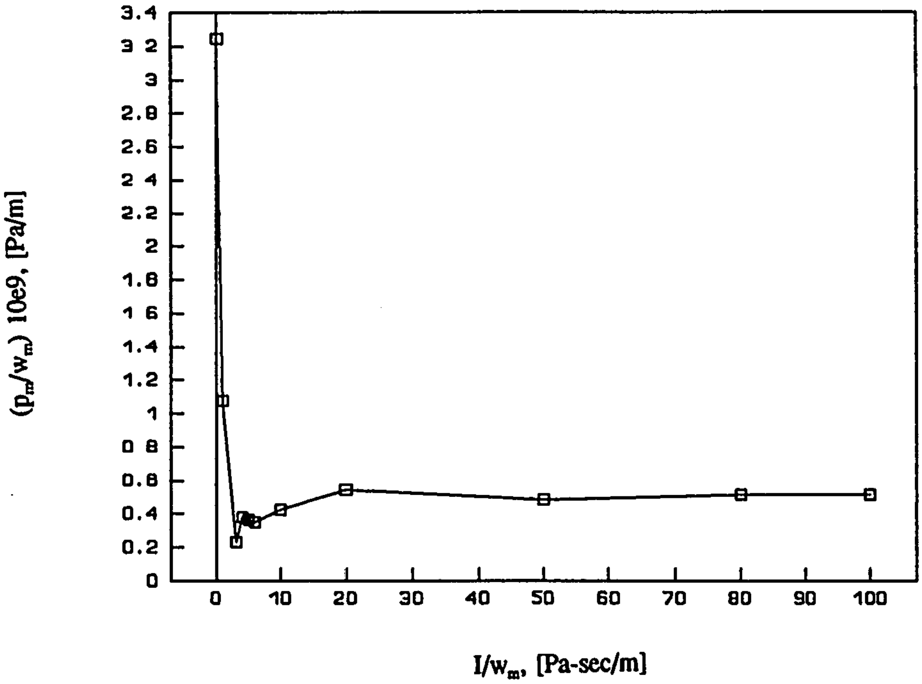

Figure 3. p_m/w_m vs I/w for $E_c/E_f=1$.

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can be ignored. The ratio of the period of loading T_2 to the half natural period of the structure T/2 provides a basis to define the loading range.

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CHAPTER III

FEA MODELING OF THE PROBLEM

Modeling

Structure

The sandwich plate consists of isotropic facings (2mm thickness aluminum) and a thick isotropic core. The elastic modulus of aluminum is 72GPa. The sandwich plate was modeled with plate elements for the facings and 3-D solid elements for the core. As shown in Figure 4, there is an overlap of 1mm between the facing and the core. This is because the nodes of the plate element are on the mid-plane of the facing which is 1mm from the surface of the facings. The 3-D solid elements and the plate elements are connected at the nodes to form the sandwich structure. The translational degrees of freedom (ux, uy and uz) of the plate and the 3-D solid are coupled at these nodes.

Elements

The 3-D solid element has eight nodes and each node has three degrees of freedom, displacements (ux, uy and uz). The plate element has 4 nodes and each node has 6 degrees of freedom (displacements ux, uy, uz and rotations rotx, roty and rotz).

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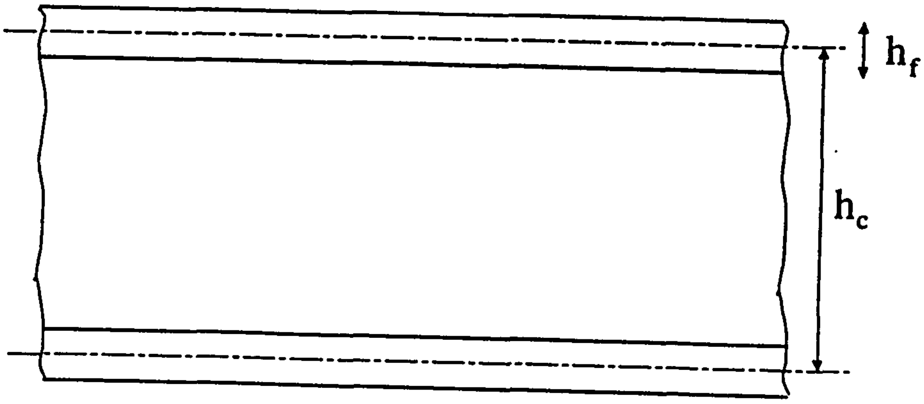

Figure 4. Square Sandwich Plate.

For a quarter sandwich plate the ratio of the side of the 3-D solid element to (h_c-h_f) is 3.

The size (a) of the sandwich plate is twenty four times the ratio of the side of the 3-D solid element to (h_c-h_f). The different sizes of the sandwich plate studied for different core properties are shown in Table 1. The structure was modeled as a quarter taking advantage of symmetries. The maximum deflections of the full sandwich plate and quarter sandwich plate for a static pressure of 1kPa are same. Since the maximum deflections of static loading are same for both the full sandwich plate and quarter sandwich plate, the quarter sandwich plate model with the symmetry boundary conditions has been chosen for the study of the various analyses. The results for the maximum deflection of the quarter model sandwich plate, with 22mm thick core and 72GPa elastic modulus, by ANSYS and the classical plate theory for a static pressure of 1kPa are compared below for one and two layers of 3-D solid elements. For a 4*4 (16 elements) meshing the maximum deflections are given below

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Table 1

The Different Sizes and Thicknesses of Plates

a [m]	h_c [m]	(h_c-h_f) [m]	$a/2(h_c-h_f)$
0.096	0.006	0.004	12
0.192	0.010	0.008	12
0.288	0.014	0.012	12
0.384	0.018	0.016	12
0.480	0.022	0.020	12

	w_{st} ANSYS [m]	w_{st} plate [m]
1 layer of 3-D solid elements	1.978e-6	1.934e-6
2 layers of 3-D solid elements	1.981e-6	1.934e-6

The difference in the maximum deflection between one and two layer 3-D solid elements is 0.15%. This is permissible as this will not affect the results. The model with one layer of 3-D solid elements was then chosen for the analysis. The maximum deflection for a 24mm thick sandwich plate with 72GPa elastic modulus core for different numbers of elements is given in Table 2.

The deflections of the quarter model with 4*4 (16 elements) meshing when compared with the 5*5 (25 elements) meshing within 1%. Hence, the model with 4*4 meshing of the quarter plate is chosen for modeling.

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Table 2

Deflection of the Plate for Different Number of Elements

Number of elements per quarter plate	w_{st} ANSYS [m]	% Difference
9=3*3	1.925e-6	3.65
16=4*4	1.978e-6	1
25=5*5	1.998e-6	-

Boundary Conditions

The quarter plate is simply supported along the sides of the lower facing. For the quarter model symmetry boundary conditions, see Figure 5, (for symmetry along y=a/2 the degrees of freedom uy, rotx and rotz are restrained while for x=a/2, ux, roty and rotz are restrained) are also applied.

Loads

For the static linear analysis a pressure load of 1kPa is applied. A small load is applied so that the deflections are small and within the linear range. Different pressures are applied for static non-linear and dynamic analyses. The pressure is applied on the upper facing of the sandwich plate.

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Material Properties

The facings of the sandwich structure are isotropic and made of aluminum. The core is isotropic and the elastic modulus of the core has been varied from stiff to soft materials, E_c/E_f varying from 1 to 0.1389e-3. The density of the core does not depend on the elastic modulus of the material used, and is retained constant ρ_c=2700 Kg/m^3 for all the analyses. This is done because the effect of density on the sandwich plates was not of interest.

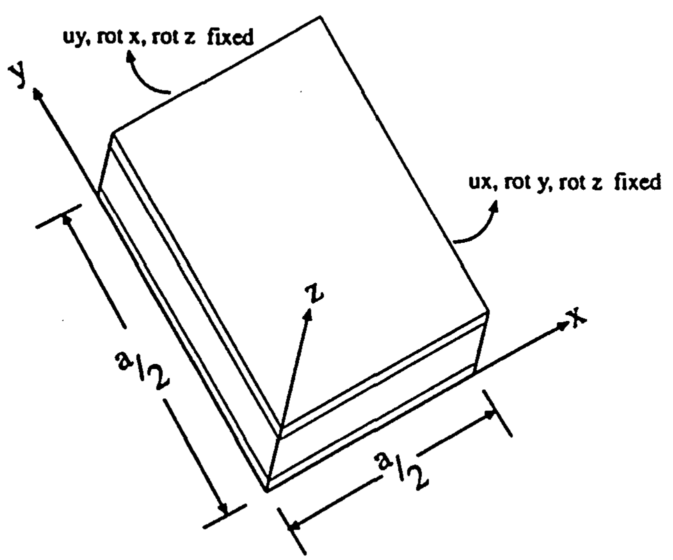

Figure 5. Plate Showing Size and Boundary Conditions.

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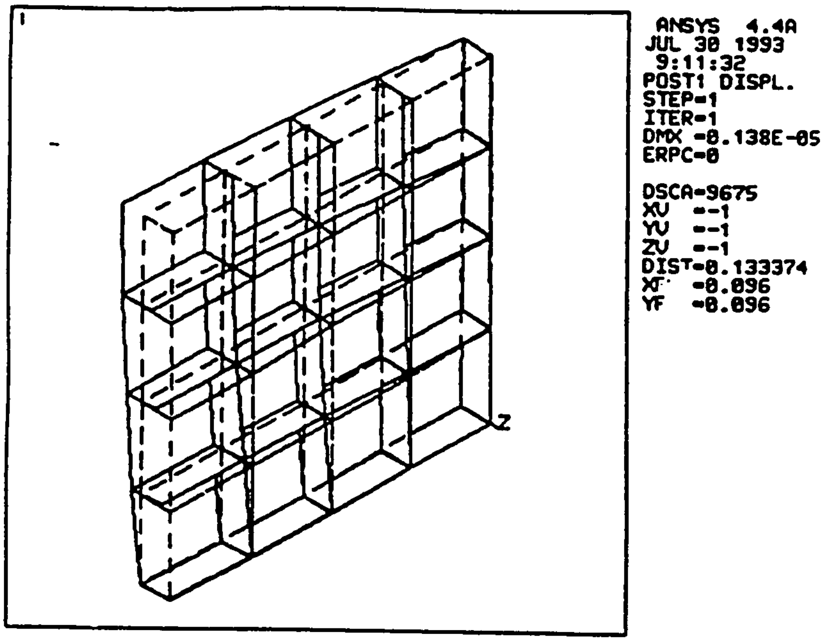

Figure 6. Meshed Quarter Sandwich Plate.

Static Analysis

Linear Analysis

The sandwich plate deformation in the ANSYS model accounts for transverse shear deformation through the solid core elements. Hence, the ANSYS model is less rigid than the classical plate theory. The linear analysis is performed to study the effect of the core paremeters on the bending stiffness of the sandwich plates, and to compare it to the bending stiffness of the equivalent classical Kirchhoff plate.

Non-linear Analysis

The non-linear static analysis is performed taking into account the large

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deflection effects, where the plate deformations are used to continuously redefine the geometry of the sandwich plate and accordingly update the stiffness matrix. During large deflections, the pressure loads remain normal to the element and follow its rotation whereas body forces and concentrated loads remain parallel to the original direction and do not rotate in the direction of the surface. Stress stiffening effects were not included in the analysis.

Modal Analysis

The modal analysis is performed on the full plate model to extract the fundamental natural frequency. When the plate is modeled as a quarter the symmetric conditions eliminate the asymmetric modes of the full plate to behave symmetrically. Hence, the frequencies of the full plate model are considered to find the natural time periods of the structure.

Dynamic Analysis

Linear Dynamic Analysis

The linear dynamic analysis is performed to relate the peak deflection to the pulse duration and intensity, and subsequently to relate the pressure and impulse ratios for the iso-response plots. For 20 integration time steps per cycle a phase shift of 1% is observed with respect to the period elongation, which is acceptable. The loading was applied in 20 integration time steps. Since the blast load rises very sharply, the rise time is $T_1=T_2/4$ for the load pulse. The ratio (N) of the loading period to the half

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natural period is varied for values ranging from 0.1333 to 133.33, for different loading ranges. The duration of integration time step δt, varies for different loading periods. For dynamic loading (N=6.6665) the peak response of the plate (a=480mm, E_c=72Gpa, p_m=1kpa,) obtained is shown in the Figure 7. For the same plate when the load is impulsive (N=1.3333) the peak response of the plate is as shown in the Figure 8. No pressure is applied, once the pressure decays to p_m/1000. For impulsive loading, when N is 0.1333 the plate takes a longer duration to attain the peak response compared to the period of loading. Depending on the nature of loading, the free vibration phase is varied (relatively longer for impulsive and shorter for dynamic and quasi-static loadings). When N is 0.1333, T_3=3.5T_2, whereas for 1.3333 > N > 133.33 T_3=1.5T_2.

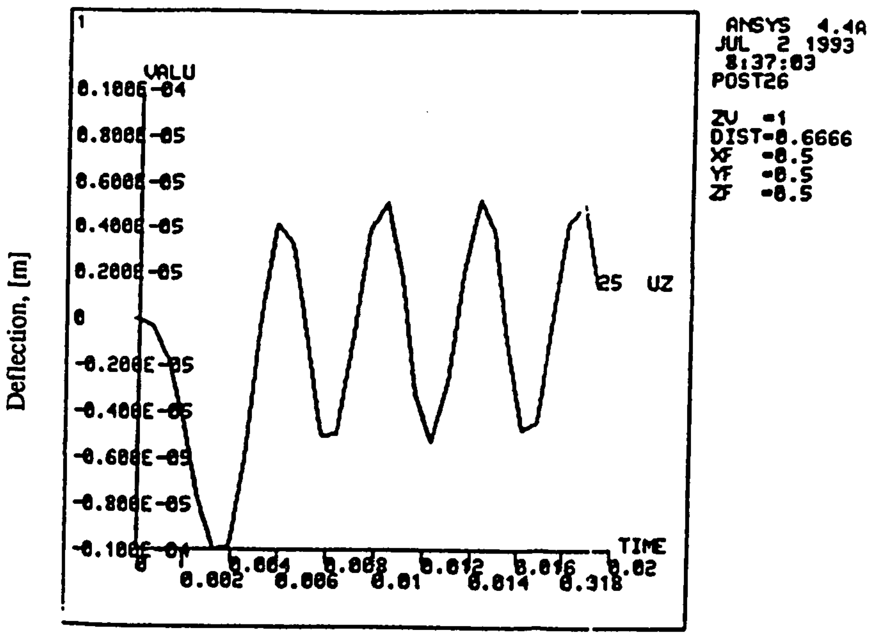

Figure 7. Response of the Centre of Plate to Dynamic Loading.

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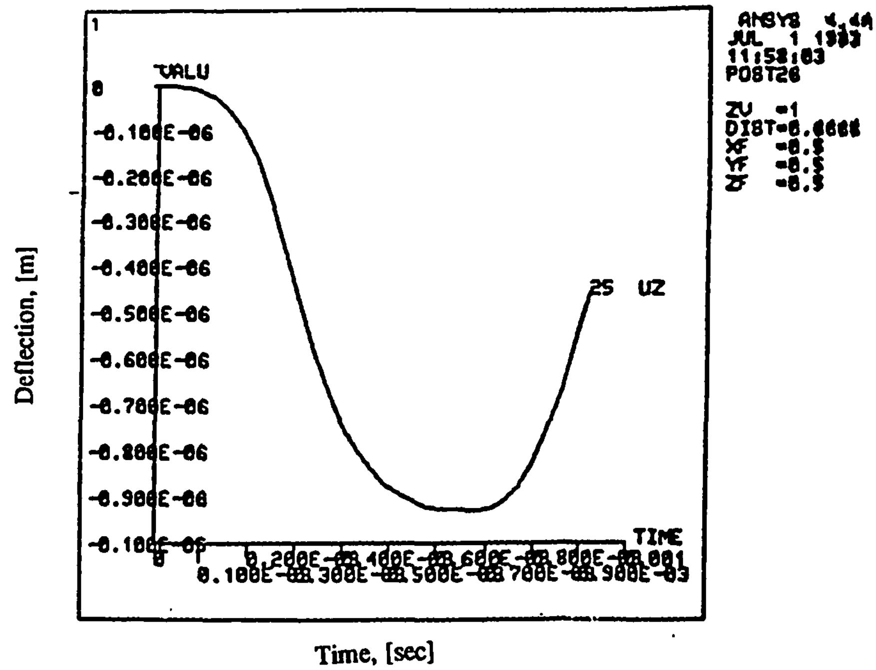

Figure 8. Response of the Centre of Plate to Impulsive Loading.

Non-linear Dynamic Analysis

For non-linear dynamic analysis the load period is divided into twenty load steps as for the linear analysis, but 5 iterations are now defined for each load step to allow for the non-linear convergence. Large deflection effects are included in the analysis. Also, comparisons with analyses that include stress stiffening effects were conducted. When the geometry of the structure changes because of the loading, the stiffness matrix is updated for every iteration. The stiffness matrix changes along with the nodal positions unlike the linear loading. This is the large deflection effect. When a structure stiffness changes due to the stress state, stress stiffening is said to occur. The effect of stress stiffening is accounted for by the generation of an additional matrix known as stress stiffness matrix.

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CHAPTER IV

PLATE RESPONSE TO PRESSURE LOAD

Static Pressure

Linear Results

The maximum deflections of the sandwich plate under the static loading are largely affected by the core shear properties. It is the core modulus and thickness which account for the shear effects. The ANSYS results for the maximum central deflection of the sandwich plate with varying E_c/E_f and h_c/h_f are tabulated in Table 3. The asymptotic nature of the deflection/plate thickness for different core moduli is shown in Figure 9.

Comparison to Thin Plate Theory

According to the Kirchhoff plate theory a line which is straight and normal to the midsurface before deformation is assumed to remain straight and normal throughout the deformation also. The thin plate theory does not include transverse shear effects and it implies infinite transverse shear rigidity. The maximum deflection of a thin rectangular plate with sides a and b under a uniform pressure p_{st} is given by [5]

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$$w_m = \frac{16}{\pi^6}\,\frac{p_{st}}{D}\sum_{m=1}^{3}\sum_{n=1}^{3}\frac{(-1)^{\frac{m-1}{2}}(-1)^{\frac{n-1}{2}}}{mn\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \tag{9}$$

$$w_m = \frac{16}{\pi^6}\,p_{st}\cdot\frac{a^4}{D}\sum_{m=1}^{3}\sum_{n=1}^{3}\frac{(-1)^{\frac{m-1}{2}}(-1)^{\frac{n-1}{2}}}{mn\left(m^2+n^2\left(\frac{a}{b}\right)^2\right)^2} \tag{10}$$

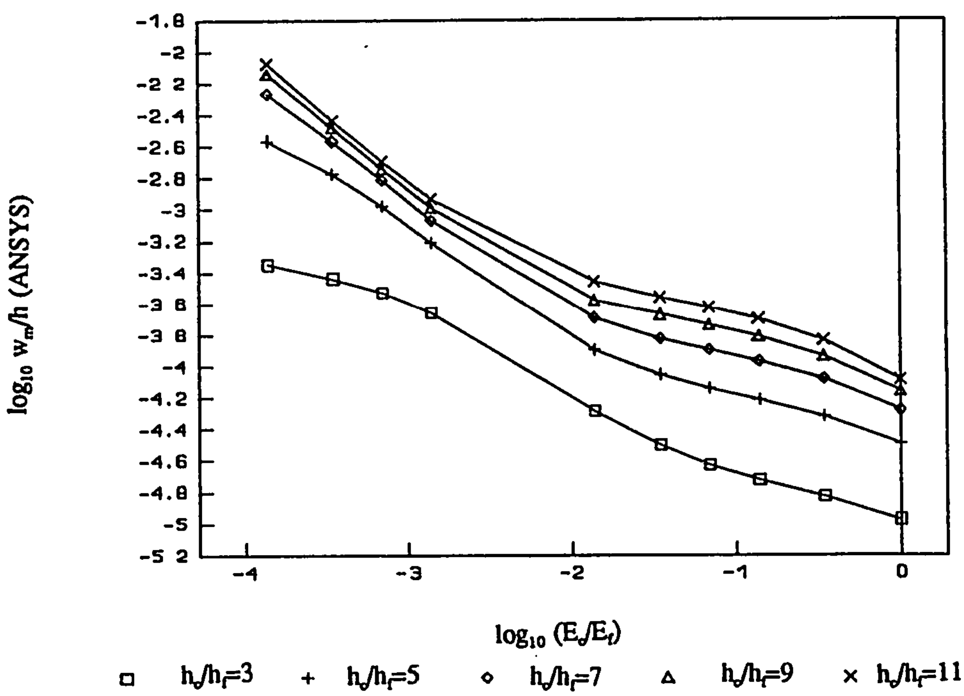

Figure 9. $\log_{10}$ w_m/h (ANSYS) vs $\log_{10}$ (E_c/E_f) for Different h_c/h_f.

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Table 3

The Maximum Deflection (ANSYS) of the Panel for Varying h_c/h_f and E_c/E_f

			w_m [10^{-6} m]		
E_c/E_f	h_c/h_f=3	h_c/h_f=5	h_c/h_f=7	h_c/h_f=9	h_c/h_f=11
1	0.0849	0.3883	0.8381	1.378	1.978
0.3472	0.1184	0.5804	1.340	2.336	3.516
0.1389	0.1512	0.7328	1.731	3.099	4.796
0.0694	0.1884	0.8662	2.025	3.633	5.660
0.0347	0.2508	1.060	2.401	4.294	6.582
0.0139	0.4143	1.540	3.256	5.337	8.336
0.1389e-2	1.747	7.419	13.74	20.58	28.02
0.6944e-3	2.354	12.63	24.46	36.42	48.8
0.3472e-3	2.915	20.29	43.63	66.60	89.25
0.1389e-3	3.563	32.78	87.18	146.1	202.8

and for a=b:

$$w_m = p_{st}\frac{a^4}{D}\ 4.055\ 10^{-3} \tag{11}$$

where, p_{st}=Applied pressure [Pa]

a=Length of the plate [m]

D=Bending stiffness of the plate [N m], defined as:

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Table 4

The Maximum Central Deflection of the Panel by Plate Theory for Varying h_c/h_f and E_c/E_f

	w_m [10^{-6} m]				
E_c/E_f	h_c/h_f=3	h_c/h_f=5	h_c/h_f=7	h_c/h_f=9	h_c/h_f=11
1	0.0767	0.3679	0.8074	1.34	1.933
0.3472	0.0974	0.5216	1.243	2.2	3.345
0.1389	0.1066	0.6018	1.501	2.767	4.361
0.0694	0.1101	0.6343	1.613	3.027	4.852
0.0347	0.1108	0.6520	1.675	3.177	5.142
0.0139	0.1113	0.6630	1.715	3.274	5.333
0.1389e-2	0.1114	0.6697	1.740	3.335	5.455
0.6944e-3	0.1114	0.6701	1.742	3.338	5.461
0.3472e-3	0.1114	0.6703	1.742	3.340	5.464
0.1389e-3	0.0114	0.6704	1.743	3.341	5.467

$$D = \frac{E_c h_c^3}{12(1-\nu_c^2)} + \frac{E_f (h_c+h_f)^3}{12(1-\nu_f^2)} - \frac{E_f (h_c-h_f)^3}{12(1-\nu_f^2)} \qquad (12)$$

and, E_c = Modulus of elasticity of the core [Pa]

E_f = Modulus of elasticity of the skin [Pa]

h_c = thickness of the core [m]

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Table 5

The Maximum Deflection Ratio (ANSYS/Plate Theory) of the Panel for Varying h_c/h_f and E_c/E_f

			$w_{m\ ANSYS}/w_{m\ plate}$		
E_c/E_f	h_c/h_f=3	h_c/h_f=5	h_c/h_f=7	h_c/h_f=9	h_c/h_f=11
1	1.1070	1.0553	1.0379	1.0284	1.0229
0.3472	1.2151	1.1127	1.078	1.0618	1.0510
0.1389	1.4182	1.2177	1.1529	1.1198	1.0996
0.0694	1.7111	1.3658	1.2551	1.200	1.1663
0.0347	2.2417	1.6260	1.4328	1.3375	1.2880
0.0139	3.667	2.2320	1.8980	1.6302	1.5630
0.1389e-2	15.36	11.0771	7.8950	6.1714	5.1366
0.6944e-3	20.7015	18.8467	14.0434	10.9102	8.9348
0.3472e-3	25.6307	30.2685	25.0397	19.9407	16.3304
0.1389e-3	31.3253	48.8927	50.0214	43.730	37.093

t_f = thickness of the skin [m]

ν_c = Poisson's ratio of the core

ν_f = Poisson's ratio of the skin

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The bending stiffness of a sandwich plate, as given by equation 12, is due to the core and the facings assuming Kirchhoff type linear deformation through the thickness. The structure is subjected to 1kPa pressure so that the deflections are small and a linear behavior can be observed. The results for the maximum defleetion by pure bending classical plate theory are tabulated in Table 4. The asymptotic nature of the maximum deflection with increasing core softness is shown in Figure 10. The maximum deflection ratio $w_{m\ ANSYS}/w_{m\ plate}$ is listed in Table 5. Graphs of $w_{m\ ANSYS}/w_{m\ plate}$ for varying core moduli and thicknesses are shown in Figures 11 and 12, respectively. It is seen that when E_c/E_f is greater than 0.0694 the maximum deflections of the sandwich panel predicted by ANSYS finite element analysis and the thin plate theory are in close agreement. The stiffness ratio of ANSYS/plate is as shown in Table 6 and Figure 13. The stiffness of the sandwich plate by ANSYS approaches the stiffness of the plate theory with the increase of the thickness and core modulus.

Non-linear Behavior

When the displacements and strains developed in a structure are small the linear deformation approximations can be used. This means that the geometry of the structure is assumed to remain unchanged. When large displacements occur the geometric non-linearity should be taken into account. The decrease or increase of displacements due to geometrical non-linearities is referred to as large displacement effects. Stresses due to membrane action which are neglected in plate pure bending increase or decrease in displacements when compared to linear approximation of small

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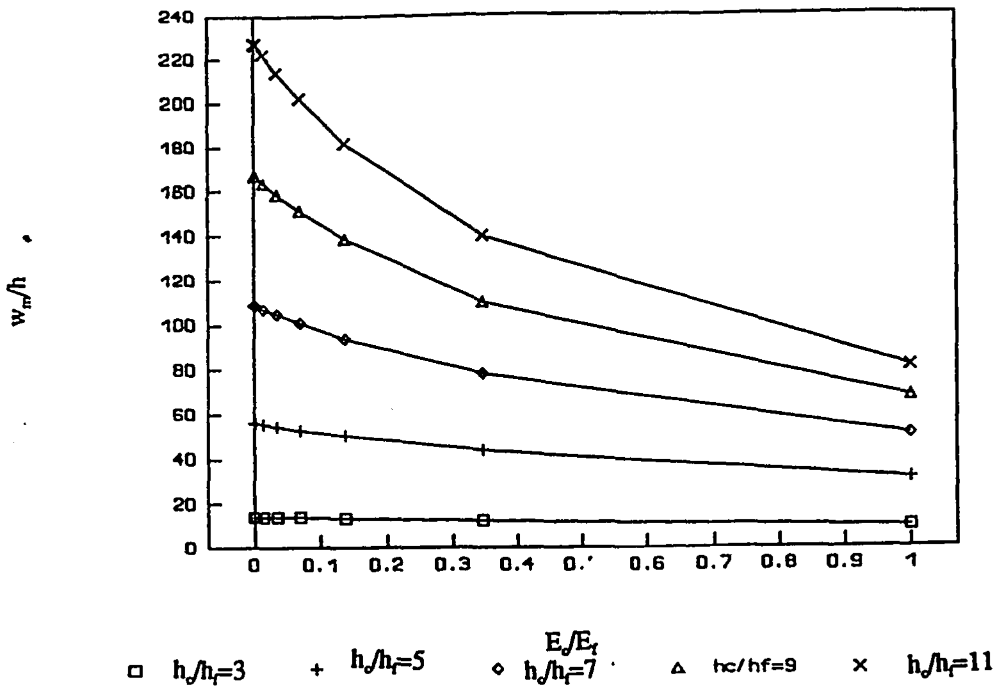

Figure 10. $w_{m\ plate}$ vs E_c/E_f for Different h_c/h_f.

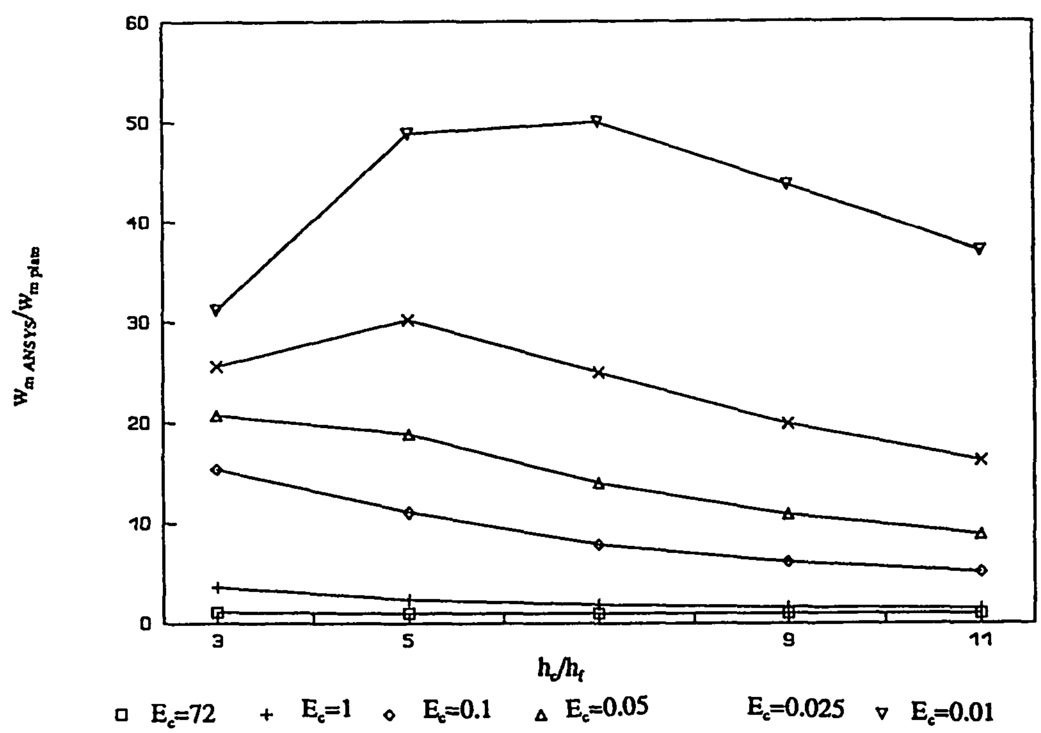

Figure 11. $w_{m\ ANSYS}/w_{m\ plate}$ vs h_c/h_f for Different E_c/E_f.

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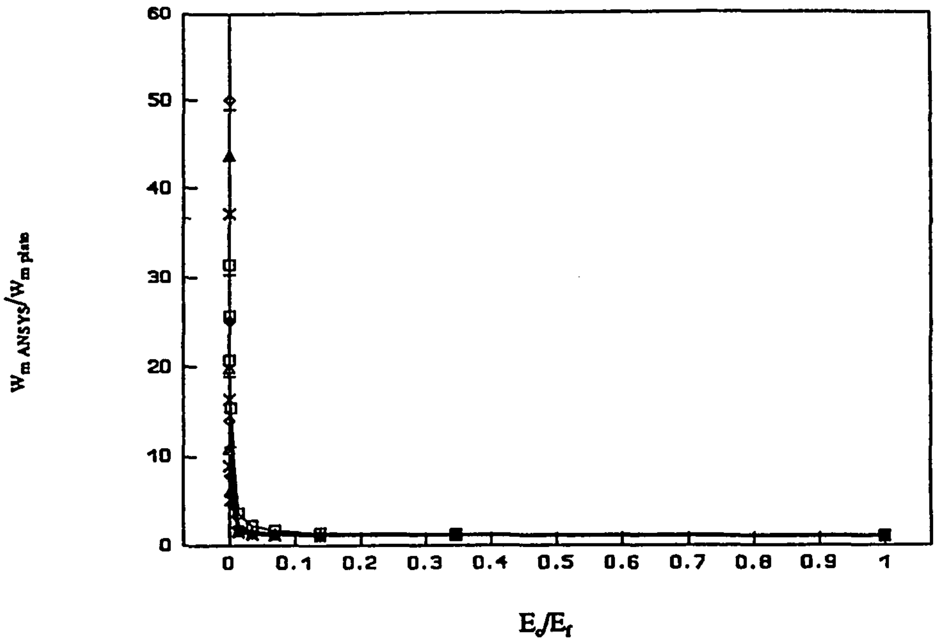
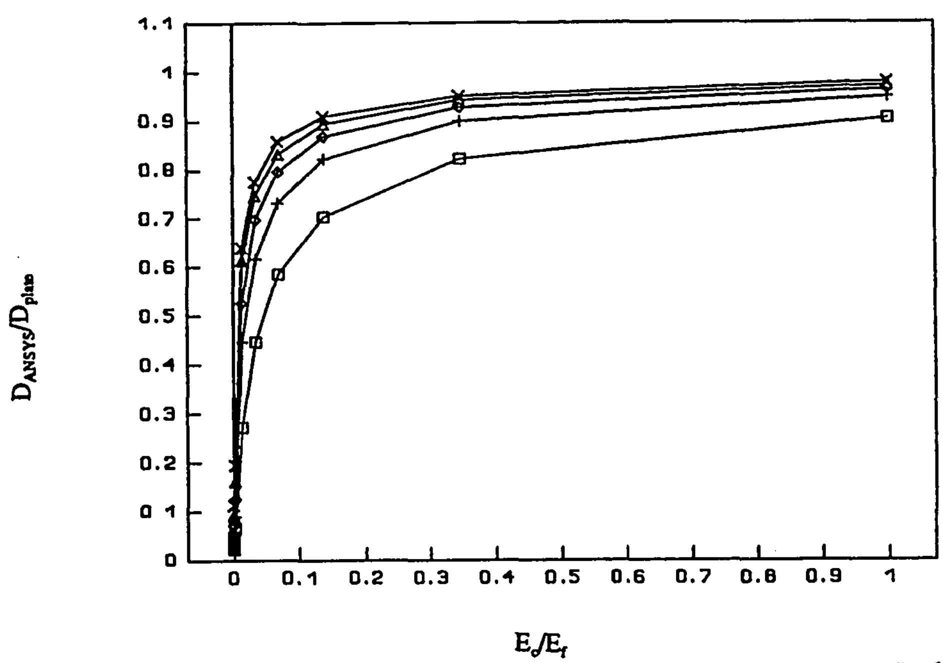


□ $h_c/h_f=3$ + $h_c/h_f=5$ ◇ $h_c/h_f=7$ △ $h_c/h_f=9$ × $h_c/h_f=11$

Figure 12. $w_{m\ ANSYS}/w_{m\ Plate}$ vs E_c/E_f for Different Thicknesses.



□ $h_c/h_f=3$ + $h_c/h_f=5$ ◇ $h_c/h_f=7$ △ $h_c/h_f=9$ × $h_c/h_f=11$

Figure 13. D_{ANSYS}/D_{plate} vs E_c/E_f for Different h_c/h_f.

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Table 6

The Stiffness Ratio (ANSYS/Plate Theory) of the Panel for Varying h_c/h_f and E_c/E_f

	D_{ANSYS}/D_{Plate}				
E_c/E_f	$h_c/h_f=3$	$h_c/h_f=5$	$h_c/h_f=7$	$h_c/h_f=9$	$h_c/h_f=11$
1	0.9033	0.9476	0.9634	0.9723	0.9776
0.3472	0.8233	0.8987	0.9276	0.9418	0.9514
0.1389	0.7051	0.8212	0.8673	0.8930	0.9094
0.0694	0.5844	0.7321	0.7967	0.8333	0.8574
0.0347	0.4461	0.6150	0.6979	0.7476	0.7764
0.0139	0.2727	0.4480	0.5268	0.6134	0.6398
0.1389e-2	0.0651	0.0902	0.1266	0.1620	0.1947
0.6944e-3	0.0483	0.0530	0.0712	0.0916	0.1120
0.3472e-3	0.0390	0.0330	0.0400	0.0501	0.0612
0.1389e-3	0.0319	0.0204	0.0200	0.0228	0.0269

displacements. The large deflection procedure accounts for any structure, translational or rotational. The stiffness of the sandwich plate is then calculated for the new geometric position. During large deflection analysis the pressure loads will remain normal to the element and will follow its rotation. w_m/h for varying E_c/E_f and h_c/h_f are shown in Figures 14 and 15. When the static maximum deflection of the sandwich

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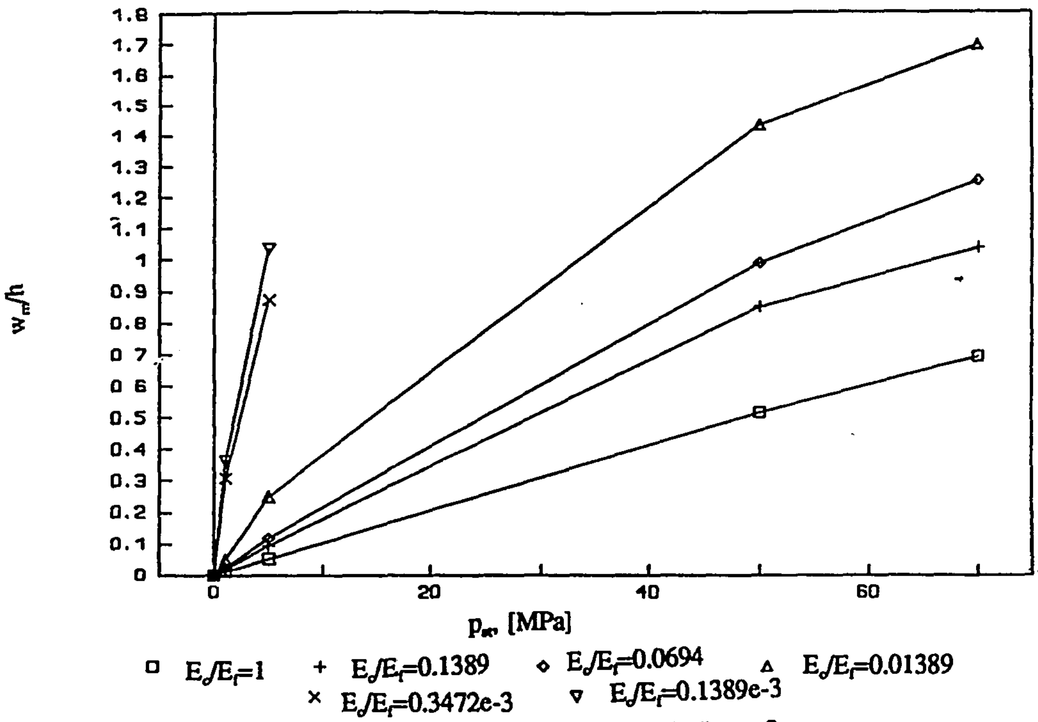

Figure 14. Large Deflection / h vs Pressure for $h_c/h_f = 3$.

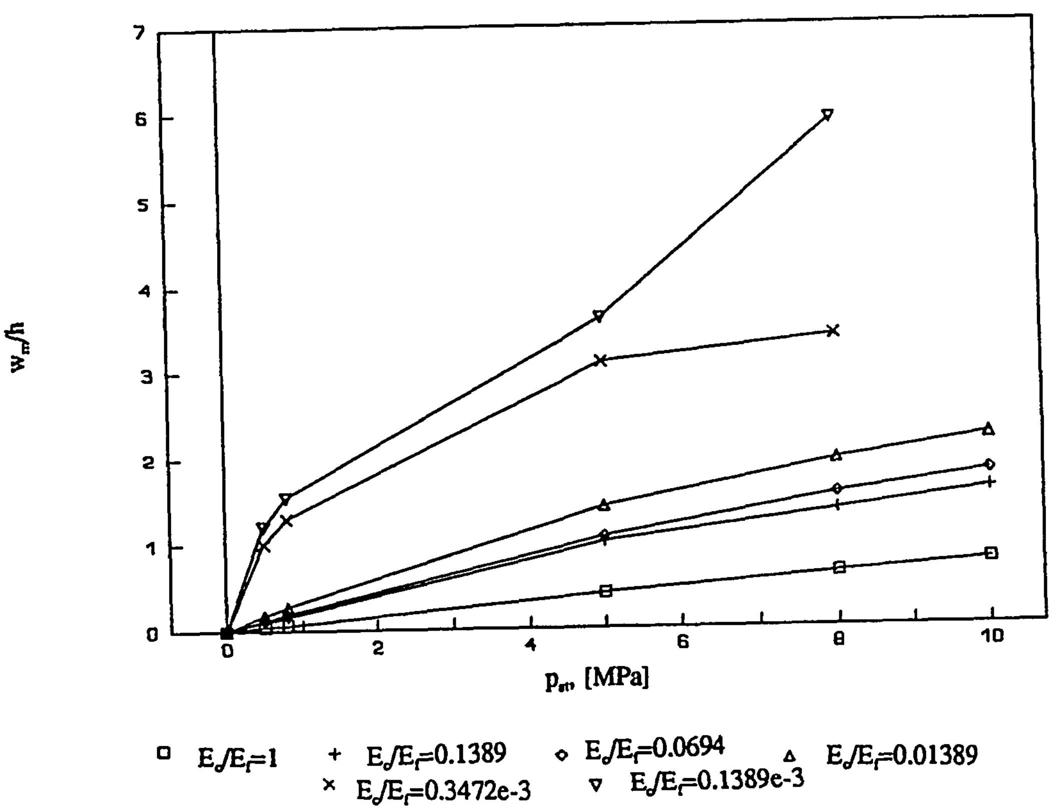

Figure 15. Large Deflection / h vs Pressure for $h_c/h_f = 11$.

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plate for large-deflection non-linear analysis is not more than 95% of the maximum deflection for the linear analysis, the sandwich plate is assumed to be behaving non-linearly.

Response to Blast Load

Modal Analysis Results

The natural frequency of a square thin plate is [6]:

$$f = \frac{\omega}{2\pi} = \frac{19.73}{2\pi a^2}\sqrt{\frac{D}{\rho_{av}h}} \tag{13}$$

where,

$$\rho_{av}h = \rho_c h_c + 2\rho_f h_f \tag{14}$$

The natural frequencies of the sandwich panel as predicted by ANSYS are smaller than those of the plate theory, because ANSYS considers the transverse shear of the solid which makes the structure more flexible. The fundamental natural frequencies obtained from ANSYS and plate theory are shown in Tables 7 and 8 respectively. The ratio of the fundamental frequency by ANSYS to the fundamental frequency by thin plate theory is shown in Table 9 and Figure 16. The f^2 ANSYS/f^2 Plate vs E_c/E_f for different h_c/h_f is shown in Table 10 and Figure 17. As the core modulus E_c increases the first modal natural frequency of the sandwich structure approaches the first mode natural frequency predicted by the thin plate theory. Also, the first modal natural

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Table 7

First Mode Natural Frequency (ANSYS) for Varying h_c/h_f and E_c/E_f

	f [Hz]				
h_c/h_f	E_c/E_f 1	E_c/E_f 0.1389	E_c/E_f 0.0139	E_c/E_f 0.1389e-2	E_c/E_f 0.1389e-3
3	3362.2	2252.2	1848.9	921.9	650.2
5	1658.9	1039.8	812.2	372.3	180.6
7	998.7	651.9	495.0	239.8	97.0
9	705.3	465.3	344.0	177.1	67.2
11	542.1	348.1	258.2	139.7	52.2

frequencies of the full plate and quarter plate models are the same, but the subsequent modes are different. This is apparently because of the asymmetric modes of the full plate which cannot be obtained by the symmetric quarter model.

Linear Dynamic Analysis

For N < 4 an impulsive response is observed, whereas for N > 18 the response approaches quasi-static realm in a fluctuating manner, Figure 18. In the range between the impulsive and quasi-static realms the response behaves dynamically. The peak response of the sandwich plate is occuring at N=7. This is due to the shape of the load. The peak pressure of the pulse and the peak of the pulse of the structure are at the maximum position when N=6. This shows that the history of loading has to be

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Table 8

First Mode Natural Frequency (Plate Theory) for Varying h_c/h_f and E_c/E_f

	f [Hz]				
h_c/h_f	E_c/E_f 1	E_c/E_f 0.1389	E_c/E_f 0.0139	E_c/E_f 0.1389e-2	E_c/E_f 0.1389e-3
3	3713.0	3150.0	3059.0	3050.0	3049.0
5	1695.0	1326.0	1263.0	1257.0	1256.0
7	1009.0	740.2	692.4	687.5	687.1
9	708.7	493.1	453.4	449.3	448.8
11	542.7	361.4	322.8	323.1	322.7

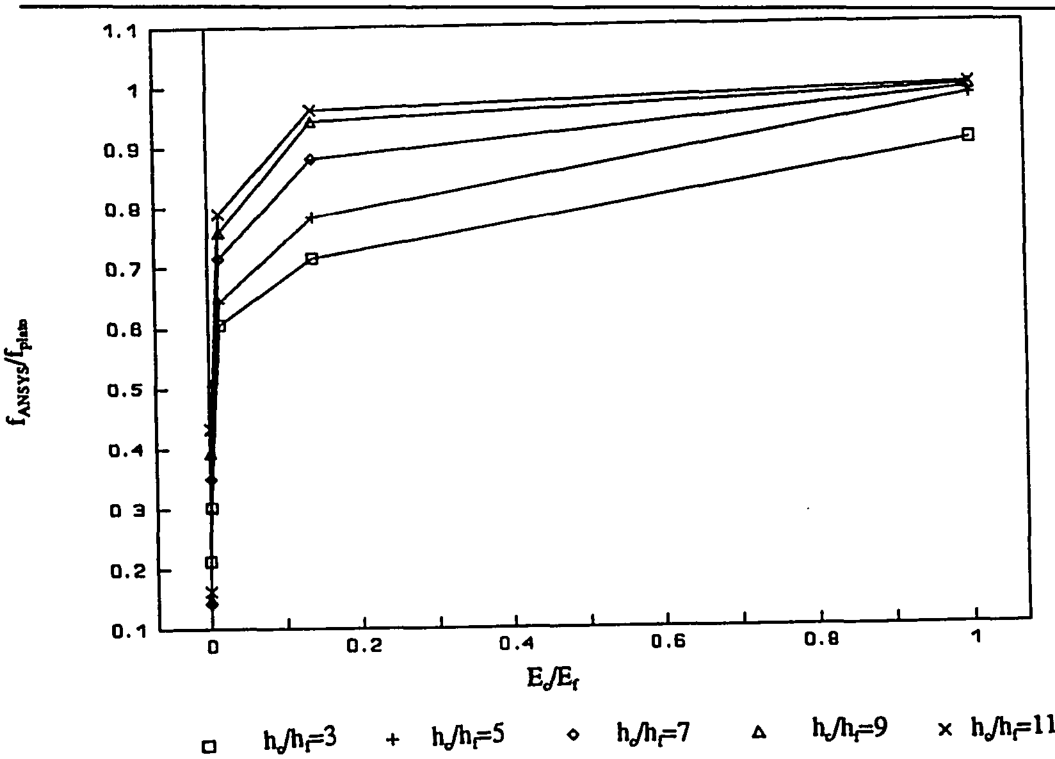

Figure 16. f_{ANSYS}/f_{Plate} vs E_c/E_f.

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Table 9

First Mode Natural Frequency ratio (ANSYS/Plate Theory) for Varying h_c/h_f and E_c/E_f

	f_{ANSYS}/f_{Plate}				
h_c/h_f	E_c/E_f 1	E_c/E_f 0.1389	E_c/E_f 0.0139	E_c/E_f 0.1389e-2	E_c/E_f 0.1389e-3
3	0.9055	0.7149	0.6044	0.3022	0.2132
5	0.9787	0.7841	0.6430	0.2962	0.1438
7	0.9898	0.8807	0.7150	0.3488	0.1411
9	0.9952	0.9436	0.7587	0.3941	0.1497
11	0.9989	0.9632	0.7901	0.4323	0.1617

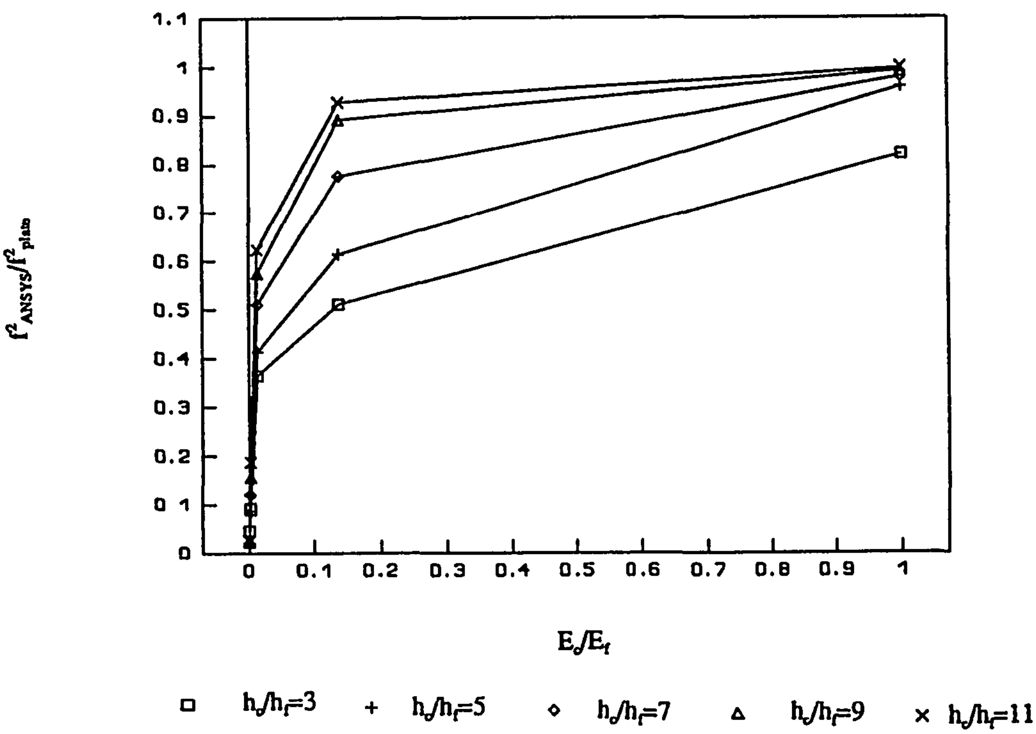

Figure 17. f^2_{ANSYS} / f^2_{Plate} vs E_c/E_f.

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Table 10

First Mode [Natural Frequency Ratio]^2 (ANSYS/Plate Theory) for Varying h_c/h_f and E_c/E_f

	f^2_{ANSYS}/f^2_{Plate}				
h_c/h_f	E_c/E_f 1	E_c/E_f 0.1389	E_c/E_f 0.0139	E_c/E_f 0.1389e-2	E_c/E_f 0.1389e-3
3	0.8199	0.5110	0.3653	0.0913	0.0454
5	0.9578	0.6148	0.4134	0.0877	0.0206
7	0.9797	0.7756	0.5112	0.1216	0.0199
9	0.9904	0.8903	0.5756	0.1553	0.0244
11	0.9978	0.9277	0.6242	0.1868	0.0261

considered for the range of N from N=7 to N=18. The effect of frequency ratio on the dynamic ratio is shown for E_c/E_f=0.1389e-3 in Figure 19. This behavior was studied also for other E_c/E_f ratios and it showed that the three realms of response (impulsive, dynamic and quasi-static) occured at almost the same N for different core moduli as shown in Figure 20. This is possibly due to the relative frequencies, N.

Iso-Response Plots

For a given pulse shape, the maximum response of a structure depends on both the peak pressure p_m and the pulse duration. One of these parameters can be substituted by the specific impulse I. The same maximum deflection w_m of the

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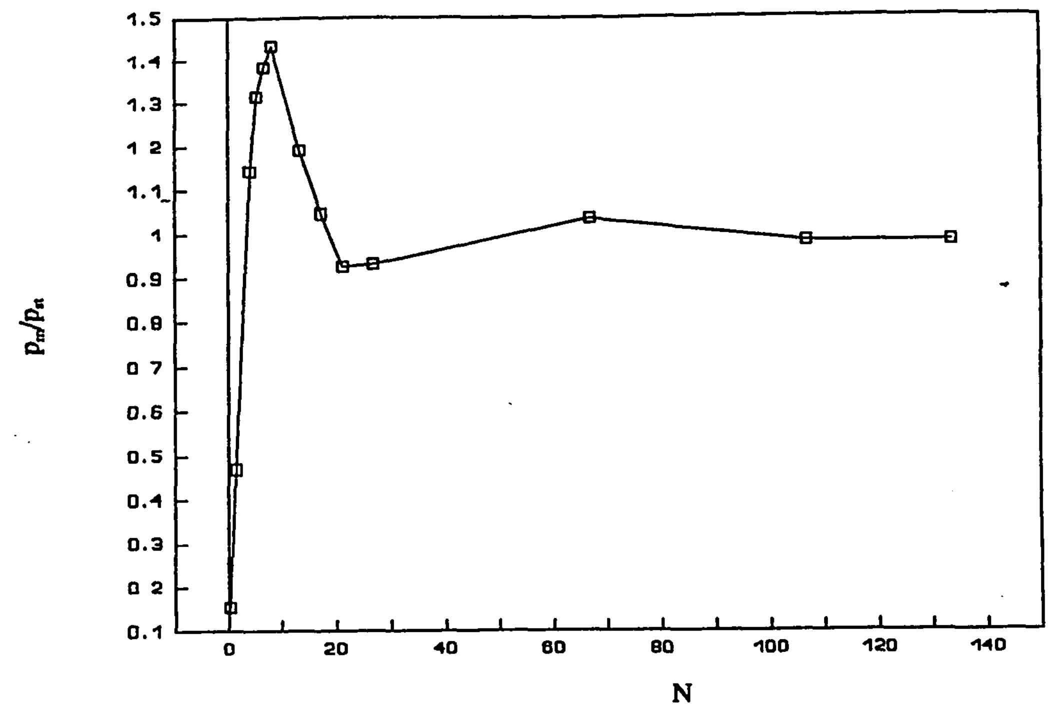

Figure 18. p_m/p_{st} vs N.

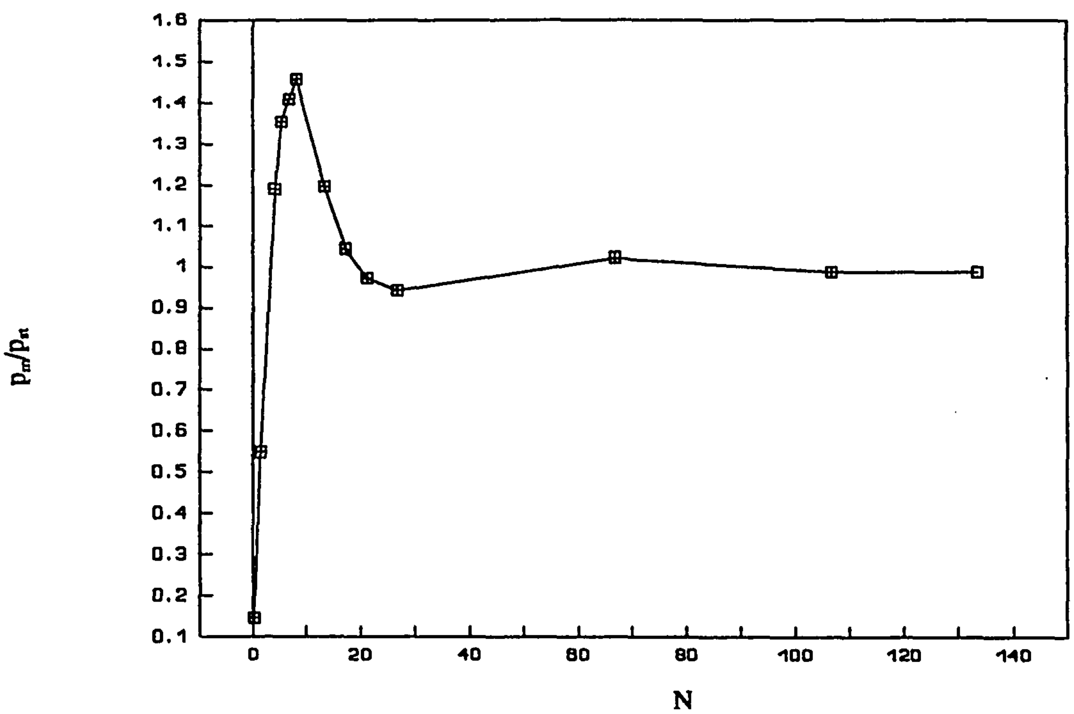

Figure 19. w_m/w_{st} vs N for E_c/E_f=0.1389e-3.

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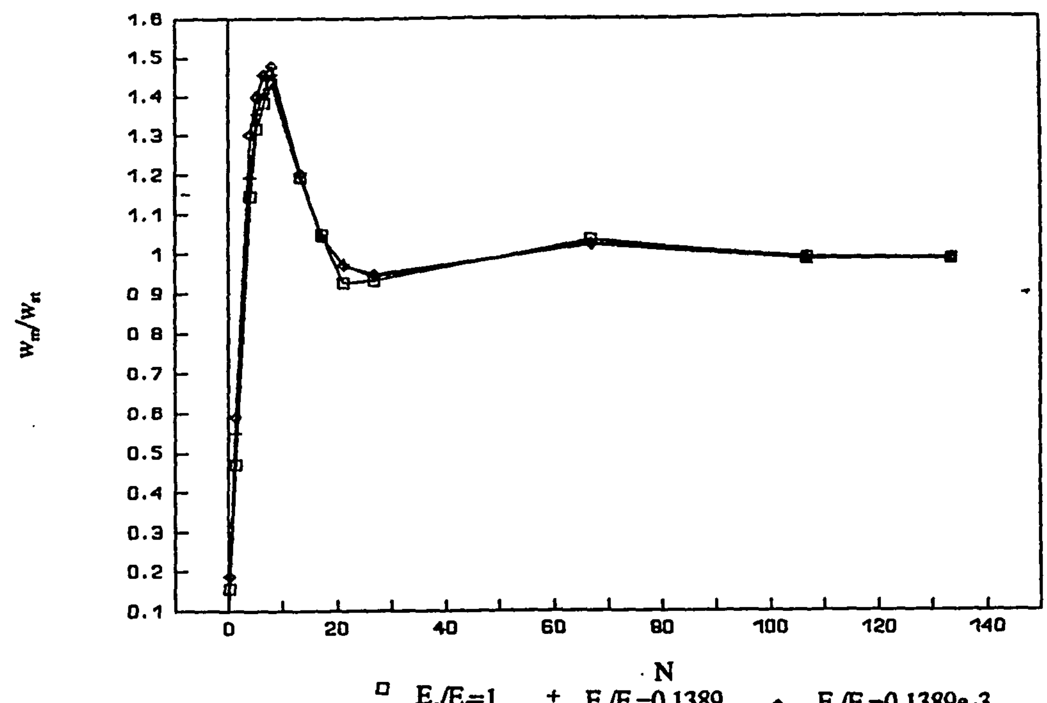

Figure 20. w_m/w_{st} vs N for Different Moduli.

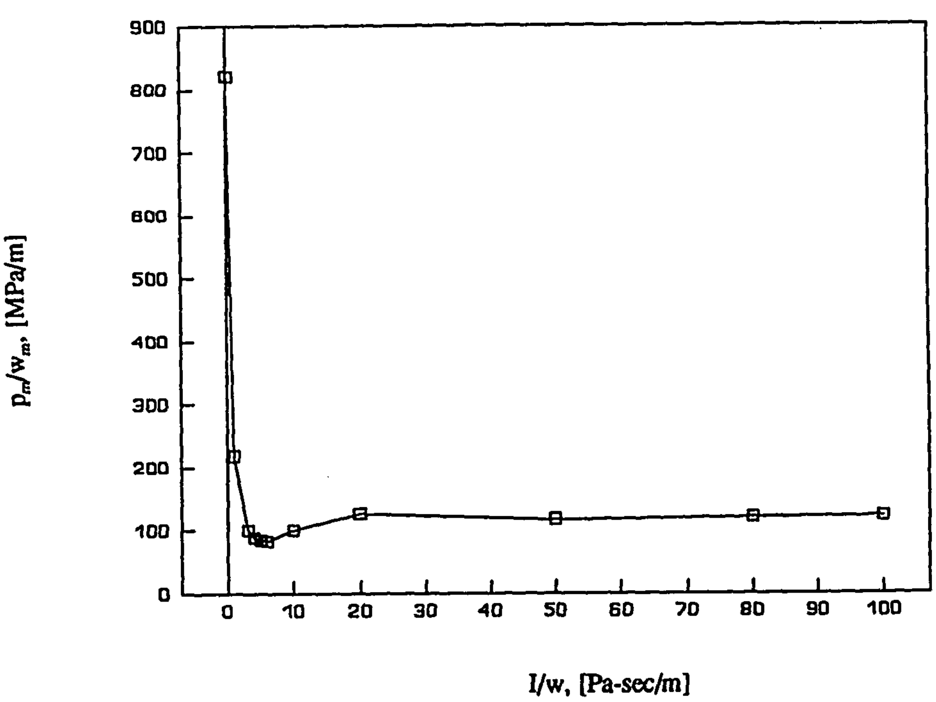

Figure 21. Dimensional Iso-response Plot for h_c/h_f =11, E_c/E_f = 0.01389.

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Table 11

Iso-response Data for $E_c/E_f = 1$

N	w_m/w_{st}	p_m/p_{st}	p_m/w_m [Pa/m]	I/w_m [Pa-sec/m]	I/I_{imp}
0.1333	0.156	6.4102	3.2425e9	46822.3	1.0000
1.3333	0.4695	2.1298	1.0767e9	154194	3.2931
3.9999	1.1435	0.8745	4.4208e8	190406.7	4.0665
5.3332	1.316	0.7598	3.8417e8	220630	4.7120
6.6665	1.3842	0.7224	3.6523e8	262125.6	5.5983
7.9998	1.4332	0.6977	3.5273e8	303809.5	6.4885
13.333	1.1916	0.8392	4.2426e8	609121.7	13.0092
26.667	0.9312	1.0738	5.4280e8	1558957.6	33.2952
66.665	1.0323	0.9687	4.8971e8	3514300	75.0561
106.66	0.9833	1.017	5.1413e8	5899383	125.9100
133.33	0.9823	1.0181	5.1466e8	7385486.4	157.7343

structure may be obtained for different combinations of p_m and I.

An iso-response curve is a plot of all the combinations of p_m and I that result in the same response. Dimensional Iso-response plot of p/w_m vs I/w_m, Figure 21, is shown for $E_c/E_f = 0.01389$. In the impulsive realm (high pressure and low impulse), the response is dominated by impulse and hence high pressure is required to provide

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Table 12

Iso-response Data for E_c/E_f =0.01389

N	w_m/w_{st}	p_m/p_{st}	p_m/w_m [Pa/m]	I/w_m [Pa-sec/m]	I/I_{imp}
0.1333	0.1460	6.8496	8.2169e8	24732.9	1.0000
1.3333	0.5485	1.8231	2.1867e8	65646.2	2.6542
3.9999	1.1910	0.8396	1.0072e8	91095.9	3.6831
5.3332	1.3531	0.7390	8.8650e7	106879.4	4.3213
6.6665	1.4071	0.7106	8.5251e7	128516.6	5.1961
7.9998	0.4563	0.6866	8.2372e7	149011.5	6.0248
13.333	1.1950	0.8368	1.0038e8	302650.1	12.2367
26.667	0.9419	1.0616	1.2735e8	767944.5	31.0495
66.665	1.0216	0.9788	1.1742e8	1769610.1	71.5488
106.66	0.9865	1.0136	1.2159e8	2931663.4	118.5239
133.33	0.9857	1.0145	1.2169e8	3668869.4	148.3396

is shown for E_c/E_f = 0.01389. In the impulsive realm (high pressure and low impulse), the response is dominated by impulse and hence high pressure is required to provide the necessary impulse that causes the maximum response w_m. In the quasi-static realm (large impulse), the response is dominated by the pressure level and p/w_m converges asymptotically towards the static level. In the dynamic realm p/w_m is smaller than the

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Table 13

Iso-response Data for E_o/E_f = 0.1389e-3

N	w_m/w_{st}	p_m/p_{st}	p_m/w_m [Pa/m]	I/w_m [Pa-sec/m]	I/I_{imp}
0.1333	0.1887	5.2994	26130128	3895.22	1.0000
1.3333	0.5884	1.6995	4730369	7051.56	1.8103
3.9999	1.3012	0.7685	3789314	16946.2	4.3505
5.3332	1.4	0.7142	3522367	21002.8	5.3919
6.6665	1.4546	0.6874	3389830	25263.4	6.4857
7.9998	1.4778	0.6776	3336670	29844.17	7.6617
13.333	1.2	0.8333	4105090	61196.22	15.7105
26.667	0.9467	1.0563	5208333	155281.25	39.8645
66.665	1.0202	0.9802	4833252	360183.66	92.4681
106.66	0.9822	1.0181	5020080	598644.57	153.6869
133.33	0.9812	1.0191	5025125	748944.72	192.2727

static level. It is observed that the dynamic response occurs for 4 > N > 18.

The impulse which has to be applied for a certain peak deflection to occur is asymptotic to a minimum value in the impulsive realm of the iso-response plot. The static pressure is the asymptotic pressure value in the quasi-static realm.

The impulse and pressure values may be divided by the asymptotic values to

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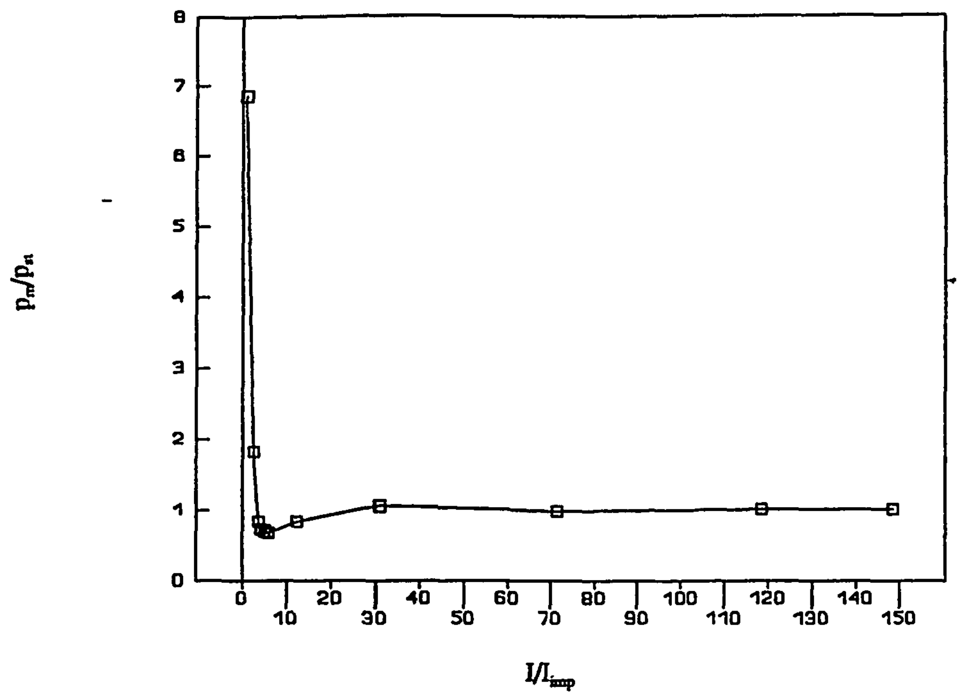

Figure 22. Iso-response Plot for $h_c/h_f = 11$, $E_c/E_f = 0.01389$.

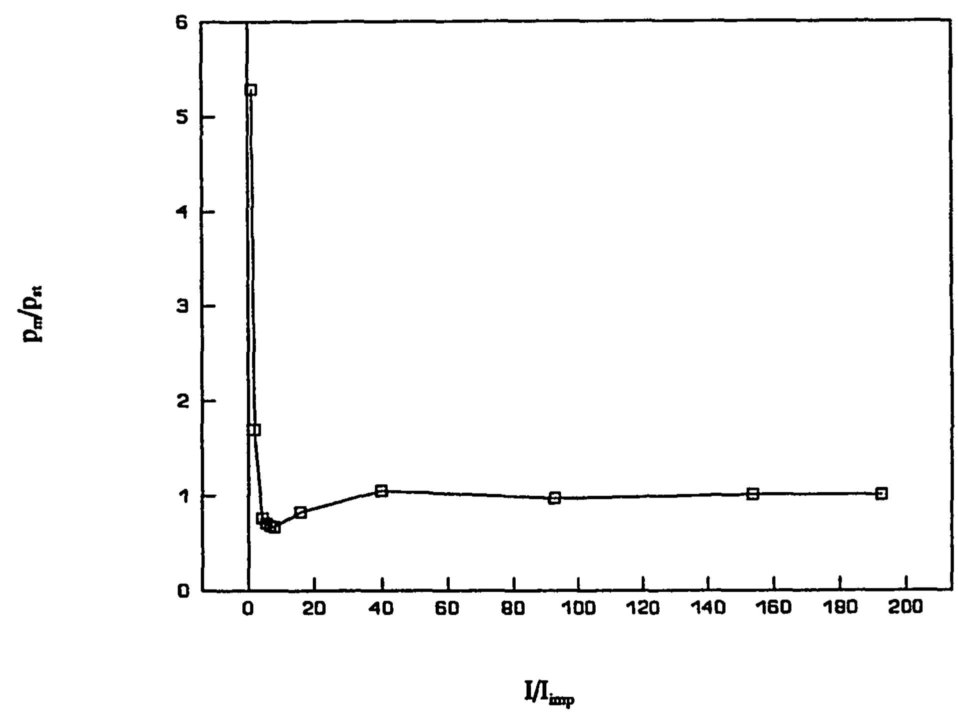

Figure 23. Iso-response Plot for $h_c/h_f = 11$, $E_c/E_f = 0.1389e\text{-}3$.

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get non-dimensional parameters. The non-dimensional impulse and pressure are then I/I_{imp} and p_m/p_{st} where I_{imp} is the minimum asymptotic value of impulse and p_{st} is thestatic pressure which are required to obtain w_m, see Figures 22 and 23.

I and p_m are the applied impulse and pressure to obtain the required deflection w_m. When $N < 4$ dynamic pressure higher than the static is needed because the dynamic loading is not acting long enough to deflect the sandwich plate. This is the impulsive realm, where it is not important how large the pressure is but the duration of time the load acts must be sufficient to provide the needed impulse for the peak response to occur. A very large pressure when applied for a very small duration may not result in any significant response at all. For $N > 18$ the load is acting for a very long interval. This is due to the fact that the load is acting long enough for the structure to obtain a peak response relative to the pressure intensity only. The intermediate range is the dynamic range, during which a lower load is required to cause the peak response. It is observed that the peak response occurs when N=6. This may be due to the pulse shape, where the peak p_m occurs relatively early and not in a sinusoidal shape as it is for the natural vibration.

Non-linear Behavior

The w_m/h for transient loading when N=6.6665 for different pressures and E_c/E_f are shown in Tables 14 and 15. It is seen (see Figures 24 and 25) that the large deflection has no effect on the deflection ratio. The ratio w_m/h for stress stiffening, linear and large deflection is same. This is different from the static loading where the

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Table 14

Non-linear Dynamic Deflection Ratio of w_m/h for $E_c/E_f = 1$ and $h_c/h_f = 11$

p	w_m/h With stress stiffening	w_m/h Linear	w_m/h Largĕ deflection
1e3	0.1273e-3	0.1141e-3	0.1273e-3
5e5	0.0659	0.0570	0.0659
8e5	0.1018	0.0912	0.1018
1e6	0.1273	0.1141	0.1273
2e6	0.2338	0.2282	0.2538
5e6	0.6162	0.5704	0.5758
8e6	0.9375	0.9128	0.9095

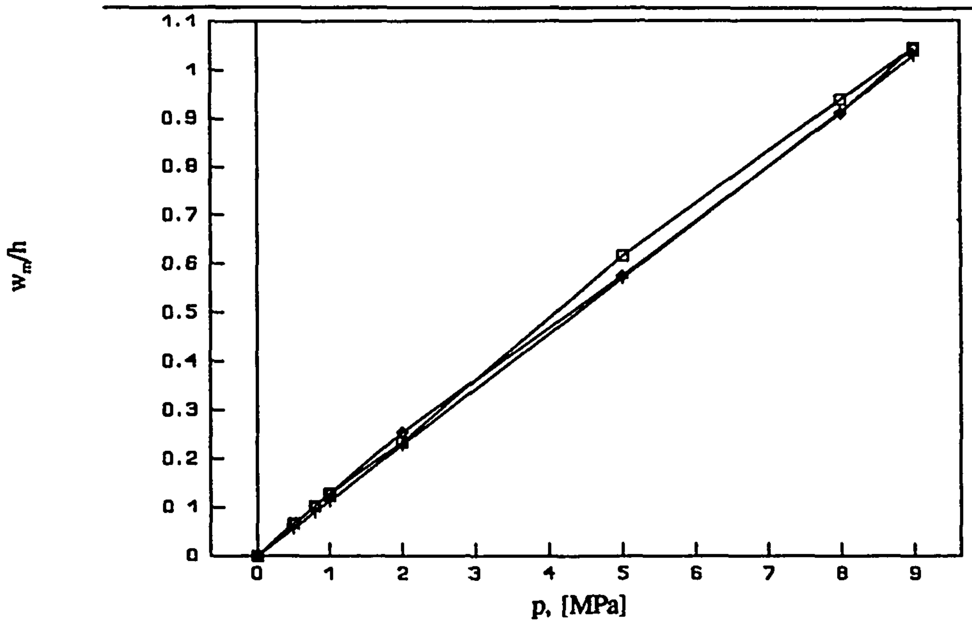

Figure 24. Non-linear Dynamic Behavior for $h_c/h_f = 11$, $E_c/E_f = 1$.

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Table 15

Non-linear Dynamic Deflection Ratio of w_m/h for E_c/E_f = 0.01389 and h_c/h_f = 11

p	w_m/h With stress stiffening	w_m/h Linear	w_m/h Large deflection
1e3	0.5483e-3	0.5408e-3	0.5483e-3
1e5	0.0492	0.05408	0.0548
5e5	0.2729	0.2704	0.2729
8e5	0.4329	0.4325	0.4329

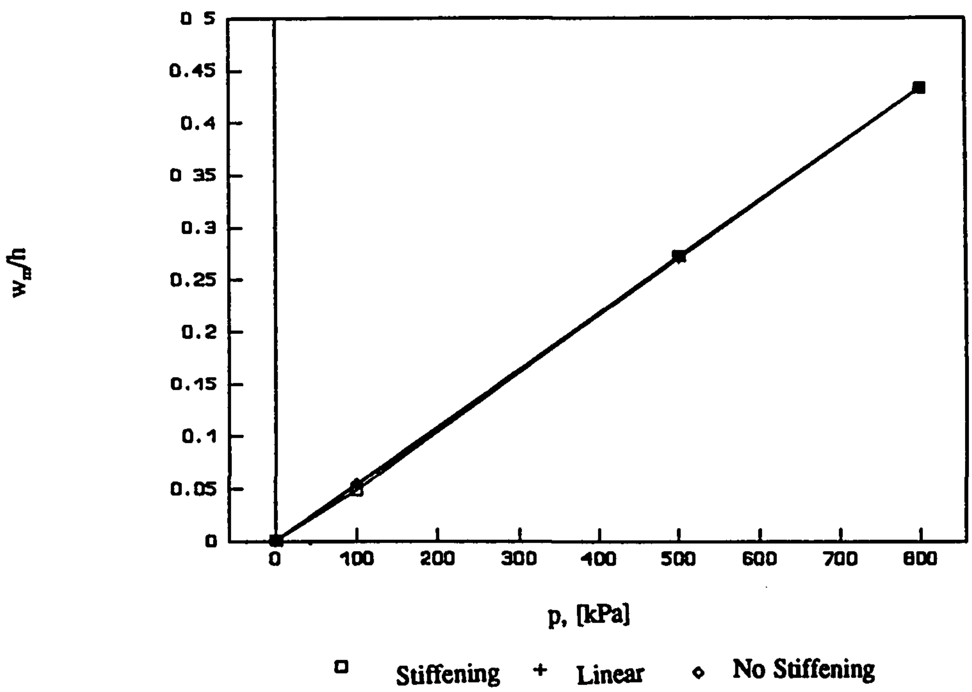

Figure 25. Non-linear Dynamic Behavior for h_c/h_f = 11, E_c/E_f = 0.01389.

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non-linearities are very clear.

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CHAPTER V

CONCLUSIONS

The response of a sandwich plate subjected to static and dynamic loading was studied. The results of the sandwich plate were compared with the thin plate theory for static loading. It was found that the deviations for soft cores is very wide. The deviations are because the sandwich plate includes transverse shear whereas the cross sections of the Kirchhoff thin plate are assumed straight after deflection. The stiffness of the sandwich plate increases with the thickness of the core. For static non-linearity the large deflections were very high as the core softness increased, especially when $E_c/E_f < 0.01389$. For non-linear transient analysis the effects of large deflection with or without stress-stiffening were generally the same which is different from the static non-linear behavior. Iso-response plots, which give the different non-dimensional values of pressure and impulse for the same peak deflection were shown. Impulsive, dynamic and quasi-static realms were determined for different core elastic moduli.

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Appendix A

Tables for Different Analyses

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Table A

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 1$ and $h_c/h_f = 11$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.8241e-4	0.8241e-4
5e5	0.4121e-1	0.4123e-1
7e5	0.5769e-1	0.5775e-1
8e5	0.6593e-1	0.66e-1
2e6	0.1648	0.1646
5e6	0.4123	0.4046
8e6	0.6593	0.6271
9e6	0.7417	0.6962
1e7	0.8241	0.7627

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Table B

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 0.3472$ and $h_c/h_f = 11$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.1465e-3	0.1465e-3
1e5	0.1465e-1	0.1465e-1
5e5	0.7325e-1	0.7329e-1
8e5	0.1172	0.1173
1e6	0.1465	0.1465
5e6	0.7325	0.6971
6e6	0.8790	0.8192
7e6	1.0255	0.9337

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Table C

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 0.1389$ and $h_c/h_f = 11$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.218e-3	0.218e-3
1e5	0.218e-1	0.2181e-1
5e5	0.1090	0.1091
8e5	0.1744	0.1560
1e6	0.218	0.2179
3e6	0.6536	0.6372
5e6	1.09	1.0111
6e6	1.3080	1.770
8e6	1.7440	1.3521
1e7	2.1800	1.5867

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Table D

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 0.0694$ and $h_c/h_f = 11$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.2358e-3	0.2358e-3
1e5	0.2358e-1	0.2359e-1
5e5	0.1179	0.118
8e5	0.1886	0.1886
1e6	0.2358	0.2355
5e6	1.1792	1.0704
8e6	1.8864	1.5387
1e7	2.3580	1.7925

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Table E

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 0.0139$ and $h_c/h_f = 11$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.3485e-3	0.3485e-3
1e5	0.3485e-1	0.3486e-1
5e5	0.1742	0.1739
8e5	0.2788	0.277
1e6	0.3486	0.3447
3e6	1.045	0.95
5e6	1.7429	1.4158
8e6	2.7888	1.9354
1e7	3.486	2.2071

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Table F

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 0.3472e\text{-}3$ and $h_c/h_f = 11$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.3718e-2	0.3718e-2
4e4	0.1487	0.1451
5e4	0.1859	0.1792
1e5	0.3718	0.3318
5e5	1.8593	1.01
8e5	2.9748	1.2958
5e6	18.593	3.09
8e6	29.75	3.38

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Table G

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for E_c/E_f = 0.1389e-3 and h_c/h_f = 11

p	w_m/h Linear	w_m/h Non-linear
1e3	0.845e-2	0.8445e-2
1e4	0.845e-1	0.830
2e4	0.1659	0.1587
4e4	0.338	0.2835
5e5	4.225	1.2054
8e5	6.76	1.5475
5e6	42.25	3.6
8e6	67.600	5.8875

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Table H

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 1$ and $h_c/h_f = 3$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.1063e-4	0.1063e-4
1e6	0.1062e-1	0.1062e-1
4e6	0.0425	0.0425
5e6	0.0531	0.0532
1e7	0.1062	0.1065
3e7	0.3186	0.3167
5e7	0.5310	0.5151
6e7	0.6372	0.6077
7e7	0.7435	0.6957

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Table I

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate to Show Large Deflection for Static Non-linearity for $E_c/E_f = 0.3472$ and $h_c/h_f = 3$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.148e-4	0.148e-4
1e6	0.148	0.148
5e6	0.074	0.0741
10e6	0.148	0.1483
30e6	0.444	0.4367
55e6	0.814	0.7581
65e6	0.962	0.8716

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Table J

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 0.1389$ and $h_c/h_f = 3$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.189e-4	0.189e-4
1e6	0.0189	0.0189
3e6	0.0567	0.0567
5e6	0.0945	0.0946
8e6	0.1512	0.1512
2e7	0.3780	0.3726
4e7	0.7560	0.7055
5e7	0.9450	0.8512
7e7	1.323	1.0387

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Table K

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate to Show Large Deflection for Static Non-linearity for $E_c/E_f = 0.0694$ and $h_c/h_f = 3$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.2355e-4	0.2355e-4
1e6	0.0235	0.0235
3e6	0.0706	0.0707
5e6	0.1177	0.1176
8e6	0.1884	0.1875
1e7	0.2355	0.2337
3e7	0.7065	0.6549
7e7	1.6485	1.2512

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Table L

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 0.0139$ and $h_c/h_f = 3$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.5178e-4	0.5178e-4
1e6	0.0517	0.0517
5e6	0.2590	0.2520
8e6	0.4142	0.3895
1e7	0.5178	0.4734
5e7	2.585	1.4312
7e7	3.619	1.6937

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Table M

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 0.3472e\text{-}3$ and $h_c/h_f = 3$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.3643e-3	0.3643e-3
5e5	0.1821	0.1724
1e6	0.3643	0.3102
5e6	1.8215	0.8740

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Table N

Ratio of Maximum Deflection to the Thickness of the Sandwich Plate
to Show Large Deflection for Static Non-linearity
for $E_c/E_f = 0.1389e-3$ and $h_c/h_f = 3$

p	w_m/h Linear	w_m/h Non-linear
1e3	0.4453e-3	0.4453e-3
3e5	0.1295	0.1335
6e5	0.2418	0.2671
1e6	0.4450	0.3641
5e6	2.066	1.0378

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Table O

Non-linear Dynamic Deflection Ratio of w_m/h for E_c/E_f = 0.1389e-3 and h_c/h_f = 11

p	w_m/h
1e3	0.885e-2
1e4	0.877e-2
4e4	0.3131
8e4	0.5408
1e5	0.6350

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Appendix B

Figures for Various Analyses

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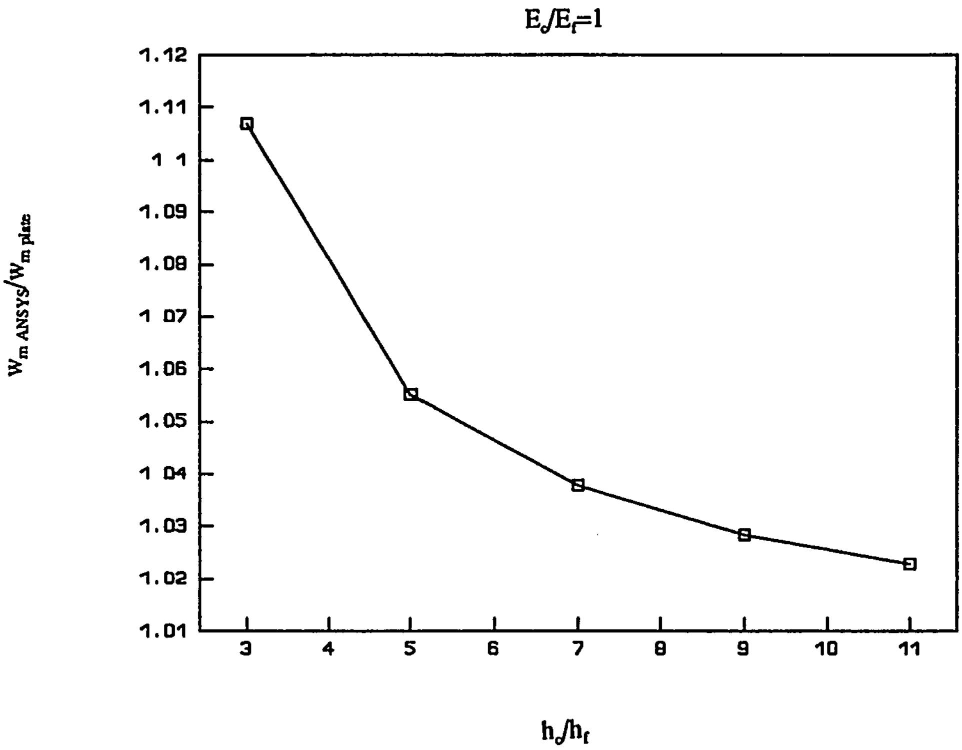

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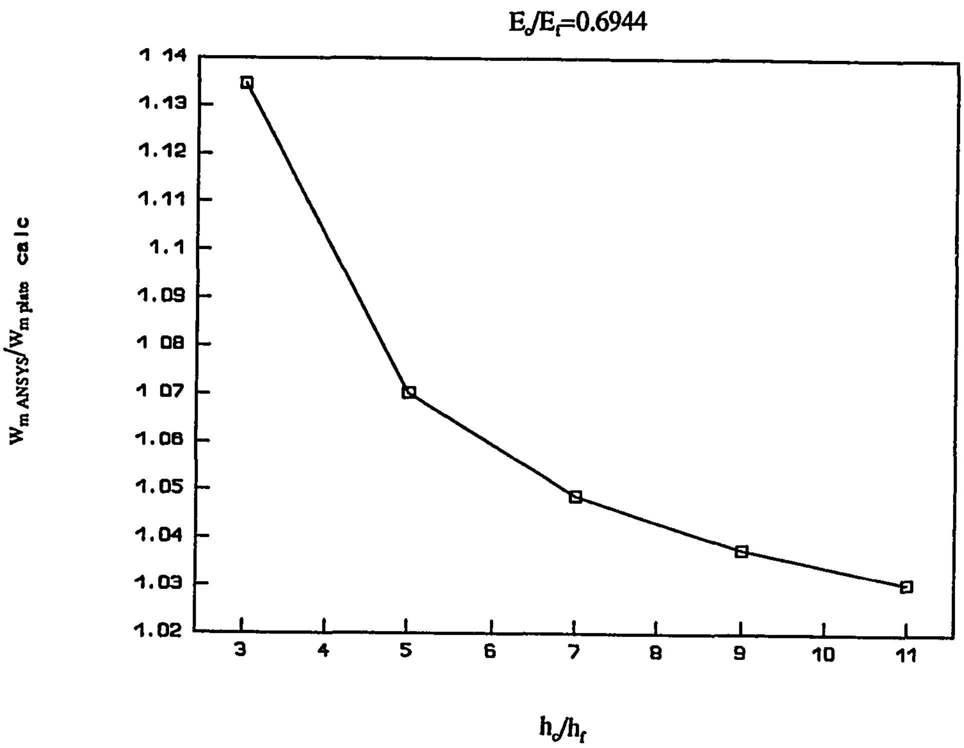

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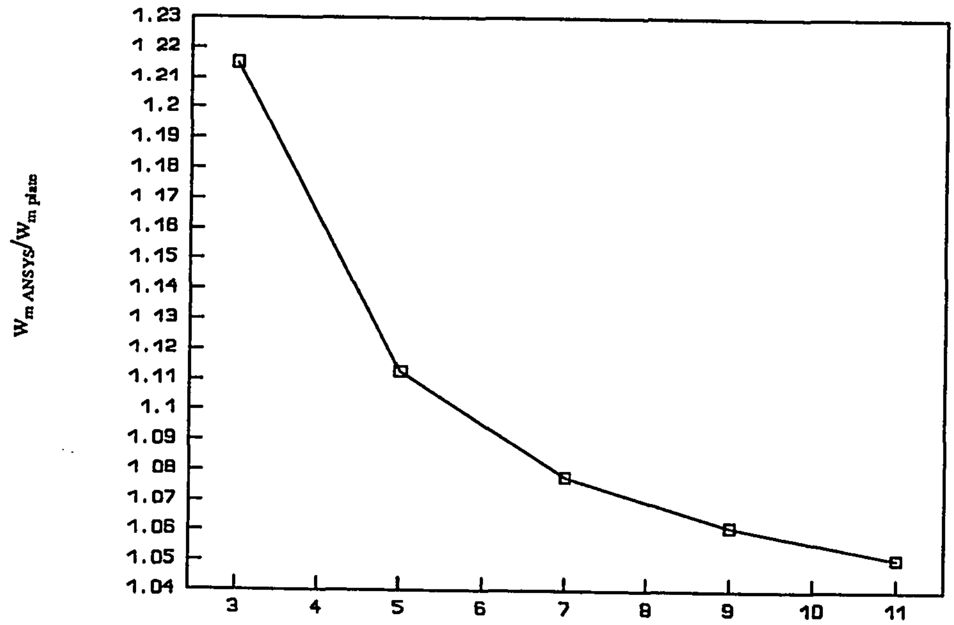
E_c/E_f=0.3472



h_c/h_f

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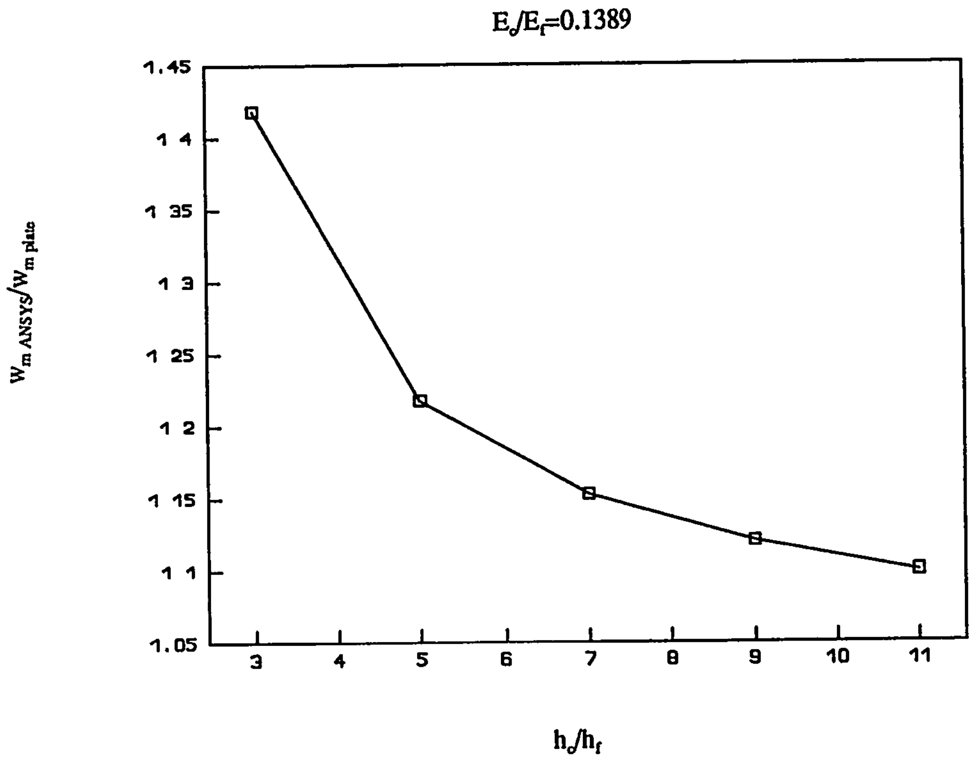

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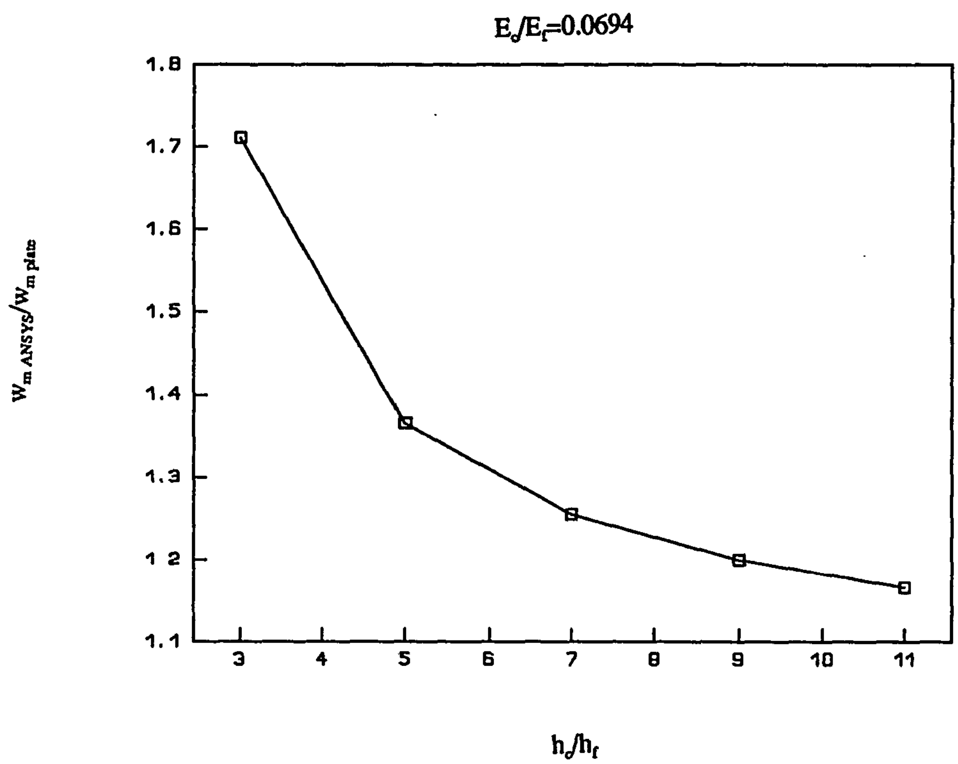

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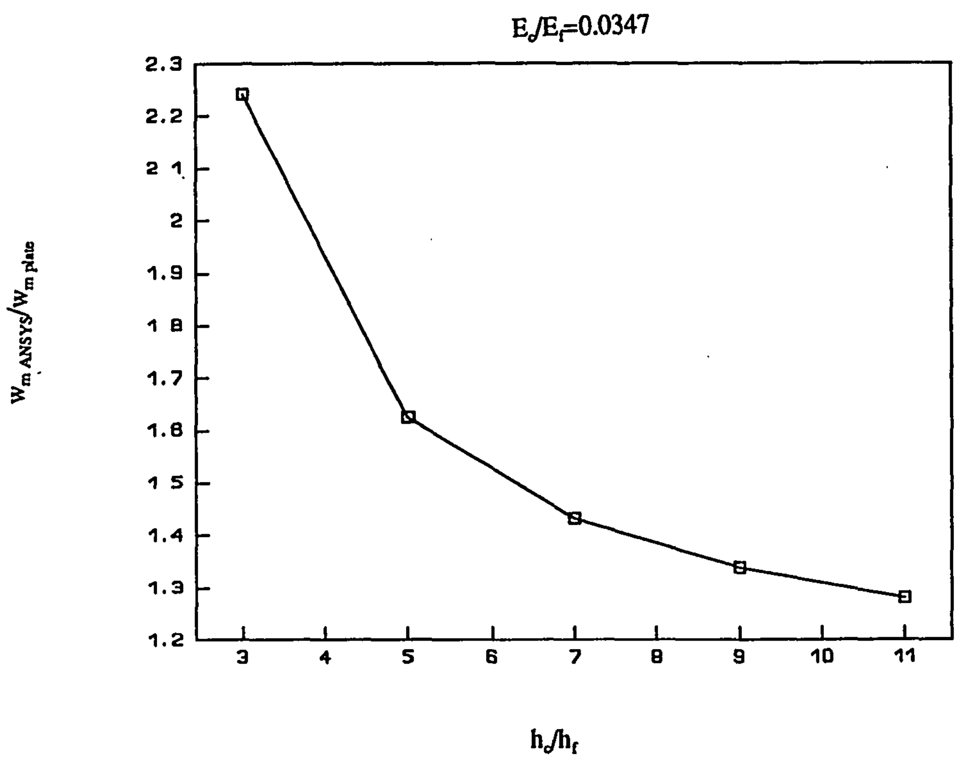

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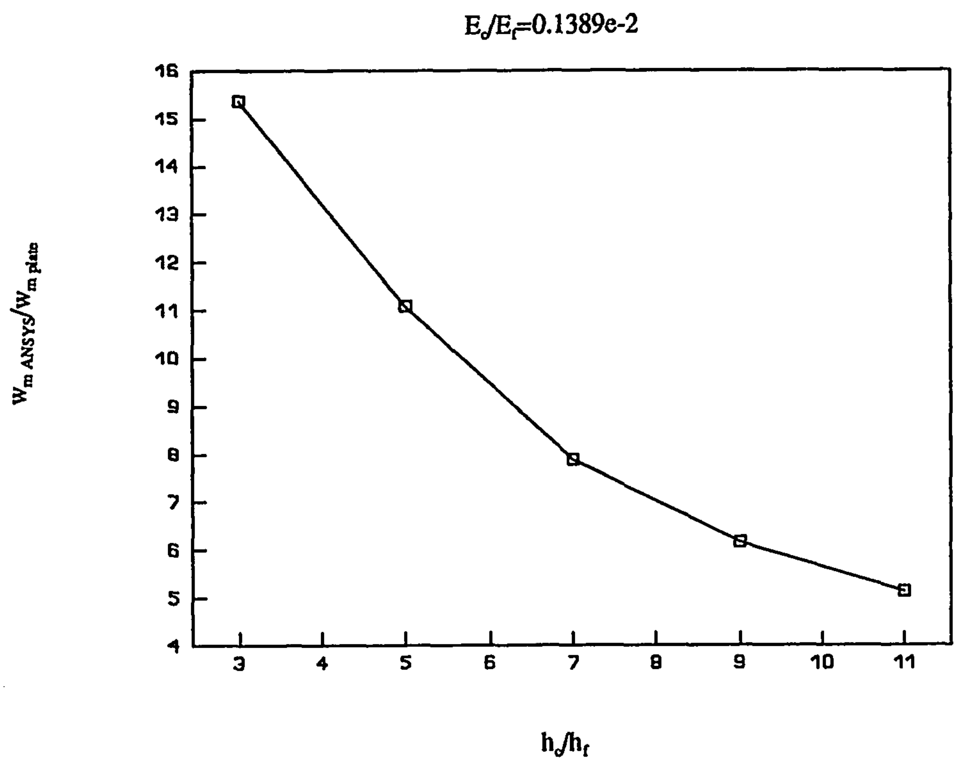

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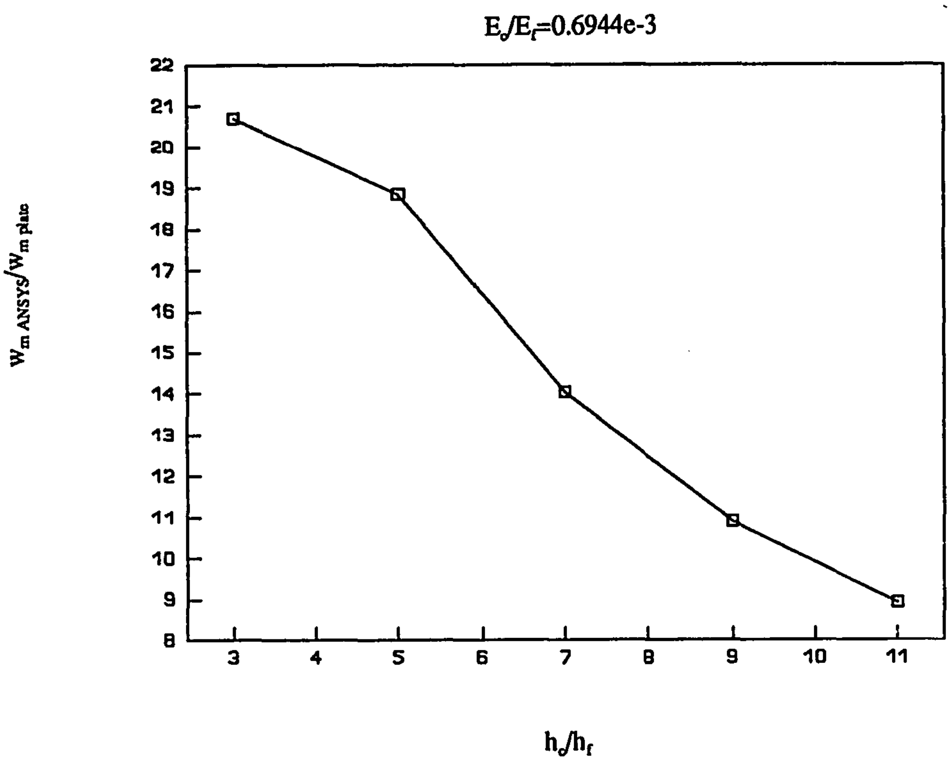

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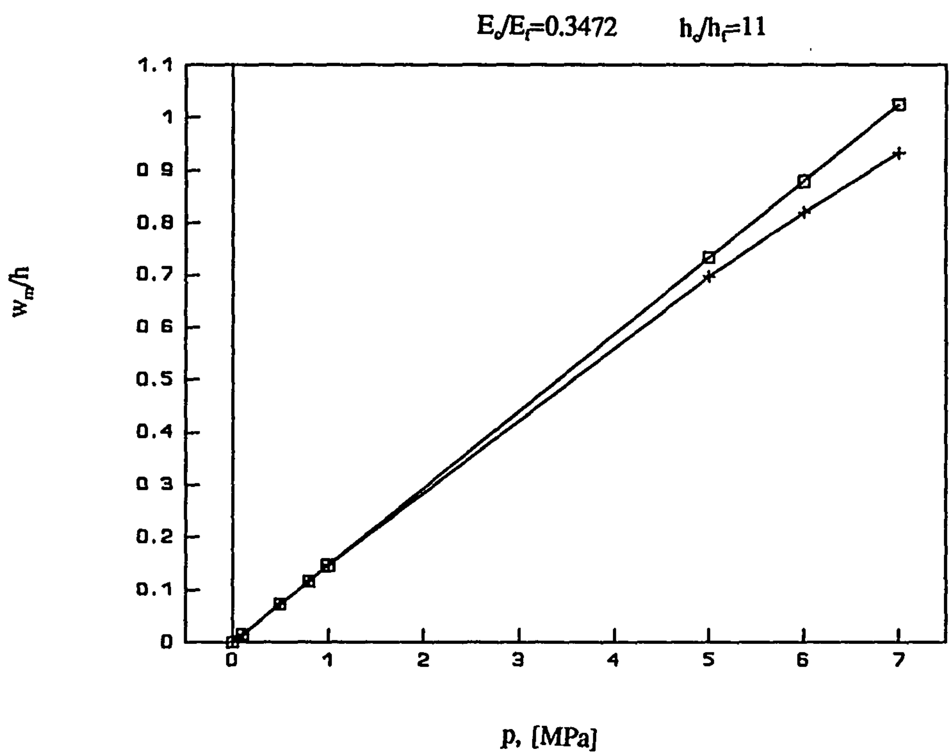

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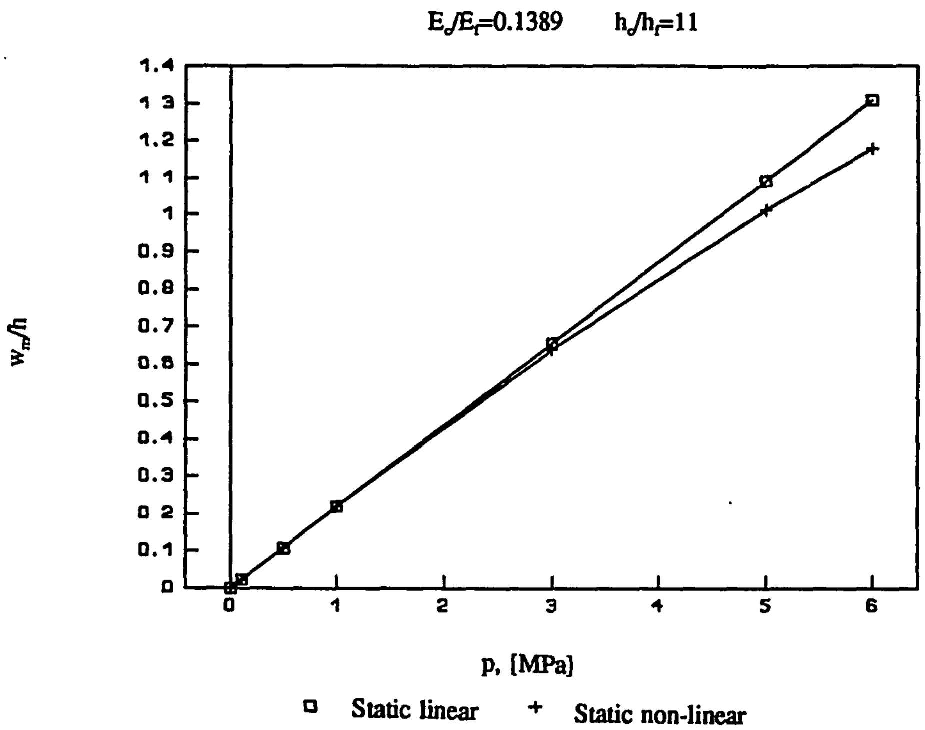

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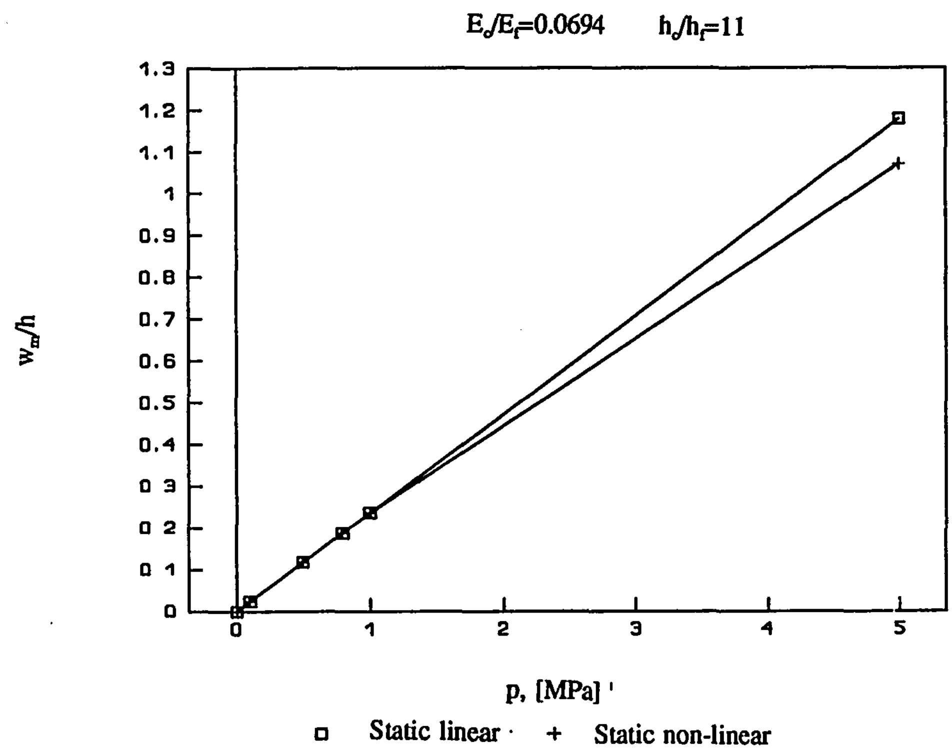

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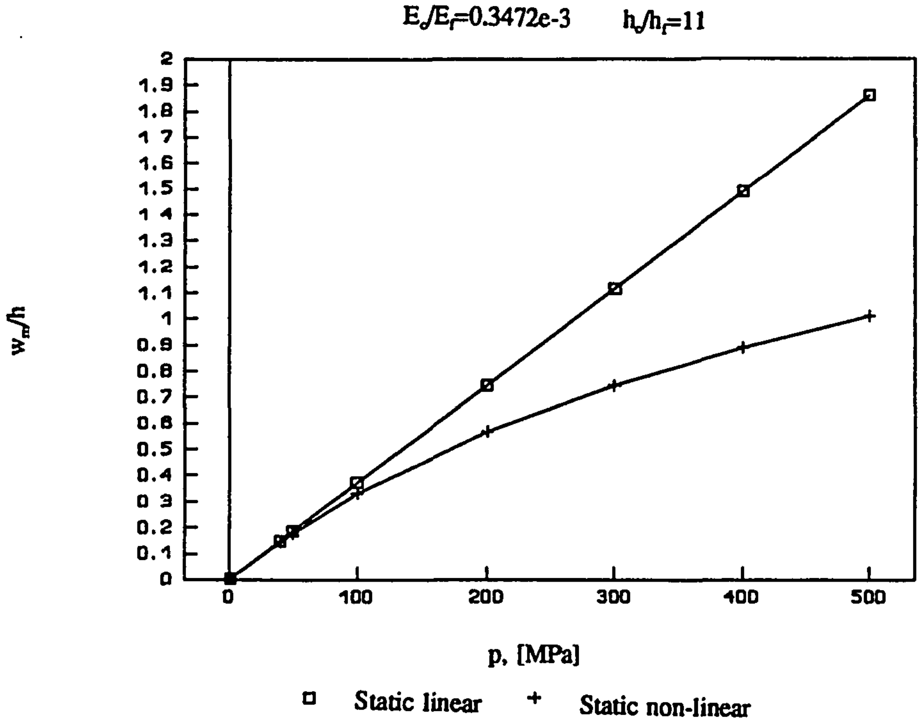

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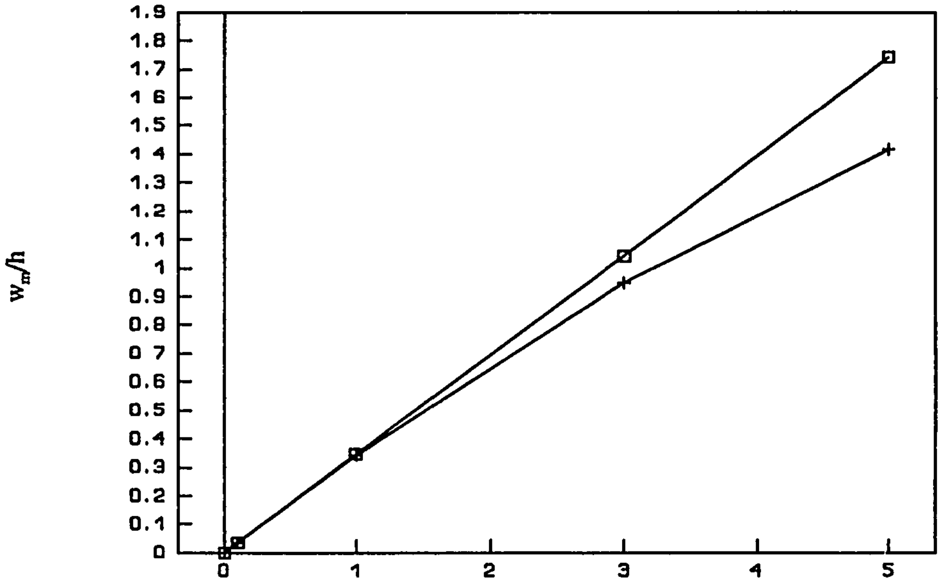
E_c/E_f=0.0139 h_c/h_f=11



p, [MPa]

□ Static linear + Static non-linear

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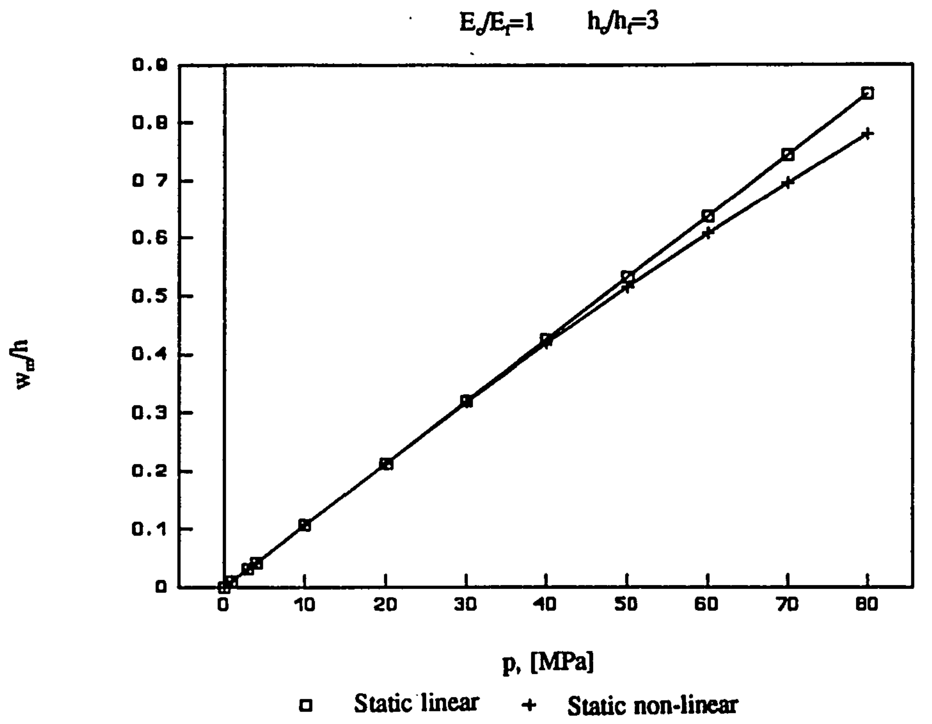

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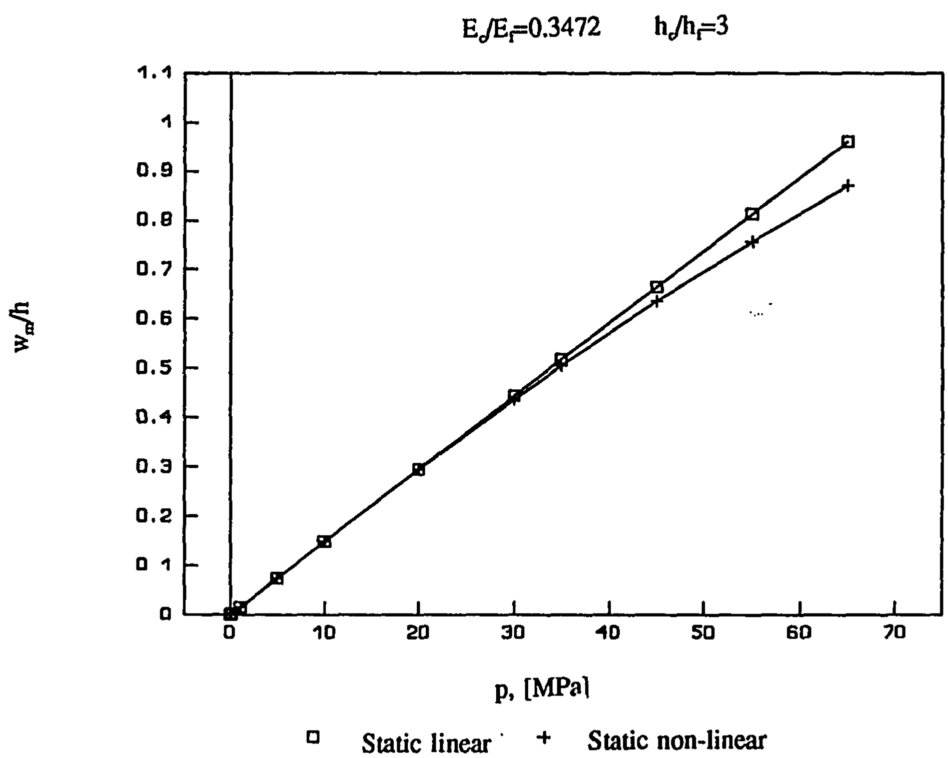

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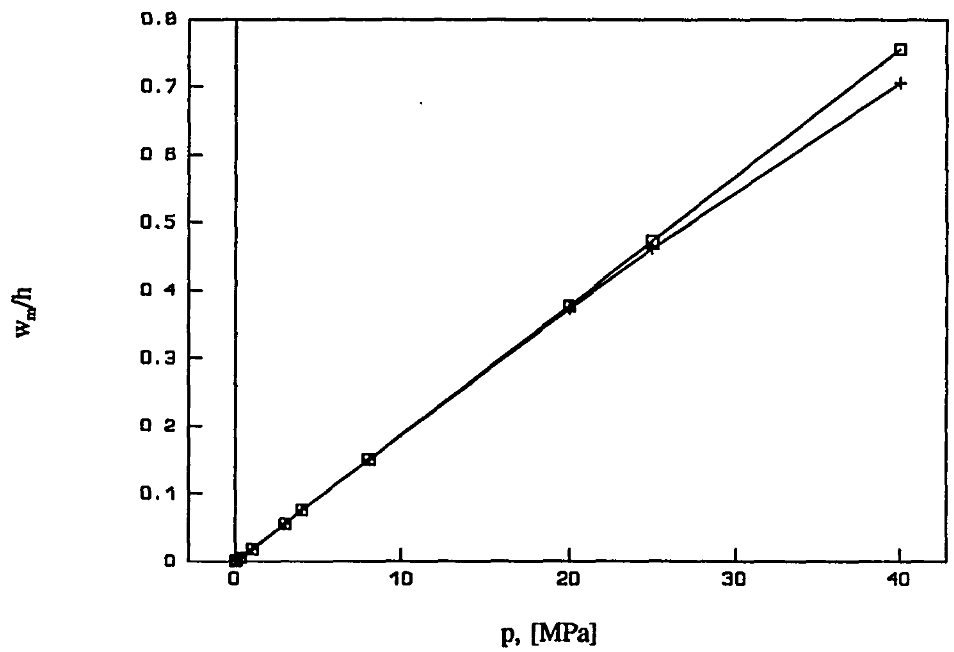
E_c/E_f=0.1389 h_c/h_f=3



□ Static linear + Static non-linear

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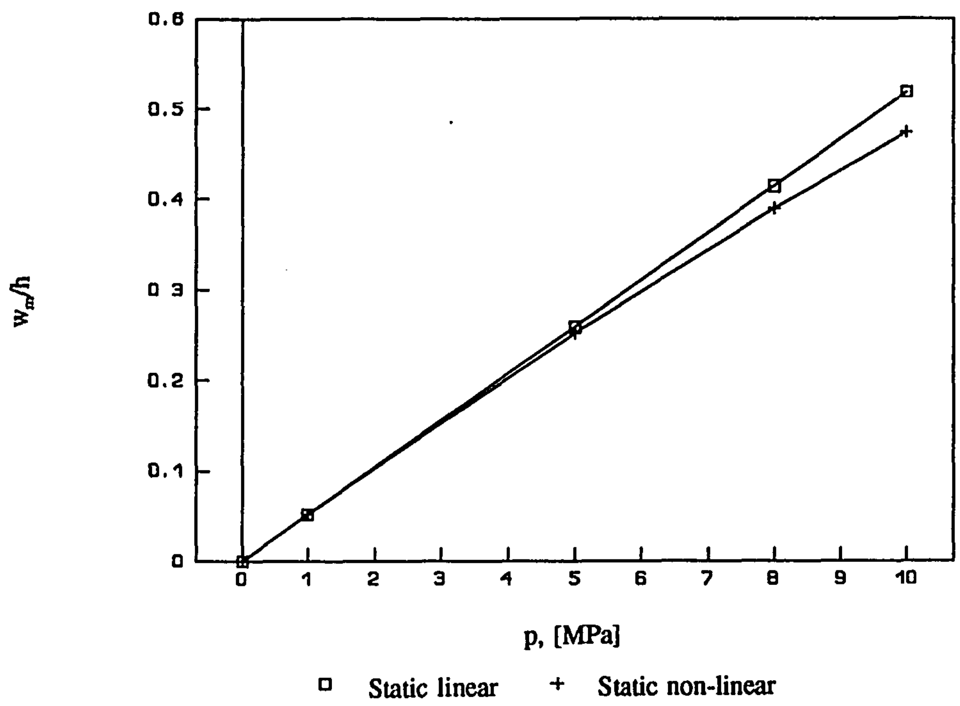
E_c/E_f=0.0139 h_c/h_f=3



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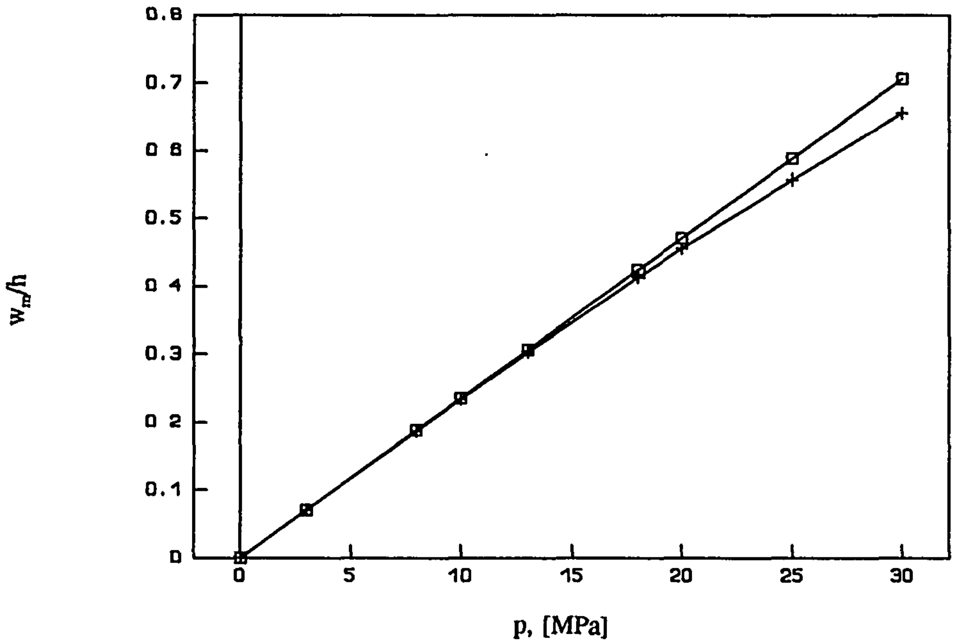
E_c/E_f=0.0694 h_c/h_f=3



□ Static linear + Static non-linear

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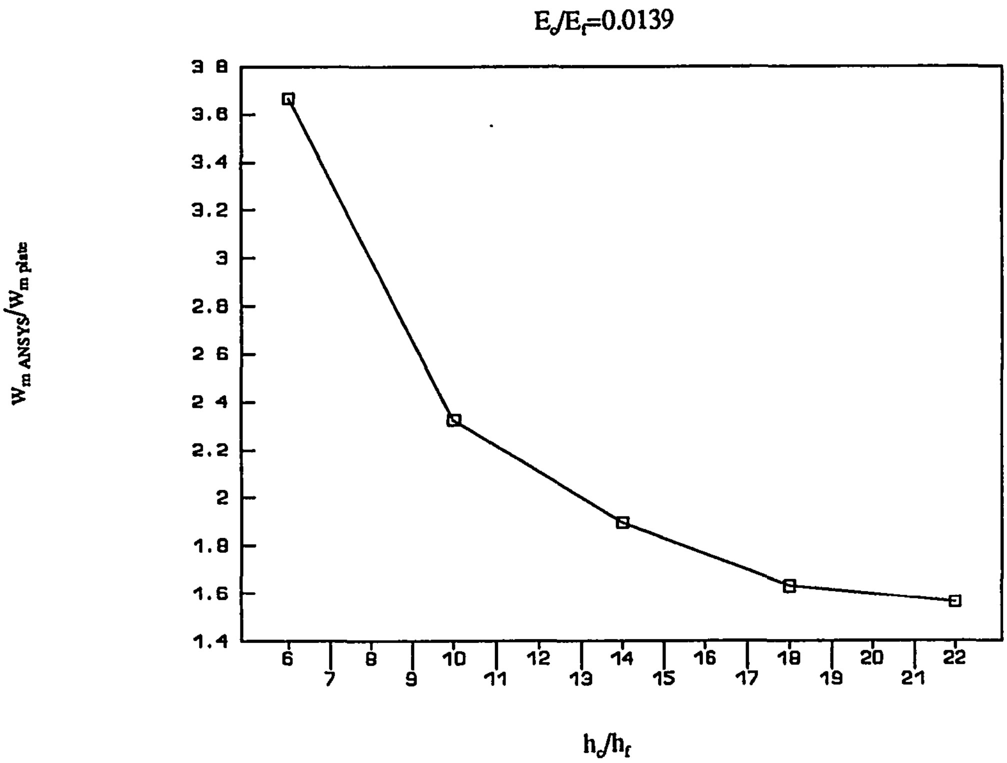

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Appendix C

ANSYS Computer Programs for Different Analyses

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```
ex/COM,ANSYS REVISION  4.4    UP437 A 16    14.2491    5/ 4/1992

/SHOW,VT340
/PREP7
/TITLE, SANDWICH PLATE WITH 48 ELEMENTS (QUARTER)
KAN,0                       *Static analysis
ET,1,63                     *Plate element type defined
ET,3,45                     *3-D solid element type defined
KAY,6,1                     *Large deflection
EX,1,72E9                   *Facing modulus of elasticity [pa]
NUXY,1,0.335                *Facing poisson ratio
DENS,1,2700                 *Facing density [kg/m^3]
R,1,0.002                   *Facing thickness [m]
EX,3,10E9                   *Core modulus of elasticity [pa]
NUXY,3,0.335                *Core poisson ratio
DENS,3,2700                 *Core density [kg/m^3]
N,1,,,11E-3                 *Top facing nodes
N,5,0.24,,11E-3
FILL
NGEN,5,5,1,5,1,,0.06
TYPE,1
MAT,1
REAL,1
E,1,2,7,6                   *Top facing plate elements
EGEN,4,1,1,1,1
EGEN,4,5,1,4,1
NGEN,2,200,1,25,1,,,-22E-3  *Bottom facing nodes
TYPE,1
MAT,1
REAL,1
EGEN,2,200,1,16,1           *Bottom facing elements
NGEN,2,300,1,25,1           *Core nodes
NGEN,2,200,201,225,1
TYPE,3
MAT,3
E,301,302,307,306,401,402,407,406     *Core elements
EGEN,4,1,33
EGEN,4,5,33,36,1
D,201,UZ,0,,205,1                     *Boundary conditions (bottom)
D,401,UZ,0,,405,1
D,205,UX,0,,225,5,ROTY,ROTZ           *Symmetry conditions (quarter plate)
D,405,UX,0,,425,5
D,221,UY,0,,225,1,ROTX,ROTZ
```

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```
D,421,UY,0,,425,1
D,206,UZ,0,,221,5
D,406,UZ,0,,421,5
D,5,UX,0,,25,5,ROTY,ROTZ
D,21,UY,0,,25,1,ROTX,ROTZ
D,305,UX,0,,325,5
D,321,UY,0,,325,1
CP,1,UX,1,301                    *Coupling of plate and core element nodes
CPSGEN,4,1,1,1,1
CPSGEN,5,5,1,4,1
CP,50,UY,1,301
CPSGEN,5,1,50,50,1
CPSGEN,4,5,50,54,1
CP,100,UZ,1,301
CPSGEN,5,1,100,100,1
CPSGEN,5,5,100,104,1
CP,150,UX,202,402
CPSGEN,3,1,150,150,1
CP,157,UX,206,406
CPSGEN,4,1,157,157,1
CPSGEN,4,5,157,160,1
CP,250,UY,202,402
CPSGEN,4,1,250,250,1
CP,260,UY,206,406
CPSGEN,5,1,260,260,1
CPSGEN,3,5,260,264,1
CP,350,UZ,207,407
CPSGEN,4,1,350,350,1
CPSGEN,4,5,350,353,1
EP,1,2,1000,,16                *Pressure applied on the upper facing
AFWRITE
FINISH
/exe
/input,27
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```
ex/COM,ANSYS REVISION  4.4    UP437 A 16     14.2491     5/ 4/1992

/SHOW,VT340
/PREP7
/TITLE, SANDWICH PLATE WITH 48 ELEMENTS (QUARTER)
KAN,2                *Modal analysis
ET,1,63              *Plate element type defined
ET,3,45              *3-D solid element type defined
KAY,1,-1             *Full subspace iteration used
KAY,2,2              *Expand first mode each load step
KAY,3,2              *Print first two reduced mode shapes
KAY,7,2              *Use subspace iteration to extract first two modes
EX,1,72E9            *Facing modulus of elasticity [pa]
NUXY,1,0.335         *Facing poisson ratio
DENS,1,2700          *Facing density [kg/m^3]
R,1,0.002            *Facing thickness [m]
EX,3,10E9            *Core modulus of elasticity [pa]
NUXY,3,0.335         *Core poisson ratio
DENS,3,2700          *Core density [kg/m^3]
N,1,,,11E-3          *Top facing nodes
N,5,0.24,,11E-3
FILL
NGEN,5,5,1,5,1,,0.06
TYPE,1
MAT,1
REAL,1
E,1,2,7,6                    *Top facing plate elements
EGEN,4,1,1,1,1
EGEN,4,5,1,4,1
NGEN,2,200,1,25,1,,,-22E-3       *Bottom facing nodes
TYPE,1
MAT,1
REAL,1
EGEN,2,200,1,16,1            *Bottom facing elements
NGEN,2,300,1,25,1            *Core nodes
NGEN,2,200,201,225,1
TYPE,3
MAT,3
E,301,302,307,306,401,402,407,406      *Core elements
EGEN,4,1,33
```

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```
EGEN,4,5,33,36,1
D,201,UZ,0,,205,1                  *Boundary conditions (bottom)
D,401,UZ,0,,405,1
D,205,UX,0,,225,5,ROTY,ROTZ            *Symmetry conditions (quarter plate)
D,405,UX,0,,425,5
D,221,UY,0,,225,1,ROTX,ROTZ
D,421,UY,0,,425,1
D,206,UZ,0,,221,5
D,406,UZ,0,,421,5
D,5,UX,0,,25,5,ROTY,ROTZ
D,21,UY,0,,25,1,ROTX,ROTZ
D,305,UX,0,,325,5
D,321,UY,0,,325,1
CP,1,UX,1,301                      *Coupling of plate and core element nodes
CPSGEN,4,1,1,1,1
CPSGEN,5,5,1,4,1
CP,50,UY,1,301
CPSGEN,5,1,50,50,1
CPSGEN,4,5,50,54,1
CP,100,UZ,1,301
CPSGEN,5,1,100,100,1
CPSGEN,5,5,100,104,1
CP,150,UX,202,402
CPSGEN,3,1,150,150,1
CP,157,UX,206,406
CPSGEN,4,1,157,157,1
CPSGEN,4,5,157,160,1
CP,250,UY,202,402
CPSGEN,4,1,250,250,1
CP,260,UY,206,406
CPSGEN,5,1,260,260,1
CPSGEN,3,5,260,264,1
CP,350,UZ,207,407
CPSGEN,4,1,350,350,1
CPSGEN,4,5,350,353,1
EP,1,2,1000,,16                    *Pressure applied on the upper facing
AFWRITE
FINISH
/exe
/input,27
```

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```
/SHOW,VT340
/PREP7
/TITLE, RECTANGULAR PANEL WITH 48 ELEMENTS (QUARTER)
KAN,4                    *Non-linear Transient Dynamic Analysis
ET,1,63                  *Facing element defined as plate element
EX,1,72E9                *Facing modulus of elasticity (constant)
KAY,5,2                  *Initial velocity and acceleration defined zero
KAY,6,1                  *Large deflection analysis
KAY,8,1                  *Stress stiffening included
KAY,9,1                  *Full Newton-Raphson
KAY,10,0                 *In-memory wavefront equation solution
NUXY,1,0.335             *Facing Poisson ratio
DENS,1,2700              *Facing density [kg/m^3]
R,1,0.002                *Thickness of the facing [m]
ET,3,45                  *Core element defined as 3-D solid element
EX,3,72E9                *Core modulus of elasticity (variable)
NUXY,3,0.335             *Core Poisson ratio
DENS,3,2700              *Core density [kg/m^3]
N,1,,,11E-3
N,5,0.24,,11E-3
FILL
NGEN,5,5,1,5,1,,0.06
TYPE,1
MAT,1
REAL,1
E,1,2,7,6                          *Top facing plate elements
EGEN,4,1,1,1,1
EGEN,4,5,1,4,1
NGEN,2,200,1,25,1,,,-22E-3         *Bottom facing nodes
TYPE,1
MAT,1
REAL,1
EGEN,2,200,1,16,1                  *Bottom facing elements
NGEN,2,300,1,25,1
NGEN,2,200,201,225,1
TYPE,3
MAT,3
E,301,302,307,306,401,402,407,406  *Core elements
EGEN,4,1,33
EGEN,4,5,33,36,1
D,201,UZ,0,,205,1                  *Boundary conditions (bottom)
```

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D,401,UZ,0,,405,1
D,205,UX,0,,225,5,ROTY,ROTZ          *Symmetry conditions (quarter plate)
D,405,UX,0,,425,5
D,221,UY,0,,225,1,ROTX,ROTZ
D,421,UY,0,,425,1
D,206,UZ,0,,221,5
D,406,UZ,0,,421,5
D,5,UX,0,,25,5,ROTY,ROTZ
D,21,UY,0,,25,1,ROTX,ROTZ
D,305,UX,0,,325,5
D,321,UY,0,,325,1
CP,1,UX,1,301                 *Coupling of plate and 3-D solid element nodes
CPSGEN,4,1,1,1,1
CPSGEN,5,5,1,4,1
CP,50,UY,1,301
CPSGEN,5,1,50,50,1
CPSGEN,4,5,50,54,1
CP,100,UZ,1,301
CPSGEN,5,1,100,100,1
CPSGEN,5,5,100,104,1
CP,150,UX,202,402
CPSGEN,3,1,150,150,1
CP,157,UX,206,406
CPSGEN,4,1,157,157,1
CPSGEN,4,5,157,160,1
CP,250,UY,202,402
CPSGEN,4,1,250,250,1
CP,260,UY,206,406
CPSGEN,5,1,260,260,1
CPSGEN,3,5,260,264,1
CP,350,UZ,207,407
CPSGEN,4,1,350,350,1
CPSGEN,4,5,350,353,1
EP,1,2,0,,16                  *Element pressure (top)
ITER,5,,
AFWRITE
FINISH
/PREP6
NTABLE,2                      *Transient loading
NSTEPS,30
FILL,1,1,30,1,3.067e-5,3.067e-5   *Table of time
FILL,2,1,5,1,200,200          *Linear rise (5 steps)
EXP,2,6,20,1,1000,-0.4605,2.3025  *Exponential decay (15 steps)
```

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```
FILL,2,21,30,1,0,0          *Zero load
LGR1,TIME,1
EP,1,2,2,,16,1              *Pressure applied on the upper facing
XVAR,1
PLVAR,2
LFWRITE
FINISH
/EXE
/INPUT,27
```

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BIBLIOGRAPHY

Frostig, Y.(1990). Bending of Sandwich Beams with Transversely Flexible Core. AIAA Journal. Vol. 28. No. 3, March 1990.

Kanematsu, H. H., Hirano, Y., and Iyama, H. (1988). Bending and Vibration of CRFP-Faced Rectangular Sandwich Plates. Composite Structures, Vol. 10, pp. 145-163.

Monforton, F. R., and Ibrahim, I. M. (1977). Modified Stiffness Formulation of Unbalanced Anisotropic Sandwich Plates. Int. Jour. Mech. Sci. pp. 335-43

Rao, K. M. (1985). Buckling Analysis of Anisotropic Sandwich Plates Faced with Fiber-Reinforced Plastics. AIAA Journal, 23, 1247-53

Reismann, H. (1988). Elastic Plates - Theory and Application. John Wiley & Sons Inc., Section 5.1.1.

Szilard, R. (1974). Theory and Analysis of Plates. Prentice Hall Inc., Section 4.1.

Watanabe, N., Miyachi, K., and Daimon, M. (1993). Stiffness and Vibration Characteristics of Sandwich Plates with Anisotropic Composite Laminates Face Sheets. AIAA Journal, 1323-CP.

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