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Empirical Essays on Inflation and Economic Growth

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EMPIRICAL ESSAYS ON INFLATION AND ECONOMIC GROWTH

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This dissertation collects three empirical essays on inflation and economic growth. The first essay examines the impact of inflation uncertainty on the level of inflation. Uncertainty is measured by the conditional variance of inflation, and inflation is modeled as a GARCH-in-mean process with a two-regime Markov-switching coefficient on uncertainty. Using a Bayesian estimator with the Markov Chain Monte Carlo approach, we find that the impacts of uncertainty on inflation are statistically significant and have different signs in different regimes for the U.S. post-war data. The regime switching nature of the inflation process can explain the contradictory theoretical predictions and empirical evidence shown in the existing literature.

In the second essay, we develop a time-varying parameter model with survey information to forecast future inflation rates. To capture the inflation dynamics, we first specify quarterly U.S. inflation as an AR(2) process with time-varying unobservable parameters. The model is estimated by the Kalman filter algorithm. We then examine survey data information by combining Survey of Professional Forecasters (SPF) forecasts into the model. Compared to the survey data and the classical ARIMA models, the time-varying parameter models significantly reduce out-of-sample prediction errors. We find that the improvement lies over the time-

varying feature of the models and that including survey data does not significantly improve predictive ability, indicating that the survey data do not contain too much information beyond realized inflation rates.

The third essay re-examines the convergence hypothesis by revising a four-step procedure of the panel unit root test suggested by Evans and Karras (1996). We use data on output for 24 OECD countries over 40 years. We incorporate the spatial autoregressive error structure into a fixed-effect panel model to account for spatial dependence that may induce significant size distortion of the conventional panel unit root tests. Bootstrap procedures are employed to find related critical values. In contrast to the results obtained from the test that does not consider the spatial error structure, our results indicate that there is no output convergence among the OECD countries.

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CHAPTER I

INTRODUCTION

In applied econometrics, economists always strive to find an empirical econometric model that is able to approximate the realistic economy. Then, relying on the model, we can make statistical inference, verify certain economic theory, evaluate economic policy, or forecast future evolution of the economy. Therefore, a lot of methodological advances in econometrics have emerged in order to provide us tools that can depict the real world more accurately and/or be applicable with less restrictive assumptions.

In this dissertation, we present three empirical essays that benefit from recent developments in econometrics. The essays apply new econometric tools to the fields in macroeconomics. In the first essay, the Bayesian simulation techniques enable us to better understand the impact of inflation uncertainty under the regime switching context. The application of the Kalman filter to the state-space model allows us to produce more accurate predictions on inflation in the second essay. In the third essay, we re-examine the convergence hypothesis in a more robust way, by applying the bootstrap procedure to a panel unit root test that accounts for spatial dependence among data.

The first essay examines the impacts of inflation uncertainty on the level of inflation. It is a puzzle in the studies of the relationship between inflation uncertainty and inflation that two different theoretical analyses predict contradictory signs of the impacts of inflation uncertainty. Evidence on both predictions has been found in related empirical studies. This puzzle has motivated us to use the regime switching model to investigate the problem.

We adopt a Generalized Autoregressive Conditional Heteroskedasticity in mean (GARCH-in-mean) model with a two-regime Markov-switching coefficient on uncertainty that is measured by the conditional variance of inflation. Using a Bayesian estimator with the Markov Chain Monte Carlo (MCMC) approach, we find that the impacts of uncertainty on inflation are regime switching and have significantly different signs in different regimes for the U.S. post-war data. The regime that exhibits a negative impact represents the periods when the Fed is inflation-intolerant. The regime with a positive impact indicates that the Fed is inflation-accommodative. Therefore, it is the regime switching nature of the inflation process that produces the puzzling empirical results.

The second essay is concerned with inflation prediction. In this essay, we develop a time-varying parameter state-space model with survey forecasts to forecast inflation. To forecast inflation, especially for the short run inflation rates, it is crucial to appropriately capture the inflation dynamics. We therefore set the parameters in the model to be time-varying to better capture the inflation dynamics. Recent empirical studies also suggest that survey measures frequently beat other existing economic and econometric models in inflation prediction. Motivated by this finding, we incorporate the Survey of Professional Forecasters (SPF) forecasts into the model, which enables us to fully explore information a forecaster conditional on. The model is estimated by using the Kalman filter algorithm. Compared to the survey measure and the classical ARIMA models, the time-varying parameter models with and without survey information significantly reduce out-of-sample prediction errors. The empirical results also indicate that the incorporation of the survey forecasts does not significantly improve the predicative ability of the time-varying parameter model.

Finally, the third essay re-examines the well-know convergence hypothesis in macroeconomics. The convergence hypothesis has been extensively studied in the literature. Recently using variant panel unit root tests is very popular. However, Baltagi, Bresson, and Pirotte (2007) investigate popular panel unit root tests and find that when the data are generated with spatial dependence, the conventional tests are all over-sized. This motivates the current study to re-examine the convergence hypothesis by revising a four-step procedure of the panel unit root test suggested by Evans and Karras (1996). We use data on output for 24 OECD countries over 40 years. We incorporate the spatial autoregressive error structure into a fixed-effect panel model to account for spatial dependence that may induce significant size distortion. In addition, we bootstrap related critical values, considering that the finite sample properties of the test may be different from its asymptotic counterparts. In contrast to the results obtained from the test that does not consider the spatial error structure, our results indicate that there is no output convergence among the OECD countries.

CHAPTER II

THE IMPACTS OF INFLATION UNCERTAINTY ON INFLATION: EVIDENCE FROM A MARKOV-SWITCHING GARCH-IN-MEAN MODEL

2.1 Introduction

The relationship between the level of inflation and its uncertainty has been a hot area of macroeconomic study. Since the early 1970s, economists have noticed that there is a positive relationship, i.e. higher uncertainty of the inflation process is associated with a higher inflation (e.g., Okun, 1971). Cukierman and Meltzer (1986) show that when the public has limited information about the preference of the monetary authority, the central bank has an incentive to take advantage of this asymmetric information. Because unanticipated price change has real effects, the central bank would "create large positive surprises when he cares most about stimulation." (p.1122) The Cukierman-Meltzer hypothesis implies a positive impact of uncertainty on inflation.

However, Holland (1995) argues that it is possible that there is a negative causal effect of inflation uncertainty on the level of inflation. When inflation uncertainty increases, the monetary authority would contract money supply in order to eliminate possible welfare loss. In this case, inflation uncertainty has a negative impact on inflation. Therefore, these two types of theories predict different signs of the effect of inflation uncertainty on inflation.

Empirical studies have reported contradictory outcomes for the U.S. data. Grier and Perry (1998) find that the impact of uncertainty on inflation is negative, while Karanasos et al. (2004) find a positive impact. On the other hand, Baillie et al.

(1996) provide evidence that the effect of uncertainty on inflation is insignificant. One may ask the question: What is the reason behind these seemly contradictory results? This essay attempts to answer this question.

We re-examine the effect of inflation uncertainty on inflation for the postwar U.S. data by modeling inflation as a regime switching Generalized Autoregressive Conditional Heteroskedasticity in mean (GARCH-in-Mean) process. Owyang (2001) claims that inflation is "a result of the process governing the monetary policymaker's preference." (p.41) He shows that the preference of the monetary authority is embodied in the weight put on the inflation target. This preference is not observable by the public. When an accommodative central banker is in charge, monetary policy may focus more on stimulating employment. When an inflation-intolerant central bank is in control, the policy maker is "less willing to trade off inflation for small gain in unemployment." (p.41)

In our GARCH-in-Mean model, the effect of uncertainty on inflation is measured by the coefficient on the conditional standard deviation in the mean equation. The novel point is that this effect is Markov-switching. The Markov-switching model proposed by Hanmilton (1989) is suitable for the present study because it models the regime shift as an unobservable state variable.

A two-regime switching model is considered. If both theoretical predictions are supported, the monetary authority puts different efforts on fighting against inflation in different regimes. One of the two regimes corresponds to the Cukierman-Meltzer hypothesis in which the monetary authority is inflation accommodative. The effect of uncertainty on inflation in this regime is positive. The other regime corresponds to Holland's argument that the monetary authority is inflation intolerant, and therefore has a negative impact of uncertainty on inflation.

For a Markov-switching GARCH-in-Mean (MS-GARCH-M) model, the conditional variance depends on all previous unobservable states. This makes the conventional likelihood estimation infeasible. Therefore, this paper proposes a Bayesian estimator using the Markov Chain Monte Carlo (MCMC) approach. Specifically, a hybrid Gibbs sampler is used to generate random samples. That is, the griddy Gibbs sampler is combined with the standard Gibbs sampler to generate random draws for the parameters in the variance process, whereas other parameters are generated directly by using the standard Gibbs sampler algorithm.

Using the U.S. quarterly GDP deflator data from 1947Q1 to 2006Q1, we find that the impacts of uncertainty on inflation are statistically significant and have different signs in different regimes. The regimes identified in the data are generally consistent with previous studies on monetary regimes. The regime switching nature of the inflation process can explain the mixed evidence shown in the existing literature. Previous studies that do not take this state dependence issue into account would find different results, depending on the sample period selections.

This chapter proceeds as follows. Section 2.2 specifies the model. Section 2.3 outlines the estimation procedure. Section 2.4 shows the empirical results and relates the findings to previous studies. Section 2.5 concludes.

2.2 The Model

In empirical studies, it is very popular to measure uncertainty by using various

GARCH models, developed by Engle (1983) and further generalized by Bollerslev (1986).[1] A GARCH model generates the time-varying conditional variance of unpredictable residuals based on available information, which is consistent with theoretical models' requirements.

There are two main approaches to testing the relationship between inflation uncertainty and the level of inflation. Both approaches use a time-varying conditional variance or standard deviation as the measure of inflation uncertainty.

The first one, introduced by Grier and Perry (1998), is a two-step technique. Grier and Perry first generate a conditional variance series as a result of estimating GARCH models. Then, they conduct Granger-causality tests in a VAR of the generated series and the level of inflation. Conrad and Karanasos (2005) and Fountas and Karanasos (2004) also employ this approach. The advantage of this approach is that it can test the lagged effect between inflation and uncertainty. But Karanasos, et al. (2004) point out a logic contradiction in the two-step method: the two-step method first generates conditional variance assuming no correlation between inflation and its variance, while the second step tests the Granger-causality between them.

The second approach, including studies by Bailie et al. (1996), Karanasos, et al. (2004), Hwang (2001), and Grier, et al. (2004), examines the link between inflation and its uncertainty using GARCH-in-mean models. This approach has the advantage that it can test the effect of uncertainty on the level of inflation within a single model. Thus, it does not have the logical contradiction problem.

We propose a MS-GARCH(1,1)-M model for the inflation process. The

[1] Other measures include the moving standard deviation and the dispersion among individual survey forecasts. They are criticized for not distinguishing unpredictable uncertainty from variability.

model belongs to the second approach. This approach is chosen because it is straightforward to extend a conventional GARCH-in-mean model to incorporate the Markov-switching effect of uncertainty, although the estimation procedure is complicated. Moreover, this model does not suffer from the logic contradiction.

For T observations of inflation, the model is:

(1) $$\pi_t = \beta_0 + \sum_{i=1}^{p} \beta_i \pi_{t-i} + \rho(S_t)\sqrt{h_t} + \sqrt{h_t}\,\varepsilon_t,$$

(2) $$h_t = c + \phi h_{t-1} + \lambda h_{t-1}\varepsilon_{t-1}^2,$$

(3) $$\rho(S_t) = (1 - S_t)\rho_0 + S_t\rho_1,$$

$$c > 0, \phi \geq 0, \lambda \geq 0, \phi + \lambda < 1, \text{for } t \text{ from } 1 \text{ to } T,$$

where π_t is the inflation rate, h_t is the conditional variance, ε_t is a Gaussian white noise with mean zero and unit variance, and p is the autoregressive lag length. The parameter ρ depends on an unobservable state variable S_t. The common nonnegative and stationary restrictions are imposed to the GARCH process. We assume that the state variable S_t can take value either 0 or 1, representing a two-regime switching in the mean equation. The state variable S_t follows a first order Markov process. The transition of the process from one state to the other is governed by the following transition probabilities:

$$\Pr(S_t = 1 \mid S_{t-1} = 0) = e_{01},$$

$$\Pr(S_t = 0 \mid S_{t-1} = 1) = e_{10},$$

where the two subscripts of each transition probability denote from state i (the first subscript) to state j (the second subscript).

We directly introduce the conditional standard deviation in the mean equation.[2] The specification thus allows for testing the impact of uncertainty on inflation within a single model. The model implies that the level of inflation is affected by inflation uncertainty. In addition, the parameter of the conditional standard deviation is Markov-switching, which differentiates our model to the previous literature. The presence of the state-dependent parameter, according to the two theoretical predictions, reflects the change in the central bank's preference. Recall that the two theoretical explanations imply that there are changes in the monetary authority's preference, which would induce different effects of uncertainty on inflation in different regimes. The unobservable state variable is designed to capture this regime shift.

One should note that, if in different regimes inflation uncertainty has a different impact on inflation, mixed empirical evidence is possible when ignoring shifts in regime. The effect can be negative, if in most sample period the central bank is implementing anti-inflationary policy. In contrast, positive effect would appear if the output expansion incentive is dominating. Statistically, the effect of uncertainty on inflation follows a mixed distribution. The mixed distribution consists of two distributions with different means: one is positive and the other is negative. One distribution can dominate the other.[3] The estimate without taking into account regime shifts depends on the sample period.

The current model nests the conventional GARCH model as well as the

[2] For the current study, we limit our primary interest on the impact of inflation uncertainty by not introducing lagged inflation level term into the variance equation. This specification guarantees the nonnegativity of the conditional variance.

[3] "Dominance" here means that the probability of an observation being generated from a distribution is greater than the probability of from another distribution.

simple MS-GARCH model. If the parameters in the model are constant across regimes, the model shows a conventional GARCH-M process. If the coefficient of the conditional standard deviation term in the mean equation is insignificant, the model implies uncertainty has no direct effect on inflation.

The specification of the mean equation is similar to the model estimated in Owyang (2001) and Owyang and Ramey (2004).[4] In their model, the inflation rate depends on the Markov-switching preference of the central bank and the structural parameter.

Relating Owyang's model to the two seemingly contradictory theoretical explanations, one can test the following hypotheses. The first is that, if there are monetary regime shifts, the impact of uncertainty on inflation changes across regimes. The second is that in one regime the impact of uncertainty on inflation is significantly positive, whereas in the other regime the impact is significantly negative.

Tests for these hypotheses focus on the value of ρ. First, to confirm whether there are regime shifts between state 1 and state 0 is equivalent to testing whether ρ_1 and ρ_0 are significantly different, where ρ_i, for *i*=0 or 1, represents the value of ρ in state *i*. Second, one can test $\rho_1 < 0 < \rho_0$, indicating two different impacts of inflation uncertainty on the level of inflation.

2.3 Estimation Approach

We adopt a regime switching GARCH model to model the inflation process. The relatively complicated statistical model poses difficulties in estimation due to the

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[4] See equation (5) in Owyang (2001) and Owyang and Ramey (2004).

so-called path dependence problem. Therefore, we propose a Bayesian simulation-based estimator for the model. One of the recently developed Markov Chain Monte Carlo (MCMC) approaches, the hybrid Gibbs sampling method, is applied to obtain the simulated sample for estimation.

In contrast to classical estimation, a parameter in Bayesian estimation is treated as a random variable. A researcher's belief about the parameter prior to observing the sample data is summarized in the prior distribution. After the data are available, one can obtain the posterior distribution of the parameter by updating information contained in the data according to the Bayes rule.

Making statistical inference is based on the posterior distribution. The point estimate of the parameter is the posterior mean and the associated standard error is the posterior standard deviation. Calculating these moments requires integration through the posterior distribution. Numerical integration methods such as Monte Carlo integration are often applied because it is generally difficult to find a close-form solution. A Gibbs sampler can be used to generate random draws of the parameter. The posterior mean and the standard error are estimated by using the sample mean and the sample standard deviation.

In this paper, we use a hybrid Gibbs sampler to generate random samples of the model's parameters. The posterior mean and standard error are obtained by using the sample mean and the sample standard deviation.

2.3.1 The Path Dependence Problem

In a regime switching GARCH model, as is presented in this paper, the maximum likelihood estimation (MLE) approach is difficult to implement. This is because the conditional variance depends on the entire past history of the latent state variables. The problem, referred to as the path dependence problem, was first pointed out by Hamilton and Susmel (1994).

Consider the two-regime switching GARCH model. The likelihood function (ignoring the constant term) for the model is:

$$(4) \qquad f(\boldsymbol{\theta} \mid \boldsymbol{\pi}) = \prod_{t=1}^{T} h_t^{-1/2} \exp\left(-\frac{1}{2h_t}(\pi_t - (\beta_0 + \sum_{i=1}^{p} \beta_i \pi_{t-i}) - \rho(S_t)\sqrt{h_t})^2\right),$$

where $\boldsymbol{\theta} \equiv [\ \beta_0, \beta_1, ..., \beta_p, \rho_1, \rho_0, e_{00}, e_{11}, c, \phi, \lambda\ ]$, $\boldsymbol{\pi}$ denotes a column vector containing sample observations of inflation.[5] If the state variables were observable, they would simply be dummy variables so that we can maximize the likelihood function to obtain estimates.

However, the difficulty here is that the state variables are not observable. The common technique to deal with a latent variable in the MLE is to integrate with respect to it throughout all possible paths. Note that the conditional variance at time t (h_t) depends on h_{t-1} and the latent state variable, S_t, which can take the value 1 or 0 with some probabilities. The lagged conditional variance, h_{t-1}, again depends on h_{t-2} and S_{t-2}, and so on. This dependence structure indicates that the conditional variance at time t depends on the entire past history. In this case, 2^t possible paths have to be accounted to construct the likelihood function for the t-th observation. This is almost impossible to compute given current computer capacity. Hamilton and Susmel circumvent the problem by considering only a lower order ARCH process.

Nonetheless, under a Bayesian framework, it is much easier to construct the

[5] Throughout this dissertation, a boldface letter denotes a vector or matrix.

likelihood function. One can use simulation method to integrate out the state variables. In addition, because the Gibbs sampler works based on the distribution conditional on other state variables, the latent state variables can be generated one by one. The Gibbs sampler repeatedly samples one state variable from the distribution conditional on other state variables. The state value for each sample observation is obtained after *T* draws. The latent state variables thus become "observable" and we now can treat them as dummy variables. Consequently only one path has to be considered to construct the likelihood function.

2.3.2 The Gibbs Sampler[6]

In order to estimate the parameters in the proposed model, one needs an integral with respect to a high-dimensional joint distribution function. In this case, MCMC approaches such as a Gibbs sampler allow one to construct a Markov chain random sample and use the sample to conduct numerical integration.

The Gibbs sampling, or Gibbs sampler, is based on the Clifford-Hammersley theorem.[7] A complicated multivariate distribution can be completely characterized by information contained in the full set of conditional distributions. This property enables one to sample the joint distribution by sequentially sampling from the conditional distributions that are lower-dimensional functions. In addition, the Gibbs sampler is particularly suitable for the current study in that it can deal with the path dependence problem.

[6] Bauwens, Lubrano, and Richard (2000) and Tsay (2002) contain more elaborate presentations of the Gibbs sampler and the griddy Gibbs discussed in this subsection. More technical introduction about MCMC can be found in Tierney (1994)

[7] Geman and Geman (1984) give the name "Gibbs".

A griddy Gibbs sampler is combined into the standard Gibbs procedure in this essay. Because in equation (4) the variance parameter of the distribution is time-varying, the conditional distribution of the GARCH parameters (c , ϕ , and λ in equation (2)) does not correspond to any known standard distribution. One can not directly generate draws for them. We use the griddy Gibbs sampler to deal with the problem. The griddy Gibbs sampler proposed by Tanner (1996) is a sampler for a univariate density and is known for its high flexibility. In essence, a griddy Gibbs sampler draws random numbers for a variable by inverting its empirical cumulative probability distribution.[8]

2.3.3 Estimation of the MS-AR-GARCH-M Model Using Gibbs Sampler

We outline a Bayesian simulation-based estimator of the MS-GARCH-M model. The derivation is closely related to Tsay (2002). The model here, by including autoregressive terms, extends Tsay's (2002) model in which the conditional standard deviation is the only regressor in the mean process. Consequently in the sampling scheme in subsection 2.3.3.2, step (c) is new.

Decompose the high-dimensional joint distribution of $\boldsymbol{\theta}$ by partitioning $\boldsymbol{\theta}$ into 4 blocks: the autoregressive coefficients, $\boldsymbol{\beta} \equiv [\beta_0\ \beta_1 \ldots \beta_p]'$; the in-mean coefficients, $\boldsymbol{\rho} \equiv [\rho_1(S_t = 1)\ \rho_0(S_t = 0)]'$; the transition probabilities, $\boldsymbol{e} \equiv [e_{00}\ e_{11}]'$; and the GARCH parameters, $\boldsymbol{\alpha} \equiv [c\ \phi\ \lambda]'$. In addition, we treat all unobservable state variables, S_t, for t from 1 to T, as parameters to be estimated, stacked as $\boldsymbol{S} \equiv [S_1 \ldots S_T]'$.[9] The approach only requires that we sequentially sample from the

[8] See Bauwens and Lubrano (1998) for the application to conventional GARCH models.
[9] This is referred to as data augmentation in the MCMC literature.

following conditional posterior distributions:

$$f(e\,|\,\theta,S,\pi)\,;\,f(S\,|\,\theta,\pi)\,;\,f(\beta\,|\,\theta_{-\beta},S,\pi)\,;\,f(\rho\,|\,\theta_{-\rho},S,\pi)\,;\,f(\alpha\,|\,\theta_{-\alpha},S,\pi)\,,$$

where θ_{-i} $(i=\beta,\rho,e,\alpha.)$ denotes the parameter vector excluding parameter block i. The posterior mean and standard error are obtained by using the sample mean and the sample standard deviation.

2.3.3.1 The Prior

Bayesian estimation requires one to give prior distributions for parameters. For the parameters that we can draw directly, we use conjugate priors;[10] for the GARCH parameters that we need to use the griddy Gibbs sampler, assume uniform distributions. Further assuming that all blocks of parameters are independent, we have the following prior distributions:

$$\beta \sim MultiN(\beta_0,\Sigma_0)\ ;\ \rho_i \sim N(\rho_{i0},\sigma_{i0}^2)\ ;\ e_{kk} \sim Beta(\gamma_{k0},\gamma_{k1})\ ;\ \alpha_j \sim U(a_j,b_j),$$

for i=1, 0, k=0,1, j=1, 2, 3. β_0 and Σ_0 are the mean vector and variance-covariance matrix of the prior multi-normal distribution of β. ρ_{i0} and σ_{i0}^2 are the hyper-parameters of the prior distribution of ρ_i. γ_{k0} and γ_{k1} are the hyper-parameters of the prior beta distribution of e_{kk}, the probability of transferring from state k to state k. a_j and b_j are the hyper-parameters of the prior uniform distribution of the j-th element in α.

[10] A conjugate prior has the property that the prior and posterior distributions belong to the same family of distributions. For the derivations of posterior estimator under conjugate priors, please refer to DeGoot(1970, Chapter 9).

2.3.3.2 Sampling Scheme

We sketch the sampling scheme here. Details about derivations can be found in Appendix A.

(a) Starting from arbitrary starting values of other parameters, sample the transition probability e_{kk} from the following beta distribution,

$Beta(n_{kk} + \gamma_{kk}, n_{ki} + \gamma_{ki})$, for i, k=1,0, $i \neq j$,

where n_{ki} is the number of transitions from k to i.

(b) Use the single-move method proposed by Carlin, Polson, and Stoffer (1992) to generate the unobservable vector $\boldsymbol{S}$.

(c) Draw the autoregressive coefficients in $\boldsymbol{\beta}$ from the multivariate normal distribution with variance $\boldsymbol{\Sigma}_1^{-1} = \sum_{t=1}^{T} \boldsymbol{y}_t^* \boldsymbol{y}_t^{*\prime} + \boldsymbol{\Sigma}_0^{-1}$ and mean $\boldsymbol{\beta}^* = \boldsymbol{\Sigma}_1 (\boldsymbol{Y}^{*\prime} \boldsymbol{\pi}^* + \boldsymbol{\Sigma}_0^{-1} \boldsymbol{\beta}_0)$, where $\boldsymbol{\pi}^* = [\pi_1^* \ldots \pi_T^*]'$, $\boldsymbol{Y}^* = [\boldsymbol{y}_1^* \ldots \boldsymbol{y}_T^*]$. π_t^* and $\boldsymbol{y}_t^*$ are obtained by the following transformations: $\pi_t^* = (\pi_t - \rho(S_t)\sqrt{h_t})/\sqrt{h_t}$, $\boldsymbol{y}_t^* = [1\, \pi_{t-1} \ldots \pi_{t-p}]'$.

(d) Draw the state-dependent coefficient ρ_i (i=1 or 0) from the normal distribution with variance $\sigma_{i*}^2 = (n_i + 1/\sigma_{i0}^2)^{-1}$ and mean $\rho_i^* = \sigma_{i*}^2 (n_i \overline{\pi_i}^{**} + \rho_{i0} / \sigma_{i0}^2)^{-1}$, where n_i is the number of data points in state i and $\overline{\pi_i}^{**}$ is the average over the number of data points in state i.

(e) Draw the GARCH coefficient α_j, for j=1, 2, 3, by using the griddy Gibbs sampler.

Repeat n times steps (a) ~ (e). In each step the parameter vector is updated with generated draws. With the generated sample, one can estimate the parameters by using sample averages. The standard errors can be obtained by using the sample standard deviations.[11] Notice here in order to obtain the random sample for the joint distribution, we discard the first *m* draws as the "burn-in" sample.[12]

Step (b) shows how the approach solves the path dependence problem. The single-move approach draws one state variable each time, treating all the other state variables as known. One can compute the conditional posterior probability of $S_t=1$, $P(S_t = 1 \mid \pi, S_{-t}, \theta)$. Then the state variable can be generated by comparing the probability to a random draw from the (0, 1) uniform distribution. The state value for each sample observation is obtained after *T* movements. To construct the likelihood function with these known state variables is straightforward.

2.4 Empirical Results

2.4.1 Data Description and Preliminary Estimation

The data used are the U.S. quarterly GDP deflator from 1947Q1 to 2006Q1, drawn from the website of the Federal Reserve at St. Louis.[13] Inflation is measured by annualizing the log difference of the data. Both the augmented Dickey-Fuller test (ADF test) and the Philip-Perron test (PP test) reject the null that there is a unit root in the inflation process. In the following empirical estimation, we impose the stationary restriction for the sampling scheme and the prior.

The whiteness of residuals and the Akaike Information Criterion (AIC)

[11] We also try using every 10 draws to estimate the parameters. The results do not change substantially.
[12] The theory only claims that the Markov chain sample asymptotically converges to the joint distribution. In practice the first *m* draws in the sampler are dropped and only the remaining sample draws are used to make inferences. The dropped *m* draws are referred to as the "burn-in" sample.
[13] We choose quarterly data because the relatively lower frequency can respond to the criticism that the GARCH-in-mean type method ignores the possible lagged effect of monetary policy.

indicate that the lag length is 4. Therefore we estimate an AR(4) model for the inflation process. The estimation results are shown in Table 1. The Q-statistics for squared residuals at lag 6, 8, and 12 are highly significant, indicating a GARCH effect in the second moment.

Table 1

Results of the AR(4) Model

Variable	Constant	AR(1)	AR(2)	AR(3)	AR(4)
Coefficient	β_0	β_1	β_2	β_3	β_4
Estimate	0.033*	0.574*	0.152*	0.142**	-0.040
Standard Error	(0.006)	(0.065)	(0.073)	(0.073)	(0.064)
Q-statistics					
Lag	6	8	12		
Residual	2.384	10.912	17.586		
Squared Residual	132.443*	152.600*	166.226*		

Note: "*" indicates statistical significance at the 5% level. "**" indicates statistical significance at the 10% level. The results are from an AR(4) model of U.S. quarterly inflation measured by annualizing the log difference of the GDP deflator. The sample ranges from 1947Q2 to 2006Q1.

2.4.2 Results from GARCH Model without Regime Switching

Before we present results from the Markov-switching GARCH-in-mean model, it is suggestive to consider the conventional GARCH model without regime switching. These estimates provide the initial picture of the problem and make the regime switching estimates comparable. In this subsection, we estimate a conventional AR(4)-GARCH(1,1)-M model without considering possible regime switching. The model is estimated by the maximum likelihood technique.

Table 2 shows the estimates from the AR(4)-GARCH-M model without regime switching. In the mean equation, all autoregressive parameters are significant at conventional significance levels. In the variance equation, all parameters are significant and the estimates show that the volatility of inflation is persistent. The insignificance of the Q-statistics of residuals and residuals squared for various lag lengths indicates that the specification is acceptable.

It is notable that in this conventional GARCH model the impact of uncertainty on inflation is significant and negative, with a value equal to -0.38. The result is in favor of Holland's argument. Similar conclusions are also drawn in Grier and Perry (1998) and Grier, et al. (2004). The negative sign of the estimate is found in most previous studies, despite the fact that some of them report the effect is insignificant or weak. Holland (1995), using survey based measures, finds a weakly negative effect of inflation uncertainty on inflation. Hwang (2001) and Conard and Karanasos (2005) report a negative but insignificant impact of uncertainty. A sharp contrast to this is that Fountas, et al. (2004) report a significant and positive effect of uncertainty.

2.4.3 Results from the Markov-switching GARCH Model

The MS-GARCH(1,1)-M model for U.S. inflation is estimated by using the proposed Bayesian estimator.

We choose zero mean and diffuse priors for the parameters in the mean equation, suggested by Chib and Greenberg (1994). The prior for $\boldsymbol{\beta}$ is a multi-normal distribution with mean $\boldsymbol{0}_{5\times 1}$ and variance $10 * \boldsymbol{I}_{5\times 5}$, where $\boldsymbol{0}_{5\times 1}$ denotes a column vector with five zero elements and $\boldsymbol{I}_{5\times 5}$ denotes a 5-by-5 identity matrix. The prior for ρ_i is a normal distribution with mean 0 and variance 10.

Table 2

Results of the AR (4)-GARCH-M Model

Variable	Constant (in mean)	AR (1)	AR (2)	AR (3)	AR (4)	S.D. (in mean)
Coefficient	β_0	β_1	β_2	β_3	β_4	ρ
Estimate	- 0.004	0.381*	0.183*	0.145**	0.198*	-0.380*
Standard Error	(0.017)	(0.075)	(0.083)	(0.080)	(0.070)	(0.132)
Variable	Constant (in variance)	GARCH	ARCH			
Coefficient	c	ϕ	λ			
Estimate	$9.670*10^{-6}$*	0.669*	0.279*			
Standard Error	$(4.555*10^{-6})$	(0.076)	(0.098)			
Q-statistics						
Lag	6	8	12			
Residual	5.462	6.910	15.043			
Squared Residual	6.388	6.645	12.655			

Note: "*" indicates statistical significance at the 5% level. "**" indicates statistical significance at the 10% level. The results are obtained by using the MLE for an AR(4)-GARCH-M model of US quarterly inflation measured by annualizing the log difference of the GDP deflator. The sample ranges from 1947Q2 to 2006Q1.

In the prior beta distribution for e_{kk}, we set the first parameter as 95 and the second as 5. The specified values reflect the prior belief that monetary regime should not change too often. Given the values, the mean value of the beta distribution is

0.95, which implies that the expected duration of staying any state would be five years. We also try using 5 and 95 as the parameter values and the empirical results do not change substantially. This means regime shifts happen frequently, with expected duration being one quarter.

The prior distributions for the parameters, $c, \phi, \text{and}\, \lambda$, are uniform distributions. The hyper-parameters in these uniform distributions constitute the grid interval in the griddy sampler. In a griddy Gibbs sampler, the choice of grid interval constitutes some empirical difficulties. It should cover sufficient support of the distribution that contributes to major part of the integration for probability density function (PDF). A simple way to "tune" the interval is that, before fixing the interval and run thousands iterations, one can run the program several times with fewer iterations and plot histograms to find the proper interval.

The grid interval choice for the variance parameters is initially based on the nonnegative and stationary restrictions. After diagnosing the histograms obtained from the results of the "pre-running" test, we "tune" the uniform intervals to the following values: 0 and $5*10^{-5}$ for c, 0.4 and 0.9 for ϕ, and 0 and 0.5 for λ. It turns out that these specifications do cover most part of the density support, indicating that the specified values are satisfactory. Figure 1 shows the histograms of all parameters that we apply the griddy Gibbs sampler. From the figure, one can see that for each parameter the specified interval has covered most part of its PDF. The graph for each parameter does not show many probability clusters at the edges of the interval or too concentration of nonzero values inside the interval.

The initial value of the conditional variance is set as the sample variance and the pre-sample error is zero. The first value of the state variable is generated from a Bernoulli distribution with equal probability. Initial values for the coefficients are set to be the MLE estimates in Table 2. We run the Gibbs sampler with 9000 iterations and use the last 3000 iterations as an approximation to the joint distribution.

To determine the convergence of the Markov chain, we use the method of monitoring the Cumulative Sum (*CUSUM*) statistics suggested by Yu and Mykland (1998):

$$CUSUM_t = (\frac{1}{t}\sum_{i=1}^{t}\theta^{(i)} - \mu_\theta)/\sigma_\theta,$$

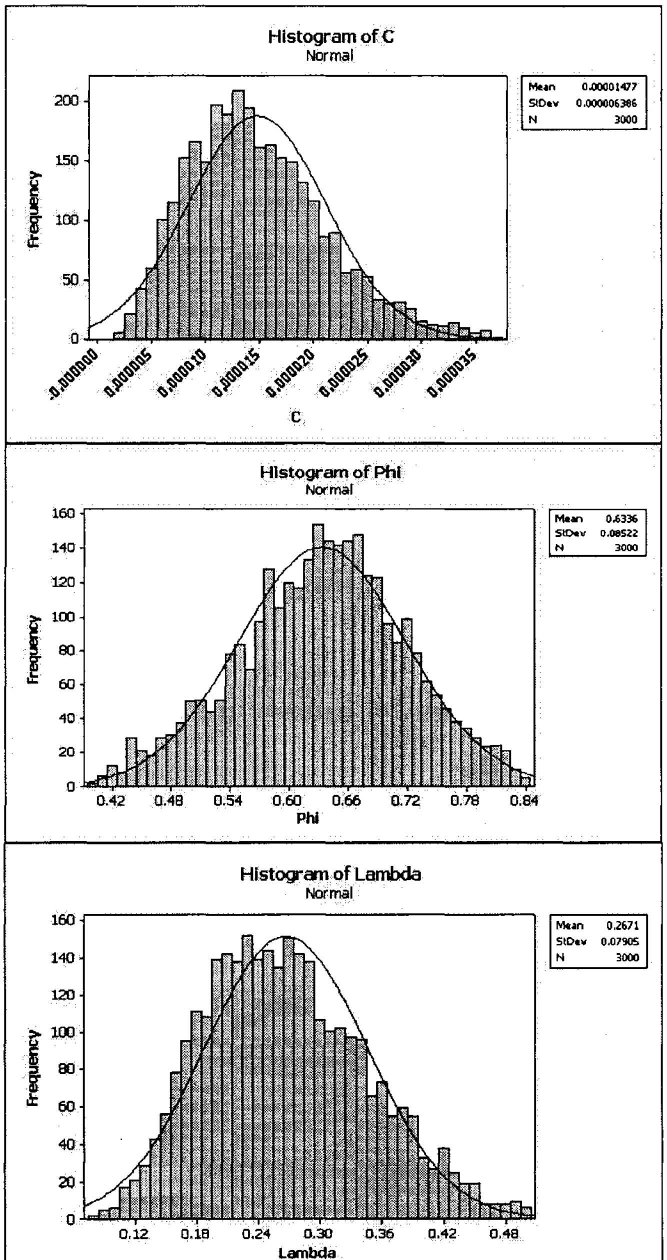
where $\theta^{(i)}$ is the i-th draw, and $\mu_\theta, \sigma_\theta$ are the empirical mean and standard deviation. Yu and Mykland suggest that the graph of the statistics against t will converge to zero smoothly if the Markov chain achieves convergence.

Figure A.1 in Appendix A plots the *CUSUM* statistics for all parameters. All graphs show that the statistics smoothly converge to zero after at most 5000 iterations. So we use the last 3000 iterations to make inference.

Table 3 reports the estimates obtained from the MCMC sample. The estimates are computed by average over the sample and the standard errors are the sample standard deviations. All estimates are significant at least at the 10% level. The estimates for the autoregressive parameters and the GARCH parameters are similar to the ones obtained in the model without switching regime.

Figure 1

Histograms of c, ϕ (Phi), and λ (Lambda)



Note: The graph is based on the last 3000 iterations of 9000 iterations for the AR(4)-MS-GARCH(1,1)-M model of quarterly U.S. inflation: 1947Q2~2006Q1.

The effect of uncertainty on inflation under each regime is captured by the ρ_i

coefficients. The most striking finding is that the estimates show consistency with the theoretical predictions. Both ρ_1 and ρ_0 are significantly differ from zero, indicating that inflation uncertainty has an impact on inflation level. However, they have opposite signs.

First, we conduct the difference test between the two estimates using the Gibbs sample. The difference of the two parameters can be tested by comparing the difference between sample means of the two parameters. The value of the *t*-statistic, 4.3, is significant at any conventional significance level. The rejection of the null that ρ_1 and ρ_0 are equal indicates that there are regime changes in the effect of inflation uncertainty on inflation.

Second, the one-side tests for ρ_1 and ρ_0 are conducted. The hypothesis that ρ_1=0 against ρ_1<0 is rejected at 5% level. This indicates that in regime 1 inflation uncertainty negatively affects inflation level. According to Holland's argument, the monetary authority, as uncertainty prevails, conducts tight policy by contracting money supply growth since it is more concerned with inflation control. On the other hand, the Cukierman-Meltzer hypothesis is supported in regime 0, indicated by the rejection of the null that ρ_0=0 against ρ_0>0.

The opposite effects of uncertainty in different regimes confirm the conjecture made in the introduction part. Previous apparently contradictory results are obtained by not considering regime shifts in the inflation process. These existing empirical results are an outcome of a mixed distribution that consists of two distributions. One has a positive mean, whereas the other has a negative mean. The conclusion depends on which regime prevails during the sample period. It is not surprising that different sample periods and approaches generate different conclusions.

Table 3

Results of the AR (4)-MS-GARCH(1,1)-M Model

Variable	Constant (in mean)	AR (1)	AR (2)	AR (3)	AR (4)	S.D. in mean (state 1)
Coefficient	β_0	β_1	β_2	β_3	β_4	ρ_1
Estimate	0.011*	0.350*	0.142*	0.145*	0.156*	-0.608*
Standard Error	(0.003)	(0.084)	(0.075)	(0.073)	(0.070)	(0.332)
Variable	Constant (in variance)	GARCH	ARCH	Pr $(S_t=0\|S_{t-1}=0)$	Pr $(S_t=1\|S_{t-1}=1)$	S.D. in mean (state 0)
Coefficient	c	ϕ	λ	e_{00}	e_{11}	ρ_0
Estimate	$1.477*10^{-5*}$	0.634*	0.267*	0.932*	0.964*	0.997*
Standard Error	$(6.386*10^{-6})$	(0.085)	(0.079)	(0.023)	(0.012)	(0.423)

Note: "*" indicates statistical significance at the 5% level. The estimates are based on the sample from the last 3000 iterations out of 9000 iterations. The results are obtained by using a Gibbs sampling algorithm for an AR(4)-MS-GARCH(1,1)-M model of U.S. quarterly inflation measured by annualizing the log difference of the GDP deflator. The sample ranges from 1947Q2 to 2006Q1.

The Bayesian estimator also gives the estimated posterior probabilities of being in a regime, conditional on the sample. In Figure 2, we plot the probabilities of being in state 1 against time. State 1, corresponding to the negative impact of uncertainty, prevails in most periods, as one can see in Figure 2. Therefore, the negative effect is more likely to appear if one ignores the regime shifts. This can explain the fact that previous studies tend to report a negative sign of the impact of inflation uncertainty.

Careful examination of Figure 2 provides more insights on the issue of regime shifts. Although the model is simple, it shows regime shifts that roughly correspond to monetary regime changes documented in the literature (for example, Romer and Romer, 2004; and Owyang and Ramey, 2004). In particular, regime shifts found in this paper are very similar to the findings in Owyang and Ramey (2004).

First, during most time periods, the probabilities of being stat 1 are very high. According to Holland's argument, the negative sign in this state suggests that the Federal Reserve is constantly implementing anti-inflationary policy. Owyang and Ramsey (2004) also indicate that a regime associated with anti-inflationary policy is predominating.

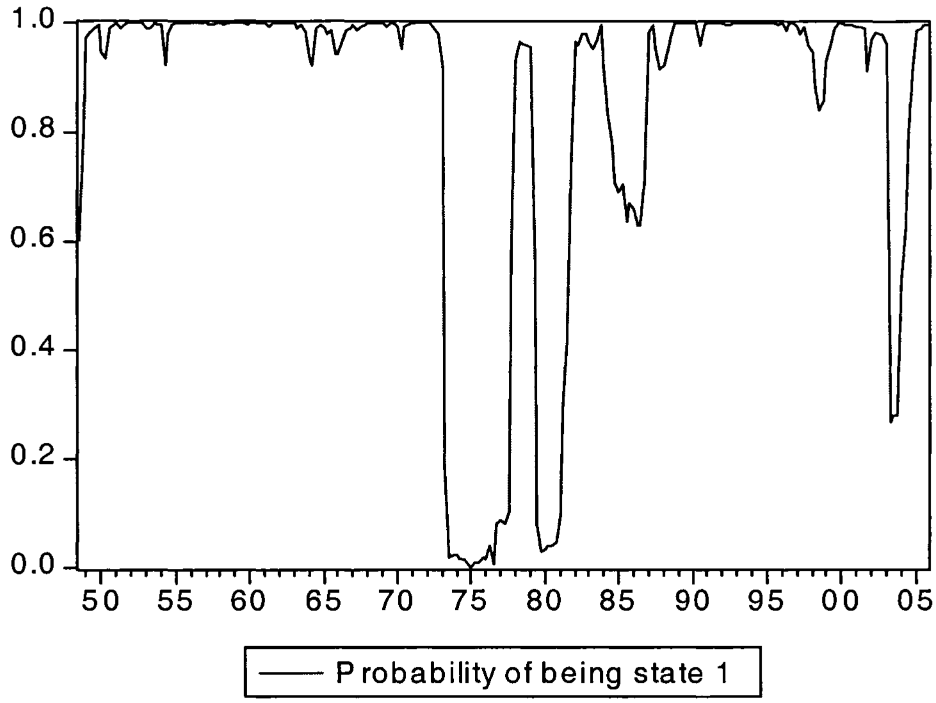
Second, for the past five decades, there are only a few regime shifts, most happened in the 1970s. For the later 1970s, under the regime of "Burns" and "Miller", monetary policy on inflation control is generally considered as slack in the literature, though the inflation rates were very high partially due to the two oil shocks.[14] Clarida, et al. (2000) give convincing evidence and argument that the slack monetary policy in the 1970s plays a more crucial role than the two oil shocks in accounting for rising inflation. The relatively frequent regime shifts in the 1970s (under the "Burns" regime and the "Miller" regime) suggest that monetary policy during this period is more volatile than it in other stable periods (the "Greenspan" regime and the "Martin" regime, i.e., before the 1970s and after the 1970s). This

[14] Because the chairmanship of the Federal Reserve is often found to be closely related to monetary policy change, it is common to use the chairman's name to label a monetary regime.

indicates the ambivalent attitude of the Federal Reserve facing rises in both inflation and unemployment. The timing of regime shifts in the 1970s in this paper coincides with the dates in Owyang and Ramey (2004). But we do not find the change in monetary policy during the period around 1969. In addition, we find that in 2002, there was a moderate change of monetary policy from tight to slack. We infer that the change in 2002 may reflect the change in policy after the most recent recession in 2001.

Figure 2

Probabilities of Being State 1



Note: The curve shows the probability of an observation being generated from state 1. The horizontal axis is time, whereas the vertical axis represents probability. State 1 corresponds to the negative impact of uncertainty on inflation.

Third, in the middle of 1980, a sharp change from state 0 to state 1 happened. After 1982, the probability of being state 1 is decreasing. These changes capture monetary policy changes during the "Volcker" regime in which there were an initial short-lived disinflation policy and moderate expansion policy thereafter. There is a consensus about monetary policy changes in the "Volcker" regime. Changes in the monetary stance found in Owyang and Ramey for this period are very similar to our finding. Both Clarida, et al. (2000) and Bernanke and Mihov (1998) point out that monetary policy during the "Volcker" regime significantly differs with policy in other periods. Our finding provides further evidence about the change.

Finally, note that the model does not identify any regime change in the early 1950s, although the level of inflation during this period is even higher than the level in the 1970s. Owyang and Ramey (2004) find that this period belongs to the anti-inflationary regime. Romer and Romer (2004) point out that "policymakers in this period were emphatic that higher inflation would not increase output and employment in the long run; indeed, they believed that its long-run effects were negative." (p. 135) Therefore, monetary policy in this period is considered as tight in the literature. The sustained high probability of being state 1 during this period shows that the model correctly identifies monetary policy in this period as anti-inflationary.

2.5 Concluding Remarks

In this paper, we examine the effect of inflation uncertainty on inflation by introducing regime switching in a GARCH-in-Mean model. According to theoretical predictions, changes in the preference of the monetary authority are the source of regime shifts in the impact of inflation uncertainty. To overcome difficulties induced by the regime switching feature, we use a Bayesian estimator to estimate the model. A Gibbs sampler algorithm is applied.

The estimation results show that in a prevailing regime the impact is negative, which supports Holland's argument. In the other regime which often happened in the 1970s, the impact is positive. This is consistent with the Cukierman and Meltzer's prediction. Therefore, the results are able to explain existing empirical evidence that is rather mixed. The mixture of distributions for the state-dependent variable can generate opposite impacts. The results also indicate that the model can capture regime shifts closely related to monetary policy changes documented in the literature.

CHAPTER III

A TIME-VARYING PARAMETER MODEL FOR INFLATION PREDICTION

3.1 Introduction

Inflation is a crucial economic indicator. For both policymakers and the public, it is very important to predict future economic status based on current information to help improve their decision making process. A central bank that takes stabilizing inflation as one of policy objectives may need estimates of future inflation to design monetary policy. Because almost all transactions are measured in nominal term, investors participating in financial markets, firms setting prices, customers deciding time to buy certain goods, and workers negotiating wages need inflation forecasts to calculate possible loss caused by inflation. Therefore, one of the most active fields in economic research is inflation prediction and a lot of prediction approaches have been applied.

This chapter proposes a time-varying parameter model to capture the dynamic characteristics in the inflation process. We also combine survey forecasts into the model in order to fully explore the information set that a forecaster conditions on. We specify quarterly U.S. inflation as an AR(2) process with time-varying unobservable parameters and the survey forecast is combined as a regressor in the AR model.

Compared with other forecasting approaches, our approach has two distinguishing features. First, we allow time-varying parameters in a state-space model to better capture inflation dynamics. This is motivated by the fact that the previous literature has adopted Markov-switching models to account for possible parameters instability. In addition, some economic theories imply that parameters of the inflation process are time- or state- dependent. Instead of setting arbitrarily the number of states and fixing parameters within regimes, we allow changes in parameters to be fully determined by data. This goal is achieved by estimating the state-space model with the Kalman filter algorithm.

Second, we examine survey data information by extending the state-space model with survey data. Previous literature has indicated that survey measures beat other conventional approaches to inflation prediction (see Ang, Bekaert, and Wei, 2007), which implies that survey forecasts may contain some useful information in forecasting inflation. Thereby, the model combines information contained in both the classical ARMA model and survey forecasts. This essay is, to our knowledge, the first attempt to use a time-varying parameter model with survey data to forecast inflation.

We conduct out-of-sample prediction for the time-varying parameter models both with and without survey information, the survey approach, and the ARIMA models. For all models, one-year-ahead inflation rates are predicted over 1985Q1~2004Q4 as well as 1995Q1~2004Q4. We compare mean prediction errors (MPEs), root mean squared prediction errors (RMSPEs), and mean squared prediction errors (MSPEs) to evaluate the forecast accuracy. We find that both time-varying parameter models significantly reduce prediction errors. Compared to the errors of survey data, the prediction errors of the time-varying parameter model with survey information are reduced by as much as 22.14% for the post-1985 sample and 21.15%

for the post-1995 sample and the prediction errors of the time-varying parameter model without survey information are reduced by 20.48% for the post-1985 sample and 20.94% for the post-1995 sample.

The results also indicate that although including survey forecasts into the time-varying parameter model helps to reduce the prediction errors, the effect of survey information is very weak. For all models, we conduct pairwise statistical tests for equal mean squared prediction errors (MSPEs). The results show no statistical difference between the two time-varying parameter models. The ineffectiveness of including survey data suggests that the predictive ability of the model comes from the time-varying parameter characteristic. The survey data do not contain too much information beyond realized inflation rates. As a result, we conclude that valuable information, which would be helpful for forecasting inflation, can be lost by restricting parameters constant over time.

This chapter will proceed as follows. In section 3.2, we briefly review existing approaches to inflation prediction. In section 3.3, we propose our methodology. Data issues are discussed in section 3.4. Section 3.5 gives our empirical results. Section 3.6 concludes.

3.2 Related Literature

There are various studies about modeling the inflation process, among which many recent empirical papers are concerned with non-linear behaviors and structural changes.[15] The non-linear phenomenon raises the possibility that parameters of a

[15] See, for example, Kim (1993), Garcia and Perron (1996), and Evans and Wachtel (1993), among others.

model may change over time. Garcia and Perron (1996), using a three-state Markov-switching model, find that each state occurred once in the inflation process, indicating some structural shifts. Kim (1993) also uses a Markov-switching model and shows that the mean and volatility of inflation have three major structural changes for the U.S. GDP deflator data from 1950Q1 to 1990Q4.

The use of non-linear Markov-switching models in the previous literature has been strongly motivated by parameter instability of the inflation process (See Evans and Wachtel, 1993). However, whether a Markov-switching model is appropriate remains as an open question because there are no strong tests in the existing literature for selecting non-linear models.

In addition, one of possible caveats in those regime switching models is that the number of possible states is set in advance. The regime selection may be based on the implications of economic theory so that the model may have an explicit economic interpretation. But the selection may be somehow arbitrary from the data-driven point of view. The arbitrariness in setting the states may actually distort information in the data. In prediction exercises, this may generate poor forecasts. Therefore, for completely capturing the inflation dynamics it seems natural to let parameters be time-varying.

Some economic theories also imply that the inflation process presents non-linear behaviors and is time-varying. Standard sticky price theories suggest that the speed of inflation adjustment should be quicker than the speed of deflation because the downward sticky price and wage will slowdown the downward change process of inflation. This indicates that the dynamics of inflation is asymmetric, which has been observed in the U.S. data.

Models of incomplete nominal adjustments show that the inflation process is time-dependent and state-dependent. For instance, Burstein (2006) develops a state-dependent pricing model in which output and inflation respond to the size of the growth rate of money asymmetrically. Since the size of the money growth rate which reflects monetary policy can be time-varying, the inflation dynamics would be time-varying. Lancing (2006), introducing boundedly-rational expectations into a standard New-Keynesian Phillips Curve, further shows that the time-varying inflation dynamics can also be driven by non-monetary shocks propagated via the expectation feedback mechanism.

Many inflation prediction techniques have been developed in the literature and the performance of these techniques is compared. One of remarkable results in the recent prediction approach comparison is that the survey approach stands out to be successful, suggesting that survey data contain useful predictive contents that should be considered in a forecasting model. Although slightly mixed evidence has been revealed, methods using survey data frequently dominate over other alternatives. Fama and Gibbons (1984), Grant and Thomas (1999), Thomas (1999), Mehra (2002), Hueng and Wang (2005), and Ang, et al. (2007) consistently find that survey outperforms other alternatives.

In the comprehensive investigation of existing inflation prediction models, Ang, et al. (2007) classify four approaches: the classical ARIMA model, the econometric framework derived from theoretical models such as the Phillips Curve, models using term structure information, and survey measures. Ang, et al. (2007)

investigate all four methods and find that using survey data alone beats all other kinds of forecasts. The empirical success of survey in forecasting reflects the possibility that either existing econometric models inappropriately capture the inflation process, or they fail to incorporate current information completely, or both.

In practice, survey forecast is generally viewed as a proxy for expected inflation. The theory on expected inflation suggests that a forecasting model taking expected inflation into account may gain better forecasting ability. The theory on the Expectations-Augmented Philips Curve [16] implies that expected inflation has an impact on the true inflation process. In the time-inconsistency analysis such as Barro and Gordon (1983), the rational expectation of the public generates the inflationary bias. These theories imply that expected inflation (hence the survey data) should contain important information for future inflation movements. According to these theoretical results, a forecasting model using survey information is more appropriate and may generate better forecasts.

3.3 Methodology

3.3.1 A Time-varying Parameter Inflation Prediction Model with Survey Information

We specify the inflation process in the following state-space form:

(5) $$\begin{aligned}\pi_t &= \mu_t + a_{1t}\pi_{t-1} + \ldots + a_{pt}\pi_{-pt} + s_t\pi^e_{t|t-1} + w_t \\ &= \boldsymbol{X}_t\boldsymbol{A}_t + w_t,\end{aligned}$$

[16] See King (2000) for a discussion on the history and debates of the Philips Curve.

$$= \left(1\, \pi_{t-1} \ldots \pi_{t-p}\ \pi^{e}_{t|t-1}\right) \begin{pmatrix} \mu \\ a_1 \\ \ldots \\ a_p \\ s \end{pmatrix}_t + w_t$$

$$(6) \qquad \begin{pmatrix} \mu \\ a_1 \\ \ldots \\ a_p \\ s \end{pmatrix}_t = \begin{pmatrix} f_{0,0} & 0 & \ldots & 0 \\ 0 & f_{1,1} & \ldots & 0 \\ & & \ldots & \\ 0 & \ldots & & f_{p+1,p+1} \end{pmatrix} \begin{pmatrix} \mu \\ a_1 \\ \ldots \\ a_p \\ s \end{pmatrix}_{t-1} + \begin{pmatrix} v_1 \\ v_2 \\ \ldots \\ v_{p+2} \end{pmatrix}_t ,$$

or equivalently,

$$(7) \qquad \boldsymbol{A}_t = \boldsymbol{F}\boldsymbol{A}_{t-1} + \boldsymbol{v}_t ,$$

where π_t is inflation at time t, $\pi^{e}_{t|t-1}$ is the expected inflation rate based on information available at time t-1, μ is an intercept, a_{it}, for i=1, 2, .., p, is the AR parameter at time t on the i-th lagged inflation rate, $\mathbf{A}_t = [\mu\, a_1\, a_2 \ldots a_p\; s]'_t$, is an unobservable vector of parameters that are changing overtime, $\boldsymbol{F}$ is a $(p+2)\times(p+2)$ parameter matrix to be estimated, and w_t , $\boldsymbol{v}_t$ are error terms. Further assume that w_t ~i.i.d. N(0, R), $\boldsymbol{v}_t$ ~i.i.d. N(0, $\boldsymbol{Q}$), and $w_t, \boldsymbol{v}_t$ are not correlated.

Equation (5) models inflation as an autoregressive process with p lags. The optimal lag length is determined by using the Schwartz Information Criterion (SIC). The motivation of using the AR(p) model is that both Ang, et al. (2007) and Hueng and Wang (2005) show that, except the approach using survey data, the ARIMA model produces the second best forecasts of future inflation.[17] Their results indicate

[17] Ang, et al. (2007) actually find that the best ARMA model for forecasting inflation is the ARMA(1,1) model, instead of the AR(2) model. Nonetheless, the estimation results in this essay shows that using the AR(2) model with time-varying parameters outperforms the survey data approach, which is better than the classical ARMA(1,1) model with fixed coefficients.

that past inflation contains important information that helps to predict the future inflation rate. In addition, we incorporate expected inflation (measured as survey data) into the AR(p) process.

The specification embodies two contributions of the current work. First, as in equation (6), we model the autoregressive parameters of the inflation process unobservable and time-varying. We incorporate time-varying parameters to precisely capture the inflation dynamics, in order to address the caveats of Markov-switching models in which parameters within a regime are fixed and the exogenous number of possible states is set in advance. Instead, our model sets coefficients to change over each period. The Kalman filter allows data to determine whether the coefficients should be changed in order to gain better prediction power.

The second purpose of the essay is to examine survey information by combining survey data into the inflation process, considering that the previous literature often finds that survey forecasts tend to beat other alternatives. We choose to use the Survey of Professional Forecasters (SPF). Since the interviewees in the SPF are professionals that specialize in inflation prediction, the data should contain the most complete information available at the time the inflation rate being forecasted. Presumably this information, updated through the Kalman filter, would be helpful in improving the predictive accuracy.

In practice, the survey measure is a very commonly used proxy for expected inflation. Hence the model presented here provides a non-structural way of investigating the role of expected inflation in determining inflation. Note that most theoretical studies on the Expectations-Augmented Philips Curve imply that the parameter of inflation expectation is unity.[18] Although this restriction is implied if the agent in the economy is rational and forward-looking, it does "not appear to be supported by the data. Alternatively, if one assumes that workers and firms do not form their expectations rationally, one is resting the theory on irrationality." (Romer, 2000, Chapter 5, p.251)

Recent developments in economic theories have provided support for Romer's view. Carroll (2003) develops an interesting story of the formation of inflation expectation. Carroll shows that, at any time point, only part of the public is updating their information about the economic status through media coverage. Consequently, aggregate inflation expectation in the economy is partially rational or backward-looking. The story further implies that the parameter of expected inflation should be time-varying because at each time point the portion of the public that is covered by the news press is different and the effect of aggregate inflation expectation on inflation should be changing.

3.3.2 Estimation Method

Equations (5) and (6) correspond to the observation equation and the state equation in the state-space model, respectively. The model can be estimated with the Kalman Filter. Here we briefly introduce the Kalman filter procedure. For details about the state-space model and the Kalman Filter, see Hamilton (1994) and Kim and Nelson (1999).

The Kalman filter is a recursive algorithm for updating linear projection. It

[18] An example would be Gali, and Gertler (1999).

involves two steps: the prediction step and the updating step. First, form prediction of $\boldsymbol{A_t}$ based on information available at t-1,

$$\boldsymbol{A}_{t|t-1} = \boldsymbol{F}\boldsymbol{A}_{t-1} .$$

The related variance-covariance matrix of forecasting $\boldsymbol{A_t}$ conditional on information available at time t-1 is

$$\boldsymbol{P}_{t|t-1} = E[(\boldsymbol{A}_t - \boldsymbol{A}_{t|t-1})(\boldsymbol{A}_t - \boldsymbol{A}_{t|t-1})'] = \boldsymbol{F}\boldsymbol{P}_{t|t-1}\boldsymbol{F}' + \boldsymbol{Q} .$$

Second, when time t comes, inflation at time t is revealed. Then one can calculate the prediction error of the predicted inflation rate at time t based on information available at time t-1,

$$\eta_{t|t-1} = \pi_t - \pi_{t|t-1} = \pi_t - \boldsymbol{X}_t\boldsymbol{A}_{t|t-1} ,$$

and the related conditional variance of the prediction error,

$$f_{t|t-1} = \boldsymbol{X}_t\boldsymbol{P}_{t|t-1}\boldsymbol{X}_t^{'} + R .$$

Since the observed inflation rate at t and the related prediction error provide us new information, we update our prediction of $\boldsymbol{A_t}$,

$$\boldsymbol{A}_{t|t} = \boldsymbol{A}_{t|t-1} + \boldsymbol{K}_t\eta_{t|t-1} ,$$

and the variance-covariance matrix becomes,

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_t\boldsymbol{X}_t\boldsymbol{P}_{t|t-1} ,$$

where $\boldsymbol{K}_t = \boldsymbol{P}_{t|t-1}\boldsymbol{X}_t^{'}f_{t|t-1}^{-1}$.

The Kalman gain, $\boldsymbol{K_t}$, is the weight put on the prediction error from time t-1 to time t.

The parameters in the model can be estimated by using the MLE. With a starting vector of the unobservable parameters, one can obtain all estimates of the time-varying parameters.[19]

3.3.3 Measuring Predictive Accuracy

In this paper, we conduct out-of-sample prediction exercises. As a common measure of the short run inflation rate in the literature, we forecast the future annual inflation rate. The annual inflation rate is defined as:

$$\pi_{t,t+4} = \sum_{i=1}^{4} \pi_{t+i} ,$$

where $\pi_{t,t+4}$ is the annual inflation rate from time t to time t+4.

We estimate the model using a recursive scheme. A recursive scheme fixed the starting point of the sample. Every time a new observation is added into the sample. The model is re-estimated and one-year-ahead predictions are calculated. We also estimate the time-varying parameter model without survey to find out the effect of including survey data.

Three classical econometrical models, the ARMA(1,1), the AR(2) model, and the random walk model, are also estimated for comparison purpose. This selection is based on the fact that previous papers often show that the classical ARIMA model is the best model of inflation prediction among all economic and econometrical models.

To measure the performance of our model, we adopt MPE, RMSPE and MSPE as the measures of predicative ability. We also report the relative RMSFE in the paper. The survey data approach serves as the benchmark model. The MPE and RMSPE are defined as:

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[19] Since the state vector is unobservable, the Kalman filter estimation needs a "wild guess" of the starting vector. Then, the state vector will quickly converge to the true value because the algorithm updates the estimates directly based on the prediction error.

$$MPE = \frac{1}{T}\sum_{t=1}^{T}(\pi_t - \hat{\pi}_t),$$

$$RMSPE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\pi}_t - \pi_t)^2} \quad ,$$

where π_t is the annual inflation rate and $\hat{\pi}_t$ is the one-year-ahead inflation forecast.

In order to examine the results from a statistical perspective, we evaluate the models by conducting pairwise tests for MSPEs. Since forecasts are made based on estimated parameters, the impact of uncertainty introduced by the estimates should be considered. The procedures of forecast evaluation recommended by West (2006) are adopted. Specifically, we test the null hypothesis,

$$E(\hat{\pi}_{1t} - \pi_t)^2 - E(\hat{\pi}_{2t} - \pi_t)^2 = 0,$$

against

$$E(\hat{\pi}_{1t} - \pi_t)^2 - E(\hat{\pi}_{2t} - \pi_t)^2 > 0,$$

where $\hat{\pi}_{it}$, i=1,2, denotes the forecast of the inflation rate at time t from model i.

Two cases should be considered:

(a) If two models are nonnested, regress $(\hat{\pi}_{1t} - \pi_t)^2 - (\hat{\pi}_{2t} - \pi_t)^2$ on a constant and examine the standard t-statistic. However, a heteroskedasticity and autocorrelation consistent estimator of the standard error should be used (Newey and West, 1987).

(b) If two models are nested, according to Clark and West (2005), regress $(\hat{\pi}_{1t} - \pi_t)^2 - ((\hat{\pi}_{2t} - \pi_t)^2 - (\hat{\pi}_{1t} - \hat{\pi}_{2t})^2$ on a constant and examine the t-statistic. Again, a heteroskedasticity and autocorrelation consistent estimator of the standard error is used.

3.4 The Data

In this essay, we use log difference of the GDP deflator data observed at the first quarter of 2006 to measure inflation.

The expected inflation rate from survey data are obtained using the median forecasts of the SPF data, available at the website of the Federal Reserve Bank of Philadelphia[20], ranging from the fourth quarter of 1968 to the fourth quarter of 2005. Although the CPI forecast data are also available in the SPF survey, we do not use CPI because the data only start from 1981. The survey is conducted once a quarter and professional forecasters from leading forecasting firms provide their forecasts of major economic indicators including the GDP deflator and CPI. Details about the SPF data can be found in Croushere (1993).

There are at least two other major survey datasets available: the Michigan Survey and the Livingston Survey. Nonetheless, we believe that the SPF data contain the most complete information predicators conditional on because the forecaster are professionals and the SPF gives monetary award to the best forecasters. Carroll (2003) carefully compares the three types of survey data and finds that SPF forecasts are more rational than the other two. Therefore, the SPF data should be more suitable for the current study that intends to incorporate more information.

The SPF dataset gives the GDP deflator forecasts for proceeding quarter, current quarter, and consecutive four following quarters from current quarter. The one-year-ahead inflation forecast of the survey data is constructed by taking the log difference of the expected GDP deflator. We notice that there are some differences

[20] The SPF previously is collected by the National Statistical Association and the National Bureau of Economic Research. Currently it is conducted by the Federal Reserve Bank of Philadelphia.

between the realized GDP deflator of the current quarter and the corresponding forecast in the survey data. We use the survey forecast of current quarter to calculate the expected inflation rate because it reflects information about the current inflation rate for the forecasters.

Table 4 provides basic data descriptive statistics. The first column shows the statistics for the entire sample. The second and third columns correspond to the out-of-sample forecast horizons in this essay. From the table, one can observe the stylized facts of the inflation process: inflation rates are deceasing and the process is becoming stable in the 1980s and the 1990s. The table shows that both mean and standard deviation are deceasing along the three samples. For the entire sample, the maximum value occurred during one of the oil crises in the 1970s. The minimum value is observed after 1995.

Table 4

U.S. GDP-deflator-measured Inflation Data Descriptive Statistics

Sample Period	1968Q4-2005Q4	1985Q1-2004Q4	1995Q1-2004Q4
Mean	0.0406	0.0238	0.0191
Median	0.0318	0.0209	0.0172
Maximum	0.1217	0.0479	0.0378
Minimum	0.0078	0.0078	0.0078
Std. Dev.	0.0249	0.0096	0.0073

Note: Data are measured in annual rate with quarterly frequency.

3.5 Results

We conduct out-of-sample prediction of the annual inflation rate for the time-varying parameter models with and without survey information, the AR(2) model, the ARMA (1,1) model, the random walk model, and the survey data. The SIC shows that the optimal lag length for the AR model is 2 lags. We also use 2 lags for the time-varying parameter models. In line with Ang, et al. (2007), we start out-of-sample forecasts from 1985Q1 and 1995Q1. The one-year-ahead forecast at time t only uses information available at t. The forecasts end in the fourth quarter of 2004.

We conduct the ADF unit root tests prior to the estimation. The results show that the inflation process is stationary. Nevertheless, since whether inflation is stationary is a disputable conclusion in the literature, we also consider the random walk model with drift in which the one-unit-root restriction is imposed.

We first report the MPEs in Table 5. For the whole forecast period from 1985Q1 to 2004Q2, except the two time-varying parameter models, all models over-predict the inflation rates (with negative MPE values). This is due to the impacts of the high inflation rates in the 1970s. On the contrary, the two time-varying parameter models under-predict inflation (with positive MPE values). In addition, the two time-varying models have MPEs that are closer to zero than MPEs from other models except the random walk model. The two time-varying parameter models are not so vulnerable to the impacts of the high inflation rates in the 1970s, because they focus more on the most recent changes in inflation. And this is also true for the random walk model that only uses the current observation to forecast. The random walk model generates MPEs closer to zero than those from all other models, suggesting that the model is likely to produce "unbiased" forecasts on average. Nonetheless, the random walk model also leads to great volatile of forecasts, as will be seen when we compare the forecast accuracy in terms of MSPE and RMSPE.

Splitting the forecast period into two parts further strengthens our argument. For the period 1985Q1-1994Q4, all models over-predict inflation; for the period 1995Q1-2004Q4, the survey data, the ARMA(1,1) model, and the AR(2) model still over-predict inflation with decreasing MPE values, whereas the random walk model and the two time-varying parameter models under-predict inflation.

Table 5

Mean Prediction Errors (MPE)

	TVPwS	TVPw/oS	Survey	ARMA(1,1)	AR(2)	RW
MPE (1985Q1-2004Q4)	0.0652	0.0775	-0.4246	-0.3934	-0.6072	-0.0050
MPE (1985Q1-1994Q4)	-0.0685	-0.0401	-0.7130	-0.5637	-0.7829	-0.0805
MPE (1995Q1-2004Q4)	0.2075	0.2020	-0.1032	-0.1999	-0.4024	0.0763

Note: "TVwS" and "TVw/oS" stand for the time-varying model with and without survey information, respectively. "RW" means the random walk model. The results are scaled by multiplying 100.

We show the RMSPEs in Table 6. The results are consistent with our expectation. Among all models over the two sample periods, the time-varying parameter model with survey information generates the best forecasts for future one year inflation because it takes advantages of (1) the Kalman filter that captures the inflation dynamics and (2) the most complete information set by including survey data. Compared to the prediction errors of the survey data, the prediction errors of this model are reduced by as much as 22.14% for the post-1985 sample and 21.15% for the post-1995 sample.

Table 6

Root Mean Squared Prediction Errors (RMSPE)

		TVPwS	TVPw/oS	Survey	ARMA(1,1)	AR(2)	RW
1985Q1-2004Q4	RMSPE	0.0065	0.0066	0.0083	0.0091	0.0089	0.0076
	Relative RMSPE	77.86%	79.52%	100%	110.42%	107.36%	91.47%
	Percentage	-22.14%	-20.48%	100%	10.42%	7.36%	-8.53%
1995Q1-2004Q4	RMSPE	0.0056	0.0057	0.0071	0.0087	0.0073	0.0072
	Relative RMSPE	78.85%	79.06%	100%	121.09%	102.47%	98.76%
	Percentage	-21.15%	-20.94%	100%	21.09%	2.47%	-1.24%

Note: Survey is the benchmark model. Relative RMSPE is the ratio between the RMSPE of the model and that of the benchmark model (survey). Percentage = ((RMSPE of the model - RMSPE of Survey)/RMSPE of survey)*100%. "TVwS" and "TVw/oS" stand for the time-varying model with and without survey information, respectively. "RW" means the random walk model.

The time-varying parameter model without survey information also shows much higher accuracy than the survey data: it reduces prediction errors by as much as 20.48% for the post-1985 sample and 20.94% for the post-1995 sample. The predictive content in the survey data does help to predict the future inflation rates in the time-varying parameter model, but they are not as informative as expected. Including the survey data only reduces aggregate forecast errors by 2% over 1985Q1-2004Q4 and less than 1% over 1995Q1-2004Q4. This is also confirmed by checking the coefficients associated with survey: in most cases the values are very close to zero.

According to previous discussions, we believe that the survey data contain little information beyond the past values of inflation if one adopts the Kalman filter in the time-varying parameter model to fully explore the inflation dynamics. The improvement in accuracy attributes to the time-varying characteristic of the model.

It can be inferred that for other alternative approaches, including the empirically successful ARIMA models, a large portion of information is lost when the parameters are restricted to be fixed. The reason that the survey data contribute little, we argue, is that most of the professional forecasters are forecasting inflation using the classical ARMA model, because the method is computationally inexpensive and often found in the literature that it outperforms other models with rich economic contents. This finding, combining with the finding in Carroll (2003) and Ang, et al. (2007) that the SPF forecasts are the most accurate forecasts among all three survey measures, indicates that it is very common that forecasters are "pervasive sticky" as in Mankiw and Reis (2002, 2006) or "backwards-looking."

When evaluating the performances of the ARMA (1,1) model and the AR(2) model, we find that they generally perform worse than the survey data, a result consistent with the findings in Ang, et al. (2007). The performances of those two classical models, compared to those of the time-varying parameter models, are even worse. This again confirms that the restrictions on parameters lose valuable predicative contents, especially information about the inflation dynamics. A deviation of current study to Ang, et al. (2007) is that we find that the AR(2) model outperforms the ARMA(1,1) model. However, as we will see, there is actually no statistical difference between the two models in terms of MSPE.

It is notable that the random walk model is better than the survey approach and the other two ARMA models, but it is worse than the two time-varying parameter models. This tells us that it is important to emphasize the current changes in inflation, as is in the time-varying parameter models. The random walk model purely relies on the current observation to make predication. It cuts the noise introduced by previous observations especially inflation rates in the 1970s so as to generate more accurate inflation forecasts than those from survey and other ARMA models. It also loses useful information in the previous observations so that it predicts worse than the two time-varying parameter models.

We then conduct pairwise *t*-tests for equal MSPEs from the models and Tables 7 and 8 summarize the results. Tables 7 and 8 clearly show that the time-varying parameter models with and without survey information statistically beat the other three alternatives. Compared to the time-varying model without survey information, the time-varying model with survey information does not significantly reduce prediction errors. The contribution of survey information is ignorable.

In addition, we find that the survey approach, the random walk model, the ARMA(1,1) model, and the AR(2) model almost have no statistical difference in predicative ability.[21] Hence, the claim that survey outperforms other approaches is doubtable. In fact, the random walk model is even better (not statistically) than the survey approach when we simply compare RMSPEs.

We also plot the forecast series from the models and the inflation series in Figure 3. In the graphs, all models (except the random walk model which fluctuates

[21] In most cases the ARMA(1,1) model is worse than other models.

around the inflation series in a volatile way) more or less capture the dynamics of the inflation process because the curves of the forecast series look like shifting the inflation series to the right. This suggests that there are lagged responses of the models to changes in inflation. The models need some time to learn these changes. The gap between a model's forecast series and the true inflation series reflects the learning lags.

Table 7

Pairwise Tests of MSPE (Post 1985)

Predication Sample Period 1985Q1-2004Q4 H_0: $MSPE_{model\ 1}$-$MSPE_{model\ 2}$=0 H_1: $MSPE_{model\ 1}$-$MSPE_{model\ 2}$>0					
Model 1 / Model 2	TVPwS	RW	Survey	AR(2)	ARMA(1,1)
TVPwS	1.11 (0.135)	1.96* (0.026)	4.28* (0.000)	6.43* (0.000)	3.06* (0.002)
TVPw/oS		1.73* (0.044)	1.79* (0.039)	6.36* (0.000)	2.88* (0.003)
RW			0.71 (0.2311)	1.74* (0.043)	3.35* (0.001)
Survey				0.94 (0.175)	0.96 (0.169)
AR(2)					0.49 (0.310)

Note: The tests are testing the null hypothesis that the squared prediction errors from two models have equal mean value. "TVwS" and "TVw/oS" stand for the time-varying model with and without survey information, respectively. "RW" means random walk. Each entry contains *t*-statistic and *p*-value (in parentheses). "*" indicates statistical significance at the 5% level.

However, as we see in Figure 3, a distinct characteristic is that in general the time-varying parameter models produce forecasts closer to the inflation process, especially prior to 1999. The responses of the two time-varying parameter models to changes in inflation are much faster than the survey data, indicated by the smaller gaps between the two time-varying parameter models' series and the inflation series. The time-varying parameters models have better ability to capture the dynamics of the inflation process because the Kalman filter continuously updates the parameters based on current forecast error. The classical ARMA models will take longer time to reflect the change because they put equal weight on each forecast error even it may occur at the very beginning of the sample. The fixed parameter restriction will lose information about current inflation movements.

Table 8

Pairwise Tests of MSPE (Post 1995)

Predication Sample Period 1995Q1-2004Q4 H_0: $MSPE_{model\ 1}$-$MSPE_{model\ 2}$=0 H_1: $MSPE_{model\ 1}$-$MSPE_{model\ 2}$>0					
Model 1 / Model 2	TVPwS	RW	Survey	AR(2)	ARMA(1,1)
TVPwS	0.67 (0.251)	1.90* (0.032)	2.15* (0.017)	4.97* (0.000)	2.69* (0.004)
TVPw/oS		1.87* (0.035)	1.30** (0.097)	4.91* (0.000)	2.66* (0.004)
RW			0.07 (0.472)	0.08 (0.468)	3.89* (0.000)
Survey				0.18 (0.429)	1.06 (0.149)
AR(2)					1.46** (0.075)

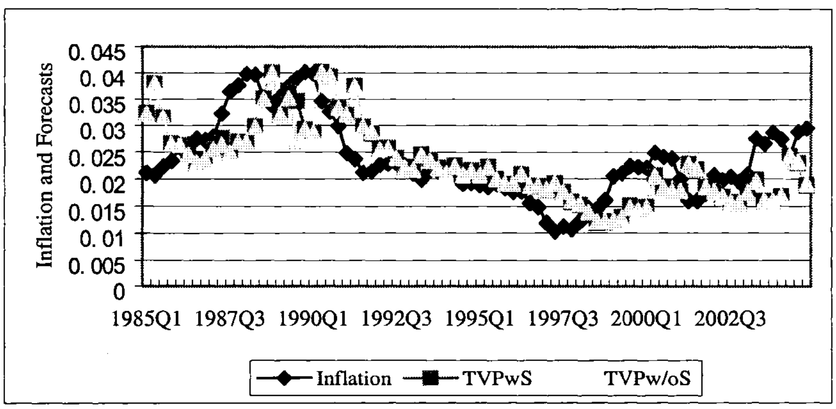
Note: The tests are testing the null hypothesis that the squared prediction errors from two models have equal mean value. "TVwS" and "TVw/oS" stand for the time-varying model with and without survey information, respectively. "RW" means random walk. Each entry contains *t*-statistic and *p*-value (in parentheses). "*" and "**" indicate statistical significance at the 5% level and the 10% level, respectively.

One can observe in Figure 3 that the two time varying parameter models (with and without inflation survey data) are generating almost the same forecast series. The ineffectiveness of survey data in time-varying model shows that the predictive power of the model comes from in the time-varying feature of the model.

Figure 3

Forecasts and Inflation (Post 1985)

(a)



(b)

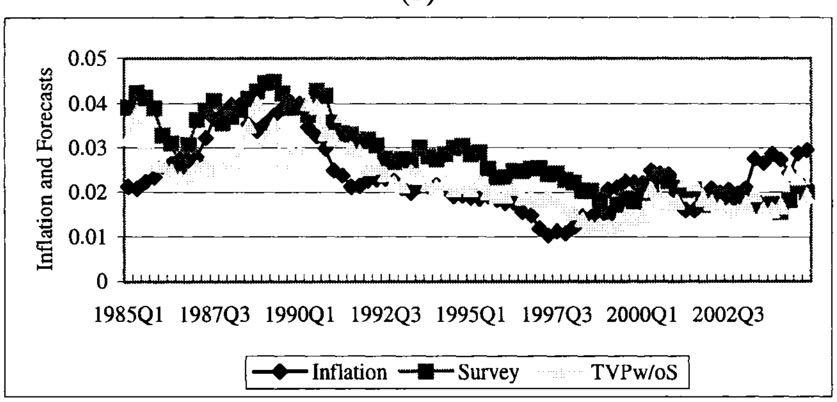
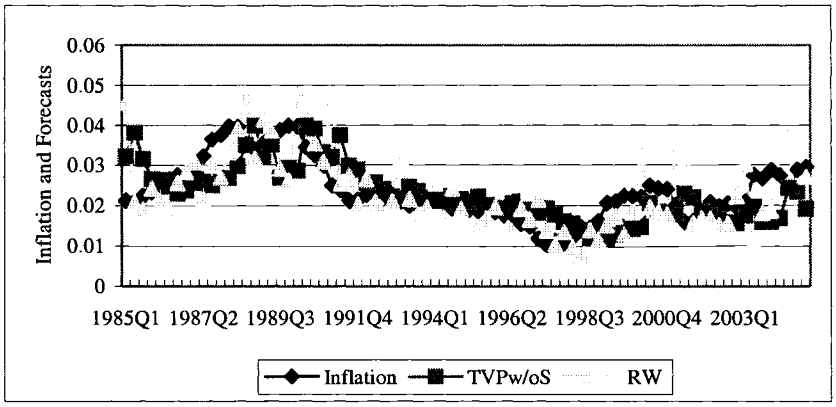
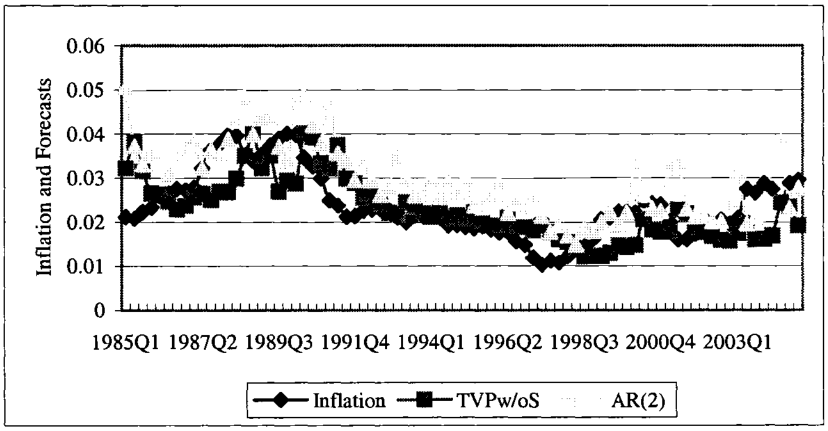


Figure 3—Continued

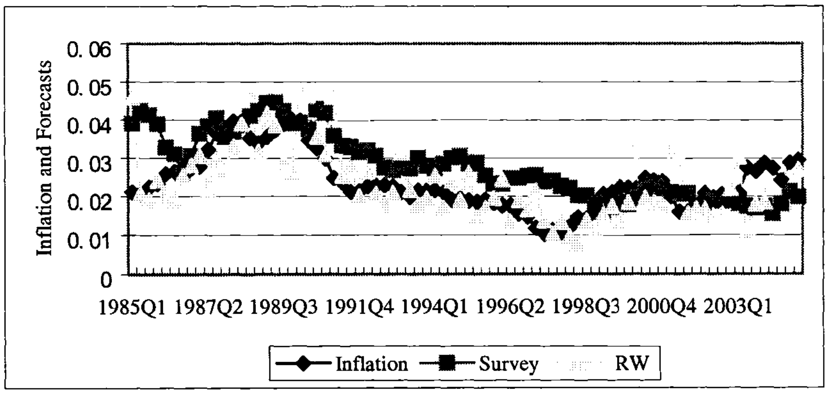
(c)



(d)



(e)



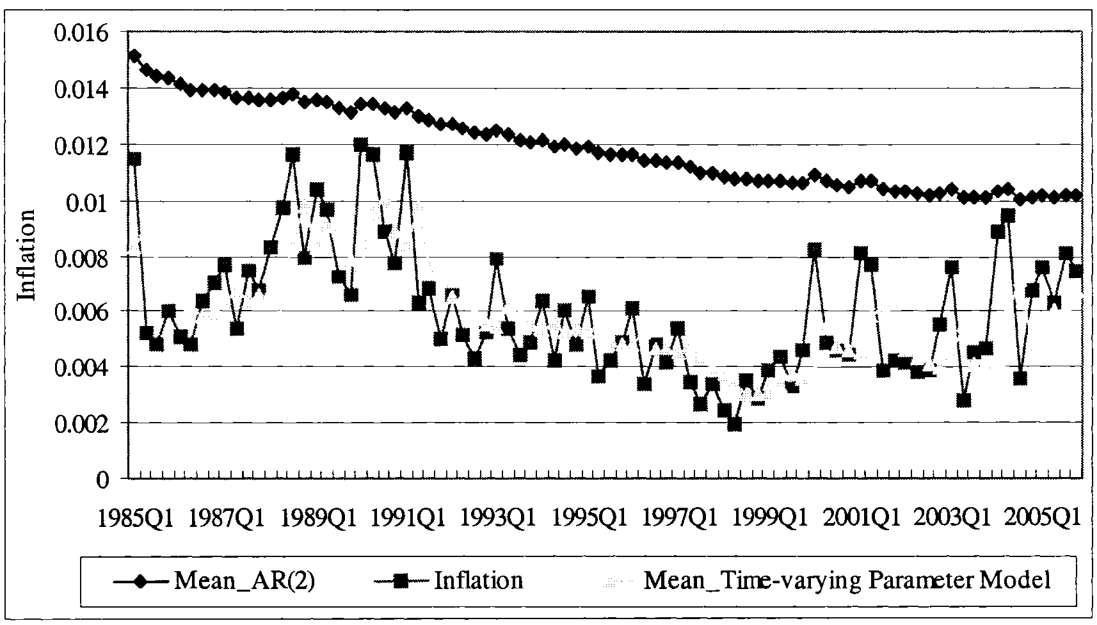
Note: The graphs plot the annualized inflation series and the forecast series from the survey approach, the AR(2) model, the random walk model (RW), the time-varying parameter model without survey (TVPw/oS), and the time-varying parameter model with survey (TVPwS). The forecast period is from 1985Q1 to 2004Q4.

Finally, we also observe in Figure 3 that the AR(2) model tends to over-predict the inflation rates, especially for the early 1980s. This is because the ARMA models put equal weight on each forecast error during the entire sample period, while ignoring the current changes in the inflation series.

To more clearly see the impacts of restricting the coefficients, we plot the inflation series and the implied mean series (unconditional mean) from the AR(2) model and the time-varying parameter model without survey information in Figure 4.

Figure 4

Inflation and Predicted Mean



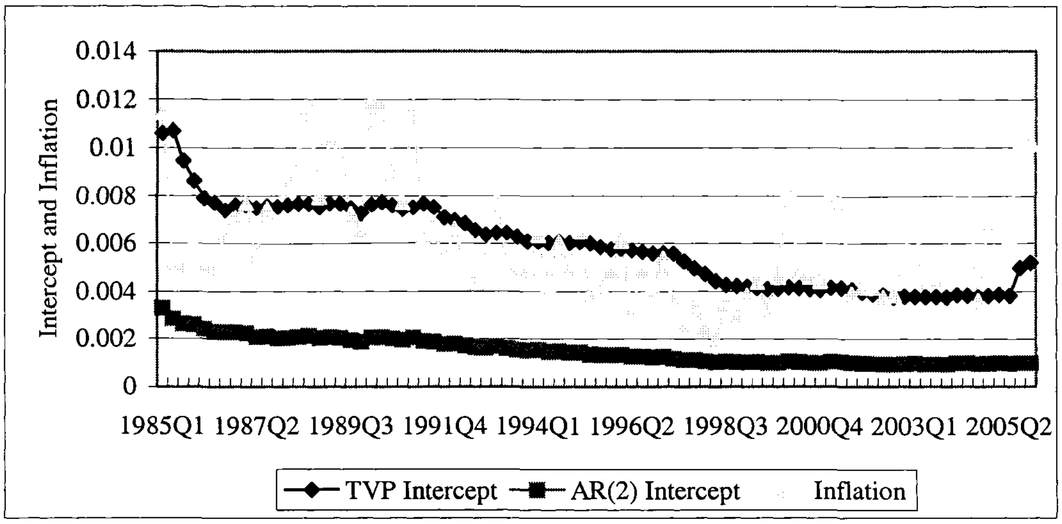
Note: The graph plots the quarterly inflation series and the predicted mean series from the AR(2) model and the time-varying parameter model without survey information from 1985Q1 to 2005Q4.

From Figure 4, we find that the AR(2) model has larger predicted mean values for the whole sample period, although the series are decreasing along with the inflation series. This reflects that the AR(2) model exaggerates the impacts of the high inflation rates during the 1970s and ignores valuable information about current changes in inflation. In contrast, the time-varying parameter model quickly captures changes in inflation by changing its coefficients. The time-varying parameter model produces predicted means very close to the inflation series. This is due to the fact that the model puts higher weights on the most recent prediction errors. Therefore, we argue that the time-varying parameter model has superior ability in forecasting inflation, especially for short run inflation.

To view the changes in the parameters of the inflation process, we also plot the intercept series in Figure 5 and the series of the sum of the AR(1) and AR(2) coefficients in Figure 6.

Figure 5

Inflation and Intercepts

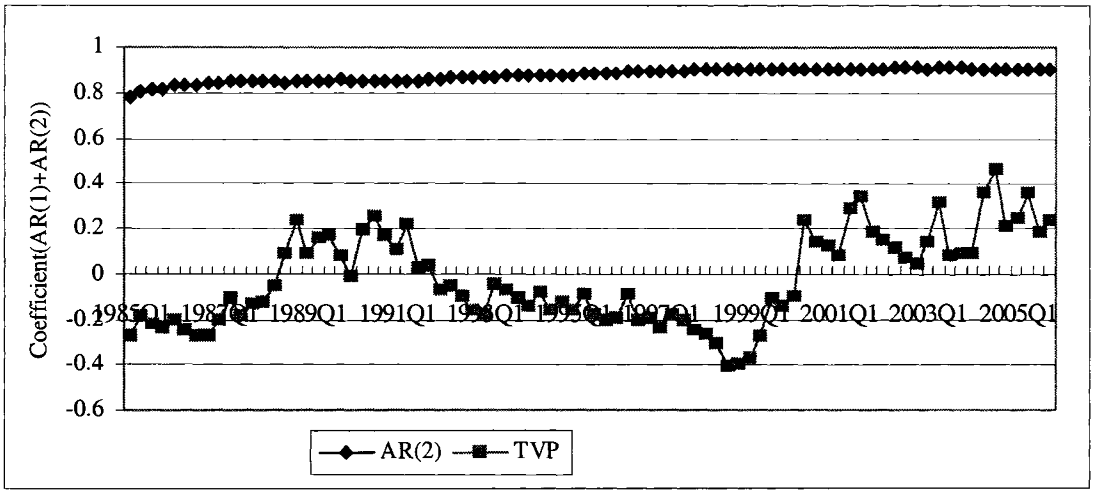


Note: The graph plots the quarterly inflation series and the intercept series from the AR(2) model, and the time-varying parameter model (TVP) without survey information from 1985Q1 to 2005Q4.

Figure 5 shows that the time-varying parameter model has a decreasing intercept series that captures the trend of inflation, while the AR(2) model produces an intercept series that is much lower than the inflation series, with values slightly decreasing.

Figure 6

Coefficients of the Time-varying Parameter Model and the AR(2) Model



Note: The graph plots the coefficient series (AR(1)+AR(2)) from the AR(2) model and the time-varying parameter model (TVP) without survey information from 1985Q1 to 2005Q4.

In Figure 6, we find that for the time-varying parameter model, while its intercept captures the trend of the inflation series, its coefficient sum series captures the dynamics of inflation. One can observe that the coefficient sum series from the time-varying parameter model varies in the pattern shown in the inflation series. The series do not show the sign that inflation is non-stationary. Consider the coefficient sum series of the AR(2) model. The series is increasing, indicating the inflation is converging to a non-stationary pattern.

3.6 Conclusion

We have proposed a time-varying parameter model with survey information to forecast inflation in the United States. The distinctive characteristics of our model include allowing the model parameters to be time-varying and combining survey information. We conducted out-of-sample prediction of the annual inflation rates over the periods from 1995Q1 to 2004Q4 and from 1985Q1 to 2004Q4 for the following models: the time-varying parameter models with and without survey information, the survey data, the ARMA(1,1) model, the AR(2) model, and the Random Walk model. We evaluated the forecasting performances of the models by comparing MPE, RMSPEs, and *t*-tests for equal MSPE.

The results indicate that both time-varying models significantly reduce prediction errors. Comparing the predicative ability of the time-varying parameter models with and without survey information, we find that the survey data do not help in improving the predicative accuracy. This suggests that the previous literature that treats model parameter as constant loses valuable information that would be helpful for capturing the inflation dynamics.

CHAPTER IV

CONVERGENCE REVISITED: EVIDENCE FROM A BOOTSTRAP PANEL UNIT ROOT TEST UNDER SPATIAL DEPENDENCE[22]

4.1 Introduction

Stemming from the neoclassical Solow growth model (Solow, 1956) of long-run growth, the convergence hypothesis has been extensively studied. The issue is of interest because it is pertained to the influence of policy on economic growth.

From a theoretical point of view, standard neoclassical models assume that economic growth is an inexorable, exogenous process and the gaps of output per capita among countries, due to differences of initial capital, would vanish sooner or later. The convergence process thus is characterized as absolute convergence, which means that groups of countries would share the same steady-state characteristics, and therefore converge to the same long-run growth path.

Nevertheless, recent endogenous growth theories (i.e. Romer, 1986, 1990; Rebelo, 1991; and Lucas, 1988) believe that economic growth is endogenous and that institutional factors could have profound impacts. That is, two countries may have identical long-run growth rates but different output levels, which is referred to as conditional convergence.

This essay proposes an empirical analysis to test the convergence hypothesis among 24 OECD countries over 1953-2000. We revise the panel unit root test proposed by Evans and Karras (1996) to empirically test the convergence hypothesis.

[22] This chapter is a modification of Liu and Ruiz (2006). The use of the paper is under the permission of Ruiz and the publisher.

In particular, the analysis specifically considers cross-sectional correlation among the countries, which, according to Baltagi, et al. (2007), makes conventional panel unit root tests lose statistical power and suffer from size distortion.

This study contributes to the literature by applying the spatial error model to account for cross-sectional dependence in the panel unit root test. Although previous empirical studies in spatial econometrics show that economic growth engines are affected by spatial factors, studies that use panel unit root tests to test the convergence hypothesis under the spatial dependence context still remain scarce. Motivated by the four-step procedure proposed by Evans and Karras (1996) and further refined by Gaulier, Hurlin and Jean-Pierre (1999), this study revises their procedure by explicitly taking into account not only heterogeneity in the sample, but also cross-sectional dependence that is generally ignored in related studies. A fixed-effect panel is estimated and the effect of spatial dependence is captured by the spatial dependent error structure.

Another contribution of the essay is that we apply bootstrap procedures to find critical values. Although most panel unit roots tests have the advantage over the univariate unit root tests that the statistics are asymptotically normal, it is a major concern in the literature that the finite sample properties of the test statistics should be examined due to the limited sample size. We consider two resampling schemes and show that tests based on asymptotic distribution can produce misleading results.

The empirical results suggest that there is no output convergence among OECD countries. We compare our results to the convergent results obtained by the panel unit root test suggested by Evans and Karras (1996). The finding confirms the conclusion by Baltagi, et al. (2007) that failing to consider spatial dependence makes conventional panel unit root tests over-sized.

The essay proceeds as follows: section 4.2 briefly reviews related literature. A revised panel unit root test is proposed in section 4.3; section 4.4 presents the results; and section 4.5 concludes.

4.2 Literature Review

For more than a decade, a large number of theoretical and empirical studies have investigated the origins of economic growth and convergence. The convergence hypothesis is implied by the neoclassical theory. However, endogenous growth theories reject the absolute convergence hypothesis. The well-known neoclassical Solow model (Solow, 1956) treats technology as an exogenous source of long-run economic growth. Accordingly, countries with different initial capital converge to the same long run steady state, characterized as absolute convergence.

In contrast, endogenous growth theories believe that the sources of long-run economic growth such as knowledge spillovers (Romer, 1986), incomplete market competition of R&D products (Romer, 1990), non-diminishing marginal return of the "core capital" (Rebelo, 1991), or human capital accumulation (Lucas, 1988), are endogenous. These models suggest that policy can adjust the engine of economic growth and hence convergence is not likely to happen or it is conditional. The discrepancies among theories spark a large number of empirical studies focusing on the convergence hypothesis.

Several empirical techniques to test for convergence have been developed based on the neoclassical growth model and mixed evidence is found. The first approach is to use cross-sectional data to test β-convergence, which means poor countries grow faster than rich ones and σ-convergence, which is characterized by the diminishing income variance between poor and rich countries. The typical literature of this sort includes Baumol (1986) and Barro and Sala-I-Martin (1995), both of which support the convergence hypothesis.

The literature of another approach, first introduced by Bernard and Durlurf (1995), considers a stochastic convergence process (from a time-series point of view) and proposes that convergence is true if the series of output difference between two countries is stationary. Therefore, the convergence test can be interpreted as testing unit root in the series. In addition, a zero long-run mean value of the series indicates absolute convergence while a nonzero mean value implies conditional convergence.

Due to advances in the econometric techniques, some panel data approaches have been adopted to address the convergence hypothesis. Among those methods, the extensions of variant panel unit root tests have been proposed as more powerful alternative approaches than those based on individual time series (unit roots tests). These panel unit root tests, such as Levin, Lin and Chu (2002), Im, Pesaran and Shin (2003), and so on, increase the power of the test for convergence by the square root of the number of the cross-sectional units. Evans and Karras (1996) propose a four-step procedure to test convergence based on the time series interpretation of convergence and they find strong evidence on conditional convergence for U.S. states and a sample of 54 countries.

One major criticism of the conventional unit root tests is their insufficiency of modeling cross-sectional dependence and heterogeneity among data. If the i.i.d. (independent and identically distributed) and homogeneity assumptions are violated, these techniques suffer from significant size distortions that do not disappear with simple demeaning. A simulation studied by Baltagi, et al. (2007) has shown that the panel unit root tests that do not consider cross-sectional dependence are over-sized up to 20% by the presence of cross-sectional correlation.

Some studies have attempted to consider heterogeneity in the data sample. Barro (1991) and Armstrong (1995) include continent dummies into their model to control for differences among country groups. Gaulier, et al. (1999) consider the heterogeneous nature among countries and extend the Evans and Karras's method by using a fixed-effect dynamic panel model. Heterogeneity among countries is captured by the country-specific parameters. They find absolute convergence among European countries and conditional convergence among OECD countries, as well as no convergence among world countries.

Nevertheless, most studies do not consider possible cross-sectional dependence that may generate misleading conclusion. In this case, the error term may not be i.i.d. Pesaran (2004) tests cross-sectional correlation among countries, using the Penn World table data, 1971-2000. He finds strong cross-sectional dependence among countries within the same continent and weaker but significant dependence across continents (except Middle East and African countries). Given cross-sectional correlation and interactions, which in particular are common among countries, conventional panel unit root tests are misspecified.

Recent econometric advances have proposed two new approaches to release the restrictive i.i.d. error assumption. One of them, proposed by Pesaran (2007), assumes an unobservable common factor in the error term to capture cross-sectional correlation. He suggests that an international average can be used as a proxy to capture the unobservable common factor. Alternatively, Baltagi et al.(2007) suggest that one can use spatial econometric models to address the problem and this is adopted by the present paper.

In spatial econometric empirical papers, it has been found that the engine of economic growth is affected by the spatial factors. Coe and Helpman (1995) and Keller (2002) find that, for OECD countries, the R&D spillover effect is huge and depends on the geographic distance. Lall and Yilmaz (2001) find that human capital levels are spatially interdependent. These results are crucial because they indicate that the generally accepted i.i.d. error structure assumption is actually invalid and the conventional regression models are possibly misspecified. It will be more appropriate that one takes the spatial effect into account in testing the convergence hypothesis.

4.3 Methodology

Evans and Karras (1996) propose a general four-step procedure to test the convergence hypothesis primarily based on the time series representation of the convergence process. However, when contemporaneous cross-sectional correlation appears the procedure is problematic from both economic and econometric standpoints.

Therefore, we introduce the spatial autoregressive error process into the Evans and Karras's model to capture the effect of spatial dependence.[23] Although spatial models are not new in the convergence literature, especially in regional growth studies, they are only applied to the cross-sectional representation of the convergence process and limited in regional data. The present essay applies a panel unit root test with spatially correlated errors to the problem of output convergence in country level data. This section describes the methodological issues: subsection 4.3.1 briefly introduces the definition of stochastic convergence under the panel context; the four-step procedure proposed by Evans and Karras (1996) is reviewed in subsection 4.3.2; subsection 4.3.3 discusses the spatial model used in the study and its economic implications; the revised four-step procedure is summarized in subsection 4.3.4; a preliminary pre-test of cross-sectional dependence and data issue are given in subsections 4.3.5 and 4.3.6, respectively.

4.3.1 The Definition of Stochastic Convergence

The concept of stochastic convergence is introduced by Bernard and Durlurf (1995). If the series of the output differences between two countries are stationary, we say that the two countries are stochastically convergent. If the series converges to zero, convergence is absolute; otherwise we say it is conditional.

Mapping the time series interpretation of the convergence process into the panel context, Evans and Karras (1996) develop a four-step panel unit root test procedure. Gaulier, et al. (1999), by introducing fixed-effect parameters, extend this approach to consider the heterogeneous nature among countries.

[23] In this paper, we do not consider autocorrelation among error terms in time dimension because it can be eliminated by including sufficient lag length.

Convergence, according to Evans and Karras (1996), is defined as the deviation of output per capita of a sample country from the international average approaches a constant value as time goes to infinity. Hence, under the panel context, N economies converge if and only if expected GDP per capita cross economies differences are stationary:

$$\lim_{p \to \infty} E_t (y_{it+p} - \bar{y}_{t+p}) = \mu_i ,$$

where y_{it} is log GDP per capita for country i at time t, $\bar{y}_t$ is the international average at time t, p is the lag length, i=1, 2, ..., N. Convergence to be absolute or conditional depends on whether $\mu_i = 0$ for all i or $\mu_i \neq 0$ for some i.

The data generating process for N countries can be assumed as:

$$(8) \qquad \Delta(y_{it} - \bar{y}_t) = \alpha_i + \rho_i (y_{it-1} - \bar{y}_{t-1}) + \sum_{j=1}^{p} \gamma_{ij} \Delta(y_{it-j} - \bar{y}_{t-j}) + u_{it} ,$$

where α_i is the time-invariant parameter for individual country i, p is the lag length, ρ_i and γ_i are parameters to be estimated, and u_{it} is the i.i.d. error term of country i at time t. Equation (8) essentially is a typical function form of the Augmented Dickey-Fuller test under the panel context.

The existence of unit root can now be characterized by testing the null that all parameters ρ_i's are equal to zero. The N economies are said to be convergent if the parameters, ρ_i's, are less than zero. The type of the convergence process, if any, is characterized by testing the individual-specific parameters, α_i's, which are pertaining to the unconditional mean of the series, are equal to zero.

4.3.2 Conventional Four-step Procedure (Evans and Karras, 1996)

The following four-step procedure of the panel unit root test for convergence is proposed by Evans and Karras (1996):

(1) Apply ordinary least squares (OLS) to equation (8) to obtain, σ_i , the estimate of standard deviation. To control for heterogeneity across individuals, the series is normalized by dividing it with the estimated standard error.

(2) Using ordinary least squares, obtain the parameter estimate $\hat{\rho}$ and its t-ratio, $t(\rho)$, by estimating

$$\Delta(y_{it} - \bar{y}_t)/\sigma_i = \alpha_i + \rho(y_{it-1} - \bar{y}_{t-1})/\sigma_i + \sum_{j=1}^{p} \gamma_{ij}\Delta(y_{it-j} - \bar{y}_{t-j})/\sigma_i + u_{ij}/\sigma_i$$

as a panel.

(3) If the t-ratio exceeds an appropriately chosen critical value, reject H_0: ρ_i = 0 in favor of H_a: ρ_i <0. If not, H_0 may hold.

Evans and Karras (1996, p.260-263) show that the test H_0: ρ_i = 0 against H_a: ρ_i<0 in (8), which indicates that there is a unit root in the series, is equivalent to testing ρ =0 against not all of them are zeros.

Although most panel unit root test statistics have asymptotic normal distribution, it is believed that their finite sample properties may be different. As in the case of Evans and Karras, asymptotic normality of the test statistic relies on the requirement that both N and T go to infinity. In fact, Evans and Karras use Monte Carlo simulation to approximate the distribution of the statistics.

(4) If H_0 can be rejected, calculate the F-ratio:

$$\Phi(\hat{\alpha}_i) = 1/(N-1) \times \sum_{i=1}^{N} t(\hat{\alpha}_i)^2 ,$$

where $t(\hat{\alpha}_i)$ is the t-ratio of α_i , obtained by applying OLS to equation (8) for economy i.

Evans and Karras (1996, p.263) derive the F-ratio based on the standard joint hypothesis test approach in OLS estimation. If the F-ratio exceeds an appropriately chosen critical value, infer that convergence is conditional. If not, convergence may be absolute.

4.3.3 Spatial Effect

For both econometric and economic theory, it is too restrictive to assume no cross-sectional dependence in testing convergence hypothesis. Economically, the restriction implies that the economies are closed, which is obviously inappropriate to understand the convergence process, considering the continuing process of globalization around the world in recent decades. Today's global economy has shown a strong trend of globalization and barriers to international trade and factor mobility have been significantly reduced. Indeed, the integration of financial markets and the tighter connections among countries produce higher rates of factor exchanging, larger trade volume, and faster knowledge diffusion. Econometrically, the presence of cross-sectional dependence leads to the non-spherical error structure that results invalid statistical inferences.

Under the open economy context, the economies may tend to be convergent. First, the convergence process would be accelerated when human capital and physical capital move among countries in response to differentials in remuneration rates. Second, under the open economy context, another possibility for poor countries to converge towards the richer ones is through technology diffusion or knowledge spillover.

It is also possible that, under the open economy context, countries are divergent. For example, a group of countries that benefit each other at the local level may produce divergence compared to countries outside the group, despite convergence among the countries within the group. Two countries may compete for customers, nature resources, or human resources in global markets. In this case the two countries may diverge due to the competition.

According to Baltagi, et al. (2007), the power and size of panel unit root tests without considering cross-sectional dependence will be greatly impaired. As a matter of fact, previous studies have suggested that the OECD country group is cross-sectional dependent when conducting simple OLS estimation (Pesaran, 2004, for example). In order to establish more robust results, it is necessary to consider cross-sectional dependence.

Modeling cross-sectional dependence by taking advantage of the spatial econometric technique is appealing because the dependence among countries is related to location and distance.[24] In this essay, cross-sectional dependence can be called spatial effect. First, to what degrees the factors emphasized by neoclassical and endogenous theories, including technology, factor mobility, culture, and institutions, are correlated are associated with geographical locations. Second, barriers to trade, to technology spillover, to capital mobility, and to migration are generally set up in country level and their free mobility is confined within a country. Third, distance also affects the degree of interactions among countries. Fourth, the cross section unit in

[24] Pesanran (2007) proposes to use a random common factor to capture cross-sectional dependence. We prefer the spatial error model because the dependence channels among countries implied by theory are strongly spatially related. In addition, some papers that use this method assume that the loading of the common factor is homogeneous across countries, which would be eliminated by demeaning the series.

the data is generally defined in terms of location.

In order to capture the spatial effect, following suggestions by Baltagi, et al. (2007), we assume that the error structure to be spatially autoregressive:

(9) $\boldsymbol{u}_t = \lambda \mathbf{W}_n \boldsymbol{u}_t + \boldsymbol{\varepsilon}_t$,

where $\boldsymbol{u}_t$ is the $N\times1$ error vector in period t=1,...,T, λ is the spatial autoregressive parameter, ε_t is an $N\times1$ i.i.d. error with mean zero and variance σ^2. Equation (9), called spatial autoregressive error (SAE), is widely used in spatial studies.[25]

The spatial error specification is to capture contemporaneous cross-sectional dependence across countries. This specification implies that the innovations of all countries containing unmodeled factors that influence current output are interrelated according to the orders of contiguity.

Note that this error structure makes some traditional panel unit root tests, which are based on the combination of individual unit root tests,[26] invalid, because estimating single series fails to account the spatial effect and will produce incorrect standard errors. The model should be jointly estimated to avoid incorrect standard error.

The $N\times N$ $\boldsymbol{W}$ matrix, referred to as the spatial weight matrix in the literature, has to be pre-specified due to identification problem. Choice of the $\boldsymbol{W}$ matrix is always subject to disputation. The pre-specified weight matrix should be exogenous and invariant over time, which precludes using variable such as trade or FDI

[25] Baltagi, et al. (2007) investigate three different types of spatial error specification. In practice, there is no general rule to determine which one is the best. We choose the spatial autoregressive specification since it is the most widely adopted approach in spatial empirical work. More importantly, it implies the global effect, which means one country's change can affect all other countries with various degrees determined by the weight matrix (see Anselin, 2003).

[26] For example, the I.P.S. test.

(Foreign Direct Investment) as spatial weights. The most popular way is to assign the weight value based on simple contiguity check. Specifically, we specify the weight matrix in the following way:

$w_{ij} = 1$, if the two countries are located within the same continent;

$w_{ij} = 0$, otherwise.

As a common technique in spatial econometrics, the final weight matrix, to ensure that the result is spatial stationary, has been row-standardized in estimation.

In specifying the weight matrix, continent location of countries is served as the selection criterion. The idea is that countries within one continent are similar to each other in terms of variables such as social status, culture, technology, and climate. A commonly observable phenomenon is that countries located within the same continent have a strong trend of integration: European countries have formed European Union; North American countries have the market integration agreement (NAFTA); Australia and New Zealand are well known to be closely related; similar agreement can also be found in the Asian countries. Empirical evidence has also indicated the existence of growth clubs or clusters and those clubs and clusters are generally confined within a continent. (Durlauf and Johnson 1995; Quah 1997) Pesaran (2004) has found weak, though significant, correlations across continents but strong cross-sectional dependence among countries located within the same continent.

Given the first order spatial autoregressive error specification, the OLS method is inappropriate. The true full ($NT \times NT$) variance-covariance matrix is:

$$\boldsymbol{\Omega} = \sigma^2 (\boldsymbol{I}_T \otimes ((\boldsymbol{I}_N - \lambda \boldsymbol{W}_{N\times N})^{-1} (\boldsymbol{I}_N - \lambda \boldsymbol{W}'_{N\times N})^{-1})).$$

We should point out that in our model we consider two possibilities of heterogeneity. First, the presence of α_i's parameters, isolating the effect of omitted variables, indicates the different structural characteristics of the economies. Second, the variance-covariance matrix has not only non-zero off-diagonal elements but also variant diagonal elements (heteroskedasticity). Unless the number of neighbors is constant for each observation, the diagonal elements will not be constant. (see Anselin, 1988, 2003)

4.3.4 A Revised Four-step Procedure

The test procedure follows Evans and Karras (1996) and Gaulier, et al. (1999). However, since they do not consider the non-i.i.d. error structure, we extend their models by incorporating spatial dependence. Due to the presence of the spatial effect, we correspondingly modify the Evans and Karras's four-step procedure from the following perspectives: (1) we abandon the normalization procedure in the first step since the OLS estimates cannot provide correct standard errors; (2) we jointly estimate the fixed-effect dynamic panel by maximum likelihood other than OLS which produces biased estimator; (3) instead of the F-statistic suggested by Evans and Karras, we use the standard F-test in the fixed-effect panel model because the retrieved fixed-effect parameters from the demeaned equation are inconsistent when T is finite. In addition, we are concerned with the finite sample properties of the test statistics. We hence use the bootstrap method to generate related critical values for all tests. The procedure consists of 4 steps.

First, we determine the lag length by estimating equation (8) for each country with OLS. This is essentially the ADF unit root test, given the function form of equation (8). The lag length is chosen based on the SIC criterion. In contrast to Evans and Karras (1996) that collect the estimated standard error of u to normalize the original series, we directly use the true series in the following steps. The OLS residuals cannot be used to correctly estimate the variance-covariance matrix due to the presence of spatial correlation.

Second, we jointly estimate

$$(10) \qquad \Delta(y_{it} - \bar{y}_t) = \alpha_i + \rho(y_{it-1} - \bar{y}_{t-1}) + \sum_{j=1}^{p} \gamma_{ij} \Delta(y_{it-j} - \bar{y}_{t-j}) + u_{ij},$$

$$\boldsymbol{u}_t = [u_{1t} \dots u_{Nt}]', \ \boldsymbol{u}_t = \lambda \boldsymbol{W}_n \boldsymbol{u}_t + \boldsymbol{\varepsilon}_t$$

as a panel.

Here we impose the same lag length to the panel, which is the largest length found in individual series tests. Although it seems more efficient to estimate with different lag lengths, differences in lag length naturally generate an unbalanced panel structure of the model, which makes the estimation step greatly complicated.[27] Allowing longer lag length does not pose too much trouble in that our interest lies over the coefficients ρ and α, while using a parsimonious model may suffer from the omitted variable bias.

As is shown in Elhorst (2003), it is possible to estimate equation (10) by maximum likelihood with sufficient long T, based on the variance-covariance matrix. After demean (over time dimension) the series to remove the country-specific fixed effect, the log likelihood function can be derived as

$$-NT \frac{Ln(2\pi\sigma^2)}{2} + T \sum_{i=1}^{N} Ln(1 - \lambda\omega_i) - \frac{1}{2\sigma^2} \sum_{t=1}^{T} \boldsymbol{e}_t' \boldsymbol{e}_t ,$$

[27] Gaulier, et al. (1999) estimate this panel by the LSDV (Least Square Dummy Variable) estimator without imposing lag length. Estimating such a dynamic panel with the LSDV method will produce biased estimates.

where $e_t = (I - \lambda W)[(z_t - \bar{z}) - (X_t - \bar{X})\theta]$, $z_t = \Delta(y_{.,t} - \bar{y}_t)$, X_t is the vector of all right hand side explanatory variables, and θ is a coefficient vector. $\bar{X}$ and $\bar{z}$ are the averages of X and z over time for each country. ω's are the characteristic roots of the standardized weight matrix.

Third, we conduct t-test of ρ to determine whether the process is convergent or not. One can draw the conclusion that convergence happens based on the rejection of the null hypothesis that all ρ's are zero without identifying if the convergence is absolute or conditional.

Our limited panel sample size requires us to take small sample performance of the test into account. In the present study, we use the bootstrap method to find critical values. Specifically, we consider two ways to resample estimated residuals: the i.i.d. resampling scheme and the block resampling scheme. In the first way, we resample the residuals from both time and cross section dimensions, under the assumption that the residuals are i.i.d. In the second way, following Maddala and Wu (1999), we resample the residuals in the time dimension with the cross section index fixed, in order to preserve the contemporaneous cross-sectional dependence structure. We generate bootstrap sample under the null hypothesis and calculate the statistic to obtain its distribution. The bootstrap procedures are discussed in Appendix B.

The two different resampling schemes are considered in order to find out the robustness of our results. We should notice that the block resampling scheme allows for cross-sectional dependence, but it does not mean that it is superior to the i.i.d. resampling scheme. On one hand, if the residuals are i.i.d., it has been proved that the block resampling scheme provide less accurate approximations to the true distribution, compared to the i.i.d. resampling scheme.(see Lahiri, 2003) On the other hand, under the cross-sectional context, the block resample scheme is expected to be better than the first scheme. However, how well the second scheme can approximate the true distribution is an unanswered question.

In addition, the block resampling scheme can further model the underlying cross-sectional dependence, in the case that our spatial error model is not sufficient to capture all cross-sectional correlation among the residuals. In section 4.4, we show that our empirical results are robust to the two resampling schemes.

Fourth, using a F-test to test the restrictions that the fixed effects, α_i's, jointly are equal to zero. If they are zero, then all country's output per capita will converge to an international long run level, which supports the absolute convergence hypothesis. If not, conditional convergence is expected.

Evans and Karra (1996) derive the F-statistic based on the t-value of the fixed-effect parameters. The fixed-effect parameters can be retrieved from the demeaned equation, but these estimates are inconsistent and estimated standard errors are incorrect due to the dynamic panel structure with spatial errors. Hence associated t-values are no longer correct.

Instead of using the F-statistic suggested by Evans and Karras, we use the standard F-statistic in the fixed-effect panel model. The test statistics is derived based on comparison between the restricted model and the unrestricted model. Under the null, the model is restricted with all unit-specific parameters being zero. We use the following F-ratio to test the null hypothesis that all fixed-effect parameters are equal to zero:

$$F(N-1, NT-N-K)=\frac{(RRSS-URSS)/(N-1)}{URSS/(NT-K-N)},$$

where K is the number of regressors, RRSS is the restricted residual sum of squared computed from the spatial error model without fixed effect, URSS is the unrestricted residual sum of squared calculated from the fixed-effect model with spatial error (equation (10)). Under the conditional convergence hypothesis, all fixed-effect parameters will be different from zero. Therefore, rejection of the null hypothesis can be viewed as evidence on conditional convergence.

4.3.5 Preliminary Test

To statistically confirm the existence of cross-sectional dependence in the sample, we conduct a LM test proposed by Breusch and Pagan (1980). The test statistic is

$$LM = \sqrt{\frac{1}{N(N-1)}}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(T\hat{\delta}_{ij}^{2}-1),$$

where $\hat{\delta}_{ij}$ is the sample estimate of correlation coefficient of residuals:

$$\hat{\delta}_{ij}=\frac{\sum_{t=1}^{T}e_{it}e_{jt}}{\sqrt{\sum_{t=1}^{T}e_{it}^{2}\sum_{t=1}^{T}e_{jt}^{2}}}$$

and e_{it} is the OLS estimate of the error term u_{it}, computed from the four-step approach suggested by Evans and Karras.

Under the null hypothesis that there is no cross-sectional dependence, the test statistic is asymptotically standard normal. The value of the LM test equals 18.97, which is strongly significant, suggesting that OLS errors are cross-sectionally correlated. This preliminary test confirms that we should take spatial effect into account.

4.3.6 The Data

The data used in this study are drawn from the Penn World Table (PWT), constructed by Heston, Summers, and Aten. We use annual time series data from 1953-2000 of per capita GDP - adjusted PPP with 1996 constant price - to conduce the empirical research. We take logarithm before conducting the empirical study. The sample includes the following countries: Australia, Austria, Belgium, Canada, Switzerland, Denmark, Spain, Finland, France, United Kingdom, Greece, Ireland, Iceland, Italy, Japan, Korea, Republic of Mexico, Netherlands, Norway, New Zealand, Portugal, Sweden, Turkey, and the United States.

4.4 Empirical Results

We conduct the ADF test for the individual series and the SIC's show that each series has lag length no more than two. Due to the initial condition for estimation, the sample period for regression is adjusted to 1956-2000.

Table 9 reports the results in the fixed-effect model with spatial error. According to the bootstrap critical values of the spatial autocorrelation parameter λ, we find that it is significant at the 1% level. This again confirms that there is notable spatial effect among OECD countries.

The spatial autoregressive parameter λ has a negative value. The interpretation of the negative value is not straightforward. It does not necessarily mean that all unmodeled factors in the error process are negatively cross-sectionally correlated. To see this, note that since $|\lambda|<1$, the inverse of the variance-variance matrix can be represented in the Leontief expansion:

$$(\boldsymbol{I}_N - \lambda \boldsymbol{W}_{N\times N})^{-1} = I + \lambda W + \lambda W^2 + \ldots$$

Obviously, every country's errors will be affected. The complete variance-covariance matrix will be a complex matrix.[28] The negative value will generate a complicated checkerboard pattern of correlation: some off-diagonal elements will be negative while others may be positive. As we mentioned above, if countries benefit each other with knowledge spillover, then they may be positively correlated. On the other hand, if countries are competing for resources, costumers, or foreign investment, they may be negatively correlated.

Table 9

Estimates of the Fixed-effect Panel Model with Spatial Dependence

Parameter	Estimate	Standard Error	*t*-Statistics	Critical Values		
				1%	5%	10%
ρ	-0.022	0.005	-4.083	-5.56	-4.85	-4.50
				-6.66	-5.65	-5.11
λ	-0.068*	0.040	-1.710	-1.82	-0.74	-0.12
				-1.62	-0.53	0.60
σ: 0.025						

Note: "*" means statistical significance at the 5% level. For each statistic the table shows two sets of critical values: entries in the upper cells are obtained by not keeping the cross-section structure (under the i.i.d. resampling scheme). Entries in the lower cells are obtained by keeping the cross-section structure (under the block resampling scheme). The critical values are obtained by 3000 bootstrap iterations.

Because of the notable spatial process underlying the true DGP, inferences

[28] For detailed discussions about the structure of the variance-covariance matrix, see Anselin (2003).

from previous studies might not be reliable. This is indeed the case for the present study. In Table 9 the t-statistic value associated with ρ is not significant, according to the bootstrap critical values. This conclusion is robust to the two different resampling schemes. Therefore the process characterized by equation (8) contains one unit root. Failure to reject the null hypothesis enables us to conclude that there is no convergence for our sample OECD countries.

One should note that the results show that it is important to consider the small sample properties of the test statistic. If we conduct the test based on the asymptotic distribution of the test statistic, misleading conclusions will be drawn. We will incorrectly reject the unit root null hypothesis under the asymptotic test.

This no convergence conclusion is in sharp contrast to the studies by Evans and Karras (1996) and Gaulier, et al. (1999) in which conditional convergence is found in their data sample. For comparison purpose, we also re-estimate the model using the Evan and Karras's four step approach. We report the results in Table 10.

In this case, the results are the opposite. We find that the t-ratio is significant, indicating that the null hypothesis is rejected. Comparing the two different conclusions, we find the problem indicated by Baltagi, et al. (2007) that the unit root test ignoring the underlying spatial correlation process tends to incorrectly reject the null hypothesis when the true DGP contains a unit root.

It should be noted that, in the fixed-effect model without spatial error, even we consider cross-sectional dependence in a very general form by using the block resampling scheme to keep the cross-sectional residual structure, the conclusion do not change. This fact is in line with the finding in Baltagi et al. (2007). Baltagi et al.

(2007) point out that, when the sample data are correlated in a spatial form, the panel unit root tests which take cross-sectional dependence into account are still over-sized, although their performance is better than the tests that assume independence in the sample. In Table 10, the critical values at all levels decrease when we resample residuals in the way that keeps the cross-sectional dependence structure, but we still reject the null hypothesis of no convergence.

Table 10

Estimates of the Fixed-effect Panel Model without Spatial Dependence

Parameter	Value	Standard Error	t-Statistics	Critical Values		
				1%	5%	10%
ρ	-0.036*	0.0067	-6.065	-4.02	-3.51	-3.15
				-4.50	-3.88	-3.50
σ: 1.012						

Note: "*" means significance at the 1% level. The model is estimated using the LSDV method. For each statistic the table shows two sets of critical values: entries in the upper cells are obtained by not keeping the cross-section structure (under the i.i.d. resampling scheme). Entries in the lower cells are obtained by keeping the cross-section structure (under the block resampling scheme). The critical values are obtained by 3000 bootstrap iterations.

The divergence may be due to the great structural disparity, including very different culture, institutional setup, as well as structural breaks. Indeed, the sample countries experienced very different development history and paths. For example, the United States maintained a very strong economic status for the whole sample period, due to strong domestic demand and high productivity. Japan and Korea relied on export for economic growth and they started their rapid-growth periods during the 1970s. These differences may contribute to the divergence among countries.

The divergence may also be attributed to the inter-continental divergence. There are subgroups of the sample countries closely related and interacted, for instance, countries in European Union and North America. Homogeneity within those subgroups actually will increase heterogeneity among groups in that the inter-group output differences are increasing. Then the divergence is not a surprise. As an example, consider the knowledge spillover effect among countries. The knowledge spillover effect (and also migration, homogeneity of culture and political setup, and trade, etc.) generally becomes weaker with distance going up. Firms located within the same continent are more likely to benefit each other in a quicker way. As countries located within the same continent become more homogenous in the knowledge level, the inter-continental differences are growing. Therefore, this may contribute to inter-continental divergence. On the other hand, knowledge accumulation in one region finally improves productivity of all firms wherever they are located. Thus, a global geographic spillover effect may contribute to inter-continental convergence. The final effect of knowledge spillover should be the combination of the two effects. Our results suggest that the inter-continental divergence effect may be more crucial.

To this point, it is not necessary to conduct the fourth step because we find that the sample countries are divergent. However the fourth step can tell us that there are indeed great disparities among the sample countries. Therefore, we still estimate the pooled panel model with spatial error and calculate the F-statistic to test if the unit-specific parameters are different. The calculated F-statistic value is 2.43. The bootstrap critical values at 1%, 5%, and 10% are respectively 2.48, 2.10, and 1.93 under the i.i.d. resampling scheme. Under the block resampling scheme, the critical values at 1%, 5%, and 10% are respectively 2.86, 2.23, and 1.92. We reject the null hypothesis that all fixed-effect parameters are zero at the 5% level under the two sets of critical values. Therefore, we do find that there is considerable heterogeneity among sample countries.

4.5 Concluding Remarks

This study tests the convergence hypothesis by taking advantage of a new panel unit root test. The study investigates output convergence of log GDP per capita among 24 OECD countries over 48 years. A fixed-effect panel model is estimated. In particular, considering cross-sectional dependence that might produce invalid statistical inferences in standard panel unit root tests, we revise the four-step procedure proposed by Evans and Karras to accommodate a spatial autoregressive error structure. In addition, we bootstrap related critical values, considering the limited sample size of our data.

The empirical results show that output is diverging among OECD countries. We also compare our results to the results obtained by applying the conventional approach. We find that the conventional approach suggests that output is converging. According to the simulation results by Baltagi, et al. (2007), the conventional approach is inappropriate in studying the convergence hypothesis because the test is over-sized with cross-sectional dependence.

Appendix A

Derivations of the Posterior Distributions of the Parameters in the Markov-switching GARCH-in-Mean Model in Chapter 2

In this appendix, we provide more details about how to sequentially sample the parameters in the MS-AR-GARCH-M model. The model consists of the following equations:

$$\pi_t = \beta_0 + \sum_{i=1}^{p} \beta_i \pi_{t-i} + \rho(S_t)\sqrt{h_t} + \sqrt{h_t}\varepsilon_t,$$

$$h_t = c + \phi h_{t-1} + \lambda h_{t-1}\varepsilon_{t-1}^2,$$

$$\rho(S_t) = (1-S_t)\rho_0 + S_t\rho_1,$$

$$\Pr(S_t = 1 \mid S_{t-1} = 0) = 1 - e_{00}$$

$$\Pr(S_t = 0 \mid S_{t-1} = 1) = 1 - e_{11},$$

$$c > 0, \phi \geq 0, \lambda \geq 0, \phi + \lambda < 1, \text{for } t \text{ from 1 to } T,$$

There are 4 blocks in the parameter vector

$$\boldsymbol{\theta} \equiv [\beta_0, \beta_1, \ldots, \beta_p, \rho_1, \rho_0, e_{00}, e_{11}, c, \phi, \lambda],$$

which includes the autoregressive coefficients, $\boldsymbol{\beta} \equiv [\beta_0\ \beta_1 \ldots \beta_p]'$; the in-mean coefficients, $\boldsymbol{\rho} \equiv [\rho_1(S_t = 1)\ \rho_0(S_t = 0)]'$; the transition probabilities, $\boldsymbol{e} \equiv [e_{00}\ e_{11}]'$, where the two subscripts of each transition probability denote from state i (the first subscript) to state j (the second subscript); and the GARCH parameters, $\boldsymbol{\alpha} \equiv [c\ \phi\ \lambda]'$. In addition, the unobservable state variables, S_t, for t from 1 to T, are treated as parameters to be estimated, stacked as $\boldsymbol{S} \equiv [S_1 \ldots S_T]'$. The approach requires that we sequentially sample from the following (posterior) conditional distributions:

$$f(\boldsymbol{e} \mid \boldsymbol{\theta}, \boldsymbol{S}, \boldsymbol{\pi})\ ;\ f(\boldsymbol{S} \mid \boldsymbol{\theta}, \boldsymbol{\pi})\ ;\ f(\boldsymbol{\beta} \mid \boldsymbol{\theta}_{-\beta}, \boldsymbol{S}, \boldsymbol{\pi})\ ;\ f(\boldsymbol{\rho} \mid \boldsymbol{\theta}_{-\rho}, \boldsymbol{S}, \boldsymbol{\pi})\ ;\ f(\boldsymbol{\alpha} \mid \boldsymbol{\theta}_{-\alpha}, \boldsymbol{S}, \boldsymbol{\pi}),$$

where $\boldsymbol{\pi}$ denotes a column vector containing sample observations of inflation. $\boldsymbol{\theta}_{-i}$

($\boldsymbol{i} = \boldsymbol{\beta}, \boldsymbol{\rho}, \boldsymbol{e}, \boldsymbol{\alpha}.$) denotes the parameter vector excluding the parameter block i.

The prior distributions are:

$\boldsymbol{\beta} \sim MultiN(\boldsymbol{\beta}_0, \boldsymbol{\Sigma}_0)$; $\rho_i \sim N(\rho_{i0}, \sigma^2_{i0})$; $e_{kk} \sim Beta(\gamma_{k0}, \gamma_{k1})$; $\alpha_j \sim U(a_j, b_j)$,

for i=1, 0; k=0,1; j=1, 2, 3. $\boldsymbol{\beta}_0$ and $\boldsymbol{\Sigma}_0$ are the mean vector and variance-covariance matrix of the prior multi-normal distribution of $\boldsymbol{\beta}$. ρ_{i0} and σ^2_{i0} are the hyper-parameters of the prior distribution of ρ_i. γ_{k0} and γ_{k1} are the hyper-parameters of the prior beta distribution of e_{kk}, the probability of transferring from state k to state k. a_j and b_j are the hyper-parameters of the prior uniform distribution of the j-th element in $\boldsymbol{\alpha}$.

Given the prior distributions, the detailed sampling scheme for the model is the following:

1. Sampling the transition probabilities

The posterior distribution of e_{kk} (k=0, 1) is

$f(e_{kk} \mid \boldsymbol{\theta}_{-e_{kk}}, \boldsymbol{S}, \boldsymbol{\pi}) \propto f(\boldsymbol{S} \mid \boldsymbol{\theta}) f(e_{kk})$[29],

where $f(e_{kk})$, is the prior distribution of e_{kk}, and the assumption that $\boldsymbol{S}$ is independent of $\boldsymbol{Y}$ (the column vector of data) is applied.

(A.1) $f(\boldsymbol{S} \mid \boldsymbol{\theta}) f(e_{kk}) = \prod_{t=1}^{T} f(S_{t+1} \mid S_t, \boldsymbol{\theta}) = e_{11}{}^{n_{11}} (1-e_{11})^{n_{10}} e_{00}{}^{n_{00}} (1-e_{00})^{n_{01}}$,

where n_{ij} is the number of transitions from i to j.

Given the prior beta distributions, the posterior distribution functions are

(A.2) $f(e_{kk} \mid \boldsymbol{\theta}_{-e_{kk}}, \boldsymbol{S}, \boldsymbol{\pi}) \propto f(\boldsymbol{S} \mid \boldsymbol{\theta}) f(e_{kk}) \propto e_{kk}{}^{n_{kk}+\gamma_{kk}-1} (1-e_{kk})^{n_{kj}+\gamma_{kj}-1}$,

where k, j=0, 1 and $k \neq j$. The posterior distributions for the transition probabilities

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29 The sign $\propto$ means "is proportional to". In Bayesian econometrics, the constant term in the posterior distribution is normally ignored because it does not affect the final results. The expression that follows this sign is the posterior distribution excluding constant terms.

have a beta density kernel. One can generate the transition probabilities from the following beta distributions:

$e_{kk} \sim Beta(n_{kk} + \gamma_{kk}, n_{kj} + \gamma_{kj})$.

2. Sampling the state variables $\boldsymbol{S}$

We use the single-move sampling method initially proposed by Carlin, Polson, and Stoffer (1992). Yoo (2004) has used this method in estimating an ARMA-GARCH model with regime switching in the mean of the ARMA process.

The following lemma is drawn from Yoo (2004), detailed proof can be also found in that paper.

Lemma For the model with equation (1) and (2), the conditional distribution of S_t is given by

(A.3) $$f(S_t \mid \boldsymbol{\theta}, \boldsymbol{\pi}, \boldsymbol{S}_{-S_t}) \propto f(S_{t+1} \mid S_t, e_{kk}) f(S_t \mid S_{t-1}, e_{kk}) f(\boldsymbol{\pi} \mid \boldsymbol{S}, \boldsymbol{\theta})$$
$$\propto f(S_{t+1} \mid S_t, e_{kk}) f(S_t \mid S_{t-1}, e_{kk}) \prod_{t=j}^{T} h^{-1/2}{}_t \exp(-0.5\varepsilon_t^2).$$

In equation A.3, the first two terms can be calculated from the transition probabilities. The last term is the sample likelihood function from *j* to *T*.

Therefore, the conditional posterior probability of $S_t = 1$ is given by

(A.4) $$P(S_t = 1 \mid \boldsymbol{\pi}, \boldsymbol{S}_{-t}, \boldsymbol{\theta}) = P(S_t = 1 \mid S_{t-1}, S_{t+1}) f(\boldsymbol{\pi} \mid \boldsymbol{S}_{-t}, S_t = 1, \boldsymbol{\theta}) /$$
$$[P(S_t = 1 \mid S_{t-1}, S_{t+1}) f(\pi \mid \boldsymbol{S}_{-t}, S_t = 1, \boldsymbol{\theta}) + P(S_t = 0 \mid S_{t-1}, S_{t+1}) f(\boldsymbol{\pi} \mid \boldsymbol{S}_{-t}, S_t = 1, \boldsymbol{\theta})]$$

Then, the state variable can be generated by comparing the value of $P(S_t = 1 \mid \boldsymbol{\pi}, \boldsymbol{S}_{-t}, \boldsymbol{\theta})$ with a random draw from the (0, 1) uniform distribution. If the value is greater than the random draw, set the state at time *t* as 1. The procedure can be implemented backwardly with updating the state variables one by one.

3. Sampling the autoregressive coefficients in $\boldsymbol{\beta}$

The posterior distribution of $\boldsymbol{\beta}$ can be generated by the transformation:

(A.5) $$\frac{\pi_t - \rho(S_t)\sqrt{h_t}}{\sqrt{h_t}} = \frac{\beta_0 + \sum_{i=1}^{p} \beta_i \pi_{t-i}}{\sqrt{h_t}} + \varepsilon_t, \text{ or } \pi_t^* = \boldsymbol{\beta}' \boldsymbol{y}_t^* + \varepsilon_t$$

where $\boldsymbol{y}_t^* = [1\, \pi_{t-1} \ldots \pi_{t-p}]'$.

The regression becomes a linear regression model with standard normal error. Then the parameter vector $\boldsymbol{\beta}$ follows a multivariate normal distribution with

variance $\Sigma_1^{-1} = \sum_{t=1}^{T} \boldsymbol{y}_t^* \boldsymbol{y}_t^{*'} + \Sigma_0^{-1}$ and mean $\boldsymbol{\beta}^* = \Sigma_1 (\boldsymbol{Y}^{*'} \boldsymbol{\pi}^* + \Sigma_0^{-1} \boldsymbol{\beta}_0)$,

where $\boldsymbol{\pi}^* = [\pi_1^*, \ldots, \pi_T^*]'$, $\boldsymbol{Y}^* = [y_1^*, .., y_T^*]'$.

Now one can generate $\boldsymbol{\beta}$ through this multivariate distribution.

4. Sampling ρ_i

The posterior distribution of ρ_i can be generated through the transformation:

(A.6) $$\frac{\pi_t - (\beta_0 + \sum_{i=1}^{p} \beta_i \pi_{t-i})}{\sqrt{h_t}} = \rho_i + \varepsilon_t, \text{or } \pi^{**}{}_t = \rho_i + \varepsilon_t$$

The regression again becomes a linear regression model with standard normal error. Then the parameter ρ_i follows a normal distribution with variance $\sigma_{i*}^2 = (n_i + 1/\sigma_{i0}^2)^{-1}$ and mean $\rho_i^* = \sigma_{i*}^2 (n_i \overline{\pi_i}^{**} + \rho_{i0} / \sigma_{i0}^2)^{-1}$, where n_i is the number of data points in state *i* and $\overline{\pi_i}^{**}$ is the average over the data points in stat *i*. ρ_i can be generated from this normal distribution.

5. Sampling the coefficients in the variance equation: $\boldsymbol{\alpha}$

The posterior distribution of the GARCH process can be written as:

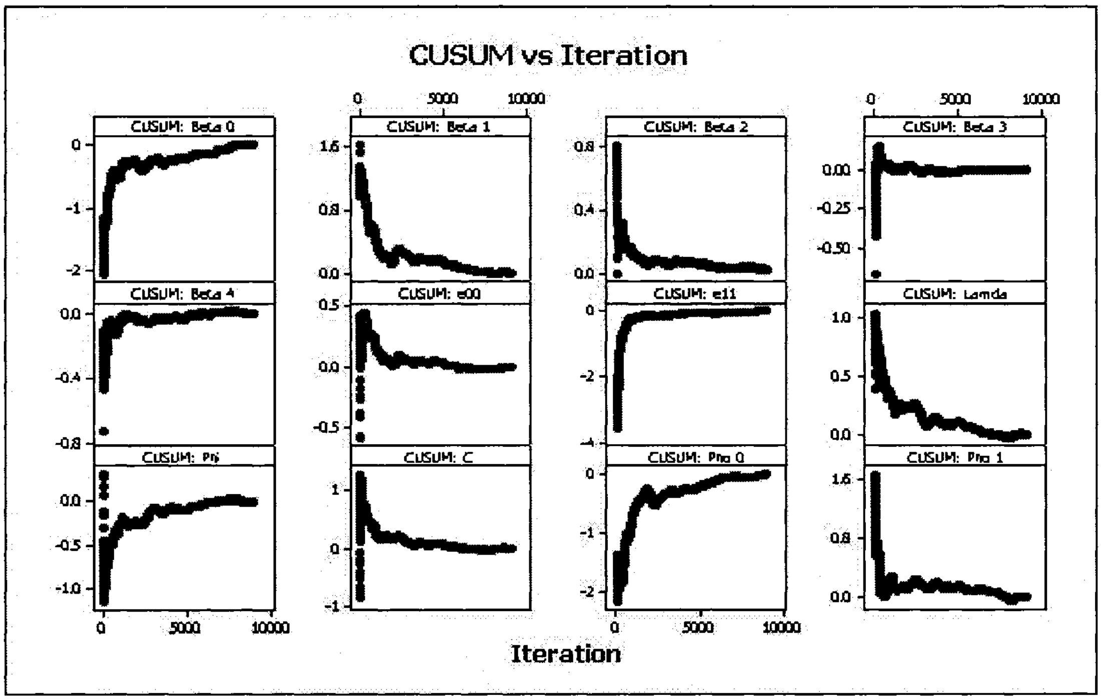
(A.7)

$$f(\alpha_j \mid \boldsymbol{\pi}, \boldsymbol{\theta}_{-\alpha_j}, \boldsymbol{S}) \propto f(\alpha_j) \prod_{t=1}^{T} h_t^{-1/2} \exp(-\frac{1}{2h_t} (\pi_t - (\beta_0 + \sum_{i=1}^{p} \beta_i \pi_{t-i}) - \rho(S_t)\sqrt{h_t})^2)$$

where α_j j=1,2,3, denotes the j-th element in $\boldsymbol{\alpha}$ and $f(\alpha_j)$ is the prior distribution of α_j. Because the conditional variance is time-varying, function A.7 does not correspond to any known distribution. We can use the griddy Gibbs sampler to draw parameters in $\boldsymbol{\alpha}$ one by one. Function A.7 can be evaluated over the posterior distributions' support to obtain their approximations. Once the approximation of a parameter is available, the random draws of that parameter can be generated by inverting the approximated CDF.

Figure A.1

CUSUM Plot



Notes: The graph shows the *CUSUM* statistics of all variables against iteration. Results are based on 9000 Gibbs sample draws for the MS-AR(4)-GARCH(1,1)-M model of quarterly U.S. inflation data: 1947Q2-2006Q1.

Appendix B

Bootstrap Procedures in Chapter 4

In Chapter 4, by bootstrap methods, we find the critical values of the t-tests of ρ and λ, as well as the F-test of α's. The procedures are the following:

First, we estimate the corresponding models and store the residuals and estimates.

Second, we resample the stored residuals and use these resampled residuals and estimated parameter values in the first step to generate new data. Here we impose the null hypothesis to generate new data. For instance, when bootstrapping the critical values of ρ, we let the DGP contain one unit root (by setting ρ as 0). To resample residuals, we consider two ways: (1) we resample all residuals together assuming they are generated from an i.i.d. distribution; (2) we resample residuals by fixing the cross section dependence structure. The results are robust to the two resampling approaches. In addition, when sampling the residuals, we generate the initial values of data by the "moving block" method, following Wu and Wu (2001).

Third, we re-estimate the model using the new data and calculate the corresponding statistics.

Fourth, we repeat 3000 times steps 2-3 and store all calculated values of the statistics.

The critical values are found at the corresponding quantiles of the ordered values of the statistics.

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