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APPLICATIONS OF TIME-VARYING-PARAMETER MODELS TO  
ECONOMICS AND FINANCE

by

Peng Huang

A Dissertation  
Submitted to the  
Faculty of The Graduate College  
in partial fulfillment of the  
requirements for the  
Degree of Doctor of Philosophy  
Department of Economics  
Advisor: C. James Hueng, Ph.D.

Western Michigan University  
Kalamazoo, Michigan  
August 2006

# APPLICATIONS OF TIME-VARYING-PARAMETER MODELS TO ECONOMICS AND FINANCE

Peng Huang, Ph.D.

Western Michigan University, 2006

This dissertation focuses on applying time-varying-parameter models to the field of financial and monetary economics. The first two essays analyze the cross-sectional returns on the U.S. stock market by emphasizing the dynamics of risk loadings. The third essay studies the impact of a tight monetary policy on weak currencies during financial crises by examining the time-varying relationship between interest rates and exchange rates.

Motivated by the pricing errors found in small size and low book-to-market ratio portfolios in the Fama-French three-factor model, the first essay proposes a time-varying four-factor model. As small size and low book-to-market ratio firms are more sensitive to the risk related to innovations in the discount rate, the model incorporates a new risk factor to capture the information about the discount-rate risk for which the Fama-French three factors cannot fully account. In addition, the investors' learning process mimicked by the Kalman filter procedure is used to model the evolution of risk loadings. The results indicate that the model outperforms the Fama-French three-factor model in explaining the cross-sectional returns by substantially reducing pricing errors.

The second essay analyzes the risk-return relationship in a capital asset pricing model (CAPM) with a time-varying beta estimated by adaptive least squares (ALS) based

on Kalman foundations. The results show the presence of a significant and positive risk-return relationship in the up market and the presence of a significant and negative risk-return relationship in the down market. In comparison with the model that assumes a constant beta, the CAMP with a time-varying beta reduces unexplained returns and improves the accuracy of the estimated risk-return relationship.

The third essay investigates the use of interest rates as a monetary instrument to stabilize exchange rates in the Asian financial crisis. Since previous studies suggest that the interest-exchange rate relationship may vary within, or across, regimes, a time-varying-parameter model with generalized autoregressive conditional heteroskedastic (GARCH) disturbances is used to estimate the impact of raising interest rates on exchange rates. The empirical evidence shows that an increase in interest rates leads to currency depreciation during certain periods of financial crises.



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Peng Huang

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## CHAPTER I

### INTRODUCTION

Examining relationships between variables is the major way for researchers to test economic hypotheses. The traditional time-series analysis typically assumes relationships between variables are constant. However, recent studies show that a time-series econometric model that ignores the evolution of relationships between variables may generate poor forecasts or misleading results. Thus, this dissertation focuses on investigating some dynamic relationships related to financial and monetary economics by utilizing time-varying-parameter models.

To explain the cross-section of average returns on the U.S. stock market, the first essay investigates common risk factors by proposing a time-varying four-factor model. Empirical evidence indicates that the Fama-French three-factor model cannot adequately explain the cross-sectional returns because significant pricing errors are observed in small size and low book-to-market ratio portfolios. The essay investigates this problem by concentrating on specific characteristics of these portfolios. Previous studies assert that small size and low book-to-market ratio portfolios are more sensitive to the risk associated with innovations in the discount rate. Since the discount rate is an average of interest rates over time, a change in interest rates can lead to a change in the discount rate. Therefore, the TERM factor, defined as the yield spread between 10-year government bond and 3-month Treasury bill, is included in the model to capture the information about the discount-rate risk for which the common Fama-French three factors cannot fully account. Moreover, the risk loadings on common risk factors are found to exhibit time-

variation in previous literature. However, the Fama-French three-factor model assumes risk loadings to be constant over time. To replicate the movements of risk loadings more accurately, the Kalman filter procedure is used to proxy the investors' learning process of unobservable risk loadings.

To evaluate performance, the time-varying four-factor model and the Fama-French three-factor model are estimated with 25 size and book-to-market double-sorted portfolios from the period 1963:7-2004:12, respectively. The empirical evidence indicates that the time-varying four-factor model outperforms the Fama-French three-factor model in explaining the cross-sectional returns by substantially reducing both the individual and the aggregate pricing errors. Experiments are also conducted on portfolios sorted by industries and 25 size and book-to-market double-sorted portfolios prior to 1963. The results further confirm that the time-varying four-factor model remarkably reduces pricing errors when compared to the Fama-French three-factor model.

Using the daily data from the S&P 500 stocks, the second essay analyzes the risk-return relationship in a capital asset pricing model (CAPM) with a time-varying beta (risk). Adaptive least squares (ALS) with Kalman foundations is used to capture the dynamics of betas since empirical evidence finds that betas tend to be time-variant. Due to the use of realized returns, the risk-return relationship is examined under an up market and a down market. The results show that a significant and positive risk-return relationship exists when the market excess return is positive and a significant and negative risk-return relationship exists when the market excess return is negative. For comparison purposes, the model based on the assumption of a constant beta is also examined. The estimation results show that, in comparison with the constant beta model,

the CAMP with a time-varying beta generates abnormal returns that are not statistically different from zero. Additionally, in terms of the realized market excess return, the time-varying beta CAPM enhances the accuracy of estimates of the risk-return relationship.

Finally, the third essay investigates the effectiveness of using interest rates to stabilize exchange rates in the Asian financial crisis. Previous studies suggest that the relationship between interest rates and exchange rates may vary over time. Therefore, instead of assuming a constant interest-exchange rate relationship during arbitrarily chosen periods, this essay allows the relationship to be totally determined by the data. A time-varying-parameter model with generalized autoregressive conditional heteroskedastic (GARCH) disturbances is employed to estimate the impact of raising interest rates on exchange rates. With the weekly data from four East Asian countries: Indonesia, South Korea, the Philippines, and Thailand, our results show that raising interest rates leads to currency depreciation in some periods of the Asian financial crisis. This is in favor of the revisionist view that a tight monetary policy has a perverse impact on exchange rates during crisis periods.

In summary, these three essays concentrate on dynamic relationships in the field of financial and monetary economics. The first two essays stress that failing to take account of the time-evolution of risk loadings in an asset pricing model could lead to significant pricing errors because the wedge between constant risk loadings estimated with ordinary least squares (OLS) and real ex ante investors' expectations of risk loadings. The third essay implies that arbitrarily assuming a constant interest-exchange rate relationship may lose some important information about the dynamics of the relationship within, or across, regimes. To solve these problems, this dissertation



employs time-varying-parameter models, such as, the state-space model estimated with the Kalman filter and ALS based on Kalman foundations, to proxy investors' time-evolving expectations and estimate time-varying relationships. The first two essays prove that a time-varying-parameter model improves the estimates of risk loadings by significantly reducing pricing errors. The third essay finds that an increase in interest rates has a significant impact on exchange rates only during certain periods of the financial crisis. The results of all three essays emphasize the importance of recognizing time-varying relationships in economic and financial studies. The results also suggest that time-varying-parameter models are useful and effective methodologies to capture the dynamics of time-varying relationships.

## CHAPTER II

### A TIME-VARYING MULTI-RISK-LOADINGS MODEL

#### 2.1 Introduction

The cross-section of average returns of the stock market has been the focal point of finance for many years. The most influential asset-pricing model in the 1990s is the three-factor model proposed by Fama and French (1993, 1996). In this model, Fama and French use the market excess return, the difference between the returns on small-size portfolios and big-size portfolios (SMB), and the difference between the returns on high book-to-market ratio (B/M) and low B/M portfolios (HML) to mimic common risk factors in the returns on stocks. Their results indicate that the three-factor model is more successful at explaining the average returns than the Sharpe-Lintner CAPM.

Although the Fama-French three-factor model has generated impressive performance, empirical evidence indicates that it is still not able to completely capture the cross-sectional returns,<sup>1</sup> especially the returns of smallest size (small) or lowest B/M (growth) portfolios in the 5 by 5 size and B/M double-sorted portfolios. In fact, Fama and French (1993) estimation results show that during the sample from 1963:7 to 1990:12, there are still three portfolios of which the pricing errors<sup>2</sup> are significantly different from zero. Two of them belong to growth portfolios. Recent studies show that if the sample extends to include the data after 1990, the evidence becomes more apparent. Petkova

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<sup>1</sup> The most well-known problem with the Fama-French three-factor model is the momentum effect (Jegadeesh and Titman [1997]).

<sup>2</sup> To test asset pricing models, excess returns of portfolios are regressed on explanatory variables of an asset model under the time-series framework. The estimated intercept is regarded as the pricing error. According to Merton (1973), the pricing errors in a well-specified asset pricing model should not be statistically different from zero.

(2006) uses the data from 1963:7 to 2001:12 to estimate the Fama-French three-factor model. The results illustrate that 6 out of 25 portfolios have pricing errors significantly different from zero and 4 of them locate at lowest B/M or smallest size quintile. Adrian and Franzoni (2005) report very close results using quarterly data from 1963 to 2004. Among 9 total small and growth portfolios, 5 have pricing errors distinguishable from zero. Even studies that use the data outside the United States display similar outcomes. Chiao and Hueng (2005) estimate the Fama-French three-factor model for 5 by 5 size and B/M double-sorted portfolios with the Japanese stock market data. They find that 6 out of 9 small and growth portfolios have pricing errors different from zero.

The above evidence causes us to doubt the explanatory power of the Fama-French model for cross-sectional returns. In this paper, we argue that this problem might arise from two different sources. The first source is concerned with the empirical evidence that risk loadings vary over time. Previous studies, such as Harvey (1989), Ferson and Harvey (1991, 1993), and Jagannathan and Wang (1996), demonstrate that  $\beta$  in the CAPM tends to be volatile through time. Fama and French (1997) and Ferson and Harvey (1999) show that risk loadings on the market excess return, HML, and SMB exhibit strong time variation. However, the Fama and French (1993, 1996) three-factor model assumes that risk loadings are constant over time. Thus, the inability to account for time-variation in risk loadings could lead to significant pricing errors. Moreover, as pricing errors are mostly found to be significant in small or growth portfolios, we conjecture that the second source of pricing errors may be related to some specific features of such portfolios. According to Cornell (1999), both small and growth portfolios generate cash flows in the distant future and therefore are more sensitive to risk associated with variations in the

discount rate. Not fully conveying such information into the model could also result in mispricing in returns of small or growth portfolios.

As we conjecture that the Fama-French three-factor model can not completely account for time-varying risk loadings and information related to changes in the discount rate, we propose a new time-varying four-factor model in this paper. In contrast with the Fama-French three-factor model, there are two main changes for this new model. The first is that the concept of learning about time-varying risk loadings is introduced into the model. Adrian and Franzoni (2004, 2005) assert that the main reason for the failure of the unconditional CAPM is that the model does not mimic the investors' learning process. They argue that the unobservability of time-varying  $\beta$  will induce the investors' learning process. However, the ordinary least squares (OLS) time-series regression cannot successfully mimic the investors' learning process, which leads to the difference between the investors' true expectation of  $\beta$  and  $\beta$  estimated with OLS. Thus, the authors initial the Kalman filter procedure to model the movements of  $\beta$  and replicate the investors' learning process. Their empirical results show that  $\beta$  estimated by the Kalman filter significantly reduces pricing errors when compared to the unconditional CAPM.

Inspired by the Adrian and Franzoni paper, we innovate by applying the Kalman filter to the case of multi-risk loadings. Similarly, we assume that risk loadings are mean-reverting and have an autoregressive structure. We expect that the learning process mimicked by the Kalman filter could capture the dynamics of risk loadings and provide better estimates for investors' expectations than the conventional OLS regression. Thus, it will eventually help reduce pricing errors for a model with multi-risk loadings. Unlike Adrian and Franzoni's model, our model does not use any state variables. We argue that

arbitrarily constructing risk loadings as a function of several state variables might lead to the loss of information during the process of estimation.

The second change of the model in this paper is that in addition to the common Fama-French three-factors, there is a new factor in the model. The new factor, TERM, is defined as the yield spread between the 10-year government bond rate and 3-month Treasury bill rate. Cornell (1999) claims that small or growth portfolios whose cash flows occur in a long duration are more sensitive to risk related to changes in the discount rate. He argues that not completely capturing information related to changes in the discount rate is likely to generate significant pricing errors in small or growth portfolios. Campbell and Vuolteenaho (2004) also suggest that small and growth portfolios are more sensitive to the discount-rate news because of high discount-rate beta for these portfolios in the sample after the 1960s. Thus, the major reason to include TERM into the model is that we expect the term spread between long-term and short-term bonds to carry information about risk related to changes in the discount rate beyond the Fama-French three factors.

Many studies have proven that term spread contains information about movements in interest rate, such as Campbell and Shiller (1991) and Diebold, Rudebusch and Aruoba (2003). According to these studies, the term spread represents a good proxy for the shifts in interest rate. Since the discount rate is an average of interest rates over time, a change in interest rates can lead to a change in discount rate. Thus, we use TERM as a variable to capture information about risk related to changes in the discount rate. Chen, Roll and Ross (1986) argue that the discount rate varies with the term spread across different maturities. Furthermore, there is evidence that implies the Fama-French

three factors can not fully account for risk information contained in the term spread. Petkova (2006) shows that only a small fraction of innovations in the term spread can be explained by Fama-French three factors. Adrian and Franzoni (2005) confirm that the market excess return and HML only used together with the term spread as state variables can improve the tests of the CAPM. Both of these studies imply that the term spread probably conveys important information beyond the Fama-French three factors. Thereby, including TERM in the model seems to be a potential way to reduce pricing errors for small and growth portfolios. Moreover, as we know, the econometricians' information set is smaller than the investors' information set. Under the framework of the learning process mimicked by the Kalman filter, adding TERM into the model may efficiently extend the econometricians' information set, and therefore make estimates of risk loadings more close to the investors' ex ante expectations.

To evaluate the performance of the time-varying four-parameter model, U.S. stock market data covering the period from 1963:7 to 2004:12 are used in this paper. The assets tested are 5 by 5 size and B/M double-sorted portfolios.<sup>3</sup> The Fama-French three-factor model is estimated for comparison purposes. In addition, the time-varying three-factor model estimated with the Kalman filter and the four-factor model estimated with OLS<sup>4</sup> are examined to analyze the sole contribution of the learning process and TERM, respectively. This paper focuses on checking individual pricing error and aggregate pricing error generated by each model. The estimation results indicate that the both the

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<sup>3</sup> Professor Kenneth French kindly provides the data on his website.

<sup>4</sup> The three risk factors in the time-varying three-factor model estimated with the Kalman filter are the same with the Fama-French three factors. The four-factor model estimated with OLS includes TERM in addition to the Fama-French three factors.

Kalman filter and the TERM factor can partially reduce pricing errors, which confirms our two conjectures mentioned above.

The most impressive result arises from the combination of the Kalman filter and TERM since the time-varying four-factor model remarkably reduces both individual and aggregate pricing errors relative to the Fama-French three-factor model. For individual portfolios with pricing errors statistically different from zero in the Fama-French model, their significant pricing errors almost disappear in the time-varying four-factor model. Moreover, the time-varying four-factor model achieves a great reduction in the aggregate pricing error. The root mean squared error (RMSE) and the composite pricing error (CPE), two measures for the aggregate pricing error, reduce by 60 percent and 50 percent, respectively, in the time-varying four-factor model when compared to the Fama-French three-factor model. In sum, the time-varying four-factor model significantly diminishes pricing errors not only specific to small or growth portfolios, but overall pricing errors across 25 portfolios. The results imply that this model has a better ability in explaining the cross-section of average returns than the Fama-French three-factor model.

In order to examine the explanatory power the time-varying four-factor model and to check for possible data mining problems, we also use portfolios that are sorted by different characteristics rather than size and B/M to estimate the model. Our results show that, for industry-sorted portfolios, both the individual and aggregate pricing errors are greatly reduced by the time-varying four-factor model compared to the Fama-French three-factor model. Furthermore, we use the data prior to 1963 to form the size and B/M double-sorted portfolios and evaluate the performances of the time-varying four-factor model. Again, the time-varying four-factor model outperforms the Fama-French three-

factor model by greatly reducing both the individual and the aggregate pricing errors. We conclude that the success of the time-varying four-factor model lies in the fact that it successfully mimics the investors' evolutionary learning process of time-varying risk loadings and incorporates more information of risks related to changes in the discount rate into the estimation process.

The remainder of the paper is organized as follows. The four-factor time-varying model is introduced in section 2.2. Section 2.3 contains the description of the data and empirical estimation. In section 2.4, the empirical results are reported and analyzed. Section 2.5 contains concluding remarks.

## 2.2 The Time-Varying Four-Factor Model

### 2.2.1 The TERM factor

Since significant pricing errors are mostly found in small or growth portfolios, we question whether the Fama-French three factors have adequate ability in explaining the returns of such portfolios. It is natural for us to start investigating this problem by concentrating on specific characteristics of small and growth portfolios. Previous studies indicates that both small and growth portfolios are more sensitive to changes in the discount rate. Based on the relationship between risk and duration for projects, Cornell (1999) provides a possible reason to explain the sensitivity of small and growth portfolios to changes in the discount rate. He stresses that the relatively higher risk of long-term projects arises from variation in the discount rate rather than variation in cash flows. As small and growth firms usually generate cash flow in a longer duration, their returns are likely to respond more strongly to shocks in the discount rate compared with large and



value firms.<sup>5</sup> This is very similar to the situation of long-term bonds, which, because of their longer duration, are more sensitive than short-term bonds to shocks to the discount rate.

A recent study by Campbell and Vuolteenaho (2004) provides further evidence to support that small and growth portfolios respond more strongly to changes in the discount rate. They decompose the market beta of one portfolio into the cash-flow beta and the discount-rate beta. Their results indicate that the discount-rate betas of small and growth portfolios are larger than those of large and value portfolios after the 1960s. This means that small or growth portfolios are likely to be more sensitive to the discount-rate news in this period. Campbell and Vuolteenaho (2004) ascribe the relatively higher discount-rate betas of small and growth portfolios to the long duration of cash flows, future investment opportunities, and dependence on external fund raising. Like Cornell (1999), they think that small and growth firms with negative current cash flows but valuable future investment opportunities react more greatly to the discount-rate news. Moreover, in line with Perez-Quiros and Timmermann (2000), Campbell and Vuolteenaho argue that small and young firms with little collateral rely more heavily on external financing, such as bank loans, because they don't have easy access to other credit sources. Therefore, these firms are more sensitive to interest costs and external financial conditions.

Based on the above arguments, we conclude that the information about shocks in the discount rate plays an important role in explaining returns of small or growth portfolios. The unexplained returns of small or growth portfolios in Fama-French three-factor model cause us to doubt whether the three factors fully carry such information.

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<sup>5</sup> Large firms refer to firms with biggest size and value firms refer to firms with highest book-to-market ration (B/M).

Actually, Cornell (1999) implies that taking account of changes in the discount rate may improve the tests of an asset-pricing model and help to explain some anomalies in returns, especially for small or growth firms. This inspires us to select some variables that are able to capture information about changes in the discount rate. In this paper, we choose the term spread between 10-year government bond rate and 3-month Treasury bill rate (TERM) as an additional risk factor to the Fama-French three-factor model.

It is well-known that the term spread is a variable that contains abundant information about changes in interest rate. Campbell and Shiller (1991) document that the yield spread between long-term and short-term bonds contains information about future movements in interest rate. Diebold, Rudebusch and Aruoba (2003) assert that the slope factor, which is highly correlated (0.98) with the yield spread between 10-year bond and 3-month Treasury bill, responds significantly to innovations in federal funds rate. Since the discount rate is an average of interest rates over time, changes in interest rates will affect the value of the discount rate, which eventually influences stock prices and returns. Therefore, the term spread seems to be a good candidate to capture information about risk related to changes in the discount rate. Chen, Roll and Ross (1986) claim that the discount rate changes with the term spread across different maturities. Fama and French (1993) suggest that risk in bond returns arises from unexpected changes in interest rates and that the term spread is a good variable to represent risk related to innovations in the discount rate. They construct a risk factor for the bond market by the spread between returns on long-term and short-term government bonds. Their results show that long-term bonds are more sensitive than short-term bonds to the term spread. Fama and French ascribe this to the ability of the term spread in conveying information

about risk related to shocks to the discount rate. Due to the long-term bonds having a relatively longer duration, the results obtained by Fama and French (1993) are consistent with Cornell's (1999) conclusion that projects with a longer duration are more sensitive to shocks to the discount rate.

The reason for adding the term spread into the Fama-French model is not just dependent on its ability to carry information related to changes in the discount rate. In fact, another important reason is that some indirect evidence shows that the Fama-French three factors can not fully account for the information contained in the term spread. Although Petkova (2006) illustrates that SMB is positively and significantly related to the term spread, her results indicate that only very small portion (about 5 percent) of surprise in the term spread can be explained by the Fama-French three factors. Adrian and Franzoni (2005) show that only after the market excess return and HML are combined with the term spread as state variables for  $\beta$ , their conditional CAPM greatly improves. Both these studies hint that term spread carries information beyond the Fama-French three factors.

Furthermore, we also focus on the predictive power of the term spread for cross-sectional stock returns. Previous studies illustrate that the term spread does exhibit the explanatory power in different asset pricing models whenever it is used directly as a factor in a model (Chen, Roll, and Ross [1986], Petkova [2006], and Aretz, Bartram, and Pope [2004]) or used as a state variable for other variables (Campbell and Vuolteenaho [2004] and Adrian and Franzoni [2005]). Essentially, TERM seems to be an appropriate variable to proxy risk related to shocks to the discount rate for the stock market.

### 2.2.2 The time-varying four-factor model based on the Kalman Filter

The time-varying parameter model has a wide application in estimating unobservable variables. It has been used to estimate time-varying relationships and proxy agents' time-evolving expectations. The basic tool to deal with the time-varying parameter model is the Kalman filter. The Kalman filter involves Bayes' rule, and it updates the time-varying parameters through learning from prediction errors. A time-varying parameter model based on the Kalman filter has been widely used to capture the dynamics of time-varying variables.

Substantial finance literature, such as Harvey (1989), Ferson and Harvey (1991, 1993), and Jagannathan and Wang (1996), has shown that  $\beta$  in a CAPM varies through time. These studies emphasize that the OLS regression is not a suitable methodology for estimating time-varying risk loadings. In fact, Franzoni (2002) asserts that portfolios, especially small portfolios, exhibit considerable long-run variation in  $\beta$ . Since significant pricing errors may arise from the wedge between the OLS estimators and investors' expectations, Adrian and Franzoni (2004, 2005) have been the first to introduce the Kalman filter to test the conditional CAPM. Adrian and Franzoni assume that  $\beta$  is reverting back to a slowly time-varying mean. As  $\beta$  is unobservable and wanders with time, it induces the learning process of rational investors. Ignoring the investors' learning process can lead to the difference between  $\beta$  estimated by a certain model and the investors' true expectation of  $\beta$ . Adrian and Franzoni stress that the primary source of the mispricing in the unconditional CAPM is that  $\beta$  estimated with OLS differs from investors' ex ante expectation of  $\beta$ . Therefore, they employ the Kalman filter to mimic

the investors' learning process. Their empirical results show that their learning type of CAPM outperforms the unconditional CAPM.

Inspired by Adrian and Franzoni (2004, 2005), we are the first to implement the learning type model to the case of multi-risk loadings. We hope that the Kalman filter methodology can also capture the dynamics of multi-risk loadings because Fama and French (1997) and Fesron and Harvey (1999) show that risk loadings on common risk factors are volatile through time. In this paper, time-varying risk loadings are assumed to be unobservable and mean-reverting. Each loading has an autoregressive structure. The time-varying parameter model can be represented by the following state-space form:

$$(1) \quad y_t = \mathbf{X}_t' \boldsymbol{\beta}_t + \varepsilon_t,$$

$$(2) \quad \boldsymbol{\beta}_t = \boldsymbol{\alpha} + \mathbf{F} \boldsymbol{\beta}_{t-1} + \mathbf{v}_t,$$

where  $y_t$  is a scalar and  $\mathbf{X}_t$  is a  $k \times 1$  vector.  $\boldsymbol{\beta}_t$  represents time-varying coefficients and it is a  $k \times 1$  vector. The error term  $\varepsilon_t$  is a scalar and is assumed to be iid  $N(0, R)$ .  $\mathbf{F}$  is a  $k \times k$  diagonal matrix.  $\mathbf{v}_t$  is  $k \times 1$  and is assumed to be  $N(0, \mathbf{Q})$ . Note that  $\varepsilon_t$  and  $\mathbf{v}_t$  are independent of each other. For the variance of  $\varepsilon_t$  and the covariance matrix of  $\mathbf{v}_t$ ,  $R$  is a scalar and  $\mathbf{Q}$  is a  $k \times k$  diagonal matrix.

For the time-varying four-factor model in this paper,  $y_t$  denotes the excess return ( $r_{i,t}$ ) of portfolios  $i$ , which equals the return of portfolio  $i$  minus the risk-free rate.  $\mathbf{X}_t$  represents the common risk factors of the stock market, which is  $(r_{m,t}, \text{SMB}_t, \text{HML}_t, \text{TERM}_t)$ . Here  $r_{m,t}$  denotes the market excess return and it can be calculated as the market return minus the risk-free rate. In the time-varying four-factor model,  $\boldsymbol{\beta}_t$  equals  $(\beta_{i,t}^m, \beta_{i,t}^s, \beta_{i,t}^h, \beta_{i,t}^e)'$ .  $\beta_{i,t}^m$ ,  $\beta_{i,t}^s$ ,  $\beta_{i,t}^h$ , and  $\beta_{i,t}^e$  represent the time-varying

risk loadings on the four risk factors,  $r_{m,t}$ ,  $\text{SMB}_t$ ,  $\text{HML}_t$ , and  $\text{TERM}_t$ , respectively. They are assumed to have an autoregressive structure. According to the state-space model (equations [1] and [2]), the time-varying four-factor model can be represented by the state-space form:

$$(3) \quad r_{i,t} = \mathbf{X}'_t \boldsymbol{\beta}_{i,t} + \varepsilon_{i,t}$$

$$= (r_{m,t}, \text{SMB}_t, \text{HML}_t, \text{TERM}_t) \begin{pmatrix} \beta_{i,t}^m \\ \beta_{i,t}^s \\ \beta_{i,t}^h \\ \beta_{i,t}^e \end{pmatrix} + \varepsilon_{i,t},$$

$$i = 1, 2, \dots, N, t = 1, 2, \dots, T,$$

$$(4) \quad \begin{pmatrix} \beta_{i,t}^m \\ \beta_{i,t}^s \\ \beta_{i,t}^h \\ \beta_{i,t}^e \end{pmatrix} = \begin{pmatrix} \alpha_i^m \\ \alpha_i^s \\ \alpha_i^h \\ \alpha_i^e \end{pmatrix} + \begin{pmatrix} F_i^m & 0 & 0 & 0 \\ 0 & F_i^s & 0 & 0 \\ 0 & 0 & F_i^h & 0 \\ 0 & 0 & 0 & F_i^e \end{pmatrix} \begin{pmatrix} \beta_{i,t-1}^m \\ \beta_{i,t-1}^s \\ \beta_{i,t-1}^h \\ \beta_{i,t-1}^e \end{pmatrix} + \begin{pmatrix} v_{i,t}^m \\ v_{i,t}^s \\ v_{i,t}^h \\ v_{i,t}^e \end{pmatrix},$$

where  $\varepsilon_{i,t} \sim \text{iid } N(0, R_i)$  and  $\mathbf{v}_{i,t} = (v_{i,t}^m, v_{i,t}^s, v_{i,t}^h, v_{i,t}^e)' \sim \text{iid } N(0, \mathbf{Q}_i)$ .  $\varepsilon_{i,t}$  and  $\mathbf{v}_{i,t}$  are independent of each other. They are considered as idiosyncratic shocks to portfolio  $i$  and they are uncorrelated with shocks to other portfolios.

The state-space model based on equations (3) and (4) can be solved by the Kalman filter, which consists of the prediction and updating two steps. Because risk loadings on different factors are not observable, investors need to form their expectations about risk loadings based on available information. In the prediction step, the one-period ahead forecast of a risk loading  $\beta_i$  at time  $t-1$  can be expressed as an autoregressive process conditional on  $\beta_i$  of time  $t-1$ :

$$(5) \quad \beta_{i,t|t-1} = \alpha_i + F_i \beta_{i,t-1|t-1}.$$

Based on the expectation of risk loadings, the expected excess return of portfolio  $i$  at  $t$  is :

$$(6) \quad \begin{aligned} r_{i,t|t-1} &= X_t' \beta_{i,t|t-1} \\ &= \beta_{i,t|t-1}^m r_{m,t} + \beta_{i,t|t-1}^s \text{SMB}_t + \beta_{i,t|t-1}^h \text{HML}_t + \beta_{i,t|t-1}^e \text{TERM}_t. \end{aligned}$$

After realized return  $r_{i,t}$  is observed at time  $t$ , the prediction error  $\eta_{i,t|t-1}$  can be obtained by comparing the difference between the realized  $r_{i,t}$  and expected  $r_{i,t|t-1}$ :

$$(7) \quad \eta_{i,t|t-1} = r_{i,t} - r_{i,t|t-1},$$

where  $\eta_{i,t|t-1}$  contains new information about  $\beta_{i,t}$  beyond  $\beta_{i,t|t-1}$ .

In the subsequent updating step, based on the prediction error,  $\beta_{i,t|t}$  an inference of risk loading  $\beta_{i,t}$  can be updated with information up to time  $t$ :

$$(8) \quad \beta_{i,t|t} = \beta_{i,t|t-1} + K_{i,t} \times \eta_{i,t|t-1},$$

where  $K_{i,t}$  is the Kalman gain. It determines how much weight should be assigned to the prediction error  $\eta_{i,t|t-1}$ . The Kalman gain can be described by the following equation:

$$(9) \quad K_{i,t} = P_{i,t|t-1} X_t' f_{i,t|t-1}^{-1}.$$

In practice, investors continue to adjust their inference about risk loadings through learning new information. So the dynamic process (equations [8] and [9]) tries to mimic the investors' learning process about unobservable risk loadings. After incorporating the new information from the prediction error  $\eta_{i,t|t-1}$ , the updated risk loading  $\beta_{i,t|t}$  can be used to form expectation of risk loading for time  $t+1$ ,  $\beta_{i,t+1|t}$ . Therefore, the prediction and updating two steps can be put forward continuously from time 1 to  $T$ .

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<sup>6</sup> Note that  $P_{i,t|t-1} = E[(\beta_{i,t} - \beta_{i,t|t-1})(\beta_{i,t} - \beta_{i,t|t-1})']$  and  $f_{i,t|t-1} = X_t' E[(\beta_{i,t} - \beta_{i,t|t-1})(\beta_{i,t} - \beta_{i,t|t-1})'] X_t + R_i$ .

The above content briefly introduces the main idea of how the Kalman filter can mimic the investors' learning process of time-varying risk loadings. The details of the Kalman filter methodology are found in Hamilton (1994) and Kim and Nelson (2001). The Kalman filter is a dynamic procedure that can update unobservable risk loadings by learning through prediction errors that contain new information. As risk loadings are affected by idiosyncratic shocks and wander over time, the Kalman filter seems to be a better methodology than OLS to capture dynamics in risk loadings.

Although this paper employs the Kalman filter to estimate risk loadings like Adrian and Franzoni (2005), there are several differences between our model and their model. The first difference is very apparent: there is only one risk loading in the Adrian and Franzoni model; that is,  $\beta$  from the CAPM that needs to be estimated. Our paper applies the Kalman filter process to the case of multi-risk loadings. The second difference comes from distinct assumptions about the mean of risk loadings. Adrian and Franzoni suppose the mean of  $\beta$  to be unobservable and slowly time-varying. Our model assumes that the mean of each four risk loadings is unobservable and constant. In our model, investors still need to form expectations about current levels of risk loadings and the mean of risk loading. The third difference is that Adrian and Franzoni treat  $\beta$  as a function of several state variables. Conversely, this paper doesn't use state variables to estimate risk loadings. We argue that it is more efficient and convenient to use a multi-risk loadings model rather than one single-risk loading ( $\beta$ ) model dependent on several state variables. Since we don't know the exact function form of state variables for risk loadings, an arbitrary setup might lead to information loss during the estimation process and a larger wedge between estimated risk loading and real ex ante investors'



expectations. Nonetheless, using a multi-risk loadings model like our model can skip the step of constructing risk loadings as a function of state variables.

## 2.3 Empirical Tests

### 2.3.1 Data

The data used in this paper include the monthly returns of 5 by 5 portfolios double sorted by size and book-to-market ratio (B/M) and 10 portfolios sorted by industry, compiled over the period from July 1963 to December 2004. The 25 portfolios are constructed at the end of June each year by sorting all NYSE, AMEX, and NASDAQ stocks according to two criteria: size and B/M. Both the size and B/M breakpoints are NYSE quintiles. The size breakpoints for year  $t$  are the NYSE market equity quintiles at the end of June of  $t$ . B/M for June of year  $t$  is the book equity for the last fiscal year end in  $t-1$  divided by market equity for December of  $t-1$ . The return of each portfolio is the value-weighted return of stocks that constitute that portfolio. The 25 size and B/M double-sorted portfolios are a standard set for testing asset pricing models. We chose size and B/M-sorted portfolios because empirical results show that the returns of small and growth portfolios can not be adequately explained by the Fama-French three-factor model in the sample after 1963.

Table 1 reports the average and standard deviation of month-by-month returns of 25 size and B/M-sorted portfolios. We can find evidence that there is a negative relationship between average return and size, and there is a positive relationship between average return and B/M except in one case. The exception happens at lowest B/M quintile of 5 by 5 size and B/M-sorted portfolios. Note that the average returns of growth

Table 1

## Basic Statistics of Returns (in %) for 25 Size and B/M-Sorted Portfolios

The table shows the month-to-month (1968:7–2004:12) mean and standard errors of value-weighted returns for 25 size and book-to-market ratio (B/M) double-sorted portfolios.<sup>7</sup> The 25 portfolios are constructed at the end of June each year by sorting all NYSE, AMEX, and NASDAQ stocks according to size and B/M. Both the size and B/M breakpoints are NYSE quintiles. The size breakpoints for year  $t$  are the NYSE market equity quintiles at the end of June of  $t$ . B/M for June of year  $t$  is the book equity for the last fiscal year end in  $t-1$  divided by market equity for December of  $t-1$ . The return of each portfolio is the value-weighted return of stocks that constitute that portfolio. The 25 size and B/M portfolios are constructed from the intersections of five size (five rows: from smallest size to biggest size) and five B/M (five columns: from lowest B/M to highest B/M) groups.

Mean of returns for 5 by 5 size and B/M sorted-portfolios					
	low(growth)	2	B/M	4	high(value)
small	0.490	1.166	1.253	1.458	1.530
2	0.762	1.075	1.282	1.377	1.471
size	0.832	1.160	1.130	1.267	1.456
4	0.986	0.971	1.208	1.273	1.314
big	0.873	1.037	1.010	1.051	1.080
Standard deviation of returns for 5 by 5 size and B/M-sorted portfolios					
	low(growth)	2	B/M	4	high(value)
small	8.407	7.161	6.091	5.620	5.956
2	7.656	6.200	5.384	5.196	5.794
size	7.067	5.606	5.038	4.800	5.436
4	6.304	5.376	4.987	4.723	5.440
big	4.968	4.749	4.493	4.344	4.909

<sup>7</sup> Table 1 reports the basic statistics for the data from 1968:7 to 2004:12. A time-varying-parameter based on the Kalman filter procedure needs prediction errors and variances of prediction errors to maximize its likelihood function. However, at time  $t=1$ , we don't have prior information for the time-varying coefficients and prediction errors, for example,  $\beta_{t|t-1}$  in equation (5), and  $\eta_{t|t-1}$  in equation (7). Thus, to start the Kalman filter procedure, an arbitrary initial value  $\beta_{0|0}$  and its variance  $\mathbf{p}_{0|0}$  (wild guessing) need to be set. According to Kim and Nelson (1999), as new information  $y_t$  arrives, most of the weight in the updating equation (8) is assigned to new information contained in the prediction errors. To minimize the effect of the arbitrary initial values, they suggest evaluating the likelihood function by eliminating the first several observations. Therefore, the first 5-year estimates (1963:7 to 1968:12) are eliminated to offset the effect of initial values. This paper only focuses on the test period from 1968:7 to 2004:12.

portfolios (the five portfolios in lowest B/M quintile) seem to increase with size. The observation challenges the ability of the risk factors constructed based on size and B/M in explaining returns of growth portfolios since Fama and French (1992) assert that there exists a negative relation between return and size. The finding in Table 1 is similar to the Fama and French (1993) observation that the returns of portfolios in lowest B/M quintile do not have a monotonic pattern. Their estimation results of the three-factor model show that two of growth (lowest B/M) portfolios have pricing errors different from zero. The observation here implies that risk factors constructed based on size and B/M can not fully account for the cross-section of average returns, especially growth portfolios. This is in line with previous literature that significant pricing errors generated by the Fama-French three-factor model are found in growth portfolios.

Another interesting observation in Table 1 is that for the portfolios with the same B/M, the standard error of smallest size (small) portfolios is always the biggest. This suggests that the returns of small portfolios are more volatile than the returns of portfolios with relatively large size. The similar phenomenon can be observed for the lowest B/M portfolios, that is, growth portfolios. For portfolios in the same size quintile, the standard errors of growth portfolios are always the biggest. As the values of risk factors are the same across different portfolios, relatively larger volatility in returns of small and growth portfolios might imply that the risk loadings of such portfolios tend to be more volatile. This is consistent with our second conjecture that using the time-varying risk-loadings model rather than OLS could capture the dynamics in risk loadings, particularly for small or growth portfolios.

In addition, 10 industry-sorted portfolios are chosen to test the time-varying four-factor model. Table 2 reports the average and standard deviation of month-by-month returns of 10 industry-sorted portfolios. Each NYSE, AMEX, and NASDAQ stock is assigned to 1 of 10 industry portfolios at the end of June of year  $t$  according to its four-digit SIC code at that time. The return of one industry portfolio equals the value-weighted return of stocks in that portfolio. We choose industry sorted portfolios is because we want to investigate the applicability of the time-varying four-factor model and see whether this model can explain returns of portfolios sorted by different characteristics beyond size and B/M.

Table 2

Basic Statistics of Returns (in %) for 10 Industry-Sorted Portfolios

The table presents the month-by-month (1968:7–2004:12) mean and standard error (S.D.) of value weighted returns for 10 portfolios sorted by industry. At the end of June of year  $t$ , each NYSE, AMEX, and NASDAQ stock are assigned to 1 of 10 industry portfolios according to its four-digit SIC code at that time. The return of each industry portfolio equals the value-weighted return of stocks in that portfolio. Industries from 1 to 10 refer to consumer nondurables, consumer durables, manufacturing, energy, high technology, telecommunication, wholesale and retail, healthcare, utilities and other (firms not included in the first nine industries).

Returns of 10 industry sorted portfolios										
	1	2	3	4	5	6	7	8	9	10
Mean	1.102	0.909	0.929	1.115	0.920	0.960	1.021	1.100	0.900	1.063
S.D.	4.681	5.857	4.998	5.438	7.070	4.848	5.543	5.203	4.245	5.245

Table 3 shows the basic statistics for factors used in this paper. The three common risk factors in the Fama-French model include the market excess return, SMB and HML. The market excess return is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks less the 1-month Treasury bill rate. SMB refers to the average

return of small-size portfolios minus the average return of big-size portfolios, and HML refers to the average return of high book-to-market ratio (B/M) portfolios minus the average return of low B/M portfolios.

Table 3

Basic Statistics for Factors (in %)

The table shows the month-by-month (1968:7–2004:12) mean and standard error of factors used in this paper. The market return is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks. The market excess return is the value-weighted market return minus the 1-month Treasury bill rate. SMB refers to the average return of small-size portfolios minus the average return of big-size portfolios and HML refers to the average return of high book-to-market ratio (B/M) portfolios minus the average return of low B/M portfolios. TERM refers to the yield spread between 10-year government bond and 3-month Treasury bill rate.

	Mean	Standard deviation
The market return	0.936	4.633
1-month T-bill return	0.496	0.230
The market excess return	0.440	4.649
SMB	0.040	3.331
HML	0.520	3.097
TERM	1.610	1.319

The data of 25 size and B/M-sorted portfolios, 10 industry-sorted portfolios, and the Fama-French three factors are all downloaded from Professor Kenneth French's website. He provides the description of these data in detail. The TERM factor in this paper is defined as the yield spread between 10-year government bond and 3-month Treasury bill rate. The data of these two variables come from the Federal Reserve Bank of St. Louis.

### 2.3.2 The details of the estimation procedure

Similar to that of Fama and French (1993), this paper adopts the time-series approach to test a multi-risk-loadings model. This approach originates from Jensen (1968), who first suggests using the time-series regression to test asset pricing models. For example, the equation for the Fama-French three-factor model in time-series regression is

$$(10) \quad r_{i,t} = \alpha_i + \beta_{i,t}^m r_{m,t} + \beta_{i,t}^s \text{SMB}_t + \beta_{i,t}^h \text{HML}_t + \varepsilon_{i,t},$$
$$i = 1, 2, \dots, N, t = 1, 2, \dots, T,$$

where the excess return of portfolio  $i$  is regressed on explanatory variables (risk factors) with the OLS estimation. Merton (1973) stresses that the estimated intercept  $\hat{\alpha}_i$  should not be statistically different from zero if the model can well explain the return of portfolio  $i$ . The estimated intercept is called either abnormal return because it can not be explained by factors in the model or it can be called pricing error. To check whether the pricing error  $\hat{\alpha}_i$  is indistinguishable from zero,  $t$ -statistics can be used.

Note that the time-varying four-factor model in this paper is estimated with the Kalman filter. Thus the pricing error cannot be computed as the intercept in a time-series OLS regression. Adrian and Franzoni (2005) advocate using the approach of one-period ahead forecast. This paper follows their approach. The estimation procedure for the time-varying four-factor model is described as follows. First, equations (3) and (4) will be estimated with maximum likelihood estimation (MLE). The expectation for risk loadings of time  $t+1$  based on information at time  $t$ , i.e.,  $\hat{\beta}_{i,t+1|t}^m$ ,  $\hat{\beta}_{i,t+1|t}^s$ ,  $\hat{\beta}_{i,t+1|t}^h$  and  $\hat{\beta}_{i,t+1|t}^e$ ,

can be obtained. Second, the difference between the realized return at  $t+1$  and the predicted return formed on the expected risk loadings at  $t$  will be computed:

$$(11) \quad \hat{\alpha}_{i,t+1} = r_{i,t+1} - (\hat{\beta}_{i,t+1|t}^m r_{m,t+1} + \hat{\beta}_{i,t+1|t}^s \text{SMB}_{t+1} + \hat{\beta}_{i,t+1|t}^h \text{HML}_{t+1} + \hat{\beta}_{i,t+1|t}^e \text{TERM}_{t+1}).$$

The pricing error  $\hat{\alpha}_i$  of portfolio  $i$  is defined as the time-series mean of  $\hat{\alpha}_{i,t+1}$ :

$$(12) \quad \hat{\alpha}_i = \frac{1}{T-1} \sum_{t=1}^{T-1} \hat{\alpha}_{i,t+1},$$

and the standard deviation of  $\hat{\alpha}_i$  is defined as:

$$(13) \quad \hat{\sigma}(\hat{\alpha}_i) = \sqrt{\frac{1}{(T-2)} \sum_{t=1}^{T-1} (\hat{\alpha}_{i,t+1} - \hat{\alpha}_i)^2}.$$

In a well-specified asset-pricing model, the pricing error  $\hat{\alpha}_i$  should not be different from zero. The standard  $t$ -statistics can be used to test if the pricing error of portfolios  $i$  is equal to zero:

$$(14) \quad t(\hat{\alpha}_i) = \frac{\hat{\alpha}_i}{\hat{\sigma}(\hat{\alpha}_i) / \sqrt{T-1}}.$$

In addition, we want to know the overall performance of a model. The aggregate pricing error of a set of portfolios can be a good indicator. There are two ways to measure aggregate pricing error. The first one is on the basis of the root mean squared error (RMSE), which gives equal weight on individual pricing error of portfolios from 1 to  $N$ . RMSE can be expressed as:

$$(15) \quad \text{RMSE} = \sqrt{\left(\frac{1}{N} \sum_{i=1}^N \hat{\alpha}_i^2\right)}.$$

The second way to calculate aggregate pricing errors is the composite pricing error (CPE) suggested by Campbell and Vuolteenaho (2004). CPE is computed as:

$$(16) \quad \text{CPE} = \sqrt{\hat{\alpha}' \hat{\Omega}^{-1} \hat{\alpha}},$$

where  $\hat{\alpha}$  is the  $N \times 1$  vector constructed from the individual pricing errors  $\hat{\alpha}_i$  from 1 to  $N$  and  $\hat{\Omega}$  is a  $N \times N$  diagonal matrix with return variances of each portfolio on its main diagonal. The weighting matrix,  $\hat{\Omega}^{-1}$ , plays a role in placing less weight on more volatile portfolios.

## 2.4 Estimation Results

This section reports the estimation results of the time-varying four-factor model. For comparison purposes, the results of the Fama-French three-factor model estimated with OLS are also reported. Moreover, we try to isolate the contributions of the additional risk factor, TERM, and the Kalman filter in reducing pricing errors. Therefore, the outcomes of the four-factor model estimated with OLS and the time-varying three-factor model estimated with the Kalman filter are also shown. The four-factor model estimated with OLS consists of the TERM factor as well as three common risk factors advocated by Fama and French (1993). The time-varying three-factor model includes the Fama-French three factors and it can be expressed as the state-space model like equations (1) and (2). It is also estimated with the Kalman filter.

### 2.4.1 5 by 5 size and B/M double-sorted portfolios

Table 4 demonstrates the individual and aggregate pricing errors produced by different models for 25 size and B/M-sorted portfolios. We look first at Panel A of this table. Panel A displays the estimated intercept of the Fama-French three-factor model



based on the OLS regression.  $t$ -statistics indicate that, among 25 portfolios, there are 7 portfolios of which the pricing error is different from zero at the 5 percent level. Note that 5 of them are located at the row of small (smallest size) portfolios or the column of growth (lowest B/M) portfolios. However, the total number of small and growth portfolios is only 9. The result here confirms that the returns of small and growth portfolios cannot be successfully explained by the Fama-French three-factor model. This is consistent with the findings of Petkova (2006), Adrian and Franzoni (2005), and Chiao, and Hueng (2005). The aggregate pricing error, which is measured by RMSE and CPE, is shown at the bottom of Panel A. RMSE and CPE of the Fama-French three-factor model are 0.153 and 0.423, respectively.

Fama and French (1993) show that only two of small and growth portfolios have pricing errors different from zero. However, our result, as well as the recent studies of Petkova (2006) and Adrian and Franzoni (2005), indicate that the number of small and growth portfolios with significant pricing errors in the Fama-French three-factor model increases. This difference may stem from the inclusion of the data after 1990 in recent studies. The firms in small and growth portfolios may have become more sensitive to changes in the discount rate in the last decade. Particularly, the boom of the initial public offering (IPO) of the high-tech firms in the 1990s could cause a change in the characteristics of firms that constitute the small and growth portfolios. As the high-tech firms tend to be young firms that generate cash flows in a long duration and rely more heavily on external financing, small and growth portfolios that contain a higher proportion of such firms will be more sensitive to changes in the discount rate. Thus, the

Table 4

## The Pricing Errors (in %) of 25 Size and B/M-Sorted Portfolios, 1968:7–2004:12

The table reports the pricing errors of each portfolio from 1968:7 to 2004:12. The individual pricing error equals the estimated intercept in the Fama-French three-factor model and the four-factor model estimated by OLS. For the time-varying three-factor model and the time-varying four-factor model, individual pricing error is defined as time-series mean of the difference between realized return and expected return. In the time-varying parameter models, standard errors are computed as the time-series standard deviation. *t*-statistics are given in parentheses. At the bottom of each panel, the measures of aggregate pricing error, RMSE and CPE, are reported.

Panel A: Fama-French three-factor model estimated by OLS					
	low(growth)	2	B/M	4	high(value)
small	-0.506* (-4.348)	0.039 (0.465)	0.061 (0.911)	0.220* (3.310)	0.137 (1.930)
2	-0.171* (-2.108)	-0.088 (-1.176)	0.049 (0.717)	0.077 (1.166)	0.017 (0.250)
size	-0.028 (-0.363)	0.011 (0.132)	-0.123 (-1.542)	-0.043 (-0.577)	0.003 (0.030)
4	0.180* (2.350)	-0.183* (-2.083)	-0.025 (-0.285)	0.003 (0.036)	-0.123 (-1.244)
big	0.184* (2.978)	0.031 (0.418)	-0.037 (-0.431)	-0.144 (-1.932)	-0.232* (-2.085)
RMSE		0.153		CPE	0.423
Panel B: Four-factor model estimated by OLS					
	low(growth)	2	B/M	4	high(value)
small	-0.081 (-0.456)	0.195 (1.517)	0.075 (0.726)	0.213* (2.070)	-0.008 (-0.073)
2	0.092 (0.746)	0.083 (0.724)	-0.020 (-0.195)	0.076 (0.749)	0.128 (1.221)
size	-0.002 (-0.016)	-0.002 (-0.018)	-0.111 (-0.903)	-0.065 (-0.566)	0.184 (1.386)
4	0.264* (2.244)	-0.117 (-0.861)	0.108 (0.811)	0.069 (0.572)	-0.027 (-0.174)
big	0.141 (1.477)	0.093 (0.819)	0.052 (0.397)	0.090 (0.790)	-0.087 (-0.505)
RMSE		0.115		CPE	0.373

\* Statistically significant at the 5% level.

Table 4–Continued

The Pricing Errors (in %) of 25 size and B/M-Sorted Portfolios, 1968:7–2004:12

Panel C: Time-varying three-factor model estimated by Kalman filter					
	low(growth)	2	B/M	4	high(value)
small	-0.475* (-4.234)	0.032 (0.399)	0.060 (0.953)	0.216* (3.446)	0.138* (2.000)
2	-0.113 (-1.545)	-0.023 (-0.358)	0.061 (1.020)	0.077 (1.274)	0.024 (0.381)
size	-0.006 (-0.084)	0.050 (0.664)	-0.070 (-1.035)	-0.014 (-0.213)	0.042 (0.528)
4	0.208* (2.825)	-0.086 (-1.174)	-0.001 (-0.011)	0.012 (0.160)	-0.104 (-1.119)
big	0.156* (2.690)	0.062 (0.970)	-0.012 (-0.152)	-0.102 (-1.446)	-0.197 (-1.847)
RMSE		0.137		CPE	0.389
Panel D: Time-varying four-factor model estimated by Kalman filter					
	low(growth)	2	B/M	4	high(value)
small	-0.017 (-0.151)	0.092 (1.160)	0.057 (0.904)	0.096 (1.505)	0.019 (0.280)
2	0.035 (0.484)	0.007 (0.106)	-0.069 (-1.170)	-0.015 (-0.256)	0.070 (1.103)
size	0.003 (0.038)	-0.055 (-0.732)	-0.084 (-1.242)	-0.062 (-0.946)	0.062 (0.791)
4	0.162* (2.193)	-0.038 (-0.523)	-0.012 (-0.166)	-0.006 (-0.077)	-0.054 (-0.585)
big	0.002 (0.034)	0.033 (0.508)	0.026 (0.323)	0.041 (0.589)	-0.025 (-0.237)
RMSE		0.058		CPE	0.198

\* Statistically significant at the 5% level.

increasing number of significant pricing errors in small and growth portfolios when the sample extends to include the data after 1990 could result from the inability of the Fama-French three factors to completely capture risk related to innovations in the discount rate.

Panel B exhibits the outcomes of the four-factor model estimated with OLS. After including the TERM factor into the conventional Fama-French three-factor model, we find that the number of significant individual pricing errors has been decreased greatly. Only two individual pricing errors remain significant at the 5 percent level; one of them belongs to growth portfolios. The significant unexplained returns in small or growth portfolios almost disappear after the TERM factor is considered. Since TERM proxies the risk related to innovations to the discount rate, the result here asserts that the returns of small or growth portfolios are sensitive to such risk. Furthermore, RMSE and CPE of the four-factor model estimated with OLS are 0.115 and 0.373, respectively. Compared with Panel A, the aggregate pricing error is reduced by around 25 percent. The results in Panel B confirm our conjecture that adding TERM into the conventional Fama-French model can help reduce abnormal returns in small or growth portfolios because TERM documents the information about risk related to changes in the discount rate that the Fama-French three factors cannot fully capture.

Next, we move to Panel C of Table 4, where we can see the results for the time-varying three-factor model. Among 9 small or growth portfolios, there are 5 whose pricing errors are statistically different from zero. RMSE is decreased by about 10 percent and CPE is decreased by about 15 percent with respect to the results in Panel A. Although the reduction in pricing errors is not substantial, the relatively smaller pricing errors generated by the time-varying three-factor model still implies that the learning process mimicked by the Kalman filter improves the accuracy of forecast for risk loadings relative to the unconditional model. This is consistent with the findings of Adrian and Franzoni (2005).

The results of the time-varying four-factor model are reported in Panel D of Table 4. The performance of the time-varying four-factor model is very impressive: it achieves great reduction in both individual pricing error and aggregate pricing error. The null hypothesis that the pricing error of an individual portfolio is equal to zero can only be rejected in 1 of all 25 portfolios. Apparently, the time-varying four-factor model outperforms the other three models in reducing the individual pricing error. In particular, the forecast accuracy of this model for small and growth portfolios improves significantly in contrast with the Fama-French three-factor model. Significant pricing errors almost vanish in all small or growth portfolios. The aggregate pricing error generated by the time-varying four-factor model further confirms the explanatory power of the time-varying four-factor model. RMSE of this model equals 0.058, which is reduced by over 60 percent compared to the Fama-French three-factor model. CPE is equal to 0.198 in this model. The reduction in CPE is also over 50 percent.

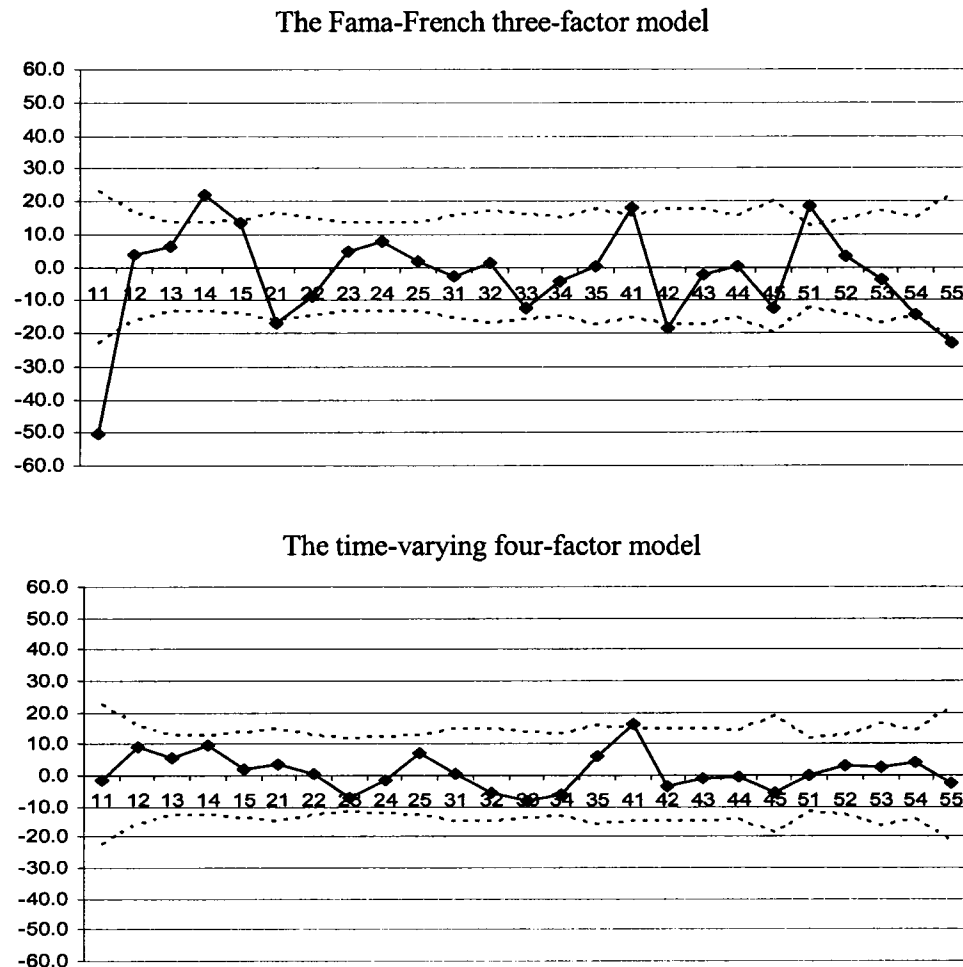
Similarly, the time-varying four-factor model also outperforms the four-factor model estimated with OLS by decreasing both the individual pricing error and aggregate pricing error. A possible interpretation is that although the additional TERM factor conveys information beyond the Fama-French three factors, the OLS regression is still not able to track time-varying risk loadings, which leads to imprecise estimates for risk loadings. Nevertheless, under the framework of the learning process, the accuracy of estimates for risk loadings improves as the Kalman filter captures the dynamics of risk loadings. Hence, the pricing errors in the time-varying four-factor model are further diminished relative to the four-factor model estimated with OLS.

Figure 1 illustrates the individual pricing errors of each 25 size and B/M-sorted portfolios produced by the Fama-French three-factor model and the time-varying four-factor model, which gives us a clearly visual comparison. In general, the absolute values

Figure 1

### Individual Pricing Errors (in basis point) of 25 Size and B/M-Sorted Portfolios

The figure shows individual pricing errors (in basis point, 1 basis point=0.01%) for the Fama-French three factor model and the time-varying four-factor model. The number on vertical axis refers to pricing errors. Each two-digit number on the horizontal axis represents a separate portfolio. The first digit denotes the size quintile (1 being the smallest and 5 the largest). And the second digit denotes the B/M quintile (1 being the lowest and 5 the highest). The dashed lines refer to two-standard error band around zero.



of pricing errors are smaller in the time-varying four-factor model than in the Fama-French model. Almost all the individual pricing errors in the time-varying four-factor model are less than 10 basis points in absolute value. Especially, the reductions in pricing errors of small portfolios (with the first digit equal to 1 on the horizontal axis) and growth portfolios (with the second digit equal to 1 on the horizontal axis) in the time-varying four-factor model are evident when compared with the Fama-French three-factor model. The most problematic portfolio for the Fama-French three-factor model is the small-growth portfolio<sup>8</sup> (Campbell and Vuolteenaho [2005] and Adrian and Franzoni [2005]). The performance of the time-varying four-factor model is so overwhelming that we conclude that this model is more successful at capturing dynamics of risk loadings and conveying more information relative to the discount-rate risk into the estimation process than the Fama-French three-factor model.

#### 2.4.2 10 industry-sorted portfolios

Although the time-varying four-factor model successfully diminishes the pricing errors for size and B/M-sorted portfolios, we are still interested in the applicability of this model. Daniel and Titman (1997) stress that it could be dangerous to examine asset pricing models only with portfolios sorted by characteristics known to be related to average returns, such as size and B/M. Therefore, we also chose 10 industry-sorted portfolios to evaluate the time-varying four-factor model. There is another reason we are concerned with industry-sorted portfolios. Fama and French (1997) find that risk

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<sup>8</sup> The small-growth portfolio refers to the portfolio with the smallest size and lowest B/M in 5 by 5 size and B/M double-sorted portfolios.

loadings of industry-sorted portfolios exhibit great time-variation, so it is difficult to estimate them precisely.

Similar to what we did with 25 size and B/M double-sorted portfolios, we compare the results of the time-varying four-factor model with the other three models for 10 industry sorted portfolios.  $t$ -statistics in Table 5 show that, for the Fama-French three-factor model, the null hypothesis that pricing error equals to zero is rejected for the second and eighth portfolios. After the three-factor model is extended to include the TERM factor, the results do not change much. The pricing errors of these two portfolios are still significantly different from zero at the 5 percent level. Then we look at the time-varying three-factor model estimated by the Kalman filter: the pricing error of the second portfolio is no longer significant and the eighth remains significant. RMSE and CPE in the four-factor model estimated by OLS are both larger than those of the Fama-French three-factor model. Conversely, the time-varying three-factor model based on the Kalman filter reduces RMSE and CPE by 20 percent, respectively, with respect to the Fama-French model. Given these results, we can conclude that the learning process mimicked by the Kalman filter plays a relatively more important role in explaining the returns of industry-sorted portfolios than the TERM factor. The possible cause is that the pricing errors of industry portfolios in the Fama-French three-factor model mainly result from the wedge between OLS estimates and true investors' expectations rather than risk associated with changes in the discount rate.

Nevertheless, the best performance still comes from the time-varying four-factor model.  $t$ -tests indicate that there is no individual pricing error statistically different from zero in this model. Almost all the absolute values of individual pricing errors in the time-



Table 5

## The Pricing Errors (in %) of 10 Industry-Sorted Portfolios, 1968:7-2004:12

The table reports the pricing errors of each portfolio from 1968:7 to 2004:12. The individual pricing error equals the estimated intercept in the Fama-French three-factor model and the four-factor model estimated by OLS. For the time-varying three-factor model and the time-varying four-factor model, individual pricing error is defined as time-series mean of the difference between realized return and expected return. In the time-varying parameter models, standard errors are computed as the time-series standard deviation. *t*-statistics are given in parentheses. At the bottom of each panel, the measures of aggregate pricing error, RMSE and CPE, are reported.

	Fama-French three-factor estimated by OLS	Four-factor model estimated by OLS	Time-varying three-factor model estimated by Kalman filter	Time-varying four-factor model estimated by Kalman filter
1	0.088 (0.686)	0.088 (0.446)	0.105 (0.966)	-0.045 (-0.415)
2	-0.373* (-2.270)	-0.613* (-2.420)	-0.288 (-1.802)	-0.188 (-1.175)
3	-0.143 (-1.522)	-0.250 (-1.728)	-0.094 (-1.132)	-0.099 (-1.198)
4	0.107 (0.546)	0.201 (0.666)	0.122 (0.656)	0.148 (0.796)
5	0.258 (1.640)	0.028 (0.115)	0.128 (0.892)	-0.082 (-0.585)
6	0.065 (0.435)	0.166 (0.726)	-0.008 (-0.056)	0.016 (0.113)
7	-0.047 (-0.354)	0.033 (0.159)	-0.010 (-0.088)	-0.082 (-0.692)
8	0.450* (2.899)	0.850* (3.574)	0.405* (2.996)	0.249 (1.836)
9	-0.184 (-1.275)	-0.167 (-0.749)	-0.134 (-0.963)	-0.039 (-0.288)
10	-0.084 (-1.042)	-0.025 (-0.197)	-0.016 (-0.224)	0.012 (0.170)
RMSE	0.223	0.356	0.178	0.121
CPE	0.054	0.136	0.038	0.018

\* statistically significant at 5% level.

varying four-factor model are smallest among the four models. RMSE and CPE in this model are reduced by around 45 percent and 40 percent with respect to the Fama-French three-factor model. Both of RMSE and CPE are also the smallest among the four models. Once again, the time-varying four-factor model outperforms the other models in explaining returns for industry-sorted portfolios.<sup>9</sup> The consistent and robust performance of the time-varying four-factor model in industry-sorted portfolios implies that this model has a wide application in explaining stock returns.

#### 2.4.3 An alternative sample period

Previous studies have intensively explored the post-1963:7 sample for the 25 size and B/M-sorted portfolios. This first reason is that the book value for firms is not generally available in the pre-1963 COMPUSTAT dataset. Second, the COMPUSTAT has a serious selection bias prior to 1963, which are tilted toward big historically successful firms (Fama and French [1992]). The third reason is that, the CAPM is usually found to fail in explaining cross-sectional returns, especially the book-to-market anomaly in the post-1963:7 sample (Adrian and Franzoni [2005]).

Since many researchers investigate asset pricing with the same dataset, data mining has become a potential problem. Ferson and Harvey (1999) suggest that out-of-sample studies might reduce the risk of data mining. Campbell and Vuolteenaho (2004)

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<sup>9</sup> The paper also experiments with 30 and 48 industry-sorted portfolios provided by Professor Kenneth French on his website. The results confirm the superiority of the time-varying four-factor model. When the Fama-French three-factor model is used, there are 6 and 9 statistically significant pricing errors in the 30 and 48 industry-sorted portfolios, respectively. When the time-varying four-factor model is used, there is no statistically significant pricing error existing in either set of portfolios. Both the RMSE and CPE are reduced by more than 30 percent.

Table 6

## The Pricing Errors (in %) of 25 Size and B/M-Sorted Portfolios, 1953:4–1963:6

The table reports the pricing errors of each portfolio from 1953:4–1963:6. The individual pricing error equals the estimated intercept in the Fama-French three-factor model and the four-factor model estimated by OLS. For the time-varying three-factor model and the time-varying four-factor model, individual pricing error is defined as time-series mean of the difference between realized return and expected return. In the time-varying parameter models, standard errors are computed as the time-series standard deviation. *t*-statistics are given in parentheses. At the bottom of each panel, the measures of aggregate pricing error, RMSE and CPE, are reported.

Panel A: The Fama-French three-factor model					
	low(growth)	2	B/M	4	high(value)
small  2  size  4  big	-0.217 (0.529)	-0.213 (0.728)	-0.088 (0.424)	0.217* (2.009)	0.159 (1.424)
	-0.333 (-1.603)	0.167 (0.967)	-0.001 (-0.005)	0.022 (0.200)	0.238* (2.007)
	0.082 (0.604)	0.091 (0.714)	-0.008 (-0.071)	0.105 (0.897)	-0.246 (-1.658)
	-0.074 (-0.635)	0.131 (1.284)	0.255* (2.167)	-0.121 (-0.902)	-0.384 (-1.872)
	-0.092 (-1.147)	-0.034 (-0.286)	0.434* (2.946)	-0.391* (-2.747)	-0.310 (-1.735)
RMSE		0.362	CPE		1.193
Panel B: The time-varying four-factor model					
	low(growth)	2	B/M	4	high(value)
small  2  size  4  big	-0.025 (-0.065)	-0.132 (-0.472)	-0.011 (-0.059)	0.058 (0.575)	0.103 (0.954)
	-0.083 (-0.417)	0.035 (0.210)	0.030 (0.281)	0.062 (0.601)	0.022 (0.194)
	0.027 (0.211)	-0.090 (-0.750)	-0.007 (-0.063)	-0.029 (-0.261)	0.072 (0.505)
	-0.017 (-0.157)	0.044 (0.448)	-0.011 (-0.099)	0.027 (0.207)	-0.037 (-0.192)
	-0.009 (-0.116)	0.007 (0.059)	0.116 (0.810)	-0.050 (-0.380)	-0.128 (-0.744)
RMSE		0.062	CPE		0.221

\* statistically significant at 5% level.

argue that the pre-1963 sample provides an opportunity for an out-of-sample test because this sample is relatively untouched in comparison with the well mined post-1963 sample. Therefore, in this subsection, we experiment with the pre-1963:7 data of the 25 size and B/M double-sorted portfolios.<sup>10</sup> Due to the availability of the bond return data, we can only extend our experiment back to 1953. Table 6 reports the results of 25 size and B/M double-sorted portfolios for the period 1953:4-1963:6. Panel (A) shows the pricing errors from the Fama-French constant loadings three-factor model and Panel (B) shows those from the time-varying four-factor model. There are five individual pricing errors that are significantly different from zero in Panel (A). Note that these significant pricing errors are not observed particularly in small or growth portfolios. Although these observations are not where the original motivation of this paper is derived from, they can be explained by the selection bias (toward big historically successful firms) prior to 1963:7. Nonetheless, the time-varying four-factor model still eliminates all these pricing errors. The model does not produce any significant individual pricing errors for all 25 portfolios. Both the RMSE and the CPE are reduced by more than 80% from the Fama-French three-factor model to the time-varying four-factor model.

## 2.5 Conclusions

The Fama-French three-factor model has had an influential impact on the development of asset pricing models. However, empirical studies show that this model cannot fully capture the cross-sectional average returns, especially small or growth portfolios. This triggers us to develop a new time-varying four-factor model. Our model

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<sup>10</sup> The data is downloaded from Professor Kenneth French's website.

has two main differences from the Fama-French three-factor model. The first one is that, rather than using the OLS estimation, a time-varying parameter model based on the Kalman filter is employed to estimate risk loadings. Since empirical evidence indicates that risk loadings vary over time, the Kalman filter is expected to enhance the accuracy in estimates of time-varying risk loadings. The Kalman filter used in this paper aims at mimicking investors' learning process. The second difference is that the TERM factor, which measures the yield spread between 10-year government bond and 3-month Treasury bill rate, is included in the model in addition to the Fama-French three factors. The TERM factor is expected to capture the risk related to changes in the discount rate for which the Fama-French three-factors can not fully account. Both Cornell (1999) and Campbell and Vuolteenaho (2004) suggest that small and growth portfolios are more sensitive to such risk.

Using data from the U.S. stock market, we first estimate the time-varying four-factor model for 25 size and B/M -sorted portfolios. To evaluate the performance of this model, we also investigate the Fama-French three-factor model estimated with OLS, the four-factor model estimated with OLS, and the time-varying three-factor model estimated with the Kalman filter. The results show that the pricing errors of 5 out of 9 small and growth portfolios are significantly different from zero in the Fama-French three-factor model, which is in line with the results of other studies. Through comparing, we find that both the four-factor model estimated with OLS and the time-varying three-factor model can partially reduce the individual and aggregate pricing errors relative to the Fama-French three-factor model. Although the improvement is not substantial, it still confirms the sole contribution of the TERM factor and the Kalman filter in reducing the pricing

errors. The combination of the TERM factor and the Kalman filter improves the outcomes remarkably. The time-varying four-factor model significantly decreases both the individual and the aggregate pricing errors for 25 size and B/M-sorted portfolios. The individual pricing errors all become insignificant from zero except one. RMSE and CPE, which measure the aggregate pricing error, reduce 60 percent and 50 percent, respectively, when compared to the Fama-French three-factor model. The great reduction in pricing errors of the time-varying four-factor model implies that this model does a good job in explaining the cross-sectional returns for size and B/M double-sorted portfolios.

To check the robustness of the results and avoid the potential data mining problem, we apply the time-varying four-factor model to industry-sorted portfolios to see whether this model has an explanatory power for the average returns of these portfolios. We compare the time-varying four-factor model with the other three models. The outcomes again illustrate that the time-varying four-factor model outperforms the other three models. In particular, the significant individual pricing errors found in the Fama-French three-factor model vanish in the time-varying four-factor model. And the time-varying four-factor model also produces aggregate pricing errors that are much smaller than the Fama-French three-factor model. An out-of-sample experiment that uses 5 by 5 size and B/M double-sorted portfolios from 1953:4 to 1963:6 is also conducted. The results again show that the time-varying four-factor model remarkably reduces both the individual and aggregate pricing errors in comparison with the Fama-French three-factor model.

We attribute the strong explanatory power of the time-varying four-factor model in explaining cross-sectional returns to two reasons: (1) the Kalman filter improves the

accuracy of the estimation of risk loadings expectation since the learning process mimicked by it captures the dynamics of risk loadings that the common OLS estimation can not; (2) the additional TERM factor contains some information related to innovations in the discount rate for which the Fama-French three risk factors can not fully account, and therefore it enhances the explanatory power of the learning model.

## CHAPTER III

### THE CONDITIONAL RISK-RETURN RELATIONSHIP WITH A TIME-VARYING-PARAMETER MODEL

#### 3.1 Introduction

The capital asset pricing model (CAPM) derived by Sharpe (1964) and Lintner (1965) is one of the most important asset pricing models to describe the risk-return relationship. Substantial empirical work has been conducted to investigate the Sharpe-Lintner CAPM. Fama and MacBeth (1973) initiated a three-step approach to test the CAPM, which has become the standard methodology in the literature. However, many empirical studies show that the Sharpe-Lintner CAPM provides an inadequate explanation of the risk-return relationship due to the lack of the evidence that indicates a statistically significant relationship between risk and return (e.g., Fama and French [1992] and He and Ng [1994]). The unsuccessful empirical performance of the CAPM causes people to cast doubts on the model. The criticism of the CAPM aims either at the theoretical foundations of the model or at the validity of testing methodologies. Many researchers argue that the empirical failure of the CAPM arises from the deficiencies of the Fama and MacBeth (1973) methodology.

The well-known Sharpe-Lintner CAPM has an equation form:

$$(17) \quad E(R_i) - R_f = \beta_i (E(R_i) - R_f).^{11}$$

This model shows us that the expected excess return of portfolio  $i$ , represented by the expected return ( $E(R_i)$ ) minus the risk-free rate ( $R_f$ ), equals  $\beta$  of portfolio  $i$  times the

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<sup>11</sup> The Sharpe-Lintner CAPM only concentrates on the first two moments of stock returns



expected market excess return, represented by the expected market return ( $E(R_m)$ ) minus the risk-free rate.  $\beta_i$  measures the systematic risk for portfolio  $i$ , which equals the covariance between the return of portfolio  $i$  and the market return divided by the variance of the market return:

$$(18) \quad \beta_i = \frac{\text{cov}(R_i, R_m)}{\sigma_{R_m}^2}.$$

From equation (17), we see that the Sharpe-Lintner CAPM focuses on the relationship between the expected return and the risk ( $\beta$ ) of portfolios. Due to the unobservability of the data for expected returns on portfolios, Fama and MacBeth (1973) use realized returns to proxy expected returns. However, Pettengill, Sundaram and Mathur (PSM) (1995) claim that the validity of the Sharpe-Lintner CAPM is not directly examined with the Fama and MacBeth (1973) methodology because realized returns rather than expected returns are used. They argue that for each portfolio, there must be a non-zero probability of which the realized return is smaller than the risk-free rate. However, the Sharpe-Lintner CAPM (equation [17]) emphasizes expected returns of portfolios, which must be greater than the risk-free rate. Thus, to solve the problem of using realized returns instead of expected returns, PSM (1995) partition the market into an up market and a down market based on the realized market excess return. With the data of realized returns, they estimate the risk-return relationship for the up market and the down market, respectively. Their results indicate that a positive risk-return relationship exists when the realized market return is greater than the risk-free rate and an inverse risk-return relationship exists when the realized market return is smaller than the risk-free rate.

Although PSM (1995) show the significant risk-return relation based on the up and down markets, they neglect the important fact that  $\beta$  in the CAPM tends to be time-variant. Similar to Fama and MacBeth (1973), PSM (1995) assume beta is constant over time. However, many studies, such as Harvey (1989), Ferson and Harvey (1991, 1993), and Jagannathan and Wang (1996), document that  $\beta$  in the CAPM shows great time-variation. Such studies cause us to question the credibility of the empirical results derived from the assumption of the constant  $\beta$  under the PSM (1995) framework.

In order to improve the accuracy of  $\beta$  estimation and derive more reliable results for the risk-return relation dependent on the up and down markets, a time-varying  $\beta$  is taken into account in this paper. Due to the unobservability of  $\beta$  in the CAPM, Adrian and Franzoni (2004, 2005) argue that an econometric model that fails to mimic the investors' learning process of time-evolving  $\beta$  might lead to inaccurate estimates of  $\beta$ . According to McCulloch (2005), adaptive least squares (ALS) with Kalman foundations provides a better way of estimating time-varying coefficients and proxying agents' time-evolving expectations by incorporating the learning process. This methodology not only nests the Kalman solution of the elementary local level model (LLM), but also proposes a simple way to setup a rigorous initialization. Hence, in attempt to capture the dynamics of  $\beta$ , we are the first to employ ALS with Kalman foundations to replicate the investors' learning process and model the movements of  $\beta$ .

To evaluate the performance of the time-varying  $\beta$  CAPM estimated with the ALS based on Kalman foundations, we apply the model to 10 industry-sorted portfolios formed by stocks listed in S&P 500. The estimation period covers from November 2, 1987 to December 31, 2003. Due to the use of the data of realized returns, this paper

examines the risk-return relationship under the up and down market conditions, respectively. In addition, we replicate the PSM (1995) model with the assumption of a constant  $\beta$  for comparison purposes. The results show that for both the time-varying  $\beta$  CAPM and the PSM model, when the realized market excess return is positive, there exists a significant and positive risk-return relationship; when the realized market excess return is negative, there exists a significant and negative risk-return relationship.

Moreover, our results suggests that the time-varying  $\beta$  estimated with ALS based on Kalman foundations is more successful at explaining the cross-sectional returns than the constant  $\beta$  estimated with OLS in the PSM model. First, the estimated intercepts, which represent the unexplained returns of a model, are found to be statistically different from zero for both the up and down markets in the PSM model. This indicates that the constant  $\beta$  estimated by the PSM model cannot fully account for the cross-sectional returns. In contrast, for our time-varying  $\beta$  CAPM based on ALS with Kalman foundations, neither of the estimated intercepts is significantly different from zero under the up and down markets. Second, the estimated value of the risk-return relation can be regarded as the price paid for the  $\beta$  risk. According to Fama and MacBeth (1973) and Isakov (1999), the estimated risk-return relation should not be statistically different from the realized market excess return. The empirical evidence shows that the estimated risk-return relationship is significantly different from the realized market excess return for the up market in the PSM model. By contrast, the estimated risk-return relation derived from the time-varying  $\beta$  based on ALS with Kalman foundations is found to be not statistically different from the realized market excess return for both the up and down markets. In

addition, the magnitude of the risk-return relation estimated by our model is closer to the realized market excess return than that of the PSM model.

In general, the results mentioned above indicate that the CAPM based on ALS with Kalman foundations outperforms the PSM model in estimating the risk-return relation for both the up and down markets. The dominant performance of our model implies that ALS with Kalman foundations successfully improves the accuracy of the estimation of  $\beta$  by mimicking the investors' learning process of the unobservable  $\beta$  for which the common OLS methodology cannot account.

The rest of this paper is organized as follows. The next section introduces a time-varying  $\beta$  CAPM estimated via the ALS with Kalman foundations and explains the risk-return relationship under the up and down market conditions. Section 3.3 describes the data and the estimation methodology. Section 3.4 reports and analyzes the estimation results. Section 3.5 provides concluding remarks.

### 3.2 A Time-Varying $\beta$ CAPM and the Asymmetric Risk-Return Relationship

#### 3.2.1 A time-varying $\beta$ CAPM based on ALS with Kalman foundations

To test the risk-return relationship in a CAPM, PSM (1995) assume that  $\beta$  is constant over time. The assumption of constant  $\beta$  has been challenged by many studies, such as Harvey (1989), Ferson and Harvey (1991, 1993), and Jagannathan and Wang (1996). All these studies indicate that  $\beta$  tends to be volatile over time. The evidence of time-varying  $\beta$  causes us to cast doubt on the results derived from the assumption of the constancy of  $\beta$  under the PSM (1995) framework. Jagannathan and Wang (1996) provide a possible economic reason to explain why  $\beta$  changes over time. They argue that the

conditional  $\beta$  and the expected market return should be correlated with each other. During the periods of recession, the expected market risk premium is high. Companies that are in relatively poor financial conditions are likely to sharply increase their financial leverages compared to other companies. Consequently, the risk level of these company increases, which means  $\beta$  rises.

As  $\beta$  in the CAPM is not observable when the CAPM holds conditionally, previous papers employ different methods to estimate time-varying  $\beta$ . Jagannathan and Wang (1996), Harvey and Campbell (1999), and Lettau and Ludvigson (2001) treat  $\beta$  as a function of state variables<sup>12</sup> in a conditional CAPM. Engle, Bollerslev, and Wooldridge (1988) model the movements of  $\beta$  in a generalized autoregressive conditional heteroskedastic (GARCH) model. Adrian and Franzoni (2004, 2005) stress that an econometric model that fails to mimic the investors' learning about  $\beta$  could lead to the wedge between the investors' inference of  $\beta$  and  $\beta$  estimated from the model. They argue that the OLS regression can not successfully capture the dynamic of  $\beta$  and suggest using the Kalman filter procedure to mimic the investors' learning process. Their estimation results show that the learning type of the CAMP outperforms the unconditional CAPM by reducing pricing errors.

Motivated by Adrian and Franzoni (2004, 2005), we use ALS with Kalman foundations proposed by McCulloch (2005) to estimate time-varying  $\beta$  in a CAPM. ALS with Kalman foundations not only incorporates the Kalman solution of the LLM<sup>13</sup> but

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<sup>12</sup> State variables refer to underlying economic variables that can capture variation in future investment opportunities.

<sup>13</sup> The local level model has a simple form and it expresses a process  $y_t$  as the sum of a Gaussian random walk  $\mu_t$  and independent Gaussian white noise  $\varepsilon_t$ : (19)  $y_t = \mu_t + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ , and (20)  $\mu_t = \mu_{t-1} + \nu_t, \nu_t \sim N(0, \sigma_\nu^2)$ , where  $\varepsilon_t$  and  $\nu_t$  are independent of each other.

also employs a simple and rigorous initialization. Therefore, it is considered to be a more accurate and elegant method to estimate time-varying parameters. In order to use ALS with Kalman foundations to estimate time-varying  $\beta$ , our conditional CAPM needs to be represented in the following state-space form:

$$(21) \quad r_{i,t} = \mathbf{x}_t \boldsymbol{\lambda}_{i,t} + \varepsilon_{i,t},$$

$$(22) \quad \boldsymbol{\lambda}_{i,t} = \boldsymbol{\lambda}_{i,t-1} + \boldsymbol{\eta}_{i,t},$$

where  $r_{i,t}$  denotes the excess return of portfolio  $i$  (the portfolio return minus the risk-free rate) at time  $t$ .  $\mathbf{x}_t = (1, r_{m,t})$  is a  $1 \times 2$  row vector in which  $r_{m,t}$  denotes the market excess return (the market return minus the risk-free rate).  $\boldsymbol{\lambda}_{i,t} = (\alpha_{i,t}, \beta_{i,t})'$  is a  $2 \times 1$  column vector in which  $\beta_{i,t}$  represents the  $\beta$  risk of portfolio  $i$  at time  $t$  and  $\alpha_{i,t}$  is a time-varying intercept.  $\varepsilon_{i,t} \sim N(0, \sigma_{\varepsilon,i}^2)$  and  $\boldsymbol{\eta}_{i,t} \sim N(0, \mathbf{Q}_{i,t})$ , where  $\sigma_{\varepsilon,i}^2$  is a scalar and  $\mathbf{Q}_{i,t}$  is a  $2 \times 2$  covariance matrix. Note that  $\beta_{i,t}$  and  $\alpha_{i,t}$  in  $\boldsymbol{\lambda}_{i,t}$  are unobservable variables and we assume they follow a random walk. The idiosyncratic shocks to portfolio  $i$ ,  $\varepsilon_{i,t}$  and  $\boldsymbol{\eta}_{i,t}$ , are independent of each other and uncorrelated with shocks to other portfolios. To obtain the estimation of the investors' expectation of  $\beta$  at time  $t$ , we assume:

$$(23) \quad \boldsymbol{\lambda}_{i,t} | \mathbf{r}_{i,t} \sim N(\mathbf{b}_{i,t}, \mathbf{P}_{i,t}),$$

where  $\mathbf{r}_{i,t} = (r_{i,1}, \dots, r_{i,t})'$ .  $\mathbf{b}_{i,t}$  is the expected value of  $\boldsymbol{\lambda}_{i,t}$  conditional on the information up time  $t$  and  $\mathbf{P}_{i,t}$  is a  $2 \times 2$  covariance matrix of  $\boldsymbol{\lambda}_{i,t}$  conditional on the information up to  $t$ .

The state-space form as shown in equations (21) and (22) can be solved by the extended Kalman filter:

$$(24) \quad \mathbf{b}_{i,t} = \mathbf{b}_{i,t-1} + f_{i,t}^{-1} (\mathbf{P}_{i,t-1} + \mathbf{Q}_{i,t}) \mathbf{x}_t' (r_{i,t} - \mathbf{x}_t \mathbf{b}_{i,t-1}),$$

$$(25) \quad \mathbf{P}_{i,t} = (\mathbf{P}_{i,t-1} + \mathbf{Q}_{i,t})(\mathbf{I} - f_{i,t}^{-1} \mathbf{x}_t' \mathbf{x}_t (\mathbf{P}_{i,t-1} + \mathbf{Q}_{i,t})),$$

$$(26) \quad f_{i,t} = \mathbf{x}_t (\mathbf{P}_{i,t-1} + \mathbf{Q}_{i,t}) \mathbf{x}_t' + \sigma_{\varepsilon,i}^2,$$

where  $\mathbf{I}$  is a  $2 \times 2$  identity matrix. From equation (24), we can see that  $\mathbf{b}_{i,t}$  is updated through the prediction error  $(r_{i,t} - \mathbf{x}_t' \mathbf{b}_{i,t-1})$ . Therefore,  $\mathbf{b}_{i,t}$  contains new information beyond  $\mathbf{b}_{i,t-1}$ . The term  $f_{i,t}^{-1} (\mathbf{P}_{i,t-1} + \mathbf{Q}_{i,t}) \mathbf{x}_t'$  in equation (24) is called the Kalman gain, which determines how much weight to be assigned to the prediction error  $(r_{i,t} - \mathbf{x}_t' \mathbf{b}_{i,t-1})$ . In practice, investors continue to adjust their inference of unobservable  $\beta$  of portfolio  $i$  through learning prediction errors. Thus, the dynamic process like equations (24), (25), and (26) can be used to mimic the investors' learning process of  $\beta$ .

To simplify the extended Kalman filter as shown by equations (24), (25), and (26) and get a rigorous long-run ALS gain coefficient, McCulloch (2005) assumes that  $\mathbf{Q}_{i,t}$  (the covariance matrix of the transition error  $\boldsymbol{\eta}_{i,t}$  in equation [22]) is directly proportional to  $\mathbf{P}_{i,t-1}$  in the spirit of Ljung (1992) and Sargent (1999):

$$(27) \quad \mathbf{Q}_{i,t} = \rho_i T_{i,t-1} \mathbf{P}_{i,t-1},$$

where  $\rho_i$  is the signal/noise ratio. It is the index of the uncertainty of the transition error ( $\mathbf{Q}_{i,t}$ ) to the measurement error per effective observation at time  $t-1$  ( $T_{i,t-1} \mathbf{P}_{i,t-1}$ ).  $T_{i,t}$  as shown by McCulloch (2005), is derived based on the Kalman solution of the LLM.<sup>14</sup> It measures the effective sample size and can be expressed as:

$$(28) \quad T_{i,t} = (1 + \rho_i T_{i,t-1})^{-1} T_{i,t-1} + 1.$$

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<sup>14</sup> See McCulloch (2005) for the details of the derivation of  $T_{i,t}$  from the LLM.

The initialization of  $T_{i,t}$  is zero. If  $\rho_i > 0$ ,  $T_{i,t}$  increases as the sample size increases.

When  $t \rightarrow \infty$ ,  $T_{i,t}$  can be determined as an asymptotic approximation:

$$(29) \quad \lim_{t \rightarrow \infty} T_{i,t} = T_i = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{\rho_i}}.$$

As we show later, the gain coefficient for ALS with Kalman foundations is the inverse of  $T_{i,t}$ .

Based on equation (27), the filter equations (24), (25), and (26) can be rearranged to a more convenient “information form”:<sup>15</sup>

$$(30) \quad \mathbf{b}_{i,t} = \mathbf{W}_{i,t}^{-1} \mathbf{z}_{i,t},$$

$$(31) \quad \mathbf{P}_{i,t} = \sigma_{\varepsilon,i}^2 \mathbf{W}_{i,t}^{-1},$$

where

$$(32) \quad \mathbf{z}_{i,t} = (1 + \rho_i T_{i,t})^{-1} \mathbf{z}_{i,t-1} + \mathbf{x}_t' r_{i,t},$$

$$(33) \quad \mathbf{W}_{i,t} = (1 + \rho_i T_{i,t})^{-1} \mathbf{W}_{i,t-1} + \mathbf{x}_t' \mathbf{x}_t.$$

Equations (30)–(33) together are called the ALS filter with Kalman foundations because  $T_{i,t}$  in this system is derived based on the Kalman solution of the LLM and it can be updated through equation (28). In this paper, we will employ the filter based on equations (30)–(33) to estimate time-varying  $\beta$ .

To see the advantages of ALS with Kalman foundations over previous ALS, we set  $\mathbf{R}_{i,t} = \frac{\mathbf{W}_{i,t}}{T_{i,t}}$ . Then equations (30)–(33) can be rearranged to the recursive least squares (RLS) form:

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<sup>15</sup> See McCulloch (2005) for the details of the rearrangement of the filter equations (24), (25), and (26).



$$(34) \quad \mathbf{b}_{i,t} = \mathbf{b}_{i,t-1} + \gamma_{i,t} \mathbf{R}_{i,t}^{-1} \mathbf{x}'_t (r_{i,t} - \mathbf{x}_t \mathbf{b}_{i,t-1}),$$

$$(35) \quad \mathbf{R}_{i,t}^{-1} = \mathbf{R}_{i,t-1}^{-1} + \gamma_{i,t} (\mathbf{x}'_t \mathbf{x}_t - \mathbf{R}_{i,t-1}^{-1}),$$

$$(36) \quad \mathbf{P}_{i,t} = \gamma_{i,t} \sigma_{\varepsilon,i}^2 \mathbf{R}_{i,t}^{-1},$$

where  $\gamma_{i,t}$  is the gain coefficient and it can be proved to be just equivalent to  $\frac{1}{T_{i,t}}$  in ALS

with Kalman foundations . As mentioned above,  $T_{i,t}$  is based on the Kalman solution of the LLM, so we know that the gain coefficient  $\gamma_{i,t}$  also nests the rigorous Kalman solution of the LLM. McCulloch (2005) asserts that  $\gamma_{i,t}$  can be estimated by maximum likelihood (ML). By contrast, previous ALS studies (e.g., Ljung [1992] and Sargent [1999]) treat  $\gamma_{i,t}$  as a constant. Moreover, the value of this gain coefficient is set arbitrarily in those studies. Hence, in comparison with previous ALS literature, the most important improvement of ALS with Kalman foundations developed by McCulloch (2005) is the time-varying gain coefficient  $\gamma_{i,t}$  derived from the Kalman solution of the LLM.

ALS with Kalman foundations based on equations (30)–(33) needs initial values for “information form,”  $\mathbf{z}_{i,0}$  and  $\mathbf{W}_{i,0}$ . According to McCulloch (2005), at time 0 there is a diffuse prior about the coefficients. Thus, all the eigenvalues of the covariance matrix  $\mathbf{P}_{i,0}$  would be infinite, which implies the elements in  $\mathbf{P}_{i,0}^{-1}$  are all zeros. Given this result, it is reasonable to initialize equations (32) and (33) with zeros:

$$(37) \quad \mathbf{z}_{i,0} = \mathbf{0}_{2 \times 1},$$

$$(38) \quad \mathbf{W}_{i,0} = \mathbf{0}_{2 \times 2}.$$

On the contrary, previous ALS studies arbitrarily setup initial values for parameters needed to be estimated. Here, we see that ALS with Kalman foundations circumvents this problem and provides a simple but rigorous initialization. This is another advantage of the algorithm over the former studies.

With the initial values from equations (37) and (38) and the updated filter based on equations (30)–(33), the log-likelihood for ALS with Kalman foundations can be determined by the equation below:

$$(39) \quad r_{i,t} | \mathbf{r}_{i,t-1} \sim N(\mathbf{x}_t \mathbf{b}_{i,t-1}, \sigma_{\varepsilon,t}^2 s_{i,t}^2),$$

where  $s_{i,t}^2 = (1 + \rho_i T_{i,t}) \mathbf{x}_t \mathbf{W}_{i,t-1}^{-1} \mathbf{x}_t' + 1$ . Given equation (39), the gain coefficient  $\gamma_{i,t}$ , that is the inverse of  $T_{i,t}$ , can be estimated by ML.

In short, compared to the former ALS proposed by Ljung (1992) and Sargent (1999), ALS with Kalman foundations proposed by McCulloch (2005) nests the rigorous Kalman solution of the LLM. It not only allows the ALS gain coefficient  $\gamma_{i,t}$  to be estimated by ML, but also provides a convenient and rigorous way to determine the initial values. Given the advantages of ALS with Kalman foundations, we believe that this approach would be more successful at improving the accuracy of estimates for conditional  $\beta$  and capturing the dynamics of  $\beta$ .

### 3.2.2 The risk-return relationship based on the up and down market conditions

According to equation (17), the Sharpe-Lintner CAPM can be expressed as:

$$(40) \quad E(r_i) = \beta_i E(r_m),$$

where  $r_i$  and  $r_m$  denote the excess return of portfolio  $i$  and the market excess return respectively. Equation (24) shows us a positive risk-return tradeoff because both the expected portfolio excess return ( $E(r_i)$ ) and the expected market excess return ( $E(r_m)$ ) should be positive, otherwise no one will buy risky assets. Given a certain positive expected market excess return, the higher the risk ( $\beta$ ) of one portfolio, the higher the expected return of this portfolio. Therefore, based on the Sharpe-Lintner CAPM, the risk-return relationship should be tested with expected returns.

However, all empirical research uses realized returns to proxy expected returns:

$$(41) \quad R_i - R_f = \beta_i (R_m - R_f) .$$

Pettengill, Sundaram and Mathur (PSM) (1995) argue that the use of realized returns instead of expected returns could be the reason that the Sharpe-Lintner CAPM fails in empirical tests. One important fact has been neglected for equation (41): during some periods, both the realized return of portfolio  $i$  and the realized market return can be less than the risk-free rate. This means that the realized excess return of one portfolio and the realized market excess return can be negative ( $(R_i - R_f) < 0$  and  $(R_m - R_f) < 0$ ), which is contrary to the assumption that both the expected return of one portfolio and the expected market return must exceed the risk-free rate in the Sharpe-Lintner CAPM.

Since different portfolios have distinct return distributions, for portfolios with a higher  $\beta$  to have higher risk, there must be some probability that the realized return of a higher  $\beta$  portfolio is less than the return of a lower  $\beta$  portfolio. If this were not the case, no one would invest in lower  $\beta$  portfolios. It is easy to understand this argument when the market is divided into the up and down two regimes. In equation (41), we can see that if the realized market return is less than the risk-free rate ( $(R_m - R_f) < 0$ ), for

portfolios with a positive  $\beta$ , the realized returns of such portfolios will be negative ( $(R_m - R_f) < 0$ ). Under this situation, the realized excess return of a higher  $\beta$  portfolio is less than the realized excess return of a lower  $\beta$  portfolio. In contrast, when the realized market return is greater than the risk-free rate, the realized excess return of a higher  $\beta$  portfolio exceeds the excess return of a lower  $\beta$  portfolio. On the basis of this analysis, PSM (1995) find that if realized returns are used, a positive risk-return relation exists when the market excess return is positive and a negative risk-return relation exists when the market excess return is negative.

Following PSM (1995), Isakov (1999) examines the risk-return relation based on the up and down markets for the Swiss stock market and obtains similar results to PSM. Fraser et al. (2004) investigate 10 industry-sorted portfolios in the U.K. stock market. With the constant  $\beta$  estimated by OLS and the time-varying  $\beta$  estimated by a GARCH model, their results show that the estimated risk-return relation is only found to be negative and significant in the down market. Sandoval and Saens (2004) examine the risk-return relation for the four main Latin American (Argentina, Brazil, Chile, and Mexico) stock markets using the same approach as PSM (1995). Their results indicate that a significant and positive risk-return relation exists in the up market and a significant and negative risk-return relation exists in the down market. With 127 U.S. industry portfolios, Galagedera and Faff (2004) analyze the risk-return relation dependent on both the magnitude of market volatility and the up and down market conditions. Their results also show the existence of the positive risk-return relation in the up market and the existence of the negative risk-return relation in the down market.

Although these papers get the similar results to PSM (1995), they don't pay attention to the reliability of the risk-return relationship estimated with the PSM model. By contrast, we argue that, in the PSM model, the risk-return relation estimated on the basis of a constant  $\beta$  could be inaccurate because substantial financial literature finds that  $\beta$  tends to be time-varying. Therefore, in this paper we not only pay attention to the sign of the estimated risk-return relation, we also emphasize analyzing both the unexplained part of returns represented by the estimated intercept and the value of the estimated risk-return relation. Furthermore, this paper distinguishes from the previous papers by emphasizing capturing the dynamic of  $\beta$  via an econometric model that is able to mimic the investors' learning process of time-evolving  $\beta$ .

Since realized returns are used in this paper to examine the risk-return relation, we define the market into the up and down regimes following the PSM (1995) methodology. If the realized market return exceeds the risk-free rate ( $R_m > R_f$ ), the market is an up market; if the realized market return is less than the risk-free rate ( $R_m < R_f$ ), the market is a down market. Given this division, the relationship between risk and return can be examined in terms of different market conditions.

### 3.3 Data and Methodology

#### 3.3.1 Data

The data used in this paper comprise the daily weekday returns of 10 industry-sorted portfolios formed by stocks in S&P 500 from November 2, 1987 to December 31, 2003. Table 7 reports the time-series means and standard deviations of the daily returns of 10 portfolios. The total number of days is 4079. We choose 385 companies listed in

S&P 500 for the dataset. These 385 companies are classified into 10 industry sectors by the Global Industry Classification Standard (GICS) code, which includes energy, material, industrials, consumer discretionary, consumer staples, health care, financials, information technology, telecom services, and utilities industries. We select these 385 companies because all have been listed in S&P 500 throughout the entire sample period. The source of the dataset is the Center of Research for Security Prices (CRSP) at University of Chicago.

Table 7  
Basic Statistics of the Returns (in %) for 10 Industry-Sorted Portfolios

The table presents the daily (November 2, 1987–December 31, 2003) means and standard deviations (S.D.) of the value weighted returns for 10 portfolios sorted by industry sectors. Total 385 companies are classified into ten industry sectors by the Global Industry Classification Standard (GICS) code. Industries from 1 to 10 refer to energy, material, industrials, consumer discretionary, consumer staples, health care, financials, information technology, telecom services and utilities industry, respectively.

Industry	1	2	3	4	5	6	7	8	9	10
Mean	0.063	0.069	0.079	0.085	0.085	0.088	0.092	0.111	0.060	0.058
S.D.	1.299	1.287	1.194	1.212	1.099	1.280	1.316	1.800	1.405	0.966

Table 8 presents the time-series means and standard deviations of the market excess return with and without considering the up and down markets. The market return used in this paper is defined as the value-weighted return on all NYSE, AMEX, and NASDAQ stocks. To measure the risk-free rate, the 1-month U.S. Treasury bill rate is used. The data for these two series are kindly provided by Professor Kenneth French on his website

Table 8

## Basic Statistics of the Market Excess Returns (in %)

The table presents the daily (November 2, 1992–December 31, 2003) means and standard deviations (S.D.) of the market excess return.<sup>16</sup> If the market return ( $R_{m,t}$ ) is greater than the risk-free rate ( $R_{f,t}$ ), the market is defined as the up market. If the market return ( $R_{m,t}$ ) is less than the risk-free rate ( $R_{f,t}$ ), the market is defined as the down market.<sup>17</sup>

	The whole period	The up market	The down market
The number of days	2814	1485	1329
Mean	0.031	0.739	−0.760
S.D.	1.064	0.734	0.781

3.3.2 Estimation procedure

The general approach of the empirical estimation used in this paper is a modification of the Fama and MacBeth (1973) in/out of sample methodology. The methodology includes two stages. The first stage is the  $\beta$  estimation period in which  $\beta$  for each of 10 industry-sorted portfolios will be estimated with a time-series regression. For the time-varying  $\beta$  CAPM, ALS with Kalman foundations is employed to estimate time-varying  $\beta$ . The second stage is the test period in which the risk-return relationship is investigated based on the up and down market conditions. A cross-section estimation will be run by regressing the return of each portfolio at time  $t+1$  against the conditional  $\beta$  estimated based on the information up to time  $t$  from the first stage. For comparison purposes, we also estimate the PSM (1995) model of which  $\beta$  is assumed to be time-invariant. In the PSM model,  $\beta$  for each portfolio will be estimated with OLS by using

<sup>16</sup> As the PSM model needs the first five-year (November 2, 1987–October 30, 1992) data to estimate  $\beta$ , we focus on the test period from November 2, 1992 to December 31, 2003.

<sup>17</sup> During this sample period, none of the daily market returns is found to be just equal to zero.

the five-year data at the first stage. This estimated  $\beta$  will proxy the  $\beta$  risk of this portfolio in the following five years. In line with PSM (1995),  $\beta$  will be updated every five years.

In this paper, we focus on analyzing the risk-return relation in a CAPM dependent on the up and down market conditions because the realized returns are used. When the realized market excess return is greater than zero, the market is defined as the up market. When the realized market excess return is less than zero, the market is defined as the down market. To estimate the risk-return relationship for the up and down markets, the cross-sectional regression will be run for each day in the second stage (the test period):

$$(42) \quad r_{i,t+1} = \varphi_{1,t}\delta + \varphi_{2,t}(1-\delta) + \varphi_{3,t}\delta \beta_{i,t+1|t} + \varphi_{4,t}(1-\delta) \beta_{i,t+1|t} + v_{i,t},$$

$$i = 1, 2, \dots, 10,$$

where  $\delta$  is the dummy variable. If  $R_{m,t} > R_{f,t}$  (the up market),  $\delta = 1$ , and if  $R_{m,t} < R_{f,t}$  (the down market),  $\delta = 0$ . According to Fama and MacBeth (1973), the time-series mean of the estimated intercepts ( $\hat{\varphi}_{1,t}$  and  $\hat{\varphi}_{2,t}$ ) and the estimated slopes ( $\hat{\varphi}_{3,t}$  and  $\hat{\varphi}_{4,t}$ ) can be computed as:

$$(43) \quad \hat{\varphi}_j = \frac{1}{T_j} \sum_{t=1}^{T_j} \hat{\varphi}_{j,t},$$

$$j = 1, 2, 3, \text{ and } 4,$$

where  $T_j$  is determined by the number of days for the up market and the down market. The estimated coefficients  $\hat{\varphi}_3$  and  $\hat{\varphi}_4$  represent the risk-return relation for the up and down markets, respectively. And the standard deviation of  $\hat{\varphi}_j$  is:

$$(44) \quad \hat{\sigma}_j = \sqrt{\frac{1}{T_j - 1} \sum_{t=1}^{T_j} (\hat{\varphi}_{j,t} - \hat{\varphi}_j)^2}.$$



With the time-series mean ( $\hat{\phi}_j$ ) and standard deviation ( $\hat{\sigma}_j$ ) of the daily coefficients estimated by the cross-section regression (equation (42)), a simple t-statistic can be used to check the significance of estimated coefficients:

$$(45) \quad t(\hat{\phi}_j) = \frac{\hat{\phi}_j}{\hat{\sigma}_j / \sqrt{T_j}}.$$

Rather than using a pooled time-series and cross-section estimation in the second stage, the methodology used here aims at addressing the problem caused by correlation of residuals in the cross-section regressions. According to Fama and French (2004) and Petersen (2005), in finance applications, residuals of a given period may be correlated across firms. Petersen (2005) asserts that the second stage in the Fama-MacBeth (1973) methodology is designed to solve this problem and he shows that estimates based on this methodology are unbiased in the presence of correlation of residuals across firms.

As the realized returns are used in this regression, we expect a positive risk-return relationship to exist in the up market and a negative risk-return relationship to exist in the down market. Thereby, the following hypotheses are tested:

$$H_0: \varphi_3 = 0 \quad \text{and} \quad H_0: \varphi_4 = 0$$

$$H_a: \varphi_3 > 0 \quad H_a: \varphi_4 < 0.$$

In addition to the risk-return relation, we also pay attention to the estimated intercepts  $\hat{\phi}_1$  and  $\hat{\phi}_2$ . Although PSM (1995) don't report the estimated intercepts, we should not neglect the important implication of estimated intercepts in an asset pricing model. Fama and MacBeth (1973) examine the risk-return relation by regressing the realized returns of portfolios on estimated  $\beta$  and argue that the estimated intercept in their regression should be equal to the realized risk-free rate in terms of the Sharpe-Lintner

CAPM. Instead in this paper we use the realized excess returns (the realized return minus the risk-free rate) of portfolios as dependents. According to the CAPM,  $\beta$  is the complete measure of risk in the market. Thus, the estimated intercepts ( $\hat{\phi}_1$  and  $\hat{\phi}_2$ ) in our regression (equation [41]) represent the returns of portfolios that cannot be explained by the  $\beta$  risk. For a well-specified asset pricing model, we would expect that neither of the estimated intercepts for the up and down markets should be statistically different from zero. In this paper we also use  $t$ -statistic as shown in equation (45) to examine the significance of the estimated intercept  $\hat{\phi}_1$  for the up market and  $\hat{\phi}_2$  for the down market.

As mentioned by Isakov (1999), another solution to analyze the reliability of results derived from different asset pricing models is to check whether the estimated risk-return relation equals to the realized market excess return. Given equation (41), we know that the realized excess return of a portfolio is the product of the  $\beta$  risk and the realized market excess return. The realized market excess return can be regarded as the risk price compensated for an investor to hold a portfolio with the  $\beta$  risk. As suggested by Fama and MacBeth (1973) and Isakov (1999), using the realized excess returns of portfolios to regress against  $\beta$ , we would expect that the estimated coefficients ( $\hat{\phi}_3$  and  $\hat{\phi}_4$ ) that denote the risk-return relation for the up and down markets should not be statistically different from the realized market excess returns of the up and down markets, respectively. Therefore, in this paper we use a two-population  $t$ -test to check whether on average the difference between the estimated coefficient and the realized market excess return is significantly different from zero.

### 3.4 Estimation Results

We report the estimation results of the PSM model that is based on the constant  $\beta$  in Table 9. From Table 9, we find that the estimated coefficient  $\hat{\phi}_3$  for the up market is significant and positive. The estimated value for  $\hat{\phi}_3$  is 0.893 percent. This means when the realized market excess return is positive, the average incremental return for per unit of risk ( $\beta$ ) is 0.893 percent per day. In other words, the estimated daily risk price paid for per unit of the  $\beta$  risk in the up market equals 0.893 percent. On the other hand, the estimated coefficient  $\hat{\phi}_4$ , which represents the risk-return relation in the down market, equals  $-0.823$  percent and it is statistically different from zero. This implies that when the realized market excess return is negative, on average, an estimated increase for per unit of  $\beta$  will lead to 0.823 percent incremental loss per day.

Table 9

#### Estimation Results of the PSM Model

The table reports the time-series mean and t-statistics for the estimated intercepts and slopes of the PSM model based on the cross-sectional regression (equation [42]):  $r_{i,t+1} = \phi_{1,t}\delta + \phi_{2,t}(1-\delta) + \phi_{3,t}\delta\beta_{i,t+1|t} + \phi_{4,t}(1-\delta)\beta_{i,t+1|t} + v_{i,t}$ , where  $\delta=1$ , if  $R_{m,t} > R_{f,t}$  (the up market) and  $\delta=0$ , if  $R_{m,t} < R_{f,t}$  (the down market).  $\beta$  in the PSM model is estimated with OLS. The estimated coefficients are in percentages. An asterisk indicates the statistical significance at the 5% level.

	The up market		The down market	
	$\hat{\phi}_1$	$\hat{\phi}_3$	$\hat{\phi}_2$	$\hat{\phi}_4$
Coefficient	-0.239*	0.893*	0.215*	-0.823*
t-Statistic	-4.807	18.785	4.094	-17.021

Table 10 presents the estimation results for the time-varying  $\beta$  CAPM dependent on the up and down market regimes. Different from the PSM model,  $\beta$  in this model is assumed to be time-variant and ALS with Kalman foundations is employed to model the time evolution in  $\beta$ . Note that in Table 10, the  $t$ -statistic for the estimated coefficient  $\hat{\phi}_3$  equals 20.558, which denotes the risk-return relation in the up market is significant and positive. Meanwhile, we find that the  $t$ -statistic for the estimated coefficient  $\hat{\phi}_4$  is  $-20.241$ . This indicates the risk-return relation in the down market is significant and negative. In addition, the estimated risk prices for the up and down markets are equivalent to 0.743 percent and  $-0.723$  percent, respectively.

Table 10

Estimation Results of the CAPM Based on ALS with Kalman Foundations

The table reports the time-series mean and  $t$ -statistics for the estimated intercepts and slopes of the time-varying  $\beta$  CAPM based on the cross-sectional regression (equation [42]):  $r_{i,t+1} = \varphi_{1,t}\delta + \varphi_{2,t}(1-\delta) + \varphi_{3,t}\delta\beta_{i,t+1|t} + \varphi_{4,t}(1-\delta)\beta_{i,t+1|t} + v_{i,t}$ , where  $\delta=1$ , if  $R_{m,t} > R_{f,t}$  (the up market) and  $\delta=0$ , if  $R_{m,t} < R_{f,t}$  (the down market).  $\beta$  is estimated with ALS with Kalman foundations. The estimated coefficients are in percentages. An asterisk indicates the statistical significance at the 5% level.

	The up market		The down market	
	$\hat{\phi}_1$	$\hat{\phi}_3$	$\hat{\phi}_2$	$\hat{\phi}_4$
Coefficient	0.027	0.743*	-0.004	-0.723*
$t$ -Statistic	0.841	20.558	-0.119	-20.241

In general, according to Tables 9 and 10, both the sign and the significance of the estimated coefficient  $\hat{\phi}_3$  for the up market and the estimated coefficient  $\hat{\phi}_4$  for the down market confirm the results obtained by PSM (1995). That is, a positive risk-return

relationship exists when the realized market excess return is greater than zero and a negative risk-return relationship exists when the realized market excess return is less than zero.

As we mention before, in order to evaluate the reliability of the estimated risk-return relation obtained from different models, the significance of the estimated intercepts is an important criterion because the estimated intercepts represent unexplained returns for a model. A well-specified asset pricing model should have estimated intercepts that are not statistically different from zero for both the up and down markets. Note that in Table 9, the estimated intercept for the up market,  $\hat{\phi}_1$ , is significantly different from zero at the 1 percent level. The negative value of  $\hat{\phi}_1$  implies that the PSM model overestimates portfolios returns in the up market. Furthermore, we see that the estimated intercept for the down market,  $\hat{\phi}_2$ , is also significantly different from zero at the 1 percent level. The positive sign of  $\hat{\phi}_2$  suggests that the PSM model tends to underestimate portfolio returns in the down market. Our argument for the significant intercepts found in the PSM model is that the constant  $\beta$  estimated with the OLS methodology is not able to capture the time-variation of  $\beta$ . The inaccurate estimate of  $\beta$  in the PSM model leads to the significant mispricing for returns on portfolios.

Now we turn to look at the estimated intercepts for the CAPM based on ALS with Kalman foundations in Table 10. Neither the estimated intercept  $\hat{\phi}_1$  for the up market nor the estimated intercept  $\hat{\phi}_2$  for the down market is significantly different from zero. The results indicate that the CAPM with ALS based on Kalman foundations can well account for the excess returns of portfolios because the unexplained returns are not statistically distinguishable from zero. Compared with the estimated intercepts that are found to be

statistically different from zero in Table 9, apparently the time-varying  $\beta$  CAPM based on ALS with Kalman foundations outperforms the PSM model. The time-varying  $\beta$  model successfully reduces the mispricing under both the up and down markets.

In addition to the estimated intercepts, the second criterion this paper uses to investigate the credibility of estimation results is to check the values of the estimated risk-return relation. As suggested by Fama and MacBeth (1973) and Isakov (1999), the estimated coefficients  $\hat{\phi}_3$  and  $\hat{\phi}_4$  for the risk-return relationship, which also proxy the price paid for the  $\beta$  risk, should not be significantly different from the realized market excess returns. In this paper, we use a two-population  $t$ -test to examine whether on average the difference between the estimated value of the risk-return relationship and the realized market excess return is statistically distinguishable from zero. Table 11 presents the results of the two population  $t$ -test for the both the PSM model and the CAPM based on ALS with Kalman foundations.

Table 11

Two-Population  $t$ -test for the Estimated Value of the Risk-Return Relationship

The table reports the results of the two-population  $t$ -test. A two-population  $t$ -test is used to check whether, on average, the difference between the estimated coefficients ( $\hat{\phi}_3$  and  $\hat{\phi}_4$  estimated by the PSM model and the CAPM based on ALS with Kalman foundations) and the realized up and down market excess returns is significantly different from zero. We examine the up and down markets, respectively. The asterisk indicates the statistical significance at the 5% level.

	The PSM model		The CAPM based on ALS with Kalman foundations	
	$\hat{\phi}_3$	$\hat{\phi}_4$	$\hat{\phi}_3$	$\hat{\phi}_4$
$t$ -Statistics	-2.999*	1.215	-0.075	-0.864

First, note that in Table 8 the realized market excess returns for the up and down markets during the test period from November 2, 1992 to December 31, 2003 are 0.739 percent and  $-0.760$  percent, respectively. For the results of the PSM model in Table 11, although the difference between the estimated coefficient  $\hat{\phi}_4$  and the realized down market excess return is not significant, the difference between the estimated coefficient  $\hat{\phi}_3$  and the realized up market excess return is found to be significantly different from zero. The results tell us that for the up market, the estimated value of the risk-return relation (0.893 percent) statistically differs from the realized market excess returns (0.739 percent) in the PSM model. This implies the PSM model is not able to accurately estimate the risk-return relation in the up market. By contrast, the results of two-population  $t$ -test in Table 11 do not show any significance for both the up and down markets in the time-varying  $\beta$  CAPM based on ALS with Kalman foundations. This suggests that the estimated values of the risk-return relation ( $\hat{\phi}_3$  and  $\hat{\phi}_4$ ) are not statistically different from the realized market excess return for both the up and down markets in the time-varying  $\beta$  CAPM.

Furthermore, compared with the estimated up market risk price (0.893 percent) and down market risk price ( $-0.823$  percent) in the PSM model, the values of the estimated risk price for the up market (0.743 percent) and the down market ( $-0.723$  percent) in our time-varying  $\beta$  CAPM are closer to the realized up market excess return (0.739 percent), and the realized down market excess return ( $-0.760$  percent), respectively. This further confirms that the risk-return estimated by the CAPM based on ALS with Kalman foundations are more accurate than those obtained by the PSM model.

In short, the estimation results support the existence of a positive risk-return relation in the up market and the existence of a negative risk-return relation in the down market. Under the framework of the time-varying  $\beta$  CAPM, neither of the estimated intercepts for the up and down markets is statistically different from zero. Moreover, the estimated values of the risk-return relation are found to be not significantly different from the realized market excess returns in the time-varying  $\beta$  CAPM. All these results confirm our postulation that ALS with Kalman foundations improves the reliability of the estimation of the risk-return relationship.

### 3.5. Conclusions

In this paper, the risk-return relation is examined by using the daily returns for 10 portfolios sorted by industry sectors in S&P 500. The estimation period is from November 1987 to December 2003. Different from the PSM (1995) model, this paper assumes  $\beta$  (risk) in the CAPM varies over time because the substantial financial literature documents  $\beta$  in the CAPM tends to be time-variant. To capture the dynamics of  $\beta$ , ALS with Kalman foundations is employed to mimic the investors' learning process about  $\beta$ . As McCulloch (2005) suggests, ALS with Kalman foundations nests the rigorous Kalman solutions of the LLM and provides simple and elegant initial values in contrast with previous ALS. Thus ALS with Kalman foundations is expected to provide better estimates for a time-varying relationship and investors' expectations of time-evolving  $\beta$ .

With the time-varying  $\beta$  estimated via the ALS with Kalman foundations, the risk-return relation is examined under the up and down markets because the realized returns are used in this paper. When the market return exceeds the risk-free rate, the



market is defined as an up market; when the market return is lower than the risk-free rate, the market is defined as a down market. For comparison purposes, we also replicate the PSM model in which  $\beta$  is assumed to be constant over time. The estimation results of the CAPM based on ALS with Kalman foundations confirms the presence of a positive risk-return relation in the up market and the presence of a negative risk-return relation in the down market, which is in line with the PSM model.

In addition, our results show that the time-varying  $\beta$  CAPM based on ALS with Kalman foundations outperforms the PSM model. In the PSM model the estimated intercepts, which represent the unexplained returns for the model, are statistically different from zero for both the up and down market. By contrast, our time-varying  $\beta$  CAPM doesn't generate any significant estimated intercepts in both the up and down markets. This indicates the  $\beta$  risk estimated by ALS Kalman foundations can well account for the cross-sectional returns. On the other hand, the values of the risk-return relation (risk price) estimated by the CAPM based on ALS with Kalman foundations are not significantly different from the realized market excess return. By contrast, the risk-return relation estimated by the PSM model is found to be significantly different from the realized market excess return in the up market. The good performance of the CAPM based on ALS with Kalman foundations implies that ALS with Kalman foundations successfully mimics the investors' learning process of time-varying  $\beta$  and therefore enhances the accuracy of the estimates for  $\beta$ . In general, our results indicate that  $\beta$  is still a good measure of the systematic risk.

## CHAPTER IV

### INTEREST RATES AND EXCHANGE RATES IN ASIAN CRISIS COUNTRIES: EVIDENCE FROM A TIME-VARYING-PARAMETER MODEL WITH GARCH DISTURBANCES

#### 4.1 Introduction

The traditional way to maintain stable exchange rates during currency crises is to increase interest rates. This policy was advocated by the International Monetary Fund (IMF) during the Asian financial crisis in the 1990s. The traditional view believes that an increase in interest rates can convey the information of the monetary authority's commitment to maintain a fixed exchange rate, raise returns for investors, make speculation less attractive, and reduce capital outflow. However, the appropriateness of this monetary policy has been argued intensively. In particular, a revisionist view has been proposed by several economists, such as Furman and Stiglitz (1998), Feldstein (1998), and Radelet and Sachs (1998). They argue that a depreciation in exchange rates can be attributed to a tight monetary policy because a interest rate hike could raise default probabilities, weaken financial position of firms that are debt constrained, and increase exchange rate risk premiums. These two opposite views about the use of interest rates as a monetary instrument to defend weak currencies have raised intense controversy.

To investigate the effectiveness of the interest rate defense, many empirical studies have been conducted to analyze the relationship between interest rates and exchange rates for the Asian financial crisis. However, the empirical evidence is mixed as well. Using data from Indonesia, Korea, Malaysia, and Thailand, Gould and Kamin

(2000) claim that higher interest rates don't have a significant impact on exchange rates during the financial crisis. Basurto and Gosh (2001) find little evidence that higher real interest rates contribute to higher risk premiums based on data from Indonesia, Korea, and Thailand. They argue that a tight monetary policy doesn't have a negative effect on exchange rates as suggested by the revisionist view. But Furman and Stiglitz (1998) show that interest hikes are associated with currency depreciation for nine emerging countries. In contrast, Dekle, Hsiao and Wang (2002) stress that a hike in interest rates stabilizes depreciating currencies in Korea, Malaysia, and Thailand during the Asian financial crisis.

A possible reason to explain the mixed evidence on the efficacy of the interest rate defense is because previous studies obtain their empirical results dependent on different sample periods and they typically assume that the relationship between interest rates and exchange rates is constant over time. For example, Gould and Kamin (1999) present their empirical evidence for Korea and Thailand based on the sample from July 4, 1997 to July 31, 1998 and evidence for Indonesia based on the sample from August 15, 1997 to July 31, 1998. In contrast, Basurto and Ghosh (2001) investigate the interest-exchange rate nexus in Indonesia, Korea, and Thailand by using data from 1990 to 2006. It is worthwhile pointing out that conclusions derived from a constant interest-exchange rate relationship based on a particular sample period could be misleading.

Some researchers have realized this problem. Baig and Goldfajn (2002) argue that proper increases in interest rates can lead to currency appreciation, but additional increases could lead to excessive risk premiums and depreciate a currency. Therefore the interest-exchange rate relationship depends on the size of a raise in interest rates and may

vary under different conditions. Cho and West (2003) also suggest the sign of the correlation between interest rates and exchange rates relies on the sizes of monetary shocks and shocks to the exchange rate risk premium and it may change over time. Baig and Goldfajn (2002) provides empirical evidence that both the significance and the sign of the correlation between interest rates and exchange rates exhibit time-variation in the mist of the Asian financial crisis for Indonesia, Korea, Malaysia, the Philippines, and Thailand. The empirical results obtained by Cho and West (2003) also suggest the correlation between interest rates and exchange rates is sample dependent and varies over time. In addition, a recent study by Caporale, Cipollini, and Demetriades (2005) asserts that the impact of interest rates on exchange rates varies across tranquil and turbulent periods for four East Asian countries (Indonesia, Korea, the Philippines, and Thailand).

Motivated by the dependence of the interest-exchange rate relationship on sample periods, this paper attempts to contribute to the literature by analyzing a time-varying relationship between interest rates and exchange rates. Since estimating a constant relationship within arbitrarily chosen periods may result in the loss of important information about the dynamics of the interest-exchange rate relationship, this paper uses a time-varying-parameter model (TVP model) with GARCH disturbances estimated via the Kalman filter to study the impact of raising interest rates on exchange rates. Compared to the methodologies employed by previous studies, the major advantage to our model is that changes in the interest-exchange rate relationship are totally determined by data. In addition, our methodology is able to account for heteroskedastic shocks to exchange rates.

Using weekly data of interest rates and exchange rates from January 1997 to December 1999, we investigate the role of interest rates in stabilizing exchange rates for Indonesia, Korea, the Philippines, and Thailand in and after the Asian financial crisis.<sup>18</sup> The confidence interval bands constructed from the conditional means and variances of the estimated time-varying coefficients are used to check the significance of the interest-exchange rate relationship. Our estimation results indicate the existence of a significant and positive relationship between interest rates and exchange rates in certain crisis periods for all four countries. Exchange rates respond most strongly to interest rate hikes in Thailand because a significant and positive impact of interest rates on exchange rates is found to exist in Thailand for 37 weeks. The significant and positive impact is shown to exist for no more than 7 weeks during the crisis period in any of the other three countries. In general our empirical results imply that an increase in interest rates can lead to currency depreciation during certain periods of the Asian financial crisis, which is in favor of the revisionist view that a tight monetary policy has a perverse impact on exchange rates in financial crises. For the periods after the Asian financial crisis, we don't find evidence that exchange rates are significantly affected by interest rates in all four countries.

The remainder of the paper is organized as follows. The next section briefly reviews the literature about the time-varying relationship between interest rates and exchange rates. Section 4.3 introduces the TVP model with GARCH disturbances. Section 4.4 describes the data. Section 4.5 presents and discusses the empirical results. Finally, Section 4.6 concludes the paper.

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<sup>18</sup> In this paper, the duration of the Asian financial crisis is defined as the period from July 1997 to December 1998.

## 4.2 Literature Review

The objective of this paper is to identify the impacts a tight monetary policy has on currency depreciation during a financial crisis. Different from previous studies, the relationship between interest rates and exchange rates is assumed to be time-variant in this paper. According to the traditional view, an interest rate hike can increase the rate of return on assets denominated by the domestic currency, which makes speculation more expensive. It can also signal the monetary authority's commitment to stabilize a depreciating currency, enhance investors' confidence, and discourage capital flight. Therefore, an increase in interest rates can be used as a monetary instrument to defend weak currencies. In contrast, as suggested by Furman and Stiglitz (1998), the exchange rate risk premiums are positively and strongly related to interest rates because an increase in interest rates could cause default on loans, increase the probability of bankruptcy among highly leveraged firms, and weaken financial sectors. According to the revisionist view, higher interest rates could destabilize and depreciate a currency in crisis episodes because it induces higher risk premiums.

The opposite traditional and revisionist views provide a possible theoretical background for the time-varying relationship between interest rates and exchange rates. Baig and Goldfajn (2002) argue that an increase in interest rates could appreciate currencies following the traditional view. But they also suggest that an additional increase in interest rates can lead to excessively high risk premiums and depreciate currencies as suggested by the revisionist view. Therefore, one would expect that the relationship between interest rates and exchange rates are sensitive to the magnitude of an

interest rate raise. The relationship could be positive or negative depending on different situations. Cho and West (2003) develop a simple structural model in which interest rates and exchange rates are driven by monetary shocks and shocks to the exchange rate risk premium. They claim that if the monetary authority raises interest rates in anticipation of depreciation, a dominant monetary shock causes a positive correlation between interest rates and exchange rates while a dominant risk premium shock causes a negative correlation. Their model implies that the sign of the interest-exchange rate correlation could vary over time due to the difference between the sizes of monetary shocks and risk premium shocks.

Some studies have already provided empirical evidence suggesting a time-varying relationship between exchange rates and exchange rates. Caporale et al. (2005) recognizes the possible change in the interest-exchange rate relationship across tranquil and turbulent periods. Dummy variables are included into their bivariate vector error correction model (VECM) to capture the shift in the relationship between interest rates and exchange rates. Based on the data from Indonesia, Korea, the Philippines, and Thailand, interest rates are found to have a positive impact on exchange rates during tranquil periods and have a negative impact on exchange rates during turbulent periods. Their results imply the effect of interest rates on exchange rates varies across different regimes.

Furthermore, the relationship between interest rates and exchange rates is also shown to be time-variant within the same regime as well. Frankel and Rose (1995) use the data from industrial countries and find that during tranquil periods, the interest-exchange rate nexus could change. With the daily data from five East Asian countries,

which include Indonesia, Korea, Malaysia, the Philippines, and Thailand, Baig and Goldfajn (2002) estimate the relationship between interest rates and exchange rates by using both the panel and time-series regressions. In order to account for possible changes in the interest-exchange rate correlation, they employ a rolling-window technique. The results of both the panel and time-series estimations show that not only the significance but the sign of the correlation vary when the estimation sample rolls forward within the period of the Asian financial crisis. Cho and West (2003) examine the relationship between interest rates and exchange rates with a vector autoregression (VAR) model for Korea, the Philippines, and Thailand. Their results also document that both the significance and the magnitude of the interest-exchange rate correlation change with different sample periods.

The results of the existing literature indicate that the interest-exchange rate nexus is sample dependent and could change over time, which motivates us to present a time-varying relationship. It is worthwhile noticing that although the previous studies recognize the time-evolution of the relationship between interest rates and exchange rates, they normally estimate a constant relation within arbitrarily chosen periods. For example, Caporale et al. (2005) use dummy variables to account for the shift of the interest-exchange rate relationship across tranquil and turbulent regimes. However, their methodology is not able to detect the possible changes within the same regime. Although Baig and Goldfajn (2002) employ a rolling-window technique, they assume the relationship between interest rates and exchange rates are constant within each “window” period. Their estimated relationship is still sensitive to the starting and ending dates of each “window”. Therefore, we argue that by focusing on a constant relationship within



arbitrarily chosen periods, one may lose important information about the dynamics of the interest-exchange rate relationship within and across regimes that might be misspecified.

To overcome this problem, this paper employs a TVP model with GARCH disturbances estimated via a Kalman filter to model the evolution of the relationship between interest rates and exchange rates. The main advantage of our model is that instead of assuming a constant relationship between interest rates and exchange rates within an arbitrarily defined sample periods, we let changes in the relationship to be fully determined by the data. The model is also capable of finding other changes in relationship that have not been reported in the existing literature. Another advantage is that the model takes into account heteroskedastic shocks to exchange rates.

In this paper, we are interested in examining the contemporaneous impact of increasing interest rates on exchange rates because Cho and West (2003) argue that participants in foreign exchange rate markets react very quickly to the interest-rate setting and exchange rates could be simultaneously determined with interest rates. The recent study by Caporale et al. (2006) also analyzes the effectiveness of raising interest rates by focusing on the contemporaneous correlation between interest rates and exchange rates. On the other hand, Dekle et al. (2002) stress that exchange rates could react to the lags of interest rates because of differences in institutional setup among countries. Previous studies such as Basurto and Ghosh (1999) and Baig and Goldfajn (2002) use the lags of interest rates to investigate the impact of a tight monetary policy on exchange rates during crisis periods. Thus, we will also analyze the lagged impact of interest rates on exchange rates.

There are two recent studies that pay attention to the time-varying relationship between interest rates and exchange rates as well. Chen (2006) uses a Markov-switching approach to study the case of the Asian financial crisis. Different from our paper, he focuses on the impact of interest rates on exchange rate volatility. Bautista (2006) estimates the interest-exchange rate correlation with a dynamic conditional correlation GARCH model (DCC-GARCH). However, he doesn't report the significant level for the estimated correlation, which leads the readers to question the reliability of his conclusion. In contrast, it is easy for us to construct confidence intervals to check the statistical significance of the time-varying relationship by using the conditional means and variances of the TVP model.

#### 4.3 TVP Model with GARCH Disturbances

As the relationship between interest rates and exchange rates has been suggested to be time-variant in previous literature, in this paper we use a TVP model with GARCH disturbances developed by Harvey et al. (1992) and Kim and Nelson (1999) to capture the dynamics of the impact of interest rates on exchange rates. This model is estimated with a Kalman filter, which has been widely used to estimate time-varying relationships. The Kalman filter procedure is believed to better account for the time-varying relationship between interest rates and exchange rates because the Kalman filter updates estimated coefficients based on available information at each point of time. In addition, instead of simply treating shocks to exchange rates as homogeneous disturbances, we assume shocks to exchange rates to be heteroskedastic in our model because foreign exchange rates normally exhibit a behavior of time-varying volatility.

In order to use the Kalman filter to estimate the time-varying relationship between interest rates and exchange rates, a TVP model with GARCH disturbances needs to be represented in a state-space form. As the standard Kalman filter procedure assumes homogeneous disturbances, the state-space model needs to be modified to incorporate the heteroskedastic shocks due to the assumption of the GARCH effects in disturbances. In order to make the standard Kalman filter operable, as suggested by Harvey et al. (1992) and Kim and Nelson (1999), the state-space form can be expressed as follows:

$$(46) \quad y_t = [\mathbf{X}_t', 1] \begin{bmatrix} \boldsymbol{\beta}_t \\ \varepsilon_t \end{bmatrix},$$

$$(47) \quad \begin{bmatrix} \boldsymbol{\beta}_t \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} \mathbf{I}_4 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_{t-1} \\ \varepsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_t \\ \varepsilon_t \end{bmatrix}.$$

Equations (46) and (47) are called the measurement equation and the transition equation, respectively. The dependent variable  $y_t$  in equation (46) is  $\Delta ex_t$ , which is the first difference of the logarithm of the nominal exchange rate ( $ex_t$ ).<sup>19</sup>  $\mathbf{X}_t$  refers to a  $4 \times 1$  column vector  $(1, \Delta ir_t, \Delta ir_{t-1}, \Delta ex_{t-1})'$  in which  $\Delta ir_t$  is the first difference of the domestic interest rate ( $ir_t$ ).  $\boldsymbol{\beta}_t$  is a  $4 \times 1$  column vector  $(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t})'$ .  $\beta_{0,t}$ ,  $\beta_{1,t}$ ,  $\beta_{2,t}$ , and  $\beta_{3,t}$  represent the time-varying coefficients on variables  $(1, \Delta ir_t, \Delta ir_{t-1}, \text{ and } \Delta ex_{t-1})$ . The variables in  $\boldsymbol{\beta}_t$  are unobservable. In equation (46)  $\varepsilon_t$  is the heteroskedastic shock to exchange rates and  $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ . In equation (47) we assume the time-varying-coefficient vector  $\boldsymbol{\beta}_t$  follows a random walk and  $\mathbf{I}_4$  is a  $4 \times 4$  identity matrix.  $\mathbf{v}_t$  is a  $4 \times 1$  column vector  $(v_{1,t}, v_{2,t}, v_{3,t}, v_{4,t})'$  and  $\mathbf{v}_t \sim N(0, \mathbf{Q})$ .  $\mathbf{Q}$  denotes a  $4 \times 4$  diagonal

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<sup>19</sup> The nominal exchange rate ( $ex_t$ ) is defined as the domestic currency with respect to the U.S. dollar.

variance-covariance matrix and its diagonal elements equal to  $\sigma_0^2$ ,  $\sigma_1^2$ ,  $\sigma_2^2$ , and  $\sigma_3^2$  respectively. The shocks,  $\varepsilon_t$  and  $\mathbf{v}_t$  in equations (46) and (47), are independent of each other.

Note that our model differs from the standard TVP model by assuming the heteroskedastic shocks ( $\varepsilon_t$ ) to exchange rates. The GARCH effect is introduced to this shock via the following equation,

$$(48) \quad \varepsilon_{t|t-1} \sim N(0, h_t).$$

The conditional variance  $h_t$  of  $\varepsilon_t$  is assumed to have a GARCH (1, 1) effect,

$$(49) \quad h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1}.$$

With conditional variance  $h_t$ , equations (46) and (47) can be represented as the following equations,

$$(50) \quad y_t = \mathbf{X}_t^{*'} \boldsymbol{\beta}_t^*,$$

$$(51) \quad \boldsymbol{\beta}_t^* = \mathbf{F}^* \boldsymbol{\beta}_{t-1}^* + \mathbf{v}_t^*,$$

where  $\mathbf{X}_t^{*'} = [\mathbf{X}_t', 1]$ ,  $\boldsymbol{\beta}_t^* = \begin{bmatrix} \boldsymbol{\beta}_t' \\ \varepsilon_t \end{bmatrix}$ ,  $\mathbf{F}^* = \begin{bmatrix} \mathbf{I}_4 & 0 \\ 0 & 0 \end{bmatrix}$ , and  $\mathbf{v}_t^* = \begin{bmatrix} \mathbf{v}_t' \\ \varepsilon_t \end{bmatrix}$ . The variance-covariance matrix of  $\mathbf{v}_t^*$  can be expressed as,

$$(52) \quad E(\mathbf{v}_t^* \mathbf{v}_t^{*'}) = \begin{bmatrix} \mathbf{Q} & 0 \\ 0 & h_t \end{bmatrix} = \mathbf{Q}_t^*.$$

Note that the heteroskedastic shocks ( $\varepsilon_t$ ) to exchange rates are included in the transition equation (equation (51)).

Equations (50) and (51) together are estimated with the Kalman Filter. The Kalman Filter is a dynamic procedure that updates unobservable time-varying-coefficient

vector  $\beta_t^*$  by incorporating prediction errors that contain the most updated information.

The distribution of  $\beta_t^*$  at time  $t-1$  is:

$$(53) \quad \beta_t^* | \psi_{t-1} \sim N(\beta_{t|t-1}^*, P_{t|t-1}^*),^{20}$$

where  $\psi_{t-1}$  denotes the information set at time  $t-1$ .  $\beta_{t|t-1}^*$  and  $P_{t|t-1}^*$  are the conditional mean vector and the conditional covariance matrix of  $\beta_t^*$  based on available information up to time  $t-1$  respectively. Assuming that  $X_t^*$  is observable at the beginning of time  $t$  and new observation  $y_t$  is made at the end of time  $t$ , the Kalman filter procedure consists of two steps: the prediction step and the updating step. In the prediction step, the expectation of  $\beta_t^*$  based on the available information up to time  $t-1$  is:

$$(54) \quad \beta_{t|t-1}^* = F^* \beta_{t-1|t-1}^*.$$

Meanwhile, the conditional variance  $P_{t|t-1}^*$  can be also obtained with the information up to time  $t-1$ :

$$(55) \quad P_{t|t-1}^* = F^* P_{t|t-1}^* F^{*'} + I_5 Q_t^* I_5',$$

where  $I_5$  is a  $5 \times 5$  identity matrix. With  $\beta_{t|t-1}^*$  and observable  $X_t^*$  at the beginning of time  $t$ , the optimal predictor of  $y_t$  is:

$$(56) \quad y_{t|t-1} = X_t^{*'} \beta_{t|t-1}^*.$$

In the following updating step, after  $y_t$  is realized at the end of time  $t$ , we can calculate the prediction error as:

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<sup>20</sup>  $P_{t|t-1}^*$  is defined as  $E[(\beta_t^* - \beta_{t|t-1}^*)(\beta_t^* - \beta_{t|t-1}^*)']$  and it is a  $4 \times 4$  diagonal variance-covariance matrix with  $p_{0,t|t-1}$ ,  $p_{1,t|t-1}$ ,  $p_{2,t|t-1}$ , and  $p_{4,t|t-1}$  on its diagonal.

$$(57) \quad \eta_{t|t-1}^* = y_t - \mathcal{Y}_{t|t-1},$$

where the prediction error  $\eta_{t|t-1}^*$  contains new information about  $\beta_t^*$  beyond  $\beta_{t|t-1}^*$ . The conditional variance of the prediction error is,

$$(58) \quad f_{t|t-1} = \mathbf{X}_t^{*'} \mathbf{P}_{t|t-1}^* \mathbf{X}_t^*.$$

According to equation (58), the source of the prediction error arises not because of uncertainty of the time-varying coefficients  $\beta_t^*$  but also because of the heteroskedastic shocks ( $\varepsilon_t$ ) to exchange rates.

Based on the prediction error,  $\beta_{t|t}^*$  denoted as the inference of  $\beta_t^*$  at time  $t$  can be updated with the information up to time  $t$ :

$$(59) \quad \beta_{t|t}^* = \beta_{t|t-1}^* + \mathbf{K}_t^* \times \eta_{t|t-1}^*,$$

where  $\mathbf{K}_t^*$  is the Kalman gain and it determines how much weigh to be assigned to the prediction error  $\eta_{t|t-1}^*$ .  $\mathbf{K}_t^*$  can be determined by the following equation,

$$(60) \quad \mathbf{K}_t^* = \mathbf{P}_{t|t-1}^* \mathbf{X}_t^* f_{t|t-1}^{-1},$$

where  $f_{t|t-1} = \mathbf{X}_t^{*'} \mathbf{P}_{t|t-1}^* \mathbf{X}_t^*$ . Meanwhile, the conditional covariance matrix of  $\beta_t^*$  is updated with new information as well,

$$(61) \quad \mathbf{P}_{t|t}^* = \mathbf{P}_{t|t-1}^* - \mathbf{K}_t^* \mathbf{X}_t^{*'} \mathbf{P}_{t|t-1}^*.$$

The dynamic process expressed by equations (59) and (60) tries to model time-evolution of the unobservable time-varying-coefficient vector  $\beta_t^*$ .

The Kalman filter briefly introduced above shows us that it continues to update the expectation and the conditional variance of the estimated time-varying coefficients based on new arrival information. We believe that this algorithm can capture the

dynamics of the time-varying relationship between interest rates and exchange rates and provide more accurate estimates. Via the Kalman filter, we can obtain the estimated parameters for the GARCH (1, 1) process ( $\alpha_0$ ,  $\alpha_1$ , and  $\alpha_2$  in equation (49)) and the conditional variance for shocks to exchange rates ( $h_t$ ). Moreover, at each point of time we get the conditional means for the time-varying coefficients ( $\beta_{t|t-1}^* = (\beta_{0,t|t-1}, \beta_{1,t|t-1}, \beta_{2,t|t-1}, \beta_{3,t|t-1})'$ ) and the conditional variances for the time-varying coefficients (the diagonal elements ( $p_{0,t|t-1}, p_{1,t|t-1}, p_{2,t|t-1},$  and  $p_{4,t|t-1}$ ) of  $P_{t|t-1}^*$ ). As mentioned by Koopman and Franses (2002), confidence interval bands for the estimated time-varying coefficients can be constructed with the conditional means and the conditional variances.

#### 4.4 Data

To study the effect of using interest rates to fight against depreciating currencies during financial crises, this paper chooses data from four East Asian countries. These countries are Indonesia, South Korea, the Philippines, and Thailand.<sup>21</sup> These four countries experienced sharp currency depreciation in the midst of the Asian financial crisis. The weekly data on nominal exchange rates and nominal interest rates are collected on each Friday from January 3, 1997 to December 31, 1999. The interest rates we utilize are defined as annual rates, which includes the Indonesia interbank call rate, the Korea call overnight rate, the Philippines interbank call rate, and the Thailand interbank

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<sup>21</sup> We choose these four countries because Indonesia, South Korea, and Thailand were the countries most affected by the Asian financial crisis. The Philippines, Malaysia, and Hong Kong were also affected heavily by the crisis. Due to the availability of the data, only the Philippines was chosen.

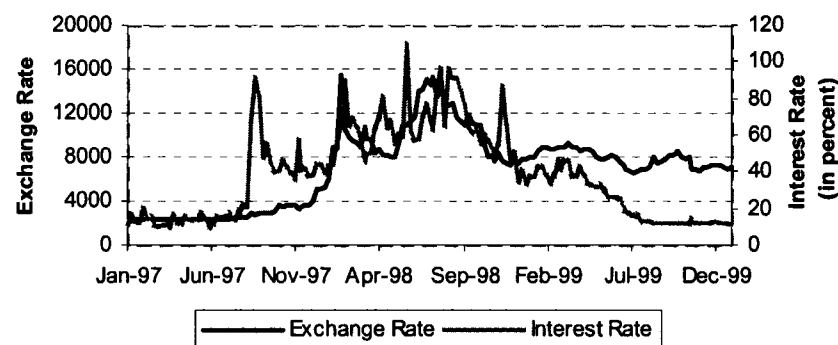
overnight rate. Because short-term nominal interest rates are widely accepted to be the most accurate indicator of the stance of monetary policy, we also choose short-term nominal interest rates to measure the monetary policy of countries that underwent currency crisis following the previous studies such as Cho and West (2003), Caporale et al. (2005), and Chen (2006). The exchange rates<sup>22</sup> are defined as the domestic currency against the U.S. dollar. The source of the data for both exchange rates and interest rates is Datastream. Figure 2 gives us a visual display of the movements of interest rates and exchange rates during the sample period.

Figure 2

#### Nominal Exchange Rates and Interest Rates (percent per annum)

Figure 2 plots the weekly data (from January 3, 1997 to December 31, 1999) of exchange rates and interest rates in levels for Indonesia, Korea, the Philippines, and Thailand.

##### (a) Interbank call rate and rupiah/dollar exchange rate, Indonesia



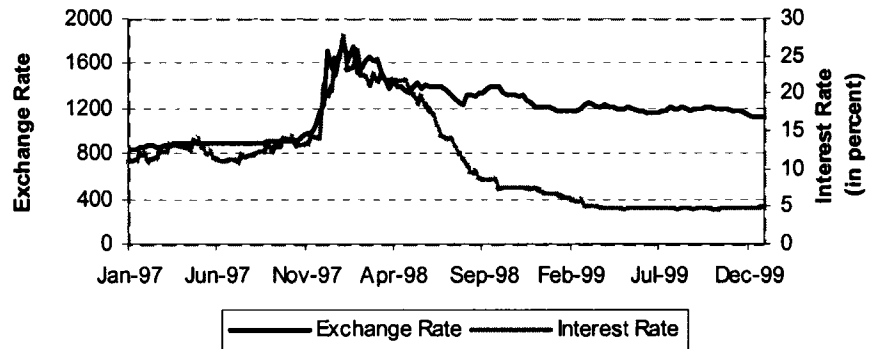
<sup>22</sup> The exchange rates used here are spot exchange rates.



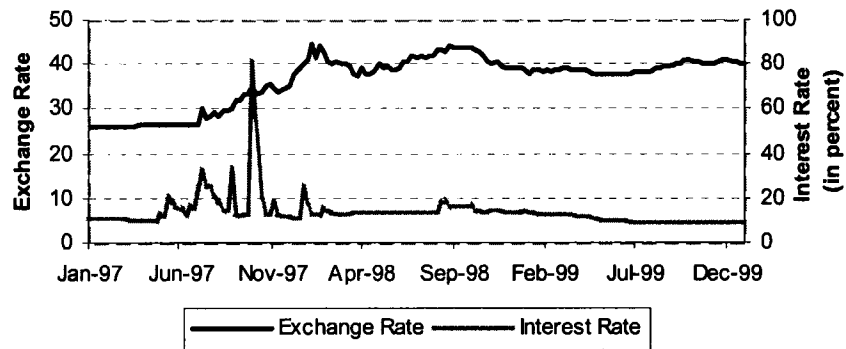
Figure 2-Continued

Nominal Exchange Rates and Interest Rates (percent per annum)

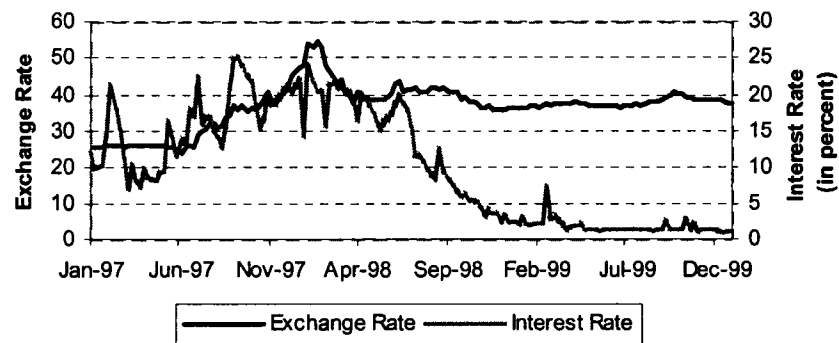
(b) Call overnight rate and won/dollar exchange rate, Korea



(c) Interbank call rate and peso/dollar exchange rate, the Philippines



(d) Interbank call rate and baht/dollar exchange rate, Thailand



From Figure 2, it is clear that domestic currencies began depreciating on July 1997 for Indonesia, the Philippines, and Thailand. For Korea, the onset of currency depreciation was December 1997. In line with Bautista (2006), we define the period of the Asian financial crisis is from July 4, 1997 to December 25, 1998. Note that during the financial crisis period, all these countries experienced sharply depreciating exchange rates. The exchange rates were volatile in these four countries until early 1999. In addition, as shown in Figure 2, the short-term interest rates were also raised dramatically in all four countries during the crisis period. This indicates that the monetary authority in each country attempted to hike interest rates to defend weak currencies. Both the exchange rates and interest rates fell back and became relatively stable after early 1999.

Unit root tests are conducted to check whether the individual exchange rate and interest rate series are integrated. Table 12 presents the results of unit root tests for exchange rates and interest rates in levels. For all four countries, we fail to reject the null hypothesis that the series has a unit root for the logarithm of exchange rates with the Augmented Dickey-Fuller (ADF) test. The KPSS test confirms the results of the ADF test for exchange rates. For interest rates, the ADF test fails to reject the null of a unit root except the Philippines. However, the results of KPSS test indicate the presence of unit roots for the Philippines interest rate at the 5% level. From Figure 2(c), we can easily see that for Philippines interest rate, there is an apparently volatile period from May 1997 to January 1998. After January 1998, the level of the interest rate in Philippine became very stable. Thus, to account for a possible structural shift in the interest rate of the Philippines, we decide to conduct unit root tests for the turbulent period (1997:5-

1998:1) and the tranquil period (1998:2-1999:12), respectively. The results of the ADF and KPSS tests for the Philippines interest rate suggest that there exists a unit root for the interest rate in the turbulent period. For the tranquil period, the results don't show the presence of a unit root for the interest rate in the Philippines.

Table 12

Unit Root Tests

Table 12 presents the results of the ADF test and the KPSS test for exchange rates and interest rates in levels. The weekly data of exchange rates and interest rates are collected for Indonesia, Korea, the Philippines, and Thailand from January 3, 1997 to December 31, 1999. The lags in the ADF test are selected by the Schwartz information criteria. The ADF test assumes a series has a unit root under the null hypothesis. For the KPSS test, the null hypothesis assumes a series is stationary. An asterisk indicates the statistical significance at the 5% level.

	ADF (no trend)	ADF (trend)	KPSS (no trend)	KPSS (trend)
Exchange rates				
Indonesia	-1.691	-1.244	0.913*	0.321*
Korea	-1.806	-1.387	0.553*	0.276*
The Philippines	-1.862	-1.007	0.983*	0.326*
Thailand	-1.957	-1.367	0.577*	0.273*
Interest rates				
Indonesia	-2.876*	-2.914	0.362	0.355*
Korea	-1.174	-2.066	0.800*	0.225*
The Philippines	-6.871*	-7.377*	0.640*	0.156*
Thailand	-1.222	-2.632	1.072*	0.236*

We further conduct the ADF and the KPSS tests to check the first difference of exchange rates and interest rates. The results show that for all four countries, both the

exchange rates and the interest rates are stationary in first differences. Moreover, we use the Johansen cointegration test to examine the existence of the cointegrated relationship between the exchange rate and interest rate for each country.<sup>23</sup> The results of the cointegration tests indicate that the exchange rate is not cointegrated with the interest rate to each country.

#### 4.5 Empirical Results

Given equations (50) and (51), we estimate the TVP model with GARCH disturbances for Indonesia, Korea, the Philippines, and Thailand. The TVP model with GARCH disturbances is estimated via maximum likelihood estimation (MLE). It is worth mentioning that in order to run the MLE methodology, we need to set initial values for the parameters of the TVP model. To offset the impact of these initial inputs, we report the estimation results from March 28, 1997 to December 31, 1999 by eliminating the first ten-week estimates.<sup>24</sup> Table 13 reports the estimated parameters of the GARCH (1, 1) process for the shocks to exchange rates. We reject the null hypothesis that the coefficient ( $\alpha_1$ ) on the ARCH term equals zero at the 5% level for all four countries. Furthermore, the estimated coefficient ( $\alpha_2$ ) on the GARCH term for each country is found to be statistically different from zero at the 5% level. The significance of the

<sup>23</sup> For the Philippines, we only test the cointegrated relationship in the turbulent period (1997:5-1998:1).

<sup>24</sup> A time-varying-parameter based on the Kalman filter procedure needs prediction errors and variances of prediction errors to maximize its likelihood function. However, at time  $t=1$ , we don't have prior information for the time-varying coefficients and prediction errors, for example,  $\beta_{t|t-1}^*$  in equation (54), and  $\eta_{t|t-1}^*$  in equation (57). Thus, to start the Kalman filter procedure, an arbitrary initial value  $\beta_{0|0}^*$  and its variance  $\mathbf{p}_{0|0}^*$  need to be set. According to Kim and Nelson (1999), as new information  $y_t$  arrives, most of the weight in the updating equation (59) is assigned to new information contained in the forecast error. To minimize the effect of the arbitrary initial values, they suggest evaluating the likelihood function by eliminating the first several observations.

estimated coefficients on both the ARCH and the GARCH terms indicates that the exchange rates in these countries exhibit strong periods of volatility, which confirms that the incorporation of the GARCH disturbances into a TVP model is appropriate.

Table 13

Estimated GARCH (1, 1) Parameters

The table reports the parameters of the GARCH (1, 1) process estimated from a TVP model with GARCH disturbances for Indonesia, Korea, the Philippines, and Thailand. The GARCH (1, 1) process is used to account for heteroskedastic shocks to exchange rates. The conditional variance of shocks to exchange rates ( $h_t$ ) is represented based on equation (49):  $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1}$ . Standard errors are reported in parentheses. An asterisk indicates the statistical significance at the 5% level.

	Indonesia	Korea	The Philippines	Thailand
$\alpha_0$	2.113 (1.154)	0.280* (0.134)	0.052* (0.024)	0.034 (0.029)
$\alpha_1$	0.354* (0.102)	0.495* (0.114)	0.304* (0.073)	0.196* (0.042)
$\alpha_2$	0.595* (0.103)	0.457* (0.110)	0.672* (0.072)	0.803* (0.042)

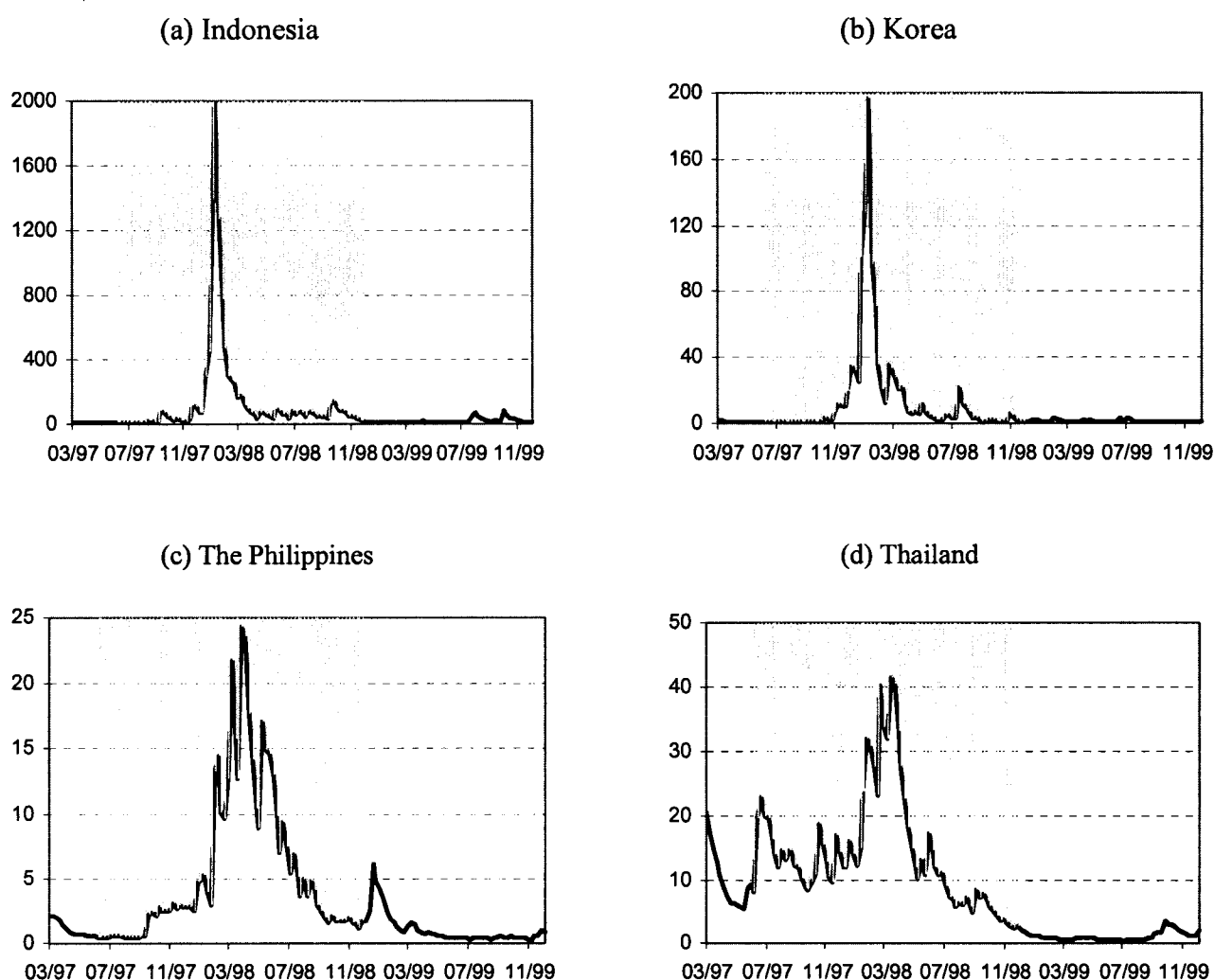
We plot the series of the estimated conditional variances ( $h_t$ ) of exchange rates for each country in Figure 3. Note that for all four countries, the conditional variances increase dramatically from July 1997 to December 1998. The results here are consistent with the fact that these four countries experienced currency crisis during this period. The magnitude of the conditional variances is small in Indonesia, Korea and the Philippines before July 1997 except Thailand. The possible explanation is that Thailand already

experienced speculative attacks in its foreign exchange rate market in 1996 and the Thai baht was under heavy pressure at the beginning of 1997.

Figure 3

### Conditional Variances of Shocks to Exchange Rates

Figure 3 displays the conditional variances ( $h_t$  in equation (49)) of shocks to exchange rates estimated via a TVP model with GARCH disturbances. The shaded areas represent the period of the Asian financial crisis from July 1997 to December 1998.



The chief goal of this paper is to identify the efficacy of using interest rates to defend exchange rate depreciation in the mist of the Asian financial crisis. We carry out this study by investigating the time-varying relationship between interest rates and exchange rates estimated via the TVP model with GARCH disturbances. We plot the estimates of the time-varying coefficients on interest rates ( $\Delta ir_t$  and  $\Delta ir_{t-1}$ ) and their 95% confidence interval bands for Indonesia, Korea, the Philippines, and Thailand. The results are reported in Figures 4, 5, 6, and 7. The estimated coefficients on  $\Delta ir_t$  and  $\Delta ir_{t-1}$  represents the contemporaneous and the lagged impacts of interest rates on exchange rates respectively.

Figure 4 shows the case of Indonesia. Based on Figure 4(a), the estimated time-varying coefficients on  $\Delta ir_t$  is found to be not significantly different from zero throughout most of the sample. The only exception is that in August 1997, estimated coefficients are found to be significant and positive for two weeks. This means during these two weeks, an increase in interest rates resulted in an increase in exchange rates simultaneously. In other words, higher interest rates cause currency depreciation. We don't find evidence that the lagged interest rates ( $\Delta ir_{t-1}$ ) in Figure 4(b) have a significant impact on exchange rates.

Next, we move to Figure 5. Figure 5 exhibits the estimated time-varying coefficients for Korea. We fail to reject that the estimated coefficients on  $\Delta ir_t$  equal zero given the 95% confidence bands in Figure 5(a). For Figure 5(b), we find that the lower 95% confidence band is above zero for the time-varying coefficients during the period from late November in 1997 to mid January in 1998. The period in which a significant and positive relationship is shown is six weeks. We also find that interest rates have a

significant and positive effect on exchange rates for one week in October 1998. This indicates that an interest-rate raise could lead to currency depreciation in the Korea in certain time of the crisis.

The estimates of the time-varying coefficients on interest rates for the Philippines are demonstrated in Figure 6. According to Figure 6(a), the contemporaneous impacts of interest rates on exchange rates are not different from zero at the 5% significant level. In Figure 6(b), we observe that the lagged interest rates have a positive impact on exchange rates that is statistically different from zero for four weeks from early July to early August in 1997. Like the cases of Indonesia and Korea, the estimation results of the Philippines confirm that exchange rates respond positively to a hike in interest rate during some periods in the financial crisis.

Then we look at the results for Thailand. In Figure 7(a), the interest rates are shown to have a significant and positive impact on current exchange rates from mid November to mid December in 1997 and from mid January to late August in 1998 in terms of the 95% confidence bands. In contrast, the impacts of the lagged interest rates on exchange rates are found to be not significantly different from zero in Figure 7(b). Compared to the previous three countries, the significant impact of interest rates on exchange rates is evident in Thailand, which spans a much longer period (37 weeks). The strong evidence of the positive effect of interest rates on exchange rate in Thailand is consistent with Cho and West (2003). Their study shows that Thailand is the only country of which the exchange rate risk premium is strongly and positively related to the interest rate among three East Asian countries (the other two are Korea and the Philippines). Therefore, one would expect an interest rate raise is most likely to



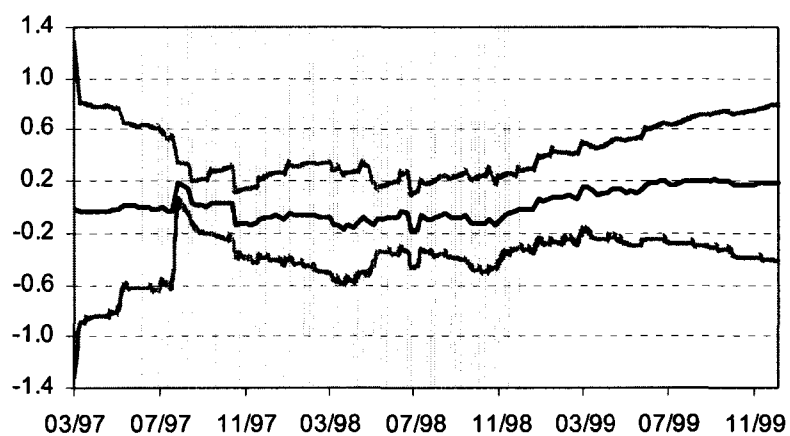
depreciate the currency in Thailand because a small hike in interest rates may induce an excessive exchange rate risk premium.

Figure 4

#### Estimated Time-Varying Coefficients (Indonesia)

The dark line represents the estimates of the time-varying coefficients and the two light lines represent the 95% confidence interval bands. The shaded areas represent the period of the Asian financial crisis from July 1997 to December 1998.

(a) The time-varying coefficients on  $\Delta i_{r_t}$



(b) The time-varying coefficients on  $\Delta i_{r_{t-1}}$

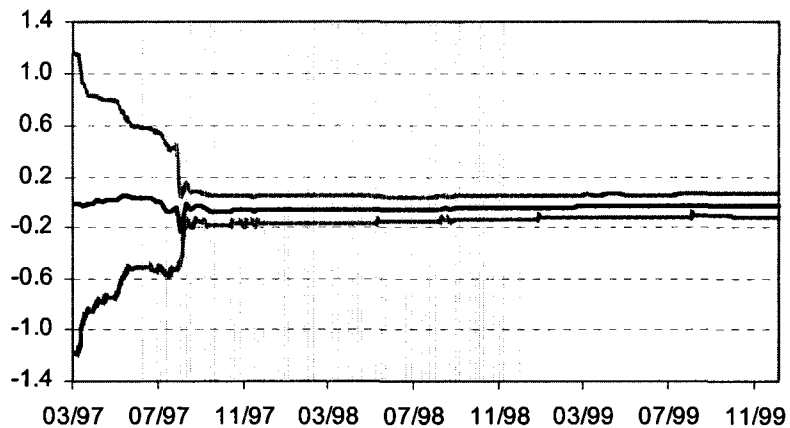
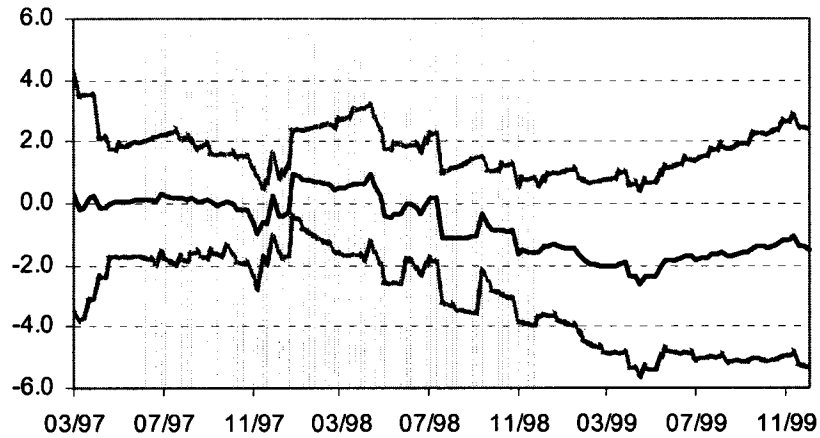


Figure 5

Estimated Time-Varying Coefficients (Korea)

The dark line represents the estimates of the time-varying coefficients and the two light lines represent the 95% confidence interval bands. The shaded areas represent the period of the Asian financial crisis from July 1997 to December 1998.

(a) The time-varying coefficients on  $\Delta ir_t$



(b) The time-varying coefficients on  $\Delta ir_{t-1}$

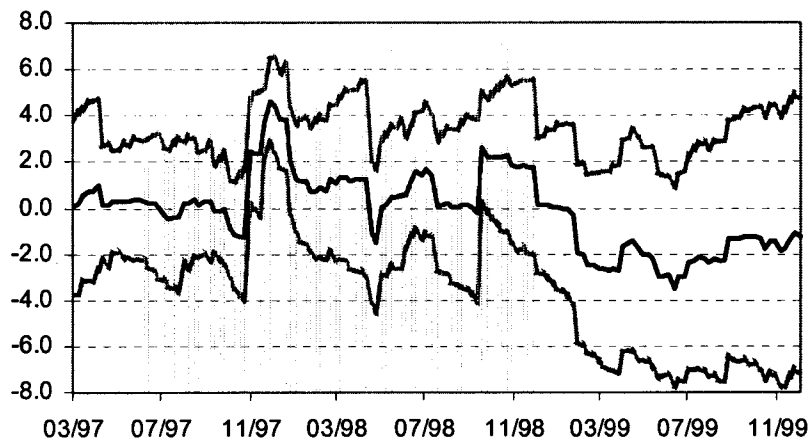
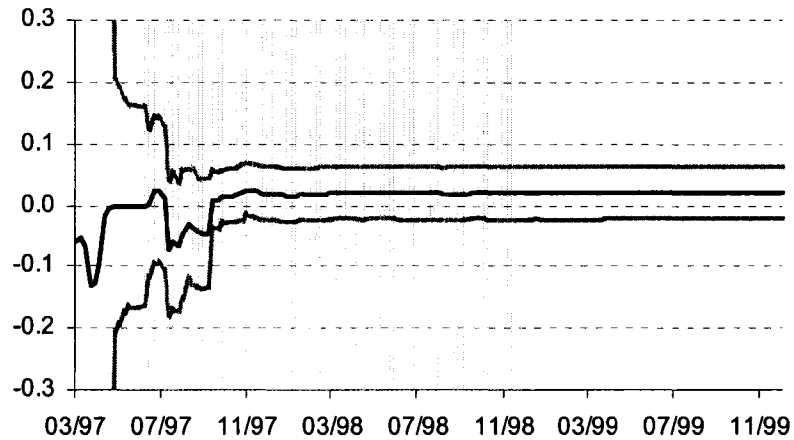


Figure 6

Estimated Time-Varying Coefficients (The Philippines)

The dark line represents the estimates of the time-varying coefficients and the two light lines represent the 95% confidence interval bands. The shaded areas represent the period of the Asian financial crisis from July 1997 to December 1998.

(a) The time-varying coefficients on  $\Delta i r_t$



(b) The time-varying coefficients on  $\Delta i r_{t-1}$

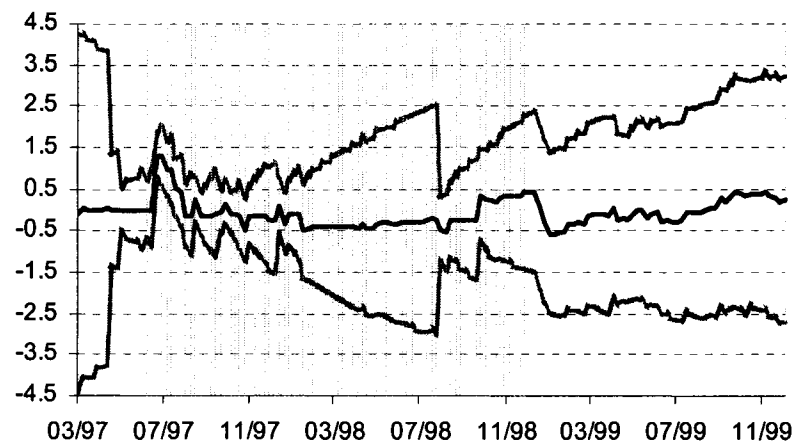
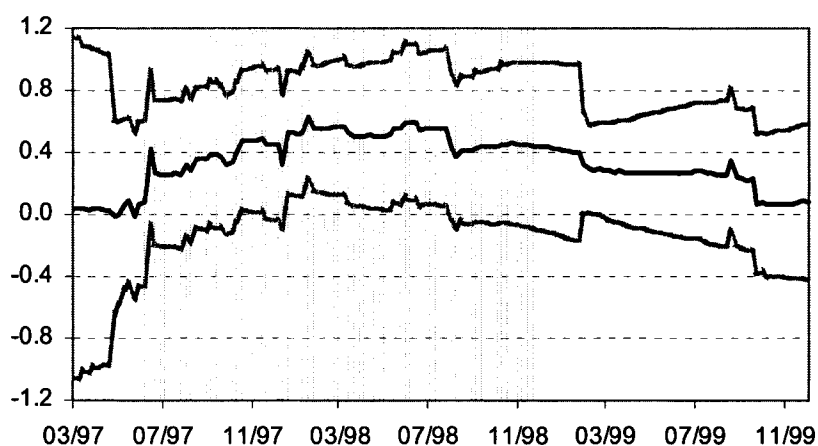


Figure 7

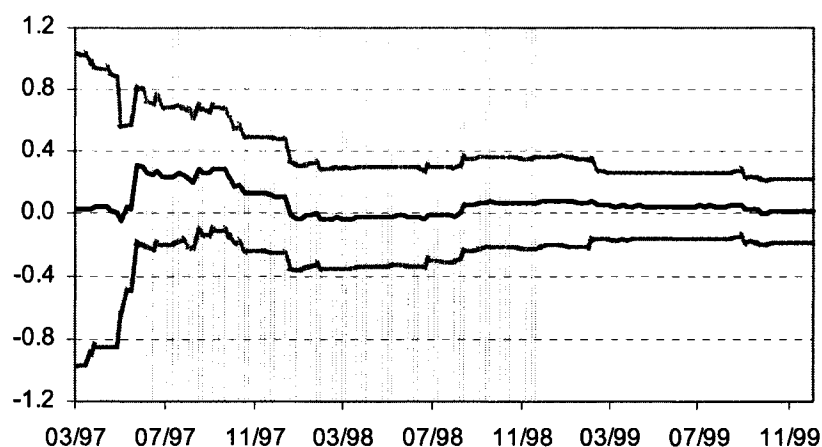
### Estimated Time-Varying Coefficients (Thailand)

The dark line represents the estimates of the time-varying coefficients and the two light lines represent the 95% confidence interval bands. The shaded areas represent the period of the Asian financial crisis from July 1997 to December 1998.

(a) The time-varying coefficients on  $\Delta i r_t$



(b) The time-varying coefficients on  $\Delta i r_{t-1}$



In summary, based on Figures 4, 5, 6, and 7, interest rates are shown to have a significant and positive impact on exchange rates during certain periods of the Asian financial crisis for Indonesia, Korea, the Philippines, and Thailand. We don't find any evidence that an increase in interest rates has a significant and negative impact on exchange rates. The results here are in line with the recent work by Caporale et al. (2005) who also document that higher interest rates cause exchange rate depreciation during the Asian financial crisis for Indonesia, Korea, the Philippines, and Thailand. In addition, our results also indicate that exchange rates don't respond significantly to changes in interest rates in the post-crisis period.

#### 4.6 Conclusions

The role of interest rates in stabilizing depreciating currencies has been one of the most controversial topics since the Asian financial crisis. Although the traditional view advocates using a tight monetary policy to defend weak currencies, the revisionist view stresses that increasing interest rates could lead to exchange rate depreciation because higher interest rates induce higher risk premiums. Many empirical studies have been conducted to examine the interest-exchange rate nexus for the East Asian countries. They normally assume the relationship between interest rates and exchange rates are time-invariant during arbitrarily chosen periods. However, both Baig and Goldfajn (2002) and Cho and West (2003) suggest the interest-exchange rate relationship could vary over time and their empirical evidence shows the relationship between interest rates and exchange rates is sample-dependent. Thus to overcome this problem, this paper proposes

using a TVP model with GARCH disturbances estimated via the Kalman filter to capture the dynamics of the relationship between interest rates and exchange rates.

Using weekly data of exchange rates and interest rates from January 1997 to December 1999, we analyze the impact of interest rates on exchange rates for Indonesia, Korea, the Philippines, and Thailand. The confidence interval bands are constructed from the conditional means and variances of the time-varying coefficients to check the significance of the effect of interest rates on exchange rates. The empirical results indicate that for all four countries, interest rates are found to have a significant and positive impact on exchange rates during certain periods of the financial crisis. The impact of interest rates on exchange rates is the most evident in Thailand. We are unable to find a significant relationship between interest rates and exchange rates in the post-crisis period. The empirical results represented here support the revisionist view that a tight monetary policy could have a perverse effect on exchange rate during currency crises.

## CHAPTER V

### CONCLUSIONS AND FINAL REMARKS

This dissertation studies the topics related to asset pricing and monetary economics. In particular, it focuses on time-varying relationships between variables. As the assumption of a constant relationship in the time-series analysis may lead to misleading results, this dissertation employs a time-varying-parameter model based on the Bayesian approach to account for dynamic relationships. The first two essays analyze portfolio returns under the risk-return framework, and the third essay analyzes the effectiveness of using interest rates to defend currency depreciation during financial crises.

The first essay proposes a time-varying four-factor model to explain the cross-section of average returns on the U.S. stock market. In addition to the Fama-French three factors, a TERM factor, defined as the yield spread between 10-year and 3-month treasury rates, is included in the model to carry the information related to the discount-rate risk for which the Fama-French three factors cannot fully account. Moreover, a time-varying-parameter model estimated with the Kalman filter is used to model the movements of the risk loadings and replicate the investor's learning process.

With 5 by 5 size and book-to-market double-sorted portfolios formed by U.S. stocks from the period 1963:7–2004:12, the estimation results show that 6 statistically significant pricing errors are found in the Fama-French three-factor model that is based on the constant risk loadings. In contrast, the time-varying four-factor model only generates 1 pricing error that is significantly different from zero out of 25 portfolios.

Additionally, the time-varying four-factor model reduces the aggregate pricing errors generated by the Fama-French three-factor model by more than 50 percent. To check the robustness of the estimation results, experiments are conducted with industry-sorted portfolios as well as size and book-to-market double-sorted portfolios prior to 1963. Again, both the individual and aggregate pricing errors in the Fama-French three-factor model are greatly reduced in the time-varying four-factor model. The empirical evidence implies that (1) the TERM factor conveys information related to shocks to the discount rate for which the Fama-French three factors cannot fully account; (2) the Kalman filter improves the accuracy of the estimates of the risk loadings since the investors' learning process mimicked by the Kalman filter captures the dynamics of the risk loadings that the common OLS estimation cannot.

The second essay investigates the risk-return relationship with a time-varying-beta CAPM. In order to capture the dynamics of betas in the CAPM, ALS with Kalman foundations is employed to proxy the investors' learning process of unobservable betas. The stocks listed in S&P 500 are used to evaluate the performance of the time-varying-beta CAPM. The PSM model based on a constant beta is also estimated for comparison purposes. The empirical results show that a positive risk-return relationship exists when the market excess return is positive and a negative risk-return relationship exists when the market excess return is negative. The results also indicate that the time-varying-beta CAPM produces abnormal returns that are not statistically different from zero while the PSM model generates significant abnormal returns. Moreover, the results find that in terms of the realized market excess return, the estimated risk-return relationship obtained from the time-varying-beta CAPM is more accurate than that of the PSM model. In



short, the second essay suggests the beta in a CAPM is still a good measure of risk since ALS with Kalman foundations is more successful at capturing the dynamics of the beta risk than the OLS regression.

Different from the first two essays, the third essay focuses on the effectiveness of using interest rates as a monetary instrument to defend depreciating exchange rates in the Asian financial crisis. As assuming a constant relationship may lose some important information about the dynamics of the interest-exchange rate relationship, this essay employs a time-varying-parameter model with GARCH disturbances to estimate the impacts of raising interest rates on exchange rates. With weekly data from four East Asian countries (Indonesia, South Korea, the Philippines, and Thailand), the empirical evidence show that for all four countries, an increase in interest rates has a significant and positive impact on exchange rates during certain periods of the Asian financial crisis. This result supports the revisionist view that a tight monetary policy will lead to currency depreciation during financial crisis periods because a hike in interest rate could induce excessive exchange rate risk premiums.

In sum, the three essays in this dissertation apply time-varying-parameter models to economics and finance. As estimation methodologies like OLS may not capture time-evolving relationships, these three essays provide evidence of the necessity of using time-varying-parameter models as an alternative to deal with time-varying relationships in the study of economics and finance.

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