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Anderson Localization Effects in the Light Transmitted through a Rough Thin Metal Film

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ANDERSON LOCALIZATION EFFECTS IN THE LIGHT TRANSMITTED THROUGH A ROUGH THIN METAL FILM

by

Naimah Mohd. Shafie

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ANDERSON LOCALIZATION EFFECTS IN THE LIGHT TRANSMITTED THROUGH A ROUGH THIN METAL FILM

Naimah Mohd. Shafie, M.A.

Western Michigan University, 1991

The theory of Anderson localization is used to study transmission enhancements in the diffusely transmitted light through a rough thin metal film. These transmission enhancements arise due to phase coherent effects associated with Anderson localization of surface polaritons and are manifested as a well-defined peak in the angular distribution of the intensity of the diffuse component of the transmitted light for light moving opposite to the specularly reflected beam.

The dependence of the widths and heights of the localization related transmission peak is investigated with computer programming and calculations by McGurn and Maradudin (1989). The peak is found to depend on the degree of surface roughness disorder and the dielectric constant of the media. We studied this transmission peak using the dielectric constants appropriate to silver, copper, and aluminum, finding the optimal condition for each material (silver, copper, and aluminum) under which the localization phenomena can be observed.

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Naimah Mohd. Shafie

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CHAPTER I

INTRODUCTION

As an introduction, we are going to review the basic concept of extended and localized states of electron wavefunctions as introduced by Anderson[1] in 1958. Following Anderson's seminal paper on localized states, a substantial amount of literature has developed on Anderson localization and the nature of localized and extended states. An example of such works can be found in a very good review written by Mott and Davis.[2]

In 1958, Anderson[1] pointed out that if the disorder arising from impurities present in a metallic system is large enough, the conduction electron wavefunctions, treated in the Fermi gas model, in such systems may become localized, i.e., the envelope of these electron wavefunctions decay exponentially away from some central point in space, i.e.,

$$|\psi(\mathbf{r})| \sim \exp(|\mathbf{r}-\mathbf{r}_0|/\xi) \qquad (1)$$

where ξ is the localization length. This is illustrated in Figure 1. What Anderson was trying to point out is that the electron wavefunctions in a random potential may be profoundly altered away from its plane wave (extended state) nature if the randomness is sufficiently strong. The traditional view regarding the nature of the electron wavefunctions in disorder systems had been that the scattering by the random potential caused the Bloch waves to lose phase coherence, on the length scale of the mean free path ℓ and remain in extended states. By retaining this phase coherence which previous theorists had ignored, Anderson was able to show that the resulting phase coherent effects could significantly modify the electron wavefunctions, turning them into localized states.

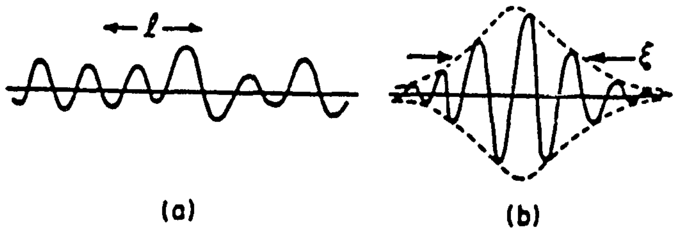

Figure 1. Typical Wave Functions of Extended and Localized States.
(a) illustrates extended state with mean free path, ℓ;
(b) localized state with localization length ξ.

Source: Lee, P. & Ramakrishnan, T. V. (1985). Disordered electronic systems. Review of Modern Physics, 57, 288.

In this paragraph we are going to give a qualitative discussion about what happens in very strong disordered systems to create localized states and how these localized states will transform into extended states as the strength of the disorder is decreased. If we go to the limit of very strong disorder, we can understand the existence of the localized state easily. In this limit, a zeroth-order description of the localized electron eigenstate would be a bound state or a localized orbital bound by a deep fluctuation in the random potential. These localized orbitals, however, are found to interact with one another by a tunnelling effect between their respective wells and this tunnelling acts as a perturbation in the system of localized electrons. The important point, demonstrated by Anderson,[1] is that admixtures of localized states created by tunnelling will not produce extended state, composed of linear combinations of infinitely many localized orbitals, for sufficiently disordered systems. The reason for this is that closely-neighboring orbitals in general have very different energies. Therefore, the admixture from perturbation theory for these states of vastly different energies is small.

On the other hand, the interaction of the orbitals which are close in energy is exponentially small because these orbitals tend to be far apart in space and their wavefunctions decay exponentially with distance from the center of their binding random fluctuation. Thus, the wavefunction will be exponentially localized in the strongly disordered limit. (However, as this disorder is decreased a transition will occur from the localized states to extended states due to the increasing probability for quantum tunnelling processes between similar energied orbitals in the system).

From our above discussion we see that in three dimensions, depending on the strength of the random disorder in our system, we can have both localized and extended states in our random system. This is not always found to be the case. Indeed, in one-dimensional systems, which are extremely susceptible to fluctuations, it is easier to establish the existence of localized states than the extended ones. Mott and Twose[3] and Borland[4] have shown that all states are localized in one dimension, no matter how weak the disorders are. In two dimensions, however, the existence or nonexistence of an extended state in the Fermi gas model of a metal has been a point of contention for many years. It is now generally believed that all states are localized in two dimensions for arbitrarily weak potentials.[5]

Now that we understand the two limits of weak and strong disorder in three-dimensional systems, the interesting question is what happens for intermediate disorder. One way of studying the effect of varying degrees of disorder is to keep the potential of the disorder fixed and to vary the energy of the electron orbitals. We have already described the low energy case (large disorder) in the previous paragraphs. As we increase the electron energy, disorder is less important. We expect the states deep in the band tails (low energies) to be localized, since these are states that are formed from localized orbitals bound in deep potential fluctuations. The states in the center of the band (intermediate energies) have the best chance of remaining extended for a moderately disordered system. Thus, as a function of energy, the states must change their character from being localized to being extended. The critical energy at which this change occurs is called the mobility edge by Mott[6] in 1967. It is so named because, if the Fermi energy lies in a region of localized states (below the mobility edge), the conductivity at zero temperature would vanish, whereas if the Fermi energy lies in a region of extended states (above the mobility edge) this configuration would give rise to a finite zero-temperature conductivity. Thus, the mobility edge marks the transition between a metal and an insulator. This is illustrated in Figure 2. An interesting question in regards to the mobility edge was whether the transition from insulator to conductor as the Fermi surface passed through the mobility edge was continuous, i.e., does the conductivity go to zero continuously in the disorder or does it drop to zero suddenly at some specific value of the disorder? Mott[7] has argued for a discontinuous transition based on the idea of Ioffe and Regal[8] that the lower limit for the mean free path in a metal is the inter atomic spacing. But more recent work of Abrahams, Anderson, Licciardello, and Ramakrishnan[5] seems to argue for a continuous drop in the electron conductivity at the mobility edge.

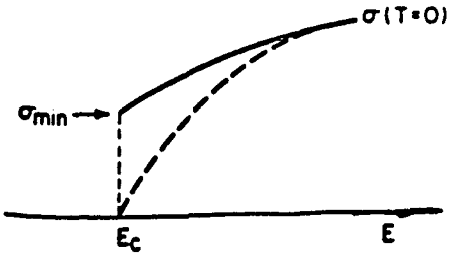

Figure 2. Schematic Illustration of the Mobility Edge E_c Which Separates Localized and Extended States. The two possibilities of a continuous or discontinuous transition with δ_{min} are shown.

Source: Lee, P. & Ramakrishnan, T. V. (1985). Disordered electronic systems. Review of Modern Physics, 57, 289.

From our previous discussions, we can now see that Anderson's original approach to the problem of localization was based on the application of perturbation theory to a system which was strongly localized by strong disorder. The arguments presented by Anderson[1] showed convincingly that states are localized by strong disorder and that such states are quite distinct from the extended states familiar from the Fermi gas theory of metals, so that something like a phase transition from extended to localized states as the disorder is increased must occur. The approximations of

Anderson's[1] original work, however, predicted a much narrower range of stability for localized states than is seen in current numerical simulations.[9] Due to this discrepancy more precise calculations are needed, and work is still proceeding to resolve the problem of precisely determining the nature of the wavefunctions of Anderson localized states.

In order to resolve the problem of the stability of localized solutions, and to determine the amount of disorder needed to localize the excitations in disordered many body systems, a number of experiments and theories have been proposed. It was not until the work of Abrahams et al.[5] that significant advances in obtaining a theoretical solution to this problem have been made. The work of Abrahams et al.[5] was based on the ideas of scaling. What they did was view the disordered conducting materials as being broken up in blocks with weak conduction between the blocks. They go through a decimation procedure where they try to combine these blocks to other blocks, observing how the conductivity changes with the decimation. After the decimation the resulting conductivity is rescaled (change the length scales which measure the material) so that the sample is readjusted to its original shape. The fixed points of this rescaling indicate the transitions of the system between the two different phases (conducting and insulating). All points on the phase diagram of conductivity versus disorder will, in the limit of an infinite number of decimation, approach either of these fixed points (insulating or conducting). The fixed point which this approach indicates the ultimate nature of the material. In one dimension and two dimensions all points for arbitrary weak disorder approach the insulating fixed point and are hence insulators.

One-dimensional scaling theory gives the well-known results obtained by Mott and Twose[3] in 1961 that all states are localized. In two dimensions, the Abrahams et al.[5] theory predicts that all states in random two-dimensional systems are also localized, no matter how weak the disorder is. In three dimensions Abrahams et al.[5] found that there is a mobility edge with energy bands of localized and extended states. In all of these considerations, however, the standard assumption that the electrons in these many-electron systems are non-interacting was made. The question as to how electron-electron interactions change the above results for the dependence of the conductivity on system disorder is an important one which has as yet to be answered satisfactorily.

It was shown by Mott in 1949[7] that Coulomb correlation in a pure (absence of disorder) electron system may lead to a metal-to-insulator transition as well. This is called the Mott-transition. If the Coulomb repulsions (electron-electron interactions) in a conducting system become strong enough the electrons will become more and more correlated, until eventually they will crystalize into the so-called Wigner crystals. These highly correlated systems, and the Wigner crystal itself, will be insulators as the electron will become trapped and unable to move in the highly correlated structures. We have seen above that, in the disordered systems, a metal-insulator transition may arise due to the Anderson localization of the electron wavefunctions. This is because the transition arises due to the localization of the wavefunctions by the disorder. We see that Coulomb correlations may cause problems in our picture of the origins of Anderson localization because both Anderson and electron correlation mechanisms may be acting to create the metal-insulator transition in disorder system with electron-electron scattering. Because of this problem, there have been many qualitative discussions of the interplay between disorder and electron-electron interactions, but until recently very few quantitative results were known. In the last few years a quantitative theory of the interacting disordered systems has begun to emerge, at least in the weakly disordered limit.[10,11,12,13]

In order to study Anderson localization effects which are not complicated, as they are in electron systems, by the interactions of the wave excitations with one another in the disordered media, people have been looking for wave excitations in which the different waves in the systems do not interact with one another. These systems will then exhibit transition systems that are truly Anderson, without having to worry about the interaction of successive waves which could create localized states via a Mott type of mechanism. In such systems, we can study a nice, clean cut Anderson localization problem, unlike the electron systems where we have to worry whether localizing transitions are coming about due to Anderson localization or because of Mott-transition mechanism.

People looked for systems with pure Anderson transitions. They found such systems in Acoustics and Optics. Acoustical and Optical systems have non-interacting excitation, but in these systems the additional problem develops as to whether one can measure a phenomenon which is directly related to Anderson localization. The effect that we are going to discuss now, is an optical effect which satisfies the criterion of being a means of measuring Anderson localization states.

It is well known that surface roughness plays a significant role in a variety of optical interactions that occur at solid surfaces. For example, it makes possible effects that cannot exist at a planar dielectric surface, such as the resonant excitation of surface electromagnetic waves (surface polariton), or surface shape resonances associated with localized protrusions or indentations on a surface, by a volume electromagnetic wave incident from the vacuum side. The excitation of these surface-localized electromagnetic modes in turn strongly enhances the electromagnetic field in the immediate vicinity of the surface, which is believed to be a major factor in the surface-enhanced Raman scattering of light from molecules adsorbed on a metal surface. In addition and of primary concern to us is the fact that these resonant interactions also give rise to general diffuse scattering of radiation observed from randomly rough surfaces or in the transmission of light through a thin surface rough film.[10,11,12,13]

The rough surfaces that give rise to the effects just mentioned and to others can be of several types. They can be deterministic and periodic, such as one-dimensional (classical) gratings or two-dimensional grating (bi-grating); they can be deterministic but not periodic, as in the case of a localized protrusion or indentation on an otherwise planar surface; or they can be random and defined only through the statistical properties of the surface profile function. We shall be interested in this later variety of such surface roughness.[14]

Recently, there has been considerable interest in the localization properties of electromagnetic waves traveling in disordered media and on rough surfaces. Propagation both in bulk media and along surfaces has been studied for the purpose of determining the effect of localization on the optical properties of bulk materials and surfaces. In this thesis we shall concentrate on effects arising from the localization of surface electromagnetic waves traveling on the randomly rough surface. The simplest of all optical interactions occurring at a randomly rough surface is the scattering of light from such a surface. It has been the subject of numerous experimental and theoretical investigations. It has been found that lowest-order perturbation theory gives an adequate description of polarized scattering from very weakly corrugated surfaces.[11]

From a theoretical standpoint, calculations related to surface polaritons have been done to determine which features in the diffuse scattering and transmission of light from randomly rough surface gratings ruled on thin metal film are caused by surface-polariton localization. It has been shown by McGurn and Maradudin[15] that the major contribution from localization is a maximum in the anti-specular region of the diffuse scattering of radiation from the surface and another maximum in the diffusely transmitted light moving antiparallel to the specularly reflected beam. The height and width of this antispecular peak and the transmission maxima can be shown to be related simply to the localization length of the surface polariton resonantly excited in the scattering process. In addition, these calculations determine the surface-polariton localization length as a function of surface roughness and polariton frequency. The above work is based on diagrammatic techniques developed by Vollhardt and Wolfle[16] in studies of the localization properties of electrons in random potential.

In this thesis we shall study the calculations done by McGurn and Maradudin[15] for transmission of light through a film with randomly rough surfaces. We shall investigate the dependence of the width and height of the localization related transmission peak. Specifically, the peak is found to depend on the degree of surface roughness disorder and the dielectric constant of the media which itself is dependent on the frequency of the scattered light and material of the film. We shall investigate the dependence on frequency of the height and width of the transmission peaks using the dielectric constants appropriate to aluminum, copper, and silver for a fixed surface roughness. We shall be interested in finding the optimal conditions under which this localization phenomenon can be observed in these materials. In Chapter II we shall discuss calculations of thin localization peaks for a system that has a dielectric medium with large dielectric losses. We shall study this peak for aluminum, copper, and silver thin films observing the frequency dependence of the height and width of the localized related transmission anomaly. In Chapter III we shall present our conclusion.

CHAPTER II

THEORY AND RESULT

A large portion of this chapter will deal with recently discovered transmission enhancements in the light diffusely transmitted through a rough metal film. These transmission enhancements arise due to phase coherent effects associated with Anderson localization of surface polaritons. This enhanced transmission manifests itself as a well-defined peak in the angular distribution of the intensity of the diffuse component of the transmitted light when the transmission angle corresponds to that of transmitted light moving in a direction directly opposite to that of the specularly reflected beam from the thin film.

This new mechanism of Anderson localization as a source for enhanced transmission peaks is found to operate on weakly rough or disordered surfaces that support localized surface polaritons. The diffuse transmission of light from such surfaces is viewed as arising from the resonant interaction of incident and transmitted light with surface polaritons moving along the rough or disordered surface. It is the phase coherent propagation of these surface polaritons along the random surface that creates the conditions necessary to localize them (i.e., the wave function of the surface polariton is confined in its motion to a finite region of the surface) and this localization of surface polaritons shows up in the diffuse transmission as a peak in the angular distribution of diffusely transmitted light.

Transmission enhancements associated with Anderson localization of surface polaritons have been observed in the transmission of light from a number of weakly rough metallic mirrors. These effects were originally predicted theoretically by McGurn and Maradudin[15] and later observed experimentally on silver and gold film by Han-Gu, Dummer, Maradudin and McGurn.[9] They found that the angular distribution of the intensity of the diffuse component of the transmitted light display, a well-defined peak in the direction of transmission, is directly opposite to the direction of specular reflection of the incident light. They predicted that the same effect should also occur when the film is made from a nearly transparent dielectric material and/or when both surfaces of the film are randomly rough. Only a few thin film structures have been investigated as of this writing and one of our goals in this thesis will be to study the effects of different materials and frequencies of light on the heights and widths of the transmission enhancement peaks.

With computer programming and calculations by McGurn and Maradudin,[15] we investigate the dependence of the widths and heights of the localization related transmission peak. Using the dielectric constants appropriate to aluminum, copper, and silver, we also investigate the dependence on frequency of the height and width of the transmission peaks. The peak is found to depend on the degree of surface roughness disorder and the dielectric constant of the media which itself is frequency and material dependent. In our work we will fix the surface roughness and specifically investigate the effects on the localization peaks of changes in the frequencies and materials of the film.

The computer programming that we used consists of a program that reads in the experimentally measured values of the dielectric constants at various wavelengths appropriate to aluminum, copper, and silver. Using the built-in functions of the IMSL[17] library we then interpolate the data sets to obtain and approximate expression for the angular frequency dependent dielectric constant at general frequencies. The dielectric constants at general frequencies and wavelengths for the different materials (i.e, aluminum, copper, silver) are then used to determine the differential transmission intensity versus transmission angle, and the localization peak widths and heights are determined from these plots. Using the peak widths and heights plots we find the optimal condition under which the localization phenomena can be observed in aluminum, copper, and silver.

Interpolation for Dielectric Constant

We will now discuss this interpolation routine in greater detail and then discuss how it is used in determining the localization peak in the differential transmission computations. In this thesis we have ten sets of experimental data on the dielectric constants for aluminum, copper, and silver (three sets of data for aluminum, five sets of data for copper, and one set of data for silver). All of these data are listed in Tables 1 through 9. In these sets of data are listed values for wavelength versus dielectric constants for the real and imaginary parts irrespective of the complex dielectric constants of these materials. It is our intent to use a spline interpolation routine to find the relationship between wavelength and the dielectric constants for general wavelength.

In the program, we use the B-Spline interpolation. In this interpolation of the data sets, we approximate the values of dielectric constants between each successive pair of nodes (points) by using polynomials of degree two. We divide the interval into a collection of subintervals and construct a different approximating polynomial on each

Table 1

Wavelength and Real and Imaginary Parts of Dielectric Constant for Aluminum (Shiles et al., 1980)

$\lambda(\mu m)$	$-\epsilon_1$	ϵ_2
3.10E+01	3.18E+04	4.02E+04
2.95E+01	3.01E+04	3.62E+04
2.70E+01	2.68E+04	3.03E+04
2.48E+01	2.43E+04	2.59E+04
2.30E+01	2.14E+04	2.24E+04
2.14E+01	1.95E+04	2.01E+04
2.00E+01	1.80E+04	1.79E+04
1.88E+01	1.66E+04	1.60E+04
1.72E+01	1.50E+04	1.38E+04
1.55E+01	1.32E+04	1.13E+04
1.41E+01	1.18E+04	9.49E+03
1.29E+01	1.05E+04	7.89E+03
1.13E+01	8.77E+03	5.94E+03
9.54E+00	6.93E+03	4.07E+03
8.27E+00	5.58E+03	2.86E+03
7.29E+00	4.51E+03	2.05E+03
6.20E+00	3.39E+03	1.39E+03
4.96E+00	2.25E+03	8.28E+02
4.13E+00	1.63E+03	5.54E+02
3.54E+00	1.24E+03	3.87E+02
3.10E+00	9.71E+02	2.80E+02
2.07E+00	4.53E+02	9.73E+01
1.55E+00	2.52E+02	4.61E+01
1.24E+00	1.54E+02	3.02E+01
8.27E-01	6.15E+01	4.56E+01
6.20E-01	5.42E+01	1.95E+01

Source: Shiles, E., Sasaki, T., Inokuti, M., & Smith, D. Y. (1980). Self consistence and sum-rule test in the Kramers-Kronig analysis of optical data: Application to aluminum. Physical Review B, 22, 1612.

Table 2

Wavelength and Real and Imaginary Parts of Dielectric Constant for Aluminum (Bennett & Bennett, 1966)

$\lambda(\mu m)$	$-\epsilon_1$	ϵ_2
3.20E+01	2.60E+04	5.56E+04
3.10E+01	2.58E+04	5.31E+04
3.00E+01	2.56E+04	5.08E+04
2.90E+01	2.54E+04	4.84E+04
2.80E+01	2.47E+04	4.59E+04
2.70E+01	2.45E+04	4.36E+04
2.60E+01	2.38E+04	4.12E+04
2.50E+01	2.36E+04	3.91E+04
2.40E+01	2.31E+04	3.64E+04
2.30E+01	2.25E+04	3.42E+04
2.20E+01	2.19E+04	3.18E+04
2.10E+01	2.10E+04	2.93E+04
2.00E+01	2.05E+04	2.71E+04
1.90E+01	1.96E+04	2.47E+04
1.80E+01	1.88E+04	2.24E+04
1.70E+01	1.80E+04	2.02E+04
1.60E+01	1.69E+04	1.79E+04
1.50E+01	1.58E+04	1.58E+04
1.40E+01	1.47E+04	1.37E+04
1.30E+01	1.37E+04	1.18E+04
1.20E+01	1.24E+04	9.88E+03
1.10E+01	1.10E+04	8.06E+03
1.00E+01	9.84E+03	6.49E+03
9.00E+00	8.41E+03	5.02E+03
8.00E+00	7.02E+03	3.72E+03

Source: Bennett, H. E., & Bennett, J. M. (1966). Optical properties of the noble metals in the infrared and far infrared. Applied Optics, 22, 1103.

Table 3

Wavelength and Real and Imaginary Parts of Dielectric Constant for Aluminum (Schulz, 1954)

$\lambda(\mu m)$	$-\epsilon_1$	ϵ_2
9.50E-01	6.92E+01	2.98E+01
9.00E-01	5.54E+01	3.02E+01
8.50E-01	4.68E+01	2.97E+01
8.00E-01	4.57E+01	2.81E+01
7.50E-01	4.75E+01	2.56E+01
7.00E-01	4.66E+01	2.17E+01
6.50E-01	4.20E+01	1.64E+01
6.00E-01	3.51E+01	1.16E+01
5.50E-01	2.77E+01	8.09E+00
5.00E-01	2.27E+01	5.95E+00
4.50E-01	1.84E+01	4.23E+00
4.00E-01	1.52E+01	3.14E+00

Source: Schulz, L. G. (1954). The optical constant of silver, gold, copper, and aluminum. _Journal Optical Society of America_, _44_, 357 & 362.

subinterval. From these we obtain the interpolation polynomial for the entire data.

The interpolation process will be done using the built-in functions of IMSL[17] library called BSNAK, BSINT, and BSVAL. The BSNAK function computes the spline knot

Table 4

Wavelength and Real and Imaginary Parts of Dielectric Constant for Copper (Schulz, 1954)

$\lambda(\mu m)$	$-\epsilon_1$	ϵ_2
9.50E-01	3.87E+01	1.62E+00
9.00E-01	3.43E+01	1.52E+00
8.50E-01	2.99E+01	1.31E+00
8.00E-01	2.57E+01	1.22E+00
7.50E-01	2.13E+01	1.11E+00
7.00E-01	1.74E+01	1.00E+00
6.50E-01	1.33E+01	9.49E-01
6.00E-01	9.40E+00	1.04E+00
5.50E-01	5.34E+00	3.48E+00
5.00E-01	5.08E+00	4.26E+00
4.50E-01	4.08E+00	3.88E+00

Source: Schulz, L. G. (1954). The optical constant of silver, gold, copper, and aluminum. Journal Optical Society of America, 44, 357 & 362.

sequences. These spline knot sequences will be used in the second built-in function of IMSL[17] library BSINT to compute the spline interpolation and returning the B-Spline coefficients. With the B-Spline coefficients and tested/general wavelength as input, we will find the approximated value of dielectric constants by using BSVAL.

Table 5

Wavelength and Real and Imaginary Parts of Dielectric Constant for Copper (Lenham, 1966)

$\lambda(\mu m)$	$-\epsilon_1$	ϵ_2
2.00E+01	1.35E+04	7.61E+03
1.80E+01	1.15E+04	6.11E+03
1.60E+01	9.00E+03	4.64E+03
1.40E+01	6.80E+03	3.36E+03
1.20E+01	5.05E+03	2.29E+03
1.00E+01	3.50E+03	1.40E+03
8.00E+00	2.20E+03	7.28E+02
6.00E+00	1.30E+03	3.24E+02
5.00E+00	1.00E+03	1.40E+02
4.00E+00	6.22E+02	8.80E+01

Source: Lenham, A. P. (1966). Applicability of the anomalous skin-effect theory to the optical constants of Cu, Ag, and Au in the infrared. Journal Optical Society of America, 56, 683.

In the program, each data set will be plotted using the built-in function of IMSL[17] library PLOTP. For plotting the program, we used the UMACH function to set the output device unit number before calling PLOTP function. The dielectric constants at tested/general wavelength for aluminum, copper, and silver are then used to determine the differential transmission intensity versus transmission

Table 6

Wavelength and Real and Imaginary Parts of Dielectric Constant for Copper (Ordal et al., 1983)

$\lambda(\mu m)$	$-\epsilon_1$	ϵ_2
6.40E-01	7.67E+00	1.70E+00
6.00E-01	5.98E+00	1.70E+00
5.60E-01	4.09E+00	2.20E+00
5.20E-01	3.71E+00	6.99E+00
4.80E-01	3.10E+00	7.01E+00
4.40E-01	2.39E+00	6.79E+00
4.00E-01	1.81E+00	5.92E+00

Source: Ordal, M. A., Long, L. L., Bell, R. J. Bell, S. E., Bell, R. R., Alexander, R. W., Jr., Ward, C. A. (1983). Optical properties of the metals Al, Co, Cu, Au, Fe, Pb, Ni, Pd, Pt, Ag, Ti, and W in the infrared and far infrared. Applied Optics, **22**, 1104.

angle, and the localization peak widths and heights are determined from these plots. We now turn to a discussion of the dependence of these features of the localization on the λ and $-\epsilon_1$, and ϵ_2 for aluminum, copper and silver.

Localization Peak

McGurn and Maradudin[15] used a geometry system as in Figure 3 in their paper. It consisted of vacuum in the

Table 7

Wavelength and Real and Imaginary Parts of Dielectric Constant for Copper (Hagemann et al., 1975)

$\lambda(\mu m)$	$-\epsilon_1$	ϵ_2
1.24E+01	4.24E+03	4.25E+03
2.48E+01	3.08E+02	6.03E+01
1.23E+01	7.17E+01	7.46E+00
8.27E-01	2.76E+01	2.74E+00
7.29E-01	1.96E+01	1.95E+00
7.08E-01	1.80E+01	1.79E+00
6.89E-01	1.63E+01	1.70E+00
6.70E-01	1.48E+01	1.69E+00
6.53E-01	1.34E+01	1.58E+00
6.20E-01	1.04E+01	1.75E+00

Source: Hagemann, H. J., Gudat, W., & Kunz, C. (1975). Optical constant from the far infrared to the X-ray region: Mg, Al, Cu, Ag, Au, Bi, C, and Al_2O_3. Journal Optical Society Of America, 65, 742.

region $x_3 > \zeta(x_1)$ and $x_3 < -d$, and a metal film characterized by an isotropic, frequency-dependent dielectric constant $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$ in the region $-d < x_3 < \zeta(x_1)$. They worked in the frequency range in which $\epsilon_1(\omega) < -1$ and $\epsilon_2(\omega) << |\epsilon_1(\omega)|$; characteristic of metal system in the visible spectrum. The surface profile function $\zeta(x_1)$ is

Table 8

Wavelength and Real and Imaginary Parts of Dielectric Constant for Copper (Ordal et al., 1983)

$\lambda(\mu m)$	$-\epsilon_1$	ϵ_2
1.00E+01	2.27E+03	1.14E+03
9.00E+00	1.99E+03	9.05E+02
8.00E+00	1.66E+03	6.72E+02
7.00E+00	1.31E+03	4.71E+02
6.00E+00	9.99E+02	3.17E+02
5.00E+00	6.95E+02	1.92E+02
4.00E+00	4.56E+02	1.05E+02
3.00E+00	2.54E+02	4.80E+01
2.00E+00	1.12E+02	1.80E+01
1.50E+00	6.37E+01	9.28E+00
1.25E+00	4.46E+01	6.56E+00

Source: Ordal, M. A., Long, L. L., Bell, R. J. Bell, S. E., Bell, R. R., Alexander, R. W., Jr., & Ward, C. A. (1983). Optical properties of the metals Al, Cu, Au, Fe, Pb, Ni, Pd, Pt, Ag, Ti, and W in the infrared and far infrared. Applied Optics, 22, 1105.

assumed to be a stationary, gaussian process defined by the properties $< \zeta(x_1) > = 0$ and $< \zeta(x_3)\,\zeta(x_1') > = \delta^2 \exp(-|x_1-x_1'|^2 / a^2)$, where the angular brackets denote an average over the ensemble of realizations of the surface profile.

Table 9

Wavelength and Real and Imaginary Parts of Dielectric Constant for Silver (Johnson & Christy, 1972)

$\lambda(\mu m)$	$-\epsilon_1$	ϵ_2
4.00E-01	4.45E+00	2.12E-01
4.25E-01	5.77E+00	2.07E-01
4.50E-01	7.01E+00	2.10E-01
4.75E-01	8.45E+00	2.96E-01
5.00E-01	9.79E+00	3.10E-01
5.25E-01	1.14E+01	3.44E-01
5.50E-01	1.30E+01	4.31E-01
5.75E-01	1.44E+01	3.89E-01
6.00E-01	1.60E+01	4.33E-01
6.33E-01	1.81E+01	5.01E-01
6.50E-01	1.94E+01	4.67E-01

Source: Johnson, P. B., & Christy, R. W. (1972). Optical constant of the noble metals. Physical Review B, 6, 4374.

The light that they used is taken to be p-polarized with the x_1x_3-plane as the plane of incidence. The magnetic field vector is $H(x,t) = (0, H_2(x_1x_3|\omega, 0) \exp(-i\omega t)$, where $H_2(x_1x_3|\omega)$ is given by

$$H_2(x_1x_3|\omega) = \exp[ikx_1 - i\alpha_0(k)\,x_3] + \int \frac{dq}{2\pi} R(q|k) \exp[iqx_1 + i\alpha_0(q)\,x_3], \quad x_3 \geqslant \zeta(x_1) \qquad (2a)$$

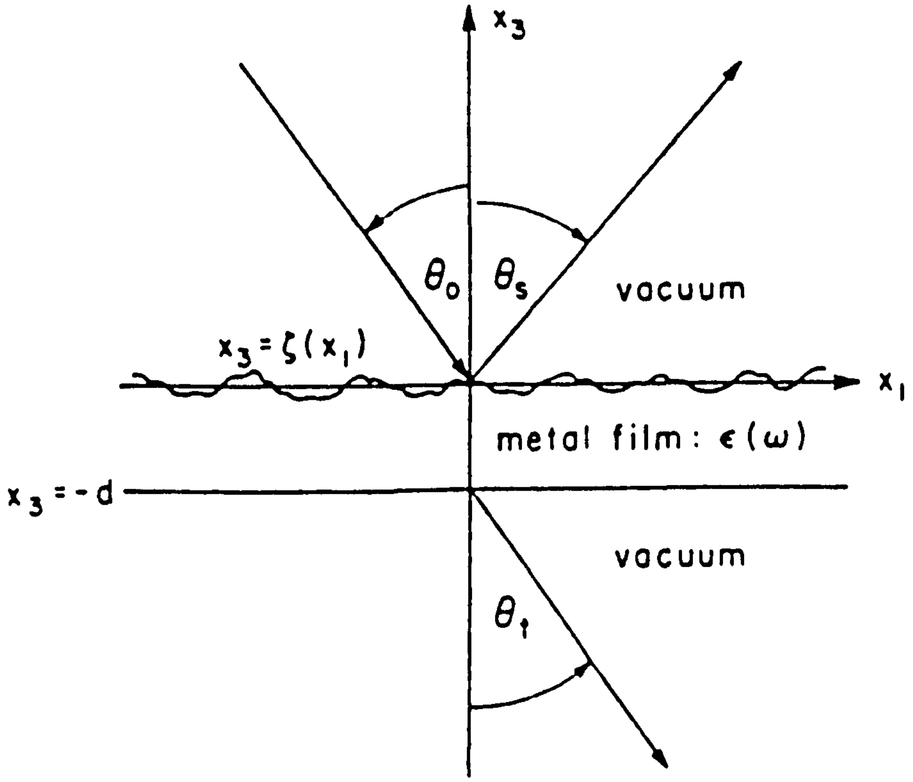

Figure 3. The Physical System Studied by McGurn and Maradudin (1989).

Source: McGurn, A. R., & Maradudin, A. A. (1989). An analogue of enhanced backscattering in the transmission of light through a thin film with a randomly rough surface. Optics Communications, 72, 280.

$$= \int \frac{dq}{2\pi} \exp(iqx_1)\{B(q|k)\exp[-i\alpha(q)x_3] + C(q|k)\exp[i\alpha(q)x_3]\}, \quad -d \leq x_3 \leq \zeta(x_1) \tag{2b}$$

$$= \int \frac{dq}{2\pi} T(q|k)\exp[iqx_1 - i\alpha_0(q)x_3], \quad x_3 \leq -d \tag{2c}$$

where

$$\alpha_0(q) = (\omega^2/c^2 - q^2)^{1/2}, \quad \mathrm{Re}\,\alpha_0(q) > 0, \quad \mathrm{Im}\,\alpha_0(q) > 0 \tag{3}$$

$$\alpha(q) = [\epsilon(\omega)\omega^2/c^2 - q^2]^{1/2}, \quad \mathrm{Re}\,\alpha(q) > 0, \quad \mathrm{Im}\,\alpha(q) > 0 \tag{4}$$

and R(q|k), B(q|k), C(q|k) and T(q|k) are determined by matching the electromagnetic boundary conditions.

From the scattered and transmitted field amplitudes in equations (2a) and (2c), respectively, they obtained the average differential reflection and transmission coefficients,

$$\frac{\partial R}{\partial \theta_s} = \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_s}{\cos\theta_0}\langle |R(q|k)|^2\rangle \Big|^{q=(\omega/c)\sin\theta_s,\, k=(\omega/c)\sin\theta_0}_{q=(\omega/c)\sin\theta_s,\, k=(\omega/c)\sin\theta_0} \qquad (5)$$

$$\frac{\partial T}{\partial \theta_t} = \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_t}{\cos\theta_0}\langle |T(q|k)|^2\rangle \Big|_{q=(\omega/c)\sin\theta_t,\, k=(\omega/c)\sin\theta_0} \qquad (6)$$

where L_1 is the length of the surface along the x_1-axis, while the angles of incidence, scattering, and transmission, θ_0, θ_s, θ_t, respectively, are defined in Figure 3.

From equation 2 and the boundary conditions at the metal surfaces, they expanded the reflection and transmission amplitudes in power of the surface profile function, $\zeta(x_1)$, and got the averaged differential reflection and transmission coefficients of the diffuse components of the reflected and transmitted light:

$$\left(\frac{\partial R}{\partial \theta_s}\right)_{\mathrm{diff}} = \frac{4}{L_1}\frac{\omega^3}{2\pi c^3}\cos^2\theta_s \cos\theta_0 \langle |G(q|k)|^2\rangle_{\mathrm{diff}} \Big|_{q=(\omega/c)\sin\theta_s,\, k=(\omega/c)\sin\theta_0} \qquad (7)$$

$$\left(\frac{\partial T}{\partial \theta_t}\right)_{\mathrm{diff}} = \frac{4}{L_1}\frac{\omega^3}{2\pi c^3}\cos^2\theta_t \cos\theta_0 \,|G_0[(\omega/c)\sin\theta_t]|^2 \langle |F(q|k)|^2\rangle_{\mathrm{diff}} \Big|_{q=(\omega/c)\sin\theta_t,\, k=(\omega/c)\sin\theta_0} \qquad (8a)$$

where

$$F(q|k) = \int \frac{dp}{2\pi} V_T^{(1)}(q|p)\, G(p|k) \tag{8b}$$

and

$$V_T^{(1)}(q|k) = 2\frac{\epsilon(\omega)-1}{\epsilon(\omega)}\hat{\zeta}(q-k)\frac{qk+\alpha_0(q)\,\alpha(k)}{\epsilon(\omega)\,\alpha_0(q)+\alpha(q)}\alpha(q)\exp\{-i[\alpha_0(q)-\alpha(q)]d\} \tag{8c}$$

where G(q|k) is the Green's function for the propagation of surface polaritons along the surfaces of the rough thin metal film.

The surface polariton Green's function G(q|k) is related to the probability of injecting a polariton of momentum k and frequency ω into the system and having it scattered by the surface roughness into a final state of momentum q and frequency ω. McGurn and Maradudin[15] have made extensive studies of G(q|k) which allowed them to determine that the polaritons in the thin film are localized. The localization of the film polaritons show up on $<|G(q|k)|^2>$ as a pole in the q+k of the form

$$< |G(q|k)|^2 > \;\alpha\; \frac{1}{(q+k)^2 + 4\Delta_{tot}^2} \tag{9}$$

where

$$\Delta_{tot} = \frac{\epsilon_2(\omega)K_{sp}}{2\epsilon_1(\omega)[\epsilon_1(\omega)+1]} + 2\pi^{1/2}a\sigma^2 C_1^2\left[\frac{\epsilon_1(\omega)-1}{\epsilon_1(\omega)}\right]^2 \tag{10}$$

Upon calculating G(q|k) and substituting into equations (7) and (8) McGurn and Maradudin[15] find for q = $^{\omega}/c$ sin θ_s and k = ω/c sin θ_i

$$\left(\frac{\partial R}{\partial \theta_s}\right)_{diff} \quad \alpha \quad \frac{1}{(q+k)_2 + 4\Delta_{tot}^2} \tag{11a}$$

$$\left(\frac{\partial T}{\partial \theta_t}\right)_{diff} \quad \alpha \quad \frac{1}{(q+k)_2 + 4\Delta_{tot}^2} \tag{11b}$$

so that at normal incidence (k=0) the full width at half maximum, $\Delta\theta$, of the backscattering and transmission peaks are

$$\Delta\theta = \frac{4\Delta_{tot}}{\omega/c} \tag{12}$$

They also calculated the peak heights and found that for the transmission peak the ratio of the localization peak height to the diffuse background intensity was given by dividing equation (21) in McGurn and Maradudin[15] by equation (19) of McGurn and Maradudin and evaluating the resulting ratio for q = 0.

We have evaluated equation (12) and the expression for the ratio of the localization peak height to the diffuse background intensity for aluminum, silver and copper. The results are shown in Figures 7, 8, 9, 10, 11, and 12 (pages 38-41).

CHAPTER III

CONCLUSIONS

We studied problems associated with the diffuse transmission of light through a thin metal film with rough surfaces. We investigated the dependence on frequency of the heights and widths of transmission peaks arising from the Anderson localization of surface polariton, using dielectric constants appropriate to aluminum, copper, and silver. These peaks were found to depend on the degree of surface roughness as well as on the frequencies of the elastically transmitted light. The computer programming that we used consists of a subroutine that reads in the experimental measured values of the dielectric constants at various wavelength appropriate to aluminum, copper and silver, and a main routine which computes the diffuse component of transmitted light through the rough film. Using the built-in function of the IMSL[17] library we interpolate experimental data sets for the dielectric constant of silver, aluminum, and copper to obtain an approximate expression for the angular frequency dependent dielectric constant of these materias at general frequencies. These dielectric constants are then used by the main routine to determine the differential transmission intensity versus transmission angle, and the localization peak widths and heights are determined from these plots. Using the peak widths and heights plots we find the optimal condition under which the localization phenomena can be observed in aluminum, copper, and silver.

From our peak width and height plots we find that the different materials behave quite differently from one another with increasing frequencies. Even though they behave differently, one general property of aluminum, copper and silver is that the real part of the dielectric constant increases as the wavelength increases. This is illustrated in Figures 4, 5, and 6. The second general property of this material is that the full width at half-maximum for the transmission peaks tends to change with respect to the imaginary part of the dielectric constant. This is so because the equation (18) in the paper by McGurn and Maradudin[15] states that the width is directly related to the imaginary part of the dielectric constant. This is illustrated in Figures 7, 8, and 9 (pages 38 and 39). The third general property of aluminum, copper, and silver is that the ratio which is the height of the enhancement peak to the diffuse background generally decreases with increased wavelength. This is correlated to the dielectric constant. It appears this decrease is associated with the

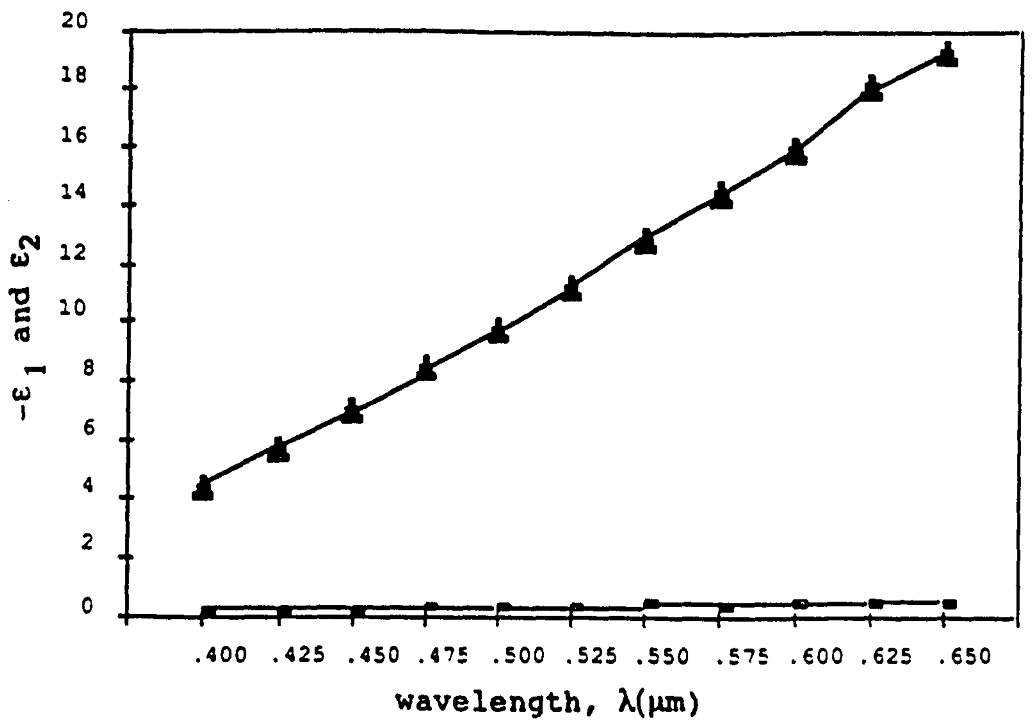

Figure 4. Frequency Dependent of Dielectric Constant for Silver.
$\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$ where ϵ_1 and ϵ_2 are the real and imaginary parts irrespectedly for silver. Curve for $-\epsilon_1$ is ▲. Curve for ϵ_2 is ■.

Source: Johnson, P. B., & Christy, R. W. (1972). Optical constants of the noble metals. Physical Review B, 6, 4374.

increase in the real part of dielectric constant with frequency increase. The larger effect in terms of the height of the peaks will occur for different values of the wavelength of the incident light. This is illustrated in Figures 10, 11, and 12 (pages 40 and 41).

Of all of the three materials, copper is the most interesting of all, since it has a very narrow peak that has a high ratio of background to peak in the visible

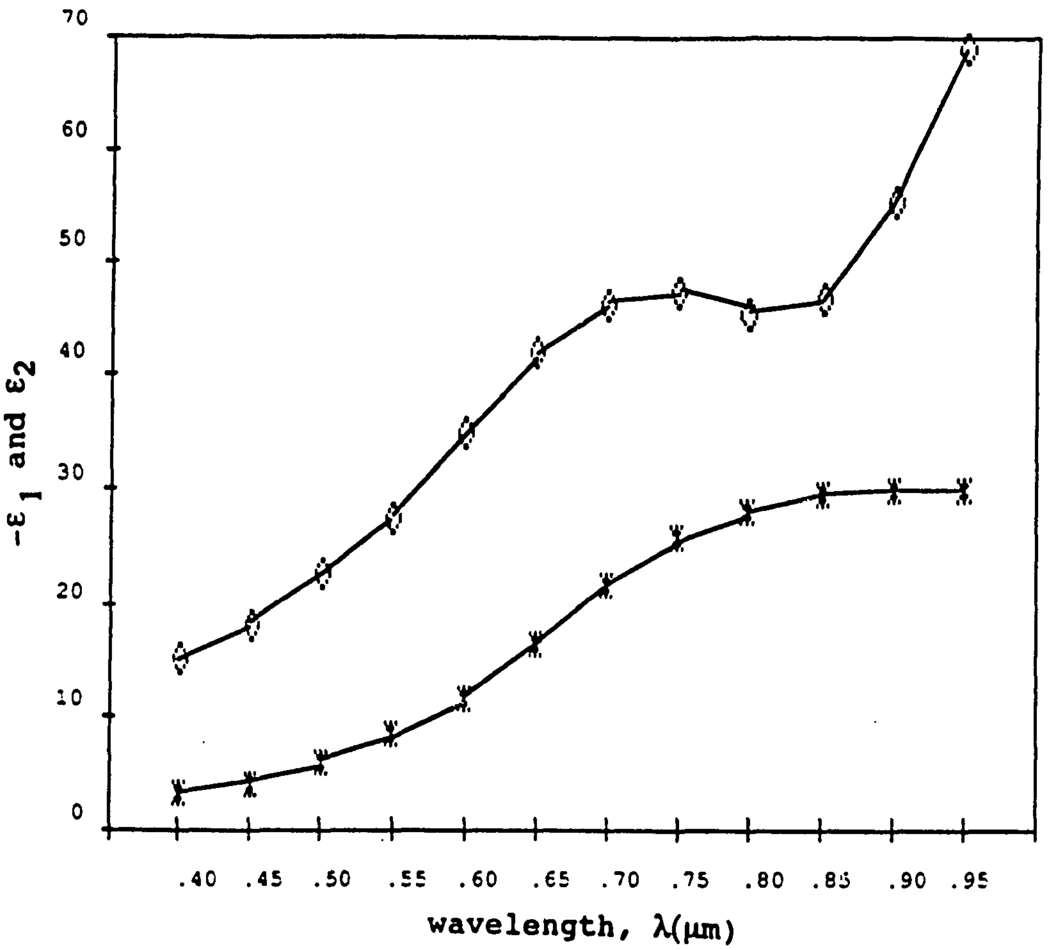

Figure 5. Frequency Dependent of Dielectric Constant for Aluminum.
$\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$ where ϵ_1 and ϵ_2 are the real and imaginary parts irrespectedly for aluminum. Curve for $-\epsilon_1$ is ◊. Curve for ϵ_2 is ✱.

Source: Schulz, L.G. (1954). The optical constants of silver, gold, copper, and aluminum. Journal Optical Society of America, 44, 357 & 362.

spectrum region. As we go up the visible spectrum region, the peak all fades out and becomes broad and loses its amplitude. This is illustrated in Figure 12 (page 41). As for aluminum and silver, they both have very broad peaks that have a high ratio of background to peak in the visible

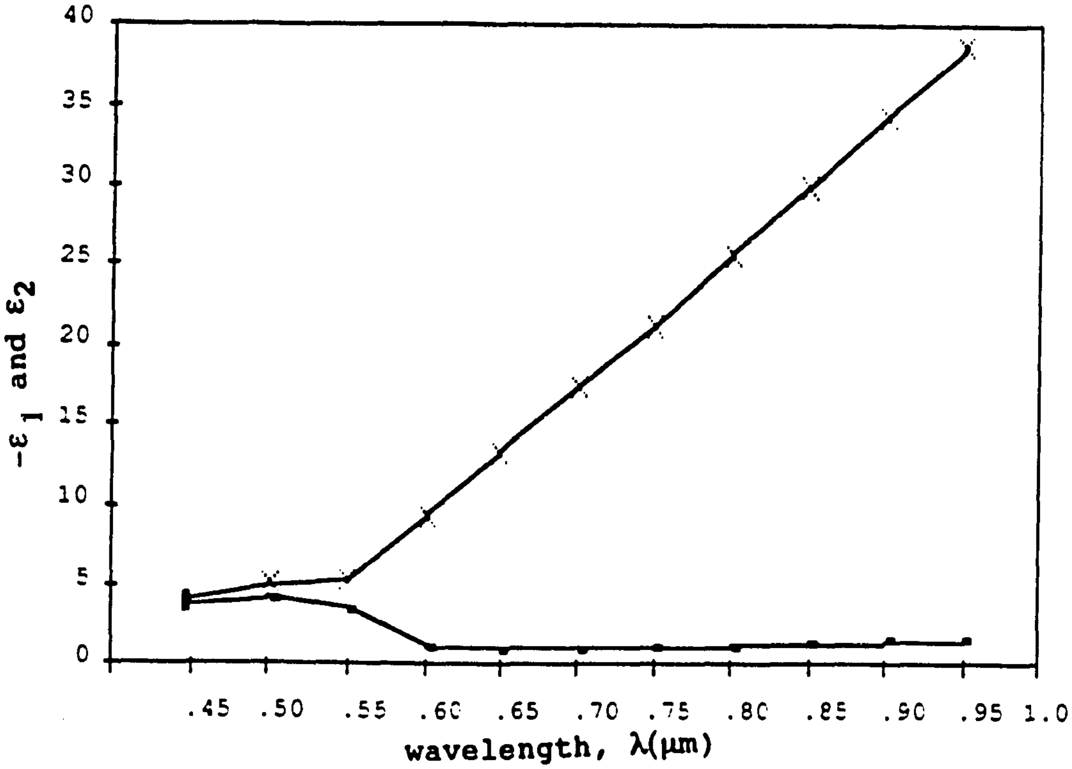

Figure 6. Frequency Dependent of Dielectric Constant for Copper. $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$ where ϵ_1 and ϵ_2 are the real and imaginary parts irrespectedly for copper. Curve for $-\epsilon_1$ is X. Curve for ϵ_2 is ■.

Source: Schulz, L.G. (1954). The optical constants of silver, gold, copper, and aluminum. Journal Optical Society of America, 44, 357 & 362.

spectrum region. As we go to increasing frequencies, the peaks are still visible and do not lose their amplitudes. This is illustrated in Figures 10 and 11 (page 40).

The physical system that we used consists of the light coming in at the normal incidence. We then measured the diffuse component transmitted light about the normal of the lower surface of the film. Eventually, this will broaden

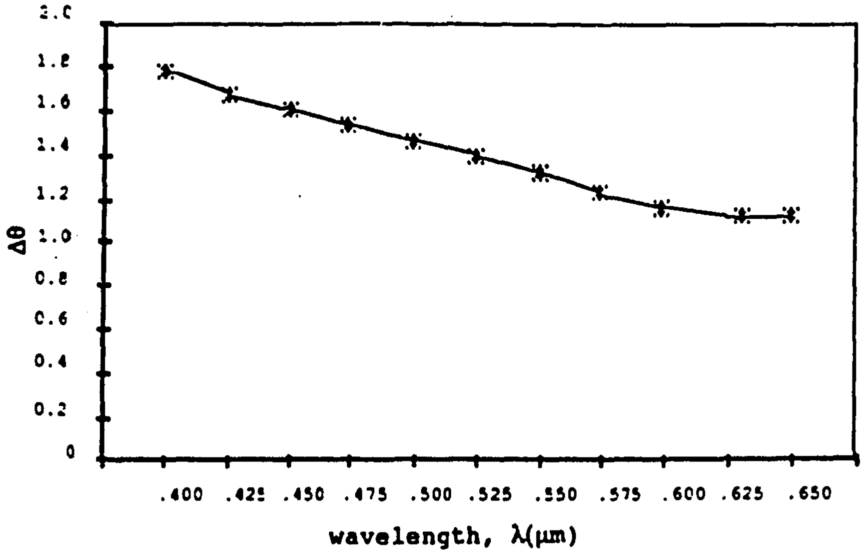

Figure 7. Full Width at Half-Maximum for the Transmission Peaks Plotted Versus Wavelength for Silver. The angle of incidence is zero degree.

out with increasing roughness. If we had perfect smooth surfaces, then the intensity of light would not have any angular distribution about the normal.

It is therefore of interest to study this enhanced transmission in the case of a thin metal film with a large amplitude, randomly rough surface by numerical simulations. Such calculations are now in progress.

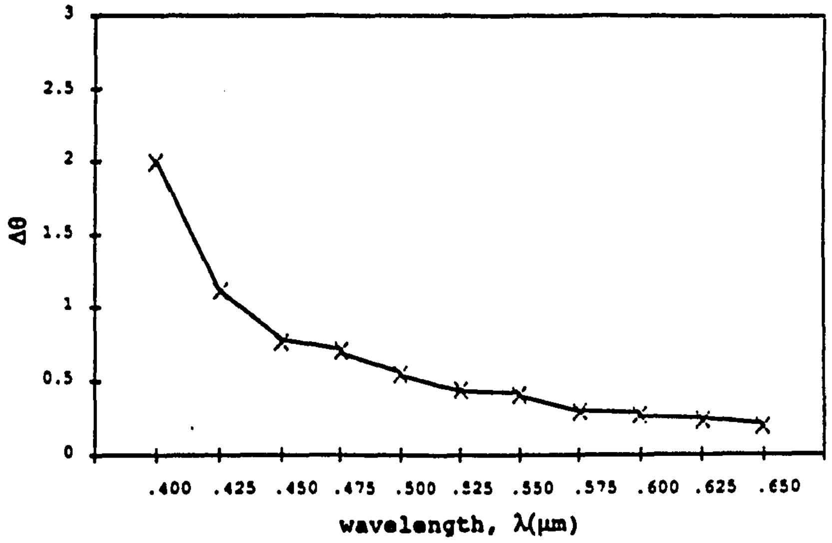

Figure 8. Full Width at Half-Maximum for the Transmission Peaks Plotted Versus Wavelength for Aluminum. The angle of incidence is zero degree.

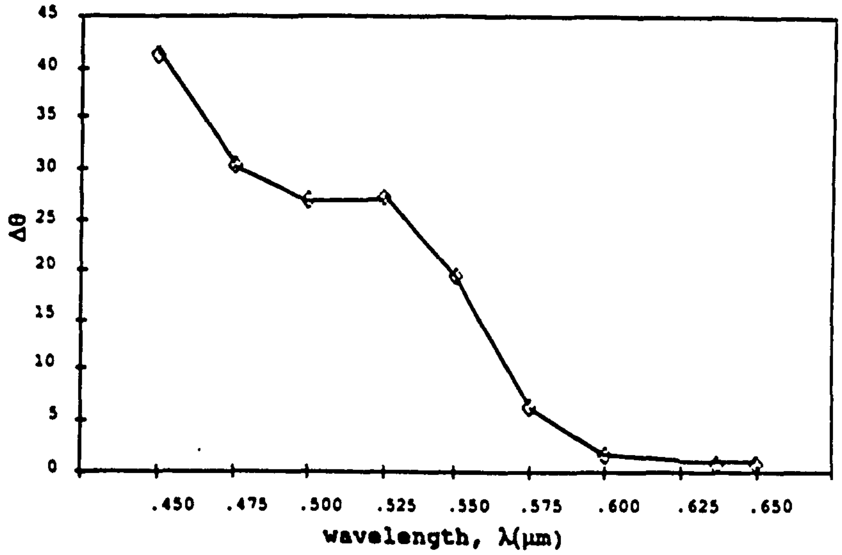

Figure 9. Full Width at Half-Maximum for the Transmission Peaks Plotted Versus Wavelength for Copper. The angle of incidence is zero degree.

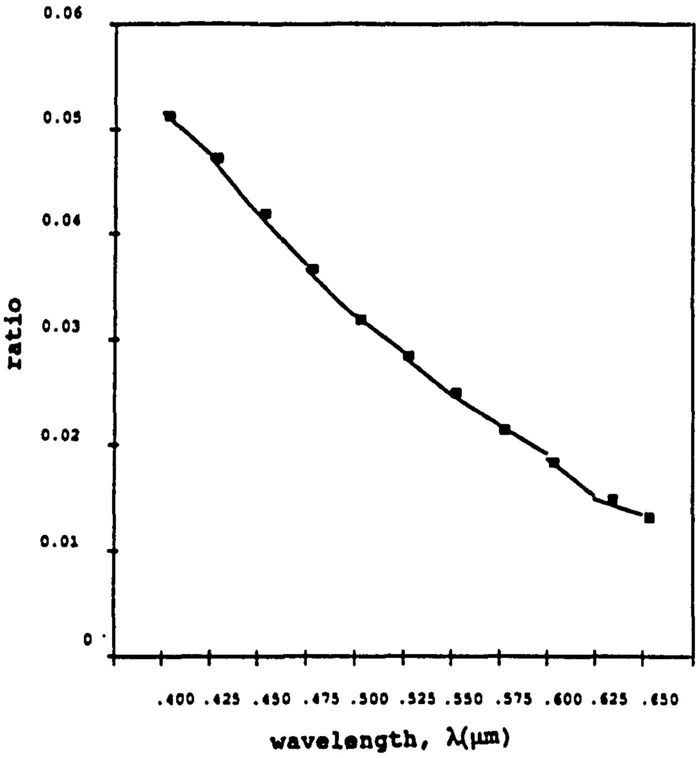

Figure 10. Ratio of the Transmission Peak Heights to the General Diffuse Background Versus Wavelength for Aluminum.

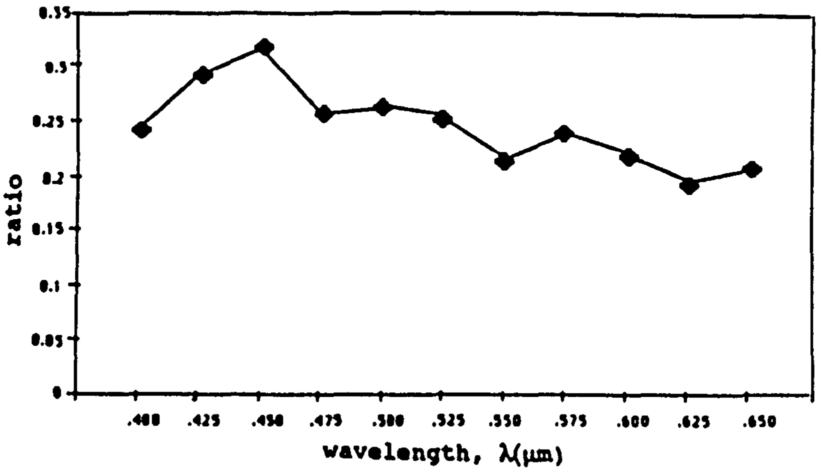

Figure 11. Ratio of the Transmission Peak Heights to the General Diffuse Background Versus Wavelength for Silver.

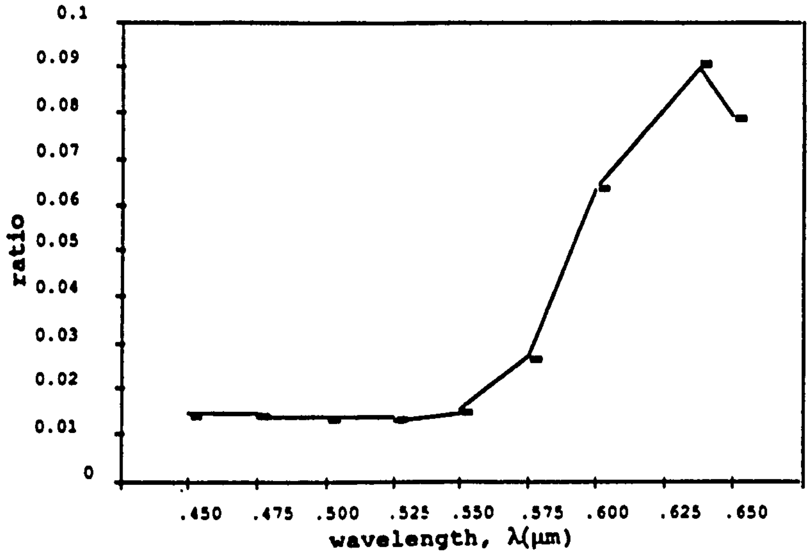

Figure 12. Ratio of the Transmission Peak Heights to the General Diffuse Background Versus Wavelength for Copper.

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