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THE EFFECTS OF HUTCHINGS' LOW STRESS ADDITION AND SUBTRACTION ALGORITHMS ON THE ACCURACY AND RATE OF PROBLEM SOLVING WITH LOW PERFORMING MATH STUDENTS

by

Stephen Edgar Hadden

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Submitted to the
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Western Michigan University
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THE EFFECTS OF HUTCHINGS' LOW STRESS ADDITION AND SUBTRACTION ALGORITHMS ON THE ACCURACY AND RATE OF PROBLEM SOLVING WITH LOW PERFORMING MATH STUDENTS

Stephen Edgar Hadden, Ed. S.
Western Michigan University, 1981

The accuracy and rate of problem solving using Hutchings' low stress addition and subtraction algorithms was investigated using four, fourth-grade low performers. Two multiple baseline designs were used, (one for addition and one for subtraction) where subjects were required to work addition and subtraction problems using the conventional method during Baseline. As Baseline for each subject stabilized, a brief training program was implemented using Hutchings' low stress addition or subtraction algorithm. Following training, subjects were required to work the addition and subtraction problems using the new algorithm. Probes were administered before and after training to measure knowledge of place value and a second probe was given at the end of the study to analyze whether the new algorithm generalized to problems of various sizes and difficulty. Results indicate that accuracy and rate of problem solving improved for all subjects after training with the new addition algorithm, but only for one subject after being trained in subtraction.
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Stephen Edgar Hadden
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vi</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Addition</td>
<td>2</td>
</tr>
<tr>
<td>Subtraction</td>
<td>5</td>
</tr>
<tr>
<td>Current Research on Hutchings' Low Stress Algorithms</td>
<td>12</td>
</tr>
<tr>
<td>II. METHOD</td>
<td>17</td>
</tr>
<tr>
<td>Subjects</td>
<td>17</td>
</tr>
<tr>
<td>Setting</td>
<td>17</td>
</tr>
<tr>
<td>Independent Variables</td>
<td>17</td>
</tr>
<tr>
<td>Dependent Variables</td>
<td>18</td>
</tr>
<tr>
<td>Experimental Design</td>
<td>19</td>
</tr>
<tr>
<td>Reliability</td>
<td>19</td>
</tr>
<tr>
<td>Materials</td>
<td>20</td>
</tr>
<tr>
<td>Procedure</td>
<td>20</td>
</tr>
<tr>
<td>Baseline</td>
<td>21</td>
</tr>
<tr>
<td>Training</td>
<td>21</td>
</tr>
<tr>
<td>III. RESULTS</td>
<td>24</td>
</tr>
<tr>
<td>Reliability</td>
<td>24</td>
</tr>
<tr>
<td>Place Value Probes</td>
<td>24</td>
</tr>
</tbody>
</table>

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LIST OF TABLES

Table 1. Percent Correct on Pre and Post-training Place Value Probes .............................................. 25
Table 2. Percent of Columns Correct on Generalization Probes ............................................................ 51
LIST OF FIGURES

Figure 1: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of addition problems for subject one .................................. 27

Figure 2: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of addition problems for subject two .................................. 30

Figure 3: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of addition problems for subject three ................................ 33

Figure 4: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of addition problems for subject four ................................ 36

Figure 5: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of subtraction problems for subject one .................................. 39

Figure 6: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of subtraction problems for subject two .................................. 42

Figure 7: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of subtraction problems for subject three ................................ 45

Figure 8: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of subtraction problems for subject four ................................ 48
CHAPTER I

INTRODUCTION

With the "New Math" being taught in the schools today stressing mathematical concepts rather than computations, students are becoming more and more detached from the basic operations of addition, subtraction, multiplication, and division. Although these are being de-emphasized in the classroom, they are still important to function in today's society.

There is an increasing amount of information that is seen as important to pass on to students at the elementary level. Therefore, anything taught at this level must be done so quickly and efficiently, one of these being the basic operations mentioned above.

Since less time is spent on the instruction of the four basic operations (algorithms) they are often not mastered. An algorithm is defined as a problem solving procedure. One reason why students fail to learn the conventional algorithms may be due to the negative effects associated with mathematics in general. Skinner (1968) states that "the figures and symbols of mathematics have become standard emotional stimuli. The glimpse of a column of figures is likely to set off not mathematical behavior, but a reaction of anxiety, guilt, or fear" (p. 18).

In 1972, Dr. Lloyd B. Hutchings developed a set of "Low Stress" algorithms for the four basic operations. He states that his low
stress algorithms "appear to permit easy mastery after brief training, provide greater computational power than conventional algorithms, and operate with much less stress on the user than conventional algorithms" (p. 219).

Other advantages of low stress algorithms include: Each skill in the entire sequence is clearly defined, discrete, and logically linked to each other skill, each new skill introduced extents previous learning be teaching the minimum set of new component skills needed to yield a maximum range of mathematical competence, each new skill builds directly and cumulatively on previous taught skills, each skill is taught in a totally visible (overt) format to allow the teacher to observe actual pupil learning and assure high quality learning by identifying and correcting error patterns before they become habit formed, and low stress algorithms are not any more difficult when the problems get very large, and the "memory load" for math facts is reduced over the conventional procedures (Alessi, 1979). When performing addition with the conventional algorithm, the student needs to memorize approximately 900 complex facts, such as 13 + 7 or 21 + 9. With Hutchings' low stress addition algorithm, the student only needs to know approximately 100 basic addition facts, such as 3 + 4 or 5 + 7. Lankford (1974) states that poor computers "often made errors when their counting or other derivations of unknown combinations became too involved for their short memory spans" (p. 29).

**Addition**

Hutchings defines the low stress algorithm for addition as follows:
The low stress addition algorithm uses a new notation, called half-space notation, to record individual steps. Half-space notation uses numerals of no more than a half-space in height to record the sum of two digits. With half-space notation, the units portion of the sum of two digits is written at the lower right of the bottom digit and the tens portion is written at the lower left of the bottom digit.

When presented with a single column of numbers, the student would...

... start at the top, add the first two digits, and record the sum of the new notation. The complete sum of each two digit addition is recorded in half-space notation, but only the ones portion of each sum is used in the next addition. The ones portion of the column sum is always the same as the ones portion of the last two digit sum. The tens portion of the column sum is always the same as the number of tens recorded at the left of the column. For a column in some multicolumn exercise, the last step, counting the tens at the left of the columns would be slightly changed; the total number of tens is no longer written in the tens place of the first column's sum but instead at the top of the next column at the left. However, the column sum for the last column in a multicolumn example is recorded in exactly the same way as the sum of a single column exercise (pp. 220-223).

Example:  
\[ \begin{array}{c} 
4 \\
26 \\
195 \\
183 \\
\hline 
23 
\end{array} \]

In this example, the student would follow this sequence:

1. Four plus two equals six.
2. Six (from the sum of four plus two) plus nine equals 15.
3. Five (from the units place in 15) plus eight equals 3.
4. The student brings down the three and adds the tens digit to complete the problem.

There have been many other methods suggested for performing addition and subtraction that are very similar to Hutchings' low stress algorithms. Bartarseh (1974) suggested a procedure to teach low
performing and educably mentally retarded students addition of multi-digit numbers and the concept of carrying. For example:

\[
\begin{array}{c}
547 \\
+ 876 \\
13 \\
12 \\
\hline
1423
\end{array}
\]

1. Write the numbers, one under the other, units under units, tens under tens, etc.

2. Underline the units column and add the numbers in that column. Write this sum with the three under the line for the units column and the one under the tens column.

3. Underline the tens column and add. Write this sum with the two under the line for the tens column and the one under the hundreds column.

4. Underline the hundreds column and add. Write the sum with the four under the hundreds and the one in the thousands column.

5. Bring down the results of the additions of the first two columns under their respective positions.

Sanders (1971) method is similar to Hutchings' low stress in that it reduces the student's "memory load." With this algorithm students add the first two addends in the column, vocalize the ones portion of the sum and hold up one finger to represent one ten (if necessary). The student continues down the column vocalizing the units and holding up respective fingers for tens. After the final two addends have been summed, the student writes the number representing the vocalized units, and counts his/her fingers to obtain the answer for the tens column.
This method along with Bartarseh's helps the student by reducing the "memory load" but they lack a complete written record as in Hutchings' algorithms. This written record is extremely useful for teachers in giving immediate feedback to students and quick identification of error patterns.

Fulkerson (1963) suggested another algorithm which he terms the "tens method." Using this procedure the student sums the first two addends, remembers the units and slashes the second addend if the sum was 10 or more. The student proceeds down the column keeping the unit sums in his/her head and slashing any number that results in a sum of 10 or more. At the end of the column the student simply writes down the final sum for units and carries the number of slashes over to the tens column. The process then repeats. For example:

\[
\begin{array}{c}
421 \\
689 \\
\hline
+ 214 \\
\hline
1324
\end{array}
\]

Again, this method does not supply a full written record as in Hutchings' low stress algorithm.

O'Malley (1969) designed an algorithm for addition that is very similar to Hutchings' method but still lacks the full written record. In this system the student would add the tens to the tens column immediately. The same would occur when adding the tens column (add the hundreds to the hundreds column immediately), hundreds column, etc.

Subtraction

Dr. Hutchings defines low stress subtraction as follows:
Low stress subtraction has a special notation used to record the steps performed in solving a subtraction problem. The notation consists of two parts. The first is to record all upward regrouping of places by a half-space "1" placed at the upper left of numerals occupying such places. For example, 643 could be recorded in regrouped form as $5 \overline{14} 3$, or $5 \overline{13} 3$, or $6 \overline{3} 3$. The second is to write the regrouped minuend (top number) directly above its subtrahend (bottom number), which helps some children in reading and organizing their work. The regrouped minuend is written in the middle rather than at the top. In subtraction, the entire minuend is regrouped and recorded before any subtraction occurs. After this regrouping, all subtraction is completed in an uninterrupted sequence (p. 231).

For example:  

```
  6 2 8 3 4  
- 5 1 1 1 2 1 4  
  4 9 8 3 7  
```

This method also offers the full written record and seems to be less confusing to children who have used it.

Sherril (1979) tested the effectiveness of two procedures, decomposition and equal addition. The decomposition approach is the method most frequently taught in the schools today. Here the student borrows from a column and renames the minuend. The equal addends approach differs from decomposition in that instead of borrowing one ten from the tens column and adding it to the ones column in the form of ten ones, the student would add ten ones to the ones column in the subtrahend and add one ten to the tens column in the minuend.

Fourty-eight subjects were randomly placed in one of two groups. One group was trained with the decomposition approach and the other group was trained with the equal addends approach. At the end of the study a 40-item post test was administered to all 48 subjects to test the...
accuracy of their problem solving. The mean number correct for the decomposition group was 31.3 while the mean number correct for the equal addends group was 25.1. The results indicate that the decomposition approach yielded greater accuracy of problem solving. The present study will attempt to demonstrate that Hutchings' low stress algorithm for subtraction is more accuracy than the decomposition method (conventional algorithm).

Brownell (1949) utilized 1,400 third-grade subjects from 41 different classrooms. These subjects were divided into three experimental centers which was dependent on the type of teaching experiences they had in grades one and two. Each of these sections were divided again with half of the subjects learning to borrow through decomposition, and the other half through equal addition. These sections were divided once more so that under each sub-section, half the students were being instructed in a meaningful manner while the other half were being instructed in a mechanical manner. Meaningful instruction means that an understanding of the subtraction process is stressed, where by mechanical instruction is simply teaching the steps involved to obtain the correct answer.

The pre-experimental period consisted of three instructional tasks; the subjects were taught basic subtraction facts, subtraction of two and three digit numbers without borrowing, and to teach addition with and without carrying. The subjects were also pretested at this time. The experimental period lasted 15 days. During this time each group of students was taught subtraction in one of four ways; equal addition meaningful, equal addition mechanical, decomposition meaningful,
and decomposition mechanical. At the end of the experimental period, subjects were administered a post-test and interviewed individually. Following this, a six week post-experimental period was implemented where no additional instruction was given. At the end of this period the subjects were administered a second post-test and a second interview.

Post-tests were administered to determine the effectiveness of instruction, and interviews were held to determine the subject's level of understanding.

Experimental findings concluded that meaningful instruction, especially in the case of decomposition, resulted in greater accuracy and rate than mechanical instruction.

Again, the present study will attempt to demonstrate that Hutchings' low stress subtraction algorithm is more accurate than the decomposition method. In addition, it will analyze whether there is an increased level of understanding, as measured percent correct and incorrect rate.

The present study will also attempt to analyze through place value probes, whether there is an increased level of understanding resulting from instruction in Hutchings' low stress subtraction algorithms, as measured by percent correct. In addition, the present study will also compare two algorithms, one being the decomposition method, and the other being Hutchings' low stress method.

Blankenship (1978) stated that from the research to date, "inversion errors are by far the most common type of all systematic errors. These errors result from subtracting the minuend from the subtrahend in subtraction borrow problems" (p. 13).
The author tested the effectiveness of demonstration and feedback on remediating systematic inversion errors. Nine learning disabled subjects were selected on the basis of being observed making inversion errors on the Key Math test and a specifically designed test which included subtraction problems with no zeros.

An A-B design was used with A being Baseline in which the students worked 45 subtraction problems ranging in difficulty. The dependent variables consisted of number of correct digit differences, percent of correctly computed problems, and number of problems computed correctly and incorrectly per minute. During each session of intervention the experimenter demonstrated one problem for each subject individually. The student was then given a problem to complete. If the problem was done correctly, the experimenter allowed the subject to go on and complete the 45 problems for the day. If the problem was incorrect, the experimenter repeated the procedure until the problem was done correctly. Subjects were then given feedback only on the first row of problems completed that day (which were similar to the ones demonstrated).

Results showed that systematic inversion errors in the problems similar to the ones demonstrated decreased from an average of 86.7 percent during Baseline to 6.7 percent during intervention. Data on the other problems showed that the demonstrations generalized; 95 percent inversion errors during Baseline to 9 percent during intervention. This study suggests that systematic inversion errors can be corrected quickly and easily. The present study will also attempt to show that a quick and easy training session will improve subtraction accuracy.
Lovitt (1968) assessed the effects of writing the answers to mathematics problems versus verbalizing the problems before making a written response on performance rate. A reversal design was used with Baseline consisting of the subject simply writing the answers to mathematics problems. During phase two the subject was required to verbalize the problem before writing the answer. The third phase was a return to Baseline. The results indicated that the subject's error rate decreased and his correct answer rate increased as a result of verbalizing the problem before writing the answer.

This study is similar to the present study in terms of the antecedent event. In the present study the subjects are trained to ask themselves if they have to borrow in order to complete the subtraction sequence.

Schwartzman (1975) designed an algorithm for subtraction which has been explained in the following way:

\[
\begin{array}{c}
A & 2 & 4 \\
B & 5 & 7 \\
C & 3 & 2 \\
D & 1 & 5 & 7 & 6 \\
E & & 4 & 8 & 0 & 6 \\
\end{array}
\]

The numbers that we want to subtract appear in rows C and D. We start subtracting in the first column at the right and take the smaller digit away from the larger no matter which is on top. If the digit in row C is smaller than the digit in row D we put our difference for that column in row A. If the digit in row D is the smaller of the two we put our difference in row E. If the two digits are equal we put a zero in row E. Now we move left to the next column. Since we just finished entering a digit in row A, we reduce the digit in row C of the column we are now in by one, then subtract as before. If we had not just put an entry in row A we would go right ahead and subtract where we are without any alteration. The process continues until we have taken the difference in every column and placed our answers in row A or row E. The final answer will
appear in row E. We go to each column in which the row E entry is missing and in that blank put the difference between the row A entry and the number ten (p. 628).

This algorithm is similar to Hutchings' subtraction algorithm in that the regrouped numbers are not crowded along the top of the problem resulting in possible confusion for the students. The algorithm does have two shortcomings: 1) It is a lengthy process, the student must work through the entire problem and then go back and subtract any numbers in row A from ten; and 2) the problem format may be difficult for children to quickly master due to many differences from the conventional algorithm.

Wenner (1977) suggested an algorithm for students having difficulty memorizing the basic subtraction facts. Instead of regrouping a number in the minuend and subtracting, the student would borrow ten ones from the next column and subtract the subtrahend from that. The student would then add the remainder to the minuend in the ones column. This sum would equal the difference. For example:

\[
\begin{array}{c}
41 \\
327 \\
189 \\
338
\end{array}
\]

1. The student would rename the two tens as one ten and ten ones. The next step would be to subtract nine from the ten ones leaving one left over. The one is then added to the seven and this sum, eight, is the difference in the ones column.

2. The student would then rename the five as four hundreds and ten tens. Next the student would subtract eight from the
ten tens leaving a remainder of two. The two is then added to the one ten and the sum, three, is the difference in the tens column.

3. Finally, one is subtracted from four with a difference of three. The answer is 338.

This algorithm would be easier to master than Hutchings' subtraction algorithm for students who do not yet know their basic subtraction facts, although it does not have the advantage of the full written record that Hutchings' algorithm offers.

**Current Research on Hutchings' Low Stress Algorithms**

Alessi (1974) investigated the effects of three independent variables: 1) Instruction in Hutchings' low stress addition algorithm, 2) token economy reinforcement contingencies, and 3) level of problem difficulty. Subjects were chosen on the basis of a high score on a basic addition facts pretest. The design consisted of a $3 \times 2 \times 3$ factorial design with subjects randomly assigned to the 18 cells. The dependent variables, correct addition and number of attempted columns were collected in one 30-minute testing session for each subject at the end of the study. Half of the children were instructed in Hutchings' low stress addition algorithm and half were instructed in the conventional addition algorithm. Testing was performed under three separate conditions of token reinforcement contingencies and with three tests fixed at different levels of problem difficulty. The results indicated that Hutchings' low stress algorithm produced higher scores for the number of columns correct and number of columns attempted.
Significant differences among means were also found for the difficulty level for number of columns correct but not for number of columns attempted. No other main or interaction effects reached a level of significance.

Zoref (1976) compared speed and accuracy between Hutchings' low stress addition algorithm and a calculator using a multi-element design. The subjects used were six fourth-grade students, three male and three female. One-half of the students were identified as low performers and the other half as high performers, based on teacher reports, classroom performance, and Metropolitan Achievement Test scores. The study looked at the effects of three independent variables: 1) Three types of calculation procedures; conventional, Hutchings' low stress addition algorithm, and pocket calculators; 2) two problem array sizes, five columns by seven rows of digits and two columns by seven rows of digits; and 3) two types of subjects with respect to math skills, high and low performers. Speed and accuracy were used as the dependent variables. A separate training session (20 minutes each) was given on each of the three types of calculation procedures. The remainder of the study was used to collect data on each calculation procedure. Results indicated that performance with Hutchings' low stress addition algorithm was the most stable and had the lowest error rate for all subjects while the conventional algorithm produced opposite results. Calculator performances decreased in accuracy as the problems increased in difficulty.

Gillespie (1976) measured preferences using the Hutchings' low stress addition algorithm and the conventional algorithm under conditions of differentially increasing response effort without reinforcement.
(Experiment I), and with reinforcement (Experiment II) using a reversal
design. Six third-grade students, three males and three females, were
chosen to participate in this study. The independent variables consisted
of Hutchings' low stress addition algorithm versus the conventional
additional algorithm, and equal response effort for both algorithms
versus differentially increased response effort of 50 percent and
100 percent for the preferred algorithm. The dependent variables were
algorithm preference, rate of columns correct and incorrect, and percent
accuracy, on 4 X 5 problem arrays. In Experiment I the subjects were
presented with three 20-minute instructional sessions on Hutchings'
low stress addition algorithm and one 20-minute session on the conven­tional algorithm, after which the pre-baseline phase began. During the
first four sessions of pre-baseline the students were forced to alternate
between the two types of algorithms. During all subsequent conditions
each subject was given a choice of which algorithm he/she preferred to
use. During Condition A (Baseline), all subjects had a choice of
which algorithm they wished to use, each with an equal response effort.
From this, a preference was established for each student. In Condi­
tion B, subjects again had a free choice. However, the preferred
algorithm established during Baseline for each subject required a
50 percent increase in problems completed (response effort) than in
Condition A. Condition C was identical to Condition B except that a
100 percent response effort over Condition A was required for the
preferred algorithm. Condition A was repeated followed by another
Condition B and a second return to Baseline (Condition A). Experiment II
was identical to Experiment I except that tokens were made contingent
on accuracy of the subjects answers in Conditions B and C, and there was only one return to Baseline. Results indicated that the Hutchings' algorithm was preferred in 20 out of 24 equal response effort and 11 out of 14 50 percent increased response effort. Algorithm preferences dissolved and were altered by reinforcement only during the 100 percent increased response effort condition.

Rudolph (1976) investigated three research questions. First, is the Hutchings' low stress addition algorithm effective in teaching "distractible" students computational skills? Second, what are the effects of a distracting versus a non-distracting environment on the subject's performance? Third, how comparable are the performances of these and other mainstreamed students when using either the low stress or current algorithm within these different environments? The independent variables selected were; the algorithm used by the student, current or Hutchings' low stress; the type of environment, distracting or non-distracting; the handicap status of the subject; and response costs for errors or no costs for errors. The dependent variables were percent of columns correct, correct rate, and incorrect rate. Four seventh-grade students were selected for the study, two from regular education classrooms and two from an emotionally impaired classroom. All were selected on the basis of teacher recommendations, Metropolitan Achievement Test scores, and Peabody Individual Achievement Test scores. Subjects also had to receive a score of 95 percent or above on a pretest of basic addition facts. A multi-element baseline within a reversal design was used where four subjects, two emotionally impaired and two regular education were exposed to Hutchings' low stress addition
algorithm and the current algorithm. A counter-balanced multi-element
design was used where distracting and non-distracting environments
were alternated. Results indicated greater accuracy using Hutchings'
low stress addition algorithm as opposed to the current algorithm.
No consistent trend was seen in the data comparing distracting and non-
distracting environments, and the changes in performance cannot be
systematically associated with handicap status.

All of these studies relate to the present study in four ways.
First, all used subjects close in age and grade level to the subjects
of the present study. Second, all trained and measured the effects of
Hutchings' low stress addition algorithm. Third, all used brief
training sessions when training the new algorithm. Fourth, all measured
accuracy as one of their dependent variables.

The present study goes beyond the previous research in one signi-
ficant way, it will attempt to increase accuracy and rate on subtraction
problems using Hutchings' low stress algorithm for subtraction. There
has been very little research completed on the effectiveness of this
algorithm on student performance and none assessing the performance
of the same subjects who have been trained to use Hutchings' addition
and subtraction algorithms. The present study will attempt to answer
the question of whether Hutchings' low stress algorithms for addition
and subtraction can increase the accuracy and correct rate of problems
completed with a 60-minute training session. In addition, this study
is designed with the objective of showing school personnel that low
performers can be quickly remediated using the new algorithms and
succeed.
CHAPTER II

METHOD

Subjects

The subjects for this study were four fourth-grade students, three male and one female. All of the subjects were placed in the Academic Adjustment Program (AAP) for approximately one-half hour per day. The subjects were chosen on the basis of three criteria; teacher recommendations, low scores on the addition and subtraction subtests of the Science Research Associates (SRA) achievement test (8, 13, 26 and 20 percentile consecutively), and a score of 80 percent or above on both an addition and subtraction basic facts pretest.

Setting

The study took place in the AAP room at an Elementary School in a rural school district near Kalamazoo, Michigan. The AAP room is a non-special education classroom where students who have deficits in specific areas receive remedial help. Approximately three sessions per week were held with each subject and lasted 10 minutes each. A total of 29 sessions were held with three subjects and 30 sessions with the fourth subject.

Independent Variables

The present study involved two independent variables: Hutchings' low stress algorithm for addition and subtraction (Hutchings', 1976).
Dependent Variables

Three dependent variables were measured: Rate of columns correct per minute, rate of columns incorrect per minute, and percent correct of columns attempted. Rate was calculated by dividing the number of columns correct or incorrect by the total time it took each subject to complete the worksheet. Except for the correct and incorrect addition rate for subject one, all calculations on the correct and incorrect rates changed after Session 11. Calculations performed on data from Sessions 1 through 11 were done by dividing the number of columns correct and incorrect by five. Occasionally a subject completed the worksheet before the 5 minutes had passed; therefore the calculation would result in a less variable and decreased correct and incorrect rate. Beginning with Session 12 (or in the case when training occurred during that session, the subsequent session), calculations performed were precise correct and incorrect rates per minute. Data for subject one are precise correct and incorrect rates per minute beginning with Session 1. Percent correct was calculated by dividing the number of columns correct by the number of columns attempted and multiplying that by 100. Columns were scored correct or incorrect by the digit in the sum corresponding to that column's addends.

Since addition problems resulted in sums of four or five digits, the first two columns in any five digit sum were counted as one and both of those digits had to be correct in order for it to be counted as a correct column.
The total number of columns for each worksheet was 12 for addition (three four-column problems), and 24 for subtraction (three eight-column problems).

**Experimental Design**

Two multiple baseline designs across subjects were used. One multiple baseline design was used to compare Hutchings' low stress addition to the conventional addition algorithm. The second multiple baseline design was used to compare Hutchings' low stress subtraction algorithm to the conventional subtraction algorithm.

**Reliability**

Reliability was taken on the subjects correct and incorrect columns. The experimenter first calculated the answers to all the problems on both the addition and subtraction worksheets. A blank piece of tracing paper was then placed over each individual worksheet so that the problems and corresponding answers could be seen through the paper. The experimenter then put an "X" over any incorrect column and circled any correct columns. An independent observer followed the same procedure with one exception. Rather than using tracing paper, the observer scored correct and incorrect columns directly on the worksheet. The reliability coefficient was calculated using the formula: 

$$\text{reliability} = \frac{\text{agreements}}{\text{agreements} + \text{disagreements}}$$
Materials

The basic facts pretest for addition and subtraction required four identical worksheets for addition, and four identical worksheets for subtraction (Appendix A).

Daily sessions required one addition and one subtraction worksheet (Appendix B). The addition worksheet consisted of three problems, each being four columns by five rows typed with an I.B.M. Selectric orator typing element with three spaces between columns and two spaces between rows. The subtraction worksheets consisted of three problems, each being eight columns by two rows. Subtraction problems were typed leaving three spaces between columns and three spaces between rows. Addition and subtraction problems were constructed using a random numbers table. One consideration was made when constructing subtraction problems, the subtrahend was kept smaller than the minuend so the problem would not result in a negative number.

Materials used for instruction consisted of two training packages, one for addition and subtraction, condensed from Hutchings' low stress training package (Appendix C and D).

Two pre and post training probes were also used from the Science Research Associates (S.R.A.) math program (Appendix E and F).

Procedure

The subjects came to the classroom at different times, therefore not all the subjects were in the AAP room at the same time. Two subjects worked with the experimenter from 10:25 - 10:35 A.M.
days a week and two subjects worked with the experimenter from 11:20 - 11:30 A.M. three days a week.

**Baseline**

Each session the subjects were given two worksheets to complete, one in addition and one in subtraction. The experimenter would begin by placing the addition worksheet face down on each subject's desk. When the subjects were seated appropriately the experimenter would say "begin" and start timing using the second hand on the classroom clock. At the end of five minutes the experimenter would say "stop" and the process would be repeated using the subtraction worksheets.

**Training**

Training began with the first subject when the Baseline measures of the dependent variables were stable for either addition or subtraction. Criterion for stability was pre-defined as being three data points varying no more than 20 percent and showing no trend for percent correct, and three data points varying no more than .8 and showing no trend for correct and incorrect rate per five minutes. This definition was thought to be stringent enough for the required stability while still allowing a range for the variability which often occurs when measuring academic performance.

Subjects were trained in using Hutchings' low stress algorithms for addition or subtraction depending on which Baseline stabilized first. Training took approximately one-half hour. The subject was taken to the other side of the room to work alone at a table with the...
experimenter. This area was separated from the other students by bookshelves and three filing cabinets.

During each subject's first training session, an SRA place value probe was administered (level M-13) in order to assess the subjects' knowledge of place value. The experimenter then took the subject through the training package. After the training was completed the experimenter assisted the subject with some examples using the newly trained low stress algorithm.

The subject was then required to use the new algorithm on subsequent worksheets. Each individual subject was assisted by the experimenter during the first session after training. Assistance consisted of asking the subject leading questions pertaining to the process involved in using the new algorithm, such as, "what would you do next?", or "where does that number go?".

Two format changes were made on the addition and subtraction worksheets that the subjects used after training. On the addition worksheets, lines were drawn between the columns in order to prevent possible confusion when writing numbers along the sides on the columns. On the subtraction worksheets, narrow horizontal boxes were drawn between the rows to serve as a prompt for the subjects to do all the renaming within the box.

All subjects were trained using the same procedure and a minimum of five data points were required before the next subject could be trained in the same algorithm.

At the end of the study the place value probe was readministered to all the subjects in order to analyze whether their place value skills
had improved due to the use of Hutchings' low stress addition and subtraction algorithms. A second probe consisting of addition and subtraction problems of varying difficulty was also given (level M-1 and M-2) and subjects were instructed to complete this using the new algorithm within an imposed 10-minute time limit. This probe was administered to analyze whether the subjects would generalize the new low stress algorithms to problems of varying sized and difficulty.
CHAPTER III

RESULTS

Reliability

A total of six reliability checks were taken during the course of the study yielding a mean percent agreement of 98 for number of columns correct and incorrect with a range of 50 to 100 percent.

Place Value Probes

Table 1 presents the results of the two place value probes given before and after training. From Table 1 it can be seen that there was no substantial increase in scores after being trained in Hutchings' low stress algorithms.

Organization of Dependent Data Presented

The results of each subject involved in this study will be presented individually. A total of two figures are included for each subject. One figure presents percent correct, correct rate, and incorrect rate for addition, while the other figure presents the same dependent variables for subtraction.

Data points included in the figures for correct and incorrect rate that were derived from dividing by the inflated time of five minutes are designated by arrows.
Table 1

Percent Correct on Pre and Post-training Place Value Probes

<table>
<thead>
<tr>
<th>Subjects</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-training:</td>
<td>41%</td>
<td>24%</td>
<td>34%</td>
<td>34%</td>
</tr>
<tr>
<td>Post-training:</td>
<td>45%</td>
<td>21%</td>
<td>14%</td>
<td>38%</td>
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</table>

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Subject One

Subject one received a score of 90 percent on the pretest for basic addition facts.

From Figure 1 it can be seen that the data for percent of columns correct was variable for the first four sessions going from zero percent, to 100 percent, to 33 percent, to 100 percent. From Session 5 through 11 the data stabilized ranging from 100 percent to 67 percent with no definite trend. From Session 13 to Session 26 (just prior to training), the data trend was steadily downward to 38 percent of columns correct in Session 26. After training, two sessions were conducted with subject one, each being 100 percent.

The number of columns completed correctly per minute (correct rate) showed a slight upward trend from Session 1 to Session 20 with a rate of 0.2 columns correct per minute and then dropped with a downward trend to a point just prior to training of 0.6 columns correct per minute. After training, two sessions were conducted resulting in an apparent upward trend to the final point of 1.0 columns correct per minute in Session 29.

The number of columns completed incorrectly per minute (incorrect rate) was very stable from Session 1 through Session 18 ranging from zero columns incorrect to 0.4 columns incorrect. The data from Session 19 through 26 was unstable ranging from zero columns incorrect per minute to 1.4 columns incorrect per minute with a point just prior to training of 1.0 columns incorrect. After training, two sessions
Figure 1: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of addition problems for subject one.
were conducted both resulting in an incorrect rate per minute of zero.

Subject Two

Subject two received a score of 98 percent on the pretest of basic addition facts.

From Figure 2 it can be seen that Baseline data for percent of columns correct showed a slight upward trend in the first four sessions to 75 percent, after which it showed a downward trend to a point just prior to training of 33 percent. After training, percent of columns correct increased immediately to 88 percent and stabilized around that point until Session 20 when it dropped to 75 percent. For the next three sessions the data showed a sharp downward trend to the lowest post-training point of 40 percent. Sessions 24 through 29 increased and stabilized with four of the five sessions being at least 13 percent higher than the highest point in Baseline (75 percent).

The number of columns completed correctly per minute showed an upward trend from Sessions 1 through 4, ranging from 0.2 to 1.8 correct columns per minute. Sessions 5 through 12 were stable ranging from 0.8 to 1.8 correct columns. After training, correct columns per minute increased steadily to 2.4 in Session 19 and then decreased steadily from Sessions 20 to 23, to 0.4. The remaining six sessions showed a somewhat stable rate ranging from 1.2 to 2.4 correct columns per minute.

The number of columns completed incorrectly per minute was variable during Baseline ranging from 2.6 to 0.6 with the highest points being the first and the last. The first session after training
Figure 2: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of addition problems for subject two.
showed a sharp decrease in rate of incorrect columns per minute. A stable rate occurred from Sessions 15 through 19 and then increased to the highest point after training of 1.0. The remaining sessions were stable with the majority of the sessions showing lower incorrect rates than those during Baseline.

Subject Three

Subject three received a score of 87 percent on the pretest of basic addition facts.

From Figure 3 it can be seen that Baseline data for percent of columns correct was highly variable ranging from 4 percent to 92 percent. No definite trend can be established for Sessions 1 through 11. Sessions 12 through 18 stabilized ranging from 42 percent to 67 percent. After training percent of columns correct increased sharply to 100 percent in Session 21. Sessions 22 through 27 remained high ranging from 88 percent to 100 percent. A sharp decrease to 56 percent occurred in Session 28 but increased to 100 percent in Session 29 (the final session).

The number of columns completed correctly per minute increased slowly and steadily throughout Baseline. The two highest points occurred in Sessions 8 and 10 (2.0 and 2.2 respectively). The three lowest points occurred in Sessions 1, 2, and 7, all being 0.4. After training the number of correct columns per minute stabilized with the extreme points being 1.6 in Session 26 and 1.0 in Sessions 21, 27, and 28.

The number of columns completed incorrectly per minute showed no trends during Baseline. The extreme points occurred in Session 11 (1.8)
Figure 3: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of addition problems for subject three.
and Session 10 (0.2). After training the number of incorrect columns per minute dropped sharply to zero then rose to 0.2, from Sessions 22 through 26. All the points except one (Session 28) were either equal to or below the lowest point that occurred during Baseline.

Subject Four

Subject four received a score of 93 percent on the pretest of basic addition facts.

From Figure 4 it can be seen that data collected during Baseline for percent of columns correct was variable with no definite trends. The highest point during Baseline occurred in Sessions 8 and 16 (75 percent). The lowest point occurred in Sessions 13, 19, and 20 (0 percent). After training percent of columns correct increased sharply to 100 percent for Sessions 26 through 28, then decreased slightly to 88 and 89 percent in Sessions 29 and 30, respectively. All points after training were above any points that occurred during Baseline.

The number of columns completed correctly per minute was again variable showing no definite trends during Baseline. The highest rate occurred in Session 16 (3.0), while the lowest rate occurred in Sessions 13, 19, and 20 (0). After training the number of correct columns per minute suggest an upward trend with the lowest point occurring in Session 26 (0.8) and the highest point occurring in Sessions 28 and 30 (1.6). All points after training were within the range of points from Baseline.
Figure 4: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of addition problems for subject four.
The number of columns completed incorrectly per minute showed a slight upward trend during Baseline although it was extremely variable. The lowest point occurred in Sessions 2 and 8 (0.6) and the highest point occurred in Sessions 13 and 14 (4.0). After training the number of incorrect columns per minute dropped sharply to zero in Sessions 26 and 28 with slight increases to 0.2 in Sessions 29 and 30.

**Subtraction**

**Subject One**

Subject one received a score of 83 percent on the pretest for basic subtraction facts.

From Figure 5 it can be seen that Baseline data for percent of columns correct is very stable although somewhat variable. No definite trend can be determined. The highest point occurred in Session 16 (58 percent) and the lowest point occurred in Session 1 (6 percent). After training the percent of columns correct increased sharply to 82 percent in Session 27 with an apparent upward trend reaching 96 percent in Session 29.

The number of columns completed correctly per minute shows a slight upward yet variable trend during Baseline with the extreme point occurring in Session 1 (0) and Session 19 (4.3). After training the number of correct columns per minute steadily increased from 1.8 in Session 27 to 4.6 in Session 29.

The number of columns completed incorrectly per minute also shows a slight yet variable upward trend. The lowest point occurred in
Figure 5: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of subtraction problems for subject one.
Session 16 (2.0) and the highest point occurred in Session 20 (6.3). After training the number of incorrect columns per minute dropped sharply to 0.4 in Session 27 and dropped once more to 0.2 in Sessions 28 and 29.

Subject Two

Subject two received a score of 96 percent on the pretest of basic subtraction facts.

From Figure 6 it can be seen that the percent of columns correct stabilized at a high percentage for Sessions 1 through 5 then dropped to the lowest point during Baseline in Sessions 6 and 7 (63 percent). Sessions 8 through 11 showed a downward trend with an increase in Session 12 to 100 percent. Sessions 13 through 16 remained at a high percentage ranging from 95 percent to 100 percent. After training the percent of columns correct remained high with the exceptions of Sessions 22 and 27 where the percentages dropped to 69 percent and 71 percent, respectively. The highest point after training occurred in Session 29 (100 percent).

During Baseline the number of columns completed correctly per minute remained stable from Sessions 1 through 11, possibly due to the decreased variability that resulted from the calculation used. The highest point occurred in Session 13 (7.7) after which the rate dropped in Sessions 13 and 16 to 4.8 and 4.6, respectively. After training the number of correct columns per minute was stable for Sessions 18 through 21 after which it dropped sharply to the lowest point of 2.2 during Session 22. Sessions 23 through 29 were extremely variable ranging from 3.0 in Session 23 to 8.0 in Session 29.
Figure 6: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of subtraction problems for subject two.
The number of columns completed incorrectly per minute was stable for Sessions 1 through 5 but then increased sharply to 1.8 in Sessions 6 and 7. Sessions 8 through 16 show a slight downward trend with the lowest points occurring in Sessions 12 and 15 (0). After training the number of incorrect columns per minute was variable yet stable with no significant changes over Baseline. The highest point occurred in Session 27 (1.4) and the lowest point occurred in Session 29 (0).

Subject Three

Subject three received a score of 87 percent on the pretest for basic subtraction facts.

From Figure 7 it can be seen that the percent of columns correct showed a steady yet variable downward trend from Sessions 1 to 14, the highest point being 54 percent in Session 5 and the lowest point being 8 percent in Session 14. Data from Session 15 increased sharply to 83 percent and the remaining Baseline sessions (10 through 21) stabilized at a high percentage. After training the percent of columns correct remained high with the highest point being 100 percent in Session 23 and the lowest point being 88 percent in Sessions 24 and 26. There appears to be no difference between the sessions after training and Sessions 15 through 21 of Baseline.

The number of columns completed correctly per minute was stable yet variable from Sessions 1 to 14. In this group the highest point occurred in Session 12 (3.0) and the lowest point occurred in Session 14 (0.7). The rate in Session 15 increased sharply to 4.0 and the remaining
Figure 7: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of subtraction problems for subject three.
Baseline sessions (16 through 21) remained at this high rate. During the first three sessions after training the number of correct columns per minute decreased from 4.8 to 2.8. In Session 28 the rate increased sharply to 7.7 correct columns per minute and then decreased to 3.0 in Session 29.

The number of columns completed incorrectly per minute steadily increased from Sessions 1 to 14. The lowest point occurred in Session 5 (2.2) and the highest point occurred in Session 14 (6.6). The incorrect rate in Session 15 dropped sharply to 0.8 and the remaining six sessions of Baseline stabilized at this level. After training the data stabilized at a low rate with the highest point occurring in Session 23 (0).

**Subject Four**

Subject four received a 97 percent on the pretest of basic subtraction facts.

From Figure 8 it can be seen that the percent of columns correct during Baseline stabilized at a high percentage. The highest point occurred in Session 5 (88 percent) and the lowest point occurred in Session 7 (63 percent). In the first session after training the percent of correct columns increased to 92 percent. With the exception of Session 14 (67 percent) all sessions after training had a percentage of 92 or higher.

The number of columns completed correctly per minute was stable throughout Baseline, possibly due to the decreased variability that resulted from the calculation used. The highest point occurred in
Figure 8: The percent correct, correct rate (rate corr.), and incorrect rate (rate incorr.) of subtraction problems for subject four.
Session 5 (4.2) and the lowest point occurred in Session 7 (3.0). After training the number of correct columns per minute increased over Baseline. All the sessions except one had rates that were higher than any point during Baseline. The highest point occurred in Session 24 (8.0) and the lowest point occurred in Session 14 (3.2).

The number of columns completed incorrectly per minute was again stable throughout Baseline with the highest point occurring in Session 7 (1.8) and the lowest point occurring during Sessions 2 and 5 (0.6). Again, this could be due to the decreased variability that resulted from the calculation used. After training the number of incorrect columns per minute decreased sharply to 0.4 in Session 13. All subsequent sessions stabilized at points lower than any points in Baseline with the exception of Session 14 which had an incorrect rate of 1.6 columns per minute.

**Generalization Probes**

Table 2 presents the results of the generalization probes for addition and subtraction. From Table 2 it can be seen that the newly trained algorithm appear to generalize quite readily to problems of varying sizes and difficulty. All percentages except one were above 80 for both operations.
Table 2

Percent of Columns Correct on Generalization Probes

<table>
<thead>
<tr>
<th>Operation</th>
<th>Subjects</th>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Addition:</td>
<td></td>
<td>89%</td>
<td>94%</td>
<td>91%</td>
</tr>
<tr>
<td>Subtraction:</td>
<td></td>
<td>72%</td>
<td>93%</td>
<td>85%</td>
</tr>
</tbody>
</table>

Note. Percent calculated with the following formula:

\[
\text{Percent} = \left( \frac{\text{# of columns correct}}{\text{# of columns attempted}} \right) \times 100
\]
CHAPTER IV

DISCUSSION

Addition

The results of the present study indicate that Hutchings' low stress algorithm for addition resulted in higher percent of columns correct and lower incorrect rates than the conventional method for all four subjects. The correct rate remained unchanged.

These results confirm previous findings (Alessi, 1974; Zoref, 1976; Gillespie, 1976; Rudolph, 1976) that Hutchings' low stress algorithm for addition is a more accurate method of computation than the conventional algorithm.

A possible reason for these findings is that although the subjects increased their accuracy using the new algorithm, they failed to increase their speed. Since the number of sessions conducted with each subject using the new algorithm ranged from 2 to 15, the subjects may not have had enough practice using the new algorithm.

Another possible explanation for the increase could be the result of novelty effects. The subjects may have responded to the new addition algorithm as a different, new, and exciting procedure to use for a boring task. This may have resulted in closer attending response to the task and in turn more errors corrected.

Three significant data points need to be discussed at this time. From Figure 2 it can be seen that there was a substantial decrease in
percent of columns correct and number of columns completed correctly per minute in Session 22. A possible explanation for this is that during that particular session the subject was emitting a high rate of off task and aggressive behaviors toward other students in the classroom. Frequent prompting was required by the experimenter to keep the subject on task. Possible antecedents for this behavior were ridicule for being "stupid" and frequent name calling from classmates.

A second point for discussion occurred during Session 12 on Figure 1. From Figure 1 it can be seen that the percent of columns correct and the number of columns correct decreased to zero. The only possible explanation that can be offered is that during this session the subject emitted many "won't do" behaviors. These behaviors consisted of talking to others, frequent complaints about the difficulty of the task, and guessing of answers. The subject completed one column out of the 12 possible during this session.

The third point for discussion occurred during Session 29 on Figure 3. From Figure 3 it can be seen that the percent of columns correct decreased to 56 and the number of columns completed incorrectly per minute increased to 0.8. Once again, the possible explanation for this decrease is that the subject emitted many "won't do" behaviors similar to those mentioned above.

Subtraction

The results of the effectiveness of Hutchings' low stress subtraction algorithm do not appear to be as clear as the results of the low stress addition algorithm. There appear to be changes in the
dependent variable measures for subject one (Figure 5), but subjects two, three, and four yielded no significant changes in accuracy or rate when using the low stress subtraction algorithm.

Two possible explanations can be offered for the increase in percent of columns correct and the decrease in the number of columns completed incorrectly per minute for subject one. First, the accuracy may have improved because the subject was required to do all regrouping first and then subtract, rather than alternating from regrouping to subtracting. This may have resulted in less confusion and therefore less errors for the subject. Once again there was no significant changes in the number of columns completed correctly per minute, which as explained earlier, may have occurred because the algorithm was new and while accuracy improved, speed did not.

The second possible explanation may have been the novelty effect noted earlier. Since the new algorithm was new and different it may have resulted in fewer errors than the conventional subtraction algorithm.

Although the percent of columns correct for subject one increased over Baseline, this apparent effect should be made with caution. The data from the sessions using the new algorithm failed to stabilize before the study was terminated. The same is true for the rate of columns correct per minute. The data seems to show an upward trend but did not become stable before the completion of the study.

Only one determination can be made about the lack of any significant effect for subjects two, three, and four. These subjects already had the skills necessary to perform subtraction with regrouping accurately and at a fairly high rate. This may explain why the subjects
performed with accuracy during Baseline thus producing a ceiling effect.

The only hypothesis that can be made about the effectiveness of the low stress algorithm is for subject four. From Figure 8 it can be seen that the percent of columns correct became very stable after training. This may be due to the newly trained algorithm or the effects of practice on accuracy. Subjects received only one size subtraction problem throughout the course of the study. By performing the same task in each session, the accuracy of problem solving for subject four may have increased due to the repetition of the task.

A series of data points measuring the percent of columns correct for subject three (Figure 7) increased significantly during Session 15 (Baseline) and remained above 80 percent for the remainder of the study. In addition, the rate of columns incorrect decreased significantly during the same session and remained below 0.8 for the remainder of the study. The apparent explanation for these changes is that during Session 14 the subject emitted many non-compliance behaviors such as refusing to complete the worksheet when asked, talking to classmates, and doodling on his worksheet. After frequent prompting by the experimenter the subject quickly completed the worksheet, subtracting the smaller number from the larger number regardless of their positions in the problem. At the end of the session the experimenter discussed this behavior with the subject. The experimenter asked the subject to work harder in the following session. The result of this discussion was a significant increase in accuracy and correct rate.
Overall, the changes in the dependent variables may have resulted from variables other than the ones discussed. For addition, changes could have resulted from the reduced short and long-term memory load since the subjects were not required to remember a long chain of addition facts while solving the problem or remember many complex addition facts over time. A second reason could be the overt format utilized with Hutchings' low stress addition algorithm. This increases the likelihood that subjects would observe and correct their mistakes.

For subtraction, the mechanics of completing all the regrouping in the middle rather than on top may reduce the likelihood of errors because the subject does not have to look over a row to find the minuend, it is placed right above the subtrahend.

Generalizing these results to other settings and subjects should be made with caution. This study included only four subjects that had been classified as "low performers" in addition and subtraction by their teachers and test results in the same areas. All were from rural middle-class backgrounds, were white, and attended a small rural elementary school. Generalizations to other populations and/or situations may not yield similar results.

There are many advantages for applying Hutchings' low stress algorithms for addition and subtraction in the classroom. These include:

1. Training can be done quickly and easily (approximately one-half hour for each algorithm).
2. Students appear to grasp the task with little difficulty.
3. There are few physical differences between the low stress algorithms and the conventional algorithm.
4. Because of the overt calculations that are made with the low stress algorithms, students and teachers can readily pinpoint and correct errors and determine developing error patterns.

The major disadvantage of using Hutchings' low stress algorithms in the classroom is that teachers may have to reconstruct worksheets and workbook pages to allow room for students to perform necessary calculations within the problem.

The experimenter suggests that future research stemming from this study should cover two important areas. The first is that the effectiveness of Hutchings' low stress subtraction algorithm should be analyzed using problems of varying sizes. Secondly, the effect of both the low stress addition and subtraction algorithms should be analyzed over time to observe whether the accuracy and rate maintain.
APPENDIX A

BASIC FACTS PRETESTS FOR
ADDITION AND SUBTRACTION
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SCORE ____
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**SCORE** ________
APPENDIX B

DAILY WORKSHEETS FOR
ADDITION AND SUBTRACTION
<table>
<thead>
<tr>
<th>DATE</th>
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<td>2 2 7 8</td>
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<tr>
<td>6 6 9 9</td>
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<tr>
<td>4 1 8 6</td>
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<td>2 8 5 7</td>
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<td>+ 4 7 5 7</td>
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<td>4 7 8 3</td>
<td></td>
</tr>
<tr>
<td>+ 1 8 5 9</td>
<td></td>
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</tbody>
</table>
\[
\begin{array}{c}
35519896 \\
-35491642 \\
\hline
87235318 \\
-48259663 \\
\hline
51169338 \\
-23643864
\end{array}
\]
APPENDIX C

TRAINING PACKAGE FOR ADDITION
I am going to show you the usual way of writing number facts and then another way of writing them.

You have all seen number facts written like this:

\[ \begin{array}{c}
7 \\
+8 \\
\hline
15
\end{array} \]

Well, they can also be written like this, using two small (half-space) numbers instead of the line and plus sign.

\[ \begin{array}{c}
7 \\
+8 \\
\hline
15
\end{array} \]

Do you still see the fifteen? (Point to both fifteens.)

I'll write the two examples next to one another.

\[ \begin{array}{c}
7 \\
+8 \\
\hline
15
\end{array} \] \[ \begin{array}{c}
7 \\
+5 \\
\hline
185
\end{array} \]

Do you all see the fifteen? (Point )

Let's look at another one. I can write "9 plus 5 is 14" like this \[ \begin{array}{c}
9 \\
+5 \\
\hline
14
\end{array} \] \[ \begin{array}{c}
9 \\
+5 \\
\hline
14
\end{array} \]

Both of these say "9 plus 5 is 14."

Tell me what these say:

\[ \begin{array}{cccccccc}
9 & 9 & 6 & 6 & 4 & 4 & 5 & 5 \\
+8 & +7 & 13 & 17 & +5 & 9 & +6 & 12 \\
\hline
17 & 17 & 13 & 17 & 9 & 12 & 162 & 7 \\
\end{array} \]

(Call on students, point to the full notation form 9 when asking.)
The little number on the right* is understood to be in the one's place, as are 9 and 8.

The little number on the left* is understood to be in the ten's place.

In other words, this is the same as this (point from "big 7" to "little 7"). And this is the same as this (point from "big one" to "little one").

Now watch me write the following facts both ways.

```
9  9  8  8  4
+7  17  13  15  9
16  17  15  19
```

Look at the last pair. Are they different from the others? Note that there is no ten's place number and (do not draw until after saying this) there is no "little one" on the left.

Let's look at another.

a) 4 Is there any ten's number here? (Do not draw box until after asking question.)

b) NO!! (repeat)

c) So will there be any little number on the left?

d) 4

(Do not draw box until after asking question.)

NO!! (repeat)
Again, \[
\begin{array}{c}
4 \\
+3 \\
\hline
7 \\
\end{array}
\quad
\begin{array}{c}
4 \\
 \quad 3 \\
\hline
7 \\
\end{array}
\]

If there is no ten's place number, there is no "little number" on the left.

Now watch me write the rest of these.

Notice \[
\begin{array}{c}
3 \\
+1 \\
\hline
4 \\
\end{array}
\quad
\begin{array}{c}
3 \\
 \quad 1 \\
\hline
4 \\
\end{array}
\]

no ten's number here so no "little number" here

but \[
\begin{array}{c}
7 \\
+3 \\
\hline
10 \\
\end{array}
\quad
\begin{array}{c}
7 \\
 \quad 3 \\
\hline
10 \\
\end{array}
\]

There is a ten's number here so there is a "little number" here

Again, notice \[
\begin{array}{c}
5 \\
+1 \\
\hline
6 \\
\end{array}
\quad
\begin{array}{c}
5 \\
 \quad 1 \\
\hline
6 \\
\end{array}
\]

There is no ten's number here so there is no "little number" here

but \[
\begin{array}{c}
8 \\
+5 \\
\hline
13 \\
\end{array}
\quad
\begin{array}{c}
8 \\
 \quad 5 \\
\hline
13 \\
\end{array}
\]

There is a ten's number here so there is a "little number" here
Now I am going to show you a special way of adding that uses only those "little numbers" on the right.

I'll say that again (repeat previous statement).

This should make your addition very easy and accurate. It is a scientific method and many scientists do addition this way. Watch.

First, do you see that an example can be just number facts piled one atop the other? (Do not point with this question.)

OK! Here we go, start at the top, writing facts as you learned and using only numbers at the right for addition.

a) Say, "The first fact we do may look a bit different because we do not have any little numbers yet." (Point)

b) Say, "This is the only time we will use two big numbers. In the rest of the example we use one little number and one big one."

c) Say, "Now, eight plus five is thirteen."

d) Write the thirteen, i.e., \( \frac{5}{3} \) in the example.

a) Say, "We've written the thirteen but we'll use only the three."

b) Draw arrow \( \frac{3}{7} \).

c) Say, "Three plus seven is ten."

d) Write the 10, i.e., \( \frac{3}{7} \) in the example.
a) Say, "We've written the ten but we'll use only the 0."

b) Draw arrow 9. 

c) Say, "Zero plus nine is nine."

d) Write 9, i.e., 9^0_9 in the example.

Say, "We've written the nine and look that's all we have this time because zero and nine is just nine. But that's OK because we only use the right-hand number anyway."

b) Draw arrow 3^9_9 .

c) Say, "Nine plus eight is seventeen."

d) Write the seventeen, i.e., 18^9_7 in the example.

Say, "We've written the seventeen but we'll use only the seven."

b) Draw arrow 6^9_9 .

c) Say, "Seven plus six is thirteen."

d) Write the thirteen, i.e., 16^7_3 in the example.

Say, "We've written the thirteen but we'll use only the three."

b) Draw the arrow 8^3_1 .

c) Say, "Three plus eight is eleven."

d) Write the eleven, i.e., 18^3_1 in the example.
a) Say, "We've written the eleven but we'll use only the one.

b) Draw arrow $71$.

c) Say, "One plus seven is eight."

d) Write the eight, i.e., $78$ in the example.

Now we're at the key part. All we've done is use number facts. We haven't done any "in your head" work.

Nevertheless, we already know the answer! Watch.

The last little number on the right is the right half of the answer.

To get the left half, we just count the little numbers on the left that we didn't use. One, two, three, four, five, there are five of the, so the first half of the answer is five. The answer is 58.

Now watch me do another. Remember we use only the right side "little numbers." We will not bother to write the arrows anymore, just say

Now the last number on the right is a 2, so the right half of the answer is a 2! We get the left half of the answer by counting the little numbers on the left that we didn't use. One, two, three, four, five. There are five of them so the left half of the answer is 5. The answer is 52.
Now say the words for these with me as I do them at the board. 
(Children do not do this.)

\[
\begin{array}{ccc}
8 & 9 & 4 \\
15 & 15 & 182 \\
17 & 27 & 65 \\
19 & 39 & 111 \\
183 & 165 & 132 \\
181 & 187 & 180 \\
184 & 190 & 187 \\
160 & 9 & 163 \\
52 & 49 & 43 \\
\end{array}
\]

Now copy these examples and do them by yourself. If you have any questions, ask me.

\[
\begin{array}{ccc}
6 & 8 & 5 \\
5 & 2 & 4 \\
9 & 7 & 9 \\
8 & 6 & 8 \\
5 & 9 & 7 \\
6 & 8 & 9 \\
+ 9 & + 5 & + 8 \\
\end{array}
\]

After most have finished, say. "Check your work with mine as I do them at the board."

After doing the examples, say, "Now let's review."
I'll write the work for another one on the board. I want someone to raise their hand and tell me what the answer is.

6
8 4 plus 9 is 13
\[\frac{1}{2}\] 3 plus 5 is 8
7 8 plus 7 is 15
5 5 plus 5 is 10
0 0 plus 9 is 9
9 9 plus 3 is 12

(Point to box.) Who will tell me what the right side of the answer is and how he got it.

(Point to box.)

(Locate correct response.) Good! That's correct. The last little number on the right becomes the right side of the answer.

Who will tell me what the left side of the answer is and how he got it. (Locate correct response.) Good! That's correct, we count up the little numbers on the left for the left side of the answer.

Now, what do you suppose we do if there is more than one column? That is, if there is another column at the left of the column you're adding. Like this

4 6
7 8
6 7
8 1
7 18

Can we still write our left-hand answer number at the bottom if there is more than one column? No, we can't?

When there's more than one column, each column can have only one number at the bottom (except for the very last column which does have the usual two).
So the single number that we put at the bottom is always the right-hand number.

\[
\begin{array}{c}
8 \\
7.5 \\
1.6 \\
1.9 \\
1.8 \\
5.8 \\
1.3 \\
3
\end{array}
\]

(Write and point)

What can we do with the left-hand number?

Would it make sense to throw it away? No, it's part of the problem. So we will put it at the very top of the next column at the left. That way we don't lose it and it's still on the left side.

Watch! (Write on board.)

Count the little number on the left with me.

One, two, three, four.

There are four of them so we write a 4 at the top of the next column.

Now, when I start adding that column I will start with the four (4) first. Let's be sure you understand. (Repeat twice from the *.)

This is called carrying, some of you already understand it. Good. Carrying is very easy.

But carrying is very important. You must never forget to carry.

Look at these examples and tell me what to write at the top of the left-hand column. (Write on board.)

\[
\begin{array}{cccccccc}
6 & 8 & 8 & 5 & 7 & 6 & 8 & 5 \\
5 & 9.7 & 7 & 6.1 & 9 & 6.2 & 1 & 8.3 \\
6 & 2.9 & 9 & 1.6 & 9 & 1.2 & 1 & 1.25 \\
8 & 2.5 & 4 & 1.0 & 1 & 1.4 & 4 & 1.75 \\
7 & 1.6 & 3 & 0.8 & 1 & 1.3 & 9 & 1.52 \\
4 & 3.0 & 4 & 1.4 & 4 & 3.5 & 6 & 1.30 \\
\end{array}
\]

(Do with volunteers from class at board.) Good, we write the left-hand answer number at the top of the next column. (Repeat three times.)

Remember though that for the last column only, the left-hand answer number is at the bottom as though it were a single column.
Now, copy these examples and do them with me.

```
7 6
5 9
8 7
6 9
8 3
9 5
```
```
7 9 8
6 8 5
4 7 6
6 9 5
8 3 9
4 2 2
```
```
8 7 7
8 7 6
5 7 6 9
8 7 6 2
8 5 7 6
8 3 9 5
```

Again, do you see that I always carry the number of tens to the top of the next column? (Point and illustrate example.) Except when there are no more columns. Then I write the number of tens on the bottom line as part of the answer. (Point and illustrate with each.)

Good! Are there any questions?

Now take these dittoed examples and do them by yourselves. If you have trouble, ask me for help.

```
7
6
8
4
7
6
9
0
```
```
6 8 7
4 8 3
6 9 5
8 7 4
```
```
7 9 8
6 8 5
4 7 6
6 9 5
8 7 6
9 5 4
```
```
+ 8 7 4
+ 5 8
+ 3
+ 4 2
```
```
4 8 7 6 8 7 6
9 8 7 6 8 7 6
9 8 7 6 8 5 6
7 9 5 6 7 9 3
8 5 2 7 4 9 8
+ 6 7 8 5 6 7 8
```

Be sure to make and place your numbers neatly!

(Allow time needed for most to finish.)

Now, I will do them. Check your work against mine.
APPENDIX D

TRAINING PACKAGE FOR SUBTRACTION
There is a special notation for the new subtraction but it's very easy. Have you noticed that regrouping work in the old subtraction is written over the top number. This regrouping work in the old way was usually written here

\[
\begin{array}{cccccc}
6 & 7 & 3 & 8 & 4 & 8 \\
- & 1 & 1 & 8 & 7 & 9 & 6 \\
\end{array}
\]

Instead, with the new notation we will write our "work," our regrouped top number, in the middle between the top and bottom numbers. That is, we will write it here

\[
\begin{array}{cccccc}
6 & 7 & 3 & 8 & 4 & 8 \\
- & 1 & 1 & 8 & 7 & 9 & 6 \\
\end{array}
\]

Solved the old way, this problem might have looked something like this

\[
\begin{array}{cccccc}
6 & 13 & 7 & 14 \\
6 & 7 & 3 & 8 & 4 & 8 \\
- & 1 & 1 & 8 & 7 & 9 & 6 \\
\end{array}
\]

\[
\begin{array}{cccc}
5 & 5 & 5 & 0 & 5 & 2 \\
\end{array}
\]
Solved the new way this problem would look like this

\[
\begin{array}{cccc}
6 & 7 & 3 & 8 \\
\hline
6 & 6 & \underline{13} & 7 \underline{14} \\
\hline
- & 1 & 1 & 8 \\
\hline
5 & 5 & 5 & 0 \end{array}
\]

Do you see that the regrouping work is written in a different place with the new way?

The regrouping of the top number, that is, the work written in the middle, will be called the "rename."

\[
\begin{array}{cccc}
6 & 7 & 3 & 8 \\
\hline
6 & 6 & \underline{13} & 7 \underline{14} \\
\hline
- & 1 & 1 & 8 \\
\hline
5 & 5 & 5 & 0 \end{array}
\]

This is the "rename"

Now you know the special notation, "renaming in the middle." You are ready to learn the rest of the new way's procedures and see some problems solved.
A Concise Statement of the Rules

1) We will call the regrouped or partially regrouped top number a "rename" and write it in the middle, that is, between the top and bottom numbers.

Now here are the two rules you have not seen before. They will seem easy after you have seen the problems solved.

2) If any of the top numbers have to be regrouped, even if only two of them,

[say, "you will see later that you cannot regroup one number. You can regroup none or you can regroup two or more."]

We will write all of them in the rename, both the regrouped numbers and those not regrouped.

3) We will write a complete rename with all of the top numbers, regrouped or not, before doing any subtraction.

Let's look at some examples showing rule 2.
In this problem all five top numbers are regrouped in the rename, and none are the same as before. 

\[
\begin{array}{cccccc}
6 & 2 & 4 & 3 & 4 \\
\hline
5 & 1 & 3 & 12 & 4 \\
\hline
- & 1 & 5 & 7 & 9 & 8 \\
\hline
4 & 6 & 6 & 3 & 6
\end{array}
\]

In this problem four of the top numbers are regrouped in the rename, but one number, 5, is the same as before. We include it in the rename even though it is not regrouped.

\[
\begin{array}{cccccc}
5 & 7 & 3 & 4 & 8 \\
\hline
5 & 6 & 1 & 3 & 18 \\
\hline
- & 1 & 6 & 4 & 5 & 9 \\
\hline
4 & 0 & 8 & 8 & 9
\end{array}
\]

In this problem three of the top numbers are regrouped in the rename, but two numbers, 8 and 2, are the same as before. We include them in the rename even though they are not regrouped.

\[
\begin{array}{cccccc}
8 & 2 & 7 & 1 & 6 \\
\hline
8 & 2 & 6 & 10 & 16 \\
\hline
- & 1 & 1 & 6 & 2 & 7 \\
\hline
7 & 1 & 0 & 8 & 9
\end{array}
\]

In this problem two of the top numbers are regrouped in the rename, but three numbers, 7, 3, and 8, are the same as before. We include them in the rename even though they are not regrouped.

\[
\begin{array}{cccccc}
7 & 3 & 8 & 2 & 4 \\
\hline
7 & 3 & 8 & 1 & 14 \\
\hline
- & 1 & 1 & 6 & 1 & 5 \\
\hline
6 & 2 & 2 & 0 & 9
\end{array}
\]

Of course when really doing a subtraction problem we do not circle any numbers in the rename. That was done in these examples just to help you find the numbers which are not regrouped. Probably you now understand
what rule 2 says, let's look at some examples showing rule 3. In fact,
let's use the problems we've already seen.

When solving any subtraction problem
which has regrouping

First write the complete rename

Then subtract

When solving any subtraction problem
which has regrouping

First write the complete rename
Then subtract

\[
\begin{array}{c}
5 \ 7 \ 3 \ 4 \ 8 \\
5 \ 6 \ 1 \ 2 \ 1 \ 3 \ 1 \ 8 \\
- \ 1 \ 6 \ 4 \ 5 \ 9 \\
\hline
4 \ 0 \ 8 \ 8 \ 9
\end{array}
\]

When solving any subtraction problem which has regrouping

\[
\begin{array}{c}
8 \ 2 \ 7 \ 1 \ 6 \\
- \ 1 \ 1 \ 6 \ 2 \ 7 \\
\hline
7 \ 1 \ 0 \ 8 \ 9
\end{array}
\]

First write the complete rename

\[
\begin{array}{c}
8 \ 2 \ 7 \ 1 \ 6 \\
8 \ 2 \ 6 \ 1 \ 0 \ 1 \ 6 \\
- \ 1 \ 1 \ 6 \ 2 \ 7 \\
\hline
7 \ 1 \ 0 \ 8 \ 9
\end{array}
\]

When solving any subtraction problem which has regrouping

\[
\begin{array}{c}
7 \ 3 \ 8 \ 2 \ 4 \\
- \ 1 \ 1 \ 6 \ 1 \ 5 \\
\hline
7 \ 1 \ 0 \ 8 \ 9
\end{array}
\]
First write the complete rename

\[ \begin{align*}
7 & \quad 3 & \quad 8 & \quad 2 & \quad 4 \\
7 & \quad 3 & \quad 8 & \quad 1 & \quad \frac{1}{4} \\
- & \quad 1 & \quad 1 & \quad 6 & \quad 1 & \quad 5 \\
\end{align*} \]

Then subtract

\[ \begin{align*}
7 & \quad 3 & \quad 8 & \quad 2 & \quad 4 \\
7 & \quad 3 & \quad 8 & \quad 1 & \quad \frac{1}{4} \\
- & \quad 1 & \quad 1 & \quad 6 & \quad 1 & \quad 5 \\
\end{align*} \]

\[ \begin{align*}
6 & \quad 2 & \quad 2 & \quad 0 & \quad 9 \\
\end{align*} \]

Good. Probably you now understand what rule 3 says. That means we're ready to see, in detail, how to solve different kinds of problems.
PROBLEMS WITH REGROUPING,
BUT NOT ACROSS ZEROS

The second kind of subtraction problem is also fairly easy. This kind includes problems like the last three problems you saw when you were learning rule 3 and rule 4. These are problems where some or all of the top numbers are regrouped, where none of the numbers you have to regroup, or borrow, across are zeros:

\[
\begin{align*}
5 & \quad 7 & \quad 3 & \quad 4 & \quad 8 \\
- & \quad 1 & \quad 6 & \quad 4 & \quad 5 & \quad 9 \\
\hline
7 & \quad 3 & \quad 8 & \quad 2 & \quad 4 \\
- & \quad 1 & \quad 1 & \quad 6 & \quad 1 & \quad 5
\end{align*}
\]

Let's do these step by step and consider everything you might think in solving each of them.

First we look at the problem to decide whether or not there will be any regrouping. Is there any top number which is smaller than the number directly beneath it. If we find even one smaller top number, there will be regrouping. Starting at the right we say is 8 smaller than 9. Yes. There will be regrouping.
Now that we know regrouping will occur, we know there must be a rename and we can begin to write it, that is, to regroup the top number. Write the 8 in the middle directly over the 9. When you do this you might think "Bring down the 8."

Can we take 9 from 8? No. Therefore we must make the top number larger. Remember there is only one way to do that. Add 10 ones to the 8 making it 18 ones, which is written 18.

Good. Now where did we get the 10 ones we added to the 8? We took ten from the 4, in the form of 1 ten. Remember that, one taken from the 4 stands for 10 added to the 8.

\[\begin{array}{cccccc}
5 & 7 & 3 & 4 & 8 \\
- & 1 & 6 & 4 & 5 & 9 \\
\hline
1 & 8 \\
\end{array}\]

[You might say, if the linguistic level of your class makes it appropriate, "this is so because any digit, in this case 4, has 10 times the place value of the digit immediately at its right, in this case 8."]
You could say, instead of thinking all this, "1 ten added to the 8 leaves 3 tens instead of 4, so I'll bring down 3 instead of 4." Write 3 in the rename.

Can you take 5 from 3? No.

So, 3 must be made larger. Take 1 ten from the top 3, at the left, and add it as 10 ones, 10, to the 3 in the middle to make 13. Now the top 3 has lost 1 (ten) so it is 2 in the rename.

Can you take 4 from 2? No.

So, the 2 must be made larger. Take 1 ten away from the top 7, at the left, and add it as 10 ones, 10, to the 2 in the middle to make 12. Now the top 7 has lost 1 (ten) so it becomes 6 in the rename.
Can you take 6 from 6? Yes.

Regrouping is not needed. Bring down the next number, 5.

Can you take 1 from 5. Yes.
The completely renamed problem looks like this and is now ready for subtraction.

Let's look at another problem.

Are any top numbers smaller than any bottom numbers? Yes.
So renaming is needed.

Bring down the 6. Can you take 7 from 6? No.
So the 6 must be made larger. Take 1 ten from the top 1, at the left, and add it as 10 ones, 10, to the 6 in the middle to make 16. Now the top 1 has lost 1 (ten), so it becomes 0 in the rename.

Can you take 2 from 0? No.

So the 0 must be made larger. Take 1 ten from the top 7, at the left, and add it as 10 ones, 10, to the 0 in the middle to make 10.

[Say, "Remember any number added to zero is unchanged, that is, the sum of zero and some number is simply that number."]

Now the top 7 has lost 1 so it becomes 6 in the rename.

Can you take 6 from 6? Yes.
Regrouping is not necessary. Bring 8 2 7 1 6 down the next number, 2.

\[
\begin{array}{r}
2&6&10&16 \\
-1&1&6&2&7 \\
\end{array}
\]

Can you take 1 from 2? Yes.

\[
\begin{array}{r}
2&6&10&16 \\
-1&1&6&2&7 \\
\end{array}
\]

Regrouping is not necessary. Bring 8 2 7 1 6 down the next number, 8.

\[
\begin{array}{r}
8&2&6&10&16 \\
-1&1&6&2&7 \\
\end{array}
\]

The completely renamed problem looks like this and is now ready for subtraction.

\[
\begin{array}{r}
8&2&6&10&16 \\
-1&1&6&2&7 \\
\end{array}
\]

Good, now do the regrouping work for the following examples. That is, write the renames for the top number. Concentrate on doing all the renames as quickly and accurately as possible. Do not do the subtraction.
\[
\begin{array}{cccc}
4 & 3 & 6 & \quad 4 & 7 & 2 & \quad 5 & 6 & 0 & 6 \\
-1 & 8 & 7 & \quad -2 & 9 & 3 & \quad -1 & 7 & 4 & 6 \\
\hline
& & & & & & & & & \\
\end{array}
\]

\[
\begin{array}{cccc}
3 & 8 & 7 & 0 & \quad 7 & 3 & 4 & 6 & 5 & \quad 7 & 6 & 3 & 2 & 4 \\
-1 & 8 & 5 & 6 & \quad -1 & 2 & 9 & 8 & 7 & \quad -1 & 8 & 0 & 0 & 6 \\
\hline
& & & & & & & & & \\
\end{array}
\]

\[
\begin{array}{cccc}
8 & 9 & 1 & 6 & 7 & 3 & 4 & 0 & 7 & 6 \\
-1 & 2 & 9 & 9 & 4 & 8 & 9 & 3 & 5 & 7 \\
\hline
& & & & & & & & & \\
\end{array}
\]

\[
\begin{array}{cccc}
8 & 4 & 3 & 7 & 6 & 4 & 0 & 5 & 9 & 2 & 3 \\
-1 & 4 & 7 & 6 & 8 & 5 & 2 & 2 & 7 & 5 & 9 \\
\hline
& & & & & & & & & & \\
\end{array}
\]
APPENDIX E

S.R.A. PLACE VALUE PROBES
(LEVEL M-13)
1. Use digits and commas to write another name for each number.
   a. eight million, four hundred fifty-six thousand, seven hundred forty-two
   b. four hundred thirty-eight million, three hundred ninety-two thousand, eight hundred thirty-seven

2. Put in (, , or =.
   a. 26 62             c. 101,001 110,100
   b. 478 874            d. 63,101,232 115,335,313

3. Write the correct numeral.
   a. 48 hundreds + 3 tens + 6 ones =
   b. 5 thousands + 71 tens + 2 ones =
   c. 15 hundreds + 12 tens + 17 ones =
   d. 3 thousands + 24 hundreds + 2 tens + 29 ones =

4. Replace blank with the correct numeral.
   a. 27 = ___ tens + 7 ones           d. 10,000,000 = ___ thousands
   b. 563 = ___ tens + 3 ones         e. 10,000,000 = ___ hundreds
   c. 4203 = ___ thousands + 12 hundreds + 0 tens + 3 ones

5. In the numeral 186,759,342
   the 4 is in the tens place and its total value is 40;
   the 5 is in the ___ place and its total value is ___;
   the 6 is in the ___ place and its total value is ___;
   the 3 is in the ___ place and its total value is ___;
   the 1 is in the ___ place and its total value is ___.

6. Put these numbers in order on the number line.
   1103 33,605 672,481 999
   1. _____  2. _____  3. _____  4. _____
APPENDIX F

POST TRAINING PROBES FOR
ADDITION AND SUBTRACTION
(LEVELS M-1 AND M-2)
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