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College Physics Students' Conceptual Understanding of Area and Volume, and Relationships Between these Concepts and Students' Understanding of Physics Concepts

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COLLEGE PHYSICS STUDENTS' CONCEPTUAL UNDERSTANDING OF AREA AND VOLUME, AND RELATIONSHIPS BETWEEN THESE CONCEPTS AND STUDENTS' UNDERSTANDING OF PHYSICS CONCEPTS

by

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Concepts such as area and volume are foundational ideas for many concepts introduced in introductory science courses. At the college level, most instructors typically assume that incoming students have already developed an understanding of these underpinning ideas. However, doubt has surfaced in recent years about students' depth of understanding and mastery of these fundamental concepts. Because deficiencies in understanding basic concepts may relate to the learning of subsequent concepts, instructors have expressed concerns about students' understanding of fundamental ideas and if the failure to understand these ideas hinders students' subsequent progress.

This study was designed to (a) investigate the nature of college physics students' understanding of the area and volume concepts and (b) to begin to inquire into the nature of the relationship between students' understanding of the area and volume concepts and their conceptualization of pressure and density.

Four hundred and thirty-one first-semester introductory physics students at Western Michigan University participated in the study. All participants completed a

paper-pencil inventory designed to evaluate a student's concepts of area and volume within a framework in which four categories of conceptual understanding were defined. Twenty-seven students participated in a follow-up clinical interview which was designed to elicit additional information about their prior understanding of area and volume. Eight of these students were interviewed a second time to determine their concepts of pressure and density and to provide insights into the link between these concepts and the students' understanding of area and volume.

Results of the analyses of the paper-pencil inventory and the area and volume interviews indicated that a majority of students entering beginning college physics courses have not developed a good conceptual understanding of area and volume and that their thinking is confined to the rote use of mathematical formulae without a supporting understanding of the concepts behind the mathematical expression. Furthermore, analysis of the pressure and density interviews provided evidence that students' understanding of area and volume, how they think and reason about these concepts, and whether they require mathematical procedures and available formulae do influence their ability to conceptualize pressure and density.

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BACKGROUND AND THE PROBLEM

Introduction

Most concepts are built upon other concepts. This is also true for scientific concepts, where advanced and more abstract concepts are built upon less abstract ones, which, in turn, depend on fundamental ones. The fundamental concepts are those describing attributes of objects and states, which can only be measured directly, but not derived from any other concepts. Examples of fundamental concepts are length and mass. Abstract concepts, on the other hand, are those which use or are based upon the fundamental or less abstract concepts. For example, volume, a low-level abstract concept, uses the basic concept of length; while density, at the next level of abstraction, uses the basic concept of mass and the lower-level abstract concept of volume. In this hierarchy of conceptual construction, a concept at a higher level of abstraction is thus built upon a number of lower hierarchical concepts. For example, acceleration, an advanced and more abstract concept, is defined as the change of velocity with respect to time. Therefore, acceleration is built upon a lower hierarchical abstract concept, velocity, and a basic concept, time. The concept of velocity, in turn, is defined as the change of a particle's position with respect to time. Thus, velocity is based upon two basic concepts, time and displacement. Here, displacement describes the particle's change in position by specifying how far and in what direction the particle has moved.

Because the degree of abstractness of concepts increases from fundamental concepts which are lowest in the hierarchy to those in the higher levels, the fundamental and less abstract concepts usually have close connections with the everyday life of human beings, while those with higher degrees of abstractness are more distant from an individual's daily experience. For example, "distance" and "time" are fundamental scientific concepts, which are also used frequently, among other concepts, in describing individuals' everyday activities (e.g., "distance" is used to describe the spatial length between two places, while "time" is also used to describe the time-interval needed to travel between them). "Wave length" and "frequency" are more abstract concepts which use the concepts of distance and time. But these concepts, wave length and frequency, are further removed from an individual's daily activity (e.g., although people watch television programs and listen to the radio, they may not know the meaning of "wave length" and "frequency"). The concept of "particle wave" is yet even more abstract; so abstract that although it uses the concepts of wave length and frequency, there is no comparable macro-physical phenomena that exists which can be observed to help conceptualize the idea. Thus, particle wave is so remote from an individual's day-to-day experiences that it hardly has any usage in everyday language.

Due to the hierarchical nature of concepts, educators assume that students must understand the fundamental and less abstract ones before learning more abstract concepts. Since the fundamental concepts can also have casual, non-scientific, and imprecise meanings used in everyday language, students often bring these casual meanings to the classroom where more formal and organized learning is expected to occur (Clement, 1982; Gunstone, 1984; Minstrell, 1982). Researchers have shown that these imprecise and non-scientific meanings often contribute to difficulties in the students' subsequent learning of more abstract ideas and scientific concepts (Champagne, Klopfer, & Anderson 1980; Clement, 1993; Driver, 1989; Mestre & Touger, 1989; Minstrell, 1989; Resnick, 1983). For example, in everyday language, "force" involves the will or intent of a living individual (Lakoff & Johnson, 1980). Thus, students who hold this casual meaning of force believe rigid objects (such as tables) cannot exert forces (Clement, 1993). This alternative conception can be a problem when the students envision Newton's third law, which states that a table exerts a force on a book that it is supporting. According to Clement, in a diagnostic test, 76% of a sample of 112 high school students indicated that a table does not exert a force on a book lying at rest on it. Their thinking is that the table is rigid and inanimate, therefore, it cannot exert a force on the book. Thus, while this alternative conception is contrary to Newton's third law, the reasoning that justifies it is the belief that force involves the will or intent of an active and living thing.

In the science education literature, student's prior knowledge, including the imprecise and non-scientific meanings of concepts, is often referred alternately as pre-concepts, misconceptions, naive schemas, or alternative frameworks (for example, see Arons, 1990; Driver, 1989; Redish, 1994; White, 1983). The typical student's prior knowledge may not be totally incorrect. Rather, it mixes many vague and imprecise meanings developed from the individual's daily experience with the meanings of scientific concepts. Thus, in most cases, this prior knowledge may provide students with incomplete, unclarified, or even a confused conceptual framework for developing an understanding of new scientific concepts and theories.

Student's difficulties in learning scientific concepts have been discussed and researched by many science educators. For example, in physics, Arons (1990) systematically summarized his own experiences and the findings of many physics education studies in his book, A Guide To Introductory Physics Teaching. This book, which calls for substantial changes in introductory physics teaching, emphasizes the importance of addressing student's "pre-concepts" or "naive theories." Arons also stresses the need for physics instructors and researchers to study and to understand the nature of college students' understanding of the prerequisite concepts used in introductory physics (such as "area" and "volume"). His concern is that the lack of understanding of these concepts "may seriously impede their grasp of the concepts and lines of reasoning that we seek to cultivate from the beginning of an introductory physics course" (Arons, 1990, p. 1).

Student's prior knowledge in science has also been investigated by many science education researchers (e.g., Bodner, 1991; Clement, 1982; Cohen, Eylon, & Ganiel, 1983; Dykstra, 1991; Erickson & Aguirre, 1984; Goldbery, & Anderson, 1989; Goldberg & McDermott, 1986; Gunstone, 1987; Heller & Finley, 1992; Hesse & Anderson, 1992; Minstrell, 1982; Schoon, 1992; Stofflett, 1993; Trowbridge &

McDermott, 1981). These investigations have produced a great deal of data that lead to the following general conclusions: (a) students hold alternative and non-scientific ideas in a wide range of science topics; (b) these alternative ideas are highly resistant to change by instruction; (c) students may hold inconsistent or even contradictory ideas by keeping specific knowledge isolated from other conflicting information; and (d) many underlying concepts and fundamental relationships that instructors assume are obvious to their students are actually not understood by the students.

Most of these studies have been rather descriptive in characterizing students' problems and difficulties in conceptual learning (Anderson, 1987). There are few studies to help single out what background knowledge contributes to students' alternative conceptions developed during formal science learning. Furthermore, the issue of how an inadequate understanding of fundamental and less abstract concepts affects the development of subsequent or more abstract ones remains largely unaddressed. Thus, these studies have provided few diagnostic-prescriptive models designed to foster the effective learning and teaching of science concepts. In order to improve science teaching and learning, especially that pertains to the development of scientific concepts, science education researchers must study the nature of science students' understanding of lower hierarchical concepts and how an understanding of lower hierarchical concepts influences the development of higher-level concepts. The present study was initiated with considerations for studying these issues. It is a beginning work of such studies.

Research Questions

This study is designed to answer questions which would help provide insight into the nature of students' prior understanding of concepts which are at the lower level of the conceptual hierarchy. The study then compares students' prior understanding between classes that require different levels of mathematics preparation. Further, the study sheds light on the nature of the relationship between students' understanding of fundamental and less abstract concepts and their ability to conceptualize subsequent and more abstract concepts.

The study uses two concepts commonly used in introductory college physics, area and volume, to investigate the nature of student's prior understanding. The reasons for choosing area and volume are threefold. The first reason is that there has been reasonable doubt among college science instructors about students' conceptual mastery of these concepts (Arons, 1990). Yet, in this author's review of literature, no research studies were located which investigated college students' prior understanding of these concepts. The second reason is that these concepts have everyday, imprecise meanings, which are not consistent with scientific uses of the concept. These everyday understandings, thus, may hinder the students' progress in learning other science concepts. The third reason of choosing these concepts is that they are foundations for many higher level science concepts (e.g., pressure, energy flux, density). Thus, deficiencies in understanding these concepts may relate to subsequent learning difficulties for students.

The study then further investigates two of the subsequent concepts, pressure, defined as force per unit area, and density, defined as mass per unit volume. These concepts are both developed in introductory college physics. The reasons to choose the concepts of pressure and density as examples of more abstract concepts in this study are threefold. The first reason is that these concepts use the concepts of area and volume. The second reason is that these concepts are at a higher level of abstraction than area and volume. The third reason is that these concepts are introduced at the beginning of most introductory college physics courses.

The overall goals of the study are to collect data which (a) would help the researcher to understand the nature of students' understanding of the concepts of area and volume and (b) help provide an initial insight into the relationship between the students' concepts of pressure and density and their understanding of the concepts of area and volume.

Based on the above goals, four research questions were asked. They are:

1. What are college science students' understandings of the concepts of area and volume?

2. What characterizes the students' difficulties with these concepts?

3. Do students in mathematically more-advanced courses differ in their initial understanding of the area and volume concepts from those in mathematically less-sophisticated courses?

4. Is there any relationship between students' ability to conceptualize pressure and density and their understanding of area and volume?

Importance of the Study

The significance of this study are both practical and theoretical. The practical significance of the study is that in recent years concerns about students' mastery of many prerequisite concepts, including area and volume, have been expressed and ideas that these concepts should be addressed in introductory college science courses have been suggested (for example, see Arons, 1990). However, these concerns and suggestions are based upon individual experience but not upon research. Therefore, the primary goal of investigating the nature of science students' understanding of the area and volume concepts is to provide research evidence which would allow instructors to decide whether or not these concepts need to be addressed during their instruction. In addition, introductory college science courses designed for students taking different curricula often have different prerequisites, e.g., algebra or calculus. Instructors may assume that students enrolled in courses with more-mathematical prerequisites will have better understanding of basic concepts than those enrolled in courses with less prerequisites. This is an assumption and is not based upon research. Therefore, a comparison of the level of students' prior conceptual understanding of area and volume in science courses with different prerequisites can provide useful information to instructors.

The theoretical significance of this study is that science educators have agreed that in order to achieve effective and efficient science learning and teaching, researchers must try to understand not only the initial state of students' knowledge but also the influence of this prior knowledge on the development of scientific concepts (Mestre & Touger, 1989). For the purpose of providing useful diagnostic-prescriptive models to foster the effective learning and teaching of science concepts, researchers should first provide information on how understanding of less abstract concepts affects the development of more abstract concepts. Logically, the first step of providing such information is to gain insight into the nature of students' thinking about these related concepts. Thus, the second goal of this study is to begin to explore the nature of the relationship between students' understanding of area and volume and their conceptualization of subsequent physics concepts.

CHAPTER II

THEORETICAL FRAMEWORK

Introduction

The primary task of this chapter is to develop a theoretical framework which can be used to characterize a student's conceptual understanding. The first section of the chapter will discuss what a concept is, what it means to understand a concept, how people demonstrate their understanding of a concept, and the method of evaluating individuals' conceptual understanding. In the second section, a theoretical framework for human concept representation and a model developed from this framework for interpreting individuals' conceptual understanding will be introduced. An empirical justification for the use of this model will also be provided. Based upon this theoretical model for concept interpretation, the last section of the chapter will present a working framework to characterize a person's understanding of the area, volume, pressure, and density, concepts which are under investigation in this study.

Conceptual Understanding and Evaluation

Concepts and Conceptual Understanding

Webster's (1989) dictionary defines a concept as a mental image of a thing formed by a generalization from particulars, or an idea of what a thing in general is to be. In cognitive science, this mental image or idea formed by a generalization from natural phenomena is described as knowledge representation (Shoben, 1988). Thus, a concept is a representation of a generalized idea of a class of objects, chain of events, or types of natural process. For example, "apple" is a concept for a class of objects -- a group of fruits with delicate flesh, smooth skin, and sweet taste. "Science" as a concept is referring to a chain of events describes the process and results of studying nature. And, "velocity" as a concept is specifying a physical process defines the rate of change of an object's position with respect to time.

A concept's meaning includes two main constituents: (1) the natural phenomena it represents (that is the nature of the objects, events, or processes the concept describes); and (2) how it fits into a person's mental structure (or what it pictures in a person's mind). Here, a mental structure is defined by Piaget (1970) as a system of transformation by which a person interrelates, transforms, and interprets information. It is, thus, an internally organized whole which transcends or goes beyond mere factual knowledge.

Box 1 in Figure 1 illustrates concept representation. On the left side is the "natural phenomenon," which is a realistic position that natural phenomenon is independent of human existence (note: this is the author's philosophical perspective of "natural phenomenon" used in this study). When a natural phenomenon is identified and a concept describing it is defined or constructed by consensus within a specific human community, a standard description of that event is provided. Since this

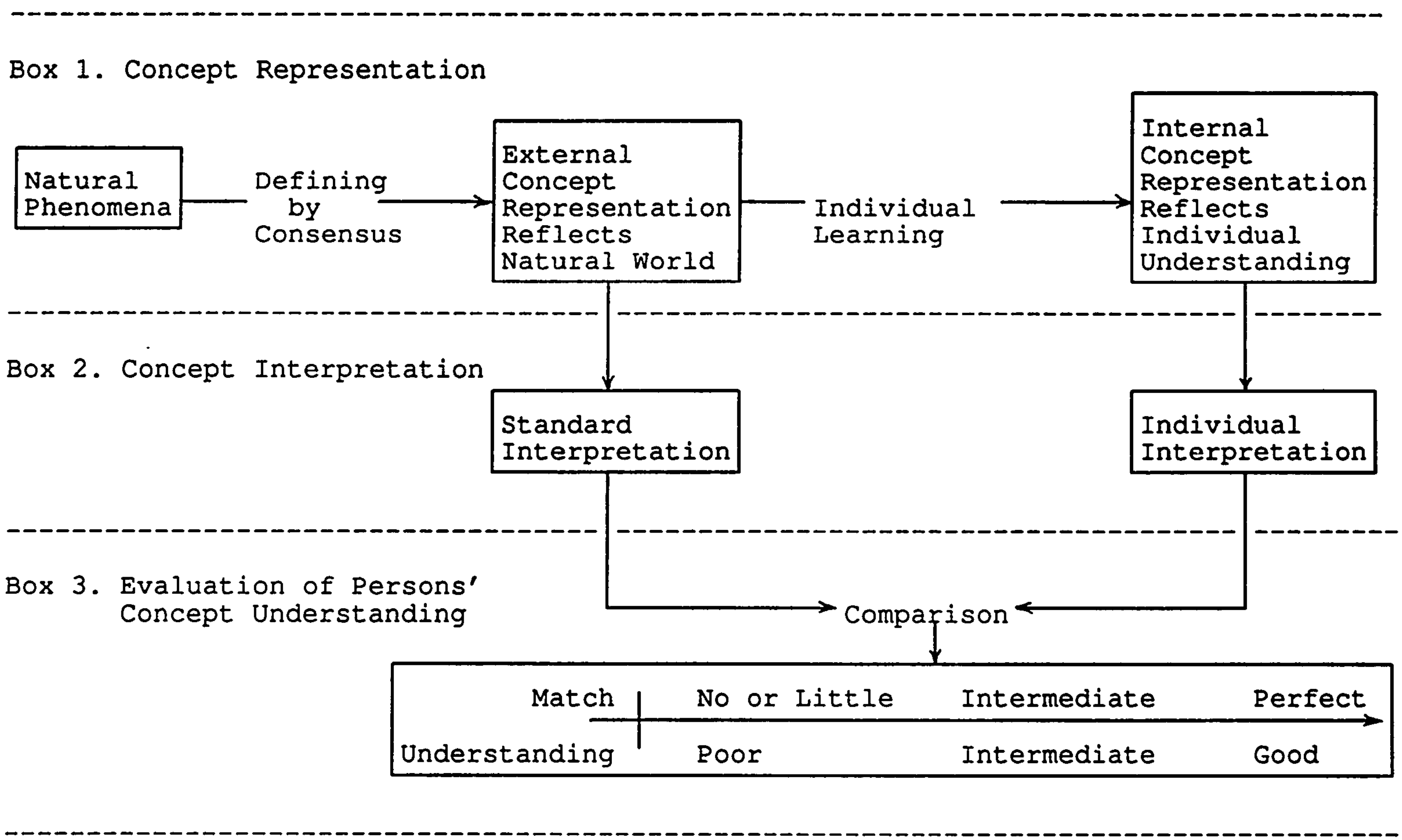


Figure 1. Concept Representation, Interpretation, and Evaluation of Individuals' Understanding.

standard description is established by a community, it is then independent of and external to particular learners. In Box 1 of Figure 1, this standard description is depicted as "external concept representation."

External concept representation is the first part of a concept's meaning -- the natural phenomena it represents. This representation is what is typically given in dictionaries, stated in textbooks, and frequently used as a standard explanation for natural phenomena by experts.

As a learner acquires the external representation of a concept, his or her mental image of that concept is influenced by the learner's prior experience and perceptions of the world (Driver & Bell, 1986; Mestre & Touger, 1989; Resnick, 1983). The concept is internalized by the learner when it is processed using his or her existing mental structure (Piaget, 1977) and the meaning of the concept is now assimilated into the knowledge base of the person. Thus, a concept's meaning to a person is a joint-product of the concept's external representation and the person's internal mental structure. It may no longer be necessarily identical to the concept's external meaning. On the right side of Box 1 of Figure 1, this mental image of a concept in a particular person is depicted as "internal concept representation."

Internal concept representation is the second constituent of a concept's meaning -- a person's mental image of what the concept represents. Because this internal representation is characterized by a particular person's mental structure and represents an image of a concept based upon his or her internal knowledge base, it is now what actively functions within the person. In other words, this can be referred to as the person's conceptual understanding.

Evaluation of Conceptual Understanding

Obviously, a person's conceptual understanding, or the internal mental representation of concepts can only be discerned through his or her description and use of the concepts. Usually, this involves using symbols, drawings, and language, or by examining a person's application of concepts to everyday or academic tasks. This demonstration of concept understanding is referred to by Reif (1987) as a person's concept interpretation. The right side of Box 2 in Figure 1 depicts concept interpretation by individual persons, while the left side shows the standard interpretation which should match the accepted description of a concept.

Thus, a person's understanding of a concept is demonstrated primarily by his or her interpretation of the concept. Therefore, one way to determine a person's understanding is possible by a comparison of his or her interpretation with the commonly accepted, or standard interpretation which experts feel best reflect the external representation of the concept. Box 3 of Figure 1 shows that a perfect match between a person's interpretation and the standard interpretation indicates a good understanding, while a poor or no match evidences a poor understanding. Obviously, intermediate matches will indicate varying degrees of understanding.

Concept Representation and Concept Interpretation

To conduct a comparison between a person's interpretation of a concept with a standard one, the constituents of an interpretation must be first identified and defined. In other words, what an interpretation consists of must be elucidated. This section will illustrate such components of such an interpretation.

A Framework of Human Concept Representation

Smith & Medin (1981) in their summary of research on human concept representation state that there are three prominent forms of concept representation used to describe a concept. They are:

General Definition

The general definition of a concept is an abstraction from a set of features which are individually necessary and jointly sufficient for describing the concept. For example, "motion," in physics, is generally defined as the movement of an object which changes its position in space with respect to a point of reference. A general definition is normally (a) precise and concise, (b) explicit in identifying the central features, and (c) generalizable to all its instances. However, a general definition may be difficult for people to understand, simply because it is too compact to translate all its meaning to specific instances.

Specific Examples

Specific examples of a concept are special cases of the concept categorized by featural correlations and recognizable concept instances. For example, linear movement of a particle is a special case of motion. It describes the motion of a particle moving in one dimension. Circular motion is another special case of motion. It describes the motion of a particle moving in a path with a constant radius and constantly changing direction. Specific examples are usually closely related to real situations of a concept and, thus, easy for people to understand. However, because each example can only be used for the special case, it is constrained to a special instance of a concept and has limited generalizability. Consequently, although a set of specific examples can implicitly represent the central features of a concept, it may not represent the entire range of the concept due to the existence of various non-categorizable instances.

Associated Features

Associated features of a concept describe the concept by a set of features which are related to and have been frequently encountered in various instances of the concept. For example, "motion" is frequently associated with a moving body's mass, speed, change in speed, change in direction, and a set of external forces exerted on the body by other parties. This form of concept representation identifies the important features of a concept by their frequency of occurrence. The representation is thus imprecise and nonformal, and while factual, may be inadequate and non-cohesive to the central features of a concept.

Scientific Concept Interpretation

Corresponding to the forms of concept representation described above -- general definition, specific examples, and associated features, Reif (1987) and Reif and Allen (1992) have developed a parallel frame for describing human interpretation of scientific concepts. According to them, the following three modes of interpretation are most essential in describing a person's understanding of scientific concepts.

Formal Definition

This is similar to concept representation by general definition in that a scientific concept can be interpreted by a formal definition. The formal definition of a concept is an abstraction explicitly specifying the meaning attributed to the name of the concept. It can often, but not necessarily, be expressed as a mathematical statement. For example, the concept of acceleration in physics is defined as the rate of change of a particle's velocity with respect to time. It is mathematically stated as $a = dv/dt$ (a is the vector acceleration and dv/dt is the derivative of the vector velocity with respect to time t).

A formal definition usually consists of (a) a conceptual declaration of a concept it describes, and (b) the operational meaning of this concept. Here, the conceptual declaration interprets what the concept describes by indicating an explicit set of its characterizing features. The operational meaning of the concept, on the other hand, interprets how the concept is described by associating its characterizing features with a mathematical expression. Therefore, a formal definition not only specifies what natural aspect or phenomenon it refers to, but also details how this natural aspect or phenomenon is described. For example, in acceleration, the rate of change in velocity with respect to time conceptually declares what acceleration describes -- how fast a moving body changes its speed and direction. At the same time, it also indicates how, or by what operation, the changes in speed and direction are described -- they are measured in unit-intervals of time.

Classified Standard Cases

Comparable to concept representation by specific examples, a scientific concept can also be interpreted by using classified standard cases. Classified standard cases are derivations from a concept's formal definition, which pertain to special cases identified by criteria of featural resemblances and associated constraints. For example, "linear motion," "circular motion," "constant-speed motion," and "accelerated motion" are all special cases of the general concept of motion.

Classified standard cases normally have expressions in mathematics. Thus, their operational meanings and procedures are usually explicit and, therefore, they often provide direct and easier starting points for interpreting a concept. However, because the essential features of the concept are implicit, the general definition of the concept may not be obvious. Because each standard case's applicability depends on a specific instance, a concept may be misinterpreted by matching a standard case with an incorrect specific instance.

Associated Features

Scientific concepts can also be interpreted by associated features. Associated features of a concept are those that relate to the concept, even if they are not directly derivable from the concept's formal definition. For example, "force" and "mass" are associated features for the concept of acceleration. They are not derivable from the definition of acceleration, nevertheless, they are related to the concept (Newton's second law -- the acceleration of a particle is proportional to the net external force exerted on it and inversely proportional to its mass).

Associated features describe a concept by matching a specific situation of the concept with other available knowledge fragments (Reif, 1987), but in a rather nonformal fashion. The process involves more automatic recognition than explicit invocation of a definition of the concept. It is, thus, usually a fast and effortless way of providing indirect descriptions for the concept. This form of concept interpretation has three major flaws: (1) Because these knowledge fragments are casual and non-cohesive, they do not necessarily provide a good and adequate base for depicting the concept's general definition; (2) Because they may reflect common-sense knowledge about the concept, they may not distinguish it from other similar concepts; and (3) They are often not accompanied by well-specified conditions where they do or do not apply.

A Model of Scientific Concept Interpretation

The above description of scientific concept interpretation suggests a model for interpreting a person's scientific conceptual understanding. The model was developed by Reif and Allen (1992), where they propose two main components: (1) "main interpretation knowledge," which is the knowledge required by formal definition, classified standard cases, and associated features; (2) "conditional knowledge," which is the knowledge of the applicability conditions and application methods associated with each category of the main interpretation knowledge. Figure 2 depicts this model.

An important requirement for a good scientific interpretation proposed by this model is the ability to use more than one form of the main interpretation knowledge in a complementary fashion, whenever such a use is needed to facilitate an effective and efficient interpretation. The rationale is that for a specific concept, a full representation must be capable of describing all its instances. In realistic situations, some instances may be easily and fully interpreted by using one form of the main interpretation knowledge (formal definition, classified standard cases, and associated features) alone. However, there are other instances where only one form of the main interpretation knowledge may not be sufficient and effective for interpreting the concept, and thus, two or more forms of the main interpretation knowledge are required. For example, the concept of acceleration is generally defined as a particle's rate of change in velocity with respect to time, including the rate of change in direction as well as the rate of change in speed. For a special case of constant-speed

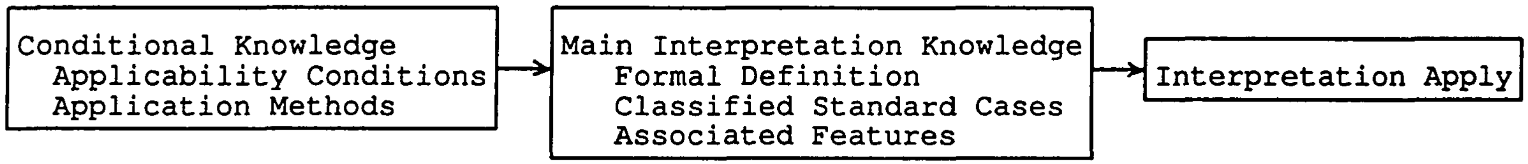


Figure 2. Model for Scientific Concept Interpretation.

circular motion, the acceleration is simplified as the rate of change in direction only. Conveniently, a derivation for this classified standard case readily yields the relationship that the rate of change in direction is equal to the moving body's speed-squared divided by the radius of curvature of its path, and is directed towards the center of curvature. This example shows the ease and explicitness of the interpretation by using the form of classified standard cases alone. However, when a circular motion has a non-constant speed, there is no comparable classified standard case. To facilitate a sufficient and effective interpretation, then, the general definition and an associated feature, acceleration's vector property, must be used to derive the following rule. First, according to the vector property of acceleration, an associated feature of the concept, a particle's acceleration can be described by splitting it to its tangential and radial components. Here, the tangential component describes the particle's change in speed, and the radial component the change in direction. Second, by using the formal definition of acceleration, the tangential component or the particle's change in speed is explained as the change in speed divided by time. The radial component or the rate of change in direction of travel, according to the classified standard case of circular motion, is still the moving body's speed-squared divided by the radius of curvature of its path. Third, by the associated feature of the vector property, the overall acceleration vector is computed as the sum of its component vector accelerations. Thus, in this case, the formal definition, an associated feature, and a classified standard case of acceleration are used jointly for a complete interpretation of the acceleration concept.

Empirical Justification for the Use of the Model

Reif & Allen (1992) used this model to examine both expert scientists' and novice students' interpretations of the acceleration concept. Their results, which are presented below, provide evidence for the model's ability to categorize conceptual understanding through concept interpretation.

Expert Scientists' Interpretation

The study shows that for individual expert scientists:

1. All of the categories of main interpretation knowledge for acceleration are used jointly in a complementary fashion to achieve a complete and effective interpretation.

2. Each category of the main interpretation knowledge used is carefully applied with appropriate applicability conditions and application methods.

3. Their use of knowledge shows a high degree of coherence. Their use of words and terms attributed to this knowledge is also precise and accurate.

Such an interpretation, therefore, demonstrates how an expert scientist typically understands the acceleration concept.

Novice Students' Interpretation

In contrast, for novice students, Reif and Allen (1992) report that:

1. All the categories of main interpretation knowledge for acceleration are also

used. However, these forms of interpretation knowledge are usually applied disjointly. In other words, individual students do not tend to connect different categories of main knowledge together in their interpretation.

2. The students often associate inappropriate applicability conditions and/or application methods with the main interpretation knowledge that they use. This deficiency in associating relevant conditional knowledge with main interpretation knowledge leads to many misinterpretations of the concept.

3. The students' lack of coherence in their knowledge of the concept is flawed because of deficient applicability conditions and improper application methods, which contribute to their incorrect interpretation. Also, the students' use of words and terms attributed to their knowledge are often imprecise, ambiguous, and inconsistent.

Thus, novice students' interpretations reflect lesser degrees of conceptual understanding of acceleration when compared to experts in physics.

Framework for Characterizing Students' Understanding of the Concepts Under Investigation

The concepts targeted for investigation in the present study are area, volume, pressure, and density. Using the model of scientific concept interpretation described in the previous section, a person's conceptual understanding can be characterized by the form(s) of interpretation knowledge he or she uses to describe and explain these concepts. Interpretation knowledge includes both main interpretation knowledge and the associated conditional knowledge, which can be further divided into applicability conditions and application methods (see Figure 2).

Students' Interpretation Versus Standard Interpretation

The method used in this study to characterize a person's conceptual understanding is to compare a student's interpretation of the concept with commonly accepted or standard interpretation that a community of experts uses to describe the concept (see Boxes 2 and 3 in Figure 1). For area, volume, pressure, and density, formal definitions can be found in most introductory physical science and mathematics textbooks (e.g., Metcalfe, Williams, & Castka, 1982; Tillery, 1992; Lehrman & Swartz, 1969). These standard interpretations in various forms are also well established in the literature and generally agreed upon by expert scientists and mathematicians (Arons, 1990; Shroyer & Fitzgerald, 1986). Standard interpretations are compared to students' interpretations, and thus allow one to characterize student conceptual understanding in these areas.

Standard Interpretation Knowledge of the Target Concepts

Table 1 lists the standard knowledge used to interpret the target concepts. The first column lists the common labels of the concepts. The second column states the main interpretation knowledge for each concept. This knowledge is in the form of a formal definition, exemplars of certain classified standard cases, and important associated features. The third and fourth columns give the necessary conditional knowledge associated with each form of the main interpretation knowledge.

Table 1

Knowledge for Interpreting the Concepts of Area and Volume

Concept	Main Interpretation Knowledge	Conditional Knowledge	
		Applicability Conditions	Application Methods
Area	Formal Definition		
	Measurement of surface extent in square units	Any closed surface or 2-dimensional space	Count unit-squares by using a grid, or by formulas as short-cuts
	Classified Standard Cases		
	Length x Width	Any rectangular surface	Multiply values of length and width
	and/or		
	Other formulas	Other Regular shapes with measurable dimensions	Compute area according to appropriate mathematical formulas
	Associated Features		
	Area has additive property	Any surface	Manipulate the area of a figure by adding/subtracting sub-units of area
	and		
	Area is conserved when a surface is rearranged into other shapes	Any surface	

Table 1 -- Continued

Concept	Main Interpretation Knowledge	Conditional Knowledge	
		Applicability Conditions	Application Methods
Volume	Formal Definition Measurement of space extent in cubic units	Any closed 3-dimensional space	Count unit-cubes by using formulas as short-cuts
	Classified Standard Cases Length x Width x Height	Any rectangular space	Multiply values of length, width, and height
	and/or Other formulas	Other shapes with measurable dimensions	Compute volume according to appropriate mathematical formulas
	Associated Features Volume has additive property	Any space	Manipulate the volume of space by adding/subtracting sub-units of volumes
	and Volume is conserved if a space is rearranged into other shape	Any space	

Table 1 -- Continued

Concept	Main Interpretation Knowledge	Conditional Knowledge	
		Applicability Conditions	Application Methods
Pressure	Formal Definition Strength of a force within a given surface (or force each unit-square of the surface bears)	Any force over any surface	Find the perpendicular component of the exerting force, then divide it by the area.
	Classified Standard Cases F/Area, where F is perp-endicular to the surface	Perpendicular force only	
	Associated Features Pressure is a force exerted on a defined 2D surface, instead of a point.	Any force and associated surface	
Density	Formal Definition Amount of matter which occupies an unit cubic-space (thus, it describes how closely matter is arranged in mass in space.)	Any object of uniform matter	Divide the mass of the object by the volume space it occupies

Table 1 -- Continued

Concept	Main Interpretation Knowledge	Conditional Knowledge	
		Applicability Conditions	Application Methods
Density	Associated Features Density is mass within a defined 3D space	Any object	

Specifically, the third column indicates the applicability conditions, which specify when the interpretation knowledge should be applied, and the fourth column identifies appropriate application methods, which detail how the knowledge should be applied.

Categories of Conceptual Understanding

According to the model, conceptual understanding can be differentiated and characterized by (a) the form(s) of main interpretation knowledge an individual is able to use, with appropriate conditional knowledge, for describing a concept; and (b) the ability to use more than one form of the main interpretation knowledge in a complementary fashion, whenever such a use is needed to facilitate an interpretation. The reasons for such a differentiation of conceptual understanding are of both theoretical and empirical.

Theoretically speaking, an individual who has not formally studied science can also develop non-formal ideas about scientific concepts (Driver & Bell, 1986; Driver & Easley, 1978; Helm & Novak, 1983; Resnick & Chi, 1988). In other words, they may be able to describe or interpret concepts by using some of the associated features, but not by classified standard cases or formal definition. For example, individuals who have not had formal training in physics normally can relate "acceleration" to motion and speed in a rather casual, imprecise, and non-formal way (Dykstra, 1991; McDermott 1984), indicating their knowledge of some associated features of acceleration. However, this knowledge is not scientifically organized or defined. Thus, interpreting scientific concepts by using associated features reveals a preliminary understanding. When students receive formal science or mathematics instruction, a formal definition, specific classified cases, and important associated features are usually studied. In the beginning, a formal definition is usually less meaningful than the associated features and the relationships expressed for classified standard cases (Bareiss, 1989). This is because a formal definition normally refers to general situations, thus, it is more abstract and less comprehensible than specially derived expressions for particular situations (Arons, 1990), and is often constructed after learning of a variety of tangible particulars (Rutherford, 1993). For example, in acceleration, students normally can make sense of the concept and its computation methods in classified cases (e.g., constant acceleration motion) before they can articulate a more general notion (e.g., non-constant accelerations). Therefore, students who have had some formal training are normally first capable of interpreting a concept by using classified standard cases in addition to associated features, but are not necessarily capable of applying the formal definition to specific cases in a meaningful way. In other words, interpreting scientific concepts by using classified standard cases and associated features demonstrates an emerging or developing understanding. Typically, as formal training advances, students will be expected to be able to articulate the concept by using the formal definition, associated features, and the classified standard cases (as short-cuts for derivations of the formal definition) whenever they are needed. Thus, the ability to demonstrate the use of all three forms of main interpretation knowledge is evidence of a higher hierarchy of understanding that can be labeled as good conceptual understanding.

On the other hand, empirically, Reif (1989) has shown that a good conceptual understanding is demonstrated by an interpretation in which all forms of interpretation knowledge are used in complementary ways and lower levels of understanding is evidenced by the lack of ability to apply all the forms of interpretation knowledge. According to Reif, formal definition is usually not used by individuals at an intermediate level, while a yet lower level of understanding is indicated by only using associated features of the concept.

According to the forms of main interpretation knowledge and appropriate conditional knowledge an individual is able to use to interpret a concept, this study describes students' conceptual understanding by four categories. Figure 3 further defines and illustrates these four categories of understanding. The lowest hierarchy of conceptual understanding is labeled "preliminary understanding." It is defined as a category where a student is only able to describe a concept using its associated features. The next hierarchy of understanding is labeled "emerging understanding." In this intermediate category, a student defines a concept using its classified standard cases. (S)he is not able to connect this definition with either a formal definition or any important associated features of the concept. The next intermediate category is labeled "developing understanding." At this stage, a student is able to interpret a concept using classified standard cases and associated features, with appropriate conditional knowledge. Yet, uses of formal definition are not apparent. The last category, labeled "good conceptual understanding," is the highest hierarchy of understanding where a student is able to coherently apply all three forms of main interpretation knowledge,

Category of Understanding	Preliminary Understanding	Emerging Understanding	Developing Understanding	Good Conceptual Understanding
Forms of Main Interpretation Knowledge Demonstrated	Associated Features	Classified Standard Cases	Associated Features + Classified Standard Cases	Associated Features + Classified Standard Cases + Formal Definition

Note: Main interpretation knowledge must be used with appropriate conditional knowledge.

Figure 3. Categories of Conceptual Understanding.

or formal definition, classified standard cases, and associated features, with correct conditional knowledge.

CHAPTER III

STUDY DESIGN AND METHODOLOGY

Summary of Research Questions and Theoretical Framework

The Research Questions

The purpose of this study is (a) to describe the nature of students' prior understanding of the concepts of area and volume, and (b) to begin to explore the nature of the relationship between the understanding of these concepts and the subsequent physics concepts of pressure and density. Four specific research questions are addressed:

1. What are college science students' understandings of the concepts of area and volume?

2. What characterizes students' difficulties with these concepts?

3. Do students in mathematically more-advanced courses differ in their initial understanding of the area and volume concepts from those in mathematically less-sophisticated courses?

4. Is there any relationship between students' ability to conceptualize pressure and density and their understanding of area and volume?

The Theoretical Framework

A theoretical model for characterizing a student's conceptual understandinghas been developed by Reif (1987) and Reif and Allen (1992) based upon work in knowledge representation from the cognitive sciences. In this model, a person's understanding of scientific concepts is identified as the person's ability to correctly and coherently use one or more of the three essential modes of concept interpretation, namely, formal definition, classified standard cases, and associated features. The formal definition of a concept is a verbal statement or a mathematical expression which explicitly specifies the meaning attributed to the concept. Thus, it is generalizable to all instances of the concept. Classified standard cases are derivations from a concept's formal definition, which pertain to special cases identified by criteria or sets of constraints. Associated features of a concept are those that relate to the concept, but they may not be directly derivable from the concept's formal definition. In other words, associated features are knowledge fragments, and are often not accompanied by well-specified conditions that can be used to interpret the concept. Reif and Allen (1992) labeled knowledge required by formal definition, classified standard cases, and associated features as "main interpretation knowledge." They also maintain that the correct use of main interpretation knowledge requires appropriate "conditional knowledge," which specifies where (applicability conditions) and how (application methods) the main interpretation knowledge may be applied.

Using this model for interpreting individuals' scientific concepts, a person's conceptual understanding of area, volume, pressure, and density, in the present study, can be characterized by the form(s) of main interpretation knowledge a student uses to describe the concepts. Thus, this study developed a working framework for evaluating a student's conceptual understanding by identifying the form(s) of main interpretation knowledge and associated conditional knowledge a student employs to describe a concept. Based on the form(s) of main interpretation knowledge used, four specific categories of conceptual understanding were defined (see Figure 3 on page 31). These four specific categories are: (1) preliminary understanding, where only associated features are used; (2) emerging understanding, where classified standard cases are solely used with appropriate conditional knowledge; (3) developing understanding, where classified standard cases and associated features are used correctly with appropriate conditional knowledge; and (4) good conceptual understanding, where all three forms of main interpretation knowledge (formal definition, classified standard cases, and associated features) are correctly and consistently used with appropriate conditional knowledge.

Study Design

Overview

This study used first-semester introductory physics students as subjects. These subjects were asked to participate in one or both of two data collection procedures. The first was to complete a paper-pencil instrument and the second was to take part in clinical interviews. The paper-pencil instrument was administered to all subjects of the study, while the clinical interviews were only conducted with selected students. Because there were no suitable instruments or prior interview schedules available, both the paper-pencil instrument and the clinical interviews were specifically designed to collect the desired data. More details about the students in the study, data collection techniques, and the design of the paper-pencil instrument and the interviews are described in the following sub-sections.

The Sample

The subject sample for this study consisted of students enrolled in all first-semester introductory physics courses offered at Western Michigan University at Kalamazoo, Michigan, in the Fall semester of 1992. The participants were selected because it was felt that they were likely to represent a typical group of university students who take introductory physics. The introductory physics courses offered at Western Michigan University are Physics 107 (Elementary Physics) which is a one-semester survey course for students who are not majoring in science, Physics 113 (General Physics I) which is the first semester of a two-semester algebra-based course for students who are primarily majoring in sciences other than physics or engineering, and Physics 205 (Mechanics and Heat) which is the first semester of a two-semester calculus-based course for physics and engineering students. Western Michigan University is an emerging research university evolving from its role as a teacher preparation college to an educational institution with an extensive and wide-ranging undergraduate and graduate programs. Thus, it seems reasonable to assume that these physics students are typical of most introductory physics students throughout the United States.

Data Collection Techniques

Two data collection techniques were used in this study. First, a paper-pencil instrument was used to elicit from students their main interpretation knowledge and conditional knowledge by asking them to describe and interpret the concepts of area and volume. Second, two clinical interviews were used to elicit information about students' thinking of the area and volume concepts and their relationship to the development of pressure and density concepts. Specifically, the first interview was used to obtain additional insight into a student's interpretation of the area and volume concepts, probing further both the forms of main interpretation knowledge they are able to apply and the coherence of their knowledge when they are asked to apply their knowledge in various situations. This interview was conducted immediately after the students completed the paper-pencil instrument. The second interview was conducted immediately after the students had studied the concepts of pressure and density in their physics classes. This interview was used to elicit students' main interpretation knowledge of the pressure and density concepts, with appropriate conditional knowledge, and hopefully to gain some insights into how these concepts are related to the students' prior understanding of the area and volume concepts. The particular emphasis in the second interview was to obtain information that would provide a framework for understanding how a student's concepts of area and volume influence the development of the pressure and density concepts.

Design of the Paper-Pencil Instrument

A review of the literature and available assessment instruments did not indicate any suitable instruments that would be useful in this study. Therefore, a paper-pencil inventory was developed by the author to elicit students' prior understanding of area and volume. This inventory was titled Knowledge of Area and Volume Inventory (see Appendix A).

The Inventory Structure

The inventory structure was designed after consulting the research of Hestenes, Wells, and Swackhamer's (1992) and Hestenes and Wells's (1992) about the development of paper-pencil instruments for assessing students' conceptual understanding in introductory physics. Hestenes, Wells, and Swackhamer developed and validated an instrument to probe and assess the commonsense beliefs of students on Newtonian mechanics concepts prior to any formal instruction. Hestenes and Wells developed and validated a second instrument for assessing students' understanding of basic mechanics concepts after instruction. Based upon an item analysis and examination on the overall structures of the instruments, both of these studies provided useful ideas and recommendations for developing similar assessment instruments for use with introductory physics students. In particular, they used a scaffolded structure, meaning that items are arranged in a sequence of progressive difficulty for each concept. This technique was considered especially useful for eliciting information from students about their knowledge and conceptual understanding. Thus, this technique was very useful in this study to help determine a student's forms of main interpretation knowledge and associated conditional knowledge about area and volume. Therefore, this scaffolded structure was employed in designing the inventory instrument.

The inventory is structured in the following way: the first half of the inventory covers the area concept, while the second half elicits students thinking about the volume concept. For each of these two concepts, items are arranged in a sequence which requires the students to use different form(s) of main interpretation knowledge and appropriate conditional knowledge required at each category of understanding. Specifically, students are first asked to provide verbal statements describing their ideas about the concepts. This is followed by items that require them to either assign numerical values or explain their thinking about problems involving specific areas or volumes of various shapes. These shapes range from simple and regular two-dimensional and three-dimensional geometric figures to more complicated and irregularly-shaped objects. The questions are presented in an array of progressive complexity and, thus, elicit from students different forms of main interpretation knowledge and conditional knowledge. Each item requires students to provide explanations as well as specific answers.

The Inventory Items

During the development of the inventory, two documents were especially helpful to construct inventory items. They were the measurement units from Michigan State University's Middle Grades Mathematics Project (Shroyer & Fitzgerald, 1986) and the Second National Assessment in Mathematics -- Area and Volume Portion (Hirstein, 1981). The Middle Grades Mathematics Project was developed as a special series of innovative textual materials for teaching elementary and secondary students mathematics. It was designed to help students develop a deeper and more meaningful conceptual understanding of basic mathematics ideas. In particular, the measurement units used visual and graphical modules to help students progress in developing a more meaningful understanding of the concepts of area, volume, and mass. In addition, it contains many review problems, practice exercises, and unit tests, reflecting different levels of understanding of these concepts. The Second National Assessment in Mathematics -- Area and Volume Portion analyzed data collected from students' scores on a national mathematical assessment test. This assessment test was administered to middle-school and high-school students to evaluate their mathematics performance in area and volume concepts. The Second National Assessment in Mathematics -- Area and Volume Portion also surveyed students' understanding and their misconceptions related to area and volume, and discussed methods for evaluating students' conceptual understanding of these concepts. In constructing the inventory items for this study, items were first selected from the measurement units of the

Middle Grades Mathematics Project and then revised or rewritten based on the needs of this study and the recommendations from these documents.

During development, the Knowledge of Area and Volume Inventory was also reviewed and evaluated by the members of the author's dissertation committee, a professor of physics, and a number of graduate teaching assistants who taught introductory physics at Western Michigan University. Their critical comments were also used to polish the wording, phrasing, and drawings for items in the inventory.

After this initial development phase, a test administration was conducted with 17 students from one section of the algebra-based introductory physics offered at Western Michigan University in the Summer 1992. The purpose of this test administration was to determine its ability to be understood by students and to elicit the type of information required in the study. This resulted in another revision of the inventory where three items were removed to reduce the time needed to complete the inventory and six items were changed from a multiple-choice to a free-response format.

Table 2 summarizes the Knowledge of Area and Volume Inventory (see Appendix A) items, together with the standard interpretation knowledge (see Table 1 in Chapter II) required to answer each item. There are a total of 15 items in the inventory, three of which have sub-items. The first eight items cover the area concept while the remaining seven items relate to the volume concept. Specifically, Item 1 asks students to define or describe area in their own words. Item 2 requires students to assign numerical values to areas of six different figures and to explain the methods

Table 2

List of Inventory Items

Description of Item	Item Number	Main Interpretation Knowledge	Conditional Knowledge	
			Applicability Conditions	Application Methods
Define/describe area	1	The measurement of extent of a 2-D space in square-units	All closed 2-D spaces	
Assign area values to:				
Simple rectangles	2-i, 2-ii	Length X Width	Rectangular surfaces	Measure the length and width, then multiply.
Figures of joint rectangles of different sizes	2-iii, 2-iv	L X W for rectangle and Area's additive property	Surfaces formed by sub-units of rectangles	Adding or subtracting areas of sub-units found by using formulae
Figures of joint regular-shapes with measurable edges	2-v	Math formulas for rectangles and triangles and Area's additive property Or	Surfaces formed by rectangular or triangular sub-units	Adding or subtracting areas of sub-units calculated by using formulae

Table 2 -- Continued

Description of Item	Item Number	Main Interpretation Knowledge	Conditional Knowledge	
			Applicability Conditions	Application Methods
Assign area values to:				
Figures of joint regular-shapes with measurable edges	2-v	Counting of unit-squares enclosed in the figure	Any closed 2-D figure	Use grid to find the equivalent number of squares within the border of the figure
Irregularly shaped figures	2-vi	Find the number of unit-squares within the figure	Any closed 2-D figure	Use grid to count the equivalent number of squares enclosed within the figure
Compare two figures to detect wrong ideas that:				
One dimensional length equals to area	3-i	Area is a 2-D measurement with square-units	Any 2-D surface	

Table 2 -- Continued

Description of Item	Item Number	Main Interpretation Knowledge	Conditional Knowledge	
			Applicability Conditions	Application Methods
Compare two figures to detect wrong ideas that:				
Perimeter equals to area	3-ii	Area is the number of unit-squares enclosed within the perimeter	Any closed surface	
Area is dependent of the shape of a figure	3-iii, 3-iv	Area's additive property makes a figure rearrangeable in shape	Any 2-D space	
Compute the amount of money needed to carpet a rectangular room-floor of given dimensions	4	Area can be find by formula L X W, or, by counting the unit squares	Rectangular floors	Multiply the length with the width of the floor
			Any floor	Count the unit-squares in the floor

Table 2 -- Continued

Description of Item	Item Number	Main Interpretation Knowledge	Conditional Knowledge	
			Applicability Conditions	Application Methods
Cut a large square into smaller ones and find the number of smaller squares	5	Area measures number of unit-squares a figure encloses; Cutting down a large square does not change the total area; Total Area = (Length of Side)^2	Any square	Method 1: Find area of the big square; find area of the smaller square; divide the first area by the second area. Method 2: Divide the large square's edge by that of the smaller square's, square the result.
Define/describe volume	9	The measurement of extent of a 3-D space in cubic-units	All closed 3-D spaces	

Table 2 -- Continued

Description of Item	Item Number	Main Interpretation Knowledge	Conditional Knowledge	
			Applicability Conditions	Application Methods
Assign volume values to:				
simple rectangular box/block	10-i	L X W X H	Rectangular boxes	Measure the length and width and height, then multiply them together.
Object of joint rectangular blocks of different sizes	10-ii	L X W X H for blocks and volume's additive property	Objects formed by sub-units of rectangular blocks	Adding or subtracting volumes of sub-units found by using formulas
block of unit-cubes	11	L X W X H	Rectangular blocks with marked units along each edge	Find the number of units along each edge, then multiply them together.
		Or		
		count the number of unit-cubes in the block	Rectangular blocks with marked unit-cubes	

Table 2 -- Continued

Description of Item	Item Number	Main Interpretation Knowledge	Conditional Knowledge	
			Applicability Conditions	Application Methods
Assign volume values to:				
irregular container filled by small cubes	12	Volume measures the number of unit-cubes within a container, Volume of a cube = $(\text{length of an edge})^3$, and volume's additive property	Any container	Find volumes of small cubes by using formula, then add them together.
Explain how to find volume of an irregularly shaped solid	13	Volume is the equivalent number of unit-cube space occupied by a solid; volume can be found by reshaping the solid.	Any closed 3-D space	Find the equivalent number of unit-cubes in the solid

Table 2 -- Continued

Description of Item	Item Number	Main Interpretation Knowledge	Conditional Knowledge	
			Applicability Conditions	Application Methods
Cut a large cube into smaller ones and find the number of smaller cubes	14, 15	Volume measures number of unit-cube space an object occupies; cutting down a large cube does not change the total volume; Total Volume = (Length of an edge)^3	Any cubic object	Method 1: Find volume of the big cube; find volume of the smaller cube; divide the big volume by the small volume. Method 2: Divide the large cube's edge by that of the smaller cube's, cube the result.

they used to obtain those values. These six figures include two rectangles (Items 2-i and 2-ii), two complex figures with rectangular sub-units (Items 2-iii and 2-iv), one progressively more complicated figure with multiple geometric sub-units (Item 2-v), and one irregular figure whose edges are not straight but arcuate (Item 2-vi). Assigning numerical values to the areas in item 2 requires different forms of main interpretation knowledge. For example, formulae for classified standard cases may be used to answer Items 2-i and 2-ii, while formulae must be used with an associated feature of area, the additive property, to answer Items 2-iii and 2-iv. Furthermore, a deeper understanding of the formal definition is needed to obtain numerical answers for Items 2-v and 2-vi by counting unit squares within each figure. Item 3 asks students to compare areas of paired figures. This item helps expose misconceptions about area, such as using one dimensional length for area or thinking of area as being equal to the perimeter. Items 4 through 8 require students either to solve problems or answer questions related to area. These items are used to elicit additional information about a student's related knowledge about area, such as area's associated features other than the additive property.

Volume items followed a similar pattern to the area items. Item 9 asks students to define or describe volume in their own words. Items 10-i, 10-ii, and 11 require students to assign numerical values to volumes of three different objects and to explain the methods they used to obtain their values. These three objects include one rectangular block (Item 10-i), one complex object with sub-units of rectangular blocks (Item 10-ii), and one rectangular solid divided into unit cubes (Item 11). Assigning numerical values to the volumes of these objects requires different forms of interpretation knowledge about volume. For example, formulae for classified standard cases may be the most effective method to obtain a numerical value for Item 10-i, while formulae have to be used with an associated feature of volume, the additive property, to answer Item 10-ii. Item 12 requires students to estimate the volume of an irregular container, while Item 13 elicits from students their ideas about finding the volume of an irregular solid whose boundary is neither rectangular nor spherical. To answer Items 12 and 13 require the knowledge of volume's formal definition. Items 14 and 15 require students to solve problems related to volume and are used to elicit additional information about a student's related knowledge, such as the space-filling/space-taking property of a 3-D object (an associated feature of volume).

Design of the Interviews

Two interviews were conducted with students. The first was used to further ascertain students understanding of area and volume while the other was to determine students' concepts of pressure and density. The area and volume interviews were designed primarily to further probe an interviewee's existing knowledge. This includes their main interpretation knowledge, their conditional knowledge, and their consistent use of this knowledge in different situations. In addition, the area and volume interviews was also designed to elicit information from interviewees about their misconceptions and conceptual difficulties with these concepts. Similarly, the pressure and density interviews were designed to obtain information about students' interpretation knowledge of pressure and density and to probe the nature of the link between a student's concepts of pressure and density and his or her prior understanding of area and volume.

The techniques used to develop the interview questions followed those of Anderson's (1992). Specific protocols (see Appendix F) were used to guide each interview. These included a pre-instruction conversation to engage and gain the confidence of the interviewee, explain the purpose of the interview, obtain permission to audio-tape the interview, and to request that the student should "think aloud" about the concept and questions during the interview.

Materials (see lists in Appendix F) that might be useful during an interview, such as scratch paper, a ruler, and a box of wooden cubes, were collected. Props (see the lists in Appendix F) that were used to illustrate objects in interview questions, such as a piece of irregularly-shaped paper and a rectangular box, were also available to help the interviewer raise and explain interview questions.

Last, specific lead questions for each interview (see Appendix F) were developed to initiate the conversation and to elicit information from students about their knowledge and understanding of these concepts. For example, when interviewing students about their understanding of area, the interviewer would show the students a piece of irregularly-shaped paper (see Appendix G) and asked: "How would you measure the size of this irregularly-shaped figure?" The interviewees' answers as well as their justifications for the answers (the students were asked to talk out loud while thinking about and answering the question) would provide deeper insights into the students conceptual knowledge. Follow up questions were developed on the spot to probe any vague or non-specific answers. For example, was a student's thinking tied to use of formulae, or did it indicate that a student knew that the area is the number of unit squares within the figure?

Methodology

Subjects

The initial population of this study consisted of all 489 students who enrolled in Physics 107, 113, and 205 at Western Michigan University in the semester of Fall 1992. From this population, 431 students agreed to participate in the study by signing the consent forms (see Appendix C) giving permission to use the information provided. Thus, the research population consisted of 431 students. Figure 4 provides a profile of these students.

A total of 27 students were interviewed to further explore their understanding of area and volume. The selection for these interviewees was based on a survey of the students' answers to the inventory. The criteria for the selection were that: (a) these interviewees should represent all three classes, Physics 107, Physic 113, and Physics 205; and (b) that these interviewees should represent the range of understanding the area and volume concepts. Thus, among these 27 interviewed students, 14 were from Physics 107, 8 were from Physics 113, and 5 were from Physics 205; according to the students' understanding of area, 2 of these students were in the category of preliminary

Class	Physics 107				
Number of Students	171				
Top Four Curriculum Preferences	* Aviation Science & Tech. * Speech Pathology * Business/Economics * University Curriculum				
University Status	Fresh	Soph.	Junior	Senior	Gr
Number of Students Percent in Class	84 49%	42 25%	37 22%	7 4%	1 0%
Prior Physics Course Taking	Yes		No		
	High-Schl	College			
Number of Students Percent in Class	91 53%	10 6%	70 41%		
Math Background	Pre-Algbr	Algebra	Calculus		
Number of Students Percent in Class	90 53%	47 27%	34 20%		
GPA Level	2.0 <	2.0-2.9	3.0-4.0	N/A	
Percent Students At: High-Schl	1%	37%	57%	5%	
Percent Students At: WMU	2%	19%	24%	55%	

Class	Physics 113				
Number of Students	131				
Top Four Curriculum Preferences	* Biomedical/Pre-medical Sci. * Engineering Graphics * Industrial Design * Manufactural Engineering				
University Status	Fresh	Soph.	Junior	Senior	Gr
Number of Students Percent in Class	12 9%	42 32%	50 38%	23 18%	4 3%
Prior Physics Course Taking	Yes		No		
	High-Schl	College			
Number of Students Percent in Class	66 50%	12 9%	53 41%		
Math Background	Pre-Algbr	Algebra	Calculus		
Number of Students Percent in Class	20 15%	48 37%	63 48%		
GPA Level	2.0 <	2.0-2.9	3.0-4.0	N/A	
Percent Students At: High-Schl	0%	24%	73%	3%	
Percent Students At: WMU	1%	55%	33%	11%	

Class	Physics 205			
Number of Students	129			
Top Four Curriculum Preferences	* Mechanical Engineering * Paper Sci. & Engineering * Computer Science * Physics/Geology Ed.			
University Status	fresh	Soph.	Junior	Senior
Number of Students Percent in Class	2 2%	74 57%	44 34%	9 7%
Prior Physics Course Taking	Yes		No	
	High-Schl	College		
Number of Students Percent in Class	84 65%	23 18%	22 17%	
Math Background	Pre-Algbr	Algebra	Calculus	
Number of Students Percent in Class	1 1%	10 8%	118 91%	
GPA Level	2.0 <	2.0-2.9	3.0-4.0	N/A
Percent Students At: High-Schl	0%	20%	76%	4%
Percent Students At: WMU	3%	44%	38%	15%

Note: All information was self-reported.

Figure 4. Description of Subjects.

understanding, 3 in the category of emerging understanding, 12 in the category of developing understanding, and 10 in the category of good conceptual understanding; and according to their understanding of volume, 15 were in the category of preliminary understanding, 7 in the category of developing understanding, and 5 in the category of good conceptual understanding (no students were identified in the category of emerging understanding). Tables 3 and 4 illustrate the number of interviewees in each category of understanding as well as their class. A profile of these 27 interviewed students is provided in Figure 5.

Eight of the above 14 selected interviewees from Physics 107 were interviewed a second time to gather additional information and insights into their concepts of pressure and density and the nature of the link of these concepts with the students' prior understanding of area and volume. The remaining 6 students were not interviewed because they were not available. There were two reasons for only selecting Physics 107 students for this second interview. The first reason was that "pressure" and "density" are taught in Physics 107, Physics 113, and Physics 205 at different places in the curricula. Professors used different instructional materials and strategies for developing these concepts. Only interviewing students from one class reduces the risk of such inconsistent background. The second reason was that Physics 107 class contained students who represented the whole range of students' understanding of area and volume. There was no need to interview students from all three classes. Furthermore, because Physics 107 introduces the concepts of pressure and density much earlier in the curriculum than do Physics 113 and Physics 205,

Table 3

Categories of Understanding of Interviewees for the Area Concept

Category of Conceptual Understanding	Number of Students In Physics Class 107	113	205	Total	Number of Interviewees In Physics Class 107	113	205	Total
Preliminary Understanding	11	5	2	18	2	0	0	2
Emerging Understanding	8	9	2	19	1	2	0	3
Developing Understanding	110	76	68	254	5	4	3	12
Good Conceptual Understanding	42	41	57	140	6	2	2	10

Table 4

Categories of Understanding of Interviewees for the Volume Concept

Categories of Conceptual Understanding	Number of Students In Physics Class 107	113	205	Total	Number of Interviewees In Physics Class 107	113	205	Total
Preliminary Understanding	35	24	9	68	8	5	2	15
Emerging Understanding	0	0	0	0	0	0	0	0
Developing Understanding	72	67	70	209	2	3	2	7
Good Conceptual Understanding	42	30	34	106	4	0	1	5

Class	Physics 107				
Number of Students	14				
Curriculum Preferences (Number of Students)	* Aviation Sci. & Tech. (8) * Speech Pathology (2) * Business/Economics (2) * University Curriculum (2)				
University Status	Fresh	Soph.	Junior	Senior	Gr
Number of Students	6	5	3	0	0
Prior Physics Course Taking	Yes		No		
	High-Schl	College			
Number of Students	7	0	7		
Math Background	Pre-Algbr	Algebra	Calculus		
Number of Students	1	13	0		
GPA Level At:	2.0 <	2.0-2.9	3.0-4.0	N/A	
High School	0	8	6	0	
Western Mich. Univ	0	2	3	9	

Class	Physics 113				
Number of Students	8				
Curriculum Preferences (Number of Students)	* Biomedical Sci. (3) * Engineering Graphics (2) * Industrial Design (2) * Manufactural Eng. (1)				
University Status	Fresh	Soph.	Junior	Senior	Gr
Number of Students	2	1	3	2	0
Prior Physics Course Taking	Yes		No		
	High-Schl	College			
Number of Students	6	0	2		
Math Background	Pre-Algbr	Algebra	Calculus		
Number of Students	0	6	2		
GPA Level At:	2.0 <	2.0-2.9	3.0-4.0	N/A	
High School	0	2	6	0	
Western Mich. Univ	0	3	3	2	

Class	Physics 205			
Number of Students	5			
Curriculum Preferences (Number of Students)	* Mechanical Eng. (3) * Paper Sci. & Eng. (1) * Computer Science (1)			
University Status	fresh	Soph.	Junior	Senior
Number of Students	1	2	2	0
Prior Physics Course Taking	Yes		No	
	High-Schl	College		
Number of Students	5	0	0	
Math Background	Pre-Algbr	Algebra	Calculus	
Number of Students	0	1	4	
GPA Level At:	2.0 <	2.0-2.9	3.0-4.0	N/A
High School	0	0	2	3
Western Mich. Univ	0	4	0	1

Note: All information was self-reported.

Figure 5. Description of Interview Subjects.

interviewing students from all three classes may induce the possible contaminating effects occurring in the time interval between interviews. For example, students may talk to each other about the interview. Using only Physics 107 students, thus, reduces the possibility of such contamination.

Permission for the Study From HSIRB at WMU

The application for permission to use physics students in this study was filed with the Human Subjects Institutional Review Board (HSIRB) at Western Michigan University in August 1992. To fulfill the board's requirements, a cover letter to students (Appendix B) was written and distributed with the Knowledge of Area and Volume Inventory (Appendix A), informing students of the purpose of the study, how data would be used, and asking them to participate. A consent form (see Appendix C) was also prepared and distributed with the Inventory requesting students' signatures granting permission to use the data from their completed inventory and any interview tapes, if they were interviewed. Permission from the HSIRB for using physics students in this study was obtained in September, 1992 (see Appendix H).

Data Collection

Administration of the Knowledge of Area and Volume Inventory

The Knowledge of Area and Volume Inventory was administered to all subjects during the first week of class in the Fall semester of 1992. There were three scheduled sections of Physics 107, two sections of Physics 113, and one section of Physics 205. Prior to the beginning of the course, the researcher contacted the instructors of each section of the courses to explain the purpose of the study, the administrative details, and to ask for their permission to administer the Inventory near the end of a lecture session on a particular day (10-15 minutes were sufficient for students to complete the Inventory). To provide for consistent procedures for the administrating the inventory, Administration Instructions for the Knowledge of Area and Volume Inventory (see Appendix E) was developed and followed during each administration.

The Inventory was administered in a typical lecture-room setting to the students in each of the six sections who attended class on September 10th or 11th. The Inventory was introduced and administered to the students by the researcher in five classes. In one evening class (Physics 107), it was given to the students by the designated instructor of the class.

Each student taking the Inventory was given a plastic ruler marked in metric units and transparent grids with centimeter-squares. Pencils were provided upon request. Students were also allowed to use calculators if they had them available, since they were judged to provide no meaningful advantages.

Interviews for Students' Area and Volume Concepts

The area and volume interviews were scheduled during the second and third weeks of the semester. All the interviews were conducted in the same conference room in Everett Tower on Western Michigan University's campus under carefully controlled and uniform circumstances. Only the interviewer and the interviewee were present during each interview. Each interview took up to 30 minutes and each interview was audio-taped.

A protocol (see Appendix F) was used to guide each interview. In the beginning of an interview, the researcher engaged the student in casual conversation to relax the student and gain their confidence. The researcher then explained the purpose of the interview, again requested the student's permission to audio-tape the interview, and asked the student to "think aloud" while answering interview questions. During the interview scratch paper, pencils, a ruler, a transparent grid, and a box of wooden cubes were used by the researcher or the student as needed. Props, including a piece of regular white paper, a piece of paper that was cut into an irregular shape (see Appendix G), a rectangular box, a small balloon, a lump of clay, and a plastic bag, were used to help illustrate interview questions.

The lead questions (see Appendix F) were used to help begin the questioning and to provide uniformity to each interview. Because students' replies to these questions would affect any subsequent questions, the interviewer followed the students' lead and designed further questions base upon the students' initial answers. Students were also asked to comment further on their answers to specific responses in the Inventory and in each case to justify why they said what they did.

Interviews About Pressure and Density Concepts

The pressure and density interviews with students were conducted one to three weeks after the students were interviewed for their area and volume concepts and after the pressure and density concepts were taught in Physics 107 class. All these interviews were conducted under carefully controlled and uniform circumstances in a general physics laboratory in Rood Hall on Western Michigan University's campus. Only the interviewer and the interviewee were present during each interview. Each interview took up to 30 minutes and was audio-taped.

The process for each of these interviews followed a similar pattern to the area and volume interviews. A protocol (see Appendix F) was used to guide each interview. In the beginning of an interview, the researcher again engaged the student in casual conversation to reduce the student's anxiety and to gain the student's confidence. The researcher then explained the purpose of the interview, again requested the student's permission to audio-tape the interview, and asked the student to "think aloud" while answering interview questions. The lead questions (see Appendix F) were used to help begin the questioning and to provide uniformity to each interview. During the interview scratch paper, pencils, and a ruler were used by the researcher or the student as needed. Props, including a metal block and three paired objects (one pair having the same volume and the same shape but different mass, another having the same mass and the same shape but different volume, and the other having the same mass and the same volume but different shape) were used to help illustrate interview questions.

CHAPTER IV

ANALYSIS AND FINDINGS

Introduction

The research population of this study consisted of 431 first-year college physics students. Four types of data were collected from this population. First, data describing students' background in physics, mathematics, and their university status were obtained for all 431 students. Next, data concerning the initial status of each student's understanding of area and volume were determined using the Knowledge of Area and Volume Inventory administered to all students in the study population. Then, data eliciting additional information about students' prior understanding of area and volume were obtained from twenty-seven interviews with individual students. Lastly, additional data probing students' understanding of the pressure and density concepts were obtained in eight additional student interviews.

The task of this chapter is to present and analyze these data using the framework of student conceptual understanding developed by Reif and Allen (1992). In this framework, a person's understanding of a concept is described in terms of the main interpretation knowledge and appropriate conditional knowledge used by the person. Depending upon the forms of main interpretation knowledge and conditional knowledge employed by a person to explain a concept, his or her conceptual understanding is classified in one of four categories. These categoriesare: (1) preliminary understanding, (2) emerging understanding, (3) developing understanding, and (4) good conceptual understanding. These categories of conceptual understanding are defined and illustrated in Figure 3 on page 31.

The data describing each student's background in physics, mathematics, and their university status were examined and presented in Chapter III. Figure 4 on page 53 provides a brief profile of the research population's background in physics, mathematics, the students' current university status, and their curriculum preferences.

The first section of this chapter presents the data collected from the Knowledge of Area and Volume Inventory. These data were used to categorize the students' conceptual understanding of area and volume and to identify student conceptual difficulties. The second section of this chapter presents the data and analysis from the area and volume interviews. The last section of the chapter describes the data and analysis from the pressure and density interviews. These data were used to determine the status of each interviewee's understanding of pressure and density and to relate this understanding to each student's concepts of area and volume.

Inventory Data and Analysis

Overview

Students' answers to each question in the Inventory were analyzed to place the students in a particular category of conceptual understanding for area and volume. In this process, each student's answers and responses to the items related to area were analyzed to identify the type of main interpretation knowledge and appropriate conditional knowledge used by the student to answer a set of scaffolded questions about area. After identifying their main interpretation knowledge and appropriate conditional knowledge, students were placed in a category of conceptual understanding using previously established criteria. Similarly, a student's answers and responses to the volume items on the inventory were also analyzed to identify the student's main interpretation knowledge and appropriate conditional knowledge for volume. Students were then placed in an appropriate category of conceptual understanding.

The inventories were then analyzed a second time to identify student misconceptions or specific difficulties with the area and volume concepts. This analysis was depended on the students' explanations or justifications for their answers to the Inventory items. Misconceptions and specific difficulties were described for each category of conceptual understanding.

Student Understanding of Area

Student understanding of the concept of area is described as the forms of main interpretation knowledge and appropriate conditional knowledge a student is able to use to answer the area questions in the Inventory. A student's interpretation knowledge was identified by specific criteria written prior to the analysis of the Inventory data. Depending on the forms of main interpretation knowledge used in a student's responses on the Inventory, each student was placed in a specific category of understanding based upon previously established criteria.

Criteria for Identifying Students' Main Interpretation Knowledge of Area

Identifying students' main interpretation knowledge of area was determined using the following criteria:

Formal Definition. The formal definition used in this study is that area is the measurement of the extent of a closed two-dimensional space measured in unit-squares. This formal definition applies to any closed 2-dimensional figure and operationally requires that one count the equivalent number of unit-squares within that figure.

The criteria for determining a student's knowledge of and ability to use this formal definition are:

1. Assigning correct area values to figures in Items 2-i through 2-vi (see Appendix A) and clearly indicating that these values were obtained by using a grid (to count the equivalent number of unit-squares). "Clearly" means the student has provided explicit evidence of using methods that include counting unit-squares. This evidence includes written statements, drawings, or mathematical procedures. Or:

2. Assigning correct area values to figures in the items indicated above and describing or defining the concept of area in Item 1 by explicitly using all the following indicators: (a) area is a measurement, (b) area measures the extent of a 2-D

surface, and (c) the measurement of area is in unit-squares.

Classified Standard Cases. A classified standard case for a rectangle is that area equals the figure's length times width. This classified standard case applies to all rectangular 2-dimensional objects and operationally requires the student to measure the length and width of the rectangle in a common unit and to find the product of these measurements.

The criteria for determining a student's knowledge of and ability to apply this classified standard case are:

1. Assigning correct area values to figures in Items 2-i through 2-vi and providing evidence that these values were obtained by applying the formula "A = L x W." Or:

2. Assigning correct values to figures in Items 2-i to 2-iv and describing or defining the concept of area in Item 1 by the correct formula.

Associated Features. One important associated feature of area is that the total area does not change when a figure is reconfigured (i.e., conservation of area). This associated feature applies to all two-dimensional surfaces and means that areas of complex regular figures can be determined by adding together the areas of smaller sub-units.

A student's knowledge of and ability to apply this associated feature were determined by examining the student's explanations for their answers to Items 2-iii and 2-iv. The criterion used to identify this associated feature was the ability to determine the individual areas of sub-units and then add them together to determine the total area of the figure.

Students may also use other vague and erroneous associated features to describe the area concept. For example, they may relate area to: (a) a region or a space, (b) the size of an object, (c) perimeter, or (d) a multiplication operation. A student's use of these associated features to explain the concept of area was determined by examining the student's explanations to the questions in Items 1 to 3.

Categories of Conceptual Understanding of Area

After a student's main interpretation knowledge and appropriate conditional knowledge of area were identified using the above criteria, the student was assigned to a category in the hierarchy of conceptual understanding. These categories are:

Preliminary Understanding. This is the lowest category of conceptual understanding, where a student describes area using only associated features. These students apparently have little understanding or only a vague idea of what area is or how area values are assigned to surfaces. Student answers may also demonstrate various misconceptions about area, such as regarding area as perimeter, or as the product of the lengths of all sides of the figure.

Emerging Understanding. This is an intermediate category of conceptual understanding, where a student defines area using only a particular classified standard case, such as "A = L x W." Students in this category are able to calculate a rectangle's area by applying the formula "A = L x W" but are not able to formally define area or apply the additive property to determine the area of a complex figure composed of two or more simple rectangles.

Developing Understanding. This is another intermediate category of conceptual understanding, where a student interprets area by using classified standard cases and an associated feature, the additive property. Formal definitions are not stated or applied at this stage. Specifically, students are able to use mathematical formulae such as "A = L x W" to calculate rectangular areas, and are able to use area's additive property to determine the area of complex figures composed of two or more geometric sub-units. Students at this stage are not able to determine areas of irregular figures. They typically think that one needs formulae to calculate areas of these figures, but they admit that they do not know the formulae.

Good Conceptual Understanding. This is the highest category of conceptual understanding. At this stage, a student conceptually understands area as the number of unit-squares within any closed two-dimensional figure. Thus, students are able to correctly and coherently apply the three forms of main interpretation knowledge (formal definition, classified standard cases, and associated features) with appropriate conditional knowledge. Students area able to use a grid to determine the area of figures which cannot be readily computed by formulae. These students also use formulae for classified standard cases, apply area's additive property when appropriate, and explicitly demonstrate that they understand that formulae are only short-cut methods for counting unit-squares.

Results

Figure 6 presents a summary of the analysis of participant responses to determine the category of conceptual understanding of area for the students in this study. Significant results are:

1. One hundred and forty (32.5%) students in all classes were rated as having good conceptual understanding. Among them, forty-two (24.6%) were enrolled in Physics 107, forty-one (31.3%) in Physics 113, and fifty-seven (44.2%) in Physics 205.

2. Two hundred and fifty-four (58.9%) students were in the category of developing understanding. Of these students, one hundred and ten (64.3%) were enrolled in Physics 107, seventy-six (58.0%) in Physics 113, and sixty-eight (52.7%)

Category of Conceptual Understanding	Class and Number of Students in Classes			
	Phys. 107	Phys. 113	Phys. 205	All Classes
Preliminary Understanding	11 (6.4%)	5 (3.6%)	2 (1.6%)	18 (4.2%)
Emerging Understanding	8 (4.7%)	9 (6.9%)	2 (1.6%)	19 (4.4%)
Developing Understanding	110 (64.3%)	76 (58.0%)	68 (52.7%)	254 (58.9%)
Good Conceptual Understanding	42 (24.6%)	41 (31.3%)	57 (44.2%)	140 (32.5%)
Total Number of Students	171 (100%)	131 (100%)	129 (100%)	431 (100%)

Figure 6. Students' Understanding of the Area Concept.

in Physics 205.

3. Nineteen (4.4%) students were placed in the category of emerging understanding. Among them, eight (4.7%) were enrolled in Physics 107, nine (6.9%) in Physics 113, and two (1.6%) in Physics 205.

4. The remaining eighteen (4.2%) students were in the category of preliminary understanding. Among them, eleven (6.4%) were enrolled in Physics 107, five (3.6%) in Physics 113, and two (1.6%) in Physics 205.

Students' Understanding of Area. These results indicate that nearly one-third of the students (32.5%) from all classes (one-fourth in Physics 107, one-third in Physics 113, and nearly one-half in Physics 205) had attained good understanding of the area concept which they demonstrated by using appropriate main interpretation knowledge (formal definition, classified standard cases, and associated features) and associated conditional knowledge.

More than one-half of the students (58.9%) in all classes were able to calculate the area of simple and complex geometric figures by applying correct formulae and by adding areas of sub-units of more complex figures. These students demonstrated their ability to use classified standard cases and an associated feature of area (the additive property for complex figure). However, they did not demonstrate the more general understanding that area is the number of unit squares within a surface.

Overall 4% of the students (5% in Physics 107, 7% in Physics 113, and less than 2% in Physics 205) could only apply a classified standard case (area equals length times width) to calculate the area of a rectangle. These students were only able to demonstrate a rudimentary conceptual understanding of area.

Another 4.2% of the students (6% in Physics 107, 4% in Physics 113, and less than 2% in Physics 205) could only define area using less specific associated features, such as area is a region or the size of an object, and evidenced misconceptions, such as thinking that the perimeter of a figure or the product of the lengths of three or four sides equals its area. These students were only able to demonstrate a preliminary, but less than adequate, conceptual understanding of area.

Comparison of Students' Understanding of Area Between Courses. The percentage of students in each category of conceptual understanding for area shows a similar pattern across all three physics courses in the study. In each course, the largest percentage of students are in the category of developing understanding while less students are in the category of good conceptual understanding. The percentages of students in the categories of preliminary and emerging understanding are both comparably small. This indicates that students' initial conceptual understanding of area follows a similar trend in the three physics courses, although students in the more advanced class have a larger percentage of their population at the stage of good conceptual understanding.

Student Understanding of Volume

Student understanding of the concept of volume is also described as the forms of main interpretation knowledge and appropriate conditional knowledge a student is able to use to answer the volume questions in the Inventory. A student's interpretation knowledge was again identified by established criteria written prior to the analysis of the Inventory. Depending on the forms of main interpretation knowledge and appropriate conditional knowledge used in a student's responses, he or she was placed in a specific category of conceptual understanding based upon previously established criteria.

Criteria for Identifying Students' Main Interpretation Knowledge of Volume

Identifying students' main interpretation knowledge and appropriate conditional knowledge of volume was determined by using the following criteria:

Formal Definition. The formal definition of volume used in this study is that volume is the measurement of the extent of a closed three-dimensional space in unit-cubes. This formal definition applies to all three-dimensional objects and operationally requires that one count the equivalent number of unit-cubes within that object.

The criteria for determining a student's knowledge of and ability to use this formal definition are:

1. Assigning correct volume values to objects in Items 10-i, 10-ii, and 11 and clearly indicating these values were obtained by counting the equivalent number of unit-cubes. "Clearly" means the student has provided explicit evidence of using methods involving the counting of unit cubes. This evidence includes written

statements, drawings, or mathematical procedures. Or:

2. Assigning correct volume values to objects in the items indicated above and describing or defining the concept of volume in Item 9 by using all the following indicators: (a) volume is a measurement, (b) volume measures the extent of a 3-dimensional space, and (c) the measurement of volume is given in unit-cubes. Or:

3. A response to Item 13 indicated the idea of counting the number of unit-cubes. This response includes written statements, drawings, or explanations for arithmetic operations that indicate that the object can be broken down into unit-cubes and counting the number of cubes.

Classified Standard Cases. A classified standard case of volume for a regular three-dimensional object is that its volume equals the product of the length, width, and height of the object. This classified standard case applies to all rectangular three-dimensional objects and operationally requires a students to measure the length, width, and height of an object in a common unit and find the product of these measurements.

The criteria for determining a student's knowledge of and ability to apply this classified standard case are:

1. Assigning correct volume values to objects in Items 10-i and 11 and clearly indicating that these values were obtained by using the formula "V = L x W x H." "Clearly" means the student has provided explicit evidence of using the formula. Or:

2. Assigning correct volume values to objects in Items 10-i and 11 and describing or defining the concept of volume in Item 9 by specifying the correct

formula. Or:

3. A response to Item 13 indicated the idea of using formulae. This response includes written statements, drawings, or explanations for arithmetic or other operations (e.g., using the method of liquid-displacement and calculating the volume by using formulae).

Associated Features. One important associated feature of volume is that the total volume does not change when an object is reconfigured (i.e., conservation of volume). This associated feature applies to all three-dimensional objects and means that volume of a complex regular object is the sum of the volumes of its smaller sub-units.

A student's knowledge of and ability to apply this associated feature were determined by examining the student's explanation for his or her answers to Items 10-ii and 12. The criterion used to identify this associated feature was specific evidence of adding together the volumes of smaller sub-unit's of an object to determine its total volume.

Students may also use other vague and erroneous associated features to describe the concept of volume. For example, they may relate volume to: (a) an object or a space, (b) the size of an object, (c) the surface area of an object, or (d) a multiplication operation. A student's knowledge of and ability to apply these associated features were also demonstrated if a student used these ideas in his or her answers to the questions in Items 9 to 13.

Categories of Conceptual Understanding of Volume

After a student's main interpretation knowledge and appropriate conditional knowledge of volume were identified using the above criteria, the student was assigned to a category in the hierarchy of conceptual understanding. These categories are:

Preliminary Understanding. This is the lowest category of conceptual understanding, where a student typically describes volume using only less specific associated features. These students apparently have constructed little understanding of the concept or only have a vague idea of what volume is or how volume values are assigned to three-dimensional objects. Student answers may also demonstrate various misconceptions about volume, such as regarding volume as surface area, or as the product of the area of the faces of a three-dimensional object.

Emerging Understanding. This is an intermediate category of conceptual understanding, where a student defines volume using only a particular classified standard case, such as "V = L x W x H" for regular three-dimensional objects. Students in this category are typically able to calculate a solid object's volume using a formula but are not able to formally define volume or apply the additive property to determine the volume of a complex regular object composed of two or more simple rectangular sub-units.

Developing Understanding. This is another intermediate category of conceptual understanding, where a student interprets volume by using classified standard cases and an associated feature, the additive property. Formal definitions are not stated or applied. Students at this stage are able to use mathematical formulae such as "V = L x W x H" to calculate the volume of rectangular solids, and are able to apply volume's additive property to calculate volumes of more complex objects. But, students at this stage are still unable to determine volumes of irregular objects. They typically think that formulae are needed to determine the volumes of objects, but they admit that they do not know the formulae.

Good Conceptual Understanding. This is the highest category of conceptual understanding. At this stage a student conceptually understands volume as the number of unit-cubes within a three-dimensional object. Students are able to correctly and coherently apply the three forms of main interpretation knowledge (formal definition, classified standard cases, and associated features) with appropriate conditional knowledge. They also use the idea of counting or estimating the number of unit cubes within irregular objects when they cannot be computed by using formulae. These students use formulae for classified standard cases, apply volume's additive property when appropriate, and explicitly demonstrate that they understand that formulae are only short-cut methods for counting unit-cubes.

Results

Figure 7 presents a summary of the analysis of participant responses to determine the category of conceptual understanding of volume for the students in this study. Significant results are:

1. One hundred and six (27.7%) students in all classes were rated as having good conceptual understanding. Among them, forty-two (28.2%) were enrolled in Physics 107, thirty (24.8%) in Physics 113, and thirty-four (30.1%) in Physics 205.

2. Two hundred and nine (54.6%) students were in the category of developing understanding. Of these students, seventy-two (48.3%) were enrolled in Physics 107, sixty-seven (55.4%) in Physics 113, and seventy (61.9%) in Physics 205.

3. No students demonstrated conceptual understanding in the category of emerging understanding.

4. The remaining sixty-eight (17.8%) students were rated as having preliminary understanding. Among them, thirty-five (23.5%) were enrolled in Physics 107, twenty-four (19.8%) in Physics 113, and nine (8.0%) in Physics 205.

Category of Conceptual Understanding	Class and Number of Students in Classes			
	Phys. 107	Phys. 113	Phys. 205	All Classes
Preliminary Understanding	35 (23.5%)	24 (19.8%)	9 (8.0%)	68 (17.8%)
Emerging Understanding	0 (0.0%)	0 (0.0%)	0 (0.0%)	0 (0.0%)
Developing Understanding	72 (48.3%)	67 (55.4%)	70 (61.9%)	209 (54.6%)
Good Conceptual Understanding	42 (28.2%)	30 (24.8%)	34 (30.1%)	106 (27.7%)
Total Number of Students	149 (100%)	121 (100%)	113 (100%)	383 (100%)

Figure 7. Students' Understanding of the Volume Concept.

Students' Understanding of Volume. The above results indicate that over one-fourth (27.7%) of the students from all classes (one-fourth in Physics 113, one-third in Physics 107, and one-third in Physics 205) were able to demonstrate good understanding of the volume concept by demonstrating appropriate main interpretation knowledge (formal definition, classified standard cases, and associated features) and appropriate conditional knowledge.

More than one half (54.6%) of the students (nearly one-half in Physics 107, more than one-half in Physics 113 and in Physics 205) were able to calculate the volume of simple and complex geometric objects by applying correct formulae and by adding volumes of sub-units of more complex objects. While these students were able to demonstrate their ability to use classified standard cases and an associated feature of volume (the additive property for complex object), they did not demonstrate the more general understanding that volume is the number of unit cubes contained in a three-dimensional object.

Overall about one-fifth (17.8%) of the students (nearly one-fourth in Physics 107, one-fifth in Physics 113, and less than 10% in Physics 205) could only describe volume using associated features, such as volume is an object or the size of an object, and evidenced misconceptions, such as thinking that the surface area or the product of the areas of three faces equals its volume. These students were only able to demonstrate a preliminary, but less than adequate, conceptual understanding of volume.

Comparison of Students' Understanding of Volume Between Courses. The percentage of students in each category of understanding for volume shows a similar pattern across all types of physics courses. In each course, the largest percentage of students are in the category of developing understanding while less students are in the category of good conceptual understanding. A relatively small percentage of students are in the category of preliminary understanding while no students were categorized as attaining emerging understanding. This indicates that students' initial conceptual understanding of volume follows a similar trend in the three physics courses, although students in the more advanced class have a larger percentage of their population at the stage of good conceptual understanding.

Student Reasoning Difficulties and Misconceptions About Area

Students' reasoning difficulties and misconceptions about area were determined by analyzing students' main interpretation knowledge and conditional knowledge (applicability conditions and application methods). Specifically, each student's responses to the area items, particularly their justifications and explanations for these answers, or lack of answers, were examined to determine their misconceptions, inconsistent ideas, and reasoning difficulties. Table 5 displays the results of this analysis.

Students With Preliminary Understanding of Area

Eighteen students in the category of preliminary understanding typically

Table 5

Student Misconceptions, Inconsistent Ideas, and Reasoning Difficulties With the Area Concept

Category of Understanding	Form(s) of Main Interpretation Knowledge Applied	Misconceptions, Inconsistent Ideas, and Reasoning Difficulties	Number of Students in Physics			
			107	113	205	All Classes
Preliminary Understanding	Only Unimportant Associated Features	1. Area is the size of an object, no application methods.	5			5
		2. Area is perimeter.	5	3	1	9
		3. Area is volume.	1			1
		4. Area is the multiplication of the lengths of all sides.		1		1
		5. Combination of the above.		1	1	2
		Total number of students	11	5	2	18
Emerging Understanding	Classified Standard Case (Length x Width)	For complex regular figures:				
		1. Area is perimeter.	2	3		5
		2. Area equal to 1-D length.	1			1
		3. Area is the multiplication of the lengths of all sides.	1	1	1	3
		4. Area does not exist or is infinity	1			1
		5. Have no apparent idea of any operational methods.	3	5	1	9
		Total number of students	8	9	2	19

Table 5--Continued

Category of Understanding	Form(s) of Main Interpretation Knowledge Applied	Misconceptions, Inconsistent Ideas, and Reasoning Difficulties	Number of Students in Physics			
			107	113	205	All Classes
Developing Understanding	Classified Standard Case (Length x Width) and An Important Associated Feature (the additive property)	1. Indicate formulas are needed to find areas of irregularly shaped figures.	53 (48%)	46 (61%)	40 (59%)	139 (55%)
		2. Advanced math is required, but "I don't know that math."	34 (31%)	19 (25%)	28 (41%)	81 (32%)
		3. Do not believe area of an irregular figure can be calculated or determined.	23 (21%)	11 (14%)		34 (13%)
		Total number of students	110	76	68	254
		Also students demonstrated*: Compare areas of two figures by calculations, but not by counting the number of unit squares.	58 (53%)	32 (42%)	25 (37%)	115 (45%)

* This is a sub-set of students who are in the category of developing understanding.

Table 5--Continued

Category of Understanding	Form(s) of Main Interpretation Knowledge Applied	Misconceptions, Inconsistent Ideas, and Reasoning Difficulties	Number of Students in Physics			
			107	113	205	All Classes
Good Conceptual Understanding	Formal Definition and Classified Standard Case (Length x Width) and An Important Associated Feature (Area's additive property)	No conceptual difficulties were identified. Students' reasoning is not bound to mathematics formulae.				
		Total number of students	42	41	57	140
Total number of students in all categories			171	131	129	431

described area using associated features. For example, five of these students indicated that area is the size of an object and demonstrated their preliminary understanding by presenting little evidence of understanding how to determine the area of a surface.

Two misconceptions about area were identified. The first misconception is that area equals the perimeter of a figure. Nine students indicated this misconception by using perimeters when reporting area values. These students' description or definition of area were also ambiguous and unclear. For example, they wrote that area is "inside of a given object," or "the surface you want to work with," or "space that is two-dimensional."

The second misconception that area is equivalent to volume was demonstrated by one student. Nevertheless, it is interesting to note that this student consistently defined area as "Area = width x height x length of an object," and always applied this idea to all questions and calculations involving area. In addition, another student related area to the product of the lengths of the four sides of a rectangle, while two others used a combination of addition and multiplication operations to determine the area of a surface.

Students With Emerging Understanding of Area

Nineteen students were classified in the category of emerging understanding and were able to define area as length times width. They assigned correct area values to simple rectangles but did not demonstrate an understanding of area's additive property. Nine of these students did not know how to determine the area of complex figures composed of two or more rectangular sub-units. One student expressed the idea that the area of a complex figure cannot be determined. The remaining nine at this stage demonstrated their inconsistent ideas about area. Among these nine students, who defined area as length times width, five numerically assigned areas to complex regular figures using perimeters, one used the length of the longer side as "area," and three related area to the product of the lengths of all sides of a complex regular figure.

Students With Developing Understanding of Area

Two hundred and fifty-four students were in the category of developing understanding. These students' conceptual difficulties were the result of an incomplete understanding of the implications of the formal definition that area measures the number of unit squares within a given surface. Their lines of reasoning about area are depended upon mathematical formulae. Of these students, one hundred and thirty-nine (55%) thought the area of an irregular figure required a special formula and indicated that they did not know it. Eighty-one (32%) students thought that knowledge of advanced mathematics would allow one to determine the areas of these irregular figures, but admitted that they did not know "that mathematics." Thirty-four (13%) students did not think that the area of an irregular figure could be numerically determined because there was no definitive formulae available.

When asked to compare areas of two figures, a sub-set of one hundred and fifteen (45%) students in the category of developing understanding did not reshape the complex figures or count the numbers of square-units within the figures to determine which figure had a larger area. Instead, they used formulae to calculate and then decide if the areas of the figures are equal or not.

Students With Good Conceptual Understanding of Area

One hundred and forty students demonstrated a good prior conceptual understanding of area as the number of unit squares enclosed within a figure. These students had little difficulty in assigning numerical values to all regular and irregular figures. Their reasoning of area is not bound by mathematical formulae and they were able to reason using the meaning of the concept's definition that area equals the number of unit squares within a surface.

Student Reasoning Difficulties and Misconceptions About Volume

Students' reasoning difficulties and misconceptions about volume were determined by analyzing students' main interpretation knowledge and conditional knowledge (applicability conditions and application methods). Specifically, each student's responses to the volume items, particularly their justifications and explanations for these answers, or lack of answers, were examined to determine their misconceptions, inconsistent ideas, and reasoning difficulties. Table 6 presents the results of this analysis.

Students With Preliminary Understanding of Volume

Sixty-eight students in the category of preliminary understanding described

Table 6

Student Misconceptions, Inconsistent Ideas, and Reasoning Difficulties With the Volume Concept

Category of Understanding	Form(s) of Main Interpretation Knowledge Applied	Misconception, Inconsistent Ideas, and Reasoning Difficulties	Number of Students in Physics			
			107	113	205	All Classes
Preliminary Understanding	Only Unimportant Associated Features	1. Volume is the size of an object, no application methods.	16	13	6	35
		2. Volume is surface area.	13	8	2	23
		3. Volume is the product of the areas of three faces of the object.	6	3	1	10
		Total number of students	35	24	9	68
Emerging Understanding	Classified Standard Case (L x W x H)	(No students were classified in this category)				

Table 6--Continued

Category of Understanding	Form(s) of Main Interpretation Knowledge Applied	Misconceptions, Inconsistent Ideas, and Reasoning Difficulties	Number of Students in Physics			
			107	113	205	All Classes
Developing Understanding	Classified Standard Case (Length x Width x Height) and An Important Associated Feature (Volume's additive property)	1. Indicate formulas are needed to find volumes of irregularly shaped objects.	36 (50%)	58 (87%)	44 (63%)	138 (66%)
		2. Think advanced math exists, but "I don't know that math."	23 (32%)	9 (13%)	26 (37%)	58 (28%)
		3. Do not believe volume can be computed or measured if an object is irregularly shaped.	13 (18%)			13 (6%)
		Total number of students	72	67	70	209

Table 6--Continued

Category of Understanding	Form(s) of Main Interpretation Knowledge Applied	Misconceptions, Inconsistent Ideas, and Reasoning Difficulties	Number of Students in Physics			
			107	113	205	All Classes
Good Conceptual Understanding	Formal Definition and Classified Standard Case (Length x Width x Height) and An Important Associated Feature (Volume's additive property)	No conceptual difficulties were identified. Students' reasoning is not bound to mathematical formulae.				
		Total number of students	42	30	34	106
Total number of students in all categories			149	121	113	383

volume using associated features. For example, thirty-five of these students indicated that volume is the size of an object and presented little evidence of understanding how to determine the volume of a three-dimensional object. Two misconceptions about volume were also identified with students at this stage. The first misconception is that volume is equivalent to the surface area of the object. This misconception was indicated by 23 students. For example, one student responded to Item 13 by stating: "cut it into more regular pieces and measure the surface areas and add them up."

The second misconception is that volume equals the product of the area of the three visible faces of an object in a two-dimensional drawing. This idea was used by ten students and was apparently a variant of the first misconception of volume using multiplication.

Students in the Category of Developing Understanding

Two hundred and nine students were in the category of developing understanding. Among these students, one hundred and thirty-eight thought that a special formula was required to determine the volume of an irregular object. Fifty-eight students thought that knowledge of advanced mathematics would allow one to calculate the volumes of these irregular objects. However, these students stated that they did not know the mathematics. The remaining thirteen students indicated that volume of an irregular object cannot be numerically determined because there was no definitive formula available. Overall, student conceptual difficulties at this stage appear to be due to a failure to understand that volume measures the number of unit-cubes enclosed within a three-dimensional object. Thus, their reasoning is bound to mathematical formulae.

Students With Good Conceptual Understanding of Volume

One hundred and six students demonstrated a good prior conceptual understanding of volume as the number of unit cubes enclosed within a three-dimensional object. These students had little difficulty in determining numerical volumes to regular and irregular objects. Their reasoning about volume is not bound to mathematical formulae and they are able to reason using the implication of the formal definition that volume equals the number of unit-cubes within a three-dimensional object.

Area and Volume Interview Data and Analysis

Overview

A total of twenty-seven students were interviewed to further explore their understanding of the area and volume concepts. These interviewees were selected to represent the range of students' conceptual development from preliminary to good understanding. Participant selection was based on an initial survey of the Inventory and the willingness of the students to participate in this phase of the study. The interviews were conducted one to two weeks after the administration of the Inventory and each interview, which took approximately thirty minutes, was audio-taped.

The interview questions were designed to obtain additional insights and information about a student's understanding of and reasoning about the concepts of area and volume. A general question such as "how would you describe the size of this surface?" was used to begin the interview. Once an interviewee answered this lead question (e.g., "length times width"), the interviewer followed that question with further questions such as "is this applicable to all surfaces, for example, a square, a circle, and even this surface with irregular boundaries?" These questions were used to further probe the student's main interpretation and conditional knowledge to help understand the status of that person's conceptual understanding. The interviewer then asked additional questions to clarify an interviewee's written responses to Inventory items. For example, these questions might ask a student to explain what (s)he was thinking when they responded with a question mark to an Inventory item. This information was used to further elucidate each interviewee's original answers, lines of reasoning, and difficulties with these concepts.

Before the analysis, audio-tapes of each interview were transcribed. These transcripts were analyzed to identify propositional statements, and these, in turn, were used to clarify each person's main interpretation knowledge and conditional knowledge for area and volume. Once the status of the students' main interpretation and conditional knowledge were determined, a second analysis was conducted to identify incomplete or incorrect ideas about area and volume. Next, the ideas of interviewees in each category of conceptual understanding were examined to determine if common conceptual difficulties and lines of reasoning could be identified.

Interviewees' Conceptual Difficulties With Area

Table 7 provides an overview of the results of the analysis of these interviews with the concept of area. The following sections discuss the nature of interviewees' understanding of, inconsistent ideas about, and conceptual difficulties with area.

Interviewees in the Category of Preliminary Understanding

Two students in the category of preliminary understanding were interviewed. These student described the concept of area using associated features and were unable to correctly determine area. The following interview excerpt illustrates their understanding of and difficulty with the concept.

Interview Excerpt. This student responded to Item 1 in the Inventory, which asked students to define or describe area in their own words, with: "Area is the total inside space of an object." These student assigned area values to the simple and complex regular figures in Item 2 by adding together the lengths of all sides of the figures.

Researcher: In this area and volume inventory you did a week ago, you wrote here: "Area is the total inside space of an object." Can you explain more to me about what you meant?

Student: ... I mean area is space... but it doesn't have (a) height and...so it's space...on a table or in a piece of paper or something. You know, like these shadows (pointed to the shaded areas of the figures in Items 2-i to 2-iv), they take up space, area is space.

Researcher: So, area is the same as surface space. Is that what you mean?

Table 7

Results of the Analysis of the Area Interviews

Category of Understanding	Number of Interviewees and Class	Interviewees' Interpretaion Knowledge			Conceptual Difficulties
		Main Interpretation Knowledge	Application Methods	Applicability Conditions	
Preliminary Understanding	2 - Phys.107	Area is two-dimensional space (associated feature)	Perimeter (addition of the lengths of all sides)	To all two-dimensional figures	Incorrect conditional knowledge (mis-match between main interpretation knowledge and application method)
Emerging Understanding	1 - Phys.107	1. Length times width (a classified standard case) and 2. Area is perimeter (a misconception)	A = L x W Calculating perimeter	To simple rectangles To complex figures	Co-existence of inconsistent ideas (area is length times width, yet it also equals perimeter)
	1 - Phys.113	1. Length times width (a classified standard case) and 2. Inconsistent ideas	A = L x W Addition and multiplication	To simple rectangles To complex figures	Consistent methods for determining area of complex and irregular figures are not yet developed
	1 - Phys.113	Length times width (classified standard case)	A = L x W	To simple rectangles	Lack of confidence in one's own math ability.

Table 7--Continued

Category of Understanding	Number of Interviewees and Class	Interviewees' Interpretaion Knowledge			Conceptual Difficulties
		Main Interpretation Knowledge	Application Methods	Applicability Conditions	
Developing Understanding	3 - Phys.107 3 - Phys.113 2 - Phys.205	1. Length times width (a classified standard case) and 2. The additive property (an important associated feature)	Total area is the sum of areas of smaller sub-units determined by "L x W"	To simple and complex figures	Believe advanced mathematics would allow one to calculate areas of irregular figures but admit that they do not know that mathematics.
	2 - Phys.107 1 - Phys.113 1 - Phys.205	same as above	same as above	same as above	Do not believe area of irregular figures can be determined by any means.
Good Conceptual Understanding	6 - Phys.107 2 - Phys.113 2 - Phys.205	Area is the number of unit squares within a two-dimensional figure (the formal definition)	Use a grid or formulae as short-cut ways to count the number of unit squares	To all two-dimensional figures	No conceptual difficulties identified. Reasoning is not bound by mathematical formulae.

Student: Yeah, in a way. It tells you how large (a) space is. Like on here (the Inventory), they take areas, and this one (Item 2-ii) is bigger than this one (2-i), because it has a larger area, it takes more space.

Researcher: So, you mean area measures the size of a flat space. Am I interpreting you right?

Student: Yeah!

Researcher: How do you calculate area, then?

Student: Like I did here (Item 2-1), ... I added up (the) sides. This (Item 2-ii), too. This one (Item 2-iii) has actually two parts, the square here and this part (an attached rectangle). I added up the sides of the square, it's 18, and then I did the same here (to the rectangle), it's 16. Then I added them up, it's 34. And I did the same for this one, too (Item 2-iv).

Researcher: I see. Why do you have to separate it into two parts and then find areas and then add them up?

Student: ... it has to be a square or a rectangle or something. Because... it's easier to figure out the area of this square and, the rectangle (of the figure in Item 2-iii). You can't do (it) for the whole shadow (the total area of the figure in Item 2-iii), because this part (the unit that would make the complex figure to a regular rectangle) is missing.

Discussion. This student described area as a measurement of space within a two-dimensional figure. This indicates that the student has developed correct main interpretation knowledge of the concept. However, this knowledge was followed with incorrect conditional knowledge where the perimeter was used to calculate area. In other words, the students did not use a correct application method that matched the main interpretation knowledge previously stated. What is especially interesting is that this student did not use the perimeter of a complex figure to measure its total area.

Instead, the student separated this figure into two smaller rectangular sub-units, applied the perimeter method for calculating area to each of these sub-units, and then summed up these two results to determine the total area of the complex figure. In this procedure the student used the length of the common side twice in the calculation, indicating that the applicability condition attached to the student's main interpretation knowledge was simple rectangles but not complex figures. Thus, this student continued to demonstrate consistency with the prior notion of perimeter as equivalent to area and with the mis-match between the main interpretation and conditional knowledge.

The second interviewee in the category of preliminary understanding also described area as a two-dimensional space (correct main interpretation knowledge). However, this was again followed with incorrect conditional knowledge (perimeter) to determine area.

Overall, the interviews with students in the category of preliminary understanding demonstrated that these students have begun to develop correct main interpretation knowledge and confirmed their placement into the category of preliminary understanding based on the analysis of their responses to the Inventory. In addition, the interviews revealed that the conditional knowledge held by these students was incorrect (the perimeter is used as a measure of area). This inconsistency between the students' correct main interpretation knowledge and incorrect conditional knowledge was further demonstrated by the students' failure to use appropriate application methods. Thus, this misunderstanding of conditional knowledge evidently hindered a complete development of the area concept. Their conceptual difficulty, therefore, is predominantly due to their understanding deficiency in conditional knowledge.

Interviewees in the Category of Emerging Understanding

The three interviewees in the category of emerging understanding were able to use mathematical formulae (for classified standard cases) to calculate areas of simple regular figures in the Inventory. However, these students were not able to determine the area of complex regular figures composed of two or more smaller rectangular sub-units. The following interview excerpts illustrate three different types of conceptual difficulties displayed by these students.

The First Excerpt and Discussion. This student described area as "the inside of a given object," and correctly assigned area to the regular figures in the Inventory by using the appropriate formula. However, the student assigned area to the complex regular figure in Item 2-iii by adding together the lengths of all sides (the perimeter of the figure), and left Items 2-iv, 2-v (complex regular figures), and 2-vi (irregular figure) unanswered. At this point in the interview, the researcher asked the student to further explain what (s)he was thinking when determining the area for the figure in Item 2-iii (a figure composed of two rectangles).

Student: I took all sides and added them together... It's an old way of doing it... Because I don't recall any formulas for this kind of thing. I mean if you told me (a) formula, I'd work on that. But you didn't, so I did it my way.

Researcher: That's fine. How about the rest of these (figures in Items 2-iv through 2-vi)?

Student: ... I didn't do them. It's too much work. I don't like math.

Researcher: If I ask you to do this one (Item 2-vi, an irregular figure) now, how will you do it?

Student: ... If (I) had a piece of string, I'd just hold the string down and go around it until it comes back (She showed how to go around the border and come to a close), and then see how long it (the border measured by string) was.

This student understood that the area of a rectangle could be calculated by multiplying its length by its width. However, when complex regular figures composed of two or more rectangles were encountered, the student was not able to use the formula together with the additive property to determine the area. This indicated that the student did not understand the additive property of compound areas, but had attained a knowledge of classified standard cases. Therefore, in contrast to the students in the category of preliminary understanding, this student's concept of area is at the emerging stage where knowledge of classified standard cases has been developed. Further, the interview revealed that the reason for the student used the perimeter to determine the area of complex regular figures was because perimeter was held as "an old way" for calculating area. This was explained by the student: "It's an old way of doing it... If you told me (a) formula, I'd work on that. But you didn't, so I did it my way." Thus, perimeter was thought to be a legitimate and alternative method for determining the area. In other words, the student has begun to develop correct main interpretation knowledge, but does not completely understand the conditional knowledge. Therefore, when a formula was available, the student was able to apply the main interpretation knowledge with appropriate application methods. But when formulae were not available, incorrect methods were used as compatible alternatives. This co-existence of inconsistent ideas of the application methods indicates that the student's understanding of the concept of area is incomplete, and the conceptual difficulty is a deficiency in conditional knowledge.

The Second Excerpt and Discussion. This student wrote in the Inventory: "Area is the total surface around a given object." The student answered Items 2-i and 2-ii by multiplying the length by width, and answered Items 2-iii through 2-vi by using different and inconsistent methods. When asked to explain the thinking while answering these last two items, the student said:

> I multiplied length (by) width for these two (Items 2-i and 2-ii) because they are rectangles, and that's the right formula for them. This one (Item 2-iii, a complex regular figure) and this one (2-iv, a complex regular figure)... have more than two dimensions, so you'd have to multiply all of them (lengths of three sides in Item 2-iii, additions of the lengths along each dimension and multiply the results in Item 2-iv)... This one (Item 2-v) has too many dimensions, so I guessed (the) length and width of the darkened area... I knew it was a rough estimate, but that's how much I could do... This (Item 2-vi, an irregular figure) doesn't even have (any) dimensions, so I gave it a rough guess... I don't know how you'd do it, this's my best guess.

This student understood that length times width is "the right formula" for determining the area of rectangular surfaces, demonstrating an emerging understanding of the concept. However, when complex regular figures were encountered, the student failed to apply the additive property to determine the area. Instead, (s)he multiplied together all the lengths or dimensional lengths of the figures, in a rather arbitrary way, to determine the area for these surfaces. Furthermore, for irregular figures, the student "guessed the length and width of the darkened area" and multiplied them together for the area. These arbitrary and inconsistent procedures indicate that this student neither understood the additive property of area, nor had developed any specific application methods for determining the area of complex regular and irregular figures. Thus, this student's understanding of the concept of area is emerging, because when a formula was available, the student was able to apply main interpretation knowledge with appropriate application methods. In contrast to the previous interviewee, this student's difficulty was not due to any co-existence of inconsistent ideas. Instead, the predominant difficulty for this student seems to be the lack of commitment to any particular ideas. This was evidenced when formulae were not available, arbitrary methods were used as means to obtain an answer but they were not viewed as compatible alternatives. Therefore, this student's conceptual difficulties are the lack of understanding of the additive property, a deficiency in conditional knowledge, and a lack of commitment to single definitive ideas.

The Third Excerpt and Discussion. This student wrote on the Inventory that area "means the total space an object takes up," and answered Items 2-i and 2-ii (simple rectangles) by using the correct formula (A = L x W). The student did not answer any of the questions in Items 2-iii through 2-vi (complex regular and irregular figures). In the interview, the researcher asked why (s)he did not answer Items 2-iii to 2-vi. The student was not willing to explain much but simply replied:

> I haven't learned it yet. I don't have any science, geometry, and algebra... I don't know.

Like the previous interviewees in the category of emerging understanding, this student uses a classified standard case of area, A = L x W, for rectangles and was able to use this formula to obtain correct answers for them. This indicates that the student has developed an emerging understanding. Similar to the previous interviewees at this stage, this student was unable to determine the area of complex regular and irregular figures. But in contrast, the student neither held any inconsistent ideas as compatible application methods, nor was willing to accept other ideas as alternatives. This was evidenced by the student's explanation: "I haven't learned it (method for determining the area of complex regular figures) yet." The explanation also implied that the student was committed to one method (length times width) and simply admitted that further learning would be needed for complex figures. Thus, this student's understanding of the concept of area is emerging but incomplete, because the conditional knowledge (application methods) is incomplete. This lack of complete conditional knowledge blocks the development of the student's concept of area and predictably results in difficulties in conceptual understanding.

In addition, the student's excuse for the inability to answer questions about the areas of complex and irregular figures was: "I don't have any science, geometry, and algebra..." This indicates that the student is not confident of his or her science and mathematical background and, thus, it prevents the students from trying to develop further understanding.

Overall Discussion. Interviewees in the category of emerging understanding have developed correct main interpretation knowledge about the concept of area. In contrast to students in the category of preliminary understanding, these students have also attained correct conditional knowledge for classified standard cases. However, these students' understanding was deficient because the additive property of area was not understood. Thus, the students were only able to calculate the area of simple rectangles, but were unable to determine the area of complex regular figures. Furthermore, the interviews with students at this stage of understanding revealed that some students used the perimeter of complex and irregular figures as a measure of area, indicating that they simultaneously held inconsistent ideas, and that these methods were used as compatible alternatives. Other students seemed to lack a commitment to consistent application methods, because they used arbitrary methods to obtain answers to problems of area with complex figures. Yet other students committed to one method (length times width) and simply admitted that further learning would be needed for determining areas for complex figures. Thus, these students' conceptual difficulties at the stage of emerging understanding were consistent in the interviews with the students' lack of understanding displayed on the Inventory.

Interviewees in the Category of Developing Understanding

Twelve students in the category of developing understanding were interviewed to determine their understanding of and difficulties with the concept of area. These students were able to calculate areas of simple and complex regular figures by using formulae (for classified standard cases) together with application of the additive property (an associated feature). But they were not able to determine the area for irregular figures. The following two interview excerpts illustrate their concept of area.

The First Excerpt and Discussion. When completing the Inventory, this student answered Items 2-i through 2-v (simple or complex regular figures) by using correct formulae and the additive property, but did not answer Item 2-vi (an irregular figure). In the interview, the student was asked to explain why Item 2-vi was not answered.

Student: I don't know how to calculate area for abstract figures (figures with irregular boundaries)... I couldn't come up (with) any idea, I've never done this sort of calculation... I suppose there is a formula for it (the area of an irregular figure) or calculus may be the tool (to determine the area for these figures). But I don't know for sure. I don't know calculus.

Researcher: Can we find area without math?

Student: ...That (would) be weird. To me, area... is a math term. Without math, how can we find area?

This student has developed correct main interpretation knowledge and appropriated conditional knowledge using classified standard cases of area and the additive property. However, this understanding of the concept of area is at the developing stage because the full implications of the formal definition are not understood. This was evidenced when the student stated that (s)he did not know "how to calculate area for abstract figures," indicating that the idea of area as the number of unit-squares within a closed surface was lacking. In contrast to students in the lower two categories of understanding, this student did not exhibit any inconsistent ideas between main interpretation and conditional knowledge. Instead, the student's conceptual difficulty is largely due to an incomplete understanding of the implications of the formal definition of the concept. Here the concept of area is still linked to mathematical operations or procedures. This is demonstrated by the student in the interview when (s)he explained: "I suppose there is a formula for it (the area of an irregular figure) or calculus may be the tool (to determine the area)." This difficulty is further displayed when the student explained his or her concept and reasoned: "Area... is a math term. Without math, how can we find area?" Thus, this student's conceptual difficulty is due to the lack of understanding that area is fundamentally the number of unit-squares within a closed surface and the reasoning deficiency that the determination of area is limited to mathematical procedures.

The Second Excerpt and Discussion. The student in this interview answered Items 2-i through 2-iv (simple or complex regular figures) by using correct formulae and the additive property. The student also answered Items 2-v (a transitional figure between complex regular and irregular figures) and 2-vi (an irregular figure), but did not explain how the answers were obtained. In the interview, the researcher asked the student to explain.

Researcher: You had answers to these two questions (Items 2-v and 2-vi), but you had no explanation of how you arrived at these numbers. Can you explain them to me now?

Student: ...I guessed... Because there is not a formula to my knowledge of how to find the area of this figure (Item 2-v). This one (Item 2-vi) is (the) same as that one... For sure area is here. But I doubt there is a way (to determine the area). The area can't be calculated exactly. I can only think (of) guessing it.

Researcher: What do you mean by guessing?

Student: Approximation... Because you can't be exact, there is not a way to be exact. Specially this one (Item 2-vi)... It's a hand-sketched one, there's not a way to be exact (for determining the area).

Like the student in previous interview, this one has also developed correct main interpretation knowledge and appropriated conditional knowledge using classified standard cases of area and the additive property. But the understanding of the concept of area is limited because the implications of the formal definition are not understood. The student does not know how to determine the area of irregular figures, indicating that the idea of unit-squares within a figure has not developed. This interviewee's conceptual difficulty is not due to inconsistent ideas between main interpretation and conditional knowledge, but is due to the reasoning that area determination is tied to mathematical formulae. In contrast to the previous student, this student does not think areas for irregular figures are possible, because "there is not a formula to my knowledge of how to find the area of this figure (whose boundary is irregular)." Further, the student reasoned: "For sure area is here. But I doubt there is a way (for determining the area)... The area can't be calculated exactly." This indicated a belief that since formulae for areas of irregular figures did not exist, the area of these figures could not be determined.

Overall Discussion. Interviewees in the category of developing understanding were able to calculate the area of simple and complex regular figures by using formulae and by apply the additive property, indicating main interpretation and conditional knowledge for both classified standard cases and associated features. However, these students were not able to determine the area for irregular figures, indicating a lack of understanding of the unit-square approach. In contrast to students in the lower two categories of understanding, these students did not evidence any inconsistent ideas between their main interpretation and conditional knowledge. Instead, they demonstrated difficulties with the concept of area since their thinking was bound by mathematics or available formulae. Thus, they could not determine the area of figures with irregular boundaries. As the interviews demonstrated, eight of these students believed that area of these figures exists and that formulae or mathematics may exist to determine it. Another four students believed that these areas could not be determined by any means. All these beliefs are linked to the students' lack of complete understanding of the formal definition, because their thinking is confined to mathematical formulae rather than to a generalized idea of area as the number of unit-squares within a figure.

Students in the Category of Good Conceptual Understanding

Ten students in the category of good conceptual understanding were interviewed. These students demonstrated their mastery of the concept of area by counting the number of unit-squares within a closed two-dimensional figure, and they were able to determine the areas of regular and irregular figures. The following interview excerpt illustrate these students' conceptual understanding and lines of reasoning.

Interview Excerpts. A student correctly answered all the Inventory items pertaining to area and left a question mark next to the answer to Item 2-vi (an irregular figure) as a response to the request for an explanation of the method used. In the interview, the researcher asked the student to explain what that question mark meant, the student said:

> ...What I really did was put that sheet (transparent grid) over... The first thing I did was (I) counted up whole squares. And, then I counted up...like...what I saw, like this one and this one (partial squares), (they) looked like (they) made (up) a whole square... That's just counted one (whole square).

Another student interviewed was asked to explain his idea about area.

Student:	Area is the space that an object occupies... It is length times width, or (in) other words it is the total space within a defined region such as a square... It is a measurement in uniform square units of a two-dimensional space.
Researcher:	How do you exactly find an area, then?
Student:	Using a grid (I) was able to visualize uniform square units of any shape, and rearrange it to make it easier to count.
Researcher:	You mean it's not necessary to use a formula or some kind of math?
Student:	No, it's not.

Discussion. These students as well as other interviewees in the category of good conceptual understanding demonstrated their understanding of the concept of area by counting the number of unit-squares within a closed two-dimensional figure. This indicates a deeper understanding of the concept of area. These students were also able to use appropriate formulae and apply the additive property as short-cut ways for counting unit-squares within regular figures, indicating their understanding of classified standard cases and associated features. Thus, they have developed a meaningful conceptual understanding where all forms of main interpretation and the associated conditional knowledge are correctly understood and used. This more complete understanding was demonstrated in the interviews, as shown by the above excerpts. For example, one student explicitly explained how to use a grid to count the number of whole squares and to estimate the number of square-units made up by partial squares within an irregular figure. Another student indicated that the area of a rectangle could be determined by finding the product of its length and width, but the use of formula was not necessary because it was only a short-cut way for counting the unit-squares within the figure. The students further stated explicitly that mathematical formulae were not necessary to determine the area of any figure. Thus, unlike students in the other categories of understanding, these students did exhibit a fundamental understanding of main interpretation and conditional knowledge and did not exhibit difficulties with the area concept. Instead, these students' understanding was more complete and their thinking and reasoning are not confined to mathematics or available formulae.

Interviewees' Conceptual Difficulties With Volume

Table 8 provides an overview of the results of the analysis of the interviews with students about their concept of volume. The following sections discuss the nature

Table 8

Results of the Analysis of the Volume Interviews

Category of Understanding	Number of Interviewees and Class	Interviewees' Interpretaion Knowledge			Conceptual Difficulties
		Main Interpretation Knowledge	Application Methods	Applicability Conditions	
Preliminary Understanding	2 - Phys.107 1 - Phys.113	Volume is three-dimensional space (associated feature)	Surface area of an object	To all three-dimensional objects	Incorrect conditional knowledge (mis-match between main interpretation knowledge and application method)
	3 - Phys.107 1 - Phys.113 2 - Phys.205	same as above	Multiply the areas of three faces of an object	To regular three-dimensional objects	same as above
	3 - Phys.107 3 - Phys.113	same as above	None	None	Conditional knowledge is not yet developed
Emerging Understanding	0 (No one was placed in this category.)				

Table 8--Continued

Category of Understanding	Number of Interviewees and Class	Interviewees' Interpretaion Knowledge			Conceptual Difficulties
		Main Interpretation Knowledge	Application Methods	Applicability Conditions	
Developing Understanding	2 - Phys.107 3 - Phys.113 2 - Phys.205	1. Length times width times height (a classified standard case) and 2. The additive property (an important associated feature)	Total volume is the sum of volumes of smaller sub-units determined by using "L x W x H"	To simple and complex three-dimensional objects	Believe advanced mathematics would allow one to calculate volumes of irregular objects and admit that they do not know that mathematics.
Good Conceptual Understanding	4 - Phys.107 1 - Phys.205	Volume is the number of unit cubes within an three-dimensional object (formal definition)	Use formulae as short-cut ways to count the number of unit cubes	To all three-dimensional objects	No conceptual difficulties identified. Reasoning is not limited to mathematical formulae.

of the interviewees' understanding of, inconsistent ideas about, and conceptual difficulties with volume.

Interviewees in the Category of Preliminary Understanding

Fifteen students were interviewed in the category of preliminary understanding. These students described or defined volume using associated features, such as "volume is three-dimensional space," or "it measures the size of an object." They were unable to correctly calculate the volume of regular solid objects. The following interview excerpts illustrate the three types of conceptual difficulties exhibited by these students.

The First Excerpt and Discussion. This student described volume as three-dimensional space, and calculated the volume in Item 10-i (a drawing of a rectangular prism) using the surface area. In the interview, the student was asked to further explain the concept of volume.

Student: (Volume) is how much of something can fit into a container, say, liquid in a jar, milk in a gallon bottle... the amount of something.

Researcher: How can you figure out the exact volume of an unmarked bottle?

Student: Find (the) area of (the) side panel and (the) bottom panel and add (them) together.

Researcher: Does that also apply to other shaped containers? For example, a regular box?

Student: Yeah, (I) think so. Any container. It's the area around it (that) makes up the volume.

This student had developed a vague idea of volume by associating it with "the amount of something," such as "liquid in a jar, milk in a gallon bottle." This indicates that the student has begun to develop correct, although incomplete, main interpretation knowledge of volume. However, to determine the volume of an object, the student stated that the procedure was to "find (the) area of (the) side panel and (the) bottom panel, and add (them) together," because the volume is "the area around it (the object)." This indicates that the student's conditional knowledge associated with the main interpretation knowledge is incorrect. Here, surface area is used as the application method for determining the volume of three-dimensional objects. Thus, this student demonstrated an inconsistency between the main interpretation and conditional knowledge, and this is the basis of the student's apparent difficulty with the concept of volume.

The Second Excerpt and Discussion. This student described volume as a measure of the space enclosed by a three-dimensional object. While the student demonstrated correct main interpretation knowledge, (s)he used the product of the areas of the three visible faces of that object in a two-dimensional drawing to calculate the volume. In the interview, the student was asked to further explain the idea of volume as well as the application method.

Researcher: You write here (in the Inventory) that volume is "amount of space inside something." Can you explain to me more about your idea?

Student: Sure. (reading Item 9 aloud) ...That's what I meant, amount of space inside something... Like in this box (Item 10-i), volume is the total space inside.

Researcher: Okay. How do you come up with this number (the student's answer to Item 10-i, which required students to assign volume to a regular block)?

Student: There are six units (unit-squares within one of the faces) here, twelve here (within another face of the block), and ...eight here (within the top face)... Then, I timed them up and it ended up (as) five hundred seventy-six units.

This student explained that volume is "the total space inside" a three-dimensional object, indicating that (s)he has developed an initial understanding of the main interpretation knowledge. However, the student's procedure for determining the volume of a rectangular block was to multiply the areas of the three visible faces of the block in a two-dimensional drawing. This, of course, is an incorrect application method. Thus, like the previous student interviewed, this student's difficulty with the concept of volume was also due to an inconsistency between the main interpretation knowledge and conditional knowledge and a failure to conceptualize the three-dimensional unit-cubes behind the two-dimensional unit-squares of each visible face.

The Third Excerpt and Discussion. The student in this interview described volume as "the amount of space something can hold" or "how much to fill a 3-D space." Thus, the student's main interpretation knowledge was apparently correct. However, this student did not answer any of the Inventory items that requested numerical answers for volume. In the interview, the researcher asked the student to elaborate on how to numerically assign volumes to three-dimensional objects. The student reluctantly volunteered:

> I don't know. I don't remember any of my volume formulas... I mean I don't know what to tell you at this point. I don't like math.

Like the previous interviewees in the same category, this student demonstrated correct main interpretation knowledge for the volume concept, but in contrast, did not exhibit any conditional knowledge. This indicates that the student has not developed functional application methods that match his or her main interpretation knowledge. Thus, the student's understanding of the concept of volume is at an initial stage where conditional knowledge is lacking. This lack of complete conditional knowledge blocks the development of the student's concept of volume and predictably results in difficulties in thinking about or accurately determining the volume of a three-dimensional object.

In addition, this student thinks that volume is primarily related to a mathematical formula. Yet, (s)he has a negative attitude towards mathematics (shown by the statement: "I don't like math."). This negative attitude apparently prevents the student from trying to develop further insights about volume.

Overall Discussion. Interviews with students in the category of preliminary understanding demonstrated that these students have developed a vague concept and understanding of volume using only associated features. Thus, the interviews confirmed that their initial placement into the category of preliminary understanding was valid. Furthermore, the interviews also revealed that these students' main interpretation knowledge is correct, but that their conditional knowledge is either incorrect or lacking. Here, three students interviewed used the surface area of an object to calculate volume; the other six multiplied the areas of three faces of the object in a two-dimensional drawing to determine the volume, and yet another six did not determine volume because they had not developed appropriate application methods. This inconsistency between the main interpretation and conditional knowledge evidently hindered the students' development of the concept of volume. Therefore, the students' conceptual difficulties at the stage of preliminary understanding are due to a deficiency in conditional knowledge which is essential to the continual development of conceptual understanding.

Interviewees in the Category of Developing Understanding

Seven interviewees in the category of developing understanding of volume were able to calculate volumes of regular three-dimensional objects by using mathematical formulae (for classified standard cases) and to add volumes of smaller regular sub-units (applying the additive property) to obtain the volume of more complex ones. However, they were unable to determine volumes for irregular three-dimensional objects. The following interview excerpt illustrates these students's concept of and reasoning about volume.

Interview Excerpt. This student used the word "displacement" to describe volume in his answer to Inventory Item 9, which asked students to describe or define volume in their own words. The student correctly assigned volumes to regular three-dimensional objects in Inventory Items 10 and 11 by using the formula "V = L x W x H," and responded to Item 13, which asked for an explanation of how one could find the volume of an irregularly-shaped solid, by "the method of water displacement."

In the interview, the researcher asked the student to describe the concept of volume.

Student: Volume is the entire space contained in a 3-D limited region. It can be filled with something, or (it) can be empty.

Researcher: How do you fix a value to this space?

Student: (To) figure out the volume... You'd need some type of math formulas...

Researcher: Can we do it without using any math formula?

Student: Ohm... No. Because we need some type of formula to work out the calculation. You know, length times width times height for a box, and another formula for a ball or a pyramid. ......I know I can check those formulas out in my math book.

Researcher: In Item 13 of this inventory, we asked you to explain how to find the volume of an irregularly-shaped solid. You said to use the method of water displacement. Can you think of any other methods?

Student: ...Probably by breaking it down to measurable sectors, so the dimensions can be taken, and then figure out each sector's volume.

Researcher: How?

Student: ...Subdivide the solid into normal shapes so their volumes can be easily worked with, and then add up all volumes.

Discussion. This student understood that volume "is the entire space contained in a 3-D limited region... it can be filled with something, or (it) can be empty." The student was also able to use appropriate formulae and the additive property to determine volumes for more complex three-dimensional objects. This indicates that the student has developed correct main interpretation and conditional knowledge using appropriate classified standard cases and associated features. However, this understanding of the concept of volume is at the developing stage because the full implications of the formal definition are not understood. This was evidenced by the student's inability to determine volumes of irregular solids. In contrast to students who have preliminary understanding, this student did not exhibit any inconsistent ideas between main interpretation and conditional knowledge. However, (s)he did demonstrate difficulties with the concept of volume and the student's thinking was linked to mathematics or available formulae. This was clear in the student's statements that "(to) figure out the volume, ...you'd need some type of math formulas," and "because we need some type of formula to work out the calculation (for determining volume)." Even when thinking about the volume of an irregular solid, the student's procedure was to "subdivide the solid into normal shapes so their volumes can be easily worked with (by using formulae), and then add up all volumes (of these 'normal shapes')."

Overall, this student as well as the other interviewees in the category of developing understanding demonstrated that they have developed correct main interpretation and conditional knowledge which are based on classified standard cases and the additive property. This confirmed their placement in the developing category, because they were unable to explain how to solve the problem for the volume of irregular objects by counting the number of unit cubes within the object. In other words, these students still lacked a deeper understanding of the implications of the formal definition of volume. Furthermore, these interviews also demonstrated that the students' thinking and reasoning about volume were limited to mathematics or available formulae. Thus, the students' understanding is confined to mathematics rather than based on a generalized idea of volume as the number of unit-cubes within a three-dimensional object.

Interviewees in the Category of Good Conceptual Understanding

Five students in the category of good conceptual understanding were interviewed. These students understood the implications of the formal definition of volume as the number of unit cubes contained in a three-dimensional object, and demonstrated this understanding in various ways by indicating that formulae are short-cuts for counting unit-cubes. The following interview excerpt illustrates how these students conceptualized volume and how they used that understanding to reason about volume.

Interview Excerpt. This student described the concept of volume in the Inventory as "the measure, in cubic units, of how much a certain space contains," and correctly answered all the volume items. The student responded to Item 13 (which asked students to explain how one could determine the volume of an irregularly-shaped object) by stating: "Using Archimedes' principle (meaning the water displacement method), displace a measured amount of water (or other liquid) and obtain a measure of the volume of water displaced."

In the interview the researcher asked the student to explain his or her thinking when speaking about volume.

Student: Volume to me represents a 3-D space, measured in uniform cubed-units.

Researcher: What do you mean by uniform cubed-units?

Student: How many uniform cubes a space can hold. So, a volume of twenty-four cube-feet represents a space (that) can be occupied by matter, which can be as big as twenty-four cube-feet.

Researcher: Why does water displacement work for the problem in Item 13 in this Inventory, do you think?

Student: Because the water it (the solid) displaced occupies the same amount of space, and water is liquid, it can fill up any container. So the volume can be measured easily.

Researcher: What do you think about the role of mathematical formulas in assigning volume values? Do we always need them or can we do without them?

Student: ...Math makes it easier, that's all. But we can do without math. Say, for the solid, if it can be ground up, then we can reshape it into equal cubes, uniform cubes, and see how many cubes we get...

Discussion. This student stated that volume was how many uniform cubes a three-dimensional space could hold and indicated that formulae were short-cut ways of counting the unit-cubes, showing an understanding of the fundamental idea of volume. This demonstrated the student's more complete main interpretation knowledge of volume and, thus, confirmed the student's good conceptual understanding. In addition, the interview demonstrated that the student's concept of volume is not limited to mathematical formulae for classified standard cases. This was clear in the student's statements about the role of mathematics: "Math makes it easier, that's all. But we can do without math. Say, for the solid, if it can be ground up, then we can reshape it into equal cubes, uniform cubes, and see how many cubes we get." Thus, the student's thinking and reasoning about volume are not confined by mathematics but are based on the idea of unit-cubes contained by a three-dimensional object.

Overall, interviewees in the category of good conceptual understanding demonstrated their concept of volume as the number of unit-cubes, indicating a deeper understanding of the volume concept. These student also indicated that mathematical formulae were short-cut methods for counting unit-cubes for regular objects, indicating their understanding of classified standard cases. Thus, these students have developed a meaningful concept where all forms of main interpretation and associated conditional knowledge are understood and used. This more complete understanding was also confirmed in the interviews. For example, in the above excerpt the student explicitly explained the meaning of volume by the number of unit-cubes. (S)he also indicated that the volume of a regular object could be determined by using appropriate formulae, but the use of formulae was not necessary because it was only a short-cut way for counting the unit-cubes within the object. Thus, these students did not exhibit any difficulties with the volume concept; rather, their conceptual understanding is more complete and their thinking and reasoning processes are not confined to mathematical procedures or available formulae.

Pressure and Density Interview Data and Analysis

Overview

Eight students, who previously participated in the area and volume interviews, were also interviewed a second time to determine their conceptual understanding of pressure and density and to begin to investigate the relationship between area and pressure and volume and density. All of these students were from the Physics 107 class and represented the range of conceptual understanding of area and volume. The interviews were conducted after the concepts of pressure and density were introduced in the Physics 107 course. Each interview, which took approximately thirty minutes, was audio-taped and then transcribed.

The questions in the second interview were structured to gather additional information about a student's concepts of pressure and density and to investigate the nature of the link between these concepts and the student's understanding of area or volume. Each of these interviews was initiated with a general question such as "would you explain to me what pressure (or density) means to you?" Once an interviewee replied to this lead question, the researcher followed the answer with further questions based upon the response. These questions were used to further elicit from students their understanding of pressure or density, especially regarding its relationship to his or her understanding of area or volume.

After the audio-tapes of each interview were transcribed, they were analyzed to identify propositional statements, and these, in turn, were used to establish each person's main interpretation and conditional knowledge for pressure and density according to previously established criteria. Once the students' concepts of pressure and density were determined, a second analysis of the transcripts was conducted to identify the links between each student's understanding of pressure or density and his or her understanding of area or volume.

Student Concepts of Pressure

A student's concept of pressure is operationally defined as the forms of main interpretation knowledge and appropriate conditional knowledge s(he) was able to demonstrate in response to the appropriate questions in the pressure interview. A student's main interpretation knowledge was identified using the following criteria which were established prior to the analysis of the interview data.

Criteria for Identifying Students' Main Interpretation Knowledge of Pressure

Identifying students' main interpretation knowledge for pressure was determined using the following criteria:

Formal Definition. The formal definition used in this study is that pressure is the ratio between the normal force and the area on which that force is exerted. Thus pressure is the force per unit area. This definition applies to all forces over any surface and operationally requires one to determine the ratio between the magnitude of a perpendicular force and the area on which it is applied.

The criteria for determining a student's knowledge of and ability to use this formal definition are: (a) explicitly stating or mentioning that pressure is force per unit area, or (b) indicating or demonstrating that pressure is the force uniformly distributed on a surface.

Classified Standard Cases. A classified standard case for a perpendicular force exerted on a closed two-dimensional figure is that pressure is the net force divided by area (P = F / A). This classified standard case applies to all forces and two-dimensional surfaces that are perpendicular to each other. The associated application method is to divide the net force by the area.

The criterion for determining a student's knowledge of and ability to use this classified standard case is that (s)he must explicitly state or mention that pressure is force divided by area.

Associated Features. Associated features of pressure used in this study includes: (a) force, (b) area, and (c) that pressure is a derived quantity related to the force and area.

The criteria for determining a student's knowledge of these associated features are one or more of the following: (a) indicating or demonstrating the idea that pressure is force, (b) demonstrating the idea that pressure is force which is applied to an area, and (c) demonstrating the idea that pressure is a derived quantity related to both the force and area.

Results

Table 9 provides an overview of the results of the analysis of the interviews with students about their concept of pressure and the nature of the links between their conceptual understanding of pressure and their understanding of area. The following sections discuss the interviewees' main interpretation and conditional knowledge for pressure and how this understanding may be connected to the student's understanding of the area concept.

Student A. Based upon the analyses of the Inventory and the area interview, this student's understanding of area was classified at the stage of preliminary understanding and his or her conceptual difficulties with area were identified as an inconsistency between the main interpretation and conditional knowledge. This student used an incorrect application method (perimeter) to determine area.

The student's concept of pressure is demonstrated in the following interview excerpt.

Researcher: How do you define the concept of pressure?

Student: Force divided by area.

Researcher: What do you mean by force divided by area? Can you explain more about your thinking? I mean, why is force divided by area?

Student: So we can tell how strong a force is on a surface.

Researcher: Can't you know that by the force itself? Why do we need pressure?

Table 9

Results of the Analysis of the Pressure Interviews

Student	Understanding of Area	Conceptual Difficulties with Area	Concept of Pressure: Main Interpretation Knowledge	Concept of Pressure: Application Methods	Concept of Pressure: Applicability Conditions	Ideas About or Difficulties with Pressure that are Linked to the Understanding of Area
A	PU*	Incorrect application method - perimeter is used to determine area	1. Pressure is force divided by area (Classified standard case) 2. Pressure is an evenly distributed force	Force divided by perimeter	To all active forces and all surfaces	The greater the perimeter (area), the smaller the pressure (when force is constant). Main interpretation knowledge of pressure is correct but the application method is affected by an incorrect idea of area.
B	DU**	Mathematics or formulae must be used to determine area	Pressure is passive force (associated feature)	None	None	Pressure is force and it is not related to area.
C	DU**	No confidence in one's own math ability to determine irregular area	Pressure is force exerted over an area (classified standard case)	Force divided by area	To all regular surfaces	The greater the area, the smaller the pressure (with constant force). But the idea of uniform distribution of pressure has not developed.

Table 9--Continued

Student	Understanding of Area	Conceptual Difficulties with Area	Concept of Pressure: Main Interpretation Knowledge	Concept of Pressure: Application Methods	Concept of Pressure: Applicability Conditions	Ideas About or Difficulties with Pressure that are Linked to the Understanding of Area
D	DU**	The area of an irregular figure cannot be determined	Pressure is force exerted over an area (classified standard case)	Force times area	To all regular surfaces	Pressure on an irregular surface may be measured by devices. Does not think pressure can be calculated because of an inability to determine irregular area.
E	DU**	Area for an irregular surface can only be determined by advanced mathematics	Pressure is force per unit-area (formal definition)	Force divided by area	To all surfaces	Pressure is uniformly distributed on a surface, but the idea of pressure as force per unit area is not verbalized.
F	GCU***	None	Pressure is force exerted on things (Associated feature)	Force times area	To any surface	Not available

Table 9--Continued

Student	Understanding of Area	Conceptual Difficulties with Area	Concept of Pressure			Ideas About or Difficulties with Pressure that are Linked to the Understanding of Area
			Main Interpretation Knowledge	Application Methods	Applicability Conditions	
G, H	GCU***	None	Pressure is force on an unit-square area (formal definition)	Force divided by area	To all surfaces	Pressure is force per unit area. This full conceptualization is enhance by their previous understanding of area as the number of unit squares.

* Preliminary Understanding
** Developing Understanding
*** Good Conceptual Understanding

Student: Because (pause) pressure is different from force. Force is (an) acting thing. Pressure is force divided by area. So, if a greater area is used to absorb all the force, then less of the force will distribute to each amount of the smaller area (that) supports the force.

Thus, this student's concept of pressure is apparently correct and reasonably well developed, because the main interpretation knowledge of pressure as force divided by area and the meaning of this ratio is understood. However, when the student was asked to determine the pressure of an iron block on a table, (s)he massed the block, measured the perimeter of the surface that was in contact with the table, and calculated pressure by dividing the mass by the perimeter.

Researcher: Why do you divide the mass by the length of the side (perimeter)?

Student: Because pressure is force divided by area. Weight is (the) force here, it's acting on the table. And this (the perimeter) tells the amount of area the force is working on. So, force divided by area, it's pressure.

Researcher: What if we put this side down (re-sets the iron block on another side that has a smaller area)?

Student: Take (the) weight divided into the new area. It's (the area is) much smaller (the student is roughly measuring the perimeter using a ruler)... So the pressure will be stronger.

The student continues to use the perimeter to calculate the area of a surface, and this incorrect method for determining area is also used to calculate pressure. This procedure indicates that the student's conditional knowledge of pressure is incorrect and is directly caused by the student's incorrect understanding of area.

The connection between the student's concept of pressure and the understanding of area is interesting. Here, the student's incorrect method for determining pressure seemed to make sense to him or her because it appears to verify his or her correct main interpretation knowledge. Earlier in the interview, the student demonstrated an understanding that the same force over a larger area results in a smaller pressure. Since a larger area also tends to have a larger perimeter, the student reasoned that the larger perimeter (the student's definition of area) resulted in a smaller pressure. Thus, this reasoning supported the student's understanding of the main interpretation knowledge, so the idea of perimeter as a measure of area did not seem to hinder the student's conceptual development of pressure at this stage. In other words, the student's understanding of the main interpretation knowledge for pressure is not yet affected by his or her incorrect understanding of area, but of course the associated conditional knowledge is affected.

Student B. This student's concept of area was classified at the developing stage based on the analysis of the Inventory. The student's difficulties with the area concept, based upon the analysis of the area interview, was due to the failure to understand area as the number of unit squares within a closed two-dimensional figure and the limitation of thinking and reasoning about area using only mathematical formulae. The student further believes that the area of irregular figures must be determined using advanced mathematics.

The following excerpt from the pressure interview displays the student's understanding of the pressure concept.

Researcher: How do you define the concept of pressure?

Student: Pressure? Uhmm... I think (pause) is the amount of force that could be applied to something.

Researcher: Okey. Then, what is the difference between force and pressure?

Student: Force, you have to apply more work to, while something can already have pressure with not much work... So, we do work when we push. But pressure, like the air has pressure, but (it) doesn't move anything. So, no work is done.

Apparently, this student's concept of pressure was restricted to a passive force while the term, force, alone applied to active forces. This indicates that the student is relating pressure with an associated feature, force, but has not begun to develop the idea that pressure is a concept different from the force and that it is related to area. This is further demonstrated in the interview when the researcher asked about the relationship between pressure and area.

Researcher: So, you mean pressure is a type of force... Do you think pressure has anything to do with area?

Student: Area? Uhmm... I don't think pressure has anything to do with area. (pause) For sure pressure is against (the) area (of surfaces). But it's basically force, except it doesn't move things... You need things to feel pressure, and everything has area. But pressure is always there, like the air (has pressure), (no matter) you got things (areas) or not.

The student has not conceptualized pressure as a new physical entity relating the force with area. As his or her explanation of pressure indicated, the student's concept of pressure is intuitive and not differentiated from the concept of force. Thus, at the current time, this student is not connecting the concept of area with his or her idea of pressure (as force). Since in the student's mind the pressure is independent of area, the relationship between these two concepts for this student cannot be studied any further.

Student C. This student's understanding of area was also classified at the developing stage based upon the analysis of the Inventory. Like the previous interviewee, this student's conceptual difficulties with area were also due to a failure to understand the implications of the formal definition of area as the number of unit squares within a closed figure. In addition, the student indicated in the area interview that formulae must exist for calculating areas of all regular and irregular figures, but that (s)he did not know those for irregular figures.

The student's concept of pressure was demonstrated in the pressure interview. The following excerpt illustrates that.

Researcher: How do you define the concept of pressure?

Student: ... I think it's force exerted over an area.

Researcher: How do you calculate it, then?

Student: ... It's force divided by area.

Researcher: What do you mean by force divided by area? I mean, what does pressure mean to you when you say it's a force divided by an area?

Student: It means, I think, for same force, (the) smaller (the) area, (the) greater the pressure. Because pressure is force exerted over a whole area, if the area becomes smaller, the pressure will be greater, until (the area is reduced) to a point, then it's (the pressure is) the force itself."

Researcher: I see. Do you think the force is working on the whole area evenly? I mean, for example, do two equal areas within the surface get equal share of the force?

Student: (It) must be so. But I'm not sure (it) has to be even on the entire surface. (It is an) interesting question though. (Laughed.)

The excerpt demonstrates that this student has developed correct main interpretation knowledge that pressure is the force exerted on a surface and then associated this definition with the appropriate formula, P = F / A. However, the student did not understand pressure's property of uniform distribution over the surface. This indicates that the idea of pressure as force per unit area is not fully conceptualized, and that this understanding is related to the student's understanding of area where classified standard cases are used and the thinking is restricted to formulae.

Student D. Like the previous two interviewees, this student was also classified in the category of developing understanding for his or her concept of area. The area interview with him/her also indicated that this student did not think that the area of irregular surfaces could be calculated, because no formulae were available.

The student's concept of pressure is illustrated in the following interview excerpt.

Researcher: Can you describe what pressure is to you?

Student: Pressure is force that one thing exerts on another one.

Researcher: So, it's force. Then, what's difference between pressure and force?

Student: Pressure relates to area, force doesn't.

Researcher:	How is pressure relate to area?
Student:	It's like (the) pressure the tape recorder exerts on the table (pause) is the area of the bottom of the recorder (pause) times the force gravity.
Researcher:	Okey, look, I now put the tape recorder down so it has a bigger contact area with the table. What do you think about the force and pressure it exerts on the table now? Are they changed or not?
Student:	The force should be (the) same... The pressure should be changed. It should be different because the area's different. (Laugh, indicating lack of confidence about what he/she just said.)

This student has apparently begun to understand that pressure is the force on a surface, indicating a development of the correct main interpretation knowledge. However, the student believes that pressure is equal to the product of force and area, indicating that an appropriate application method is not developed. To probe the linkage between the student's thinking of pressure and the understanding of area, the researcher asked the student to explain how to determine the pressure on an irregular surface. The student responded by:

> It's a question. (laugh)... I'm not sure. (Then paused to think.) I know we won't figure out the area, but we need it to figure out pressure. Uhmm... Probably we can't (calculate pressure). Maybe those physics devices can take readings? (The student seems thinking about it and searching for an answer, then, gave up.) I don't know, I've never had a problem like this.

Thus, as the student's statement indicates, (s)he was unable to determine area for an irregular surface and did not believe such area can be determined. As a result of this understanding of area, the student was not able to think of a method to determine the pressure exerted on this surface. Therefore, this student's difficulties with the area concept apparently influenced his or her understanding of pressure, and the idea of pressure as force per unit area will be predictably difficult for the student to conceptualize because of his or her incomplete understanding of how to calculate area.

Student E. This student's understanding of area was also rated at the developing stage according to the analysis of the Inventory, and his or her difficulties with area were identified from the area interview as the inability to determine the area of an irregular surface. Furthermore, this student's thinking and reasoning about area appeared to be influenced by the belief that irregular areas can only be determined by advanced mathematics.

The student's concept of pressure is demonstrated by his or her answers to the interview questions illustrated in the following excerpt.

Researcher: Can you define the concept of pressure?

Student: Yeah. Pressure is force per unit area.

Researcher: Can you explain more about it?

Student: O.K. Pressure, to me, is (a) force (that) exerts equally on a certain area. See, if a force is acting on an area, the force is distributing to the whole region, so each smaller part (of the area) gets an equal proportion (of the force).

The student's explanation indicates that his or her understanding of pressure is correct where the idea of pressure as a force uniformly distributed on a surface is clearly understood. According to this criteria, the student has developed a good understanding of the pressure concept because the implications of the formal definition of pressure as a force uniformly distributed on a surface is well understood. The student, however, is not able to explicitly verbalize the idea of pressure as force per unit square, and this appears to be consistent with the student's inability to think about area as unit squares.

Student F. This student's understanding of area was classified at the stage of good conceptual understanding based on the analysis of the Inventory. The student also demonstrated that his or her reasoning about area was not confined to mathematics or formulae and did not exhibit any conceptual difficulties with area, according to the area interview.

Despite a good conceptual understanding of area, the student was not able to demonstrate a significant development of the pressure concept. This is illustrated by the following interview excerpt.

Researcher: Can you tell me what pressure is to you?

Student: (Hesitating, then slowly) Pressure is force exerted on (the) top of something.

Researcher: So, it's force?

Student: (Pause, and then unwillingly) Pretty much. I don't know. I don't get time to study (it)...

Researcher: That's O.K. Just tell me as much as you can... Have you learned how to calculate pressure?

Student: (Thinking.) Force times area, maybe? I don't know for sure. As I told you, I don't get to study this yet.

Researcher: That's fine. So, you think pressure also relates to area, besides force?

Student: Uhmm... Maybe. Is it?

Similar to Student B, this interviewee has not developed an appropriate concept of pressure. As the student explained, (s)he has not spent any time to study the concept. Therefore, his or her concept of pressure remains intuitive and it is not differentiated from the concept of force. In other words, although this student's understanding of area is at the stage of good conceptual understanding, (s)he is not connecting the area concept with his or her intuitive idea of pressure (force). Therefore, the relationship between the two concepts for this student cannot be studied at this time.

Students G and H. These two students' concept of area was rated at the stage of good conceptual understanding by the analysis of the Inventory, and they did not exhibit any conceptual difficulties with area. Their thinking and reasoning were not limited to mathematical procedures or formulae.

The students' understanding of the pressure concept are also well developed. This was demonstrated in the interviews with them when the researcher asked the students to describe their idea of pressure. One student described:

> Pressure is force exerted over certain area, which (is) averaged out to each unit square.

The other student explained:

> Pressure is how strong a force is on an area. It's force, uniform force exerted on an unit square within an area.

Thus, these students evidenced a deeper understanding of the pressure concept where the idea of force per unit area is comprehended and operationalized. In addition, the fact that these students explicitly used the term unit square in their description and explanation of pressure further indicates that their thinking about pressure was influenced by their thinking about area where the idea of unit square is fundamental.

Overall Discussion

Interviews with students about their concept of pressure show that among eight interviewees, three have developed a deeper conceptual understanding of pressure since the concept is understood as force per unit area. Three other students have developed an incomplete conceptual understanding of pressure since their main interpretation knowledge is based on the classified standard case that pressure is the force divided by area, but not the implication of the formal definition that pressure is force per unit area. Two of these students also demonstrated incorrect conditional knowledge. The remaining two students were unable to distinguish between pressure and force. Therefore, they have not developed a significant understanding of the concept of pressure.

The pressure interviews also demonstrated some significant connections between these students' concept of pressure and their conceptual understanding of area. Specifically, of the three students who have developed a deeper understanding of the pressure concept, two (Students G and H) were rated as having good conceptual understanding of area as the number of unit squares within a closed figure. These students also evidenced a well developed concept of pressure as they explicitly verbalized it as force per unit area. The students' explanations about their idea of pressure clearly indicate a connection between their understanding of pressure and their understanding of area since both concepts are described using the term unit squares. The third student (Student E) was classified at the stage of developing understanding for his/her area concept. In the pressure interview, the student indicated that pressure was uniformly distributed force on a surface and was quantitatively determined by dividing the force by the area. Thus, the student's concept of pressure was well developed. However, the idea of pressure as force per unit area was not explicitly verbalized by the student. This again indicates a similarity between the student's thinking about area and the thinking about pressure because the idea of unit area is not used when describing both the concepts.

Among the three students who have attained correct main interpretation knowledge of pressure and the classified standard case that pressure is force exerted on an area, one (Student A) was placed in the category of preliminary understanding of area and two (Students C and D) have achieved developing understanding. Despite their different levels of understanding of the area concept, each of these students has developed a generalized idea of pressure that when an exerted force remains unchanged and the area is increased, the pressure will be decreased. Yet, the interviews again revealed that these students' understanding of and difficulties with the concept of area did influence their conceptualization of pressure. In particular, Student C whose thinking and reasoning about area is bound to formulae is also unable to conceptualize pressure as an uniformly distributed force on a surface or think about pressure as force per unit area. Rather, the student's approach to the pressure concept is to link it directly with formulae, which is consistent with his or her understanding of the area concept. Student D does not believe that an irregular area can be determined because no formulae is available. This student indicated a similar idea in the pressure interview that pressure on an irregular surface could not be calculated because the area could not be determined. Lastly, Student A holds an incorrect conditional knowledge for area where the perimeter is used to calculate areas. The student brings this incorrect method to the pressure concept (force divided by perimeter) and it seems to make sense because perimeter increases when area increases and, thus, the idea that the greater the area (perimeter) the smaller the pressure appears to be verified. Therefore, this incorrect understanding of the area concept is apparently connected to the student's incorrect understanding of pressure, and therefore is affected by his or her understanding of area.

The two remaining students interviewed (Students B and F) have not developed a significant understanding of the pressure concept. They do not even distinguish between pressure and force or think that pressure is related to area. Therefore, the relationship between these students' conceptual understanding of area and their concept of pressure is not able to be studied at this time.

Student Concepts of Density

A student's concept of density is operationally defined as the forms of main interpretation knowledge and appropriate conditional knowledge (s)he was able to use to answer the density questions in the interview. A student's main interpretation knowledge was identified by specific criteria which were established prior to the analysis of the interview data.

Criteria for Identifying Students' Main Interpretation Knowledge of Density

Identifying students' main interpretation knowledge for density was determined using the following criteria:

Formal Definition. The formal definition used in this study is that density is the mass per unit volume of an object. This formal definition implies that density is an identifying property of a material and operationally requires one to determine the mass and volume of an object and then to calculate the density by dividing the mass by the volume.

The criteria for determining a student's knowledge of and ability to use this formal definition are: (a) explicitly stating or mentioning that density is mass per unit volume, or (b) indicating or demonstrating the idea that density is the amount of mass contained in any unit volume of an object.

Classified Standard Cases. A classified standard case for density is mass divided by volume (D = M / V). This classified standard case applies to all materials uniformly distributed in three-dimensional objects.

The criterion for determining a student's knowledge of and ability to use this classified standard case is that they must explicitly state or mention that density is mass divided by volume.

Associated Features. Associated features of density used in this study include: (a) mass, (b) volume, and (c) that density is a derived quantity related to mass and volume.

The criteria for determining a student's knowledge of these associated features are one or both of the following: (1) indicating or demonstrating the idea that an object's density involves mass, (2) demonstrating the idea that density of an object involves mass and volume.

Results

Table 10 provides an overview of the results of the analysis of the interviews with students about their concept of density and the nature of the links between their conceptual understanding of density and their understanding of volume. The following sections discuss each interviewee's main interpretation and conditional knowledge for density and whether or not this understanding appears to be linked to the student's conceptual understanding of volume.

Student AA. Based upon an analysis of the Inventory, this student's conceptual understanding of volume was classified at the stage of preliminary understanding. A follow-up interview for volume further revealed that this student's conceptual

Table 10

Results of the Analysis of the Density Interviews

Student	Understanding of Volume	Conceptual Difficulties with Volume	Concept of Density			Ideas About or Difficulties with Density that are Linked to the Understanding of Volume
			Main Interpretation Knowledge	Application Methods	Applicability Conditions	
AA	PU*	Incorrect application method - surface area is used to determine volume	Density is a property of material. It describes the compactness of matter in space.	Mass divided by volume	To all materials	Conceptual understanding of density is linked to and supported by main interpretation knowledge for volume, but not affected by the conditional knowledge for volume.
BB	PU*	Incorrect application method - product of the areas of three faces is used to determine volume	Density is a property of material. It describes the massiveness of materials.	Same as above	Same as above	Same as above

Table 10--Continued

Student	Understanding of Volume	Conceptual Difficulties with Volume	Concept of Density			Ideas About or Difficulties with Density that are Linked to the Understanding of Volume
			Main Interpretation Knowledge	Application Methods	Applicability Conditions	
CC DD	PU*	Application method is not yet developed	Density is mass per unit volume, and it is a physical property of a material.	Mass divided by volume	To all materials	Main interpretation knowledge for volume plays an important role in the conceptualization of density, but conditional knowledge for volume does not have a significant impact on it.
EE	DU**	Thinking is restricted to mathematics and formulae	Density is a property of material. It equals the amount of mass contained in a given volume.	Same as above	Same as above	Same as above

Table 10--Continued

Student	Under-standing of Volume	Conceptual Difficulties with Volume	Concept of Density: Main Interpretation Knowledge	Concept of Density: Application Methods	Concept of Density: Applicability Conditions	Ideas About or Difficulties with Density that are Linked to the Understanding of Volume
FF GG HH	GCU***	None	Density is mass per unit volume, and it is a physical property of all materials.	Mass divided by volume	To all materials	Understanding of and reasoning about density is connected to the conceptual understanding of volume.

* Preliminary Understanding
** Developing Understanding
*** Good Conceptual Understanding

difficulties were due to an inconsistency between main interpretation and conditional knowledge. This student used an incorrect application method (surface area) to determine volume.

The following excerpt from the density interview illustrates the student's thinking about density.

Researcher:	Would you describe to me what density means to you?
Student:	I think density deals with (pause) how dense, how hard an object is.
Researcher:	What do you mean? You mean density tells us how hard something is?
Student:	Yeah, basically. (pause) We know things can be dense or not so dense. Dense things, like this iron bar, is heavy and hard. And things like these styrofoams are light and soft. We can actually see holes in them, they are not packed closely.
Researcher:	What are not packed closely?
Student:	Mass.
Researcher:	I see. Now, I have two bars on the table, they have same volume and look the same, but this one is heavier than that one. Do you think they have the same density?
Student:	No... This one (the heavier one) should have (a) higher density.
Researcher:	Why?
Student:	(Because) it's heavier, it's denser.
Researcher:	What about these two. They weigh the same and look the same (one is bigger than the other). Do they have same density?
Student:	Uhmm... (I) don't think so. They are made of different materials.

Researcher: So, what does density depend on? I mean how do we calculate density?

Student: Uhmm... I think it's weight divided by volume... That's what we did in the experiment (The student has done a density experiment in the previous week in Physics 107 laboratory)...

Researcher: Do you get same density for all of them (the students calculated density for three differently shaped objects made of lucite)?

Student: (I) don't remember. (I) think they are pretty close for the plate and (the) cylinder. But our number for the little solid was off. We probably screwed up somewhere, (because) they should come up real close, they (are) made of same type of material...

Researcher: Why is same material having same density?

Student: That's what we learned. (Because) they (the objects) are made of same material, so the (mass) is basically packed the same way, right?

Researcher: Why is mass packed the same way then the density is the same?

Student: So (a) same amount of mass (would) be packed in (a) same amount of space.

This student believes (main interpretation knowledge) that density describes the compactness of material in a given space, indicating a beginning understanding of density as an identifying physical property of a material. The student's conditional knowledge of density is also correct, where the application method of dividing the mass of an object by its volume is articulated and used. Thus, this student demonstrates a reasonably well developed understanding of the density concept.

The student's spontaneous recall of the formula for density demonstrates that his or her conceptualization was more conceptual than operational. For example, the student consistently described his or her concept of density using the idea of the compactness of material on a micro level rather than the idea of how it is computed in a macro level. In addition, the student did not verbalize the idea of density as mass per unit volume, although (s)he understood that the same amount of mass of the material would occupy the same amount of space. This indicates that density is more directly conceptualized as a property of materials rather than the operational derivation of mass per unit volume. Therefore, the student's concept of density is linked to his or her main interpretation knowledge of volume; however, his/her incorrect conditional knowledge of volume did not seem to significantly affect the student's conceptualization of density.

Student BB. Like the previous interviewee, this student was also categorized as having a preliminary understanding of volume based upon the analysis of the Inventory. The follow-up interview revealed that this student's conceptual difficulties with volume were also due to an inconsistency between main interpretation and conditional knowledge. This student used the product of the areas of the three visible faces of a three-dimensional object in a two-dimensional drawing to determine volume.

In the density interview the student demonstrated a similar conceptual understanding as illustrated by the following:

Researcher: Would you describe to me what density means to you?

Student: (Thinking, then slowly) Mass divided by volume.

Researcher: What does that mean? What does it tell us? Let's say, a mass divided by its volume always ends up with a number. What does that number tell us?

Student: It tells (pause) how heavy the material is, compare to other types, you know. Iron is heavier than wood, wood is heavier than styrofoam, and so forth.

Researcher: So, you mean it relates to a kind of material and decided by mass and volume?

Student: Yeah. Every material has density, (because) the molecules in it are packed in a certain way. Different materials have different densities, the molecules made up them are different.

This student has also developed a good understanding of the main interpretation knowledge of density as a measure of the massiveness of a material. The conditional knowledge that density is determined by dividing the mass by the volume is also used. Thus, this student's conceptualization of density is reasonably complete.

The student explained the idea of a connection between mass and volume by stating that density is a physical property describing the compactness of matter in space rather than indicating that it is the result of a computation of dividing the mass by the volume. This demonstrated that the student's thinking about density was primarily conceptual. Similar to the previous student, this student did not explicitly verbalize the idea of density as mass per unit volume in the interview, indicating again that density is thought about as a property of materials rather than as a derived quantity. Thus, this student's concept of density is supported by his/her main interpretation knowledge of volume, but his/her incorrect conditional knowledge of volume did not appear to affect the student's conceptual development of density.

Student CC and DD. These students were categorized at the level of preliminary understanding of volume based on the Inventory. Their main interpretation knowledge was that volume is a measure of three-dimensional space, but the follow-up interviews about volume revealed that they did not know how to calculate volume for any objects. Thus, these students' incomplete concept of volume was apparently related to their lack of appropriate conditional knowledge.

In the density interviews, the students demonstrated an ability to articulate the idea of density as mass per unit volume. This is shown in the following excerpts.

When the researcher asked them to describe the concept of density, Student CC said:

> (Density is) the mass of an object divided by its volume. It tells the way the matter is put together. For instance, this solid (an aluminum block) is made of a type of matter (that its constituents are) closely attached together, the density is high, since there is more mass in every smaller volume.

Student DD explained:

> We did the density experiment last week. I know what we (were) supposed to do was to verify the density of lucite was constant... What we did was to compare the weight (meaning mass) to volume... So, for each tiny volume the weight (mass) should be the same... For all the pieces (of the objects used in the experiment), we (were) supposed to get same density, because we know they (are) all made of lucite.

These students articulated their main interpretation knowledge of density as mass per unit volume and understood that it was a unique physical property of a material. They also expressed their conditional knowledge of density as mass divided by volume. Thus, these students have developed a more complete understanding of the density concept than Students AA and BB.

What is especially interesting about these students' conceptualization of density is that they did not previously demonstrate a good conceptual understanding of volume as the number of unit cubes. Neither did they know how to calculate volume. In other words, although the students understood that volume measured the amount of three-dimensional space, complementary conditional knowledge of volume was lacking. Nevertheless, these students were able to verbalize the idea of density as mass per volume or mass of equal volumes of a specific material, demonstrating a developing conceptual understanding of density. This nonparallel development between the students' concepts of volume and density indicates that while main interpretation knowledge of volume plays an important role, the conditional knowledge of volume does not seem to have a meaningful impact on students' conceptualization of density. This conclusion was further supported by Student DD's statements in the following excerpt.

> Researcher: You just said something about mass of each tiny volume. What do you mean by "each tiny volume"?
>
> Student: ...By dividing mass by volume, we break down the mass to see how much (mass) would be inside of each segment of base volume...

Apparently, the student understood the meaning of ratio and, therefore, was able to conceptualize density as mass "inside of each segment of base volume." Thus, the fact that (s)he did not think about volume as the number of unit cubes or know how to calculated it did not affect her development of the density concept.

Student EE. This student's conceptual understanding of volume was at the developing stage based upon the analysis of the Inventory, and his or her difficulties with the volume concept were identified from the volume interview as the inability to determine the volume of irregular objects. The student's thinking and reasoning about volume appeared to be influenced by the belief that irregular volumes can only be determined by using advanced mathematics.

The student's concept of density is demonstrated by the following interview excerpt.

Researcher:	Would you describe to me what density means to you?
Student:	Density, for the word is defined in (a) dictionary, means how much matter can fit in a solid. I mean, matters are structured differently inside...
Researcher:	I see. But how do you know the density of a material? How do you decide it?
Student:	(Using the iron bar on the table as an example.) Take the mass on this beam balance, and calculate the volume, and then divide the mass by the volume.
Researcher:	What does it mean when you divide the mass by the volume? Why do we do this?
Student:	That's how you figure out the density of this solid. See how much matter is in a certain volume.

This student's main interpretation knowledge is that density equals the amount of mass contained in a given volume, inferring that density is a property of a material. The student's conditional knowledge is also correct and the density of an object is calculated by dividing the mass by its volume. Thus, like the previous interviewees, this student has conceptualized density reasonably well.

This student's concept of density is articulated as the definition given in dictionaries rather than the formula in physics textbooks. Thus, the student conceptualized density as the amount of matter that filled a three-dimensional space before (s)he continued to quantify this idea as an object's mass divided by its volume. In contrast to his or her understanding of volume (where thinking was restricted to mathematical formulae), this student's thinking about density is not limited by computational methods. Thus understanding is more conceptual than computational. This indicates that the student's conceptualization of density is reinforced by his or her main interpretation knowledge of volume as a generalized measure of the extent of a three-dimensional space, but not meaningfully affected by the conditional knowledge of how to compute it.

Students FF, GG, and HH. These students were all rated as having a good conceptual understanding of volume based on the Inventory and the volume interviews. The students' main interpretation knowledge of volume were stated in terms of the number of unit cubes, and they knew that formulae were short-cut ways to obtain this number. These students did not exhibit any conceptual difficulties with volume, and their thinking and reasoning were not limited to mathematical procedures or formulae.

These students' understanding of the density concept was also well developed. This was demonstrated in the interviews with them when the researcher asked the students to describe their idea of density. Student FF described:

> Density is a property of matter. It's mass per cube centimeter. Some materials are (more) massive than others. Their atoms and molecules are joined together closely. Others' may have more space in between (the atoms and molecules).

Student GG stated:

> (Density is) how dense or how soft (meaning less dense, the student was looking at an iron bar and a styrofoam ball when talking about density; the student interchangeably used hard and dense for the iron bar and soft and sparse for the styrofoam ball to refer the high and low densities) a thing is. It's mass of a certain matter (that) occupied a three-dimensional space in a cube meter type of (measurement)...

Student HH explained:

> Density, to me, is how much mass something contains. What I'm saying is (that) a certain thing is made of a certain type of matter, so (a) same amount of mass is contained in every same amount of volume... Two different things (materials) can't have same density, that's for sure.

Thus, these students evidenced a good understanding of the main interpretation knowledge of density. Here, both the idea of density as a physical property of a material and density as a computational quantity of mass per unit volume are comprehended and operationalized.

The density interviews confirmed the students' good conceptual understanding of volume and demonstrated a deeper understanding of the density concept. The fact that these students explicitly expressed that the density of an object is quantitatively measured by the mass of a unit volume infers a direct connection between the students' conceptual understanding of density and their understanding of volume. Here, they also used the idea of unit volume in their thinking about density, and it parallels their thinking about volume as unit cubes. Thus, these students' conceptual understanding of and the pattern of thinking about volume appear to linked to their more meaningful conceptualization of density.

Overall Discussion

Interviews with eight students show that all of these students have developed various degrees of understanding of main interpretation knowledge about density. They demonstrated the idea of density as a measure of the compactness of matter in space and explicitly indicated that same type of material has the same density. Five of them were able to express the idea of density as mass per unit volume. Furthermore, all these students demonstrated a conditional knowledge that density was obtained by use of the formula, D = M / V.

The interviews also revealed that the students' conceptualization of density was linked to and supported by their generalized idea of the main interpretation knowledge of volume as a measure of a three-dimensional space. The interviews, however, did not show any apparent connection between the students' conceptual understanding of density and their conditional knowledge for volume. This nonparallel development of the density concept and conditional knowledge for volume is demonstrated by Students AA, BB, CC, and DD. Students AA and BB were not able to state correct conditional knowledge for volume, yet they were able to conceptualize density as a physical property of material describing its compactness in space. Students CC and DD had not developed appropriate application methods for calculating volume, nevertheless, they explicitly verbalized the idea of density as mass per volume, indicating a richer understanding of density.

In contrast, interviews with Students FF, GG, and HH showed a connection between the students' good conceptual understanding of volume and their thinking about density. These students apparently used their idea of volume as the number of unit cubes to understand the concept of density, because the idea of density as mass of a unit volume was expressed.

CHAPTER V

CONCLUSIONS

This chapter presents an overview of the study, provides conclusions based upon the evidence, suggests implications for introductory physics instruction, and makes recommendations for future studies.

Overview of the Study

The Problem and the Research Questions

Concepts such as area and volume are foundational ideas for many concepts introduced in introductory science courses. At the college level, most instructors typically assume that incoming students have already developed an understanding of these underpinning ideas. However, doubt has surfaced in recent years about students' depth of understanding and mastery of fundamental concepts, including area and volume. Because deficiencies in understanding these basic concepts may relate to the learning of subsequent concepts, instructors have expressed concerns about students' understanding of fundamental ideas and if the lack of understanding of these ideas hinders students' progress in learning subsequent concepts.

This study was designed to (a) investigate the nature of college physics students' understanding of the area and volume concepts and (b) to begin to inquire into the nature of the relationship between students' understanding of the area and volume concepts and their conceptualization of pressure and density.

The study addressed four specific research questions. They are:

1. What are college science students' understandings of the concepts of area and volume?

2. What characterizes students' difficulties with these concepts?

3. Do students in mathematically more-advanced courses differ in their initial understanding of the area and volume concepts from those in mathematically less-sophisticated courses?

4. Is there any relationship between students' ability to conceptualize pressure and density and their understanding of area and volume?

Design and Methods

This study used Reif and Allen's (1992) Model of Scientific Concept Interpretation to characterize students' conceptual understanding. This model describes conceptual understanding by examining a person's main interpretation knowledge using one or more of the three essential modes of concept interpretation, namely, formal definition, classified standard cases, and associated features. Correct use of each of the modes of main interpretation knowledge is judged by examining the associated conditional knowledge used by the student which specifies the particular application methods and applicability conditions used in specific circumstances.

Students' conceptual understanding in this study was evaluated by their knowledge of and ability to use one or more of the modes of main interpretation knowledge and associated conditional knowledge for a particular concept. Based upon the modes of main interpretation knowledge used, four specific categories were defined to classify conceptual understanding. The categories are: (1) preliminary understanding, (2) emerging understanding, (3) developing understanding, and (4) good conceptual understanding.

The research population consisted of 431 first-year college physics students at Western Michigan University. They were from three types of physics courses: (1) a one-semester conceptual course for students who are not majoring in science, (2) a first-semester algebra-based course for students who are primarily majoring in sciences other than physics or engineering, and (3) a first-semester calculus-based course for physics and engineering students.

Four types of data were collected from this population. First, data describing a student's background in physics, mathematics, and their university status were obtained for each student. Next, data concerning the initial status of each student's conceptual understanding of area and volume were collected using a paper-pencil instrument titled Knowledge of Area and Volume Inventory. Then, data eliciting additional information about students' prior understanding of area and volume were obtained from twenty-seven interviews with individual students. Lastly, data probing students' concepts of pressure and density and if they are linked to students' conceptual understanding of area or volume were obtained in eight additional student interviews. These latter interviews occurred after the concepts of pressure and density were introduced to the students in their respective courses.

Data collected from the paper-pencil inventory and the clinical interviews were analyzed using Reif and Allen's model for interpreting conceptual understanding. In particular, the Inventory data were analyzed to classify each student in one of the four categories of conceptual understanding for area and volume, and to identify student difficulties associated with each stage of understanding. The area and volume interview data were used to confirm the classification of students by the Inventory and to further identify student conceptual difficulties by analyzing each interviewee's main interpretation and conditional knowledge. Finally, the pressure and density interview data were used to establish each interviewee's main interpretation and conditional knowledge for these concepts and to probe the nature of the link, if any, between a student's idea of pressure and density and his or her conceptual understanding of area or volume.

Conclusions

The following conclusions emerge from the analysis of the data and are organized around the four research questions.

What Are College Science Students' Understandings of the Concepts of Area and Volume?

The answers to this research question are based upon the results of the analyses of the Knowledge of Area and Volume Inventory and the subsequent interviews of selected students representing each category of conceptual understanding.

Students' Understandings of Area

About thirty percent of the students participating in this study attained a good conceptual understanding of area prior to their enrollment in an introductory physics course. These students understand that area measures the extent of two-dimensional surfaces and can articulate that it is determined by counting the number of unit squares. They also understand that the number of unit squares enclosed in a surface can be obtained by using a grid or calculated mathematically using an appropriate formula. Thus, these students enter their introductory college physics classrooms with a good conceptual understanding and the ability to articulate and justify their thinking about area.

Sixty percent of students enrolling in these introductory physics courses have not conceptualized area as well as those in the category of good conceptual understanding. These students, classified in the developing category, understand that area measures the extent of two-dimensional surfaces, however, they have not yet developed the idea that area is measured by counting the number of unit squares within a closed figure. These students tend to rely on mathematical formulae to calculate area, but do not comprehend that formulae are only short-cut ways to count the unit squares within two-dimensional figures. In addition, these students are unable to determine the area of irregular figures for which formulae are not readily available. Thus, these students come to their introductory college physics classrooms with a concept of area which is not as fundamental as that of students with the good conceptual understanding. Their concept of area is confined to mathematical procedures and their problem-solving ability is limited to situations that involve readily available formulae and regular two-dimensional figures. These formulae are often viewed by the students as "revealed wisdom" or a magical relationship that provides answers. However, these students often do not comprehend what those answers mean in a concrete way.

In addition to these students, four percent of the student population are at the emerging stage of understanding area. These students comprehend area as a measure of the extent of two-dimensional surfaces. They know how to manipulate simple formulae for calculating the area of regular figures. However, they have failed to conceptualize the idea of counting unit squares and they are unable to use the additive property to determine the area of more complex but regular figures. Thus, these students are entering their introductory physics courses with a shallower understanding of area and their problem-solving ability is limited to the mechanical use of available formulae.

Another four percent of the population are only able to relate area to the preliminary idea of two-dimensional surfaces, a multiplication operation using a formula, or a generalized notion of the size of an object. While their ideas are not necessarily incorrect, they are vague and lack operational definition. In other words, these students enter their introductory physics classrooms with a general, yet vague, and non-specific idea about area. Operationally, they are unable to calculate area or they use related concepts, such as perimeter, to measure area. Therefore, they have not developed any depth to their concept of area that allows them to think of area as a measurement made of unit squares.

The participants in this study were students enrolled in first-semester introductory physics courses offered at Western Michigan University. Since Western Michigan University is a state-supported emerging research university with many undergraduate and graduate programs, the students in this study are likely to represent many students who take introductory physics at different colleges and universities across the United States. In addition, the participants in this study represented a typical range of students enrolling in a series of introductory-level physics courses. They are therefore likely to represent many beginning science students in many colleges and universities. Thus, it seems reasonable to conclude that:

1. Over fifty percent of students who are entering beginning college physics courses have not developed good conceptual understanding of area and that their thinking is limited to the manipulation of mathematical formulae without deep understanding of its implications. Thus, the majority of students think about area as a measurement of two-dimensional spaces that is obtained as a result of a mathematical operation. These students do not understand why they are using the formulae and are not able to interpret the meaning behind the numerical results of these calculations. And,

2. Almost ten percent of students have not yet reached the level of developing understanding of the area concept. Many are only able to use area formulae

mechanically and others are not able to explicitly relate area to the measurement of a two-dimensional space. In other words, about ten percent of students in introductory college physics courses may need immediate remedial help to constructing a working definition of area, if such conceptual understanding is a prerequisite for these courses.

Students' Understanding of Volume

Thirty percent of the students in the sample population of this study had attained a good conceptual understanding of volume prior to enrolling in their introductory physics courses. These students understand that volume is the space occupied by a three-dimensional object and that it is measured by counting unit cubes. These students know how to measure volume by determining the number of unit cubes inside a three-dimensional object. They also know that mathematical formulae are short-cut ways to count these unit cubes. Thus, these students are entering their introductory college physics classrooms with a good conceptual understanding and the ability to articulate and justify their thinking about volume.

Fifty percent of the students enrolled in introductory physics courses have not conceptualized volume as well as their classmates classified at the stage of good conceptual understanding. These students, classified in the developing stage, understand that volume measures the extent of three-dimensional space, however, they have not developed the basic concept that volume is the number of unit cubes contained in a three-dimensional object. Thus, these students tend to rely on mathematical formulae to calculate volume, but do not understand the fundamental idea that counting unit cubes lies behind the use of formulae. Furthermore, these students are unable to determine volume for irregularly-shaped objects for which formulae are not readily available. Thus, these students come to their introductory college physics classrooms with a concept of volume that is not as insightful as that of students' in the good conceptual understanding category. Their concept of volume is confined to mathematical procedures and their problem-solving ability is limited to their recall and manipulation of mathematical formulae. These formulae are often viewed as magical relationships that automatically provide answers, but they do not fully understand what these answers mean.

About twenty percent of the students relate volume to the generalized idea of space, a multiplication operation using formula, or a vague idea of the size of an object. While these ideas are not incorrect, they are vague and lack operational definition. These students enter their introductory physics classrooms classified at preliminary stage of understanding volume. Operationally, these students are unable to calculate volume or they use related concepts, such as surface area, to measure volume. Therefore, they have not developed much insight about volume as an idea that will permit them to comprehend volume as a measurement concept based upon counting unit cubes.

Because the students in this population are likely to represent a typical group of beginning students in many colleges and universities across the United States who take introductory physics, it seems reasonable to conclude that:

1. Over fifty percent of students who enroll in beginning college physics

courses have not yet constructed the good conceptual understanding of volume and their thinking is limited to the manipulation of mathematical formulae without an understanding of what these formulae represent. This means that the majority of students think about volume as a measurement of three-dimensional space that is obtained as a result of a mathematical operation. These students do not understand why they are using the formulae and are not able to interpret the meaning behind the numerical results that emerge from these calculations. And,

2. About twenty percent of students have not reached the level of developing understanding of volume. These students do not clearly think about volume as a measurement of three-dimensional space. Therefore, this twenty percent of students in introductory college physics courses may need remedial help to develop a fundamental concept of volume, if such conceptual understanding is a prerequisite for future work in these courses.

What Characterizes Students' Difficulties With the Concepts of Area and Volume?

The answers to this research question are based upon the results of the analyses of the Knowledge of Area and Volume Inventory and the area and volume interviews of selected students.

Student Conceptual Difficulties With Area

One-third of the students in this study had already attained a good conceptual understanding of area and did not demonstrate any conceptual difficulties. Their thinking and reasoning are largely conceptual, meaning that their thinking processes are supported by the idea of area as the number of unit squares contained in a two-dimensional surface.

Sixty percent of the students at the developing stage understand area as a measurement of a two-dimensional surface, but do not understand that it is fundamentally a count of unit squares. They also demonstrate a range of conceptual difficulties which are related to the rote use of mathematics or formulae. Their thinking about area is flawed because they believe that area can only be determined through mathematical manipulation and they think that these formulae produce correct results without a conceptual understanding to support the mathematical process.

Four percent of the students at the emerging stage had begun to develop the idea of area as a measurement of the extent of a two-dimensional surface. Their conceptual difficulties are related to their failure to understand area as an idea related to measuring surfaces rather than the result of a mathematical calculation without consideration of what it means.

Another four percent of the students at the preliminary stage only related area to vague ideas about surfaces. While they have the initial notion of area as a surface extent, these students have conceptual difficulties that are primarily due to their failure to conceptualize area as an idea about the measurement of that surface.

Based upon the above, this study concludes that:

1. Students' conceptual difficulties with area at the developing and emerging

stages are characterized by their failure to understand and fully operationalize the definition of area. These students think about area within the limitations of their ability to calculate area. Their reasoning is, therefore, confined to their knowledge of and ability to use mathematical formulae rather than supported by the basic idea of area as the number of unit squares within a two-dimensional surface.

2. Students at the preliminary stage of understanding of area do not know how to calculate the area of a surface, although they have the initial notion of area as a surface extent. These students' difficulties are fundamentally conceptual and are characterized by their inability to think about area as a measurement of that surface.

Student Conceptual Difficulties With Volume

Thirty percent of the participants in this study have developed a good conceptual understanding of volume as the number of unit cubes contained in a three-dimensional object. These students did not demonstrate any conceptual difficulties.

Fifty percent of the students at the developing stage understand volume as a measure of space within a three-dimensional object, but do not understand that it is basically determined by counting the unit cubes within that object. These students demonstrated a series of conceptual difficulties related to the students' rote use of mathematical operations. Their thinking about volume is deficient because they do not believe that volume can be determined unless mathematical manipulations are employed and they think that these mathematical manipulations produce correct results without understanding the basis behind the process.

Twenty percent of the students at the preliminary stage related volume to vague ideas about spaces. While they have the initial notion of volume as the extent of a three-dimensional space, these students have conceptual difficulties that are primarily due to their failure to conceptualize volume as an idea about the measurement of that space.

Based upon the above, this study concludes that:

1. Students' conceptual difficulties with volume at the developing stage are characterized by their failure to understand and fully operationalize the definition of volume. These students only think about volume within the limitations of formulae and their ability to calculate a product which represents volume. Their reasoning is, therefore, confined to their knowledge of and ability to use mathematical formulae rather than supported by the basic idea of volume as the number of unit cubes within a three-dimensional object.

2. Students at the preliminary stage do not know how to calculate volume as a measurement of a three-dimensional space. While these students have the initial notion of volume as the extent of a three-dimensional space, they have not conceptualized volume as the measurement of that space. Therefore, they have conceptual difficulties that are due to their failure to think about volume as the measurement of a three-dimensional space.

Do Students in Mathematically More-Advanced Courses Differ in Their Initial Understanding of the Area and Volume Concepts From Those in Mathematically Less-Sophisticated Courses?

The answers to this research question are based upon the results of the analysis of the Knowledge of Area and Volume Inventory.

Understanding of the Concept of Area

The student population's prior understanding of area follows a similar trend in each of the introductory physics courses. The largest group of students is made up of students with a developing understanding who have conceptualized area as the measurement of a two-dimensional surface but have not fully understood the implications of this measurement as a count of the equivalent unit squares within that surface. The second largest group of students is made up of those who have developed a good conceptual understanding, in that they comprehend the fundamental idea of area as a counting of unit squares. A smaller group of students have an emerging understanding and think of area as the measurement of two-dimensional surfaces. Another small group of students have a preliminary understanding of area in which they can only relate area to surface extent.

A small improvement in the level of students' conceptual understanding of area is exhibited as the mathematical prerequisites increase for each course. Specifically, the percentage of students with good conceptual understanding increases as one moves from courses with fewer-mathematical prerequisites to those with more-mathematical prerequisites, while the percentage of students with developing understanding decreases. The percentage of students with preliminary understanding also decreases in these courses while the percentage of those with emerging understanding did not exhibit a particular change. However, this improvement did not change the student population's overall understanding of area in each of the introductory physics courses where the largest group of students are still at the developing understanding stage.

Based upon the findings for the research population, this study concludes that students in introductory courses with more-mathematical prerequisites are not fundamentally different in their conceptual understanding of area from students in those courses with fewer-mathematical prerequisites. The percentages of students at each stage of conceptual understanding of area, the way these understandings are verbalized, and the nature of students' conceptual difficulties are similar in physics courses with different mathematical prerequisites.

Understanding of the Concept of Volume

The students' initial understanding of volume follows a same trend in the introductory physics courses that was shown for the area concept. The largest group of students is made up of students with a developing understanding who conceptualize volume as the measurement of a three-dimensional space but who do not fully understand that measurement as a count of the equivalent number of unit cubes contained in that space. The next largest group consists of those with good conceptual understanding of volume. These students reason about volume as the number of unit cubes contained in a three-dimensional object. A smaller group of students consist of those who can only relate volume to generalized and vague ideas about three-dimensional space.

Small improvements in the levels of conceptual understanding of volume are demonstrated by students as one progresses from courses with fewer-mathematical prerequisites to those with more prerequisites. The improvement are based upon increases in the percentage of students with the developing understanding from courses with fewer-mathematical prerequisites to those with more-mathematical prerequisites, while the percentage of students at the preliminary stage decreases across these courses. The percentage of students at the good conceptual understanding stage did not exhibit a meaningful change. However, this improvement did not change the student population's overall understanding of volume in each of the physics courses where the largest group of students are still at the level of developing understanding.

Based upon the findings, this study concludes that students in introductory physics courses with more-mathematical prerequisites are not fundamentally different in their initial conceptual understanding of volume from those in courses with fewer-mathematical prerequisites. The percentages of students at each stage of conceptual understanding of area, the way these understandings are demonstrated, and the nature of students' conceptual difficulties are similar in beginning physics courses, although these courses may have different mathematical prerequisites.

Is There Any Relationship Between Students' Ability to Conceptualize Pressure and Density and Their Understanding of Area and Volume?

The answers to this research question are based upon the results of the pressure and density interviews, which occurred after the Physics 107 students had received instruction in these concepts.

Relationships Between Students' Understanding of the Area Concept and Their Conceptualization of Pressure

The purpose of this part of the study was to begin to investigate the nature of relationship between a student's concepts of pressure and his or her prior understanding of area. Six students, with varying yet fundamentally correct concepts of pressure, exhibited links between their ability to conceptualize pressure and their prior conceptual understanding of area. This linkage did not depend upon mathematical formulae, but rather the ability to use and verbalize the concept of area as the number of unit squares. Specifically, the ability to operationally define area directly influenced the students' development of their concept of pressure. The general notion of area as a measurement of a surface appeared less important than the computational processes they were able to apply to the concept of pressure.

Based upon these findings, this study concludes that beginning college physics students' conceptual understanding of area is linked to their ability to subsequently conceptualize pressure. This linkage appears to be related to a student's operational definition of area more than to their idea of area as the measurement of a two-dimensional space. They approach the pressure concept in a similar way to that of area. That is, they reason about pressure as they do about area using formulae rather than using ideas that support these formulae. More importantly, they bring similar procedures for calculating area to the pressure concept even when these methods are incorrect. Thus, a student's concept of pressure is influenced by his or her prior understanding of the area concept.

Relationships Between Students' Understanding of the Volume Concept and Their Conceptualization of Density

Connections between students' ability to conceptualize density and their conceptual understanding of volume were investigated in this study. Eight students were interviewed and each demonstrated various levels of conceptual understanding for density. Their ability to conceptualize density was linked to their understanding of the volume concept. Specifically, the linkage seems relate to a student's understanding of volume as the measurement of a three-dimensional space. A student's ability to compute volume did not appear to play the same role here as it did for area in understanding of pressure. In addition, the relationship between the concepts of density and volume did not seem to be at the macro level where density is viewed as the ratio of the mass to volume for large scale objects, often summarized by the formula, D = M / V. Rather, students appeared to think about density on a micro level where these students visualized materials as consisting molecules and atoms which are arranged together in various ways so that a unit volume may contain different numbers of these fundamental particles.

Based upon the findings from the density interviews, this study concludes that:

1. Beginning college physics students' conceptual understanding of volume is linked to their ability to develop a density concept. This link appears to relate to a student's ability to conceptualize volume as the measurement of a three-dimensional space rather than to an operational definition of volume that (s)he can specifically use to calculate volume or density.

2. Students approached the density concept on a micro level where they visualize materials as composed of molecules and atoms packed together in various ways. This micro-level approach to the density concept does not seem to depend on a student's ability to calculated volume, but rather to be related to the more generalized idea of volume as a derived measurement of a given space. Thus, the link between students' conceptualization of density and their ability to think of volume as a concept of three-dimensional space appears to help them conceptualize density as the amount of fundamental particles packed in that space.

Implications for Introductory Physics Instruction

College and high-school science instructors should be informed that almost two-thirds of their students may not understand or think about area and volume using the idea of unit squares or cubes. While these students may be able to calculate area and volume using readily available formulae for regular objects, they have difficulties in extrapolating to special cases involving more complex or irregularly-shaped objects.

Although area and volume are not directly taught in many introductory physics courses, students' ability to construct subsequent scientific concepts such as pressure and density, which are based upon these underpinning concepts, appear to be linked. Thus, science instructors are well advised to know that many of their students may lack a fundamental understanding of the area and volume concepts and that they need additional opportunities to develop them.

Students' difficulties with the concepts of area and volume are often due to their failure to understand the distinction between the words that label and the definitions that describe these concepts and the ideas that the terms or definitions represent. Helping students to develop an understanding of what the terms or definitions imply and how they relate to the fundamental ideas that these concepts represent is important. Thus, instructors should provide students with learning opportunities to construct a richer understanding of underpinning concepts that form the foundation for learning other concepts. Diagnostic work at the beginning of a semester and remedial work at appropriate times may be a good way to help students construct a deeper understanding of these underpinning ideas and avoid problems when concerning subsequent concepts in the conceptual hierarchy.

Recommendations for Future Research

First, this study has identified some casual connections between students' conceptualization of pressure and density and their prior understanding of area and volume. However, since the design of this study did not allow the researcher to investigate the specific nature of the relationships between ideas in a conceptual hierarchy, additional research is recommended to determine the nature of the links between various fundamental ideas and the derived concepts which are built upon these antecedent ideas.

Second, an interesting preliminary finding from this study is that a student's ability to conceptualize density appears to depend upon the students ability to visualize matter at the micro level than at the macro level. At the micro level an object is visualized as consisting of very small fundamental particles (molecules and atoms) that are packed together in various ways within a given space. The macro level, on the other hand, views large scale objects in terms of their mass and the space that mass occupies. Since most text books introduce the concept of density at the macro level, instructors may assume that their students should first encounter the density concept at the macro level, and later students should be introduced to density at the micro level. However, in this study, students appeared to conceptualize density first at the micro level. Is this typical? Should students have the opportunity to visualize matter at the micro level first? Was the influence of the micro perspective due to the influence of a prior instruction about density? More research is needed to shed light on these questions.

Lastly, since students' conceptualization of density appears to be influenced by their generalized idea of volume and not by how it is calculated, questions about the nature of relationships between ideas in the conceptual hierarchy are raised. Can higher-level concepts be developed without an operational definition and understanding of the computational processes by which the lower-level concepts are calculated? Again, additional research is recommended to explore the nature of the relationships between these and high-level concepts in this and other conceptual hierarchies.

APPENDICES

Appendix A

Knowledge of Area and Volume Inventory

Name: ____________________

Class: ____________________

Date: ____________________

KNOWLEDGE OF AREA AND VOLUME INVENTORY

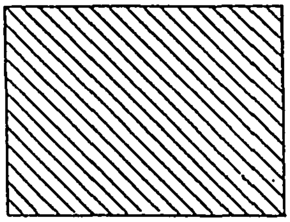
DIRECTIONS:

Individually complete the following inventory. You may use the ruler or transparent grid provided, or your own calculator. **PLEASE SHOW ALL YOUR WORK IN THE MARGINS.** This includes formulas, graphs, calculations, or words to explain your thinking.

1. Define or describe in your own words the idea of AREA. (In other words, what does AREA mean to you?)

2. **Assign a value of area to each of the following figures with the units you use. Explain how you obtained this value.**

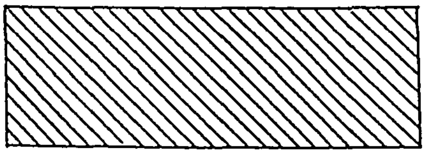
(i)



Area = ________

Explain:

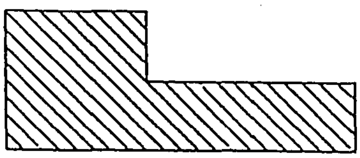
(ii)



Area = ________

Explain:

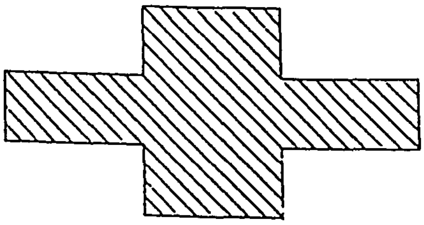
(iii)



Area = ________

Explain:

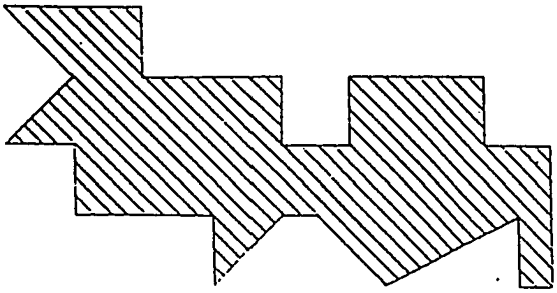
(iv)



Area = ________

Explain:

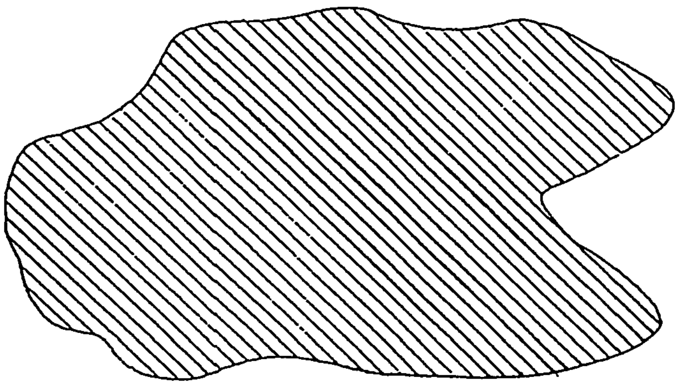
(v)



Area = ________

Explain:

(vi)



Area = ________

Explain:

3. Which figure has the larger area, a or b, or they are the same?

i)

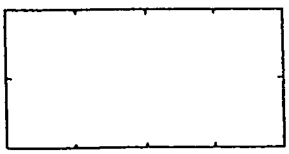


Figure a

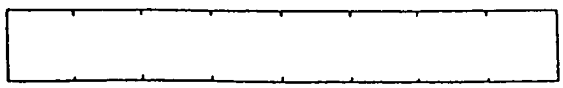


Figure b

Your answer: ___________

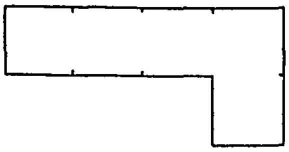
ii) 

Figure a

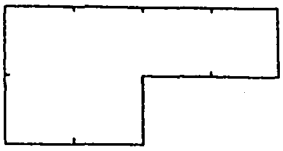


Figure b

Your answer: __________

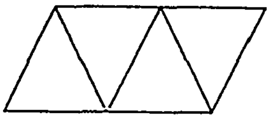
iii) 

Figure a

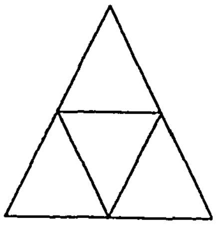


Figure b

Your answer: __________

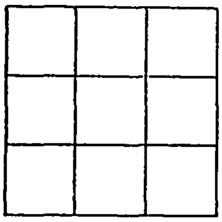
vi) 

Figure a

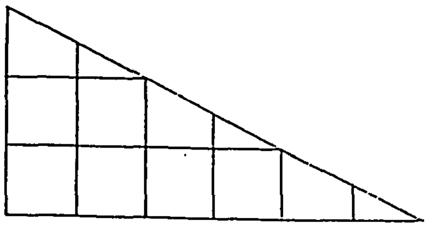
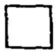


Figure b

Your answer: __________

4. One square of carpet as showing in figure (a) costs $10. How much does it cost to carpet a room whose floor plan is represented by the rectangle in figure (b)? Show your work.



(a)

(b)

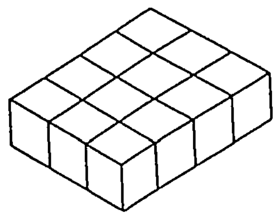
Your answer: __________

5. If you cut a square with an edge of 18 units into smaller squares, each with an edge of 2 units, how many smaller squares (of 2 units/side) will you have? **Show your work and explain your thinking.**

Your answer: __________

6. How many exterior surfaces does a cube have? __________

7. The surface area of a rectangular solid is the total area on all exterior surfaces of the solid.
What is the surface area of the solid block shown at the right? **Show your work.**



Your answer: __________

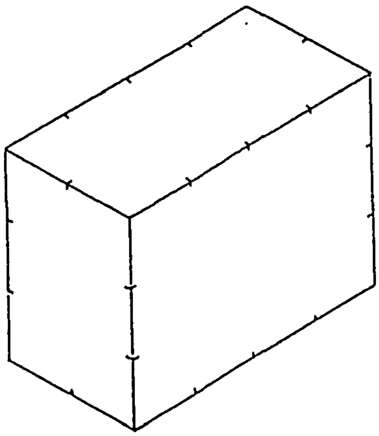
8. What is the surface area of a cube with an edge of 10 units? **Show your work.**

Your answer: __________

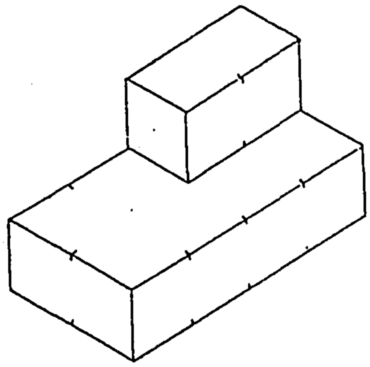
9. Define or describe in your own words the idea of VOLUME. (In other words, what does VOLUME mean to you?)

10. Assign a value to the volume of each of the following figures with the units you use. Explain how you obtained your answer.

i)



ii)



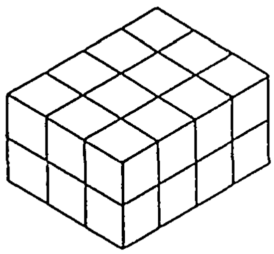
Volume = ________

Explain:

Volume = ________

Explain:

11. What is the volume of the solid block shown at the right? **Show your work.**



Your answer: __________

12. An irregular container can be just filled by 24 white cubes plus 12 black cubes. All white cubes have the same dimensions of 2 centimeters on each edge. All black cubes have 3 centimeters on each edge. What's the volume of this container? **Show your work or explain your thinking.**

Your answer: __________

13. Explain how you might find the volume of an irregularly shaped solid? More than one method is possible and anyone of them is acceptable.

14. If you cut a cube with dimensions of 8 units on each edge into cubes with edges of 2 units, how many 2-unit cubes will you have? **Show your work or explain your thinking.**

Your answer: __________

15. A large box of popcorn sells for 80¢. If you wanted to sell a smaller box one half as large in each dimension, what is a fair price to charge for the smaller box? **Explain your thinking and show your work.**

Your answer: __________

Appendix B

Letter to Students

Department of Physics

Kalamazoo, Michigan 49008-5151
616 387-4940

WESTERN MICHIGAN UNIVERSITY

September 1, 1992

Dear Student,

I am Jiang Yu from the Department of Physics at Western Michigan University. In this Fall semester I am conducting a research project in physics education which is designed to determine the nature of college students' understandings of certain underpinning concepts and the role these understandings play in students' study of subsequent physics concepts at WMU. The inventory accompanying the information form on the next page is part of this research project.

I need your help and are requesting that you participate by completing this information form and the attached inventory now. The data obtained from your completion of the inventory will be analyzed to yield the information needed for the determination of students' understandings of the underpinning concepts. Later on during the semester, I will interview some (about 5%) of you for the information leading to the role these understandings play in students' study of subsequent physics concepts. All information obtained will be strictly confidential and no one will be identified nor will the data be released for any other purposes.

There are no known hazards or risks to you and your participation is on a volunteer base. The information provided by you will be important in helping physics instructors in their future class planning and teaching. If you have any questions at any time, feel free to call me at 387-7619. Your assistance in this study is deeply appreciated.

Sincerely,

Jiang Yu

Jiang Yu
Department of Physics
Western Michigan University
Kalamazoo, MI 49008-5151

Appendix C

Consent Form

CONSENT FORM FOR PHYSICS "UNDERPINNING" STUDY

This study involves obtaining information from students in introductory physics classes at Western Michigan University during the Fall of 1992. Students will be requested to complete a one page information sheet, and an inventory of their understanding of area and volume concepts. A small percentage (about 5%) of students will be asked to participate in a interview with the researcher. All participation is voluntary.

The information sheet consists of student name, major, his/her approximate high-school and WMU G.P.A., and his/her previous and current experience in mathematics and physics in high school and college.

The inventory consists of a series of items about the concepts of area and volume. Information obtained from the inventory will be analyzed to determine the level of a student's understanding of area and volume.

The interview will be conducted to obtain insight into the participants' thinking, understanding, and conception of area and volume, as well as the role these understandings play in a participant's conceptualization of subsequent physics ideas. The interviews will be audio-taped for latter analysis.

There are no known risks to participants. The results of this study will hopefully determine participants' understandings of the underpinning concepts of area and volume, and hence will help physics instructors in their future class planning and teaching.

The administration of the inventory along with the information sheet, and the conduction of the interviews will be done by Jiang Yu, telephone number 387-7619. If participants have questions, they may call her or contact her advisor Dr. Robert H. Poel, telephone number 387-3337.

I have read the foregoing information and understand it. I also understand that I am free to withdraw consent and discontinue participation in this study at any time. Refusal to participate in this study will have no effect on my grade in this physics class.I have been informed that identifiable audio-tapes and information obtained in this research study (information sheet, inventory and interview) are confidential, that it will be destroyed at the conclusion of the study, and that I will not be identified by name in any way. I agree that this information may be used for research purposes.

Participant's Name (Print) ______________________________

Participant's Signature _________________________________

Date ________________________

Appendix D

Information About You

INFORMATION ABOUT YOU

Your Name is ________________________________.

The current physics course(s) you are taking is (are)

(lecture)__________________, (lab)__________________.

Your current major is ________________________________.

Your university status is (circle one) freshman, sophomore, junior,

senior, or other __________________ (please specify).

Your approximate high-school G.P.A. is __________________.

Your approximate WMU G.P.A. is ______________________.

Have you taken any physics courses before? Yes _____ No _____

If yes, at (circle one or both) college, or high school.

If at college, please list the courses: __________________,

__________________, __________________,

__________________, __________________.

Which high-school mathematics courses have you taken? Please

list them: ______________________, ______________________,

______________________, ______________________,

______________________, ______________________.

Are you taking any math courses this semester? Yes ____ No ____

If yes, please list the courses: ______________________,

______________________,

______________________.

Have you taken any college level math courses? Yes ____ No ____

If yes, please list the courses: ______________________,

______________________, ______________________,

______________________, ______________________.

THANK YOU! PLEASE GO ON TO THE INVENTORY.

Appendix E

Administration Instructions for Knowledge of Area and Volume Inventory

ADMINISTRATION INSTRUCTIONS
FOR
KNOWLEDGE OF AREA AND VOLUME INVENTORY

THE INSTRUCTOR WILL NEED:

1. Sufficient sets of the inventory, information sheet, consent form, and cover letter for each student in the class.
2. Sufficient plastic rulers and transparent grids.
3. Supply of spare pencils.
4. A copy of this instruction sheet.

THE STUDENT WILL NEED:

1. An inventory booklet accompanied by a student information sheet, consent form, and cover letter.
2. A pencil or a pen.
3. The transparent grid and ruler.
4. A calculator (optionally supplied by student).

INSTRUCTIONS:

After distributing all the materials listed above to the students, read aloud to the students the following instructions:

THE INVENTORY YOU HAVE BEEN GIVEN IS PART OF A RESEARCH PROJECT IN PHYSICS EDUCATION. AS THE COVER LETTER INDICATES, YOUR VOLUNTARY PARTICIPATION IS IMPORTANT, NOT ONLY TO THIS RESEARCH STUDY, BUT ALSO TO THE IMPROVEMENT OF THE COURSE OF STUDY AND INSTRUCTION IN THIS COURSE. PLEASE READ AND SIGN THE CONSENT FORM, FILL OUT THE INFORMATION SHEET, AND THEN COMPLETE THE INVENTORY.

YOU WILL NEED TO USE YOUR OWN PENCIL OR A PEN. YOU WILL ALSO NEED THE TRANSPARENT GRID AND RULER PROVIDED. YOU MAY USE THEM WHEREVER IT IS APPROPRIATE. PLEASE DO NOT WRITE OR DRAW ANYTHING ON THE TRANSPARENT GRID. YOU MAY ALSO USE YOUR OWN CALCULATOR, ALTHOUGH ONE IS NOT NECESSARY TO COMPLETE THE INVENTORY. IF YOU NEED TO BORROW A PENCIL, I HAVE SOME SPARE ONES.

WHEN YOU ARE FINISHED, PLEASE RETURN THE SIGNED CONSENT FORM, INFORMATION SHEET, INVENTORY BOOKLET, AND THE TRANSPARENT GRID AND RULER TO ME.

YOU MAY BEGIN NOW.

Appendix F

Interview Schedules and Questions

INTERVIEW PROTOCOLS

CLINICAL INTERVIEWS ABOUT AREA AND VOLUME

Procedure of an Interview

1. Converse a little (no more than three minutes) with the student before beginning an interview with him/her. The purpose of the conversation is to warm up with the student so that (s)he will think and talk freely. The conversation can be about weather, the student's major and interest, their classes, etc.

2. Explain the purpose of the interview to the student. Emphasize that I am not looking for right or wrong answers. What I like to know is their thinking and any difficulties with the concepts.

3. Show the tape recorder to the student, explain its purpose, how the tapes will be used and treated, and ask for the student's permission to use it.

4. Start interview questions, tell the student that I want him/her to think freely and "think aloud".

5. Follow the student to get more information of his thinking, but control the course so as to stay with that relevant to thinking of area and volume.

Materials and Props to Take With

1. a tape recorder, tapes, paper and pencil, a ruler
2. a piece of regular white xerox paper (8.5" X 11")
3. a piece of paper cut into an irregular figure
4. a rectangular box, a small balloon, a lump of clay, a plastic bag
5. a grid, a box of wooden cubes

Lead Questions of Interviews

1. Area:

-- How would you describe the size of a piece of paper?

-- What measurements could you use to describe the size of this piece of paper (a piece of regular one)? How about this one (the piece that has been cut into irregular shape)? (Does the student know the term area, the idea of L x W.)

-- Do you know any other terms that could be used to describe the size of a piece of paper? What do they mean to you?

-- How could you find the size of an irregularly shaped figure like this one? (Does the student know that it could be reshaped to a more regular figure and calculate? Does (s)he know it could have a grid overlapped on and the area equals to the number of unit squares counted?)

-- How many dimensions do you think this piece of paper have? Why? (first a piece of regular paper, then the piece that has been cut into irregular shape.)

-- Do you think the size of this piece of paper have anything to do with its dimensions? Why or why not? (show him/her the regular paper first, then the irregular one)

-- Do you recognize this (the student's Knowledge of Area and Volume Inventory)?
Can I ask you questions about it? (This should lead to specific questions I want to ask him/her about based on the student's answers to the inventory items.)

2. Volume:

-- How would you describe the size of this box? What about this balloon?

-- What measurement would you use to generally describe how large a container is? (Knowing the term volume, the idea of L X W x H.)

-- Do you know any other terms that could be used to describe the size of a solid or a box? What do they mean to you?

-- How could you find the size of an irregularly shaped solid? For examples, this lump of clay? (Does the student know that the clay could be reshaped to a more regular figure and then calculate?

-- How could you find the size of this plastic bag? Does (s)he know the plastic bag could be filled with water or even small wooden cubes, and then figure out their volume?)

-- How many dimensions do you think this box have? Why? How about the balloon? Why? How about the plastic bag? Why?

-- Do you think the size of this box have anything to do with its dimensions? Why or why not? How about the balloon? How about the bag?

-- Can I ask you questions about the inventory you did, again? (This should lead to specific questions I want to ask about based on the student's answers to the inventory items.)

CLINICAL INTERVIEWS ABOUT DENSITY

Procedures of Interviews

1. Converse a little (no more than three minutes) with the student before beginning an interview with him/her. The purpose of the conversation is to warm up with the student so that (s)he will think and talk freely. The conversation can be about weather, their classes, homework load, etc.

2. Explain the purpose of the interview to the student. Emphasize that I am not looking for right or wrong answers. What I like to know is their thinking.

3. Show the tape recorder to the student, explain its purpose, how the tapes will be used and treated, and ask for the student's permission to use it.

4. Start interview questions, tell the student that I want him/her to think freely and "think aloud".

5. Follow the student to get more information of his thinking, but control the course so as to stick with that relevant to thinking and ideas about density and its relationship to volume.

Materials and Props to Take With

1. a tape recorder, tapes, paper and pencil, a ruler
2. three pairs of objects, one pair have same size (volume) and same shape but different weight, another same weight and same shape but different size (volume), the other same weight and same size (different looking but actually same in volume) but different shape.

Lead Questions of the Interviews

-- Do you remember the experiment you did last week? (The experiment was "Density of Lucite".)
Can you describe the experiment to me?

-- What were you trying to measure in that experiment? (density of lucite)

-- What were you trying to verify in that experiment? (Density is independent of shape and size.)
What was your result from the experiment? How do you think of that?

-- Can you describe to me what DENSITY means to you? What does it measure or describe of a material? (What is this student's understanding of the term and the concept of density?)

4

-- What does density depend on, you think? (shape, size, weight, type of material)

-- Look, I have a pair of solids here, they have same shape and same size but different weight, do you think they have same density or not? Why or why not?

-- What about this pair, they have same weight and same shape but one is big and the other is small? Do they have same density?

-- Now look at the pair, they have same weight and same volume, I can tell you they do, but they are differently shaped. Do they have same density? Why or why not?

-- Follow the student once he/she gets on to the topic, let the student lead me. (What is the role of this student's understanding of volume in his/her understanding of density?)

CLINICAL INTERVIEWS ABOUT PRESSURE

Procedures of Interviews

1. Converse a little (no more than three minutes) with the student before beginning an interview with him/her. The purpose of the conversation is to warm up with the student so that (s)he will think and talk freely. The conversation can be about weather, their classes, homework load, etc.

2. Explain the purpose of the interview to the student. Emphasize that I am not looking for right or wrong answers. What I like to know is their thinking.

3. Show the tape recorder to the student, explain its purpose, how the tapes will be used and treated, and ask for the student's permission to use it.

4. Start interview questions, tell the student that I want him/her to think freely and "think aloud".

5. Follow the student to get more information of his thinking, but control the course so as to stick with that relevant to thinking and ideas of pressure and its relationship to area.

Materials and Props to Take With

1. a tape recorder, tapes, paper and pencil, a ruler
2. a piece of metal block, rectangular shape

Lead Questions of the Interviews

-- Have you studied PRESSURE in your lecture? (Yes. They have just finished the pressure concept in lecture.)
Could you describe to me what PRESSURE means to you? (What is this student's thinking about the term and the concept of pressure?)

-- When you think about pressure, what picture do you have in your mind? What about it? What does it depend on? (What conceptions are there in the student's mind about pressure?)

-- How do you relate pressure with force? (Does the student confuse about the two?)

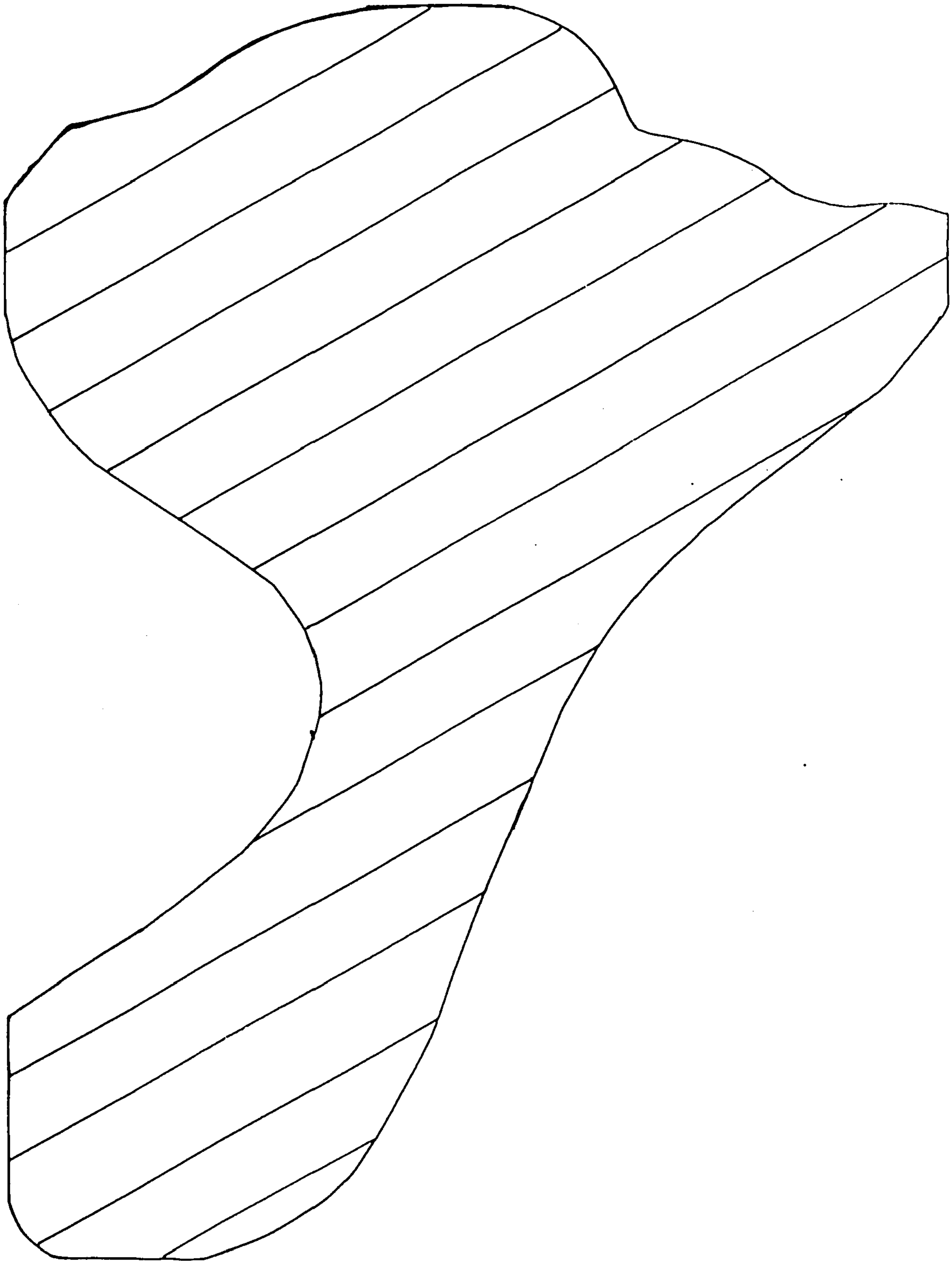
-- I have this metal block on the table as you can see. Do you think the block exert force on the table? Is there pressure? If yes, then ask: Now I put it in a different way so that the smaller face is set on the table. Do you think the force exerted by the block to the table is changed? Why or why not? How about pressure, is it changed?

-- Now look, the metal block has the pressure on the contact surface of the table as you say, if I circle a small area of the contact surface, like this (circle a small area within the contact surface), do you think the circled area feels the same pressure as the whole contact surface does? Why or why not?

-- Follow the student's thinking, let he/she leads me to more information probing. (What role does this student's understanding of area have on his/her understanding of pressure?)

Appendix G

An Irregular Paper Used in the Interviews



Appendix H

HSIRB Permission Letter

Human Subjects Institutional Review Board

Kalamazoo, Michigan 49008-3899

WESTERN MICHIGAN UNIVERSITY

Date: September 9, 1992

To: Jiang Yu

From: Mary Anne Bunda, Chair Mary Anne Bunda

Re: HSIRB Project Number 92-08-07

This letter will serve as confirmation that your research protocol, "Investigation of College Physics Students' Understanding of "Underpinning" Concepts of "Area" and "Volume" has been approved after full review by the HSIRB. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the approval application.

You must seek reapproval for any change in this design. You must also seek reapproval if the project extends beyond the termination date.

The Board wishes you success in the pursuit of your research goals.

xc: Poel, Center for Science Education

Approval Termination: September 9, 1993

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