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**TRCLC 14-10**
**August 31, 2015**

# Innovative Park-and-Ride Management for Livable Communities

## FINAL REPORT

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Transportation Research Center
*for Livable Communities*

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**16. Abstract**

Park-and-ride (P&R) has been recognized as an effective way to tackle the challenge of the last-mile problem in public transportation, i.e., connecting transit stations to final destinations. Although the design and operations of P&R facilities have been extensively investigated, there is a pressing need for a theoretically sound methodology for planning and managing P&R facilities. It is critically important to investigate where P&R facilities should be strategically located and how often transit service should be provided such that the net social benefit can be maximized.

This project proposes an integrated planning methodology for locating P&R facilities and designing transit services simultaneously to promote public transportation and reduce traffic externalities in urban areas. The optimal P&R facility and transit service design problem is formulated as a mathematical program with complementarity constraints, and a solution algorithm based on the active-set approach is used to solve the optimal design problem effectively. A numerical example is employed to demonstrate that the optimal design shifts commuters from the automobile mode to transit and P&R modes and, hence, improves the net social benefit dramatically. The study provides a heretofore missing theoretical framework for integrated planning of P&R facilities and transit services.


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# Chapter 1: Introduction

Public transportation yields significant positive impacts in reducing traffic congestion, providing alternative means of travel, and contributing to better quality of life in urban areas (Vuchic, 2005). The Utah Transit Authority (UTA) has progressively invested in public transportation over the past 15 years, especially in mass transit systems. As a result, public transportation ridership in Utah has increased 79% since 1999, amounting to almost 43 million boardings in 2012 (UTA, 2013). This growth has significantly outpaced growth in population and vehicle miles traveled (VMT). Moreover, nationally, young travelers have been driving less and using public transportation more in recent years. From 2001 to 2009, the average number of VMT by 16- to 34-year-olds dropped by 23%, while the annual number of passenger miles per capita traveled on transit systems increased by 40% (Davis et al., 2012). Despite the increasing importance of public transportation, literature on transit facility site selection, planning and operations is rather limited.

Park-and-ride (P&R) describes an operation in which commuters, traveling by private vehicles, gather at a common site that enables them to transfer to public transportation (Noel, 1988). Figure 1 illustrates two typical P&R lots operated by UTA's TRAX light rail system in Salt Lake City, Utah. The operation allows commuters to use the automobile or transit in the geographic area to which it is best suited. Private vehicles are used to travel to P&R facilities located in low-density suburban or urban fringe areas where (fixed-route) transit services are not justified; transferring to transit allows commuters to avoid traffic congestion and high parking cost in reaching major activity centers (e.g., central business districts). Since its first introduction in Detroit in the 1930s (Bullard and Christiansen, 1983), P&R has been recognized as an effective way to promote public transportation and reduce traffic externalities in urban areas (e.g., Bolger et al., 1992; Niblette and Palmer, 1993).

P&R management has become increasingly important because more resources have been invested in high-quality transit services in recent years. The number of fixed-guideway transit options (e.g., commuter, heavy and light rail systems) in the U.S. has almost tripled over the last

three decades (APTA, 2013). Indeed, UTA completed the FrontLines 2015 program two years ahead of schedule in 2013, which added 70 miles of passenger rail and more than doubled its existing 64-mile rail network in seven years (UTA, 2013). Furthermore, many states are actively deploying managed lanes coupled with express bus services in those lanes (FHWA, 2008). For example, an 800-mile regional managed-lane network is planned in the San Francisco Bay Area in California. The network will serve a high volume of express buses, whose implementation cost is expected to be $3.4 billion between 2015 and 2035 (MTC, 2008). Well-planned and managed P&R facilities are critical to the success of such high-quality transit services.

Although the design and operations of P&R facilities have been extensively investigated, to our surprise, there is a lack of theoretically sound guidance for where to locate them, an important aspect of P&R planning. Some agencies have provided criteria for selecting P&R facility locations (see, e.g., Burns, 1979; Fradd and Duff, 1989; Spillar, 1997; ASSHTO; 2004, FDOT, 2012). However, this approach often produces confusing and even contradictory suggestions because the criteria are based primarily on experiential evidence (Holguin-Veras et al., 2012). The research literature does not offer much help either, although previous studies have been conducted on optimal locations of P&R facilities. Some studies (e.g., Horner and Groves, 2007; Farhan and Murray, 2008; Aros-vera et al., 2013), have not considered how commuters react to the provision of P&R facilities, while others (e.g., Sargious and Janarthanan, 1983; Wang et al., 2004; Holguin-Veras et al., 2012), have focused on highly simplified settings, such as a linear city. On the other hand, the attractiveness of P&R services not only relies on strategically deployed P&R facilities but also depends on the level of transit services offered through these P&R facilities. For example, in the San Francisco Bay Area, some P&R lots are oversubscribed while others are nearly empty and travelers would rather drive to work than use a poorly located and served P&R lot (Shirgaokar and Deakin, 2005).




(a) UTA P&R Lot at TRAX Millcreek Station (photo by Ziqi Song)



(b) UTA P&R Lot at TRAX Meadowbrook Station (photo by Ziqi Song)

**FIGURE 1 Typical P&R Lots**

As P&R facilities should be carefully planned and integrated into a multimodal transportation system, a systems approach is needed to consider the interactions of multiple transportation modes and commuters' choice of mode, P&R lot, and travel route. The objective of this study is to develop a theoretically sound methodology for locating P&R facilities and designing transit service frequency simultaneously to promote public transportation and reduce traffic externalities in urban areas. The remainder of this report is organized as follows. Chapter 2 presents the problem statement and describes feasible flow distributions across a multimodal network with P&R facilities. Chapter 3 formulates a user equilibrium (UE) model in the multimodal network. Chapter 4 proposes the optimal P&R facility and the transit service design problem. Chapter 5 investigates the solution algorithm, followed by a numerical example in Chapter 6. Last, Chapter 7 concludes the report.

## Chapter 2: Problem Statement

Consider a multimodal transportation network $G = (V, L)$, where $V$ and $L$ denote the sets of nodes and directed links, respectively. The link set consists of four mutually exclusive subsets, namely, road link set, $L_R$, transit link set, $L_T$, boarding link set, $L_P$, and alighting link set, $L_E$. Road and transit links are physically separated links and are connected by boarding and alighting links. For the sake of modeling, P&R facilities are assumed to be located on boarding links. Therefore, boarding links are also termed P&R candidate links and the two terms are used interchangeably hereinafter. Figure 2 shows an example multimodal network in which four types of links are present.

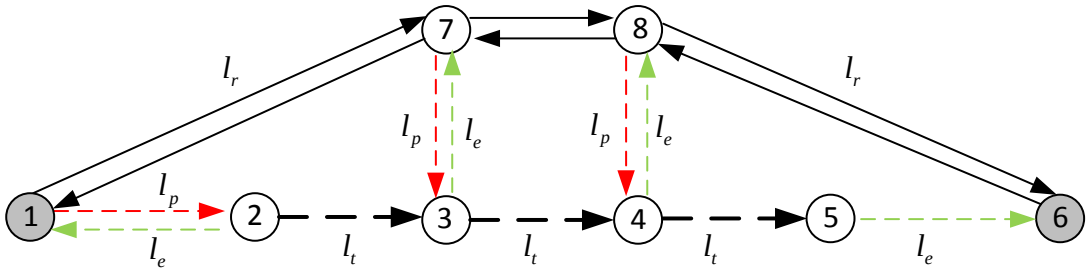



**FIGURE 2 An Example Network**

Commuters have the flexibility to choose one of three travel modes available in this multimodal network, that is, automobile, $a$, transit, $t$, or P&R, $p$. The set of available modes is denoted as $M$, and $M = \{a, t, p\}$. It is assumed that commuters' mode choices can be captured by a multinomial logit model. It is further assumed that users for any origin-destination (OD) pair have access to all three travel modes. On the other hand, the aggregate travel demand for each OD pair is assumed to be given and fixed.

Transit users employ three types of links in sequence (boarding, transit, and alighting links) to complete a commuter trip. Travel times on these three types of links are assumed to be flow-independent. Users pay the transit fare, $\tau_{ij}$, on transit links, $(i, j) \in L_T$, and also experience

waiting time, $\psi_{ij}$, on boarding links, $(i,j) \in L_P$. The average waiting time on a boarding link depends on the transit frequency of the transit line that serves the transit link connecting to this boarding link. Assume $N$ transit lines with the transit frequency of line $n$ denoted as $f_n$. The waiting time can be calculated as $\psi_{ij}(f_n) = 1/(2f_n)$, implying that passengers' arrival follows a uniform distribution and the headway between transit vehicles is constant (De Cea and Fernandez, 1993; Wu et al., 1994; Li et l., 2007). P&R users start their trips in private vehicles and park their vehicles on P&R candidate links to access public transportation. They may need to pay a parking fee $\beta_{ij}$ on link $(i,j) \in L_P$ depending on the P&R lot location. Travelers who choose to use private vehicles travel on road links only. The travel time functions of road links are assumed to be separable and follow the Bureau of Public Roads (BPR) function. Apart from peak-hour traffic congestion, travelers generally also face high parking costs if they choose to park their vehicles in the downtown area. Since the downtown parking fee is only charged once at the travelers' destination, it can be treated as an OD-specific charge and is denoted as $\mu_{ij}^w$ for $(i,j) \in L_R$. If the destination node of OD pair $w$ coincides with the head node of link $(i,j) \in L_R$, that is, $d(w) = j$, then $\mu_{ij}^w \geq 0$; otherwise, $\mu_{ij}^w = 0$.

Generally, the construction cost of P&R facilities and operating cost of transit services are relatively high. Therefore, it is of critical importance to investigate, in a given multimodal transportation network, where P&R facilities should be strategically located and how often transit service should be provided such that the net social benefit can be maximized.

## 2.1 Feasible Region

To properly describe users' travel behavior in a multimodal network with P&R facilities, it is critical to define the feasible flow patterns first. In this study, we adopt link-based variables to represent the feasible flow region to avoid the cumbersome path enumeration. Let $A$ be the node-link incidence matrix for the network. $E^w$ denotes an "input-output" vector specifying the origin and destination of OD pair $w$ in the flow balance constraints, and it has exactly two non-zero components: one has a value of 1 corresponding to the origin node and the other one has a

value of -1 in the component for the destination. $x^{w,m}$ is a vector and its components, $x_{ij}^{w,m}$, are passenger flow on link $(i,j)$ for OD pair $w$ using travel mode $m$. For each OD pair $w$, the total passenger demand is fixed and denoted as $D^w$, while the demand for each travel mode $m$ is not fixed and denoted as $d^{w,m}$. A binary variable, $y_{ij} \in \{0,1\}$, is used to show whether a P&R lot is built on a candidate link $(i,j) \in L_P$. Given the locations of P&R facilities, the feasible region can be represented as the following linear system:

$$Ax^{w,m} = E^w d^{w,m} \qquad \forall w \in W, m \in M \qquad (1)$$
$$\sum_{m \in M} d^{w,m} = D^w \qquad \forall w \in W \qquad (2)$$
$$x_{ij}^{w,m=a} \geq 0 \qquad \forall (i,j) \in L_R \qquad (3)$$
$$x_{ij}^{w,m=a} = 0 \qquad \forall (i,j) \in L_T \cup L_P \cup L_E \qquad (4)$$
$$x_{ij}^{w,m=t} \geq 0 \qquad \forall (i,j) \in L_T \cup L_P \cup L_E \qquad (5)$$
$$x_{ij}^{w,m=t} = 0 \qquad \forall (i,j) \in L_R \qquad (6)$$
$$x_{ij}^{w,m=p} = 0 \qquad \forall (i,j) \in L_P, o(w) = i \qquad (7)$$
$$x_{ij}^{w,m=p} \geq 0 \qquad \forall (i,j) \in L_P, o(w) \neq i \qquad (8)$$
$$x_{ij}^{w,m=p} = 0 \qquad \forall (i,j) \in L_R, d(w) = j \qquad (9)$$
$$x_{ij}^{w,m=p} \geq 0 \qquad \forall (i,j) \in L_R, d(w) \neq j \qquad (10)$$
$$x_{ij}^{w,m=p} \geq 0 \qquad \forall (i,j) \in L_E, d(w) = j \qquad (11)$$
$$x_{ij}^{w,m=p} = 0 \qquad \forall (i,j) \in L_E, d(w) \neq j \qquad (12)$$
$$\sum_{w \in W} x_{ij}^{w,m=p} \leq P_{ij}^0 + y_{ij} P_{ij}^P \qquad \forall (i,j) \in L_P \qquad (13)$$

Equation (1) is the flow balance constraint, and equation (2) implies that the sum of demand for each mode is the total demand, which is given. Equations (3) – (4) indicate that users choosing the automobile mode only drive on road links. In contrast, equations (5) – (6) show that transit users do not use road links.

The rest of the equations above are intended to capture the behaviors of P&R users accurately. P&R users have access to all four types of links in the multimodal network. However, they have a set of specific restrictions in route choices and these restrictions are represented mathematically as constraints in the feasible region. First, P&R users cannot exclusively use public transportation for the entire trip; if they do, they would be categorized as transit users. This can be enforced by requiring the flow of the P&R mode to be zero if the tail node of a boarding link

coincides with the trip origin, $o(w)$, as shown in equations (7) – (8). In other words, the initial portion of a P&R trip should be carried out by private vehicle. Second, an entire P&R trip cannot only use road links; if it did, it would be an automobile trip. Equations (9) – (10) are introduced to restrict the flow of the P&R mode on road links whose head nodes coincide with the trip destination, $d(w)$, to be zero. Third, once P&R users park their vehicle at a P&R facility, they should not be able to switch back to road links. This rule can be realized by prohibiting P&R users from accessing alighting links unless they approach the end of their trips, that is, the head node of the alighting link coincides with the trip destination, $d(w)$, as shown in equations (11) – (12). Fourth, the number of P&R users who transfer from private vehicles to public transportation at P&R candidate links should not exceed the parking lot capacity on these corresponding links. To be more realistic, it is assumed that all P&R candidate links have small initial parking capacity, $P_{ij}^0$, representing the limited on-street parking spots. The capacity of P&R facilities is a pre-defined number of parking spots, $P_{ij}^P$. Thus, the total parking capacity on candidate links is $P_{ij}^0 + y_{ij}P_{ij}^P$. Equation (13) shows the parking capacity constraint. Note that the parking capacity constraint only applies to P&R users and transit users can still access boarding links without any restriction. The feasible region will be referred to as $\Phi$ hereinafter.

## Chapter 3: User Equilibrium Problem in a Multimodal Network with P&R Facilities

This chapter formulates the UE problem in a multimodal network with P&R facilities using a mathematical program and provides equivalent conditions. The mathematical program can be formulated as follows:

$$P1: \min_{(x,d)} \sum_{(i,j)\in L_R} \int_0^{v_{ij}} t_{ij}(\omega)d\omega + \sum_{w\in W}\sum_{m=\{t,p\}}\sum_{(i,j)\in\{L_T\cup L_P\cup L_E\}} t_{ij}\, x_{ij}^{w,m} + \sum_{w\in W}\sum_{m\in\{t,p\}}\sum_{(i,j)\in L_T} \tau_{ij}\, x_{ij}^{w,m}$$
$$+ \sum_{w\in W}\sum_{m=p}\sum_{(i,j)\in L_P} \beta_{ij} x_{ij}^{w,m} + \sum_{w\in W}\sum_{m=\{tp\}}\sum_{(i,j)\in L_P} \psi_{ij}(f_n) x_{ij}^{w,m} + \sum_{w\in W}\sum_{m=a}\sum_{(i,j)\in L_R} \mu_{ij}^{w} x_{ij}^{w,m}$$
$$+ \frac{1}{\theta}\sum_{w\in W}\sum_{m\in M}(lnd^{w,m} + \alpha^m - 1)d^{w,m}$$
$$\text{s.t. } (x,d) \in \Phi.$$

**Theorem 1**. The optimal solution $(x,d)$ to problem P1 satisfies the UE conditions in a multimodal network.

**Proof.** Derive the KKT conditions for problem P1 as follows:

$$t_{ij}(v_{ij}) + \left(\rho_i^{w,m} - \rho_j^{w,m}\right) \geq 0 \quad \forall w \in W, m = p, (i,j) \in L_R, d(w) \neq j \tag{14}$$
$$x_{ij}^{w,m}\left[t_{ij}(v_{ij}) + \left(\rho_i^{w,m} - \rho_j^{w,m}\right)\right] = 0 \quad \forall w \in W, m = p, (i,j) \in L_R, d(w) \neq j \tag{15}$$
$$t_{ij}(v_{ij}) + \left(\rho_i^{w,m} - \rho_j^{w,m}\right) + \mu_{ij}^{w} \geq 0 \quad \forall w \in W, m = a, (i,j) \in L_R \tag{16}$$
$$x_{ij}^{w,m}\left[t_{ij}(v_{ij}) + \left(\rho_i^{w,m} - \rho_j^{w,m}\right) + \mu_{ij}^{w}\right] = 0 \quad \forall w \in W, m = a, (i,j) \in L_R \tag{17}$$
$$t_{ij} + \left(\rho_i^{w,m} - \rho_j^{w,m}\right) + \tau_{ij} \geq 0 \quad \forall w \in W, m = \{t,p\}, (i,j) \in L_T \tag{18}$$
$$x_{ij}^{w,m}\left[t_{ij} + \left(\rho_i^{w,m} - \rho_j^{w,m}\right) + \tau_{ij}\right] = 0 \quad \forall w \in W, m = \{t,p\}, (i,j) \in L_T \tag{19}$$
$$t_{ij} + \left(\rho_i^{w,m} - \rho_j^{w,m}\right) + \beta_{ij} + \psi_{ij}(f_n) + \pi_{ij} \geq 0 \quad \forall w \in W, m = p, (i,j) \in L_P \tag{20}$$
$$x_{ij}^{w,m}\left[t_{ij} + \left(\rho_i^{w,m} - \rho_j^{w,m}\right) + \beta_{ij} + \psi_{ij}(f_n) + \pi_{ij}\right] = 0 \quad \forall w \in W, m = p, (i,j) \in L_P \tag{21}$$

$$\pi_{ij}\left[\sum_{w\in W} x_{ij}^{w,m=p} - c_{ij}^{0} + y_{ij}c_{ij}^{P}\right] = 0 \quad \forall (i,j) \in L_P \tag{22}$$

$$\sum_{w\in W} x_{ij}^{w,m=p} \le c_{ij}^{0} + y_{ij}c_{ij}^{P} \qquad \forall (i,j) \in L_P \tag{23}$$

$$t_{ij} + \psi_{ij}(f_n) + \left(\rho_i^{w,m} - \rho_j^{w,m}\right) \ge 0 \quad 0 \quad \forall w \in W, m = t, (i,j) \in L_P \tag{24}$$

$$x_{ij}^{w,m}\left[t_{ij} + \psi_{ij}(f_n) + \left(\rho_i^{w,m} - \rho_j^{w,m}\right)\right] = 0 \quad \forall w \in W, m = t, (i,j) \in L_P \tag{25}$$

$$t_{ij} + \left(\rho_i^{w,m} - \rho_j^{w,m}\right) \ge 0 \quad \forall w \in W, m = t, (i,j) \in L_E \tag{26}$$

$$x_{ij}^{w,m}\left[t_{ij} + \left(\rho_i^{w,m} - \rho_j^{w,m}\right)\right] = 0 \quad \forall w \in W, m = t, (i,j) \in L_E \tag{27}$$

$$t_{ij} + \left(\rho_i^{w,m} - \rho_j^{w,m}\right) \ge 0 \quad \forall w \in W, m = p, (i,j) \in L_E, d(w) = j \tag{28}$$

$$x_{ij}^{w,m}\left[t_{ij} + \left(\rho_i^{w,m} - \rho_j^{w,m}\right)\right] = 0 \quad \forall w \in W, m = p, (i,j) \in L_E, d(w) = j \tag{29}$$

$$\frac{1}{\theta}(lnd^{w,m} + \alpha^{m}) + \lambda^{w} - E^{w}\rho^{w,m} = 0 \quad \forall w \in W, m \in M \tag{30}$$

$$\pi_{ij} \ge 0 \quad \forall (i,j) \in L_P \tag{31}$$

$$(x,d) \in \Phi$$

where $\rho^{w,m}$ is the multiplier associated with equation (1), whose component $\rho_i^{w,m}$ is called the node potential of node $i$ for OD pair $w$ traveling on mode $m$. $\lambda^{w}$ and $\pi_{ij}$ are the multipliers associated with equations (2) and (13), respectively. $\pi_{ij}$ can be further interpreted as the extra overflow charge that arises when the demand for a P&R lot exceeds its parking capacity.

Let $\Psi_{r+}^{w,m}$ and $\Psi_{r0}^{w,m}$ be the sets of links along utilized and unutilized paths, respectively, for travelers for OD pair $w$ using mode $m$. $c_r^{w,m}$ denotes the generalized travel cost along path $r$ between OD pair $w$ for mode $m$. When the flow distribution reaches equilibrium within each transportation mode, the equilibrium travel cost for OD pair $w$ via mode $m$ is denoted as $C^{w,m}$. If a path, $r$, is utilized by travelers for OD pair $w$ using mode $m$, the flow on each link along the path must be positive, that is, $x_{ij}^{w,m} > 0$, $\forall \Psi_{r+}^{w,m}$.

For travelers choosing the automobile mode, if link $(i,j)$ is utilized, that is, $x_{ij}^{w,m=a} > 0$, equations (16) and (17) reduce to $t_{ij}(v_{ij}) + \left(\rho_i^{w,m=a} - \rho_j^{w,m=a}\right) + \mu_{ij}^{w} = 0$. Summing this equation along a utilized path, $r$, yields the following,

$$c_{r+}^{w,m=a} = \sum_{(i,j)\in\Psi_{r+}^{w,m=a}} (t_{ij}(v_{ij}) + \mu_{ij}^{w}) = -\sum_{(i,j)\in\Psi_{r+}^{w,m=a}} (\rho_i^{w,m=a} - \rho_j^{w,m=a}) = \rho_{d(w)}^{w,m=a} - \rho_{o(w)}^{w,m=a}$$

Therefore, the generalized cost of every utilized path for automobile users is the same and equals to $\rho_{d(w)}^{w,m=a} - \rho_{o(w)}^{w,m=a}$ for OD pair $w$. Similarly, if link $(i,j)$ is unutilized, that is, $x_{ij}^{w,m=a} = 0$, then $t_{ij}(v_{ij}) + (\rho_i^{w,m=a} - \rho_j^{w,m=a}) + \mu_{ij}^{w} \geq 0$. We have $c_{r0}^{w,m=a} = \sum_{(i,j)\in\Psi_{r0}^{w,m=a}} (t_{ij}(v_{ij}) + \mu_{ij}^{w}) \geq \rho_{d(w)}^{w,m=a} - \rho_{o(w)}^{w,m=a}$. This indicates that all utilized paths between OD pair $w$ for automobile users have the same generalized travel cost, that is, $\rho_{d(w)}^{w,m=a} - \rho_{o(w)}^{w,m=a} = C^{w,m}$, and the generalized travel cost of every unutilized path is higher than or equal to $C^{w,m}$.

Using a similar argument, the generalized travel cost along a utilized path for transit users between OD pair $w$ can be derived from equations (18) – (19) and (24) – (27) as follows:

$$c_{r+}^{w,m=t} = \sum_{(i,j)\in\Psi_{r+}^{w,m=t}} t_{ij} + \sum_{(i,j)\in\Psi_{r+}^{w,m=t}\cap L_T} \tau_{ij} + \sum_{(i,j)\in\Psi_{r+}^{w,m=t}\cap L_P} \psi_{ij}(f_n) = \rho_{d(w)}^{w,m=t} - \rho_{o(w)}^{w,m=t}$$

For an unutilized path, the generalized travel cost is $c_{r0}^{w,m=t} = \sum_{(i,j)\in\Psi_{r0}^{w,m=t}} t_{ij} + \sum_{(i,j)\in\Psi_{r0}^{w,m=t}\cap L_T} \tau_{ij} + \sum_{(i,j)\in\Psi_{r0}^{w,m=t}\cap L_P} \psi_{ij}(f_n) \geq \rho_{d(w)}^{w,m=t} - \rho_{o(w)}^{w,m=t}$.

For P&R users, it follows from equations (14) – (15), (18) – (21), and (28) – (29) that the generalized travel cost along a utilized path between OD pair $w$ is as follows:

$$c_{r+}^{w,m=p} = \sum_{(i,j)\in(\Psi_{r+}^{w,m=p}\cap L_R)\&d(w)\neq j} t_{ij}(v_{ij}) + \sum_{(i,j)\in\Psi_{r+}^{w,m=p}\cap L_T} (t_{ij} + \tau_{ij})$$
$$+ \sum_{(i,j)\in(\Psi_{r+}^{w,m=p}\cap L_P)\&o(w)\neq i} [t_{ij} + \psi_{ij}(f_n)] + \sum_{(i,j)\in\Psi_{r+}^{w,m=p}\cap L_E} t_{ij} = \rho_{d(w)}^{w,m=p} - \rho_{o(w)}^{w,m=p}$$

Similarly, for an unutilized path, we have the generalized cost $c_{r0}^{w,m=p} \geq \rho_{d(w)}^{w,m=p} - \rho_{o(w)}^{w,m=p}$.

Rewrite equation (30) and it is equivalent to the following multinomial logit model:

$$\frac{d^{w,m}}{D^{w}} = \frac{\exp(-\theta C^{w,m} - \alpha^{m})}{\sum_{m\in M}\exp(-\theta C^{w,m} - \alpha^{m})}$$

where $\theta$ and $\alpha^m$ are the dispersion parameter and mode-specific constant, respectively. Thereby, we prove that the optimal solution $(x, d)$ to the P1 problem satisfies the UE conditions in the multimodal network.

## Chapter 4: Optimal P&R Facility and Transit Service Design Problem

The objective of this study is to propose an integrated framework for locating P&R facilities and optimizing transit services simultaneously to achieve the maximum net social benefit. A mathematical programming approach is adopted to identify the optimal design, that is, optimal locations of P&R facilities and transit frequency, and in the meantime ensures that flow distributions follow the UE principle. The P1 problem provides a tool for obtaining user-equilibrium flow distributions in a multimodal network with pre-defined P&R facility locations and transit service frequency. By incorporating the KKT conditions for problem P1 as constraints, user-equilibrium flow distributions are guaranteed and we thus formulate the optimal design problem as a mathematical program with complementarity constraints (MPCC).

With the logit-based mode choice model, the expected indirect utility received by a randomly sampled individual can be expressed as (Williams, 1977; Small and Rosen, 1981; Yang, 1999; Wu et al., 2011),

$$E\left[\max_{m\in M}(-u^{w,m}+\varepsilon^{w,m})\right]=\frac{1}{\theta}\ln\left[\sum_{m\in M}\exp(-\theta c^{w,m}-\alpha^{m})\right]$$

This measure has an economic interpretation related to consumer surplus. Based on the representative consumer theory of the logit model (Oppenheim, 1995), the total expected indirect utility received by travelers can be expressed as $\frac{1}{\theta}\sum_{w\in W}\ln[\sum_{m\in M}\exp(-\theta c^{w,m}-\alpha^{m})]D^{w}$. Revenues, such as parking fees and transit fares, are a transfer from users to the operator and do not represent either a social gain or a social cost. Therefore, the net social benefit is the sum of user benefit and revenue, excluding amortized investment and operating costs of P&R facilities, and variable transit operating cost. $\gamma_{ij}$ denotes the summation of amortized investment and operating costs of the P&R facility located on link $(i,j)\in L_P$. $\kappa_n$ represents the variable cost of per-unit transit service for transit line $n\in N$. The maximization of the net social benefit is equivalent to the minimization of its negative value (i.e., total expected social cost). The optimal P&R facility and transit service design problem is formulated as the following mathematical program P2:

$$P2: \min_{(x,d,y,f,\rho,\pi,\lambda)} -\frac{1}{\theta}\sum_{w\in W}\ln\left[\sum_{m\in M}\exp(-\theta c^{w,m}-\alpha^{m})\right]D^{w}$$
$$-\sum_{w\in W}\sum_{m=\{t,p\}}\sum_{(i,j)\in L_T}\tau_{ij}x_{ij}^{w,m}-\sum_{w\in W}\sum_{m=p}\sum_{(i,j)\in L_P}\beta_{ij}x_{ij}^{w,m}$$
$$-\sum_{w\in W}\sum_{m=a}\sum_{(i,j)\in L_R}\mu_{ij}^{w}x_{ij}^{w,m}+\sum_{(i,j)\in L_P}y_{ij}\gamma_{ij}+\sum_{n\in N}f_n\kappa_n$$

s.t. (14) – (31)

$$(x,d)\in\Phi$$

$$y_{ij}\in\{0,1\} \qquad \forall(i,j)\in L_P \qquad (32)$$

$$f_n\in\mathbb{Z}^{+} \qquad \forall n\in N \qquad (33)$$

where the objective function is the negative value of net social benefit. This formulation involves two vectors of binary or integer decision variables (i.e., $y_{ij}$ and $f_n$), which generally make the problem more difficult to solve, especially for large-scale networks. To remove integer decision variables, we introduce a set of binary variables, $z_n^b$, to replace the transit frequency, $f_n$, as follows:

$$f_n = 1+\sum_{b=1}^{B}2^{(b-1)}z_n^b$$

The range of frequency that can be represented by the above expression is 1 to $2^B$. For example, it is possible to use three binary variables, that is, $B=3$, to represent the transit frequency $f_n = 1+z_n^1+2z_n^2+4z_n^3$ ranging from 1 to 8. The parameter $B$ is a pre-specified value. Let the maximum possible transit frequency be $f_{max}$; then we have $B=\lfloor\log_2 f_{max}\rfloor$. By adding more binary variables (i.e., increasing $B$), the model can handle higher transit frequency if necessary. Note also that the binary requirement for $y_{ij}$ and $z_n^b$ can be written in the form of complementarity constraints as well. For example, the following constraints ensure that $y_{ij}$ is either 0 or 1 for each P&R candidate link $(i,j)$,

$$0\le y_{ij}\le 1$$
$$y_{ij}(1-y_{ij})=0.$$

Based on the preceding discussions, we can reformulate the optimal P&R facility and transit service design problem as an MPCC without any binary or integer variables, as follows:

$$P3: \min_{(x,d,y,f,\rho,\pi,\lambda)} -\frac{1}{\theta}\sum_{w\in W} ln\left[\sum_{m\in M}\exp(-\theta c^{w,m}-\alpha^{m})\right]D^{w}$$

$$-\sum_{w\in W}\sum_{m=\{t,p\}}\sum_{(i,j)\in L_T}\tau_{ij}x_{ij}^{w,m}-\sum_{w\in W}\sum_{m=p}\sum_{(i,j)\in L_P}\beta_{ij}x_{ij}^{w,m}$$

$$-\sum_{w\in W}\sum_{m=a}\sum_{(i,j)\in L_R}\mu_{ij}^{w}x_{ij}^{w,m}+\sum_{(i,j)\in L_P}y_{ij}\gamma_{ij}+\sum_{n\in N}\sum_{b\in B}(1+\sum_{b=1}^{B}2^{(b-1)}z_n^b)\kappa_n$$

s.t. (14) – (31)

$(x,d)\in\Phi$

$0\le y_{ij}\le 1 \qquad \forall(i,j)\in L_P \qquad (34)$

$y_{ij}(1-y_{ij})=0 \qquad \forall(i,j)\in L_P \qquad (35)$

$0\le z_n^b\le 1 \qquad \forall n\in N, b\in B \qquad (36)$

$z_n^b(1-z_n^b)=0 \qquad \forall n\in N, b\in B \qquad (37).$

## Chapter 5: Solution Algorithm

As formulated, P3 is an MPCC, a class of problems difficult to solve because it violates the Magasarian-Fromovitz constraint qualification (MFCQ) and its feasible region is also non-convex. The KKT conditions may not hold for MPCC problems; therefore, standard algorithms for nonlinear programming problems are generally not effective in solving MPCC problems. In this study, we focus on finding "strongly stationary" solutions, which are defined in Zhang et al. (2009). We extend the active set algorithm (ASA) developed in Zhang et al. (2009) to solve the integrated planning of P&R facility and transit service problem.

Instead of solving an MPCC directly, the ASA solves two simpler problems sequentially. The first problem involved is a restricted version of problem P3, which generates information to assess the current design of P&R location and transit frequency. The second problem is a sub-problem that provides potential direction for updating the current design. Given an initial feasible design for P&R facility locations and transit frequency, the set of P&R candidate links, $L_p$, can be divided into two complementary sets $\Omega_{y,0} = \{(i,j) \in L_p : y_{ij} = 0\}$ and $\Omega_{y,1} = \{(i,j) \in L_p : y_{ij} = 1\}$. Similarly, the frequency set can be divided into $\Omega_{z,0} = \{(b,n) \in \Omega_{z,0}, b \in B : z_n^b = 0\}$, and $\Omega_{z,1} = \{(b,n) \in \Omega_{z,1}, b \in B : z_n^b = 1\}$. A restricted version of the P&R design problem can be formulated as follows:

$$P4: \min_{(x,d,y,z,\rho,\pi,\lambda)} -\frac{1}{\theta}\sum_{w\in W} ln\left[\sum_{m\in M}\exp(-\theta c^{w,m} - \alpha^m)\right] D^w$$
$$-\sum_{w\in W}\sum_{m=\{t,p\}}\sum_{(i,j)\in L_T}\tau_{ij}x_{ij}^{w,m} - \sum_{w\in W}\sum_{m=p}\sum_{(i,j)\in L_P}\beta_{ij}x_{ij}^{w,m}$$
$$-\sum_{w\in W}\sum_{m=a}\sum_{(i,j)\in L_R}\mu_{ij}^{w}x_{ij}^{w,m} + \sum_{y\in Y}y_{ij}\gamma_{ij} + \sum_{n\in N}\sum_{b\in B}(1 + \sum_{b=1}^{B}2^{(b-1)}z_n^b)\kappa_n$$

s.t.
$$y_{ij} = 0 \qquad \forall (i,j) \in \Omega_{y,0} \tag{38}$$
$$y_{ij} = 1 \qquad \forall (i,j) \in \Omega_{y,1} \tag{39}$$
$$z_n^b = 0 \qquad \forall (b,n) \in \Omega_{z,0}, b \in B \tag{40}$$

$$z_n^b = 1 \qquad \forall (b,n) \in \Omega_{z,1}, b \in B \qquad (41)$$

$(x,d) \in \Phi$

*and* (14) to (31).

The purpose of solving the restricted problem is to assess the current design and obtain information regarding updating the P&R facility design and transit frequency (i.e., sets $\Omega_{y,0}$, $\Omega_{y,1}$, $\Omega_{z,0}$, and $\Omega_{z,1}$) to achieve higher social benefit. Let $(x^*, d^*)$ be the solution to the multimodal UE problem P1 given a feasible design $(y^*, z^*)$. Fracchinei and Pang (2003) demonstrate that multiplier vectors $(\rho^*, \pi^*, \lambda^*)$ must exist so that $(x^*, d^*, y^*, z^*, \rho^*, \pi^*, \lambda^*)$ is also optimal for problem P4. As a result, instead of solving P4 directly, we can obtain the optimal solution to P4 by solving a corresponding P1 problem. Let $\delta_{ij}$, $\sigma_{ij}$, $\chi_n^b$, and $\zeta_n^b$ denote the Lagrange multipliers associated with constraints (38) − (41), respectively. The values of these multipliers estimate changes in the objective function value of problem P4. For example, if $\delta_{ij} < 0$ for $(i,j) \in \Omega_{y,0}$, this suggests that moving link $(i,j)$ to the complementary set $\Omega_{y,1}$ may reduce the objective function value and improve the social benefit. On the other hand, if $\sigma_{ij} > 0$ for $(i,j) \in \Omega_{y,1}$, it may be beneficial to shift link $(i,j)$ from $\Omega_{y,1}$ to $\Omega_{y,0}$. Similarly, $\chi_n^b$ and $\theta_n^b$ provide information on updating sets $\Omega_{z,0}$ and $\Omega_{z,1}$. We thus formulate the second simpler problem to automate the process of set adjustment:

$$P5: \min_{(g,h,\bar{y},\bar{z})} \sum_{(i,j)\in\Omega_{y,0}} \delta_{ij} g_{ij} + \sum_{(b,n)\in\Omega_{z,0}} K^b \chi_n^b h_n^b - \sum_{(i,j)\in\Omega_{y,1}} \sigma_{ij} g_{ij} - \sum_{(b,n)\in\Omega_{z,1}} K^b \zeta_n^b h_n^b$$

s.t.

$$\bar{y}_{ij} = g_{ij} \qquad \forall (i,j) \in \Omega_{y,0} \qquad (42)$$

$$\bar{y}_{ij} = 1 - g_{ij} \qquad \forall (i,j) \in \Omega_{y,1} \qquad (43)$$

$$\bar{z}_{ij}^b = h_n^b \qquad \forall (b,n) \in \Omega_{z,0} \qquad (44)$$

$$\bar{z}_{ij}^b = 1 - h_n^b \qquad \forall (b,n) \in \Omega_{z,1} \qquad (45)$$

$$\sum_{b=1}^{B} h_n^b \le 1 \qquad \forall n \in N \qquad (46)$$

$$g_{ij} \in \{0,1\} \qquad \forall (i,j) \in L_P \qquad (47)$$

$$h_n^b \in \{0,1\} \qquad \forall (b,n) \in \Omega_{z,0} \cup \Omega_{z,1}, b \in B \qquad (48)$$

$$\sum_{(i,j)\in\Omega_{y,0}} \delta_{ij} g_{ij} + \sum_{(b,n)\in\Omega_{z,0}} \chi_n^b h_n^b - \sum_{(i,j)\in\Omega_{y,1}} \sigma_{ij} g_{ij} - \sum_{(b,n)\in\Omega_{z,1}} \theta_n^b h_n^b \ge \varphi \qquad (49)$$

The variables $g_{ij}$ and $h_n^b$ are introduced to indicate whether to move the corresponding design variable to the complementary set. The decision variables $\bar{y}$ and $\bar{z}$ denote the updated design of P&R locations and transit frequency. Equation (46) implies that only one digit of the frequency variable can be changed at a time for one transit line to prevent too much fluctuation in iterations. Note that the multipliers generated by the CONOPT solver (Drud, 1994) are linear with respect to its digit $b$. To ensure that changes are always made to the smallest digit possible, a vector of constant $K^b$ is introduced as a weighting factor.

The Lagrange multipliers only estimate the changes in the objective function value of the design problem P4 and thus adjusting the design accordingly may not lead to an actual reduction in net social costs, which can be verified by solving the multimodal UE problem P1 with the resulting design. The parameter $\varphi$ is initially set to $-\infty$ and the objective function value of problem P5 is denoted by $\bar{\varphi}$. When the current updated design cannot lead to a real reduction, constraint (49) is introduced to force the program to generate a different design. The parameter $\varphi$ is updated by $\varphi = \bar{\varphi} + \varepsilon$, where $\varepsilon$ is a sufficient small positive number. The optimal objective function value of problem P5 is strictly greater than the current value $\bar{\varphi}$ and subsequently the program generates a distinct new location design. Problem P5 is solved iteratively until either a valid new design is found or the optimal objective function value reaches zero.

The ASA solution procedure can be summarized as follows:

Step 0: Choose an initial feasible design $(y_{ij}, z_n^b)$ and solve problem P1. Initialize sets $\Omega_{y,1}$, $\Omega_{y,0}$, $\Omega_{z,0,}$ and $\Omega_{z,1}$.

Step 1: Solve problem P4 and denote the optimal objective function value as $SC$. Obtain Lagrange multipliers, $\delta_{ij}$, $\sigma_{ij}$, $\chi_n^b$, and $\zeta_n^b$. Set $\varphi = -\infty$.

Step 2: Solve problem P5 and denote the optimal objective function value and optimal solutions as $\bar{\varphi}$ and $(\bar{y}_{ij}, \bar{z}_{ij}^b)$, respectively. If $\bar{\varphi} < 0$, proceed to step 3. Otherwise, stop and $(\bar{y}_{ij}, \bar{z}_{ij}^b)$ is the best design found.

Step 3: Solve problem P1 with $(\bar{y}_{ij}, \bar{z}_{ij}^b)$. Calculate the total expected social travel costs and denote it as $\overline{SC}$. If $\overline{SC} > SC$, set $\bar{\varphi} = \bar{\varphi} + \varepsilon$ and proceed to step 2. Otherwise, update the current design $(y_{ij}, z_n^b)$ and sets $\Omega_{y,1}$, $\Omega_{y,0}$, $\Omega_{z,0}$, and $\Omega_{z,1}$ accordingly. Proceed to step 1.

## Chapter 6: Numerical Example

Consider the multimodal transportation shown in Figure 3, which contains 21 nodes and 52 links. There are six OD pairs and the passenger demand table is shown in Table 1. Origins and destinations are highlighted in grey in Figure 3. Travel times on road links are assumed to follow the BPR function while travel times on boarding, transit, and alighting links are assumed to be fixed. For simplicity, all road links are assumed to have the same free-flow travel time of 5 and link capacity of 10. All boarding and alighting links have the same fixed travel time of 1, and all transit links have a fixed travel time of 10 except link (17,18), which is assumed to have a fixed travel time of 20. The waiting time experienced by transit and P&R users on a boarding link is determined by the transit frequency of the transit line serving the transit link that is immediately connected to the boarding link. Two transit lines are seen in Figure 3. Line 1 consists of transit links (9,10), (10,11), (11,12), (12,13), and (13,14), while Line 2 includes (15,16), (16,17), and (17,18). The variable operating cost of one unit of transit service per hour for these two transit lines is 50 and 100, respectively. The amortized investment and operating costs of a P&R parking facility is assumed to be 10. The on-street parking spots and P&R parking facility capacity are 0.1 and 3, respectively. Dispersion parameter $\theta$ in the logit model is assumed to be 0.1. Three mode-specific parameters associated with automobile, transit, and P&R modes (i.e., $\alpha^a, \alpha^t$, and $\alpha^p$) are assumed equal to 0, 1, and 2, respectively, which implies that users prefer the automobile mode the most and the P&R mode the least provided all else is equal. To highlight the impacts of P&R location and transit frequency design, P&R parking and downtown parking fees and transit fares (i.e., $\beta$, $\mu$, and $\tau$) are assumed to be zero in this example. GAMS (Rosenthal, 2012) and CONOPT solver (Drud, 1994) are used to implement the ASA solution procedure.

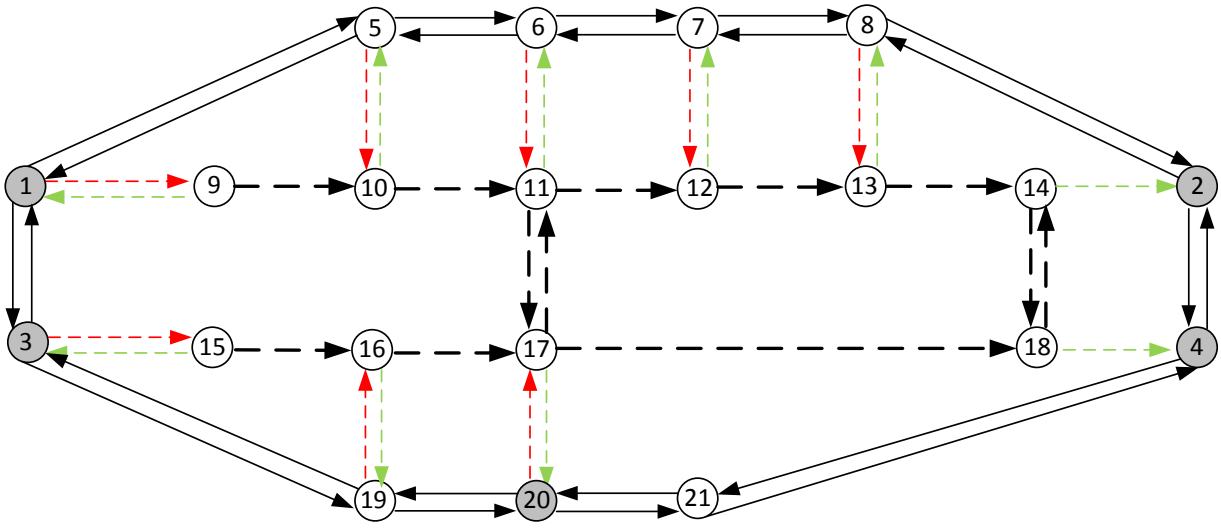



**FIGURE 3 A Multimodal Transportation Network**

**TABLE 1 Aggregate Passenger Demand Table**

| Origin | Destination | |
|---|---|---|
| | 2 | 4 |
| 1 | 30 | 15 |
| 3 | 10 | 10 |
| 20 | 15 | 10 |

The results in Table 2 are from our GAMS implementation on a Dell XPS computer with a 2.3 GHz processor and 6.0 GB RAM. Table 2 indicates that four out of eight P&R candidate facilities are deemed to be beneficial in terms of net social benefit maximization. It also shows that two P&R facilities are so popular that overflow charges have to be introduced to keep the P&R users at these two facilities under capacity. On the other hand, the optimal transit frequency vectors for transit lines 1 and 2 are $(1, 1, 0)$ and $(1, 1, 0)$, respectively. Therefore, the optimal transit service frequency on both lines is identical and $f_{1,2} = 4$ units per hour.

Table 3 further illustrates the impacts of optimal P&R facility and transit service design on social cost as well as on different user groups. The status quo condition (i.e., no P&R facility and no transit optimization) is used as a base scenario to evaluate the performance of the resulting optimal design. Overall, the optimal design reduces the social cost from 6431.46 to 4541.52, a

reduction of 29.38%. The optimal design also suppresses auto travel demand across all six OD pairs and encourages travelers to shift to transit and P&R modes. In particular, users choosing the P&R mode surge dramatically due to construction of P&R facilities and improvements in transit service. Furthermore, equilibrium travel costs for all three modes in the optimal design scenario are less than those under the status quo condition. Therefore, we actually identify a Pareto-improving strategy (Song et al., 2009; Wu et al., 2011; Lawphongpanich and Yin, 2011), even though this is not the explicit objective of the optimal design problem.

**TABLE 2 Summary of Optimal P&R Facility Location Design**

| P&R Candidate Links | Optimal Design | P&R Flow | Overflow Charge | Utilization Rate (%) |
|---|---|---|---|---|
| (1, 9) | Do not build | 0.00 | NA | NA |
| (3, 15) | Build | 3.10 | 0.14 | 100 |
| (5, 10) | Build | 1.77 | 0.00 | 57 |
| (6, 11) | Do not build | 0.00 | NA | NA |
| (7, 12) | Do not build | 0.00 | NA | NA |
| (8, 13) | Do not build | 0.00 | NA | NA |
| (19, 16) | Build | 2.20 | 0.00 | 71 |
| (20, 17) | Build | 3.10 | 1.08 | 100 |

**TABLE 3 Comparison between Status Quo and Optimal Design**

| Scenario | Status Quo | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| OD | Automobile Mode | | | Transit Mode | | | P&R Mode | | |
| | Cost | Flow | Percentage | Cost | Flow | Percentage | Cost | Flow | Percentage |
| (1, 2) | 86.99 | 18.51 | 61.68 | 82.00 | 11.21 | 37.38 | 108.90 | 0.28 | 0.93 |
| (1, 4) | 91.50 | 7.60 | 50.64 | 82.00 | 7.22 | 48.16 | 108.90 | 0.18 | 1.20 |
| (3, 2) | 91.99 | 4.95 | 49.52 | 82.00 | 4.95 | 49.48 | 111.02 | 0.10 | 1.00 |
| (3, 4) | 86.23 | 3.92 | 39.22 | 72.00 | 5.99 | 59.85 | 103.63 | 0.09 | 0.93 |
| (20, 2) | 74.28 | 6.64 | 44.25 | 62.00 | 8.34 | 55.57 | 109.74 | 0.03 | 0.17 |
| (20, 4) | 68.51 | 3.42 | 34.20 | 52.00 | 6.56 | 65.59 | 99.74 | 0.02 | 0.20 |
| Social Cost | 6431.46 | | | | | | | | |
| Scenario | Optimal Design | | | | | | | | |
| OD | Automobile Mode | | | Transit Mode | | | P&R Mode | | |
| | Cost | Flow | Percentage | Cost | Flow | Percentage | Cost | Flow | Percentage |
| (1, 2) | 65.12 | 16.86 | 56.20 | 59.50 | 10.89 | 36.28 | 65.25 | 2.25 | 7.51 |
| (1, 4) | 70.03 | 5.67 | 37.78 | 59.50 | 5.97 | 39.82 | 55.25 | 3.36 | 22.41 |
| (3, 2) | 70.12 | 4.08 | 40.81 | 59.50 | 4.34 | 43.43 | 59.64 | 1.58 | 15.76 |
| (3, 4) | 64.41 | 3.10 | 30.99 | 49.50 | 5.06 | 50.64 | 49.64 | 1.84 | 18.38 |
| (20, 2) | 51.07 | 6.62 | 44.13 | 39.50 | 7.75 | 51.63 | 54.50 | 0.64 | 4.24 |
| (20, 4) | 45.36 | 3.40 | 33.97 | 29.50 | 6.10 | 61.02 | 44.50 | 0.50 | 5.01 |
| Social Cost | 4541.52 | | | | | | | | |

## Chapter 7: Concluding Remarks

This report proposes an integrated planning framework to strategically locate P&R facilities and optimize transit service frequency. In a multimodal transportation network with P&R facilities, P&R users' route choice behavior is explicitly modeled and the multimodal UE is formulated as a link-based mathematical programming model. An MPCC problem is formulated to identify the optimal P&R facility and transit service design. A numerical example is used to demonstrate the effectiveness of a modified ASA solution procedure. The results show that the optimal design improves the net social benefit dramatically and encourages commuters to shift from the automobile mode to the transit and P&R modes. Note also that the optimal design problem reduces the equilibrium travel costs for all users in the system and provides a new direction of exploring Pareto-improving strategies.

This study is timely and much needed as many states are expanding their public transportation options dramatically. The proposed modeling framework provides practitioners with an effective tool to determine the optimal locations of P&R facilities as well as transit frequency. By imposing a budget constraint, the framework can also be used to prioritize candidate P&R projects. In addition, the framework can be used to evaluate current P&R practices quantitatively. The framework can also be expanded to consider other operation strategies (e.g., congestion pricing) to achieve a higher level of social welfare.

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