



8-8-2017

15-10 Development of Multi-class, Multi-criteria Bicycle Traffic Assignment Models and Solution Algorithms

Seungkyu Ryu
Utah State University

Anthony Chen
Utah State University

Keith Christensen
Utah State University

Keechoo Choi
Utah State University

Follow this and additional works at: <https://scholarworks.wmich.edu/transportation-reports>

 Part of the Transportation Engineering Commons

WMU ScholarWorks Citation

Ryu, Seungkyu; Chen, Anthony; Christensen, Keith; and Choi, Keechoo, "15-10 Development of Multi-class, Multi-criteria Bicycle Traffic Assignment Models and Solution Algorithms" (2017). *Transportation Research Center Reports*. 18.

<https://scholarworks.wmich.edu/transportation-reports/18>

This Report is brought to you for free and open access by the Transportation Research Center for Livable Communities at ScholarWorks at WMU. It has been accepted for inclusion in Transportation Research Center Reports by an authorized administrator of ScholarWorks at WMU. For more information, please contact wmu-scholarworks@wmich.edu.



TRCLC 15-10
August 8, 2017

Development of multi-class, multi-criteria bicycle traffic assignment models and solution algorithms

FINAL REPORT

Seungkyu Ryu
Anthony Chen, Ph.D.
Keith Christensen, Ph.D.
Keechoo Choi, Ph.D.
**Department of Landscape Architecture
and Environmental Planning**
Utah State University
Logan, UT 84322



Transportation Research Center
for Livable Communities

Western Michigan University | University of Texas at Arlington | Utah State University | Wayne State University | Tennessee State University

**Technical Report
Documentation Page**

1. Report No. TRCLC 15-10	2. Government Accession No. N/A	3. Recipient's Catalog No. N/A	
4. Title and Subtitle Development of multi-class, multi-criteria bicycle traffic assignment models and solution algorithms		5. Report Date August 31, 2015	
		6. Performing Organization Code N/A	
7. Author(s) Seungkyu Ruy, Anthony Chen, Jacqueline Su, and Keith Christensen		8. Performing Org. Report No. N/A	
9. Performing Organization Name and Address Landscape Architecture and Environmental Planning Utah State University Logan, UT 84322		10. Work Unit No. (TRAIS) N/A	
		11. Contract No. TRCLC 15-10	
12. Sponsoring Agency Name and Address Transportation Research Center for Livable Communities (TRCLC) 1903 W. Michigan Ave., Kalamazoo, MI 49008-5316		13. Type of Report & Period Covered Final Report 4/1/2015 - 8/30/2017	
		14. Sponsoring Agency Code N/A	
15. Supplementary Notes			
16. Abstract Cycling is gaining popularity both as a mode of travel in urban communities and as an alternative mode to private motorized vehicles due to its wide range of benefits (health, environmental, and economical). However, this change in modal share is not reflected in current transportation planning and travel demand forecasting modeling processes. The existing practices to model bicycle trips in a network are not sophisticated enough to describe the full cyclist experience in route decision-making. The purpose of this paper is to develop a multi-class and multi-criteria bicycle traffic assignment model that not only accounts for multiple user classes by acknowledging that there are different types of cyclists with varying levels of biking experience, but also for relevant factors that may affect each user classes' behavior in route choice decisions. The multi-class, multi-criteria bicycle traffic assignment model is developed in a two-stage process. The first stage examines key criteria to generate a set of non-dominated paths for each user class, and the second stage determines the flow allocation to each user's set of efficient paths. Numerical experiments are then conducted to demonstrate the two-stage approach for the multi-class, multi-criteria bicycle traffic assignment model.			
17. Key Words Traffic assignment; multi-objective shortest path; multiclass; cyclist route choice; bicycle		18. Distribution Statement No restrictions.	
19. Security Classification - report Unclassified	20. Security Classification - page Unclassified	21. No. of Pages	22. Price N/A

Disclaimer

The contents of this report reflect the views of the authors, who are solely responsible for the facts and the accuracy of the information presented herein. This publication is disseminated under the sponsorship of the U.S. Department of Transportation's University Transportation Centers Program, in the interest of information exchange. This report does not necessarily reflect the official views or policies of the U.S. government, or the Transportation Research Center for Livable Communities, who assume no liability for the contents or use thereof. This report does not represent standards, specifications, or regulations.

Acknowledgments

This research was funded by the US Department of Transportation through the Transportation Research Center for Livable Communities (TRCLC), a Tier 1 University Transportation Center.

Table of Contents

Chapter 1: Introduction	2
Chapter 2: Methodology	5
2.1 Multiple User Classes	6
2.2 Criteria Affecting Cyclists' Route Choice	6
2.3 Multi-Objective Shortest Path Procedure	11
2.4 Path-Size Logit Stochastic Traffic Assignment	12
Chapter 3: Solution Procedure	14
3.1 Stage 1: Multi-Objective Route Generation	15
3.2 Stage 2: Customized Path-based Algorithm	17
Chapter 4: Numerical Results	18
4.1: Description of the Network and Scenarios	19
4.2: Characteristics of the Winnipeg Network	21
4.3: Route Generation Results from Stage One	23
4.4: Bicycle Traffic Assignment Results from Stage Two	24
Chapter 5: Concluding Remarks	31
References	33

Chapter 1: Introduction

The recent rise in cycling in many cities can be attributed to municipal efforts to promote the health, environmental, and economical benefits of non-motorized modes. Cycling's increased mode share consequently leads to a higher demand for reliable bicycle traffic assignment methodology. Unfortunately, there is only a limited quantity of tools and methods available for modeling bicycle trips in a network. Only a few research efforts focus on network analysis for bicycle trips (e.g., Klobucar and Fricker, 2007; Broach et al., 2011; Mekuria et al., 2012). While these methods do provide pioneering efforts to develop traffic assignment methods for bicycle trips, they are too simplistic. Given an origin-destination (O-D) trip table that describes the travel demand pattern within a study area, the traffic assignment problem is to determine the flows by assigning the O-D trip table to routes in a transportation network according to some behavioral route choice rules. However, current methods are based simply on the all-or-nothing (AON) assignment method using a single attractiveness measure such as distance, safety, or a composite measure of safety multiplied by distance. This is problematic because cyclists travel not only on one route, but on many different routes based on different levels of biking experience and different preferences using different combinations of criteria for selecting a cycle route. The AON simplistic modeling of cyclists' route choice will affect the bicycle traffic assignment results and may influence investment decisions for bicycle infrastructures. Therefore, it is imperative to incorporate heterogeneous cyclist route choice behaviors in the bicycle traffic assignment model in order to enhance the accuracy of bicycle traffic forecasts.

The route choice model for bicycles is much more complex than the model for private motorized vehicles because there many influential factors affecting cyclist route choice decisions.

According to empirical studies on bicycle route analysis, cyclists choose routes based on any number of criteria that may include distance, number of intersections, road grade, bicycle facility, and safety. In identifying the factors that affect cyclist route choice decisions, Stinson and Bhat (2003), Hunt and Abraham (2007), and Broach et al. (2011) discovered travel distance/time was significant while Hopkinson and Wardman (1996), Akar and Clifton (1996), Dill and Carr (2009) and Winters et al. (2011) revealed safety was likewise influential. Sener et al. (2009) confirmed that the travel distance/time and safety were important factors in cyclist

route choice. Mekuria et al. (2012) suggested that stress is an important factor in cyclist trip-making behavior. Handy and Xing (2011) analyzed the key factors in commuting trips in six small U.S cities, while Heinen and Handy (2012) compared the factors with respect to health, environmentally friendliness, and travel enjoyment in bicycle cities like Davis in the United States and Delft in the Netherlands. Using GPS tracking data, Hood et al. (2011) developed a path-size logit (PSL) model (Ben-Akiva and Birelaire, 1999) as a cyclist route choice model and performed the bicycle traffic assignment on a pre-enumerated path set generated by the doubly stochastic method (Bovy and Fiorenzo Catalano, 2007). Menghini et al. (2008) also adopted a PSL model for traffic assignment on a pre-generated path using a breadth-first search link elimination approach. On the other hand, Ryu et al. (2015) developed a two-stage bicycle traffic assignment model. The first stage considers two key criteria (e.g., distance related attributes and safety related attributes) to generate a set of non-dominated (or efficient) paths, while the second stage determines the flow allocation to the set of efficient paths.

While it is important to analyze the various criteria that affect cyclist decision making, it is also critical to consider multiple user classes in a bicycle traffic assignment model. According to a study on Portland cyclists (Geller 2006), residents can be categorized into four types of cyclists: “The Strong and the Fearless,” “The Enthused and the Confident,” “The Interested but Concerned,” and “No Way No How.” Each group has distinct relationships and attitudes with bicycle transportation that may affect their preferences in route choice.

Consequently, the purpose of this research is to build on these existing studies by developing a multi-class and multi-criteria bicycle traffic assignment model that explicitly considers multiple user classes and multiple criteria affecting cyclist route choice decisions for estimating bicycle volumes on a transportation network. The multi-class component aims to model the different types of cyclists by segmenting them into multiple user classes according to the cyclists’ characteristics, while the multi-criteria component aims to model the relevant factors (e.g., least elevation gain route, shortest distance route, safest route, least accident route, bike friendly route, lowest pollution route, route with green space, etc.) that affect each user class’s behavior in making route choice decisions. By integrating both the multi-criteria and multi-class components

into the model, this research seeks to gain a more comprehensive understanding of cyclist decision making and of bicycle network analysis.

The overall procedure for developing the multi-class and multi-criteria bicycle traffic assignment model follows a two-stage process (Ryu et al., 2015). The first stage considers key criteria (e.g., one or more factors relevant to each user class) to generate a set of non-dominated (or efficient) paths for each user class, while the second stage determines the flow allocation to each user's set of efficient paths. Specifically, the multiple objective shortest path problem based on relevant key attributes is developed in Stage 1 to generate the efficient paths for each user class, and the path-size logit (PSL) stochastic traffic assignment method is adopted in Stage 2 to determine the flow allocations in a network. Numerical experiments are conducted to demonstrate the two-stage approach for the multi-class, multi-criteria bicycle traffic assignment.

The remainder of this paper is organized as follows. After the introduction, the multiple bicycle user classes and criteria are described, followed by the presentation of the two-stage traffic assignment procedure, a numerical experiment to demonstrate the features and applicability of the proposed two-stage procedure, and some concluding remarks.

Chapter 2: Methodology

This section describes the methodology for modeling the multi-class, multi-criteria bicycle traffic assignment procedure as shown in Figure 1. There are two stages in this procedure: (1) route generation for determining individual route choice sets based on the relevant criteria for each user class, and (2) traffic assignment for allocating flows to routes of each user class. The multi-class, multi-criteria bicycle traffic assignment model assigns the bicycle O-D matrices of multiple user classes (assumed to be given from the mode choice step of a four-step travel demand forecasting model) based on the path-size logit (PSL) stochastic traffic assignment model using the relevant individualized route sets to obtain the bicycle traffic flow pattern on the network.

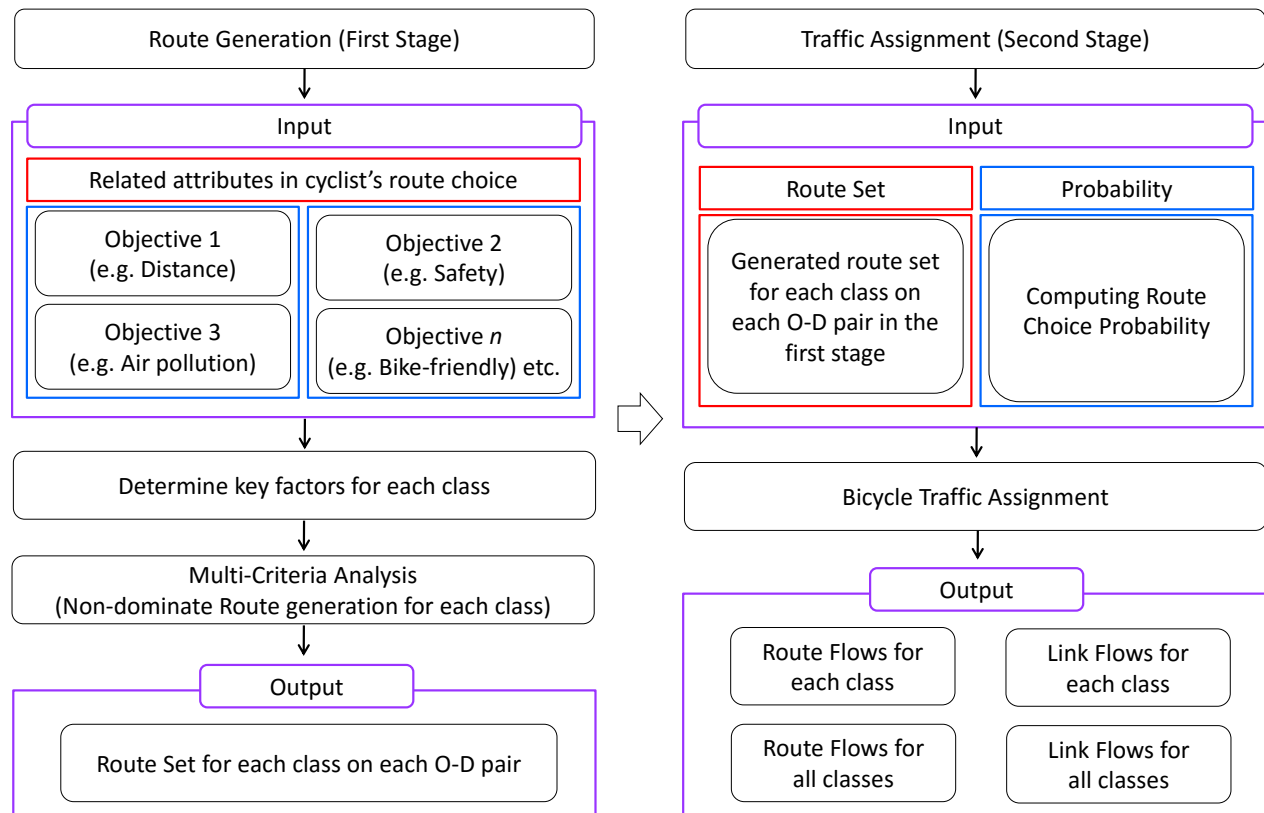


Figure 1. Multi-class, multi-criteria bicycle traffic assignment procedure

The following subsections describe the multiple user classes, the multiple criteria affecting cyclists' route choice decisions, the multi-objective shortest path algorithm, and the PSL stochastic traffic assignment model for flow allocations to the efficient paths.

2.1 Multiple User Classes

Based on the Portland study (Geller, 2006), it has been suggested that there are four types of transportation cyclists as indicated in Figure 2. These four types of cyclists are: (1) strong and fearless, (2) enthused and confident, (3) interested but concerned, and (4) no way no how. Strong and fearless cyclists represent less than 1% of the population; they are the rare daily commuters who “*will ride regardless of roadway conditions.*” Enthused and confident cyclists represent 7% and are semi-regular cyclists who are “*comfortable sharing the roadway with automotive traffic, but they prefer to do so operating on their own facilities*” (e.g., bicycle lanes and bicycle boulevards). The interested but concerned cyclists, who represent 60% of the population, are irregular cyclists who are “*curious about cycling*” but are concerned with riding a bicycle. Lastly, no way no how travelers represent 33% and are simply “*not interested in bicycling at all, for reasons of topography, inability, or simply a complete and utter lack of interest*”. The study also noted that “*the separation between these four broad groups is not generally clear-cut*”. However, this classification with percentage to each user class serves as a good foundation to develop a multi-class version of the multi-criteria bicycle traffic assignment model.

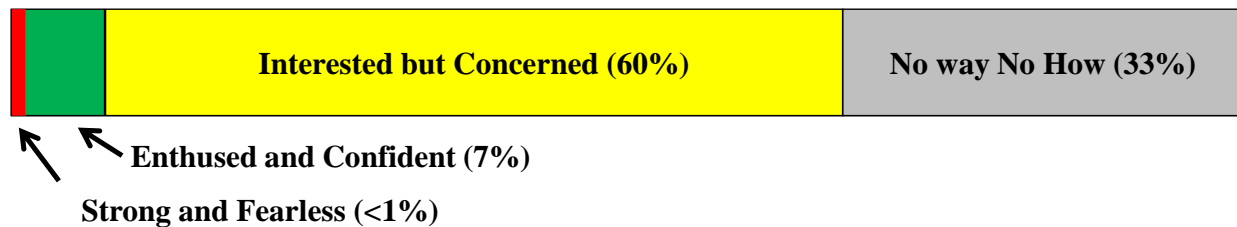


Figure 2. Four types of cyclists in Portland (Geller, 2006)

2.2 Criteria Affecting Cyclists' Route Choice

Empirical studies on bicycle route choice analysis indicate that cyclists choose routes based on a number of criteria. Examples of key criteria include travel distance or time (Stinson and Bhat, 2003; Hunt and Abraham, 2007; Broach et al., 2011), safety (Hopkinson and Wardman, 1996; Akar and Clifton, 2009; Dill and Carr, 2003; Winters et al., 2011), stress (Mekuria et al. 2012), travel distance/time and safety (Sener et al., 2009), etc. Willis et al. (2015) summarized the influential factors that may affect bicycle travel with 24 relevant papers published between 2005 and 2015. Route planners acknowledge the diversity and quantity of influential factors by

providing a variety of bicycle routes that optimize different factors (e.g., least elevation gain route, shortest distance route, safest route, least accident route, bike friendly route, lowest pollution route, route with green space, etc.) to serve the needs of different cyclists.

In this study, three key criteria (e.g., route distance related attributes, route safety related attributes, and route pollution related attributes) are adopted to develop the multi-class, multi-criteria bicycle traffic assignment model. These criteria are composed of many factors; it encompasses the relevant factors identified by the literature for modeling route choice decisions for each cyclist class. For example, criteria related to route safety incorporates many of the cyclist safety concerns that [Hopkinson and Wardman \(1996\)](#), [Akar and Clifton \(2009\)](#), [Dill and Carr \(2003\)](#) and [Winters et al. \(2011\)](#) uncovered in their research regarding route choice. Figure 3 provides a summary of the different factors by organizing them into four groups and showing how the factors contribute to the three key criteria used to model cyclists' route choice decisions for different user classes. The four factor category groups include (a) motorized traffic related data (e.g., traffic volume, proportional of heavy vehicles, speed limit, etc.) used in the [Highway Capacity Manual \(HCM, 2010\)](#), (b) network topology (e.g., link distance, slope, intersection configuration, etc.), (c) bicycle facility (e.g., bike lane, bike path, bike parking, etc.), and (d) user preferences (e.g., road cognition, environmental impact, bike friendliness, etc.). These factors are further applied into the three key criteria (distance-related attributes, safety-related attributes, and air pollution related attributes) to determine cyclist route choice decisions for different user classes. The details of these three key criteria are described in the following subsections.

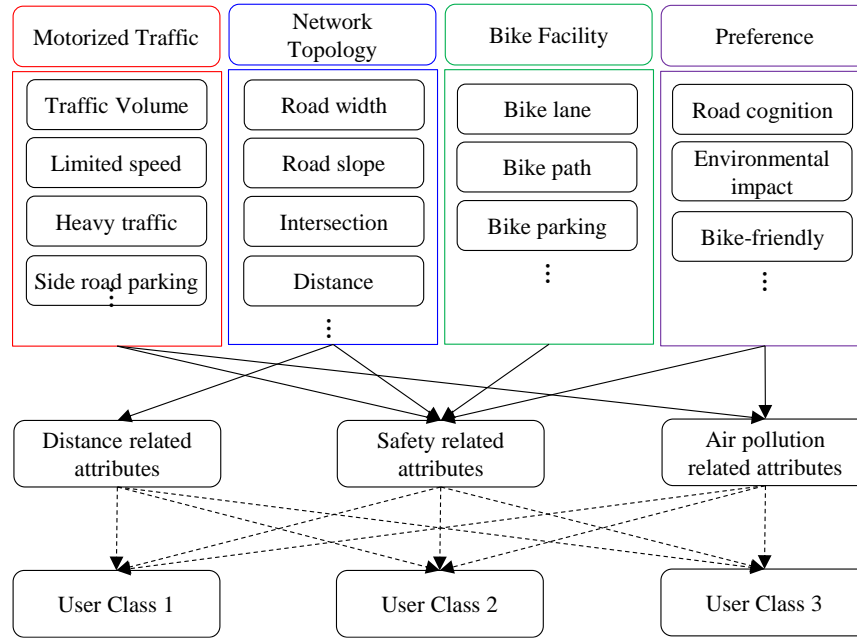


Figure 3. Three key criteria affecting cyclists' route choice decisions

2.2.1 Route Distance

As a composite measure, route distance is composed of both the sum of link distances along the route and the turning movement penalties (or delays) at intersections that the route passes through. Intersection delays are especially significant for cyclists; they have been shown to be a major deterrent against route choice. To address the unit incompatibility problem between link length and intersection turning movement penalty (link length measures length in meters while intersection penalty measures time in seconds), penalty is converted to an equivalent distance unit with an appropriate conversion factor. The route distance can be computed as follows:

$$d_k^{rs} = \sum_{a \in A} l_a \delta_{ka}^{rs} + \sum_{a \in IN_i} \sum_{b \in OUT_i} cf_i^t d_i^t \delta_{ka}^{rs} \delta_{kb}^{rs}, \quad rs \in RS, k \in K_{rs}^m \quad (1)$$

where d_k^{rs} is the distance (in meter) on path k connecting O-D pair rs ; l_a is the length (in meter) on link a ; δ_{ka}^{rs} (δ_{kb}^{rs}) is the path-link indicator; 1 if link a (b) is on path k between O-D pair rs and 0; cf_i^t is the penalty conversion factor to an equivalent distance unit (in meter/second) for turning movement t at intersection i ; d_i^t is the penalty (in second) of turning movement t at intersection i ; A is the set of links; IN_i and OUT_i are the sets of links terminating into and originating out of

intersection i ; RS is the set of O-D pairs; and K_{rs}^m is the set of paths connecting O-D pair rs of class m . The route distance in Eq. (1) can be computed by summing the link distances (first term) and the intersection penalties (second term) caused by turning movement from link a to link b of intersection i that comprise of that path. Note that the first term can further include other attributes such as penalty for links with elevation gain, restriction on gradient, or any attribute that has an impact on the physical geometry of the link. On the other hand, the second term can further include signalized delays at intersections. Note that using two consecutive path-link indicators $\delta_{ka}^{rs}\delta_{kb}^{rs}$ (i.e., link a and link b along path k between origin r and destination s), correct turning movement penalty (left, through, and right) can be appropriately added to the route cost without the need to expand the network to represent turning movements for all approaches of each intersection (Chen et al., 2012).

2.2.2 Route Bicycle Level of Service (BLOS)

The safety aspect of bicycle facilities (or the suitability for bicycle travel) can be assessed by a variety of different measures. Lowry et al. (2012) recently reviewed thirteen methods used in the research community and found that most measures score the perceived safety of bicycle facilities by using a set of variables to represent conditions of the roadway and environment that affect a cyclist's comfort level. To account for the different attributes contributing to the safety of bicycle routes in this paper, we decided to use the Highway Capacity Manual's (2010) bicycle level of service (BLOS) measure as a surrogate measure. The BLOS measure is a reasonable bicycle safety measure to use because it is considered to be a state-of-the-art method and thus widely used across the United States as a guide for bicycle facility design. It should be noted that the BLOS measure is not the only measure of bicycle safety and that other bicycle safety measures can be easily substituted into our proposed framework for modeling cyclists' route choice behavior. The route BLOS measure is a composite measure based on the average segment bicycle score on a route ($ABSeg$), the average intersection bicycle score on a route ($ABInt$), and the average number of unsignalized conflicts/driveways per mile on a route ($Cflt$). Based on the HCM (2010), the route BLOS can be computed as follows.

$$BLOS_k^{rs} = 0.200 \cdot (ABSeg_k^{rs}) + 0.030 \cdot (\exp(ABInt_k^{rs})) + 0.050 \cdot (Cflt) + 1.40, \quad rs \in RS, k \in K_{rs}^m \quad (2)$$

where $BLOS_k^{rs}$ is the bicycle level of service on path k between O-D pair rs ; $ABSeg_k^{rs}$ is the length weighted average segment bicycle score on path k between O-D pair rs ($ABSeg_k^{rs} = \left(\sum_{a \in A} l_a \cdot Bseg_a \cdot \delta_{ka}^{rs} \right) / \left(\sum_{a \in A} l_a \cdot \delta_{ka}^{rs} \right)$); l_a is the link length (in meter); $ABInt_k^{rs}$ is the average intersection bicycle score on path k between O-D pair rs ($ABInt_k^{rs} = \sum_{i \in I} \sum_{a \in IN_i} \sum_{b \in OUT_i} IntBLOS_i \delta_{ka}^{rs} \delta_{kb}^{rs} / N_k^{rs}$); $Clft_k^{rs}$ is the number of unsignalized conflicts per km; N_k^{rs} is the number of intersections on path k between O-D pair rs .

Note that the segment and intersection bicycle scores ($Bseg_a$ and $IntBLOS_i$) provided in Eqs. (3) and (4) are calibrated based the volume and speed of motorized vehicles, the width configuration of bicycle facilities, pavement conditions, number of intersections, etc. The derived BLOS score is a relative measurement without score unit to evaluate the comfortableness on cycling route. The details of the BLOS development can be found in [NCHRP Report 616 \(Dowling et al. 2008\)](#).

$$BSeg_a = 0.507 \ln \left(\frac{v_a}{4 \cdot PHF_a \cdot La_a} \right) + 0.199 F_{s_a} (1 + 10.38 \cdot HV_a)^2 + 7.066 \left(\frac{1}{PC_a} \right)^2 - 0.005 (We_a)^2 + 0.76 \quad (3)$$

$$IntBLOS_i = -0.2144 \cdot Wt_i + 0.0153 \cdot CD_i + 0.0066 \left(\frac{Vol15_i}{L_i} \right) + 4.1324 \quad (4)$$

where

PHF_a : peak hour factor of link a	Wt_i : width of outside through lane plus paved shoulder (including bike lane where present) of intersection i
HV_a : proportion of heavy motorized vehicles of link a	CD_i : crossing distance, the width of the side street (including auxiliary lanes and median) of intersection i
We_a : average effective width on outside through lane of link a (m)	$Vol15_i$: volume of directional traffic during a 15 minute period of intersection i
F_{s_a} : effective speed factor on link a	L_i : total number of directional through lanes of intersection i
La_a : total number of directional through lanes on link a	
v_a : directional motorized vehicle volume on link a (vph)	

PC_a : *FHWA's* five point pavement surface condition rating on link a

To calculate segment and intersection LOS scores, it requires not only the volume and speed of motorized vehicles, which are obtained exogenously by solving the multi-class traffic assignment problem with multiple vehicle types, but also detailed network topology information (e.g., pavement surface condition, average effective width of outside through land, crossing distance, etc.).

2.2.3 Route Pollution

In some cities where air quality is inadequate, cyclists may prefer a route that avoids pollution. For simplicity, we choose carbon monoxide (CO), which has been shown as an important indicator for the level of atmospheric pollution, as a representative attribute of air quality. In addition, there exist empirical functional expression and data availability for computing the network-wide CO pattern. However, other pollutants can be modeled in a similar manner (see [Pankow *et al.* \(2014\)](#) for a more detailed evaluation of cyclists' exposure to traffic related air pollution). In this study, the route pollution is computed as follows:

$$CO_k^{rs} = \sum_{a \in A} g_a \cdot \delta_{ka}^{rs}, \quad \forall k \in K_{rs}^m, rs \in RS \quad (5)$$

where g_a is the amount of CO pollution in grams per hour (g/h) on link (or segment) a . To estimate the amount of CO pollution, we adopt the nonlinear macroscopic model of [Wallace *et al.* \(1998\)](#):

$$g_a(v_a) = 0.2038 \cdot t_a(v_a) \cdot \exp\left(\frac{0.7962 \cdot l_a}{t_a(v_a)}\right) \quad (6)$$

where v_a is the motorized vehicle volume on link a ; $t_a(v_a)$ is the link travel time (in minutes); and l_a is the link length (in meters). The above CO emission has also been adopted in [Yin and Lawphongpanich \(2006\)](#), [Nagurney *et al.* \(2010\)](#), [Chen and Xu \(2012\)](#), [Chen and Yang \(2012\)](#), [Ng and Lo \(2013\)](#), [Xu *et al.* \(2013, 2015\)](#), and [Szeto *et al.* \(2014\)](#).

2.3 Multi-Objective Shortest Path Procedure

The three key criteria identified in Section 2.2 will be used in the multi-objective shortest path procedure to generate non-dominated (or efficient) paths relevant to each user class. The solution procedure for multiple objective shortest path problems involves the generation of a set of non-dominated (or Pareto) paths because there may not be a single optimal path that dominates all other paths in all objectives. This detail makes the solution procedure for the multi-objective shortest path problem distinct from that of the single objective shortest path problem. In the literature, there are several solution procedures that have been developed for solving the multi-objective shortest path problem, including the label correcting approach (Skriver and Andersen, 2000), the label setting approach (Tung and Chew, 1992), the ranking method (Climaco and Martins, 1982), and the two-phase method (Ulungu and Teghem, 1995). In addition to generating a set of efficient paths, the multi-objective shortest path procedure needs to handle a non-additive route cost structure (i.e., the route cost is not a simple additive sum of the link attributes). Of the objectives (or criteria) considered for bicycle route generation, route BLOS is non-additive. It is a composite measure based on the average segment bicycle score ($ABSeg$ given in Eq. (3)) on a route, the average intersection bicycle score ($ABInt$ given in Eq. (4)) on a route, the average number of unsignalized conflicts/driveways per mile on a route ($Cflt$), and the route-specific constant (1.40). These four terms ($ABSeg$, $ABInt$, $Cflt$, and 1.40) are non-additively combined to calculate the route BLOS. Of the four multi-objective shortest path methods mentioned above, the label correcting approach, the label setting approach, and the ranking method are not directly applicable for solving non-additive shortest path problems with multiple objectives. In this paper, we adopted the two-phase procedure developed by Ulungu and Teghem (1995) for solving the multi-objective shortest problem with non-additive route cost structure to generate a set of efficient paths. Note that Ehrgott et al. (2012) also adopted the two-phase procedure for solving the bi-objective cyclist route choice model. The overall two-phase procedure is described in Section 3.

2.4 Path-Size Logit Stochastic Traffic Assignment

In the stochastic traffic assignment problem, route overlapping is one of the major concerns in modeling route choice decisions (see Prashker and Bekhor (2004) and Chen et al. (2012) for a detailed description of the different approaches for handling the route overlapping problem). In this paper, the path-size (PS) factor is adopted to handle the route overlapping problem due to its

simplicity and relatively better performance compared to other closed-form models (e.g., cross-nested logit (CNL) model and paired combinatorial logit (PCL) model). The PS factor accounting for different path sizes is determined by the length of links within a path and the relative lengths of paths that share a link as follows:

$$PS_k^{rs} = \sum_{a \in k} \left(\frac{l_a}{L_k^{rs}} \right) \cdot \left(\frac{1}{\sum_{l \in K_{rs}^m} \delta_{la}^{rs}} \right), \quad \forall rs \in \text{RS}, k \in K_{rs}^m \quad (7)$$

where PS_k^{rs} is the PS factor of path k between O-D pair rs ; l_a is the length of link a ; and L_k^{rs} is the length on path k between O-D pair rs . Paths with a heavy overlapping with other paths have a smaller PS value, while paths that are more distinct have a larger PS value. For other functional forms of the PS factor, see [Bovy et al. \(2008\)](#) and [Prato \(2009\)](#). With the derived PS value in Eq. (7), the PS-logit (PSL) probability for the stochastic traffic assignment problem can be expressed as

$$P_k^{rs} = \frac{PS_k^{rs} \cdot \exp(U_k^{rs})}{\sum_{j=1}^n PS_j^{rs} \cdot \exp(U_j^{rs})}, \quad \forall rs \in \text{RS}, k \in K_{rs}^m \quad (8)$$

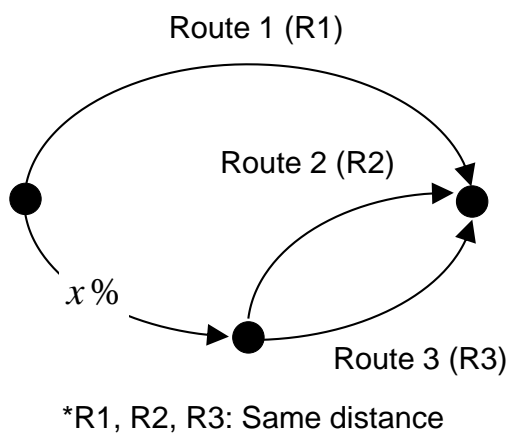
where U_k^{rs} is the utility of path k between O-D pair rs . A possible way to define the utility is as follows:

$$U_k^{rs} = - \left((d_k^{rs})^\alpha \cdot (BLOS_k^{rs})^\beta \cdot (CO_k^{rs})^\gamma \right), \quad \forall rs \in \text{RS}, k \in K_{rs}^m \quad (9)$$

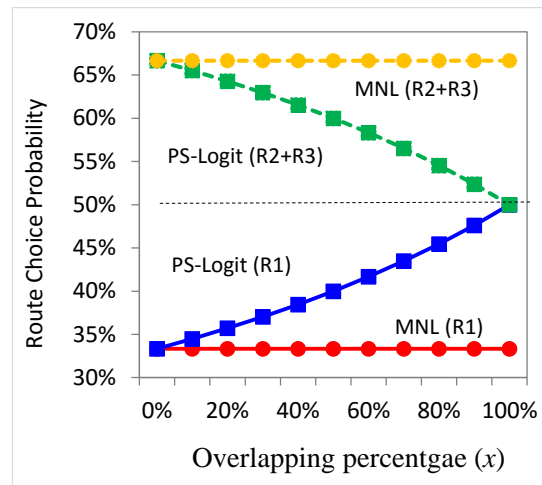
where α , β and γ are parameters of the utility function. Figure 4 provides an illustration of how the PSL model resolves the route overlapping problem using the loop-hole network. This network, which is shown in Figure 4(a), consists of three routes. Route 1 (R1) is an independent route (i.e., no overlapping with other routes), while Route 2 (R2) and Route 3 (R3) share an overlapping percentage x between the two routes. For illustration purposes, all three routes have the same distance. The route choice probability with different percentages of route overlapping is shown in Figure 4(b). As can be seen, the PSL model gives the same choice probability as the multinomial

logit (MNL) model when there are no route overlaps ($x=0$). In this case, the independence assumption (i.e., the three routes are distinct without any overlap) is fully satisfied, and the PSL model degenerates to the MNL model at $x=0$. However, when there are route overlaps ($x>0$), the PSL choice probability of the two overlapping routes (R2+R3) becomes smaller with an increasing x value (i.e., shown in the green line), which is more reasonable compared to the constant MNL choice probability results (i.e., R1 shown in the red line and R2+R3 shown in the yellow line) for all x values.

The PSL stochastic traffic assignment model is used to allocate the multi-class O-D demands based on different types of cyclists described in Section 2.1 using the combined utilities of multiple criteria via the PSL probability expression in Eq. (8).



(a) Loop-hole network



(b) Route choice probability

Figure 4. Illustration of the PSL model in resolving the route overlapping problem

Chapter 3: Solution Procedure

The overall procedure for solving the multi-class and multi-criteria bicycle traffic assignment model follows a two-stage process, which is described in Figure 1. The first stage generates a set of non-dominated, efficient paths for each user class by inputting multiple criteria (route distance, route level of service, route air pollution) in a multiple objective shortest path algorithm. The second stage then produces a complete bicycle flow pattern on the network by adapting a path size logit (PSL) stochastic multi-class traffic assignment model that determines the flow allocations to

the efficient paths generated in Stage 1. This section describes the details of the two-stage multi-class, multi-criteria bicycle traffic assignment procedure.

3.1 Stage 1: Multi-Objective Route Generation

In Stage 1, the two-phase procedure developed by [Ulungu and Teghem \(1995\)](#) is adopted to solve the multi-objective shortest problem with non-additive route cost structure. In the first phase, the possible routes are generated using one of the objectives, while the second phase determines the efficient routes (or non-dominated routes) relative to the remaining objectives. The overall two-phase procedure is described in Figure 5.

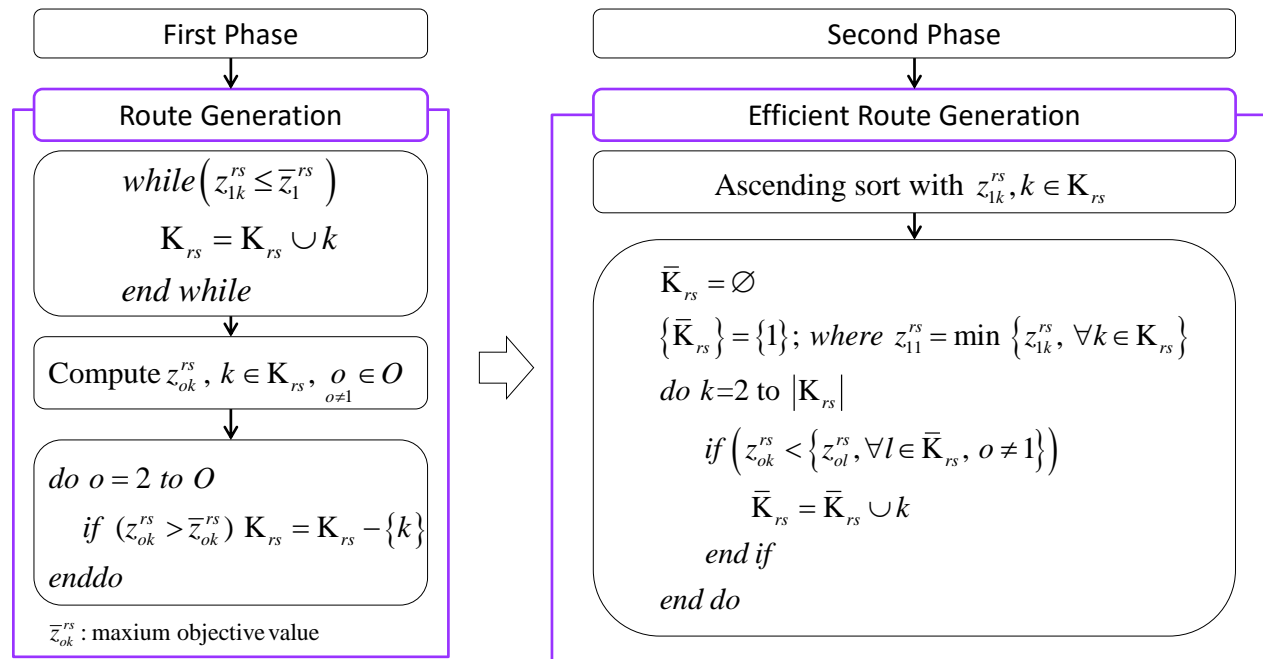


Figure 5. Two-phase procedure for generating efficient routes

Using the three key criteria as an example for illustration purposes, the first phase uses the distance-related attributes (i.e., link distance and intersection delay) to generate a set of realistic routes without exceeding the maximum allowable bound. The corresponding safety-related attributes (route BLOS) and pollution-related attributes (route CO) are also computed. If the routes are higher than the threshold values (e.g., \bar{z}_{1k}^{rs} , \bar{z}_{2k}^{rs} and \bar{z}_{3k}^{rs}) in the other objectives, then the routes are excluded from the route set. With these generated routes sorted in an ascending order, the first route in the set is the route with the minimum distance route and serves as the first efficient route

with the minimum distance. Then, the next route is compared to the routes in the efficient route set to determine whether it satisfies the non-dominated route condition. If the route is satisfied, the route will remain in the efficient route set. The process is repeated for the remaining routes in the set. A pseudo code of the two-phase procedure is provided to generate efficient routes for all origin-destination (O-D) pairs and all user classes as follows:

```

do  $rs=1$  to RS
  do  $m=1$  to M
     $\mathbf{K}_{rs}^m = \emptyset$  // Initialize route set
    while ( $z_{1k}^{rs} \leq \bar{z}_{1k}^{rs}$ )
       $\mathbf{K}_{rs}^m = \mathbf{K}_{rs}^m \cup k$  // Generate all possible routes for the first objective
    end while
    if ( $z_{2k}^{rs} > \bar{z}_2^{rs}$ ) or ( $z_{3k}^{rs} > \bar{z}_3^{rs}$ )  $\mathbf{K}_{rs}^m = \mathbf{K}_{rs}^m - k$ 
    end if // Exclude dominated routes by comparing with other objectives
    do  $n=1$  to Criteria # -1
      Ascending order with  $z_{1k}^{rs}$ 
       $\bar{\mathbf{K}}_{rs}^m = \{1\}$  // Initialize efficient route set with the first route
      do  $k=2$  to  $|\mathbf{K}_{rs}^m|$  // Update efficient route set with other routes
        do  $l=1$  to  $|\bar{\mathbf{K}}_{rs}^m|$ 
          if ( $z_{n+1,k}^{rs} < z_{n+1,l}^{rs}$ )  $\bar{\mathbf{K}}_{rs}^m = \bar{\mathbf{K}}_{rs}^m \cup k$ 
          else  $\bar{\mathbf{K}}_{rs}^m = \bar{\mathbf{K}}_{rs}^m - \{k\}$ 
          end if
        end do
      end do
    end do
  end do
end do

```

Each user class has its own efficient route set that considers the tradeoffs among the multiple criteria that are important to the users in each class. The route generation procedure extends the two-phase multi-objective shortest path procedure to determine multiple efficient path sets for multiple user classes. To reduce the intensive memory requirements of storing efficient paths, a universal efficient path set is designed to store the efficient paths for all user classes without the need to separately store efficient paths for each individual user class (i.e., an efficient path can be shared or used by multiple user classes). A binary (true/false) indicator in each user class is used to determine the individual class efficient path set out of the universal efficient path set. This

simple scheme can help reduce the memory requirements by eliminating the storage of redundant efficient paths for each user class. Figure 6 provides an example of the individual class efficient path set and the universal efficient path set for all classes with three criteria obtained from the route generation procedure in Stage 1.

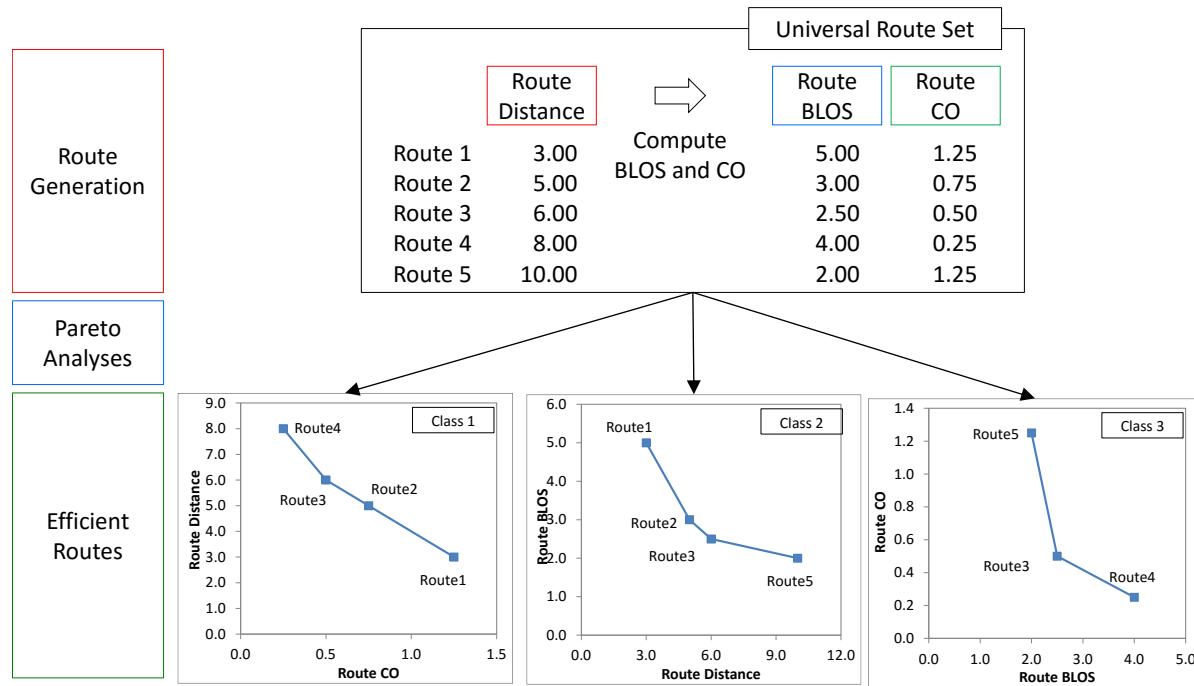


Figure 6. Example of individual class efficient route set and the universal efficient route set

3.2 Stage 2: Customized Path-based Algorithm

In Stage 2, a customized path-based algorithm is developed for solving the PSL stochastic multi-class and multi-criteria bicycle traffic assignment model. The overall flowchart is provided in Figure 7. The main steps include: (1) computing the path-size factor and utility for each efficient path identified in stage one for each user class, (2) calculating the path probability based on the PSL model for each user class, (3) assigning the demand to the efficient paths according the PSL probabilities for each user class, and (4) outputting the bicycle flow pattern on the network, which includes individual class route and link flows as well as aggregate route and link flows of all classes.

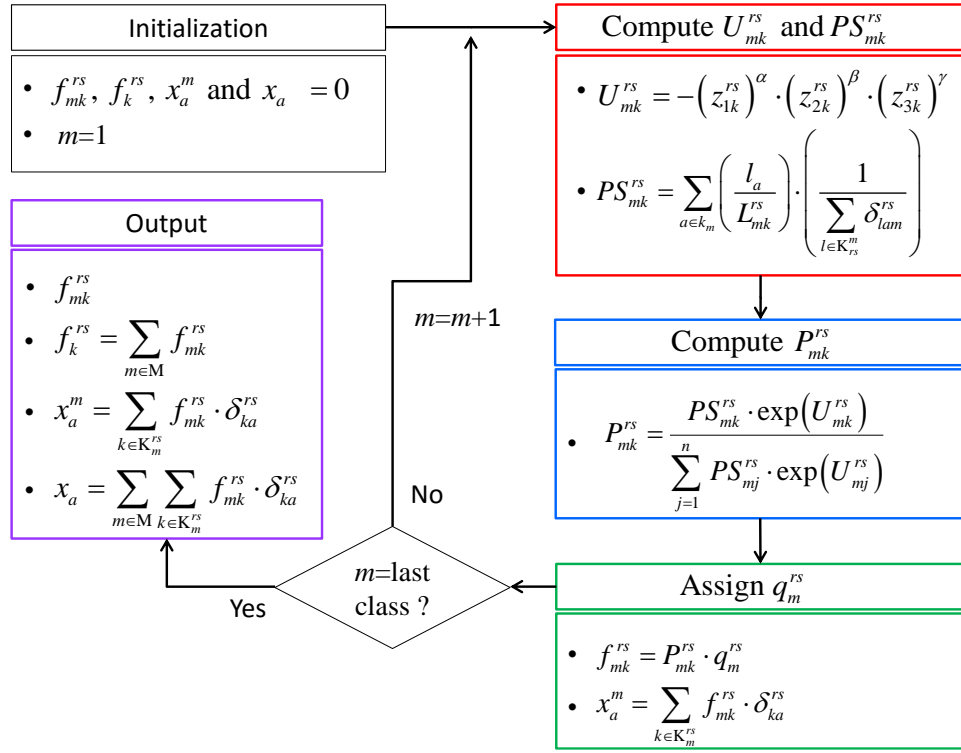


Figure 7. Path-based bicycle traffic assignment procedure

4 NUMERICAL RESULTS

To demonstrate the multi-class, multi-criteria bicycle traffic assignment problem, three classes of cyclists are adopted to develop the numerical experiments for examining the effects of multiple user classes and multiple criteria on the bicycle traffic assignment results. The three cyclist classes are as follows: the “strong and fearless” cyclist class (who compose of less than 1% of the population), the “enthused and confident” cyclist class (7% of the population), and the “interested but concerned” cyclist class (60% of the population). The “no way no how” cyclist class, who compose of 33% of the population, is not included in the numerical experiments because this user class does not consider cycling as a potential mode. The two-stage bicycle traffic assignment procedure is coded in Intel Visual FORTRAN XE and runs on a 3.60GHz processor and 16.00GB of RAM. The total computational efforts require 603 seconds, about 95% of which is spent in the first stage.

4.1 Description of the Network and Scenarios

A real network in the City of Winnipeg, Canada is used to demonstrate the applicability of the two-stage procedure for performing the multi-class, multi-criteria bicycle traffic assignment problem. Figure 8 provides an illustration of the Winnipeg network, which consists of 154 zones, 1,067 nodes, 2,555 links (1,943 links without centroid connectors), and 4,345 O-D pairs for motorized vehicles. The network structure, O-D trip table for motorized vehicles, and link performance parameters are from the Emme/4 software (INRO Consultants, 2013). The bicycle network is assembled based on information obtained from the City of Winnipeg (2013). 541 of the 2,555 links are bikeways. The bicycle O-D demand is created based on the gravity model with the gamma impedance function using 2006 census data (City of Winnipeg, 2006). Note that trip lengths greater than 10 km are excluded in generating the skim trees for the gravity model. To create the multi-class bicycle O-D trip tables, the bicycle O-D demand is segmented into the three user classes mentioned above (i.e., strong and fearless cyclists, enthused and confident cyclists, and interested but concerned cyclists). Table 1 provides a summary of the generated bicycle O-D demand for each user class and the total demand for bicycle trips. Figure 9 presents the trip length frequency distribution (TLFD) for the bicycle trips using route distance to define the trip categories. As can be seen, the majority of the bicycle trips are between 2 to 7 km in length, which is in accordance with the values observed in Washington, D.C. (2012).



Figure 8. Winnipeg network with bike lanes

Table 1. Generated bicycle demand for each user class

Class #	Type	Proportion	Total demand
1	Strong and Fearless	1.5%	82.0
2	Enthused and Confident	10.3%	573.9
3	Interested but Concerned	88.2%	4919.1
Total		100.0%	5575.0

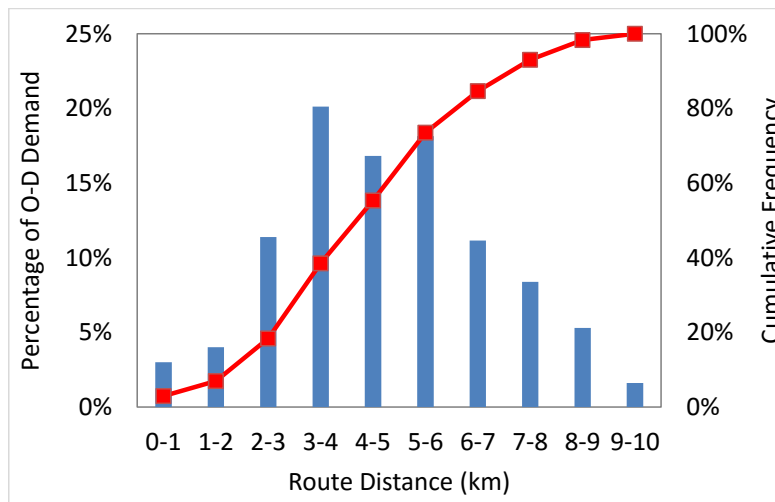


Figure 9. Bicycle trip length frequency distribution

Two scenarios are set up to examine the effects of using different number of criteria in the utility function on the multi-class bicycle traffic assignment model. Table 2 provides a summary of the two scenarios. Scenario 1 assumes all user classes adopt two criteria for the utility function, but the two criteria are different for each user class. On the other hand, Scenario 2 assumes the following: Class 1, the strong and fearless cyclist class, is only concerned with route distance; Class 2, the enthused and confident cyclist class, uses both route distance and route BLOS; and Class 3, the interested but concerned cyclist class, adopts all three criteria (route distance, route BLOS, and route CO) for route choice decisions.

Table 2. Summary of criteria used for the utility function of each user class in the two scenarios

	Scenario 1			Scenario 2		
	Class 1	Class 2	Class 3	Class 1	Class 2	Class 3
Route Distance	X	X		X	X	X
Route BLOS		X	X		X	X
Route CO	X		X			X

4.2 Characteristics of the Winnipeg Network

Figure 10 shows the characteristics of the Winnipeg network that are used to compute the three key route choice criteria described in Section 2.2. Figure 10(a) depicts the link length distribution used for computing the three route choice criteria; Figure 10(b) and Figure 10(c) plot the motorized volume and speed distributions obtained from the multi-class motorized vehicle traffic assignment results provided by the Emme/4 software (INRO Consultants, 2013); Figure 10(d) and Figure 10(e) show the computed bicycle segment and intersection LOS distributions based on Eqs. (3) and (4) from HCM (2010); and Figure 10(f) plots the link CO distribution based on the nonlinear macroscopic model of Wallace et al. (1998) given in Eq. (6). A segment with a high motorized vehicle volume typically gives a higher BLOS value, while links with a larger effective width on the outside lane typically gives a lower BLOS value. In addition, a segment with a high motorized vehicle volume typically yields a large value for CO due to the congestion effect. These

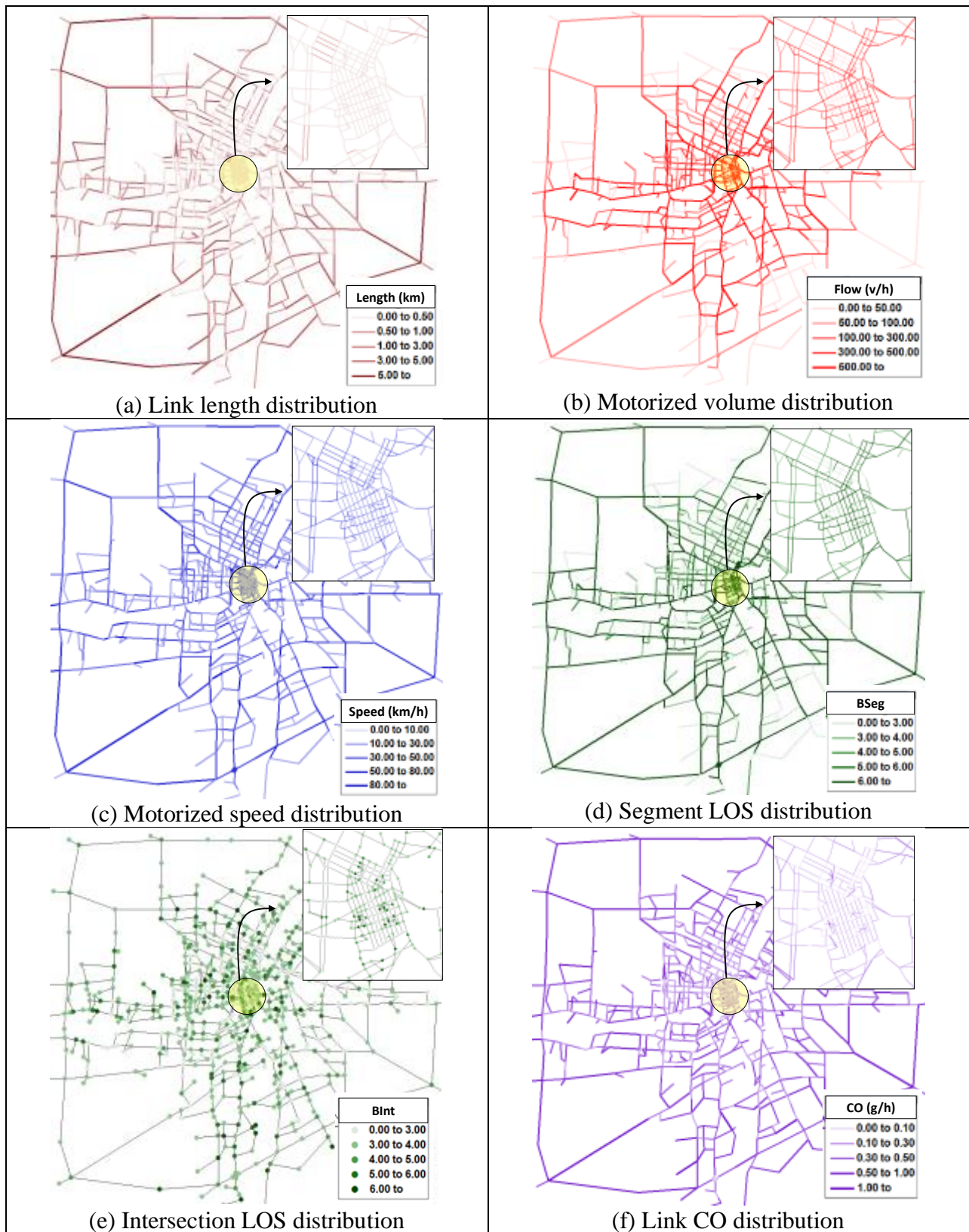


Figure 10. Characteristics of the Winnipeg network

characteristics of the Winnipeg network serve as the input factors for calculating the three route criteria: route distance, route BLOS, and route pollution.

4.3 Route Generation Results from Stage One

Based on the characteristics of the Winnipeg network shown in Figure 10, Stage 1 uses the two-phase procedure to generate a set of efficient routes for each user class according to the criteria adopted in the two scenarios. In this study, we assume an upper bound for each criteria (i.e., 10 km for route distance, 7 for route BLOS, and 25 CO g/h for route pollution) to generate the efficient bicycle routes.

Figure 11 provides a sample of the results of the route distribution using three criteria for Class 3 in Scenario 2 and a comparison of the total number of efficient routes between the two scenarios. Specifically, Figures 11(a), (b) and (c) show the route distribution using distance, BLOS, and CO, respectively, while Figure 11(d) compares the total number of efficient routes for each user class in each of the two scenarios. For Class 3 in Scenario 2, the total number of efficient routes is 50,994 with an average of 6.92 routes per O-D pair (there are 7,368 O-D pairs overall in the Winnipeg network). Longer distance O-D pairs typically have more efficient routes, while shorter distance O-D pairs have less efficient routes. In terms of the route distribution, most routes are between 5 to 8 km in terms of distance, 3 to 4 for BLOS values, and 5 to 8 CO g/h for pollution. As for the comparison between the two scenarios, the number of efficient routes depends on the number of criteria and the specific criteria used to generate the efficient routes.

In Scenario 1, all three user classes use two criteria with different combinations of criteria (e.g., route distance and route pollution for Class 1, route distance and route BLOS for Class 2, and route BLOS and route pollution for Class 3) as shown in Table 2, but the numbers of efficient routes generated are quite different as shown in Figure 11(d). In Scenario 2, it is clear that as the number of criteria increases, the number of efficient routes increases. This is generally expected for the multi-objective optimization problem (i.e., the number of non-dominated solutions increases exponentially as the number of criteria increases). Between the two scenarios, users from Class 1 have the least number of efficient routes (with either using route distance only as in Scenario 2 or using both route distance and route pollution as in Scenario 1). On the other hand, users from Class

3 have the most number of efficient routes with using either all three route criteria as in Scenario 2 or just two criteria (i.e., route BLOS and route pollution) as in Scenario 1.

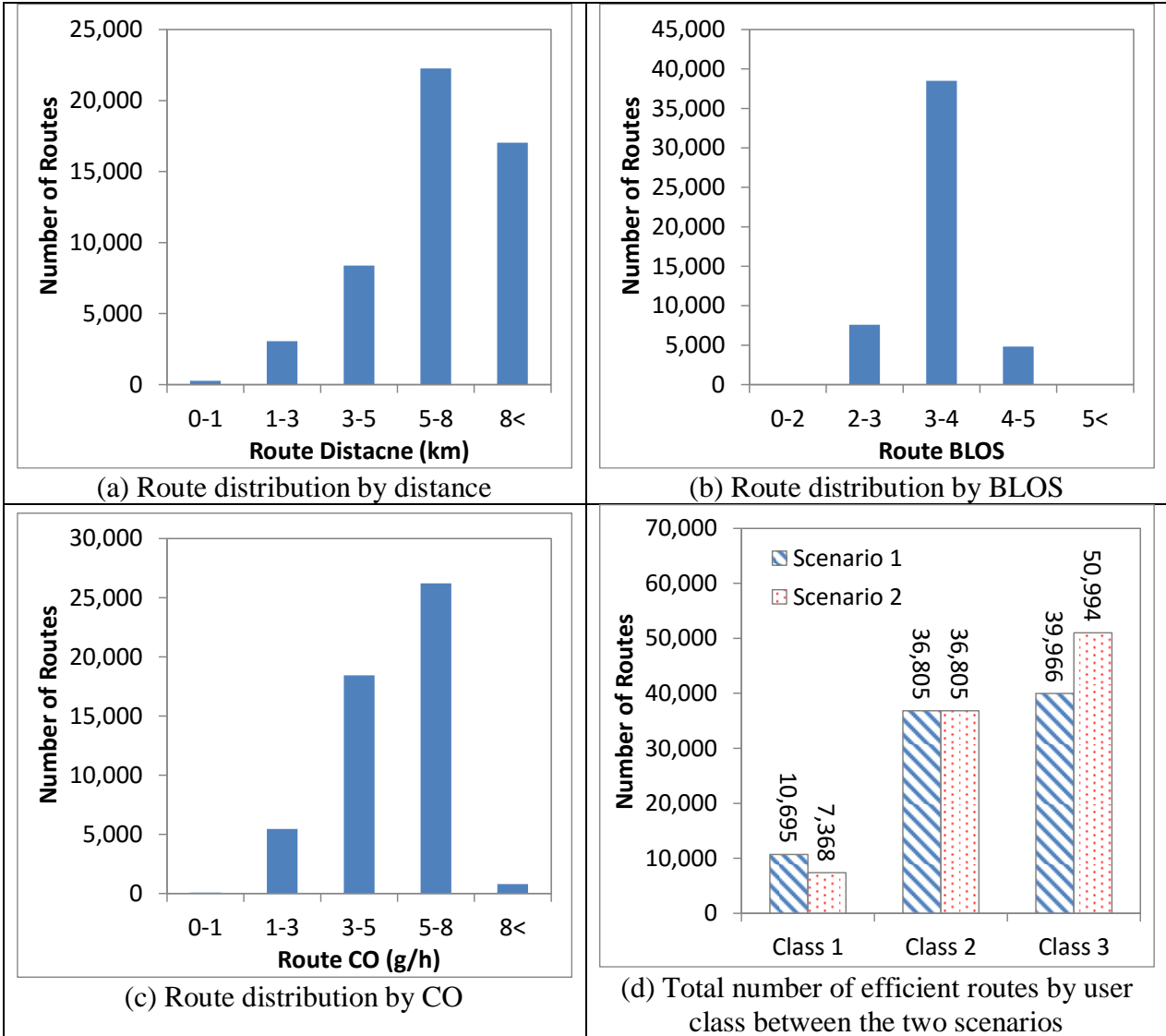


Figure 11. Route distribution by route criterion for class 3 of scenario 2 and total number of efficient routes by user class between the two scenarios

4.4 Bicycle Traffic Assignment Results from Stage Two

Using the efficient routes generated from the first stage for all user classes, we perform the customized path-based algorithm for assigning the multi-class bicycle O-D trip tables to the network according to the PSL stochastic loading method. In this study, the following parameters

are used for the utility function in Eq. (9): $\alpha = 0.862$; $\beta = 0.117$; (these two values are obtained from Kang and Fricker, 2013), and $\gamma = 0.05$ (this value is assumed). Figure 12 depicts the link flow pattern of each user class for both scenarios. Note that the magnitude of the link flow is color coded and represented by the thickness of the line. For the link flow pattern of Class 1 in Scenario 1, the total number of efficient routes (using the criteria route distance and route pollution) is 10,695. Conversely, in Scenario 2 (which uses route distance as the sole criterion), the total number of efficient routes is 7,368. Therefore, the link flow patterns between the two scenarios are quite different since different numbers and route utilities are being used to assign the O-D demand of Class 1. On the other hand, Class 2 users of both scenarios use the same two objectives (route distance and route BLOS) to compute the route utilities, and consequently yield the same link flow pattern. As for Class 3, the two scenarios adopt different objectives (i.e., route BLOS and route pollution for Scenario 1 and all three route criteria for Scenario 2) and generate different numbers of efficient routes (See Figure 11(d)). However, the resulting link flow patterns are visually similar as this class has the largest amount of O-D trips (88% of total demand or 4919 trips out of 5575 trips) compared to 656 trips or less than 12% in Class 2 and Class 3 (See Table 1).

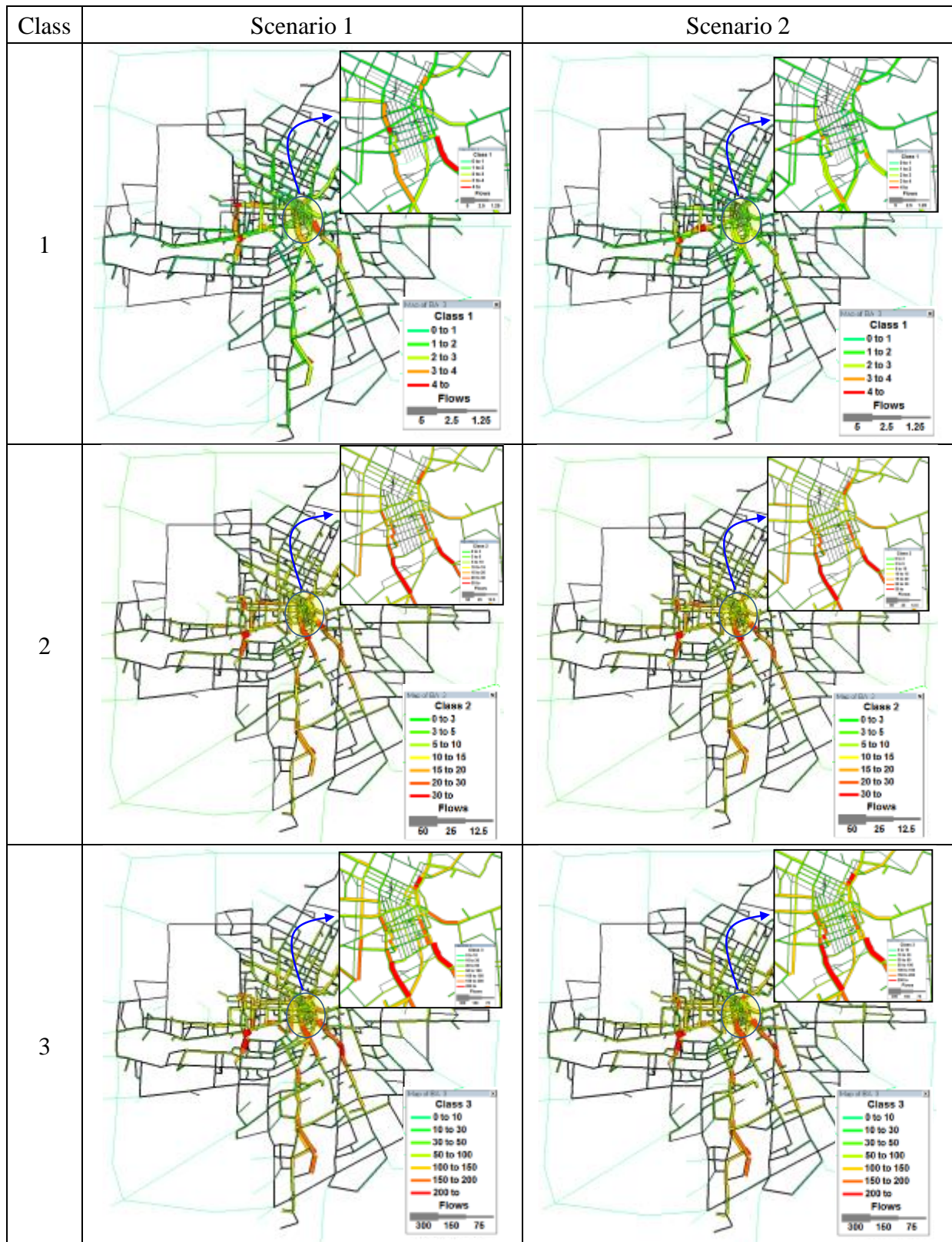


Figure 12. Link flow pattern of each user class for both scenarios

For the aggregate network measures, Table 3 provides the average traveled distance, the average traveled BLOS, and the average traveled CO for each user class computed according the following equations:

$$\text{Average traveled distance: } ATD^m = \sum_{rs \in RS} \sum_{k \in K_{rs}^m} d_k^{rs} f_{mk}^{rs} / \sum_{rs \in RS} \sum_{k \in K_{rs}^m} f_{mk}^{rs}, \quad \forall m \in M \quad (10)$$

$$\text{Average traveled BLOS: } ATB^m = \sum_{rs \in RS} \sum_{k \in K_{rs}^m} BLOS_k^{rs} f_{mk}^{rs} / \sum_{rs \in RS} \sum_{k \in K_{rs}^m} f_{mk}^{rs}, \quad \forall m \in M \quad (11)$$

$$\text{Averaged traveled CO: } ATC^m = \sum_{rs \in RS} \sum_{k \in K_{rs}^m} CO_k^{rs} f_{mk}^{rs} / \sum_{rs \in RS} \sum_{k \in K_{rs}^m} f_{mk}^{rs}, \quad \forall m \in M \quad (12)$$

Table 3. Average traveled distance, BLOS and CO for each user class and all user classes

		Class 1	Class 2	Class 3	All
Scenario 1	Route distance (km/h)	4.808	5.129	5.423	5.384
	Route BLOS	3.863	3.612	3.566	3.575
	Route CO (g/h)	3.999	4.221	4.335	4.318
Scenario 2	Route distance (km/h)	4.787	5.129	5.136	5.130
	Route BLOS	3.847	3.612	3.639	3.640
	Route CO (g/h)	4.022	4.221	4.198	4.198

* *bold and red fonts indicate the criteria used for the specific user class*

A cursory glance at Table 3 would reveal several obvious patterns in aggregate network measures. Firstly, the table shows that route distance seems to have a higher impact when comparing the two scenarios (e.g., Class 1 and Class 3). This is particularly obvious in Scenario 2: Class 1, which uses route distance as its only criterion, has the lowest average traveled distance among the three user classes and in both scenarios. Lastly, the table shows a positive-correlated relationship between route distance and route CO. Minimizing route distance implicitly reduces the value of route CO (see Eqs. (5) and (6)).

A closer inspection of Table 3 would reveal the effects of using multiple criteria in the calculation of the aggregate network measures. The effect can be readily observed in Scenario 1 by examining the values for route BLOS and route CO for Class 2 and 3. Since Class 2 focuses on minimizing route distance and route BLOS while Class 3 focuses on minimizing route BLOS and route CO, we might expect that Classes 2 and 3 would have lower values for their respective criteria of focus. However, Table 3 shows that Class 3's value for route CO is higher than Class 2's value for route

CO even though route CO was minimized in Class 3 and not in Class 2. These unexpected results may be attributed to the multi-criteria aspect of the calculation process. It is likely that the weighting between the two assigned criteria is influencing the results. In the case of Class 3's relatively high route CO value, route BLOS was given more weight than route CO during the calculation process. Thus, we can conclude that the flow patterns are sensitive to the different combinations of criteria.

For the disaggregate analysis, we examine the effect of multi-class and multi-criteria considerations on the route choice probabilities. The user classes considered in the analysis include single class and multiple classes. For the single user class, two different utility functions are used for comparison; the first utility function uses two criteria (route distance and route BLOS), and the second utility function uses three criteria (route distance, route BLOS, and route pollution). For the multiple user classes, we continue to use the setup from the two scenarios. For demonstration purposes, we use O-D pairs (5-2) and (43-4) to respectively represent a short O-D pair and a long O-D pair in the Winnipeg network. Figure 13 shows three major efficient routes for each O-D pair and the route choice probabilities. For both O-D pairs, Route 1 is the shortest-distance route among three efficient routes, while the other two are efficient routes (but these routes do not necessarily have the best value in the other two criteria). For the short O-D pair (5-2), Figure 13(a) shows that the single user class with a bi-criteria utility function assigns a higher probability for all three routes compared to those of the single user class with a three criteria utility function and both scenarios of the multiple user classes. The reason is that the number of efficient routes generated for the short O-D pair using the single user class with bi-criteria utility function is much less compared to the other cases. Therefore, it assigns a higher probability to these efficient routes. Figure 13(b) shows that there is less disparity in the assigned probabilities to the three major efficient routes for the long O-D pair (43-3) compared to the short O-D pair (5-2). Also, Scenario 2 assigns a higher probability to Route 1 since it only uses the route distance as the objective for generating efficient routes, while Scenario 1 considers both route distance and route pollution.

From Figure 13(c), we can observe that the route choice probabilities of each class are significantly different in the multi-class analysis. Cyclists from Class 1 travel only on the shortest route in both scenarios. Although Scenario 1 considers two objectives, the network generates only one efficient route because both objectives (i.e., route distance and route CO) are highly correlated. There are a few notable differences in route choice probabilities within individual user classes. In the long O-

D pair (43-4) analysis, the probabilities between Routes 1 and 3 for Class 2 cyclists in both scenarios differ by 4.4 percentage points. For Class 3 cyclists, there is little variance in route choice probability for all three routes in Scenario 1. However, in Scenario 2, Class 3 cyclists experience greater variance in route choice probability; the probabilities between Routes 2 and 3 differ by 5.1 percentage points.

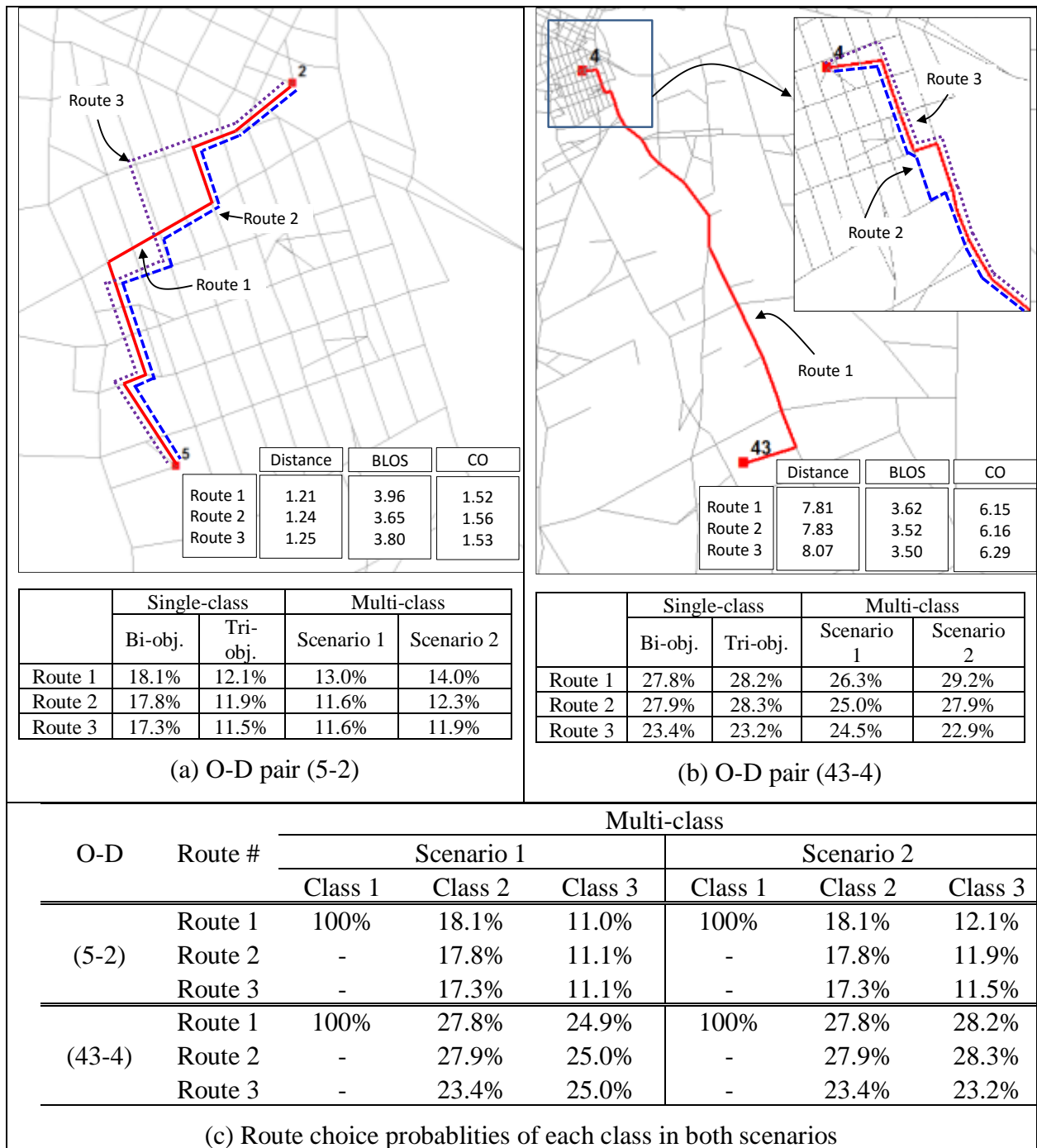


Figure 13. Effect of multi-class and multi-criteria considerations on route choice probabilities

5 CONCLUDING REMARKS

In this paper, we present the development of a multi-class, multi-criteria bicycle traffic assignment model that explicitly considers multiple user classes and multiple criteria affecting cyclist route choice decision-making for estimating bicycle volumes on a transportation network. The multi-class component incorporates defined cyclist classes with differing levels of cycling experience and interest, while the multi-criteria component incorporates relevant factors that affect each user class' behavior in route choice decision-making. The overall procedure for developing the multi-class and multi-criteria bicycle traffic assignment model follows a two-stage process. The first stage considers key criteria (e.g., one or more factors relevant to each user class) to generate a set of non-dominated (or efficient) paths for each user class, while the second stage determines the flow allocation to each user's set of efficient paths using a path size logit model. After the development of the model, we tested the model on a real network in Winnipeg, Canada, to demonstrate the applicability of the model.

The results of the Winnipeg experiment reveal that the integration of multiple user classes and multiple criteria into the bicycle traffic model yield variable outcomes. There are three main reasons that explain the variability in outcomes. First, each user class has different route choice preferences that affect the attributes used in the analysis. Second, the route choice probabilities are highly sensitive to the number of criteria used in the analysis (e.g., two objectives in the Scenario 1 and three objectives in the Scenario 2). Also, the aggregate network measures for route distance, route BLOS, and route CO are highly sensitive to the number of criteria used in the utility function, to the weighting of each criterion in the calculation process, and to each specific user class. Finally, the flows patterns between the single class model and multi-class models are significantly different

because the single class model is incapable of using different combinations of criteria to match the specific preferences of each user class.

This paper is based on three key criteria: route distance-related attributes, route safety-related attributes, and route pollution-related attributes. While the route distance attribute is fairly straightforward, we had to choose surrogate measures for the route safety and route pollution criteria. For our analysis, we chose to use route BLOS as a surrogate measure for modeling cyclists' perception of safety and route CO as a surrogate measure for air pollution. There are other possibilities for surrogate measures; for example, it may be helpful to consider measures such as the bicycle compatibility index ([Harkey et al. \(1998\)](#)) or route stress ([Mekuria et al., 2012](#)) as a substitute for perception of safety. Other criteria, such as route cognition based on the concept of space syntax ([Raford et al., 2007](#)) from the field of urban planning, may also provide more insight. More numerical tests should be conducted with different network topologies, bicycle facilities, and cyclist characteristics. Note that the current two-stage bicycle traffic assignment model did not consider the effect of congestion (i.e., link travel times are independent of flows). It would thus be necessary to consider a flow-dependent model to capture the effects of congestion and safety in terms of motorized traffic in the bicycle traffic assignment procedure. In addition, the two-stage approach could be extended to consider other travel choice dimensions (e.g., mode choice, destination choice, and travel choice). One example is to consider mode choice in addition to route choice in a multi-modal road network ([Li et al., 2015](#)). Destination choice and travel choice could also be considered in a similar manner to create different combined travel demand models involving non-motorized modes.

References

1. 2010 Highway Capacity Manual. 2010. Transportation Research Board, Washington D.C.
2. Akar, G., Clifton, K J. 2009. Influence of individual perceptions and bicycle infrastructure on decision to bike. *Transportation Research Record* 2140: 165-171.
3. Ben-Akiva, M., Bierlaire, M., Discrete choice methods and their applications to short term travel decisions. *Handbook of Transportation Science*. R.W. Hall ed., Kluwer Publishers, 1999
4. Bovy, P., Bekhor, S., Prato, G. 2008. The factor of revisited path size: Alternative derivation. *Transportation Research Record* 2076: 132–140.
5. Bovy, P., Fiorenzo-Catalano, S. 2007. Stochastic route choice set generation: Behavioral and probabilistic foundations. *Transportmetrica* 3(3): 173-189.
6. Broach, J., Gliebe, J., Dill, J. 2011. Bicycle route choice model developed using revealed preference GPS data. Presented at the 90th Annual Meeting of the Transportation Research Board, Washington, D.C.
7. Chen, A., Pravinongvuth, S., Xu, X., Ryu, S., Chootinan, P. 2012. Examining the scaling effect and overlapping problem in logit-based stochastic user equilibrium models. *Transportation Research Part A* 46(8): 1343-1358.
8. City of Winnipeg. 2013a. Winnipeg cycling map.
9. <http://www.winnipeg.ca/publicworks/MajorProjects/ActiveTransportation/maps-and-routes.stm> Accessed Jan 13, 2014.
10. City of Winnipeg. 2006. 2006 Mobility by Neighborhood.xls in 2006 Census.
11. <http://winnipeg.ca/census/2006/Selected%20Topics/default.asp>. Accessed Jan 13, 2014.
12. Climaco J C N., Martins, E Q V. 1982. A bicriterion shortest path problem. *European Journal of Operational Research* 11(4): 399-404.
13. Dill, J., Carr, T. 2003. Bicycle commuting and facilities in major US cities: if you build them, commuters will use them. *Transportation Research Record* 1828: 116-123.
14. Dowling, R G., Reinke, D B., Flannery, A., Ryus, P., Vandehey, M., Petritsch, T A., Landis, B W., Roupail, N M., Bonneson, J A. 2008. Multimodal Level of Service Analysis for Urban Streets Multimodal Level of Service Analysis for Urban Streets. National Cooperative Highway Research Program, Report 616.
15. Ehrgott, M., Wang, J., Raith, A., van Houtte, A. 2012. A bi-objective cyclist route choice model. *Transportation Research Part A* 46(4): 652-663.
16. Emme4 software. 2013. INRO Consultants.
17. http://www.inro.ca/en/products/emme/trial_req.php. Accessed Jan 13, 2013
18. Geller, R. 2006. Four types of cyclists. Portland Bureau of Transportation, Portland, OR.
19. <http://www.portlandoregon.gov/transportation/article/264746> Accessed Dec. 13, 2009
20. Handy, S., Xing, Y., 2011. Factors correlated with bicycle commuting: A study in six small U.S. cities. *International Journal of Sustainable Transportation*, 5(2): 91-110.
21. Harkey, D L, Reinfurt, D W., Knuiman, M. 1998. Development of the bicycle compatibility index. *Transportation Research Record* 1636: 13-20.
22. Heinen, E., Handy, S. 2012. Similarities in attitudes and norms and the effect on bicycle commuting: Evidence from the bicycle cities Davis and Delft. *International Journal of Sustainable Transportation*, 6(6): 257-281.
23. Hood, J., Sall, E., Charlton, B. 2011. A GPS-based bicycle route choice model for San Francisco, California. *Transportation Letters: The International Journal of Transportation Research* 3(1): 63-75.

24. Hopkinson, P., Wardman, M. 1996. Evaluating the demand for cycle facilities. *Transport Policy* 3(4): 241-249.
25. Hunt, J D., Abraham, J E. 2007. Influences on bicycle use. *Transportation* 34(4): 453-470.
26. Kang, L., Fricker, J. 2013. A bicycle route choice model that incorporate distance and perceived risk. Presented at the 92nd Annual Meeting of the Transportation Research Board, Washington, D.C.
27. Klobucar, M., Fricker, J. 2007. Network evaluation tool to improve real and perceived bicycle Safety. *Transportation Research Record* 2031: 25-33.
28. Li Z.C., Yao M.Z., Lam W.H.K., Sumalee A., Choi K., 2015. Modeling the effects of public bicycle schemes in a congested multi-modal road network. *International Journal of Sustainable Transportation* 9(4): 282-297.
29. Lowry, M., Callister, D., Gresham, M., Moore, B. 2012. Assessment of community-wide bikeability with bicycle level of service. *Transportation Research Record* 2314: 41-48.
30. Mekuria, M., Furth, P., Nixon, H. 2012. Low-stress bicycling and network connectivity. Mineta Transportation Institute, San José State University
31. Menghini, G., Carrasco, N., Schussler, N. 2010. Axhausen K.W. Route choice of cyclists in Zurich. *Transportation Research Part A* 44(9): 754-765.
32. Prashker, J.N., Bekhor, S. 2004. Route choice models used in the stochastic user equilibrium problem: A review. *Transport Reviews* 24(4): 437-463.
33. Prato, C.G. 2009. Route choice modeling: past, present and future research directions. *Journal of Choice Modelling* 2(1): 65-100.
34. Raford, N., Chiaradia, Alain, J., Gil, J. 2007. Space syntax: The role of urban form in cyclist route choice in Central London. *Transportation Research Board 86th Annual Meeting*, Washington, DC, USA, 21-25 January 2007.
35. Ryu, S., Chen, A., Choi, K., 2015. **A two-stage bicycle traffic assignment model**. Paper presented at the *94th Transportation Research Board Annual Meeting*, Jan. 11-15, 2015, Washington D.C., USA.
36. Sener, I., Eluru, N., Bhat, C R. 2009. An analysis of bicycle route choice preferences in Texas, US. *Transportation* 36(5): 511-539.
37. Skriver A J V., Andersen, K A. 2000. A label correcting approach for solving bicriterion shortest-path problems. *Computers & Operations Research* 27(6): 507-524.
38. Stinson, M A., Bhat, C R. 2003. An analysis of commuter bicyclist route choice using a stated preference survey. *Transportation Research Record* 1828: 107-115.
39. Tung, C T., Chew, K L. 1992. A multicriteria Pareto-optimal path algorithm. *European Journal of Operational Research* 62(2): 203-209.
40. Ulungu E L., Teghem, J. 1995. The two phases method: an efficient procedure to solve bi-objective combinatorial optimization problems. *Foundations of Computing and Decision Sciences* 20(2): 149-65.
41. Wallace, C.E., Courage, K.G., Hadi, M.A., Gan, A.G. 1998. *TRANSYT-7F user's guide*. University of Florida, Gainesville.
42. Washington D.C. 2012. Visualization: 2012 biking by the numbers
43. <http://jdantos.wordpress.com/2013/01/01/visualizations-2012-biking-by-the-numbers/trip-length-frequency/> Accessed Jan. 9, 2014
44. Willis, D., Manaugh, K., Geneidy, A. 2015. Cycling under influence: Summarizing the influence of perceptions, attitudes, habits, and social environments on cycling for transportation. *International Journal of Sustainable Transportation*, 9(8): 565-579.

45. Winters, M., Davidson, G., Kao, D., Teschke, K. 2011. Motivators and deterrents of bicycling: comparing influences on decisions to ride. *Transportation* 38(1): 153-168.