The Effect of Fines on Relative Bonded Area

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The Effect of Fines on Relative Bonded Area

A Paper Technology Thesis Submitted to the Faculty of Western Michigan University Kalamazoo, Michigan

by
Richard G. Bowler
September 1965
Acknowledgement

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Introduction

Bonding characteristics of the fibers in a sheet of paper have long been, and still are, difficult to isolate in a manner which will give exact quantitative information on the degree of bonding. It is possible to indicate with strength tests different levels of bonding. Other factors -- fiber strength, fiber length, cell wall thickness, etc., -- also enter into values obtained in these tests and make the assignment of exact meaning to bonding impossible. The subject of a major part of this literature search deals with measurement of another bonding characteristic, relative bonded area or simply bonded area. This again is not an exact measure of bonding and, here, only its merits as a measure of a characteristic of bonding are treated. The treatment is further limited for the most part to discussion of the optical method initiated by Parsons.
Literature Search
Origin of the Optical Method

Parsons, (1) in 1942, experimented with a method of determining a characteristic of bonding, bonded area, via optical measurements. He used light scattering coefficients to determine that part of the surface area of a sheet which is in optical contact.

The light scattering coefficient is defined as the limiting of the relative light energy per unit thickness, scattered backwards from an imaginary layer of material as the thickness of the layer becomes very small.

Charts which allow the evaluation of the scattering coefficient, S, can be obtained by equations. The Kubulka Munk chart is an aid used for this evaluation. Only two optical measurements are necessary, \( R_0 \) and \( C_0 .89 \). \( R_0 \) is the reflectance of a single sheet backed by a material of zero reflectance. \( C_0 .89 \) is the Tappi opacity.

The actual S values are not read directly from the chart, but they are evaluated by dividing a value designated SX (on the chart) by the sheet thickness. However, since S would sometimes be meaningless, as in the case of a bulky sheet which is dry pressed to a fraction of its thickness, a value called the specific scattering coefficient \( S' \) is used. This value is obtained by dividing SX by \( W \), the basis weight.

Davis (2) stated that light scattering coefficients should be proportional to total surface area per mass of the particles of material. Using this and other assumptions, Parsons reasoned
that bonded area could be determined by means of a plot, $S$ vs. specific bonded area.

Primary Assumptions in the Employment of the Optical Method

The basic assumptions (3) of the method are these: (a) If a pulp is not made in such a manner as to prevent fiber to fiber bonding, $S'$ is a linear plot increasing directly with the area. (b) $S'$ in the sheet is a linear relation to the area not in optical contact, following the same function assumed in (c). The principle involved is that light is refracted or scattered only at interfaces involving contact between two media of differing refractive index. In paper, of course, this is largely the cellulose-air surface.

The assumption that all of the original surface area of the fibers in a handsheet which does not scatter light must be involved in interfiber bonding has been challenged frequently. Ratliff, (3) in 1949, stated that the distance between fiber and optical bonding was reasonably close, and even if this distance was not the same, it should be correct to believe that any unobstructed fibrillae coming within distance of optical contact would be drawn close enough to bond by surface forces. In 1955, Haselton (4) reasoned that since bonding distances are known to be 1-5Å and optical contact can involve a maximum separation of 400-500Å, the optical determination of bonded area could be in error on the
high side. This matter is still argued.

Kallmes (7) (8) (9) refers to two schools of thought -- those who believe that only a small part of the optical bonded area is hydrogen bonded, and those who contend that if two fibers are close enough to be optically bonded, the surface tension forces developed in the last stage of drying will cause complete bonding.

Validity of Optical Methods

Haselton investigated the methods of measuring relative bonded area via scattering coefficients. (4) He made use of the Brunauer Emmet and Teller method for computation of surface area with gas absorption. The technique was first used on cellulosic material by Emmet and DeWitt (11) with much promise. This was verified by Haselton, who obtained data showing a good linear relationship between the scattering coefficient method of bonded area determination and the Brunauer Emmet and Teller (10) area of the sheet. He further indicated that the scattering coefficient method on water dried sheets gives a direct relation to the unbonded area, regardless of whether the area has been altered by beating or wet pressing.

Investigations of Solvent Systems Used in Zero Bonded Area Sheets

In more work on this project, Haselton (4) also explored the use of butanol and benzene dry sheets to facilitate a condition of zero bonded area. He also examined values obtained by specially spray dried fibers from a water suspension.
The butanol sheets were made by successively (a) forming sheets in a standard mold using water, (b) replacing the water with acetone, and (c) replacing the acetone with butanol.

The benzene sheets were formed in the same manner, only benzene was used as the replacement agent in step (c).

The spray dried fibers were formed by spraying 0.8 gram of oven dry fiber at 20 per cent consistency over 55 square feet of horizontal polyethylene and allowing drying to take place with the contact between fibers therefore reduced to a minimum.

Data on this operation showed that $S^i$ values for benzene bonded sheets were high, with butanol sheets and spray dried sheets following in magnitude of this value. Of course, regular water bonded sheets had lower scattering coefficients than any of the solvent used to give zero bonded paper.

To explain the facts, the theory proposed was that the water bonded sheets give both high internal and external bonding (fibril to fibril bonding and fiber to fiber bonding); therefore, they exhibit the lowest scattering coefficient.

The spray dried fibers have a great deal of internal bonding (the fibrils bond to each other causing the individual fibers to close up), but no fiber to fiber bonding. This leads to very high valued scattering coefficients.

The sheets formed with butanol solvent have little fiber to fiber bonding, and, because the polarity of butanol is not nearly as great as that of water, there is little internal bonding or
closing up of the fiber. This leads to an abnormally high value for specific scattering coefficient and abnormally low values for relative bonded area.

The benzene dried sheets, because the solvent is least polar of those mentioned, have very little external bonding compared to the water dried fibers or the butanol fibers, and they have even less internal bonding than the butanol fibers. Therefore, they could be expected to exhibit especially high values for scattering coefficients.

Because of the above, it is incorrect to take the differences between the bonded areas of a test sheet and the bonded area of a butanol or benzene sheet (as determined by the optical method) to be the external bonded area. One would obtain high values. Haselton found the correlation factor to be 0.67 on a specific pulp between relative bonding area, using, in one case, butanol sheets as the zero bond sheet and, in another case, spray dried fibers as the zero bond sheet.

T = Test sheet formed in water and allowed to dry in mat
B = Benzene formed "O bond" sheets
Bu = Butanol formed "O bond" sheets
S = Spray dried fibers

<table>
<thead>
<tr>
<th>Internal Bonding</th>
<th>External Bonding</th>
</tr>
</thead>
<tbody>
<tr>
<td>T &gt;&gt; S &gt;&gt; Bu &gt; B</td>
<td>T &gt;&gt; Bu &gt; B &gt;&gt; S</td>
</tr>
</tbody>
</table>
The Effect of Different Beating Intervals on the Standard Zero Bond Sheet

It is known that the degree of swelling of fibers is dependent upon the amount of beating; (5)(6) therefore, the assumption of a constant correction factor for the butanol specific scattering coefficient may be in error. Ingmansen (12) decided to attempt obtaining extrapolated values for S' at different refining intervals, using a technique relating S value to tensile strength at a given beating time for various degrees of wet pressing.

In this study, a linear relationship between the S values of unbonded fibers dried with butanol and the refining time was obtained. A degree of external bonding became apparent since some of the sheets showed considerable tensile strength. This is thought to be due to the condensation of water vapor by cool air surroundings, which the evaporation of butanol created.

Finally it was found that the total area of the spray dried fiber remains constant, and it is independent of the degree of refining. It is shown that the specific area of the fiber increases in the water swollen state with the degree of refining. Therefore, upon drying, these fibrils must bond to the parent fiber again.

The Role of the Lumen in Optical Analysis

It was also hypothesized by Ingmansen that most of the large internal area of the fiber will not scatter light because of the
small distances of separation; but, at the very minimum, it would be expected that the lumen area of the fiber and perhaps some of the longest pores, would scatter light.

Kallmes, (7) in a recent paper, criticizes the optical method because in his investigation he found that not enough emphasis is placed on the effect of the lumen as a source of light scattering. This, he contends, gives high $S'$ values.

**Comparison of Optical Method to Determine Relative Bonding Area**

Ingmansen (12) used the following methods to evaluate relative bonding area; (a) Parson's original method, (b) Haselton's method involving use of corrective values for solvent dried pulps, (c) a refinement of Haselton's method, (d) extrapolation of $S'$ value to zero tensile.

All four calculations show the same tendency for the rate of development of relative bonding area to decrease with increasing refining time. Each improvement of the estimate of relative bonded area generally lowers the degree of bonding, and the most accurate of the estimates based on the concept of dry fiber area gives relative bonding area 2 per cent lower than the original optical method of Parson's.
Short Method of Finding S' of Unbonded Fibers

A quick method of determining scattering coefficients for dry but unbonded fibers is advocated by Ingmansen. (12) By measuring specific scattering coefficients and tensile strength for unbeaten handsheets at several moderate values of wet pressure, and then extrapolating to zero tensile strength, a reasonably accurate value of this coefficient can be obtained by which total dry specific surface and constants to convert total bonding area at any beating interval may be obtained.

In using this extrapolation to zero tensile strength, the assumption is made that SX is proportional to specific fiber surface and the subsequent values for water dried handsheets are taken as being proportional to the effective bonded area.
Speculations on the Role of Fines

"In the case of the unclassified pulp (made into handsheets) where large amounts of fines were known to be present, these fines have apparently bonded on the surface of parent fibers with the result that the dry fiber surface remains essentially constant."

With unclassified and classified pulps (in pulp form), one observes the same specific volume at the same refining intervals, but the rate of increase and the level of specific surface was much higher for the unclassified pulps because of the presence of fines.

Ingmansen concludes, for pulp in his study, at a given total bonded area, that the strength of paper is independent of the degree of fibrillation or the amount of fines. In addition to this, he states, "The only major role of fibrillation and fines in producing strength is to provide greater surface tension (the Campbell Effect) to draw fibers into close enough proximity for bonds to be established.... It appears that fines are not so effective as an equivalent surface area of fibrils."

Somewhat conversely, Brown, (13) Higgins and Harrington, (14) and others (15)(16)(17) found that fines control a given property of the whole pulp.

Natural vs. "Manufactured" Short Fibers

The characteristics of short fiber fractions depend on whether the short fibers were "manufactured" by cutting the long
fibers or if they originated as such from the wood. This has been noted first by Brown and later by Steenberg, Landgreen and Wahren, (18) Stephenson, Roschier, (20) and Higgins and Harrington. (21) All found differences in chemical and physical properties between the natural fractions and those produced by beating.

**Factors affecting Scattering Coefficient in Various Pulp Fractions**

Marton and Alexander (21) point out that the scattering coefficient depends both on the total area of the scattering surfaces and the size and distribution of voids in the paper structure. The size and distribution of voids is also a function of density.

In their work, Marton and Alexander investigated the properties of Bauer McNett fractions of draft, neutral sulfite semi-chemical and groundwood pulps from poplar. Specific fractions under consideration were those retained on 20, 35, 65 and 150 mesh screen and also that material which passes through the 150 mesh screen (fines).

They found that for chemical pulps density remains constant and therefore increasing fineness of the fraction should increase $S'$ correspondingly and thus decrease relative bonded area. For mechanical pulps a sharp increase in $S'$ is observed since density and specific surface both increase with fraction number.
Density and Scattering Coefficients

Marton also found that to exhibit behavior typical of normal beating, the scattering coefficient of a mechanical pulp must be measured at a density high enough to achieve maximum scattering of light. Until this density is reached the scattering coefficient will show anomalous behavior.

Fines and Tensile Strength

From his data Marton concluded that, concerning tensile strength, fines have great influence for mechanical pulps. In fact, the breaking length derived from the whole pulp is about the same as that resulting from the short fiber fraction of this pulp.

For chemical pulps it is a different story. Fines of the full chemical pulps are considerably weaker than any of the longer fractions. The whole pulp has greater tensile strength than any one of its fractions therefore fines or water soluble portions of the pulp must enhance the bonding between fibers.

It was further postulated from this work that the difference in behavior between soft and hard chemical pulps is an inherent one due to the severity of the cook. The tensile strength of the high yield sulfite whole pulp is about equal to that of the fractions retained on the 20 and 35 mesh screens. The short fiber fractions appear to contribute little to tensile strength.
Arlov, (17) in explanation of the short fiber vs, long fiber effect on tensile strength, states that the long fiber is more firmly anchored in the sheet than the short one and when under load and long fiber makes better use of its intrinsic strength to distribute the external tension over a greater number of bonds. This theory explains well the decreasing breaking length with increasing fraction number.

The fact that fines play a more important role in the tensile behavior of a groundwood handsheet is understandable if one thinks of the long fibers of the groundwood as essentially sticks being stiff, unfibrillated etc., while the fines because of their increased surface area contribute to bonding. (21)
Experimental Program - Results
Object: To study the effect of fines on paper tensile strength and bonded area.

Materials and equipment used:

Materials

- Weyerhauser bleached sulfite pulp
- Deionized water
- Formaldehyde
- Muslin cloth, clamps

Equipment

- Pilot beater and refiner
- British disintegrator
- Bauer McNett fiber classifier
- TAPPI sheet machine
- Vacuum pump
- Filter paper
PROCEDURE

The pulp was prepared by combined action from the pilot beater and the pilot refiner. The treatment was quite severe and almost 60 per cent of the material treated passed through a 150 mesh screen in the Bauer McNett classifier. This material was collected on large muslin cloth which was supported below the classifier tanks with clamps and wire. The material collected on the muslin was to serve as the fines for this study. Fibers retained in the classifier tanks were combined and became the pulp with 0 per cent fines.

A problem occurred in the classification. The hard water from the tap left a residue on each fraction of the collected pulp. It was therefore necessary to use deionized water for all of the several classifications and the subsequent operations with the pulp.

After all of the necessary classification for the study had been completed, handsheets were formed for opacity, tensile, and brightness tests. In order to use the quick method (explained in the literature search and in more detail later in the theory) for determining the relative bonded area it was necessary to perform corresponding tensile opacity and brightness tests on each sheet. Therefore the TAPPI method for optical sheet formation could not be used. Because of this and because it was necessary to know quite exactly the fines concentration, a
rather unorthodox method of sheet formation was employed. A filter paper was placed over the TAPPI sheetmold. The sheets were all formed on filter paper causing all of the fines in the fiber suspension to be retained in the sheet. Also it was hoped that all of the needed tests could be performed on each of the individual sheets.

In an attempt to avoid the problem of prolonged drainage time and irregular sheet formation brought about by a high fines concentration a vacuum of 20 inches of mercury was used to provide additional sheetmold suction for every sheet.

After the vacuum formation each sheet was carefully separated from the filter paper, placed between blotters (three blotters on each side of the sheet) and pressed for five minutes. The sheets were removed from the press, the wet blotters were replaced with dry ones, and a second pressing, identical to the first one, was effected. A third pressing with fresh blotters, but for only two minutes, completed the pressing cycle. As indicated on the data page, there were three pressing cycles (one at each of three wet pressures 0, 100, and 300 p.s.i.) Immediately after the final pressing in each cycle, the sheets were taken to the standard conditioning room where they remained for approximately twenty hours before testing.
All tensile strength results were weight corrected to correlate with the TAPPI standard 1.2 gram handsheets. The actual weight per sheet averaged 1.6 grams. This weight seemed to accent changes in tensile which turned out to be small with the employed wet press variation. Opacity and brightness tests were run according to TAPPI Standards. The scattering coefficient was obtained from the Kubelka-Munk chart.
Theory

Only a comparison of relative bonded area was necessary, therefore the quick method of Ingmanson, involving extrapolation of the scattering coefficient to zero tensile with wet pressure serving as a parameter, was used. (See the data pages in the appendix.)

$S_0$ represents the scattering coefficient at 0 tensile or the scattering coefficient of a 0 bond sheet. Nine pounds per fifteen millimeters width was selected as the base tensile strength for comparison of relative bonded areas. $S_u$ represents, in general, the scattering coefficient of a water dried sheet which gives a tensile strength of nine pounds per fifteen millimeters width.

According to Ingmanson:

1) $S_u = K A_u - 1$
   where $A_u$ = the exposed or unbonded surface of a water dried sheet
   $i$ and $K$ = constants

2) $S_0$ was calculated as follows:
   (a) The slope of scattering coefficient vs. tensile strength was determined at each fines concentration from graph (I)
   (b) $S_u$ was determined from graph (I)
   (c) Then $S_0 = -$slope 9.00 + $S_u$ at nine
(3) Likewise
\[ S_0 = KA_0 + 1 \]
where \( A_0 \) = the unbonded area of a zero bond fibers which had been water dried
1 and \( K \) = constants

(4) Let \( A_b = A_0 - A_u \)
and (1) and (2)

\[ A_o = \frac{S_o - 1}{K} \quad \text{and} \quad A_u = \frac{S_u - 1}{K} \]

\[ A_b = \frac{S_o - 1}{K} - \left[ \frac{S_u - 1}{K} \right] = \frac{S_o - S_u}{K} \]

(5) \[ \frac{A_b}{A_o} = \frac{S_o - S_u}{S_o} = \text{relative bonded area} \]

\[ -B_a = \frac{-s}{S_{ub}} \]

where \( -B_a \) = change in relative bonded area

\( -s \) = change in scattering coefficient

\( S_{ub} \) = scattering coefficient of a water-dried unbonded sheet
Results

Upon completion of the calculations (page 26) it is seen that relative bonded area decreased with increasing fines concentration from 0 - 30 per cent but that the decrease stopped at 30 per cent and there was no further drop in relative bonded area when 60 per cent fines was used.

Discussion

It can be seen from the data tables in the appendix that tensile strength did not increase with wet pressure in all cases as predicted in the literature, scattering coefficient, likewise, did not decrease with increasing wet pressure. However when one plots scattering coefficient on the Y axis and tensile strength on the X axis, see graph (I) a reverse slope is attained as predicted by Ingmanson. Therefore this method can be applied in this study.

It can be seen from graph (II) that relative bonded area decreased from 0-30 per cent fines concentration but from 30-60 per cent relative bonded area did not change significantly. Therefore assuming, first, that the data is not faulty, what happened?

It is true that in order to realize lower relative bonded area by the optical method brightness and/or opacity must decrease at a decreasing rate with increasing tensile strength from 0 - 60 per cent fines using wet pressure as the parameter. This comes from Ingnmansons' theory explained and derived in this report.
From this it must be concluded that brightness and/or opacity have decreased at a decreasing rate from 30-60 per cent in this study. This can be verified from the data pages, from table X and from graph II.

For every fines concentration it can be seen from table (I) that opacity is lower at the highest tensile strength than it is at the lowest tensile strength. Brightness varies in the same way except with the 30 per cent concentration of fines where it is exactly the same at the extremes of tensile.

It is also apparent from Table I that sheet weight corrections will accent opacity and brightness changes at 0 per cent fines. By comparing the figures and the slopes in graph I from 0 through 60 per cent fines it can be seen that opacity and brightness decrease with increasing tensile strength most sharply at 0 per cent fines. The rate of decrease is much less sharp at 30 per cent fines than at 0 per cent and is about the same as that at 60 per cent. This can be reasoned not only by scrutinizing table I but also by again comparing slopes in graph I. It should now be clear how the data obtained lead to the results of this study or the trend in graph I.

But now why did the data behave the way it did? Can a consistent theory be developed to explain a decrease in relative bonded area from 0-30 per cent fines and no further decrease thereafter?
To attempt such theorizing one should recall that increasing optical contact signifies the presence of more medium having one refractive index and therefore decreasing opacity. Then one might proceed to accept that from 0-30 per cent fines concentration this effect occurred and was indeed pronounced because the fines with their small size served to plug up the optical voids between the fibers. In the jump from 30 to 60 per cent concentration there is no change in the rate at which opacity decreased with increasing tensile. This could be explained by allowing that the increased fines concentration from 0 to 30 to 60 per cent just meant more fines attaching to the fibers and to fines already plugging interfiber optical voids. It is correct to think that the true relative bonded area would still be increasing but optical instruments might not show this. The limit of their sensitivity is 5000 Angstroms. The actual separation allowed by a hydrogen bond is three to five Angstroms. Therefore the optical method of Ingmanson might be effective in picking up interfiber voids where there is no bonding involved but it would have no way to register the bonding of fines in excess of those needed to plug up the interfiber voids.

Two more points per line on graph I would have had given the results more authority. The data on pages 27-29 could have been more complete. Many tensile samples broke in the clamp and were not recorded. In some instances it was only possible to get one brightness reading per sheet. Indeed, the mortality
rate of the sheets was high considering that six were formed for each wet pressure in each fines concentration used. However, more test sheets and more points for the graph were expensive in terms of man hours and proved to be too much of a luxury for the time allotted. This author is also aware of the possible error introduced by using the method of sheet formation described in the procedure. Any strain in the wet sheet could have introduced quite an error in the tensile strength test. However it can be seen from table I that opacity and brightness behaved as required at the extremes of tensile strength to allow Ingmanson's optical method to be used for the determination of relative bonded area.

Conclusions

1. Theory can be fitted to the data obtained indicating that relative bonded area as optically measured could decrease to a minimum with addition of fines to a point where additional fines would cause no further decrease.

2. The method of determining relative bonded area is tricky, time consuming, insensitive and gives results almost opposite of what one would usually expect.

3. Relative bonded area might be more accurately determined by gas absorption.
Recommendations for future work

Work done by Haselton, (4) indicates that gas absorption may be a precise method of determining relative bonded area. Good correlation between the Bauer-Emmit-Teller determination of specific surface area and the Hawkins-Jura determination give satisfactory results when applied to absorption data on cellulose material. Nitrogen and Butane gases each gave good correlation between these methods of specific surface area determination. Carbon dioxide reacted chemically with cellulose and could not be used since gas absorption determinations demand that only physical absorption occur. The calculations are simple consisting mostly of graphing and algebra. However, a knowledge of surface phenomenon as exists in a good physical chemistry course is helpful if not absolutely necessary.

This author would not recommend this procedure for undergraduate thesis work because sophisticated equipment and extreme conditions which demand a great deal of time and care are involved. For instance, the accuracy of the gas absorption method depends on the use of an oil diffusion silicone pump backed by a rotary pump to attain pressures at $10^{-4}$ millimeters mercury. Very low temperatures are also needed which call for the use of liquid nitrogen - $195^\circ C$ and liquid oxygen - $182.9^\circ C$.

In short then, it is the opinion of this author that work done on a graduate level using the gas absorption method of determination would lead to a deeper understanding of how fines effect relative bonded area than is possible by the optical method.
BIBLIOGRAPHY


Appendix
Calculations

Calculation of slopes from graph (I)

\[ M = \text{slope} \]

\[ M_0 = \frac{1.43 - 1.24}{0.19} = 0.19 \]

\[ M_{30} = \frac{1.25 - 1.19}{0.06} = 0.06 \]

\[ M_{60} = \frac{1.03 - .98}{0.05} = 0.05 \]

Calculation of \( S_0 \) from \( M \) and tensile reference point point 9.00 # /15mm - graph (I)

\[ y = mx + b \]

\[ y = b - mx \]

\[ b = y + mx = S_0 \]

\[ S_0 = 1.19 + (0.19)9 \]

\[ = 1.19 + 1.71 = 2.90 \]

\[ S_{30} = 1.19 + (.06)9 \]

\[ = 1.19 + 0.54 = 1.73 \]

\[ S_{60} = .98 + (0.05)9 \]

\[ = .98 + .45 = 1.43 \]

Calculation of RBA from \( S_0 \) and \( S \) at 9.00 # /15mm tensile

\[ RBA_0 = \frac{S_0 - S_{90}}{S_0} = \frac{2.90 - 1.19}{2.90} = 0.590 \]

\[ RBA_{30} = \frac{S_{30} - S_{930}}{S_{30}} = \frac{1.73 - 1.19}{1.73} = 0.311 \]

\[ RBA_{60} = \frac{S_{60} - S_{960}}{S_{60}} = \frac{1.43 - .98}{1.43} = 0.314 \]
<table>
<thead>
<tr>
<th>Sheet</th>
<th>Tensile Readings #/15 mm</th>
<th>Average Tensile Sheet</th>
<th>Tensile Corr. for 1.2 gm Sheet</th>
<th>Average Tensile 0% fine</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>1.61</td>
<td>109 110 118 124</td>
<td>11.56</td>
<td>2.61</td>
</tr>
<tr>
<td>B</td>
<td>1.68</td>
<td>123 114 118 127</td>
<td>11.70</td>
<td>2.30</td>
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<td>C</td>
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<td>D'</td>
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### 100 psi

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<th>Brightness Readings</th>
<th>Average Opacity Sheet</th>
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<th>S' Sheet</th>
<th>S' O% Fine</th>
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### 320 psi

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Scattering Coefficient vs Tensile Strength

- □ -- 0% Fines
- △ -- 30% Fines
- ○ -- 60% Fines
II RELATIVE BONDED AREA VS % FINES

RELATIVE BONDED AREA

% FINES
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