Variance Analysis of Basis Weight Variation on the Pilot Machine

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VARIANCE ANALYSIS OF BASIS WEIGHT
VARIATION ON THE PILOT MACHINE

by

Timothy A. Armstrong

A Thesis submitted to the
Faculty of the Department of
Paper Science and Engineering
in partial fulfillment
of the
Degree of Bachelor of Science

Western Michigan University
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ABSTRACT

The purpose of this study was to determine the significance of the variations of the pilot machine of the Department of Paper Science and Engineering at Western Michigan University by variance analysis conducted on basis weight profiles obtained from the Industrial Nucleonics scanning basis weight gauge. The profiles obtained from the basis weight gauge were subjected to a computer program which computed the cross-direction, machine-direction, and random component variations and determined F-ratios. The F-ratio shows the significance of the component variation compared to the random component variation. Cross-direction variation was found to decrease with speed. Machine-direction variation showed no significant trend and the random variation decreased with speed. It was found that the F-ratio comparing the machine-direction component to the random component was significant at the one per cent confidence level and should be lessened if better operational efficiency for the pilot machine is desired.
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</table>
Basis weight uniformity is a very important aspect of the fourdrinier paper-making process. It has been shown that variations within the process reduce basis weight uniformity and therefore the quality of the product. Any reductions in the variations bring about increased efficiency and uniformity. Variance analysis is a very useful statistical tool for locating the significant variations in a process. Once the significant variations are located, work may be concentrated on reducing them.

It is the purpose of this thesis to analyze basis weight profile data from the pilot machine of the Department of Paper Science and Engineering by variance analysis to determine the significance of variations within the process.
IMPORTANCE OF UNIFORMITY

Burkhard and Wrist (1) point out that paper is influenced by every stage of its process and is a valuable record of the operation. The quantity and quality of the paper and the speed and efficiency at which it is produced gives the producer the performance of the operation. The single most important criteria affecting performance is basis weight and moisture uniformity, and uniformity influences nearly every other factor in the process. Wet web strength is a function of basis weight so a wide range of basis weight variation increases the possibility of a break or "downtime". There will be less load on the drying section of the paper machine if there is less basis weight variation. Total variation tends to increase with speed so by reducing the variation a higher speed is possible.

Doering (2) explains that many benefits may be obtained from the fourdrinier paper-making process by sharper basis weight control. Sharper control brings about a closer distribution around the basis target and can result in a reduction in materials usage as shown in Figure 1. The operating target is 30 pounds per ream and control by manual sampling maintained a lower process limit of 28.4 pounds per ream. Sharper control by
Figure 1. Optimum raw material usage
continuous basis weight measurement allowed the operating target to be lowered to 29.3 pounds per ream and still maintain the lower limit of 28.4 pounds per ream. The net result is a reduction of 0.7 pounds per ream or 2.3 per cent of the original materials usage. In addition there is less production below 28.4 pounds per ream. This reduction in materials usage brings about the potential for a speed increase since more material can be dried by the same dryer conditions. Doering points out that approximately a one per cent increase in end moisture yields the potential for a three to fifteen per cent increase in speed, depending upon the machine.

A reduced dryer load and lower steam consumption can also result from reduced materials usage.

NATURE OF VARIATIONS

The fourdrinier paper-making process has three basic component variations which are of concern (1). These component variations are the cross-machine component, machine-direction component, and the random component. DeWitt (3) states that cross and machine direction components should be the only concern for reduction if they are significant as shown in Figure 2. The random component is very difficult to isolate and reduce.

Cross-direction components are affected basically
Figure 2. Sheet variations are two directional.

Figure 3. Types of machine direction variation.
by the slice adjustment. However, internal fittings, vanes, and guides in the headbox may also contribute. They are constant with time and vary with position across the sheet. (1).

Machine-direction components are of three types as illustrated by Figure 3. They are long term drift, long term (low frequency), and short term (high frequency) variations. They all add up to make the total machine-direction component and are dependent upon time but independent upon position across the sheet. The machine-direction component may be periodic or aperiodic, depending upon the source. Periodic variations can usually be attributed to rotating elements and/or electrical feedback. Aperiodic fluctuations are usually caused by drifts and random fluctuations in consistency and flow. (1).

Random variations are neither constant with time or position. They may be attributed to local fluctuations in flow of stock or in consistency and can also be a measure of large scale turbulence in the headbox. (1).

MANUAL VS. CONTINUOUS SAMPLING

Because of the variations in the fourdrinier paper-making process, sampling becomes a major problem to insure that a representative sample is obtained.

The total manual sampling error may be represented
by three different variables (3). Distribution error, $E_{SD}$, has to do with the point at which the sample is taken. Handling and preparation error, $E_{SP}$, has to do with cutting the sample and keeping it under the correct humidity conditions. Instrument error, $E_I$, is the error in the measuring device itself. Total sampling error, $E_T$, then becomes:

$$E_T = \sqrt{E_{SD}^2 + E_{SH}^2 + E_I^2}$$

A manual end-of-reel sample provides very little information about the overall variations in the machine direction and gives only a representative cross-direction sample if it truly represents the average profile as shown in Figure 4.

A scanning gauge measures continuously and also moves across the sheet as the paper moves past. Its data or profile is representative of the machine-direction variations over the entire width of the sheet and the profile also contains the cross-direction variations. One scan is more representative than one end-of-reel manual sample. However, the sheet is scanned many times during a reel so the entire reel is monitored. A much more representative data set is obtained by scanning and it is obtained nondestructively. Both of these characteristics are conducive to the use of variance analysis.
Figure 4. Manual end-of-reel vs. continuous sampling
VARIANCE ANALYSIS

Analysis of variance is a useful statistical tool for identifying significant component variations in a process. From Streit (5), variance analysis minimizes time to time variations and attempts to measure variations present at a virtual instant of manufacturing time.

From Ferber's text on Marketing Research (4), variance analysis determines the significance of observed relationships between two or more sets of sample data or more than two statistics. Burkhard and Wrist (1) point out that this lends itself very well to a several component system since the total variance of a system equals the sum of its component variances.

Ferber (4) outlines the procedure involved in using variance analysis. It begins with obtaining the data and segregating the total variance into its component variances. From DeWitt (3), fifteen or more diagonal scans of a continuous basis weight gauge are required to obtain enough representative data so that meaningful results may be obtained. This data is then divided into cross-machine segments and entered into a matrix. The cross-direction and machine direction component variances may be determined as well as the random component. Ferber (4) explains that sampling variance is taken to indicate the effect of randomness on the data. Signifi-
icance of the component variance is then found by compar-paring them to the random variance and is the F-ratio. If the component variance exceeds the random variance by a greater amount than should be expected merely from sampling variations, then that component is said to be a significant factor in the data and should be reduced. Usually the F-ratio is compared to a null hypothesis value or a value that takes into account the particular sample size and is the probability value of a given confidence level that one would obtain if the component had no significance. Burkhard and Wrist (1) have found from their analysis of paper machine trials that a com-
ponent variance is significant at a one per cent con-
fidence level and this would be the null hypothesis.
STATEMENT OF THE OBJECTIVES OF THE STUDY

Variance analysis of basis weight profiles is by no means unique. However, to the authors knowledge, it has never been conducted on a pilot machine.

The pilot machine of the Paper Science and Engineering Department at Western Michigan University is a very important "laboratory" tool. Many paper and allied companies rent time for the use of the facilities to conduct research. It would be economically unfeasable to shutdown a production machine for research trials. The pilot machine provides a representative replacement of a production machine. The results obtained from the pilot machine should reflect the results that would be obtained on a large scale production machine, and should be highly reliable since many important decisions may be made because of them.

Because of the use of the pilot machine, its variability should be kept at a minimum. The objective of this study was to determine if the variability was great enough to require a further study to reduce the variations present.
The pilot machine of the Paper Science and Engineering Department at Western Michigan University was used to collect the data for this thesis. An Industrial Nucleonics basis weight and moisture gauge was the source of the data collected. The basis weight profiles are recorded on an X-Y recorder on the control panel for the instrument. Each complete scan is recorded. Twenty scans were obtained for each of the seven sets of data. The first four and the last set of data were obtained during thesis trials of fellow students. The remaining two sets were obtained during machine trials conducted by the Hercules Corporation.

Figure 5 shows one scan of the basis weight gauge. The scales are calibrated in direct reading units for the process \((6)\). In this case each horizontal line represents \(0.2\) pound \((25 \times 38-500)\) basis weight unit and each of the \(n\) major horizontal lines represent one pound \((25 \times 38-500)\) basis weight unit. These values are determined by the scale of basis weight units that is being used during operation. All data for this thesis was obtained on the same scale. One recorder for the basis weight gauge records the highest to lowest value per traverse of the gauge and shows the weighted average for the scan. This is shown in Figure 6. By comparing the record with the scale, values per line may be obtained.
FIGURE 5
PROFILE FROM BASIS WEIGHT GAUGE

FIGURE 6
SEGMENT OF RECORDER CHART SHOWING RANGE AND WEIGHTED AVERAGE PER SCAN

5 (#) / 25 units = .2 (#) / unit
Once this is found, data may be taken off of the profiles. At each major vertical line across the profile a data point was obtained. The number of data points for each set of data varied from nine to fifteen. Each of the twenty profiles for each set of data was subjected to the same process of entering the data points into a matrix.

Each of the matrices was analyzed by the BASIC computer program (Appendix) written by the author from a procedure outlined by Burkhard and Wrist as shown in Appendix 2. This program calculated the average of the rows and columns and also the grand average of the data. From these values the component variances were calculated and compared. The results were tested to the one per cent significance suggested by Burkhard and Wrist and either accepted or rejected.

The output of the program printed the average of the columns and rows, the grand average, the three component variances, the F-ratio, and whether they were acceptable or non-acceptable.

Data of varying basis weights and speeds were obtained to find if any trend could be determined contributed by these factors.

Profiles and data as entered into the matrices are in Appendixes 3 and 4.
The characteristics of the various Trials are presented in Table I. There were four different basis weights (25 x 36-500) of 44, 48, 52, and 83 pounds used and essentially three different speeds of 78, 88, and 120 feet per minute. The dimensions of the matrices and the null hypothesis values are given.

Table II gives the various cross-direction component (CDC) variances, machine-direction component (MDC) variances, random component (RC) variances, and the total variances and Table III gives the standard deviations for the data. The total variances were obtained by taking the sum of the component variances. Burkhard and Wrist found CDC to decrease with speed, MDC to have no significant trend compared to speed, and RC to show a definite upward trend with increased speed.

In this study CDC variances were found to follow the trend as stated above. The variances ran from 1.02 pounds$^2$ at 78 feet per minute (fpm) through 0.29 pounds$^2$ at 88 fpm to 0.14 pounds$^2$ at 120 fpm. A more common term, standard deviation, shows the same trend as the values decreased from 1.01 pound at 78 fpm through 0.54 pound at 88 fpm to 0.37 pound at 120 fpm.

MDC also followed the results found by Burkhard and Wrist. No significant trend was found since high and low values
# TABLE I
## CHARACTERISTICS OF TRIALS

<table>
<thead>
<tr>
<th>Trial</th>
<th>Basis Weight (lbs)</th>
<th>fpm</th>
<th>Dimensions of Matrix</th>
<th>Null Hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rows</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>48</td>
<td>80</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>II</td>
<td>83</td>
<td>80</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>III</td>
<td>52</td>
<td>78</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>IV</td>
<td>52</td>
<td>78</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>V</td>
<td>44</td>
<td>88</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>VI</td>
<td>44</td>
<td>88</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>VII</td>
<td>44</td>
<td>120</td>
<td>20</td>
<td>18</td>
</tr>
</tbody>
</table>

|       |                    |     | Columns               |                 |
|       |                    |     |                       |                 |
|       |                    |     |                       |                 |
|       |                    |     |                       |                 |
|       |                    |     |                       |                 |


# TABLE II
## VARIANCES

(pounds)$^2$

<table>
<thead>
<tr>
<th>Trial</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>Total Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CDC</td>
<td>MDC</td>
<td>RC</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0.41</td>
<td>0.42</td>
<td>1.11</td>
<td>1.94</td>
</tr>
<tr>
<td>II</td>
<td>0.59</td>
<td>1.90</td>
<td>3.16</td>
<td>5.65</td>
</tr>
<tr>
<td>III</td>
<td>1.02</td>
<td>1.37</td>
<td>1.14</td>
<td>3.53</td>
</tr>
<tr>
<td>IV</td>
<td>0.28</td>
<td>0.63</td>
<td>0.30</td>
<td>1.21</td>
</tr>
<tr>
<td>V</td>
<td>0.29</td>
<td>1.43</td>
<td>0.65</td>
<td>2.37</td>
</tr>
<tr>
<td>VI</td>
<td>0.29</td>
<td>0.77</td>
<td>0.32</td>
<td>1.38</td>
</tr>
<tr>
<td>VII</td>
<td>0.14</td>
<td>1.38</td>
<td>0.15</td>
<td>1.67</td>
</tr>
</tbody>
</table>
were obtained at each speed range. At 38 \text{fpm} values of 0.77 \text{pounds}^2 \text{ and } 1.43 \text{pounds}^2 \text{ were obtained or standard deviations of 0.36 \text{ pound} \text{ and } 1.19 \text{ pound}. At 80 \text{fpm} a wider range was obtained with values of 0.42 \text{ pounds}^2 \text{ and } 1.90 \text{ pounds}^2 \text{ of standard deviations of 0.65 \text{ pound} \text{ and } 1.38 \text{ pound}.}

However, the RC showed a definite decrease with increased speed, contrary to the reported findings above. The lowest value of 0.15 \text{pounds}^2 \text{ was obtained at } 120 \text{fpm} \text{ while the highest value of } 3.16 \text{pounds}^2 \text{ was obtained at } 80 \text{fpm}. Burkhard and Wrist stated that basis weight variation is characteristic of a particular paper machine. This may explain why the contradicting values.

Table IV shows the same results with the coefficient of variation, a better statistical tool for comparison than variance and standard deviation.

No significant trends of variation were to be found with respect to basis weight as shown by Figure 7 where basis weight is plotted versus variance.

Only four values were found to be significant and all were F-ratios comparing $S_2/S_3$. They were on Trials IV, V, VI, and VII as shown in Table V. Trial IV gave an F-ratio of 2.13 compared to the null hypothesis value of 2.01. Trails V, VI, and VII were all compared to a null hypothesis value of 2.04 and the values obtained were 2.18, 2.42, and 9.39 respectively. All of these F-ratios
were comparing the machine-direction component to the random component. This would suggest that the machine direction component should be reduced. The trials were in chronological order and Table V would suggest that these ratios were progressively worse with time.
-19-

**TABLE III**

**STANDARD DEVIATIONS**

(pound)

<table>
<thead>
<tr>
<th>Trial</th>
<th>CD</th>
<th>MD</th>
<th>RD</th>
<th>Total</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>.641</td>
<td>.648</td>
<td>1.055</td>
<td>1.394</td>
<td>47.913</td>
</tr>
<tr>
<td>II</td>
<td>.768</td>
<td>1.380</td>
<td>1.780</td>
<td>2.380</td>
<td>83.35</td>
</tr>
<tr>
<td>III</td>
<td>1.01</td>
<td>1.171</td>
<td>1.069</td>
<td>1.880</td>
<td>51.991</td>
</tr>
<tr>
<td>IV</td>
<td>.530</td>
<td>.794</td>
<td>.548</td>
<td>1.100</td>
<td>50.61</td>
</tr>
<tr>
<td>V</td>
<td>.539</td>
<td>1.196</td>
<td>.806</td>
<td>1.546</td>
<td>43.10</td>
</tr>
<tr>
<td>VI</td>
<td>.539</td>
<td>.878</td>
<td>.566</td>
<td>1.175</td>
<td>44.41</td>
</tr>
<tr>
<td>VII</td>
<td>.374</td>
<td>1.175</td>
<td>.387</td>
<td>1.294</td>
<td>44.18</td>
</tr>
</tbody>
</table>

**TABLE IV**

**COEFFICIENT OF VARIATION**

(per cent)

<table>
<thead>
<tr>
<th>Trial</th>
<th>CD</th>
<th>MD</th>
<th>RD</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.34</td>
<td>.35</td>
<td>2.21</td>
<td>2.92</td>
</tr>
<tr>
<td>II</td>
<td>0.92</td>
<td>1.66</td>
<td>2.14</td>
<td>2.86</td>
</tr>
<tr>
<td>III</td>
<td>1.94</td>
<td>2.26</td>
<td>2.06</td>
<td>3.62</td>
</tr>
<tr>
<td>IV</td>
<td>1.04</td>
<td>1.57</td>
<td>1.08</td>
<td>2.17</td>
</tr>
<tr>
<td>V</td>
<td>1.25</td>
<td>2.77</td>
<td>1.86</td>
<td>3.57</td>
</tr>
<tr>
<td>VI</td>
<td>1.21</td>
<td>1.96</td>
<td>1.28</td>
<td>2.65</td>
</tr>
<tr>
<td>VII</td>
<td>0.85</td>
<td>2.66</td>
<td>0.88</td>
<td>2.93</td>
</tr>
</tbody>
</table>
Figure 7. Basis weight vs. variance
<table>
<thead>
<tr>
<th>Trial</th>
<th>$S_1/S_3$</th>
<th>Null</th>
<th>Accept</th>
<th>$S_2/S_3$</th>
<th>Null</th>
<th>Accept</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.37</td>
<td>1.99</td>
<td>Reject (x)</td>
<td>0.38</td>
<td>2.16</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>0.19</td>
<td>1.99</td>
<td></td>
<td>0.60</td>
<td>2.16</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>0.89</td>
<td>1.97</td>
<td></td>
<td>1.20</td>
<td>2.01</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>0.94</td>
<td>1.97</td>
<td></td>
<td>2.13</td>
<td>2.01</td>
<td>x</td>
</tr>
<tr>
<td>V</td>
<td>0.44</td>
<td>1.98</td>
<td></td>
<td>2.18</td>
<td>2.04</td>
<td>x</td>
</tr>
<tr>
<td>VI</td>
<td>0.90</td>
<td>1.98</td>
<td></td>
<td>2.42</td>
<td>2.04</td>
<td>x</td>
</tr>
<tr>
<td>VII</td>
<td>0.93</td>
<td>1.98</td>
<td></td>
<td>9.39</td>
<td>2.04</td>
<td>x</td>
</tr>
</tbody>
</table>
The cross-direction variations of the pilot paper machine are not significant to the operation of the machine. However, the machine-direction variations seem to be getting worse with time and concentration should be devoted to reducing the variations if overall operational efficiency is desired.

Cross-direction component variation was found to decrease with speed while machine-direction variation showed no trend. The random component variation tended to decrease with speed which contradicted the findings of Burkhard and Wrist but may be inherent with the pilot machine.

During the course of obtaining data it was observed that the speed fluctuated as much as plus or minus three or four feet per minute. This could be a definite factor contributing to the variation in the machine-direction component. However, fluctuations of this type would probably affect the random component unless the fluctuation was periodic or dependent upon time.

The automatic control just recently installed may help the system. Signals received by the Industrial Nucleonics basis weight gauge are compared to a target and the stock flow valve is operated accordingly. The one set of data obtained after the control was installed had the same amount of variation as before. After more
knowledge of its operation is obtained, the benefits of having automatic control may be obtained.
RECOMMENDATIONS

The variations of the pilot machine basically are not significant enough to seriously produce erroneous data. The machine direction component was found to be significant on the latter trials and may be looked at with concern. If it is consistently getting worse as may be suggested from the data, then some effort may be required to reduce the variation to maintain good performance.

The results obtained with and without automatic control should be examined to determine if this type of control is beneficial at the pilot level. Better tuning of the controller may be a goal.
LITERATURE CITED

1) Burkhard and Wrist- TAPPI 37(12):613,(1954)
6) Instruction Manual- Industrial Nucleonics Corp. 1966
COMPUTER PROGRAM FOR CALCULATING VARIANCES

10 REM VARIANCE ANALYSIS PROGRAM FOR BASIS WT. PROFILES
20 DIM A(20,20), B(20), C(20), D(20)
30 PRINT "TYPE VALUES FOR M=# ROWS & N=# COLUMNS",
40 INPUT M,N
50 MAT READ A(M,N)
60 FOR S=1 TO N
70 LET B(S)=0
80 LET X=0
90 FOR T=1 TO M
100 LET X=X + A(T,S)
110 NEXT T
115 FOR U=1 TO M
120 LET B(U)= X/M
130 NEXT S
140 FOR U=1 TO M
150 LET C(U)=0
160 LET X=0
170 FOR V=1 TO N
180 LET Y=Y + A(U,V)
190 NEXT V
200 LET C(U)= Y/N
210 NEXT U
220 LET N1=M*M
230 LET X1=0
240 LET X2=0
250 FOR S=1 TO N
260 LET X=0 + D(S)
270 NEXT S
280 LET X2=X1/2
290 LET T=0
300 FOR G=1 TO N
310 LET T=T + C(G)-X2
320 NEXT G
330 PRINT "S1=S2/S3/S4/S5"
340 END
477 NEXT U
478 PRINT "X2="; X2
480 LET F1=S1/S3
490 LET F2=S2/S3
500 IF (F1*100)<1 THEN 530
510 PRINT "F1= "; F1, "NOT ACCEPTABLE"
520 GO TO 540
530 PRINT "F1= "; F1, "ACCEPTABLE"
540 IF (F2*100)<1 THEN 570
550 PRINT "F2= "; F2, "NOT ACCEPTABLE"
560 GO TO 515
570 PRINT "F2= "; F2, "ACCEPTABLE"

APPENDIX 2
PROCEDURE FOR CALCULATING VARIANCES

\[ S_1 = \frac{\sum_{p=1}^{m} \frac{(X_{p,s} - \bar{X})^2}{m-1}}{n} \]

\[ S_2 = \frac{\sum_{s=1}^{n} (X_{s} - \bar{X})^2}{n-1} \]

\[ S_3 = \sum_{p,s} \frac{(X_{p,s} - X_{p,s} - X_{,s} + \bar{X})^2}{(m-1)(n-1)} \]

Where:

n = number of positions across a strip
m = number of strips
\( \bar{X} \) = grand average
\( X_{p,s} \) = hold \( s \) while varying \( p \) through range (1 - m)
\( X_{,s} \) = hold \( p \) while varying \( s \) through range (1 - n)
\( X_{p,s} \) = vary \( p \) and \( s \) through range of matrix
Appendixes III and IV may be found in the original copy on file at the Department of Paper Science and Engineering.