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A Statistical Analysis of Strength Characteristics of Handsheets

Edward J. Horvath
Western Michigan University

A STATISTICAL ANALYSIS OF STRENGTH CHARACTERISTICS OF HANDSHEETS

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Edward J. Horvath

Edward J. Horvath
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ABSTRACT

A literature survey was made in order to provide an outline of fundamental statistical definitions, formulas and methods which are used by the paper technician. These methods, if properly applied, would furnish valuable information as to the accuracy of test values, significance of differences and merits of various operations.

A demonstration is made of the application of statistical methods to test data obtained from samples of unbleached kraft pulp in the form of wet laps, air-dry pulp and soaked pulp. The analysis was devoted to the study of strength variations, uniformity and duplicability of handsheets produced from the three samples.

Introduction

Statistics is the science which involves the study of populations, variations and methods for the reduction and interpretation of data (5). This survey of literature concerning the statistical analysis of the strength characteristics of handsheets has been undertaken in an effort to compile an abstract of statistical methods that can be applied to the testing of pulp and paper. In addition, the objective of this literature investigation has been to provide demonstrations of the applications of these methods to various test data and their subsequent evaluation.

Evolution of Statistics

The first fundamental laws of statistics were developed by Blaise Pascal and Pierre de Format in the year 1654 (13). These Frenchmen worked on the theory of probability in connection with a gambling problem. Since that time, men in all branches of science have made efforts to develop methods for the reduction of enormous quantities of meaningless data. The discoveries of these men have been grouped into the form of a new science, the science of statistics.

Like all other sciences, statistics is in a stage of rapid evolution. During the past twenty years more and more new statistical techniques have been developed. The advent of World War II created new industrial problems. With increased production, better methods for the control of product quality were required. It was during these war years that statistics gained widest recognition in the field of quality control. By means of statistical inference the quality of products were retained at a definite level and the actual production processes were controlled.

Application in the Paper Industry

The paper industries have also begun to utilize the benefits of this elaborate mechanism. The increasing application of statistical methods to test evaluation and quality control are indicated by the numerous recent publications pertaining to this subject.

The variability of paper, the wide range of test results and the poor precision of evaluation have led to the adoption of statistical techniques. These techniques enable us to do the following: (1) to determine the precision of a set of tests, (2) to determine the minimum number of tests necessary to obtain a result having a derived degree of accuracy, (3) to determine whether observed differences in results are significant, (4) to reduce a large mass of test values to comprehensible form, (5) to determine reproducibility of test procedures, (6) to select the best testing procedures and apparatus, (7) to find the factors that influence variability.

In order to apply the statistical methods for the interpretation of test results, it is not necessary to have a strong background in mathematics. However, some of the fundamental definitions, nomenclature and statistical methods should be studied thoroughly. These fundamentals, which shall be presented on the subsequent pages, have been obtained from (5), (9), and (12).

Fundamental Definitions

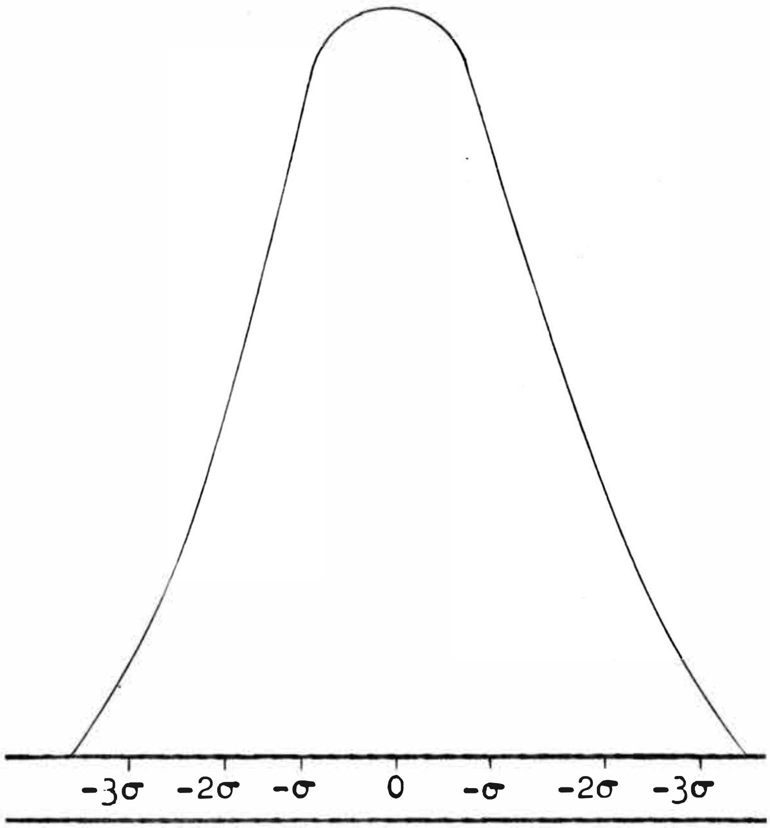
A universe is any large group of related individuals. These individuals form the population. This can be illustrated simply by examining a large set of bursting strength values of a sample of paper. The whole group of values represents the universe, while the individual test values symbolize the population.

A normal universe is one in which the population is distributed according to the Gaussian or normal law of error. This law relates the frequency of occurrence of a measurement to the amount by which it varies from the population average. The symmetrical, bell-shaped curve in Figure 1 is a graphical representation of the normal law of error. The distribution of values about the population mean 0 is measured in units of standard deviation σ on the abcissa.

This graph shows that values which deviate slightly from the population average occur more frequently than values that deviate considerably from the average. Since sixty-eight percent of the area under the curve is enclosed between $\pm\sigma$, the probability is that two out of every three values will deviate from the population average by less than one standard deviation. The different probabilities of occurrence have also been established for the limits $\pm 2\sigma$ and $\pm 3\sigma$. These probabilities are of vital importance in the applications of statistical methods.

Fig. 1

The Normal Law of Error



A sample is a relatively small collection of the population of a universe. Simple sampling is the method by which a representative portion of the population is withdrawn for analysis.

A histogram is a graph which is used to display the frequency of occurrence of values in a set of data.

Statistical Nomenclature

Some of the important symbols that are used in statistical methods are listed below.

n = the number of test values in a sample.

x, y = the individual variables in the sample.

$\bar{x}$ = the arithmetic mean.

$\bar{\bar{x}}$ = the average of a group of averages.

d = the deviation of any test from the average of the population.

Σd^2 = the sum of the squares of these deviations.

m = the maximum error.

σ = the standard deviation of a large sample.

σ' = the standard deviation of an average estimated from σ and n.

σ'' = the standard deviation of an average estimated from a sample of similar averages.

V = the coefficient of variation.

r_{xy} = the coefficient of correlation.

Outline of Methods

The arithmetic mean is the most common statistic used.

$$\bar{x} = \frac{\Sigma x}{n}$$

Statisticians have found that the arithmetic mean fails to give full information about a sample because of the large distribution of values about it.

The median has provided us with a more effective method which can be used for the representation of a large set of data. It is defined as the middle measurement if all values are arranged in increasing order of magnitude. The distribution of the deviations from the median are less than the corresponding deviations from the arithmetic mean.

The standard deviation is obtained by the formula

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

This is one of the most important statistical methods employed in analyses. The standard deviation can be used to measure the precision of a single test result and the dispersion of values about the average or median.

The probable error is also a measure of the dispersion of values about the average or median. This is not the "most probable error," but it is the error on either side of which the mean deviation is likely to fall. The numerical value of the probable error is 0.6745σ.

The value of the maximum error is $\pm 3\sigma$. This can be justified by examining the graph of the normal law of error. Ninty-nine percent of the area is enclosed between the curve and the units $\pm 3\sigma$ on the abscissa. This means that ninty-nine percent of the values will fall within the limits $\pm 3\sigma$ and only one percent will exceed these limits.

The standard error of an average, hereafter referred to as the standard deviation of an average, can be calculated from the following equation

$$\sigma' = \frac{\sigma}{\sqrt{n}}$$

This equation provides us with a method for determining the number of tests required to produce an average, the maximum error of which will fall within a predetermined limit.

The standard deviation of the averages of a group of samples can be determined by the following method

$$\sigma'' = \sqrt{\frac{\Sigma d^2}{n}} \qquad (d = \bar{\bar{x}} - \bar{x})$$

By a comparison of σ'' and σ', the sources of error can be revealed by the use of this statistic. Under ideal conditions σ'' would approximately equal σ'.

The coefficient of correlation can be used to correlate the variations found in two different types of test data.

$$r_{xy} = \frac{\frac{\Sigma(xy)}{n} - \bar{x}\bar{y}}{\sigma_x \sigma_y}$$

For example, this statistic can be used to study the effect of beating time upon the various strength factors of handsheets.

The significance of differences of average values is also an important factor which must be considered. Many times, test values are not rejected when they should be. This invariably causes a misinterpretation of data. Determinations which exceed $3\sigma'$ are to be considered significant and should be rejected. The "t" test is even more exact than the preceding method for the determination of significant differences, but this test will not be discussed here.

<u>Applications</u>

The actual application of the foregoing procedures can be more easily understood by examining a few illustrative examples. The applications which shall be presented here have been obtained from: (a) a study of the precision of two folding endurance testers (6); (b) a study of the effect of different couching methods on handsheets (9); (c) a study of laboratory pulp tests (5); (d) a study of the effect of beating time and drying methods on handsheet characteristics (9); (e) a comparison of strength tests of kraft liner board (10); (f) a study of the effect of a digester circulating system on pulp quality (5).

A Study of Two Folding Endurance Testers

Laboratories are often faced with the problem of purchasing new equipment for paper testing. Usually, there is a choice between two or more different types of equipment and a decision must be made concerning the type of machine that should be purchased. The precision of the machines is one of the most important factors which must be considered before this decision is made. Statistical methods can enable us to determine the precision and the best testing apparatus.

A statistical analysis was made to determine the precision of Schopper and MIT fold testers. This analysis involved several experiments.

The first experiment was conducted in order to estimate the reproducibility of the two machines on a single sample of paper. The fold strips were cut in machine and across direction. Two hundred tests were made on each machine. The average, standard deviation and coefficients of variation were computed from the test data. Frequency diagrams were also plotted from the results produced by the machines.

A comparison of the frequency curves revealed that there were no apparent trends in folding level resulting from position within samples. There was also an approximate agreement with the normal or Gaussian distribution. The coefficient of variation of the MIT was found to be slightly higher than the corresponding coefficient of the Schopper. Since this was not considered sufficient evidence to make a definite estimation of the precision of the two machines, folding tests were made on three more samples of paper. The results of the three newly derived coefficients of variation were found to be very close in numerical value. This lead to the conclusion that neither of the two machines were more accurate in precision than the other.

A second analysis was made to determine if the precision of results obtained from the MIT changed with the material tested. A large number of tests were made on three different papers that varied in furnish and processing. The data obtained was evaluated by the various statistical methods as in the preceding example.

The coefficients of variation of each type of paper were analyzed and were found to be relatively consistent. This suggested that the coefficients of variation were fairly constant from batch to batch and were independent of machine or across direction. Then coefficients of the three different grades of paper were compared and it was discovered that there was a change in variation. It was therefore concluded that the MIT could be used for control work and for the prediction of the reproducibility of results on any grade of paper.

The Effect of Couching on Sheet Properties

An analysis was made in order to determine what type of couching process gave the most uniform and duplicable bursting strength results. Six sets of handsheets were made on the same machine and each set was couched in a different way. Otherwise, all conditions were made as constant as possible. Bursting strength tests were made on the handsheets and the results were subjected to the various statistical calculations.

Since there was only a relatively small difference in the values of the coefficients of variation, the sheets were considered to be of equal uniformity. The duplicability of results was determined by observing the ratios of V to V' (V' being the symbol which represents the calculated coefficient of variation). If the uniformity of the handsheets had been the only factor which caused the lack of duplicability of results, the values of the ratio V to V' would have scattered about 1. However, the values of the ratios were much higher than 1 except when the handsheets were couched by a heavy couch roll and a flat plate pressure of 5 lbs./sq.in.. It was therefore concluded that these two methods produced handsheets which could be more easily duplicated.

A Study of Laboratory Pulp Tests

A study of laboratory tests was made to determine if the bleach value of unbleached sulphite pulps was related to the strength of the pulp. The usual laboratory beater-run method was employed. Samples were taken from the beater at definite time intervals and the bleachability (permanganate number) was determined at each interval. Handsheets were also made from each sample and subjected to burst and tear tests. The values obtained from these tests were recorded as burst value, tear value and pulp value. Five hundred and forty of these test values were reduced to forty-six simple statistics and were analyzed.

The values of the arithmetic mean were investigated and it was found that these values implied that some pulp strength was lost in the bleaching process. The standard deviations indicated that bleached pulps were less variable, or that they lend themselves to testing so that the inherent variabilities were smaller. The coefficient of correlation of the bleach value to each of the other tests showed that there was a higher degree of association between pulp value and burst value than between pulp value and tear value. The positive signs of each of the coefficients of correlation suggested that the relationships were direct. Therefore, it was concluded that the pulp strength increased with increasing bleach value.

The Effect of Drying Methods on Sheet Characteristics

An analysis of bursting strength values was made so that an estimate of the effect of beating time on sheet characteristics could be obtained. A sample of sulphite pulp was milled for 44 and 66 minutes. Handsheets were formed, dried and tested. The various statistical values were calculated from the test data.

Inspection of these values revealed that the probable error of the mean increased with decreasing beating time. The value of the probable error was much larger for the 44-minute milled sample than the 66-minute sample. This indicated that the 66-minute pulp was more uniform and produced a handsheet of better formation. A comparison of the maximum error of the averages also provided sufficient evidence for the conclusion that the 66-minute sheets could also be more readily reproduced.

A similar analysis was made to study the effect of different drying methods on sheet properties. Handsheets were formed from five different pulp samples and dried in the vacuum oven, electric oven and loft. The results obtained were tested as in the above example.

By taking the sum of the squares of the probable errors of the sheet averages, it was found that the precision of the data obtained from the sheets dried in the ovens was slightly better than those dried in the loft. But since the trend of precision of the results was only slightly in favor of the ovens, it was concluded that these exceptions indicate the effect of formation differences.

The variability of the sheets among themselves was determined by the use of the ratio of the standard deviation of the sheet averages to the direct average of the bursting strength values. It was found that the trend for least variability was in the sheets dried in the electric ovens.

A Study of Strength Tests on Kraft Liner Board

The bursting strength values obtained from two kraft liner boards were analyzed statistically. It was desired to know if the liners were normal with respect to strength properties. A series of one hundred tests were made. The statistical values were computed and frequency diagrams were plotted.

Since the arithmetic average did not provide enough information concerning the spread of values, it was necessary to calculate the standard deviation. After this value was derived, the deviations from the mean were investigated in order to determine whether any of the values were significantly different. No significant differences were found. All values were within the range of 3σ. Therefore, it was concluded that the results and the two samples of liner board were perfectly normal.

The Effect of a Change of Cooking Procedure on Pulp Quality

An analysis was made to determine the effect on pulp strength and uniformity due to the installation of a circulating system in a sulphite digester. Forty-nine sets of pulp tests were made on pulps cooked before the circulating system was installed. The same number of tests were also made after its addition to the digester.

By comparing the means, it was found that the values were higher for the circulated series. The standard deviations were also larger for this series. Therefore, since the standard deviation is a measure of the variability, it was concluded that the variability of the handsheets was decreased by the installation of the circulating system in the digester.

The statistical emphasis was then placed on the differences between averages in order to lay the foundation for the determination of any improvement of pulp quality by the circulating system. To do this, it was necessary to determine if the differences were significant or if they were just a result of normal variation. The standard errors were calculated and it was found that they were from four to six times their respective standard errors. Therefore, they were considered reliable.

It was concluded from the above analysis that: (1) the installation of the circulating system caused an increase in pulp strength, (2) using the circulating system caused the pulp to be more uniform with respect to strength variations from cook to cook.

LITERATURE CITED

1. Bicking, C. A., Pulp Paper Mag. Canada 49, No. 3: 181-188 (Feb. 1948).

2. Carrier, N. H., and Young, J. H., World's Paper Trade Rev., 105, No. 20: 1593-1602 (May 15, 1936).

3. Hughson, G. D., Pulp Paper Mag. Canada 51, No. 8: 85-86 (July,1945).

4. Mode, E. B., "Elements of Statistics," 2nd ed., New York, N. Y., Prentice Hall Inc., 1942.

5. Pascoe, T. A., and Fraser, M., Paper Trade J. 112, No. 12: 39-44 (Mar. 20, 1941)

6. Reitz, L. K., and Sillay, F. J., Paper Trade J. 126, No. 17: 54-58 (Apr. 22, 1948).

7. Renner, M. S., Tech. Assoc. Papers 31: 318-332 (June, 1950).

8. Liebert, E., Das Papier 4, No. 17/18: 344-351 (Sept., 1950).

9. Simmonds, F. A., and Doughty, R. H., Paper Trade J. XCVII, No. 25: 32-37 (Dec. 21, 1933).

10. Toulouse, J. H., TAPPI 33, No. 1: 44-48 (Jan., 1950).

11. Warburton, J. A., Pulp Paper Mag. Canada 51, No. 8: 83-84 (June, 1950).

12. Youden, W. J., "Statistical Methods for Chemists," 1st ed., New York, N. Y., John Wiley & Sons, Inc., 1951.

13. Collier's Encyclopedia. 1st edition. Section on Statistics.

LABORATORY EXPERIMENTAL PROBLEM

Introduction

Quality control technicians in the paper industry have found that the application of statistical methods to strength test data have enabled them to reveal numerous variations and phenomenon. One such phenomena involves the variation of the strength of handsheets with changes in moisture. It is commonly known throughout the paper industry that pulp in the form of wet laps will lose some strength characteristics as the laps dry out. My experimental work has been devoted to the verification of this phenomena by statistical methods and to the study of changes produced by soaking pulp. In addition, I have made a statistical comparison of the uniformity and duplicability of the handsheets which were formed for the analysis.

Preparation And Testing of Handsheets

A representative sample was taken from wet laps of unbleached kraft pulp. The laps were allowed to become air-dry and a second sample was removed. The air-dry pulp was then soaked for twenty-four hours before the last sample was taken. These pulp samples were beaten for six minutes and handsheets were formed on the Noble and Wood Sheet Machine. All conditions were maintained as uniformly as possible during sheet formation, in order to prevent the introduction of obscuring variables. The sheets were then dried for three minutes at a temperature of 250°F and subjected to the various strength tests. The statistical values were calculated and the resulting data analyzed.

Analysis of Bursting Strength Results

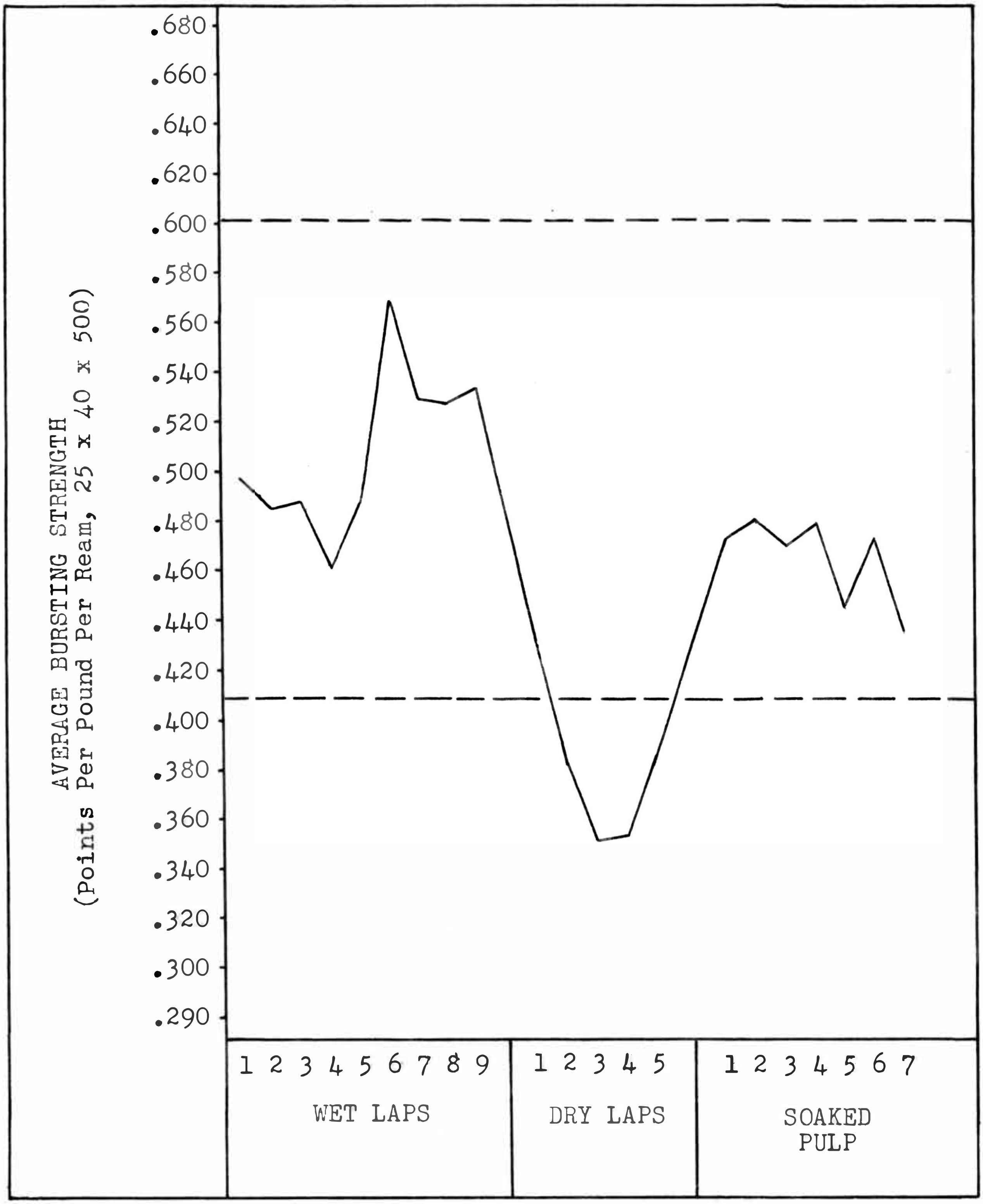
The bursting strength results were arranged into three groups corresponding to the treatment to which the original pulp was subjected. The analysis of the calculated statistical values was divided into three parts.

(1.) A Study of Strength Variation

The first portion of the analysis was concerned with the determination of the strength variation produced by forming handsheets from wet laps, air-dry pulp and soaked pulp. In order to facilitate this determination, it was necessary to establish tolerance limits. The bursting strength results obtained from the wet laps were assumed to be the standard values to which all others could be compared. Maximum and minimum values from the average were calculated by employing $\pm 3\sigma$ limits. Since $\pm 3\sigma$ includes 99.73% of all the values scattered about the average, any values that exceeded these limits were considered to be significantly different. The limiting values of the results obtained from the wet laps were found to be 0.506 $\pm$0.096.

A chart was constructed in order to simplify the analysis. This graphical representation is presented in Figure 1. The abscissa is divided into subsets of handsheets, while the ordinate is divided into values of average bursting strength. The values which were plotted were the average of twelve tests. Limiting lines were drawn at the average burst values of 0.602 and 0.410.

Fig. 1



AVERAGE BURSTING STRENGTH
(Points Per Pound Per Ream, 25 x 40 x 500)
.680
.660
.640
.620
.600
.580
.560
.540
.520
.500
.480
.460
.440
.420
.400
.380
.360
.340
.320
.300
.290
1 2 3 4 5 6 7 8 9
WET LAPS
1 2 3 4 5
DRY LAPS
1 2 3 4 5 6 7
SOAKED PULP

It was found that the handsheets formed from the air-dry pulp yielded average values that exceeded the lower tolerance limit. Therefore, there was sufficient evidence to conclude that the pulp lost strength after drying. On the other hand, the handsheets formed from soaked pulp yielded values which were within the lower limits. This indicated that by soaking the pulp a portion of the original strength was regained.

(2.) A Study of the Uniformity of Handsheets

The second portion of the analysis was devoted to a study of the uniformity of the handsheets formed from the three samples of pulp. A study of this type involved a comparison of the probable errors of the averages of the sheet averages. These values are presented in Table I. Inspection of these data reveal that the probable errors of the averages obtained from wet laps and air-dry pulp were higher than for the soaked pulp. This indicated that the sheets formed from soaked pulp were more uniform. Since all conditions were held as constant as possible during sheet formation, it could be said that the sheets formed from soaked pulp had a 1.82 better formation than those obtained from air-dry pulp and 1.91 better formation than sheets obtained from wet laps. (1.82 and 1.91 being the ratios of 0.020 and 0.021 to 0.011 respectively.) Although the probable error of the results from the wet laps was 0.001 higher than the value of the sheet formed from air-dry pulp, it was not considered to be sufficient evidence to indicate a significant difference in uniformity.

(3.) A Study of Duplicability of Handsheets

The last portion of the analysis was concerned with a study of the duplicability of handsheets. This required a comparison of the spread of the bursting strength values obtained from each sample about the average bursting strength. In order to facilitate this comparison, three histograms were made. Figures 2, 3 and 4 presented on the following pages are graphical representations of the frequency and distribution of values obtained from wet laps, air-dry pulp and soaked pulp respectively. The abscissa of each graph is divided into units of standard deviation arranged about the mean. The ordinate is divided into units which represent the frequency of occurrence.

Inspection of Figure 2 revealed that the relative spread of values for the wet lap sample was exceedingly large. Many of the values extended to as far as ±4σ and ±5σ. It was found that the frequency of occurrence did not increase uniformly as the average value was approached. These factors all lead to the conclusion that the sheets produced from wet laps were not easily duplicated.

The curve in Figure 3 resembled the normal cure of error with respect to the spread of values about the mean. Only a few values were found to exceed ±3σ. Although this factor showed that the spread of bursting strength values was relatively uniform, it was found that values that differed slightly from the average occurred about as frequently as values which differed considerably. This was sufficient evidence to prove that handsheets produced from air-dry pulp were not easily duplicated.

It was found that the curve plotted employing the results obtained from soaked pulp closely approximated the normal curve of error. All values were found to be within $\pm 3\sigma$ and values occurred more frequently about the average bursting strength. It was therefore concluded that handsheets produced from soaked pulp were not only more uniform, but were more readily duplicated than handsheets from wet laps and air-dry pulp.

TABLE I.

Samples	Sets Six Sheets	Average Bursting Strength Points/lb./Ream 25 x 40 x 500	Standard Deviation o	Standard Deviation of Average for One Set o'	Standard Deviation of Average of Sample Averages o''	Probable Error of Average of Sample Averages
	1	.497	.033	.0095		
	2	.485	.029	.0084		
	3	.488	.027	.0078		
Wet	4	.461	.053	.0153		
Laps	5	.488	.061	.0176	.031	.021
	6	.568	.032	.0092		
	7	.528	.031	.0090		
	8	.526	.050	.0145		
	9	.532	.029	.0084		
	Average	.506		.0110		
	1	.433	.021	.0061		
Air-dry	2	.378	.025	.0072		
Pulp	3	.351	.022	.0064	.030	.020
	4	.353	.019	.0055		
	5	.383	.034	.0098		
	Average	.380		.0070		
	1	.473	.027	.0078		
	2	.479	.040	.0116		
Soaked	3	.470	.031	.0090		
Pulp	4	.477	.030	.0087	.016	.011
	5	.446	.026	.0075		
	6	.473	.028	.0081		
	7	.436	.023	.0066		
	Average	.465		.0080		

Fig. 2

Caption

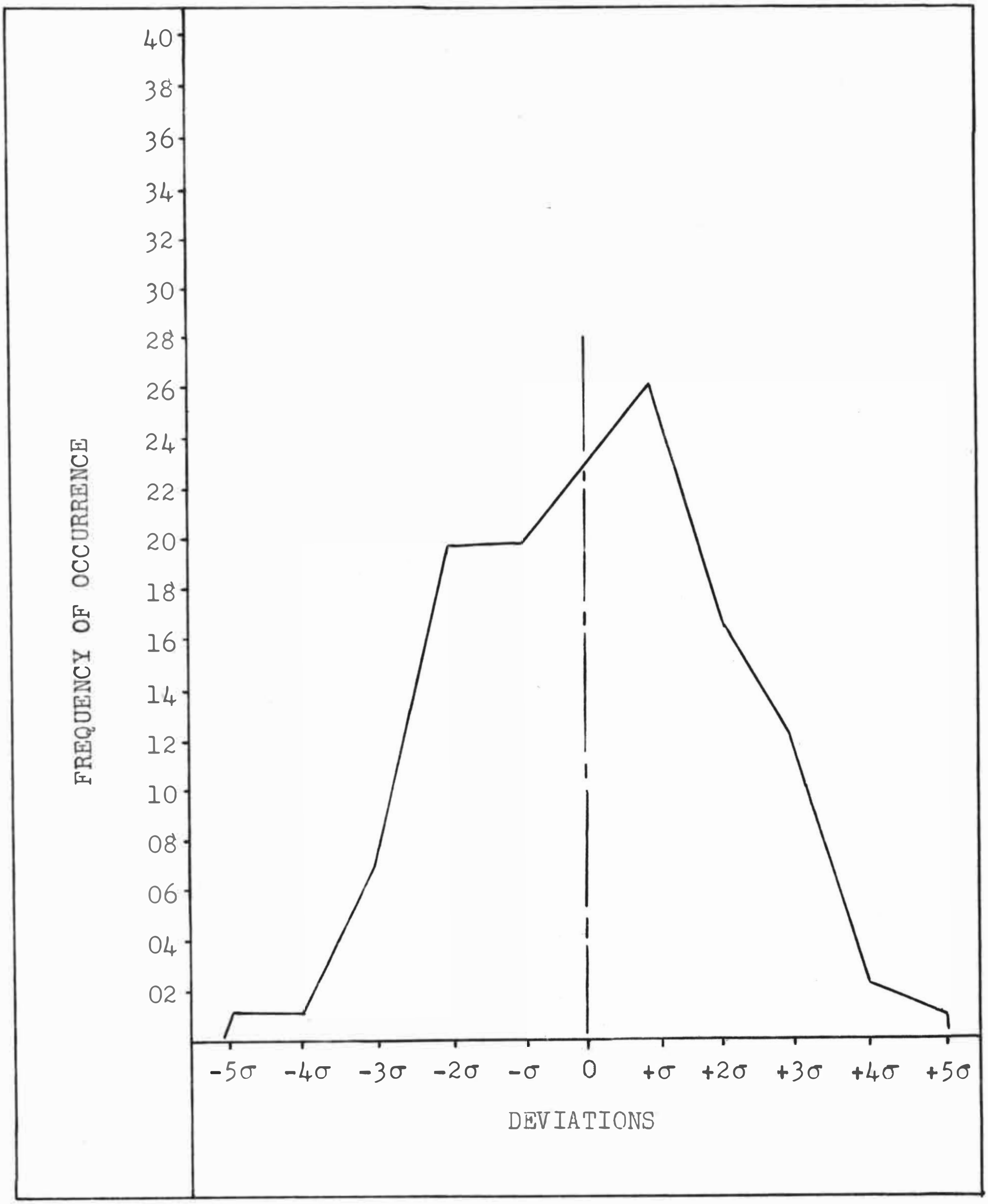


Fig. 3

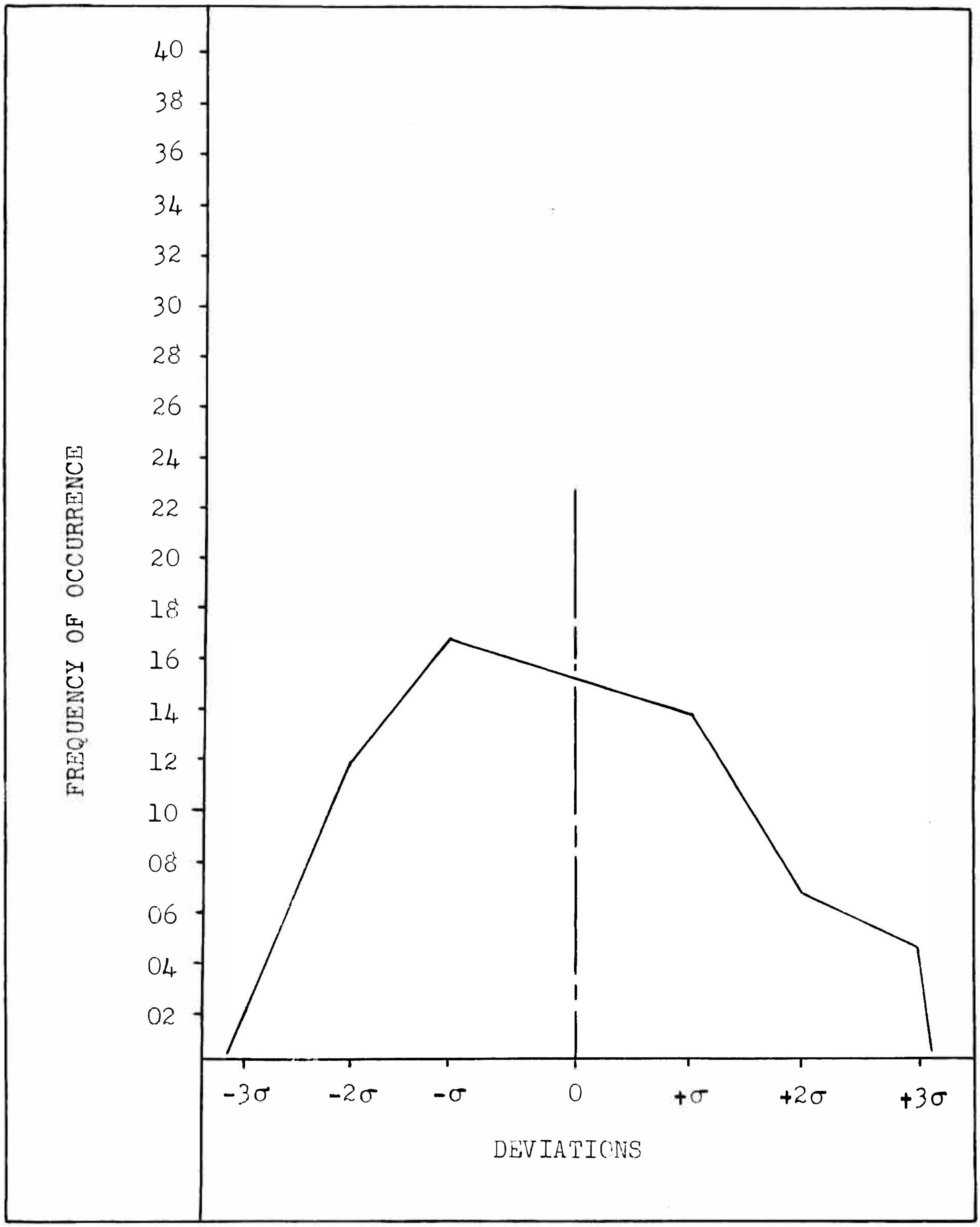
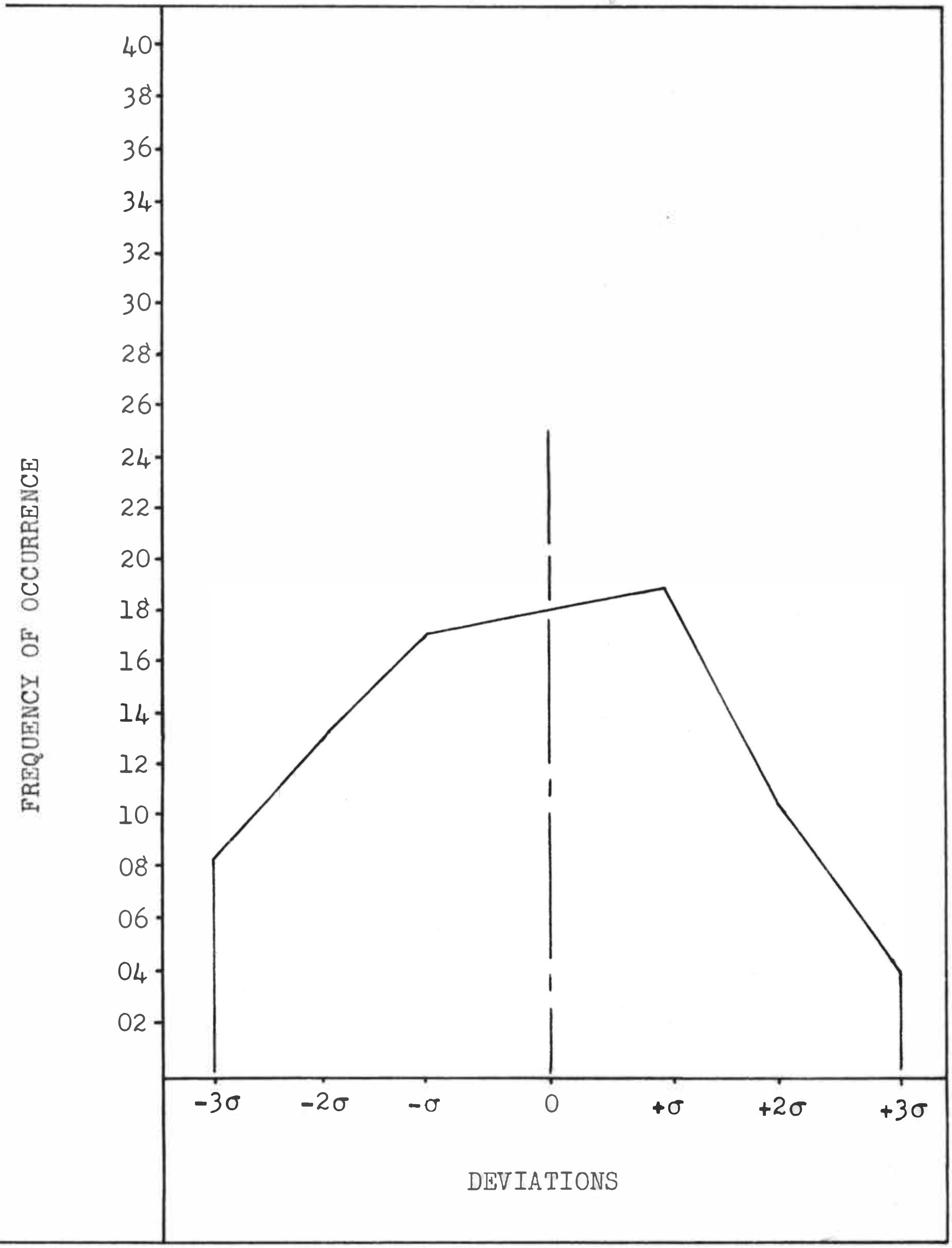

FREQUENCY OF OCCURRENCE
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
08
06
04
02
-3σ
-2σ
-σ
0
+σ
+2σ
+3σ
DEVIATIONS

Fig. 4 Caption



Summary

Handsheets fromed from wet laps, air-dry pulp and soaked pulp were tested. The statistical values of the bursting strength results were calculated for each of the three groups of sheets. The analysis of these values was devoted to the study of the strength variations, uniformity and duplicability of these handsheets.

By establishment of tolerance limits, it was shown that the pulp lost strength after losing moisture. It was also found that the strength was increased by soaking pulp, but only to a portion of the original strength of the wet laps.

A study concerning the probable errors of the averages of burst results for each group of handsheets revealed that the uniformity of the handsheets produced from soaked pulp was far greater than those obtained from wet laps and air-dry pulp.

The last portion of the analysis was concerned with the study of the distribution of values. It was found that the plotted values obtained from the soaked pulp formed a curve that approximated the normal curve of error more closely than the other results. It was therefore concluded that soaked pulp yielded values which were not only more uniform, but more easily duplicated.