Bayesian analysis is a statistical method that uses probabilities to measure uncertainty about unknown data in order to draw a proper inference. The analysis of complex statistical models using Bayesian inference has become more prominent in recent years. This is partly due to the increasing availability of computing power. Another factor is the discovery of a number of simulation techniques that made Bayesian inference possible for complex models. However, many of these techniques are computationally demanding. This research attempted to enhance Bayesian applications by using the ParInt software package.

Bayesian models analysis often requires the evaluation of complicated multidimensional integrals. For higher-dimensional non-linear models, the only practical methods for analysis are based on stochastic simulation techniques such as Monte Carlo (MC), quasi-Monte Carlo (QMC) and Markov chain Monte Carlo (MCMC) techniques. These are known to be computationally intensive, with some analyses requiring days of CPU time on powerful computers.

The ParInt software package has been developed at Western Michigan University. The system is designed for solving integration problems numerically via high performance computing. ParInt uses multiple solution methods including adaptive domain partitioning, recursive region from priority queue, and split region. It evaluates new subregions and update results in insert new subregions into priority queue.

The purpose of ParInt is to approximate multivariate integrals of the form:

\[ f = \int f(x) \, dx \]

The output contains an approximation \( Q \) and an absolute error estimate \( \varepsilon \), satisfying \( |Q - f| \leq \varepsilon \), where \( \varepsilon \) is the absolute and the relative tolerance error, respectively.

ParInt applies an adaptive region subdivision algorithm (Figure 2) that divides the domain D into subregions (Figure 3) for evaluation.

![Figure 1: ParInt Architecture](image1.png)

![Figure 2: Adaptive Integration Algorithm](image2.png)

![Figure 3: Subdivision of integration domain](image3.png)

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Results

We selected several applications from various areas to be implemented in C and evaluated using ParInt:

- Application Data
  - Medical Data
    - Birth weight Data: predicting low birth weight, linked to several causes.
    - Heart transplant data: survival of patients on the waiting list for the Stanford heart transplant program.
    - Photocarcinogen data: mice with tumors survival time.
    - Cross-classification in health-care data of schizophrenic patients.

- Climate Data
  - Global weather: an estimate of the joint probability density function (PDF) for uncertain climate system properties.
  - Tomato data: forecasting tomato intensity.

- Economics Data
  - Various examples from Econometrics [5–6].

- General Statistics
  - Bayesian analysis of a linear model with simulated data.
  - Multivariate logistic distribution.
  - Multivariate normal.
  - Nonlinear regression.

Conclusion & Future work

Bayesian models require solving high-dimensional multivariate integration problems.

- We implemented a number of Bayesian problems using ParInt.
- ParInt computations deliver accurate and efficient results. Running the applications on Thor reduces the execution time considerable.
- We plan on adding tools for enhancing Bayesian inference solutions in ParInt.
- We will continue to investigate the role of High Performance Computing in analyzing complicated Bayesian models.

References