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The Application of Linear Programming in the Design of Least Cost Furnishes for Paper Making Systems

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THE APPLICATION OF LINEAR PROGRAMMING IN THE DESIGN OF LEAST COST FURNISHES FOR PAPER MAKING SYSTEMS

by

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ABSTRACT

Linear programming is a mathematical technique used to optimize complex physical processes. Due to rising pulp costs, the need to minimize the costs of raw materials in paper furnishes is acute. This paper studies the potential of linear programming as a tool to design furnishes meeting designated paper specifications while minimizing raw material costs. The flexibility of linear programming is discussed and the applicability of linear models describing multi-component furnishes with their effect on paper properties is investigated. Some paper properties were found to fit well with the linear model. Some of these were opacity, smoothness, tensile, mullen, and tear. At the same time, other properties such as porosity and fold exhibited distinctly nonlinear characteristics. There was a direct relation between the linear blending characteristics of a property and the success or failure with which the linear programming method could be used to design a specific furnish to meet the requirements of that property. Filler in the furnish presents more problems to accurate linear programming results. There are strong indications, however, that experience and careful tuning of the filler's coefficients in the linear programming equations can overcome most of the initial shortcomings found in this report.

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INTRODUCTION

This investigation evaluates the applicability of linear programming techniques for the design of least cost furnishes for paper manufacture. The linear programming procedure designs pulp blends that will meet grade specifications while minimizing the raw materials cost for paper production. To use this technique, a crucial concession must be made. The mathematical basis of linear programming solves problems in such a way that as two elements are mixed, the resultant properties of that blend must change in direct proportion to the ratio of the elements. This is called the linearity assumption. Paper is a highly complex physical and chemical substance. The factors influencing a single measurable property of paper are far too numerous and interrelated to be mathematically simulated in a simple linear equation. For this reason linear programming must predict changes in properties from differing paper blends on a very empirical basis.

There are, therefore, two primary questions to be answered in this report. First, how well does linear blending hold up for different physical and optical properties of paper. Second, if linear blending is valid, can it be used along with linear programming techniques to accurately predict paper qualities for least cost furnish.

HISTORICAL BACKGROUND

Linear Programming

Linear programming is a mathematical technique used to optimize complex physical processes. Any process which receives inputs, performs an activity on them, thus producing outputs, may be adaptable to a linear programming analysis. Linear programming requires that the activity must be described by a system of linear equations or inequalities. Furthermore, more variables must exist than there are equations to provide alternative solutions for optimization. All variables used in the linear equations must be positive so that a valid solution with no negative yields or requirements can result. This means the process has an irreversible activity. Finally, a directing force, known as the functional equation, must represent the value or commodity to be maximized or minimized for optimization (1).

Linear programming utilizes principles of applied statistics with all computations conforming to normal operations in matrix algebra. This property makes linear programming very compatible with computers (2). Therefore, optimal analysis is quite practical for very complicated systems involving hundreds of variables and equations.

The formulation of linear expressions describing a given process is straight forward in theory, but practical problems rarely lend themselves to simple analysis. The greatest single shortcoming of

linear programming is that a process must conform to the linear relations of first order equations. Real activities tend to be more complicated and can be defined only with more sophisticated relationships such as second and third order or even exponential equations. Despite the limitation of linear programming to first order relationships, some compensations can be made. For instance, where non-linear characteristics are exhibited by a system, intermediate line segments can approximate the curve and be applied in the linear program analysis. Although this method is not extremely versatile nor precise, an analyst familiar with the system being optimized, can choose a linear approximation that is adequate in most situations to generate valid solutions. However, great care must be exercised in using this procedure as poor approximations can lead to gross errors in the solution. It is the responsibility of the programmer to incorporate his own understanding of the real process capacity and other limitations that could not be included in the mathematical representation of the system (3).

Another shortcoming of linear programming is that it accounts only for those costs that vary directly with throughput in the optimization analysis. Therefore, capital costs, fixed costs such as taxes, insurance, labor, and depreciation costs cannot be included within the network of the linear program itself (4).

Linear programming may require large amounts of computer time

for problems involving hundreds of equations. Computing time will increase approximately with the square of the number of equations. Computation time can be decreased by combining some of the equation's variables to reduce the matrix size, but this will also cause a loss of flexibility and control in the program.

Linear programming also has a tendency to continue optimizing profits for increments as small as a fraction of a cent. For this reason, the data used in the linear programming model of the system must be as accurate as possible. Errors within the linear equations that describe the system can lead to fictitious solutions to the optimization problem (5). Therefore, every solution to a linear programming problem must be carefully examined for its reasonableness. If a problem does exist within the mathematical model of the process, the bug is usually very hard to find and correct as the computational path to an optimized solution is very complex.

Linear programming is the general title for several different types of optimizing algorithms. The Simplex algorithm developed by Dr. George Dantzig in 1947 is the most commonly used today by engineers (5). Some fundamental procedures used when applying the Simplex algorithm to practical problems will be discussed now.

Each optimization problem must be described in terms of the interaction of different process variables. The variables can represent machine hours, money, the portion of a blend, or any other facet of production that the program may be dealing with. All variables are restricted to positive values.

Each variable has a coefficient in the linear equation. The coefficients may have positive or negative values. The proper pairing of the appropriate coefficient value with a given variable is vital for the accuracy of the optimization routine.

The Simplex algorithm can only solve linear equalities. A simple method exists for the conversion of inequalities to equalities without changing the basic characteristics of the original inequality. To each inequality, a new and unique variable is added whose value, like all other variables must be positive. If a specific inequality's "variable" side of the equation is less than or equal to the given constant, the coefficient to the new variable is a positive one and vice versa. This new variable can now take on values to make up the difference or slack between the "variable side" of the equation and the constant term. These new variables are referred to as slack variables (5).

A system of linear equations implies additivity. Therefore, the profit of the n^{th} unit is the same as the first and the amount of each resource required is the same for the first as the n^{th} unit. As a direct result of the linearity assumption, the amount of resource consumed is a linear function of the level of activity.

Once the problem has been mathematically interpreted, there will be a system of simultaneous equations or constraints, as they are called

by Llewellyn, of the form:

$$a_{11} X_1 + a_{12} X_2 + \ldots + a_{1n} X_n = b_1$$
$$a_{21} X_1 + a_{22} X_2 + \ldots + a_{2n} X_n = b_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1} X_1 + a_{m2} X_2 + \ldots a_{mn} X_n = b_m$$

such that: $X_j \geq 0 \qquad j = 1,n$

$b_k \geq 0 \qquad k = 1,m$

$n > m$

"n" is greater than "m" in all cases as this is a fundamental prerequisite for linear programming. It provides the multiplicity of soluctions from which the optimum solution may be selected. Given this necessary condition, "m" variables can be solved at one time with the "m" equations. The rest of the variables, "n" - "m", are arbitrarily set to zero. This method will give a basic solution. As can be deduced, the optimum solution to a linear programming problem can have only as many variables included as there are constraints or equations. The function of the Simplex aglorithm then is to find that set of "n" variables and the correct value for each of those variables which will optimize the problem (5).

If a linear programming problem is to be optimized, some specific function must define the level of optimization. This is done with a first order equation called the objective. The objective function is of the general form:

$$Z = C_1 X_1 + C_2 X_2 + \ldots\ldots\ldots + C_n X_n$$

$$\text{Such that: } X_j \geq 0 \qquad j = 1,n$$

In this equation, the coefficients to the variables, C_k, represent the contribution that each variable makes toward the total optimization of the function. X_j represents the quantity of that specific product to be utilized in the solution.

For example, if this were a profit maximization problem, C_k would be given as the profit for each unit manufactured of the k^{th} product. X_k would be a part of the solution from the simultaneous equations and tell how many of the k^{th} product should be manufactured. The solution to a linear programming problem will be those "m" variables with their respective values which, when substituted into the objective function, will give the greatest value for "Z" (5).

Obviously, an optimum solution can be found if all possible combinations of the "n" variables are taken "m" at a time and solved for the system of simultaneous equations. These equations are actually solved as an M x N matrix using principles of matrix algebra. However, solving all of the possible combinations of "n" variables is far too cumbersome a procedure for practical application. For example, a problem with 10 equations and 20 variables would have 184,756 possible solutions to calculate. For this reason, the Simplex method was developed. The key to this method is that it provides a means of determining the change in profit a particular selection of variables will cause before the actual solution is calculated. This enables the program to introduce

only those variables to an existing solution which will most greatly increase the profit in the new solution and eliminate those variables in the present solution which contribute the least to the total profit. This capability in the Simplex algorithm shortens the computational effort of solving that same 10 X 20 problem to 1/92,378 the computational time (5).

Linear Blending

Blending several different types of pulp to produce the optimum furnish for a specific grade of paper is common practice in paper mills today. The two dominant considerations when blending different fiber types are the resultant paper properties of the mixed pulp and the cost per ton of the mixture. The objective of pulp blending then is to meet the grade specifications at the least possible cost in raw materials.

Of course, it isn't practical to make handsheets of every different possible combination of various pulps to determine the paper properties for each blend. The realistic alternative to this is to devise a method of predicting what changes will occur in paper characteristics when specific amounts of different fibers are mixed. One such method is called the linear blending theory. As Nordeman describes linear blending theory with respect to burst properties (given a burst factor for a pure pulp at a specific freeness for a hardwood and a softwood) for all blends of two pulps, the burst strength should be on

a straight line between the original burst-freeness points at a distance in proportion to the blend ratio from the softwood burst point (6). The use of the linear blending theory is very straight forward. For example, given burst strengths C_1 and C_2 for two different pulps, then the resultant burst strength for a blend of the two is:

$$C_1 X_1 + C_2 X_2 = 100 C_3$$

Where X_1 and X_2 are the percentage of their respective pulps in the mixture and C_3 is the resultant burst strength.

The application of linear blending theory as a tool for predicting paper properties certainly appeals in an empirical sense but the reliability of this method is highly questionable. Nordeman felt that a linear relationship was valid for burst strength as he concluded that the linear blending theory allows one to determine mathematically which combination of pine and hardwood freeness and burst levels will allow maximum utilization of the hardwood (6). This provides good empirical evidence that some degree of linearity does exist in burst properties when blending two pulps.

Caution must be exercised however to avoid generalization. The fiber binding systems that form a paper mat are very complex and heterogenous. The simplicity of a first order mathematical equation could not possibly describe the many facets of fiber and fiber-bonding characteristics which produce a given property such as burst. But as Skalicky points out, there is no gain in defining physical relations

within a sheet because for the majority of tests, the force which acts on the sample can be accurately determined but the system upon which it acts cannot be expressed physically (7). For this reason, empirical relationships are the most feasible mathematical approach to the simulation of paper properties.

In support of the linear blending theory, Brecht stated that if the properties of pulp are similar, the curve characterizing the mixing properties is almost always linear (8). On the other hand, he adds that if the measured properties are dissimilar, the mixing curve is rarely linear. In the case of nonlinear blending characteristics, different properties are influenced more or less by the dominant component in the paper. For example, absorbancy, air permeability, brightness and brightness reversion are all influenced more heavily by the pulp with the smaller value in these properties. Tear strength, stiffness and opacity, however, are affected much more by the component with the larger value (8).

Jamieson has pointed out that the scattering and absorption coefficients of a pulp are linearally dependent on those properties of each of its components. This is shown by (9):

$$\left(\frac{K}{S}\right)_{\text{Blend}} = \frac{K_1\ X_1 + K_2\ X_2 + \ldots}{S_1\ C_1 + S_2\ C_2 + \ldots}$$

K = Absorption coefficient

S = Scattering coefficient

X_n = Percentage of each component pulp

Some of Parson's work with optical properties of pulp blends indicates that the scattering coefficient is not always linearly related to the scattering coefficient of its components (10). But these tend to be exceptional cases involved with the fraction of fines in the pulp. Parsons stated that a given fraction of pulp scatters or absorbs light independently of its surroundings. Therefore, even in a mixture of different pulps, each component retains its original optical properties (10).

Page has found that the tensile properties of a sheet of paper also conforms to an empirical relationship of the form (11).

$$\frac{1}{T} = \frac{9}{8\,Z} + \frac{12\,A\,d\,g}{b\,P\,L\,(RBA)}$$

T = Finite span tensile strength

Z = Zero span tensile strength

A = Mean fiber cross sectional area

d = Density

g = Acceleration

b = Shear strength per unit area of fiber-fiber bonds

P = Perimeter of average fiber cross section

L = Mean fiber length

RBA = Fraction of fiber surface bonded in sheet

This is not the simple linear relationship desired. However, it does offer credibility to the premise that empirical relationships can describe a property resulting from complex interactions of fibers in paper.

Alin and Ruvo have done extensive research with polynomial representation of pulp mixture properties (12). The simplest expressions describing paper properties as a function of furnish make-up are linear of the form:

$$F^{i} = \sum A_{j}^{i} X_{j}$$

Where $\sum X_{j} = 1$

A_{j}^{i} = Coefficient of the i^{th} property of the j^{th} component

The values for the coefficients were derived experimentally with the application of the least squares method to the data. Almin evaluated second order equations of the form:

$$F = \sum_{j=1}^{S} A_j X_j + \sum_{j=1}^{S-1} \sum_{k>j}^{S} B_{jk} X_j X_k$$

The sign of the "B" coefficients will indicate whether the nonlinear effect when blending is synergistic or antagonistic. The investigation indicated that second order equations best describe tear strength and scattering coefficient. However, they pointed out that, if the range of validity were restricted, linear approximations can be acceptable. Third and higher order equations gave no significant increase to the accuracy of the equations (12).

Fillers generally improve the optical appearance of pulp and are detrimental toward strength characteristics. Almin and Ruvo noted a pronounced negative dependence of breaking length on clay content (12). Skalicky found that tear strength linerally decreased with increasing ash content. He also demonstrated that opacity increased linearly with

increasing filler content which agrees with earlier discussions on the effect of individual components on the optical properties of paper (7).

EXPERIMENTAL DESIGN

Introduction

The preceding literature survey indicated that there are two areas for experimental research if a thorough knowledge of linear programming and its possible use in furnish design is to be evaluated. Initially, a study of linear blending would produce data that could be a valuable resource later when applying linear programming to pulp blending. Secondly, a test of the linear programming procedure would indicate how accurately a furnish can be designed to meet specifications with the linear programming cost optimization method.

Since the success or failure of linear programming in paper furnish design is totally dependent on the linear change in paper properties as one pulp is blended with another, a complete understanding of linear blending is necessary if any intelligent interpretation of linear programming results can be made. Therefore, a large portion of the experimental work was devoted toward generating more concise information on linear blending with the pulps that would later be used for the linear programming study. The results of this aspect of the experiment will be discussed in detail directly following the explanation of experimental procedure. Graphs showing the change in specific properties as one pulp is blended with another are included with this paper in the Appendix.

The experimental test of linear programming consists of two parts. The first section deals with pulp blending while the second section also

uses filler in the system. Klondyke water washed clay was used in this experiment.

Experimental Design

The pulps selected for this experiment were Rayonier bleached kraft softwood, Weyerhaeuser bleached kraft hardwood and groundwood manufactured by Kimberly Clark. The softwood and hardwood pulps were prepared at several different freenesses in small laboratory valley beaters, according to Tappi Standard T200 05-70. The softwood samples were refined to freenesses of 500 CSF, 400 CSF, 300 CSF, and 200 CSF. The hardwood samples were refined to freenesses of 500 CSF, 400 CSF, and 300 CSF. Later in the experiment, when the groundwood became available, it was disintegrated and refined very lightly in a valley beater for fifteen minutes.

From these softwood and hardwood samples, handsheets were made in the Noble and Wood Handsheet machine. The dry sheet target basis weight was 60.5 grams per square meter or 2.5 grams per handsheet. Not only were the handsheets 100 percent of each pulp and freeness, but blending studies were initiated at this point by combining different pulps (two at a time) in ratios of 90% - 10%, 80% - 20%, 60% - 40%, 40% - 60%, 20% - 80%, and 10% - 90%. The blending procedure was accomplished by bringing both of the pulp slurries to the same consistency, and then mixing them volumetrically in the sheet mold.

One of the pulp blends was a softwood pulp of 500 CSF mixed with

another softwood pulp at 200 CSF. In a similar way, the 500 CSF hardwood pulp was blended with the 300 CSF hardwood pulp. To measure the change in paper properties as pulps of more contrasting characteristics are blended, the 400 CSF softwood pulp was blended with the 500 CSF hardwood pulp and the 300 CSF softwood pulp was blended with the 300 CSF hardwood pulp.

A study of filler addition on paper properties was also started in the handsheet phase of the experiment. To accomplish this, clay was added to the pulps at 10%, 20%, 50%, 100%, and 200% of the total dry sheet weight (60.5 g/m^2). The retention in the sheet mold is very low so the actual amount of ash in the sheets never exceeded 15%. The pulps selected for this study were the 500 CSF softwood, the 400 CSF hardwood, and the groundwood pulps. It should be noted here that the groundwood sheet could not be made with filler additions of greater than 50% due to lack of strength in the sheet.

Ten handsheets of each set were made and placed in the controlled temperature and humidity room. Of the ten original handsheets made, eight sheets were selected for actual paper testing on the basis of weight and the visual appearance of the sheet. Altogether, there were eight sets of handsheets made from single pulps at specific freenesses. Four groups of blended handsheets were made each consisting of six sets of handsheets. Three groups of filled handsheets were made with each containing five sets of handsheets with the exception of the groundwood group as mentioned previously. This makes a total of forty-five sets

of handsheets with eight handsheets to a set (total of three hundred eighty handsheets tested).

At the onset of this experiment, nine tests were chosen to measure and describe various physical and optical properties of the sheet. The tests were Martin Sweets brightness, Sheffield smoothness, Sheffield porosity, tensile, burst, tear, fold, and density.

As these are all established tests for paper, little need be said about them individually except where some specification or comment may pertain to the results of the tests.

With one exception, each sheet in a given sample set was measured once for a given property. The results for all eight sheets in a set were averaged to give a test value for the property being measured. Large data tables are a cumbersome and often difficult way to convey information, so the results of these tests have been portrayed graphically. Several representative graphs have been included in the body of this report to illustrate a specific type of trend or result. The rest of the graphs are included in the Appendix of this paper. The graphs are in three major groups consisting of those that measure a change in a specific property of the paper against a change in the freeness of a given pulp, the change in a property as one pulp is blended into another, and the change in a property as clay is added to the sheet.

Sheffield smoothness required a greater number of tests on each

handsheet because of a large variability of smoothness values on a single sheet. To accomplish this, five readings were taken on each handsheet and the average value for each sheet was recorded then all the sheets of the sample set were averaged. There are forty smoothness readings for each sample set value or point on the smoothness graphs. The result of this increased sample size was consistent trends on all three types of graphs for Sheffield smoothness.

Once the test data was collected, the results were compiled and the information was fed into the computer for the printer-plotter drawn graphs which make up the Appendix of this report. Through the points on these graphs, least square lines were drawn of either first or second degree depending on the configuration of the points.

The least squares fits for the data points on the'property vs. Canadian Standard Freeness' graphs were used to create the data log for all the pulps used in this experiment. This data log was in turn used in the linear programming equations. An example of this is shown on the following page in Table 1.

TABLE OF EXPERIMENTAL VALUES FOR PULPS TESTED

Test	Groundwood	500 SWK	400 SWK	300 SWK	200 SWK	500 HWK	400 HWK	300 HWK
Opacity	96.8	67.4	66.2	66.1	66.3	79.6	78.6	78.0
Brightness	54.6	83.0	81.2	79.5	77.7	84.5	82.1	80.0
Smoothness	241	340	322	306	290	292	283	274
Porosity	68	72	45	29	18	117	77	49
Tensile	3.3	10.3	10.5	10.7	11.0	5.7	7.0	7.4
Mullen	5.6	38.2	40.0	40.4	40.7	16.8	20.0	22.3
Fold	1	550	561	540	524	8	14	26
Tear	15	88	81	74	71	57	55	49

Table 1: Showing derived coefficient values for all pulps used in Linear Programming experiment.

SIMULTANEOUS EQUATIONS FOR LINEAR PROGRAMMING

Opacity $\leq$ $96.8X_1+67.4X_2+66.2X_3+66.1X_4+66.3X_5+79.6X_6+78.6X_7+78.0X_8$

Brightness $\leq$ $54.6X_1+83.0X_2+81.2X_3+79.5X_4+77.7X_5+84.5X_6+82.1X_7+80.0X_8$

Smoothness $\geq$ $241X_1+340X_2+322X_3+306X_4+290X_5+292X_6+283X_7+274X_8$

Porosity $\geq$ $68X_1+72X_2+45X_3+29X_4+11.0X_5+5.7X_6+77X_7+49X_8$

Tensile $\leq$ $3.3X_1+10.3X_2+10.5X_3+10.7X_4+11.0X_5+5.7X_6+7.0X_7+7.4X_8$

Mullen $\leq$ $5.6X_1+38.2X_2+40.0X_3+40.4X4+40.7X_5+16.8X_6+20.0X_7+22.3X_8$

Fold $\leq$ $1X_1+550X_2+561X_3+540X_4+524X_5+8X_6+14X_7+26X_8$

Tear $\leq$ $15X_1+88X_2+81X_3+74X_4+71X_5+57X_6+55X_7+49X_8$

Figure 1: Showing simultaneous equations used for Linear Programming where X_1, X_2, ...are equal to the percent of their corresponding pulps (see Table I) in the blend.

With the final values of all the pulps for the eight tests determined, the set of simultaneous equations needed for the linear programming could be constructed as shown in Figure 1. With the paper specifications inserted into the data along with cost information, the data table is complete and ready to run on the computer.

Although the paper specifications were arbitrary, the cost information, for the sake of authenticity, had to be based on current pulp prices and the cost of refining had to be ascertained in a realistic manner. Pulp prices were obtained from local mills. Groundwood was selling for about $225 per ton, bleached kraft softwood for about $310 per ton, and bleached kraft hardwood sold for about $325 per ton. References for the power requirements of stock treatment came from two sources: a private communication with the Mead Corporation and a presentation prepared by D. W. Danforth of the Pulp and Paper Research Center, Inc. The horsepower figures are from a study of a Sprout-Waldron pressurized disc refiner treating softwood pulp of 3.6% - 4.15% consistency, at a rate of one hundred gallons per minute. The treatment of hardwood in this experiment was assumed to require approximately three quarters the power of the softwood to effect the same incremental drop in freeness. The cost computations are shown in detail in Appendix 1. The cost of power data used in this model was derived from a steam cost study which indicated that a million BTU's of steam cost one dollar. One kilowatt-hour has a heat value of ten thousand BTU's. The value of one KWH is then, one one-hundredth times one dollar or one cent per KWH. These cost figures along with the computations

needed to derive them are shown in Appendix 1.

Because of the dramatic difference between the nature of filler clay and fibrous pulps, the treatment of filler information is somewhat different than the method used to derive constants for pulps. The retention of filler by each individual pulp is generally proportional to the amount of filler added to the sheet mold. This relationship is shown in Figure 2. However, the actual retention for a particular blend of different pulps may be quite different from that expected with either pulp. A good example of this fact can be seen on the graph showing the relationship between percent clay added and percent clay retained. At the 100% addition level, more than twice as much clay is retained by the hardwood as the softwood. Furthermore, the amount of clay retained by the groundwood cannot even be read on the chart as it exceeds more than 15% clay retained at less than the 50% addition level. When the linear program specifies a pulp blend between the hardwood and the softwood pulps of various freenesses with a certain level of filler, the amount of filler to be added to the sheet mold must be compensated to account for the different retention properties of the pulps in the furnish. If, for example, a 45% softwood, 45% hardwood and 10% filler blend was needed, the most direct method of determining the amount of filler to add would be to find that level of addition where the average amount of filler retained in the sheet for the hardwood and softwood components was 10%. This occurs graphically at the 213% addition level as illustrated in Figure 2. Of course, as

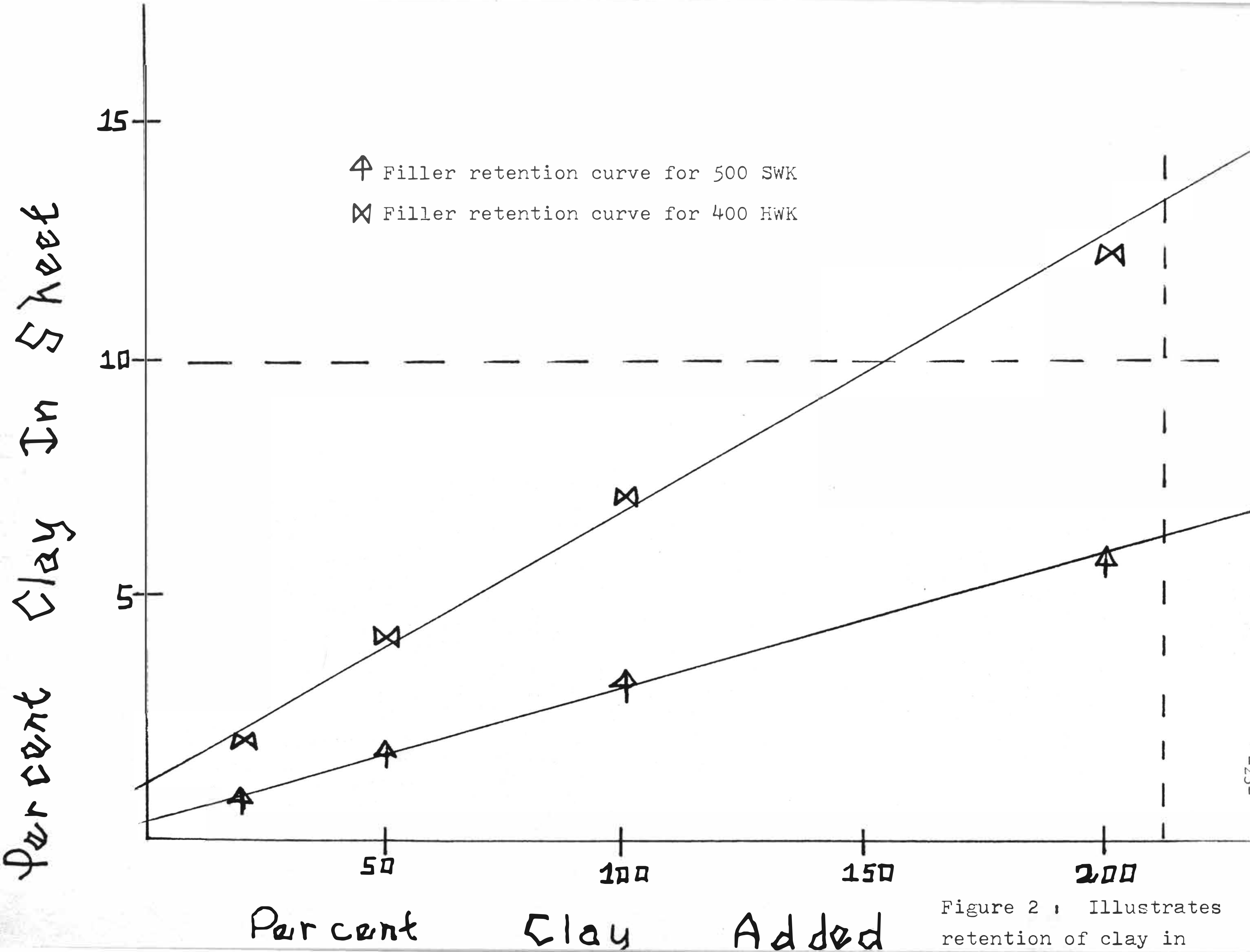


Figure 2 : Illustrates retention of clay in

more pulp components are specified, the problem of determining filler addition levels multiples but the same principle applies.

The second special consideration a filled sheet must be given in linear programming is concerned with the derivation of the constants for a filler in the simultaneous equations of a linear programming problem. All of the coefficients for pulp properties in the linear equations are based on actual data obtained from handsheets made from 100% of each specific pulp. This is obviously impossible with a filler since it must depend on a fibrous base sheet for its cohesion and strength. Therefore, an estimate of a "100% sheet of filler" must be made based on extrapolations of data collected from tests on the filled handsheets. An encouraging aspect of this procedure is that as far as the filler additions were carried, most of the properties in the filled sheets increased or decreased in a linear fashion with respect to the amount of filler added. Since most of the filler retention levels will be within those limits actually tested in the filled handsheets, this makes the linearity assumption quite valid over the range it will be used.

The actual change in the properties of the handsheets was noted from the zero addition level to the 100% addition level. This procedure was done graphically using the least squares lines instead of data points for added experimental accuracy. An example of this procedure can be seen in Figure 3 showing the change in burst strength as more filler is added to the base sheet. In this case, the softwood lost

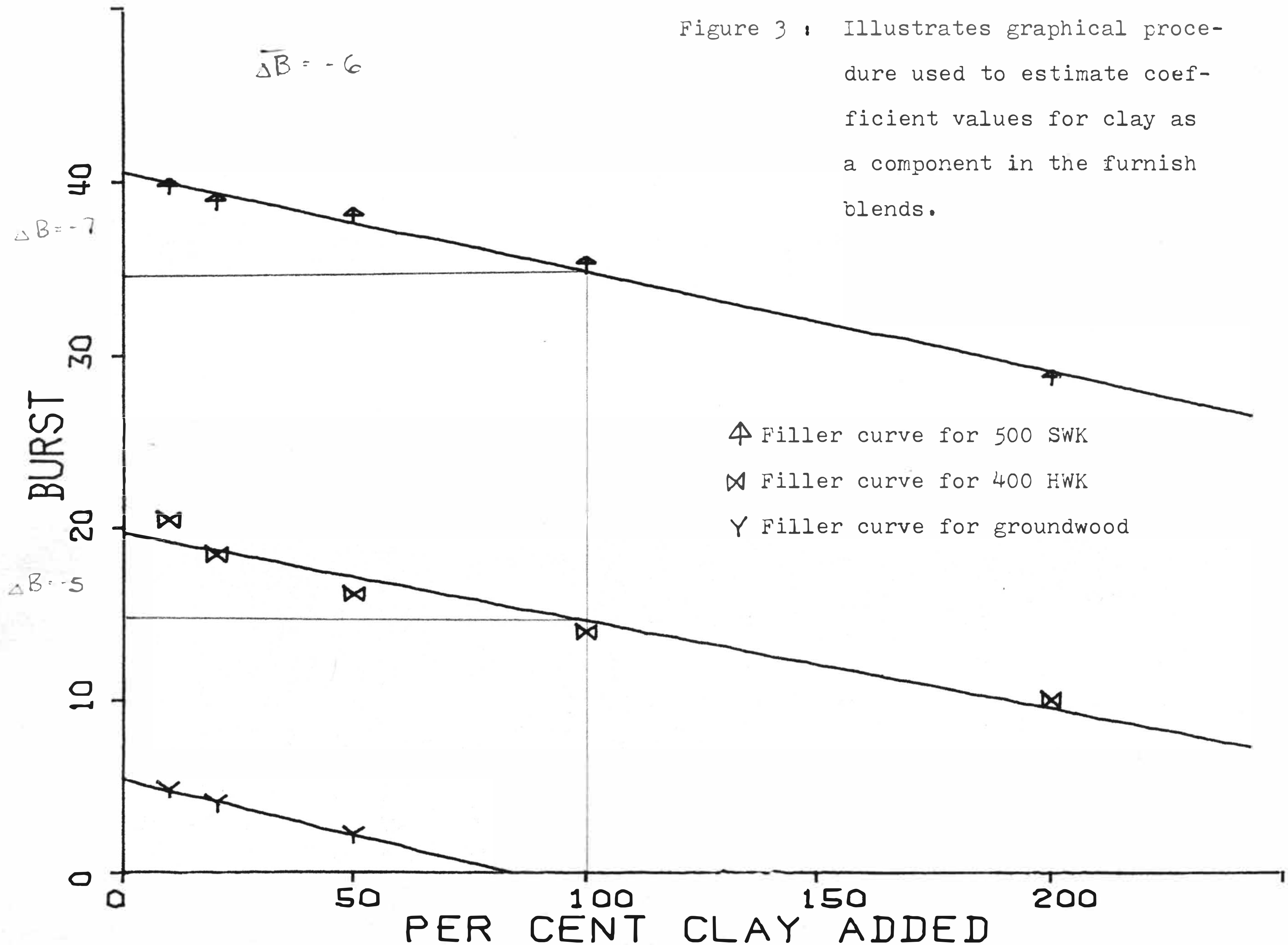


Figure 3 : Illustrates graphical procedure used to estimate coefficient values for clay as a component in the furnish blends.

seven points of burst while the hardwood lost five points as the filler addition level increased from 0% to 100%. The actual amount of filler in the softwood and hardwood sheets is 3.2% and 6.9% respectively. Using this information, the predicted total loss in burst strength as the percent filler in the sheet climbs to 100% is two hundred nineteen pounds per square inch for the softwood and seventy-two pounds per square inch for the hardwood. Of course, this is a physical impossibility since burst cannot have a negative value. At higher retention levels than those tested the effects of filler on burst strength exhibits curvilinear characteristics. Therefore, the estimated coefficients are reasonable approximations to the loss in burst strength with filler additions at the levels in which we are interested.

Depending on the relative amount of hardwood and softwood, the loss in burst due to the filler component should be biased proportionally toward the pulp type of the greatest amount. To arrive at the final coefficient value to be entered into the simultaneous equations of the linear programming problem, the loss in burst should be subtracted from a reasonable estimate of the burst strength of these pulps that will make up the furnish of the optimum paper. An example of filler coefficients for a sheet with equal amounts of hardwood and softwood pulps can be seen in Figure 4. This procedure can be approached by a successive approximation method or it may be reasonably accurate using an intelligent guess. It may be good to point out again that linear programming is a valuable tool in furnish design but the solutions are only as good as the data supplied to the linear equation.

ADJUSTED FILLER COEFFICIENTS FOR LINEAR PROGRAMMING*

Test	Filler
Opacity	160
Brightness	820
Smoothness	+36
Porosity	+84
Tensile	-30
Mullen	-110
Fold	-2500
Tear	32

*Based on approximately 50% - 50% addition of softwood and hardwood pulps.

Figure 4: List of estimated coefficient values for Klondyke clay. These values were used in the linear programming trial.

The last phase of the experiment was an actual test run of the linear programming accuracy in designing a pulp furnish. Two of the furnish mixtures utilized all the hardwood, softwood, and groundwood samples tested in the base data study. The third furnish was generated with the groundwood sample replaced by the filler-water washed Klondyke clay. Although the clay had a very large economic advantage, it was highly detrimental to the sheet's strength properties which limited the clay's usefulness as a component in the furnish. Nevertheless, 15% clay in the sheet for a 60.5 grams per square meter basis weight seemed to be a reasonable limit for filler.

Using the output of the linear programming routine, the furnishes were blended with the specified pulps and made into 60.5 gram per square meter handsheets for testing. These three sets of handsheets were tested for their physical and optical properties.

The analysis of the test results is included in the discussion of the results.

PRESENTATION

OF

RESULTS

RESULTS AND MATHEMATICAL ANALYSIS OF LINEAR PROGRAMMING STUDY

Table II: Lists results of linear programming portion of the experimental procedure.

RUN	TEST	SPECIFIED	PREDICTED	ACTUAL	ACTUAL ERROR	% ERROR
Pulp Blend 1	Opacity	85	85	89.1	4.1	4.6
51.4% 400 HWK	Brightness	72	72.1	60.6	-11.4	-15.9
35.0% Grwd.	Smoothness	280	270	294	24	8.9
7.2% 300 HWK	Porosity	70	70.1	62	- 8.1	-11.6
5.5% 400 SWK	Tensile	5.0	5.9	6.2	0.3	5.1
	Mullen	11	16	15.7	- 0.3	- 1.9
	Fold	40	40	8	-32	-80
	Tear	25	42	44	2	4.8
Pulp Blend 2	Opacity	85	85	90.6	5.6	6.6
50.8% 400 HWK	Brightness	72	72	57.6	-14.4	-20
35.7% Grwd.	Smoothness	280	276	279	3	1.1
13.5% 300 HWK	Porosity	70	70	62	- 8	-11.4
	Tensile	5.0	5.7	6.3	0.6	10.5
	Mullen	11	15	15.6	0.6	4.0
	Fold	7	11	7	- 4	-36
	Tear	25	40	40	0	0
Filler Blend 1	Opacity	80	82.3	83	0.7	0.9
48.7% 500 SWK	Brightness	83.5	83.5	77.9	- 5.6	- 6.7
40.5% 500 HWK	Smoothness	270	288	294	6	2.1
10.8% Clay	Porosity	115	92	80	-12	-13
	Tensile	3.0	4.1	5.5	1.4	34
	Mullen	5	13.5	16.7	3.2	23.7
	Fold	1	1.1	11	9.9	900
	Tear	32	69.4	93	23.6	34

WESTERN MICHIGAN UNIVERSITY LIN PROG RESULTS

PROG. ID. =PULP1 9EQS. AND 8NON-BASIS VARS.

MAXIMIZED VALUE OF FUNCTIONAL= -0.28870587E+05

MAXIMIZED SOLUTION

VAR. NAM.	UNIT CST	NO UNITS	L.L. VAR.	LOW LIMIT	TOP L. VAR	TOP LIMIT
SLACKVAR#1	0.0	234.3	SLACKVAR#7	-3.1	SLACKVAR#2	********
REAL VAR#1	-27.0	10.8	SLACKVAR#2	-242.3	SLACKVAR#2	********
SLACKVAR#3	0.0	727.9	SLACKVAR#7	-0.9	REAL VAR#3	3.2
SLACKVAR#4	0.0	2346.5	SLACKVAR#2	-0.9	SLACKVAR#7	7.4
SLACKVAR#5	0.0	108.4	SLACKVAR#2	********	SLACKVAR#7	7.2
SLACKVAR#6	0.0	752.5	SLACKVAR#2	********	SLACKVAR#7	1.9
REAL VAR#2	-327.2	48.7	REAL VAR#3	-348.7	SLACKVAR#7	-141.1
SLACKVAR#8	0.0	3738.9	REAL VAR#3	-1.4	SLACKVAR#7	4.0
REAL VAR#6	-312.2	40.5	SLACKVAR#7	-777.6	REAL VAR#3	-274.5

SOLUTION SET FOR FILLED FURNISH

Table III .

#9

Run 3

PROG. ID. =PULP1   9EQS. AND 8NON-BASIS VARS.

MAXIMIZED VALUE OF FUNCTIONAL= -0.28180868E+05

MAXIMIZED SOLUTION

VAR. NAM.	UNIT CST	NO UNITS	L.L. VAR.	LOW LIMIT	TOP L. VAR	TOP LIMIT
SLACKVAR#1	0.0	17001.5	REAL VAR#3	-0.9	SLACKVAR#2	********
REAL VAR#1	-226.3	35.7	SLACKVAR#2	-312.6	REAL VAR#3	-48.7
REAL VAR#8	-312.9	13.5	REAL VAR#4	-319.7	REAL VAR#6	-311.4
REAL VAR#7	-312.5	50.8	REAL VAR#6	-313.4	REAL VAR#5	-298.7
SLACKVAR#5	0.0	73.3	REAL VAR#6	-3.2	REAL VAR#2	3.5
SLACKVAR#6	0.0	417.1	SLACKVAR#2	********	REAL VAR#3	0.6
SLACKVAR#3	0.0	1320.9	REAL VAR#2	-0.2	REAL VAR#6	0.7
SLACKVAR#8	0.0	1491.1	REAL VAR#6	-0.4	REAL VAR#3	0.3
SLACKVAR#7	0.0	398.3	SLACKVAR#2	********	REAL VAR#3	0.0

SOLUTION SET FOR PULP 2 SPECIFCATIONS

Table IV

4

#2

RUN 2

PROG. ID. =PULP1 9EQS. AND 8NON-BASIS VARS.

MAXIMIZED VALUE OF FUNCTIONAL= -0.28235002E+05

MAXIMIZED SOLUTION

VAR. NAM.	UNIT CST	NO UNITS	L.L. VAR.	LOW LIMIT	TOP L. VAR	TOP LIMIT
REAL VAR#1	-226.3	36.0	REAL VAR#2	-298.1	SLACKVAR#7	-48.7
REAL VAR#3	-327.7	5.5	REAL VAR#2	-328.3	SLACKVAR#7	-317.8
REAL VAR#8	-312.9	7.2	REAL VAR#4	-316.7	REAL VAR#2	-312.3
REAL VAR#7	-312.5	51.4	REAL VAR#2	-313.2	REAL VAR#4	-308.5
SLACKVAR#5	0.0	88.8	REAL VAR#6	-2.7	REAL VAR#2	2.8
SLACKVAR#6	0.0	507.2	REAL VAR#4	********	SLACKVAR#7	0.6
SLACKVAR#3	0.0	1064.2	REAL VAR#2	-0.1	REAL VAR#4	0.3
SLACKVAR#8	0.0	1658.6	REAL VAR#6	-0.3	SLACKVAR#7	0.3
SLACKVAR#1	0.0	1243.3	SLACKVAR#7	-0.9	REAL VAR#2	1.3

SOLUTION SET FOR PULP 1 SPECIFICATIONS

Table V.

#1

Run 1

PROG. ID. =PULP1 9EQS. AND 8NON-BASIS VARS.

ITER. NO.	VALUE OF FUNCTION	VARIABLE OUT	VAR. IN
1	0.50727004E+05	SLACKVAR#1	REAL VAR#4
2	0.43941069E+05	SLACKVAR#3	REAL VAR#5
3	-0.91497109E+04	REAL VAR#4	REAL VAR#1
4	-0.26624111E+05	SLACKVAR#9	REAL VAR#4
5	-0.27128595E+05	SLACKVAR#7	REAL VAR#3
6	-0.31443850E+05	REAL VAR#4	REAL VAR#2

DUAL ALGOR. DETECTED INCONSISTENT MATRIXCHECK DATA AND RESUBMIT

COLUMN= 1

ROW= 1 0.00000000E+00
ROW= 2 0.00000000E+00
ROW= 3 0.00000000E+00
ROW= 4 0.38763591E+01
ROW= 5 0.12328367E-01
ROW= 6 0.55196060E+00
ROW= 7 -0.30312625E+00
ROW= 8 0.65089124E+01
ROW= 9 -0.59868546E+00
ROW= 10 -0.55709070E+00
ROW= 11 0.20299149E+01
ROW= 12 0.60099244E+00
ROW= 13 0.00000000E+00

ERROR PRINTOUT WHEN PAPER REQUIREMENTS CANNOT BE MET BY CURRENT PULP SELECTION

Table VI .

COLUMN= 2

ROW= 1 0.00000000E+00
ROW= 2 0.00000000E+00
ROW= 3 0.00000000E+00
ROW= 4 0.00000000E+00
ROW= 5 -0.74555779E+16
ROW= 6 -0.34283762E+15
ROW= 7 -0.74555779E+16
ROW= 8 -0.34283762E+15
ROW= 9 -0.34283762E+15
ROW= 10 -0.34283762E+15
ROW= 11 -0.74555779E+16
ROW= 12 -0.34283762E+15
ROW= 13 -0.74555779E+16

COLUMN= 3

ROW= 1 0.00000000E+00
ROW= 2 0.00000000E+00
ROW= 3 0.00000000E+00
ROW= 4 0.00000000E+00
ROW= 5 -0.13038325E+12

DISCUSSION OF RESULTS

Linear Blending

The literature survey revealed that there was quite a bit of interest in the prospects of linear blending, but little precise information was available concerning the specific blending characteristics of two pulps with respect to a certain property. Several generalizations found in the literature are noteworthy and should be mentioned here again. This first axiom of pulp blending should probably be called the golden rule of linear blending. The more similar two pulps are, with respect to one property, the more linear will be the transition of that property from one value to the other as one pulp is blended into the other. In addition to this, if two pulps are blended together that are widely differing with respect to a property, the greatest deviation from a linear blending characteristic will occur when the pulps are blended in 50% - 50% proportion. Finally, in the case of nonlinear blending characteristics, different properties are influenced more or less by the dominant component in the paper. For example, absorbency, permeability, brightness and brightness reversion are all influenced more heavily by the pulp with the smaller value in these properties. Tear strength, stiffness and opacity however, are affected much more by the component with the larger value.

There are four blending curves for each property in the study of

linear blending. Regretfully, the groundwood pulp was not available for this study as its arrival was delayed due to transportation difficulties. As mentioned previously, two of the blends are between the same pulp at different freenesses; in one case, hardwood and the other case softwood. Each blend has six data points and each point was determined by eight tests. Therefore each blending characteristic curve was generated by not less than forty eight individual paper tests. Two of the blending characteristic curves are studies of blending similar pulp types into one another with the expectation of a linear transition. The other two blending curves for each property are blend studies of relatively different pulps. In this case, characteristic blending curves are expected to be less linear due to greater divergence in sheet properties between hardwood and softwood.

Opacity is a model example of linear blending characteristics in all four blend studies. This supports Parson's statement concerning the scattering properties of a pulp and the linear relation to the scattering coefficients of its components. A graph showing the blend characteristic curves is in Figure 5.

Brightness does not illustrate a distinct linear or nonlinear trend. This is due to the fact that the brightness of the pulps was so close that instrument error along with the differences between one set of sheets and the next became more significant than the incremental changes in brightness as one pulp is blended into another.

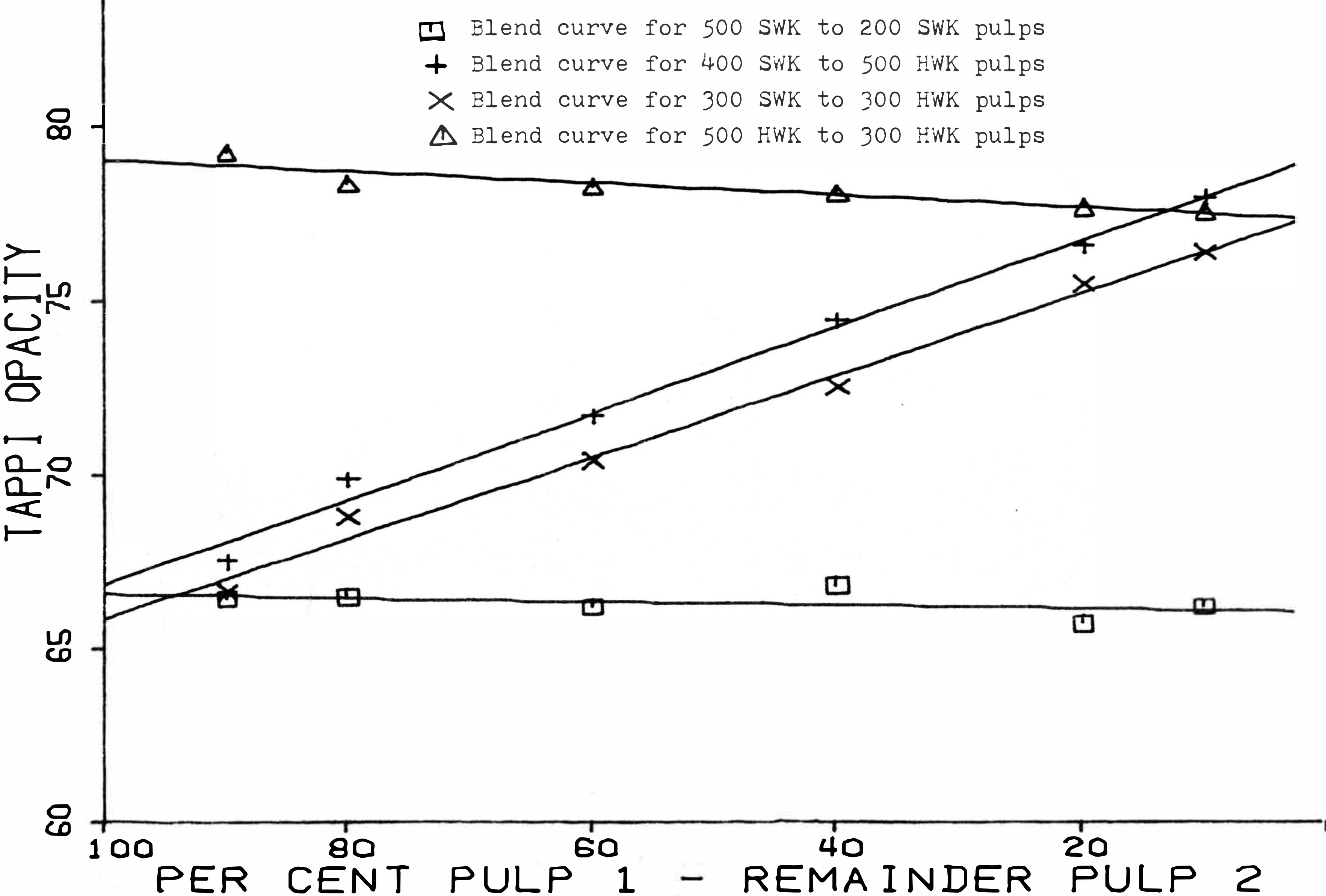


Figure 5 : Illustrates linear blending characteristics of opacity.

Smoothness is another good example of linear blending for all four pulp blends except that, even with forty measurements per point on the graph, the varience of measurement was still large. Distinct linear trends are present in all the blending curves. This could be due largely to the fact that the difference in smoothness between all the various pulps was not great therefore, the golden rule of linear blending applies.

Porosity is a direct contrast to smoothness. Measurements are consistant and the blending characteristic is distinctly curvilinear. The line drawn through the data points is again a least squares fit but this time it is a second degree polynomial. This can be seen very readily on the sample graphs in Figure 6. If this graph can be considered a reliable forecaster, particular problems can be expected when the linear program uses linear equations to predict the value of a multi-component furnish on the basis of porosity. The obvious conclusion from this data is that the porosity specification as well as the porosity coefficient for each of the pulps must be chosen with care or the porosity requirement could prove detrimental to accurate furnish design.

Tensile shows good linear characteristics in all four blend curves. This is highly desirable because in most manufacturing situations, the sheet strength requirements are critical in both paper manufacture and end use. Therefore, a good predicter is essential for strength specifications. Again, significant variance in the handsheet quality has produced some random nonlinearity among some of the points. These inconsistencies are however, well within the limits of reasonable error.

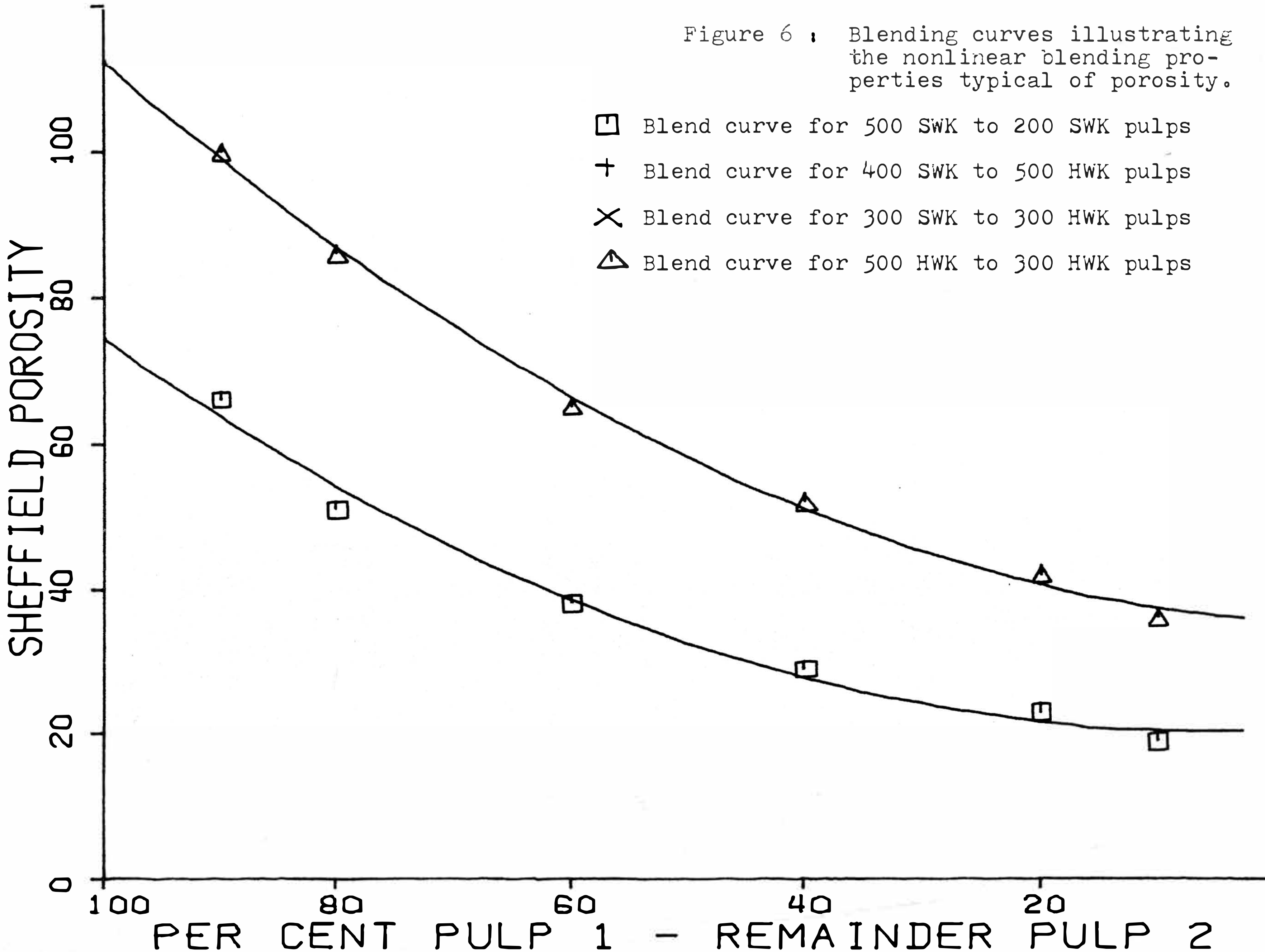


Figure 6 : Blending curves illustrating the nonlinear blending properties typical of porosity.

Burst also exhibits strong linear characteristics on all four blend studies. Here, too, handsheet variations contributed to random deviation from a perfectly linear blending transition. The deviation of points from the least squares line is more pronounced for the burst result than the tensile but the variance between individual paper tests is greater for burst than the variance in the tensile tests.

Fold blending characteristics are the most complex of all the tests. The hardwood - hardwood and softwood blends show nearly linear transition as one freeness pulp replaces another but the net change in fold for either test is not extremely large. When large fold strength is exhibited, extremely large test variance occurs. Since the softwood pulps have a much stronger fold than hardwood there is much more deviation from the least squares line for the softwood - softwood blend than the hardwood - hardwood blend. There is a distinct linear relationship for each of these blend curves. The softwood - hardwood blends show definite curvilinear trends. The change in fold strength is much greater as the hardwood is blended into the softwood. In the case of the 300 SWK - 300 HWK blend, it can clearly be seen that the greatest deviation from a linear characteristic curve occurs close to the 50% - 50% blend as is common for nonlinear blending characteristics. The most outstanding feature of the fold blending data is the diverse nature of all the least squares lines for the blending characteristics. This makes the fold test difficult to use conveniently in a linear programming application as prediction errors are inevitable when using linear approximations to define such erratic trends.

Tear strength is the last test used in this linear blending investigation. Consistent curvilinear trends occur in all four blend characteristic curves. As seen before, the greatest deviation from a linear blend occurs in the 50% - 50% blends. Since the curvilinear blending characteristic is consistant and not extremely curved, this test should prove to be a useful predicting parameter.

Density was a test that was to be used but the erratic data collected indicated that the particular sheet making process (Noble and Wood) did not produce sheets with consistent density. Therefore, the density test was not a useful parameter for specifying sheet properties in this study, but it should be reconsidered when a different sheet making process is employed.

Linear Programming

A summary of the results for the linear programming portion of this experiment is displayed in Table II. As expected, certain tests proved to behave in a more predictable manner than others. It is readily apparent that when pulps with widely divergent properties are mixed, distinct nonlinear trends take place with several properties. Examples of this can be seen in pulp blends one and two for the fold test. On the other hand, results with less error between the predicted and actual property values such as in opacity can be observed on tests that demonstrated more linear transitions upon mixing. This discussion will concern itself with the reliability of predicted values for

individual tests when using linear programming to design paper furnishes. Where large errors exist between actual and predicted test values, explanations will be made and possible solutions will be suggested.

The primary difficulty in a thorough analysis of this data lies in the absence of blending data between groundwood and one of the two bleached kraft pulps. This represents a significant void in the blending data prepared for this experiment. Since the greatest differences, both physically and optically, for the test pulps lies between the groundwood and the kraft pulps, a blend study of one of those mixes would have given good reference data. The reason for this obvious shortcoming is the delayed arrival of groundwood pulp during the experimental stage of this thesis.

Pulp Blend 1 and Pulp Blend 2 are similar in composition. Pulp Blend 1 has 36% groundwood, 51.4% 400 CSF hardwood kraft, 7.2% 300 CSF hardwood kraft and 5.5% 400 CSF softwood kraft. Pulp Blend 2 has 35.7% groundwood, 50.8% 400 CSF hardwood kraft and 13.5% 300 CSF hardwood kraft. The blending characteristics and the accuracy of predicted property values for both of these pulps should be quite similar. Therefore, a comparison of the results of Pulp Blend 1 and Pulp Blend 2 should indicate the reproducibility of these blending techniques.

The filled blend involves several important changes as compared to the first two pulp blends. These are the pulp mixture and the method of deriving coefficients used for calculating the filled handsheets' paper

characteristics. The composition of the filled blend is 48.7% 500 CSF softwood kraft, 40.5% 500 CSF hardwood kraft and 10.8% Klondyke water washed clay. Because of these differences, the results of the pulp blend with filler included will be treated separately from the first two pulp blends.

Opacity was expected to conform well to linear blending principles. Although the linear programming results indicate 4.6% and 6.6% error in the opacity value for the two pulp blends, the important consideration is that the error is approximately constant. This indicates that an adjustment of an opacity coefficient in the linear equations may result in a reduction of the error. An increase in the opacity value for groundwood from 96.8 to 106 or 110 may eliminate future error in opacity predictions. The filled blend showed a negligible error of 0.9% in the linear programming estimate of paper qualities. This is certainly encouraging but it should not be considered conclusive as more trials are required to make the results credible.

Brightness test results did not show nearly the accurate test results found in opacity. Errors of 15.9% and 20% were calculated between the predicted and actual brightness of the sheet. Obviously, the presence of groundwood in the blend lowers brightness more dramatically than original tests indicated. Again both errors are quite similar indicating that a change in coefficient values of the linear equations could correct it. Lowering the groundwood brightness coefficient from 54.6 to

approximately 30 may give more realistic results for future blends of these approximate proportions. The filled blend had a more accurate brightness prediction than the groundwood blends. The error was 6.7% .

The smoothness test showed a significant difference between the error of Pulp Blend 1 and Pulp Blend 2. They were 8.9% and 1.1% respectively. This can be attributed to the fact that smoothness measurement had a large variability for individual tests. Therefore, average values tend to be unreliable and very hard to reproduce. The smoothness parameter may not be a valid test to use with linear programming unless a sheet making procedure is devised to make sheets with more uniform smoothness. Although the smoothness error is only 2.1% for the filled blend, there is no more justification to keep this test as a blending parameter than with the two pulp blends.

Porosity has approximately equal errors for both pulp blends. Of 11.6% and 11.4%. The difficulty in analyzing this test lies in identifying the misleading coefficient. The nonlinearity of the blending curves (see Appendix 3) indicates that the linear assumption will give higher predicted values than actual test results. This is exactly what happened for both pulp blends. The solution to this difficulty is to lower the porosity value of one or more of the pulp components. The selection of values that should be lowered is the responsibility of the programmer. The error for the filled pulp blend is unsatisfactory at 13%. Again the compensation in the porosity values must be done by the programmer and is somewhat ambiguous.

Tensile strength appears to have a large percentage error but the actual error is 0.3 kilograms and 0.6 kilograms for Pulp Blend 1 and Pulp Blend 2 respectively. The weak groundwood did not diminish the strength of the pulp blends as much as linear blending would predict. These are small actual errors, however, and a minor adjustment to the groundwood tensile value could negate this problem. The same difficulty, in principle, exists in the filled blend only to a larger degree. The detrimental effect of filler on tensile strength has clearly been overestimated. Increasing clay's coefficient for tensile strength from -30 to -20 may produce a more realistic estimate for the linear equations.

The mullen test was considerably more successful in this first run of linear programming trials than the tensile test. The errors for the mullen test were -1.9% and 4.0% for Pulp Blend 1 and Pulp Blend 2 respectively. This is satisfactorily accurate. There is a significant accuracy problem with the filled blend however. As with the tensile test, the detrimental effect of clay on mullen strength was overestimated. An increase in the clay's mullen coefficient is needed. A change from -110 to -80 may be appropriate.

The fold test results clearly reveal that this property is linear programming's Achilles' heel. Due to the distinct curvilinear nature of fold's blending characteristics, linear approximations do not tend to be reasonably close to actual test results. Adjustments to fold coefficients would be never ending and could often be misleading. These

test results indicate that it may be inadvisable to use the fold test as a contributing parameter for the design of the furnish with linear programming.

The tear test had exceptionally good results with Pulp Blends 1 and 2 with errors of 4.8% and 0% respectively. This property exhibits curvilinear blending characteristics as can be seen in Appendix 3. Therefore, it is unusual that a linear approximation would generate such good results. More test cases should be run before final conclusions are made concerning the suitability of the tear requirements for linear blending. The filled pulp blend did have the unacceptable error of 34% in its estimate of tear strength. Again, the detrimental effect of filler on tear strength was overestimated and some adjustments could be made on the filler's tear coefficient. Changes to the tear coefficient should be made with the knowledge that all the nonlinearity problems that applied to the tear test for the pulp blends are also valid for the filled blend.

CONCLUSION

Linear programming has shown good promise for application in the design of paper furnishes to meet designated requirements while optimizing the cost of raw materials. Pulp blends are more straightforward than blending a filler component in addition to pulps. Except for a few tests, the properties of a filled sheet seem predictable but more coefficient tuning is necessary for filled sheets due to the indirect method used to arrive at coefficient values for the filler component. An important by-product was discovered when running the linear programming routine with arbitrary sets of requirements. When specifications have been set to such values that no combination of the pulps and filler available will meet those specifications, a processing error is detected by the linear programming method and the appropriate message is sent with the printout. This can be a very efficient means of surveying paper property limitations with existing raw materials available when designing new grades of paper.

Concerning the pulp blend results of the linear programming study, those properties that exhibited linear blending characteristic curves also showed good correlation with the predicted results from the linear programming solution. Those tests include opacity, tensile, mullen, and tear. Conversely, porosity and fold showed distinct curvilinear blending characteristics and gave poor results in the error analysis of the linear programming study.

The filled furnish had significantly more error in several of its predicted paper properties than the pulp blends. This is due directly to poor choice of coefficients in the linear equations. There was good evidence in the study that as the filler content in the paper increases, most paper properties change in a predictable manner. Furthermore, this change is linear in most cases.

LITERATURE CITED

1. Greenwald, D. U., "Linear Programming", 1st ed., New York, Ronald Press Company, 1957.

2. Froster, B.B., Tappi 52 (9):1658-60,(Sept. 1969).

3. Norris, W.E., Chem. Eng. (Dec. 1, 1969).

4. Teichroew, D., "An Introduction to Management Science", New York, John Wiley and Sons Inc., 1964.

5. Llewellyn, R.W., "Linear Programming", 1st ed., New York, Holt Rinehart Winston, 1964, pp. 1-130.

6. Nordeman, D.R., Paper Trade Journal, 44-45, (March 20, 1972).

7. Skalicky, C., Papir Celul. 22 (8) 209-214, (1967).

8. Brecht, Svensk Papperstid 66 (5) 159-67, (March 15, 1963).

9. Jamieson, A.G., Pulp and Paper Mag. Can. 68 (12), T612-18, (Dec. 1967).

10. Parsons, S.R., Paper Trade Journal 115 (25) 34-42, (Dec. 1942).

11. Page, D.H., Tappi 52 (4) 674-81, (April, 1969).

12. Almin, K. E., and Ruvo, A.D., Svensk Papperstid 70 (24) 864-50, (Dec. 31, 1967).

13. Lamer, K., Wochbl, Papierfabcik 94 (13) 460-62, (July 15, 1966).

APPENDIX I

CALCULATION OF POWER COSTS FOR PULPS OF VARIOUS FREENESSES

Pulp & Freeness	HPD/T		Power Cost	Total Cost
Groundwood	7.5		$1.34	$226.34
500 CSF SWK	12.5		2.24	327.24
400 CSF SWK	15.1		2.70	327.70
300 CSF SWK	17.7	$\times\ 0.179\ \frac{\$}{\text{HPD}}\ =$	3.17	328.17
200 CSF SWK	20.3		3.63	328.63
500 CSF HWK	12.0		2.15	312.15
400 CSF HWK	14.1		2.52	312.52
300 CSF HWK	16.6		2.89	312.89

Steam Cost - $1.00/M BTU

10,000 BTU = 1 KWH $\qquad \frac{10{,}000\ \text{BTU}}{1\ \text{KWH}}$

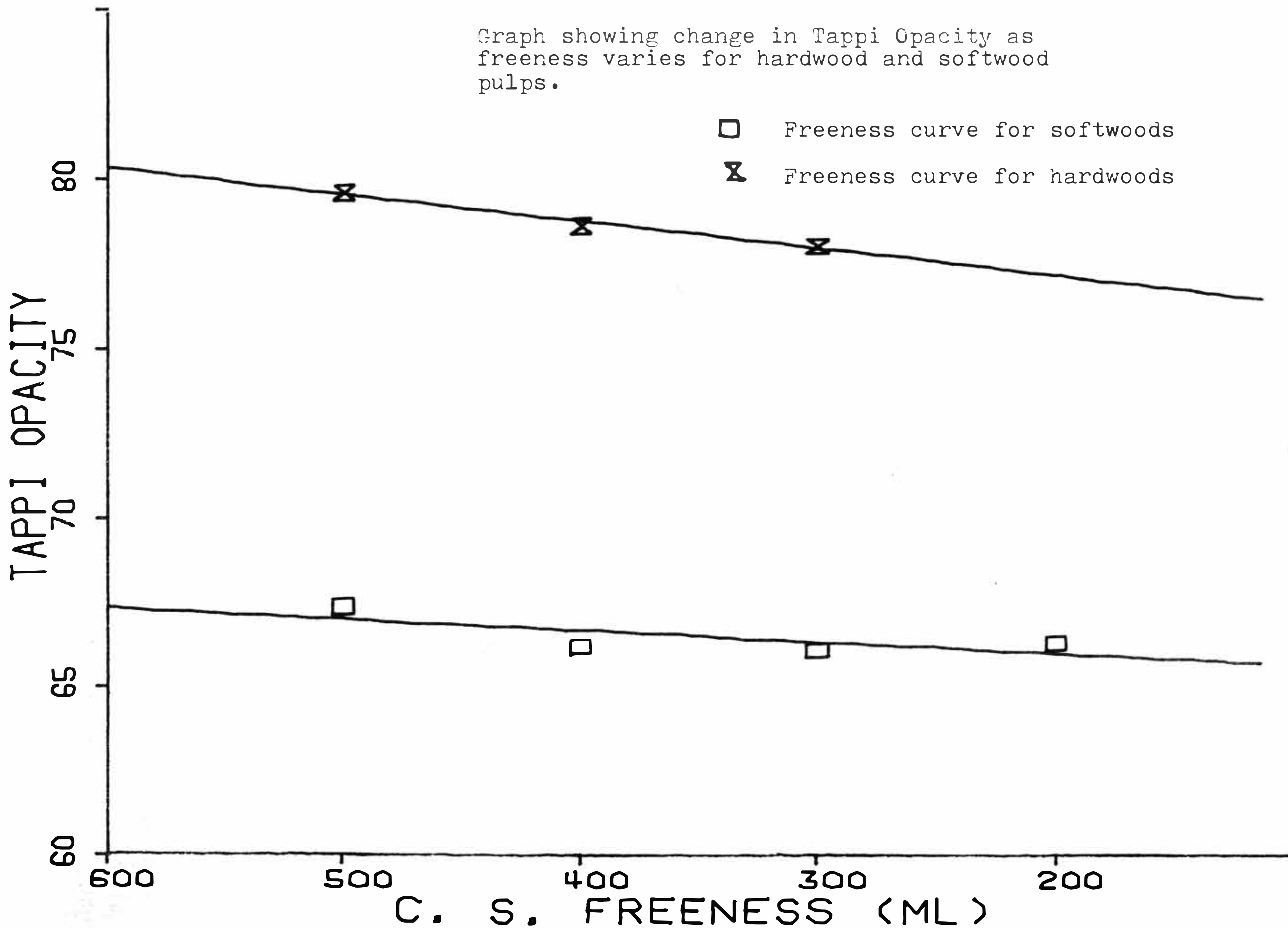
.7457 KWH = 1 Hp $\qquad \frac{.7457\ \text{KWH}}{1\ \text{Hp}}$

Steam Cost: $\frac{\$1.00}{\text{M BTU}} \times \frac{10{,}000\ \text{BTU}}{1\ \text{KWH}} = \frac{1¢}{1\ \text{KWH}}$

Incremental Power Cost: $\frac{1\ \text{HPD}}{\text{T}} \times \frac{.7457\ \text{KWH}}{1\ \text{HP}} \times \frac{24\ \text{H}}{\text{D}} \times \frac{\$0.01}{1\ \text{KWH}} = \frac{\$0.179}{\text{T}}$

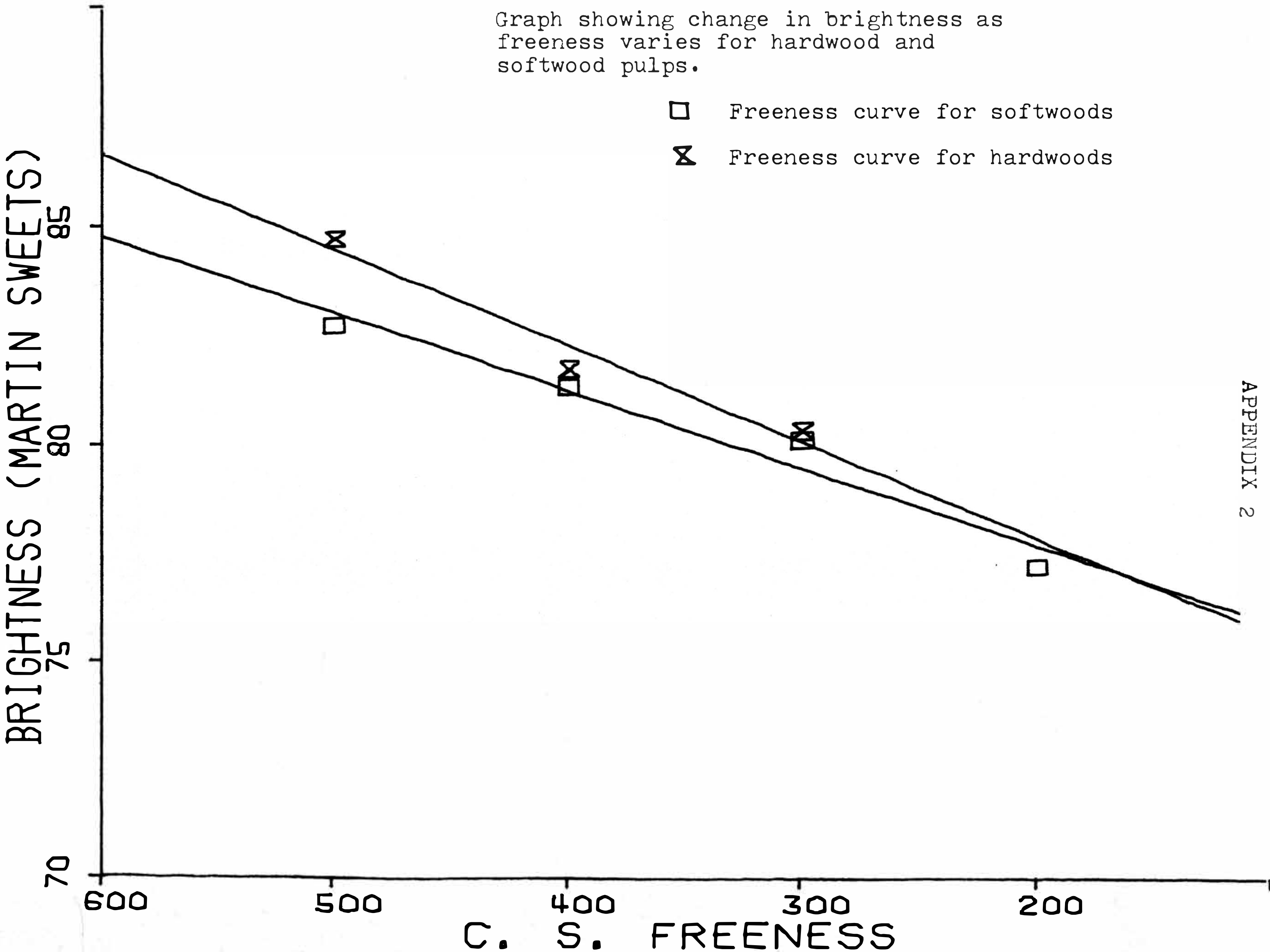
$$\frac{\text{HPD}}{\text{T}} \times 0.179\ \frac{\$}{\text{HPD}} = \frac{\$}{\text{T}}$$

APPENDIX 2

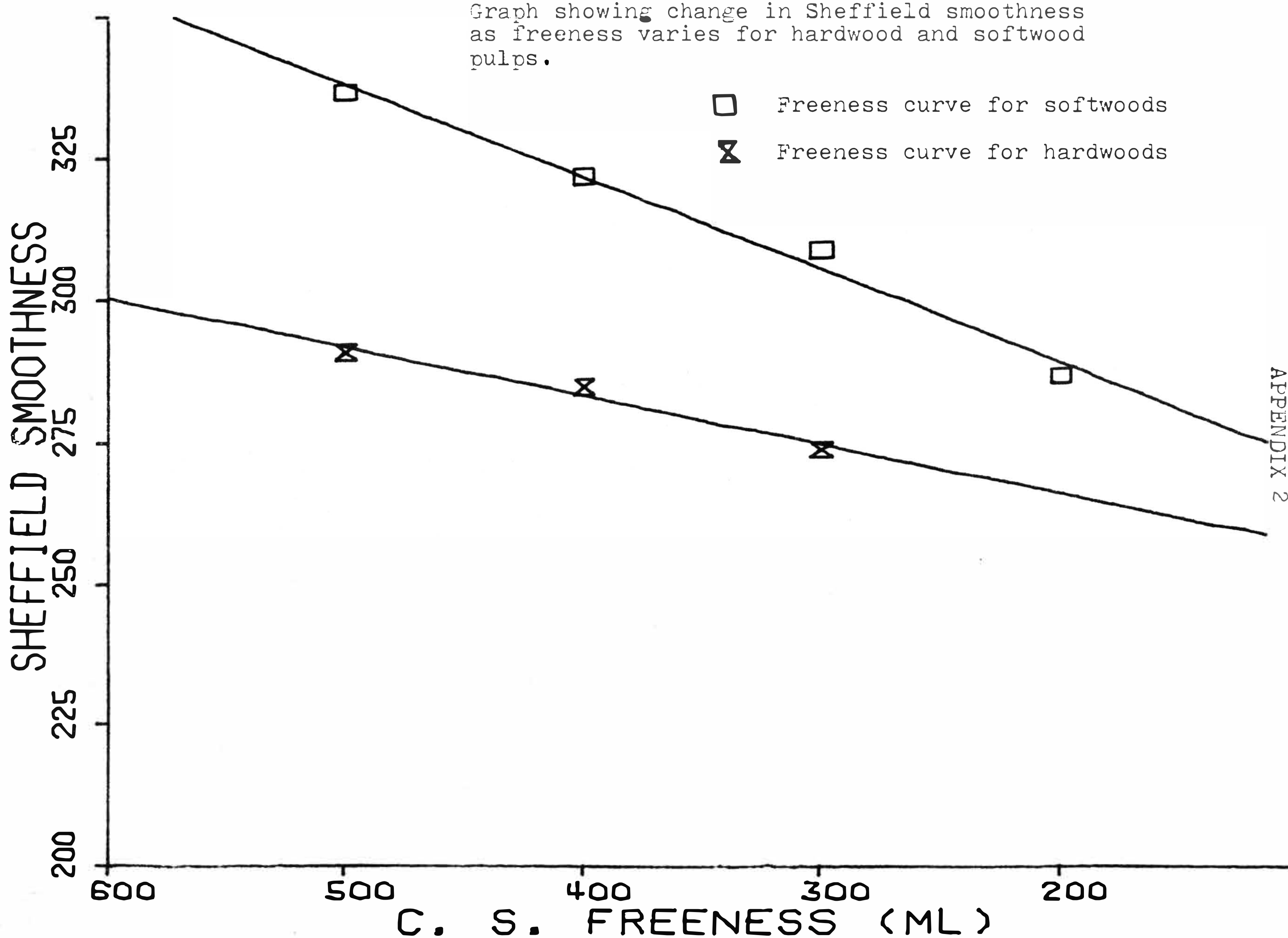


Graph showing change in Tappi Opacity as freeness varies for hardwood and softwood pulps.
Freeness curve for softwoods
Freeness curve for hardwoods
TAPPI OPACITY
80
75
70
65
60
600
500
400
300
200
C. S. FREENESS (ML)

APPENDIX 2

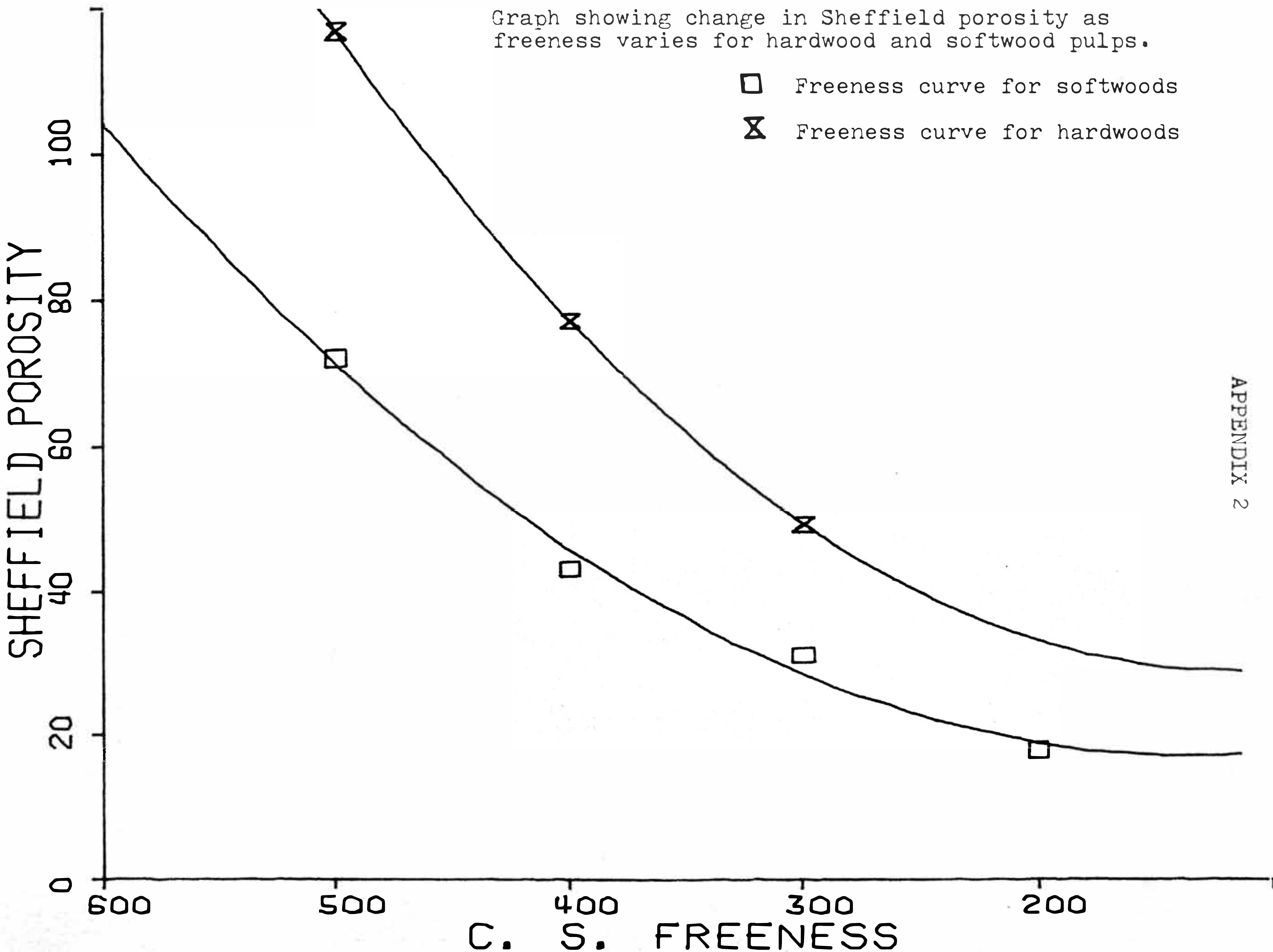

Graph showing change in brightness as freeness varies for hardwood and softwood pulps.
Freeness curve for softwoods
Freeness curve for hardwoods
BRIGHTNESS (MARTIN SWEETS)
85
80
75
70
600
500
400
300
200
C. S. FREENESS

APPENDIX 2

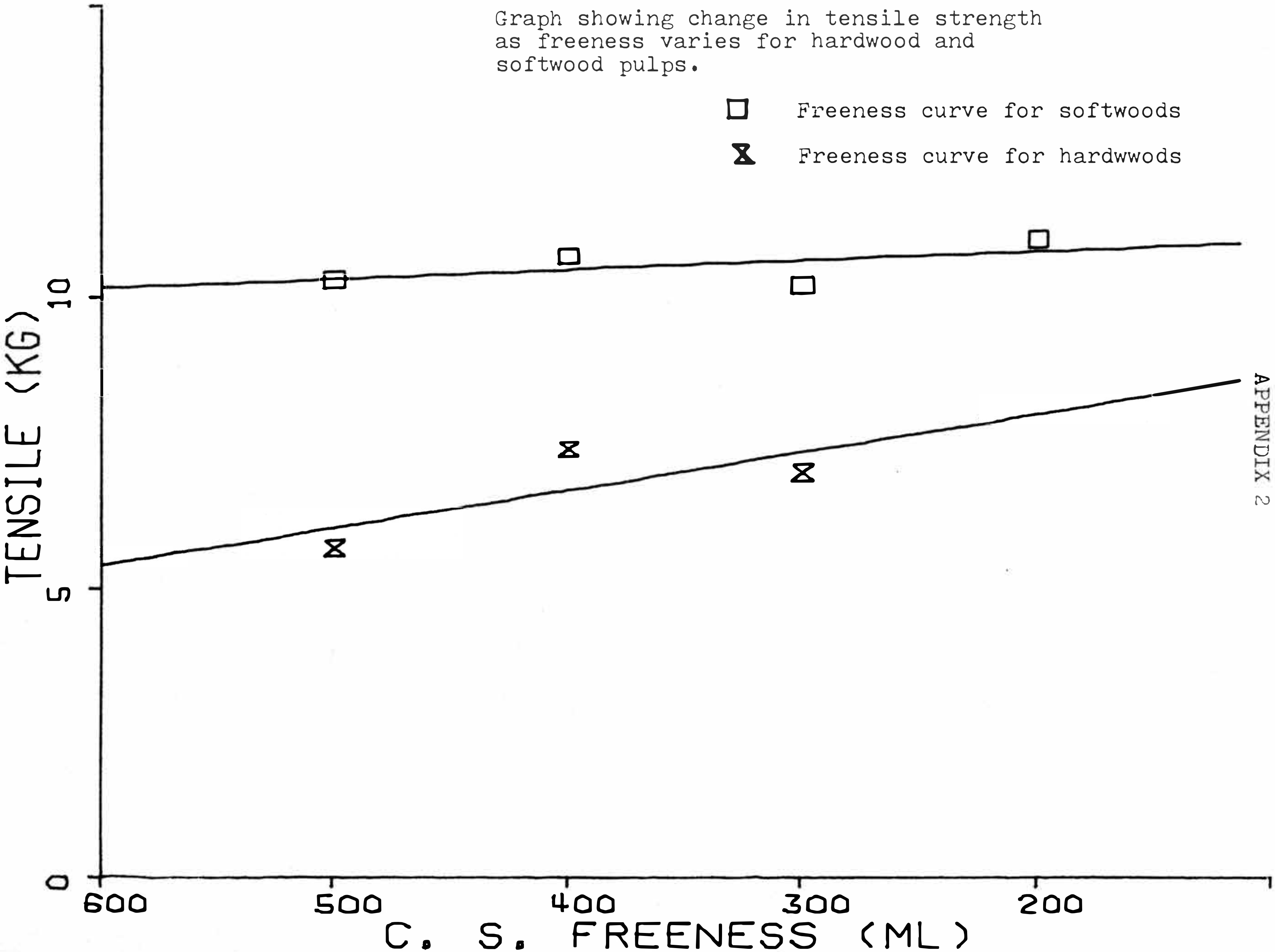


Graph showing change in Sheffield smoothness as freeness varies for hardwood and softwood pulps.

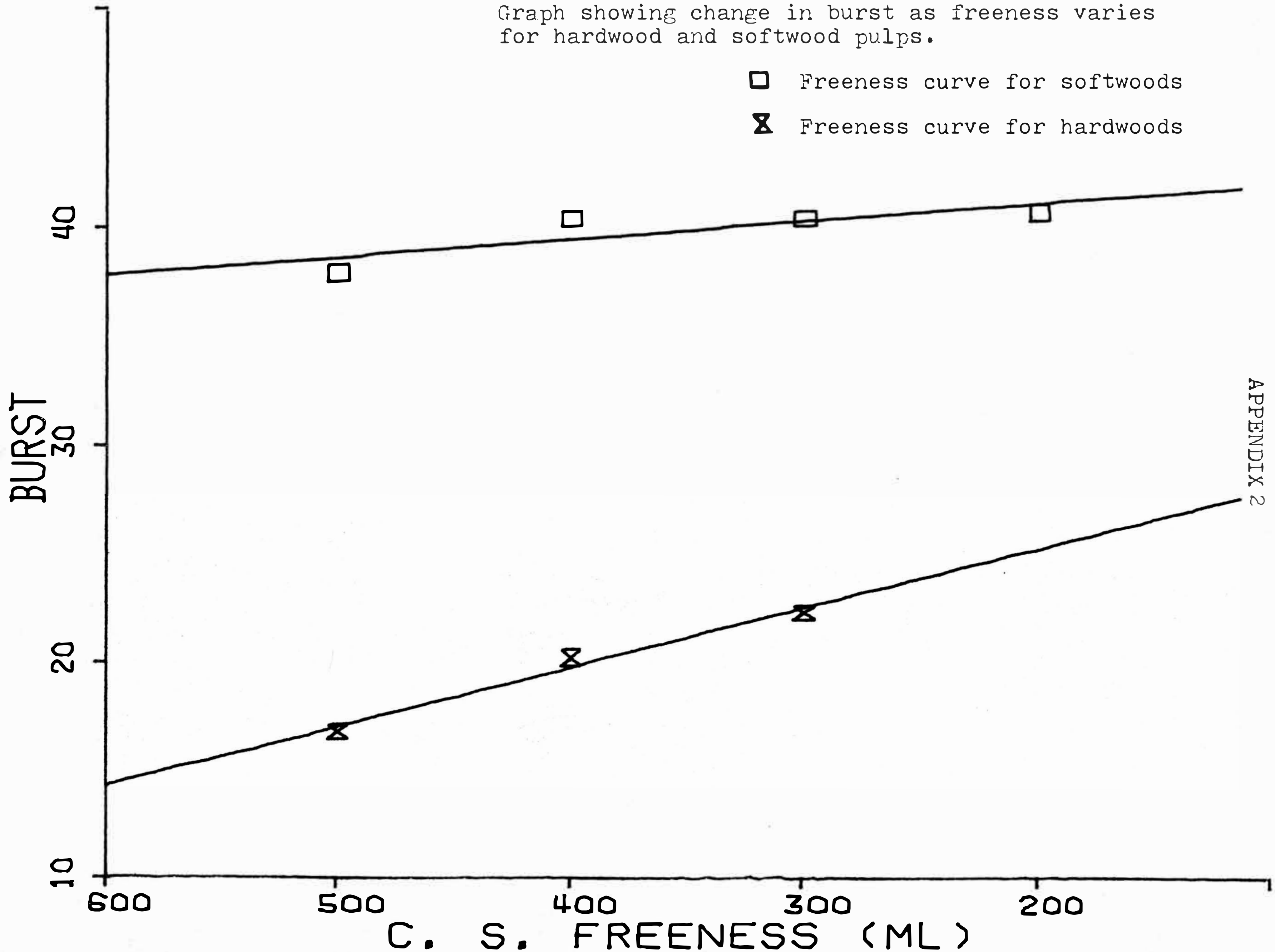
APPENDIX 2


Graph showing change in Sheffield porosity as freeness varies for hardwood and softwood pulps.
Freeness curve for softwoods
Freeness curve for hardwoods
SHEFFIELD POROSITY
0
20
40
60
80
100
600
500
400
300
200
C. S. FREENESS

APPENDIX 2

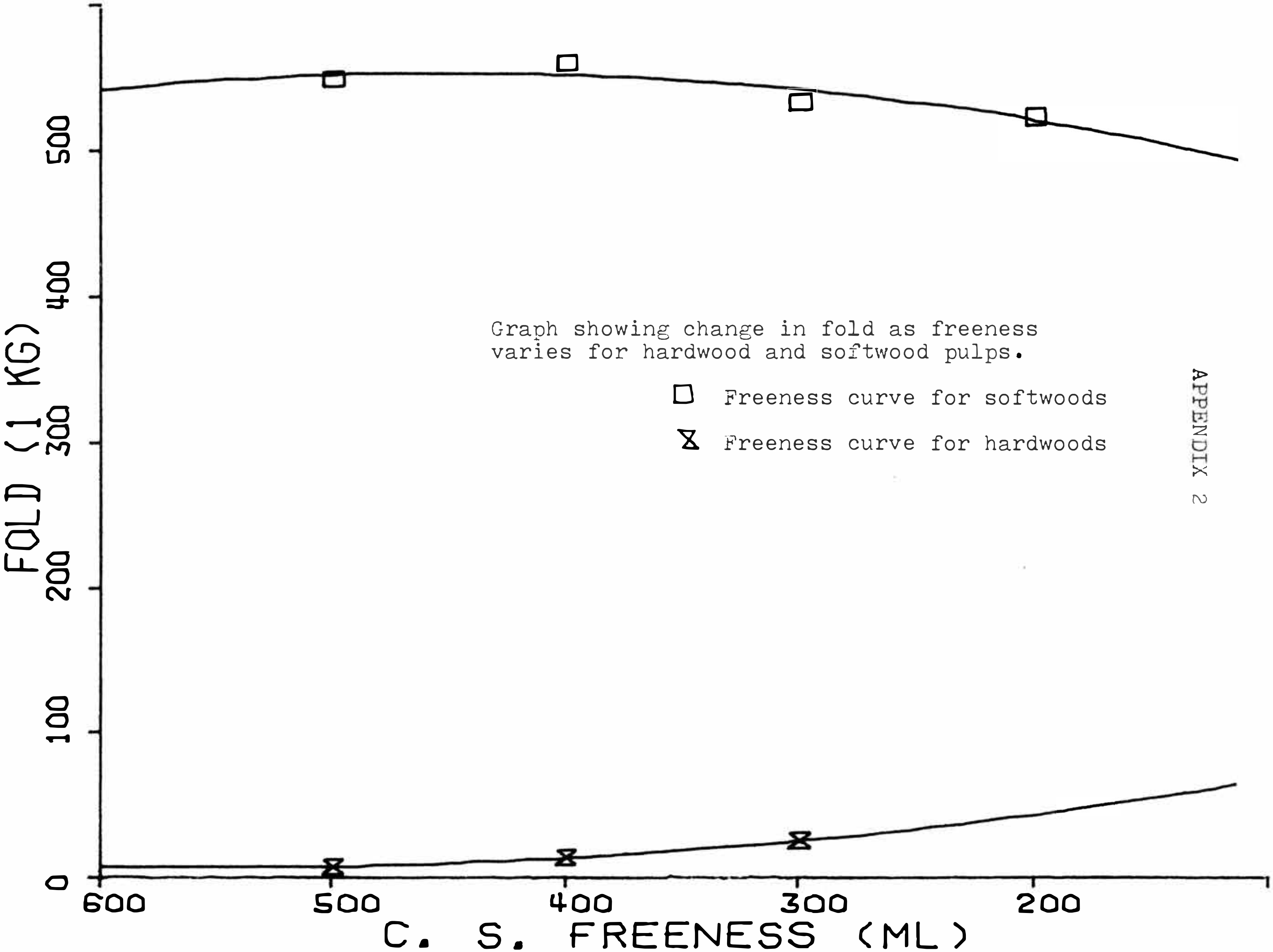


Graph showing change in tensile strength as freeness varies for hardwood and softwood pulps.

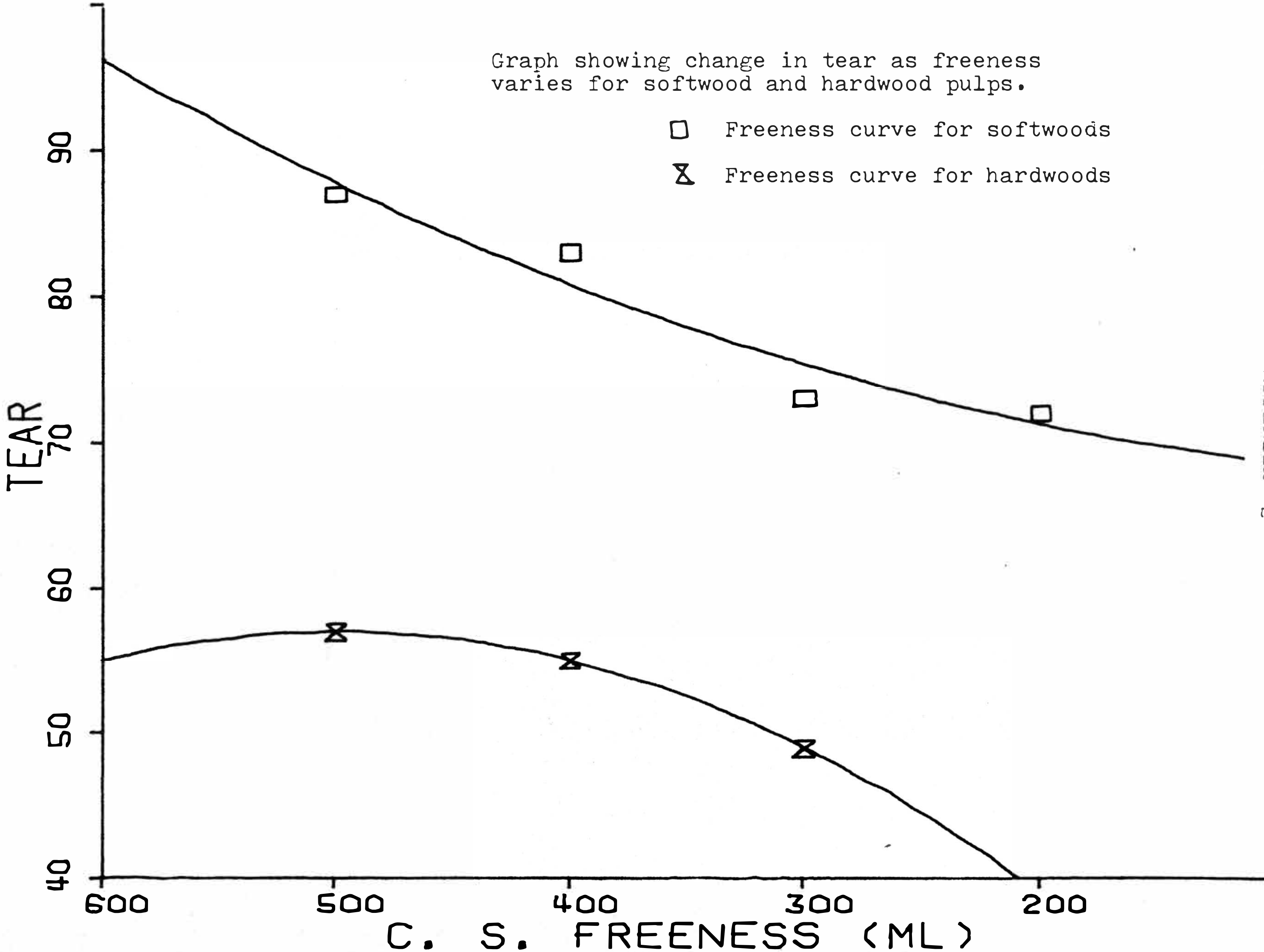


Graph showing change in burst as freeness varies for hardwood and softwood pulps.

APPENDIX 2

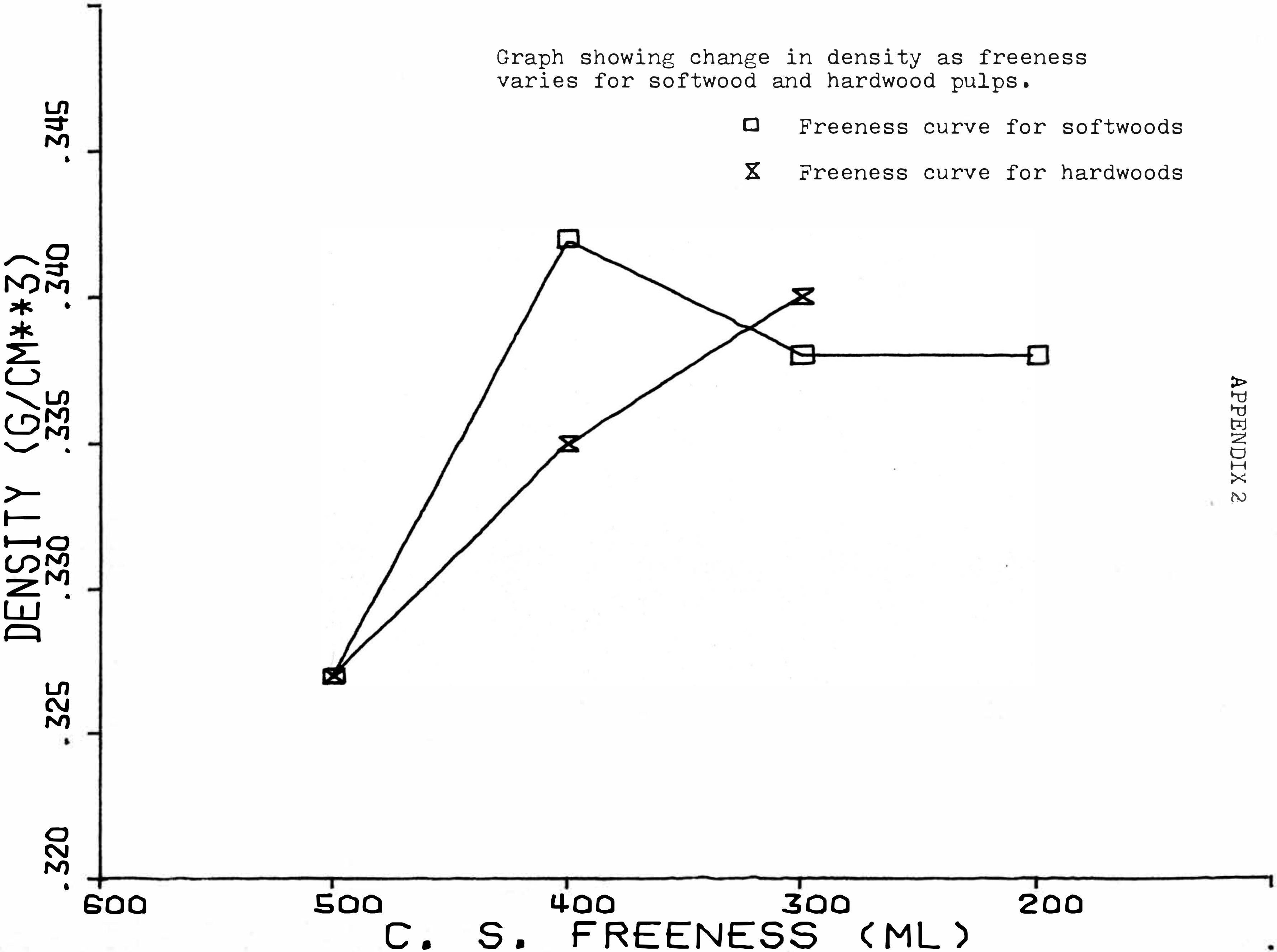

FOLD (1 KG)
0
100
200
300
400
500
600
500
400
300
200
C. S. FREENESS (ML)
Graph showing change in fold as freeness varies for hardwood and softwood pulps.
Freeness curve for softwoods
Freeness curve for hardwoods

APPENDIX 2



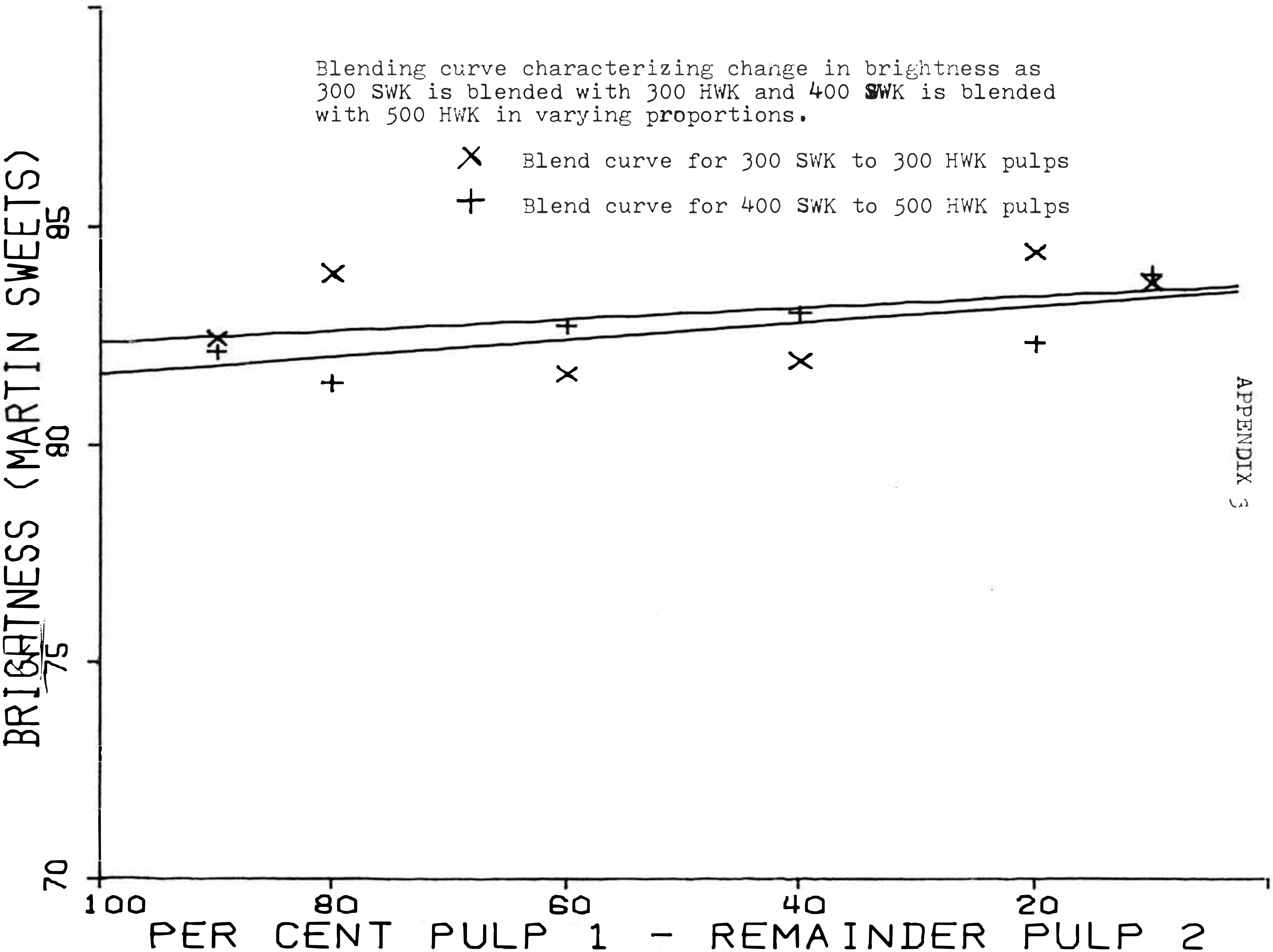
Graph showing change in tear as freeness varies for softwood and hardwood pulps.
Freeness curve for softwoods
Freeness curve for hardwoods
TEAR
90
80
70
60
50
40
600
500
400
300
200
C. S. FREENESS (ML)

APPENDIX 2

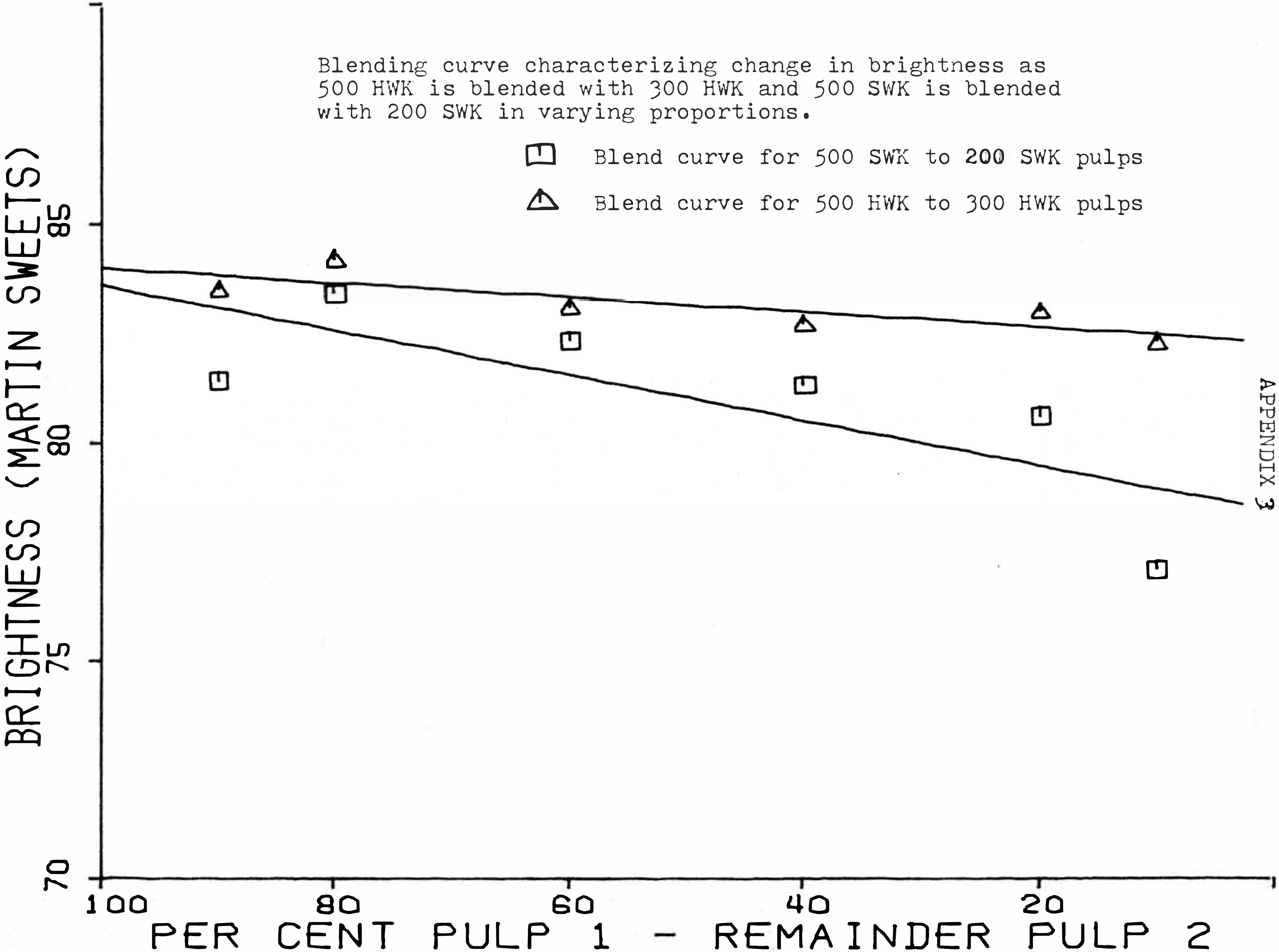


Graph showing change in density as freeness varies for softwood and hardwood pulps.
Freeness curve for softwoods
Freeness curve for hardwoods
DENSITY (G/CM**3)
.320
.325
.330
.335
.340
.345
600
500
400
300
200
C. S. FREENESS (ML)

APPENDIX 3

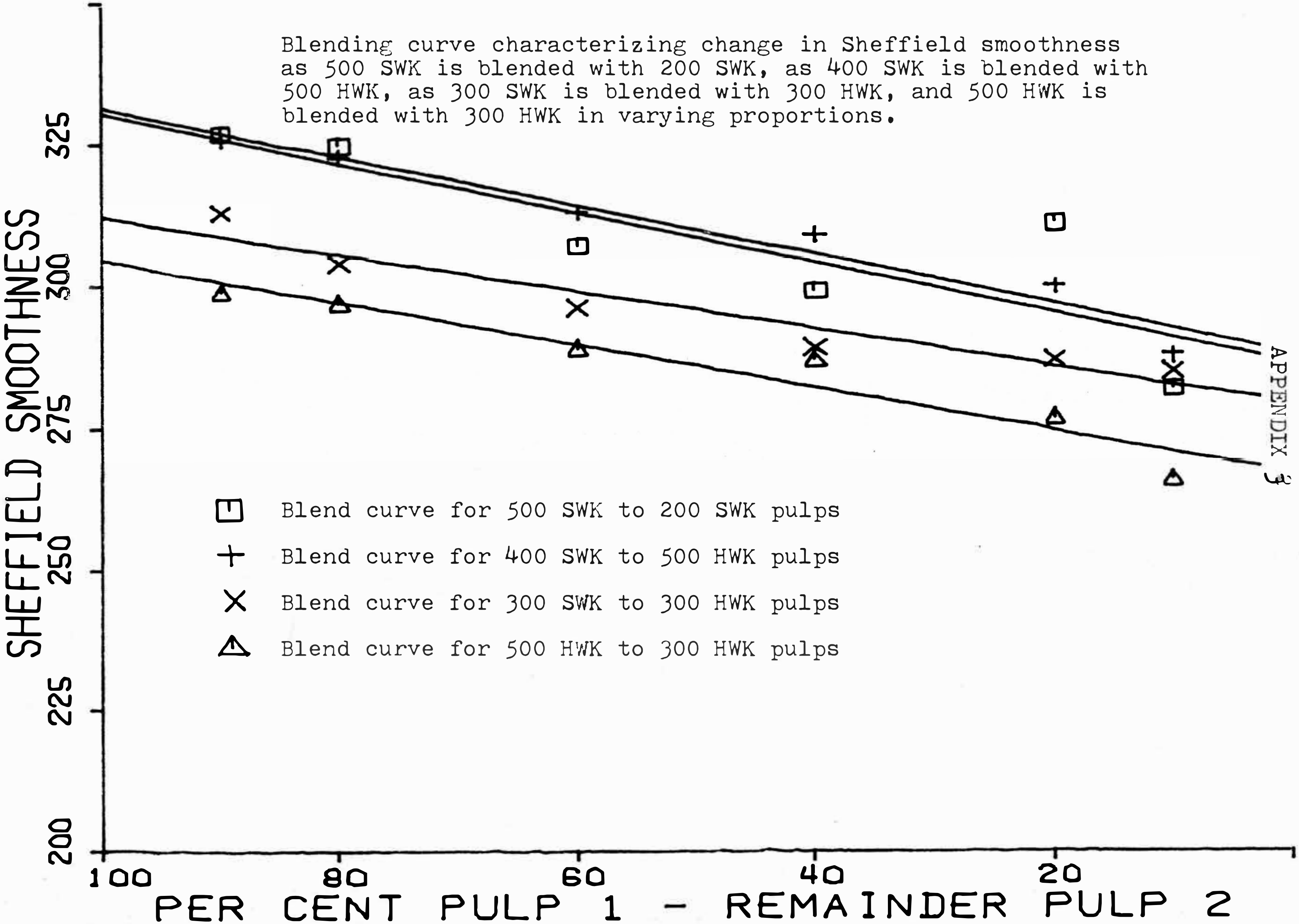


Blending curve characterizing change in brightness as 300 SWK is blended with 300 HWK and 400 SWK is blended with 500 HWK in varying proportions.
X Blend curve for 300 SWK to 300 HWK pulps
+ Blend curve for 400 SWK to 500 HWK pulps
BRIGHTNESS (MARTIN SWEETS)
85
80
75
70
100
80
60
40
20
PER CENT PULP 1 - REMAINDER PULP 2

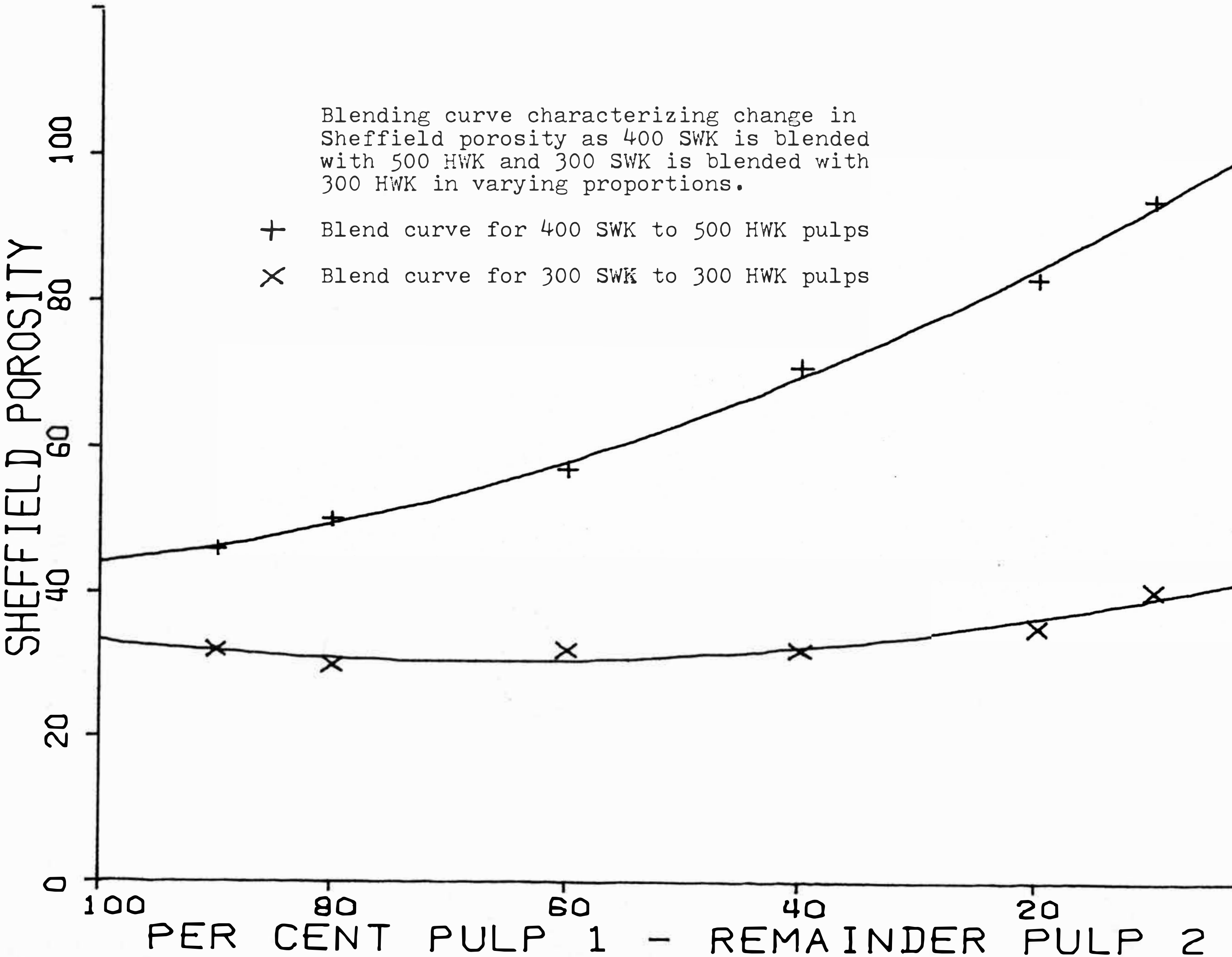

Blending curve characterizing change in brightness as 500 HWK is blended with 300 HWK and 500 SWK is blended with 200 SWK in varying proportions.
Blend curve for 500 SWK to 200 SWK pulps
Blend curve for 500 HWK to 300 HWK pulps
BRIGHTNESS (MARTIN SWEETS)
85
80
75
70
100
80
60
40
20
PER CENT PULP 1 - REMAINDER PULP 2

APPENDIX 3

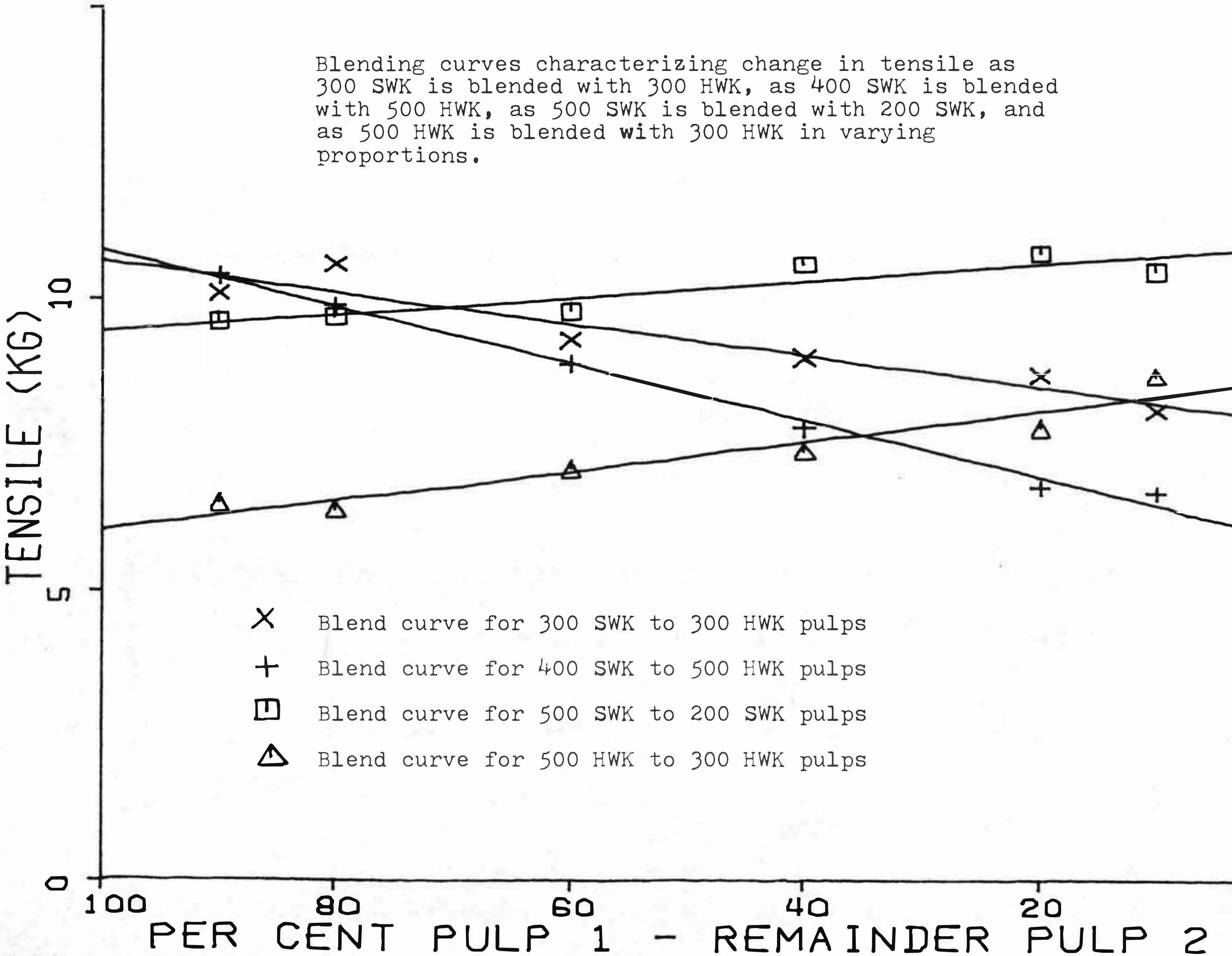
APPENDIX 3

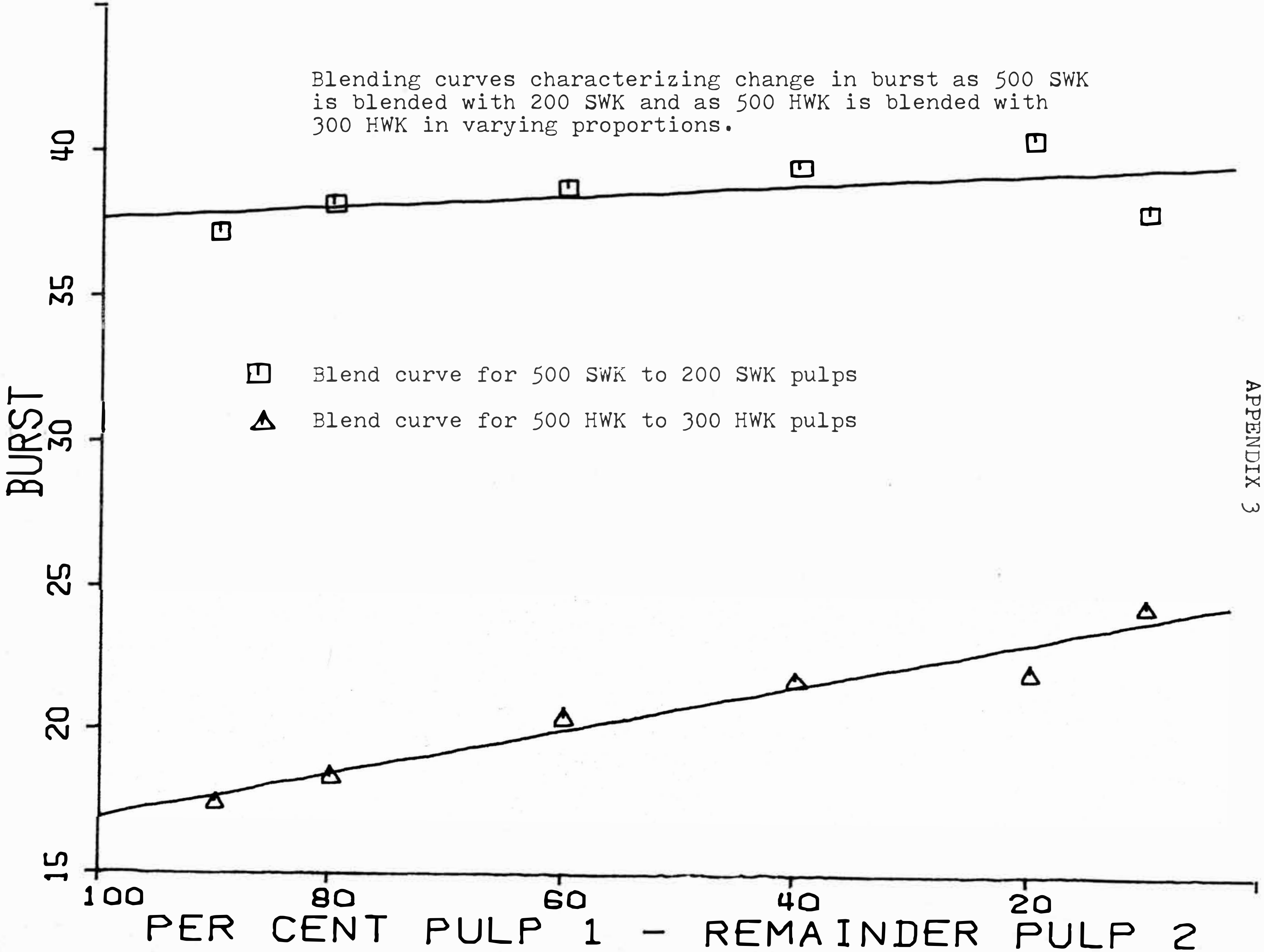


Blending curve characterizing change in Sheffield smoothness as 500 SWK is blended with 200 SWK, as 400 SWK is blended with 500 HWK, as 300 SWK is blended with 300 HWK, and 500 HWK is blended with 300 HWK in varying proportions.
SHEFFIELD SMOOTHNESS
325
300
275
250
225
200
100
80
60
40
20
PER CENT PULP 1 - REMAINDER PULP 2
Blend curve for 500 SWK to 200 SWK pulps
Blend curve for 400 SWK to 500 HWK pulps
Blend curve for 300 SWK to 300 HWK pulps
Blend curve for 500 HWK to 300 HWK pulps


Blending curve characterizing change in Sheffield porosity as 400 SWK is blended with 500 HWK and 300 SWK is blended with 300 HWK in varying proportions.
+ Blend curve for 400 SWK to 500 HWK pulps
× Blend curve for 300 SWK to 300 HWK pulps
SHEFFIELD POROSITY
0
20
40
60
80
100
100
80
60
40
20
PER CENT PULP 1 - REMAINDER PULP 2

APPENDIX 3

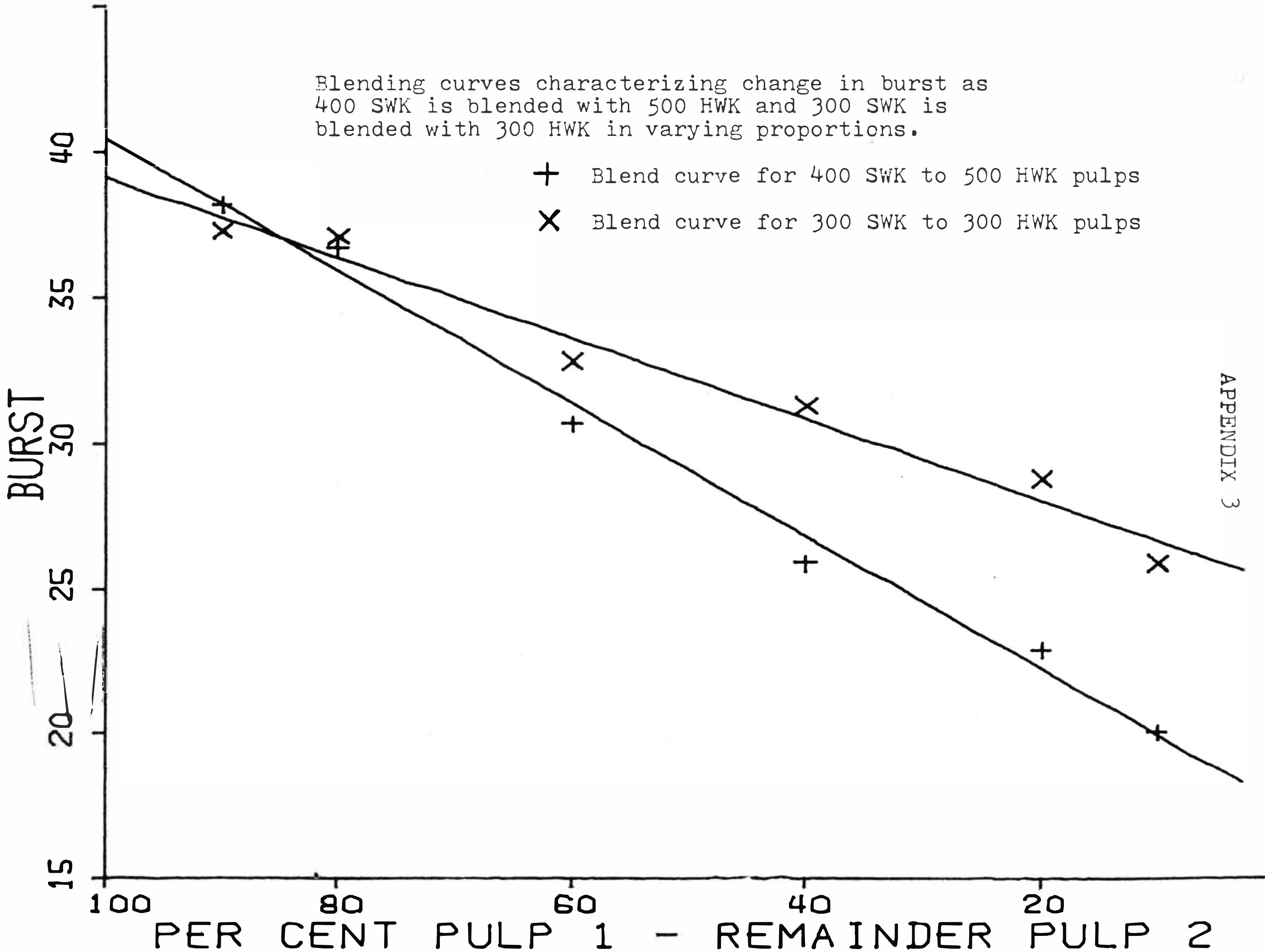

Blending curves characterizing change in tensile as 300 SWK is blended with 300 HWK, as 400 SWK is blended with 500 HWK, as 500 SWK is blended with 200 SWK, and as 500 HWK is blended with 300 HWK in varying proportions.
TENSILE (KG)
0
5
10
100
80
60
40
20
PER CENT PULP 1 - REMAINDER PULP 2
Blend curve for 300 SWK to 300 HWK pulps
Blend curve for 400 SWK to 500 HWK pulps
Blend curve for 500 SWK to 200 SWK pulps
Blend curve for 500 HWK to 300 HWK pulps



Blending curves characterizing change in burst as 500 SWK is blended with 200 SWK and as 500 HWK is blended with 300 HWK in varying proportions.
Blend curve for 500 SWK to 200 SWK pulps
Blend curve for 500 HWK to 300 HWK pulps
BURST
15
20
25
30
35
40
100
80
60
40
20
PER CENT PULP 1 - REMAINDER PULP 2

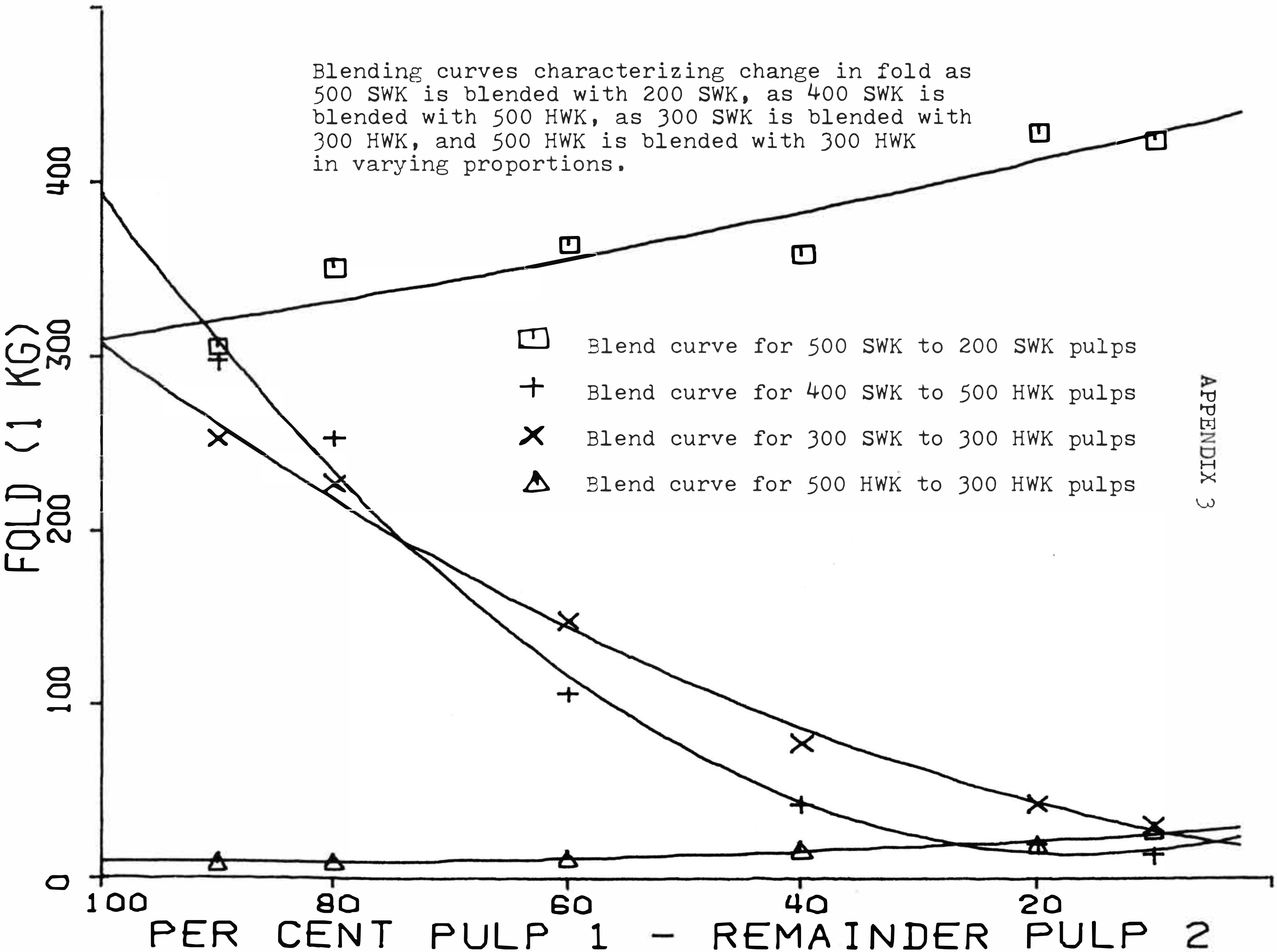
APPENDIX 3

APPENDIX 3

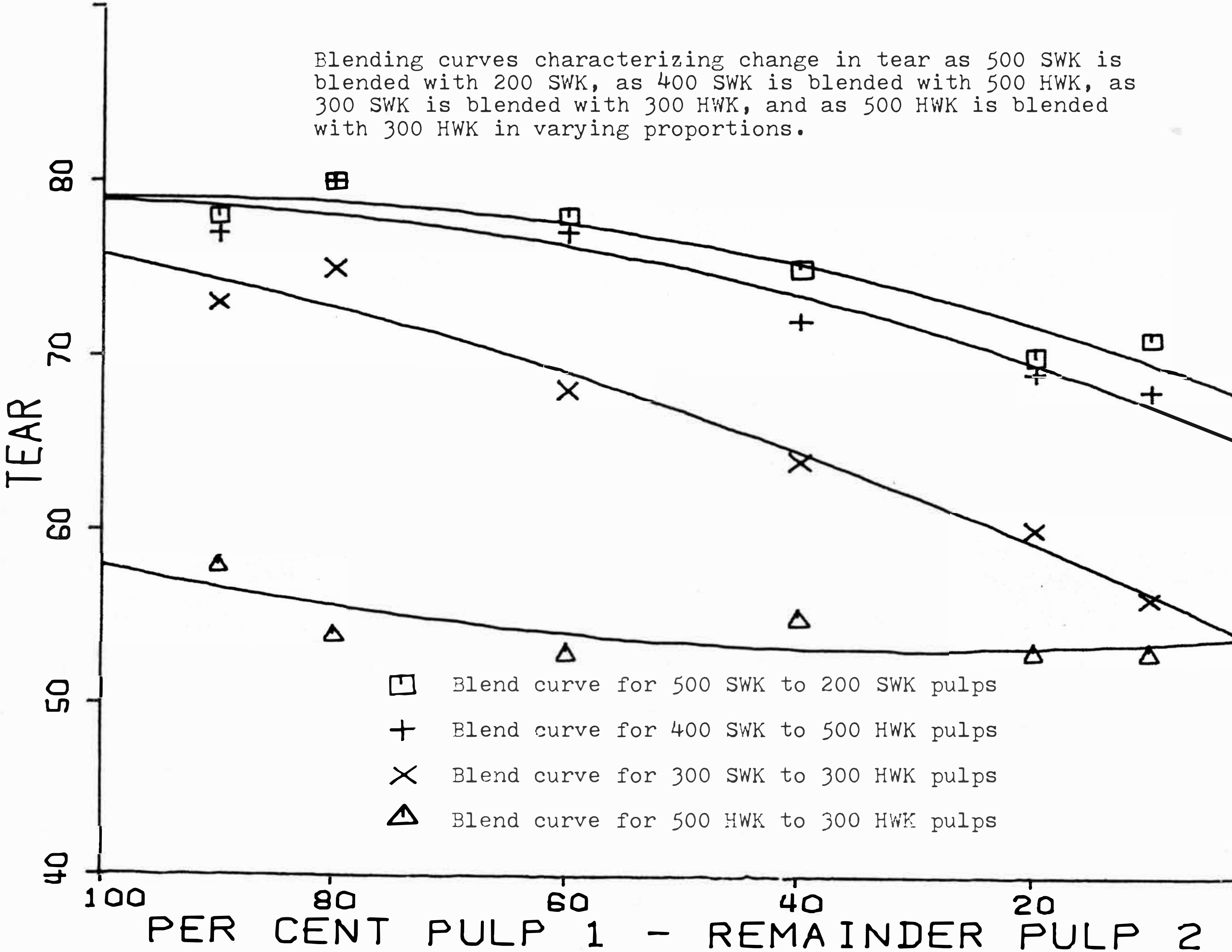


Blending curves characterizing change in burst as
400 SWK is blended with 500 HWK and 300 SWK is
blended with 300 HWK in varying proportions.
+ Blend curve for 400 SWK to 500 HWK pulps
X Blend curve for 300 SWK to 300 HWK pulps
BURST
40
35
30
25
20
15
100
80
60
40
20
PER CENT PULP 1 - REMAINDER PULP 2

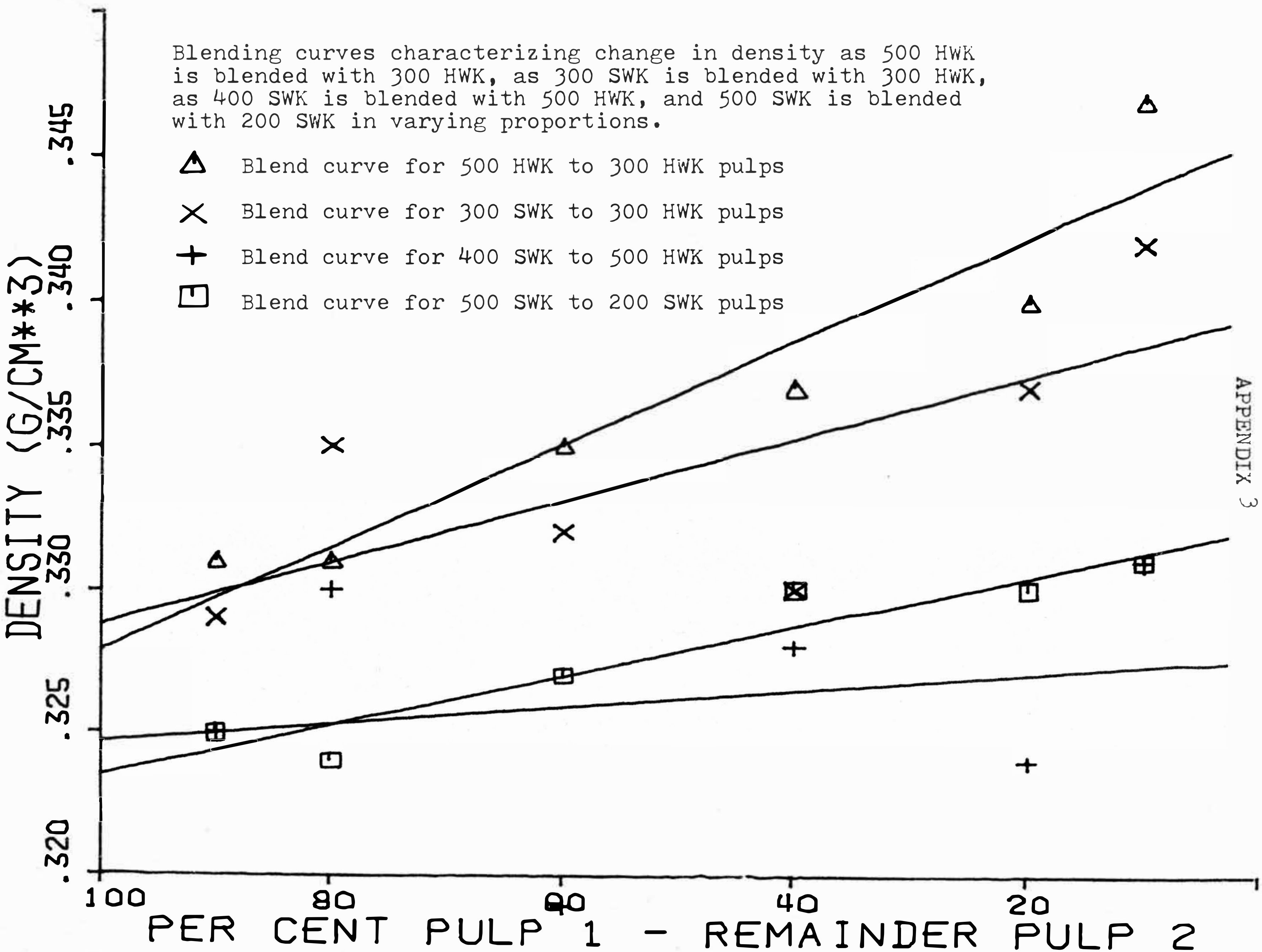
APPENDIX 3


Blending curves characterizing change in fold as 500 SWK is blended with 200 SWK, as 400 SWK is blended with 500 HWK, as 300 SWK is blended with 300 HWK, and 500 HWK is blended with 300 HWK in varying proportions.
Blend curve for 500 SWK to 200 SWK pulps
Blend curve for 400 SWK to 500 HWK pulps
Blend curve for 300 SWK to 300 HWK pulps
Blend curve for 500 HWK to 300 HWK pulps
FOLD (1 KG)
0
100
200
300
400
100
80
60
40
20
PER CENT PULP 1 - REMAINDER PULP 2

APPENDIX 3

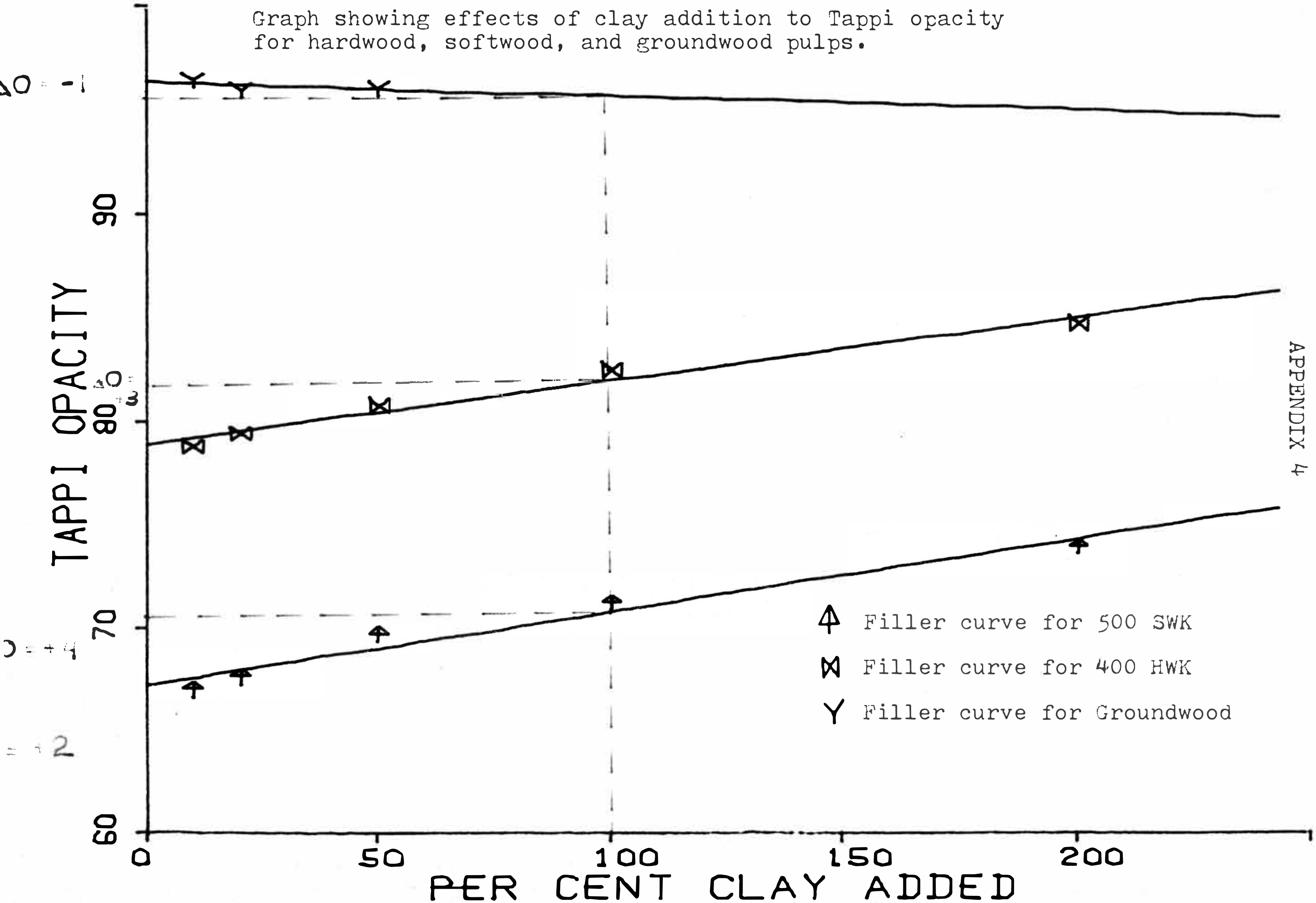

Blending curves characterizing change in tear as 500 SWK is blended with 200 SWK, as 400 SWK is blended with 500 HWK, as 300 SWK is blended with 300 HWK, and as 500 HWK is blended with 300 HWK in varying proportions.
TEAR
80
70
60
50
40
Blend curve for 500 SWK to 200 SWK pulps
Blend curve for 400 SWK to 500 HWK pulps
Blend curve for 300 SWK to 300 HWK pulps
Blend curve for 500 HWK to 300 HWK pulps
100
80
60
40
20
PER CENT PULP 1 - REMAINDER PULP 2

APPENDIX 3

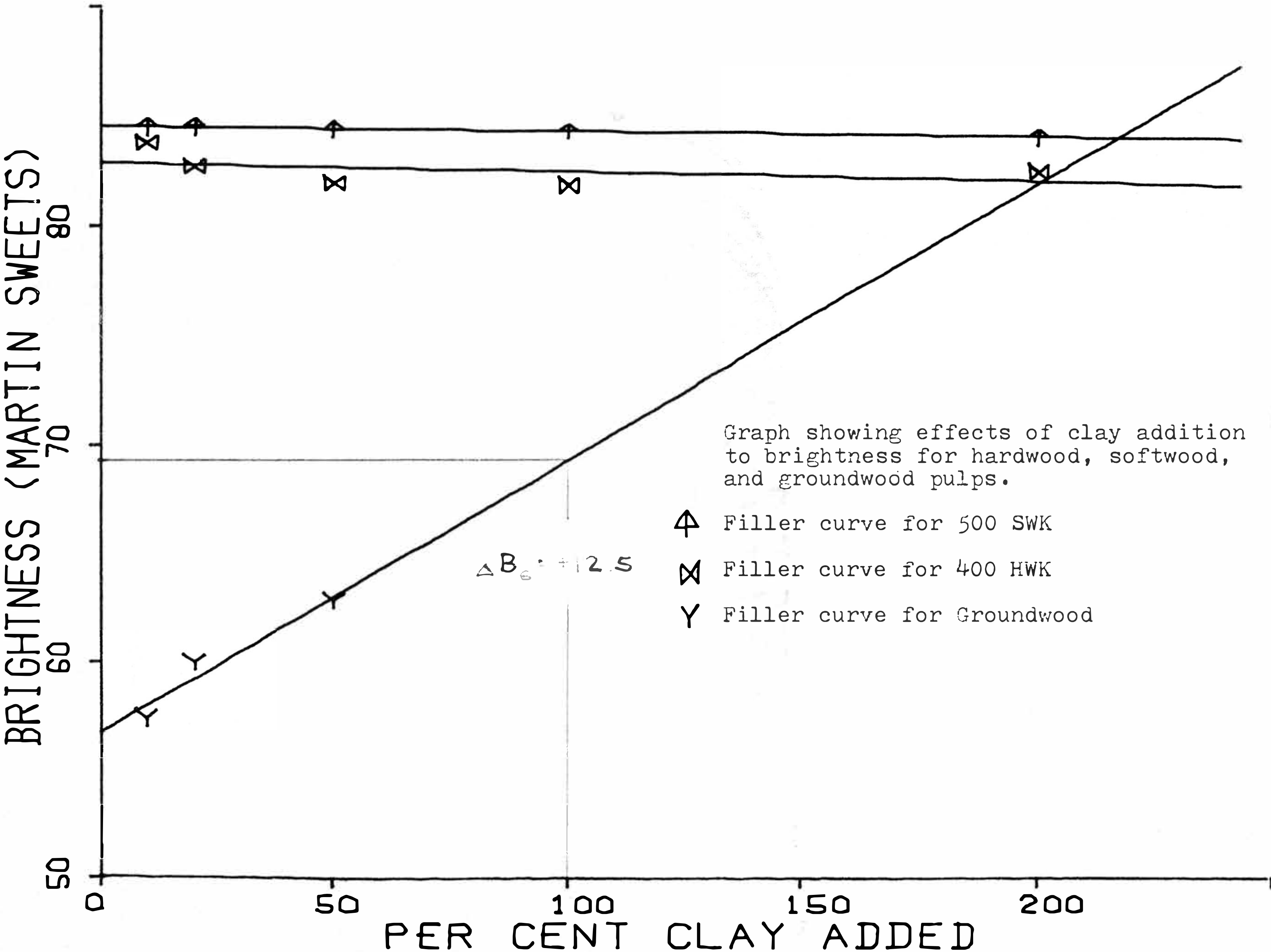

Blending curves characterizing change in density as 500 HWK is blended with 300 HWK, as 300 SWK is blended with 300 HWK, as 400 SWK is blended with 500 HWK, and 500 SWK is blended with 200 SWK in varying proportions.
Blend curve for 500 HWK to 300 HWK pulps
Blend curve for 300 SWK to 300 HWK pulps
Blend curve for 400 SWK to 500 HWK pulps
Blend curve for 500 SWK to 200 SWK pulps
DENSITY (G/CM**3)
.320
.325
.330
.335
.340
.345
100
80
60
40
20
PER CENT PULP 1 - REMAINDER PULP 2

APPENDIX 4

Graph showing effects of clay addition to Tappi opacity
for hardwood, softwood, and groundwood pulps.

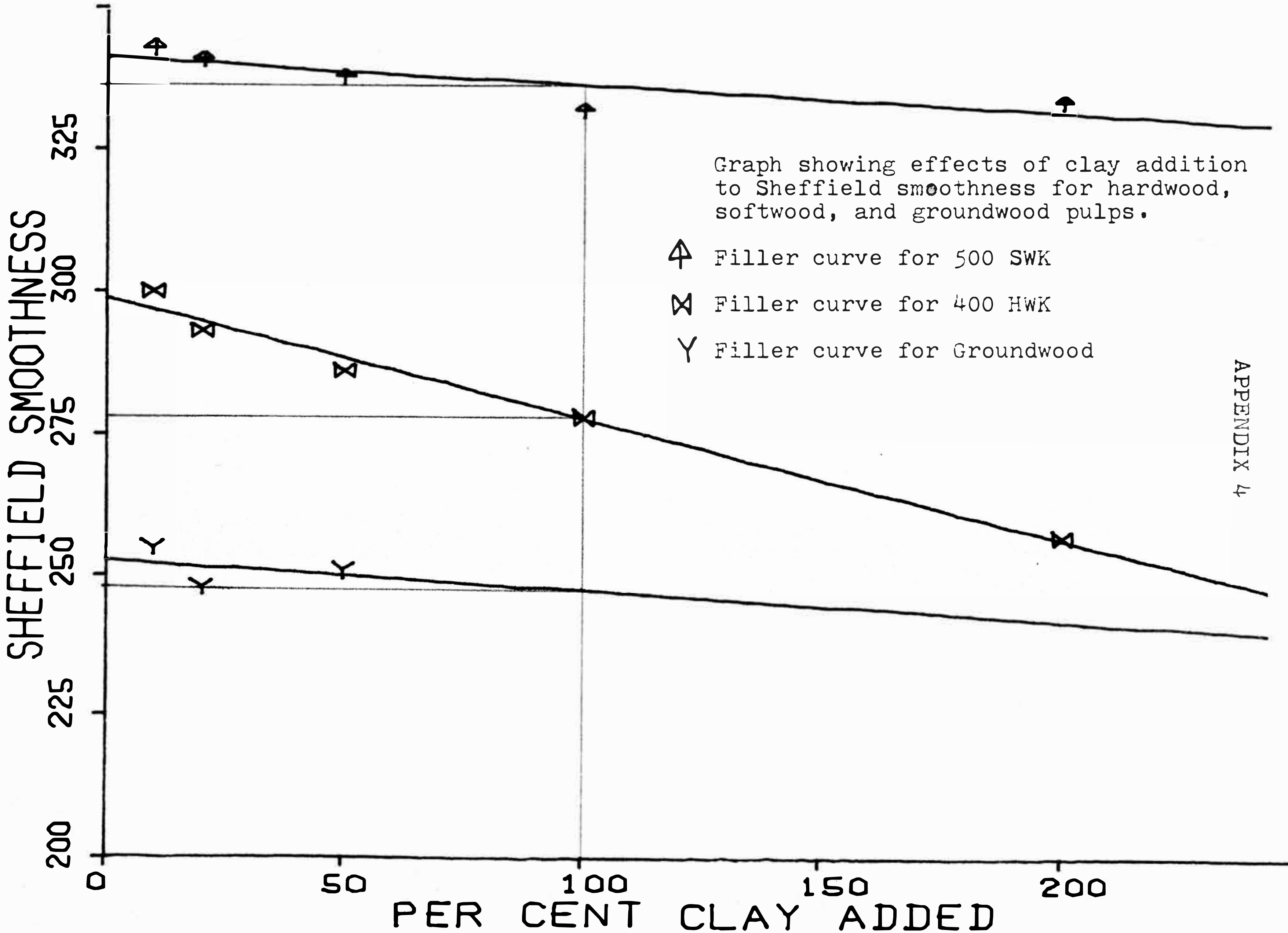


APPENDIX 4



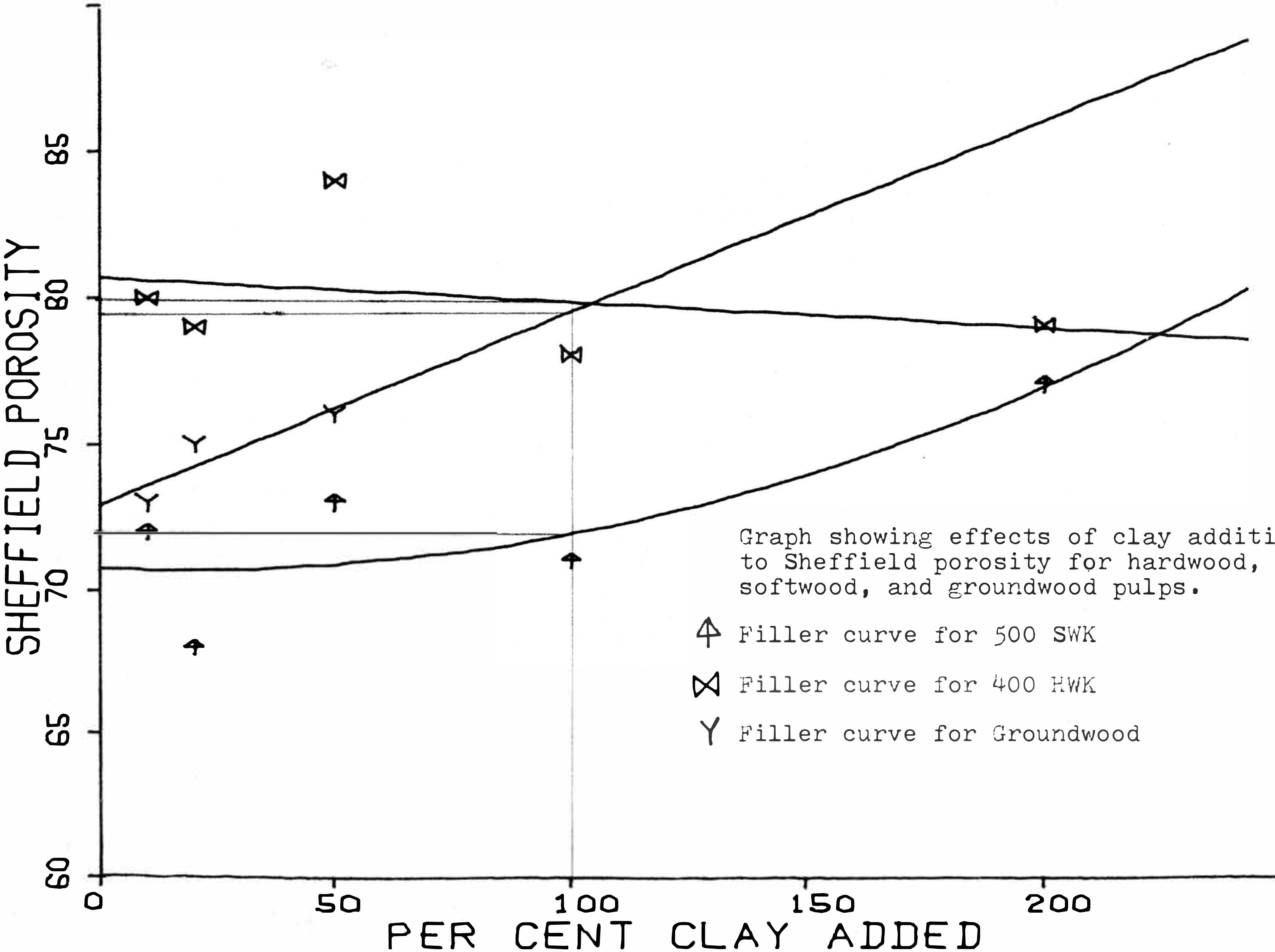
Graph showing effects of clay addition to brightness for hardwood, softwood, and groundwood pulps.

APPENDIX 4



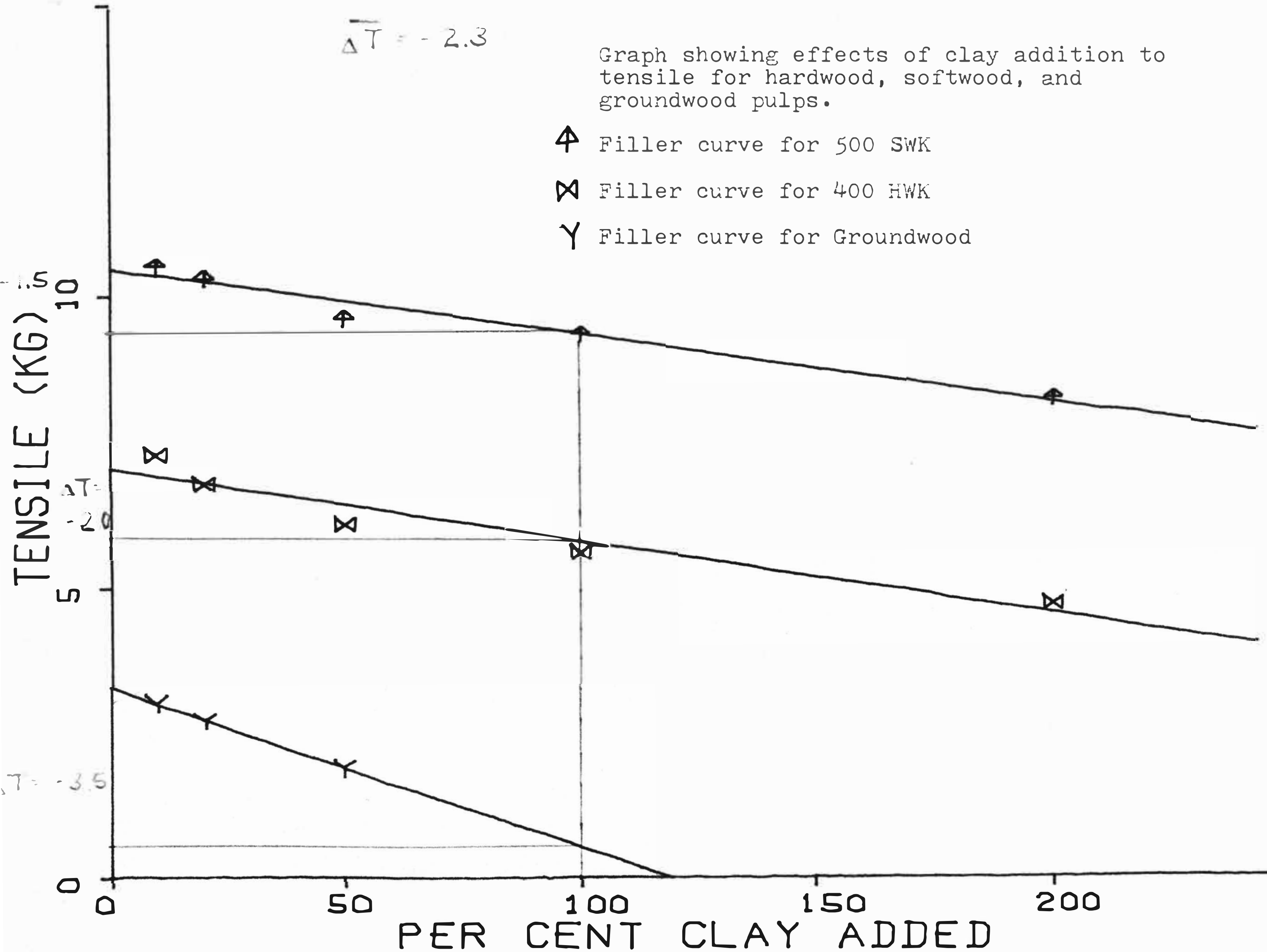
Graph showing effects of clay addition to Sheffield smoothness for hardwood, softwood, and groundwood pulps.
Filler curve for 500 SWK
Filler curve for 400 HWK
Filler curve for Groundwood
SHEFFIELD SMOOTHNESS
200
225
250
275
300
325
0
50
100
150
200
PER CENT CLAY ADDED

APPENDIX 4



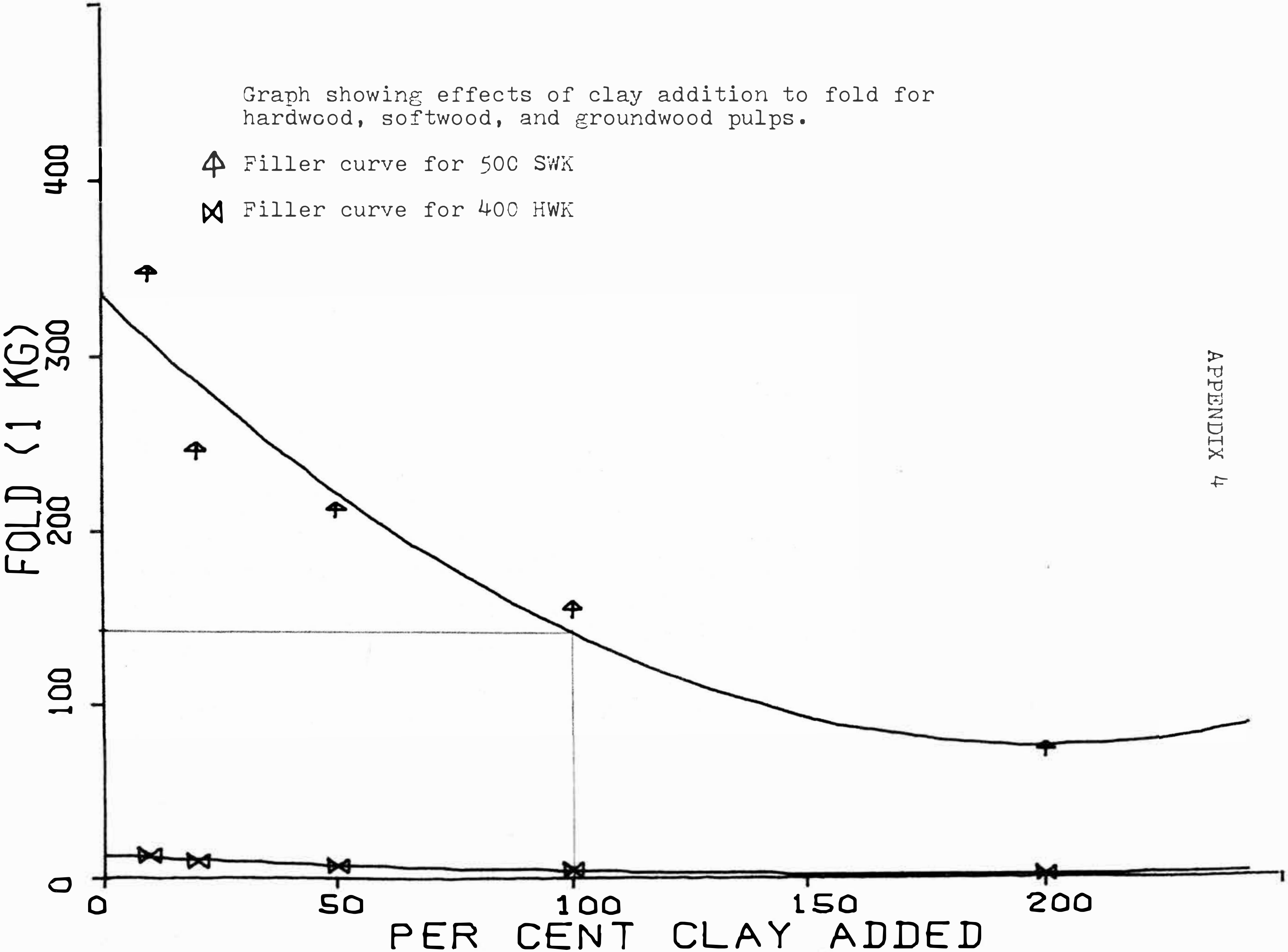
Graph showing effects of clay addition to Sheffield porosity for hardwood, softwood, and groundwood pulps.

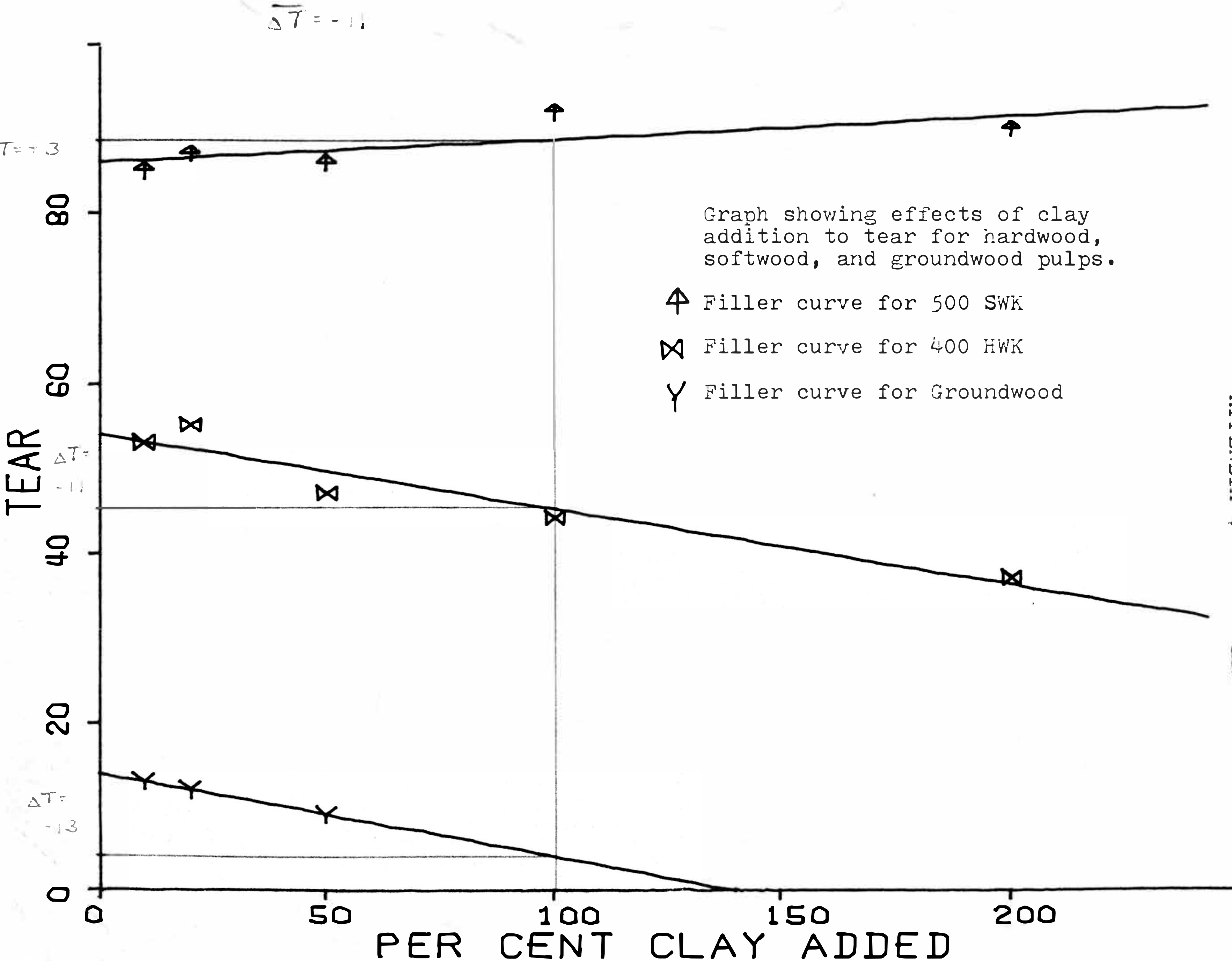
APPENDIX 4



Graph showing effects of clay addition to tensile for hardwood, softwood, and groundwood pulps.
Filler curve for 500 SWK
Filler curve for 400 HWK
Filler curve for Groundwood
ΔT = -2.3
-1.5
ΔT = -2.0
ΔT = -3.5
TENSILE (KG)
0
5
10
0
50
100
150
200
PER CENT CLAY ADDED

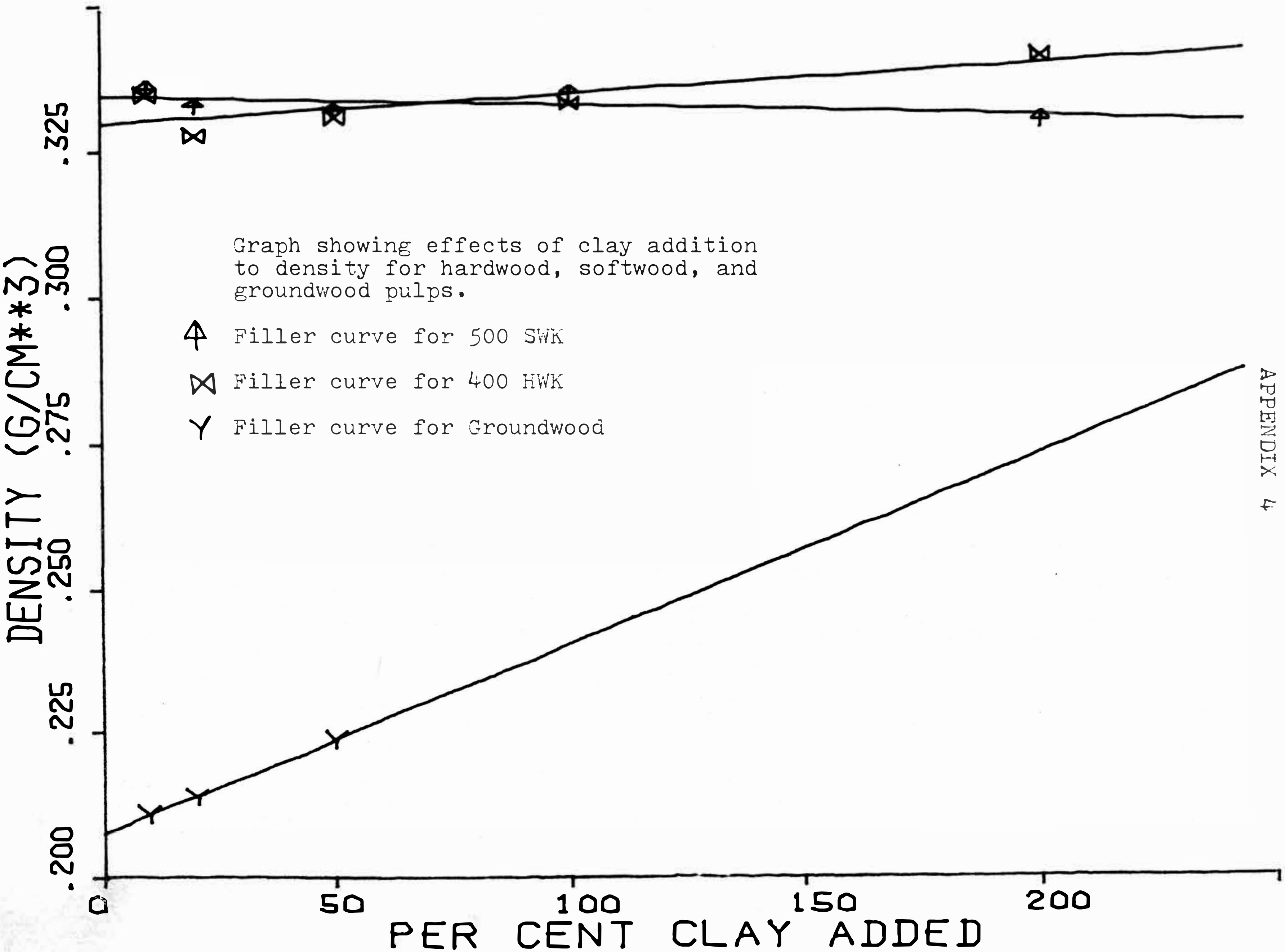
APPENDIX 4


Graph showing effects of clay addition to fold for hardwood, softwood, and groundwood pulps.
Filler curve for 500 SWK
Filler curve for 400 HWK
FOLD (1 KG)
400
300
200
100
0
0
50
100
150
200
PER CENT CLAY ADDED



ΔT = -11
T = +3
80
60
40
20
0
TEAR
ΔT = -11
ΔT = -13
0
50
100
150
200
PER CENT CLAY ADDED
Graph showing effects of clay addition to tear for hardwood, softwood, and groundwood pulps.
Filler curve for 500 SWK
Filler curve for 400 HWK
Filler curve for Groundwood

APPENDIX 4


Graph showing effects of clay addition to density for hardwood, softwood, and groundwood pulps.
Filler curve for 500 SWK
Filler curve for 400 HWK
Filler curve for Groundwood
DENSITY (G/CM**3)
.200
.225
.250
.275
.300
.325
0
50
100
150
200
PER CENT CLAY ADDED

APPENDIX 4