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A Study of Methods Used to Determine Fiber Length and Fiber Length Distributions by Optical Means

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A Study of Methods Used to Determine Fiber Length and Fiber Length Distributions by Optical Means.

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November 20, 1952
Russell Larson

A Study of Methods Used to Determine Fiber Length and Fiber Length Distribution by Optical Means

ABSTRACT

Various methods of slide preparation and fiber measurement are discussed in this literature survey. It is generally agreed that the time required to complete an individual test of fiber length has been an important factor in preventing a wider application of fiber length measurements in stock preparation control. Most authors use a method whereby a slide containing a number of fibers is projected and the measurements are made directly on the screen.

A method of measuring fiber length and fiber length distribution by a projection arrangement is described and the operating procedure given. The relation between pulp at different degrees of freeness which were run through the Bauer-MacNett classifier is given. The distribution of the fiber length was plotted by several graphical methods. Actual data are presented to show the usefulness of a projection arrangement in pulp refining.

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A Study of Methods Used to Determine Fiber Length and Fiber Length Distribution by Optical Means

Object of Survey

The objective of the literature survey of this thesis project is to review each method now used to determine fiber length and, to evaluate which method or combination of methods is best suited as a stock preparation control test.

Introduction

At present there are two different procedures for obtaining an estimate of the average fiber length of a sample of pulp. One is the optical and the other is the mechanical test. The optical test consists of making a fiber suspension in water, preparing a slide from this suspension, and by means of a projector making an actual count of the number of fibers that fall within certain length groups. The mechanical test also requires an aqueous suspension of fibers. They are separated into several length groups by means of a series of screens of graduated openings through which part of the fiber suspension passes.

Different authors have presented varied opinions of the relative merits of the different fiber length procedures. James d'A Clark (1) suggests that the only meaningful fiber length result is the one obtained by a combination of the mechanical and optical methods.

Characteristics of Classifier Method

In Graff and Miller's (2) review of the method of Steinschneider, Kross, and Imgrund, it was noticed that screen classification does not separate beaten fibers according to their actual lengths chiefly due to the roughened fiber side walls and their adhering to each other by fibrils. Classification does give a measure of the degree of roughening, which is important in connection with strength properties of paper.

Characteristics of a Projection Arrangement.

In Graff and Miller's (loc. cit.) review of Schulze's method it was noted that it was extremely difficult to measure the fibers by the use of a microscope because of the natural tendency of the fibers to curl and take on many varied shapes. Schulze preferred the use of a projection method whereby a microscope was placed horizontally and the projection made downward on a white sheet of paper where the measurement could be made easier.

As to a method developed by Bergman and Backman, the reviewers Graff and Miller (loc. cit.) commented that it represents a simple method for determining fiber length using a minimum amount of equipment. A thin suspension of fibers on a slide was made and projected at 70 diameter onto a piece of white paper. Measurement was made directly with a perimeter ruler (a map measure) or a pencil drawing was prepared of their projection and set aside to be measured at any convenient time.

Minimum Length of a Fiber to be Measured

In reporting average fiber length, authors have given varied opinions as to the minimum length of fiber that was considered practical to measure. Some measured the fines only as short as five-tenths of a millimeter while others thought it wise to consider the material down to two-hundredths of a millimeter. It brings up the question as to where it is more logical to stop. Possibly measurements to the nearest two-hundredths would tend to be more accurate. However the time element enters in as well as the question as to whether one intends to run a length determination for a research laboratory or for a quality control department. In either case it should be done with the least possible amount of time. One-tenth of a millimeter is the shortest material retained by a 150 mesh screen in a pulp classifier, and this is about the finest mesh screen practicable to use. Fibers shorter than this are classified by most users of the classifiers to be flour or debris. The inclusion of fine material in an estimate of fiber length by visual means accounts for nearly one-half of the operating time.

Requirements for Fiber Selection and Measurement

John H. Graff (3) set up some requirements for the microscopic determination of fiber dimensions. First, the sample must be a true distribution of all the fibers present. Second, the fibers should be lined up parallel so that every fiber present, long or short, is accounted for. Third, a standard number of measurements must be carried out within the time allotted and should show the least probable error. Fourth, the method must be accurately reproducable. Finally, the frequency distribution of the measurements taken must be expressed in relative distribution by weight as well as by number.

Most authors have set up similar requirements, with the exception of the methods used in slide preparation.

Slide Preparation

Many ways of diluting, staining and placing a sample of fibers onto a slide have been presented. Most authors begin with a suspension of pulp in water.

Graff and Feavel (4) proposed a method whereby a pulp suspension was prepared which gave 25 fibers to a drop. One drop of this suspension was placed on the end of several slides using a dropper having an opening of five millimeters. The water was then evaporated and two drops of iodine-iodide-calcium chloride stain were placed on the fibers.

After about two minutes the excess was removed. The fibers were then straightened, and laid parallel using a dissecting microscope and a dissecting needle. The fibers were then covered with a cover glass and the fibers were measured by a projection method which will be explained later.

Clark (5) also prepared a dilute suspension of pulp (one-tenth of a gram per liter) and transferred four milliliters of this suspension, using a six millimeter pipette with a rubber bulb at one end, into a special cell. Two to three milliliters of a one-half percent solution of locust bean gum plus a few drops of formaldehyde were then added to the cell. The function of the gum was twofold: it dispersed and also immobilized the suspended fibers which made the procedure convenient and more accurate. The formaldehyde was added as a preservative. This cell was then placed in a projection unit.

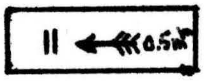
In the method developed by Fyfe (6), three drops of a slurry containing 75 fibers per drop were transferred to a clean microscopic slide. The water was allowed to evaporate and two drops of Herzberg stain were placed on the fibers and allowed to stand for one minute. Under a Greenough-type microscope, at ten diameter, the fibers were straightened and aligned, by the use of a dissecting needle, in an unused area of the slide. A cover glass was placed over the straightened and aligned fibers. A cover glass was also placed over the short fiber fragments that were left in the original two drops of Herzberg stain. The slide with the fibers so arranged was then projected onto a screen for measurement.

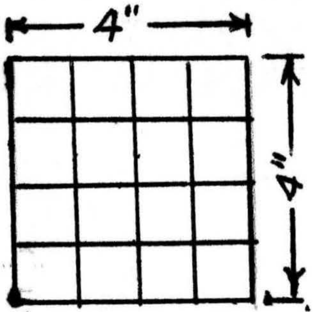
Sieber (7) described a method developed by Steinschneider, Kross, and Imgrund. A suspension of fibers was made in a gelatin solution so that the mixture was liquid when hot and solid when cool. The mixture was then spread uniformly over the slide while hot; upon cooling, the slide was ready to be placed in a projection apparatus, for projection and measurement.

Slide Projection

The method most frequently used for slide projection was one whereby a slide containing fibers was projected onto a white screen and measurement made directly. The type of projector did not affect the accuracy as long as the lens gave a clear magnified image of the fibers. The magnification used ranged from 70 diameter to as high as 400 diameter.

The most convenient method of getting the proper magnification (5), whereby a direct measurement of the fibers was made, called for adjusting the magnification such that a slide containing a pair of parallel lines five tenths of a millimeter apart exactly coincided with two sides of a four inch square on the screen.





Therefore a fiber measured to be four inches on the screen was one-half millimeter in length.

Slide Projection

Graff and Feavel (loc. cit.) used a projection microscope from which the microscope tube was removed. In place of an objective, the microscope was equipped with a projection lens. The microscope, and light source, were placed in a boxlike table equipped with a reflecting mirror. On the top of the table box and above the reflecting mirror was a ground glass on which a series of lines were drawn, at seven and one-half millimeter intervals, representing the actual space between the lines of one-tenth of a millimeter at 75 diameter. Above the ground glass was a hood, which made possible the observing and reading of the dimensions of the projected fibers without disturbance from the light in the laboratory room.

Fyfe (loc. cit.) used a modified microprojector to project an image of the fibers onto a screen. The screen was placed 28 inches from the microprojector and had a calibrated scale of concentric circles in one-half millimeter divisions. The center or smallest circle of this scale was subdivided into one-tenth millimeter divisions by means of dots which radiated from the center of the circle in eight equally spaced strokes. The magnification onto the screen was 50 diameter. The field of view was seven millimeters in diameter which permitted the measuring of extremely long fibers without having to fix reference points along the fibers.

Details of Fiber Measurement

Measurement of the image of the fibers on a screen can be done by various methods. One of the methods used (2,7) which did not consume much time made use of a perimeter ruler (map measure). The ruler had a calibrated scale, from which the length of the fibers in metric units, was calculated through the use of a conversion factor.

Clark (loc. cit.) described a method used to measure the lengths of each fiber using a piece of semitransparent manifold paper, about eight by ten and one-half inches, which had been ruled lengthwise with a parallel series of light pencil lines, one-fourth of an inch apart, and with a pair of heavier parallel lines at the top and bottom of the sheet exactly ten inches apart. On the screen was a four inch square subdivided into one inch squares (as previously mentioned). Beginning with the upper left-hand square, ruled on the screen, the paper was moved so that the tip of a fiber coincided with the point at the left-hand corner of the manifold paper where the lines intersected. The paper was then moved so that the light parallel line coincided as nearly as possible with the fiber being measured. When the fiber curved away from the line, the point of a sharp hard pencil was rested at the place on the light line where the curvature started and, using the pencil point as a pivot, the paper was pushed or pulled until more of the fiber coincided with the line; then the pencil point was moved to the place where the fiber curved off again, etc. In this way, the image of any fiber, no matter how curled, was accurately straightened out. After one fiber was finished a mark was placed on the light line, and the next fiber was measured. This was done for every fiber that had an end lying in the top right-hand square and which had more than one-half of its length inside the four inch square. Similarly it was done for the other one inch squares omitting images of two millimeters or less, corresponding to those shorter than one-tenth of a millimeter.

Arithmetic and Weighted Average Lengths

The average fiber length may be computed by dividing the total length of all the fibers measured by the number of fibers measured. This gives the numerical average fiber length. This numerical result is dependent upon the length considered to be the minimum recorded length. The calculated number of fibers less than one-half millimeter long in a pulp may be as high as 85 percent of the total number of fibers; this may be however, only twelve percent by weight (2).

Reed and Clark (1) found that in their classifier experiments the weighted average fiber length by weight in each fraction was eight percent greater than the arithmetical average fiber length.

Clark (loc. cit.) developed an equation whereby the numerical average fiber length could be converted to weighted average fiber length by weight.

Conclusion

Most authors have agreed that a report of numerical fiber length of pulp, especially if debris is included is of no value. Fiber length measurements should be as accurately as is practicable, a measure of the weighted average fiber length by true weight.

The measurement of the lengths of fibers may be accomplished far more rapidly by the use of a projection method than with a microscope.

-The End-

Russell Larson

Literature Cited

1. Reed, Arthur E., and Clark, James d'A., Tappi 33, no. 6:294-298 (June 1950).
2. Graff, John H., and Miller, B.W., Paper Trade J. 109, no. 6:31 (1939).
3. Graff, John H., "Pulp and Paper Microscopy," The Institute of Paper Chemistry, Appleton, Wisconsin, 1942,p.5.
4. Graff, John H., and Feavel, John, Paper Trade J. 118, no. 7:140, 142-145 (Feb. 17, 1944).
5. Clark, James d'A., Paper Trade J. 115, no. 26:36-42 (Dec. 24, 1942).
6. Fyfe, Thomas, Tappi 34, no. 7:306-310 (July, 1951).
7. Sieber, Rudolph, "Die Chemisch-Technischen Untersuchungs-Methoden der Zellstoff und-Papier Industrie," Springer-Verlag, Berlin, 1943, p.392.
8. Montigny, R. de, and Zbrowski, P., Paper Trade J. 123, no. 19: 42-48 and 123, no. 22: 29-34 (Nov. 7 and 28, 1946).

Outline of Proposed Laboratory Experiments

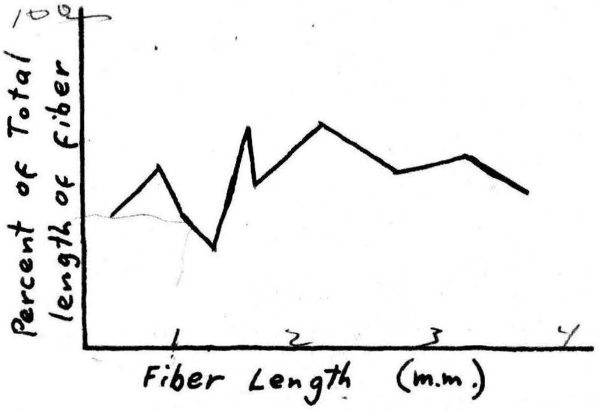
Equipment and Methods to be Used.

A Bausch and Lomb triple purpose microprojector will be used to project an image of the fibers onto a screen. The magnification at the screen will be 25.4 times, or such that one millimeter on the slide will equal four inches when projected onto the screen.

Slides will be prepared from a water suspension of fibers. A dropper will be used to transfer the suspension onto the slide such that each slide will contain 40 to 50 fibers. The water will be evaporated, Herzberg stain will be used, and a cover glass will be placed over the fibers. After one minute the excess of stain will be removed. The slide will then be placed in the projector and projected onto the screen, where the fibers will be measured using a Keuffel and Esser map measurer for determination of the length of irregular and curved lines. The results will be recorded on a fiber length frequency chart similar to the one described by Thomas Fyfe.(6) A minimum of 100 fibers will be counted from each sample and fibers less than 0.25 millimeters in length will be regarded as debris.

Objective of Work.

To obtain useful results it was decided that the author work with fractions of fibers to be supplied by David J. Kraske. Reference can be made to Kraske's thesis for the manner in which the samples were chosen. Measurements will be made of these fibers and the average fiber length as well as the distribution will be recorded. A series of graphs will be made plotting the percent of total length of the fiber against the fiber length ranges.



Weyerhouser standard bleached kraft pulp will be used. A summary of the planned experimental work is as follows:

1. A numerical average fiber length and fiber length distribution of whole pulp (not fractionated); unbeaten and beaten to different degrees of freeness. (500, 400, 300, and 200 freeness).
2. A numerical average fiber length and fiber length distribution of fractions of pulp obtained from Bauer-Mac Nett classifier. *
3. A numerical average fiber length and fiber length distribution of fibers retained on a fiber length index grid as produced by the Hermann Manufacturing Company in Lancaster, Ohio.

* Refer to Kraske's thesis for details concerning wire mesh to be used.

PART II

EXPERIMENTAL WORK

A Study of Methods Used to Determine Fiber Length and Fiber Length Distribution by Optical Means.

Experimental Program

As was stated previously in this paper, the purpose of this thesis was to find which method or combination of methods used to determine fiber length and fiber length distribution was best suited as a stock preparation control test.

Fractions of fibers, supplied by D. J. Kraske, were used for the experimental work. The fiber samples were taken from the fiber suspensions used by Kraske in making handsheets. This was done to obtain useful results which would correlate with the physical tests made on the handsheets. The fiber suspension was agitated and a test tube was immersed to get a representative sample. This sample was then diluted such that one drop of the slurry contained ten to twenty fibers. Six to seven drops of this slurry, after thorough mixing, were transferred onto a microscopic slide using a six inch length of four millimeter glass tubing fitted with a rubber dropping bulb.

The water was allowed to evaporate from the surface of the slide by heating on a hot plate held at seventy degrees Centigrade. The slides were tapped with a needle and then one drop of a one-half percent solution of locust bean gum was added. By the addition of the gum, a better distribution of the fibers on the slide was obtained. After drying, two drops of "C" stain were applied and a cover glass was placed over the fibers. The fibers were allowed to take up the stain, and after one minute, the excess stain was removed with a blotter. The fibers were then ready to be measured.

Measuring the Fiber Length

A Bausch and Lomb triple purpose micro-projector was used to project horizontally the image of the fibers onto a white screen where the measurements were made. The magnification was adjusted by placing a Bausch and Lomb calibrated microscopic slide onto the stage of the projector. The distance between the projector and the screen was changed until the one millimeter divisions on the slide coincided with two lines drawn two inches apart on the screen. The slide containing the fibers was placed onto the stage of the projector and the images of the fibers were measured using a Keuffel and Esser map measurer, designed to measure the length of curved and irregular lines. The map measurer gave the readings in inches. To get the correct values in millimeters, the observed reading was divided in half. Thus, a fiber measured to be six inches on the screen was actually three millimeters in length.

The lengths of all the fibers were recorded on a fiber length frequency chart.* A minimum of one hundred fibers was measured in order to get the true fiber length distribution of the fraction. All whole fibers, broken fibers and fiber fragments were measured. Fibers less than 0.25 millimeters in length were regarded as negligible debris.

Tabular Presentation

The results of all fiber length measurements are summarized in

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* Table V shows a typical example of a fiber length frequency chart.

Table I and Table II. These tables represent in tabular form all the measurements that were made during the experimental work. In Table I the whole pulp and the fractions retained by the various screens are compared at the different degrees of freeness. In Table II the same data are re-arranged to give a comparison of the fiber length distribution of the whole pulp at different degrees of freeness. Each fraction is individually compared at the corresponding different degrees of freeness.

Graphical Presentation and Analysis

It was decided to present graphically the results obtained on West Coast bleached kraft pulp at 400 Canadian freeness; the four hundred freeness being very common in commercial practice. Figure I was plotted to show the relation between the percent of total number of fibers and the fiber length intervals. The fractions retained on the sixteen and twenty mesh screens took the shape of normal distribution curves. The curves representing the whole pulp, and the fraction retained on the two hundred mesh screen were positively skewed, showing the predominance of short fibers. The curve representing the fraction retained on the ten mesh screen was negatively skewed which showed that the majority of the fibers fell within the longer length intervals.

Figure II was plotted similar to Figure I except that the percent of total length of fiber, instead of percent of total number of fibers, was plotted against the length intervals. The values for each point were calculated by multiplying the average value of each 0.5 millimeter interval length by the number of fibers in that interval. This figure divided by the total length of all the fibers gave the values which were plotted on the graph.

By analysis of figure II it was noticed that the curve representing the whole pulp took the shape of a normal distribution curve, showing that plotting the percent of total length of fiber against the length intervals has the effect of minimizing the presence of the shorter fibers.

Figure III expresses the same data, except that the cumulative length percent is plotted against the fiber length interval. Thus the percent of the total length of fiber retained by the two hundred mesh screen which was two millimeters or less in length was nineteen percent.

From analysis of these graphs it was also noticed that there was not a clear separation of the fibers between the sixteen mesh screen and the twenty mesh screen using the Bauer-MacNett classifier. There was noticed a trend for the length of the fibers retained on the sixteen mesh screen to approach the distribution that was found to be retained by the twenty mesh screen as the freeness was lowered. No explanation for this behavior could be found.

Numerical versus Weighted Average Fiber Length

The numerical average fiber length, the weighted average fiber length by weight and the fiber length range were calculated for whole pulps (not fractionated), either unbeaten or beaten to different degrees of freeness namely 600, 500, 400, 300, and 200 ml. Canadian freeness.

Furthermore, the same calculations were made after measuring samples of fibers at each freeness which were run through the Bauer-MacNett classifier and retained on screens of 10, 16, 20, and 200 mesh.

The numerical average fiber length was computed by dividing the total length of all the fibers measured by the number of fibers measured (see Table VI). The weighted average fiber length was determined as follows. It was assumed that all the fibers were arranged side by side in order of increasing length. Furthermore it was assumed that the fibers increased uniformly in length. Two and one-half percent of the fibers on each end of the distribution were considered as stray fibers and neglected. The average of the next five percent was taken as two sides of a trapezoid. The length of the vertical line which passes through the centroid of the trapezoid was calculated by a formula developed by Clark (5). This gave the weighted average fiber length by area, and if all the fibers are assumed to be of uniform density this can be regarded as the weighted average fiber length by weight.

Table VI shows a sample calculation of the numerical and weighted average fiber length. All data used in Table VI were taken from the sample at 400 freeness which was retained by the twenty mesh wire of the Bauer-MacNett classifier.

A summary of the average fiber length data and the range of fiber length is tabulated in Table III, which compares the whole pulp and the different fractions at the different levels of freeness. The data from Table III was re-arranged to produce Table IV which shows a comparison of the average fiber length and range for each fraction at different levels of freeness.

A graphical representation of fiber length range and average fiber length of fractions was shown in figures IV and V. By analysis of figure IV it was evident that both the numerical and the weighted average fiber lengths decreased with increasing mesh of wire. The range of the fiber length, as shown in figure V, was nearly the same for each fraction as the freeness was lowered. The numerical average fiber length for the whole pulp was lowered by 0.75 millimeter by beating from a freeness of 750 ml. down to a freeness of 200 ml. The weighted average fiber length by weight was decreased by 0.50 millimeter. The weighted average gave a preferable answer because it minimized the effect of the short fibers.

Conclusions

It was found that the method described is workable and can be used as a stock preparation control test. By compiling the data such as shown in Figure III. The data could then be used to compare one beater run with another.

One drawback is the amount of time involved in preparing and counting 100 fibers. One and one-half hours was the average time required to prepare a slide and count the fibers from one sample. Possibly this method could be used for more applications if the time element could be shortened.

ADDENDUM

FIBER LENGTH INDEX GRID

As the name indicates the fiber length index is the weight of pulp after starting with ten grams, that is retained on the blades of a fiber length index grid.

It was decided to measure the average fiber length of the pulp which was retained on a fiber length index grid as manufactured by the Hermann Manufacturing Company in Lancaster, Ohio.

Ten grams of a pulp slurry were separately run onto the grid at different levels of freeness, namely 750, 600, 500, 400, 300, and 200 ml. The work was carried out according to the procedure set up by de Montigny and Zborowski (8). The weight of the fibers which were retained by the grid were tabulated in Table X-1.

Uniform samples were taken from the pulp remaining on the grid at 750, 400, and 200 ml. Canadian freeness. Measurements were made using the projection arrangement to get the average fiber lengths. Table X-2 shows the results which were obtained as compared with average fiber length data of the whole pulps at the corresponding freeness.

Analysis

It was found that by starting with ten grams of pulp that this did not always give a separation of the long fibers from the shorter ones.

The weight that was retained on the grid showed a decline as the freeness of the stock was lowered.

In studying the results of the average fiber lengths of the fractions retained on the grid it was noticed that satisfactory selectivity of retention of long fibers was obtained at the lower freeness levels. The quantity of stock retained at these levels of freeness ranged from two to two and one-half grams as compared to 5.65 grams retained at 750 ml. freeness. The results of the measurements of the fiber lengths obtained at the higher freeness levels did not conform with the expected results.

It was assumed that the reason we did not get the expected results at the higher levels of freeness was due to the probability of the fibers not being caught by the blades but by the previously deposited fibers. It was proposed that by trying to keep a constant weight of stock retained on the grid and varying the amount of stock to begin with that this may be the solution to the problem of getting better selectivity of retention of long fibers.

THE END

Russell Larson

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June 4, 1953

FIGURE I

DISTRIBUTION CURVES -- GROUPS ARRANGED IN 0.5 mm INTERVALS

2.

Percent of Total Number
of Fibers vs
Fiber Length Interval

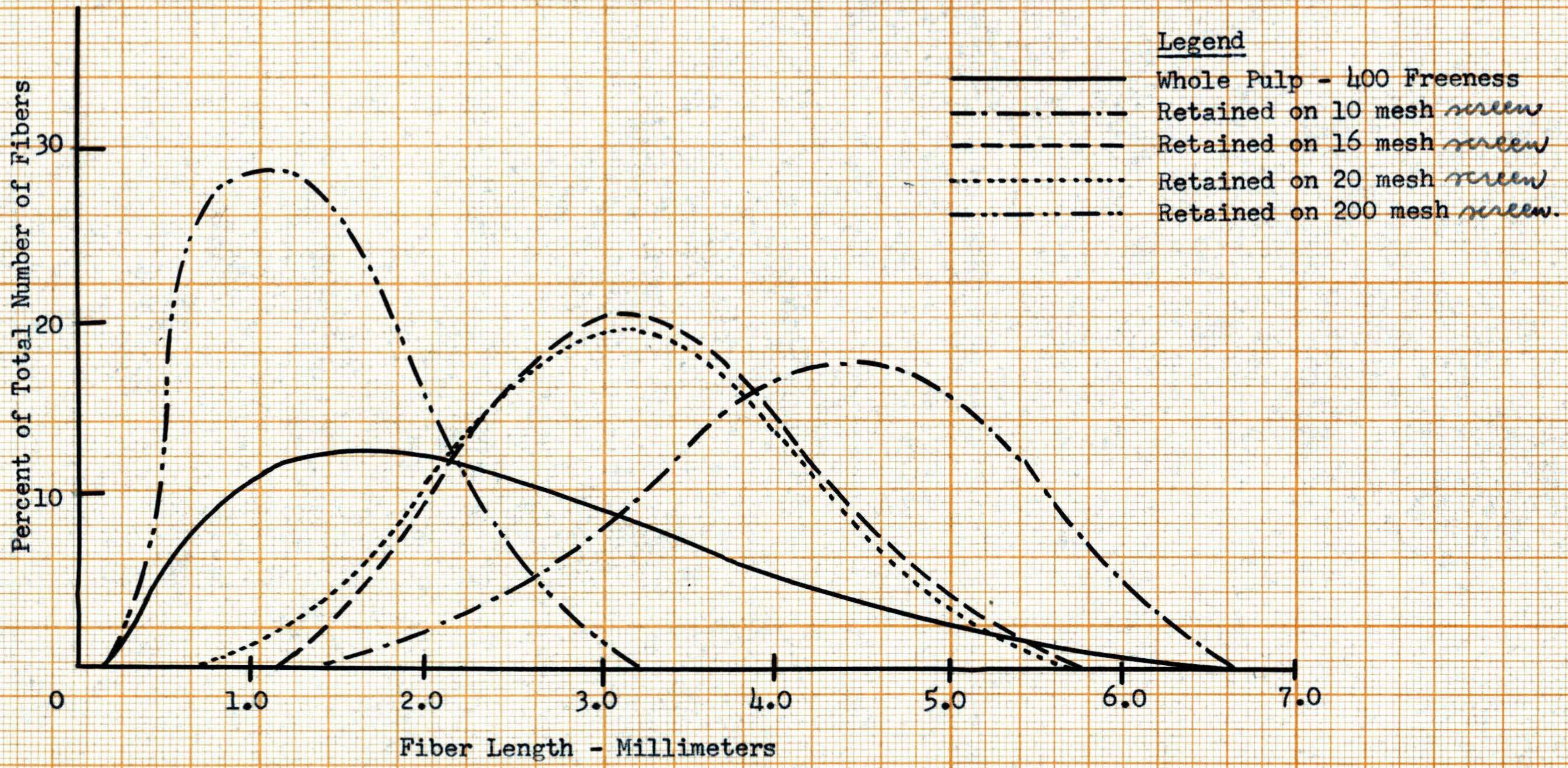


FIGURE I

FIGURE 21

DISTRIBUTION CURVES -- GROUPS ARRANGED IN 0. 5 mm INTERVALS

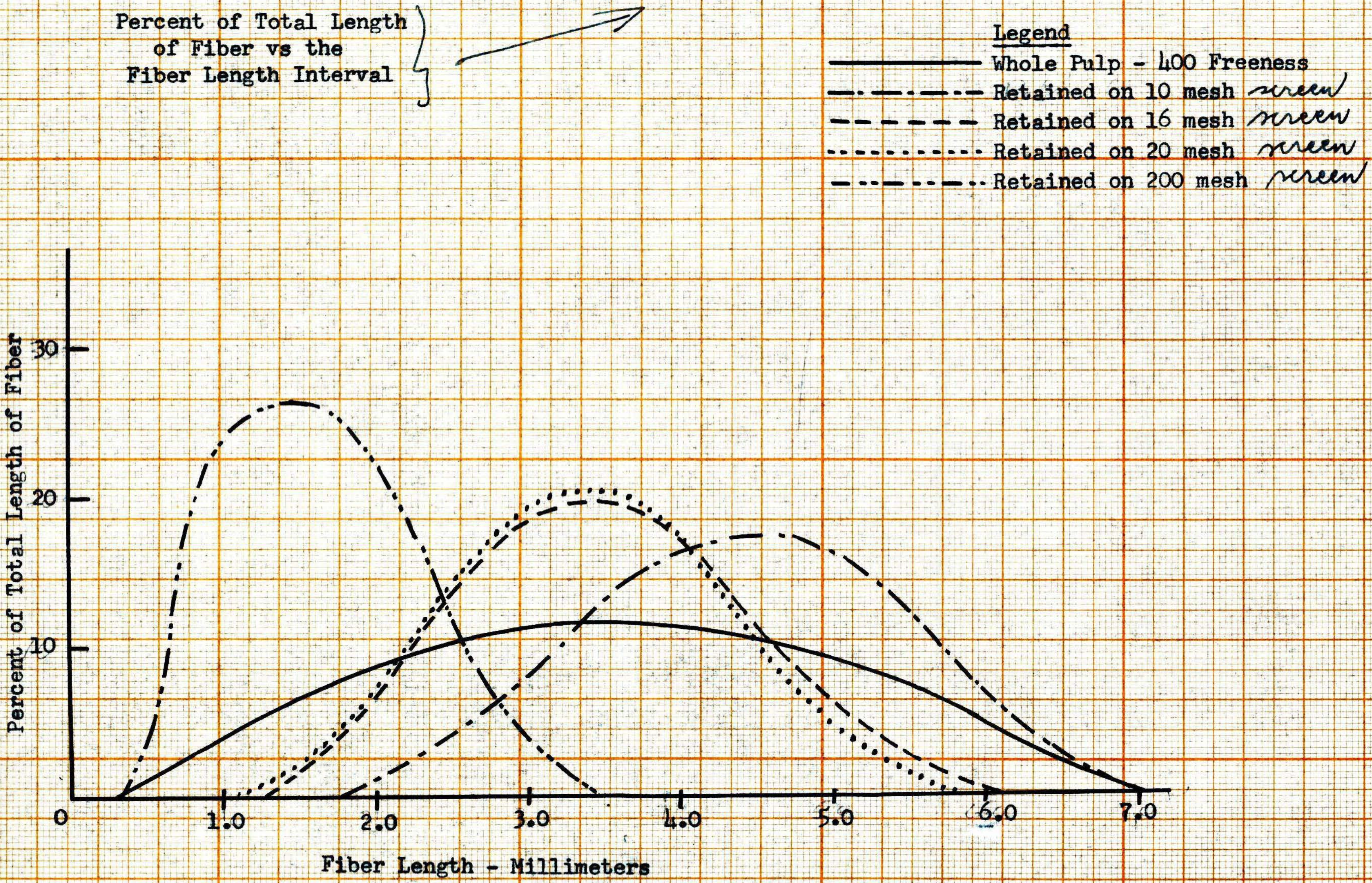


FIGURE II

FIGURE III

DISTRIBUTION CURVES -- GROUPS ARRANGED IN 0.25 mm INTERVALS

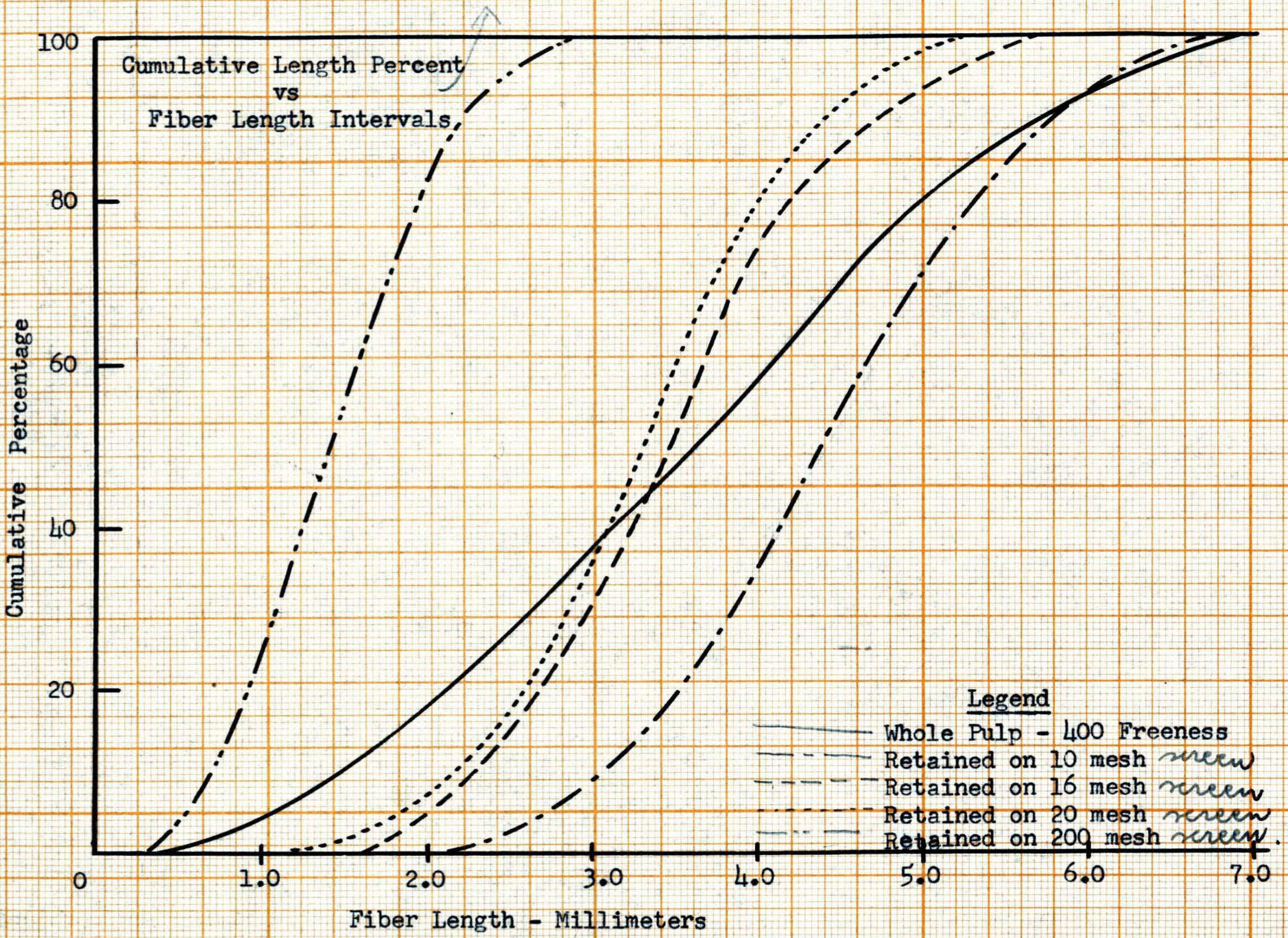


FIGURE III

FIGURE IV

GRAPHICAL REPRESENTATION OF RANGE OF FIBER LENGTH AND AVERAGE FIBER LENGTH OF FRACTIONS

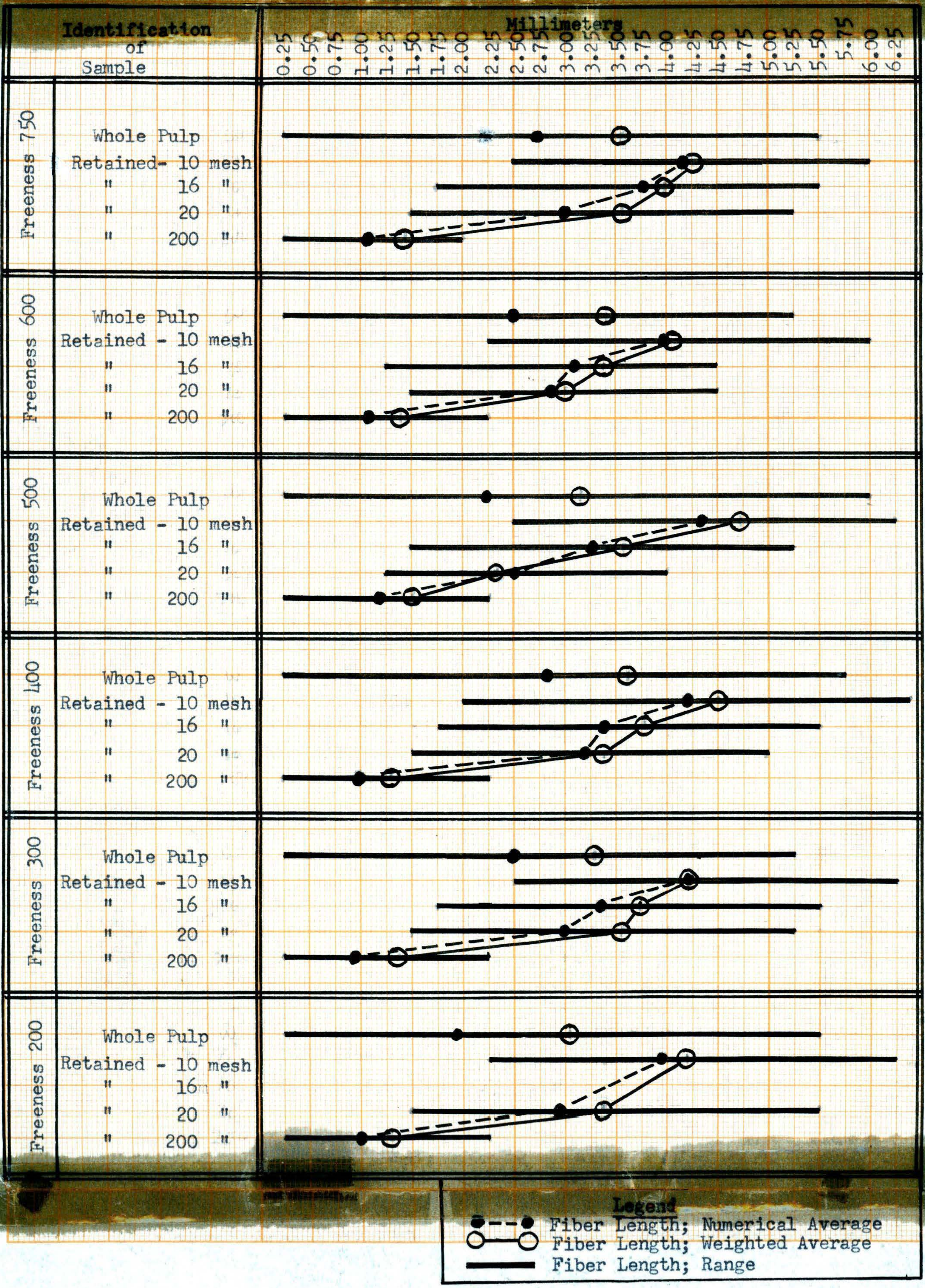
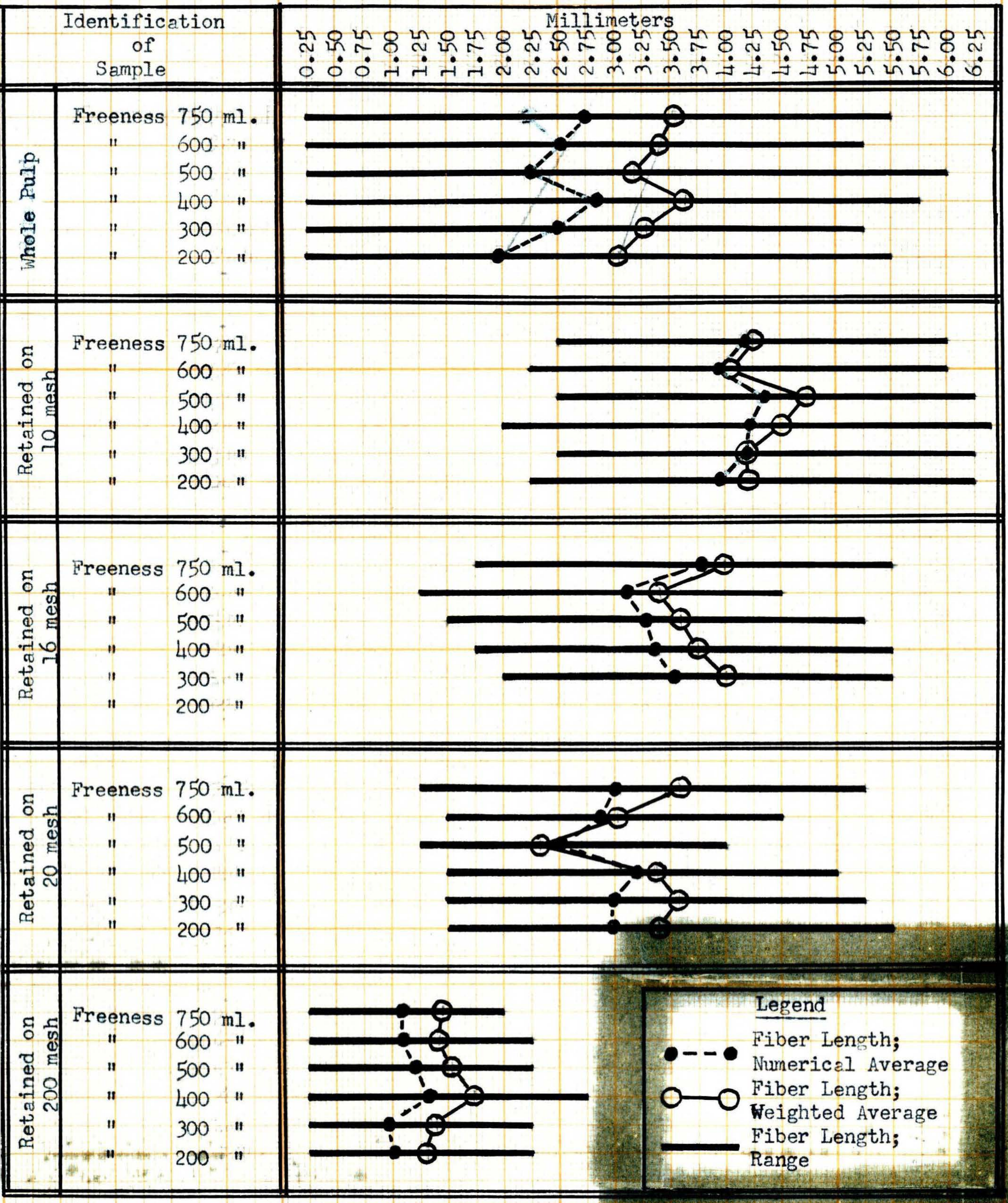


FIGURE V

GRAPHICAL REPRESENTATION OF RANGE OF FIBER LENGTH AND AVERAGE FIBER LENGTH OF FRACTIONS



TRACE [illegible] Type

TABLE I

SUMMARY OF FIBER LENGTH DISTRIBUTION DATA

IDENTIFICATION OF SAMPLE		COUNT OF FIBERS IN GROUPS; MEASUREMENTS IN MILLIMETERS																											TOTAL COUNT
		0.25-0.50	0.51-0.75	0.76-1.00	1.01-1.25	1.26-1.50	1.51-1.75	1.76-2.00	2.01-2.25	2.26-2.50	2.51-2.75	2.76-3.00	3.01-3.25	3.26-3.50	3.51-3.75	3.76-4.00	4.01-4.25	4.26-4.50	4.51-4.75	4.76-5.00	5.01-5.25	5.26-5.50	5.51-5.75	5.76-6.00	6.01-6.25	6.26-6.50	6.51-6.75	6.76-7.00	
FREENESS 750	FRACTION																												
	WHOLE PULP	3	4	13	6	11	9	5	6	4	5	11	4	7	6	8	4	7	5	7	1	5	0	1	0	1	0	0	133
	RETAINED ON 10 MESH	0	0	0	3	1	1	0	0	0	5	2	6	7	9	12	11	9	9	7	6	8	6	4	0	0	1	0	106
	" " 16 "	0	0	1	0	0	0	4	4	4	6	10	7	9	8	8	7	9	4	11	4	5	0	1	1	0	0	1	104
	" " 20 "	1	0	0	0	4	2	4	5	15	13	12	9	8	6	7	7	4	0	3	3	1	1	0	0	0	0	0	105
	" " 200 "	10	9	18	13	10	5	3	1	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	73
FREENESS 600																													
	WHOLE PULP	2	3	10	7	9	7	11	6	9	2	9	5	2	4	2	3	9	2	1	3	1	0	1	1	1	0	0	110
	RETAINED ON 10 MESH	0	0	0	1	2	2	2	1	2	4	6	7	9	7	9	5	13	8	11	6	3	3	3	0	1	0	0	104
	" " 16 "	0	2	3	3	6	2	5	2	8	8	10	13	10	7	13	7	8	1	1	2	1	1	0	1	1	0	0	115
	" " 20 "	0	0	3	2	0	2	6	6	13	12	21	13	4	4	7	3	3	0	0	1	0	0	0	0	0	0	0	100
	" " 200 "	11	12	17	12	7	9	6	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	79
FREENESS 500																													
	WHOLE PULP	8	6	9	5	9	7	9	5	5	2	4	1	7	11	6	8	2	3	5	1	2	1	4	1	0	0	0	121
	RETAINED ON 10 MESH	0	0	0	0	0	0	0	0	1	4	5	6	9	5	15	16	12	11	9	10	2	3	5	5	1	2	1	122
	" " 16 "	0	0	0	0	2	4	6	9	17	12	13	12	5	8	13	10	3	7	8	3	2	1	0	0	0	0	0	135
	" " 20 "	0	0	0	0	10	10	19	15	14	12	8	9	9	5	5	2	1	1	1	0	0	0	0	0	0	0	0	121
	" " 200 "	10	16	19	15	12	16	8	3	3	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	104
FREENESS 400																													
	WHOLE PULP	4	5	14	9	4	5	9	8	8	4	6	9	3	4	6	3	6	3	7	3	1	3	1	1	1	0	1	126
	RETAINED ON 10 MESH	0	0	0	2	0	1	0	2	4	2	5	5	5	12	15	11	6	9	11	10	8	5	3	2	3	1	0	122
	" " 16 "	0	0	0	1	0	1	2	6	8	9	11	6	13	9	11	7	4	3	4	1	2	1	0	0	0	0	0	99
	" " 20 "	0	1	0	0	2	5	3	7	9	6	15	12	16	10	9	9	4	4	1	1	2	1	0	0	0	0	0	117
	" " 200 "	3	10	14	19	7	6	10	6	2	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	81
FREENESS 300																													
	WHOLE PULP	3	10	8	3	7	10	7	8	12	12	5	7	12	5	4	2	5	2	3	2	0	0	1	2	0	0	0	130
	RETAINED ON 10 MESH	0	0	0	1	2	1	0	3	3	2	2	5	7	15	15	13	9	10	11	6	7	4	10	3	1	0	0	130
	" " 16 "	0	0	0	0	0	0	3	6	7	11	13	8	11	11	14	6	6	6	6	4	4	1	2	0	1	0	0	122
	" " 20 "	0	0	0	0	1	5	11	16	20	9	10	10	13	8	9	1	8	3	1	4	2	0	0	0	0	0	0	131
	" " 200 "	20	15	18	14	10	4	5	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	91
FREENESS 200																													
	WHOLE PULP	10	11	13	12	5	4	9	5	7	7	4	9	5	2	1	3	1	0	1	1	1	1	0	0	0	0	0	112
	RETAINED ON 10 MESH	0	0	0	2	0	1	0	0	1	6	5	6	9	9	18	9	8	6	1	6	8	3	0	2	1	0	0	101
	" " 16 "	NONE RETAINED BY 16 MESH																											
	" " 20 "	0	0	0	1	0	8	6	7	17	8	13	12	10	5	7	5	5	2	2	1	2	0	0	0	0	0	0	111
	" " 200 "	15	12	15	21	12	6	4	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	88

TOTAL 3193

TABLE II

SUMMARY OF FIBER LENGTH DISTRIBUTION DATA

IDENTIFICATION OF SAMPLE		COUNT OF FIBERS IN GROUPS; MEASUREMENTS IN MILLIMETERS																											TOTAL COUNT
		0.25 - 0.50	0.51 - 0.75	0.76 - 1.00	1.01 - 1.25	1.26 - 1.50	1.51 - 1.75	1.76 - 2.00	2.01 - 2.25	2.26 - 2.50	2.51 - 2.75	2.76 - 3.00	3.01 - 3.25	3.26 - 3.50	3.51 - 3.75	3.76 - 4.00	4.01 - 4.25	4.26 - 4.50	4.51 - 4.75	4.76 - 5.00	5.01 - 5.25	5.26 - 5.50	5.51 - 5.75	5.76 - 6.00	6.01 - 6.25	6.26 - 6.50	6.51 - 6.75	6.76 - 7.00	
WHOLE PULP	FREENESS 750	3	4	13	6	11	9	5	6	4	5	11	4	7	6	8	4	7	5	7	1	5	0	1	0	1	0	0	133
	" 600	2	3	10	7	9	7	11	6	9	2	9	5	2	4	2	3	9	2	1	3	1	0	1	1	1	0	0	110
	" 500	8	6	9	5	9	7	9	5	5	2	4	1	7	11	6	8	2	3	5	1	2	1	4	1	0	0	0	121
	" 400	4	5	14	9	4	5	9	8	8	4	6	9	3	4	6	3	6	3	7	3	1	3	1	1	1	0	1	126
	" 300	3	10	8	3	7	10	7	8	12	12	5	7	12	5	4	2	5	2	3	2	0	0	1	2	0	0	0	130
	" 200	10	11	13	12	5	4	9	5	7	7	4	9	5	2	1	3	1	0	1	1	1	1	0	0	0	0	0	112
RETAINED ON 10 MESH	FREENESS 750	0	0	0	3	1	1	0	0	0	5	2	6	7	9	12	11	9	9	7	6	8	6	4	0	0	1	0	106
	" 600	0	0	0	1	2	2	2	1	2	4	6	7	9	7	9	5	13	8	11	6	3	3	3	0	1	0	0	104
	" 500	0	0	0	0	0	0	0	0	1	4	5	6	9	5	15	16	12	11	9	10	2	3	5	5	1	2	1	122
	" 400	0	0	0	2	0	1	0	2	4	2	5	5	5	12	15	11	6	9	11	10	8	5	3	2	3	1	0	122
	" 300	0	0	0	1	2	1	0	3	3	2	2	5	7	15	15	13	9	10	11	6	7	4	10	3	1	0	0	130
	" 200	0	0	0	2	0	1	0	0	1	6	5	6	9	9	18	9	8	6	1	6	8	3	0	2	1	0	0	101
RETAINED ON 16 MESH	FREENESS 750	0	0	1	0	0	0	4	4	4	6	10	7	9	8	8	7	9	4	11	4	5	0	1	1	0	0	1	104
	" 600	0	2	3	3	6	2	5	2	8	8	10	13	10	7	13	7	8	1	1	2	1	1	0	1	1	0	0	115
	" 500	0	0	0	0	2	4	6	9	17	12	13	12	5	8	13	10	3	7	8	3	2	1	0	0	0	0	0	135
	" 400	0	0	0	1	0	1	2	6	8	9	11	6	13	9	11	7	4	3	4	1	2	1	0	0	0	0	0	99
	" 300	0	0	0	0	0	0	3	6	7	11	13	8	11	11	14	6	6	6	6	4	4	1	2	0	1	0	0	122
	" 200	NONE RETAINED BY 16 MESH																											—
RETAINED ON 20 MESH	FREENESS 750	1	0	0	0	4	2	4	5	15	13	12	9	8	6	7	7	4	0	3	3	1	1	0	0	0	0	0	105
	" 600	0	0	3	2	0	2	6	6	13	12	21	13	4	4	7	3	3	0	0	1	0	0	0	0	0	0	0	100
	" 500	0	0	0	0	10	10	19	15	14	12	8	9	9	5	5	2	1	1	1	0	0	0	0	0	0	0	0	121
	" 400	0	1	0	0	2	5	3	7	9	6	15	12	16	10	9	9	4	4	1	1	2	1	0	0	0	0	0	117
	" 300	0	0	0	0	1	5	11	16	20	9	10	10	13	8	9	1	8	3	1	4	2	0	0	0	0	0	0	131
	" 200	0	0	0	1	0	8	6	7	17	8	13	12	10	5	7	5	5	2	2	1	2	0	0	0	0	0	0	111
RETAINED ON 200 MESH	FREENESS 750	10	9	18	13	10	5	3	1	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	73
	" 600	11	12	17	12	7	9	6	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	79
	" 500	10	16	19	15	12	16	8	3	3	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	104
	" 400	3	10	14	19	7	6	10	6	2	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	81
	" 300	20	15	18	14	10	4	5	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	91
	" 200	15	12	15	21	12	6	4	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	88

TOTAL 3193

TABLE III
SUMMARY OF AVERAGE FIBER LENGTH DATA AND RANGE OF FIBER LENGTH

IDENTIFICATION OF SAMPLE		AVERAGE FIBER LENGTH		RANGE OF FIBER LENGTH MILLIMETERS
		NUMERICAL AVERAGE MM.	WEIGHTED AVERAGE MM.	
FREENESS 750	WHOLE PULP	2.74	3.56	0.25 — 5.50
	RETAINED ON 10 MESH	4.24	4.25	2.50 — 6.00
	" " 16 "	3.78	4.00	1.75 — 5.50
	" " 20 "	3.02	3.58	1.25 — 5.25
	" " 200 "	1.10	1.46	0.25 — 2.00
FREENESS 600	WHOLE PULP	2.54	3.44	0.25 — 5.25
	RETAINED ON 10 MESH	3.98	4.06	2.25 — 6.00
	" " 16 "	3.12	3.42	1.25 — 4.50
	" " 20 "	2.82	3.01	1.50 — 4.50
	" " 200 "	1.10	1.42	0.25 — 2.25
FREENESS 500	WHOLE PULP	2.26	3.19	0.25 — 6.00
	RETAINED ON 10 MESH	4.38	4.74	2.50 — 6.25
	" " 16 "	3.28	3.60	1.50 — 5.25
	" " 20 "	2.52	2.33	1.25 — 4.00
	" " 200 "	1.20	1.52	0.25 — 2.25
FREENESS 400	WHOLE PULP	2.84	3.65	0.25 — 5.75
	RETAINED ON 10 MESH	4.23	4.52	2.00 — 6.50
	" " 16 "	3.37	3.75	1.75 — 5.50
	" " 20 "	3.21	3.38	1.50 — 5.00
	" " 200 "	1.33	1.74	0.25 — 2.75
FREENESS 300	WHOLE PULP	2.51	3.30	0.25 — 5.25
	RETAINED ON 10 MESH	4.23	4.23	2.50 — 6.25
	" " 16 "	3.54	4.07	2.00 — 5.50
	" " 20 "	3.00	3.57	1.50 — 5.25
	" " 200 "	0.98	1.36	0.25 — 2.25
FREENESS 200	WHOLE PULP	1.99	3.05	0.25 — 5.50
	RETAINED ON 10 MESH	3.98	4.24	2.25 — 6.25
	" " 16 "	—	—	NO FIBERS RETAINED
	" " 20 "	3.00	3.39	1.50 — 5.50
	" " 200 "	1.04	1.32	0.25 — 2.25

TABLE IV
SUMMARY OF AVERAGE FIBER LENGTH DATA AND RANGE OF FIBER LENGTH

IDENTIFICATION OF SAMPLE		AVERAGE FIBER LENGTH		RANGE OF FIBER LENGTH MILLIMETERS
		NUMERICAL AVERAGE M.M.	WEIGHTED AVERAGE M.M.	
WHOLE PULP	FREENESS 750 ml.	2.74	3.56	0.25 MM. — 5.50 MM.
	" 600 "	2.54	3.44	0.25 MM. — 5.25 MM.
	" 500 "	2.26	3.19	0.25 MM. — 6.00 MM.
	" 400 "	2.84	3.65	0.25 MM. — 5.75 MM.
	" 300 "	2.51	3.30	0.25 MM. — 5.25 MM.
	" 200 "	1.99	3.05	0.25 MM. — 5.50 MM.
RETAINED ON 10 MESH	FREENESS 750 ML.	4.24	4.25	2.50 MM. — 6.00 MM.
	" 600 "	3.98	4.06	2.25 " — 6.00 MM.
	" 500 "	4.38	4.74	2.50 " — 6.25 "
	" 400 "	4.23	4.52	2.00 " — 6.50 "
	" 300 "	4.23	4.23	2.50 " — 6.25 "
	" 200 "	3.98	4.24	2.25 " — 6.25 "
RETAINED ON 16 MESH	FREENESS 750 ML.	3.78	4.00	1.75 MM. — 5.50 MM.
	" 600 "	3.12	3.42	1.25 " — 4.50 "
	" 500 "	3.28	3.60	1.50 " — 5.25 "
	" 400 "	3.37	3.75	1.75 " — 5.50 "
	" 300 "	3.54	4.01	2.00 " — 5.50 "
	" 200 "	—	—	NO FIBERS RETAINED
RETAINED ON 20 MESH	FREENESS 750 ML.	3.02	3.58	1.25 MM. — 5.25 MM.
	" 600 "	2.82	3.01	1.50 " — 4.50 MM.
	" 500 "	2.52	2.33	1.25 " — 4.00 "
	" 400 "	3.21	2.38	1.50 " — 5.00 "
	" 300 "	3.00	3.57	1.50 " — 5.25 "
	" 200 "	3.00	3.39	1.50 " — 5.50 "
RETAINED ON 200 MESH	FREENESS 750 ML.	1.10	1.46	0.25 MM — 2.00 MM.
	" 600 "	1.10	1.42	0.25 " — 2.25 "
	" 500 "	1.20	1.52	0.25 " — 2.25 "
	" 400 "	1.33	1.74	0.25 " — 2.75 "
	" 300 "	0.98	1.36	0.25 " — 2.25 "
	" 200 "	1.04	1.32	0.25 " — 2.25 "

TABLE V

METHOD OF CLASSIFYING FIBER LENGTH OBSERVATIONS

~~Sample of Data~~ -- Freeness 400 ml. -- Fraction Retained on 20 mesh ~~wire~~ screen.

Fiber Length Range ; mm.	Fiber Count				Mean of Range	Number of fibers	Total Length	Cumulative Length
0.25 -- 0.50					0.38	0	0	0
0.51 -- 0.75	/				0.63	1	0.63	0.63
0.76 -- 1.00					0.88	0	0	0.63
1.01 -- 1.25					1.13	0	0	0.63
1.26 -- 1.50	//				1.38	2	2.76	3.39
1.51 -- 1.75	𝍸				1.63	5	8.15	11.54
1.76 -- 2.00	///				1.88	3	5.64	17.18
2.01 -- 2.25	𝍸	//			2.13	7	14.91	32.09
2.26 -- 2.50	𝍸	////			2.38	9	21.42	53.51
2.51 -- 2.75	𝍸	/			2.63	6	15.78	69.29
2.76 -- 3.00	𝍸	𝍸	𝍸		2.88	15	43.20	112.49
3.01 -- 3.25	𝍸	𝍸	//		3.13	12	37.60	150.09
3.26 -- 3.50	𝍸	𝍸	𝍸	/	3.38	16	54.10	204.19
3.51 -- 3.75	𝍸	𝍸			3.63	10	36.30	240.49
3.76 -- 4.00	𝍸	////			3.88	9	34.90	275.39
4.01 -- 4.25	𝍸	////			4.13	9	37.20	312.59
4.26 -- 4.50	////				4.38	4	17.52	330.11
4.51 -- 4.75	////				4.63	4	18.52	348.63
4.76 -- 5.00	/				4.88	1	4.88	353.51
5.01 -- 5.25	/				5.13	1	5.13	358.64
5.26 -- 5.50	//				5.38	2	10.76	369.40
5.51 -- 5.75					5.63	0	0	369.40
5.76 -- 6.00					5.88	0	0	369.40
6.01 -- 6.25	/				6.13	1	6.13	375.53

Total Number of Fibers Measured --------- 117

Range of Distribution --1.25 --5.50 mm.

TABLE VI

CALCULATION OF NUMERICAL AND WEIGHTED AVERAGE FIBER LENGTH

Sample: 400 ml. Canadian Freeness
Fraction Retained on 20 mesh wire

Numerical Average Fiber Length.

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$\bar{X}$ = Numerical average fiber length.

$\sum_{i=1}^{n} X_i$ = The sum of all fiber length values from X_1 to X_N Inclusive.

n = Total number of fibers measured.

Data from TABLE V:

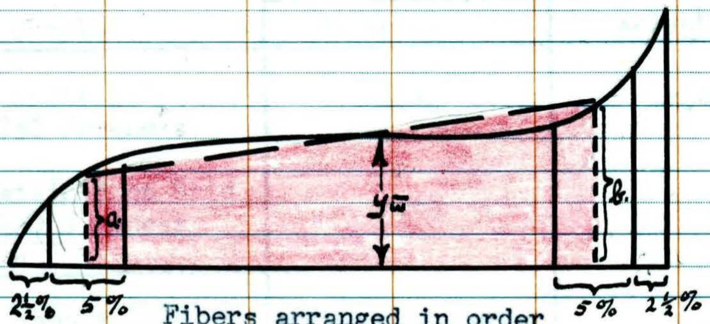
$\sum_{i=1}^{n} X_i$ = 375.53 mm.

n = 117 fibers

Calculations:

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{375.53}{117} = 3.21 \text{ millimeters}$$

Weighted Average Fiber Length.(Trapezoid Method)



Fibers arranged in order of increasing length

Data from TABLE V:

a = 1.67 mm.

b = 4.63 mm.

* Formula:

$$y_{\bar{\omega}} = \frac{2}{3}\,\frac{a^2+b^2+ab}{a+b}$$

$y_{\bar{\omega}}$ = weighted average fiber length by area.

a = average length of lower five percent of fibers.(see diagram)

b = average length of upper five percent of fibers.

Calculations:

$$y_{\bar{w}} = \frac{2}{\,}\;\frac{(1.67)^2+(4.63)^2+(1.67\times 4.63)}{1.67+4.63}$$

$y_{\bar{w}}$ = 3.38 Millimeters

* Formula used was developed by Clark (5). .05

Note: two and one-half percent of the fibers on both extreme ends were neglected.

3.38
3.21
.17

8%

$338\,\overline{)17.000}$
1690
348

TABLE X - 1

FIBER LENGTH INDEX GRID

(Summary of data)

Freeness	Quantity of Whole Pulp Used	Quantity Retained on Grid
750 ml.	10 grams	5.65 grams
600 ml.	10 grams	3.30 grams
500 ml.	10 grams	3.00 grams
400 ml.	10 grams	2.67 grams
300 ml.	10 grams	2.40 grams
200 ml.	10 grams	2.14 grams

TABLE X - 2

A COMPARISON OF AVERAGE FIBER LENGTH OF FIBERS RETAINED ON INDEX GRID vs. THE AVERAGELENGTH OF THE WHOLE PULPS

Identification of Sample	Average Fiber Length mm.		Range mm.	Quantity Retained on Grid
	Numerical	Weighted		
Whole Pulp 750 Freeness	2.74	3.56	0.25 -- 5.50	xx
Retained on Grid 750 Freeness	2.64	3.66	0.25 -- 6.25	5.65 gram
Whole Pulp 400 Freeness	2.84	3.65	0.25 -- 5.75	xx
Retained on Grid 400 Freeness	2.59	3.53	0.25 -- 5.75	2.67 gram
Whole Pulp 200 Freeness	1.99	3.05	0.25 -- 5.50	xx
Retained on Grid 200 Feeeness	2.42	3.34	0.25 -- 7.00	2.14 gram

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