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# Evaluating the Use of the Math Reasoning Inventory for Improvement in Fraction Instruction

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EVALUATING THE USE OF THE MATH REASONING INVENTORY  
FOR IMPROVEMENT IN FRACTION INSTRUCTION

by

Kathy Huffman Boyer

A dissertation submitted to the Graduate College  
in partial fulfillment of the requirements  
for the degree of Doctor of Education  
Special Education and Literacy Studies  
Western Michigan University  
August 2014

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## EVALUATING THE USE OF THE MATH REASONING INVENTORY FOR IMPROVEMENT IN FRACTION INSTRUCTION

Kathy Huffman Boyer, Ed.D.

Western Michigan University, 2014

In an attempt to close the mathematic achievement gap between students from the United States and students from other countries, a new national set of standards, the Common Core State Standards for Mathematics, was developed and adopted by the state of Michigan in 2010. These standards emphasize mathematical reasoning and application, rather than the previous emphasis on performing calculations. Unfortunately, teachers generally have had little training in how to assess students' mathematical reasoning, how to teach mathematical reasoning, or how to provide remediation to those students who show need. The purpose of this study was to see if use of the new online Math Reasoning Inventory (MRI) developed by Marilyn Burns assisted four volunteer fourth and fifth grade teachers from a local public school in assessing students' conceptual knowledge of fractions and in targeting instruction about fractions. This study employed a mixed-design using both qualitative and quantitative measures. The four teachers were observed at the beginning of the unit on fractions in order to determine which reasoning strategies they modeled for their students to use in solving fraction problems. These teachers were observed a second time after they had been trained to administer the MRI near the beginning of the unit on fractions and again after the teachers had administered the MRI midway through the fraction unit in order to see if teachers' strategy modeling increased.

Teachers administered the MRI again at the end of the fraction unit and students' pre- and posttest data were compared in order to assess whether or not there was an increase in students' use of reasoning strategies. Finally, the teachers completed a survey in which they evaluated the usefulness of the inventory. Results showed few significant differences in student achievement or in teachers' use of mental reasoning strategies in instruction as a result of using the Math Reasoning Inventory.

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Kathy Huffman Boyer

## TABLE OF CONTENTS

ACKNOWLEDGMENTS .....	ii
LIST OF TABLES .....	vi
LIST OF FIGURES .....	vii
CHAPTER	
I. INTRODUCTION .....	1
Current Status.....	4
Study Purpose and Significance.....	11
II. REVIEW OF RELEVANT LITERATURE .....	13
Misconceptions About Fractions .....	15
Teaching Practices That Promote Conceptual Mathematic Reasoning in Struggling Learners.....	19
Assessment of Math Reasoning .....	26
Summary .....	32
III. METHODOLOGY .....	34
Design .....	34
Participants.....	36
Setting .....	37
Instrumentation .....	38
Observation Protocol .....	38
Math Reasoning Inventory.....	39

Table of Contents—Continued

CHAPTER		
	Teacher Survey .....	39
	Procedures .....	40
	Data Analysis .....	44
	Math Reasoning Inventory .....	44
	Observation Protocol .....	45
	Teacher Survey .....	45
	Summary .....	46
IV.	RESULTS .....	47
	Research Question One: Effects of MRI on Student Achievement .....	47
	Summary of Data on Student Achievement .....	54
	Research Question Two: Changes in Teaching Practices .....	54
	Summary of Data on Teaching Practices .....	61
	Research Question Three: Identifying Students with Misconceptions .....	62
	Research Question Four: Identifying Needed Interventions .....	62
	Overall Teacher Input .....	63
	Summary of Results .....	65
V.	DISCUSSION .....	67
	Summary of Study Problem and Methodology .....	67
	Summary of Findings .....	71
	Implications .....	76
	Limitations .....	78



Table of Contents—Continued

CHAPTER

Future Directions ..... 80

Final Thoughts ..... 81

REFERENCES ..... 83

APPENDICES

A. Human Subjects Institutional Review Board Letter of Approval ..... 90

B. Recruitment Poster ..... 92

C. Observation Protocol ..... 94

D. Teacher Survey ..... 96

## LIST OF TABLES

1. Comparison of Pretest-Posttest Total Correct Answers from Four Teacher's Classrooms .....	48
2. Comparison of Pretest-Posttest Total Correct Answers from Fifth Grade Students.....	49
3. Comparison of Pretest-Posttest Total Correct Answers from Fourth Grade Students.....	50
4. Comparison of Pretest-Posttest Appropriate Reasoning Strategy Responses from Four Teachers' Classrooms.....	51
5. Percentages of Specific Reasoning Strategies Used by Students in Each Participant's Classroom .....	53
6. Classroom Discussion Data .....	58
7. Numbers of Specific Reasoning Strategies Modeled by Teachers in Each Participant's Classroom .....	60
8. Numerical Results of Teacher Survey of the Math Reasoning Inventory .....	64

## LIST OF FIGURES

1. Minutes of teacher direct instruction observed.....	55
2. Minutes of classroom discussion .....	56
3. Minutes of individual work time.....	56
4. Minutes of group work time .....	57

## **CHAPTER I**

### **INTRODUCTION**

Today's student faces intense global competition upon entering the workforce. Increasing educational rigor by preparing all students for postsecondary training has been proposed as a way to ensure that students from the United States will be ready to face this economic reality. Unfortunately, teachers generally have had little training in how to provide this rigor, how to assess student learning, and how to provide interventions to those students who show need (Elmore, 2002; Yoon, Duncan, Lee, Scarloss, & Shapley, 2007). This is especially true in the area of mathematics and particularly problematic when it comes to assisting students who have difficulties with mathematics.

This call for increased rigor is not without basis. Possession of a bachelor's degree is seemingly prerequisite to obtaining a well-paying job. In 2012, persons with a bachelor's degree had median weekly earnings of \$1,066, compared to \$652 for those with a high school diploma and \$471 for those without a high school diploma. Additionally, having a degree provides a considerable advantage to even obtaining employment: Currently a 4.5% unemployment rate exists for those with at least a bachelor's degree compared to an 8.3% unemployment rate for those with only a high school diploma and a 12.4% unemployment rate for those with no high school diploma (United States Department of Labor, 2013). Therefore, setting the groundwork for success in higher education has become a national priority. On the White House website (n.d.), President Barack Obama challenges every American to complete at least one year

of college or postsecondary training. He also sets the goal for the United States that by the year 2020, America will have the highest proportion of college graduates in the world (White House, n.d., Higher Education, para.3). ACT, a nonprofit education organization that has conducted extensive research, agrees with this plan:

The goal of ensuring that all high school graduates are ready for college and career is the right one for the United States. . . . The relationships between skills development, workforce productivity, and economic growth demand that high school graduates be college and career ready so they can acquire the requisite skills and knowledge they will need to meet the demands of the changing and increasingly competitive global economy. College and career readiness is an essential step for the longer-term economic viability of the United States. (ACT, 2011, p. 4)

Statistics regarding achievement in the subject of mathematics indicate that meeting Mr. Obama's goal will be a significant challenge. According to one study, students from the United States came in 28th place worldwide in performing calculations and solving applied mathematics problems (Norris, 2004). Colleges, particularly community colleges, report high numbers of incoming freshmen requiring remedial mathematics classes. In a follow-up study of 2003-2004 first-time postsecondary students conducted by the U.S. Department of Education (2013b), 42% had enrolled in a remedial math class (*An Overview of Classes Taken*, Table 2-B). *The Nation's Report Card* (U.S. Department of Education, 2013a) reveals 25% of 12th grade students were below basic level as measured by the National Assessment of Educational Progress (NAEP); for students with disabilities, 74% were below basic level. Equally troubling is the finding that fewer than 10% of the students in the lowest level college remedial math classes go on to graduate (Sherer & Grunow, 2010).

Not surprisingly then, success in advanced high school mathematics classes has been shown to be a significant predictor of successful college completion. In a study

conducted by the U.S. Department of Education (USDE), it was found that “finishing a course beyond the level of Algebra 2 (for example, trigonometry or pre-calculus) more than doubles the odds that a student who enters postsecondary education will complete a bachelor’s degree” (USDE, 1999, Selected Findings). In response to this, some states have raised the number and rigor of math classes required for high school graduation. In Michigan, for example, students are now required to take a math class in each year of high school. The content of Algebra II must also be mastered as a prerequisite for obtaining a high school diploma. At least 38 other states have also increased the number and rigor of math classes required for graduation (Reys, Dingman, Nevels, & Teucher, 2007).

Some students, particularly those with disabilities, have difficulty meeting these increased standards. Thurlow, Sinclair, and Johnson (2002) found that 36% of students who drop out of school have learning disabilities. Moreover, persons with disabilities have a 14.5% unemployment rate (United States Department of Labor, 2014, Table A-6). Another study found that 7% of high school students have a significant disability in one area of mathematics (Geary et al., 2009). Provisions of the No Child Left Behind Act of 2001 (NCLB, 2002) as well as those in the Individuals with Disabilities Improvement Education Act (IDEA) of 2004 require that students with disabilities must not only have access to, but also must progress in the general curriculum. Schools, therefore, need to take steps to ensure that students with disabilities make adequate yearly progress (AYP) in mathematics and language arts, as measured by standardized assessments, or face loss of federal funds.

Difficulty passing these standardized assessments in mathematics is also present prior to the high school level. In 2013, USDE reported that 26% of eighth graders nationally scored below the basic level in math. In Michigan 30% were below this level. For fourth graders, numbers of students below basic level were 17% nationally and 23% in Michigan. Math difficulties are present even in earliest grades (USDE, 2013a). Gersten and Chard (1999) found that some kindergarteners, particularly those from low socioeconomic homes, had little to no basic number sense, a necessary prerequisite for success in any aspect of math.

### **Current Status**

In light of these and other equally dismal measures, several governmental initiatives have been implemented. In 2006, then-president Bush created the National Mathematics Advisory Panel (NMAP) with the charge to “rely upon the ‘best available scientific evidence’ and recommend ways to ‘foster greater knowledge of and improved performance in mathematics among American students’” (NMAP, 2008, p. xiii). To this end, task groups were created to analyze the best available evidence in each of five areas: Conceptual Knowledge and Skills, Learning Processes, Instructional Practices, Teachers and Teacher Education, and Assessment. Additionally, three subcommittees were formed, one to set the standards of evidence to be considered by each task group, one to examine instructional materials, and one to conduct a survey of practicing algebra teachers. The Panel as a whole examined over 16,000 research publications and policy statements, heard testimony from 110 experts, received input from 160 interested individuals and organizations, and analyzed surveys from 743 algebra teachers (NMAP,

2008, chapter 1, p. 1). Unfortunately, even with this extensive investigation, the Panel could make few conclusive recommendations, and these were mainly in the form of suggestions for specific areas for further research and instruction.

Among the recommendations of the Panel were for more formal instruction in number sense, conceptual knowledge of fractions, and using formative assessments. Specifically, one recommendation was for teachers to promote number sense through more explicit instruction in computational estimation, in composing and decomposing numbers, in the basic meaning of operations, and in comparing magnitudes of numbers (NMAP).

Another finding was based on interviews with Algebra I teachers: “Difficulty with the learning of fractions is pervasive and is an obstacle to further progress in mathematics. . . . Instruction focusing on conceptual knowledge of fractions is likely to have the broadest and largest impact on problem-solving performance.”(NMAP, 2008, p. 28). Wu (2001) agrees: “The failure rate in algebra will continue to be high unless we radically revamp the teaching of fractions and decimals” (p. 1). To further confirm the relationship of fraction knowledge to math achievement, Bailey, Hoard, Nugent, and Geary (2012) conducted a study with sixth and seventh grade students. Controlling for intelligence and working memory, performance on fraction comparison tests in the sixth grade was found to be a significant predictor to math achievement in the seventh grade.

The National Mathematics Advisory Panel also reports that formative assessments, defined as “the ongoing monitoring of student learning to inform instruction,” were found to have significant benefit to students (NMAP, 2008, p. 46). A strategy reported as “promising” was to use technology to conduct these assessments in



order to provide teachers with immediate information on needed whole class and individual student interventions. While the use of formative assessments benefits students of all ability levels, the Panel recommends more frequent assessment for students who struggle with mathematics. This allows teachers to better adapt instruction based on student progress (NMAP, 2008, p. 47). The Panel also recommends that research be conducted regarding “dynamic assessments involving ‘think alouds’ . . . and . . . the impact they have on helping teachers improve their effectiveness” (NMAP, 2008, p. 47).

Seeing a need to standardize and improve curriculum and thus better prepare students for higher education and the workforce, the National Governors Association and the Council of Chief State School Officers joined forces in 2009 in another governmental initiative. Out of this collaboration came the Common Core State Standards for Mathematics (CCSSM; National Governors Association, 2010). Forty-three states, including Michigan, have adopted these standards, which emphasize an understanding of math concepts over simply performing arithmetic operations. Along with the skill standards for each grade level, there are also eight Standards for Mathematical Practice, which cross all grade levels. These practice standards list the proficiencies and processes that teachers should employ as a part of any mathematics instruction: an emphasis on reasoning, communicating, and modeling mathematical ideas.

On a local level, many school districts have implemented response to intervention (RTI) programs in order to provide increasing intensities of support to struggling students. Although every school’s program is unique, the basic RTI model is a tiered instructional intervention system based on frequent assessment of educational progress and use of assessment data in a problem-solving model. “At its most basic level, RTI is

about informed instructional decisions aimed at improving learning outcomes” (Riccomini & Smith, 2011, p. 7). Under an RTI program, students take a brief assessment (screener) at various intervals throughout the year, which measures mastery of select grade level standards. Students who do not show mastery receive some level of intervention.

The intervention levels in an RTI model are known as tiers. All students receive Tier 1 instruction, which is the core grade-level instruction. Based on screener data, a teacher may see need to make whole-class Tier 1 interventions or provide Tier 2 level interventions for certain students. Tier 2 interventions are generally provided as additional instructional time in a small group setting. Students who show need for even more intensive intervention receive Tier 3 interventions, usually including more frequent additional instruction and in a smaller group setting than in Tier 2. In some districts, Tier 3 interventions are conducted in special education settings. According to Riccomini and Smith (2011), approximately 80% of students will have educational needs met through Tier 1 instruction, 15% may need Tier 2 interventions, and 5% may need Tier 3 interventions (pp. 8-9).

To ensure these interventions are effective, additional assessments (formative, progress monitoring) are regularly conducted. In this way, movement through tiers is fluid—students who make adequate progress with Tier 2 interventions move back to Tier 1. Likewise, students who do not make adequate progress given Tier 2 supports receive more intensive Tier 3 interventions. Under the Individuals with Disabilities Education Improvement Act of 2004 (IDEA 2004), districts can use a child’s failure to respond to an intensive researched-based intervention as part of an evaluation to determine whether

that child has a learning disability. Because it is more proactive in providing assistance to struggling students, the RTI model is seen as a better alternative to the “wait and fail” model previously used to identify students with learning disabilities.

Most RTI programs have been implemented in the area of reading; however, the Institute of Educational Sciences (IES) published a list of eight recommendations for assisting students who struggle with mathematics. Among these are to use a universal screening for students in need of interventions, explicit instruction in problem-solving, and verbalizing math thought processes (Gersten et al., 2009).

One of the challenges faced in implementing the IES recommendations is finding easy to use and affordable universal math screeners. The National Center on Response to Intervention (2011) website defines screening instruments as:

Brief assessments that are valid, reliable, and evidence-based. They are conducted with all students or targeted groups of students to identify students who are at risk of academic failure and, therefore, likely to need additional or alternative forms of instruction to supplement the conventional general education approach.  
(Screening Tools Chart)

One of these math screeners, the DELTA test, is used by several Michigan school districts. Developed by Michigan’s Ottawa Area Intermediate School District, the DELTA test assesses mastery of critical computational Common Core State Standards from the previous grade level (Ottawa Area Intermediate School District, 2014). While this test adequately identifies those students who need interventions for weaknesses in computational skills, it does little to identify students who are in need of interventions in the areas of problem-solving or the understanding of and communication of math concepts.

Since problem-solving and higher level understanding of concepts is a focus—both of the movement toward success in higher education as well as in the new Common Core State Standards—it is important for schools to find a screener to identify students in need of interventions in these areas. While national assessment programs aligned with the CCSSM are in the process of being developed, they will not be available until the 2014-2015 school year. Moreover, assessments developed by the Smarter Balanced Assessment Consortium (SBAC), the association in which the State of Michigan is a member, do not appear to be practical for screening purposes. Computer-based formative, summative, and interim assessments are part of this program, but information from these is reported to take weeks to become available (Smarter Balanced Assessment Consortium, n.d., Frequently Asked Questions section, question 8). This is obviously too long a time to wait for information on needed interventions.

Recent National Council Teachers of Mathematics (NCTM) calls for research had over 300 submissions, but only “a handful of submissions were primarily focused on issues related to assessment (and only a few of those were accepted)” (Tarr et al., 2013, p. 350). This dearth of mathematic assessments is also reflected on The National Center on Response to Intervention (2011) website, which provides reviews of mathematics screeners and progress monitoring assessments. Of those which have been rated as effective, further examination shows most to be similar to the DELTA test in mainly assessing computational skills. Only two seem to measure application of mathematical concepts. The highest rated, STAR by Renaissance Learning, costs \$1,799 for software along with a \$9 per student fee (Screening Tools Chart). This is far too costly for most districts. The other, the AIMSweb Mathematics and Applications (M-CAP) by Pearson,

is more affordable at \$4 per student (Screening Tools Chart). While data management is web-based, teachers must score tests and input data. Furthermore, the word problems included on the test only appear to assess whether the student can perform the appropriate calculations, rather than demonstrate deeper understandings of math concepts (AIMSweb, 2010, Sample Pages).

Perhaps the most promising assessment is the Math Reasoning Inventory (MRI). Not included in the National Center on Response to Intervention reviews, the MRI is a web-based formative assessment developed in 2012 by Marilyn Burns at Scholastic to assess mathematic thinking skills of fourth through eighth graders based on CCSSM content prior to sixth grade (MRI, 2012, Overview: About the Assessment, para. 1). The MRI is a computational and computer-based test with three sections: whole numbers, decimals, and fractions. Each section contains a four-question written computational test, which takes about 5 to 10 minutes and can be administered in a group setting. Each assessment also includes an individual face-to-face interview reported to take 10 minutes per student where each student solves 10 to 12 mental math problems and is asked how he or she was able to solve them. The interviewer selects the choice that most closely matches the student's reported strategy from a list on a computer screen. Four types of reports can be immediately generated: individual, group, item analysis, and assessment review. A considerable advantage over other assessments is that the MRI has been funded by the Bill and Melinda Gates Foundation so as to be free of cost to teachers and districts.

Besides identifying students in need of math reasoning interventions, the MRI was designed to inform teaching practices. This follows a National Mathematics Advisory Panel finding: "Teachers' regular use of formative assessments improves their

students' learning, especially if teachers have additional guidance on using the assessment results to design and individualize instruction" (NMAP, 2008, p. 47). In the case of the MRI, teachers can use knowledge of student thought processes to plan instruction and determine which students would benefit from interventions by verbalizing math processes or in choosing appropriate problem-solving strategies, both recommendations of the Institute of Educational Sciences for math RTI programs.

### **Study Purpose and Significance**

The purpose of this study was to conduct a comprehensive pilot investigation of the fraction portion of the Math Reasoning Inventory in order to determine what, if any, effects that training teachers to conduct the MRI and then actually administering the MRI to their students had on fraction instruction in regular classroom settings. Specifically, the efficacy of the MRI as a screener to identify students in need of intervention as well its use as a formative assessment to inform teaching practices was examined. Using a mixed-design study combining observations, questionnaires, and pre- and posttests, the overarching question guiding the study was: What effect does the Math Reasoning Inventory (MRI) have on student achievement in fractions? As a component of this, the study examined the usefulness of the MRI in assisting teachers to identify students' misconceptions in reasoning about fractions. Further, once teachers administered the MRI, this study examined whether their teaching practices changed as a result and whether teachers were able to use the MRI to identify needed interventions in teaching the conceptual aspects of fractions.

This pilot study was conducted as a part of the School Improvement Plan for the upper elementary building in a large school district in southwest Michigan. While this district is in its fourth year of having a math RTI program, district administrators as well as teachers there have been concerned with persistent achievement deficits across all tier levels in in the area of fractions. Given the Common Core emphasis on conceptual understanding, seeking ways to assess and provide interventions for these deficits is essential. This study was an attempt to determine whether the Math Reasoning Inventory could fit these needs.

## **CHAPTER II**

### **REVIEW OF RELEVANT LITERATURE**

Recent studies have attempted to identify common characteristics of students who have math learning disabilities as well as to determine how these disabilities develop, knowledge useful for the RtI program's goal of preventing math disabilities. Gersten, Jordan, and Flojo (2005) studied research on young children and found that one of the factors that contributes to math difficulties is lack of number sense. Acknowledging there is no commonly agreed upon definition, they describe number sense as the ability to work fluently and flexibly with numbers as well as the ability to check the reasonableness of an answer. According to the National Math Advisory Panel (2008), "Poor number sense interferes with learning algorithms and number facts and prevents use of strategies to verify if solutions to problems are reasonable" (p. 27).

In 2009, Geary et al. conducted a longitudinal study of 306 kindergarteners. As these students were assessed yearly through third grade, students with math disabilities and those with low achievement in math were found to have low scores on number line and counting knowledge, indicating poor number sense. Bryant, Bryant, and Hammill (2000) examined previously identified characteristics of older students with math weaknesses and presented a list of 33 of the characteristics to 391 special education teachers. These teachers were asked to rate these behaviors in their 870 students who had been identified as having learning disabilities in math. Difficulty solving word problems, difficulty solving multi-step problems, and difficulty understanding the language of



mathematics were the three most common characteristics noted. Another highly identified behavior in the group with math disability was that students did not realize that their answers were “unreasonable” and thus did not go back and make corrections, again pointing to lack of number sense.

This lack of number sense is particularly problematic when it comes to fractions. In light of the Math Panel’s findings (2008) that lack of fraction knowledge impedes future progress in math and that focus on instruction of fraction concepts has the broadest impact on problem-solving, the Institute of Educational Sciences (IES) studied research related to fraction instruction and published a guide, *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade* (Seigler et al., 2010). This guide lists recommendations for fraction instruction:

1. Build on students’ informal understanding of sharing and proportionality to develop initial fraction concepts.
2. Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.
3. Help students understand why procedures for computations with fractions make sense.
4. Develop students’ conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.

5. Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them. (p. 11)

Recommendation 5 further states, "Greater understanding of fractions, knowledge of students' conceptions and misconceptions about fractions, and effective practices for teaching fractions are critically important for improving classroom instruction" (Seigler et al., 2010, p. 9). This recommendation forms the basis for this study.

In reviewing literature related to conceptual fraction instruction, three categories of studies were considered. Research regarding misconceptions about fractions is the first. The second category includes studies that report on effective instructional practices for the struggling learner, with a focus on those that address fraction misconceptions in a discussion format. The third section considers assessment of conceptual fraction knowledge and concludes with research related to the Math Reasoning Inventory.

### **Misconceptions About Fractions**

The IES guide for fraction instruction lists some common student misconceptions about fractions from the reviewed literature. Among these were:

Not viewing fractions as numbers at all, but rather as meaningless symbols that need to be manipulated in arbitrary ways to produce answers that satisfy a teacher . . . Focusing on numerators and denominators as separate numbers rather than thinking of the fraction as a single number . . . Confusing properties of fractions with those of whole numbers. (Seigler et al., 2010, pp. 6-7)

These IES conclusions are based on numerous studies. Peck and Jencks (1981) identified some of these same misconceptions when assessing sixth grade students' understanding of fractions. When asked to draw representations of fractions, students did not draw equal sized pieces. Moreover, some of these students could correctly use an

algorithm to compare fractions, but were unaware of the inaccuracy of their supporting drawings. Other students failed to consider the relationship of the numerator to the denominator when comparing fractions and reasoned that the fraction with the most pieces was the largest. When adding fractions with unlike denominators, some students used incorrect algorithms to solve the problems, and then drew incorrect drawing to support their answer. Over a 30-week period, Behr, Wachsmuth, and Post (1985) gave 8 fourth and fifth graders six numerals with which to construct fractions that would add up to 1. These students were next given the task to construct two fractions whose sum would be close to, but not equal to 1. Students were asked to estimate and were given 1 minute for each task. Behr et al. found that despite the fact that students had considerable experience adding and comparing fractions prior to being given these tasks, 49% of the students' answers indicated that they either had no concept of the magnitude of fractions or that they had no knowledge of the addition of fraction algorithm, incorrectly adding both the numerator and denominator (p. 128).

In a study with third and fourth graders, Mack (1995) found that while students had some informal knowledge of fractions and often could correctly solve some verbally presented problems involving fractions, they made errors when given the same problems represented symbolically. Students made common errors, such as adding both the numerator and the denominator. The most frequent error Mack found, however, was that students overgeneralized whole number knowledge and mixed symbolic representations of whole and fractions. In many cases, students expressed the numerator as the number of *wholes*, rather than as the number of parts of the whole. Mixed numbers were treated the same way—when asked to write  $1 \frac{1}{3}$  students wrote  $\frac{1}{3}$ , reasoning that the numerator

was a whole number and the denominator meant another third (p. 433). The ability to correctly solve verbally presented mixed number, but not those presented symbolically, extended to subtraction problems as one student answered that 1 minus  $\frac{4}{5}$  was  $\frac{3}{5}$  since he said the 1 represented  $\frac{1}{5}$  (p. 434). Other mixing of the concepts of wholes and fractions was demonstrated by a student who said that  $\frac{1}{4}$  plus  $\frac{2}{4}$  equaled 3 (p. 437). Tzur (1999) found that children also had difficulty with the concept of improper fractions. While two fourth grade students were able to create and name non-unit fractions using a computer program, when presented with models of  $\frac{9}{8}$ , they initially named the fraction as  $\frac{9}{9}$  to fit their previously learned conceptions (p. 390).

Roddick and Silvas-Centeno (2007) conducted an exercise of middle school students solving division of fraction problems, using pattern blocks. Besides noting that students did not understand the relationship between the numerator and denominator, these students also did not recognize the need to create equal sized pieces when dividing a whole into parts. Further complicating the efforts to remediate these misconceptions was the fact that students reverted to using partially learned algorithms in solving the problems. This use of incorrect algorithms was also noted by Siegler and Pyke (2013). In a study comparing sixth and eighth graders with low and high mathematics achievement, they found that use of incorrect algorithms was more common in multiplication and division of fractions than in addition and subtraction. This was largely attributed to the student using previously learned whole number algorithms to solve fraction problems. Siegler and Pyke also noted that low-achieving students viewed the numerators and denominators as separate numbers when comparing fraction sizes.

Somewhat surprisingly, some of these same misconceptions are held by practicing or prospective elementary teachers. Ward (1999) asked her junior and senior elementary education majors to work in groups to arrange fractions in ascending order using dominoes to represent the fractions. In the discussion that followed, some students insisted that they had to do the more laborious task of finding common denominators, rather than using mental pictures or knowledge of benchmark fractions to complete the arrangement. This likely indicates a lack of conceptual fraction knowledge. Other groups insisted that the larger the denominator, the smaller the fraction—completely ignoring the relationship of the numerator to the denominator. Still others insisted that a fraction with a zero as both numerator and denominator was equal to 1, thus being the largest fraction in the group!

Similar errors were found by Matthews and Ding (2011). In a study of 163 education majors, they found that many students solved addition problems by adding both the numerator and denominator, again disregarding the numerator–denominator relationship. Other common errors were failing to consider benchmark fractions when comparing fractions and the inability to find equivalent fractions.

According to Wu (2011), many of these teachers' misconceptions occur because they themselves were never given adequate and accurate instruction in the conceptual aspects of fractions. He further states that while teachers learn how to perform computations with fractions, few learn the mathematic principles behind fractions. This causes teachers to teach fractions in a way that is incomprehensible to students. As an example, he points out that teachers simultaneously introduce fractions to their students as parts of a whole, as ratios, and as division—all without giving conceptual

explanations. This causes students to “believe that there is a mysterious quantity called fraction that possesses three totally unrelated properties and then also asks them to compute with this mysterious quantity in equally mysterious ways” (Wu, 2011, p. 375).

These observations are not new. In 1990, Ball conducted a longitudinal study of 252 elementary and secondary math teachers’ understandings of mathematics as they transitioned from college students to practicing teachers. She found that most teachers could accurately perform calculations, but when asked to select a word problem that represented division of fractions, only 30% chose the correct answer on a multiple-choice test. Further, because more than one answer could be selected, of those who chose the correct answer, 30% of the teachers also chose the word problem that represented *multiplication* of fractions (p. 454). Because of this, Ball believes that teachers need to have a deep conceptual knowledge of mathematics themselves. This is necessary, she says, because teachers of mathematics must also be able to “interpret and appraise students’ ideas, helping them to extend and formalize intuitive understandings and challenging incorrect notions” (p. 458). Although Ball’s work predates the development of the Common Core State Standards for Mathematics (CCSSM), the teaching skills she outlines are especially relevant considering the conceptual focus of the CCSSM.

### **Teaching Practices That Promote Conceptual Mathematic Reasoning in Struggling Learners**

Certain instructional practices have been shown to help low-achieving students develop mathematic reasoning skills. The National Mathematics Advisory Panel (2008) recommended that struggling learners receive “some explicit systematic instruction that includes opportunities for students to ask and answer questions and think aloud about the

decisions they make while solving problems” (p. 49). Following this, Gersten et al. (2009) were charged by the Institute of Education Science (IES) to review research to further identify practices that help students who have low math achievement. The resulting guide, *Assisting Students Struggling with Mathematics: Response to Intervention (RTI) for Elementary and Middle Schools*, recommended effective practices for students who need additional math assistance in an RTI setting. Of the eight recommendations made, one echoes NMAP recommendations for discussion as a component of mathematics instruction: “Instruction during the intervention should be explicit and systematic. This includes . . . verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review” (Gersten et al, 2009, p. iii).

*Discussion* as part of mathematics instruction is viewed as critical enough that three of the six National Council of Teachers of Mathematics (NCTM) Standards for Teaching Mathematics relate to this issue. According to Standard 2,

The teacher of mathematics should orchestrate discourse by

- posing questions and tasks that elicit, engage, and challenge each student’s thinking;
- listening carefully to students’ ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students’ ideas;

- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate. (NCTM, 2014b, Standard 2)

In searching for empirical studies in a review of literature from 1990-2008, Misquitta (2011) found only 10 that covered fraction instruction for struggling students that were from peer-reviewed sources, met NCTM standards, and were conducted in U.S. classroom settings (p. 111). Of these, only four focused on fraction concepts, rather than computation, and in only two of these did interventions prove effective. Both effective intervention studies (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Jordan, Miller, & Mercer, 1999) used an instructional sequence known as CRA, where students transitioned from use of Concrete manipulatives to Representational models, such as drawings and number lines, to use of Abstract symbols. The Butler et al. study also provided explicit instruction and frequent feedback.

More recently, Fuchs et al. (2013) conducted a study of a fraction intervention with 259 fourth grade at-risk students. In this investigation, a control group received instruction using a standard math program that focused on area models of fractions and computation. The experimental group received the same classroom instruction, but also received a 12-week intervention that focused on a measurement model of fractions three times per week for 30 minutes. In this intervention, the tutor introduced a concept, “providing explanations in simple language, requiring students to repeat explanations in their own words, and checking for understanding frequently (to reduce the language demands of instruction)” (Fuchs et al., 2013, p. 697). Students then worked in groups



where they practiced strategies and applied the concepts they had been taught. Leadership of the group was rotated and each student was held responsible for working the problem. Their findings: “By any standard, the effects of intervention designed to foster measurement interpretation of fractions for AR [at-risk] fourth graders were strong, with the achievement gap for AR learners substantially narrowed or eliminated” (Fuchs et al., 2013, p. 698).

To evaluate a discussion-based mathematics intervention program that covered fractions as well as other content, Ketterlin-Geller, Chard, and Fien (2008) conducted a study of 51 fifth grade students who received low scores on mathematics achievement tests. They split these students into three groups, two of which received interventions outside of their regular math class. One of the experimental groups received an extended session of the regular math curriculum, while the other received a specific intervention called Knowing Mathematics. Knowing Mathematics, a program in which students review and reinforce key math concepts by reading specially designed problems in a comic book format, is based on class discussion focusing on students’ mistakes and misconceptions. While both intervention groups made greater gains on achievement tests than did the control group, Ketterlin-Geller et al. predicted that the students who received the Knowing Mathematics intervention would be better prepared for future advanced mathematics.

Other studies of a more qualitative nature have identified effective instructional programs or methods focusing on fraction interventions for struggling students. Some of these programs have been developed and field tested in classroom settings. Pitsolantis and Osana (2013) created a fraction unit that uses three stages of instruction. They call

these stages “sites,” or points in time where concepts could be linked to symbols or rules to make mathematics meaningful (p. 20). In the first stage, mathematic symbols were linked to manipulatives, drawings, or real-world models during a class discussion. Next, the teacher linked the underlying fraction concept to a computational procedure. Again, drawings, manipulatives, or real-world models were used. Finally, teachers encouraged students to use models to check the reasonableness of their answers. Pitsolantis and Osana reported improved understanding of fraction concepts in their fifth and sixth grade students by this linking of concepts to models.

Another teacher used clocks as fraction models in an afterschool intervention with a struggling student, Lucia, and her friend, Kayla (Chick, Tierney, & Storeygard, 2007). Although Chick initially thought Lucia was the weaker math student, she found that Lucia could more easily use the clock models to demonstrate conceptual knowledge and perform calculations than could Kayla. Chick discovered that even though Kayla had performed better on previous tests, she did so because she had merely memorized algorithms. “This episode reveals the importance of encouraging and understanding student reasoning. . . . If Candace had relied only on the class written work, she might not have understood the girls’ true strengths and weaknesses” (Chick et al., 2007, pp. 56-57).

Bray (2011, 2013) also advocates using student reasoning to guide instruction. In a 2011 case study of teacher knowledge and beliefs, Bray found variations in ways four teachers in an urban setting handled student misconceptions in class discussion. Three of the teachers were reluctant to discuss misconceptions, thinking that students would be confused and that bringing up misconceptions would cause embarrassment. The other teacher purposely planned discussions around misconceptions, believing that this was

essential for increasing student knowledge. This teacher also created a classroom culture in which making and discussing mistakes was viewed as a part of the learning process, thus increasing student engagement. Bray (2013) reports that by choosing mathematically rich problems, anticipating and planning for errors, and planning for and guiding discussions, students' conceptual understanding can increase. Additionally, Bray found that teachers who themselves had deeper conceptual knowledge of mathematics were more skilled in leading productive class discussions

Creating a classroom climate that encourages children to discuss their mathematical thinking is the focus of a 2008 article by Kline. Based on numerous observations of elementary teachers in classroom settings, Kline recommends that teachers allow for the diversity of learners by allowing students adequate time to process their thoughts, encouraging students to question one another's solutions and strategies, and allowing students to struggle. A further recommendation concerns use of student misconceptions in the course of class discussion. "One of the single most powerful ways to make an impact on young children's thinking is by accepting incorrect answers or ideas as a natural part of doing mathematics and pursuing them in the same ways as correct solution" (p. 146). Kline concludes by suggesting that teachers who continuously reflect on their own teaching practices that center on classroom discussion can encourage students to take risks and increase their abilities to think mathematically (p. 151).

Identifying student reasoning and using this knowledge to inform instruction is central to a mathematics instructional approach known as Cognitively Guided Instruction (CGI). Based on problem solving, teachers who use this approach believe that even very young children have innate abilities to solve mathematical problems prior to receiving

formal instruction. Although there is no prescribed way to teach using CGI, Carpenter, Fennema, Loef Frank, Levi, and Empson (1999) found several common characteristics in teachers using this method. One is that teachers base instruction on carefully selected word problems and provide manipulatives or “tools” to the students to aid them in solution. Another is that students are encouraged to communicate their solutions through discussion, writing, or drawing pictures. Teachers are then able to use this knowledge of student thinking to further develop students’ conceptual understanding, all while carefully creating an environment that encourages student engagement. Early research on CGI (Peterson, Carpenter, & Fennema, 1989) compared teaching methods of 20 first grade teachers and found that those teachers who used discussion and questioning of students’ problem solving had students who scored higher on problem-solving tasks than did students of teachers who merely explained how to complete problems and then checked problems nonverbally.

Empson and Levi (2011) further extended CGI principles into teaching fraction concepts. Believing that children have the innate ability to find equal shares, this becomes the starting point for fraction instruction. Empson and Levi found that by carefully crafting word problems, even very young children can solve those involving multiplication and division of fractions. The key to success, they say, is that teachers listen carefully to the students’ ideas. This allows students to feel that their ideas have merit and thus encourages involvement and willingness to take risks. First grade classroom teachers who piloted some of Empson and Levi’s early work on fractions reported amazement about the conceptual understanding of fractions demonstrated by their students. As one such teacher said of her former fifth grade students, “I don’t think

they understood as profoundly what fractions were about as these first graders do now. It just became a bunch of symbols to them at that point, and they knew how to manipulate the symbols, but I don't think they really knew what they were doing" (Empson & Levi, 2011, p. 232).

In order to develop the conceptual thinking required in the Common Core Curriculum, then, teachers must not only develop their own fraction understandings, but must also be skilled at developing those of their students. This calls for teachers to teach differently than they themselves were taught.

The teacher's role has changed. Teachers must listen carefully, understand children's thinking, and evaluate whether that thinking is logical. Then, on the basis of this analysis, teachers must formulate new questions or tasks to dispel misconceptions, deepen understanding, move children to more efficient or sophisticated methods, or challenge them to consolidate their procedures. (Flowers, Kline, & Rubenstein, 2003, p. 300)

### **Assessment of Math Reasoning**

Assessment of a student's mathematical reasoning is the key to discovering and providing interventions for misconceptions. The National Council of Teachers of Mathematics (2014a) believes that assessment "should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done *to* students; rather, it should also be done *for* students, to guide and enhance their learning" (para. 1). Assessment is also a critical part of the RTI process:

Following the RTI model in mathematics requires that classroom teachers, who are charged with providing high quality initial instruction to all children; interventionists, who are charged with closing gaps; and special educators, who are charged with providing students with compensatory skills and strategies, understand how to assess for key mathematical ideas and how to translate the diagnostic information into meaningful learning experiences that connect with the underlying structures of numbers. (Koeller, Colesman, & Risley, 2011, p. 55)

According to the Formative Assessment for Students and Teachers State Collaborative on Assessment and Student Standards (FAST SCASS, 2012), there are many types of assessments: summative, formative, progress monitoring, universal screening, diagnostic, interim, and curriculum embedded; however, for purposes of this study, only universal screening assessments and formative assessment will be discussed.

Use of a universal screening assessment is the first step in identifying students who are in need of intervention in an RTI program. Not coincidentally, to “screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk” is also the first recommendation in the IES Practice Guide: *Assisting Students Struggling with Mathematics: Response to Intervention (RTI) for Elementary and Middle Schools* (Gersten et al., 2009, p. 13). By definition, a universal screening instrument is a brief assessment given to all students at regular intervals, generally two to three times a year in a group setting. This screening instrument also needs to be valid and reliable and have a focus on critical content standards (Hughes & Dexter, n.d.). The IES Practice Guide reports research on effective screeners focusing on number sense for the primary grades. For upper grades, however, they could find few studies of screeners which met their recommendations of a focus on mathematical understanding as well as on computation. Given this recommendation as well as the emphasis of the Common Core State Standards for Mathematics on solving problems and communicating mathematical ideas, the task of finding a universal screening instrument that assesses conceptual reasoning becomes an important one.

Also vital to the RTI process, as well as for any mathematics instruction, is the use of formative assessment. In a report prepared for the Council of Chief State School

Officers, FAST SCASS (2012) defines formative assessment as “a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students’ achievements of intended instructional outcomes” (p. 5). The importance of formative assessment is asserted in a NCTM position paper:

Through formative assessment, students develop a clear understanding of learning targets and receive feedback that helps them to improve. In addition, by applying formative strategies such as asking strategic questions, providing students with immediate feedback, and engaging students in self-reflection, teachers receive evidence of students’ reasoning and misconceptions to use in adjusting instruction. By receiving formative feedback, students learn how to assess themselves and how to improve their own learning. At the core of formative assessment is an understanding of the influence that assessment has on student motivation and the need for students to actively monitor and engage in their learning. The use of formative assessment has been shown to result in higher achievement. The National Council of Teachers of Mathematics strongly endorses the integration of formative assessment strategies into daily instruction. (NCTM, 2013, p. 1)

Sammon, Kobett, Heiss, and Fennell (1992) suggest that teachers can incorporate this formative assessment into daily instruction through observation and asking probing questions, with individual student interviews, and by analyzing problem solving tasks.

Stylianou, Kenney, Silver, and Alacaci, (2000) advocate asking students to explain their solutions to problems. “Looking at how students justify and make sense of their answers can yield information that is lost when students give only a numerical answer” (p. 141).

Despite the importance of formative assessment, however, research has found that many teachers have not developed skills in using these techniques of formative assessment or in how to use assessment results to inform instruction. The Consortium for Policy Research in Education (CPRE) developed an online assessment called the Teacher Analysis of Student Knowledge (TASK) to evaluate teachers’ abilities to use assessments

to guide students toward learning targets. In this assessment, teachers were presented with a problem and a set of student responses from one of five grade-level-appropriate content areas. Teachers used this information to complete seven steps; among them was to answer the problem, explain the conceptual knowledge needed for the student to solve the problem, decide if each of the student responses had mathematic validity, comment on the conceptions or misconceptions shown in four of the students' solutions, and suggest the next instruction steps for two students who demonstrated incorrect or less sophisticated responses. The teacher's appraisals of student answers were then rated across what were considered four domains of formative assessment: teacher knowledge of the mathematical concepts behind the tasks, teachers' identification of student conceptions and misconceptions, teachers' knowledge of underlying learning targets, and teachers' ability to make appropriate instructional decisions based on individual student needs.

Results of the TASK field test suggest that teachers need more assistance in both conducting formative assessment as well as using results to inform instruction. While almost all 376 teachers (98%) in the field test of the grades 3-5 fraction content area could correctly answer the problem, only 50% of the teachers were able to identify the conceptual knowledge needed by the students in order to solve it (Supovitz, Ebby, & Sirinides, 2013, p. 11). When analyzing the student responses, 80% made computational or general analyses, rather than identifying underlying student conceptions or misconceptions (p. 16). Only 18% of the teachers gave responses that dealt with student misconceptions or with moving a student to a more sophisticated conceptual understanding when identifying next instructional steps for individual students (p. 25).



Across the domains examined on the TASK, there were more procedural responses than any other category. In fact, with the exception of Analysis of Student Thinking in fractions, the procedural responses outnumbered conceptual and learning trajectory responses combined. Given the emphasis in the Common Core State Standards on rigor as a balance between conceptual and procedural understanding, this suggests that there is a great deal of room for growth in teacher capacity to identify, interpret, and respond to students' conceptual understanding. (Supovitz et al., 2013, p. 27)

Clearly, then, there is a need for formative assessment tools that assist the teacher in developing the ability to identify student misconceptions as well as to target interventions based on the underlying concepts behind these misconceptions.

In 2012, the Math Reasoning Inventory (MRI) was released. According to the Technical report of the MRI,

The purpose of the Mathematics Reasoning Inventory (MRI) assessment tool is to make teachers' classroom instruction more effective. Specifically, the online formative assessment is primarily designed to serve as a tool for conducting face-to-face interviews that focus on core numerical reasoning strategies and understandings. Because students respond to questions by explaining their thought processes, the interviewer can record both the students' accuracy and the strategies they used to solve problems. This information is translated into rich, descriptive narratives for each individual as well as an entire class to help guide the teacher's next steps. (Wilmot, 2012, p. 4)

The MRI measures concepts of whole number, decimal, and fractions based on content from the Common Core State Standards covered prior to sixth grade. Developed by six mathematics education experts over a 3-year period, questions assess what are considered to be "Essential Understandings." In the area of fractions, these understandings are that the student uses the relationship between numerators and denominators when comparing fractions, applies concepts of whole number operations to operations with fractions, shows knowledge of equivalence, can relate fractions to decimals, can solve applied problems, and can estimate (p. 5). Four reports are instantly generated at completion of the assessment: two individual student reports and two whole-class reports; each of these

reports lists the appropriate or inappropriate strategies students used to solve the problem. Although the MRI was designed as a formative assessment to inform teaching practices, the student interview is reported to take only 10 minutes, which may make the MRI useful as a universal screener as well.

Prior to release of the MRI, Wilmot (2012) produced a Technical Report of the Math Reasoning Inventory. In it she details the development process and reports reliability and validity measures. Throughout the 3 years of development, questions were field tested by a total of 81 teachers with a diverse group of 525 students. Feedback from these teachers was used to eliminate and improve questions and improve some of the online functions. Finally, information from a teacher survey was used to verify usefulness of the MRI as a tool to assist teachers in improving curriculum and instruction. As part of this survey, teachers were asked to comment on what they had learned about their students. Many of these responses related to the student's dependency on algorithms to solve problems and an accompanying disregard for whether or not the answer was reasonable. Related to this, teachers realized that they could not assume that fluency with algorithms equated to conceptual understanding, even when the student's answer was correct. Other teachers saw the need to ask more probing questions in order to identify areas where interventions were needed (pp. 40-41). These survey comments indicate that the Math Reasoning Inventory may be a valuable tool in assisting teachers to identify students' misconceptions and to suggest areas of needed interventions.

## Summary

Today's teacher of mathematics faces several challenges. Developing conceptual knowledge of fractions has been shown to improve students' problem-solving skills and performance in higher math classes, yet studies have shown that some teachers themselves do not have a solid conceptual knowledge of fractions or of how to teach them. Much of this can be attributed to the fact that many teachers were taught under a model that equates computational fluency with math proficiency. Given the more conceptual basis of the Common Core State Standards for Mathematics, procedural knowledge alone is no longer enough.

Another challenge faced by today's teacher is that of differentiating instruction. Under an RTI program, teachers are expected to provide a quality research-based education to all students while identifying and correcting students' misconceptions and provide interventions to those students who struggle. Because students benefit from instruction delivered in a discussion-based format that allows for "thinking aloud," teachers need to employ good questioning techniques and encourage students to share their thought processes and justify their answers. Teachers also need to incorporate formative assessment as a standard instructional practice. Studies have shown, however, that teachers do not necessarily possess these necessary skills. Moreover, even when teachers are able to identify student conceptions and misconceptions, studies have shown that they are not always able to use this knowledge to inform their teaching practices.

The Math Reasoning Inventory (MRI) was designed to help teachers meet these challenges. This online assessment provides teachers with problems and probing questions to present to their students; responses are categorized in terms of whether the

student used appropriate or inappropriate strategies to solve the problem. This information can be used by the classroom teacher to target needed whole class or individual student strategy instruction. Teachers who took part in the MRI field testing reported that the MRI would help them change teaching practices and point to interventions needed by students. Research needs to be done to see if the MRI is indeed a tool that assists classroom teachers in identifying needed interventions and in developing a wider repertoire of teaching practices.

## **CHAPTER III**

### **METHODOLOGY**

#### **Design**

“Learning is too complex a phenomenon to be the sole province of any one discipline, theoretical perspective, or research method” (Bell, 2004, p. 243). With this in mind, I employed a mixed-design study using qualitative and quantitative methods in order to conduct a preliminary evaluation of the Math Reasoning Inventory (MRI). Specifically, I used a mixed-method design called the triangulation design. In this design, qualitative and quantitative data are collected concurrently and are given the same weight. Further, the triangulation design is time-efficient and is often the choice of researchers who, like me, are new to mixed-method design (Creswell & Plano-Clark, 2007). The validating quantitative data model is a variant of the triangulation design and was chosen for this study in order to better verify research results by combining the types of data, as well as to improve some of the weaknesses inherent in using one method alone.

In qualitative research, a multiple-case study approach uses several data sources to provide in-depth description of the cases (Creswell, 2007; Marshall & Rossman, 2011). In this study, the cases to be considered are four elementary teachers. Additionally, participants in multiple-case studies are a bounded by a process, phenomenon, or an activity (Creswell, 2007; Marshall & Rossman, 2011), which, in this investigation, is the administration of the Math Reasoning Inventory. Further, case

studies can be used to provide in-depth, detailed evaluations (Marshall & Rossman, 2011). These evaluative data were collected by observing teachers' math instruction and through open-ended questions on the concluding survey. The MRI Item Analysis Report also provided qualitative data.

A multiple-case study design was also employed to produce one of the quantitative measures. Using a multiple-case study within-subject A-B design (Poling, Methot, & LeSage, 1995), baseline measures from observations were taken (A), followed by measures taken after treatments (B). In this instance, the treatments were the training for and first administration of the MRI. This is described in further detail in a subsequent section. Another of the quantitative measures was the result from a pretest–posttest design, using measures from the MRI Group and Item Analysis Reports. Finally, quantitative data from a teacher survey were collected at the conclusion of the study.

To summarize, the purpose of this mixed-methods study using a design known as the triangulation design was to concurrently gather quantitative and qualitative data in order to evaluate the effectiveness of the Math Reasoning Inventory (MRI) in improving teaching practices and, as a result, student achievement. Pretest and posttest data, as well as quantitative data gathered in observations and surveys, explored the relationship between teacher behavior and the MRI. Qualitative data were concurrently gathered through observation, MRI item analysis, and open-ended survey questions in order to give a more detailed evaluation of the effectiveness of the Math Reasoning Inventory.

The overarching question guiding this study was: What effect does the Math Reasoning Inventory (MRI) have on student achievement in fractions? Subquestions to be addressed were: Did teachers find the MRI useful in identifying students with

misconceptions in reasoning about fractions? Did use of the MRI change teacher instructional practices? Did use of the MRI suggest any needed interventions?

### **Participants**

Since the Math Reasoning Inventory assesses Common Core State Standard knowledge that students should acquire prior to the sixth grade (MRI, 2012), a targeted convenience sample of four volunteer classroom teachers—two fourth grade teachers and two fifth grade teachers—were recruited from one upper elementary building where I am also employed. Following the approval of the Human Subjects Institutional Research Board (see Appendix A), recruitment posters (see Appendix B) were placed in the office and in teacher workrooms. Five teachers responded—three from fifth grade and two from fourth grade. The first two teachers from each grade level to volunteer were given and subsequently signed an informed consent document. All four volunteers were veteran teachers, with years of experience ranging from 6 to 36 years and a combined teaching experience of 71 years.

Due to recommendations from the authors of the Math Reasoning Inventory that the inventory be used with fourth to eighth grade students, third grade classroom teachers from this building were excluded. Likewise, teachers who do not give math instruction or teachers who were not employed in this same building were also excluded from the study.

## Setting

This study took place in a geographically large district in Southwest Michigan, which has areas ranging from suburban to rural. This district has one high school, one middle school, one third through fifth grade building, and two early elementary buildings, as well as a variety of alternative education settings. The district is quite racially homogenous: 86% of the approximately 3,000 students are white, 5% are multiracial, 4% are African American, 3% are Hispanic, and 1% is Asian (Michigan Department of Education [MDE], 2014b). The district is also relatively affluent with only 22% of students qualifying for free and reduced lunch (MDE, 2014a).

The study participants teach in the district's third through fifth grade building, which has an enrollment of approximately 600 students in 25 general education and 3 special education classrooms. Contractually, general education classroom caseloads can only be 27 students; however, most classes have 24 or 25 students. In its fourth year with a four-tiered RTI program, this building also employs a full-time reading interventionist as well as me, a full-time math interventionist, who provide Tier III interventions. Tier II interventions are provided by classroom teachers, a part-time interventionist, and two paraprofessionals. Special education is considered as Tier IV.

While this district boasts high scores on the state mathematics test, there are some strands that are shown to be persistent areas of weakness. One of these is fractions. To identify those specific Common Core State Standards of concern, students take a math screener, the Delta test, as part of the RTI process. Published by the Ottawa Area Intermediate School District (OAISD), the Delta test assesses what are considered to be critical standards from the previous grade (OAISD, 2014). Comparison of fractions,



addition and subtraction of fractions, and identification of fractions on a number line have been shown to be areas of struggle for the 3 years this district has administered the Delta test. Since these are Tier I, or core grade-level instructional issues, the district agreed to fund the substitute teacher costs incurred in this pilot study of the fraction portion of the MRI as part of the building's school improvement plan and provided support in terms of access to teachers and facilities.

## **Instrumentation**

### **Observation Protocol**

Participating teachers were each observed in three half-hour sessions to assess how time was spent during math class. I created an observation protocol (see Appendix C) to facilitate data collection. This document has four sections. At the top are spaces to record general information, such as the date and length of the observation, the specific fraction topic, and the number of students in attendance. At the center of the form are spaces to tally the number of minutes spent in each of four typical math activities: direct teacher instruction, group discussion, individual student work time, and group (or partner) work time. Additionally, spaces are left to tally data specific to discussion: the numbers of students who gave explanations or asked questions, the number of any misconceptions addressed, and the number of probing questions asked by the teacher. (I defined probing questions as those asking students to explain or justify their answers, rather than questions where teachers are merely seeking the right answer.) The lower part of the form leaves spaces to tally the number of times the teacher models strategies assessed on the MRI, whether considered appropriate or not appropriate. These are:

comparing mentally, using the relationship between numerator and denominator (appropriate) or converting to common denominator (inappropriate); computing mentally, using knowledge of equivalence, extending knowledge of whole number operations to fractions, using benchmark fractions (appropriate), or using standard algorithm or finding exact answer (inappropriate); and applying understanding by modeling with mathematics or showing understanding of equivalence (appropriate). Finally, there is space to record any additional observations.

### **Math Reasoning Inventory**

Participating teachers administered the fraction portion of the MRI, which is accessed online, to their students near the beginning of the unit on fractions and again at the end of that unit. The Group Report and Item Analysis Report were used to compare pretest and posttest quantitative data; the Item Analysis Report lists the percentage of the students who correctly answered each question, while the Group Report lists the percentages of students who used each strategy as noted above, whether appropriate or inappropriate, in the categories of comparing mentally, computing mentally, or applying understanding. The Item Analysis Report was also used to determine the exact reasoning strategy used by students who answered each question correctly. These reports were accessed online immediately following administration of the MRI.

### **Teacher Survey**

Following the second administration of the MRI, teachers were asked to evaluate the MRI by completing a survey (see Appendix D). This included five questions rated on

a 5-point Likert-type scale, as well as four open-ended questions or prompts. The teachers were asked to *strongly agree* to *strongly disagree* to the following statements:

The inventory was easy to administer.

- The inventory changed the way I teach.
- The inventory identified the students who need interventions.
- The inventory identified areas where I need to make interventions.
- I recommend that use of this inventory be continued and expanded.

Open-ended questions were:

- What, if anything, did you find most useful about this inventory?
- What, if anything, did you find least useful about this inventory?

Prompts asked teachers to discuss anything new they learned as a result of giving the inventory and to comment on anything else they would like to share.

### **Procedures**

After obtaining informed consent from volunteers, I assigned each participant teacher a letter for identification purposes to allow for as much anonymity as possible. All subsequent reports refer to the participant by letter. It was also necessary that participating teachers send passive consent letters to parents of all of their students to both inform parents of this study and to allow parents to exempt their children from the assessment. By parent request, a total of 10 students were excluded from the study.

The first data collection occurred through observation of each teacher for 30 minutes during one of the initial lessons on fractions. The previously discussed observation protocol was used to record the data. While the intention was that I be as

much a nonparticipant as possible, I am a colleague as well as being known to the students; some of the students in each of the participants' classrooms also receive additional daily math interventions from me. The timeframe for these first observations was different for the fourth and fifth grade teachers due to the curriculum sequence of the respective grades. Fifth grade began the fraction unit in September and fourth grade in April. These first observations provided the baseline data to contrast with any possible differences in math instruction after training for and administration of the Math Reasoning Inventory.

The next activity was training the teachers to administer the MRI. This took place during a 3-hour period of release time during the school day. Again, the timeline was different for fourth and fifth grade teachers so that two training sessions were necessary. The MRI website provided all of the necessary training materials: a written and video overview of the MRI, administration guidelines, written questions and interview prompts, a PDF document to be printed as question cards for students, and an extensive library of videotaped student responses to allow teachers to practice identifying and selecting students' reasoning strategies (MRI, 2014).

The training began with a viewing of seven short video clips featuring Marilyn Burns, a well-known math educator and author of the MRI, who introduced and discussed the purpose of the Math Reasoning Inventory. Next, we viewed some video clips of teachers conducting MRI assessment interviews with students. This was followed by reading the guidelines for conducting student interviews along with viewing video clips to demonstrate these procedures. We then explored the 12 actual online assessment screens the teachers would be using for administration of the MRI. Each of these question

screens was divided into four sections. The first lists the question prompt with a view of the corresponding student card. The next section has answer bubbles for the teacher to select whether the student's answer was correct or incorrect, or whether the student self-corrected or did not answer. Another section has a question prompt asking the student to explain how he or she arrived at the answer, followed by six strategies that the teacher can choose from based on the student response. Finally, there is a space for the teacher to type in any notes. Once the teachers were comfortable with the assessment screens, we viewed video segments of students answering several of the questions. These videotape segments were used until there was agreement between the teachers and me as to the strategy the student explanation demonstrated. Next, we looked at samples of the four types of reports that are immediately available after the assessment. The Individual Report lists the strategies used by one student in the areas of comparing mentally, computing mentally, and applying understanding. The Assessment Review gives more detail of an individual student's answers for each question. Similarly, the Group Report lists the strategies used by the entire class in the above areas of mentally comparing, computing, or applying understanding, while the Item Analysis lists the entire class's answers and strategies for each of the questions. Our training ended with teachers logging into the MRI website and entering the names of their students whose parents allowed participation in the study.

At least one week after the MRI training, I again observed each teacher for a 30-minute period during a fraction lesson. My intent was to see if teachers delivered more of their instruction in a discussion format to identify students' understandings and misconceptions. I was also interested in seeing whether the teacher began to model some

of the reasoning strategies that were outlined during the training sessions. The observation protocol was used to record the data.

Midway through the fraction unit, teachers administered the Math Reasoning Inventory to students in their classes. Each had a half-day release time from their regular teaching duties to complete this. A computer, question cards, and a private space outside the classroom were provided for this purpose. Following suggestions of authors of the MRI, the teachers selected students with varied achievement levels as they assessed as many students as were possible in the given time. While my intention was that most of the students in each class would be assessed, the teachers found that the student interview took much longer than 10 minutes, as reported on the MRI website. The fifth grade teachers were able to assess only eight students each. One fourth grade teacher assessed seven students, the other five. I accessed the Group Report and the Item Analysis online after each assessment.

My final classroom observation was conducted at least 2 weeks after the teacher gave the MRI. I was interested to see whether teachers changed their instruction, perhaps as a result of any information learned from assessing their students. I again recorded all data on an observation protocol.

At the end of the unit on fractions, teachers had another half day of release time to conduct the MRI posttest with the students who had pretest scores. The procedures were the same as in the pretest. Finally, using the teacher survey, teachers evaluated the MRI as a conclusion to their participation in the study.

## Data Analysis

### Math Reasoning Inventory

Pretest and posttest scores available immediately after each administration of the Math Reasoning Inventory were used to determine any effects of the Math Reasoning Inventory on student achievement. Pretest and posttest scores of total correct questions as reported on each of the MRI Item Analysis reports were compared, using paired  $t$  tests on SPSS. First, mean scores for each individual teacher's students were analyzed, followed by an analysis of combined fifth grade scores and combined fourth grade scores. In order to determine any effects on the number of appropriate reasoning strategies used by the students between pretests and posttests, it first became necessary to compute the mean number of appropriate strategies used by students in each of the teachers' classes as reported on the Item Analysis. These means were calculated by dividing the number of appropriate strategies used by students by the total number of students who had corrected answered questions. Once the mean reasoning strategies scores for each category were computed, paired  $t$  tests were again calculated for each teacher's class. The null hypothesis was that there would be no statistical difference between the scores on the pretest and posttest on any of the paired  $t$  tests, with  $p = .05$ . The Group Report was used to determine the frequency, reported as percentages, of any student using any strategy in the question categories above. Again, pretest and posttest comparisons for each teacher were depicted graphically.

## **Observation Protocol**

Data from the observation protocol provided most of the measures to determine whether teachers' instructional behaviors changed due to the influence of the Math Reasoning Inventory. Quantitative data from observations were analyzed and depicted graphically. Minutes spent in each of the four typical math activities—teacher instruction, teacher-led discussions, individual student work time, and student group work time—were recorded for each teacher, using a graph appropriate for a multiple-baseline A-B within-subject design (Poling et al., 1995). The first observation provided the baseline information (A). This was followed by the treatments (B), consisting of MRI training and MRI administration. The purpose was to determine whether the MRI affects the time spent in discussion versus in direct instruction, as well as in group work versus student individual work time. Also depicted graphically were behaviors noted in classroom discussion: amount of student participation as measured in frequency of asking questions and providing explanations, and teacher frequency of asking probing questions or correcting misconceptions. Qualitative data were reported as a narrative and include further details on any classroom discussion, strategies modeled, and other behaviors of interest.

## **Teacher Survey**

The teacher survey was used to gather overall teacher perceptions of the Math Reasoning Inventory as well as to provide answers to the research questions of whether the MRI would help teachers identify students needing interventions or whether it would help them to determine any interventions or changes that needed to be made in their



teaching practices. In order to analyze the survey results, the four teachers' scores were combined into an average score for each of the questions. Once again, the open-ended qualitative questions were reported as a narrative.

### **Summary**

The purpose of this mixed-methods pilot study of the fraction portion of the Math Reasoning Inventory was to evaluate the effectiveness of the MRI in improving teaching practices and in any resulting changes student achievement in the classrooms of two fourth grade and two fifth grade teachers in the elementary school where I am employed. Quantitative and qualitative data were concurrently gathered from pretests, posttests, observations, and surveys. Statistical analysis was conducted on pretest and posttest scores; other quantitative data were depicted in graph or table form. Qualitative data from observations and surveys were reported as a narrative. Specific data analyses and their results are reported in the following chapter.

## **CHAPTER IV**

### **RESULTS**

The purpose of this study was to conduct a comprehensive pilot investigation of the fraction portion of the Math Reasoning Inventory (MRI) in order to determine what effects, if any, that training teachers to conduct the MRI and then actually administering the MRI to their students had on fraction instruction and subsequent student achievement in regular classroom settings. The overarching question guiding this study was: What effect does the Math Reasoning Inventory have on student achievement in fractions? Subquestions addressed were: Did use of the MRI change teacher instructional practices? Did teachers find the MRI useful in identifying students with misconceptions in reasoning about fractions? Did use of the MRI suggest any needed interventions?

This chapter reports data related to each of the questions in turn. For each, quantitative data will be reported first, followed by discussion of any qualitative data. The final section will report general teacher comments about the Math Reasoning Inventory taken from the Teacher Survey (Appendix D).

#### **Research Question One: Effects of MRI on Student Achievement**

Data from the MRI pretests and posttests were used to determine any possible effects on student achievement from teacher training for and administration of the MRI. Information from the Item Analysis and the Group Report was available online immediately after each testing session and was used to gather the quantitative data to

answer the research question: What effect does the Math Reasoning Inventory have on student achievement in fractions?

The first three analyses examined whether there were significant differences between pretests and posttests in total correct answers. In the first analysis, information from each classroom taken from the pretest and posttest data recorded in the Item Analysis were compared, using paired  $t$  tests in SPSS. The Item Analysis reports the percentage of students who correctly answered, incorrectly answered, or did not answer each of the 12 questions. Table 1 shows paired  $t$  tests results of the percentages of correct responses to the 12 questions ( $n = 12$ ) from students tested in the classes of each of the four participant teachers. Although results for all four teachers' students are reported in this same table, results for each teacher will be discussed individually.

Table 1

*Comparison of Pretest-Posttest Total Correct Answers from Four Teacher's Classrooms*

Teacher	Pretest		Posttest		$n$	95% CI for Mean Difference	$p$	$t$
	$M$	$SD$	$M$	$SD$				
A	69.92	24.74	74.00	23.59	12	-16.15, 7.98	.47	-.75
B	54.17	41.06	70.83	27.87	12	-32.31, -1.02	.04*	-2.35
C	73.00	24.20	71.58	25.14	12	-11.88, 14.71	.82	.25
D	58.50	30.29	57.25	27.73	12	-13.46, 15.96	.86	.19

\*  $p < .05$ .

For Teacher A, there was no significant difference in the total scores between the pretest ( $M = 69.92$ ,  $SD = 24.74$ ) and the posttest ( $M = 74.00$ ,  $SD = 23.59$ ) scores,  $t(12) = -.75$ ,  $p = .47$ . For Teacher B, there is a significant difference between total pretest

( $M = 54.17$ ,  $SD = 41.06$ ) and posttest scores ( $M = 70.83$ ,  $SD = 27.87$ ),  $t(12) = -2.35$ ,  $p = .04$ . However, Cohen's effect size value  $d = .19$  suggests low practical significance. For Teacher C, there is again no difference between pretest ( $M = 73.00$ ,  $SD = 24.20$ ) and posttest total scores ( $M = 71.58$ ,  $SD = 25.14$ ),  $t(12) = .25$ ,  $p = .82$ . Likewise, for Teacher D, no difference exists between pretest ( $M = 58.50$ ,  $SD = 30.29$ ) and posttest total scores ( $M = 57.25$ ,  $SD = 27.73$ ),  $t(12) = .19$ ,  $p = .86$ .

In order to see whether a significant difference existed between pretest and posttest results at the fifth grade level, the percentages of correct answers of the 16 students from the two fifth grade teachers were combined for each of the 12 questions ( $n = 12$ ) and a paired  $t$  test was again conducted. Table 2 shows the results of this test. For the combined fifth grade classroom scores, there was no significant difference in the total scores between the pretest ( $M = 71.50$ ,  $SD = 22.64$ ) and the posttest ( $M = 72.75$ ,  $SD = 23.84$ ) scores,  $t(12) = -.26$ ,  $p = .80$ .

Table 2

*Comparison of Pretest-Posttest Total Correct Answers from Fifth Grade Students*

	Pretest		Posttest		$n$	95% CI for Mean Difference	$p$	$t$
	$M$	$SD$	$M$	$SD$				
Combined total scores	71.50	22.64	72.75	23.84	12	-11.73, 9.23	.80	-.26

\*  $p < .05$ .

A third analysis was made to see whether a significant difference between pretest and posttest total correct responses exists at the fourth grade level. The percentages of correct answers from the 12 students of the two fourth grade classrooms were combined

for each of the 12 questions ( $n = 12$ ) and a paired  $t$  test was again conducted. Table 3 shows the results of this test. There was no significant difference between the fourth grade pretests ( $M = 56.75$ ,  $SD = 33.55$ ) and posttests ( $M = 62.08$ ,  $SD = 24.96$ ) scores,  $t = -1.01$ ,  $p = .34$ .

Table 3

*Comparison of Pretest-Posttest Total Correct Answers from Fourth Grade Students*

	Pretest		Posttest		$n$	95% CI for Mean Difference		$p$	$t$
	$M$	$SD$	$M$	$SD$					
Combined total scores	56.75	33.55	62.08	24.96	12	-16.99, 6.32	.34	-1.01	

\*  $p < .05$ .

For purposes of this study, finding a significant increase in students using appropriate reasoning strategies was more important than finding a significant increase in total test scores. The fourth analysis compared reasoning strategies used by students in pretests and posttests, also using a paired  $t$  test. Data presented in the Item Analysis first had to be manipulated to make it usable for this purpose. The Item Analysis reports the strategies used by students who correctly answered each question as a fraction. As an example, the Item Analysis may report that 5 of 6 students have used an appropriate reasoning strategy for a particular question, while 1 of 6 students used a strategy considered inappropriate. The fraction of each strategy considered appropriate was converted to a percent, which was then used for the paired  $t$  test scores for each question. The first three questions required mental comparison of fractions. For these questions, the strategy that was considered appropriate and was reported by the participating teachers

was using the relationship between the numerators or denominators to compare. Reported strategies considered inappropriate were guessing or converting to a common denominator. Next came six questions requiring mental computation of fraction problems. For this question category, reported strategies considered appropriate were using benchmark fraction to estimate or extending knowledge of whole number operations to those of fractions. Reported strategies considered inappropriate were finding the exact answer when asked to estimate or using the standard algorithm to compute. Finally, there were three problems requiring applying understanding of fractions. For these questions, the reported strategies considered appropriate were applying understanding of equivalence or in modeling with mathematics to solve the problem. There were no reported inappropriate strategies. Results from this analysis are reported in Table 4. Again, while data for all teachers are reported in this table, results for each teacher will also be discussed separately.

Table 4

*Comparison of Pretest-Posttest Appropriate Reasoning Strategy Responses from Four Teachers' Classrooms*

Teacher	Pretest		Posttest		<i>n</i>	95% CI for Mean Difference	<i>p</i>	<i>t</i>
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>				
A	69.17	32.55	75.25	24.68	12	-23.82, 11.65	.47	-.76
B	72.92	44.54	97.92	7.22	12	-54.52, 4.52	.09	-1.86
C	61.17	42.27	51.50	45.06	12	-10.00, 29.34	.30	1.08
D	84.72	25.09	93.74	11.87	12	-18.46, .41	.06	-2.11

For Teacher A, there was no significant difference in the use of appropriate reasoning strategies used by the students between the percentages for the pretest

( $M = 69.17$ ,  $SD = 32.55$ ) and the posttest ( $M = 75.25$ ,  $SD = 24.68$ ),  $t(12) = -.76$ ,  $p = .47$ .

Likewise, there was no significant difference for the percentages of correct strategies used by students for Teacher B pretest ( $M = 72.92$ ,  $SD = 7.22$ ) and posttest ( $M = 97.92$ ,  $SD = 7.22$ ),  $t(12) = -1.86$ ,  $p = .09$ ; for Teacher C pretest ( $M = 61.17$ ,  $SD = 42.27$ ) and posttest ( $M = 51.50$ ,  $SD = 45.06$ ),  $t(12) = 1.08$ ,  $p = .30$ ; or for Teacher D pretest ( $M = 84.72$ ,  $SD = 25.09$ ) and posttest ( $M = 93.74$ ,  $SD = 11.87$ ),  $t(12) = -2.11$ ,  $p = .06$ .

In order to determine the frequency with which students in each of the classrooms used particular strategies, the total percentages of students using each specific reasoning strategy were gathered from the Group Report for each teacher. The Group Reports list total percentages of any student using any of the reasoning strategies used to correctly answer questions in each of three categories: mental comparison of fractions, mental computation of fractions, and applying understanding of fractions. For example, if all of the students tested by one of the participant teachers used the relationship between the numerator and denominator as a strategy to correctly answer one or more of the questions requiring comparison of fractions, this would be reported as 100%; if none of the students used this strategy to solve any of the comparison problems, it would be reported as 0%. However, since any student who correctly answered an application problem also used a correct strategy of modeling with mathematics or applying understanding of equivalence, results have already been reported in previous analyses. Therefore, Table 5 reports only the percentages of total student use of each appropriate and inappropriate reasoning strategy in the question categories of comparing or computing from each teacher's pretest and posttest, as well as the change in the percentage of students using each strategy. As can be seen, the greatest changes were in the student use of standard algorithms for

estimation tasks and in changing to common denominators when comparing fractions, although changes were both positive and negative.

Table 5

*Percentages of Specific Reasoning Strategies Used by Students in Each Participant's Classroom*

Question Category	Teacher	Pretest	Posttest	Percent Change
Comparing Mentally (Appropriate)	A	50	62	+16
Uses relationships between numerators and denominators to compare	B	100	100	0
	C	38	29	-11
	D	86	100	+14
Comparing Mentally (Not Appropriate)	A	62	38	-24
Converts to common denominators to compare	B	0	60	+60
	C	100	86	-14
	D	0	0	0
Computing Mentally (Appropriate)	A	62	62	0
Uses benchmark of $\frac{1}{2}$ or 1 to make estimates	B	100	75	-25
	C	75	57	-18
	D	44	71	+27
Extends understanding of operations with whole numbers to operations with fractions	A	50	50	0
	B	50	100	+50
	C	38	71	+33
	D	43	43	0
Computing Mentally (Not Appropriate)	A	62	50	-12
Figures out exact answer when asked to estimate	B	0	0	0
	C	88	100	+12
	D	0	0	0
Uses standard algorithm to make estimates	A	25	50	+25
	B	25	0	-25
	C	38	100	+62
	D	0	0	0



### **Summary of Data on Student Achievement**

Five of the paired  $t$  tests regarding total test scores show no significant difference between the pretests and posttests of the MRI; for these, the null hypothesis cannot be rejected. The  $t$  test for Teacher B showed a statistically significant difference; however, the effect size showed low practical significance. For the four pretest–posttest comparisons of reasoning strategies, there was no significant difference shown and, again, the null hypothesis cannot be rejected. Therefore, the conclusion is that teacher training for use of and subsequent administration of the Math Reasoning Inventory had no significant impact on student achievement in fractions.

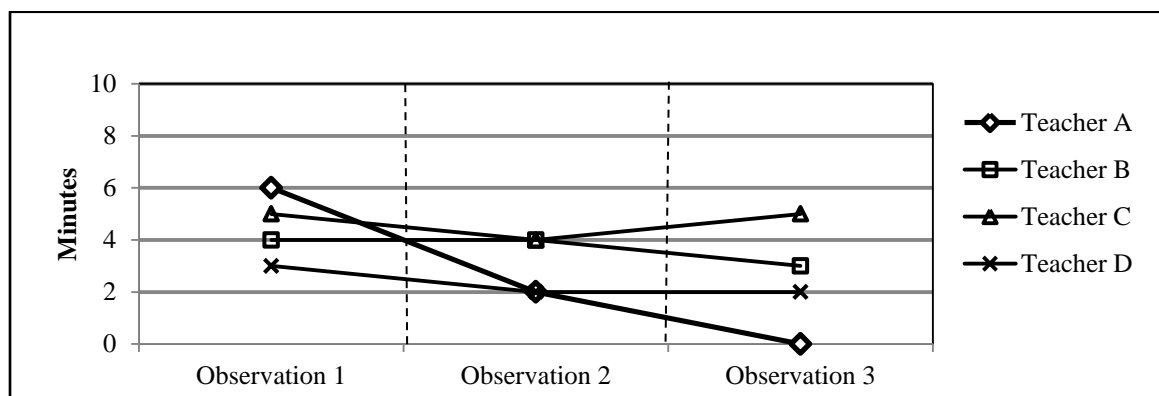
### **Research Question Two: Changes in Teaching Practices**

Data from classroom observations and the Teacher Survey were used to answer the question: Did use of the Math Reasoning Inventory change teacher instructional practices? Observational data documented any changes in how time was spent during math instruction as well as specific factors present in any whole class discussion (see the Observation Protocol in Appendix D). Observational data also noted any of the MRI reasoning strategies modeled by the teachers as well as other pertinent data. Finally, the Teacher Survey reported each individual teacher's perceptions of the MRI's effect on his or her instructional practices.

The first of the quantitative data for the three observations made of each of the four teachers are shown using multiple-baseline A-B within-subject graphs (Figures 1-4). In a multiple-baseline graph, *A* represents the baseline data, which in this study is shown using a differentiated line for each of the participants. The *B* represents the treatments,

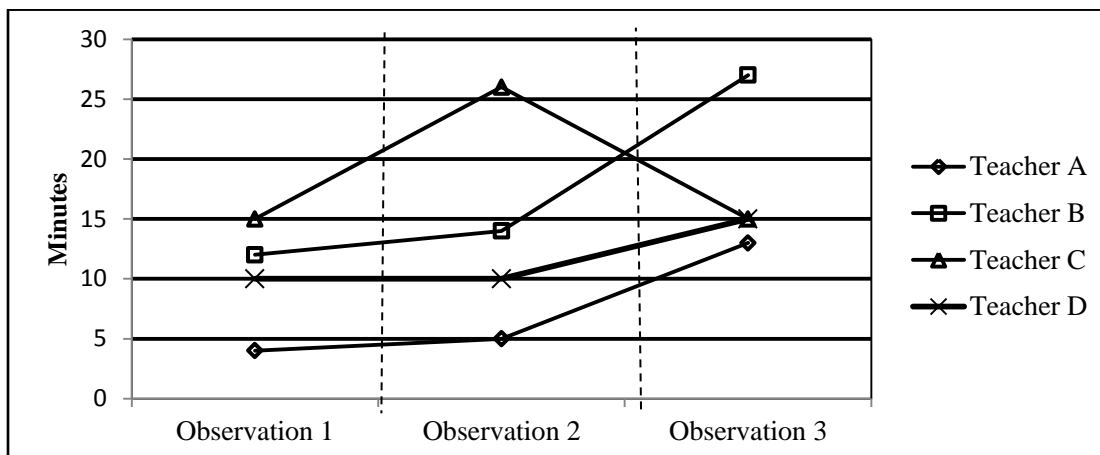
which in this study are the MRI training and the first administration of the MRI assessment and are represented by dotted vertical lines. Each of the observations was 30 minutes in length.

Figure 1 shows the number of minutes that the teacher used direct instruction during the 30-minute observation period. For purposes of these observations, I defined direct instruction as the teacher standing in front of the classroom and giving procedural directions. An example of this would be teaching standard computational algorithms. My reason for this measure was to determine whether the time spent giving direct instruction would decrease, given the focus on assessing student thinking modeled by the MRI.



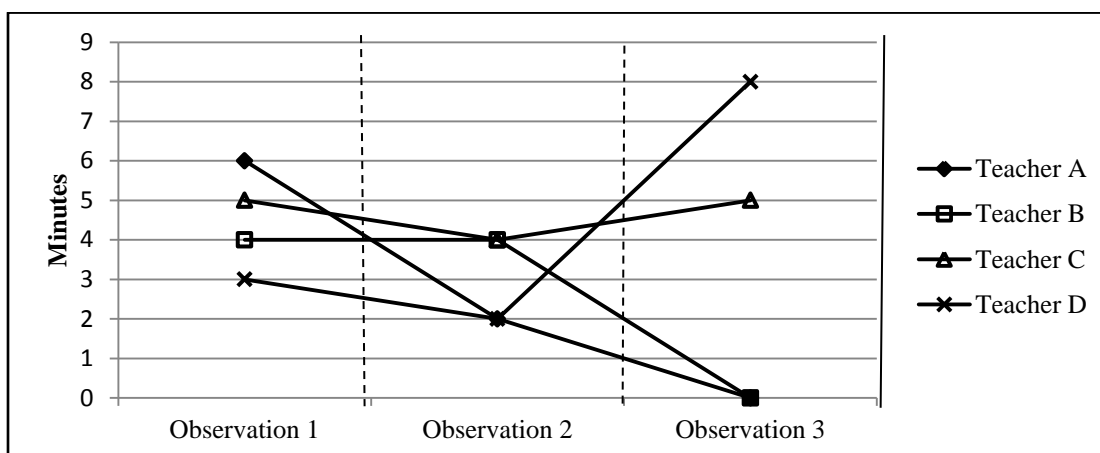
*Figure 1.* Minutes of teacher direct instruction observed. Observation 1 provided baseline data. Observation 2 occurred after training for the administration of the Math Reasoning Inventory, which is represented by the left dotted line. Observation 3 occurred after administration of the Math Reasoning Inventory pretest, which is represented by the right dotted line.

Figure 2 shows the number of minutes used in whole-class discussion. My intention for obtaining this measure was to see if teachers spent more time in classroom discussion as a result of the MRI training and assessment.



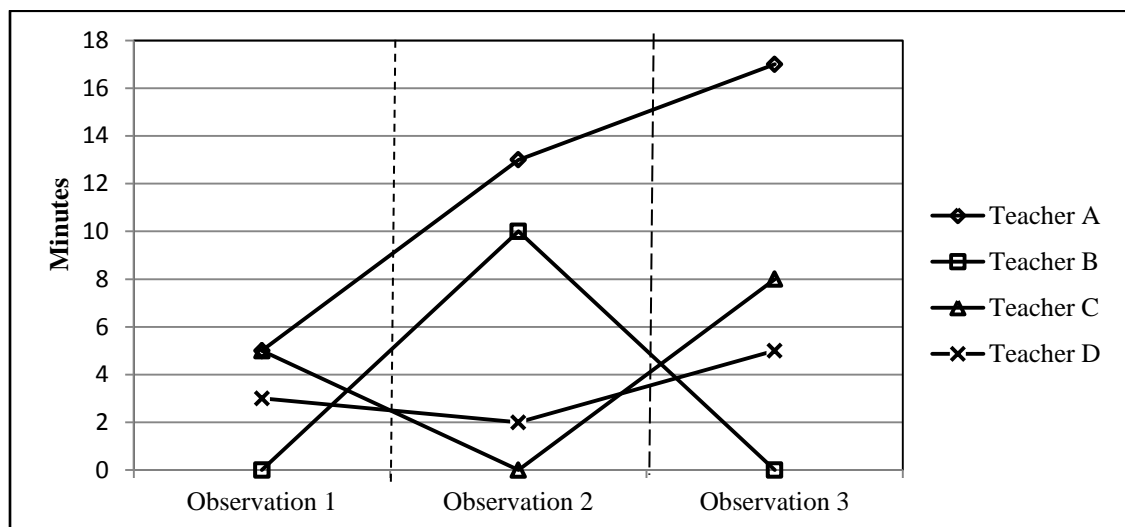
*Figure 2.* Minutes of classroom discussion. Observation 1 provided baseline data. Observation 2 occurred after training for the administration of the Math Reasoning Inventory, which is represented by the left dotted line. Observation 3 occurred after administration of the Math Reasoning Inventory pretest, which is represented by the right dotted line.

Figure 3 shows the amount of classroom time used for students to complete individual work. The intent for this was to see if student collaboration would increase, and thus time given for individual work would decrease as a result of the MRI.



*Figure 3.* Minutes of individual work time. Observation 1 provided baseline data. Observation 2 occurred after training for the administration of the Math Reasoning Inventory, which is represented by the left dotted line. Observation 3 occurred after administration of the Math Reasoning Inventory pretest, which is represented by the right dotted line.

Figure 4 shows the minutes of group or partner work time during the 30-minute observational period. As noted above, I wanted to determine whether student collaboration would increase as a result of training for and administration of the Math Reasoning Inventory.



*Figure 4.* Minutes of group work time. Observation 1 provided baseline data. Observation 2 occurred after training for the administration of the Math Reasoning Inventory, which is represented by the left dotted line. Observation 3 occurred after administration of the Math Reasoning Inventory pretest, which is represented by the right dotted line.

Within the classroom discussion, I also recorded frequencies of certain behaviors and occurrences. Specifically, I wanted to know if student participation would increase, as measured by the number of questions the students asked or by the number of times the student provided explanations as to how they solved problems. This was differentiated from the student merely answering a question, which was not recorded. I also wanted to see if teachers changed discussion behaviors due to the influence of the MRI. To this end, I recorded the number of probing questions asked by the teacher, defined as those

attempting to assess students' conceptions and misconceptions, rather than those asking for a correct answer. I also recorded the number of any student misconceptions addressed by the teacher. This was differentiated from the teacher merely correcting a procedural error, which again was not recorded. These data are reflected in Table 6.

Table 6

*Classroom Discussion Data*

Question Category	Teacher	Observation 1	Observation 2	Observation 3
Number of students who asked questions	A	5	4	3
	B	3	0	0
	C	0	0	0
	D	0	0	0
Number of students who gave explanations	A	3	7	0
	B	2	7	12
	C	3	8	7
	D	0	3	4
Number of teachers' probing questions	A	3	7	0
	B	3	0	8
	C	0	4	2
	D	3	3	4
Number of misconceptions addressed	A	4	4	8
	B	3	1	6
	C	6	4	5
	D	1	4	1

As can be seen in Table 6, there were few changes in the number of students who asked questions for any of the teachers. In fact, there were few students who asked questions. Teacher B showed the greatest increase as well as the greatest number of students who gave explanations of strategies used for solving problems, followed by Teacher C. There were no consistent trends in the number of probing questions asked by

any of the teachers. Teachers A and C addressed the most misconceptions; Teacher A and Teacher B showed the greatest increase in addressing misconceptions.

Finally, observations were made as to the ways and the number of times that teachers modeled any of the reasoning strategies which were assessed in the Math Reasoning Inventory. Teacher A used a number line to model equivalence and compare fractions during the second observation. For the third observation, colored tiles were used to model multiplication of fractions and to show equivalence. Teacher B used clocks, money, cooking, and other real-life examples to model understanding of fractions and equivalence during the first observation. In the second observation, use of benchmark fractions to estimate was modeled with fraction tiles. For the third observation, Teacher B drew models to extend knowledge of whole number operations to those of fractions by comparing repeated addition and multiplication, but used the standard algorithm to multiply fractions. In this observation, the teacher also drew models to show mixed numbers and improper fractions. Teacher C drew models and used number lines to demonstrate equivalence in the first observation. In the second observation, Teacher C compared amounts of money to benchmark fractions in order to estimate. During observation three, this teacher and drew models showing multiplication of fractions. Finally, Teacher D compared whole number addition and subtraction to those operations with fractions in the first and second observations and in the third observation extended knowledge of whole number multiplication as repeated addition to multiplication of fractions. Teacher D also drew models of fractions in each of the observations and modeled equivalence in the third observation by showing models of mixed numbers and improper fractions. None of the teachers were observed modeling the strategy of using

the relationship between the numerator and denominator to compare fractions, or used thinking aloud to demonstrate any of the strategies. The numbers of times each of the teachers modeled these strategies are shown in Table 7.

Table 7

*Numbers of Specific Reasoning Strategies Modeled by Teachers in Each Participant's Classroom*

Modeling Strategy	Teacher	Observation 1	Observation 2	Observation 3
Use of benchmark of $\frac{1}{2}$ or 1 to estimate	A	0	0	0
	B	0	2	0
	C	0	1	0
	D	0	0	0
Extending understanding of operations with whole numbers to operations with fractions	A	0	0	0
	B	0	0	4
	C	0	0	3
	D	1	3	6
Use of equivalence to compute	A	0	2	1
	B	2	0	0
	C	3	0	0
	D	0	0	1
Use of standard algorithm to make estimates	A	0	0	0
	B	0	0	2
	C	0	0	0
	D	0	0	0
Otherwise modeling with mathematics	A	0	4	3
	B	2	0	4
	C	6	1	2
	D	1	1	1

Teachers completed a survey at the conclusion of their participation in the study.

In order to obtain the participant teachers' perceptions of whether or not use of the Math

Reasoning Inventory changed their own teaching practices, this survey asked teachers to rate the following statement on a 5-point scale from *strongly agree* to *strongly disagree*: “The inventory changed the way I teach.” Two participants gave the rating of 3, or neutral, while two of the participants gave a score of 2, or disagree. This gave a total mean score of 2.5 for this item. Overall, teachers did not feel that the MRI changed their instructional practices.

### **Summary of Data on Teaching Practices**

Data from observations and teacher perceptions as measured on the Teacher Survey were used to determine whether there were any differences in teacher behaviors as a result of the teachers being trained for or administration of the MRI. Results of the observations showed the amount of classroom discussion increased in three of the four teachers, as did the amount of time spent in group work. The amount of time spent in direct instruction stayed the same; the amount of time given for individual work increased for two teachers and decreased for two. No consistent increases in student participation in discussion, in teachers’ use of probing questions, or in misconceptions addressed were noted. There were also few changes in teachers modeling reasoning strategies measured in the MRI. Finally, the Teacher Survey reflected the teachers’ perception that the MRI did not change their teaching practices. Therefore, data indicate that there were few changes in teachers’ instructional behaviors due to training for or administration of the Math Reasoning Inventory.



### **Research Question Three: Identifying Students with Misconceptions**

Information from the Teacher Survey completed at the conclusion of the study was also used to the answer to the research question: Did teachers find the MRI useful in identifying students with misconceptions in reasoning about fractions? The survey asked participant teachers to rate from *strongly agree* to *strongly disagree* with the following statement: “The inventory identified the students who need interventions.” Two teachers gave a rating of 3, or neutral; one teacher gave a rating of 4, or agree; and one teacher gave a score of 2, or disagree. This gives a total mean score of 3. This score may be somewhat inaccurate, however, as one comment on the open-ended question section made by a teacher who gave a neutral rating was that the MRI identified students who had no strategies to solve problems. This would seem to indicate that, at least for this teacher, the MRI identified students in need of strategy instruction.

### **Research Question Four: Identifying Needed Interventions**

Information from the Teacher Survey was also used to answer the research question: Did use of the MRI suggest any needed interventions? The specific survey statement that related to this question was: “The inventory identified areas where I need to make interventions.” Teachers were asked to rate this statement on a 5-point scale of *strongly agree* to *strongly disagree*. Three participants gave a score of 3, or neutral, while one gave a score of 4, or agree. This gave a mean score of 3.25. Interestingly, however, was that two of the participants who gave a score of neutral also wrote in the open-ended question section that one of the most useful aspects of the Math Reasoning Inventory was

that it showed areas where possible interventions could be made. This would seem to contradict the ratings given by these two teachers.

### **Overall Teacher Input**

The Teacher Survey was the final data collection instrument. After giving the Math Reasoning Inventory a second time, teachers were asked to complete a survey in order to give their opinions of the MRI itself. Five statements were rated on a 5-point Likert-like scale, with *strongly agree* scoring 5 points and *strongly disagree* scoring 1 point. Results of this portion of the survey are reported in Table 8; responses to three of the statements were discussed above. The first statement, “The inventory was easy to administer,” was the only statement to get a mean positive rating of 4 (from scores of 3, 5, 4, 4). While seemingly easy to administer, however, most teachers did not feel that use of the inventory should be continued and expanded, with a mean score of 2.5 from scores of 2, 3, 4, 1.

There were also two open-ended questions on the Teacher Survey. These were: “What, if anything did you find most useful about this inventory?” “What, if anything, did you find least useful about this inventory?” Finally, teachers were asked to provide responses to the following: “Please discuss anything new you learned as a result of giving this inventory.” “Please comment on anything else you would like to share about this inventory.”

Table 8

*Numerical Results of Teacher Survey of the Math Reasoning Inventory*

Survey statements	Teacher A	Teacher B	Teacher C	Teacher D	<i>M</i>
The inventory was easy to administer.	3	5	4	4	4
The inventory changed the way I teach.	2	3	3	2	2.5
The inventory identified the students who need interventions.	3	3	4	2	3
The inventory identified areas where I need to make interventions.	3	3	4	3	3.25
I recommend that use of this inventory be continued and expanded.	2	3	4	1	2.5

There were some common themes in the teacher comments. Teachers were unanimous in their opinions that the MRI took much too long to administer. One of the teachers felt that asking fewer questions would have provided the same information. Related to this were statements that, due to the length of time needed for each student interview, any future use of the MRI should be reserved for students receiving Tier II or III interventions. Another common observation was that use of the MRI was interesting in that it was able to show how students think: students who were more able had several strategies with which to solve problems, while students who struggle had few to none.

Two of the teachers made comments that bear further comment. The MRI allows use of paper and pencil on the last two questions requiring application of fraction

concepts. One of the teachers wrote that paper and pencil should be allowed for all of the questions. Given that 9 of the 12 questions on the MRI are designed to measure *mental* reasoning in comparing or computing with fractions, it would seem that this teacher did not fully comprehend one of the main concepts behind the MRI. Another teacher wrote that while training was given in how to administer the MRI, no training was given in how to use the results. This comment will be addressed in the next chapter.

### **Summary of Results**

This pilot investigation of the fraction portion of the Math Reasoning Inventory was conducted in order to determine what effects training teachers to conduct the MRI and administering the MRI to their students had on fraction instruction and resulting student achievement. Data were presented to answer each of the research questions in turn. Pretest and posttest data from the MRI website were used to answer the question regarding the effect of the MRI on student achievement. Data taken primarily from observations, as well as from one question from the Teacher Survey, were used to examine any changes in teaching practices resulting from use of the MRI. Information from the Teacher Survey also provided answers as to whether the MRI was helpful to the teachers in identifying students who needed interventions or in suggesting needed interventions. Finally, data from the Teacher Survey gave the overall perspectives of the participant teachers regarding the Math Reasoning Inventory. In this pilot study, it would seem that the Math Reasoning Inventory had little effect on student achievement in the area of fractions or on teachers' instructional practices. The inventory also did not seem to prove useful as a screener to identify those students needing interventions or as a tool

for identifying what interventions need to be made. The next chapter will discuss implications of these findings and suggestions for future research.

## CHAPTER V

### DISCUSSION

#### Summary of Study Problem and Methodology

Today's student faces intense global competition upon entering the workforce. Increasing educational rigor by preparing all students for postsecondary training has been proposed as a way to ensure that students from the United States will be ready to face this economic reality. Few students graduate with the skills necessary for success in college level mathematics classes, however. In a study of first-time postsecondary students conducted by the U.S. Department of Education (2013b), 42% had enrolled in a remedial math class (*An Overview of Classes Taken*, Table 2-B). Further, the United States Department of Education (USDE) reported that 34% of 12th grade students score below basic level in mathematics assessments in 2013. These difficulties with mathematics obviously exist prior to high school: USDE also reported that 26% of eighth graders nationally scored below the basic level in math. In Michigan 30% were below this level. For fourth graders, numbers of students below basic level were 17% nationally and 23% in Michigan (U.S. Department of Education, 2013a).

In light of these and other equally dismal measures, several governmental initiatives have been implemented. In 2006, then-President Bush created the National Mathematics Advisory Panel (NMAP) with the charge to “rely upon the ‘best available scientific evidence’ and recommend ways to ‘foster greater knowledge of and improved performance in mathematics among American students’” (NMAP, 2008, p. xiii). The

Panel as a whole examined over 16,000 research publications and policy statements, heard testimony from 110 experts, received input from 160 interested individuals and organizations, and analyzed surveys from 743 algebra teachers (NMAP, 2008, chapter 1, p. 1).

One of the NMAP findings was based on interviews with Algebra I teachers: “Difficulty with the learning of fractions is pervasive and is an obstacle to further progress in mathematics. . . . Instruction focusing on conceptual knowledge of fractions is likely to have the broadest and largest impact on problem-solving performance” (NMAP, 2008, p. 28). Wu (2001) agrees: “The failure rate in algebra will continue to be high unless we radically revamp the teaching of fractions and decimals” (p. 1). To further confirm the relationship of fraction knowledge to math achievement, Bailey et al. (2012) conducted a study with sixth and seventh grade students. Controlling for intelligence and working memory, performance on fraction comparison tests in the sixth grade was found to be a significant predictor to math achievement in the seventh grade.

In an attempt to increase the rigor necessary to prepare students for algebra and higher level mathematics classes, the State of Michigan made curricular changes. A new national set of standards, the Common Core State Standards for Mathematics (CCSSM; National Governors Association, 2010), was adopted in 2010. These standards emphasize mathematical reasoning and application, rather than the previous emphasis on performing calculations. Another curricular change was raising the number and rigor of the mathematics classes required for graduation. Students must now take a math class in each year of high school and must master Algebra II content in order to receive a diploma. Some students, particularly those with disabilities, have difficulty meeting these

increased standards. Thurlow et al. (2002) found that 36% of students who drop out of school have learning disabilities. Another study found that 7% of high school students have a significant disability in one area of mathematics (Geary et al., 2009).

In order to provide support to struggling students, some school districts have implemented response to intervention (RTI) programs. Although every school's program is unique, the basic RTI model is a tiered instructional system based on frequent assessment of educational progress and use of these assessment data to provide increasing intensities of interventions as needed. Tier I and Tier II interventions are generally provided by classroom teachers. Unfortunately, these teachers have had little training in how to assess student learning and how to provide interventions to those students who struggle (Elmore, 2002; Supovitz et al., 2013; Yoon et al., 2007). Teachers have also had little training in how to provide the more conceptual math instruction required by the Common Core curriculum. This lack of training has proven to be a problem in the district where I am employed as a math interventionist: scores on standardized tests show that many students have persistent deficits in fraction skills despite having received 4 years of interventions under an RTI program.

In 2012, Marilyn Burns at Scholastic released the Math Reasoning Inventory (MRI), a web-based formative assessment developed to assess mathematic thinking skills of fourth through eighth graders based on CCSSM content prior to sixth grade (MRI, 2012, Overview: About the Assessment, para. 1). Teachers can use this knowledge of student thought processes to plan instruction and determine which students would benefit from interventions. The interventions that are suggested through use of the MRI are verbalizing math processes and modeling appropriate problem-solving strategies, which



are both recommended by the Institute of Educational Sciences for RTI programs in mathematics.

The purpose of this study was to conduct a comprehensive pilot investigation of the fraction portion of the MRI as a part of our district's school improvement plan. I specifically wanted to determine what effects that training teachers to conduct the MRI and then actually administering the MRI to their students had on fraction instruction and resulting student achievement in regular classroom settings. Additionally, I examined the efficacy of the MRI as a screener to identify students in need of intervention as well its use as a formative assessment to inform teaching practices.

This study used a method called the triangulation design, which involved a concurrent collection of both quantitative and qualitative data. Four volunteer teachers—two fifth grade and two fourth grade—from the school where I am employed were the participants. To begin, I made observations of each of the teachers at the start of the fraction unit in order to collect baseline data, which I recorded on an observation protocol. Each teacher then received a 3-hour training session to learn how to administer the MRI. I made a second observation to see if teachers began to model any of the reasoning or assessment strategies they had learned from the MRI training. Midway through the fraction unit, teachers administered the MRI as a pretest to some of the students in their classrooms. Following this, I made a third observation to see if teachers used the assessment data to correct any misconceptions held by students or whether they used assessment techniques. At the end of the fraction unit, the teachers administered the MRI to their previously tested students as a posttest. I compared the pretest and posttest

results, using a paired  $t$  test in SPSS. Finally, the participant teachers completed a survey in which they evaluated the MRI.

The overarching question guiding this study was: What effect did the MRI have on student achievement in fractions? Subquestions addressed were: Did use of the MRI change teacher instructional practices? Did teachers find the MRI useful in identifying students with misconceptions in reasoning about fractions? Did use of the MRI suggest any needed interventions?

### **Summary of Findings**

Because I had high hopes that the MRI would solve some of my district's issues related to fraction instruction and resulting student achievement in the area of fractions, results of this study were somewhat disappointing. The findings will be reported by each research question; the final section will report the overall opinions of the participating teachers regarding the MRI.

Results from the MRI pretest and posttest were the most surprising. Because the MRI pretest was given midway through the fraction units and the posttest at the end of the unit, I expected that the total test scores would increase, with or without the influence of the MRI, due to students having more exposure to the curriculum. This was not the case. In fact, the total test scores for two of the teachers actually showed a decrease from the pretest to posttest. Only one of the teachers had pretest to posttest scores that were significantly higher, but the effect size for this showed low practical significance. I also expected that the fourth grade scores would show a dramatic increase since 10 out of 12 questions on the MRI are based on fourth grade Common Core standards; one question is

based on fifth grade standards and one on third grade standards. Combining the total correct scores by grade level again showed no statistically significant differences; gains for fourth grade were slightly over 5%, while fifth grade showed a less than 1% gain. I had hoped that the students' use of appropriate mental reasoning strategies would increase since this is the focus of the MRI, but again, this was not the case. None of the teachers showed significant increases in their students using appropriate strategies. Examining the classroom percentages of students using each reasoning strategy showed no discernable patterns, other than two classrooms' increased use of standard algorithms, considered to be an inappropriate strategy. I speculate that this is due to students having exposure to the standard algorithms just before the posttest, as these are introduced at the end of the fraction unit. I have no speculations as to the reasons for any of the other data.

The overarching question guiding this study was: What effect did the MRI have on student achievement in fractions? Given the data from the pretests and posttests, I was not able to reject the null hypothesis. Therefore, the conclusion is that teacher training for use of and subsequent administration of the Math Reasoning Inventory had no significant impact on student achievement in fractions.

The MRI was designed to be used as a formative assessment, one that informs teaching practices. My assumptions that I would observe changes in teachers' instructional behaviors due to the introduction to the MRI were also proven to be incorrect. My second observation was conducted shortly after the MRI training session. I expected that the teachers would initiate more class discussion through use of probing questions, if for no other reason than they knew I was observing them and watching for changing in their teaching practices. (I defined probing questions as those designed to

assess students' thought processes, rather than those seeking a correct answer.) This expectation was not supported by the data. While two of the teachers asked more probing questions than in the baseline observation, the two other teachers asked fewer. There was also little change in the total amount of time spent in classroom discussion: three teachers increased discussion time, and one teacher showed no increase. I had also hoped that teachers would begin to model more reasoning strategies through thinking aloud, or showing strategies that promote mental math. Once again, more strategies were modeled by two of the teachers, while two teachers modeled fewer. None of the teachers modeled strategies through thinking aloud, however. Finally, I hoped to see more collaboration among the students through use of group or partner work. Again, two of the teachers showed an increase, while two showed a decrease in group work time. Throughout these observations, no patterns were noted. No one teacher or grade level showed consistent increases or decreases in observed behaviors.

Once the teachers had administered the MRI pretest to their students, I expected that teachers would use resulting information to inform teaching practices. One behavior I expected to see was that teachers would begin to address some of the students' misconceptions about fractions. There were slight increases in addressing misconceptions for three of the four teachers. Also, since teachers were not able to administer the MRI to all of the students in their classes in the time allotted, I had hoped that teachers would ask more probing questions and increase collaboration and discussion among students in an effort to formatively assess more of their students. Data for these behaviors were again mixed: two of the teachers asked more probing questions, while two asked fewer; discussion time increased for three teachers while decreasing for one teacher; and group

work time increased for three teachers and decreased for one. Once again, no patterns were noted along individual teacher or grade level lines.

The final measure taken to assess any changes in teacher behaviors as a result of training for and administration of the MRI was to ask the teachers themselves. This was done through a survey completed by the teachers at the conclusion of their participation in the study. None of the teachers felt that the MRI changed the way they taught. One reason for this may be explained by one teacher's comment that, while there was training given in how to give the MRI, there was no training given in how to use the results. Also, the main purpose of the MRI, assessing students' ability to mentally solve problems, may not have been clear to one of the teachers who felt that students should have been able to use a paper and pencil for more of the questions on the test.

To summarize, the second question guiding this study was: Did use of the MRI change teacher instructional practices? Given the observational data as well as the opinions of the teachers themselves, the answer to this question would appear to be "no." The implications of this finding will be discussed further.

Along with use as a formative assessment to inform teaching practices, I had hoped that the MRI would prove useful as a universal screener. A universal screener is generally defined as a brief assessment given to all students at regular intervals. According to the National Center on Response to Intervention (2011), the purpose of a screener is "to identify students who are at risk of academic failure and, therefore, likely to need additional or alternative forms of instruction to supplement the conventional general education approach" (Screening Tools Chart). Although our district already uses a math screener, it primarily measures the procedural standards of math. Of interest to our

district was to find a quick assessment to identify students in need of interventions for the more conceptual aspects of math. On the teacher survey instrument completed at the end of their participation in this study, teachers were asked to rate the usefulness of the MRI for this purpose. Overall, teachers gave this a neutral rating, although one of the teachers felt that the MRI was able to identify students who had no strategies for solving problems. However, while most teachers were neutral in their opinions as to the ability of the MRI to identify students in need of interventions, they were unanimous in their opinion that the MRI took much too long to administer. Data support this opinion: While the MRI website claims that each assessment interview takes about 10 minutes, in reality each interview took the participating teachers an average of approximately 25 minutes. This is much too long for use as a universal screener; therefore, use of the MRI is impractical for this purpose.

The third question for this study was: Did teachers find the MRI useful in identifying students with misconceptions in reasoning about fractions? Once again, given the survey comments and ratings, it would seem that the MRI is not helpful to my district's teachers for the purpose of identifying students who need interventions.

Teachers were also asked to give their opinions as to the ability of the MRI to identify areas where interventions need to be made. Once again, participant teachers were generally neutral in their opinions. However, two of these same teachers wrote that one of the most useful aspects of the MRI was that it showed areas where possible interventions could be made. This seeming discrepancy may be attributed to the participating teachers' need for more training in the use of formative assessment, as mentioned above.

The fourth question for this study was: Did use of the MRI suggest any needed interventions? Although the results from the survey were somewhat contradictory, it would seem that teachers did not see the MRI as a tool for suggesting interventions for the conceptual aspects of fraction instruction.

Finally, the teacher survey also asked for overall opinions regarding the MRI. More than one teacher made the observation that it was interesting to see differences in the strategies used by students to answer MRI questions: more able students used several strategies to solve problems, while students who struggle used few or no strategies. This was not enough to recommend the MRI, however. Although most participating teachers felt that the MRI was easy to administer, few thought that use of the MRI should be continued or expanded in our district, other than for students already identified as needing Tier II or Tier III math interventions.

Given the constraints of this study, student achievement did not improve in a significant way as a result of use of the MRI, nor did teacher practices change in any appreciable way. The MRI was not seen as useful to general education classroom teachers as a screener to determine whether students need interventions in the area of fraction instruction or as a tool for suggesting where interventions are needed. Implications of these findings will be discussed further.

### **Implications**

The Technical Report of the Math Reasoning Inventory (Wilmot, 2012) reported 21 pages of mostly positive feedback from the 81 teachers who participated in the field test of the MRI. Among the comments were those in which the teachers said the MRI

showed the need to ask more probing questions in order to identify areas where interventions were needed. Several other teachers commented that the MRI made them aware of what reasoning strategies they needed to teach (Wilmot, 2012, pp. 40-41). Given these many positive comments, probably the most striking finding from this study was that the participating teachers did not seem to use the formative assessment data from the MRI to change their teaching practices in any significant way. This fact, along with the teacher comment that there was no training in how to use MRI test results, suggests that teachers may need more professional development in how to conduct formative assessments as well as how to interpret and use the information to inform instruction.

Research has shown that this need for additional training is not unique to teachers in my district. The Consortium for Policy Research in Education (CPRE) developed an online assessment called the Teacher Analysis of Student Knowledge (TASK) to evaluate teachers' abilities to use assessments to guide students toward learning targets. In this assessment, teachers were presented with a problem and a set of student responses from the third to fifth grade content area of fractions. Teachers were to use this data set to evaluate the student responses in terms of conceptual knowledge and suggest interventions for those students who had incorrect or less sophisticated responses. Of the 376 teachers who field tested the TASK, only 50% were able to identify the conceptual knowledge needed by the student to solve the problem (Supovitz et al., 2013, p. 11) and only 18% of these teachers were able to suggest interventions or next instructional steps (p. 25). The conclusion of the CPRE was that "given the emphasis in the Common Core State Standards on rigor as a balance between conceptual and procedural understanding, this suggests that there is a great deal of room for growth in teacher capacity to identify,



interpret, and respond to students' conceptual understanding" (Supovitz et al., 2013, p. 27).

Perhaps one of the factors that impacted the study participants' use of formative assessment knowledge was that a 3-hour training session covering the MRI was not long enough to impact teacher behaviors. In an Institute of Educational Sciences study, any professional development of less than 14 hours was shown to have no significant effect on student achievement (Yoon et al., 2007, p. 10). Providing this time to deliver a quality professional development covering formative assessment may be one key to improving fraction instruction and student achievement. This is especially important as classroom teachers in our district are being expected to take more responsibility for providing Tier II, or strategic, interventions.

### **Limitations**

The most obvious limitation of this study is the small number of participants. Because this pilot study of the MRI was funded as a part of the building's school improvement plan, the 3 half days of substitute fees for four teachers needed to be approved by the state. This not only impacted the number of participants, but limited the number of students each teacher was able to assess. Since the MRI website reports that the interviews take about 10 minutes for each student, I assumed that in a 3-hour period, 15 to 18 students could be assessed, for a total of 60 to 74 students. This was a gross overestimate. In the case of the fifth grade teachers, each was able to assess only 8 students in the allotted time; one fourth grade teacher was able to assess only 5 students,

and the other 7. This means that the MRI test results in this study are based on assessments of only 28 students.

Another limitation of the study concerns the observational data. Each observation was conducted for only 30 minutes, providing only brief snapshots of the classroom instruction. Also, because I was the only observer and made no video recordings, verification of the data was not possible. It was difficult to make split second judgments as to what behaviors I was observing: Was the teacher asking a probing question as opposed to simply asking for a correct response? Was the teacher modeling reasoning strategies or providing direct instruction? Was the student providing an explanation or simply giving a correct answer? I have at least 25 years of experience observing individual students in classroom settings as a part of special education eligibility evaluations. However, I found that this is quite different from observing whole-classroom behaviors and interactions. My focus of attention had to be much wider, while at the same time keeping an eye on the clock to record the amount of time spent in each activity.

A possible confounding factor to this study was our district's adoption of a new textbook series. In professional development trainings at the beginning of the school year, an outside consultant advised teachers to follow the teacher's manual and present every lesson in order. This new series claims to be tied to Common Core State Standards and includes some lessons of a more conceptual nature than was true in the previous text series. Thus, it was difficult to determine whether any changes in instruction were due to teachers following the instructional practices recommended by the textbook company or from information gathered by the MRI.

### **Future Directions**

Despite the disappointing results of this pilot study in the use of the MRI as a tool for improving fraction instruction and resulting student achievement, I feel that there is still a great deal of merit in the MRI. I plan to further explore the use of the MRI with my students who have been identified as needing Tier III, or intensive interventions. I see my students daily in small group settings; therefore, the issue of the time needed for the MRI interviews is not a concern. These smaller groups are also more conducive to having class discussions in which 100% of the students participate, thus making it possible to identify more student conceptions and misconceptions and apply information gleaned from the MRI. Further, this study examined only the fraction portion of the MRI. I also plan to explore the use of the whole number portion of the MRI, which assesses students' abilities to perform mental addition, subtraction, multiplication, and division calculations. This could provide information as to my students' understandings of the base ten number system as well as their abilities to estimate.

Another action I plan to take is to more thoroughly analyze the MRI posttest Item Analysis Report from each of the participating teacher's classroom. I will note which questions were missed most often and examine what strategies were used to solve each question and question type. Each teacher will receive a report of my findings and recommendations for his or her class.

As I attempted to identify reasons that the teachers who participated in this study seemingly did not use information obtained by the MRI to inform their teaching, I concluded that this was due to a need for more professional development in the area of formative assessment. This in turn caused me to question one of my initial assumptions in

planning this study, that once teachers identified a student's misconception, they would immediately know how to provide an intervention. As I reflected on why I had made this assumption, I came to the realization that, as a special education teacher, much of my training and expertise is in the use of formative assessments. In fact, use of formative assessments is more or less the model for providing special education services. First, we identify the students' areas of need, and then we provide remediation. This leads to what I consider needs to be an important future direction: There needs to be a greater partnership between special educators and general educators in using formative assessment data to design interventions and differentiate instruction, particularly in the implementation of an RTI plan.

### **Final Thoughts**

Today's teacher of mathematics faces several challenges. Developing conceptual knowledge of fractions has been shown to improve students' problem-solving skills and performance in higher math classes, yet studies have shown that some teachers themselves do not have a solid conceptual knowledge of fractions or of how to teach them. Given this more conceptual basis of the Common Core State Standards for Mathematics, instruction in computational procedures alone is not enough. Another challenge faced by today's teacher is that of differentiating instruction. Under an RTI program, teachers are expected to provide a quality research-based education to all students while identifying and correcting students' misconceptions and provide interventions to those students who struggle. To be most successful at this, teachers need to incorporate formative assessment as a standard instructional practice.

This study examined the ability of the fraction portion of the Math Reasoning Inventory to help classroom teachers from my district to meet these challenges. Results were somewhat disappointing: use of the MRI had little effect on student achievement in the area of fractions or on teachers' instructional practices. The inventory did not seem to prove useful as a screener to identify those students needing interventions or as a tool for identifying what interventions need to be made. While this suggests that classroom teachers need more professional development in the area of formative assessment, it also points to an opportunity for special education teachers to share their expertise in formative assessment with classroom teachers. Through this general and special education partnership, classroom teachers can learn to set appropriate learning targets for students and can learn to provide interventions for those students not meeting the targets. This is the purpose of an RTI math program.

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**Appendix A**

**Human Subjects Institutional Review Board  
Letter of Approval**

## WESTERN MICHIGAN UNIVERSITY



Human Subjects Institutional Review Board

Date: August 7, 2013

To: Shaila Rao, Principal Investigator  
Kathy Huffman Boyer, Student Investigator for dissertation

From: Amy Naugle, Ph.D., Chair

Re: HSIRB Project Number 13-08-03

This letter will serve as confirmation that your research project titled "Evaluating the Use of the Math Reasoning Inventory for Improvement in Fraction Instruction" has been **approved** under the **expedited** category of review by the Human Subjects Institutional Review Board. The conditions and duration of this approval are specified in the Policies of Western Michigan University. You may now begin to implement the research as described in the application.

Please note: This research may **only** be conducted exactly in the form it was approved. You must seek specific board approval for any changes in this project (e.g., ***you must request a post approval change to enroll subjects beyond the number stated in your application under "Number of subjects you want to complete the study."*** Failure to obtain approval for changes will result in a protocol deviation. In addition, if there are any unanticipated adverse reactions or unanticipated events associated with the conduct of this research, you should immediately suspend the project and contact the Chair of the HSIRB for consultation.

**Reapproval of the project is required if it extends beyond the termination date stated below.**

The Board wishes you success in the pursuit of your research goals.

**Approval Termination: August 7, 2014**

**Appendix B**  
**Recruitment Poster**

## **Volunteers Needed**

To help me with my dissertation, I need

**Two fourth grade teachers**

and

**Two fifth grade teachers**

to administer and evaluate the **fraction portion** of the new online Math Reasoning Inventory (MRI) developed by Marilyn Burns.

If you agree to participate, you will need to sign a consent form. The parents of your students will also have the opportunity to decline to have their child tested. You will have three half days of release time: one to be trained in administering the MRI, one to administer a MRI pretest and one to administer a MRI posttest to the students in your class. Additionally, I will observe your math class three times during your fraction unit. Finally, you will be asked to complete a brief survey at the conclusion of my study. Your responses will be anonymous with no identification of your name or grade level. Kathy Boyer will be the only one who will know your identity.

**Interested, have questions?--Contact Kathy Boyer**

**Thank you for your consideration.**



**Appendix C**  
**Observation Protocol**

Date: \_\_\_\_\_

Teacher \_\_\_\_\_

Length of observation \_\_\_\_\_ Fraction topic: \_\_\_\_\_

Number of students in attendance \_\_\_\_\_

Behavior Noted	Number Observed
Minutes of teacher direct instruction	
Minutes of teacher- lead discussion:	
• Number of students who gave explanations	
• Number of students' misconceptions addressed	
• Number of students who asked questions	
• Number of questions asked by teacher	
Minutes of student individual work time	
Minutes of student group work time	

Reasoning Strategies Modeled (Number Observed)				
Comparing Mentally		Computing Mentally		Applying Understanding
Appropriate	Not Appropriate	Appropriate	Not Appropriate	Appropriate
Uses relationship between numerator and denominator	Converts to common denominator	Uses knowledge of equivalence	Uses standard algorithm	Models with mathematics
		Extends knowledge of whole number operations to fractions	Finds exact answer to estimate	Understands equivalence
		Uses benchmark fractions to estimate		

**Other observations:**

**Appendix D**  
**Teacher Survey**

Please give your opinion regarding the Math Reasoning Inventory.  
Thank you for your participation in this study.

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1. The inventory was easy to administer.	5	4	3	2	1
2. The inventory changed the way I teach.	5	4	3	2	1
3. The inventory identified the students who need interventions.	5	4	3	2	1
4. The inventory identified areas where I need to make interventions.	5	4	3	2	1
5. I recommend that use of this inventory be continued and expanded.	5	4	3	2	1

What, if anything, did you find most useful about this inventory?

What, if anything, did you find least useful about this inventory?

Please discuss anything new you learned as a result of giving this inventory.

Please comment on anything else you would like to share about this inventory.