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Inference on Differences in k Means for Data with Excess Zeros and Detection Limits

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INFERENCE ON DIFFERENCES IN K MEANS FOR DATA WITH EXCESS ZEROS AND DETECTION LIMITS

by

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Many data have excess zeros or unobservable values falling below detection limit. For example, data on hospitalization costs incurred by members of a health insurance plan will have zeros for the percentage who did not get sick. Benzene exposure measurements on petroleum refinery workers have some exposures fall below the limit of detection. Traditional methods of inference like one-way ANOVA are not appropriate to analyze such data since the point mass at zero violates typical distribution assumptions.

For testing for equality of means of k distributions, we will propose a likelihood ratio test that accounts for excess zeros or detection limits. We will conduct simulations to study finite sample properties of the proposed procedure on both Log-normal distribution and Gamma distribution. One imputation method will be proposed as an alternative approach.

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Haolai Jiang

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

1.1 Data with Excess Zeros

Data sets with excess zeros are commonly seen in present world. Lachenbruch (2002) indicates that data sets with excess zeros have been recognized to pose potential problems since the mid 1960s or earlier.

There are series of literature citing examples from the real world.

Example 1: (Lachenbruch, 2002) (Hospitalization costs) Data are from studying hospitalization costs in a health insurance plan. Most of the members (90%) of an insurance plan do not have hospitalization costs in a year. The data set contains significant amount of zeros, and some non-zero positive numbers. It may be valuable to determine if the new policy will have impact on the costs and the proportion of people who have been hospitalized.

Example 2: (Neuhauser et. al., 2005) (DNA methylation data) MethyLight technology can be used to measure DNA methylation data. When the test region is not or partially methylated the result is undetectable which is marked as a zero. In contrast, regions that show methylation will have positive values. DNA methylation data with MethyLight data have excess zeros and positive non-zero part.

The above examples offer a peek of the cases in the real world. Traditional methods, such as Analysis of Variance, t test and Kruskal-Wallis test, are not appropriate to analyze such data since the distribution of data violates the typical assumptions.

Two density curves shown in Figure 1.1 provide visualization of the shape of data with excess zeros, positive non-zero part and also with possible detection limit.

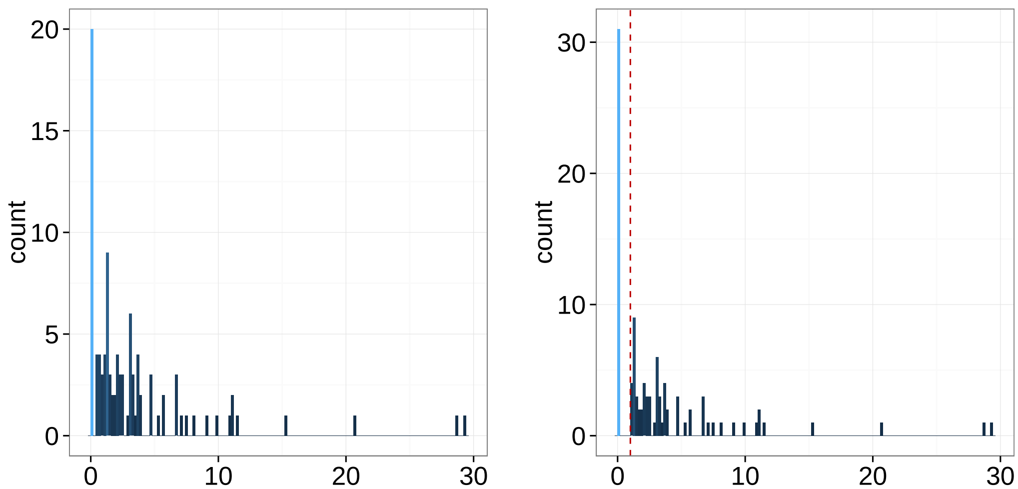


Figure 1.1: Demonstration of Data with and without Detection Limit

The two curves in Figure 1.1 are generated by assuming that the non-zero part follows Log-normal distribution, with sample size 100, mean and standard deviation of log scale both 1, and the probability of obtaining zero 0.1. The second plot sets up a detection limit of value 1 with a dashed line. We can see clearly that there is a high spike in the distribution at value zero. The count of zeros is even higher for the second plot since additional zeros being added due to the unobservable values falling below detection limit (DL).

Data with excess zeros can be categorized into two cases: one is that the positive part is discrete, and the other one is that the positive part is continuous. Zero-inflated Poisson model (ZIP) is widely used under the assumption that the positive values following a Poisson distribution. For continuous positive part, Log-normal distribution, Gamma distribution and Weibull distribution are often considered. In this dissertation I primarily consider Log-normal distribution as the one for the positive values.

Two-part model is often used due to the mixture of excess zeros and positive part. In general, if the $h(x)$ is the probability density function ($p.d.f.$) of the distribution of the positive values. The $p.d.f.$ for the mixture distribution (Lachenbruch,

2002) is

$$f_i(x) = p_i^{\delta} \times [(1 - p_i) \times h_i(x)]^{1-\delta}$$

where $\delta = 1$ if $x = 0$, $\delta = 0$ if $x > 0$, p_i is the proportion of zeros and i is the group indicator.

It is apparently not appropriate to assume that such data follow a normal distribution. In the case of two independent populations with hypothesis $H_0 : p_1 = p_2$ and $h_1(x) = h_2(x)$ v.s. $H_1 : p_1 \neq p_2$ or $h_1(x) \neq h_1(x)$, Lachenbruch (2001) compares the performance of the two-part models with traditional normal test, Wilcoxon test, and Kolmogrov-Smirnov test. He concludes that the two-part models outperform and at least perform equally well in most of the situations with only one exception.

1.2 Data with Excess Zeros and Detection Limit

In many practical cases, due to the limitation of measurement accuracy there are zeros which are not true zeros. These are unobservable positive values, less than the detection limit (DL). Therefore, in those cases the zeros come from two parts: one is the true zeros, and the other is the values below DL which are unobservable. There are many examples in the literature well demonstrate such situations. Chu (2005) describes that "This is a relevant problem in many cancer prevention trials and HIV studies."

Example 3: (Aertker and Zaccaro, 2011) (H1N1 vaccination data) Data is the antibody titers in responses to H1N1 vaccination. Zeros are recorded if the antibody titers fall below the lower limit of detection (LLOD). Observations below the LLOD consist of values which are true zeros and nonzero values but are below the LLOD.

Example 4: (Taylor et. al, 2001) (Time-weighted average occupational exposure data) Data are the time-weighted average (TWA) benzene exposure measurements from 38 petroleum refinery workers. Workers' schedules require operators work in a contamination free environments. Five workers' exposures fall below the limit of detection (LOD).

Example 5: (Chu et. al, 2005) (Hepatocarcinogenesis data) Hepatocellular carcinoma (HCC) is one of the most common cancers induced by aflatoxin. This type of cancer is inhibited by oltipraz. Hydroxylated metabolite aflatoxin M_1 (AFM_1) and non-toxic aflatoxin-mercapturic acid (AFB-NAC) are two important factors of oltipraz. The measurement of AFM_1 and AFB-NAC can have excess values near zero. Moreover, the measurements are often left censored due to some values falling below DL.

As introduced before, in the case of data with excess zeros and detection limit, in addition to the true zeros there is another source of zeros: positive unobservable values falling below detection limit (DL). Assuming the observed values are from Log-normal distribution, Chu et. al (2006) shows that the $p.d.f.$ of the mixture distribution is

$$f_i(x) = [p_i + (1 - p_i) \times \Phi(\frac{y_L - \mu_i}{\sigma_i})]^{\delta}[\frac{1 - p_i}{\sigma_i} \times \phi(\frac{y_i - \mu_i}{\sigma_i})]^{1-\delta}$$

where y_i is the logarithm of the original values from Log-normal distribution, $y_L$is the logarithm of original detection limit, μ_i and σ_i are the mean and standard deviation of log scale, $\Phi(\cdot)$ and $\phi(\cdot)$ are the cumulative distribution function ($c.d.f.$) and $p.d.f.$ of standard normal distribution, respectively. From the above $p.d.f.$, we see that assuming the positive distribution and the detection limit are known, we could obtain the expected probability of the values falling below the detection limit. Adding the probability of true zeros, the total probability of zeros is $p_i + (1 - p_i) \times \Phi(\frac{y_L - \mu_i}{\sigma_i})$.

1.3 Alternative Distributions for Positive Part

Mixture models are typically used to account for the data set with excess zeros and possible detection limit. Such models study zeros part and positive part with different distributions. Apart from using non-parametric methods for the values of positive part, Log-normal distribution is the typical choice for simulating the continuous positive part in many papers. Examples include Zhou et. al. (1997), Taylor et. al. (2001), Chu et. al. (2005, 2006), Nie et. al. (2006), and Daoud (2007).

However, even though Log-normal distribution has nice features that simplifying the calculation procedures, values of the positive part may arise from a distribution other than Log-normal.

As Figure 1.2 shows, Gamma and Weibull distributions could have similar theoretical density curves as Log-normal distribution. Even though the density curves of the three distributions are not always similar, it is still worthwhile considering the situations where their density curves are actually similar and therefore, motivating the consideration of assuming distributions other than Log-normal distribution for the positive part.

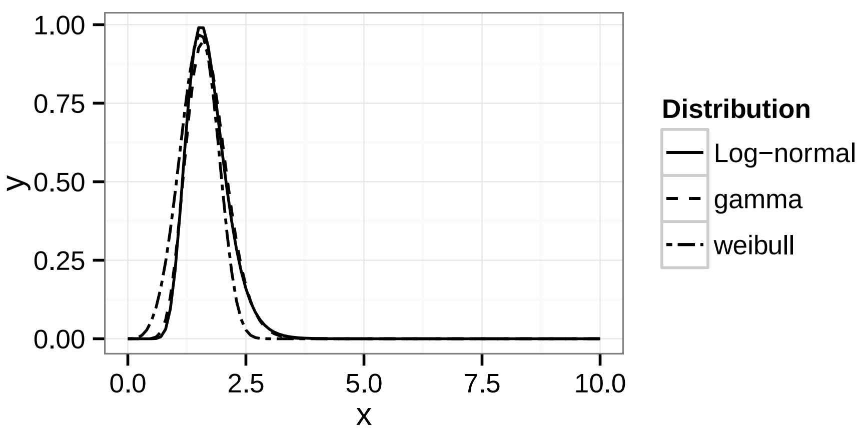


Figure 1.2: Demonstration of Distribution Similarities

1.4 Motivation and Dissertation Structure

Daoud (2007) proposes a two-part likelihood ratio test and a two-part Wald test for comparing p and $h(x)$ simultaneously for K ($K \geq 3$) independent populations. Lachenbruch (2001) mentions the potential generalization of two-part models to multiple populations without much details. Zhou and Tu (1999) proposes a likelihood ratio test and a Wald test for comparing overall means of K independent populations. However, none of them considers the detection limit. Chu et. al (2004, 2005 and 2006) considers data with excess zeros and possible detection limits in a series of paper. However, they only focus on the cases of one and two independent populations.

Motivated by the fact that such type of data are readily available as well as the related studies, I propose a likelihood ratio test for K ($K \geq 3$) independent populations with a detection limit. The simulation and formula derivation will be based on $K = 3$ independent populations, but it could be easily extended to other cases. Furthermore, the robustness and sensitivity of the proposed test will be shown under various combinations of the parameters and under different distributions through simulation studies.

Furthermore, if the number of values below DL could be reasonably estimated, we may remove the detection limit and convert the situation to a typical two-part model. We would like to tentatively explore the imputation techniques for the undetectable values and make recommendation based on that.

This dissertation is organized as follows. Chapter Two gives the review of the relevant literature. We will introduce the proposed likelihood ratio test for K ($K \geq 3$) independent populations in Chapter Three. Simulation studies will be conducted in Chapter Four. Chapter Five will explore imputation methods for the values below DL. And Chapter Six will include conclusions and potential future research directions.

CHAPTER II

LITERATURE REVIEW

Lachenbruch (2001) proposes a two-part model with density as $f_i(x) = p_i^{\delta} \times [(1 - p_i) \times h_i(x)]^{1-\delta}$ where $\delta = 1$ if x is 0 and p_i is the probability of zeros. The model assumes that data has a clump at zero and the non-negative part is continuously distributed. Since the probability density function has two parts, the null hypothesis of testing two independent populations also includes two parts: $H_0 : p_1 = p_2$ and $h_1(x) = h_2(x)$. For testing the null hypothesis, it includes a chi-square distribution with 1 degree of freedom (d.f.) for equality of proportion of zeros and a chi-square distribution with 1 d.f. for equality of non-negative distribution. He states that when the sample size is large enough, the test statistic is a summation of two components and follows a chi-square distribution with 2 d.f.. Lachenbruch (2001) introduces the formula of the test statistic:

$$\chi^2_{(2)} = \left[\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\bar{p}(1-\bar{p})(\frac{1}{n_1} + \frac{1}{n_2})}}\right]^2 + U^2$$

where $\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$. He introduces several options for U, including z test, t test, Wilcoxon test and Kolmogorove-Smirnov test.

Under alternative hypothesis, the test statistic follows a non-central chi-square distribution with 2 d.f. and a non-centrality parameter λ, which is defined by

$$\lambda = \frac{(p_1 - p_2)^2 n_1 n_2}{\bar{p}(1-\bar{p})(\frac{1}{n_1} + \frac{1}{n_2})} + \frac{(\mu_1 - \mu_2)^2}{\sigma^2(\frac{1}{n_1(1-p_1)} + \frac{1}{n_2(1-p_2)})}$$

Lachenbruch fixes sample size $n_1 = n_2 = 100$, $\sigma = 1$ and $\mu = 0$, and assumes that the non-negative part of data follows a Log-normal distribution and uses Wilcoxon as the choice of U in the test statistic. The predicted power varies from 0.378 to 0.952 over different combinations of p_1, p_2, $\mu_1 - \mu_2$. The predicted powers are calculated based on the non-central chi-square distribution with 2 d.f.. The observed powers are generated from the Monte Carlo simulations.

Lachenbruch (2001) compares the two-part model, proposed in another paper in 2001 , with other competitor models. These competitor models include Smirnov test, the t-test with equal variances, and the Wilcoxon rank sum test. Binomial test for proportion of zeros is used for all two-part models. In the paper, the author tries to address three questions: whether those tests retain size 0.05 for type I error probability; whether they have the appropriate null distribution; and comparing the observed powers of the tests in various situations.

Lachenbruch runs simulations in various situations and finds that if the populations differ only in the proportions, but not in the conditional distributions, all of the tests perform poorly with low power. If the populations differ only in the means of the Log-normal distributions, but not the proportion of zeros, the picture is unclear, and the results depend on the proportion of zeros. When differences occur in both the proportion of zeros and the conditional distributions, Smirnov test and Wilcoxon rank sum test perform well.

Unlike the regular two-part zero-inflated model $f_i(y) = (p_i)^{\delta}[(1-p_i) \times g_i(y)]^{1-\delta}$, Chu, Nie, & Cole (2006) considers the differences between true zeros and the left-censored values due to the detection limits. The authors use a mixture model in the form of $f_i(y) = [p_i + (1-p_i) \times G_i(y_L)]^{\delta}[(1-p_i) \times g_i(y)]^{1-\delta}$, where $i = 1, 2$, y_L is the detection limit, p_i is the proportion of true zeros and $g_i(.)$, $G_i(.)$ are the p.d.f and c.d.f., respectively.

The hypothesis of primary interested in Chu et. al. (2006) is $H_0 : \mu_1 = \mu_2$ vs. $\mu_1 \neq \mu_2$. They derived formulas of sample size and power under the assumptions of equal variances and unequal variances separately. The authors use a Bernoulli/Log-normal mixture model to apply on an example. They specify the percentage of true zeros and the detection limit. They also use the Maximum Likelihood Estimates

(MLEs) from the data to set up p_1, p_0, μ_1, μ_0, σ_0 and σ_1.

Chu et al., (2006) then conducts two sets of Monte Carlo simulations under different combinations of the probability of true zeros and percentage of the left-censored values, under the assumptions of equal and unequal variances separately. For each condition, 20,000 trials are generated.

The authors conclude that under equal variances assumption, the results show the expected power, 0.8, if assuming $p_1 \neq p_0$. The sample size is overestimated when $p_1 = p_0$. Under the assumption of unequal variances, the empirical power is slightly under the expected power, 0.8. The sample size is overestimated.

Nie, Chu, & Cole (2006) indicates that when conducting a medical research it is often assumed that, in the data a proportion of individuals coming from a Log-normal distribution, while the remaining individuals have true zeros because they were not exposed to the disease. Due to the detection limit, there may be some exposed individuals whose values fall below the detection limit, which are indistinguishable from the individuals with true zeros. Therefore, the observed data follow the mixture distribution with a probability density function

$$f_i(y) = \left[p + (1-p) \times \Phi\left(\frac{y_L - \mu_i}{\sigma_i}\right)\right]^{\delta} \times \left[\frac{1-p}{\sigma_i} \times \phi\left(\frac{y_L - \mu_i}{\sigma_i}\right)\right]^{1-\delta}$$

where y is the nature logarithm of the measurement, $\Phi(.)$ and $\phi(.)$ are the standard normal cumulative distribution function (c.d.f.) and probability density function (p.d.f.), respectively. y_L is the detection limit on the log scale, and $i = 1$ indicates the treatment group while $i = 0$ indicates control group.

Nie et. al (2006) provides a general approach for sample size estimation for comparing the mean differences of two independent groups with equal proportion of true zeros, which is $p_0 = p_1 = p_2$.

Under the equal variances case, $\sigma_0 = \sigma_1 = \sigma$, via maximum likelihood estimates (MLEs), they propose a formula for the optimal sample size by

$$n = min_{Q_0}\{[\sqrt{\Sigma_0}\Phi^{-1}(1 - c^{-1}\alpha) + \sqrt{\Sigma_1}\Phi^{-1}(1-\beta)]^2(\mu_1 - \mu_0)^{-2}\}$$

where $n = n_0 + n_1$, $Q_0 = \frac{n_0}{n}$, Σ_0 is the variance of $\sqrt{n}(\hat{\mu}_0 - \hat{\mu}_1)$ under null hypothesis, $H_0 :\ \mu_0 = \mu_1$, Σ_1 is the variance of $\sqrt{n}(\hat{\mu}_0 - \hat{\mu}_1)$ under alternative hypothesis, $H_1 :$ $\mu_0 \neq \mu_1$ and $c = 1$ or 2 indicating one-sided or two-sided test. The likelihood ratio test (LRT) follows a standard chi-square distribution with one degree of freedom.

1. Monte Carlo simulations are conducted for comparing the proposed sample size calculation to the naïve method which is simply inflating the sample size to compensate the information loss through left-censoring. Simulations are based on different combinations of the parameters: the proportion of true zeros, $P = 0.4$, 0.2, and probabilities of values under detection limit for control and treatment groups (0.1, 0.03), (0.2, 0.075), (0.3, 0.13), (0.4, 0.197), respectively. By setting significance level $\alpha = 0.05$ and the desired power $\gamma = 0.8$, the results indicate that the naïve approach tends to consistently underestimate the sample size. The observed power of the proposed method yields higher powers than those from the naïve approach. It is observed that the sample size increases much when the censored proportion increases. Nie et al. (2006) claims that this is due to the small decreases of the Fisher information for μ as the censoring proportion increases.

Neuhauser and Jockel (2005) builds a two-part permutation test based on Lanchenbruch's two-part model. Lanchenbruch's two-part model is in the form of $X^2 = B^2 + W^2$, where B is the Binomial test for testing the proportion of zeros and W is Wilcoxon sum rank test. The second part could use other tests, such as student's t test, Kolmogorov-Smirnov test and so on. One of the reasons to use Wilcoxon sum rank test is "Non-parametric tests based on ranks are more appropriate for non-normally distributed data such as microarry data" according to the authors.

The authors carry out the permutation test based on the sum statistics X^2 since it is not necessary to determine the permutation distributions of the summands B^2 and W^2 to calculate the p-values of the univariate tests related to B^2 and W^2. The authors use a simple random sample of $20,000$ permutations to determine the p-values of the two-part permutation test.

Using a simulation study to actual microarray data, the results show that the p-values of two-part permutation test are smaller than Lanchenbruch's two-part model, which corresponds to a higher power.

Neuhauser and Jockel claim that the two-part permutation test has several advantages: "First, it avoids the use of any asymptotic distribution and , therefore, can safely be applied in case of small sample sizes that are common in microarry experiments. Second, it reduces without any loss of power to the exact Wilcoxon test if there were no zero values. Thus, it can be used in routine analysis. Third the permutation test opens the possibility to use other tests to construct the two-part test."

However, the proposed two-part permutation test has one disadvantage, namely, it can be computer-intensive if the sample size is not too small.

Taylor, Kupper, Rappaport and Lyles (2001) compares censored Log-normal mixture model (mixture model) and left-truncated normal distribution and finds the Maximum Likelihood estimates of μ_y and σ_y^2 , where y is the logarithm of the values from Log-normal distribution, are the same for both the mixture model and the left-truncated normal distribution. Therefore, they emphasizes: "observations falling below the LOD contribute no information toward the estimation of μ_y and σ_y^2". LOD is the acronym of limit of detection. The quoted statement fails when the LOD varies across groups.

The estimated proportion of true zeros, denoted $\hat{\omega}$, has a upper limit n_0/n, where n_0 is the number of zeros and n is the total sample size. The value of $\hat{\omega}$ falls below zero when $n_0 < n\Phi(\hat{\epsilon})$, where $\Phi(.)$ is the c.d.f. of standard normal distribution, and $\hat{\epsilon} = \frac{LOD-\hat{\mu}_y}{\hat{\sigma}_y^2}$. Taylor et.al. (2001) proposes a mixture model with restriction of $\hat{\omega} > 0$. They tests $H_0 : \mu_x \geq C$ v.s. $H_1 : \mu_x \leq C$, where x follows a Log-normal distribution and C is a constant, by Likelihood Ratio (LR) test with test statistic $\hat{T} = 2\{lnL_m(\hat{\tau}_1) - lnL_m(\hat{\tau}_0)\}$, where $\hat{\tau} = \{\hat{\omega}, \hat{\mu}_y, \hat{\sigma}_y^2\}$. Taylor et. al. claims that $\hat{T}$ asymptotically follows a 50-50 mixture of a chi-square distribution with 1 degree of freedom and a point mass at zero. Additionally, they uses a two-step test for testing the appropriateness of censored model. First step tests $H_0 : \omega = 0$ v.s. $H_1 : \omega > 0$, and then the next step tests the sample mean.

Taylor et.al. (2001) conducts a simulation study for the one-sided LR test under different settings for comparing the type I error and power of censored model, mixture model, restricted mixture model and two-step procedure. They concludes that "Asymptotically, imposing the restriction $\hat{\omega} \geq 0$ has no consequence on ML estimation or hypothesis testing for a mixture model with true $\omega > 0$". And they also conclude that "Test accuracy can be improved somewhat by performing a two-step procedure where one reduces to the censored data model if an initial one-sided LR test of $H_0 : \omega = 0$ v.s. $H_1 : \omega > 0$ is not rejected at the 0.05 significance level."

Zhou and Tu (1999) proposes a likelihood ratio test for testing the overall mean equality of several independent populations that contain zeros and values from Log-normal distribution. Their null hypothesis of interest is $H_0 : M_1 = \cdots = M_K$, where $M_j = E(Y_{ij})$, and Y_{ij} are random sample from jth population,for $j = 1, \ldots, K$. Since there is no closed-form solution for the maximum likelihood estimators under the null hypothesis H_0, they use numerical method, Davidon-Fletcher-Powel (DFP) algorithm, for finding the maximum likelihood estimates (MLEs). In the paper they conclude that the proposed test performs very well in terms of power and Type I error.

Daoud (2007) proposes two-part likelihood ratio test (LRT) and two-part Wald test for testing hypothesis of K ($K \geq 3$) independent populations.

$$H_0 : p_1 = \cdots = p_k \quad \cap \quad m_1 = \cdots = m_k$$

vs.

$$H_1 : p_i \neq p_{i'} \quad \cup \quad m_j \neq m_{j'}$$

for some $i \neq i'$ or $j \neq j'$, where p_i and m_j are the proportion of zeros and the mean of Log-normal distribution, respectively. Daoud's Two-part likelihood ratio model tests the proportion of zeros, p_i, and the Log-normal part separately by the regular LRT. Each part of LRT follows chi-square distribution with $k-1$ degrees of freedom asymptotically and the two parts are independent. Combining two LRTs together, the overall two-part LRT test statistics asymptotically follows a chi-square distribution with $2(k-1)$ degrees of freedom. The same concepts apply to two-part Wald test. Daoud (2007) concludes that the two-part LRT and two-part Wald test perform very well in terms of size and power of test, compared to the existing tests under large sample. However, both tests do not perform well under small sample size.

Chu et al (2008) proposed a Bayesian approach to estimate the treatment effects for data with excess zeros and detection limits. They compare the performance of the proposed approach with maximum likelihood method via Markov chain Monte Carlo (MCMC) method on small to moderate sample size. They conclude that both approaches provided similar inference of the real example in the article. However, the paper indicates that "simulation studies with small-to-moderate sample sizes suggest that Bayesian approach provides better coverage probabilities than the frequentist approach via the maximum likelihood method since it is 'exact' conditional on the data without relying on asymptotic approximation or 'plug-in' principle."

Helsel and Cohn (1988) introduce two methods called LR and MR to estimate μ and σ^2 for data with single and multiple detection limits, respectively. They conduct simulation study to compare Mean Square Error (MSE) and bias with other commonly used methods, and concluded that MR is a "robust and efficient method." Due to the nature of the method, MR method was later named Regression on Order Statistics (ROS).

CHAPTER III

METHODOLOGY

The proposed model is based on a likelihood ratio test for testing the hypothesis

$$H_0 : M_1 = M_2 = \cdots = M_k \; vs. \; H_1 : M_i \neq M_{i'} \; for \; some \; i \neq i'$$

where M_i is the mean of the positive part of ith population $i = 1 \cdots k$.

We can apply different distributions here. If we assume the positive parts are from Log-normal distribution with μ_i and σ_i as mean and standard deviation on the log scale, then $M_i = e^{\mu_i + \frac{\sigma_i^2}{2}}$. When we assume σ_i is equal for all independent populations, it is equivalent to test $H_0 : \mu_1 = \mu_2 = \cdots = \mu_k \, vs. H_1 : \mu_i \neq \mu_{i'} \, for \, some \, i \neq i'$.

If the positive parts are from Gamma distribution with α as shape parameter and β as scale parameter, the mean is $M_i = \alpha\beta$. On the other hand, if we use Weibull distribution with scale parameter λ and shape parameter k to simulate positive values,the mean is $M_i = \lambda\Gamma(1 + \frac{1}{k})$, where $\Gamma(\cdot)$ is gamma function.

Now we use Log-normal distribution for testing the hypothesis in the above equation. Assuming the standard deviation, σ, and the probability of true zeros, p, are equal for all populations, the probability density function of one population is

$$f(y_i|\theta^\star) = \{P + (1-P) \times \Phi(\frac{y_L - \mu_i}{\sigma})\}^{m_i} \times \prod_{j=m_i+1}^{n_i} \{\frac{1-P}{\sigma} \times \phi(\frac{y_{ij} - \mu_i}{\sigma})\}$$

where $\theta^\star = (\mu_i, \sigma, P)$ are the unknown parameters vector, y_L is the detection limit in the log scale, $\phi(\cdot)$ and $\Phi(\cdot)$ are the $p.d.f.$ and $c.d.f.$ of standard normal distribution. Without loss of generality, we assume that the first m_i values in the ith population are below the DL y_L and let n_i be the total sample size of the ith population. From the above $p.d.f.$, all parameters are continuous, so it is apparently that the following likelihood function is continuous and defferentiable.

Let y_{ij} ($i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n_i$) be the Log-transformed values for the jth observation in the ith population. Since we assume the positive part following a Log-normal distribution, y_{ij} follows a normal distribution. The likelihood function for k populations is

$$\begin{aligned} L_m(\mu_1, \mu_2, \ldots, \mu_k, \sigma, P,) &= \prod_{i=1}^{k} \left\{ P + (1-P) \times \Phi\left(\frac{y_L - \mu_i}{\sigma}\right) \right\}^{m_i} \times \\ &\quad \prod_{j=m_i+1}^{n_i} \left\{ \frac{1-P}{\sigma} \times \phi\left(\frac{y_{ij} - \mu_i}{\sigma}\right) \right\} \end{aligned}$$

Similar to Taylor et al. (2001) and Nie et al. (2006) for two independent sample case, we derive the equations to obtain the maximum likelihood estimate (MLE) of $\theta = (\mu_1, \ldots, \mu_k, \sigma, P)$. We need to solve the following equations,

$$\begin{aligned} \frac{m_i \times (1-P)\phi(\varepsilon_i)}{\sigma \times [P + (1-P) \times \Phi(\varepsilon_i)]} + \frac{\sum_{j=m_i+1}^{n_i}(y_{ij} - \mu_i)}{\sigma^2} &= 0 \\ \sum_{i=1}^{k} \left\{ \frac{m_i \times (1-P) \times (-\varepsilon_i) \times \phi(\varepsilon_i)}{P + (1-P) \times \Phi(\varepsilon_i)} - \frac{n_i - m_i}{\sigma} + \frac{\sum_{j=m_i+1}^{n_i}(y_{ij} - \mu_i)^2}{\sigma^3} \right\} &= 0 \\ \sum_{i=1}^{k} \left\{ \frac{m_i \times [1 - \Phi(\varepsilon_i)]}{P + (1-P) \times \Phi(\varepsilon_i)} - \frac{n_i - m_i}{1-P} \right\} &= 0 \end{aligned}$$

where $\varepsilon_i = \frac{y_L - \mu_i}{\sigma}$, $\sigma_1 = \sigma_2 = \cdots = \sigma_k = \sigma$ and $i = 1, 2, \ldots, k$.

There exists no closed-form solution for MLEs, so we rely on numerical methods to find the solution. Among numerous numerical methods, we purposely choose L-BFGS-B method since "L-BFGS-B is a limited-memory algorithm for solving large nonlinear optimization problems subject to simple bounds on the variables." (Zhou et. al., 1997). L-BFGS-B method is an extension of Limited-memory Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. The objective of algorithm L-BFGS-B is to minimize a nonlinear function of n variables (Zhou et. al., 1997).

The nuisance parameters in our likelihood function needed to be constrained when searching for optimal values: the standard deviation $\sigma \geq 0$ and the probability of zeros $0 < p < 1$.

H_0 is equivalent to $H_0' : \mu_1 = \mu_2 = \cdots = \mu_k$ when $\sigma_1 = \sigma_2 = \cdots = \sigma_k = \sigma$. We are only interested in parameter μ_i . We treat parameters σ and P as nuisance parameters. Let the parameters under the null hypothesis be $\theta_0 = (\mu, \mu, \ldots, \mu, \sigma, P)$, and the parameters under the alternative hypothesis be $\theta = (\mu_1, \ldots, \mu_k, \sigma, P)$. The likelihood function under H_0 is

$$\begin{aligned} L_m(\mu, \mu, \ldots, \mu, \sigma, P,) &= \prod_{i=1}^{k} \left\{ P + (1-P) \times \Phi\left(\frac{y_L - \mu}{\sigma}\right) \right\}^{m_i} \times \\ & \prod_{j=m_i+1}^{n_i} \left\{ \frac{1-P}{\sigma} \times \phi\left(\frac{y_{ij} - \mu}{\sigma}\right) \right\} \end{aligned}$$

Let $\varepsilon = \frac{y_L - \mu}{\sigma}$. To find the MLE under H_0, similarly we need to solve the following equations

$$\begin{aligned} \frac{m_i \times (1-P)\phi(\varepsilon)}{\sigma \times [P + (1-P) \times \Phi(\varepsilon)]} + \frac{\sum_{j=m_i+1}^{n_i} (y_{ij} - \mu)}{\sigma^2} &= 0 \\ \sum_{i=1}^{k} \left\{ \frac{m_i \times (1-P) \times (-\varepsilon) \times \phi(\varepsilon)}{P + (1-P) \times \Phi(\varepsilon)} - \frac{n_i - m_i}{\sigma} + \frac{\sum_{j=m_i+1}^{n_i} (y_{ij} - \mu)^2}{\sigma^3} \right\} &= 0 \\ \sum_{i=1}^{k} \left\{ \frac{m_i \times [1 - \Phi(\varepsilon)]}{P + (1-P) \times \Phi(\varepsilon)} - \frac{n_i - m_i}{1-P} \right\} &= 0 \end{aligned}$$

The likelihood ratio is

$$\lambda(\theta) = \frac{L_m(\mu, \mu, \ldots, \mu, \sigma, P,)}{L_m(\mu_1, \mu_2, \ldots, \mu_k, \sigma, P,)}$$

And the test statistics $T = -2 \times ln\lambda(\theta)$ follows a χ^2 distribution with degrees of freedom $k-1$. k is number of independent populations which is three in the present study.

CHAPTER IV

SIMULATIONS AND RESULTS

In this chapter, we will conduct a set of Monte Carlo simulations by the proposed Likelihood ratio model in Chapter 3. Type I error and statistical power will be evaluated and compared. We will treat standard deviation, σ, and the probability of true zeros, p, as nuisance parameters which are not of our primary interest in this study. Our primary interest is on mean comparison of the positive part for K ($K \geq 3$) independent populations. Each combination of parameters will run 5000 simulations to obtain respective observed size and power.

As will show below, there are over 10000 different combinations in the total of observed size and power, and each combination will run 5000 simulations, it is computation intensive and requires large computing resources. Due to the heavy computation requirement, we run all the programs on Amazon Elastic Compute Cloud (EC2). The server on Amazon EC2 has 16 CPUs, 30GB of Memory, and 2x160 GB SSD Storage. All the R codes are paralleled to utilize all 16 CPUs simultaneously on server. The computation time is significantly reduced by conducting parallel computation with cloud computing, compared to using personal computer. All the program codes are written in RStudio version 0.98.1049 using R version 3.1.1 on Linux Ubuntu (64-bit) 14.04LTS platform.

4.1 Log-normal Distribution

4.1.1 Settings

Specifically, in the simulation study of this chapter we compare the equality of the means in log scale of positive part in three independent populations. For comparing type I error we compare the combinations of parameters μ and σ in log scale, p, σ, Detection limit (DL) y_L and group size n. The range of parameters μ and σ in log scale is from 0.5 to 5, incremented by 0.5. The probability of true zeros p has range $\{0.1, 0.2, 0.3, 0.4, 0.5\}$. We set the probability under DL as $\{0.01, 0.1, 0.2, 0.3, 0.4\}$ and group size as $\{100, 200, 500\}$, so that the actual values of y_L vary by different combination of μ and σ, with small, medium and large sample. The significance level is $\alpha = 0.05$. Under such setting, there will be $10 \times 10 \times 5 \times 5 \times 3 = 7500$ different combinations of μ, σ, p, y_L and n.

As introduced before, the mean and standard deviation in the log scale, μ and σ, range from 0.5 to 5 with increment of 0.5. The actual mean and standard deviation of Log-normal distribution are $e^{\mu+\frac{\sigma^2}{2}}$ and $\sqrt{(e^{\sigma^2-1})e^{2\mu+\sigma^2}}$, respectively. Therefore, the range of mean for the Log-normal distribution is $[1.868, 39824784]$, and the range of standard deviation for the Log-normal distribution is $[1.284, 6.48 \times 10^{12}]$. We think the above range is sufficiently large for the case of present study.

For observed power, all the settings keep the same as in observed size with exception of μ. We keep all μ identical for observed size, but we vary μ for comparing observed power. The differences in μ are decided by the convention introduced by Cohen (1988). Small, medium and large effect size are 0.2, 0.5 and 0.8, respectively. By the formula of $\frac{|\max \mu \text{ difference}|}{\sigma}$ = effect size, we can calculate the maximum difference of μ among all populations. Since we use three independent populations in the simulations, without loss of generality we set the difference between $\mu_1$and μ_2 equals to the maximum difference from the convention, and μ_3 is set to equal to μ_1, μ_2 and $\frac{\mu_1+\mu_2}{2}$ in three combinations, respectively. However, the above procedure for power will no longer guarantee μ to fall in the range of 0.5 and 5.

Since we should set different values of μ to obtain observed power, considering the effect size convention and the selection of values of mean for the third population, there are at least $7500 \times 3 = 22500$ combinations to obtain observed power. In order to save resources and for the sake of parsimony, we categorize each of the parameters μ, σ, p into small, medium and large size. $\mu = \{0.5, 1.0, 1.5, 2.0\}, \sigma = \{0.5, 1.0, 1.5, 2.0\}, p = \{0.1, 0.2, 0.3\}$ are set as small; $\mu = \{2.5, 3.0, 3.5\}, \sigma = \{2.5, 3.0, 3.5\}, p = 0.3$ are set as medium; and $\mu = \{4.0, 4.5, 5.0\}, \sigma = \{4.0, 4.5, 5.0\}, p = \{0.4, 0.5\}$ are set as large. Therefore, under each n and y_L, we choose different combinations of μ, σ and p with observed sizes between 0.49 and 0.51.

There are two steps to generate the data. First, based on the sample size we generate number of true zeros for each population by Binomial distribution. Second, the rest of the data for each population are generated from a Log-normal distribution. We will later use data generated from Gamma distribution to evaluate the performance of the proposed model. Third, values below the DL are censored and combined with the true zeros as the total number of zeros. The restricted Maximum likelihood Estimates (MLEs) under $H_0 : \mu_1 = \mu_2 = \mu_3$ and unrestricted MLEs under $H_1 : \mu_i \neq \mu_j,\ for\ some\ i, j$ are calculated using L-BFGS-B method with parameter restrictions in OPTIM function in R software.

4.1.2 Results

Tables 4.1-4.3 show the observed size from simulation for $n = 100, 200$ and 500. The full table of observed size consists of 500 rows, we show a random sample of 50 rows as an example. The full tables are Table 7.1-7.3 in Appendix A.

Each row in the tables shows mean μ in log scale, standard deviation σ in log scale, probability of true zeros p, and the observed sizes under different detection limits. For example, $DL0.01$ stands for the probability of a positive values falling under DL, which is $P(y \leq DL) = 0.01$, where y is the random variables from Log-normal distribution. The means of three populations are equal for size calculation, so we show only one μ instead of three.

The standard error of observed size in the present study is $se = \sqrt{\frac{0.05(1-0.05)}{5000}} = 0.00308$. We consider the observed sizes within $3 \times se$ as an acceptable range, which is $0.05 \pm 0.0092 \approx [0.041, 0.059]$. From Tables 4.1-4.3 and, we see a general pattern that as $P(y \leq DL)$ increases, the observed size becomes larger. However, the pattern is clear if comparing sizes of column $DL0.001$ and column $DL0.4$. This finding concurs with the expectation because when $P(y \leq DL)$ becomes large, there are more positive values falling below DL, which means we have less available positive observed information. The result, observed size, will become less accurate.

When comparing across three population sizes, $n = 100, 200$ and 500, we found under the same combination of μ, σ and p, large group size will tend to have a better observed size with minor fluctuations, which is close to 0.05. This finding is intuitive that more data are available, more accurate the test will be.

From Table 4.1, we found the observed sizes tend to be large when the probability of true zeros $p = 0.4$ or 0.5. This finding implies that when group sizes are small, $n = 100$ for example, large p will reduce the number of observed positive values, and the situation becomes even worse under large values of $P(y \leq DL)$. The sizes are out of the normal range $[0.041, 0.059]$ if $P(y \leq DL) = 0.4$, such as rows $1, 2, 5, 12, 14, 18$ and 20. Table 7.1 of Appendix A with complete cases also supports the above finding. This finding is relaxed a little when the group size increases as in Table 4.2 and 4.3. We see that some cases with $p = 0.3$ or 0.2 have observed sizes out of the normal range, such as rows $9, 12,$ and 37 in Table 4.2 and rows 27 in Table 4.3. Table 7.2 and 7.3 in Appendix A have more rows of this situation.

Looking further on finding the combinations of parameters that have large observed sizes, we found that $P(y \leq DL), n$ and p are three dominant factors. From Tables 4.1-4.3 and Tables 7.1-7.3, observed sizes are out of the range for the combinations of large $P(y \leq DL)$, small n and large p. In those cases, the observed sizes are bad. Moreover, we find that when p, n and $P(y \leq DL)$ fixed, the combinations of small μ and large σ tend to have worse observed size than the combinations of large μ and small σ.

In summary, across all 7500 combinations of $\mu, \sigma, p, P(y \leq DL)$ and n, the proposed likelihood ratio test maintains good observed sizes. The observed sizes increases as $P(y \leq DL)$ increases and it decreases as group size increases. And

$P(y \leq DL), n$ and p are three dominant parameters in that large $P(y \leq DL)$, small n and large p will always lead to bad observed sizes. And also, the combinations of large μ and small σ tend to have better sizes than those of small μ and large σ.

Table 4.1: Example of Observed Size Comparison for n=100

	μ	σ	p	DL0.01	DL0.1	DL0.2	DL0.3	DL0.4
1	0.5	0.5	0.5	0.051	0.057	0.053	0.060	0.065
2	0.5	1.0	0.4	0.055	0.059	0.053	0.062	0.064
3	0.5	2.0	0.3	0.049	0.051	0.059	0.054	0.061
4	0.5	3.0	0.2	0.056	0.056	0.054	0.055	0.054
5	1.0	2.0	0.5	0.055	0.054	0.055	0.057	0.072
6	1.0	3.0	0.3	0.053	0.059	0.058	0.060	0.056
7	1.0	4.0	0.1	0.052	0.054	0.055	0.059	0.055
8	1.5	1.0	0.2	0.053	0.057	0.064	0.060	0.061
9	1.5	2.5	0.5	0.055	0.062	0.059	0.066	0.058
10	1.5	3.5	0.2	0.053	0.057	0.054	0.061	0.057
11	1.5	4.5	0.2	0.054	0.056	0.056	0.053	0.054
12	2.0	1.5	0.5	0.053	0.053	0.066	0.060	0.070
13	2.0	2.0	0.1	0.048	0.052	0.054	0.055	0.054
14	2.0	2.0	0.4	0.049	0.056	0.058	0.063	0.060
15	2.0	3.0	0.3	0.054	0.057	0.055	0.058	0.058
16	2.0	3.5	0.1	0.050	0.058	0.054	0.055	0.061
17	2.0	3.5	0.3	0.050	0.051	0.062	0.060	0.058
18	2.0	4.0	0.4	0.055	0.051	0.060	0.061	0.065
19	2.5	1.0	0.1	0.053	0.052	0.052	0.060	0.053
20	2.5	1.5	0.4	0.051	0.053	0.056	0.061	0.063
21	2.5	2.0	0.4	0.051	0.060	0.060	0.058	0.063
22	2.5	2.5	0.1	0.052	0.052	0.053	0.051	0.056
23	2.5	2.5	0.5	0.054	0.062	0.063	0.066	0.067
24	2.5	4.5	0.4	0.055	0.059	0.057	0.064	0.065
25	3.0	2.0	0.1	0.046	0.057	0.056	0.057	0.056
26	3.0	3.0	0.2	0.049	0.054	0.058	0.062	0.055
27	3.0	3.5	0.4	0.050	0.059	0.060	0.061	0.063
28	3.0	4.0	0.3	0.055	0.055	0.060	0.058	0.057
29	3.5	1.5	0.3	0.050	0.059	0.057	0.060	0.057
30	3.5	1.5	0.4	0.054	0.061	0.058	0.060	0.062
31	3.5	2.5	0.3	0.055	0.055	0.058	0.060	0.059
32	3.5	4.0	0.2	0.048	0.053	0.056	0.060	0.059
33	3.5	4.5	0.1	0.055	0.051	0.055	0.062	0.053
34	4.0	2.0	0.5	0.062	0.060	0.059	0.065	0.065
35	4.0	2.5	0.2	0.050	0.057	0.058	0.055	0.055
36	4.0	2.5	0.3	0.050	0.056	0.054	0.059	0.054
37	4.0	4.0	0.5	0.052	0.058	0.056	0.065	0.069
38	4.0	5.0	0.2	0.055	0.051	0.060	0.054	0.060
39	4.0	5.0	0.5	0.052	0.055	0.059	0.061	0.059

40	4.5	0.5	0.5	0.050	0.057	0.060	0.063	0.069
41	4.5	1.5	0.5	0.055	0.059	0.063	0.064	0.066
42	4.5	3.0	0.2	0.057	0.052	0.062	0.056	0.061
43	4.5	3.0	0.3	0.051	0.061	0.056	0.056	0.056
44	4.5	3.0	0.4	0.053	0.060	0.065	0.058	0.059
45	5.0	0.5	0.2	0.058	0.057	0.055	0.060	0.057
46	5.0	0.5	0.3	0.050	0.051	0.060	0.056	0.057
47	5.0	1.5	0.3	0.051	0.054	0.063	0.059	0.056
48	5.0	3.0	0.5	0.054	0.053	0.060	0.062	0.069
49	5.0	3.5	0.1	0.056	0.054	0.060	0.054	0.059
50	5.0	4.0	0.3	0.052	0.059	0.051	0.057	0.060

Table 4.2: Example of Observed Size Comparison for n=200

	μ	σ	p	DL0.01	DL0.1	DL0.2	DL0.3	DL0.4
1	0.5	0.5	0.3	0.053	0.048	0.053	0.055	0.059
2	0.5	2.0	0.5	0.050	0.059	0.059	0.058	0.055
3	0.5	3.0	0.2	0.051	0.048	0.051	0.056	0.056
4	1.0	1.0	0.5	0.050	0.050	0.056	0.062	0.058
5	1.0	1.5	0.1	0.050	0.052	0.050	0.060	0.052
6	1.0	2.0	0.1	0.050	0.048	0.045	0.052	0.057
7	1.0	2.0	0.5	0.052	0.050	0.051	0.058	0.055
8	1.0	2.5	0.1	0.051	0.050	0.054	0.052	0.054
9	1.0	2.5	0.2	0.048	0.048	0.050	0.052	0.060
10	1.0	4.0	0.4	0.052	0.052	0.053	0.050	0.052
11	1.0	5.0	0.2	0.051	0.055	0.059	0.053	0.059
12	1.5	0.5	0.3	0.050	0.049	0.057	0.055	0.060
13	1.5	3.5	0.2	0.048	0.054	0.053	0.059	0.055
14	1.5	4.0	0.4	0.054	0.045	0.053	0.062	0.055
15	2.0	1.5	0.5	0.051	0.052	0.059	0.058	0.055
16	2.0	2.0	0.1	0.056	0.054	0.051	0.052	0.053
17	2.0	4.5	0.4	0.057	0.052	0.050	0.050	0.055
18	2.0	5.0	0.5	0.049	0.054	0.054	0.055	0.061
19	2.5	0.5	0.1	0.049	0.057	0.057	0.055	0.050
20	2.5	1.0	0.1	0.053	0.057	0.051	0.055	0.056
21	2.5	1.0	0.3	0.056	0.052	0.050	0.048	0.054
22	2.5	1.0	0.5	0.046	0.054	0.057	0.055	0.059
23	2.5	1.5	0.3	0.049	0.056	0.058	0.055	0.057
24	2.5	2.0	0.1	0.053	0.060	0.056	0.060	0.052
25	2.5	2.0	0.2	0.051	0.052	0.055	0.057	0.054
26	2.5	3.0	0.5	0.055	0.056	0.054	0.056	0.060
27	2.5	3.5	0.5	0.052	0.051	0.060	0.060	0.060
28	2.5	4.5	0.4	0.049	0.054	0.058	0.053	0.055
29	2.5	4.5	0.5	0.056	0.060	0.058	0.055	0.053
30	3.0	2.0	0.2	0.054	0.063	0.055	0.051	0.054
31	3.0	2.5	0.2	0.048	0.050	0.055	0.055	0.056
32	3.0	3.0	0.5	0.049	0.053	0.052	0.059	0.060
33	3.0	4.5	0.3	0.053	0.057	0.050	0.053	0.051
34	3.0	5.0	0.5	0.049	0.049	0.052	0.058	0.058
35	3.5	2.0	0.4	0.052	0.048	0.055	0.054	0.056
36	3.5	4.5	0.2	0.054	0.049	0.050	0.053	0.054
37	3.5	4.5	0.3	0.046	0.051	0.055	0.052	0.063
38	4.0	0.5	0.4	0.053	0.049	0.056	0.057	0.050
39	4.0	1.5	0.1	0.049	0.053	0.055	0.053	0.052
40	4.5	1.0	0.5	0.049	0.052	0.059	0.060	0.056
41	4.5	3.5	0.1	0.050	0.057	0.054	0.055	0.054
42	4.5	3.5	0.2	0.051	0.051	0.055	0.054	0.054
43	4.5	3.5	0.5	0.054	0.052	0.053	0.057	0.058
44	4.5	4.0	0.2	0.051	0.048	0.054	0.052	0.052
45	4.5	4.5	0.4	0.048	0.055	0.060	0.052	0.052

46	5.0	1.0	0.1	0.045	0.049	0.052	0.052	0.058
47	5.0	2.5	0.3	0.053	0.051	0.058	0.052	0.052
48	5.0	2.5	0.4	0.052	0.053	0.052	0.055	0.053
49	5.0	2.5	0.5	0.052	0.052	0.059	0.056	0.056
50	5.0	3.5	0.3	0.056	0.050	0.051	0.055	0.052

Table 4.3: Example of Observed Size Comparison for n=500

	μ	σ	p	DL0.01	DL0.1	DL0.2	DL0.3	DL0.4
1	0.5	1.0	0.5	0.052	0.056	0.054	0.046	0.053
2	0.5	3.5	0.4	0.052	0.047	0.055	0.055	0.053
3	0.5	4.0	0.5	0.056	0.048	0.052	0.054	0.049
4	1.0	1.0	0.4	0.050	0.046	0.057	0.053	0.053
5	1.0	1.5	0.4	0.050	0.053	0.051	0.050	0.056
6	1.0	3.5	0.1	0.052	0.053	0.058	0.049	0.050
7	1.0	4.0	0.4	0.055	0.055	0.051	0.056	0.052
8	1.0	4.5	0.5	0.053	0.048	0.049	0.053	0.055
9	1.5	0.5	0.5	0.056	0.056	0.058	0.049	0.054
10	1.5	1.5	0.4	0.049	0.048	0.051	0.052	0.054
11	1.5	2.0	0.3	0.055	0.058	0.055	0.052	0.053
12	1.5	2.5	0.1	0.045	0.046	0.048	0.049	0.049
13	1.5	4.5	0.1	0.048	0.048	0.051	0.048	0.050
14	1.5	5.0	0.1	0.056	0.047	0.054	0.054	0.048
15	2.0	2.0	0.1	0.051	0.050	0.048	0.048	0.050
16	2.0	2.0	0.2	0.051	0.049	0.048	0.054	0.053
17	2.0	2.5	0.2	0.051	0.055	0.050	0.053	0.053
18	2.0	4.5	0.3	0.049	0.053	0.049	0.051	0.046
19	2.0	4.5	0.4	0.049	0.055	0.053	0.048	0.057
20	2.5	1.0	0.5	0.045	0.048	0.050	0.058	0.053
21	2.5	2.5	0.1	0.052	0.050	0.049	0.054	0.054
22	2.5	2.5	0.4	0.057	0.047	0.052	0.053	0.050
23	2.5	4.0	0.2	0.046	0.053	0.051	0.050	0.048
24	2.5	5.0	0.1	0.046	0.051	0.053	0.054	0.047
25	2.5	5.0	0.2	0.045	0.049	0.057	0.055	0.053
26	3.0	1.5	0.4	0.050	0.048	0.054	0.055	0.055
27	3.0	2.5	0.2	0.054	0.049	0.052	0.053	0.061
28	3.0	3.0	0.1	0.049	0.054	0.048	0.053	0.055
29	3.0	3.0	0.5	0.047	0.049	0.050	0.052	0.053
30	3.0	3.5	0.4	0.049	0.049	0.052	0.055	0.055
31	3.5	2.0	0.3	0.048	0.049	0.051	0.053	0.053
32	3.5	4.0	0.1	0.050	0.050	0.049	0.052	0.049
33	3.5	5.0	0.3	0.049	0.050	0.051	0.054	0.050
34	3.5	5.0	0.4	0.054	0.051	0.054	0.051	0.049
35	4.0	1.0	0.5	0.054	0.052	0.052	0.055	0.057
36	4.0	4.5	0.5	0.057	0.049	0.052	0.053	0.053
37	4.5	0.5	0.2	0.050	0.049	0.051	0.050	0.054
38	4.5	1.0	0.5	0.051	0.058	0.051	0.051	0.052
39	4.5	1.5	0.4	0.048	0.048	0.050	0.053	0.054
40	4.5	3.0	0.1	0.053	0.050	0.052	0.057	0.058
41	4.5	3.5	0.3	0.050	0.053	0.056	0.056	0.054
42	4.5	4.5	0.4	0.050	0.050	0.048	0.050	0.058
43	4.5	4.5	0.5	0.050	0.051	0.054	0.052	0.055
44	4.5	5.0	0.2	0.047	0.053	0.049	0.052	0.050
45	5.0	1.5	0.2	0.048	0.050	0.056	0.049	0.056

46	5.0	1.5	0.4	0.047	0.052	0.059	0.054	0.053
47	5.0	3.0	0.5	0.052	0.053	0.053	0.056	0.054
48	5.0	4.0	0.2	0.048	0.049	0.047	0.052	0.055
49	5.0	4.0	0.5	0.051	0.056	0.049	0.052	0.053
50	5.0	4.5	0.5	0.047	0.046	0.049	0.054	0.053

As introduced in section 4.1.1, there are 7500 different combinations of parameters for observed size calculation. However, there will be much more combinations to vary mean μ for observed power calculation. In order to save resources, we categorize μ, σ and p into small, medium and large, and select the combinations where they have observed size around 0.05 to see those parameter combinations performances on observed power.

The following Table 4.4 is the partial table of observed power with $n = 100, P(y \leq DL) = 0.1$. The observed powers under other combinations of parameters are Table 7.4-7.17 in Appendix B. We only show μ_1, the largest mean of three μ's, in the table, but one can indirectly calculate the smallest μ using Cohen (1988)'s convention and σ. However, we set the third mean, μ_3, to be equal to either largest mean μ_1, smallest mean μ_2 or $\frac{\mu_1+\mu_2}{2}$. We find that the observed power is the smallest out of three situations when $\mu_3 = \frac{\mu_1+\mu_2}{2}$. In order to be conservative, under each combination of μ_1 and μ_2 we select the power with $\mu_3 = \frac{\mu_1+\mu_2}{2}$ only.

From Table 4.4 and Table7.4-7.17 we find that generally speaking the proposed likelihood ratio test have good powers under large effect size while setting the size around 0.05. More specifically, all but a few parameter combinations yielded powers above 0.8. The powers decreases when effect size decreases and $P(y \leq DL)$ increases. The powers for small effect size are bad, usually below 0.3 for small and medium group size, $n = 100$ or 200. When $n = 500$ the power could reach above 0.4 when $P(y \leq DL) = 0.4$, and for smaller $P(y \leq DL)$ the power could reach above 0.6.

In summary, we follow the convention of Cohen (1988) to set $0.2, 0.5$ and 0.8 as small, medium and large effect size. From Table 4.4 and Tables 7.4-7.17 in Appendix B we find the proposed likelihood ratio test performs very well on observed power when maximum difference in mean, $max|\mu_1 - \mu_2| = 0.8\sigma$. When n is small, $P(y \leq DL)$ is large and $max|\mu_1 - \mu_2| = 0.2\sigma$, the power is bad. However, power could reach above 0.6 even though $max|\mu_1 - \mu_2| = 0.2\sigma$ and $P(y \leq DL)$ is large.

Table 4.4: Observed Power with Various Effect Size for n=100

μ_1	σ	p	LargeEffect	MediumEffect	SmallEffect
0.5	0.5	0.4	0.886	0.506	0.134
2.5	0.5	0.5	0.807	0.421	0.118
2.0	1.0	0.1	0.995	0.776	0.189
2.5	1.0	0.3	0.943	0.603	0.142
4.0	1.0	0.2	0.976	0.685	0.153
0.5	1.5	0.4	0.891	0.513	0.126
1.0	1.5	0.3	0.948	0.613	0.144
2.5	1.5	0.5	0.911	0.130	0.531
2.5	1.5	0.3	0.728	0.984	0.165
3.0	1.5	0.2	0.978	0.690	0.150
1.5	2.0	0.3	0.985	0.182	0.733
1.5	2.0	0.2	0.806	0.996	0.192
3.5	2.0	0.5	0.911	0.136	0.566
3.5	2.0	0.2	0.800	0.996	0.197
4.5	2.0	0.2	0.979	0.693	0.159
5.0	2.0	0.1	0.993	0.771	0.182
3.5	3.5	0.4	0.897	0.515	0.131
4.5	3.5	0.2	0.976	0.679	0.163
2.0	4.5	0.3	0.953	0.598	0.134
3.5	4.5	0.3	0.949	0.599	0.142
4.5	4.5	0.4	0.891	0.514	0.134
0.5	5.0	0.2	0.977	0.678	0.155
1.0	5.0	0.2	0.975	0.673	0.153
1.5	5.0	0.1	0.994	0.778	0.181
3.0	5.0	0.1	0.994	0.775	0.180
4.5	5.0	0.2	0.974	0.677	0.160

4.2 Gamma Distribution

The above conclusions are drawn based on the the assumption that data are from a Log-normal distribution. This is a common assumption in many literature. However, there are a few other probability distributions that could have very similar density shapes to Log-normal distribution, as an example, Gamma distribution. The following a few figures show the similarity of Log-normal and Gamma distributions under certain values of parameters. We set the coefficient of variation to be equal in order to match the parameters.

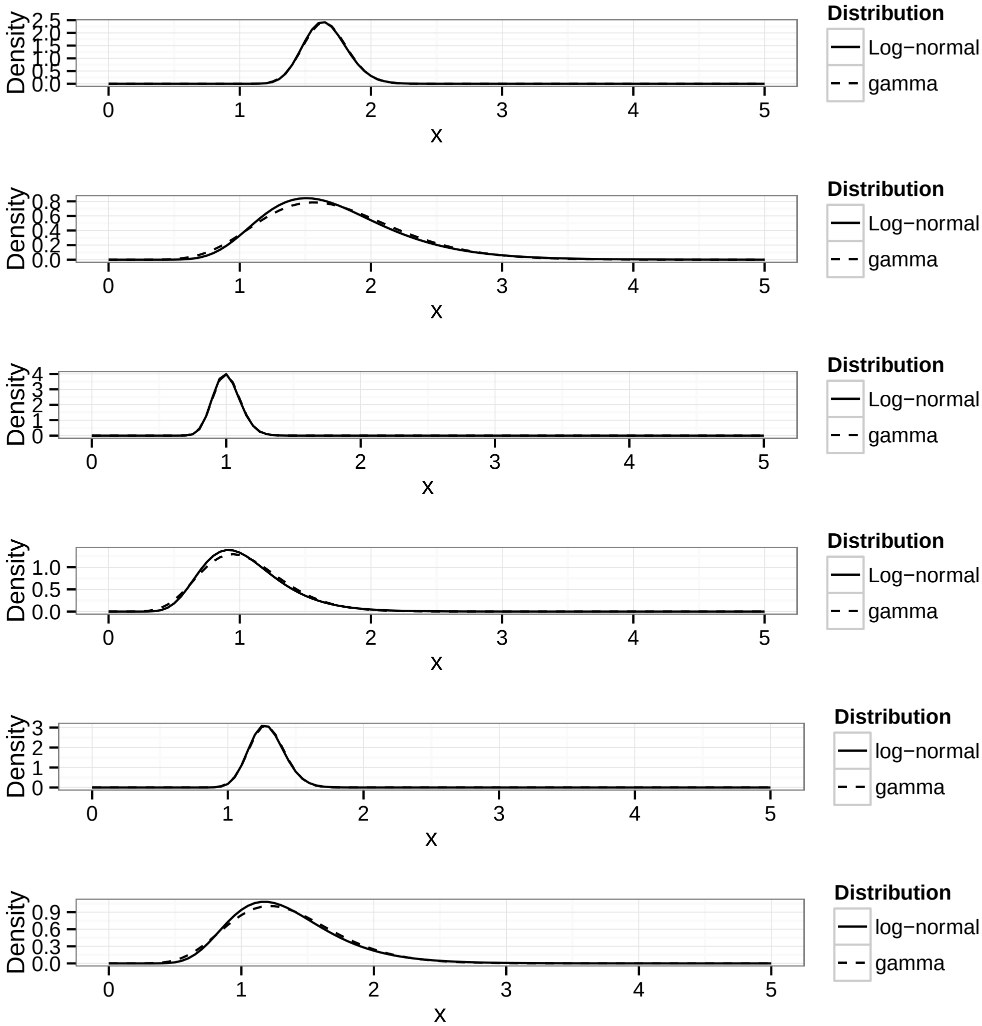


Figure 4.1: Similarity Between Log-normal and Gamma Distribution

Figure 4.1 includes the density curves of Log-normal distributions with parameters $(\mu, \sigma) = (0.5, 0.1)$, $(0.5, 0.3)$, $(0, 0.1)$, $(0, 0.3)$, $(0.25, 0.1)$ and $(0.25, 0.3)$, respectively. And it also includes the density curves of Gamma distribution with shape parameters α and scale parameter s, $(\alpha, s) = (99.5, 0.017)$, $(10.62, 0.16)$, $(99.5, 0.01)$, $(10.62, 0.10)$, $(99.5, 0.013)$, and $(10.62, 0.126)$, respectively, with the form of $p.d.f$ as $f(x; \alpha, s) = \frac{1}{\Gamma(\alpha)s^{\alpha}} x^{\alpha-1} e^{-\frac{x}{s}}$. We match parameters of Log-normal distribution with Gamma distribution by matching the square of Coefficient of Variation. The details are as follow:

The mean and variance of Long-normal distribution are $e^{\mu+\frac{\sigma^2}{2}}$ and $e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$, $CV^2 = \frac{var(Y)}{(E(Y))^2} = e^{\sigma^2} - 1$, and the mean and variance of Gamma distribution are $\alpha\beta$ and $\alpha\beta^2$, $CV^2 = \frac{var(Y)}{(E(Y))^2} = \frac{1}{\alpha}$, where α is the shape parameter and β is the scale parameter. By matching CV^2, we have the corresponding parameters of Gamma distribution.

$$\begin{cases} \alpha &= \frac{1}{e^{\sigma^2}-1} \\ \beta &= e^{\mu+\frac{\sigma^2}{2}}(e^{\sigma^2}-1) \end{cases}$$

The shape parameter α is related to σ only, and we know that the shape of Gamma distribution will be strictly decreasing when $\alpha \leq 1$. So by the above formula, in order to have similarity with Gamma distribution, σ has to meet the criterion $\alpha = \frac{1}{e^{\sigma^2}-1} > 1 \Rightarrow \sigma < 0.83$.

We are interested in if the likelihood ratio test proposed in Chapter Three perform well when the data are from a Gamma distribution, instead of a Log-normal distribution. In another word, we would like to see the robustness of the proposed likelihood ratio test, which is generated based on Log-normal distribution, using data from a Gamma distribution. The results are as follows using the parameters from above figure.

Table 4.5 shows the observed size comparison between Log-normal distribution and Gamma distribution. The first five columns are the parameters of Log-normal and corresponding Gamma distribution. The last four columns are sizes of Long-normal and Gamma distribution under$P(y_i \leq DL) = 0.1$ and 0.4, respectively. We see that sizes of both distributions are close and it is hard to decide which is constantly better than the other one. The majority of them are within the normal range $[0.041, 0.059]$ for $P(y_i \leq DL) = 0.1$. And there are more sizes out of the range when the detection limit becomes larger, $P(y_i \leq DL) = 0.4$.

On the observed statistical power side, we use sample size $n = 100$ and DL is $P(y_i \leq DL) = 0.1$ in Table 4.6 and we sort it ascendingly by σ and μ_1, descendingly by μ_2, and We see that both distributions perform well on large effect size and poor

Table 4.5: Size Comparison of Log-normal vs Gamma Distributions. $n = 100$

	μ	σ	p	shape	scale	Lnorm0.1	Gamma0.1	Lnorm0.4	Gamma0.4
1	0.00	0.1	0.1	99.50	0.010	0.059	0.053	0.057	0.052
2	0.00	0.1	0.1	99.50	0.010	0.052	0.054	0.060	0.057
3	0.00	0.1	0.1	99.50	0.010	0.059	0.057	0.066	0.052
4	0.00	0.3	0.1	10.62	0.099	0.058	0.056	0.057	0.052
5	0.00	0.3	0.1	10.62	0.099	0.054	0.055	0.054	0.056
6	0.00	0.3	0.1	10.62	0.099	0.057	0.054	0.064	0.052
7	0.25	0.1	0.3	99.50	0.013	0.053	0.051	0.059	0.059
8	0.25	0.1	0.3	99.50	0.013	0.052	0.050	0.056	0.055
9	0.25	0.1	0.3	99.50	0.013	0.059	0.056	0.068	0.063
10	0.25	0.3	0.3	10.62	0.126	0.060	0.062	0.060	0.054
11	0.25	0.3	0.3	10.62	0.126	0.057	0.054	0.066	0.058
12	0.25	0.3	0.3	10.62	0.126	0.057	0.055	0.069	0.059
13	0.50	0.1	0.5	99.50	0.017	0.058	0.056	0.054	0.063
14	0.50	0.1	0.5	99.50	0.017	0.062	0.060	0.052	0.065
15	0.50	0.1	0.5	99.50	0.017	0.062	0.053	0.065	0.063
16	0.50	0.3	0.5	10.62	0.162	0.052	0.057	0.057	0.061
17	0.50	0.3	0.5	10.62	0.162	0.059	0.065	0.064	0.060
18	0.50	0.3	0.5	10.62	0.162	0.056	0.059	0.069	0.064

on small effect size. Another finding is that Long-normal distribution is sensitive to the probability of true zero p. The power drops when p increases. However, Gamma distribution does not show this pattern. In addition, the power changes only slightly for Gamma distribution on the same level of effect size when p increases. But power of Log-normal changes much more as p increases. We think the reason is that the scale parameter β changes much smaller compared to the changes of μ. For example, from row $10-18$ in the table, β changes 0.0003 when μ changes 0.03, and tiny changes offset the increment of p.

In summary, if the true data are from a Gamma distribution, the proposed Log-normal based likelihood ratio test still maintain relatively good observed size. The test have high powers for both distributions on large effect size. On the same level of effect size, the test on Gamma distribution tends to maintain the power well as p increases.

Table 4.6: Observed Power Comparison of Log-normal vs Gamma Distributions

	μ_1	μ_2	σ	p	scale2	lnorm	gamma
1	0.00	-0.08	0.1	0.1	0.0093	0.994	0.993
2	0.00	-0.08	0.1	0.3	0.0093	0.946	0.996
3	0.00	-0.08	0.1	0.5	0.0093	0.818	0.994
4	0.00	-0.05	0.1	0.1	0.0096	0.769	0.774
5	0.00	-0.05	0.1	0.3	0.0096	0.593	0.777
6	0.00	-0.05	0.1	0.5	0.0096	0.433	0.775
7	0.00	-0.02	0.1	0.1	0.0099	0.189	0.183
8	0.00	-0.02	0.1	0.3	0.0099	0.139	0.180
9	0.00	-0.02	0.1	0.5	0.0099	0.114	0.179
10	0.25	0.17	0.1	0.1	0.0120	0.995	0.944
11	0.25	0.17	0.1	0.3	0.0120	0.948	0.946
12	0.25	0.17	0.1	0.5	0.0120	0.810	0.946
13	0.25	0.20	0.1	0.1	0.0123	0.778	0.608
14	0.25	0.20	0.1	0.3	0.0123	0.598	0.607
15	0.25	0.20	0.1	0.5	0.0123	0.431	0.622
16	0.25	0.23	0.1	0.1	0.0127	0.173	0.146
17	0.25	0.23	0.1	0.3	0.0127	0.145	0.150
18	0.25	0.23	0.1	0.5	0.0127	0.118	0.143
19	0.50	0.42	0.1	0.1	0.0154	0.993	0.812
20	0.50	0.42	0.1	0.3	0.0154	0.944	0.823
21	0.50	0.42	0.1	0.5	0.0154	0.821	0.816
22	0.50	0.45	0.1	0.1	0.0158	0.771	0.444
23	0.50	0.45	0.1	0.3	0.0158	0.601	0.431
24	0.50	0.45	0.1	0.5	0.0158	0.424	0.443
25	0.50	0.48	0.1	0.1	0.0163	0.183	0.117
26	0.50	0.48	0.1	0.3	0.0163	0.143	0.105
27	0.50	0.48	0.1	0.5	0.0163	0.114	0.121
28	0.00	-0.24	0.3	0.1	0.0775	0.992	0.988
29	0.00	-0.24	0.3	0.3	0.0775	0.944	0.990
30	0.00	-0.24	0.3	0.5	0.0775	0.824	0.989
31	0.00	-0.15	0.3	0.1	0.0848	0.779	0.738
32	0.00	-0.15	0.3	0.3	0.0848	0.585	0.736
33	0.00	-0.15	0.3	0.5	0.0848	0.436	0.741
34	0.00	-0.06	0.3	0.1	0.0928	0.187	0.164
35	0.00	-0.06	0.3	0.3	0.0928	0.140	0.168
36	0.00	-0.06	0.3	0.5	0.0928	0.119	0.164
37	0.25	0.01	0.3	0.1	0.0995	0.993	0.911
38	0.25	0.01	0.3	0.3	0.0995	0.946	0.922
39	0.25	0.01	0.3	0.5	0.0995	0.808	0.907
40	0.25	0.10	0.3	0.1	0.1089	0.787	0.537
41	0.25	0.10	0.3	0.3	0.1089	0.590	0.528
42	0.25	0.10	0.3	0.5	0.1089	0.431	0.521
43	0.25	0.19	0.3	0.1	0.1191	0.175	0.129
44	0.25	0.19	0.3	0.3	0.1191	0.148	0.130
45	0.25	0.19	0.3	0.5	0.1191	0.110	0.128
46	0.50	0.26	0.3	0.1	0.1278	0.993	0.758
47	0.50	0.26	0.3	0.3	0.1278	0.943	0.748
48	0.50	0.26	0.3	0.5	0.1278	0.807	0.753
49	0.50	0.35	0.3	0.1	0.1398	0.770	0.387
50	0.50	0.35	0.3	0.3	0.1398	0.591	0.394
51	0.50	0.35	0.3	0.5	0.1398	0.433	0.394
52	0.50	0.44	0.3	0.1	0.1530	0.181	0.112
53	0.50	0.44	0.3	0.3	0.1530	0.141	0.111
54	0.50	0.44	0.3	0.5	0.1530	0.105	0.114

CHAPTER V

IMPUTATION METHODS

When non-detect values exist, a typical way is simply substituting all values below DL with $\frac{DL}{2}$, $\frac{2DL}{3}$, $\frac{DL}{\sqrt{2}}$, or DL. However, Chu et al. (2008) points out that such substitution has two potential draw backs: (1) some of these values may come from a special distribution, and (2) it fails to take left censoring models into account. Moreover, Krishnamoorthy et al. (2009) indicates that such approach is the worst with respect to mean squared error criterion.

In our study, we propose a new imputation method to obtain the distribution parameters. However, there are two primary challenges need to be addressed in the study:

1. Estimate number of values below the DL, and separate them from true zeros. Notice that we are not able to distinguish true zeros from the values below DL in the original observations.

2. Impute the values below the DL, "fill in" the observed values, and finally obtain the parameter estimates.

We first look at one population case. Let us recall that there are two parts of the observations: one is observed positive values, and the other part is zeros, and we are not able to distinguish the values below DL from all zeros.

Our ideas of the imputation method could be demonstrated by the following steps:

1. Obtain initial estimates $\hat{\mu}_0$ and $\hat{\sigma}_0$ from observed positive values.

2. Fit a Log-normal distribution $f_0(\cdot)$ in terms of $\hat{\mu}_0$ and $\hat{\sigma}_0$.

3. Calculate the probability of $f_0(\cdot)$ under DL, $\hat{p}_+$.

4. Calculate the expected number of values falling below DL, based on the equation of $\frac{\hat{p}_+}{1-\hat{p}_+} = \frac{\hat{n}_+}{n-m} \Rightarrow \hat{n}_+ = (n-m) \times \frac{\hat{p}_+}{1-\hat{p}_+}$, where $\hat{n}_+$is the estimated number of values falling below DL, n is the total group sample size, and m is the number of observed zeros.

5. Generate $\hat{n}_+$ values from $f_0(\cdot)$ falling below DL by rejection method.

6. Combine $\hat{n}_+$ new generated values with the $n-m$ observed values as a "complete" data set. Calculate updated estimated $\hat{\mu}_1$ and $\hat{\sigma}_1$.

7. Fit an updated Log-normal distribution $f_1(\cdot)$ in terms of $\hat{\mu}_1$ and $\hat{\sigma}_1$.

8. Repeat Steps 4 - 6 until the $\hat{\mu}_i - \hat{\mu}_{i-1}$ and $\hat{\sigma}_i - \hat{\sigma}_{i-1}$ converge.

The above algorithm has a natural drawback of overestimating the number of values under DL since the initial estimates $\hat{\mu}_0$ and $\hat{\sigma}_0$ are expected to be greater than the true values. After each iteration, the estimates of mean μ will be "dragged" smaller and standard deviation σ will be larger and hence $\hat{n}_+$ increases. Essentially, it is highly likely that the number of values under DL is overestimated and ultimately all zeros (both true and not-true) will be treated as values under DL. This results in underestimate of μ and overestimate of σ.

The R codes for the above algorithm is in Appendix C.

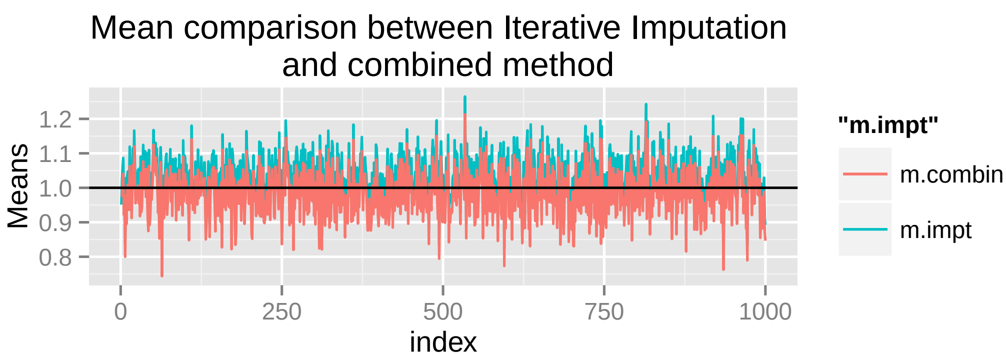
Another imputation method is called Regression on Order Statistics (ROS). It is first introduced by Helsel and Cohn (1988) as MR method, and after series of modifications, the ROS formed. ROS method use the unobserved values collectively to obtain the summary statistics, not assigning them to individual observations. More powerfully, ROS could address the data with multiple detection limit. However, such case is not under consideration in the present study.

ROS assumes the number of values under DL is known and replaces those values by the DL. In the present study, the issue is that we do not know that number. In order to implement ROS, we use the estimate from Step 4 above, $\hat{n}_+$, to be the estimate of number of values below DL. However, $\hat{n}_+$ has different values for each iteration, and it tends to be under-estimated in the initial iteration and over-estimated in the last iteration. In the present study, we use $\hat{n}_+$ from the last iteration.

The following Figures 5.1 - 5.3 show the estimated μ and σ compared to their true values. The DL is always set to be $P(y \leq DL) = 0.1$. The true values of mean μ, standard deviation σ, and probability of true zeros p are listed in the figure captions respectively. There are totally 1000 iterations and under each iteration the estimate of μ and σ will be obtained with a group of size $n = 100$. The pink line, *m.combin*, stands for the combination of the proposed imputation method in this chapter and ROS, and the blue line, *m.impt*, stands for the estimates from using the proposed method alone.

We see from the figures that *m.impt* tends to have unstable estimates, both on mean and standard deviation.

In order to make the conclusion more comprehensive, we need to see the comparison under different combinations of μ, σ and p. Our current recommendation is that when the sample size is more than 100, Regression on Order Statistics (ROS) using the estimate of number of values under DL from the proposed imputation method is recommended for obtaining summary statistics for the data with undetectable values.



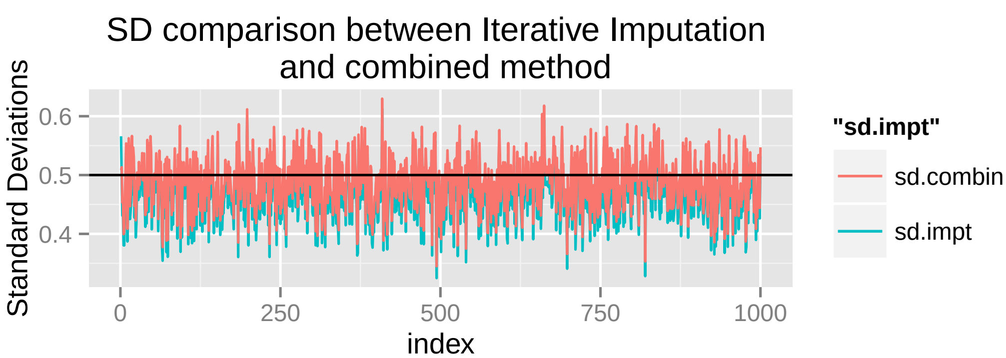
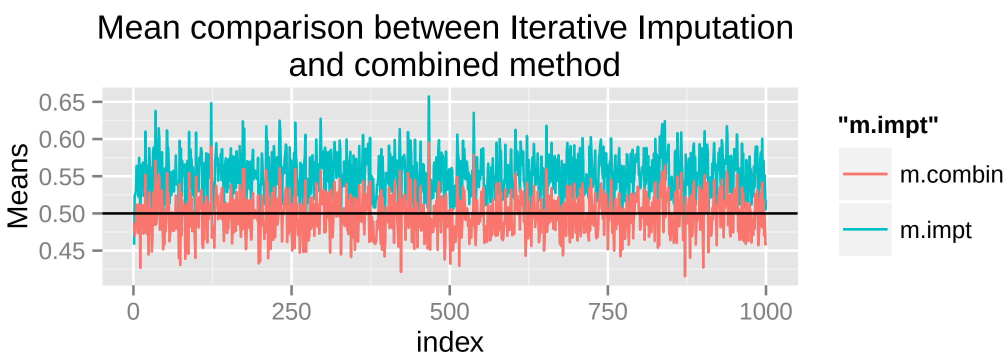


Figure 5.1: Estimates under $\mu = 1$, $\sigma = 0.5$, $p = 0.5, n = 100$



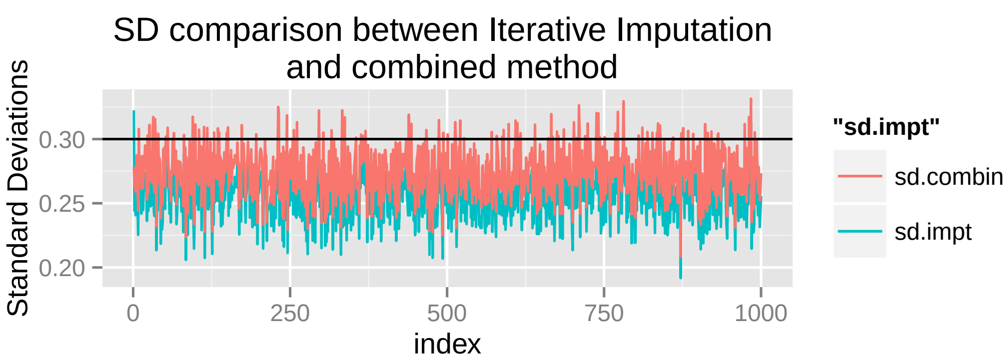


Figure 5.2: Estimates under $\mu = 0.5, \sigma = 0.3, p = 0.1, n = 100$

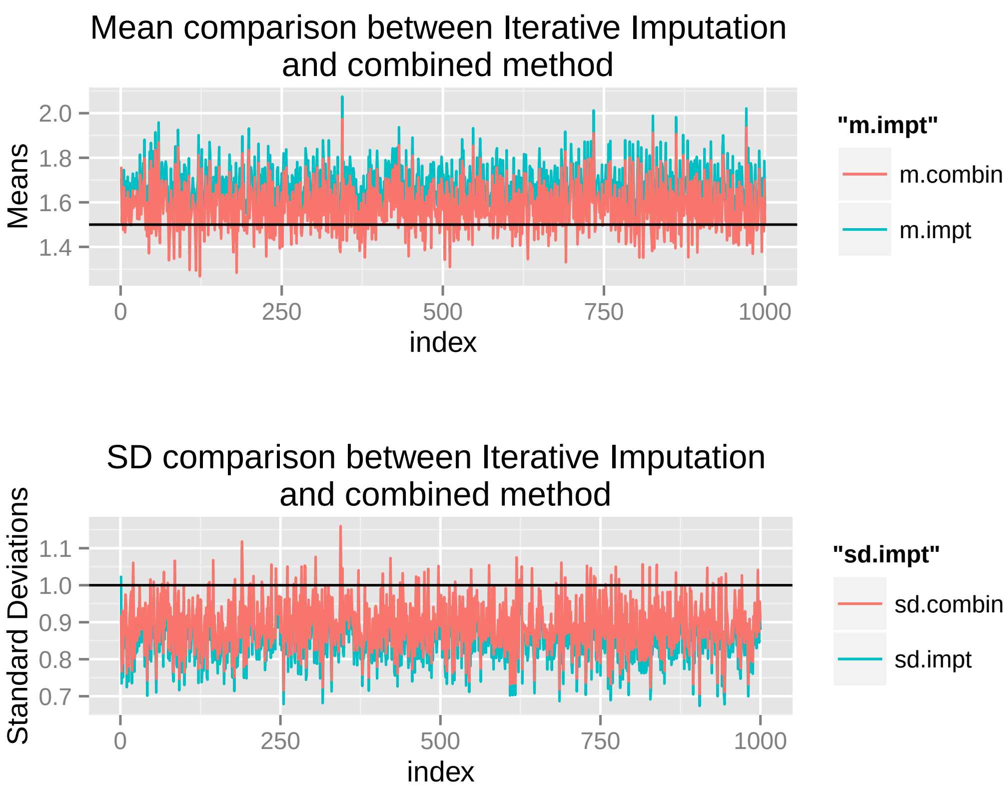


Figure 5.3: Estimates under $\mu = 1.5, \sigma = 1, p = 0.3, n = 100$

CHAPTER VI

CONCLUSIONS

The present study proposed a likelihood ratio test for K ($K \geq 3$) independent populations with a detection limit. The simulation and formula derivation will be based on $K = 3$ independent populations. Furthermore, the robustness (observed power) and sensitivity (observed size) of the proposed test was shown under various combinations of the parameters and under Log-normal distribution and Gamma distribution through simulation studies.

We found that generally speaking the proposed likelihood ratio test maintains good observed sizes. The observed sizes increases as $P(y \leq DL)$ increases and it decreases as group size increases. And $P(y \leq DL), n$ and p are three dominant parameters that large $P(y \leq DL)$, small n and large p will always lead to bad observed sizes. Moreover, the combinations of large μ and small σ tend to have better sizes than those of small μ and large σ. As for the observed power side, the proposed likelihood ratio test performs very well on observed power when maximum difference in mean, $max|\mu_1 - \mu_2| = 0.8\sigma$. When n is small, $P(y \leq DL)$ is large and $max|\mu_1 - \mu_2| = 0.2\sigma$, the power is bad. However, power could reach above 0.6 even though $max|\mu_1 - \mu_2| = 0.2\sigma$ and $P(y \leq DL)$ is large.

When data is actually from a Gamma distribution with a similar density curve as Log-normal distribution, the proposed Log-normal based likelihood ratio test still maintain relatively good observed size. The test have high powers for both distributions on large effect size. On the same level of effect size, the test on Gamma distribution tends to maintain the power well as p increases.

The imputation methods may be another direction to test the hypothesis with data with excess zeros and detection limits. Our recommendation is that the combination of the proposed imputation method and Regression on Order Statistics outperform the proposed imputation method alone.

The work presented in the current study could be extended in the following directions:

First, evaluate and improve the test performance under small group size ($n < 100$).

Second, investigate the performance of the test under large detection limit case, for example $P(y \leq DL) \geq 0.5$.

Third, if the underlying distribution is other than Log-normal or Gamma distribution, the performance of the test.

Fourth, derive a likelihood ratio test directly on Gamma distribution.

Lastly, more sophisticated imputation methods may be developed.

APPENDIX A
Tables of Observed Size

Table 7.1: Observed Size Comparison for n=100

	Mu	sigma	p	DL0.01	DL0.1	DL0.2	DL0.3	DL0.4
1	0.5	0.5	0.1	0.050	0.054	0.055	0.063	0.051
2	0.5	0.5	0.2	0.051	0.053	0.058	0.056	0.055
3	0.5	0.5	0.3	0.055	0.062	0.057	0.054	0.060
4	0.5	0.5	0.4	0.057	0.050	0.059	0.060	0.064
5	0.5	0.5	0.5	0.051	0.057	0.053	0.060	0.065
6	0.5	1.0	0.1	0.056	0.055	0.049	0.056	0.058
7	0.5	1.0	0.2	0.050	0.056	0.059	0.057	0.062
8	0.5	1.0	0.3	0.057	0.051	0.061	0.063	0.058
9	0.5	1.0	0.4	0.055	0.059	0.053	0.062	0.064
10	0.5	1.0	0.5	0.057	0.059	0.066	0.064	0.062
11	0.5	1.5	0.1	0.054	0.055	0.060	0.060	0.053
12	0.5	1.5	0.2	0.057	0.053	0.057	0.054	0.056
13	0.5	1.5	0.3	0.050	0.057	0.058	0.059	0.060
14	0.5	1.5	0.4	0.055	0.049	0.058	0.058	0.061
15	0.5	1.5	0.5	0.053	0.052	0.065	0.062	0.060
16	0.5	2.0	0.1	0.054	0.056	0.048	0.054	0.052
17	0.5	2.0	0.2	0.047	0.059	0.055	0.057	0.059
18	0.5	2.0	0.3	0.049	0.051	0.059	0.054	0.061
19	0.5	2.0	0.4	0.054	0.056	0.053	0.059	0.065
20	0.5	2.0	0.5	0.052	0.052	0.058	0.064	0.066
21	0.5	2.5	0.1	0.055	0.054	0.058	0.054	0.054
22	0.5	2.5	0.2	0.057	0.054	0.059	0.055	0.057
23	0.5	2.5	0.3	0.047	0.051	0.055	0.054	0.058
24	0.5	2.5	0.4	0.053	0.056	0.063	0.065	0.067
25	0.5	2.5	0.5	0.053	0.062	0.060	0.072	0.065
26	0.5	3.0	0.1	0.051	0.057	0.050	0.057	0.057
27	0.5	3.0	0.2	0.056	0.056	0.054	0.055	0.054
28	0.5	3.0	0.3	0.050	0.054	0.060	0.063	0.062
29	0.5	3.0	0.4	0.057	0.059	0.055	0.058	0.067
30	0.5	3.0	0.5	0.054	0.058	0.063	0.064	0.069
31	0.5	3.5	0.1	0.047	0.051	0.058	0.059	0.055
32	0.5	3.5	0.2	0.052	0.059	0.059	0.056	0.057
33	0.5	3.5	0.3	0.057	0.053	0.058	0.061	0.061
34	0.5	3.5	0.4	0.048	0.059	0.057	0.060	0.063
35	0.5	3.5	0.5	0.057	0.058	0.063	0.058	0.062
36	0.5	4.0	0.1	0.059	0.055	0.055	0.054	0.056
37	0.5	4.0	0.2	0.055	0.052	0.056	0.058	0.063

38	0.5	4.0	0.3	0.058	0.056	0.060	0.056	0.053
39	0.5	4.0	0.4	0.053	0.060	0.063	0.059	0.062
40	0.5	4.0	0.5	0.053	0.055	0.059	0.068	0.062
41	0.5	4.5	0.1	0.049	0.059	0.050	0.056	0.046
42	0.5	4.5	0.2	0.049	0.051	0.057	0.055	0.054
43	0.5	4.5	0.3	0.051	0.059	0.053	0.058	0.059
44	0.5	4.5	0.4	0.049	0.053	0.055	0.063	0.055
45	0.5	4.5	0.5	0.054	0.057	0.065	0.063	0.064
46	0.5	5.0	0.1	0.057	0.055	0.053	0.056	0.052
47	0.5	5.0	0.2	0.051	0.050	0.057	0.061	0.057
48	0.5	5.0	0.3	0.052	0.053	0.054	0.062	0.061
49	0.5	5.0	0.4	0.056	0.058	0.059	0.062	0.058
50	0.5	5.0	0.5	0.046	0.056	0.063	0.062	0.061
51	1.0	0.5	0.1	0.056	0.054	0.050	0.055	0.054
52	1.0	0.5	0.2	0.050	0.059	0.052	0.057	0.061
53	1.0	0.5	0.3	0.054	0.054	0.063	0.055	0.061
54	1.0	0.5	0.4	0.057	0.057	0.059	0.056	0.059
55	1.0	0.5	0.5	0.049	0.054	0.061	0.065	0.059
56	1.0	1.0	0.1	0.051	0.054	0.050	0.050	0.051
57	1.0	1.0	0.2	0.053	0.059	0.059	0.053	0.057
58	1.0	1.0	0.3	0.053	0.060	0.058	0.058	0.062
59	1.0	1.0	0.4	0.053	0.057	0.060	0.057	0.060
60	1.0	1.0	0.5	0.056	0.057	0.061	0.067	0.067
61	1.0	1.5	0.1	0.057	0.058	0.055	0.057	0.064
62	1.0	1.5	0.2	0.054	0.055	0.057	0.059	0.057
63	1.0	1.5	0.3	0.050	0.050	0.053	0.062	0.062
64	1.0	1.5	0.4	0.053	0.057	0.058	0.064	0.062
65	1.0	1.5	0.5	0.059	0.061	0.061	0.067	0.072
66	1.0	2.0	0.1	0.053	0.054	0.055	0.058	0.061
67	1.0	2.0	0.2	0.054	0.056	0.057	0.060	0.058
68	1.0	2.0	0.3	0.053	0.063	0.055	0.065	0.061
69	1.0	2.0	0.4	0.052	0.057	0.056	0.056	0.066
70	1.0	2.0	0.5	0.055	0.054	0.055	0.057	0.072
71	1.0	2.5	0.1	0.054	0.056	0.051	0.051	0.054
72	1.0	2.5	0.2	0.055	0.059	0.061	0.054	0.056
73	1.0	2.5	0.3	0.053	0.057	0.057	0.061	0.058
74	1.0	2.5	0.4	0.053	0.057	0.056	0.059	0.064
75	1.0	2.5	0.5	0.050	0.063	0.068	0.063	0.064
76	1.0	3.0	0.1	0.047	0.056	0.058	0.062	0.057
77	1.0	3.0	0.2	0.044	0.051	0.055	0.055	0.060
78	1.0	3.0	0.3	0.053	0.059	0.058	0.060	0.056
79	1.0	3.0	0.4	0.050	0.055	0.063	0.065	0.062
80	1.0	3.0	0.5	0.055	0.052	0.063	0.060	0.062
81	1.0	3.5	0.1	0.053	0.060	0.057	0.057	0.058
82	1.0	3.5	0.2	0.050	0.054	0.061	0.056	0.057
83	1.0	3.5	0.3	0.054	0.054	0.055	0.056	0.060
84	1.0	3.5	0.4	0.053	0.055	0.058	0.061	0.060
85	1.0	3.5	0.5	0.050	0.056	0.064	0.057	0.065

86	1.0	4.0	0.1	0.052	0.054	0.055	0.059	0.055
87	1.0	4.0	0.2	0.052	0.051	0.058	0.057	0.059
88	1.0	4.0	0.3	0.046	0.056	0.059	0.061	0.059
89	1.0	4.0	0.4	0.053	0.054	0.059	0.060	0.056
90	1.0	4.0	0.5	0.056	0.052	0.061	0.062	0.067
91	1.0	4.5	0.1	0.052	0.053	0.055	0.060	0.053
92	1.0	4.5	0.2	0.053	0.054	0.062	0.061	0.058
93	1.0	4.5	0.3	0.054	0.052	0.063	0.062	0.058
94	1.0	4.5	0.4	0.049	0.055	0.063	0.061	0.060
95	1.0	4.5	0.5	0.054	0.052	0.061	0.062	0.059
96	1.0	5.0	0.1	0.052	0.056	0.061	0.064	0.058
97	1.0	5.0	0.2	0.052	0.049	0.059	0.059	0.060
98	1.0	5.0	0.3	0.055	0.052	0.060	0.052	0.063
99	1.0	5.0	0.4	0.051	0.060	0.063	0.053	0.061
100	1.0	5.0	0.5	0.057	0.060	0.064	0.069	0.064
101	1.5	0.5	0.1	0.054	0.059	0.058	0.052	0.052
102	1.5	0.5	0.2	0.054	0.055	0.059	0.056	0.063
103	1.5	0.5	0.3	0.047	0.053	0.057	0.061	0.059
104	1.5	0.5	0.4	0.049	0.057	0.054	0.059	0.065
105	1.5	0.5	0.5	0.055	0.056	0.065	0.065	0.064
106	1.5	1.0	0.1	0.053	0.057	0.054	0.050	0.054
107	1.5	1.0	0.2	0.053	0.057	0.064	0.060	0.061
108	1.5	1.0	0.3	0.051	0.055	0.057	0.060	0.058
109	1.5	1.0	0.4	0.050	0.055	0.057	0.058	0.071
110	1.5	1.0	0.5	0.056	0.058	0.062	0.066	0.062
111	1.5	1.5	0.1	0.049	0.053	0.057	0.055	0.060
112	1.5	1.5	0.2	0.051	0.056	0.060	0.058	0.063
113	1.5	1.5	0.3	0.057	0.060	0.062	0.061	0.058
114	1.5	1.5	0.4	0.055	0.059	0.060	0.065	0.068
115	1.5	1.5	0.5	0.054	0.055	0.058	0.061	0.062
116	1.5	2.0	0.1	0.052	0.056	0.054	0.059	0.055
117	1.5	2.0	0.2	0.056	0.050	0.058	0.061	0.054
118	1.5	2.0	0.3	0.056	0.049	0.054	0.064	0.055
119	1.5	2.0	0.4	0.057	0.054	0.056	0.061	0.068
120	1.5	2.0	0.5	0.054	0.062	0.060	0.060	0.071
121	1.5	2.5	0.1	0.052	0.056	0.061	0.054	0.056
122	1.5	2.5	0.2	0.049	0.057	0.061	0.057	0.058
123	1.5	2.5	0.3	0.053	0.055	0.057	0.055	0.065
124	1.5	2.5	0.4	0.060	0.055	0.059	0.062	0.064
125	1.5	2.5	0.5	0.055	0.062	0.059	0.066	0.058
126	1.5	3.0	0.1	0.052	0.059	0.058	0.058	0.059
127	1.5	3.0	0.2	0.058	0.060	0.058	0.057	0.058
128	1.5	3.0	0.3	0.052	0.053	0.060	0.064	0.062
129	1.5	3.0	0.4	0.048	0.055	0.056	0.063	0.065
130	1.5	3.0	0.5	0.057	0.057	0.064	0.061	0.061
131	1.5	3.5	0.1	0.055	0.052	0.067	0.053	0.061
132	1.5	3.5	0.2	0.053	0.057	0.054	0.061	0.057
133	1.5	3.5	0.3	0.055	0.053	0.058	0.056	0.063

134	1.5	3.5	0.4	0.047	0.060	0.064	0.058	0.067
135	1.5	3.5	0.5	0.049	0.059	0.064	0.067	0.066
136	1.5	4.0	0.1	0.054	0.053	0.058	0.056	0.058
137	1.5	4.0	0.2	0.049	0.054	0.054	0.061	0.056
138	1.5	4.0	0.3	0.055	0.062	0.063	0.053	0.059
139	1.5	4.0	0.4	0.051	0.053	0.065	0.061	0.062
140	1.5	4.0	0.5	0.050	0.066	0.059	0.060	0.068
141	1.5	4.5	0.1	0.053	0.055	0.055	0.059	0.055
142	1.5	4.5	0.2	0.054	0.056	0.056	0.053	0.054
143	1.5	4.5	0.3	0.049	0.057	0.053	0.053	0.059
144	1.5	4.5	0.4	0.051	0.051	0.058	0.061	0.062
145	1.5	4.5	0.5	0.049	0.053	0.059	0.064	0.068
146	1.5	5.0	0.1	0.051	0.048	0.054	0.058	0.057
147	1.5	5.0	0.2	0.053	0.054	0.056	0.052	0.059
148	1.5	5.0	0.3	0.053	0.053	0.058	0.057	0.056
149	1.5	5.0	0.4	0.055	0.055	0.061	0.060	0.068
150	1.5	5.0	0.5	0.050	0.055	0.057	0.064	0.065
151	2.0	0.5	0.1	0.052	0.051	0.060	0.056	0.054
152	2.0	0.5	0.2	0.054	0.061	0.063	0.050	0.062
153	2.0	0.5	0.3	0.053	0.060	0.051	0.063	0.059
154	2.0	0.5	0.4	0.048	0.055	0.066	0.066	0.064
155	2.0	0.5	0.5	0.054	0.058	0.061	0.061	0.062
156	2.0	1.0	0.1	0.049	0.049	0.056	0.055	0.058
157	2.0	1.0	0.2	0.051	0.055	0.058	0.060	0.056
158	2.0	1.0	0.3	0.052	0.057	0.061	0.056	0.061
159	2.0	1.0	0.4	0.055	0.059	0.056	0.058	0.070
160	2.0	1.0	0.5	0.053	0.057	0.057	0.062	0.063
161	2.0	1.5	0.1	0.048	0.056	0.054	0.055	0.051
162	2.0	1.5	0.2	0.045	0.059	0.051	0.060	0.063
163	2.0	1.5	0.3	0.053	0.052	0.057	0.052	0.059
164	2.0	1.5	0.4	0.053	0.055	0.056	0.064	0.064
165	2.0	1.5	0.5	0.053	0.053	0.066	0.060	0.070
166	2.0	2.0	0.1	0.048	0.052	0.054	0.055	0.054
167	2.0	2.0	0.2	0.048	0.059	0.058	0.062	0.065
168	2.0	2.0	0.3	0.049	0.055	0.057	0.063	0.057
169	2.0	2.0	0.4	0.049	0.056	0.058	0.063	0.060
170	2.0	2.0	0.5	0.054	0.054	0.054	0.056	0.059
171	2.0	2.5	0.1	0.052	0.052	0.053	0.058	0.055
172	2.0	2.5	0.2	0.048	0.058	0.057	0.054	0.056
173	2.0	2.5	0.3	0.051	0.053	0.056	0.060	0.065
174	2.0	2.5	0.4	0.056	0.058	0.063	0.057	0.064
175	2.0	2.5	0.5	0.052	0.060	0.059	0.063	0.062
176	2.0	3.0	0.1	0.048	0.055	0.053	0.056	0.054
177	2.0	3.0	0.2	0.054	0.051	0.053	0.054	0.057
178	2.0	3.0	0.3	0.054	0.057	0.055	0.058	0.058
179	2.0	3.0	0.4	0.055	0.057	0.057	0.068	0.064
180	2.0	3.0	0.5	0.049	0.055	0.058	0.059	0.076
181	2.0	3.5	0.1	0.050	0.058	0.054	0.055	0.061

182	2.0	3.5	0.2	0.047	0.053	0.056	0.058	0.057
183	2.0	3.5	0.3	0.050	0.051	0.062	0.060	0.058
184	2.0	3.5	0.4	0.053	0.057	0.061	0.058	0.063
185	2.0	3.5	0.5	0.054	0.060	0.063	0.062	0.066
186	2.0	4.0	0.1	0.049	0.053	0.059	0.056	0.062
187	2.0	4.0	0.2	0.050	0.052	0.054	0.057	0.061
188	2.0	4.0	0.3	0.055	0.052	0.058	0.064	0.064
189	2.0	4.0	0.4	0.055	0.051	0.060	0.061	0.065
190	2.0	4.0	0.5	0.052	0.056	0.061	0.065	0.067
191	2.0	4.5	0.1	0.058	0.060	0.058	0.060	0.056
192	2.0	4.5	0.2	0.050	0.055	0.059	0.059	0.051
193	2.0	4.5	0.3	0.052	0.050	0.056	0.058	0.060
194	2.0	4.5	0.4	0.055	0.056	0.064	0.060	0.064
195	2.0	4.5	0.5	0.051	0.056	0.065	0.065	0.071
196	2.0	5.0	0.1	0.042	0.055	0.057	0.058	0.056
197	2.0	5.0	0.2	0.054	0.061	0.059	0.057	0.058
198	2.0	5.0	0.3	0.051	0.053	0.054	0.059	0.055
199	2.0	5.0	0.4	0.051	0.054	0.061	0.058	0.058
200	2.0	5.0	0.5	0.056	0.062	0.054	0.056	0.069
201	2.5	0.5	0.1	0.052	0.054	0.054	0.056	0.059
202	2.5	0.5	0.2	0.054	0.057	0.063	0.060	0.056
203	2.5	0.5	0.3	0.049	0.055	0.057	0.057	0.059
204	2.5	0.5	0.4	0.049	0.054	0.058	0.058	0.060
205	2.5	0.5	0.5	0.050	0.049	0.059	0.062	0.072
206	2.5	1.0	0.1	0.053	0.052	0.052	0.060	0.053
207	2.5	1.0	0.2	0.053	0.057	0.053	0.057	0.065
208	2.5	1.0	0.3	0.054	0.049	0.056	0.056	0.059
209	2.5	1.0	0.4	0.057	0.058	0.054	0.063	0.063
210	2.5	1.0	0.5	0.054	0.059	0.061	0.064	0.064
211	2.5	1.5	0.1	0.052	0.055	0.057	0.059	0.058
212	2.5	1.5	0.2	0.050	0.057	0.057	0.056	0.055
213	2.5	1.5	0.3	0.050	0.050	0.057	0.060	0.059
214	2.5	1.5	0.4	0.051	0.053	0.056	0.061	0.063
215	2.5	1.5	0.5	0.056	0.050	0.064	0.071	0.061
216	2.5	2.0	0.1	0.057	0.055	0.055	0.057	0.057
217	2.5	2.0	0.2	0.049	0.055	0.059	0.059	0.062
218	2.5	2.0	0.3	0.052	0.054	0.060	0.063	0.064
219	2.5	2.0	0.4	0.051	0.060	0.060	0.058	0.063
220	2.5	2.0	0.5	0.058	0.056	0.064	0.069	0.061
221	2.5	2.5	0.1	0.052	0.052	0.053	0.051	0.056
222	2.5	2.5	0.2	0.047	0.059	0.063	0.057	0.062
223	2.5	2.5	0.3	0.055	0.060	0.059	0.060	0.058
224	2.5	2.5	0.4	0.051	0.058	0.057	0.067	0.065
225	2.5	2.5	0.5	0.054	0.062	0.063	0.066	0.067
226	2.5	3.0	0.1	0.055	0.054	0.057	0.064	0.056
227	2.5	3.0	0.2	0.046	0.052	0.058	0.059	0.058
228	2.5	3.0	0.3	0.051	0.063	0.063	0.056	0.056
229	2.5	3.0	0.4	0.056	0.057	0.059	0.060	0.058

230	2.5	3.0	0.5	0.047	0.053	0.061	0.061	0.063
231	2.5	3.5	0.1	0.058	0.059	0.056	0.052	0.059
232	2.5	3.5	0.2	0.053	0.058	0.060	0.058	0.053
233	2.5	3.5	0.3	0.047	0.063	0.054	0.057	0.063
234	2.5	3.5	0.4	0.050	0.056	0.070	0.059	0.062
235	2.5	3.5	0.5	0.053	0.059	0.068	0.058	0.062
236	2.5	4.0	0.1	0.051	0.054	0.056	0.057	0.058
237	2.5	4.0	0.2	0.055	0.053	0.058	0.060	0.059
238	2.5	4.0	0.3	0.049	0.054	0.053	0.059	0.059
239	2.5	4.0	0.4	0.054	0.065	0.060	0.061	0.061
240	2.5	4.0	0.5	0.052	0.056	0.065	0.062	0.063
241	2.5	4.5	0.1	0.050	0.058	0.053	0.056	0.056
242	2.5	4.5	0.2	0.052	0.056	0.052	0.059	0.054
243	2.5	4.5	0.3	0.055	0.061	0.058	0.061	0.063
244	2.5	4.5	0.4	0.055	0.059	0.057	0.064	0.065
245	2.5	4.5	0.5	0.055	0.059	0.059	0.066	0.062
246	2.5	5.0	0.1	0.052	0.057	0.062	0.052	0.057
247	2.5	5.0	0.2	0.055	0.053	0.059	0.065	0.061
248	2.5	5.0	0.3	0.053	0.061	0.061	0.063	0.063
249	2.5	5.0	0.4	0.049	0.057	0.062	0.065	0.058
250	2.5	5.0	0.5	0.057	0.057	0.061	0.059	0.070
251	3.0	0.5	0.1	0.052	0.058	0.053	0.058	0.059
252	3.0	0.5	0.2	0.055	0.056	0.059	0.057	0.062
253	3.0	0.5	0.3	0.051	0.059	0.059	0.060	0.062
254	3.0	0.5	0.4	0.055	0.056	0.055	0.066	0.057
255	3.0	0.5	0.5	0.046	0.051	0.062	0.061	0.065
256	3.0	1.0	0.1	0.045	0.052	0.046	0.051	0.053
257	3.0	1.0	0.2	0.057	0.056	0.057	0.060	0.056
258	3.0	1.0	0.3	0.054	0.054	0.056	0.064	0.065
259	3.0	1.0	0.4	0.051	0.058	0.061	0.059	0.064
260	3.0	1.0	0.5	0.054	0.058	0.059	0.066	0.064
261	3.0	1.5	0.1	0.052	0.057	0.056	0.053	0.058
262	3.0	1.5	0.2	0.053	0.049	0.061	0.061	0.056
263	3.0	1.5	0.3	0.049	0.057	0.057	0.056	0.058
264	3.0	1.5	0.4	0.053	0.056	0.055	0.050	0.060
265	3.0	1.5	0.5	0.057	0.061	0.064	0.071	0.068
266	3.0	2.0	0.1	0.046	0.057	0.056	0.057	0.056
267	3.0	2.0	0.2	0.052	0.057	0.056	0.058	0.056
268	3.0	2.0	0.3	0.051	0.059	0.061	0.053	0.064
269	3.0	2.0	0.4	0.056	0.057	0.052	0.066	0.064
270	3.0	2.0	0.5	0.054	0.063	0.061	0.063	0.066
271	3.0	2.5	0.1	0.053	0.059	0.054	0.056	0.059
272	3.0	2.5	0.2	0.053	0.053	0.061	0.057	0.063
273	3.0	2.5	0.3	0.057	0.056	0.060	0.059	0.062
274	3.0	2.5	0.4	0.048	0.059	0.058	0.058	0.057
275	3.0	2.5	0.5	0.051	0.054	0.058	0.059	0.066
276	3.0	3.0	0.1	0.052	0.054	0.059	0.056	0.054
277	3.0	3.0	0.2	0.049	0.054	0.058	0.062	0.055

278	3.0	3.0	0.3	0.055	0.057	0.056	0.058	0.060
279	3.0	3.0	0.4	0.050	0.056	0.059	0.061	0.065
280	3.0	3.0	0.5	0.055	0.057	0.072	0.063	0.061
281	3.0	3.5	0.1	0.050	0.054	0.058	0.052	0.058
282	3.0	3.5	0.2	0.049	0.053	0.054	0.059	0.058
283	3.0	3.5	0.3	0.057	0.055	0.053	0.057	0.057
284	3.0	3.5	0.4	0.050	0.059	0.060	0.061	0.063
285	3.0	3.5	0.5	0.050	0.060	0.069	0.061	0.066
286	3.0	4.0	0.1	0.055	0.057	0.059	0.055	0.054
287	3.0	4.0	0.2	0.049	0.056	0.057	0.060	0.056
288	3.0	4.0	0.3	0.055	0.055	0.060	0.058	0.057
289	3.0	4.0	0.4	0.048	0.054	0.058	0.062	0.060
290	3.0	4.0	0.5	0.052	0.057	0.060	0.066	0.067
291	3.0	4.5	0.1	0.052	0.053	0.056	0.056	0.058
292	3.0	4.5	0.2	0.054	0.054	0.059	0.055	0.050
293	3.0	4.5	0.3	0.049	0.060	0.066	0.057	0.059
294	3.0	4.5	0.4	0.050	0.055	0.059	0.064	0.064
295	3.0	4.5	0.5	0.056	0.057	0.058	0.063	0.067
296	3.0	5.0	0.1	0.054	0.050	0.054	0.056	0.061
297	3.0	5.0	0.2	0.046	0.057	0.063	0.065	0.057
298	3.0	5.0	0.3	0.048	0.058	0.065	0.059	0.063
299	3.0	5.0	0.4	0.054	0.058	0.056	0.054	0.061
300	3.0	5.0	0.5	0.053	0.055	0.054	0.059	0.064
301	3.5	0.5	0.1	0.053	0.061	0.057	0.058	0.054
302	3.5	0.5	0.2	0.056	0.056	0.061	0.057	0.053
303	3.5	0.5	0.3	0.048	0.054	0.061	0.053	0.058
304	3.5	0.5	0.4	0.051	0.059	0.058	0.064	0.059
305	3.5	0.5	0.5	0.057	0.055	0.062	0.061	0.064
306	3.5	1.0	0.1	0.050	0.054	0.055	0.060	0.055
307	3.5	1.0	0.2	0.048	0.052	0.057	0.064	0.065
308	3.5	1.0	0.3	0.052	0.053	0.064	0.059	0.057
309	3.5	1.0	0.4	0.053	0.054	0.067	0.057	0.059
310	3.5	1.0	0.5	0.058	0.061	0.063	0.061	0.063
311	3.5	1.5	0.1	0.055	0.055	0.058	0.057	0.060
312	3.5	1.5	0.2	0.057	0.054	0.059	0.051	0.055
313	3.5	1.5	0.3	0.050	0.059	0.057	0.060	0.057
314	3.5	1.5	0.4	0.054	0.061	0.058	0.060	0.062
315	3.5	1.5	0.5	0.057	0.059	0.059	0.062	0.062
316	3.5	2.0	0.1	0.055	0.051	0.054	0.061	0.053
317	3.5	2.0	0.2	0.049	0.049	0.061	0.058	0.058
318	3.5	2.0	0.3	0.053	0.053	0.062	0.060	0.056
319	3.5	2.0	0.4	0.049	0.053	0.065	0.065	0.072
320	3.5	2.0	0.5	0.054	0.050	0.061	0.061	0.068
321	3.5	2.5	0.1	0.048	0.058	0.055	0.054	0.054
322	3.5	2.5	0.2	0.063	0.057	0.054	0.061	0.060
323	3.5	2.5	0.3	0.055	0.055	0.058	0.060	0.059
324	3.5	2.5	0.4	0.050	0.060	0.061	0.058	0.066
325	3.5	2.5	0.5	0.060	0.054	0.057	0.059	0.068

326	3.5	3.0	0.1	0.050	0.052	0.052	0.055	0.055
327	3.5	3.0	0.2	0.043	0.059	0.055	0.057	0.059
328	3.5	3.0	0.3	0.051	0.059	0.052	0.057	0.058
329	3.5	3.0	0.4	0.054	0.054	0.062	0.064	0.063
330	3.5	3.0	0.5	0.053	0.060	0.060	0.065	0.063
331	3.5	3.5	0.1	0.052	0.053	0.056	0.056	0.058
332	3.5	3.5	0.2	0.051	0.055	0.062	0.056	0.055
333	3.5	3.5	0.3	0.050	0.058	0.057	0.062	0.060
334	3.5	3.5	0.4	0.060	0.050	0.057	0.063	0.061
335	3.5	3.5	0.5	0.051	0.059	0.063	0.061	0.064
336	3.5	4.0	0.1	0.055	0.055	0.057	0.052	0.053
337	3.5	4.0	0.2	0.048	0.053	0.056	0.060	0.059
338	3.5	4.0	0.3	0.053	0.054	0.058	0.056	0.060
339	3.5	4.0	0.4	0.054	0.056	0.056	0.056	0.065
340	3.5	4.0	0.5	0.056	0.055	0.056	0.067	0.065
341	3.5	4.5	0.1	0.055	0.051	0.055	0.062	0.053
342	3.5	4.5	0.2	0.054	0.061	0.056	0.059	0.064
343	3.5	4.5	0.3	0.050	0.050	0.057	0.054	0.058
344	3.5	4.5	0.4	0.055	0.057	0.056	0.066	0.063
345	3.5	4.5	0.5	0.049	0.053	0.066	0.061	0.062
346	3.5	5.0	0.1	0.052	0.054	0.056	0.051	0.054
347	3.5	5.0	0.2	0.055	0.060	0.054	0.053	0.055
348	3.5	5.0	0.3	0.050	0.059	0.052	0.057	0.058
349	3.5	5.0	0.4	0.047	0.052	0.058	0.066	0.063
350	3.5	5.0	0.5	0.049	0.057	0.064	0.061	0.067
351	4.0	0.5	0.1	0.051	0.053	0.058	0.055	0.055
352	4.0	0.5	0.2	0.048	0.055	0.055	0.060	0.054
353	4.0	0.5	0.3	0.049	0.060	0.054	0.055	0.059
354	4.0	0.5	0.4	0.055	0.056	0.055	0.063	0.065
355	4.0	0.5	0.5	0.053	0.059	0.064	0.061	0.063
356	4.0	1.0	0.1	0.057	0.057	0.062	0.052	0.055
357	4.0	1.0	0.2	0.053	0.049	0.054	0.057	0.059
358	4.0	1.0	0.3	0.051	0.057	0.058	0.063	0.061
359	4.0	1.0	0.4	0.059	0.059	0.059	0.062	0.062
360	4.0	1.0	0.5	0.055	0.052	0.061	0.064	0.063
361	4.0	1.5	0.1	0.052	0.057	0.056	0.054	0.053
362	4.0	1.5	0.2	0.054	0.051	0.057	0.056	0.058
363	4.0	1.5	0.3	0.044	0.054	0.061	0.063	0.057
364	4.0	1.5	0.4	0.048	0.054	0.063	0.063	0.061
365	4.0	1.5	0.5	0.054	0.057	0.060	0.056	0.064
366	4.0	2.0	0.1	0.051	0.054	0.057	0.056	0.058
367	4.0	2.0	0.2	0.053	0.053	0.057	0.055	0.059
368	4.0	2.0	0.3	0.055	0.062	0.055	0.063	0.063
369	4.0	2.0	0.4	0.047	0.062	0.069	0.058	0.060
370	4.0	2.0	0.5	0.062	0.060	0.059	0.065	0.065
371	4.0	2.5	0.1	0.047	0.053	0.060	0.059	0.054
372	4.0	2.5	0.2	0.050	0.057	0.058	0.055	0.055
373	4.0	2.5	0.3	0.050	0.056	0.054	0.059	0.054

374	4.0	2.5	0.4	0.053	0.057	0.060	0.061	0.066
375	4.0	2.5	0.5	0.055	0.060	0.053	0.071	0.070
376	4.0	3.0	0.1	0.048	0.055	0.056	0.057	0.062
377	4.0	3.0	0.2	0.052	0.054	0.049	0.059	0.065
378	4.0	3.0	0.3	0.053	0.054	0.060	0.058	0.064
379	4.0	3.0	0.4	0.056	0.054	0.054	0.055	0.066
380	4.0	3.0	0.5	0.052	0.057	0.057	0.068	0.065
381	4.0	3.5	0.1	0.050	0.060	0.058	0.057	0.059
382	4.0	3.5	0.2	0.049	0.057	0.056	0.058	0.058
383	4.0	3.5	0.3	0.057	0.055	0.068	0.065	0.059
384	4.0	3.5	0.4	0.052	0.054	0.054	0.061	0.063
385	4.0	3.5	0.5	0.048	0.055	0.054	0.066	0.061
386	4.0	4.0	0.1	0.054	0.054	0.055	0.059	0.056
387	4.0	4.0	0.2	0.057	0.051	0.056	0.062	0.061
388	4.0	4.0	0.3	0.049	0.057	0.061	0.058	0.066
389	4.0	4.0	0.4	0.055	0.054	0.061	0.056	0.061
390	4.0	4.0	0.5	0.052	0.058	0.056	0.065	0.069
391	4.0	4.5	0.1	0.048	0.054	0.055	0.053	0.051
392	4.0	4.5	0.2	0.054	0.060	0.064	0.057	0.060
393	4.0	4.5	0.3	0.051	0.059	0.059	0.058	0.058
394	4.0	4.5	0.4	0.051	0.054	0.054	0.063	0.057
395	4.0	4.5	0.5	0.051	0.059	0.061	0.062	0.062
396	4.0	5.0	0.1	0.060	0.058	0.055	0.058	0.058
397	4.0	5.0	0.2	0.055	0.051	0.060	0.054	0.060
398	4.0	5.0	0.3	0.052	0.057	0.055	0.049	0.056
399	4.0	5.0	0.4	0.051	0.055	0.056	0.059	0.061
400	4.0	5.0	0.5	0.052	0.055	0.059	0.061	0.059
401	4.5	0.5	0.1	0.050	0.056	0.054	0.056	0.052
402	4.5	0.5	0.2	0.053	0.054	0.068	0.056	0.058
403	4.5	0.5	0.3	0.051	0.058	0.056	0.059	0.061
404	4.5	0.5	0.4	0.059	0.054	0.064	0.062	0.067
405	4.5	0.5	0.5	0.050	0.057	0.060	0.063	0.069
406	4.5	1.0	0.1	0.051	0.053	0.060	0.055	0.062
407	4.5	1.0	0.2	0.051	0.053	0.056	0.057	0.062
408	4.5	1.0	0.3	0.052	0.056	0.057	0.059	0.061
409	4.5	1.0	0.4	0.054	0.054	0.059	0.059	0.060
410	4.5	1.0	0.5	0.051	0.053	0.058	0.063	0.065
411	4.5	1.5	0.1	0.054	0.057	0.061	0.057	0.051
412	4.5	1.5	0.2	0.054	0.059	0.053	0.055	0.057
413	4.5	1.5	0.3	0.053	0.051	0.062	0.058	0.060
414	4.5	1.5	0.4	0.051	0.055	0.062	0.061	0.058
415	4.5	1.5	0.5	0.055	0.059	0.063	0.064	0.066
416	4.5	2.0	0.1	0.054	0.057	0.054	0.052	0.041
417	4.5	2.0	0.2	0.054	0.045	0.054	0.056	0.061
418	4.5	2.0	0.3	0.054	0.052	0.053	0.052	0.061
419	4.5	2.0	0.4	0.048	0.053	0.058	0.058	0.065
420	4.5	2.0	0.5	0.050	0.057	0.066	0.062	0.062
421	4.5	2.5	0.1	0.050	0.055	0.052	0.056	0.056

422	4.5	2.5	0.2	0.057	0.052	0.058	0.057	0.056
423	4.5	2.5	0.3	0.054	0.057	0.061	0.054	0.063
424	4.5	2.5	0.4	0.051	0.063	0.059	0.059	0.060
425	4.5	2.5	0.5	0.058	0.055	0.064	0.066	0.064
426	4.5	3.0	0.1	0.050	0.056	0.060	0.057	0.062
427	4.5	3.0	0.2	0.057	0.052	0.062	0.056	0.061
428	4.5	3.0	0.3	0.051	0.061	0.056	0.056	0.056
429	4.5	3.0	0.4	0.053	0.060	0.065	0.058	0.059
430	4.5	3.0	0.5	0.059	0.053	0.057	0.059	0.066
431	4.5	3.5	0.1	0.049	0.054	0.054	0.059	0.061
432	4.5	3.5	0.2	0.053	0.050	0.054	0.056	0.058
433	4.5	3.5	0.3	0.055	0.055	0.054	0.057	0.058
434	4.5	3.5	0.4	0.053	0.057	0.054	0.059	0.059
435	4.5	3.5	0.5	0.055	0.054	0.060	0.065	0.064
436	4.5	4.0	0.1	0.051	0.053	0.059	0.060	0.059
437	4.5	4.0	0.2	0.054	0.057	0.056	0.052	0.058
438	4.5	4.0	0.3	0.050	0.051	0.054	0.060	0.056
439	4.5	4.0	0.4	0.055	0.057	0.058	0.063	0.064
440	4.5	4.0	0.5	0.054	0.055	0.061	0.063	0.069
441	4.5	4.5	0.1	0.057	0.054	0.051	0.059	0.056
442	4.5	4.5	0.2	0.047	0.052	0.051	0.054	0.060
443	4.5	4.5	0.3	0.055	0.058	0.058	0.060	0.061
444	4.5	4.5	0.4	0.058	0.047	0.060	0.065	0.062
445	4.5	4.5	0.5	0.056	0.057	0.059	0.060	0.064
446	4.5	5.0	0.1	0.053	0.052	0.053	0.055	0.055
447	4.5	5.0	0.2	0.048	0.050	0.055	0.054	0.059
448	4.5	5.0	0.3	0.057	0.055	0.058	0.058	0.061
449	4.5	5.0	0.4	0.055	0.052	0.058	0.062	0.063
450	4.5	5.0	0.5	0.058	0.058	0.064	0.066	0.063
451	5.0	0.5	0.1	0.052	0.051	0.060	0.057	0.060
452	5.0	0.5	0.2	0.058	0.057	0.055	0.060	0.057
453	5.0	0.5	0.3	0.050	0.051	0.060	0.056	0.057
454	5.0	0.5	0.4	0.051	0.058	0.062	0.065	0.062
455	5.0	0.5	0.5	0.055	0.065	0.065	0.070	0.063
456	5.0	1.0	0.1	0.049	0.052	0.057	0.052	0.058
457	5.0	1.0	0.2	0.055	0.053	0.058	0.058	0.060
458	5.0	1.0	0.3	0.052	0.053	0.059	0.066	0.057
459	5.0	1.0	0.4	0.056	0.061	0.056	0.063	0.063
460	5.0	1.0	0.5	0.053	0.054	0.062	0.068	0.071
461	5.0	1.5	0.1	0.049	0.053	0.053	0.059	0.053
462	5.0	1.5	0.2	0.058	0.057	0.060	0.054	0.057
463	5.0	1.5	0.3	0.051	0.054	0.063	0.059	0.056
464	5.0	1.5	0.4	0.053	0.057	0.056	0.063	0.060
465	5.0	1.5	0.5	0.054	0.053	0.059	0.065	0.062
466	5.0	2.0	0.1	0.058	0.048	0.052	0.057	0.053
467	5.0	2.0	0.2	0.050	0.053	0.052	0.056	0.059
468	5.0	2.0	0.3	0.057	0.056	0.056	0.061	0.060
469	5.0	2.0	0.4	0.056	0.059	0.058	0.058	0.064

470	5.0	2.0	0.5	0.053	0.055	0.059	0.060	0.065
471	5.0	2.5	0.1	0.048	0.059	0.048	0.056	0.057
472	5.0	2.5	0.2	0.054	0.055	0.058	0.063	0.059
473	5.0	2.5	0.3	0.056	0.057	0.060	0.056	0.060
474	5.0	2.5	0.4	0.048	0.052	0.060	0.062	0.062
475	5.0	2.5	0.5	0.055	0.058	0.052	0.065	0.069
476	5.0	3.0	0.1	0.046	0.054	0.053	0.059	0.054
477	5.0	3.0	0.2	0.052	0.053	0.055	0.058	0.062
478	5.0	3.0	0.3	0.052	0.059	0.052	0.062	0.056
479	5.0	3.0	0.4	0.052	0.053	0.055	0.060	0.061
480	5.0	3.0	0.5	0.054	0.053	0.060	0.062	0.069
481	5.0	3.5	0.1	0.056	0.054	0.060	0.054	0.059
482	5.0	3.5	0.2	0.049	0.059	0.056	0.060	0.053
483	5.0	3.5	0.3	0.050	0.051	0.054	0.054	0.056
484	5.0	3.5	0.4	0.051	0.053	0.057	0.064	0.063
485	5.0	3.5	0.5	0.059	0.058	0.056	0.061	0.066
486	5.0	4.0	0.1	0.052	0.053	0.057	0.062	0.057
487	5.0	4.0	0.2	0.046	0.056	0.064	0.062	0.057
488	5.0	4.0	0.3	0.052	0.059	0.051	0.057	0.060
489	5.0	4.0	0.4	0.051	0.054	0.053	0.055	0.062
490	5.0	4.0	0.5	0.054	0.055	0.059	0.062	0.059
491	5.0	4.5	0.1	0.051	0.058	0.056	0.057	0.054
492	5.0	4.5	0.2	0.048	0.054	0.054	0.058	0.060
493	5.0	4.5	0.3	0.047	0.056	0.055	0.056	0.057
494	5.0	4.5	0.4	0.056	0.053	0.058	0.058	0.061
495	5.0	4.5	0.5	0.055	0.052	0.058	0.067	0.065
496	5.0	5.0	0.1	0.058	0.056	0.064	0.055	0.055
497	5.0	5.0	0.2	0.052	0.058	0.060	0.063	0.059
498	5.0	5.0	0.3	0.055	0.059	0.056	0.059	0.054
499	5.0	5.0	0.4	0.056	0.063	0.058	0.061	0.059
500	5.0	5.0	0.5	0.055	0.056	0.062	0.063	0.062

Table 7.2: Observed Size Comparison for n=200

	Mu	sigma	p	DL0.01	DL0.1	DL0.2	DL0.3	DL0.4
1	0.5	0.5	0.1	0.051	0.049	0.052	0.051	0.053
2	0.5	0.5	0.2	0.054	0.053	0.049	0.056	0.055
3	0.5	0.5	0.3	0.053	0.048	0.053	0.055	0.059
4	0.5	0.5	0.4	0.056	0.056	0.057	0.052	0.056
5	0.5	0.5	0.5	0.059	0.052	0.051	0.053	0.059
6	0.5	1.0	0.1	0.050	0.050	0.054	0.053	0.058
7	0.5	1.0	0.2	0.053	0.050	0.053	0.051	0.055
8	0.5	1.0	0.3	0.056	0.057	0.055	0.054	0.054
9	0.5	1.0	0.4	0.047	0.056	0.053	0.061	0.053
10	0.5	1.0	0.5	0.050	0.052	0.048	0.060	0.058
11	0.5	1.5	0.1	0.055	0.058	0.055	0.055	0.050
12	0.5	1.5	0.2	0.057	0.048	0.052	0.052	0.052
13	0.5	1.5	0.3	0.054	0.054	0.055	0.054	0.054
14	0.5	1.5	0.4	0.053	0.051	0.052	0.053	0.061
15	0.5	1.5	0.5	0.052	0.055	0.053	0.054	0.054
16	0.5	2.0	0.1	0.049	0.053	0.054	0.058	0.050
17	0.5	2.0	0.2	0.048	0.054	0.050	0.059	0.049
18	0.5	2.0	0.3	0.052	0.053	0.061	0.049	0.057
19	0.5	2.0	0.4	0.058	0.055	0.054	0.052	0.056
20	0.5	2.0	0.5	0.050	0.059	0.059	0.058	0.055
21	0.5	2.5	0.1	0.049	0.054	0.055	0.050	0.052
22	0.5	2.5	0.2	0.053	0.054	0.054	0.056	0.054
23	0.5	2.5	0.3	0.049	0.055	0.057	0.058	0.061
24	0.5	2.5	0.4	0.050	0.055	0.057	0.061	0.055
25	0.5	2.5	0.5	0.053	0.059	0.058	0.065	0.060
26	0.5	3.0	0.1	0.052	0.054	0.056	0.052	0.050
27	0.5	3.0	0.2	0.051	0.048	0.051	0.056	0.056
28	0.5	3.0	0.3	0.048	0.050	0.053	0.050	0.060
29	0.5	3.0	0.4	0.051	0.050	0.056	0.055	0.055
30	0.5	3.0	0.5	0.052	0.055	0.058	0.054	0.060
31	0.5	3.5	0.1	0.050	0.053	0.049	0.047	0.051
32	0.5	3.5	0.2	0.054	0.047	0.054	0.053	0.051
33	0.5	3.5	0.3	0.051	0.052	0.052	0.053	0.050
34	0.5	3.5	0.4	0.052	0.051	0.057	0.057	0.058
35	0.5	3.5	0.5	0.050	0.050	0.052	0.067	0.062
36	0.5	4.0	0.1	0.052	0.050	0.055	0.050	0.054
37	0.5	4.0	0.2	0.052	0.055	0.046	0.054	0.051
38	0.5	4.0	0.3	0.051	0.051	0.052	0.051	0.055
39	0.5	4.0	0.4	0.054	0.056	0.057	0.053	0.049
40	0.5	4.0	0.5	0.048	0.053	0.054	0.057	0.051
41	0.5	4.5	0.1	0.046	0.049	0.055	0.051	0.053
42	0.5	4.5	0.2	0.052	0.052	0.053	0.057	0.056
43	0.5	4.5	0.3	0.052	0.055	0.052	0.055	0.053
44	0.5	4.5	0.4	0.046	0.059	0.052	0.055	0.054
45	0.5	4.5	0.5	0.052	0.053	0.053	0.058	0.058

46	0.5	5.0	0.1	0.051	0.052	0.055	0.054	0.054
47	0.5	5.0	0.2	0.046	0.053	0.058	0.054	0.056
48	0.5	5.0	0.3	0.053	0.052	0.051	0.058	0.055
49	0.5	5.0	0.4	0.053	0.051	0.053	0.054	0.061
50	0.5	5.0	0.5	0.055	0.051	0.056	0.061	0.059
51	1.0	0.5	0.1	0.049	0.052	0.053	0.054	0.056
52	1.0	0.5	0.2	0.052	0.054	0.051	0.051	0.056
53	1.0	0.5	0.3	0.052	0.053	0.055	0.055	0.055
54	1.0	0.5	0.4	0.054	0.057	0.056	0.058	0.052
55	1.0	0.5	0.5	0.055	0.052	0.055	0.057	0.060
56	1.0	1.0	0.1	0.053	0.053	0.046	0.056	0.053
57	1.0	1.0	0.2	0.049	0.058	0.054	0.057	0.053
58	1.0	1.0	0.3	0.048	0.050	0.053	0.052	0.056
59	1.0	1.0	0.4	0.057	0.055	0.058	0.065	0.058
60	1.0	1.0	0.5	0.050	0.050	0.056	0.062	0.058
61	1.0	1.5	0.1	0.050	0.052	0.050	0.060	0.052
62	1.0	1.5	0.2	0.050	0.055	0.050	0.055	0.052
63	1.0	1.5	0.3	0.049	0.050	0.051	0.057	0.053
64	1.0	1.5	0.4	0.049	0.057	0.062	0.056	0.057
65	1.0	1.5	0.5	0.046	0.048	0.054	0.056	0.056
66	1.0	2.0	0.1	0.050	0.048	0.045	0.052	0.057
67	1.0	2.0	0.2	0.048	0.049	0.054	0.053	0.056
68	1.0	2.0	0.3	0.052	0.052	0.053	0.055	0.051
69	1.0	2.0	0.4	0.048	0.056	0.051	0.053	0.054
70	1.0	2.0	0.5	0.052	0.050	0.051	0.058	0.055
71	1.0	2.5	0.1	0.051	0.050	0.054	0.052	0.054
72	1.0	2.5	0.2	0.048	0.048	0.050	0.052	0.060
73	1.0	2.5	0.3	0.056	0.050	0.054	0.055	0.058
74	1.0	2.5	0.4	0.057	0.051	0.056	0.060	0.056
75	1.0	2.5	0.5	0.047	0.052	0.054	0.056	0.059
76	1.0	3.0	0.1	0.052	0.055	0.055	0.048	0.052
77	1.0	3.0	0.2	0.051	0.049	0.053	0.049	0.057
78	1.0	3.0	0.3	0.051	0.052	0.057	0.055	0.054
79	1.0	3.0	0.4	0.047	0.054	0.055	0.058	0.053
80	1.0	3.0	0.5	0.051	0.057	0.052	0.053	0.054
81	1.0	3.5	0.1	0.051	0.047	0.053	0.047	0.052
82	1.0	3.5	0.2	0.048	0.059	0.048	0.059	0.053
83	1.0	3.5	0.3	0.049	0.058	0.055	0.051	0.055
84	1.0	3.5	0.4	0.049	0.050	0.054	0.052	0.056
85	1.0	3.5	0.5	0.051	0.052	0.052	0.062	0.053
86	1.0	4.0	0.1	0.058	0.053	0.050	0.056	0.053
87	1.0	4.0	0.2	0.057	0.056	0.058	0.061	0.052
88	1.0	4.0	0.3	0.053	0.052	0.058	0.056	0.064
89	1.0	4.0	0.4	0.052	0.052	0.053	0.050	0.052
90	1.0	4.0	0.5	0.046	0.056	0.058	0.057	0.059
91	1.0	4.5	0.1	0.053	0.054	0.046	0.057	0.055
92	1.0	4.5	0.2	0.048	0.052	0.052	0.055	0.060
93	1.0	4.5	0.3	0.048	0.050	0.059	0.055	0.050

94	1.0	4.5	0.4	0.051	0.059	0.054	0.052	0.061
95	1.0	4.5	0.5	0.052	0.055	0.054	0.052	0.056
96	1.0	5.0	0.1	0.052	0.057	0.052	0.058	0.049
97	1.0	5.0	0.2	0.051	0.055	0.059	0.053	0.059
98	1.0	5.0	0.3	0.047	0.051	0.050	0.052	0.055
99	1.0	5.0	0.4	0.056	0.052	0.051	0.058	0.058
100	1.0	5.0	0.5	0.056	0.058	0.057	0.060	0.057
101	1.5	0.5	0.1	0.045	0.052	0.051	0.055	0.055
102	1.5	0.5	0.2	0.050	0.056	0.049	0.055	0.062
103	1.5	0.5	0.3	0.050	0.049	0.057	0.055	0.060
104	1.5	0.5	0.4	0.052	0.052	0.051	0.053	0.057
105	1.5	0.5	0.5	0.050	0.058	0.058	0.057	0.058
106	1.5	1.0	0.1	0.052	0.053	0.052	0.051	0.055
107	1.5	1.0	0.2	0.061	0.054	0.052	0.051	0.060
108	1.5	1.0	0.3	0.051	0.051	0.060	0.052	0.053
109	1.5	1.0	0.4	0.051	0.057	0.054	0.058	0.057
110	1.5	1.0	0.5	0.056	0.054	0.054	0.058	0.058
111	1.5	1.5	0.1	0.050	0.053	0.054	0.052	0.051
112	1.5	1.5	0.2	0.051	0.054	0.055	0.055	0.053
113	1.5	1.5	0.3	0.053	0.051	0.056	0.056	0.051
114	1.5	1.5	0.4	0.053	0.053	0.061	0.055	0.059
115	1.5	1.5	0.5	0.049	0.052	0.053	0.061	0.058
116	1.5	2.0	0.1	0.050	0.056	0.051	0.055	0.051
117	1.5	2.0	0.2	0.051	0.057	0.055	0.053	0.053
118	1.5	2.0	0.3	0.052	0.051	0.051	0.061	0.052
119	1.5	2.0	0.4	0.049	0.057	0.053	0.057	0.054
120	1.5	2.0	0.5	0.052	0.050	0.056	0.055	0.061
121	1.5	2.5	0.1	0.056	0.052	0.056	0.055	0.053
122	1.5	2.5	0.2	0.049	0.053	0.054	0.049	0.052
123	1.5	2.5	0.3	0.049	0.057	0.058	0.053	0.053
124	1.5	2.5	0.4	0.048	0.058	0.054	0.056	0.053
125	1.5	2.5	0.5	0.056	0.051	0.059	0.055	0.057
126	1.5	3.0	0.1	0.048	0.052	0.055	0.054	0.053
127	1.5	3.0	0.2	0.053	0.047	0.047	0.050	0.052
128	1.5	3.0	0.3	0.050	0.050	0.052	0.053	0.059
129	1.5	3.0	0.4	0.050	0.053	0.058	0.056	0.052
130	1.5	3.0	0.5	0.047	0.050	0.055	0.061	0.054
131	1.5	3.5	0.1	0.056	0.054	0.055	0.058	0.047
132	1.5	3.5	0.2	0.048	0.054	0.053	0.059	0.055
133	1.5	3.5	0.3	0.056	0.051	0.057	0.054	0.052
134	1.5	3.5	0.4	0.043	0.055	0.053	0.058	0.056
135	1.5	3.5	0.5	0.056	0.056	0.055	0.061	0.061
136	1.5	4.0	0.1	0.052	0.054	0.051	0.054	0.056
137	1.5	4.0	0.2	0.052	0.048	0.053	0.054	0.055
138	1.5	4.0	0.3	0.049	0.051	0.053	0.054	0.059
139	1.5	4.0	0.4	0.054	0.045	0.053	0.062	0.055
140	1.5	4.0	0.5	0.054	0.060	0.058	0.061	0.058
141	1.5	4.5	0.1	0.048	0.052	0.058	0.048	0.049

142	1.5	4.5	0.2	0.059	0.060	0.059	0.051	0.054
143	1.5	4.5	0.3	0.047	0.050	0.050	0.054	0.055
144	1.5	4.5	0.4	0.050	0.058	0.055	0.055	0.054
145	1.5	4.5	0.5	0.051	0.049	0.056	0.056	0.054
146	1.5	5.0	0.1	0.047	0.048	0.048	0.049	0.055
147	1.5	5.0	0.2	0.047	0.055	0.049	0.052	0.054
148	1.5	5.0	0.3	0.057	0.056	0.053	0.057	0.054
149	1.5	5.0	0.4	0.055	0.050	0.060	0.053	0.057
150	1.5	5.0	0.5	0.053	0.050	0.052	0.060	0.062
151	2.0	0.5	0.1	0.057	0.053	0.050	0.048	0.050
152	2.0	0.5	0.2	0.044	0.051	0.052	0.061	0.053
153	2.0	0.5	0.3	0.053	0.055	0.050	0.055	0.055
154	2.0	0.5	0.4	0.052	0.050	0.052	0.057	0.057
155	2.0	0.5	0.5	0.058	0.050	0.059	0.056	0.065
156	2.0	1.0	0.1	0.049	0.057	0.054	0.053	0.056
157	2.0	1.0	0.2	0.056	0.051	0.050	0.055	0.053
158	2.0	1.0	0.3	0.056	0.054	0.057	0.053	0.053
159	2.0	1.0	0.4	0.050	0.051	0.055	0.050	0.062
160	2.0	1.0	0.5	0.049	0.047	0.059	0.052	0.053
161	2.0	1.5	0.1	0.052	0.054	0.055	0.056	0.054
162	2.0	1.5	0.2	0.046	0.054	0.055	0.055	0.052
163	2.0	1.5	0.3	0.053	0.052	0.050	0.050	0.056
164	2.0	1.5	0.4	0.058	0.056	0.057	0.054	0.057
165	2.0	1.5	0.5	0.051	0.052	0.059	0.058	0.055
166	2.0	2.0	0.1	0.056	0.054	0.051	0.052	0.053
167	2.0	2.0	0.2	0.047	0.055	0.057	0.051	0.053
168	2.0	2.0	0.3	0.054	0.052	0.058	0.058	0.054
169	2.0	2.0	0.4	0.050	0.051	0.059	0.057	0.060
170	2.0	2.0	0.5	0.052	0.050	0.063	0.057	0.056
171	2.0	2.5	0.1	0.051	0.051	0.052	0.051	0.050
172	2.0	2.5	0.2	0.050	0.055	0.052	0.056	0.056
173	2.0	2.5	0.3	0.049	0.057	0.056	0.053	0.058
174	2.0	2.5	0.4	0.055	0.057	0.060	0.051	0.058
175	2.0	2.5	0.5	0.052	0.051	0.056	0.050	0.055
176	2.0	3.0	0.1	0.049	0.052	0.049	0.057	0.052
177	2.0	3.0	0.2	0.050	0.050	0.058	0.059	0.057
178	2.0	3.0	0.3	0.047	0.054	0.052	0.050	0.059
179	2.0	3.0	0.4	0.046	0.057	0.057	0.052	0.053
180	2.0	3.0	0.5	0.054	0.059	0.059	0.053	0.054
181	2.0	3.5	0.1	0.053	0.052	0.053	0.056	0.057
182	2.0	3.5	0.2	0.054	0.046	0.052	0.054	0.052
183	2.0	3.5	0.3	0.052	0.058	0.055	0.057	0.057
184	2.0	3.5	0.4	0.051	0.058	0.056	0.052	0.052
185	2.0	3.5	0.5	0.050	0.051	0.057	0.060	0.060
186	2.0	4.0	0.1	0.045	0.061	0.055	0.056	0.054
187	2.0	4.0	0.2	0.050	0.048	0.050	0.056	0.054
188	2.0	4.0	0.3	0.053	0.059	0.054	0.053	0.053
189	2.0	4.0	0.4	0.051	0.051	0.052	0.057	0.056

190	2.0	4.0	0.5	0.049	0.058	0.060	0.057	0.057
191	2.0	4.5	0.1	0.048	0.058	0.052	0.048	0.048
192	2.0	4.5	0.2	0.059	0.063	0.055	0.050	0.056
193	2.0	4.5	0.3	0.050	0.053	0.050	0.056	0.053
194	2.0	4.5	0.4	0.057	0.052	0.050	0.050	0.055
195	2.0	4.5	0.5	0.054	0.056	0.061	0.052	0.059
196	2.0	5.0	0.1	0.049	0.049	0.054	0.058	0.057
197	2.0	5.0	0.2	0.048	0.056	0.053	0.056	0.054
198	2.0	5.0	0.3	0.048	0.051	0.055	0.049	0.058
199	2.0	5.0	0.4	0.057	0.054	0.054	0.053	0.059
200	2.0	5.0	0.5	0.049	0.054	0.054	0.055	0.061
201	2.5	0.5	0.1	0.049	0.057	0.057	0.055	0.050
202	2.5	0.5	0.2	0.047	0.054	0.052	0.049	0.050
203	2.5	0.5	0.3	0.053	0.055	0.054	0.059	0.054
204	2.5	0.5	0.4	0.053	0.051	0.056	0.054	0.060
205	2.5	0.5	0.5	0.053	0.050	0.057	0.058	0.059
206	2.5	1.0	0.1	0.053	0.057	0.051	0.055	0.056
207	2.5	1.0	0.2	0.050	0.049	0.057	0.054	0.061
208	2.5	1.0	0.3	0.056	0.052	0.050	0.048	0.054
209	2.5	1.0	0.4	0.049	0.053	0.053	0.052	0.049
210	2.5	1.0	0.5	0.046	0.054	0.057	0.055	0.059
211	2.5	1.5	0.1	0.051	0.052	0.054	0.057	0.055
212	2.5	1.5	0.2	0.050	0.051	0.054	0.059	0.052
213	2.5	1.5	0.3	0.049	0.056	0.058	0.055	0.057
214	2.5	1.5	0.4	0.055	0.048	0.057	0.053	0.058
215	2.5	1.5	0.5	0.050	0.047	0.060	0.055	0.063
216	2.5	2.0	0.1	0.053	0.060	0.056	0.060	0.052
217	2.5	2.0	0.2	0.051	0.052	0.055	0.057	0.054
218	2.5	2.0	0.3	0.056	0.054	0.051	0.056	0.054
219	2.5	2.0	0.4	0.049	0.050	0.052	0.055	0.056
220	2.5	2.0	0.5	0.049	0.049	0.054	0.059	0.055
221	2.5	2.5	0.1	0.051	0.054	0.049	0.048	0.056
222	2.5	2.5	0.2	0.056	0.050	0.054	0.054	0.051
223	2.5	2.5	0.3	0.051	0.057	0.053	0.051	0.058
224	2.5	2.5	0.4	0.050	0.054	0.051	0.056	0.057
225	2.5	2.5	0.5	0.055	0.052	0.058	0.055	0.055
226	2.5	3.0	0.1	0.048	0.058	0.052	0.055	0.049
227	2.5	3.0	0.2	0.058	0.051	0.042	0.058	0.047
228	2.5	3.0	0.3	0.052	0.051	0.052	0.055	0.055
229	2.5	3.0	0.4	0.054	0.057	0.057	0.052	0.060
230	2.5	3.0	0.5	0.055	0.056	0.054	0.056	0.060
231	2.5	3.5	0.1	0.045	0.046	0.049	0.056	0.052
232	2.5	3.5	0.2	0.054	0.048	0.058	0.048	0.049
233	2.5	3.5	0.3	0.049	0.053	0.055	0.055	0.055
234	2.5	3.5	0.4	0.051	0.053	0.056	0.053	0.054
235	2.5	3.5	0.5	0.052	0.051	0.060	0.060	0.060
236	2.5	4.0	0.1	0.053	0.049	0.051	0.050	0.053
237	2.5	4.0	0.2	0.054	0.056	0.056	0.050	0.053

238	2.5	4.0	0.3	0.051	0.056	0.056	0.060	0.057
239	2.5	4.0	0.4	0.055	0.052	0.053	0.055	0.050
240	2.5	4.0	0.5	0.050	0.056	0.053	0.057	0.064
241	2.5	4.5	0.1	0.053	0.052	0.048	0.052	0.044
242	2.5	4.5	0.2	0.050	0.054	0.057	0.058	0.056
243	2.5	4.5	0.3	0.051	0.053	0.057	0.055	0.055
244	2.5	4.5	0.4	0.049	0.054	0.058	0.053	0.055
245	2.5	4.5	0.5	0.056	0.060	0.058	0.055	0.053
246	2.5	5.0	0.1	0.050	0.059	0.058	0.053	0.050
247	2.5	5.0	0.2	0.055	0.055	0.051	0.053	0.058
248	2.5	5.0	0.3	0.055	0.057	0.057	0.056	0.054
249	2.5	5.0	0.4	0.051	0.051	0.056	0.057	0.060
250	2.5	5.0	0.5	0.050	0.054	0.056	0.055	0.053
251	3.0	0.5	0.1	0.053	0.057	0.049	0.052	0.052
252	3.0	0.5	0.2	0.048	0.057	0.051	0.052	0.057
253	3.0	0.5	0.3	0.047	0.047	0.051	0.054	0.057
254	3.0	0.5	0.4	0.051	0.051	0.052	0.054	0.054
255	3.0	0.5	0.5	0.055	0.050	0.056	0.050	0.055
256	3.0	1.0	0.1	0.046	0.054	0.058	0.055	0.052
257	3.0	1.0	0.2	0.047	0.049	0.062	0.057	0.049
258	3.0	1.0	0.3	0.051	0.055	0.054	0.056	0.055
259	3.0	1.0	0.4	0.049	0.054	0.055	0.056	0.055
260	3.0	1.0	0.5	0.049	0.052	0.058	0.063	0.057
261	3.0	1.5	0.1	0.049	0.047	0.053	0.056	0.051
262	3.0	1.5	0.2	0.054	0.055	0.053	0.056	0.063
263	3.0	1.5	0.3	0.048	0.051	0.053	0.059	0.053
264	3.0	1.5	0.4	0.056	0.053	0.060	0.050	0.059
265	3.0	1.5	0.5	0.049	0.054	0.050	0.051	0.063
266	3.0	2.0	0.1	0.057	0.051	0.051	0.054	0.049
267	3.0	2.0	0.2	0.054	0.063	0.055	0.051	0.054
268	3.0	2.0	0.3	0.051	0.047	0.053	0.055	0.056
269	3.0	2.0	0.4	0.056	0.051	0.054	0.060	0.053
270	3.0	2.0	0.5	0.050	0.051	0.052	0.054	0.056
271	3.0	2.5	0.1	0.049	0.053	0.056	0.056	0.052
272	3.0	2.5	0.2	0.048	0.050	0.055	0.055	0.056
273	3.0	2.5	0.3	0.054	0.056	0.053	0.051	0.052
274	3.0	2.5	0.4	0.051	0.052	0.055	0.054	0.060
275	3.0	2.5	0.5	0.053	0.060	0.055	0.056	0.051
276	3.0	3.0	0.1	0.052	0.055	0.059	0.054	0.049
277	3.0	3.0	0.2	0.052	0.049	0.058	0.054	0.055
278	3.0	3.0	0.3	0.054	0.055	0.051	0.056	0.056
279	3.0	3.0	0.4	0.055	0.048	0.056	0.056	0.054
280	3.0	3.0	0.5	0.049	0.053	0.052	0.059	0.060
281	3.0	3.5	0.1	0.048	0.057	0.053	0.050	0.054
282	3.0	3.5	0.2	0.050	0.051	0.058	0.056	0.054
283	3.0	3.5	0.3	0.051	0.055	0.053	0.057	0.055
284	3.0	3.5	0.4	0.051	0.058	0.055	0.057	0.060
285	3.0	3.5	0.5	0.051	0.055	0.052	0.055	0.055

286	3.0	4.0	0.1	0.049	0.053	0.052	0.051	0.056
287	3.0	4.0	0.2	0.049	0.057	0.054	0.053	0.054
288	3.0	4.0	0.3	0.055	0.056	0.052	0.056	0.055
289	3.0	4.0	0.4	0.053	0.054	0.059	0.055	0.047
290	3.0	4.0	0.5	0.051	0.053	0.055	0.054	0.055
291	3.0	4.5	0.1	0.047	0.053	0.053	0.054	0.054
292	3.0	4.5	0.2	0.050	0.048	0.048	0.054	0.054
293	3.0	4.5	0.3	0.053	0.057	0.050	0.053	0.051
294	3.0	4.5	0.4	0.048	0.049	0.056	0.053	0.053
295	3.0	4.5	0.5	0.049	0.048	0.051	0.056	0.063
296	3.0	5.0	0.1	0.050	0.046	0.053	0.050	0.051
297	3.0	5.0	0.2	0.055	0.048	0.060	0.056	0.057
298	3.0	5.0	0.3	0.051	0.052	0.053	0.049	0.059
299	3.0	5.0	0.4	0.047	0.049	0.057	0.053	0.053
300	3.0	5.0	0.5	0.049	0.049	0.052	0.058	0.058
301	3.5	0.5	0.1	0.050	0.051	0.055	0.052	0.056
302	3.5	0.5	0.2	0.054	0.051	0.054	0.050	0.051
303	3.5	0.5	0.3	0.053	0.050	0.053	0.055	0.060
304	3.5	0.5	0.4	0.051	0.053	0.059	0.058	0.064
305	3.5	0.5	0.5	0.054	0.061	0.050	0.054	0.052
306	3.5	1.0	0.1	0.048	0.051	0.045	0.049	0.049
307	3.5	1.0	0.2	0.049	0.048	0.061	0.054	0.057
308	3.5	1.0	0.3	0.050	0.056	0.054	0.061	0.057
309	3.5	1.0	0.4	0.048	0.059	0.049	0.058	0.061
310	3.5	1.0	0.5	0.058	0.047	0.058	0.057	0.056
311	3.5	1.5	0.1	0.052	0.055	0.053	0.055	0.052
312	3.5	1.5	0.2	0.050	0.051	0.053	0.053	0.050
313	3.5	1.5	0.3	0.057	0.056	0.056	0.056	0.059
314	3.5	1.5	0.4	0.049	0.049	0.063	0.056	0.053
315	3.5	1.5	0.5	0.051	0.048	0.051	0.061	0.058
316	3.5	2.0	0.1	0.050	0.051	0.060	0.053	0.056
317	3.5	2.0	0.2	0.054	0.054	0.050	0.055	0.055
318	3.5	2.0	0.3	0.051	0.056	0.054	0.055	0.059
319	3.5	2.0	0.4	0.052	0.048	0.055	0.054	0.056
320	3.5	2.0	0.5	0.050	0.057	0.056	0.054	0.062
321	3.5	2.5	0.1	0.055	0.054	0.052	0.054	0.057
322	3.5	2.5	0.2	0.052	0.048	0.054	0.051	0.051
323	3.5	2.5	0.3	0.050	0.048	0.047	0.054	0.052
324	3.5	2.5	0.4	0.053	0.057	0.052	0.052	0.062
325	3.5	2.5	0.5	0.049	0.057	0.049	0.056	0.054
326	3.5	3.0	0.1	0.050	0.056	0.051	0.052	0.053
327	3.5	3.0	0.2	0.053	0.051	0.052	0.049	0.052
328	3.5	3.0	0.3	0.056	0.055	0.047	0.057	0.058
329	3.5	3.0	0.4	0.048	0.050	0.059	0.055	0.052
330	3.5	3.0	0.5	0.052	0.052	0.052	0.055	0.059
331	3.5	3.5	0.1	0.049	0.049	0.054	0.054	0.052
332	3.5	3.5	0.2	0.046	0.053	0.048	0.048	0.052
333	3.5	3.5	0.3	0.054	0.047	0.055	0.059	0.055

334	3.5	3.5	0.4	0.048	0.054	0.056	0.054	0.049
335	3.5	3.5	0.5	0.050	0.054	0.054	0.058	0.059
336	3.5	4.0	0.1	0.052	0.060	0.049	0.050	0.051
337	3.5	4.0	0.2	0.051	0.052	0.061	0.053	0.051
338	3.5	4.0	0.3	0.052	0.055	0.052	0.055	0.056
339	3.5	4.0	0.4	0.051	0.055	0.056	0.052	0.057
340	3.5	4.0	0.5	0.056	0.051	0.051	0.058	0.058
341	3.5	4.5	0.1	0.052	0.049	0.052	0.053	0.049
342	3.5	4.5	0.2	0.054	0.049	0.050	0.053	0.054
343	3.5	4.5	0.3	0.046	0.051	0.055	0.052	0.063
344	3.5	4.5	0.4	0.047	0.051	0.052	0.058	0.053
345	3.5	4.5	0.5	0.051	0.051	0.057	0.054	0.063
346	3.5	5.0	0.1	0.050	0.048	0.056	0.059	0.053
347	3.5	5.0	0.2	0.048	0.049	0.055	0.058	0.051
348	3.5	5.0	0.3	0.047	0.047	0.049	0.054	0.055
349	3.5	5.0	0.4	0.058	0.047	0.055	0.055	0.061
350	3.5	5.0	0.5	0.056	0.053	0.050	0.064	0.055
351	4.0	0.5	0.1	0.048	0.052	0.055	0.056	0.054
352	4.0	0.5	0.2	0.048	0.050	0.054	0.055	0.049
353	4.0	0.5	0.3	0.052	0.054	0.055	0.053	0.054
354	4.0	0.5	0.4	0.053	0.049	0.056	0.057	0.050
355	4.0	0.5	0.5	0.052	0.053	0.057	0.056	0.061
356	4.0	1.0	0.1	0.053	0.047	0.054	0.060	0.059
357	4.0	1.0	0.2	0.049	0.052	0.047	0.047	0.059
358	4.0	1.0	0.3	0.051	0.046	0.058	0.057	0.054
359	4.0	1.0	0.4	0.046	0.057	0.058	0.056	0.059
360	4.0	1.0	0.5	0.051	0.055	0.057	0.051	0.054
361	4.0	1.5	0.1	0.049	0.053	0.055	0.053	0.052
362	4.0	1.5	0.2	0.054	0.057	0.054	0.050	0.063
363	4.0	1.5	0.3	0.051	0.053	0.055	0.053	0.057
364	4.0	1.5	0.4	0.049	0.049	0.054	0.059	0.057
365	4.0	1.5	0.5	0.059	0.057	0.054	0.055	0.052
366	4.0	2.0	0.1	0.055	0.054	0.044	0.057	0.052
367	4.0	2.0	0.2	0.054	0.051	0.056	0.058	0.050
368	4.0	2.0	0.3	0.049	0.049	0.055	0.052	0.053
369	4.0	2.0	0.4	0.054	0.051	0.051	0.052	0.056
370	4.0	2.0	0.5	0.055	0.052	0.059	0.058	0.059
371	4.0	2.5	0.1	0.049	0.060	0.046	0.054	0.057
372	4.0	2.5	0.2	0.050	0.056	0.055	0.051	0.054
373	4.0	2.5	0.3	0.054	0.050	0.054	0.058	0.052
374	4.0	2.5	0.4	0.054	0.050	0.056	0.053	0.057
375	4.0	2.5	0.5	0.050	0.056	0.057	0.055	0.052
376	4.0	3.0	0.1	0.053	0.056	0.050	0.054	0.057
377	4.0	3.0	0.2	0.051	0.046	0.056	0.057	0.056
378	4.0	3.0	0.3	0.051	0.054	0.053	0.057	0.057
379	4.0	3.0	0.4	0.050	0.046	0.051	0.053	0.064
380	4.0	3.0	0.5	0.050	0.053	0.049	0.055	0.060
381	4.0	3.5	0.1	0.052	0.051	0.054	0.054	0.051

382	4.0	3.5	0.2	0.052	0.055	0.054	0.058	0.048
383	4.0	3.5	0.3	0.052	0.054	0.054	0.055	0.049
384	4.0	3.5	0.4	0.053	0.059	0.055	0.057	0.055
385	4.0	3.5	0.5	0.051	0.057	0.058	0.054	0.057
386	4.0	4.0	0.1	0.056	0.050	0.058	0.050	0.050
387	4.0	4.0	0.2	0.056	0.055	0.058	0.054	0.049
388	4.0	4.0	0.3	0.052	0.046	0.054	0.049	0.052
389	4.0	4.0	0.4	0.060	0.058	0.058	0.049	0.061
390	4.0	4.0	0.5	0.055	0.051	0.065	0.061	0.053
391	4.0	4.5	0.1	0.055	0.055	0.052	0.057	0.050
392	4.0	4.5	0.2	0.052	0.055	0.055	0.056	0.050
393	4.0	4.5	0.3	0.052	0.050	0.061	0.053	0.049
394	4.0	4.5	0.4	0.056	0.054	0.055	0.056	0.055
395	4.0	4.5	0.5	0.054	0.059	0.054	0.057	0.063
396	4.0	5.0	0.1	0.053	0.053	0.056	0.054	0.054
397	4.0	5.0	0.2	0.051	0.049	0.051	0.053	0.054
398	4.0	5.0	0.3	0.052	0.060	0.059	0.056	0.054
399	4.0	5.0	0.4	0.056	0.052	0.052	0.050	0.055
400	4.0	5.0	0.5	0.053	0.058	0.052	0.056	0.058
401	4.5	0.5	0.1	0.043	0.054	0.052	0.049	0.055
402	4.5	0.5	0.2	0.048	0.050	0.052	0.054	0.055
403	4.5	0.5	0.3	0.055	0.049	0.051	0.057	0.059
404	4.5	0.5	0.4	0.050	0.054	0.060	0.049	0.052
405	4.5	0.5	0.5	0.048	0.051	0.054	0.050	0.056
406	4.5	1.0	0.1	0.049	0.049	0.053	0.056	0.050
407	4.5	1.0	0.2	0.051	0.050	0.053	0.057	0.053
408	4.5	1.0	0.3	0.051	0.055	0.052	0.050	0.052
409	4.5	1.0	0.4	0.053	0.057	0.050	0.057	0.051
410	4.5	1.0	0.5	0.049	0.052	0.059	0.060	0.056
411	4.5	1.5	0.1	0.057	0.058	0.051	0.050	0.057
412	4.5	1.5	0.2	0.052	0.050	0.060	0.051	0.055
413	4.5	1.5	0.3	0.051	0.055	0.054	0.057	0.054
414	4.5	1.5	0.4	0.053	0.052	0.052	0.059	0.055
415	4.5	1.5	0.5	0.050	0.052	0.056	0.062	0.060
416	4.5	2.0	0.1	0.046	0.046	0.052	0.050	0.058
417	4.5	2.0	0.2	0.055	0.051	0.052	0.055	0.058
418	4.5	2.0	0.3	0.051	0.053	0.053	0.052	0.057
419	4.5	2.0	0.4	0.051	0.050	0.055	0.057	0.056
420	4.5	2.0	0.5	0.054	0.052	0.056	0.054	0.052
421	4.5	2.5	0.1	0.052	0.056	0.048	0.052	0.053
422	4.5	2.5	0.2	0.054	0.054	0.054	0.059	0.049
423	4.5	2.5	0.3	0.056	0.053	0.058	0.055	0.056
424	4.5	2.5	0.4	0.050	0.052	0.060	0.056	0.055
425	4.5	2.5	0.5	0.055	0.057	0.057	0.051	0.058
426	4.5	3.0	0.1	0.049	0.053	0.054	0.061	0.051
427	4.5	3.0	0.2	0.052	0.054	0.051	0.053	0.052
428	4.5	3.0	0.3	0.050	0.046	0.058	0.056	0.054
429	4.5	3.0	0.4	0.052	0.051	0.051	0.060	0.056

430	4.5	3.0	0.5	0.056	0.050	0.056	0.053	0.058
431	4.5	3.5	0.1	0.050	0.057	0.054	0.055	0.054
432	4.5	3.5	0.2	0.051	0.051	0.055	0.054	0.054
433	4.5	3.5	0.3	0.056	0.057	0.052	0.055	0.053
434	4.5	3.5	0.4	0.053	0.050	0.055	0.051	0.054
435	4.5	3.5	0.5	0.054	0.052	0.053	0.057	0.058
436	4.5	4.0	0.1	0.049	0.050	0.052	0.053	0.051
437	4.5	4.0	0.2	0.051	0.048	0.054	0.052	0.052
438	4.5	4.0	0.3	0.057	0.049	0.054	0.058	0.058
439	4.5	4.0	0.4	0.047	0.051	0.052	0.053	0.055
440	4.5	4.0	0.5	0.044	0.051	0.056	0.052	0.052
441	4.5	4.5	0.1	0.049	0.049	0.046	0.049	0.054
442	4.5	4.5	0.2	0.052	0.052	0.057	0.053	0.050
443	4.5	4.5	0.3	0.055	0.050	0.054	0.052	0.057
444	4.5	4.5	0.4	0.048	0.055	0.060	0.052	0.052
445	4.5	4.5	0.5	0.047	0.052	0.055	0.058	0.058
446	4.5	5.0	0.1	0.047	0.054	0.052	0.054	0.059
447	4.5	5.0	0.2	0.050	0.053	0.055	0.054	0.060
448	4.5	5.0	0.3	0.050	0.056	0.048	0.056	0.058
449	4.5	5.0	0.4	0.051	0.056	0.055	0.053	0.054
450	4.5	5.0	0.5	0.053	0.051	0.055	0.052	0.056
451	5.0	0.5	0.1	0.041	0.050	0.049	0.058	0.056
452	5.0	0.5	0.2	0.052	0.055	0.052	0.059	0.055
453	5.0	0.5	0.3	0.048	0.056	0.051	0.056	0.050
454	5.0	0.5	0.4	0.048	0.052	0.056	0.054	0.057
455	5.0	0.5	0.5	0.051	0.048	0.057	0.057	0.052
456	5.0	1.0	0.1	0.045	0.049	0.052	0.052	0.058
457	5.0	1.0	0.2	0.047	0.052	0.051	0.049	0.050
458	5.0	1.0	0.3	0.053	0.052	0.053	0.056	0.053
459	5.0	1.0	0.4	0.048	0.053	0.054	0.056	0.055
460	5.0	1.0	0.5	0.050	0.052	0.052	0.062	0.058
461	5.0	1.5	0.1	0.050	0.053	0.052	0.056	0.055
462	5.0	1.5	0.2	0.050	0.050	0.057	0.051	0.058
463	5.0	1.5	0.3	0.053	0.049	0.056	0.057	0.057
464	5.0	1.5	0.4	0.051	0.050	0.054	0.060	0.052
465	5.0	1.5	0.5	0.056	0.054	0.057	0.056	0.063
466	5.0	2.0	0.1	0.044	0.052	0.055	0.055	0.055
467	5.0	2.0	0.2	0.052	0.049	0.058	0.058	0.058
468	5.0	2.0	0.3	0.053	0.055	0.057	0.051	0.054
469	5.0	2.0	0.4	0.050	0.053	0.047	0.050	0.058
470	5.0	2.0	0.5	0.051	0.055	0.054	0.057	0.059
471	5.0	2.5	0.1	0.050	0.053	0.056	0.051	0.053
472	5.0	2.5	0.2	0.049	0.045	0.052	0.056	0.052
473	5.0	2.5	0.3	0.053	0.051	0.058	0.052	0.052
474	5.0	2.5	0.4	0.052	0.053	0.052	0.055	0.053
475	5.0	2.5	0.5	0.052	0.052	0.059	0.056	0.056
476	5.0	3.0	0.1	0.051	0.051	0.051	0.049	0.054
477	5.0	3.0	0.2	0.052	0.049	0.046	0.053	0.053

478	5.0	3.0	0.3	0.056	0.055	0.055	0.052	0.052
479	5.0	3.0	0.4	0.052	0.049	0.058	0.056	0.060
480	5.0	3.0	0.5	0.049	0.052	0.056	0.052	0.059
481	5.0	3.5	0.1	0.054	0.046	0.053	0.055	0.050
482	5.0	3.5	0.2	0.053	0.045	0.051	0.053	0.053
483	5.0	3.5	0.3	0.056	0.050	0.051	0.055	0.052
484	5.0	3.5	0.4	0.054	0.057	0.054	0.053	0.059
485	5.0	3.5	0.5	0.055	0.055	0.054	0.059	0.059
486	5.0	4.0	0.1	0.052	0.049	0.054	0.057	0.053
487	5.0	4.0	0.2	0.053	0.053	0.054	0.054	0.050
488	5.0	4.0	0.3	0.047	0.053	0.049	0.061	0.062
489	5.0	4.0	0.4	0.055	0.052	0.054	0.054	0.055
490	5.0	4.0	0.5	0.048	0.053	0.061	0.055	0.058
491	5.0	4.5	0.1	0.054	0.051	0.054	0.055	0.052
492	5.0	4.5	0.2	0.050	0.058	0.047	0.056	0.054
493	5.0	4.5	0.3	0.050	0.057	0.050	0.057	0.052
494	5.0	4.5	0.4	0.056	0.051	0.050	0.052	0.053
495	5.0	4.5	0.5	0.053	0.055	0.057	0.052	0.060
496	5.0	5.0	0.1	0.051	0.049	0.053	0.055	0.052
497	5.0	5.0	0.2	0.055	0.049	0.057	0.055	0.054
498	5.0	5.0	0.3	0.053	0.050	0.056	0.056	0.054
499	5.0	5.0	0.4	0.056	0.050	0.058	0.054	0.058
500	5.0	5.0	0.5	0.049	0.054	0.047	0.058	0.058

Table 7.3: Observed Size Comparison for n=500

	Mu	sigma	p	DL0.01	DL0.1	DL0.2	DL0.3	DL0.4
1	0.5	0.5	0.1	0.050	0.051	0.053	0.049	0.050
2	0.5	0.5	0.2	0.053	0.054	0.047	0.051	0.052
3	0.5	0.5	0.3	0.055	0.056	0.052	0.057	0.051
4	0.5	0.5	0.4	0.052	0.052	0.054	0.047	0.050
5	0.5	0.5	0.5	0.047	0.052	0.051	0.054	0.052
6	0.5	1.0	0.1	0.055	0.048	0.053	0.052	0.046
7	0.5	1.0	0.2	0.051	0.052	0.054	0.052	0.049
8	0.5	1.0	0.3	0.049	0.045	0.049	0.056	0.056
9	0.5	1.0	0.4	0.053	0.054	0.054	0.054	0.049
10	0.5	1.0	0.5	0.052	0.056	0.054	0.046	0.053
11	0.5	1.5	0.1	0.050	0.057	0.051	0.047	0.054
12	0.5	1.5	0.2	0.060	0.049	0.049	0.051	0.049
13	0.5	1.5	0.3	0.051	0.049	0.054	0.050	0.051
14	0.5	1.5	0.4	0.049	0.055	0.050	0.048	0.046
15	0.5	1.5	0.5	0.049	0.056	0.049	0.059	0.052
16	0.5	2.0	0.1	0.048	0.052	0.055	0.048	0.049
17	0.5	2.0	0.2	0.049	0.046	0.049	0.055	0.061
18	0.5	2.0	0.3	0.054	0.053	0.048	0.050	0.057
19	0.5	2.0	0.4	0.050	0.047	0.048	0.063	0.051
20	0.5	2.0	0.5	0.056	0.056	0.051	0.052	0.056
21	0.5	2.5	0.1	0.051	0.050	0.046	0.053	0.047
22	0.5	2.5	0.2	0.053	0.052	0.050	0.053	0.055
23	0.5	2.5	0.3	0.049	0.055	0.050	0.053	0.053
24	0.5	2.5	0.4	0.049	0.046	0.054	0.047	0.048
25	0.5	2.5	0.5	0.054	0.048	0.053	0.054	0.054
26	0.5	3.0	0.1	0.050	0.051	0.047	0.049	0.055
27	0.5	3.0	0.2	0.044	0.051	0.047	0.055	0.050
28	0.5	3.0	0.3	0.053	0.053	0.048	0.053	0.054
29	0.5	3.0	0.4	0.055	0.055	0.054	0.055	0.054
30	0.5	3.0	0.5	0.047	0.046	0.055	0.056	0.050
31	0.5	3.5	0.1	0.046	0.048	0.055	0.055	0.049
32	0.5	3.5	0.2	0.052	0.050	0.046	0.049	0.056
33	0.5	3.5	0.3	0.048	0.052	0.048	0.052	0.052
34	0.5	3.5	0.4	0.052	0.047	0.055	0.055	0.053
35	0.5	3.5	0.5	0.049	0.053	0.050	0.046	0.056
36	0.5	4.0	0.1	0.053	0.051	0.055	0.050	0.050
37	0.5	4.0	0.2	0.046	0.054	0.055	0.052	0.050
38	0.5	4.0	0.3	0.047	0.053	0.049	0.053	0.048
39	0.5	4.0	0.4	0.049	0.050	0.051	0.050	0.052
40	0.5	4.0	0.5	0.056	0.048	0.052	0.054	0.049
41	0.5	4.5	0.1	0.047	0.050	0.054	0.051	0.053
42	0.5	4.5	0.2	0.048	0.048	0.054	0.054	0.051
43	0.5	4.5	0.3	0.052	0.050	0.053	0.050	0.052
44	0.5	4.5	0.4	0.046	0.049	0.049	0.056	0.053
45	0.5	4.5	0.5	0.050	0.052	0.054	0.058	0.054

46	0.5	5.0	0.1	0.049	0.055	0.056	0.054	0.051
47	0.5	5.0	0.2	0.053	0.051	0.052	0.048	0.048
48	0.5	5.0	0.3	0.052	0.054	0.051	0.053	0.046
49	0.5	5.0	0.4	0.049	0.056	0.048	0.050	0.054
50	0.5	5.0	0.5	0.053	0.053	0.053	0.057	0.054
51	1.0	0.5	0.1	0.045	0.048	0.050	0.053	0.044
52	1.0	0.5	0.2	0.049	0.050	0.053	0.050	0.054
53	1.0	0.5	0.3	0.048	0.053	0.050	0.053	0.054
54	1.0	0.5	0.4	0.045	0.048	0.044	0.048	0.048
55	1.0	0.5	0.5	0.051	0.048	0.053	0.052	0.059
56	1.0	1.0	0.1	0.051	0.047	0.055	0.053	0.054
57	1.0	1.0	0.2	0.053	0.052	0.053	0.048	0.050
58	1.0	1.0	0.3	0.054	0.051	0.053	0.059	0.059
59	1.0	1.0	0.4	0.050	0.046	0.057	0.053	0.053
60	1.0	1.0	0.5	0.050	0.052	0.052	0.056	0.054
61	1.0	1.5	0.1	0.046	0.053	0.049	0.049	0.052
62	1.0	1.5	0.2	0.049	0.044	0.054	0.056	0.052
63	1.0	1.5	0.3	0.052	0.044	0.049	0.053	0.058
64	1.0	1.5	0.4	0.050	0.053	0.051	0.050	0.056
65	1.0	1.5	0.5	0.053	0.046	0.051	0.055	0.052
66	1.0	2.0	0.1	0.053	0.058	0.050	0.048	0.051
67	1.0	2.0	0.2	0.050	0.052	0.051	0.051	0.053
68	1.0	2.0	0.3	0.048	0.051	0.053	0.047	0.049
69	1.0	2.0	0.4	0.054	0.048	0.050	0.051	0.053
70	1.0	2.0	0.5	0.050	0.053	0.055	0.057	0.054
71	1.0	2.5	0.1	0.046	0.052	0.052	0.042	0.047
72	1.0	2.5	0.2	0.046	0.051	0.053	0.051	0.047
73	1.0	2.5	0.3	0.049	0.049	0.053	0.053	0.047
74	1.0	2.5	0.4	0.049	0.050	0.051	0.057	0.053
75	1.0	2.5	0.5	0.043	0.053	0.050	0.050	0.053
76	1.0	3.0	0.1	0.050	0.055	0.047	0.052	0.048
77	1.0	3.0	0.2	0.048	0.051	0.049	0.054	0.050
78	1.0	3.0	0.3	0.052	0.049	0.054	0.051	0.049
79	1.0	3.0	0.4	0.053	0.049	0.050	0.053	0.059
80	1.0	3.0	0.5	0.047	0.052	0.057	0.047	0.052
81	1.0	3.5	0.1	0.052	0.053	0.058	0.049	0.050
82	1.0	3.5	0.2	0.050	0.046	0.048	0.050	0.048
83	1.0	3.5	0.3	0.052	0.056	0.052	0.050	0.053
84	1.0	3.5	0.4	0.049	0.057	0.053	0.049	0.048
85	1.0	3.5	0.5	0.048	0.053	0.052	0.047	0.053
86	1.0	4.0	0.1	0.044	0.049	0.052	0.051	0.057
87	1.0	4.0	0.2	0.049	0.053	0.047	0.056	0.046
88	1.0	4.0	0.3	0.058	0.048	0.053	0.054	0.048
89	1.0	4.0	0.4	0.055	0.055	0.051	0.056	0.052
90	1.0	4.0	0.5	0.050	0.056	0.053	0.051	0.060
91	1.0	4.5	0.1	0.042	0.048	0.048	0.051	0.055
92	1.0	4.5	0.2	0.053	0.053	0.051	0.051	0.054
93	1.0	4.5	0.3	0.048	0.052	0.057	0.046	0.055

94	1.0	4.5	0.4	0.048	0.053	0.047	0.048	0.051
95	1.0	4.5	0.5	0.053	0.048	0.049	0.053	0.055
96	1.0	5.0	0.1	0.050	0.047	0.051	0.048	0.049
97	1.0	5.0	0.2	0.048	0.055	0.047	0.048	0.055
98	1.0	5.0	0.3	0.052	0.053	0.055	0.056	0.053
99	1.0	5.0	0.4	0.041	0.051	0.056	0.050	0.052
100	1.0	5.0	0.5	0.052	0.054	0.052	0.051	0.053
101	1.5	0.5	0.1	0.049	0.055	0.055	0.057	0.054
102	1.5	0.5	0.2	0.053	0.052	0.048	0.049	0.050
103	1.5	0.5	0.3	0.051	0.049	0.053	0.047	0.050
104	1.5	0.5	0.4	0.052	0.050	0.049	0.051	0.054
105	1.5	0.5	0.5	0.056	0.056	0.058	0.049	0.054
106	1.5	1.0	0.1	0.044	0.056	0.047	0.050	0.050
107	1.5	1.0	0.2	0.052	0.049	0.055	0.054	0.054
108	1.5	1.0	0.3	0.054	0.048	0.051	0.051	0.052
109	1.5	1.0	0.4	0.045	0.054	0.048	0.056	0.054
110	1.5	1.0	0.5	0.046	0.050	0.049	0.053	0.056
111	1.5	1.5	0.1	0.053	0.049	0.050	0.053	0.052
112	1.5	1.5	0.2	0.049	0.052	0.053	0.053	0.049
113	1.5	1.5	0.3	0.044	0.045	0.055	0.056	0.049
114	1.5	1.5	0.4	0.049	0.048	0.051	0.052	0.054
115	1.5	1.5	0.5	0.052	0.045	0.054	0.048	0.051
116	1.5	2.0	0.1	0.051	0.051	0.049	0.048	0.055
117	1.5	2.0	0.2	0.044	0.048	0.052	0.052	0.049
118	1.5	2.0	0.3	0.055	0.058	0.055	0.052	0.053
119	1.5	2.0	0.4	0.052	0.049	0.052	0.057	0.055
120	1.5	2.0	0.5	0.050	0.055	0.053	0.055	0.049
121	1.5	2.5	0.1	0.045	0.046	0.048	0.049	0.049
122	1.5	2.5	0.2	0.051	0.054	0.054	0.046	0.055
123	1.5	2.5	0.3	0.052	0.051	0.055	0.052	0.054
124	1.5	2.5	0.4	0.050	0.050	0.054	0.053	0.051
125	1.5	2.5	0.5	0.051	0.053	0.049	0.057	0.053
126	1.5	3.0	0.1	0.049	0.054	0.052	0.046	0.052
127	1.5	3.0	0.2	0.047	0.048	0.055	0.053	0.051
128	1.5	3.0	0.3	0.043	0.047	0.054	0.052	0.056
129	1.5	3.0	0.4	0.054	0.053	0.054	0.048	0.052
130	1.5	3.0	0.5	0.049	0.052	0.054	0.049	0.051
131	1.5	3.5	0.1	0.052	0.050	0.052	0.049	0.053
132	1.5	3.5	0.2	0.051	0.051	0.051	0.048	0.053
133	1.5	3.5	0.3	0.047	0.048	0.052	0.059	0.056
134	1.5	3.5	0.4	0.052	0.051	0.053	0.056	0.050
135	1.5	3.5	0.5	0.050	0.050	0.052	0.053	0.058
136	1.5	4.0	0.1	0.049	0.053	0.047	0.056	0.053
137	1.5	4.0	0.2	0.052	0.050	0.046	0.047	0.054
138	1.5	4.0	0.3	0.049	0.050	0.053	0.054	0.049
139	1.5	4.0	0.4	0.052	0.049	0.052	0.050	0.055
140	1.5	4.0	0.5	0.052	0.052	0.051	0.054	0.048
141	1.5	4.5	0.1	0.048	0.048	0.051	0.048	0.050

142	1.5	4.5	0.2	0.052	0.051	0.052	0.058	0.049
143	1.5	4.5	0.3	0.054	0.053	0.053	0.052	0.055
144	1.5	4.5	0.4	0.053	0.048	0.048	0.050	0.051
145	1.5	4.5	0.5	0.045	0.051	0.048	0.053	0.060
146	1.5	5.0	0.1	0.056	0.047	0.054	0.054	0.048
147	1.5	5.0	0.2	0.051	0.052	0.051	0.054	0.049
148	1.5	5.0	0.3	0.050	0.054	0.049	0.055	0.052
149	1.5	5.0	0.4	0.051	0.053	0.051	0.053	0.055
150	1.5	5.0	0.5	0.043	0.055	0.054	0.050	0.049
151	2.0	0.5	0.1	0.049	0.053	0.060	0.054	0.052
152	2.0	0.5	0.2	0.046	0.051	0.051	0.058	0.055
153	2.0	0.5	0.3	0.051	0.052	0.051	0.049	0.057
154	2.0	0.5	0.4	0.050	0.057	0.051	0.051	0.050
155	2.0	0.5	0.5	0.051	0.055	0.053	0.049	0.051
156	2.0	1.0	0.1	0.046	0.048	0.049	0.052	0.048
157	2.0	1.0	0.2	0.045	0.048	0.050	0.051	0.054
158	2.0	1.0	0.3	0.047	0.054	0.055	0.052	0.051
159	2.0	1.0	0.4	0.045	0.054	0.050	0.052	0.057
160	2.0	1.0	0.5	0.048	0.052	0.054	0.051	0.053
161	2.0	1.5	0.1	0.054	0.049	0.047	0.055	0.055
162	2.0	1.5	0.2	0.044	0.050	0.049	0.054	0.047
163	2.0	1.5	0.3	0.051	0.050	0.048	0.051	0.050
164	2.0	1.5	0.4	0.050	0.050	0.056	0.049	0.052
165	2.0	1.5	0.5	0.050	0.054	0.047	0.058	0.055
166	2.0	2.0	0.1	0.051	0.050	0.048	0.048	0.050
167	2.0	2.0	0.2	0.051	0.049	0.048	0.054	0.053
168	2.0	2.0	0.3	0.050	0.053	0.054	0.051	0.053
169	2.0	2.0	0.4	0.047	0.052	0.052	0.048	0.054
170	2.0	2.0	0.5	0.054	0.055	0.048	0.059	0.055
171	2.0	2.5	0.1	0.052	0.056	0.053	0.051	0.054
172	2.0	2.5	0.2	0.051	0.055	0.050	0.053	0.053
173	2.0	2.5	0.3	0.052	0.055	0.045	0.053	0.057
174	2.0	2.5	0.4	0.047	0.050	0.048	0.052	0.048
175	2.0	2.5	0.5	0.050	0.057	0.046	0.051	0.054
176	2.0	3.0	0.1	0.047	0.054	0.053	0.049	0.055
177	2.0	3.0	0.2	0.051	0.050	0.054	0.050	0.055
178	2.0	3.0	0.3	0.055	0.053	0.055	0.053	0.052
179	2.0	3.0	0.4	0.052	0.053	0.049	0.054	0.052
180	2.0	3.0	0.5	0.053	0.057	0.055	0.055	0.054
181	2.0	3.5	0.1	0.051	0.055	0.050	0.050	0.054
182	2.0	3.5	0.2	0.047	0.050	0.054	0.050	0.048
183	2.0	3.5	0.3	0.045	0.054	0.050	0.050	0.052
184	2.0	3.5	0.4	0.054	0.052	0.056	0.052	0.051
185	2.0	3.5	0.5	0.050	0.050	0.054	0.055	0.055
186	2.0	4.0	0.1	0.043	0.050	0.051	0.056	0.052
187	2.0	4.0	0.2	0.053	0.047	0.050	0.057	0.047
188	2.0	4.0	0.3	0.046	0.048	0.046	0.048	0.054
189	2.0	4.0	0.4	0.046	0.050	0.051	0.052	0.052

190	2.0	4.0	0.5	0.053	0.051	0.052	0.052	0.052
191	2.0	4.5	0.1	0.051	0.053	0.049	0.048	0.055
192	2.0	4.5	0.2	0.051	0.053	0.051	0.051	0.056
193	2.0	4.5	0.3	0.049	0.053	0.049	0.051	0.046
194	2.0	4.5	0.4	0.049	0.055	0.053	0.048	0.057
195	2.0	4.5	0.5	0.045	0.049	0.049	0.052	0.058
196	2.0	5.0	0.1	0.053	0.050	0.051	0.051	0.051
197	2.0	5.0	0.2	0.053	0.051	0.046	0.053	0.055
198	2.0	5.0	0.3	0.049	0.049	0.058	0.045	0.048
199	2.0	5.0	0.4	0.046	0.044	0.056	0.050	0.053
200	2.0	5.0	0.5	0.052	0.049	0.054	0.058	0.055
201	2.5	0.5	0.1	0.044	0.055	0.053	0.050	0.055
202	2.5	0.5	0.2	0.051	0.050	0.057	0.050	0.056
203	2.5	0.5	0.3	0.047	0.047	0.052	0.050	0.052
204	2.5	0.5	0.4	0.055	0.051	0.049	0.049	0.052
205	2.5	0.5	0.5	0.053	0.051	0.057	0.056	0.056
206	2.5	1.0	0.1	0.054	0.052	0.050	0.053	0.047
207	2.5	1.0	0.2	0.053	0.048	0.050	0.051	0.052
208	2.5	1.0	0.3	0.050	0.051	0.049	0.047	0.057
209	2.5	1.0	0.4	0.053	0.055	0.053	0.053	0.053
210	2.5	1.0	0.5	0.045	0.048	0.050	0.058	0.053
211	2.5	1.5	0.1	0.054	0.049	0.046	0.048	0.052
212	2.5	1.5	0.2	0.051	0.058	0.054	0.052	0.048
213	2.5	1.5	0.3	0.050	0.050	0.056	0.050	0.056
214	2.5	1.5	0.4	0.048	0.058	0.057	0.050	0.051
215	2.5	1.5	0.5	0.052	0.048	0.053	0.053	0.049
216	2.5	2.0	0.1	0.042	0.051	0.046	0.048	0.046
217	2.5	2.0	0.2	0.050	0.046	0.050	0.051	0.052
218	2.5	2.0	0.3	0.049	0.048	0.049	0.048	0.053
219	2.5	2.0	0.4	0.053	0.050	0.051	0.055	0.048
220	2.5	2.0	0.5	0.050	0.050	0.057	0.051	0.048
221	2.5	2.5	0.1	0.052	0.050	0.049	0.054	0.054
222	2.5	2.5	0.2	0.048	0.053	0.052	0.051	0.048
223	2.5	2.5	0.3	0.052	0.051	0.047	0.052	0.055
224	2.5	2.5	0.4	0.057	0.047	0.052	0.053	0.050
225	2.5	2.5	0.5	0.060	0.055	0.054	0.055	0.051
226	2.5	3.0	0.1	0.047	0.046	0.050	0.056	0.051
227	2.5	3.0	0.2	0.048	0.051	0.057	0.049	0.049
228	2.5	3.0	0.3	0.051	0.049	0.049	0.055	0.055
229	2.5	3.0	0.4	0.053	0.060	0.050	0.052	0.054
230	2.5	3.0	0.5	0.052	0.052	0.052	0.046	0.050
231	2.5	3.5	0.1	0.050	0.056	0.056	0.053	0.057
232	2.5	3.5	0.2	0.052	0.047	0.048	0.049	0.054
233	2.5	3.5	0.3	0.049	0.053	0.054	0.052	0.050
234	2.5	3.5	0.4	0.050	0.050	0.056	0.052	0.049
235	2.5	3.5	0.5	0.048	0.048	0.053	0.054	0.056
236	2.5	4.0	0.1	0.046	0.049	0.050	0.048	0.056
237	2.5	4.0	0.2	0.046	0.053	0.051	0.050	0.048

238	2.5	4.0	0.3	0.055	0.048	0.057	0.054	0.059
239	2.5	4.0	0.4	0.056	0.049	0.053	0.048	0.053
240	2.5	4.0	0.5	0.055	0.051	0.047	0.055	0.048
241	2.5	4.5	0.1	0.047	0.055	0.050	0.048	0.051
242	2.5	4.5	0.2	0.051	0.051	0.048	0.044	0.052
243	2.5	4.5	0.3	0.049	0.048	0.049	0.049	0.052
244	2.5	4.5	0.4	0.046	0.052	0.050	0.054	0.051
245	2.5	4.5	0.5	0.048	0.058	0.048	0.048	0.052
246	2.5	5.0	0.1	0.046	0.051	0.053	0.054	0.047
247	2.5	5.0	0.2	0.045	0.049	0.057	0.055	0.053
248	2.5	5.0	0.3	0.053	0.052	0.056	0.051	0.047
249	2.5	5.0	0.4	0.053	0.056	0.047	0.058	0.051
250	2.5	5.0	0.5	0.052	0.051	0.055	0.057	0.052
251	3.0	0.5	0.1	0.051	0.048	0.045	0.058	0.050
252	3.0	0.5	0.2	0.051	0.055	0.052	0.053	0.052
253	3.0	0.5	0.3	0.045	0.047	0.048	0.051	0.049
254	3.0	0.5	0.4	0.053	0.055	0.054	0.057	0.056
255	3.0	0.5	0.5	0.048	0.058	0.049	0.055	0.052
256	3.0	1.0	0.1	0.048	0.055	0.055	0.046	0.055
257	3.0	1.0	0.2	0.049	0.056	0.051	0.051	0.052
258	3.0	1.0	0.3	0.050	0.049	0.052	0.052	0.050
259	3.0	1.0	0.4	0.051	0.053	0.052	0.051	0.054
260	3.0	1.0	0.5	0.049	0.059	0.051	0.058	0.052
261	3.0	1.5	0.1	0.053	0.049	0.050	0.053	0.050
262	3.0	1.5	0.2	0.045	0.052	0.053	0.048	0.051
263	3.0	1.5	0.3	0.052	0.053	0.052	0.055	0.053
264	3.0	1.5	0.4	0.050	0.048	0.054	0.055	0.055
265	3.0	1.5	0.5	0.048	0.052	0.055	0.056	0.060
266	3.0	2.0	0.1	0.050	0.052	0.051	0.047	0.056
267	3.0	2.0	0.2	0.044	0.050	0.051	0.050	0.056
268	3.0	2.0	0.3	0.049	0.049	0.051	0.051	0.054
269	3.0	2.0	0.4	0.049	0.053	0.052	0.048	0.058
270	3.0	2.0	0.5	0.049	0.047	0.054	0.049	0.054
271	3.0	2.5	0.1	0.045	0.055	0.057	0.054	0.053
272	3.0	2.5	0.2	0.054	0.049	0.052	0.053	0.061
273	3.0	2.5	0.3	0.053	0.048	0.052	0.052	0.052
274	3.0	2.5	0.4	0.052	0.049	0.051	0.057	0.051
275	3.0	2.5	0.5	0.056	0.050	0.053	0.048	0.051
276	3.0	3.0	0.1	0.049	0.054	0.048	0.053	0.055
277	3.0	3.0	0.2	0.050	0.049	0.053	0.050	0.052
278	3.0	3.0	0.3	0.050	0.048	0.054	0.052	0.057
279	3.0	3.0	0.4	0.053	0.051	0.052	0.060	0.052
280	3.0	3.0	0.5	0.047	0.049	0.050	0.052	0.053
281	3.0	3.5	0.1	0.049	0.050	0.048	0.051	0.051
282	3.0	3.5	0.2	0.049	0.051	0.054	0.049	0.051
283	3.0	3.5	0.3	0.053	0.054	0.052	0.058	0.049
284	3.0	3.5	0.4	0.049	0.049	0.052	0.055	0.055
285	3.0	3.5	0.5	0.049	0.054	0.047	0.050	0.059

286	3.0	4.0	0.1	0.050	0.052	0.050	0.052	0.049
287	3.0	4.0	0.2	0.049	0.052	0.046	0.056	0.047
288	3.0	4.0	0.3	0.055	0.052	0.055	0.054	0.054
289	3.0	4.0	0.4	0.052	0.052	0.054	0.049	0.053
290	3.0	4.0	0.5	0.052	0.056	0.055	0.049	0.056
291	3.0	4.5	0.1	0.051	0.055	0.049	0.048	0.052
292	3.0	4.5	0.2	0.053	0.056	0.058	0.051	0.050
293	3.0	4.5	0.3	0.050	0.051	0.052	0.053	0.051
294	3.0	4.5	0.4	0.052	0.053	0.054	0.052	0.053
295	3.0	4.5	0.5	0.048	0.050	0.055	0.055	0.052
296	3.0	5.0	0.1	0.055	0.054	0.051	0.053	0.055
297	3.0	5.0	0.2	0.051	0.050	0.049	0.052	0.051
298	3.0	5.0	0.3	0.049	0.052	0.052	0.047	0.048
299	3.0	5.0	0.4	0.051	0.050	0.053	0.053	0.053
300	3.0	5.0	0.5	0.050	0.053	0.048	0.052	0.052
301	3.5	0.5	0.1	0.052	0.057	0.054	0.050	0.049
302	3.5	0.5	0.2	0.048	0.054	0.053	0.051	0.052
303	3.5	0.5	0.3	0.052	0.049	0.049	0.050	0.057
304	3.5	0.5	0.4	0.050	0.053	0.056	0.052	0.058
305	3.5	0.5	0.5	0.045	0.052	0.048	0.053	0.055
306	3.5	1.0	0.1	0.048	0.048	0.050	0.051	0.045
307	3.5	1.0	0.2	0.043	0.052	0.051	0.051	0.058
308	3.5	1.0	0.3	0.053	0.052	0.050	0.044	0.054
309	3.5	1.0	0.4	0.050	0.047	0.055	0.047	0.051
310	3.5	1.0	0.5	0.047	0.049	0.052	0.049	0.056
311	3.5	1.5	0.1	0.049	0.054	0.050	0.051	0.050
312	3.5	1.5	0.2	0.053	0.048	0.056	0.051	0.053
313	3.5	1.5	0.3	0.048	0.050	0.053	0.055	0.051
314	3.5	1.5	0.4	0.052	0.044	0.046	0.047	0.058
315	3.5	1.5	0.5	0.053	0.051	0.048	0.057	0.056
316	3.5	2.0	0.1	0.052	0.044	0.051	0.055	0.054
317	3.5	2.0	0.2	0.049	0.048	0.056	0.052	0.055
318	3.5	2.0	0.3	0.048	0.049	0.051	0.053	0.053
319	3.5	2.0	0.4	0.051	0.054	0.045	0.055	0.057
320	3.5	2.0	0.5	0.050	0.050	0.056	0.050	0.050
321	3.5	2.5	0.1	0.047	0.048	0.058	0.052	0.052
322	3.5	2.5	0.2	0.046	0.052	0.055	0.053	0.051
323	3.5	2.5	0.3	0.052	0.050	0.049	0.058	0.047
324	3.5	2.5	0.4	0.054	0.054	0.053	0.056	0.054
325	3.5	2.5	0.5	0.054	0.053	0.055	0.048	0.051
326	3.5	3.0	0.1	0.052	0.052	0.045	0.055	0.055
327	3.5	3.0	0.2	0.044	0.054	0.051	0.053	0.049
328	3.5	3.0	0.3	0.053	0.051	0.055	0.048	0.060
329	3.5	3.0	0.4	0.047	0.051	0.049	0.053	0.050
330	3.5	3.0	0.5	0.054	0.054	0.048	0.049	0.051
331	3.5	3.5	0.1	0.050	0.050	0.048	0.045	0.055
332	3.5	3.5	0.2	0.054	0.048	0.053	0.058	0.054
333	3.5	3.5	0.3	0.047	0.050	0.057	0.051	0.050

334	3.5	3.5	0.4	0.055	0.054	0.055	0.051	0.049
335	3.5	3.5	0.5	0.054	0.052	0.047	0.056	0.055
336	3.5	4.0	0.1	0.050	0.050	0.049	0.052	0.049
337	3.5	4.0	0.2	0.056	0.054	0.053	0.053	0.049
338	3.5	4.0	0.3	0.051	0.050	0.052	0.054	0.046
339	3.5	4.0	0.4	0.051	0.047	0.053	0.052	0.058
340	3.5	4.0	0.5	0.049	0.052	0.056	0.056	0.054
341	3.5	4.5	0.1	0.046	0.053	0.050	0.052	0.051
342	3.5	4.5	0.2	0.047	0.046	0.055	0.044	0.050
343	3.5	4.5	0.3	0.050	0.050	0.042	0.053	0.050
344	3.5	4.5	0.4	0.052	0.053	0.050	0.052	0.051
345	3.5	4.5	0.5	0.056	0.050	0.055	0.059	0.048
346	3.5	5.0	0.1	0.054	0.053	0.049	0.048	0.054
347	3.5	5.0	0.2	0.052	0.054	0.052	0.050	0.053
348	3.5	5.0	0.3	0.049	0.050	0.051	0.054	0.050
349	3.5	5.0	0.4	0.054	0.051	0.054	0.051	0.049
350	3.5	5.0	0.5	0.052	0.053	0.056	0.047	0.056
351	4.0	0.5	0.1	0.051	0.046	0.053	0.049	0.050
352	4.0	0.5	0.2	0.055	0.048	0.052	0.053	0.052
353	4.0	0.5	0.3	0.056	0.053	0.053	0.046	0.055
354	4.0	0.5	0.4	0.051	0.057	0.055	0.053	0.053
355	4.0	0.5	0.5	0.051	0.054	0.052	0.051	0.051
356	4.0	1.0	0.1	0.050	0.052	0.048	0.056	0.048
357	4.0	1.0	0.2	0.052	0.051	0.052	0.054	0.049
358	4.0	1.0	0.3	0.048	0.057	0.049	0.052	0.048
359	4.0	1.0	0.4	0.055	0.051	0.054	0.048	0.055
360	4.0	1.0	0.5	0.054	0.052	0.052	0.055	0.057
361	4.0	1.5	0.1	0.056	0.056	0.053	0.056	0.054
362	4.0	1.5	0.2	0.049	0.052	0.054	0.050	0.052
363	4.0	1.5	0.3	0.052	0.049	0.050	0.055	0.048
364	4.0	1.5	0.4	0.056	0.054	0.052	0.054	0.055
365	4.0	1.5	0.5	0.050	0.049	0.053	0.051	0.052
366	4.0	2.0	0.1	0.047	0.051	0.049	0.047	0.054
367	4.0	2.0	0.2	0.043	0.047	0.047	0.057	0.043
368	4.0	2.0	0.3	0.058	0.046	0.052	0.056	0.054
369	4.0	2.0	0.4	0.050	0.050	0.050	0.050	0.049
370	4.0	2.0	0.5	0.050	0.048	0.053	0.045	0.053
371	4.0	2.5	0.1	0.055	0.055	0.053	0.056	0.050
372	4.0	2.5	0.2	0.048	0.054	0.052	0.052	0.050
373	4.0	2.5	0.3	0.048	0.051	0.055	0.050	0.050
374	4.0	2.5	0.4	0.052	0.049	0.053	0.050	0.052
375	4.0	2.5	0.5	0.055	0.051	0.051	0.055	0.049
376	4.0	3.0	0.1	0.047	0.047	0.052	0.048	0.053
377	4.0	3.0	0.2	0.050	0.053	0.053	0.052	0.053
378	4.0	3.0	0.3	0.045	0.047	0.049	0.053	0.053
379	4.0	3.0	0.4	0.051	0.051	0.050	0.054	0.047
380	4.0	3.0	0.5	0.052	0.049	0.057	0.056	0.054
381	4.0	3.5	0.1	0.047	0.054	0.052	0.053	0.053

382	4.0	3.5	0.2	0.050	0.047	0.051	0.057	0.052
383	4.0	3.5	0.3	0.050	0.045	0.048	0.048	0.050
384	4.0	3.5	0.4	0.052	0.051	0.049	0.059	0.052
385	4.0	3.5	0.5	0.051	0.050	0.053	0.052	0.053
386	4.0	4.0	0.1	0.049	0.054	0.048	0.053	0.047
387	4.0	4.0	0.2	0.055	0.051	0.048	0.049	0.047
388	4.0	4.0	0.3	0.057	0.047	0.053	0.053	0.054
389	4.0	4.0	0.4	0.049	0.051	0.052	0.050	0.047
390	4.0	4.0	0.5	0.045	0.057	0.053	0.054	0.052
391	4.0	4.5	0.1	0.052	0.051	0.059	0.053	0.052
392	4.0	4.5	0.2	0.055	0.051	0.050	0.055	0.054
393	4.0	4.5	0.3	0.048	0.048	0.051	0.051	0.058
394	4.0	4.5	0.4	0.054	0.051	0.052	0.049	0.054
395	4.0	4.5	0.5	0.057	0.049	0.052	0.053	0.053
396	4.0	5.0	0.1	0.050	0.051	0.050	0.050	0.052
397	4.0	5.0	0.2	0.049	0.047	0.057	0.052	0.047
398	4.0	5.0	0.3	0.050	0.053	0.054	0.051	0.051
399	4.0	5.0	0.4	0.048	0.056	0.052	0.049	0.046
400	4.0	5.0	0.5	0.050	0.051	0.053	0.051	0.054
401	4.5	0.5	0.1	0.049	0.056	0.054	0.051	0.054
402	4.5	0.5	0.2	0.050	0.049	0.051	0.050	0.054
403	4.5	0.5	0.3	0.051	0.050	0.053	0.052	0.057
404	4.5	0.5	0.4	0.049	0.054	0.049	0.053	0.052
405	4.5	0.5	0.5	0.048	0.054	0.057	0.056	0.054
406	4.5	1.0	0.1	0.045	0.045	0.052	0.046	0.052
407	4.5	1.0	0.2	0.049	0.049	0.048	0.051	0.051
408	4.5	1.0	0.3	0.046	0.051	0.053	0.054	0.049
409	4.5	1.0	0.4	0.055	0.044	0.044	0.055	0.055
410	4.5	1.0	0.5	0.051	0.058	0.051	0.051	0.052
411	4.5	1.5	0.1	0.044	0.047	0.049	0.052	0.058
412	4.5	1.5	0.2	0.053	0.044	0.046	0.053	0.051
413	4.5	1.5	0.3	0.049	0.048	0.051	0.045	0.054
414	4.5	1.5	0.4	0.048	0.048	0.050	0.053	0.054
415	4.5	1.5	0.5	0.049	0.050	0.054	0.052	0.048
416	4.5	2.0	0.1	0.050	0.046	0.051	0.055	0.053
417	4.5	2.0	0.2	0.053	0.054	0.056	0.054	0.049
418	4.5	2.0	0.3	0.055	0.054	0.051	0.053	0.048
419	4.5	2.0	0.4	0.052	0.052	0.052	0.060	0.047
420	4.5	2.0	0.5	0.049	0.053	0.050	0.052	0.061
421	4.5	2.5	0.1	0.048	0.055	0.049	0.053	0.051
422	4.5	2.5	0.2	0.054	0.050	0.049	0.050	0.052
423	4.5	2.5	0.3	0.049	0.049	0.055	0.050	0.056
424	4.5	2.5	0.4	0.055	0.049	0.044	0.057	0.053
425	4.5	2.5	0.5	0.048	0.051	0.056	0.053	0.055
426	4.5	3.0	0.1	0.053	0.050	0.052	0.057	0.058
427	4.5	3.0	0.2	0.053	0.052	0.049	0.048	0.054
428	4.5	3.0	0.3	0.050	0.050	0.050	0.050	0.046
429	4.5	3.0	0.4	0.046	0.057	0.047	0.050	0.052

430	4.5	3.0	0.5	0.051	0.052	0.058	0.049	0.055
431	4.5	3.5	0.1	0.051	0.054	0.048	0.049	0.047
432	4.5	3.5	0.2	0.050	0.049	0.052	0.051	0.048
433	4.5	3.5	0.3	0.050	0.053	0.056	0.056	0.054
434	4.5	3.5	0.4	0.055	0.045	0.058	0.047	0.053
435	4.5	3.5	0.5	0.053	0.053	0.061	0.053	0.048
436	4.5	4.0	0.1	0.051	0.049	0.051	0.062	0.045
437	4.5	4.0	0.2	0.050	0.050	0.052	0.057	0.050
438	4.5	4.0	0.3	0.057	0.053	0.051	0.052	0.061
439	4.5	4.0	0.4	0.048	0.053	0.052	0.051	0.054
440	4.5	4.0	0.5	0.053	0.053	0.056	0.059	0.053
441	4.5	4.5	0.1	0.050	0.054	0.049	0.058	0.055
442	4.5	4.5	0.2	0.049	0.052	0.050	0.050	0.056
443	4.5	4.5	0.3	0.048	0.049	0.051	0.047	0.053
444	4.5	4.5	0.4	0.050	0.050	0.048	0.050	0.058
445	4.5	4.5	0.5	0.050	0.051	0.054	0.052	0.055
446	4.5	5.0	0.1	0.056	0.046	0.052	0.053	0.057
447	4.5	5.0	0.2	0.047	0.053	0.049	0.052	0.050
448	4.5	5.0	0.3	0.054	0.052	0.051	0.052	0.050
449	4.5	5.0	0.4	0.051	0.049	0.050	0.054	0.051
450	4.5	5.0	0.5	0.054	0.044	0.053	0.054	0.050
451	5.0	0.5	0.1	0.047	0.056	0.053	0.054	0.055
452	5.0	0.5	0.2	0.050	0.050	0.051	0.051	0.049
453	5.0	0.5	0.3	0.051	0.050	0.050	0.052	0.054
454	5.0	0.5	0.4	0.051	0.045	0.057	0.057	0.052
455	5.0	0.5	0.5	0.051	0.052	0.050	0.057	0.049
456	5.0	1.0	0.1	0.049	0.052	0.048	0.052	0.053
457	5.0	1.0	0.2	0.049	0.048	0.049	0.054	0.048
458	5.0	1.0	0.3	0.049	0.055	0.054	0.047	0.053
459	5.0	1.0	0.4	0.052	0.048	0.054	0.054	0.056
460	5.0	1.0	0.5	0.056	0.051	0.052	0.051	0.054
461	5.0	1.5	0.1	0.053	0.053	0.057	0.052	0.056
462	5.0	1.5	0.2	0.048	0.050	0.056	0.049	0.056
463	5.0	1.5	0.3	0.053	0.049	0.050	0.058	0.052
464	5.0	1.5	0.4	0.047	0.052	0.059	0.054	0.053
465	5.0	1.5	0.5	0.053	0.054	0.047	0.061	0.056
466	5.0	2.0	0.1	0.057	0.052	0.049	0.052	0.054
467	5.0	2.0	0.2	0.050	0.055	0.052	0.056	0.054
468	5.0	2.0	0.3	0.050	0.061	0.053	0.054	0.056
469	5.0	2.0	0.4	0.046	0.048	0.056	0.053	0.055
470	5.0	2.0	0.5	0.054	0.053	0.055	0.056	0.051
471	5.0	2.5	0.1	0.051	0.053	0.049	0.051	0.047
472	5.0	2.5	0.2	0.055	0.049	0.047	0.051	0.053
473	5.0	2.5	0.3	0.050	0.050	0.050	0.047	0.054
474	5.0	2.5	0.4	0.050	0.053	0.050	0.056	0.052
475	5.0	2.5	0.5	0.051	0.047	0.053	0.059	0.059
476	5.0	3.0	0.1	0.052	0.046	0.048	0.055	0.050
477	5.0	3.0	0.2	0.046	0.049	0.048	0.054	0.046

478	5.0	3.0	0.3	0.057	0.050	0.049	0.056	0.052
479	5.0	3.0	0.4	0.051	0.051	0.049	0.054	0.048
480	5.0	3.0	0.5	0.052	0.053	0.053	0.056	0.054
481	5.0	3.5	0.1	0.051	0.048	0.057	0.051	0.055
482	5.0	3.5	0.2	0.046	0.046	0.048	0.051	0.053
483	5.0	3.5	0.3	0.050	0.044	0.051	0.052	0.047
484	5.0	3.5	0.4	0.057	0.050	0.055	0.051	0.058
485	5.0	3.5	0.5	0.052	0.050	0.056	0.055	0.053
486	5.0	4.0	0.1	0.051	0.050	0.052	0.049	0.055
487	5.0	4.0	0.2	0.048	0.049	0.047	0.052	0.055
488	5.0	4.0	0.3	0.046	0.054	0.053	0.054	0.049
489	5.0	4.0	0.4	0.053	0.049	0.050	0.058	0.053
490	5.0	4.0	0.5	0.051	0.056	0.049	0.052	0.053
491	5.0	4.5	0.1	0.051	0.052	0.048	0.049	0.051
492	5.0	4.5	0.2	0.049	0.048	0.056	0.049	0.052
493	5.0	4.5	0.3	0.050	0.056	0.052	0.039	0.052
494	5.0	4.5	0.4	0.053	0.053	0.053	0.047	0.054
495	5.0	4.5	0.5	0.047	0.046	0.049	0.054	0.053
496	5.0	5.0	0.1	0.056	0.048	0.048	0.048	0.053
497	5.0	5.0	0.2	0.047	0.045	0.052	0.057	0.059
498	5.0	5.0	0.3	0.049	0.051	0.053	0.047	0.051
499	5.0	5.0	0.4	0.046	0.054	0.046	0.053	0.053
500	5.0	5.0	0.5	0.048	0.054	0.056	0.058	0.050

APPENDIX B
Tables of Observed Power

Table 7.4: Observed Powers at Various Effect Sizes. n=100, P(y≤DL)=0.01

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
1.0	0.5	0.2	0.990	0.764	0.174
4.5	0.5	0.5	0.913	0.551	0.131
5.0	0.5	0.3	0.981	0.698	0.163
1.5	1.0	0.4	0.990	0.988	0.767
3.0	1.0	0.4	0.961	0.636	0.145
3.5	1.0	0.1	0.997	0.812	0.199
5.0	1.0	0.1	0.995	0.814	0.196
0.5	1.5	0.3	0.977	0.704	0.164
1.0	1.5	0.3	0.982	0.687	0.159
2.5	1.5	0.3	0.981	0.702	0.164
1.0	3.0	0.4	0.957	0.625	0.148
3.5	3.0	0.3	0.975	0.712	0.161
1.0	3.5	0.2	0.991	0.757	0.176
3.0	3.5	0.1	0.997	0.187	0.814
3.0	3.5	0.5	0.536	0.918	0.129
4.5	3.5	0.1	0.996	0.816	0.186
5.0	3.5	0.3	0.974	0.163	0.701
5.0	3.5	0.4	0.626	0.963	0.154
2.5	4.0	0.1	0.997	0.818	0.189
4.5	4.0	0.3	0.983	0.707	0.168
0.5	4.5	0.3	0.982	0.704	0.167
3.5	4.5	0.3	0.979	0.703	0.151
4.0	4.5	0.1	0.995	0.816	0.205
1.5	5.0	0.5	0.916	0.543	0.132
3.5	5.0	0.5	0.907	0.540	0.128
4.0	5.0	0.4	0.955	0.624	0.148

Table 7.5: Observed Power at Various Effect Sizes. n=100, P(y≤DL)=0.2

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
1.0	0.5	0.1	0.994	0.187	0.767
1.0	0.5	0.2	0.674	0.973	0.167
2.0	0.5	0.3	0.941	0.576	0.142
0.5	1.0	0.1	0.994	0.774	0.188
1.0	1.0	0.1	0.991	0.779	0.175
2.5	1.0	0.1	0.992	0.775	0.189
3.0	1.0	0.1	0.991	0.775	0.183
2.0	1.5	0.2	0.973	0.684	0.157
0.5	2.0	0.1	0.992	0.761	0.184
5.0	2.0	0.1	0.993	0.780	0.184
1.0	2.5	0.1	0.993	0.769	0.173
5.0	2.5	0.1	0.991	0.185	0.773
5.0	2.5	0.5	0.420	0.791	0.118
0.5	3.0	0.1	0.992	0.775	0.182
3.5	3.0	0.3	0.938	0.580	0.145
4.0	3.0	0.2	0.973	0.668	0.149
5.0	4.0	0.3	0.942	0.584	0.143
0.5	4.5	0.1	0.993	0.784	0.187
4.5	4.5	0.1	0.992	0.187	0.773
4.5	4.5	0.2	0.682	0.977	0.156

Table 7.6: Observed Power at Various Effect Sizes. n=100, $P(y \leq DL)=0.3$

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
1.5	0.5	0.1	0.989	0.773	0.181
2.0	0.5	0.2	0.973	0.683	0.153
1.0	1.0	0.1	0.989	0.768	0.183
1.5	1.0	0.1	0.990	0.761	0.180
3.0	1.0	0.1	0.992	0.768	0.188
4.0	1.0	0.1	0.989	0.776	0.177
2.0	1.5	0.3	0.941	0.583	0.140
3.0	1.5	0.4	0.876	0.491	0.123
3.5	1.5	0.2	0.971	0.661	0.153
4.5	2.0	0.1	0.991	0.190	0.769
4.5	2.0	0.3	0.589	0.939	0.134
1.0	2.5	0.1	0.993	0.778	0.176
2.5	2.5	0.1	0.992	0.771	0.185
2.5	3.5	0.1	0.990	0.766	0.188
3.0	3.5	0.1	0.993	0.766	0.176
3.5	4.0	0.1	0.991	0.760	0.180
1.5	5.0	0.2	0.974	0.673	0.157
2.5	5.0	0.1	0.992	0.755	0.181
3.5	5.0	0.1	0.992	0.764	0.186
4.0	5.0	0.3	0.945	0.571	0.147

Table 7.7: Observed Power at Various Effect Sizes. n=100, $P(y \leq DL)=0.4$

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
0.5	0.5	0.1	0.988	0.745	0.173
1.5	0.5	0.1	0.986	0.745	0.189
4.5	0.5	0.1	0.986	0.748	0.185
1.0	1.0	0.1	0.987	0.742	0.184
2.5	1.0	0.1	0.987	0.746	0.180
2.0	1.5	0.1	0.988	0.749	0.177
4.5	1.5	0.1	0.989	0.740	0.179
5.0	1.5	0.1	0.988	0.744	0.171
0.5	2.0	0.1	0.989	0.741	0.181
3.5	2.0	0.1	0.988	0.747	0.188
4.5	2.0	0.1	0.988	0.737	0.173
5.0	3.5	0.2	0.970	0.651	0.162
0.5	4.0	0.3	0.938	0.586	0.148
3.5	4.0	0.1	0.988	0.745	0.178
0.5	4.5	0.1	0.985	0.748	0.184
1.0	4.5	0.1	0.988	0.748	0.180
2.0	4.5	0.2	0.969	0.667	0.152
3.0	4.5	0.2	0.964	0.672	0.161
4.0	4.5	0.1	0.988	0.755	0.172
0.5	5.0	0.1	0.989	0.749	0.179

Table 7.8: Observed Power at Various Effect Sizes. n=200, P(y≤DL)=0.01

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
1.0	0.5	0.1	1.000	0.985	0.352
4.5	0.5	0.5	0.998	0.857	0.206
5.0	0.5	0.3	1.000	0.948	0.284
1.5	1.0	0.4	1.000	0.907	0.246
3.0	1.0	0.4	1.000	0.913	0.248
3.5	1.0	0.1	1.000	0.986	0.349
5.0	1.0	0.1	1.000	0.987	0.357
1.0	1.5	0.3	1.000	0.949	0.275
2.5	1.5	0.3	1.000	0.952	0.283
3.5	2.5	0.3	1.000	0.951	0.281
0.5	3.0	0.3	1.000	0.947	0.287
1.0	3.0	0.4	0.999	0.913	0.240
4.5	3.0	0.3	1.000	0.947	0.277
5.0	3.0	0.5	0.997	0.845	0.209
1.0	3.5	0.2	1.000	0.974	0.315
1.5	3.5	0.1	1.000	0.988	0.350
3.0	3.5	0.1	1.000	0.349	0.986
3.0	3.5	0.5	0.855	0.999	0.212
4.5	3.5	0.1	1.000	0.987	0.354
1.0	4.5	0.3	1.000	0.949	0.278
1.5	4.5	0.4	0.999	0.915	0.245
2.5	4.5	0.2	1.000	0.973	0.323
3.5	4.5	0.3	1.000	0.291	0.952
3.5	4.5	0.5	0.856	0.998	0.213
4.5	5.0	0.2	1.000	0.307	0.970
4.5	5.0	0.3	0.949	1.000	0.278
5.0	5.0	0.5	0.999	0.851	0.202

Table 7.9: Observed Power at Various Effect Sizes. n=200, $P(y \leq DL)=0.1$

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
3.0	0.5	0.3	0.999	0.240	0.889
3.0	0.5	0.5	0.740	0.986	0.169
4.5	0.5	0.3	0.999	0.231	0.899
4.5	0.5	0.5	0.747	0.984	0.171
0.5	1.0	0.1	1.000	0.975	0.322
1.0	1.0	0.3	0.999	0.238	0.886
1.0	1.0	0.5	0.740	0.985	0.179
3.5	1.0	0.1	1.000	0.973	0.318
5.0	1.0	0.1	1.000	0.978	0.314
0.5	3.0	0.3	0.999	0.892	0.230
3.0	3.0	0.4	0.995	0.825	0.211
4.5	3.0	0.3	1.000	0.899	0.245
5.0	3.0	0.2	1.000	0.273	0.940
5.0	3.0	0.4	0.826	0.995	0.203
1.0	3.5	0.1	1.000	0.324	0.972
1.0	3.5	0.4	0.817	0.997	0.207
3.0	3.5	0.2	1.000	0.939	0.267
3.5	3.5	0.3	0.999	0.889	0.227
2.5	4.0	0.1	1.000	0.975	0.316
5.0	4.0	0.1	1.000	0.975	0.329
1.0	4.5	0.3	0.999	0.892	0.237
3.5	4.5	0.3	0.999	0.895	0.244
1.5	5.0	0.5	0.988	0.748	0.184
3.5	5.0	0.4	0.998	0.829	0.207
5.0	5.0	0.3	0.999	0.227	0.891
5.0	5.0	0.4	0.820	0.996	0.209

Table 7.10: Observed Power at Various Effect Sizes. n=200, $P(y \leq DL)=0.2$

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
2.0	0.5	0.1	1.000	0.975	0.328
2.5	0.5	0.2	1.000	0.938	0.276
3.0	0.5	0.5	0.978	0.694	0.156
3.5	0.5	0.2	1.000	0.932	0.264
4.5	0.5	0.1	1.000	0.314	0.969
4.5	0.5	0.5	0.693	0.978	0.167
2.0	1.0	0.4	0.996	0.796	0.195
2.5	1.0	0.3	0.998	0.878	0.218
4.5	1.0	0.3	0.999	0.876	0.224
2.0	1.5	0.3	0.999	0.885	0.231
0.5	2.0	0.3	0.999	0.886	0.227
4.5	2.0	0.1	1.000	0.977	0.308
5.0	2.0	0.4	0.994	0.789	0.198
0.5	3.0	0.3	0.999	0.886	0.225
1.0	3.0	0.2	1.000	0.936	0.268
1.5	3.0	0.2	1.000	0.940	0.266
3.5	3.0	0.2	1.000	0.942	0.274
5.0	3.0	0.1	1.000	0.970	0.326
2.5	3.5	0.2	1.000	0.934	0.261
0.5	4.0	0.1	1.000	0.973	0.334
1.0	4.0	0.4	0.996	0.804	0.196
2.5	4.0	0.1	1.000	0.977	0.312
3.5	4.0	0.1	1.000	0.970	0.315
4.0	4.0	0.1	1.000	0.313	0.973
4.0	4.0	0.3	0.879	0.999	0.232
3.0	5.0	0.3	0.999	0.877	0.221
4.0	5.0	0.4	0.994	0.786	0.197

Table 7.11: Observed Power at Various Effect Sizes. n=200, $P(y \leq DL)=0.3$

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
3.0	0.5	0.5	0.980	0.695	0.158
3.5	0.5	0.2	1.000	0.936	0.261
4.5	0.5	0.5	0.979	0.702	0.162
2.0	1.0	0.4	0.994	0.799	0.190
2.5	1.0	0.3	0.999	0.872	0.230
4.5	1.0	0.3	0.999	0.871	0.212
5.0	1.0	0.2	1.000	0.939	0.271
3.0	1.5	0.4	0.995	0.788	0.182
4.0	1.5	0.2	1.000	0.933	0.266
4.5	1.5	0.1	1.000	0.973	0.317
5.0	2.0	0.4	0.996	0.797	0.184
0.5	2.5	0.1	1.000	0.971	0.326
1.5	2.5	0.2	1.000	0.934	0.261
0.5	3.0	0.3	0.999	0.878	0.224
3.5	3.0	0.2	1.000	0.930	0.272
5.0	3.0	0.1	1.000	0.972	0.322
3.5	3.5	0.2	1.000	0.933	0.269
0.5	4.0	0.1	1.000	0.975	0.314
1.0	4.0	0.4	0.995	0.798	0.196
2.5	4.0	0.1	1.000	0.973	0.314
4.0	4.0	0.1	1.000	0.972	0.328
2.0	4.5	0.4	0.994	0.801	0.186
1.5	5.0	0.1	1.000	0.972	0.316
2.0	5.0	0.3	0.999	0.874	0.226
3.0	5.0	0.3	1.000	0.879	0.220
4.0	5.0	0.4	0.996	0.790	0.188

Table 7.12: Observed Power at Various Effect Sizes. n=200, $P(y \leq DL)=0.4$

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
2.0	0.5	0.1	1.000	0.968	0.308
2.5	0.5	0.1	1.000	0.960	0.304
4.0	0.5	0.4	0.994	0.787	0.186
5.0	0.5	0.3	0.998	0.872	0.220
2.5	1.0	0.4	0.994	0.783	0.200
3.0	1.0	0.2	1.000	0.932	0.258
5.0	1.0	0.2	0.999	0.928	0.260
0.5	1.5	0.1	1.000	0.961	0.315
3.5	1.5	0.2	1.000	0.931	0.259
0.5	2.0	0.1	1.000	0.963	0.315
2.0	2.5	0.1	1.000	0.964	0.313
4.5	2.5	0.2	1.000	0.931	0.256
3.0	3.0	0.1	1.000	0.968	0.319
0.5	3.5	0.3	0.998	0.875	0.221
1.5	3.5	0.1	1.000	0.967	0.310
2.5	3.5	0.2	0.999	0.930	0.273
3.5	3.5	0.4	0.994	0.796	0.178
4.0	3.5	0.3	0.999	0.875	0.220
0.5	4.0	0.4	0.992	0.789	0.196
2.5	4.0	0.4	0.992	0.789	0.197
1.0	4.5	0.3	0.999	0.875	0.232
3.5	4.5	0.1	1.000	0.962	0.313
4.0	4.5	0.3	0.999	0.871	0.234
4.5	4.5	0.2	1.000	0.926	0.256
1.0	5.0	0.1	1.000	0.964	0.303

Table 7.13: Observed Power at Various Effect Sizes. n=500, P(y≤DL)=0.01

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
1.5	0.5	0.1	1.000	1.000	0.748
1.0	1.0	0.4	1.000	1.000	0.538
2.5	1.0	0.3	1.000	1.000	0.604
3.5	1.0	0.4	1.000	0.999	0.550
5.0	1.0	0.3	1.000	1.000	0.609
2.0	1.5	0.5	1.000	0.998	0.455
1.5	2.0	0.5	1.000	0.999	0.468
2.5	2.0	0.2	1.000	1.000	0.682
4.5	2.0	0.1	1.000	1.000	0.740
0.5	2.5	0.3	1.000	1.000	0.610
5.0	2.5	0.4	1.000	1.000	0.542
0.5	3.0	0.1	1.000	1.000	0.730
3.0	3.0	0.3	1.000	1.000	0.616
4.0	3.0	0.2	1.000	1.000	0.685
4.5	3.0	0.3	1.000	1.000	0.616
2.0	3.5	0.5	1.000	0.998	0.468
2.5	3.5	0.4	1.000	1.000	0.535
3.0	3.5	0.2	1.000	1.000	0.674
5.0	3.5	0.3	1.000	1.000	0.605
0.5	4.5	0.5	1.000	0.998	0.466
2.0	4.5	0.3	1.000	1.000	0.615
4.5	4.5	0.5	1.000	0.997	0.468
1.0	5.0	0.1	1.000	1.000	0.730
3.0	5.0	0.5	1.000	0.998	0.464
3.5	5.0	0.3	1.000	1.000	0.616
4.0	5.0	0.1	1.000	0.725	1.000
4.0	5.0	0.3	1.000	1.000	0.604

Table 7.14: Observed Power at Various Effect Sizes. n=500, $P(y \leq DL)=0.2$

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
1.0	0.5	0.2	1.000	1.000	0.583
2.5	0.5	0.2	1.000	1.000	0.587
4.5	0.5	0.3	1.000	1.000	0.516
1.5	1.0	0.5	1.000	0.985	0.364
2.0	1.5	0.4	1.000	0.996	0.443
2.5	1.5	0.3	1.000	0.999	0.515
4.5	1.5	0.5	1.000	0.989	0.363
5.0	1.5	0.2	1.000	1.000	0.581
2.5	2.0	0.5	1.000	0.987	0.358
1.0	2.5	0.4	1.000	0.996	0.422
2.0	2.5	0.4	1.000	0.997	0.423
3.0	2.5	0.5	1.000	0.985	0.367
3.5	2.5	0.3	1.000	1.000	0.512
4.5	3.0	0.1	1.000	0.685	1.000
4.5	3.0	0.3	0.999	1.000	0.511
1.5	3.5	0.1	1.000	1.000	0.691
2.5	3.5	0.4	1.000	0.996	0.434
4.0	3.5	0.5	1.000	0.986	0.375
5.0	3.5	0.5	1.000	0.988	0.373
0.5	4.0	0.4	1.000	0.997	0.444
1.5	4.0	0.3	1.000	0.999	0.516
2.0	4.0	0.4	1.000	0.995	0.425
3.5	4.0	0.1	1.000	0.678	1.000
3.5	4.0	0.3	1.000	1.000	0.508
4.5	4.0	0.2	1.000	1.000	0.600
4.5	4.5	0.4	1.000	0.998	0.428
3.0	5.0	0.4	1.000	0.997	0.445

Table 7.15: Observed Power at Various Effect Sizes. n=500, P(y≤DL)=0.2

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
1.0	0.5	0.1	1.000	0.679	1.000
1.0	0.5	0.3	0.999	1.000	0.487
5.0	0.5	0.3	1.000	0.999	0.481
2.0	1.0	0.2	1.000	1.000	0.581
2.5	1.0	0.2	1.000	0.576	1.000
2.5	1.0	0.5	0.982	1.000	0.344
3.5	1.0	0.3	1.000	0.998	0.482
0.5	1.5	0.4	1.000	0.994	0.402
1.5	1.5	0.1	1.000	1.000	0.675
3.5	1.5	0.1	1.000	1.000	0.678
5.0	1.5	0.3	1.000	0.999	0.482
4.0	2.0	0.4	1.000	0.995	0.389
0.5	2.5	0.2	1.000	1.000	0.581
5.0	2.5	0.3	1.000	0.487	1.000
5.0	2.5	0.4	0.993	1.000	0.414
2.5	3.0	0.4	1.000	0.995	0.415
4.0	3.0	0.4	1.000	0.994	0.405
4.5	3.0	0.3	1.000	1.000	0.480
0.5	3.5	0.5	1.000	0.983	0.335
2.0	3.5	0.3	1.000	0.999	0.484
2.0	4.0	0.2	1.000	1.000	0.586
2.5	4.0	0.1	1.000	1.000	0.677
3.0	4.0	0.1	1.000	1.000	0.672
5.0	4.0	0.4	1.000	0.996	0.407
2.5	4.5	0.4	1.000	0.994	0.408
4.0	4.5	0.2	1.000	1.000	0.580

Table 7.16: Observed Power at Various Effect Sizes. n=500, $P(y \leq DL)=0.3$

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
1.0	0.5	0.2	1.000	1.000	0.574
2.5	0.5	0.2	1.000	1.000	0.576
0.5	1.5	0.3	1.000	0.999	0.491
1.0	1.5	0.4	1.000	0.994	0.412
2.5	1.5	0.3	1.000	0.483	0.999
2.5	1.5	0.4	0.994	1.000	0.402
4.0	1.5	0.2	1.000	1.000	0.567
4.0	2.0	0.4	1.000	0.996	0.404
1.0	2.5	0.5	1.000	0.979	0.334
2.0	3.0	0.2	1.000	1.000	0.572
3.0	3.0	0.2	1.000	1.000	0.578
4.5	3.0	0.3	1.000	0.479	0.999
4.5	3.0	0.4	0.995	1.000	0.400
2.0	3.5	0.1	1.000	0.684	1.000
2.0	3.5	0.3	0.999	1.000	0.480
3.0	3.5	0.5	1.000	0.975	0.328
2.5	4.0	0.2	1.000	1.000	0.588
4.0	4.0	0.4	1.000	0.994	0.394
0.5	4.5	0.3	1.000	0.999	0.486
1.5	4.5	0.4	1.000	0.995	0.410
4.5	4.5	0.2	1.000	1.000	0.580
4.0	5.0	0.1	1.000	1.000	0.674

Table 7.17: Observed Power at Various Effect Sizes. n=500, $P(y \leq DL)=0.4$

Mu1	Sigma	p	LargeEffect	MediumEffect	SmallEffect
0.5	0.5	0.4	1.000	0.994	0.406
1.5	0.5	0.3	1.000	0.999	0.494
3.0	0.5	0.1	1.000	1.000	0.670
4.0	0.5	0.1	1.000	1.000	0.661
1.0	1.0	0.2	1.000	1.000	0.569
3.0	1.0	0.3	1.000	0.999	0.490
2.0	1.5	0.3	1.000	0.999	0.483
2.0	2.0	0.1	1.000	1.000	0.665
3.5	2.0	0.5	1.000	0.976	0.333
2.5	2.5	0.4	1.000	0.993	0.415
4.0	2.5	0.1	1.000	0.651	1.000
4.0	2.5	0.3	0.999	1.000	0.472
1.0	3.0	0.2	1.000	1.000	0.570
5.0	3.0	0.1	1.000	1.000	0.666
1.5	3.5	0.4	1.000	0.994	0.404
2.5	3.5	0.3	1.000	0.998	0.475
0.5	4.0	0.2	1.000	1.000	0.583
4.5	4.0	0.2	1.000	1.000	0.574
3.0	4.5	0.2	1.000	1.000	0.559
3.5	5.0	0.3	1.000	1.000	0.485
4.5	5.0	0.3	1.000	0.998	0.471
5.0	5.0	0.5	1.000	0.977	0.330

APPENDIX C
Sample R Codes

```
### Setting ###
library(knitr)
library(ggplot2)
library(xtable)
library(plyr)
library(dplyr)
library(NADA)
set.seed(19851109)
###  Demonstration of Data with Excess Zeros and DL  ###
para <- c(mu = 1, sigma = 1, p = 0.2, y_l = 1)
n <- 100
n0 <- rbinom(1, n, para[3])
y <- rlnorm(n-n0, para[1], para[2])
dat1 <- c(rep(0,n0), y)
m <- qplot(dat1, geom="histogram", binwidth = .2)
m_f <- m + geom_histogram(aes(fill = ..count..), binwidth = 0.2) +
                theme(axis.title.x = element_blank())
m_f + theme(legend.position = 'none', axis.text= element_text(size=18)
    ,
                axis.title=element_text(size = 18))

## (b) With detection limit.
y_l <- para[4]; y1 <- y
dat2 <- c(rep(0, n0 + length(y1[y1 < y_l])), y1[y1 >= y_l])
m2 <- qplot(dat2, geom="histogram", binwidth = .2)
m2_f <-m2 + geom_histogram(aes(fill = ..count..), binwidth = 0.2) +
        geom_vline(aes(xintercept=1),colour="#BB0000", linetype="
           dashed") +
        theme(axis.title.x = element_blank())
m2_f + theme(legend.position = 'none',axis.text= element_text(size=18)
```

```
    ,
                axis.title=element_text(size = 18))

###  See shape similarity  ###
mu1 = 0.5; sigma = 0.25;
shape_gamma = 1/(exp(sigma^2)-1); scale_gamma = exp(sigma^2/2)*(exp(
    sigma^2)-1)*exp(mu1)
scale_wbl <- 1/(exp(sigma^2)-1) * exp(sigma^2/2)*(exp(sigma^2)-1)*exp(
    mu1) / gamma(1+1/3.5)
ff <- ggplot(NULL, aes(x = x, linetype = g))
ff + stat_function(data=data.frame(x = c(0,10), g =factor(1)), fun =
    dlnorm,
                arg = list(meanlog = mu1, sdlog = sigma)) +
stat_function(data=data.frame(x = c(0,10), g =factor(2)), fun = dgamma
    ,
                arg = list(shape = shape_gamma, scale = scale_gamma))
                    +
stat_function(data=data.frame(x = c(0,10), g =factor(3)), fun =
    dweibull,
                arg = list(shape = 4.2, scale =scale_wbl-0.25)) +
                scale_linetype_manual(values=c(1,2,6),     labels = c(
                    "Log-normal", "gamma", "weibull"),
                name = "Distribution")

### Size calculation ###
### All Data are from Log-normal, model is lognormal ###
rm(list = ls())
# 1. give parameters under null hypothesis.
Mu <- seq(0.5,5,0.5); sigma <-seq(0.5,5,0.5)
p <- seq(0.1,0.5,0.1); k <- 3 crit_val <- qchisq(0.95, k - 1)
R=5000; n1<-n2<-n3<-100; perc <- 0.1 # probability under DL.
# Generate a matrix contains all combinations.
sim <- expand.grid(Mu1 = Mu, Mu2=Mu, Mu3=Mu, sigma = sigma, p = p)
sim.df <- sim[sim$Mu1==sim$Mu2 & sim$Mu2==sim$Mu3,]
y_L <- qnorm(perc, sim.df[,1], sim.df[, 4])
# This is the DL floating with various combination of Mu and Sigma
```

```
sim.yl <- cbind(sim.df, y_L = y_L)
sim <- unname(as.list(as.data.frame(t(sim.yl))), force=T)
get_result <- function(para_tru){
    m1_tru0 <- rbinom(1, n1, para_tru[5]) # this is number of true
        zeros
        m2_tru0 <- rbinom(1, n2, para_tru[5])
        m3_tru0 <- rbinom(1, n3, para_tru[5])
        y_L <- para_tru[6]
        x1 <- rlnorm(n1 - m1_tru0, para_tru[1], para_tru[4]); y1 <-
            log(x1)
        x2 <- rlnorm(n2 - m2_tru0, para_tru[2], para_tru[4]); y2 <-
            log(x2)
        x3 <- rlnorm(n3 - m3_tru0, para_tru[3], para_tru[4]); y3 <-
            log(x3)
        para_init <- c(median(y1), median(y2), median(y3), sd(c(y1, y2
            , y3)),
                          (m1_tru0 + m2_tru0 + m3_tru0)/ (n1 + n2 + n3
                             ))
        para_null <- c(mu = median(c(y1, y2, y3)), sigma = para_init[k
             + 1],
                 p = para_init[k + 2])
        y <- list(y1[y1 >= y_L], y2[y2 >= y_L], y3[y3 >= y_L])
        m1 <- m1_tru0 + sum(y1 < y_L)
        m2 <- m2_tru0 + sum(y2 < y_L)
        m3 <- m3_tru0 + sum(y3 < y_L)
# 3. find unrestricted max log-likelihood
log.like1 <- function(para_init, y){
        mu1 <- para_init[1]
        mu2 <- para_init[2]
        mu3 <- para_init[3]
        sigma <- para_init[4]
        p <- para_init[5]
        log.li1 <- m1 * log(p + (1 - p) * pnorm((y_L - mu1) / sigma))
            + (n1 - m1) *
                          log(1 - p) - (n1 - m1) * log(sigma) - (n1 -
                             m1) * log(sqrt(2 * pi)) -
```

```
                         sum((y[[1]] - mu1)^2) / 2 / sigma^2 +
                                   m2 * log(p + (1 - p) * pnorm((y_L
                              - mu2) / sigma)) +
                         (n2 - m2) *        log(1 - p) - (n2 - m2) *
                            log(sigma) - (n2 - m2) *
                         log(sqrt(2 * pi)) -        sum((y[[2]] - mu2
                            )^2) / 2 / sigma^2 +
                         m3 * log(p + (1 - p) * pnorm((y_L - mu3) /
                            sigma)) + (n3 - m3) *
                         log(1 - p) - (n3 - m3) * log(sigma) - (n3 -
                            m3) * log(sqrt(2 * pi)) -
                        sum((y[[3]] - mu3)^2) / 2 / sigma^2
return(-log.li1)
}
opt1 <- optim(para_init, log.like1, y = y, hessian= F,
                        lower = c(-Inf,-Inf,-Inf, 10^(-5), 10^(-5)),
           upper = c(Inf, Inf, Inf, Inf, 1 - 10^(-5)), method = "L-
              BFGS-B")
lrt1 <- -opt1$value
# 4. find max log-likelihood under null hypothesis.
log.like0 <- function(para_null, y){
        mu1 <- mu2 <- mu3 <- para_null[1]
        sigma <- para_null[2]
        p <- para_null[3]
        log.li0 <- m1 * log(p + (1 - p) * pnorm((y_L - mu1) / sigma))
           + (n1 - m1) *
               log(1 - p) - (n1 - m1) * log(sigma) - (n1 - m1) * log(
                  sqrt(2 * pi)) -
               sum((y[[1]] - mu1)^2) / 2 / sigma^2 + m2 * log(p + (1
                  - p) * pnorm((y_L - mu2) / sigma)) +
               (n2 - m2) * log(1 - p) - (n2 - m2) * log(sigma) - (n2
                  - m2) * log(sqrt(2 * pi)) -
               sum((y[[2]] - mu2)^2) / 2 / sigma^2 +        m3 * log(
                  p + (1 - p) * pnorm((y_L - mu3) / sigma)) +
               (n3 - m3) * log(1 - p) - (n3 - m3) * log(sigma) - (n3
                  - m3) * log(sqrt(2 * pi)) -
```

```
                sum((y[[3]] - mu3)^2) / 2 / sigma^2
return(-log.li0)
}
opt0 <- optim(para_null, log.like0, y = y, hessian = F, lower = c(-Inf
    , 10^(-5), 10^(-5)),
                upper = c(Inf, Inf, 1 - 10^(-5)), method = "L-BFGS-B")
lrt0 <--opt0$value
num <- (2 * (lrt1 - lrt0)) > crit_val
return(num)
}
get_size <- function(para_tru) mean(replicate(R,get_result(para_tru)))
res <- c(0, unlist(mclapply(sim, get_size, mc.cores = 16)))[-1]
sim.xtable <- cbind(sim.yl, obs_size=res)
sim.xtable <- arrange(sim.xtable, obs_size)
sim.dis <- xtable(sim.xtable, caption="Simulate␣5000␣times,␣prob<=y_L␣
    is␣0.1",
        digits=c(0,1,1,1,1,2,3,3))
print.xtable(sim.dis, type="html",file="simxtable.html")
save(sim.dis,sim.xtable,file="abc.RData")

### Graphics from Proposed Method and Combined Method ###
para_true <- c(Mu = 1.5, Sigma = 1, P = 0.3)   # parameter could be
    changed accordingly
n <- 100; perc <- 0.1
DL <- pnorm(perc, para_true[1],para_true[2])
max<- 1000
m.impt <- sd.impt <- m.combin <- sd.combin <- numeric(max)
loglike <- function(Mu, Sigma, P, y = y){
        log.li <- m * log(P + (1 - P) * pnorm(DL, Mu, Sigma)) + (n - m
            ) * log(1 - P) -(n - m) * log(Sigma) -
                (n - m) * log(sqrt(2 * pi)) -sum((y - Mu)^2) / 2 /
                    Sigma^2
return(-log.li)
}
max.iter <- 500
iter <- 0
```

```
tol <- 0.0001
logli_1 <- 0
logli_0 <- 1
for(i in 1:max){
        m00 <- rbinom(1, n, para_true[3])
        x <- rlnorm(n - m00, para_true[1], para_true[2])
        y_all <- log(x)
        y <- y_all[y_all >= DL]
        y_org <- y
        m <- m00 + sum(y_all < DL)
        Mu_hat0 <- mean(y)
        Sigma_hat0 <- sd(y)
        P_hat_0 <- m/n
        #2. get estimate of mean and sigma.
                repeat{
                        iter <- iter + 1
                        Mu_hat <- Mu_hat0
                        Sigma_hat <- Sigma_hat0
                        P_hat_plus <- pnorm(DL, Mu_hat, Sigma_hat)
                        P_hat <- P_hat_0
                        if(abs(logli_1 - logli_0)/ abs(logli_0)> tol &
                             iter < max.iter){
                                #3. impute values below DL.
                                n_to <- ceiling((n - m) * P_hat_plus /
                                     (1 - P_hat_plus))
                                n_to_impute <- min(c(n_to, m))
                                seat <- numeric(n_to_impute)
                                tt <- rnorm(1000, Mu_hat, Sigma_hat)
                                t <- tt[tt < DL]
                                seat <- t[1:n_to_impute]
                                logli_0 <- loglike(Mu_hat, Sigma_hat,
                                   P_hat_0, y)
                                y <- c(y_org, seat)
                                Mu_hat0 <- mean(y)
                                Sigma_hat0 <- sd(y)
                                P_hat_0 <- (n - length(y))/n
```

```
                                logli_1 <- loglike(Mu_hat0, Sigma_hat0
                                    , P_hat_0, y)
                        }
                else break
        }
### ROS and combined ###
        oo <- rep(F, length(y_org))
        uu <- rep(T, n_to_impute)
        y_test <- c(rep(DL, n_to_impute), y_org)
        cri <- c(uu, oo)
        myros = ros(y_test, cri)
        mean_ROS <- mean(myros)
        sd_ROS <- sd(myros)
#### Compare ###
        m.impt[i] <- Mu_hat0
        m.combin[i] <- mean_ROS
        sd.impt[i] <- Sigma_hat0
        sd.combin[i] <- sd_ROS
}
Mu_impt = mean(m.impt)
Mu_ros = mean(m.combin)
sd_impt = mean(sd.impt)
sd_ros = mean(sd.combin)
ddd <- data.frame(m.impt, m.combin,   index = seq(1, max, by = 1))
p1 <- ggplot(ddd, aes(index)) +
        geom_line(aes(y=m.impt,colour = 'm.impt')) +
        geom_line(aes(y=m.combin,colour = 'm.combin')) +
        geom_hline(yintercept = para_true[1]) +
        ggtitle("Mean comparison between Iterative Imputation \n and 
            combined method") +
        ylab('Means') +scale_shape_discrete(name="") + labs(fill="")
eee <- data.frame(sd.impt, sd.combin,   index = seq(1, max, by = 1))
p2<- ggplot(eee, aes(index)) +
        geom_line(aes(y=sd.impt, colour = 'sd.impt')) +
        geom_line(aes(y=sd.combin, colour = "sd.combin")) +
        geom_hline(yintercept = para_true[2]) +
```

```
        ggtitle('SD␣comparison␣between␣Iterative␣Imputation␣\n␣and␣
            combined␣method') +
        ylab('Standard␣Deviations')
### Draw multiple plots in ggplots, found online ###
multiplot <- function(..., plotlist=NULL, file, cols=1, layout=NULL) {
require(grid)
# Make a list from the ... arguments and plotlist
plots <- c(list(...), plotlist)
numPlots = length(plots)      # If layout is NULL, then use 'cols' to
    determine layout
if (is.null(layout)) {
# Make the panel
 # ncol: Number of columns of plots
 # nrow: Number of rows needed, calculated from # of cols
layout <- matrix(seq(1, cols * ceiling(numPlots/cols)),
                   ncol = cols, nrow = ceiling(numPlots/cols))
}
if (numPlots==1) {
print(plots[[1]])
 }
else {
# Set up the page
  grid.newpage()
 pushViewport(viewport(layout = grid.layout(nrow(layout), ncol(layout)
    )))
  # Make each plot, in the correct location
for (i in 1:numPlots) {       # Get the i,j matrix positions of the
    regions that contain this subplot       matchidx <- as.data.frame(
    which(layout == i, arr.ind = TRUE))
           print(plots[[i]], vp = viewport(layout.pos.row = matchidx$
               row,                                       layout.pos.
               col = matchidx$col))
                 }
        }
}
multiplot(p1, p2)
```

APPENDIX D
R Session Information

```
R version 3.1.1 (2014-07-10)
Platform: x86_64-pc-linux-gnu (64-bit)
locale:
[1] LC_CTYPE=en_US.UTF-8
[2] LC_NUMERIC=C
[3] LC_TIME=en_US.UTF-8
[4] LC_COLLATE=en_US.UTF-8
[5] LC_MONETARY=en_US.UTF-8
[6] LC_MESSAGES=en_US.UTF-8
[7] LC_PAPER=en_US.UTF-8
[8] LC_NAME=C
[9] LC_ADDRESS=C
[10] LC_TELEPHONE=C
[11] LC_MEASUREMENT=en_US.UTF-8
[12] LC_IDENTIFICATION=C
attached base packages:
[1] splines   stats     graphics  grDevices utils
[6] datasets  methods   base  parallel
other attached packages:
[1] knitr_1.6.18    dplyr_0.2       plyr_1.8.1
[4] NADA_1.5-6      survival_2.37-7 xtable_1.7-3
[7] ggplot2_1.0.0
loaded via a namespace (and not attached):
[1] assertthat_0.1   colorspace_1.2-4 digest_0.6.4
[4] evaluate_0.5.5   formatR_1.0      grid_3.1.1
[7] gtable_0.1.2     MASS_7.3-34      munsell_0.4.2
[10] parallel_3.1.1   proto_0.3-10     Rcpp_0.11.2
[13] reshape2_1.4     scales_0.2.4     stringr_0.6.2
[16] tools_3.1.1
```

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