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MODELING OF ENVIRONMENTALLY ASSISTED FATIGUE CRACK GROWTH
BEHAVIOR

by

Sree Phani Chandar Reddy

A dissertation submitted to the Graduate College
in partial fulfillment of the requirements
for the Degree of Doctor of Philosophy
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MODELING OF ENVIRONMENTALLY ASSISTED FATIGUE CRACK GROWTH BEHAVIOR

Sree Phani Chandar Reddy

Western Michigan University, 2015

The formation of fatigue cracks and their propagation due to cyclic loading in metals have been a concern for more than hundred years. Since fatigue failure were first reported by the railroad industry in 1840s, tremendous progress has been achieved in understanding fatigue behavior of metals. But fatigue damage is still a concern due to its complex dependency on various environmental variable like humidity, temperature, time and corrosive environment. Although numerous theories and models have been proposed in the past, the effects of environment on fatigue crack growth (FCG) is not completely understood. This dissertation aims to shed light on the effect of environment on the FCG behavior in metals through experimental techniques (Part-I) and modeling (Part-II). Using the available concepts in literature and the generated experimental data, a new fatigue model is proposed which accounts for thickness, stress ratio and environmental effects on FCG behavior.

The slow-strain-rate tests revealed that the effect of liquid-air interface on tensile behavior of Al 7075-T651 can be explained by a concept of chemical notch at the liquid-air interface. The FCG behavior of Al 7075-T651 alloy in air and in vacuum were investigated by using an earlier developed custom matrix method based on ΔK and K_{max} . The FCG data generated revealed the influence of stress ratio, (R), thickness and environment (air and vacuum) on FCG behavior. A new FCG model presented in Part-II includes

the environmental effect through change in crack tip angle. In this model the mechanical fatigue damage is based on the two parameter approach, ΔK and K_{max} . The model predictions of R-ratio, thickness and environment effects on FCG behavior in metals is compared with available data published in the literature and fairly good agreement was found.

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Chapter 1

Introduction

Mechanical components are designed to endure external applied loads during their service life. The goal of any component design is to achieve the purpose without mechanical failures. Metals tend to fail under excessive deformation or yielding. With the development of new alloys, the magnitude of deformation varies widely among metals. Materials that show large deformation before failure are said to be ductile in nature, while brittle materials show relatively low deformation as it is illustrated in, Figure 1.1 by neck formation respectively.

Figure 1.1: Schematic showing a cylindrical specimen with (a) Ductile behavior (b) Brittle behavior.

To design a component as part of an engineering structure that will be subjected to loads, it is very important to know how the material responds to these loads. Is it strong enough? Is it rigid enough to withstand the loads that will be experienced in

service? What is the maximum level of load the material can withstand before it shows unacceptable deformation?. These are some of the questions that need to be answered before a material is chosen to design the component. To answer these questions, engineers have developed a number of experimental techniques for mechanical testing of engineering materials. Among them, a tensile test is the simplest technique that would reveal the load-displacement behavior of the material.

1.1 Standard Tensile Testing

The tensile test is the most commonly used material testing technique due to its simplicity and ability to determine the most fundamental mechanical properties used in engineering design. The specimen used for such a test has usually cylindrical or rectangular cross section, which is approximately uniform over a gage length. Gage length is the region within which the elongation is measured. Figure 1.2 shows a schematic of cylindrical tension specimen, with gage length ' L_g '. The standard dimension for the specimen can be found in ASTM E8/ E8M-13a standard [1] and is dependent on the capabilities of the tension testing machine being used.

Figure 1.2: Schematic of a cylindrical tension specimen.

The tensile testing is carried out by subjecting the specimen to a gradually increasing uniaxial load until failure. This is accomplished by gripping the ends of the specimen in a tension testing machine, which gradually stretching the specimen until failure. A load cell which is attached to the test machine records the resultant load that is built-up in the material as the test machine elongates the specimen. An extensometer is attached to the specimen in the gage length region which records the elongation at the mid-section of

the gage length. Figure 1.3 shows a schematic of tension test setup of a tension specimen with an extensometer, mounted in a testing machine.

Figure 1.3: Schematic of a tension test setup.

The test specimen geometry and dimensions can vary for different laboratories resulting in different load levels and elongation measurement. To establish a standard data reporting practice, the results are reported in terms of engineering stress, σ and strain, ε . From the recorded load, stress is calculated as the ratio of load and initial cross-sectional area of the specimen, Eq. 1.1. And strain is calculated as the ratio of change in gage length over the initial gage length, Eq. 1.2. The calculated stresses are plotted versus strains, which produces the well know stress-strain curve, Figure 1.4.

$$\sigma = \frac{P}{A_i} \quad (1.1)$$

where σ is stress, P is load and A_i is initial cross-sectional area.

$$\varepsilon = \frac{\Delta L}{L_i} \quad (1.2)$$

where ε is strain, ΔL is change in gage length and L_i is initial gage length.

From the stress-strain curve, the fundamental material properties can be evaluated. The

maximum stress a material can withstand before failure is termed as ultimate strength, σ_u . The strain at fracture is termed as fracture strain, ε_f . The initial part of the stress-strain curve is considered to be elastic deformation or is said to undergo elastic strain, ε_e . A material subjected to stresses in the elastic region regains its original form when the applied stress is unloaded to zero. The slope of this linear part of the curve is termed as Young's modulus, which gives a relationship between stress and strain in elastic region, Eq. 1.3. The stress level at which the curve deviates from the linear part is termed as yielding or inelastic deformation. After the stress level crosses the yield strength, the material deforms permanently and is said to undergo plastic deformation. As a standard practice, the yield stress σ_o is determined by drawing a line parallel to elastic slope, E , using offset by 0.2 % of strain as shown in Figure 1.4.

$$\sigma = E\varepsilon \quad (1.3)$$

where σ is stress, ε is strain and E is Young's modulus

Figure 1.4: Schematic of stress vs strain graph showing ultimate strength and yield strength obtained from a tension test.

In general, when the a component is subjected to stresses below the yield strength, the material is said to deform elastically. That is, the material would regain its shape when

the applied stress level is returned to zero, in this process the material is said to have not changed its mechanical or physical characteristics. Usually it is assumed that no damage occurs when the applied stress is in the elastic region. But if a material is subjected to repeated stress below σ_o over a long period of time, would result in microscopic damage and finally failure of the component. The repeated stresses is called as cyclic stresses, Fig 1.5 and the microscopic damage is termed as fatigue of metals. Due to cyclic stresses which may be well below the yield strength of the material, microscopic cracks develop causing permanent damage to components. Fatigue in metals is considered dangerous in the design point of view due to its possible failure which may vary from few cycles to few million cycles.

Figure 1.5: Schematic showing cyclic stresses profile.

1.2 Introduction to Fatigue

Fatigue is the structural damage that occurs in a material when subjected to cyclic stresses that are below the ultimate tensile strength, or even yield strength of the material [2–4]. Fatigue failures were first experienced by the railroad industry in the 1840s, where railroad axes failed regularly at shoulders [5]. Wohler [6] in 1860s performed laboratory fatigue tests under cyclic stresses and introduced the concept of S-N diagram with fatigue limit. He established that fatigue life decreased with higher stress amplitudes, σ_a given by Eq. 1.4 and below a certain stress amplitude termed as fatigue endurance limit,

the test specimens did not fracture.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \quad (1.4)$$

A S-N curve is generated by subjecting specimens to constant cyclic stresses and the cycles to failure, N_f , are recorded. A series of such tests are performed for a range of stress amplitude levels and the results are plotted as stress amplitude, σ_a , versus cycles to failure, N_f . This plot is called as S-N diagram. Figure 1.6 shows a schematic of S-N diagram with inserts showing two levels of stress amplitude. The stress amplitude, σ_{a1} , is higher than σ_{a2} resulting in lower cycles to failure, N_{f1} compared to N_{f2} . In general, stress amplitude level at which a specimen does not fail for 10^7 cycles is accepted as fatigue endurance limit, Fig. 1.6.

Figure 1.6: Schematic showing cyclic stresses profile.

Since the discovery of fatigue damage, its analysis has been incorporated into component design and has currently become quite complex. The damage mechanisms have been studied extensively and a general fatigue process has been established [6–9].

1.2.1 Fatigue process

The fatigue process can be put in three stages. 1) crack initiation 2) crack propagation and 3) failure. The process starts with dislocation movement, eventually forming persistent slip bands that nucleate short cracks. The short cracks propagate (grow) and coalesce forming a long crack, which reduces the material's load bearing capacity. When the cracks reach a critical length (depending on the stress applied and geometry of the component), it results in fracture of the component [2–4]. Thus it was established that the fatigue analysis should include the effect of crack size. For this purpose, stress intensity factor (SIF), K , has been introduced, which takes into account the crack length while evaluating failure stress. SIF, K , would be further discussed in the later sections.

1.2.2 Fatigue life

Pre-existing flaws or sharp design features or notches may nearly eliminate the crack initiation stage of the fatigue process. For such components, the crack growth life might be a dominant portion of the total life of the component and this useful life is termed as fatigue life. An estimation of fatigue life helps in scheduling inspections in order to avoid an unexpected failure. Fatigue life can be estimated by knowing how fast a crack would grow per cycle, which is termed as the fatigue crack growth (FCG) rate, da/dN . The higher the crack growth rate, the shorter the life of the component and vice versa. The FCG rate is affected by the loading conditions, geometry and environment.

1.2.3 Loading conditions

Loading conditions describe the load or stress or stress intensity factor to which a component is subjected. Usually, the stress intensity factor, K , is defined by Eq. (1.5), as a crack driving force.

$$K = F\sigma\sqrt{\pi a} \quad (1.5)$$

where F is a geometry factor, σ is the applied stress and a is the crack length. Here we describe loading conditions in terms of loading variables such as maximum stress intensity factor, K_{max} , minimum stress intensity factor, K_{min} , stress intensity range, $\Delta K = K_{max} - K_{min}$, and stress ratio, $R = K_{min}/K_{max}$ as shown in Figure 1.7. These variables are inter-related according to Eq. (1.6). Specifying any two variables is sufficient to define the loading conditions.

$$\Delta K = K_{max}(1 - R) \quad (1.6)$$

Figure 1.7: Schematic showing loading cycles in terms of K .

In order to utilize the FCG approach in design, experimental tests need to be conducted to obtain material specific FCG data. The obtained FCG data are dependent on environment as engineering materials are subjected to different environmental conditions. This raises the question how does environment effect the material? How does the applied stress and environment combination contribute to material integrity? To understand the environment effect on crack growth behavior, researchers have investigated the effect of environment under constant load and under cyclic load conditions. These topics are further discussed in the next section.

1.3 Effect of Environment

Apart from loading conditions, environment plays a significant role in the fatigue behavior of metals. In general, most engineering components interact with air, humidity, water and salt water or a combination of very aggressive conditions like high temperature and acidic conditions. This results in degradation of materials due to corrosion and stress. The combination of applied load and exposure to corrosive environment leads to accelerated degradation in the strength of metals causing premature failure. The environment may influence the ductile properties of the metals, which can cause the metal to fail in a brittle manner, thus changing the material behavior to cracking [10]. Fracture of metals due to combination of constant stress and environment is termed as 'Stress Corrosion Cracking', SCC, or 'Environmentally Assisted Cracking', EAC. If the applied stress is cyclic, it is referred as 'Corrosion Fatigue', CF. The main difference between SCC and CF is the way loading conditions dominate in each case. In the presence of constant load, environment is known to play a determinantal effect. While in CF, the cyclic stresses and corrosive environment accelerate the cracking process and the mechanical fatigue damage [11].

1.3.1 Corrosion cracking under constant load

To evaluate a material's SCC behavior, identically pre-cracked specimens are and subjected to different constant loads while exposed to corrosive environment. Though the initial crack size is the same, depending on the magnitude of applied constant load, each specimen is subjected to different stress intensity factor, K . Since the load varies per specimen, the resulting time to failure among the specimens. The constant load is applied until fracture or until 10^4 hours if no failure is observed, as suggested by ASTM standard E1681 [12]. If the specimen is not fractured between 10^3 to 10^4 hours of exposure to corrosive environment, the corresponding SIF is considered as the stress corrosion cracking threshold SIF, K_{ISCC} . This indicates that under constant load, if the

SIF is below K_{ISCC} cracks do not grow. Figure 1.8 shows a schematic of SCC test results, where the plateau indicates K_{ISCC} . The fracture toughness, K_{IC} , in the figure implies that under constant applied load, the specimen does not fracture below this level in the absence of corrosive environment. Thus Fig. 1.8 indicates the behavior of metals under corrosive environment as this curve does not exist in its absence. Figure 1.8 shows that even under constant loads cracks do not extend if SIF is higher than K_{ISCC} . But if cyclic loads are considered, corrosion fatigue cracks tend to grow even if SIF is below K_{ISCC} .

Figure 1.8: Schematic showing effect of environment on SIF below K_{IC} .

1.3.2 Corrosion fatigue

Fatigue data in laboratory air are often used as a reference for comparing other environmental conditions. But laboratory air can also be considered as corrosive due to the presence of humidity and oxygen. To understand the relative fatigue behavior in corrosive environments, S-N diagram is generated for the respective environments and compared. Figure 1.9 shows a schematic of typical S-N diagrams obtained under three different environments. The S-N diagram for air shows only a small deviation from that of vacuum, even at long life. But this deviation may be substantial if the temperature

and humidity levels are high. However, for corrosive environment the S-N diagram shows large deviation from air or vacuum environments specifically at long fatigue life. Also, the S-N diagram for corrosion fatigue shows absence of the fatigue endurance limit, plateau at high fatigue life, which is present in air and vacuum. This suggests that, the combination of simultaneous environment and cyclic stresses causes the fatigue life to decrease continuously.

Figure 1.9: Schematic showing comparison of S-N diagram for vacuum, air and corrosive environment.

By generating the S-N diagram it is assuming that the material is free of cracks or internal flaws. But internal flaws are always present at microscopic level, thus the total life can be considered as FCG behavior influenced by environment. Even today the effects of cyclic stresses and electromechanical nature of corrosion on metals is not fully understood. The practical way to determine a material's susceptibility to corrosion assisted cracking is through experimental testing. In the last two decades, extensive investigation of test procedure resulted in the development of standard procedures and advanced techniques. The procedures are frequently reexamined for further improvement. The next chapter discusses the experimental techniques to evaluate environmental effect

on a material under monotonic (slow strain rate) and cyclic loading. A detailed review of recent developments and experimental results obtained in laboratory are also discussed.

Chapter 2

Part-I : Experimental Evaluation of Material Behavior

Experimental testing of metals in laboratories at controlled conditions is the first step in understanding a material's behavior under stress. The reason for controlled conditions is to isolate the material's behavior under specific conditions, so that materials properties and susceptibilities may be determined. Engineering materials may be exposed to a combination of various environments. Understanding the material's behavior would help in design and maintenance of the components. For this purpose extensive experimental techniques have been developed in the last 70 years [13]. Most procedures have been standardized mainly for exchange of data among laboratories. The most common and well established test procedure to evaluate the fundamental properties of metals, the tension test procedure has been widely used. The basic details of the procedure have been outlined in Chapter 1. The test procedure is relatively quick and simple, due to which it does not consider environmental effect. That is the material is not sufficiently exposed to environment. To understand the effect of environment on the fundamental properties of a material, slow strain rate (SSR) technique has been developed [14]. The idea behind a SSR test is to provide sufficient time for the environment to effect the material during tests, while the fundamental properties are evaluated. Similarly to understand the

materials FCG behavior due to fatigue, FCG data with environmental effect also needs to be experimentally determined. In this chapter, first the SSR test is considered to evaluate effect of environment on the stress-strain curve of a material, followed by evaluation of FCG behavior. Both SSR and FCG testing have been carried out in laboratory to evaluate material properties. The tests were carried out in accordance to standard procedure [13,14] and were designed to answer questions that were unanswered or were found to be ambiguous in the literature. The chapter may be considered as two sections, the first section deals with monotonically increasing SSR testing while the second section deals with FCG testing. Each section contains the corresponding standard test procedure, review of results from literature followed by tests carried out in laboratory and results obtained. Based on the finding from literature and tests results obtained in laboratory, phenomenological models were proposed and a final satisfactory model is presented in Chapter 3.

After careful consideration, aluminum 7075-T651 alloy was selected for experimental testing. Al 7075-T651 is a material that is used widely in engineering applications but showed high susceptibility to corrosion fatigue. Testing such a material in corrosive environment would mean faster material degradation. From the test data obtained the physical response of the material may be evaluated to understand the effect of environment. Al 7075-T651 is popularly used in the aircraft industry due to its high strength to weight ratio, which in-turn gives design flexibility and economic advantage over steel alloys. Test specimens were machined in WMU and the material was obtained from Schuppan & Sons as one inch strip cut from 6 inch thick plate. The chemical composition of Al 7075-T651 is presented in Table 2.1 and the tensile mechanical properties in air are presented in Table 2.2.

Table 2.1: Chemical composition of Al 7075 -T651 (max Wt. %).

Si	0.40	Fe	0.50
Cu	1.2-2.0	Mn	0.3
Mg	2.1-2.9	Others, total	0.15
Zn	5.1-6.1	Others, each	0.5
Ti	0.20	Balance	Aluminum.
Cr	0.18-0.28		

Table 2.2: Tensile mechanical properties of Al 7075 -T651.

Tensile Strength, σ_u	420 MPa
Yield Strength, σ_o	320 MPa
Young's Modulus, E	75 MPa
Elongation, ε_f	9 %

2.1 Literature Review of SSR Testing Technique

To determine a materials susceptibility to environmentally assisted degradation, ASTM standard suggests using SSR testing procedure. A uniaxial tension specimen is subjected to a slow constant rate elongation in the presence of aggressive environment until failure, while load and displacement are measured. The slow constant extension rate results almost constant slow strain rate in the gage length of the specimen, while the tests are carried out under displacement control. A schematic of SSR test setup is shown in Figure 2.1.

Figure 2.1: Schematic of Slow Strain Rate test setup.

Due to application of constant strain rate, the stress in the material increases monotonically throughout the test, while the presence of the environment influences the magnitude of resulting stress and strain to failure. By comparing the failure stress and strain levels in corrosive environment to that in laboratory air or vacuum, relative susceptibility of the environment on the material may be evaluated. A schematic of test results obtained from a SSR test procedure is shown in Fig. 2.2, where the stress-strain curve for standard tensile test in air shows higher, ultimate strength and elongation to failure (ϵ_f) compared to SSR testing in air or in corrosive environment.

Figure 2.2: Schematic of stress-strain curves produced from (a) Standard tensile test in air, (b) SSR test in air and (c) SSR test in corrosive environment.

The effect of environment in SSR test is manifested mostly with respect to the change

in ductility of the material. Under laboratory air, the percentage reduction in area of the fractured surface characterize the ductility of the material. A higher reduction in area indicate ductile behavior, while a lower reduction in area imply a brittle behavior. Figure 2.3 shows a schematic of reduction in area for various strain rates for mild steel in air and in $1MNaH_2PO_4$ [15], as strain rate decreases the percentage reduction in area decreases suggesting the material tends to show brittle behavior. Similar results were reported for aluminum alloys 7179 tested in tap water [16]. The SSR testing is influenced by the strain rate at which the tests are carried out and is dependent on the material/environment system. Too high strain rate would fail to capture the environmental effects on the material since the reaction of the material/environment system may proceed at a rate slower than the ductile cracking mechanism, while too low strain rates would result in impractically long test duration. Literature review of such SSR testing results strongly suggests that a wide variety of strain rates must be evaluated to determine the effect of strain rate on EAC behavior. The strain rate that captures the influence of environment within a reasonable testing time is usually found experimentally by trial and error and is called critical strain rate. This critical strain rate is material/environment dependent and specifically evaluated for each material/ environment system. For most common materials like steel, aluminum and other alloys the critical strain rate falls in the range of 10^{-4} to $10^{-7}/s^{-1}$, which is about from two to three orders of magnitude slower than a standard tensile test [10].

Lee et. al [17] carried out a series of SSR experiments to determine the effect of testing parameters and specimen orientation on Al 2024-T351 in 3.5% NaCl. Four strain rates were evaluated, 1×10^{-5} , 1×10^{-6} , 5×10^{-7} and 8×10^{-8} for the aluminum alloy for two loading orientations, short-transverse (ST) and longitudinal (L) directions. While both ST and L specimens showed susceptibility to NaCl, ST specimens showed the highest reduction in ductility compared to air. Comparing different strain rates, the test performed at a strain rate of $8 \times 10^{-8}/s^{-1}$ showed noticeable influence of environment on the material properties for ST specimen with approximately 70% reduction in tensile

Figure 2.3: Schematic of showing influence of Strain Rate in SSR test [15].

elongation compared to air, while only 10% reduction in area was reported for a strain rate of $1 \times 10^{-5}/s^{-1}$. These results indicate the existence of a critical strain rate for a given material/environment system. The fractographic examination shows the presence of corrosion pits which indicate intergranular corrosion (IGC) mechanism. Corrosion pits are specific sites on the material where the materials micro structure is first effected by environment. The formation of these sites is attributed to presence of dislocations or dissolution of material impurities present due to alloying. These corrosion pits act like stress raisers resulting in crack formation and leading to further stress corrosion, which is termed as IGC. In general aluminum alloys are reported to be sensitive to IGC, especially when small amounts of iron segregates to the grain boundary [10]. Under corrosive environment the iron around the grain boundary degrades resulting in formation of voids which leads to cracks along the grain boundary. As the cracks propagate the bulk of the material is exposed to environment. In the absence of stress, an oxide layer formed due to initial corrosion might protect the bulk material from further corrosion, but most engineering materials are always under some form of stress which would lead to rupture in the oxide layer. Thus in laboratory investigations, a slow strain rate is applied to induce monotonically increasing stresses in the material and to rupture the oxide layer for aggressive environmental effect. Usually, the oxide layer is brittle so increasing strain

will continuously fracture the brittle oxide layer. ST orientation specimens shows the most effect of corrosion on the material. Thus specimen with the ST orientation have been chosen for testing.

2.2 Experimental Testing of Smooth Specimen in Corrosive Environment

To investigate the environmental effect on engineering fracture strain, 7075-T651 Al samples were subjected to SSR testing. The tests were carried out in accordance with the procedure specified in the ASTM standard G129 [14]. A strain rate of $8 \times 10^{-7} \text{ sec}^{-1}$ was chosen from literature [17,18]. Specimens were monotonically loaded in air and corrosive environment at a constant elongation rate until failure. The corrosive environment used was a solution of 3.5% NaCl with pH of 2.5. The acidity of the solution was achieved by adding concentrated HCl to 3.5% NaCl solution. During the test, due to chemical reaction between 7075-T651 Al alloy and solution an oxide layer is formed on the specimen surface. The acidity of the solution helps to dissolve the oxide layer resulting in exposing a fresh metal surface to the corrosive solution during the test.

2.2.1 Specimen

Cylindrical dog-bone shaped specimens with a gage diameter of 12.5 mm were machined from a $6'' \times 1' \times 1''$ blanks cut from plate in the short transverse (ST) direction. The machining was carried out in-house. First the material was machined to a dimensions of $6'' \times 1'' \times 1''$ using a bandsaw machine, followed by turning of the bar into a cylindrical shape. This is further machined into a standard test specimen with 1'' gage length and approximately 3'' gripping length. Figure 2.4 shows schematic of the plate of material procured from Schuppan & Sons and machined into cylindrical ST oriented specimens.

Figure 2.4: Schematic of machining process for SSR testing specimen.

2.2.2 Corrosion chamber

A corrosion chamber was designed to hold the corrosion solution around the gage length of the specimen. The chamber was designed using plexi-glass so that it is transparent, leak proof and corrosion resistant. The schematic of the corrosion chamber is shown in Fig 2.5, with a center through hole to hold the specimen using O-rings.

Figure 2.5: Schematic of the corrosion chamber.

The top part of the chamber is open to air, which creates a liquid-air interface. The idea is to have the specimen exposed to both corrosive liquid and air simultaneously. Consider this as laboratory simulation of a ship in ocean where part of the ship is submerged in water while the upper part is above water interacting with air. This creates a liquid-air interface for the metal. The setup also simulates a hypothetical field situation when separate drops form on metal surface that creates a liquid-air interface. So the design of the chamber is such that part of the specimen gage length would be immersed in corrosion solution and a part of it is exposed to air, Fig. 2.6.

Figure 2.6: SSR test specimen along with corrosion chamber.

2.2.3 Test procedure

The specimen was first tested in air to create a bench mark for the material properties without the influence of corrosive environment. Next, the test was repeated with the half of the specimen gage length exposed to corrosive environment, Fig. 2.6. The corrosion solution is maintained such that the liquid-air interface is at the center of the gage length. Since the extensometer could not be mounted on the gage length of the specimen, the test was performed in displacement control such that the strain rate is maintained at 8×10^{-7} . This was achieved by utilizing the displacement vs time data recorded from strain controlled tests in air. Load, displacement and strain values were recorded using automated computer program. A uniaxial servo hydraulic MTS machine with a maximum load capacity of 80 *KN* and maximum actuator displacement of 100 *mm* was used for testing, Fig. 2.7.

Figure 2.7: Experimental setup of specimen in test machine with corrosive chamber at Fatigue and Fracture Laboratory, WMU.

2.2.4 SSR test results and discussion

From the test in air, the mechanical material properties are obtained and presented in Table 2.3.

Table 2.3: Mechanical Properties of Al 7075-T651 from laboratory testing in air

Tensile Strength, σ_u	420 MPa
Yield Strength, σ_o	315 MPa
Young's Modulus, E	75 GPa
Fracture Strain, ε_f	17.5%

Figure 2.8 shows comparison of material response for SSR test in laboratory air and corrosive solution. The yield strength and Young's modulus remain unaffected while the strain to fracture has reduced from 17.5% to about 2%. The reduction in strain values indicates that the material tends to show brittle behavior in the presence of corrosive environment.

Figure 2.8: Test results of SSR testing of Al 7075-T651 in air and 3.5% NaCl solution.

Examination of the specimen gage length showed that the specimen failed at the center of the gage length. This is expected as the center of the gage length is said to experience the uniform stress and strain. The specimen tested in NaCl solution failed at the center, which corresponds to the liquid-air interface. To confirm that the specimen failure was due to presence of liquid-air interface, two additional specimens were tested. For these two specimens the liquid-air interface was maintained such that, for one specimen the liquid -air interface was 0.25" below the center of the gage length, while the other above the center of the gage length. Figure 2.9 shows the three positions of the liquid-air interface tested. All the specimen failed at the liquid-air interface. Further microscopic observations of fracture surface, Fig. 2.10 showed two regions with contrasting surface colors and roughness. Observation of the regions indicated the outer region showed ductile like fracture (at 45°) while the inner region shows brittle fracture. The two distinctive regions indicate material failure under two different processes. Since no visible corrosion was found in the inner region, it is assumed that, only the circumferential material and

outer region were influenced by the corrosive environment.

Figure 2.9: Schematic of test setup (a) smooth specimen, (b) notch in air and (c) notch in solution.

Figure 2.10: Microscopic observation of fracture surface of Al 7075-T651 specimen tested in 3.5% NaCl & pH 2.5.

Although the inner region does not show any corrosion, the material could have been influenced such that its ductility is reduced. The corrosive solution could have influenced the circumferential material causing microscopic corrosion pits. These corrosion pits would act as notches that could act as stress risers or concentrators. Due to the

developed notches, stresses are redistributed along the gage length. And the notch-tip experiences higher stresses than any other part of the gage length. So now the question arises, can we estimate the stress concentration factor, k_t for the chemically formed notch? Stress concentration factor is a dimensionless quantity that describes the stress magnification in the presence of a notch when compared to unnotched specimens. The stress concentration factor, k_t is geometry dependent and is evaluated based on combination of notch dimensions along with specimen dimensions. A k_t of 1 is said to be for unnotched smooth specimen, higher k_t value means greater influence of notch resulting in greater stress concentration around the notch tip. To further analyze the "chemical notch" assumption, SSR tests were repeated on specimens with machined notches with different stress concentration factors (SCF). In the presence of a machined notch, the environmental effect can be considered to be dominating if the specimen fails at the liquid-air interface.

2.3 Experimental Testing of Notched Specimen in Corrosive Environment

Notches with different stress concentration factors (SCF) were machined on the gage section of smooth specimens. Stress concentration factor, k_t was varied by changing the notch-tip radius (ρ). A sharp notch-tip indicates higher k_t , which implies the specimen would fail at much lower nominal stress level. Two specimens per each k_t were machined. The position of the notch was about 0.25" from the center of gage length. The test setup was such that, one set of specimens were tested with the notches immersed in liquid while the second set of specimens were tested with the notch exposed to air. The idea behind this setup, is to investigate the specimen's fracture location. If the specimen fractured at the liquid air interface that would mean that the chemical notch is sharper than the machined notch. If the specimen fractured at the notch, the k_t of the machined notch is higher than the "chemical notch" of the interface. The test setup with three variants

is shown in Fig. 2.11, where only half of the gage length is immersed in liquid solution. The first setup shown in Fig. 2.11(a) is a smooth specimen with half the gage length immersed in the solution. The second setup depicted in Fig. 2.11(b) is a specimen with the circumferential notch in air and the lower half of the gage length immersed in the solution. The third setup in Figure 2.11(c) shows the circumferential notch in the lower half of the gage length immersed in the solution.

Figure 2.11: Schematic of test setup (a) smooth specimen, (b) notch in air and (c) notch in solution.

For the smooth specimen setup shown in Fig. 2.11(a), the failure location was always at the liquid-air interface with the failure strain, ε_f , ranging from 2% to 3%. It can be noted, that the failure strain ε_f of 17.5% was recorded for the SSRT on a smooth specimen conducted in air. Test results for specimens with notches are summarized in Figure 2.12 in terms of failure location with respect to a notch position and corresponding k_t values. The cross in Figure 2.12 indicate the notch is immersed in the liquid solution, while the circle indicates the notch is in air. All tests on notched specimen in air resulted in the failure strain ε_f between 2% and 3%, approximately the same as for liquid-air interface failures in smooth specimen. For the notched specimens with k_t less than 2, irrespective of the location of the notch in air or in solution, the failure location was at the liquid-air interface. That is, the liquid-air interface acted as a ‘chemical notch’ with an apparent

stress concentration factor higher than that created by the machined notch. On the other hand, for the notched specimens with k_t greater than 2.3, the specimens failed always at the notch location. This indicates that for k_t greater than 2.3, the effect of machined notch, located either in solution or air, was greater than the liquid-air interface effect.

Figure 2.12: Schematic of failure location with respect to k_t .

From these results it can be reasoned that the liquid air-interface acts as a ‘chemical notch’ with an apparent stress concentration factor associated presumably with a jump in surface energy of a solid at the liquid-air interface, Fig 2.13. If the stress concentration of the ‘chemical notch’ is higher than the stress concentration, k_t , of the physical notch in the specimen, the specimen would fail at the liquid-air interface and vice-versa.

Figure 2.13: Schematic of machined notch and chemical notch.

For a given notch root radius, ρ and notch depth, d , the actual k_t depends on the

notch-tip angle, 2θ . If the notch is exposed to liquid-air interface, the apparent angle 2θ changes. This is due to the fact that a small chemical notch would be introduced at the notch tip and changes the effective notch angle, Fig 2.14. With decrease in 2θ the value of k_t is increasing. Thus, the effect of aggressive environment may be considered as a notch effect with a specific stress concentration k_t , which would result in a significant reduction of failure strain in comparison to that of smooth specimen in an inert environment.

Figure 2.14: Schematic of a notch-tip exposed to liquid-air interface with chemical notch.

2.3.1 Conclusions from SSR test results

The effect of corrosive environment on the material behavior under stress can be analyzed as a formation of notch. The "chemical notch" would result in weakening the material strength around the liquid-air interface leading to fracture of the specimen with lower strain values compared to specimens in air. The environmental effect may be considered into analytical models as formation of chemical notch. The notch geometry and its k_t value depends on the environment and material of the specimen. Therefore for more aggressive material/environment system, cracks would grow faster due to fatigue. Combining the assumption of chemical notch and fatigue crack growth, a model to predict crack growth in different environments may be developed. But again as discussed in section 1.2, fatigue crack growth is influenced by various parameters and is evaluated through laboratory testing. The next section describes the standard practices to evaluate FCG behavior in different environments and also describes the experimental testing

carried out in Fatigue and Fracture Laboratory at Western Michigan University.

2.4 Literature Review of FCG Testing Techniques

2.4.1 ASTM standard procedure to evaluate FCG rate behavior : R-constant procedure

ASTM standard E647-05 [13] describes test procedures to generate fatigue crack growth rate data. The procedure starts with pre-cracking a specimen, that is the specimen in loading with a cyclic constant amplitude load until a desired crack extension from a notch is achieved, this part of the test is illustrated as 1 to 2 in Figure 2.15. ASTM standard describes the criteria for selecting the maximum load for pre-cracking. Pre-cracking is followed by load shedding procedure wherein the cyclic load is decreased continuously with increase in crack length such that a ASTM specified gradient is followed. Load shedding procedure, 2 to 3 in Figure 2.15 is carried out until threshold is achieved ($da/dN < 10^{-10} \text{m/cycle}$). After threshold is attained, the cyclic loading range and R-ratio are maintained constant till the specimen fractures, 3 to 4 in Figure 2.15. When the cyclic load is maintained constant, K_{max} and K_{min} changes as crack length increases, this part of the test generates the FGC rate data. Since the ratio of minimum and maximum load (or minimum stress over maximum stress) is maintained constant, the procedure is termed as R-constant procedure, where $R = \text{min. load}/\text{max. load}$ (or $= \text{min stress}/\text{max. stress}$).

Figure 2.15: Schematic showing R-constant test according to ASTM standard E647-05.

2.4.1.1 Representing obtained data

In general, FCG rate, da/dN , may be expressed as a function of crack tip stress intensity factor range, ΔK , for a given stress ratio, R . Expressing da/dN as a function of ΔK provides results that are independent of specimen geometry, thus enabling exchange and comparison of data obtained from a variety of specimens and loading conditions [2]. This relationship is represented as a curve on a da/dN versus ΔK plot and is called fatigue crack growth rate curve. In Figure 2.16 the curve 2-3 illustrates data obtained from load shedding and the curve 3-4 illustrates data obtained from the constant R-ratio cyclic loading. The fatigue crack growth rate curve as shown in Figure 2.16 from 3-4 can be divided into three regions: near threshold (low crack growth rate), the linear region (Paris region) and the fracture region (high crack growth rate) [2–4].

It is a common practice to express the middle region (see Figure 2.16 blue line) of the fatigue crack growth curve using the Paris equation, Eq. (2.1) [19].

$$\frac{da}{dN} = C(\Delta K)^m \quad (2.1)$$

where C and m are fitting parameters determined experimentally.

Figure 2.16: Schematic of da/dN vs ΔK curve generated from R-constant test.

The FCG curve for a specific stress ratio characterizes a material's resistance to stable crack extension under cyclic loading. Knowing the loading condition, these curves can be used to predict the life of the component. During an R-constant test the threshold, ΔK_{th} , is achieved when da/dN reaches a per-determined threshold value. In general, this value is chosen to be below 10^{-10} m/cycle as shown in Figure 2.16.

2.4.1.2 Shortcomings of R-constant test

The R-constant test requires one specimen for each stress ratio, resulting in testing multiple specimens at different stress ratios. As each specimen has to follow the above tedious procedure (tension pre-cracking, load shedding and constant load range procedure) these tests are rather long and can result in additional scatter due to multiple specimens used. While performing load shedding procedure, the load shall be decreased in small steps to avoid a load history effect on FCG in the threshold region. In addition, the threshold obtained from load shedding is affected by the rate at which load shedding is performed. In order to avoid these shortcomings associated with load shedding procedure a compression-compression pre-cracking method was adopted [20] and a testing matrix in terms of $\Delta K - K_{max}$ was used, which is discussed in the next section.

2.4.2 FCG rate testing using a single specimen

Researchers have been trying to develop new test procedures to generate da/dN vs ΔK data for different stress ratios using a single specimen and reduce the testing time. Tesch, Pippan and Docker [21] proposed a new testing procedure to determine da/dN vs ΔK curves for a wide range of stress ratios using a single specimen. In this procedure, K_{max} is maintained constant while ΔK is decreased continuously by gradually increasing K_{min} . As a result, the stress ratio is not constant but increases during the test while crack growth rate decreases. They reported that, by using successive K_{max} constant tests with stepwise increasing K_{max} , FCG rate data for a wide range of R-ratio conditions can be generated. Since the R-ratio is not maintained constant, the experimental data needs to be transformed and interpolated to obtain the FCG rate curves at constant R-ratios.

As part of the master's thesis, a custom matrix method to generate FCG rate data was developed, which does not need any transformation or interpolation [22]. The developed method is based on a two-parameter driving force approach [23–27], which utilizes a testing matrix in terms of K_{max} and ΔK values corresponding to predetermined R-ratios for which FCG data, da/dN , are recorded. In addition, the developed procedure took its idea of stepwise increasing K_{max} - constant test from Ref. [21]. The testing matrix procedure and associated advantages are described and discussed in the following section.

2.4.2.1 Advantage of using single testing method

The custom matrix method gives the advantage of choosing specific K_{max} and ΔK values that would produce FCG curves of desired R-ratios with minimum testing time. In contrast to the R-constant test where K_{max} is changed continuously, the advantage of keeping K_{max} constant is that, the magnitude of the monotonic plastic zone in-front of the crack tip is maintained constant [28]. Hence, this method gives the flexibility of changing ΔK in large steps. As only one specimen is used to generate FCG data for different R-ratios, the scatter associated with multiple specimens is reduced. In addition, the test time is reduced significantly, since the most consuming threshold testing for

different R-ratios is conducted on one but not multiple specimens.

2.4.2.2 Custom matrix testing method

The custom matrix testing method (developed in Fatigue and Fracture Laboratory) involves a series of K_{max} constant tests where ΔK is changed according to the testing matrix while K_{max} is maintained constant. The specimen is pre-cracked under compressive cyclic loading. The advantage of using compression pre-cracking is that the crack would self-arrest [3, 29] due to a decreasing crack-tip driving force, which cannot sustain crack propagation. Compressive pre-cracking results in formation of a tensile residual stress in-front of the crack tip [20, 29] and an initial crack extension of Δa_i is needed to grow a crack beyond this residual tensile stress field [29]. This crack extension is performed with an initial K_{maxi} less than the first K_{max} of the testing matrix. Data recorded for this initial crack extension may show “grow and stop” behavior. After the crack grows Δa_i beyond the region affected by residual tensile stress field as shown in the insert of Figure 2.17 the FCG is continuous using predetermined K_{max} and ΔK . A series of K_{max} constant tests are performed where ΔK is increased in steps. Each ΔK in the matrix is applied for a predetermined crack extension of Δa . Data is recorded and the next ΔK is applied. The same procedure is repeated for all the K_{max} values in the matrix.

Figure 2.17: (a) Schematic showing K_{max} constant test for ΔK decreasing and ΔK increasing procedure (b) Schematic of da/dN curve representing data obtained for initial crack extension.

2.4.2.3 Determination of $K_{max} - \Delta K$ testing matrix

The testing matrix consists of ΔK values of predetermined R-ratios for the desired/-chosen K_{max} . Hence, in order to construct the testing matrix, limits are set on K_{max} , ΔK and R to confine the testing matrix to a minimum number of ΔK values needed to generate the FCG data for predetermined R-ratios. The limits for ΔK and K_{max} are chosen based on approximate values for K_{IC} and ΔK_{TH} as illustrated in Figure 2.18.

Figure 2.18: Schematic showing area of testing matrix.

Fracture toughness, K_{IC} , can be used as an upper limit for K_{max} . As most of the loading conditions for engineering applications have $R > -1$, $2K_{IC}$ was set as the upper

limit for ΔK . Subsequently, an area of interest in terms of ΔK and K_{max} is shown in Figure 2.18, which corresponds to any $R > -1$. Similarly to include a lower load ratio such as $R = -3$ a corresponding stress ratio line is introduced in the $\Delta K - K_{max}$ plot thus creating the desired area of interest. From the area of interest, K_{max} and ΔK values are selected for the predetermined stress ratios. Thus, obtained values can be represented on a $\Delta K - K_{max}$ plot as vertical lines corresponding to constant K_{max} values and each vertical line is called a bin (Figure 2.19).

Figure 2.19: Graph showing K_{max} constant lines (vertical lines) called Bins generated from area of interest shown in Figure 2.18. Circles in the Figure 2.18 represent ΔK values for predetermined stress ratios.

The obtained bins can be represented numerically as a $K_{max} - \Delta K$ test matrix for the predetermined R-ratios, where each column represents a K_{max} constant test. Table 2.4 shows the testing matrix generated for stress ratios -1, -0.5, 0.1, upto 0.95.

Table 2.4: Test data for 3 mm thickness Al 7075-T651 ST in vacuum.

	$\Delta K [MPa\ m^{0.5}]$											
Kmax	3.75	4	5	6	8	10	12	16	20	26	31	36
R = -1	37.8	8	10	12	16	20	24	32	40	52	62	72
R = -0.5	5.6	6	7.5	9	12	15	18	24	30	39	46.5	54
R = 0	3.7	4	5	6	8	10	12	16	20	26	31	36
R = 0.1	3.3	3.6	4.5	5.4	7.2	9	10.8	14.4	18	23.4	27.9	32.4
R = 0.2	3	3.2	4	4.8	6.4	8	9.6	12.8	16	20.8	24.8	28.8
R = 0.3	2.6	2.8	3.5	4.2	5.6	7	8.4	11.2	14	18.2	21.7	25.2
R = 0.4	2.25	2.4	3	3.6	4.8	6	7.2	9.6	12	15.6	18.6	21.6
R = 0.5	1.87	2	2.5	3	4	5	6	8	10	13	15.5	18
R = 0.6	1.5	1.6	2	2.4	3.2	4	4.8	6.4	8	10.4	12.4	14.4
R = 0.7	1.12	1.2	1.5	1.8	2.4	3	3.6	4.8	6	7.8	9.3	10.8
R = 0.8	NA	NA	1	1.2	1.6	2	2.4	3.2	4	5.2	6.2	7.2
R = 0.9	NA	NA	NA	NA	NA	1	1.2	1.6	2	2.6	3.1	3.6
R = 0.95	NA	NA	NA	NA	NA	NA	NA	NA	1	1.3	1.55	1.8

This testing matrix is also called as crack propagation table (CPT). Each bin starts with a highest value of ΔK and ends with lowest value of ΔK . That is we move from the top of the bin to the bottom of the bin. At threshold, data is recorded and test moves to next test point when one of the following condition is achieved, FCG rate reaches threshold ($da/dN < 10^{-10}\ m/cycle$) or number of cycles per test point reaches 1 million or the required crack extension is achieved. When the current bin is completed the test is moved to the next bin with a higher K_{max} . Figure 2.20 shows flow chart for the test procedure.

Figure 2.20: Flow chart showing test procedure.

2.4.3 Discussion on FCG rate test data

2.4.3.1 R-ratio effect on FCG behavior

Generally, it has been observed that FCG rate increases with increase in stress ratio, R . Figure 2.21 shows FCG rate data for Al 2324 for stress ratio varying from 0.9 to -1 [30]. At a given ΔK of $10MPa\sqrt{m}$, FCG rates increase with increasing in R -ratios. For FCG threshold of approximately 10^{-11} m/cycle, ΔK_{th} (and $K_{max\ th} = \Delta K/(1 - R)$) values decreases with increase in stress ratio. Table 2.5, shows a comparison of ΔK and K_{max} values required to achieve a FCG rate (da/dN) of 10^{-10} m/cycle for various R -ratios. With increase in R -ratios, the K_{max} and ΔK values show a clear trend of decrease in stress intensity factor needed for fatigue crack propagation. Apart from R -ratio, FCG data is also effected thickness and environment.

Figure 2.21: FCG rate data for Al 2324 at various stress ratios, R [30].

Table 2.5: ΔK and K_{max} values for various stress ratios needed for FCG rate, $da/dN = 10^{-10}$ m/cycle [31].

R	$K_{max}[MPa\sqrt{m}]$	$\Delta K[MPa\sqrt{m}]$
0.1	6.47	5.82
0.3	6.77	4.74
0.5	8.66	4.33
0.8	18.6	3.72
0.85	26.5	3.85
0.90	36.5	$\tilde{3}.6$

2.4.3.2 Thickness effect on FCG behavior

The FCG rate dependence on the thickness of the specimen has been a topic with contradictory results reported in the literature. Some studies show that FCG rate is independent or shows very small dependency on the thickness. Thick specimens are considered to be under plane strain condition, that is the size of plastic zone in-front of the crack tip is small in comparison to the thickness of the specimen. For thin specimen plane stress condition prevails, that is the size of plastic zone in-front of the crack tip is comparable to thickness of the specimen [32]. Thus for thin specimen the normal stress in front of the crack tip through the thickness is approximated as zero, $\sigma_{zz} = 0$, while a non zero stress exist, through the thickness in thick specimen. The presence of additional stress in the thickness of thick specimen is considered to be one of the reasons for higher FCG rate compared to thin specimen. Also, it has been proposed that the effect of thickness on FCG rate is influenced by the size and shape of the plastic zone in-front of the crack tip. The plastic zone estimate by Irwin [33] shows that, the plastic zone size for plane stress is 3 times larger than for plane strain. Larger plastic zone in plane stress results in higher residual stresses after unloading, which is opposite for plane strain specimen. Many results on FCG published in the literature show inconsistent trends. For instance, FCG rate data for high-strength maraging steel [32] in laboratory air does not show any effect of thickness. But the FCG rate data presented ranged between 10^{-7} and 10^{-4} m/cycle, indicating that the presented FCG rate data could belong to the Paris region. Also results for mild steel [32] with various thickness ($t=3.25, 7.62$ and 25.4mm) indicate no effect of thickness on the FCG rate. Park et.al. [34] reported a small variance in the threshold for FCG rate of 304 steel. They reported that a specimen with 25 mm thickness showed a slightly higher FCG rate compared to a 3 mm thick specimen. But again the data reported tend to fall in the Paris region. Matos et.al. [35] published experimental results for FCG rate on aluminum alloy 6082-T6. The FCG rate data for three different specimen thicknesses (3, 10 and 25 mm) were generated. The results indicate that, FCG rate for thick specimen is slightly higher compared to

the other two thinner specimen. Enough data were not reported below FCG rate of 10^{-7} m/cycle to understand the thickness effect in the near threshold region. In view of the above discussion we can conclude that thickness effect could be dominant in the threshold region that is FCG rate below 10^{-8} m/cycle , but more experimental data were needed to better understand the plane stress versus plane strain FCG behavior.

2.4.3.3 Environmental effects on FCG behavior: Macroscopic observations

The FCG behavior of aluminum alloys is well known to be influenced by environment. High humidity or corrosive environments show accelerated FCG compared to fatigue crack growth in laboratory air. Fatigue crack growth tests performed in vacuum, where very low or no humidity is present shows lower crack growth rate for the same ΔK and exhibit higher threshold stress intensity factor ΔK_{th} [4] compared to tests in air. Which means a fatigue crack may initiate and grow at relatively low load levels in humid conditions. This is demonstrated in Figure 2.22 where ΔK_{th1} for NaCl is lower than ΔK_{th2} for air which is lower than ΔK_{th3} for vacuum [36]. In particular, water vapor pressure is attributed to be the primary cause which influences the FCG [37].

Figure 2.22: Fatigue crack growth rate data for aluminum 7075-T6 under different environments [38].

Hartman [39] concluded that hydrogen gas diffuses into the region ahead of crack tip which results in faster FCG rates. Several hypotheses have been proposed which support the idea of hydrogen embrittlement, where hydrogen gas is produced by the surface reaction between the metal and water vapor. The hydrogen gas diffuses into the crack tip grain boundaries and embrittles the material, thus reducing the number of cycles before a brittle crack extension. Although hydrogen embrittlement is widely accepted as the dominant mechanism for corrosive environments, other models have been suggested with the common agreement that the presence of water vapor is the primary cause for accelerated FCG. Ruiz and Elices [37] showed that by maintaining the same frequency while increasing the vapor pressure resulted in increasing the FCG for the same load levels [37, 40]. Petit et al. [41] proposed a two stage mechanism model where hydrogen embrittlement is suggested to be the primary mechanism below a critical FCG rate, while adsorption of water vapor molecules is suggested to be the primary mechanism above the critical FCG rate region. Hirose et al. [42] published SCC data at 4340 steel in three environments, distilled water, 3% NaCl and 0.1N H_2SO_4 . For all the environments, the stress level to produce a detectable crack is plotted against duration of exposure. The data in Fig. 2.23, indicate the material behavior to be identical in all three environments. But when their data are compared to SCC data for pure gaseous hydrogen environment, the material shows higher resistance to SCC in pure hydrogen environment. This indicates that hydrogen embrittlement is not the only process which effects the SCC behavior of the material.

Vehoff and Rothe [43] reported that hydrogen embrittlement in FeSi- and Ni-single crystals is effected by hydrogen pressure and temperature. They proposed that a crack extension process is due to formation of cleavage due to hydrogen embrittlement and ductile fracture due to plastic slip. At constant temperature, the crack tip angle decreased with increase in hydrogen pressure, that is the crack tip is sharper for higher hydrogen pressure compared to low hydrogen pressure. Sharper crack tip indicated faster crack propagation, that is crack propagation increases with increase in hydrogen pressure. Also

Figure 2.23: Incubation time vs apparent stress intensity for crack initiation in 4340 steels.

the surface roughness increased with decrease in hydrogen pressure, indicating change in fracture process with respect to hydrogen pressure. If pressure and temperature are varied such that the crack tip angle is maintained constant, the fracture surface was reported to have similar roughness. They concluded that crack tip angle is a useful local measure of the fracture process and is constant as long as the fracture process does not change. Also when small amounts of oxygen was introduced, the crack tip angle was reported to increase resulting in crack tip blunting. It is postulated that the introduced oxygen could have blocked the hydrogen from diffusing into the crack tip which results in shielding the crack tip from hydrogen embrittlement.

For the FCG rate reported in the literature, it has been observed that the difference in crack growth is mostly in the low crack growth region. Some researchers have reported FCG rate data in different environments, which converge at high FCG rates while showing distinct behavior in the low FCG rate region. Vasudevan and co-authors [36, 44] published results for Ti-6Al-4V which shows this convergence for stress ratio 0.1. This

may be attributed to prolonged exposure of crack front to environment in the low crack growth region compared to exposure time in the high crack growth region. Also results have been published where FCG rate curves do not converge even at high FCGR region, rather they exhibit a parallel shift when plotted as a da/dN vs ΔK curves [44]. The FCG behavior under cyclic loading is typically characterized into three types, Figure 2.24. In Type-A behavior, the effect of corrosion is time dependent process. Crack propagation is observed even below K_{ISCC} , where stress corrosion cracking is absent in static loading. The crack propagation mechanism is dominated by the duration of exposure to environment due to low cyclic loading. For this type of material/environment system, the FCG curves converge at higher FCG rates. In Type-B behavior, the effect of corrosion is stress dominated. Below K_{ISCC} , the FCG curve for corrosive environment follows the inert environment suggesting absence of corrosion effect. Above K_{ISCC} , stress corrosion cracking is observed with a sudden jump in FCG rate when the applied K_{max} is equal to K_{ISCC} . Type-C behavior is a combination of Type-A and Type-B, where the fatigue process is both time and stress dependent.

Figure 2.24: Schematic representation of three types of FCG behavior in aggressive environment [38].

Under vacuum conditions, some studies have reported re-welding of the crack surfaces. Re-welding of the crack surfaces has been widely suggested to occur in the absence of water vapor [45]. Scarlin further confirmed that oxidation of the crack tip prevents re-welding during unloading phases. Most tests performed in vacuum relied on the optical measurement of crack tip. This method only measures the surface crack length, which is usually different from the crack length in the interior of specimen [46]. Microscopic pictures of the crack suggest re-welding process in the presence of a kink i.e if the crack surface exhibits high roughness, the fracture surfaces would come in contact during the unloading cycle [45–47]. However in the presence of water vapor the surface is oxidized which prevents re-welding. While in air or corrosive environments, it is suggested that the presence of oxide layer can be explained by crack closure concept, the concept itself is widely debated. Vasudevan and Sadanandan [38] proposed that FCG may be explained using two parameters ΔK and K_{max} . They concluded that, due to the lack of a comprehensive data set for different stress ratios and environmental conditions the FCG behavior is only partially understood.

2.5 Experimental Testing of Fracture Mechanics Specimen

Experiments were performed using 7075-T651 Al alloy in ST direction (yield strength 370 MPa, ultimate strength 475 MPa, and elongation at break 9%). Compact tension specimens were machined according to ASTM standards with a notch machined through EDM, Figure 2.25. All the specimens were machined at WMU and EDM cut performed at KVCC. The dimensions of the specimen used are $T = 12.50$ mm and 3.00 mm, $W = 55$ mm and an initial crack length of approximately $a_0 = 12.50$ mm.

Figure 2.25: Schematic of compact-tension (CT) specimen.

A computer controlled MTS 810 testing machine with Automated Fatigue Crack Growth Testing software (AFCGT) - series 2001 was used. Tests in air are conducted at ambient temperature and in laboratory air. Tests in vacuum are conducted at a vacuum pressure of $< 10^{-9}$ Torr and the molecular composition of the chamber is monitored continuously using mass spectrometer. Automated crack length measurement is done using the DC Potential Drop technique (DCPD) for tests in air and vacuum. In addition, crack length is measured optically using a traveling light microscope at regular intervals of a crack extension (< 5 mm) to calibrate the DCPD crack length measurements for test in air and vacuum.

From research analysis during master's, it was found that previous loading condition influenced the FCGR due to the presence of plastic zone in-front of crack tip (detailed analysis of plastic zone and its influence on FCGR data is presented in [48]). To avoid the plastic zone effects, a crack extension (0.2 mm) criteria should be met for every change in loading condition before valid FCGR data is recorded. At threshold, if the required

crack increment of 0.2 mm was not achieved, data is recorded every 5×10^5 cycles. Initial testing showed that generating FCGR data while meeting the crack extension criteria for two stress ratios ($R=0.1$ and 0.5) required about two months of total testing time. Due to the time and geometric constraint testing has been restrained to two stress ratios, two thicknesses in two environments, air and vacuum.

2.5.1 FCG rate in ambient air

Figure 2.26 and Figure 2.27 shows FCG rate data obtained for stress ratio, $R=0.1$ and $R=0.5$ respectively. For both stress ratios, the FCG rate at threshold shows significant difference. As discussed in the literature review, most of the data reported in literature belonged to the Paris region where only a slight variance of FCG rate was reported. Current tests were carefully designed to generated FCG rate data near threshold and in Paris region. Figure. 2.26 and Fig. 2.27 show that thickness of the specimen plays a significant role in the threshold region, while the effects of thickness diminish as we enter the Paris region. The ΔK_{th} for the thin specimen is higher when compared to a thicker specimen.

Figure 2.26: Fatigue crack growth rate data generated for Al 7075-T6 for R=0.1 in laboratory air.

Figure 2.27: Fatigue crack growth rate data generated for Al 7075-T6 for R=0.5 in laboratory air.

2.5.2 FCG rate in high vacuum

Figure 2.28 and Fig. 2.29 show comparison of experimental data generated for a 12.5 mm thick specimen in ambient air and high vacuum for $R=0.1$ and $R=0.5$ respectively. For both the stress ratios, FCG rate in vacuum is lower than compared to air. Additionally, the FCG rate curves for vacuum show higher threshold ΔK compared to the results in air, but the FCG rate curves tend to merge in the Paris region ($da/dN > 10^{-7}$). This indicates that, the material exhibits higher resistance at the threshold to crack propagation in vacuum. When compared to air, FCG rate curves in vacuum show steeper slope between 10^{-9} to 10^{-7} *mm/cycle*. In case of thin specimen, FCG rate data for air and vacuum coincide indicating absence of environment effect, Fig. 2.30 and Fig. 2.31. The thickness effect is absent under vacuum conditions for $R=0.5$, Figure 2.33, both 12.5 mm and 3 mm specimen thickness data fall in a narrow scatter band. But for $R=0.1$, FCG rate data below $\Delta K = 10$ [$MPa\sqrt{m}$]^{0.5} shows upto an order of magnitude difference in FCG rate, Figure 2.32.

Figure 2.28: FCG rate data generated for 12.5 mm thickness Al 7075-T6 for $R=0.1$ in laboratory air and vacuum.

Figure 2.29: FCG rate data generated for 12.5 mm thickness Al 7075-T6 for R=0.5 in laboratory air and vacuum.

Figure 2.30: FCG rate data generated for 3 mm thickness Al 7075-T6 for R=0.1 in laboratory air and vacuum.

Figure 2.31: FCG rate data generated for 3 mm thickness Al 7075-T6 for R=0.5 in laboratory air and vacuum.

Figure 2.32: FCG rate data generated for Al 7075-T6 for R=0.1 with 12.5 mm and 3 mm thickness in vacuum.

Figure 2.33: FCG rate data generated for Al 7075-T6 for R=0.5 with 12.5 mm and 3 mm thickness in vacuum.

2.5.2.1 Conclusions of FCG tests in air and vacuum

FCG rate tests in air and vacuum showed influence of stress ratio on FCG behavior. The general trend of increase in FCG rate with increase in stress ratio is verified for Al 7075 alloy. At constant stress ratio, FCG data in both air and vacuum show higher crack growth rate in thick specimen compared to thin specimens. The thickness effect is observed to be significant mostly in the near-threshold region, da/dN less than 10^{-9} m/cycle. In case of thin specimen, the FCG rate data show very minimal effect of environment.

2.6 Conclusion : Part-I : Experimental Results

SSR testing of aluminum 7075-T651 alloys in 3.5% NaCl solution resulted in the hypothesis that a "chemical notch" is formed at the liquid-air interface. To estimate the SCF for the chemical notch, specimens with varying notch radii were machined and tested. Using the test results, the k_t of the "chemical notch" is approximated as 2.1.

From the FCG rate test data, the effect of stress ratio, thickness and environment have been established. Thick specimens show higher FCG rate compare to thin specimens. The threshold SIF range for thick specimen is lower than that for thin specimen. Further, the FCG data in air show lower threshold values and higher crack growth rate when compared to vacuum. Thus, using the experimental findings we can conclude that environmental effect may be considered as an apparent notch effect while the stress ratio and thickness effect on FCG rate behavior is due to shifting of the threshold SIF range.

Chapter 3

Part-II : Modeling of FCG Rate

Behavior with Environmental Effects

3.1 Literature Review on the Mechanical Aspect of FCG Models

Fatigue crack growth formulation has been an area of interest for researchers for the last 60 years. The main idea of formulation is to develop mathematical representation that depicts the FCG behavior of the material under consideration. In 1960s, Paris was the first to postulate the famous Paris Law that correlates FCG rate and SIF range. The correlation is presented as a power law as in Eq. 3.1.

$$\frac{da}{dN} = C\Delta K^m \quad (3.1)$$

where C and m are material dependent parameters fitted from the experimental results. The Paris equation represents FCG rate data for a specific stress ratio, R , and shows a linear relationship on a log-log plot of da/dN versus ΔK plot. With change in stress ratio the fitting parameter need to be adjusted to represent the corresponding FCG rate data. Dugdale in 1961 suggested the presence of plastic or process zone in-front of the

crack tip, which influences crack growth. He presented an expression, Eq 3.2, to calculate the extent of this process zone. Dugdale's process zone is influenced by the max stress intensity factor (SIF) and is called a monotonic or forward plastic zone. The process zone was further investigated and in 1967, Rice proposed the existence of cyclic process zone due to SIF range, Eq. 3.3

$$r_f = \frac{\pi}{8} \left(\frac{K}{\sigma_y} \right)^2 \quad (3.2)$$

$$\Delta r_p = \frac{1}{\pi} \left(\frac{\Delta K}{2\sigma_y} \right)^2 \quad (3.3)$$

In 1970s, almost 10 years after Paris law was postulated, Walker [28], Elber [49], Willenborg et al. [50] and Wheeler [51] independently proposed different models that accounted for the load ratio effects. From this time in history three branches of FCG behavior philosophies have emerged. One in the line of the model proposed by Walker, the second in the line of Elber's crack closure model and the third with residual stresses of Willenborg and Wheeler. Walker suggested that the fatigue damage is influenced by both the maximum stress and the cyclic stress given by Eq. (3.4) to correlate the effects of R-ratio on fatigue life for 7075-T6 and 2024-T3 Al alloys.

$$\sigma_W = \sigma_{max}^{(1-m)} \Delta \sigma^m \quad (3.4)$$

where m was assumed to be a material fitting parameter. Eq. (3.4) can be modified as Eq. (3.5) for fatigue crack growth correlation as follows:

$$K_W = K_{max}^{(1-m)} \Delta K^m \quad (3.5)$$

where K_W is the effective SIF proposed by Walker. However, experimental results indicate

that Eq. (3.5) correlates the data only for positive R-ratios.

Elber [49] in 1970, introduced the concept of crack closure to FCG analysis. He proposed an effective SIF range ΔK_{eff} (Eq. 3.6) instead of ΔK applied to account for the crack closure concept. Following this concept a number of semi-empirical models [52–55] and strip yield models [56,57] have been developed, mainly for variable amplitude loading.

$$K_{eff} = K_{max} - K_{op} \quad (3.6)$$

where K_{op} is the stress intensity value for the crack opening load. The above equation implies that FCG driving force is affected only by the load range between the maximum load and crack tip opening load. Crack closure is the physical contact of the crack flanks during unloading part of the cycle when $P > 0$. During reloading, crack opens only after crossing a certain load level during the loading part of the cycle. The model suggests that fatigue damage of the material in the immediate vicinity of the crack tip is generated only after the crack is fully opened. The reduction in driving force is accounted by K_{op} in Eq. 3.6, which is associated with the load required to fully open a crack. However, McClung [58] and Donald [59] showed that fatigue crack growth rate is not determined solely by K_{max} and K_{op} . Various semi-empirical crack closure models were evaluated by comparing the experimental results with the predictions from the models. Each model shows limitations with respect to crack advance size and maximum compressive load. While K_{op} in Elber's model is measured experimentally, the material constant 'm' in Walker's model is a fitting parameter. Paris law, Walker model and crack closure were developed based on constant amplitude loading, where the maximum load and load range are constant. To account for variable amplitude loading where the maximum load (or K_{max}) and load range (or ΔK) may change for each loading cycle, fatigue community developed numerous models. These models derive their fundamentals from either Walker model or Elber model. Elber model received wide attention which may be due to its physical aspect of measuring reduction in driving force.

Willenborg et. al. [50] and Wheeler [51] proposed FCG models based on residual stresses created in the plastic zone in order to reproduce the effects of overload on FCG rate. Both models propose a retardation in FCG rate if current plastic zone size is within the previously created plastic zone size due to overload and associated compressive residual stresses. The Wheeler model achieves this by the introduction of a retardation factor that must be derived empirically for a given material and loading conditions. Willenborg model proposed that the retardation in the FCG rate is due to the presence of residual compressive stresses in the plastic zone due to the previous overload. Thus, the applied SIF of the current cycle is reduced, Gallagher [60] proposed Eq. (3.7) to calculate the retardation SIF (K_{red})

$$K_{red} = K_{max}^{OL} \left(1 - \frac{a - a_{OL}}{r_{pOL}} \right)^{\frac{1}{2}} - K_{max} \quad (3.7)$$

where K_{max}^{OL} is the maximum SIF corresponding to the overload cycle.

Three different versions of the Willenborg model exist, the generalized model proposed by Gallagher and Hughes [61], the modified generalized model proposed by Brussat [52] and the Walker-Chang-Willenborg model by Walker and Chang [62]. Each model was specifically developed to address overload or underload effects. The Willenborg and the generalized Willenborg models both produce satisfactory results for overload effects [63]. The modified generalized model and Walker-Chang model provides improved accuracy for loading conditions with large compressive loads [64].

In 1995, Sadanandan and Vasudevan [25] developed a two parameter approach which suggests that any FCG rate behavior may be explained as the interplay of maximum SIF and SIF range. They suggested the representation of L-shaped curves developed when constant da/dN lines are plotted on a ΔK versus K_{max} plot, Fig. 3.1. They recognized that a crack propagates only after the applied loads cross both threshold values corresponding to max. SIF and SIF range, which are represented as $K_{max\ th}^*$ and ΔK_{th}^* respectively. These threshold values are the limiting values for $da/dN \leq 10^{-10}$ m/cycle

line on a ΔK versus K_{max} plot.

Figure 3.1: Schematic of $\Delta K - K_{max}$ curves defining the limiting values at various crack growth rates (da/dN) [38].

In 2001, Kujawski [65] proposed a two parameter driving force parameter which is similar to Walker. He proposed that the FCG driving force depends on the interplay of two damage processes, monotonic damage due to K_{max} and cyclic damage due to ΔK . Thus modified the effective K proposed by Walker as

$$K^* = K_{max}^\alpha (\Delta K^+)^{1-\alpha} \quad (3.8)$$

where α is a parameter that characterizes the apparent sensitivity of the K^* to the applied K_{max} value and ΔK^+ is the positive part of ΔK_{appl} . It was stated that the α may depend on the material. The two-parameter approach was further investigated for a number of materials and reported in [66]. The values of α for different material were reported and a good correlation for R-ratio was demonstrated. In 2012, Jones et. al. [67] proposed a crack driving force ($\Delta\kappa$) given by Eq. (3.9). The proposed equation was built on the Hartman-Schijve concept, i.e FCG rate depends on the amount by which ΔK exceeds the ΔK_{th} .

$$\Delta\kappa = (\Delta K - \Delta K_{th}) / \sqrt{(1 - K_{max}/A)} \quad (3.9)$$

where ΔK_{th} and the constant A are determined from the experimental data. The effectiveness of Eq. (3.9) was demonstrated by studying 22 materials and results were reported in [6].

In 2010, Zizi et.al. [68] proposed a FCG model based on small time scale (STS) crack increment instead of reversal-based approach. The small time scale model defines FCG in terms of time increment within a loading cycle. In this method, the FCG rate is calculated by direct time integration of crack extension. The detailed derivation of the crack growth behavior can be found in [68]. The model is based on crack tip opening displacement and Willenborg model. The underlying idea is that, the crack would advance only during the loading part of the cycle and after the stress level crosses a reference stress level. The reference stress level is calculated for each loading cycle and depends on the previous unloading cycle. The validity of the proposed methodology was verified by comparing model prediction with experimental data [68]. A comparative study of the STS model and the two parameter driving force approach showed good correlation with experimental data for both models [69].

Each model outlined in this section has its limitation and constraints, Walker model works only for positive R-ratios, Crack closure model needs K_{op} to be measured throughout the test for all the R-ratios. Although crack closure model is widely used as it effectively correlates R-ratio effect, it is still a widely debated topic. Willenborg and Wheeler models were developed to explain overload, while the modified versions work for either overload or underload. Jones et. al. Eq. (3.9) needs constants, ΔK_{th} and A to be calculated for each material. Of all the models available in the literature the FCG rate behavior modeling can be put into two categories, one which considers that FCG rate behavior is influenced by the process in-front of the crack tip like Willenborg and Walker models and the other initially proposed by Elber that considers process behind

the crack tip as suggested by crack closure model. In any case, all these models do not take into account the environmental effects on FCG rate behavior. Sadanandan and Vasudevan [38] suggested that a pure fatigue behavior is represented by $\Delta K^* = K_{max}^*$ line and any deviation from this line indicate effect of environment, but it has not been formulated to include in simulations of FCG rate behavior. Also all the above models are reversal-based approaches, that is the FCG rate is determined as the crack growth averaged over a period of loading cycles. While the STS model shows good agreement with the experimental data, it does not include the environmental effect. Thus, there is a practical need to develop a simplified universal approach that is applicable for different environments and loading conditions. In the next section a FCG correlation for R-ratio effects developed in this dissertation is presented. This correlation is an extension of Kujawski's two parameters model including environmental effects but is independent of material fitting parameter. In the later sections of the chapter, a new generalized model for the FCG behavior in various environments is presented.

3.2 R-Ratio Effect on FCG Behavior: Two Parameters Approach.

This section presents a correlation of FCG curves for different R-ratios. The proposed method of correlation is explained using FCG data for 2524 Al alloy and the consistency of the proposed method has been verified using FCG data for 10 different materials including aluminum, steel, titanium and other alloys.

3.2.1 Model derivation

It has been well established in [70] that FCG behavior is an interplay between K_{max} and ΔK . Also it is well known that for ductile materials the crack driving force is mostly effected by ΔK whereas for brittle materials by K_{max} . Thus Kujawski's [65] two-parameter driving force model is further investigated in this section and generalized for

R-ratio effects on FCG behavior.

3.2.1.1 Influence of K_{max} and ΔK parameters on FCG behavior

FCG rate data of Al 2524-T351 [21] for wide range of stress ratio (Fig. 3.2a) is transferred on to $\Delta K - K_{max}$ plot. This is done by drawing da/dN constant lines and the corresponding ΔK values are used to calculate K_{max} using Eq. (1.6). Thus obtained $\Delta K - K_{max}$ set for four da/dN values are plotted in Figure 3.2(b). The solid line indicate trend lines for one da/dN constant value. From the trend line we can say that there exist two slopes for da/dN constant line. Thus the data are replotted with two sets of best fit lines as shown in Figure 3.3. The circles represent data points while the solid lines represent best-fit lines. The intersection of the best-fit lines represents a transition boundary corresponding to stress ratio R_t . The data points above the transition boundary are represented by filled circles, while the data points below the transitions boundary are represented by open circles.

Figure 3.2: Plot of (a) $da/dN - \Delta K$ with constant da/dN lines transferred on to $\Delta K - K_{max}$ plot for Al 2524-T351.

Figure 3.3: Best fit lines for da/dN constant lines in Figure 3.2.

3.2.1.2 Crack driving force

Consider the region below the transition R_t first. To increase da/dN to next higher rate one has to either increase ΔK or K_{max} . The schematic in Figure 3.4 shows that FCG rate can be increased from $(da/dN)_1$ to $(da/dN)_2$ by increasing ΔK or K_{max} . Inspection of Fig. 3.4 indicates that the increment of ΔK needed is much less than that for K_{max} . From this we can conclude that this region is more sensitive to ΔK than to K_{max} , thus this region is called as $\Delta K_{driving}$ force region. Similarly the region above the transition R_t is found to be more sensitive to K_{max} , thus it is called as $K_{max\ driving}$ force region. This is illustrated by the schematic in Figure 3.4

Figure 3.4: Schematic of ΔK and K_{max} .

Also the region below the transition boundary belongs to R -ratios greater than the R_t -ratio of the transition boundary and the region above corresponds to the stress ratios less than the R_t -ratio. The transition boundary, R_t and the two regions corresponding to ΔK and K_{max} driving forces are illustrated by the schematic in Figure 3.5.

Figure 3.5: Schematic showing regions influenced by ΔK and K_{max} .

3.2.1.3 R-ratio correlation

As the existence of two regions with a transition boundary between them is assumed, one can hypothesize that the FCG curve for R_t is equally influenced by ΔK and K_{max} . Next attempt would be to see if both regions can be collapsed to this R_t -ratio, by doing so all three regions will be collapsed to R_t -curve.

Consider the $\Delta K_{driving}$ region and the transition boundary R_t . Using Eq. (3.8) one can write the K^* for both regions respectively as

$$K^* = \Delta K^{(1-\alpha)} K_{max}^\alpha \quad (3.10)$$

$$K^* = \Delta K_t^{(1-\alpha)} K_{max}^\alpha \quad (3.11)$$

Since the K^* is an equivalent SIF and for a constant da/dN line it should be the same for both the regions. By equating Eq. (3.10) and Eq. (3.11), also using Eq. (1.6) one gets

$$\Delta K_t^{(1-\alpha)} K_{max}^\alpha = \Delta K^{(1-\alpha)} K_{max}^\alpha \quad (3.12)$$

$$\Delta K_t^{(1-\alpha)} \frac{\Delta K_t^\alpha}{(1 - R_t)^\alpha} = \Delta K^{(1-\alpha)} \frac{\Delta K^\alpha}{(1 - R)^\alpha} \quad (3.13)$$

$$\Delta K_t = \Delta K \left(\frac{1 - R_t}{1 - R} \right)^\alpha \quad (3.14)$$

$$\Delta K_{driving} = \Delta K \left(\frac{1 - R_t}{1 - R} \right)^\alpha \text{ for } R > R_c \quad (3.15)$$

Similarly for K_{max} driving region by following the above procedure we can derive Eq. (3.16)

, where α and β are constants, which will be determined in the next section.

$$K_{max \text{ driving}} = K_{max} \left(\frac{1-R}{1-R_t} \right)^{(1-\beta)} \text{ for } R < R_c \quad (3.16)$$

3.2.1.4 Determining constants α and β

The constants α and β are determined from the slopes of the best fit lines in Figure 3.3. Using the best fit lines of the da/dN constant lines in $\Delta K - K_{max}$ plot in Figure 3.3, the slopes are extracted and averaged. The averaged slope is represented by p for region below the transition boundary and q for region above the transition boundary. α and β are determined by the following relations.

$$\alpha = \frac{p}{1-p}; \beta = \frac{q}{q-1} \quad (3.17)$$

Using the experimental FCG rate data the constants p, q and R_t are determined. For all the materials studied the value of p was found to be approximately -0.33 and q was approximated as -3.0303 . The L-shaped curves from the $\Delta K - K_{max}$ plots showed the transition boundary in terms of R-ratio to be around $R = 0.33$. Hence $R_t = 0.33$ is approximated as the transition stress ratio.

3.2.2 Model validation

The developed equations and the determined constant were applied to 12 sets of FCG rate data obtained from literature, which include aluminum [21, 36, 69], steel alloys [72], titanium alloys [73] and other alloys. For the purpose of illustration and explanation 2524 Al alloy [21] and 7075 Al [36] alloy are discussed in this section. The FCG data for Al 2524-T351 and Al 7075-T3 are plotted in Fig. 3.6(a) and Fig. 3.7(a) respectively. The FCG data collapse into two narrow scatter bands for both regions ($R > R_t$ and $R < R_t$), one is influenced by the $\Delta K_{driving}$ force while the other is influenced by $K_{max \text{ driving}}$ force. The obtained scatter bands are depicted in Figure 3.6(b) and Figure 3.7(b).

(a) FCG rate experimental data.

(b) Experimental data plotted using $\Delta K_{\text{driving}}$ and $K_{\text{max driving}}$ force equation.

Figure 3.6: FCG rate data for Al 2524-T351 [21].

(a) FCG rate experimental data.

(b) Experimental data plotted using $\Delta K_{driving}$ and $K_{max\ driving}$ force equation.

Figure 3.7: FCG rate data for Al 7075-T3.

Figure 3.8: (a) Al 2524-T351 and (b) Al 7075-T3 data collapsed to R_t using $\Delta K_{driving}$ force equation (Eq 3.19 and Eq 3.15).

The process can be further extended to collapse the data to the transition stress ratio by applying Eq. (1.6) to Eq. (3.16) which collapses all the data to one single scatter band, Figure 3.8.

$$\frac{\Delta K_{driving}}{(1 - R_t)} = \frac{\Delta K}{(1 - R)} \left(\frac{1 - R}{1 - R_t} \right)^{(1-\beta)} \text{ for } R < R_c \quad (3.18)$$

$$\Delta K_{driving} = \Delta K \left(\frac{1 - R_t}{1 - R} \right)^\beta \text{ for } R < R_c \quad (3.19)$$

Eq. 3.15 and Eq. 3.19 may be combined and presented as Eq. 3.20.

$$\Delta K^* = \Delta K \left(\frac{1 - R_t}{1 - R} \right)^\gamma \quad (3.20)$$

where $\gamma = \alpha$ for $R > R_t$ and $\gamma = \beta$ for $R < R_t$. It may be noted that the transition stress ratio R_t was found to be about $R_t = 0.33$. This was found to be consistent for aluminum alloys, Fig. 3.9, steel alloys, Fig. 3.10 and titanium alloys, Fig. 3.11. Thus one may conclude that all the FCG rate data can be collapsed to R-ratio 0.33, which corresponds

to be the transition boundary ratio R_t . By having the FCG rate data for $R_t = 0.33$, one may also use the developed equations to predict the FCG data of any R-ratio. This would help to predict the FCG rate for any R-ratio needed where experimental data is unavailable. This is done by modifying Eq. 3.20 as below.

$$\Delta K = \Delta K^* \left(\frac{1-R}{1-R_t} \right)^\gamma \quad (3.21)$$

where ΔK^* correspondences to the SIF range for R_t , $\gamma = \alpha$ for $R > R_t$ and $\gamma = \beta$ for $R < R_t$.

Figure 3.9: Stress ratio correlation of FCG data for aluminum [71].

Figure 3.10: Stress ratio correlation of FCG data for steel [72].

Figure 3.11: Stress ratio correlation of FCG data for titanium alloy [73].

3.2.3 Conclusions on correlation of R-ratio effect using two parameter approach

A correlation between FCG rates at different R-ratios has been presented. The proposed method of correlation revealed that, for all the materials studied the FCG rate curves for all R-ratios can be collapsed into two narrow scatter bands. Each band is influenced by either ΔK or K_{max} . A transition stress ratio (R_t) marks the transition boundary of the influence of ΔK or K_{max} and this R_t is found to be 0.33 for all the materials studied. Using the two parameters ΔK and K_{max} , two equations have been developed to represent FCG rate curves for various R-ratios. The two scatter bands were further collapsed into a single scatter band which corresponds to the transition stress ratio, R_t . The final correlation was represented to a single equation, which may be used to predict the FCG rate for any stress ratio. The next step of developing a more general FCG model to predict FCG per loading cycle under the influence of environment is presented in the next sections.

3.3 Literature Review : FCG Behavior with Environmental Effects

3.3.1 Microscopic observation of crack tip behavior

Laird [74] analyzed phenomenologically the FCG process through blunting-resharpening mechanism during loading and unloading, respectively. Pelloux [75], Meyen [76] and Bowles et al. [77] correlated FCG rate to formation of striations at each loading cycle. Oda et al. [78], reported microscopic FCG observations utilizing atomic force microscope (AFM) and scanning electron microscope (SEM) in Fe-Si single crystal specimens tested in vacuum and air. In these tests due to thickness of the specimen, plane stress conditions prevail. Striations with spacing Δs were observed only in air when FCG rate, da/dN , was higher than $1 \times 10^{-7} m/cycle$. The above observations regarding formation of striations

are illustrated schematically in Figure 3.12. It was noticed that striations formed by a pair of concentrated slip alternately activated at the crack tip. For FCG rates between $10^{-7} - 10^{-6} m/cycle$ the observed striations spacing were approximately equal to da/dN . For $da/dN > 10^{-6} m/cycles$, Δs spacing was smaller than da/dN indicating some contribution from K_{max} , which results in an additional crack extension associated with a static fracture mode. On the other hand, a vacuum environment facilitates multi-pairs of alternating slip that results in a lack of distinct striations on the fracture surface. In addition, multi-pairs of alternating slip in vacuum generate a larger crack tip blunting angle ($CTBA, 2\theta$) than in air, as shown by Table 3.1.

Furthermore, even in air at higher FCG rates (corresponding to the upper part of the Paris region) multi-pairs of alternating slip are activated that results in a lack of distinct striations on the fracture surface. The concentrated and multi-pair slips resemble to localized plasticity in some favorably oriented surface grains versus a gross plasticity in a smooth specimen cycled just above the fatigue limit and at a high stress range, respectively.

Figure 3.12: Schematic illustration regarding striations formation.

Table 3.1: Crack tip deformation in vacuum and air [78].

Environment	Crack tip at P_{max}	$CTOD^*$ (crack tip opening displacement)	2θ (CTBA) (crack tip blunting angle)	da/dN (crack extension per cycle)
Vacuum		$5.7\mu m$	$\sim 90^\circ$	$0.4\mu m$
Air		$6.0\mu m$	$\sim 62^\circ$	$0.96\mu m$

* CTOD was measured $10\mu m$ behind a crack tip.

Oda et al. measured the crack tip deformation during a full loading cycle using an SEM microscope as it is illustrated in Figure 3.13. An examination of the crack-tip deformation during loading and unloading shows a significant asymmetry for a given load level. For example, the crack flanks stay widely open at load level 5 during unloading and they are almost closed at load level 9 during loading. Such asymmetry can be attributed to highly concentrated dislocations around the crack tip and the associated friction and back stresses that result in generation of compressive residual stresses. It is interesting to point out that plasticity induced crack closure would result in rather symmetric crack flanks deformation when crack is open.

More recently, Pippan et al. [79] carried out extensive experimental work to understand the effect of environment at low and intermittent crack propagation rate. They suggested that the crack propagation mechanism is the formation of new surface due to crack-tip blunting during loading and resharpening during unloading. This process of crack-tip blunting is affected by the material/environment system. They performed experiments on steel and aluminum alloy in 3.5% NaCl and in vacuum (10^{-7} Torr) and compared results with tests in air. After the tests, the fracture surface was mapped to reproduce the shape of crack tip through stereophotogrammetric analyses using stereo scanning electron microscope. They reported precise measurements of crack tip opening

Figure 3.13: Schematic of crack tip displacement from SEM measurements ($da/dN \cong 2.6 \mu\text{m}/\text{cycle}$ [78]).

displacement ($CTOD$), crack extension per cycle (da/dN) and crack tip blunting angle ($CTBA$), 2θ (see Figure 3.14). Their measurements show that in corrosive environment the crack tip is sharper than in vacuum, this is evident from the measured $CTBA$, which is highest in vacuum. The reported crack extension shows accelerated crack growth in corrosive environment compare to vacuum. The trend was reported to be similar for both austenitic 220 steel and 7020 aluminum alloy, as they are presented in Table 3.2, which indicates that at the same $CTOD$ the actual crack growth rate, da/dN , is a function of $CTBA$, which in turn depends on environment.

Figure 3.14: Schematic of crack showing crack tip opening displacement (CTOD) and crack tip blunting angle (CTBA).

Table 3.2: Shows crack-tip opening displacement ($CTOD$), crack tip blunting angle ($CTBA$) and crack growth rate (da/dN) in vacuum, air and 3.5% $NaCl$ for (A) Austenitic 220 steel and (B) 7020 Al alloy [79].

Material	Environment	$CTOD^*$	$CTBA$	da/dN
		μm	2θ	$\mu m/c$
(A) Austenitic 2020 steel	Vacuum	4.6	$\sim 105^\circ$	0.5
	Air	4.3	$\sim 90^\circ$	1.0
	3.5% $NaCl$	4.7	$\sim 60^\circ$	1.2
(B) 7020 Al Alloy	Vacuum	5.0	$\sim 130^\circ$	0.3
	Air	4.3	$\sim 90^\circ$	0.7
	3.5% $NaCl$	4.7	$\sim 25^\circ$	1.7

* $CTOD$ was measured $7.5\mu m$ behind a crack tip.

For ductile materials FCG per cycle, da/dN and the crack-tip blunting radius, ρ (referred to as notch radius in case of notches), occur simultaneously. It has been shown in literature [75, 80] that there is a relationship between the striation spacing and the CTOD, where the crack-tip blunting radius is estimated to be approximately half the CTOD, $\rho = 1/2 CTOD = 1/2 \delta$, see Fig. 3.15. Other authors [81] even suggested that in the striation region FCG may be approximated as Eq. 3.22, where α is a fitting parameter

$$\frac{da}{dN} = \alpha \frac{\delta}{2} \quad (3.22)$$

Figure 3.15: Schematic of crack showing crack tip opening displacement (CTOD) and crack tip radius (ρ) in (A) vacuum and (B) chemical notch in corrosive solution.

Based on the crack tip analysis from Oda and Pippan et al., the crack tip angle is an important parameter that can be used in modeling to include the environmental effect. Sharper angle is associated with aggressive environment and high crack growth. On the other hand, a blunt angle in vacuum would result in lower crack growth rate. From the experimental results presented in section 2.2.4, it was concluded that the environmental effect can be treated as formation of "chemical notch" which magnifies the stress concentration. Oda and Pippan et al. results clearly indicate that the angle of crack tip changes with environment/material system. Thus, there is an analogy between the notch and crack-tip angles which changes the apparent stress concentration at the blunt crack tip, Fig. 3.15b. Hence, in modeling of FCGR behavior, the environmental effect may be included as change in crack tip blunting angle, while the mechanical damage due to cyclic loading is associated with a two-parameter driving force as presented in section 3.1. In the next section a new FCG model is proposed by unifying the two-parameter approach, Willenber model and the crack blunting angle. The new approach takes its idea from the STS model [68] where FCG is calculated by integrating the crack extension within a cycle. One fundamental difference from the STS model is consideration of crack tip angle which depends on loading conditions and environment/material system.

3.4 FCG Rate Model: Micro Observation Based Two Parameter Approach

3.4.1 Derivation of the model

Consider a crack of length ' a ' at time instance ' t ', say after a small time increment ' dt ' the crack length extends by ' da ' as shown in schematic (3.16), where ' δ ' and ' $\delta + d\delta$ ' are the crack tip opening displacement (CTOD) at time ' t ' and ' $t + dt$ ' respectively, ' θ ' is the angle to the crack opening flanks at CTOD point.

Figure 3.16: Schematic of crack tip at times t and $t + \delta t$.

From the schematic in Figure 3.16, we can deduce a relationship between change in crack length and change in CTOD as follows

$$da = \frac{\cot \theta}{2} d\delta \quad (3.23)$$

Considering hardening effect [82] the crack tip opening displacement [3] can be ex-

pressed as Eq. (3.24) at time, t

$$\delta_t = \frac{1}{2} \frac{K^2}{E\sigma_y} \frac{n' - 1}{n' + 1} = \frac{3}{8} \frac{K^2}{E\sigma_y} \text{ (for most metals)} \quad (3.24)$$

where n' is the strain hardening exponent, E is the Young's modulus of the material, σ_y is the yield strength of the material.

At time $(t + dt)$, where dt is a small increment in time, the CTOD ($\delta_{(t+dt)}$) can be expressed as Eq. (3.25)

$$\delta_{(t+dt)} = \frac{3}{8} \frac{(K + dK)^2}{E\sigma_y} \quad (3.25)$$

The change in CTOD from time t to $t + dt$, neglecting dK^2 term

$$d\delta = \delta_{(t+dt)} - \delta_t = \frac{3}{4E\sigma_y} K dK \quad (3.26)$$

Substituting Eq. (3.26) in Eq. (3.23) and integrating the crack extension for FCG rate

$$\begin{aligned} \int_a^{a+da} da &= \frac{\cot \theta}{2} \frac{3}{4E\sigma_y} \int_{K_{min}}^{K_{max}} K dK \\ &= \frac{\cot \theta}{2} \frac{3}{4E\sigma_y} \left(\int_{K_{min}}^{K_{ref}} K dK + \int_{K_{ref}}^{K_{max}} K dK \right) \end{aligned} \quad (3.27)$$

Where K_{ref} is the SIF associated with the compressive stress due to unloading part of previous cycle. From Willenborg model we deduce that in the current loading cycle, a crack grows only after the current stress level overcomes the compressive stress level from previous unloading cycle. Thus the integral from K_{min} to K_{ref} can be neglected. Also, it has been well established that crack grows only during the loading part of the cycle, so the unloading part of the cycle is neglected in Eq. (3.27). Neglecting the integral from K_{min} to K_{ref} and integrating gives Eq. (3.28)

$$da = C\lambda(K_{max}^2 - K_{ref}^2) \quad (3.28)$$

where $C = (\cot \theta)/2$ and $\lambda = 3/(8E\sigma_y)$

To include the asymptotic behavior of FCG rate curves near threshold and high FCG regions, a non-dimensional correction factor is introduced into Eq. (3.28) leading to Eq. (3.29).

$$da = C\lambda(K_{max}^2 - K_{ref}^2) \left[\frac{1 - \frac{\Delta K_{th}}{\Delta K}}{1 - \frac{\Delta K}{\Delta K_c}} \right] \quad (3.29)$$

Where ΔK is the current SIF range, ΔK_{th} is the threshold SIF range at FCG rate $< 10^{-10} \text{ m/cycle}$ and ΔK_c is the SIF range calculated from the fracture toughness (K_c) of the material.

3.4.1.1 Evaluation of K_{ref}

Rice [83], in 1967 introduced the occurrence of reversed plastic zone due to unloading part of the cycle given by Eq. (3.30).

$$\Delta r_p = \frac{1}{\pi} \left(\frac{\Delta K}{2\sigma_y} \right)^2 \quad (3.30)$$

According to strip yield plastic zone model, the forward plastic zone size due to loading part of the cycle was proposed by Dugdale [84] and Barenblatt [85] as Eq. (3.31).

$$r_f = \frac{\pi}{8} \left(\frac{K}{\sigma_y} \right)^2 \quad (3.31)$$

Figure 3.17: Schematic of reverse plastic zone and forward plastic zone.

Then, we assume that the crack extends only after the current stress during loading crosses the compressive stress level, created from the previous unloading cycle. Thus we replace the ΔK in Eq. (3.30) with the previous unloading part of the cycle as $\Delta K_{(i-1)}$ and SIF, K in Eq. (3.31) with K_{ref} . The idea being, at K_{ref} the forward plastic zone must equal the reversed plastic zone from previous unloading part of the cycle.

Hence at K_{ref} , $\Delta r_p = r_f$, equating Eq. (3.30) and Eq. (3.31)

$$\begin{aligned} \frac{1}{\pi} \left(\frac{\Delta K_{(i-1)}}{2\sigma_y} \right)^2 &= \frac{\pi}{8} \left(\frac{K_{ref}}{\sigma_y} \right)^2 \\ K_{ref} &= \frac{\sqrt{2}}{\pi} \Delta K_{(i-1)} \text{ for } R = 0 \end{aligned} \quad (3.32)$$

For $R \geq 0$, Eq.3.32 is modified as Eq. 3.33 which is further rearranged as Eq. 3.34

$$K_{ref} = K_{min} + \frac{\sqrt{2}}{\pi} \Delta K_{(i-1)} \text{ for } R \geq 0 \quad (3.33)$$

$$\frac{K_{ref}}{K_{max}} = R + \frac{\sqrt{2}}{\pi} (1 - R) \quad (3.34)$$

The forward plastic zone and reversed plastic zone considered to develop Eq. 3.34 are only valid for stress ratios greater than or equal to zero. To include both positive and negative stress ratios, where negative R-ratio may involve crack flanks contact, a general form of Eq. 3.34 is adapted from Kujawski [86] as Eq. 3.35. The equation considers an asymptotic value for very large negative stress ratios, as suggested by Lang [87].

$$\frac{K_{ref}}{K_{max}} = 0.25e^{1.15R} + 0.21 \quad (3.35)$$

3.4.2 Model validation

In this section model predictions are compared with FCG experimental data. The model is evaluated for various conditions like thickness effect, stress ratio effect, environment frequency effects and FCG retardation due to a single overload. Using the analytical

solution, a Matlab program was created to predict FCG behavior. The MatLab program is presented in appendix D

3.4.2.1 Model predictions for thickness effect

FCG rate data generated in laboratory at WMU for stress ratio, R , of 0.1 and 0.5 in air and vacuum for specimens with thickness 12.5 and 3 mm have been presented in section 2.5. Figure 3.18 and Fig. 3.19 show model predictions for stress ratio, $R=0.1$ and 0.5, respectively. The model shows good correlation with experimental data. In air, both stress ratios shows similar trend in FCG behavior, with thick specimen having lower threshold ΔK_{th} and higher FCG rate.

Figure 3.18: Comparison of model predictions with FCG rate data generated for Al 7075-T6 for $R=0.1$ in laboratory air.

Figure 3.19: Comparison of model predictions with FCG rate data generated for Al 7075-T6 for R=0.5 in laboratory air.

In case of tests conducted in vacuum, Fig. 3.20, FCG curve at R=0.1 for thin specimen shows lower threshold compared to thick specimen. But for R-ratio, 0.5 in vacuum very minimal effect of thickness is found as it is shown in Fig. 3.21.

Figure 3.20: Comparison of model predictions with FCG rate data generated for Al 7075-T6 for R=0.1 in vacuum.

Figure 3.21: Comparison of model predictions with FCG rate data generated for Al 7075-T6 for R=0.5 in vacuum.

In all the cases, the thickness effect is predominant for FCG rate below 10^{-8} m/cycle above which the effect of thickness is negligible. The model takes into account the effect of thickness by considering the variation of ΔK_{th} with thickness. Currently, there are no available models that predict the ΔK_{th} and is considered to be material property. The threshold SIF, ΔK_{th} , values and crack tip angle, θ , used in the model predictions are presented in Table 3.3. The threshold SIF and crack tip angle for vacuum is higher than for air, which is the general trend as discussed in section 2.4.3.3. Thus, by considering the threshold SIF as a material/environmental constant, the model shows good correlation to thickness effect in air and vacuum.

Table 3.3: Constants used in the model.

Environment	Stress ratio	Thickness	ΔK_{th}	θ
	R	[mm]	[MPa $\sqrt{m}$]	
Air	0.1	3	3.7	86°
		12.5	1.6	86°
	0.5	3	2	86°
		12.5	1	86.5°
Vacuum	0.1	3	4.8	88.5°
		12.5	6	89°
	0.5	3	2.4	89°
		12.5	2.9	89°

3.4.2.2 Model predictions for tests conducted in air

Using the analytical solution, a model was created in MatLab to predict crack growth curves. Figure 3.23 shows crack growth curves predicted by the model for Ti-6Al-4V [88] for stress ratios ranging from -5 to 0.8. The model shows fairly good correlation with experimental data. The CTBA, 2θ , increases with increasing stress ratio, while the threshold stress intensity range decreases with increasing stress ratio. The fracture toughness range, ΔK_c , was evaluated from fracture toughness of the material as $\Delta K_C = K_{IC}(1 - R)$. In this model the threshold intensity stress range, ΔK_{TH} and the fracture toughness values, K_{IC} , are determined from experimental data, while CTBA is a material/environmental parameter. The model predicted similar results for aluminum alloys, Fig. 3.22 and steel alloys, Fig 3.24.

Figure 3.22: Experimental fatigue crack growth data and model predictions for 2324 aluminum alloy [30].

Figure 3.23: Fatigue crack growth data of Ti-6Al-4V for stress ratio between -5 and 0.8.

Open symbols indicate experimental data, while solid symbols represents model predictions [73].

Figure 3.24: Experimental fatigue crack growth data and model predictions for 0.5% Cr steel alloy [72].

3.4.2.3 Model predictions for tests in vacuum

Figure 3.25 shows experimental data for three stress ratios for Al 7075-T7531 (Kirby and Beervers, 1973) in air (closed symbols) and in vacuum (open symbols). Figure 3.26 shows the model predictions by solid lines, the results indicate that the model captures the influence of stress ratio in air, Figure 3.26 (a) and also the absence of its influence in vacuum, Figure 3.26(b). It can be noted that the CTBA/2, θ in air varies from 82° to 87° with stress ratio while there is negligible variation in vacuum.

Figure 3.25: FCG rate experimental data for Al 7075-T7351 [89].

Figure 3.26: Model predictions for experimental data in Figure 3.25 for (a) air and (b) vacuum [89].

3.4.2.4 Model predictions for tests in 3.5% NaCl environment

It is widely agreed that FCG rate behavior at threshold region is predominantly influenced by corrosive environment in comparison to high crack growth rate region. This is due to the fact that at threshold, environment has ample time to weaken the material compared to damage caused due to mechanical loading. Thus, it is believed that two mechanisms influences the FCG curves, near threshold the damage is predominantly due to electro-chemical processes, while in Paris region mechanical damage is the dominating mechanism. This is reflected by the model predictions for FCG curve for AA2090-T81 [90] in 3.5% NaCl, Figure 3.27 (a). The model distinguishes the CTBA for both mechanisms

along with the threshold stress intensity range, ΔK_{TH} and critical fracture toughness range, ΔK_C . While this dual mechanism does not appear to operate in the model predictions for vacuum, Figure 3.27(b).

Figure 3.27: FCG rate experimental data [90] and model predictions for AA2090-T81 in (a) 3.5% NaCl and (b) vacuum.

3.4.2.5 Model predictions for tests in water vapor

FCG rate in particular for Al alloys is influenced by the level of humidity in air. Pao et al., [91] performed FCG experiments at different water vapor pressures. The results showed that as water vapor pressure increased, the threshold SIF decreased and the FCG rate increased for a given ΔK . The model prediction shows a good agreement with the experimental data as it is indicated in Figure 3.28 (a). The model also predicts the decrease in CTBA as water vapor pressure increases, Figure 3.28(b). That is the model suggests that the crack-tip is sharper in high humid conditions compared to low humidity. On the other hand ΔK_C values are approximately independent on humidity, Figure 3.28(c).

Figure 3.28: FCG experimental data [91] and model predictions for Al 7075-T651 for $R=0.1$, (b) crack-tip blunting angle for various water vapor pressure, (c) fracture toughness for various vapor pressure.

3.4.2.6 Model predictions for frequency effect on FCG behavior

The influence of aggressive environment on the FCG rate is governed by test frequency. Lower test frequency results in higher exposure time, leading to higher CF. Gingell et al. [92] performed FCG experiments at different frequency while exposing the specimen to corrosive environment. The model predictions shows a good agreement with experimental data, Fig. 3.29, with higher frequency showing largest angle.

	θ
Air	84°
20 Hz	68°
5 Hz	60°
2 Hz	55°
1 Hz	50°
0.5 Hz	47°

Figure 3.29: FCG experimental data and model predictions for Al 7150-T651 for various frequencies in corrosive environment [92].

3.4.2.7 Model predictions for a single overload

In this section, the developed model is used to simulate FCG behavior when a single overload is applied. Data for aluminum 2325-T39 from literature [27] with overload ratio ($OLR = K_{maxOL} / K_{maxBL}$) 1.5 and 2 are compared with model predictions. A schematic of loading cycles is presented in Fig. 3.30, where K_{maxOL} is the overload cycle and $K_{maxBL} = 10 \text{ MPa}\sqrt{m}$ is the baseload cycles. $K_{min} = 1 \text{ MPa}\sqrt{m}$ is maintained constant.

Figure 3.30: Schematic of single overload cycle.

The flow chart of the procedure is provided in Fig. 3.30. Figure 3.32 shows model predictions for OLR=1.5, where the general trend of FCG behavior after a single overload is predicted. The model predictions for OLR=2, Fig. 3.33, shows good correlation with experimental data. At the instance of overload, FCG rate increases then decreases sharply until the overload cyclic plastic zone is crossed. After which, the FCG rate gradually increases and reaches the FCG rate corresponding to baseload when the crack tip overcomes the overload monotonic plastic. This indicates the model may be used to predict overload FCG behavior. The MatLab program developed for overload predictions can be found in Appendix E. First the FCG rate for baseload is evaluated, Eq. 3.29, with K_{ref} , Eq. 3.35 calculated based on K_{maxBL} . Followed by FCG rate for overload, where K_{ref} is again calculated based on K_{maxBL} , the previous cycle. Only one overload cycle is considered, so the next cycle is again baseload. For this cycle the K_{ref} is calculated based on K_{maxOL} , this gives the FCG retardation, Fig. 3.34. The FCG rate obtained for this cycle is the minimum FCG rate after the overload cycle and the crack jump for this cycle is set to the size of cyclic plastic zone, Eq. 3.30. From this point on the K_{ref} is calculated based on the monotonic plastic zone, Eq. 3.31, in-front of the crack-tip, until the crack tip grows out of the monotonic plastic zone due to overload, Fig. 3.35. The K_{ref} is evaluated by using effective SIF, Eq. 1.5, and stress ratio Eq. 1.6, which is calculated based on remaining size of monotonic plastic zone due to overload after the crack extension from previous cyclic.

Figure 3.31: Flow chart showing Matlab program procedure to evaluate overload effect on FCG behavior.

Figure 3.32: Comparison of FCG experimental data with model predictions for OLR 1.5.

Figure 3.33: Comparison of FCG experimental data with model predictions for OLR 2.

Figure 3.34: Schematic of forward and reverse plastic zone in-front of the crack-tip after application of single overload.

Figure 3.35: Schematic of crack-tip after it passes the cyclic plastic zone due to overload. Gradual increase in FCG rate in this region.

3.4.3 Conclusions for FCG model

An experimentally motivated fatigue crack growth model with environmental effects is proposed. The proposed model has three parameters, fracture toughness range, ΔK_C , threshold stress intensity range, ΔK_{TH} and crack tip blunting angle, CTBA, 2θ . ΔK_{TH} shows dependence on stress ratio and environment, ΔK_C is independent of environment, while CTBA is associated with material/environment system. The model predictions in air showed the effect of stress ratio on FCG curves. FCG predictions for AA2090-T81 in 3.5% NaCl showed the presence of two mechanisms, which depend on the ΔK range applied. Under different water vapor pressures, the model successfully predicted the blunting of the crack tip with decreased humidity. In case of overload, the model predictions show reasonable correlation with experimental data. Thus, the proposed model predicts the FCG rate at different material/environment systems through CTBA parameter.

Chapter 4

Conclusions

- Experimental testing of Al 7075-T651 showed the influence 3.5% NaCl solution on the fundamental material properties. SSR tests with liquid-air interface lead to the understanding of a chemical notch effect at the liquid-air interface and the k_t is approximated to be 2.1.
- The FCG testing in air and vacuum showed FCG rate to be higher in thick specimens compared to thin specimens. The threshold SIF range was found to be dependent on stress ratio, thickness and environment. Under vacuum conditions the threshold SIF was found to be higher than in air, indicating the effect of environment even in laboratory air condition.
- Using the two parameter approach, a new analytical correlation was proposed to correlate R-ratio effect on FCG behavior. In contrast to the available analytical correlations, the proposed correlation is independent of fitting parameters. A "master curve" for FCG was obtained for different R-ratios. All FCG curves collapsed to $R=0.33$.
- Literature review demonstrate the existence of sharper crack tips for corrosive environment compared to vacuum. Based on the chemical notch concept, two parameter approach and the crack-tip angle, a new fatigue model is proposed. The proposed

model shows reasonable correlation with experimental data for different environments.

- The developed model accounts for overload effect and frequency effect on FCG behavior. The model is validated with number of FCG rate data conducted in Fatigue and Fracture Laboratory and taken from literature.

Chapter 5

Future Work

- The current version of the model includes a constant frequency . Including frequency as a variable would broaden the models application to service loading situations.
- The crack tip angle can be further investigated to develop an analytical solution which takes into account the loading conditions, frequency and the environment that influences the crack tip angle.
- To further validate the model, the crack tip angle for overloads in different environments and frequencies need be measured experimentally or simulated using molecular dynamics with a goal to apply for service loads.

Therefore, both experimental and analytical work in this area is highly recommended.

References

- [1] A. E8/E8M-13a, *Standard Test Methods for Testing of Metallic Materials*, ASTM International, West Conshohocken, PA.
- [2] N. E. Dowling, *Mechanical Behavior of Materials*, 3rd ed. Upper Saddle, New Jersey: Prentice Hall, 2007.
- [3] S. Suresh, *Fatigue of Metals*, 2nd ed. Cambridge, UK: Cambridge University Press, 1998.
- [4] T. Anderson, *Fracture Mechanics Fundamentals and Applications*, 3rd ed. Boca Raton, FL: CRC Press, Taylor & Francis Group, 2005.
- [5] R. Peterson, "Discussion of a century ago concerning the nature of fatigue and review of some of the subsequent researches concerning the mechanism of fatigue," *ASTM Bull.*, no. 164, p. 50, 1950.
- [6] A. Wohler, "Experiments on the strength of metals," *Engineering*, p. 160, August 23 1867.
- [7] J. Bauschinger, "On the change of the position of the elastic limit of iron and steel under cyclic variations of stress," *Mitt. Mech.-Tech Lab*, vol. 13, no. 1, 1886.
- [8] J. Ewing and J. Humfrey, "The fracture of metals under repeated alterations of stress," *Phil. Trans. Roy. Soc.*, vol. CC, p. 241, 1903.
- [9] H.J. Gough, "The fatigue of metals," Greenwood and Son, Scott., London, Tech. Rep., 1924.

- [10] V. Raja and T. Shoji, *Stress corrosion cracking. Theory and practice*. Woodhead Publishing Ltd., 2011.
- [11] R. Stephens, A. Fatemi, R. R. Stephens, and H. Fuchs, *Metal Fatigue in Engineering*, 2nd ed. New York, USA: John Wiley & Sons, Inc., 2000.
- [12] A. E1681, “Standard test method for determining a threshold stress intensity factor for environmentally-assisted cracking of metallic materials,” ASTM, West Conshohocken, PA, Tech. Rep. 984, 200.
- [13] ASTM, *Annual Book of ASTM Standards, Standard Test Method for Measurement of Fatigue Crack Growth Rates*, American Society for Testing and Materials Std., Rev. V.3.01, E-64, 1999.
- [14] *Standard Practice for Slow Strain rate Testing to Evaluate The Susceptibility of Metallic Materials to Environmentally Assisted Cracking*, ASTM G129-00 (reapproved 2013), 2013.
- [15] K. Ebtehaj, D. Hardie, and R. Parkins, “The influence of chloride - chromate solution composition on the stress corrosion cracking of mg-al alloy,” *Corrosion Science*, vol. 28, no. 8, pp. 811–829, 1988.
- [16] D. Hardie, D. Holroyd, and R. N. Parkins, “Reduced ductility of high strength aluminum alloy during or after exposure to water,” *Metal Science*, vol. 13, pp. 603–610, 1979.
- [17] Y. J. Hyun Jung Lee, youngjoo Kim and S. Kim., “Effects of testing variables on stress corrosion cracking susceptibility of al 2024-t351,” *Corrosion Science*, vol. 55, pp. 10–19, 2012.
- [18] R. Parkins, “Slow strain rate testing-25 years in slow strain rate testing for the evaluation of environmentally induced cracking.” in *Research and Engineering Applications*. ASTM STP 1210, 1993.

- [19] P. Paris, M. Gomez, and W. Anderson, “A rational analysis theory of fatigue,” *Trend Eng*, vol. 13, pp. 9–14, 1961.
- [20] J. Newman Jr., J. Schneider, A. Daniel, and D. McKnight, “Compression pre-cracking to generate near threshold fatigue-crack-growth rates in two aluminum alloys.” *International Journal of Fatigue*, vol. 27, pp. 1432–1440, 2005.
- [21] A. Tesch, R. Pippan, , and H. Doker, “New testing procedure to determine da/dn - δk curves at different, constant r -values using one single specimen.” *International Journal of Fatigue*, vol. 29, pp. 1220–1228, 2007.
- [22] D. Kujawski and P. C. R. Sree, “Generation and analysis of FCGR data using a single specimen and $K_{max} - \Delta K$ testing matrix,” *International Journal of Fatigue*, vol. 31, pp. 1638–1647, 2009.
- [23] A. Vasudevan, K. Sadananda, and N. Loaut, “Two critical stress intensities for threshold crack propagation,” *Scripta Metall.*, vol. 28, pp. 65–70, 1998.
- [24] A. K. Vasudevan, K. Sadananda, and N. Loaut, “A review of crack closure fatigue crack threshold and related phenomena.” *Material Science Engineering*, vol. 188A, pp. 1–22, 1993.
- [25] A. Vasudevan and K. Sadananda, “Classification of fatigue crack growth behavior.” *Met Trans A*, vol. 26A, pp. 1221–1234, 1995.
- [26] S. Dinda and D. Kujawski, “Correlation and prediction of fatigue crack growth for different R – ratios using K_{max} and ΔK^+ parameter.” *Eng Fract Mech*, vol. 71, pp. 1779–1790, 2004.
- [27] S. Stoychev and D. Kujawski, “Analysis of crack propagation using Δk and K_{max} ,” *International Journal of Fatigue*, vol. 27, pp. 1425–1431, 2005.
- [28] K. Walker, “The effect of stress ratio during crack propagartion and fatigue for 2024-T3 and 7075-T6 aluminum.in: Effects of environment and complex loading history

- on fatigue life.” *American Society for Testing Materials*, vol. STP 462, pp. 1–14, 1970.
- [29] D. Kujawski, D. Craig, and F. Ellyin., “An experimental technique to study the behavior of small corner cracks.” *International Journal of Fatigue*, vol. 17, pp. 253–259, 1995.
- [30] S. Stoyan, “Load interaction effects on fatigue crack growth,” Ph.D. dissertation, Western Michigan University, Kalamazoo, MI, 2005.
- [31] B. Boyce and R. Ritchie, “Effect of load ratio and maximum stress intensity on the fatigue threshold in ti-6al-4v,” *Engineering Fracture Mechanics*, vol. 68, pp. 129–147, 2001.
- [32] D. Kujawski, “Environmental crack growth behavior affected by thickness/ geometry constraint,” *Metallurgical and Materials Transactions A*, vol. 44A, pp. 1340–1352, March 2013.
- [33] G. Irwin, “Analysis of stresses and strains near the end of a crack traversing a plate,” *Journal of Applied Mechanics*, vol. 24, pp. 361–364, 1957.
- [34] H. B. Park and B. W. Lee, “Effect of specimen thickness on fatigue crack growth rate,” *Nuclear Engineering and Design*, vol. 197, pp. 197–203, 2000.
- [35] P. Matos and D. Nowell, “Experimental and numerical investigation of thickness effects in plasticity-induced fatigue crack closure,” *International Journal of Fatigue*, vol. 31, pp. 1795–1804, 2009.
- [36] E. Lee, G. Glinka, A. Vasudevan, N. Iyyer, and N. Phan, “Fatigue of 7075-t651 aluminum alloy under constant and variable amplitude loadings,” *International Journal of Fatigue*, vol. 31, pp. 1858 – 1864, 2009, fatigue Damage of Structural Materials VII. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0142112308002752>

- [37] J. Ruiz and M. Elices, "Effect of water vapour pressure and frequency on fatigue behavior in 7075-T651 aluminum alloy plate," *Acta Metallurgica*, vol. 45, pp. 281–293, 1997.
- [38] A. Vasudevan and K. Sadananda, "Classification of environmentally assisted fatigue crack growth behavior," *International Journal of Fatigue*, vol. 31, pp. 1696 – 1708, 2009, fatigue Damage of Structural Materials VII. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0142112309001236>
- [39] A. Hartman, "On the effects of oxygen and water vapor on the propagation of fatigue cracks," *International Journal of Fracture*, vol. 1, pp. 167–88, 1965.
- [40] K. Sankaran, R. Perez, and K. Jata, "Effects of pitting corrosion on the fatigue behavior of aluminum alloy 7075 – T6: modeling and experimental studies," *Materials Science and Engineering: A*, vol. 297, pp. 223 – 229, 2001. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0921509300012168>
- [41] J. Petit and C. Sarrazin-Baudoux, "An overview on the influence of the atmosphere environment on ultra-high-cycle fatigue and ultra-slow fatigue crack propagation," *International Journal of Fatigue*, vol. 28, no. 11, pp. 1471 – 1478, 2006, third International Conference on Very High Cycle Fatigue (VHCF-3) Third International Conference on Very High Cycle Fatigue. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0142112306001071>
- [42] Y. Hirose, Z. Yajima, and T. Mura, *International Journal of Shore and Polar Engineering*, vol. 4, pp. 151–155, 1994.
- [43] H. Vehoff and W. Rothe, "Gaseous hydrogen embrittlement in FeSi- and Ni-Single crystals," *Acta Metallurgica*, vol. 31, no. 11, pp. 1781–1793, 1983.
- [44] E. Lee, A. Vasudevan, and K. Sadananda, "Effects of various environments on fatigue crack growth in laser formed and IM Ti-6Al-4V alloys," *International*

- Journal of Fatigue*, vol. 27, pp. 1597 – 1607, 2005. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0142112305001842>
- [45] R. Wanhill and M. Koolloos, “Fatigue and corrosion in aircraft pressure cabin lap splices,” *International Journal of Fatigue*, vol. 23, Supplement 1, pp. 337 – 347, 2001. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0142112301001475>
- [46] A. Bonakdar, F. Wang, J. Williams, and N. Chawla, “Environmental effects on fatigue crack growth in 7075 aluminum alloy,” *Metallurgical and Materials Transactions A*, vol. 43, pp. 2799–2809, 2012. [Online]. Available: <http://dx.doi.org/10.1007/s11661-011-0810-0>
- [47] R. Chlistovsky, P. Heffernan, and D. DuQuesnay, “Corrosion-fatigue behavior of 7075-T651 aluminum alloy subjected to periodic overloads,” *International Journal of Fatigue*, vol. 29, pp. 1941 – 1949, 2007. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0142112307000333>
- [48] P. Sree, “Generation and analysis of fcg data using a single specimen and $K_{max} - \Delta K$ testing matrix,” Master’s thesis, Western Michigan University, 2008.
- [49] W. Elber, “Fatigue crack closure under cyclic tension,” *Eng. Fract. Mech.*, vol. 21, pp. 37–45, 1970.
- [50] J. Willenborg, R. Engle, and H. Wood, “A crack growth retardation model using an effective stress concept,” Wright Patterson Air force base, Dayton, USA, Report AFFEL-TM-71-1-FBR, 1971.
- [51] O. Wheeler, “Spectrum loading and crack growth,” *J. Basic Engng*, vol. 94, pp. 181–186, 1972.
- [52] G. Baudin and M. Roberst, “Crack growth model for flight type loading,” in *Proceedings of the 11th ICAF Symposium*, ser. 2.7, Noordwijkerhout, Netherlands, 1981.

- [53] D. Aliaga, A. Davy, and H. Schaff., “A simple crack closure model for predicting fatigue crack growth under flight simulation loading,” in *Proceedings of the 13th ICAF Symposium*, Pisa, 1985, pp. 605–630.
- [54] A. U. de Koning, “A simple crack closure model for prediction of fatigue crack growth rate under variable amplitude loading,” in *Fracture Mechanics*, ser. ASTM STP 743, Philadelphia, USA, 1981, pp. 63–85.
- [55] A. U. de Koning and H. H. van der Linden, “Prediction of fatigue crack growth rates under variable amplitude loading.” NLR, Amsterdam, Netherlands, Report NLR MP 81023U, 1981.
- [56] J. Newman, “A crack closure model for predicting fatigue crack growth under aircraft spectrum loading.” in *methods and models for predicting fatigue crack growth under random loading*, ser. ASTM STP 748, C. J.B.Chang, Ed., Philadelphia, USA, 1981, pp. 53–84.
- [57] J. C. Newman, “A nonlinear fracture mechanics approach to the growth of small cracks,” in *Behavior of short cracks in airframe materials*, ser. AGARD CP-328, Toronto, Canada, 1983, pp. 6.1–6.26.
- [58] R. McClung, “The influence of applied stress, crack length and stress intensity factor on crack closure.” *Metall Trans*, vol. 22A, pp. 1559–71, 1991.
- [59] K. Donald, G. Bray, and R. Bush, “Introducing the K_{max} sensitivity concept for correlating fatigue crack growth data.” *The Minerals Metals and Materials Society*, pp. 123–41, 1997.
- [60] J. Gallagher, “A generalized development of yield zone models,” Wright Patterson Air Force Base, Dayton, Report AFFDL-TR-FBR-74-28, 1974.
- [61] J. Gallagher and T. Hughes, “Influence of the yield strength on overload affected fatigue crack growth behavior of 4340 steel,” Wright Patterson Air Force Base, Dayton, Report AFFDL-TR-74-27, 1974.

- [62] J. Chang and R. Engle, “Improved damage-tolerance analysis methodology,” *J.Aircraft*, vol. 21, pp. 722–730, 1984.
- [63] T. Ghidini and C. Donne, “Prediction of fatigue crack propagation in friction stir welds,” in *Proceedings of the 4th International Symposium on Friction Stir Welding*. Park City, Utah, USA: Welding Institute TWI Abington Hall, 2003.
- [64] M. Skorupa, A. Skorupa, J. Schijve, T. Machniewicz, and P. Korbut, “Fatigue crack growth behaviour of 18g2a steel under constant amplitude loading and following a single overload,” *Arch. Mech. Engng*, vol. 47, pp. 139–163, 2000.
- [65] D. Kujawski, “A fatigue crack driving force parameter with load ratio effects,” *International Journal of Fatigue*, vol. 23, pp. S239–S246, 2001.
- [66] S. Dinda, “Correlation and prediction of fatigue crack growth rate for different r -ratios,” MS Thesis, Western Michigan University, Kalamazoo, MI, 2002.
- [67] R. Jones, L. Molent, and K. Walker, “Fatigue crack growth in a diverse range of materials,” *International Journal of Fatigue*, vol. 40, pp. 43–50, 2012.
- [68] Z. Lu and Y. Liu, “Small time scale fatigue crack growth analysis,” *International Journal of Fatigue*, vol. 32, pp. 1306–1321, 2010.
- [69] Z. Lu and Y. Liu, “A comparative study between a small time scale model and the two parameter driving force model for fatigue analysis,” *International Journal of Fatigue*, vol. 42, pp. 57–70, 2012.
- [70] R. Ari-Gur and D. Kujawski, “Estimation of rate of crack propagation without reference to closure.” Western Michigan University, Kalamazoo, MI, Technical Report, 2003.
- [71] P. C. Paris, H. Tada, and J. Donald, “Service load fatigue damage — a historical perspective,” *International Journal of Fatigue*, vol. 21,

- Supplement 1, no. 0, pp. S35 – S46, 1999. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0142112399000547>
- [72] S. Stanzl-Tschegg and B. Schönbauer, “Near-threshold fatigue crack propagation and internal cracks in steel,” *Procedia Engineering*, vol. 2, no. 1, pp. 1547 – 1555, 2010, fatigue 2010. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S1877705810001682>
- [73] A. Yuen, S. Hopkins, G. Levernt, and C. Rau, “Correlations between fracture surface appearance and fracture mechanics parameters for stage ii fatigue crack propagation in ti-6al-4v,” *Metall. Trans.*, vol. 5, pp. 1833–1842, 1974.
- [74] C. Laird, “The influence of metallurgical structure on the mechanisms of fatigue crack propagation.” *ASTM STP 415*, pp. 131–168, 1967.
- [75] R. Pelloux, “Mechanisms of formation of ductile fatigue striations.” *Transactions of the American Society for Metals.*, vol. 62, pp. 281–285, 1969.
- [76] D. A. Meyn, “Observations on micromechanisms of fatigue-crack propagation in 2024 aluminum.” *Transactions of the American Society for Metals.*, vol. 61, pp. 42–51, 1968.
- [77] C. Bowles and D. Brock, “On the formation of fatigue striations.” *International Journal of Fracture Mechanics*, vol. 8, pp. 75–85, 1972.
- [78] Y. Oda, Y. Furuya, H. Noguchi, and K. Higashida, “AFM and SEM observation on mechanism of fatigue crack growth in an fe-si single crystal.” *International Journal of Fracture.*, vol. 113, pp. 213–231, 2002.
- [79] R. Pippan, C. Zelger, E. Gach, C. Bichler, and H. Weihhandl, “On the mechanism of fatigue crack propagation in ductile metallic materials,” *Fatigue of Engineering Materials and Structures*, vol. 34, pp. 1–16, 2010.

- [80] J. Schijve, “The significance of fractography for investigations of fatigue crack growth under variable amplitude fatigue,” *Fatigue and Fracture of Engineering Materials and Structures*, vol. 22, pp. 87–99, 1999.
- [81] S. Pommior and M. Rishet, “Time derivative equations for mode i fatigue crack growth in metals,” *International Journal of Fatigue*, vol. 27, pp. 1297–1306, 2005.
- [82] C. Shih, “J-analysis estimates for strain hardening materials in antiplane shear using fully plastic solution,” *Mechanics of Crack Growth. ASTM STP 590*, pp. 3–26, 1976.
- [83] J. Rice, “Mechanics of crack tip deformation and extension by fatigue.” in *Fatigue Crack Propagation*, ser. 415. Philadelphia: American Society for Testing and Materials, 1967, pp. 247–309.
- [84] D. Dugdale, “Yielding in steel sheet containing slits,” *Journal of Strain Analysis*, vol. 8, pp. 100–104, 1961.
- [85] G. Barenblatt, “The mathematical theory of equilibrium cracks in brittle fracture,” *Advances in Applied Mechanics*, vol. 7, pp. 55–129, 1962.
- [86] H.-C. Lin and D. Kujawski, “A general equation for kop and {KPR} description,” *Engineering Fracture Mechanics*, vol. 75, no. 10, pp. 3244 – 3248, 2008. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0013794408000350>
- [87] M. Lang, “A model for fatigue crack growth, Part II: modelling,” *Fatigue and Fracture of Engineering Materials and Structures*, vol. 23, no. 7, pp. 603–617, 2000. [Online]. Available: <http://dx.doi.org/10.1046/j.1460-2695.2000.00298.x>
- [88] A. Yuen, S. Hopkin, G. Leverant, and C. Rau, “Correlations between fracture surface appearance and fracture mechanics parameters for stage ii fatigue crack propagation in ti-6ai-4v,” *Metallurgical Transactions*, vol. 5, no. 8, pp. 1833–1842, 1974. [Online]. Available: <http://dx.doi.org/10.1007/BF02644148>

- [89] B. Kirby and C. Beevers, “Slow fatigue crack growth and threshold behavior in air and vacuum of commercial aluminum alloys,” *Fatigue of Engineering Materials and Structures*, vol. 1, pp. 203–215, 1973.
- [90] D. Slavik, *Environmental Fatigue Crack Growth and Microscopic Damage Mechanisms in AA2090*. University of Virginia, 1993. [Online]. Available: <http://books.google.com/books?id=YUJuJAAACAAJ>
- [91] P. Pao, M. Gao, and R. Wei, “Environmentally assisted fatigue-crack growth in 7075 and 7050 aluminum alloys,” *Scripta Metallurgica*, vol. 19, no. 3, pp. 265 – 270, 1985. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/0036974885903102>
- [92] G. A.D.B and K. J.E., “Effect of frequency on environmentally assisted fatigue crack growth in aluminum alloy 7150-t651,” University of Cambridge, Cambridge, CB2 3QZ, UK, Tech. Rep., 1997.

Appendices

Appendix A

Aluminum Alloy and Temper Designation System

A.1 Aluminum Alloy Designation System

Aluminum alloys are categorized into number of groups based on the primary alloying element. The wrought aluminum uses 4-digit system, while the cast aluminum uses 3-digit system plus a decimal.

A.2 Wrought Aluminum Alloy Designation System

- The first digit of the 4-digits indicate the primary alloying element. It is also used to identify as the aluminum series. Table A.1
- The last two digits are arbitrary numbers simply used to distinguish between alloys of the same series. One exception to this system is the 1000 series where, the last two digits indicates the minimum percentage of aluminum above 99%.
- The second number indicates how many times the original alloy has been modified. So if the second number is '0', it means it's the originally developed alloy.

Table A.1: Primary alloying element for wrought aluminum alloy designation system

Alloy Series	Primary Alloying Element
1xxx	99.000% Minimum Aluminum
2xxx	Copper
3xxx	Manganese
4xxx	Silicon
5xxx	Magnesium
6xxx	Magnesium and Silicon
7xxx	Zinc
8xxx	Other Elements

Table A.2: Primary alloying element for cast aluminum alloy designation system

Alloy Series	Primary Alloying Element
1xx.x	99.000% Minimum Aluminum
2xx.x	Copper
3xx.x	Silicon Plus Copper and/or Magnesium
4xx.x	Silicon
5xx.x	Magnesium
6xx.x	Unused Series
7xx.x	Zinc
8xx.x	Tin
9xx.x	Other Elements

A.3 Cast Aluminum Alloy Designation System

- The first digit indicates the primary alloying element. Table ??
- The last two digits are arbitrary numbers used to distinguish between alloys of the same series.
- The number after the decimal point is either 0 or 1, indicating casting or ingot respectively.
- <http://www.esab.ca/ca/en/education/blog/understanding-the-aluminum-alloy-designation-system.cfm>

The series numbering system gives the understanding of the properties of the aluminum alloys. Specially when considering welding on these materials. The 1xxx, 3xxx

Table A.3: Basic Temper Designation

Letter	Meaning
F	As fabricated, no thermal or strain hardening employed.
O	Annealed, heat treated to improve ductility.
H	Strain hardened through cold-working. The letter 'H' is always followed by two or more digits.
W	Heat treated followed by natural aging.
T	Combination of heat treatment, strain hardening, natural and artificial aging. The 'T' is always followed by one or more digits

and 5xxx series of wrought aluminum alloys are non-weldable and are considered to be non-heat treatable. The 2xxx, 6xxx and 7xxx series wrought aluminum alloys are heat treatable and can be weldable. In case of cast alloys, 2xx.x, 3xx.x, 4xx.x and 7xx.x series are heat treatable. Heat treatment gives the alloys their optimum mechanical properties. For non-heat treatable alloys, strain hardening is employed to attain the optimum properties. Heat treatment is the process of thermally heating the alloy to elevated temperature followed by quenching in water. Quenching in water produces a supersaturated solution at room temperature. If precipitates of alloying elements are formed at room temperature it is termed as natural aging. If the precipitates are formed by maintaining the alloy at elevated temperature for a long duration of time, it is termed as artificial aging. The heat treatment and aging process is used to alter the mechanical properties of the alloys. Thus the alloy designation system is followed by temper designation system which indicates the heat treatment and aging process of the alloy.

A.4 Temper Designation System

The temper designation system is an extension of the alloy numbering system separated by a hyphen. The temper designation system consists of a series of letters and numbers. Table A.3 outlines the basic temper designations

In case of 'H' and 'T' tempers, the letters are followed by one or more digits which

indicate additional temper details. Table A.4 and Table A.6 gives addition temper details for 'H' and 'T' respectively.

Table A.4: Subdivision of H temper

H temper	Meaning
H1x	Strain hardened only.
H2x	Strain hardened and partially annealed.
H3x	Strain hardened and stabilized
H4x	Strain hardened and painted
'x'-indicates degree of strain hardening (see table A.5)	

Table A.5: Degree of strain hardening in H temper

x	Meaning
1	Quarter hardened.
4	Half hardened.
6	Three-quarters hardened
8	Fully hardened
9	Extra hardened

A.5 Example

Al 7075-T651

Al - indicates aluminum

7 - indicates a 7000 series alloy

0 - indicates its the originally developed alloy.

75 - indicates the specific alloy in the series.

T6 - indicates heat treated and artificially aged.

51 - indicates stress relieved by stretching.

Table A.6: Subdivision of T temper

T temper	Meaning
T1	Naturally aged after shaping
T2	Cold worked after natural aging
T3	Heat treated, cold worked and naturally aged
T4	Heat treated and naturally aged
T5	Artificially aged after shaping.
T6	Heat treated and artificially aged.
T7	Heat treated and overaged
T8	Heat treated, cold worked and artificially aged
T9	Heat treated, artificially aged and cold worked'
T10	Cold worked and artificially aged.
Tx51	Stress relieved by stretching.
Tx52	Stress relieved by compressing.
	'x'-indicates temper subdivision

Appendix B

Fatigue Crack Growth Test Data

Table B.1: Test data for 3 mm thickness Al 7075-T651 ST in air

R=0.1		R=0.5	
$\Delta K [MPa\ m^{0.5}]$	da/dN [m/cycle]	$\Delta K [MPa\ m^{0.5}]$	da/dN [m/cycle]
3.8	6.00E-11	2.11	5.69E-11
4.35	1.77E-10	2.41	2.29E-10
4.83	6.87E-09	2.68	4.67E-09
5.36	1.08E-08	2.98	5.43E-09
5.96	1.60E-08	3.31	5.28E-09
6.61	2.33E-08	3.67	1.18E-08
7.35	1.40E-08	4.08	1.96E-08
8.16	2.21E-08	4.53	3.32E-08
9.06	3.90E-08	5.03	4.43E-08
10.06	5.80E-08	5.59	7.46E-08
11.17	8.21E-08	6.21	1.21E-07
12.97	1.48E-07	7.2	1.65E-07
15.05	2.35E-07	8.36	2.17E-07
17.47	1.00E-06		

Table B.2: Test data for 12.5 mm thickness Al 7075-T651 ST in air

R=0.1		R=0.5	
$\Delta K [MPa\ m^{0.5}]$	da/dN [m/cycle]	$\Delta K [MPa\ m^{0.5}]$	da/dN [m/cycle]
1.37	3.09E-12	1.04	3.64377E-11
1.69	6.E-11	1.16	2.00E-10
1.88	2.69E-10	1.29	5.31E-10
2.09	6.61E-10	1.43	1.36E-09
2.32	1.29E-09	1.59	1.10E-09
2.57	1.97E-09	1.76	3.64E-09
2.86	2.07E-09	1.96	5.15E-09
3.17	4.77E-09	2.17	3.56E-09
3.52	1.73E-09	2.41	5.48E-09
3.91	1.26E-08	2.68	8.17E-09
4.34	1.29E-08	2.98	1.29E-08
4.82	9.24E-09	3.3	1.60E-08
5.95	2.42E-08	3.67	3.46E-08
7.33	3.50E-08	4.52	5.07E-08
8.14	3.25E-08	5.58	8.60E-08
9.04	8.55E-08	7.19	2.94E-07
10.04	9.34E-08		
11.14	2.36E-07		
12.94	1.78E-07		
15.01	9.51E-07		
17.43	2,25E-06		

Table B.3: Test data for 12.5 mm thickness Al 7075-T651 ST in vacuum

R=0.1		R=0.5	
$\Delta K [MPa\ m^{0.5}]$	da/dN [m/cycle]	$\Delta K [MPa\ m^{0.5}]$	da/dN [m/cycle]
5.56	7.03E-11	3.09	2.88E-10
6.17	9.20E-11	3.43	1.15E-09
6.85	6.13E-10	3.81	6.23E-10
8.46	2.78E-09	4.23	4.36E-09
9.39	6.46E-09	4.7	3.06E-09
10.43	2.58E-09	5.22	1.26E-08
11.58	2.90E-08	5.79	2.39E-08
15.61	1.71E-07	6.44	4.69E-08
		7.47	7.72E-08
		8.67	8.67E-07

Table B.4: Test data for 3 mm thickness Al 7075-T651 ST in vacuum

R=0.1		R=0.5	
$\Delta K [MPa\ m^{0.5}]$	da/dN [m/cycle]	$\Delta K [MPa\ m^{0.5}]$	da/dN [m/cycle]
4.45	5.22E-10	2.22	2.15E-10
4.45	5.22E-10	2.47	2.59E-10
4.94	5.26E-10	2.47	2.59E-10
5.49	1.29E-09	2.74	4.40E-10
6.1	3.12E-09	3.05	5.49E-10
6.78	4.09E-09	3.39	8.62E-10
7.53	9.90E-09	3.76	1.67E-09
8.36	1.43E-08	4.18	4.15E-09
9.29	1.12E-08	4.65	1.87E-08
10.32	3.38E-08	5.16	5.29E-08
11.46	5.61E-08	5.73	3.63E-08
13.31	1.58E-07	6.37	7.10E-08
17.94	5.57E-07	8.59	5.89E-08
		9.97	3.17E-06

Appendix C

Matlab Program for R-ratio Correlation

```
1 clear all
2 clc
3 clf
4
5 load('data.mat')
6 r0=TI6AL4V0; % r=0
7 r01=TI6AL4V01; % R=0.1
8 r03=TI6AL4V03; % R=0.3
9 r05=TI6AL4V05; % R=0.5
10 r07=TI6AL4V07; % R=0.7
11 r08=TI6AL4V08; % R=0.8
12 %r09=TI6AL4VR09; % R=0.9
13 r_1=TI6AL4V_1; % R=-1
14 r_3=TI6AL4V_3; % R=-1
15 r_5=TI6AL4V_5; % R=-2
16
```

```

17 % calculate kmax and store in same variable .....
18
19 r0(:,3)=r0(:,1)/(1-0);
20 r01(:,3)=r01(:,1)/(1-0.1);
21 r03(:,3)=r03(:,1)/(1-0.3);
22 r05(:,3)=r05(:,1)/(1-0.5);
23 r07(:,3)=r07(:,1)/(1-0.7);
24 r08(:,3)=r08(:,1)/(1-0.8);
25 %r09(:,3)=r08(:,1)/(1-0.9);
26 r_1(:,3)=r_1(:,1)/(1+1);
27 r_3(:,3)=r_3(:,1)/(1+3);
28 r_5(:,3)=r_5(:,1)/(1+5);
29
30
31
32 % plot data
33
34 figure(1)
35 loglog(r0(:,1),r0(:,2),'or',r01(:,1),r01(:,2),'^b',r03(:,1),r03
   (:,2),'sk',r05(:,1),r05(:,2),'sg',r07(:,1),r07(:,2),'vr',r08
   (:,1),r08(:,2),'>g',r_1(:,1),r_1(:,2),'sk',r_3(:,1),r_3(:,2),
    'om',r_5(:,1),r_5(:,2),'<b')
36 hold on
37 legend('R=0','R=0.1','R=0.3','R=0.5','R=0.7','R=0.8','R=-1','R
    =-3','R=-5')
38 type='nearest';
39 dk0=interp1(r0(:,2),r0(:,1),dadnc,type);
40 dk01=interp1(r01(:,2),r01(:,1),dadnc,type);

```

```

41 dk03=interp1(r03(:,2),r03(:,1),dadnc,type);
42 dk05=interp1(r05(:,2),r05(:,1),dadnc,type);
43 dk07=interp1(r07(:,2),r07(:,1),dadnc,type);
44 dk08=interp1(r08(:,2),r08(:,1),dadnc,type);
45 dk_1=interp1(r_1(:,2),r_1(:,1),dadnc,type);
46 dk_3=interp1(r_3(:,2),r_3(:,1),dadnc,type);
47 dk_5=interp1(r_5(:,2),r_5(:,1),dadnc,type);
48 loglog(dk0,dadnc,'sk')
49 loglog(dk01,dadnc,'sk')
50 loglog(dk05,dadnc,'sk')
51 loglog(dk07,dadnc,'sk')
52 loglog(dk08,dadnc,'sk')
53 loglog(dk_1,dadnc,'sk')
54 loglog(dk_3,dadnc,'sk')
55 loglog(dk_5,dadnc,'sk')
56
57 hold off
58
59 dk(:,1)=dk_5;
60 dk(:,2)=dk_3;
61 dk(:,3)=dk_1;
62 dk(:,4)=dk0;
63 dk(:,5)=dk01;
64 dk(:,6)=dk03;
65 dk(:,7)=dk05;
66 dk(:,8)=dk07;
67 dk(:,9)=dk08;
68

```

```

69
70 kmax(:,2)=dk_3/(1+3);
71 kmax(:,1)=dk_5/(1+5);
72 kmax(:,3)=dk_1/(1+1);
73 kmax(:,4)=dk0/(1-0);
74 kmax(:,5)=dk01/(1-0.1);
75 kmax(:,6)=dk03/(1-0.3);
76 kmax(:,7)=dk05/(1-0.5);
77 kmax(:,8)=dk07/(1-0.7);
78 kmax(:,9)=dk08/(1-0.8);
79
80 %% kmax dk plot with da/dn constant lines
81
82 kkmax=[4 25];
83 rc=0.33;
84 ddk=kkmax*(1-rc);
85
86 x=6;
87
88 i=find(dadnc==6e-9);
89 j=find(dadnc==9e-9);
90 k=find(dadnc==5e-8);
91 l=find(dadnc==2e-7);
92 figure(2)
93 set(gca,'fontname','times','fontsize',12)
94 loglog(kmax(i,1:x),dk(i,1:x),'Ok','markersize',6,'
      markerfacecolor','y')
95 hold on

```

```

96 loglog(kmax(i,x+1:end),dk(i,x+1:end),'Ok','markersize',6)
97 loglog(kmax(j,1:x),dk(j,1:x),'Ok','markersize',6,'
    markerfacecolor','y')
98 loglog(kmax(j,x+1:end),dk(j,x+1:end),'Ok','markersize',6)
99 loglog(kmax(k,1:x),dk(k,1:x),'Ok','markersize',6,'
    markerfacecolor','y')
100 loglog(kmax(k,x+1:end),dk(k,x+1:end),'Ok','markersize',6)
101 loglog(kmax(l,1:x),dk(l,1:x),'Ok','markersize',6,'
    markerfacecolor','y')
102 loglog(kmax(l,x+1:end),dk(l,x+1:end),'Ok','markersize',6)
103
104 loglog(kkmax,ddk,'-k','markersize',6)
105
106 hold off
107 axis([1 1000 1 1000])
108
109 xlabel('K_m_a_x[MPam0.5'],'fontname','times','fontsize',14);
110 ylabel('{\Delta K}[MPam0.5'],'fontname','times','fontsize',14)
111 legend('R-constant Data R < R_c ','R-constnat Data R > R_c ','
    location','northeast')
112 %text(1.1,80,'Ti-6Al-4V Constant R')
113 %text(1.1,65,'Lab air and room temp.')
114 %text(1.1,50,'A.H Noroozi, G. Glinka')
115 %text(1.1,40,'S. Lambert(2007)')
116
117 %text(20,20,['R_c =',num2str(rc)],'backgroundcolor','y')
118 %text(40,3,['da/dN =',num2str(dadnc(i))])
119 %text(40,4,['da/dN =',num2str(dadnc(j))])

```

```

120 %text(40,10,['da/dN =',num2str(dadnc(k))])
121 %text(40,15,['da/dN =',num2str(dadnc(l))])
122
123 %%
124 figure(3)
125 subplot(2,2,1)
126 loglog(r0(:,1),r0(:,2),'or',r01(:,1),r01(:,2),'^b',r03(:,1),r03
   (:,2),'sk',r05(:,1),r05(:,2),'sg',r07(:,1),r07(:,2),'vr',r08
   (:,1),r08(:,2),'>k',r_1(:,1),r_1(:,2),'sk',r_3(:,1),r_3(:,2),
    'om',r_5(:,1),r_5(:,2),'<b')
127 legend('R=0','R=0.1','R=0.3','R=0.5','R=0.7','R=0.8','R=-1','R
    =-3','R=-5','location','southeast')
128 title('Ti-6Al-4V - Glinka')
129 xlabel('DK[MPam^0.^5]');ylabel('da/dN [m/cycle]')
130 subplot(2,2,2)
131 loglog(r0(:,3),r0(:,2),'or',r01(:,3),r01(:,2),'^b',r03(:,3),r03
   (:,2),'sk',r05(:,3),r05(:,2),'sg',r_1(:,3),r_1(:,2),'sk',r_3
   (:,3),r_3(:,2),'om',r_5(:,3),r_5(:,2),'<b')
132 legend('R=0','R=0.1','R=0.3','R=0.5','R=-1','R=-3','R=-5','
    location','southeast')
133 title('Ti-6Al-4V - Glinka')
134 xlabel('K_m_a_x[MPam^0.^5]');ylabel('da/dN [m/cycle]')
135 subplot(2,2,3)
136 loglog(r05(:,1),r05(:,2),'sg',r07(:,1),r07(:,2),'vr',r08(:,1),
    r08(:,2),'>k')
137 legend('R=0.5','R=0.7','R=0.8','location','southeast')
138 title('Ti-6Al-4V - Glinka')
139 xlabel('DK[MPam^0.^5]');ylabel('da/dN [m/cycle]')

```



```

140 subplot(2,2,4)
141 loglog(r0(:,3),r0(:,2),'or',r01(:,3),r01(:,2),'^b',r03(:,3),r03
   (:,2),'sk',r05(:,1),r05(:,2),'sg',r07(:,1),r07(:,2),'vr',r08
   (:,1),r08(:,2),'>k',r_1(:,3),r_1(:,2),'sk',r_3(:,3),r_3(:,2),
    'om',r_5(:,3),r_5(:,2),'<b')
142 legend('R=0','R=0.1','R=0.3','R=0.5','R=0.7','R=0.8','R=-1','R
    =-3','R=-5','location','southeast')
143 title('Ti-6Al-4V - Glinka')
144 xlabel('DK or K_max [MPam0.5]');ylabel('da/dN [m/cycle]')
145
146 %%
147 limit=3;
148 %[slopeu,interceptu,slopel,interceptl]=calslope(limit,dk,kmax);
149 %%
150 p=-0.33;
151 alpha=p/(-1+p);
152 rc=0.33;
153
154 r03fita(:,2)=r03(:,2);
155 r03fita(:,3)=r03(:,1)*((1-rc)/(1-0.3))^alpha;
156 r05fit(:,2)=r05(:,2);
157 r05fit(:,3)=r05(:,1)*((1-rc)/(1-0.5))^alpha;
158 r07fit(:,2)=r07(:,2);
159 r07fit(:,3)=r07(:,1)*((1-rc)/(1-0.7))^alpha;
160 r08fit(:,2)=r08(:,2);
161 r08fit(:,3)=r08(:,1)*((1-rc)/(1-0.8))^alpha;
162
163 q=1/p;

```

```

164 beta=q/(-1+q);
165 r0fit(:,2)=r0(:,2);
166 r0fit(:,3)=r0(:,3)*((1-0)/(1-rc))^(1-beta);
167 r01fit(:,2)=r01(:,2);
168 r01fit(:,3)=r01(:,3)*((1-0.1)/(1-rc))^(1-beta);
169 r_1fit(:,2)=r_1(:,2);
170 r_1fit(:,3)=r_1(:,3)*((1+1)/(1-rc))^(1-beta);
171 r_3fit(:,2)=r_3(:,2);
172 r_3fit(:,3)=r_3(:,3)*((1+3)/(1-rc))^(1-beta);
173 r_5fit(:,2)=r_5(:,2);
174 r_5fit(:,3)=r_5(:,3)*((1+5)/(1-rc))^(1-beta);
175 r03fit(:,2)=r03(:,2);
176 r03fit(:,3)=r03(:,3)*((1-0.3)/(1-rc))^(1-beta);
177
178 %to collapse into single curve
179 r0fit(:,1)=r0(:,1)*((1-rc)/(1-0))^(beta);
180 r_1fit(:,1)=r_1(:,1)*((1-rc)/(1+1))^(beta);
181 r_3fit(:,1)=r_3(:,1)*((1-rc)/(1+3))^(beta);
182 r_5fit(:,1)=r_5(:,1)*((1-rc)/(1+5))^(beta);
183 r03fit(:,1)=r03(:,1)*((1-rc)/(1-0.3))^(beta);
184
185 %%
186
187 figure(7)
188 set(gca, 'fontname', 'times', 'fontsize', 16)
189
190 loglog(r0(:,1), r0(:,2), 'b<', r01(:,1), r01(:,2), 'bd', r03(:,1), r03
    (:,2), 'ro', r_1(:,1), r_1(:,2), 'c^', r_3(:,1), r_3(:,2), 'bv', r_5

```

```

        (:,1),r_5(:,2),'b.',...
191     'markersize',10,'markerfacecolor',[1 0.8 0.7])
192 hold on
193 loglog(r05(:,1),r05(:,2),'ks',r07(:,1),r07(:,2),'bp',r08(:,1),
        r08fit(:,2),'g>',...
194     'markersize',10,'markerfacecolor',[0.8 1 0.8])
195 xlabel('\Delta K [MPa0.5]', 'fontsize',16,'fontname','times
        new roman');
196 ylabel('da/dN [m/cycle]', 'fontsize',16,'fontname','times new
        roman');
197 axis([1 1000 1e-11 1e-5])
198 legend('R=0','R=0.1','R=0.3','R=-1','R=-3','R=-5','R=0.5','R=0.7
        ','R=0.8','location','southeast')
199
200 %text(1.2,5e-6,'Ti-6Al-4V','fontname','times','fontsize',14)
201 %text(1.2,2.5e-6,'Lab air and room temp.','fontname','times','
        fontsize',14)
202 %text(1.2,1.2e-6,'A.H Noroozi, G. Glinka')
203 %text(1.2,1.2e-6,'Glinka et. al. (2007)','fontname','times','
        fontsize',14)
204 %text(1.2,6e-7,'S. Lambert(2007)')
205 %text(1.2,2.5e-6,'(a)','fontname','times','fontsize',14)
206
207 hold off
208
209 %%
210 figure(8)
211 set(gca,'fontname','times','fontsize',12)

```

```

212 loglog(r0fit(:,3),r0fit(:,2),'b<',r01fit(:,3),r01fit(:,2),'bd',
           r03fit(:,3),r03fit(:,2),'ro',r_1fit(:,3),r_1fit(:,2),'c^',
           r_3fit(:,3),r_3fit(:,2),'bv',r_5fit(:,3),r_5fit(:,2),'b.',...
213         'markersize',6,'markerfacecolor',[1 0.8 0.7])
214 xlabel('K_{max driving} [MPam^{0.5}]','fontsize',14,'fontname','
           times new roman');
215 ylabel('da/dN [m/cycle]','fontsize',14,'fontname','times new
           roman');
216 axis([1 100 1e-10 1e-5])
217 legend('R=0','R=0.1','R=0.3','R=-1','R=-3','R=-5','location','
           southeast')
218
219 %text(1.2,5e-6,'Ti-6Al-4V Constant R Test Data')
220 %text(1.2,2.5e-6,'Lab air and room temp.')
221 %text(1.2,1.2e-6,'A.H Noroozi, G. Glinka')
222 %text(1.2,6e-7,'S.Lambert(2007)')
223
224 %text(1.3,2e-7,'{\it K_{max driving}} = {\it K_{max}}[(1-R)/(1-R_c)
           ]^{\it (1-\beta)}'), 'backgroundcolor',[1 0.8 0.7])
225 %text(1.3,7e-8,'R < R_c')
226 %text(1.3,3.5e-8,['R_c = ',num2str(rc)])
227 %text(1.3,2e-8,['q = ',num2str(q)])
228 %%
229 figure(9)
230 set(gca, 'fontname','times','fontsize',12)
231 loglog(r05fit(:,3),r05fit(:,2),'ks',r07fit(:,3),r07fit(:,2),'bp'
           ,r08fit(:,3),r08fit(:,2),'g>',r03fita(:,3),r03fita(:,2),'ro'
           ,...

```

```

232     'markersize',6,'markerfacecolor',[0.8 1 0.8])
233 xlabel('\Delta K_{driving} [MPa m^{0.5}]','fontsize',14,'fontname'
    , 'times new roman');
234 ylabel('da/dN [m/cycle]','fontsize',14,'fontname','times new
    roman');
235 legend('R=0.5','R=0.7','R=0.8','R=0.3','location','southeast')
236 axis([1 100 1e-10 1e-5])
237
238 %text(1.2,5e-6,'Ti-6Al-4V Constant R Test Data')
239 %text(1.2,2.5e-6,'Lab air and room temp. ')
240 %text(1.2,1.2e-6,'A.H Noroozi, G. Glinka ')
241 %text(1.2,6e-7,'S. Lambert(2007) ')
242
243 %text(1.3,2e-7,'\{ \it \Delta K_{driving} \} = \{ \it \Delta K[(1-R_c)/(1-
    R)] \}^{\alpha} ','backgroundcolor',[0.8 1 0.8])
244 %text(1.3,7e-8,'R < R_c')
245 %text(1.3,3.5e-8,['R_c = ',num2str(rc)])
246 %text(1.3,2e-8,['q = ',num2str(q)])
247 %%
248
249 figure(10)
250 set(gca,'fontname','times','fontsize',12)
251 loglog(r0fit(:,3),r0fit(:,2),'b<',r01fit(:,3),r01fit(:,2),'bd',
    r03fit(:,3),r03fit(:,2),'ro',r_1fit(:,3),r_1fit(:,2),'c^',
    r_3fit(:,3),r_3fit(:,2),'bv',r_5fit(:,3),r_5fit(:,2),'b.',...
252     'markersize',6,'markerfacecolor',[1 0.8 0.7])
253 hold on

```

```

254 loglog(r05fit(:,3),r05fit(:,2),'ks',r07fit(:,3),r07fit(:,2),'bp',
           ,r08fit(:,3),r08fit(:,2),'g>',r03fita(:,3),r03fita(:,2),'ro',
           ,...
255         'markersize',6,'markerfacecolor',[0.8 1 0.8])
256 xlabel(' \Delta K_{driving} or K_{max driving} [MPam^{0.5}] ','',
           'fontsize',14,'fontname','times new roman');
257 ylabel('da/dN [m/cycle] ','', 'fontsize',14,'fontname','times new
           roman');
258 axis([1 100 1e-10 1e-5])
259 legend('R=0','R=0.1','R=0.3','R=-1','R=-3','R=-5','R=0.5','R=0.7
           ','R=0.8','location','southeast')
260
261 %text(1.2,5e-6,'Ti-6Al-4V Constant R Test Data')
262 %text(1.2,2.5e-6,'Lab air and room temp.')
263 %text(1.2,1.2e-6,'A.H Noroozi, G. Glinka')
264 %text(1.2,6e-7,'S. Lambert(2007)')
265
266 %text(1.3,2e-7,'{\it \Delta K_{driving}} = {\it \Delta K[(1-R_c)/(1-
           R)]}^{\it \alpha}','backgroundcolor',[0.8 1 0.8])
267 %text(3.5,2e-10,'{\it K_{max driving}} = {\it K_{max}}[(1-R)/(1-R_c
           )]]^{\it (1-\beta)}','backgroundcolor',[1 0.8 0.7])
268 %text(1.3,7e-8,['R_c=',num2str(rc)])
269 %text(1.3,4e-8,['p=',num2str(p)])
270 %text(1.3,2e-8,['q=1/p=',num2str(q)])
271 hold off
272
273 %%
274 figure(11)

```

```

275 set(gca,'fontsize',16)
276 loglog(r0fit(:,1),r0fit(:,2),'b<',r01fit(:,1),r01fit(:,2),'bd',
        r03fit(:,1),r03fit(:,2),'ro',r_1fit(:,1),r_1fit(:,2),'c^',
        r_3fit(:,1),r_3fit(:,2),'bv',r_5fit(:,1),r_5fit(:,2),'b.',...
277         'markersize',10,'markerfacecolor',[1 0.8 0.7])
278 hold on
279 loglog(r05fit(:,3),r05fit(:,2),'ks',r07fit(:,3),r07fit(:,2),'bp',
        r08fit(:,3),r08fit(:,2),'g>',r03fita(:,3),r03fita(:,2),'ro',
        ...,
280         'markersize',10,'markerfacecolor',[0.8 1 0.8])
281 xlabel('\Delta K_{t} [MPam^{0.5}]');
282 ylabel('da/dN [m/cycle]');
283 axis([1 100 1e-11 1e-5])
284 legend('R=0','R=0.1','R=0.3','R=-1','R=-3','R=-5','R=0.5','R=0.7',
        'R=0.8','location','southeast')
285
286 %text(1.2,5e-6,'Ti-6Al-4V','fontname','times','fontsize',14)
287 %text(1.2,2.5e-6,'Lab air and room temp.','fontname','times','
        fontsize',14)
288 %text(1.2,1.2e-6,'A.H Noroozi, G. Glinka')
289 %text(1.2,1.2e-6,'Glinka et. al. (2007)','fontname','times','
        fontsize',14)
290 %text(1.2,6e-7,'S. Lambert(2007)')
291 %text(1.2,2.5e-6,'(b)','fontname','times','fontsize',14)
292
293 %text(1.3,2e-7,'\{it\Delta K_{driving}\} = \{it\Delta K[(1-R_c)/(1-
        R)]\}^{\{it\alpha\}','backgroundcolor',[0.8 1 0.8])

```

```

294 %text(3.5,2e-10,{\it K_{max driving}} = {\it K_{max}}[(1-R)/(1-R_c
      )]}^{\it(1-\beta)},'backgroundcolor',[1 0.8 0.7])
295 %text(1.3,7e-8,['R_t=',num2str(rc)],'fontname','times','fontsize
      ',14)
296 %text(1.3,4e-8,['p=',num2str(p)],'fontname','times','fontsize
      ',14)
297 %text(1.3,2e-8,['q=1/p=',num2str(q)],'fontname','times','
      fontsize ',14)
298 hold off

```


Appendix D

Matlab Program to Simulate FCG Model

```
1 clear all
2 clc
3
4 load 7075_T651_water_vapor.mat
5
6 expdata1=DI;
7 expdata2=Air;
8 expdata3=Pa2;
9 expdata4=Pa27;
10 expdata5=Pa47;
11 expdata6=Pa133;
12 expdata7=Pa67;
13 % Material Properties—————
14 E=70E3;
15 sy=520;
16 kc=56;
```

```

17 pressure=[101 100 2 2.7 4.7 13.3 67];
18 % Parameters
19 %tita1=60;tita2=62;tita3=80;tita4=77;tita5=76;tita6=75;tita7=70;
20 tita=[52 60 79 77 75 74 71];
21 dkth=[4.99 4.99 6 6 5.5 5.4 5.2];
22 r=0.1;
23 dk=(1-r).*kc;
24
25 %-----
26 dk=3:1:dk;
27 dk=dk';
28 kmax=dk./(1-r);
29 kmin=kmax.*r;
30
31 x=1;
32 tita=tita.*x;
33
34 dadn1=model4(dk,tita(1),dkth(1),dk,E,sy,kmax,r);
35 dadn2=model4(dk,tita(2),dkth(2),dk,E,sy,kmax,r);
36 dadn3=model4(dk,tita(3),dkth(3),dk,E,sy,kmax,r);
37 dadn4=model4(dk,tita(4),dkth(4),dk,E,sy,kmax,r);
38 dadn5=model4(dk,tita(5),dkth(5),dk,E,sy,kmax,r);
39 dadn6=model4(dk,tita(6),dkth(6),dk,E,sy,kmax,r);
40 dadn7=model4(dk,tita(7),dkth(7),dk,E,sy,kmax,r);
41
42 figure(1)
43 subplot(2,2,[1,3]);
44 set(gca,'fontname','times','fontsize',16)

```

```

45 loglog(expdata1(:,1),expdata1(:,2),'dk','markersize',10,'
    markerfacecolor','r')
46 hold on
47 loglog(expdata2(:,1),expdata2(:,2),'sk','markersize',10,'
    markerfacecolor','c')
48 loglog(expdata3(:,1),expdata3(:,2),'sk','markersize',10,'
    markerfacecolor','g')
49 %loglog(expdata4(:,1),expdata4(:,2),'^k','markersize',10,'
    markerfacecolor','k')
50 loglog(expdata5(:,1),expdata5(:,2),'^k','markersize',10,'
    markerfacecolor','y')
51 %loglog(expdata6(:,1),expdata6(:,2),'ok','markersize',10,'
    markerfacecolor','y')
52 loglog(expdata7(:,1),expdata7(:,2),'ok','markersize',10,'
    markerfacecolor','m')
53 loglog(dk,abs(dadn1),'-r','linewidth',1.5)
54 loglog(dk,abs(dadn2),'-b','linewidth',1.5)
55 loglog(dk,abs(dadn3),'-g','linewidth',1.5)
56 %loglog(dk,abs(dadn4),'-k','linewidth',1.5)
57 loglog(dk,abs(dadn5),'-k','linewidth',1.5)
58 %loglog(dk,abs(dadn6),'-k','linewidth',1.5)
59 loglog(dk,abs(dadn7),'-m','linewidth',1.5)
60 hold off
61 grid on
62 grid minor
63 axis([5 50 1e-8 3e-6])
64 legend('Distilled Water','Air','2.0 Pa','4.7 Pa','67 Pa','
    location','southeast')

```

```

65 %title('The effect of water vapor pressure on 7075-T651', 'color
    ', 'r', 'fontweight', 'bold', 'fontsize', 16, 'fontangle', 'italic')
66 xlabel('\Delta K [MPa m^{0.5}]', 'fontsize', 16, 'fontname', 'times
    ')
67 ylabel('da/dN [m/cycle]', 'fontsize', 16, 'fontname', 'times')
68 %text(20, 1e-7, ['\theta = (', num2str(x), ') \theta'], 'fontsize', 14)
69 text(6, 2e-6, '(A)', 'fontsize', 14)
70
71 subplot(2, 2, 2)
72 set(gca, 'fontname', 'times', 'fontsize', 16)
73 plot(pressure, tita, 'sk', 'markerfacecolor', 'y', 'markersize', 10)
74 xlabel('Water Vapor Pressure, Pa', 'fontsize', 16, 'fontname', '
    times')
75 ylabel('Crack Angle, \theta', 'fontsize', 16, 'fontname', 'times')
76 text(125, 75, '(B)', 'fontsize', 14)
77 grid on
78
79 subplot(2, 2, 4)
80 set(gca, 'fontname', 'times', 'fontsize', 16)
81 plot(pressure, dk, 'sk', 'markerfacecolor', 'r', 'markersize', 10)
82 xlabel('Water Vapor Pressure, Pa', 'fontsize', 16, 'fontname', '
    times')
83 ylabel('\Delta K_c [MPa m^{0.5}]', 'fontsize', 16, 'fontname', 'times
    ')
84 text(10, 51.25, ['K_c = ', num2str(kc), ' [MPa m^{0.5}]'], 'fontsize',
    , 14)
85 text(125, 51.25, '(C)', 'fontsize', 14)
86 grid on

```

```

87
88
89 % text(1.5,5e-5,['A = ',num2str(A)], 'fontsize ',14)
90 % text(1.5,5e-5,['R = ',num2str(r)], 'fontsize ',16)
91 %text(6,2e-6,['\Delta K_{th} = ',num2str(dkth,3)], 'fontsize ',16)
92 %text(30,5e-7,'67Pa', 'fontsize ',16, 'fontweight ', 'bold ')
93 %text(6,1e-6,['\Delta K_c = ',num2str(dkc,4)], 'fontsize ',16)
94 %text(6,1.5e-6,['\theta = ',num2str(tita)], 'fontsize ',16)
95 %text(6,6e-7,['\sigma_{y} = ',num2str(sy)], 'fontsize ',16)
96 %text(1.5,2e-8,['p = ',num2str(p)], 'fontsize ',14)
97 %text('Interpreter','latex','String','$$da/dN=\frac{\{3C\}\{8E\}(K^2_{\max}-K^2_{ref})}{\frac{\{left[1-\frac{\{\Delta K_{th}\}}{\{\Delta K\}}\right]\{\frac{\{\Delta K\}}{\{\Delta K_c\}}\right]\}}$$',
        'Position',[5 1e-9], 'Fontsize',12, 'fontweight','bold')
98 %text('Interpreter','latex','String','$$K_{ref}=\frac{\{\sqrt{\{2\}}\}\{\pi\}\Delta K_{\{(i-1) cycle\}}}{\{2\}}$$', 'Position',[5 1e-10],
        'Fontsize',12, 'fontweight','bold')
99 %text('Interpreter','latex','String','$$r_{ff}=\frac{\{\pi\}\{8\}\{left(\frac{\{K_{ref}\}}{\{\sigma_y\}}\right)^2\}}{\{2\}}$$', 'Position',[5 8e-12],
        'Fontsize',12, 'Fontweight','bold')
100
101 %saveas(h,'HDP08','pdf')

1 function dadn=model4(dk,tita,dkth,dkc,E,sy,kmax,r)
2 kmin=r.*kmax;
3 for i=1:length(dk)
4 %         if dk(i,1)>4
5 %             tita=88.7;
6 %             dkth=3;

```

```

7 %           $dkc=10$ ;
8 %          end
9      if i==1
10          kref(i,1)=kmin(i,1);
11      else
12          kref(i,1)=(0.25*exp(1.15*r)+0.2105)*kmax(i,1);
13          dadn(i,1)=((3*cotd(tita))/(2*8*E*sy))*(kmax(i,1)^2-kref(
              i,1)^2)*((1-dkth/dk(i,1))/(1-(dk(i,1)/dkc)));
14      end
15 end

```

Appendix E

Matlab Program to Simulate FCG Behavior for Overload

```
1 clear all
2 clc
3 close all
4
5 kmax=10;
6 kmin=1;
7 kmaxol=16;
8 sigmay=370;
9 E=72000;
10 lambda=3/(8*E*sigmay);
11 deltakth=3.5;
12 deltakc=31.5;
13 deltak=kmax-kmin;
14 deltakol=kmaxol-kmin;
15 beta=1.10;
16 c=cotd(85)/2;
```

```

17
18 a=[];
19 dadn=[];
20 krefi=((0.25*exp(0.115))+0.21)*kmax;
21 dadn(1)=c*lambda*(kmaxol^2-krefi^2)*((1-(deltakth/deltakol))
    /(1-(deltakol/deltakc)));
22 a(1)=dadn(1);
23 ap=a(1)+((kmaxol/sigmay)^2/(2*pi));
24 i=2;
25 while(1)
26
27     sigmaap=(sigmay/beta)*sqrt(2*(ap-a(i-1))/a(i-1));
28     sigma=(kmax/beta)*(1/sqrt(pi*a(i-1)));
29     sigmamin=(kmin/beta)*(1/sqrt(pi*a(i-1)));
30     sigmared=sigmaap-sigma;
31     sigmaxe=sigma-sigmared;
32     sigmine=sigmamin-sigmared;
33     if(sigmaxe<0)
34         sigmaxe=0;
35     end
36     if(sigmine<0)
37         sigmine=0;
38     end
39
40     r=sigmine/sigmaxe;
41     delk=beta*(sigmaxe-sigmine)*sqrt(pi*a(i-1));
42     kmaxe=delk/(1-r);
43     krefi=((0.25*exp(1.15*r))+0.21)*kmaxe;

```



```

44     dadn(i)=c*lambda*(kmaxe^2-krefi^2)*((1-(deltakth/delk))/(1-(
        delk/deltakc)));
45
46     a(i)=a(i-1)+dadn(i);
47     if(ap<a(i))
48         break;
49     end
50 figure(1)
51
52 semilogy(a(i)*1000,dadn(i),'ro')
53 axis([0 0.3 1e-10 1e-6])
54 hold on;
55 i=i+1;
56 end
57 am=a+0.1/1000;
58 am(1)=0.03/1000;
59 dadnm=dadn;
60 %%
61 apre=[-0.5 am(1)*1000 am(1)*1000];dadnpre=[dadnm(end) dadnm(end)
        dadnm(1)];
62 apost=[am(end)*1000 1];dadnpost=[dadnm(end) dadnm(end)];
63 %%
64 %load Al2324_OL_2data.mat
65 figure(2)
66 set(gca,'fontsize',16,'fontname','times')
67 semilogy(am.*1000,dadnm.*0.7,'-r','linewidth',4);
68 hold on
69 semilogy(data1(:,1),data1(:,2),'-ko');

```

```

70 semilogy(data2(:,1),data2(:,2),'-bo');
71 semilogy(data3(:,1),data3(:,2),'-go');
72 semilogy(apre,dadnpre.*0.6,'-r','linewidth',4);
73 semilogy(apost,dadnpost.*0.6,'-r','linewidth',4);
74 hold off
75 xlabel('a [mm]')
76 ylabel('da/dN [m/cycle]')
77 legend('Model','Exp. 1 Data','Exp. 2 Data','Exp. 3 Data',
        'location','southeast')
78 axis([-1 3 1e-11 1e-5])
79
80 text(-0.3,5e-6,'Integral Based Two Parameter Model','fontname','
        times','fontsize',16,'color','r')
81 text(2,3e-9,'Al 2324','fontname','times','fontsize',16)
82 text(2,1.5e-9,'OLR=2','fontname','times','fontsize',16)
83 text(0.2,5e-7,'K_{max OL}=20 MPa m^{0.5}','fontname','times',
        'fontsize',16)
84 text(0.5,3e-9,'K_{max BL}=10 MPa m^{0.5}','fontname','times',
        'fontsize',16)
85 text(0.5,1e-9,'K_{min}=1 MPa m^{0.5}','fontname','times',
        'fontsize',16)

```