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The Robustness of Confidence Intervals for the Mean of Delta Distribution

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THE ROBUSTNESS OF CONFIDENCE INTERVALS FOR THE MEAN OF DELTA DISTRIBUTION

by

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Mathew Anthony Cantos Rosales

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INTRODUCTION AND MOTIVATION

1.1 Background

Several techniques have been exploited in the statistical analysis of skewed data and yet, selection of the appropriate approach is usually a challenge. In addition, the problem gets more complicated when data are mixed with a non-ignorable proportion of zeros. Aitchison and Brown (1957) investigated this particular scenario and believed that the best way to handle this problem is by recognizing the dichotomy of the population into zero and nonzero parts. They proposed a two-part statistical model which they called delta distribution (Δ-distribution), for which the nonzero part is lognormally distributed. In an overview, Δ-distribution is a mixture of a lognormal distribution with mean μ and variance σ^2 of the log scale and a singular distribution of zero with probability δ. The phenomenon of clumping of zeros in the data is very common in disciplines like insurance, medical businesses, life sciences, marine sciences, engineering, etc.. Some examples are:

Example 1 **(Zhou and Tu, 2000) (Medical)**: Diagnostic testing is a costly and discretionary practice and overuse of this could lead to inappropriately high diagnostic charges. Diagnostic test charge data have two interesting characteristics. First, the data contain a good number of zero observations (patients who refuse or do not undergo diagnostic tests during a study). Second, the nonzero part of the data is positively skewed which is commonly modeled by a lognormal distribution. It is often of interest to make inference on the mean of the diagnostic test charge.

Example 2 **(Owen and DeRouen, 1980) (Industry)**: There is a law protecting workers in the United States against too much exposure to air contaminants. A legal maximum allowable safe amount of exposure to air contaminants has been set and in order to maintain this, industrial hygienists periodically check workers exposure by accompanying a randomly-selected worker during the course of the day, obtaining 10-20 short grab sample measurements of the concentration of a specified contaminant at randomly selected times. Hygienists are guided by handbooks in making statistical inference for air contaminants which assumes data are lognormally distributed. However, these handbooks do not provide methods for situations where data contain several zero observations and censored values. Based on the study conducted by Owen and DeRouen, the Δ-distribution technique is appropriate for both situations.

Example 3 **(Pennington, 1983) (Marine Science)**: Large areas are usually involved in survey sampling of multi-species of fish and plankton. With this nature, it is not surprising that a particular species occupies only a certain part of the total survey area. Hence, it is expected that a considerable proportion of zeros is present in the data which only represents unsuitable or unoccupied habitat. In addition, it is also expected that nonzero values are highly skewed to the right. These data features are typical in estimation of abundance. Pennington performed a study on abundance estimation of ichthyoplankton, for which he used

Δ-distribution.

Example 4 **(Moulton and Halsey, 1995) (Pharmaceutical)**: In Phase I and II trials of new vaccines, serum antibody concentrations which are reported as geometric mean titers of concentrations (GMC) are used to describe immunogenicity. GMC is typically analyzed under lognormality assumption. However, problems arise due to left-censored values being reported and lack of sensitivity of the analysis when the concentrations are near zero.

It is apparent that there are several instances of skewed data mixed by a substantial proportion of zeros that we have to deal with. In practice, despite the gain in popularity of delta distribution, the traditional method, which is the logarithm transformation of the raw data before proceeding to a standard normal-theory analysis, is commonly used for analyzing skewed data. In order to overcome the complication of logarithmic transformation of zero values, an additive constant c is chosen before transformation. This method is very appealing because of convenience and ease of implementation. However, this practice is prone to problems and the conclusion is greatly affected by the choice of the constant c (Berry, 1987 and Fletcher, 2008).

It is worth mentioning that all examples presented above used full assumption of lognormality of the nonzero part of the data. Previous studies conducted for this situation concluded that the mean estimator obtained for Δ-distribution is more efficient than the ordinary sample mean. However, this gain in efficiency can only be achieved when model assumptions are well established. Therefore, estimators based on Δ-distribution must be examined under not-so-perfect conditions:

- Lognormality assumption of nonzero values is not met

- Presence of contaminants
- Proportion of zeros is too high
- Variability of nonzero part is too high or too small
- Small sample size

1.2 Brief Review of Lognormal Distribution

Lognormal distribution is the probability distribution of any right skewed random variable whose logarithm is normally distributed. It is denoted as LN(μ,σ^2) where μ and σ^2 are the mean and variance of the log scale, respectively.

Suppose $W_1, ..., W_n$ follow LN(μ, σ^2) then the *probability density function* of W is given as

$$f(w, \mu, \sigma^2) = \frac{1}{w\sigma\sqrt{2\pi}} e^{-[\log(w)-\mu]^2/2\sigma^2} \quad \text{for } w > 0 \text{ and } \sigma > 0$$

with mean and variance of W expressed respectively as

$$E[W] = e^{\mu+\sigma^2/2} \tag{1.1}$$

$$Var[W] = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1) \quad . \tag{1.2}$$

It can be seen in Figure 1.1 that for a given μ, skewness is directly related to σ^2, i.e., as σ^2 increases, skewness also increases.

Maximum likelihood estimation of lognormal distribution is exactly equivalent to maximum likelihood estimation of normal distribution. This means that maximum likelihood parameter estimators of μ and σ^2 of normal distribution also hold for lognormal. Therefore,

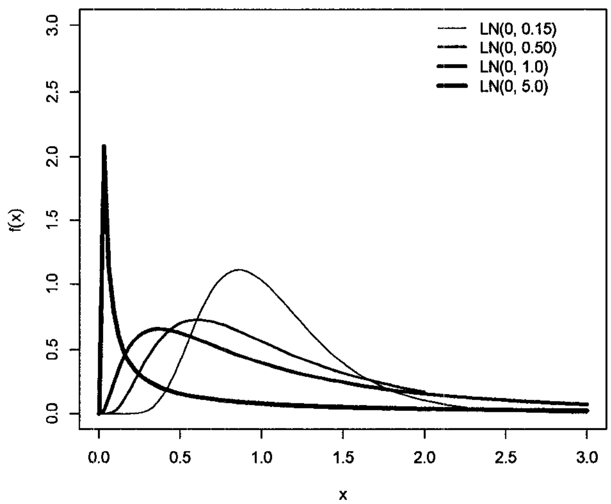

Figure 1.1: Lognormal Distribution with Fixed μ=0

$$\hat{\mu} = \frac{\sum_{i=1}^{n} \log(W_i)}{n} \tag{1.3}$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} [\log(W_i) - \mu]^2}{n} \quad . \tag{1.4}$$

1.3 The Δ-Distribution

Delta distribution is a mixture of lognormal distribution LN(μ,σ^2) and a distribution that is degenerate at zero. It is denoted as $\Delta(\delta, \mu, \sigma^2)$ where δ is the probability of zero, and μ and σ^2 are the mean and variance of the log scale of positive values, respectively. When δ is 0, Δ-distribution reduces to LN(μ,σ^2).

The formal definition of Δ-distribution is given as follows: A random variable Y has a $\Delta(\delta, \mu, \sigma^2)$ distribution if

$$
\begin{aligned}
P(Y < 0) &= 0 \\
P(Y = 0) &= \delta \\
P(Y \leq y) &= \delta + (1-\delta)F(y|\mu,\sigma^2) \text{ for } y > 0
\end{aligned}
$$

where $F(y|\mu,\sigma^2)$ is the cumulative density function of $LN(\mu,\sigma^2)$. The mean and variance of Y are:

$$
E[Y] \quad = \kappa = \quad (1-\delta)e^{\mu+\sigma^2/2} \tag{1.5}
$$

$$
Var[Y] \quad = \nu = \quad (1-\delta)e^{2\mu+\sigma^2}(e^{\sigma^2} - (1-\delta)) \tag{1.6}
$$

respectively. In general, the rth moment of Y is

$$
E[Y^r] = (1-\delta)e^{r\mu+r^2\sigma^2/2} \quad .
$$

Neither the *probability density function* nor the *probability mass function* with respect to Lebesgue measure on the real line exist for Δ-distribution.

1.3.1 Estimators of δ, κ and ν

Suppose $y_1, y_2, \ldots, y_n$ is a random sample of size n taken from delta distribution. Without loss of generality, assume the first n_0 observations are zero and the rest $n_1 = n - n_0$ observations are nonzero and let X=log(Y) for $Y > 0$. Note that n_1 follows a binomial distribution $BIN(n, 1-\delta)$. Then, the likelihood of the sample is

$$
\begin{aligned}
L(\psi, \boldsymbol{Y}) &= \delta^{n_0}[(1-\delta)f(Y_i,\mu,\sigma^2)]^{n_1} \\
&= \delta^{n_0}(1-\delta)^{n_1}\left(\frac{1}{\log Y_i\sqrt{2\pi\sigma}}\right)^{n_1} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n_1}(\log Y_i - \mu)^2\right\} \quad (1.7)
\end{aligned}
$$

where ψ is the vector of parameters. From above, it can be shown that

$$\hat{\delta} = n_0/n \tag{1.8}$$

$$\hat{\mu} = \frac{\sum_{i=1}^{n_1} \log y_i}{n_1} = \frac{\sum_{i=1}^{n_1} x_i}{n_1} = \bar{x} \tag{1.9}$$

$$s^2 = \frac{\sum_{i=1}^{n_1} (\log y_i - \hat{\mu})^2}{n_1 - 1} = \frac{\sum_{i=1}^{n_1} (x_i - \bar{x})^2}{n_1 - 1} \tag{1.10}$$

are joint complete sufficient estimators of δ, μ and σ^2, respectively. Aitchison and Brown (1957) derived minimum variance unbiased estimators of κ and ν as functions of $\hat{\delta}$, $\hat{\mu}$ and s^2. They are given as

$$\hat{\kappa} = \begin{cases} \frac{n_1}{n} e^{\hat{\mu}} G_{n_1}\left(\frac{s^2}{2}\right) & \text{if } n_1 > 1 \\ \frac{x_1}{n} & \text{if } n_1 = 1 \\ 0 & \text{if } n_1 = 0 \end{cases} \tag{1.11}$$

$$\hat{\nu} = \begin{cases} \frac{n_1}{n} e^{2\hat{\mu}} \left\{ G_{n_1}(2s^2) - \frac{n_1-1}{n-1} G_{n_1}\left(\frac{n_1-2}{n_1-1} s^2\right) \right\} & \text{if } n_1 > 1 \\ \frac{x_1^2}{n} & \text{if } n_1 = 1 \\ 0 & \text{if } n_1 = 0 \end{cases} \tag{1.12}$$

where

$$G_{n_1}(t) = 1 + \frac{n_1 - 1}{n_1} t + \sum_{j=2}^{\infty} \frac{(n_1 - 1)^{2j-1} t^j}{n_1^j (n_1 + 1)(n_1 + 3) \cdots (n_1 + 2j - 3) j!} \tag{1.13}$$

is a Bessel function often used in literature on lognormal distribution. For function $G_{n_1}(t)$, $G_{n_1}(s^2) = e^{[(n_1-1)/n_1]\sigma^2}$, so that

$$\lim_{n_1 \to \infty} G_{n_1}(s^2) = e^{\sigma^2} \quad .$$

When s^2 is relatively high, the estimator $\hat{\kappa}$ is much more efficient than the ordinary sample mean (Pennington, 1983).

The sample variance (s^2/n) is commonly used in measuring the precision of the average. However, for small to moderate sample sizes, sample variance often underestimates the true variability of the average for highly skewed distributions, such as lognormal distribution. As an alternative, Pennington (1983) derived an unbiased estimator of the variance of $\hat{\kappa}$ which is said to be more efficient than the sample variance of the average in Δ-distribution. It was expressed as

$$\hat{\nu}_{pen}(\hat{\kappa}) = \begin{cases} \frac{n_1}{n} e^{2\hat{\mu}} \left\{ \frac{n_1}{n} G_{n_1}\left(\frac{s^2}{2}\right) - \frac{n_1-1}{n-1} G_{n_1}\left(\frac{n_1-2}{n_1-1} s^2\right) \right\} & \text{if } n_1 > 1 \\ \left(\frac{x_1}{n}\right)^2 & \text{if } n_1 = 1 \\ 0 & \text{if } n_1 = 0 \end{cases} . \tag{1.14}$$

1.3.2 Previous Studies on Δ-Distribution Estimators

The ordinary sample mean is an unbiased estimator of the population mean, regardless of the distribution. But it is a known fact that the sample mean is very sensitive to very large observations, and consequently, tends to overestimate the population mean when sample size is relatively small. The magnitude of overestimation depends mainly on how extreme the large observation is and this overestimation can be very significant. In addition, the variance of the sample mean is also very sensitive to large observations and might indicate a very low precision.

Several authors studied the Δ-distribution with the aim of finding a better estimator than the ordinary sample mean. Efficiencies of Δ-distribution estimators relative to sample mean and variance have been studied and defined as

$$\text{Relative efficiency of } \hat{\kappa} = \frac{MSE(\hat{\kappa})}{MSE(\bar{x})} = \frac{E[\hat{\kappa} - \kappa]^2}{E[\bar{x} - \kappa]^2}$$

$$\text{Relative efficiency of } \hat{\nu}_{pen}(\hat{\kappa}) = \frac{MSE(\hat{\nu}_{pen}(\hat{\kappa}))}{MSE(s^2)} = \frac{E[\hat{\nu}_{pen}(\hat{\kappa}) - \nu(\hat{\kappa})]^2}{E[\hat{\nu}(\bar{x}) - \nu(\bar{x})]^2} .$$

They have proven that the estimator $\hat{\kappa}$ and its variance estimator $\hat{\nu}_{pen}(\hat{\kappa})$ are very good alternatives for the ordinary sample mean and variance, respectively, particularly in a situation when data is highly skewed and mixed with a good number of zero observations. A substantial gain in efficiency is achieved and the shortcoming of the sample mean is avoided with the use of Δ-distribution estimators. However, extra caution must be observed and model assumption must first be verified.

Smith (1988) compared the efficiency of the sample mean relative to the MVUE, $\hat{\kappa}$. It was shown that for small samples (n=15), gain in efficiency is apparent for small σ^2 and the efficiency drops as σ^2 increases. The potential gain in precision is achieved when the model is appropriate.

Robustness of the lognormal-based estimators ($\hat{\kappa}$, $\hat{\nu}(\hat{\kappa})$) was investigated by Myers and Pepin (1990) through contamination of the sample. Data in abundance estimation are usually modeled as lognormal distribution. However, they found out that 51 of the 78 samples collected were best fit by either gamma or Weibull distributions. In their simulation study, they contaminated pseudo-random lognormal observations with pseudo-random observations from gamma, Weibull or Birnbaum-Saunders distribution with the same mean and variance as the lognormal. The study showed that $\hat{\kappa}$ and $\hat{\nu}(\hat{\kappa})$ are very sensitive to violations in model assumptions. Increase in bias was noticed when there was contamination. Earlier claims were proven that in the absence of contamination, efficiency of lognormal-based estimators is directly related to sample size and inversely related to the proportion of zeros present in the data.

Pennington (1991) argued that the bias observed by Myers and Pepin is due to the fact that log(X) $\rightarrow -\infty$ as x $\rightarrow 0$ and the distributions used for contamination had high probability of values close to zero. He believes that a reasonable lower bound to the alternative distributions should be implemented. In his own simulation, all observations less than one were set to zero. His results showed that delta distribution estimators demonstrated robustness even in extreme cases.

Myers and Pepin replied back saying their results did not suffer from the limitation claimed by Pennington. They pointed out that Birnbaum-Saunders distribution has a shape similar to lognormal distribution but the Δ-distribution estimators still experienced increase in bias when contamination came from this distribution. Also, they said surveys in abundance estimation can have values much less than 1.

Syrjala (2000) demonstrated that the supposedly unbiased estimators of the delta distribution were clearly biased. The magnitude of bias increases as the population from which the samples were drawn departs from lognormality. He found out that $\hat{\kappa}$ has "curious" property. The mean estimator $\hat{\kappa}$ increases as the largest observation in the sample increases. Surprisingly, the mean estimator $\hat{\kappa}$ also increases as the smallest positive observation in the sample decreases. The estimated mean $\hat{\kappa}$ could even exceed the largest observation in the sample when the smallest positive observation is made very small (almost zero). This could happen because the estimator $\hat{\kappa}$ is a function of the sample variance of the logarithms s^2, and s^2 increases rapidly as the smallest positive observation becomes really small. He recommended that modified robust Δ-distribution estimators should be developed.

1.3.3 Interval Estimation for the Mean of Δ-Distribution

Inference about the mean is often of interest and confidence interval easily provides magnitude of an effect and gives a good overview on the possible value of the unknown parameter. There are seven existing methods of calculating the confidence interval for the mean κ that have been examined. Procedures on how to construct these intervals will be discussed in great detail in Chapter 2.

Two methods are based on the minimum variance unbiased estimators (MVUE). The first method is using the normality approximation of Aitchison and Brown's estimator for the mean $\hat{\kappa}$ and Pennington's estimator of the variance of $\hat{\kappa}$. In situations where skewness level is high, this method does not perform very well in terms of coverage probability, regardless of sample size. Also, this method was said to be the worst (Fletcher, 2008). Instead of Pennington's estimator, the second method is using Aitchison's estimator of the variance based on large sample approximation. This method has a good coverage when skewness is small but the coverage deteriorates rapidly when skewness increases (Zhou and Tu, 2000). Both methods may provide occasionally a negative lower bound which is not possible in real application.

The next two methods are based on maximum likelihood estimates(MLEs). These procedures will first calculate the interval of the log scale and then back-transform the endpoints. The third method was proposed by Zhou and Tu (2000). An adjustment to the variance estimator of the log scale was made since the MLE for σ^2 is biased. The fourth method was proposed by Fletcher (2008). The only difference between these two methods is the variance calculation. Fletcher believes that Δ-distribution should not be used in situations where there is none or only 1 positive observation in the data. Therefore, he adjusted the variance by assuming that n_1 follows a truncated binomial distribution. Both methods use the normality approximation of the MLE. The performance of these methods is about the same in terms of coverage probability. However, method 3 usually provides wider interval than method 4.

Fifth and sixth methods are both based on likelihood ratio. Method 5 uses the function $W(\theta) = 2[l(\hat{\theta}) - l(\theta)]$ that has an approximate chi-square distribution with 1 degree of freedom, χ_1^2. Through numerical search, one can obtain the interval by satisfying the equation

$$P(W(\theta) \leq \chi^2_{1,\alpha/2}) = \alpha/2 \quad .$$

The sixth method is a modification of method 5. The type of adjustment done was first introduced by Barndorff-Nielsen (1986) to help reduce the error due to normal approximation. This method provides better coverage and better balance between lower and upper error rates compared to method 5 (Tian and Wu, 2006). However, this is a very complex and computer-intensive procedure.

The last method is a bootstrap version of method 3. This method was first introduced by Angus (1994) for the mean of a lognormal distribution. The technique is to express a pivotal statistic as a function of sufficient statistics following certain distributions and then obtain the percentile-t bootstrap. This method improves coverage accuracy of method 3 and performs well on small sample sizes with high level of skewness (Zhou and Tu, 2000).

It can be noted that the first 6 methods use large sample approximation. Also, all methods are functions of $\hat{\mu}$ and s^2 which are known to be not robust.

Coverage accuracies of methods 2, 3, 5 and 7 were presented and examined by Zhou and Tu (2000). Based on their simulation, Method 7 provides the best overall accuracy. Method 3 is the best when skewness is small. Method 5 outperforms other methods for moderate to large sample size. However, they commented that these methods may not be appropriate when model assumptions are not met and further research is needed to check the robustness of these methods.

In addition, Fletcher (2008) commented: *"Further research is needed to assess the robustness of the methods to departures from the delta distribution: previous discussion of robustness in this context has tended to focus on point estimation, rather than coverage of confidence interval."*

1.4 Research Objectives

This dissertation will be developed based on the following four main objectives.

- First objective is to find a reasonably reliable interval method that does not involve computer-intensive procedures. This will be done by determining an interval that is easy to use without sacrificing coverage accuracy. Performance of existing methods will be assessed when model assumptions are not met and when contaminants are present in the sample. Based on the result of the examination, we will provide recommendations on which method is appropriate under such conditions.

- Second, to provide alternative methods that are easy to implement for situations where current methods do not perform well. The goal is to find sufficient robust estimators that can be reasonably applied into a variety of scenarios.

- Third, to provide recommendations on current software packages that can execute the methods quickly. In addition, a part of this objective is to develop computer algorithms on methods that require computational techniques, especially for bootstrap and likelihood ratio methods. Hopefully, providing the algorithm will reduce research time for others.

- Lastly, to promote the use of delta distribution. Transformation of data will not always solve the skewness problem, especially when data has clumping at zero. When used appropriately, we believe that Δ-distribution technique will provide substantial gain in accuracy.

1.5 Organization of Dissertation

Existing methods will be discussed in Chapter 2. Their performance based on previous simulation studies will be presented. Procedures on how to construct confidence interval for each method will be provided. Current software packages will be mentioned and computer algorithms will be given for methods that require intensive programming.

Chapter 3 will present the two methods we are proposing. The first proposal is based on the robust estimators for the Δ-distribution suggested by Al-Khouli (1999). The second proposal is a percentile t-bootstrap based on MLE with the assumption that n_1 follows a truncated binomial distribution.

Chapter 4 is all about simulation. It will provide the simulation framework for two simulation studies. The first simulation study is designed to check the performance of the methods in presence of outliers while the second simulation study is designed to examine the accuracy of the methods when the data depart from model assumption. Discussion and recommendation based on simulation results will also be part of this chapter.

Lastly, Chapter 5 will have the final conclusion and recommendations for future research.

Chapter 2

CONFIDENCE INTERVALS FOR THE MEAN κ

Uncertainty about the population mean κ can be assessed by using confidence interval (CI). By definition, confidence interval with 100(1-α)% confidence level (CL) is a set of parameter values which depends only upon the sample data and satisfies the condition

$$P(\kappa \in CI) = 1 - \alpha \quad .$$

Hence, the selection of a confidence level will determine how likely that the CI produced will contain the true value of κ. In practice, common choices for CL are 0.90, 0.95, and 0.99. The higher the CL, the wider the produced CI will be. The confidence limits $\hat{\vartheta}_{\alpha_1}$ and $\hat{\vartheta}_{1-\alpha_2}$ of the confidence interval are defined such that

$$P(\hat{\vartheta}_{\alpha_1} \leq \vartheta \leq \hat{\vartheta}_{1-\alpha_2}) = 1 - \alpha \quad ,$$

where $\alpha 1$ and $\alpha 2$ are the lower and upper error rates, respectively. Thus, CL is equal to 1 - α_1 - α_2. Usually, it is desired that $\alpha_1 = \alpha_2 = \alpha/2$. In this dissertation, we set the CL to be 95%. Hence, $0.025 = \alpha_1 = \alpha_2$.

In this chapter, existing methods for the construction of confidence interval for the mean κ will be provided. The discussion is mainly about how to construct the CI using a particular approach. Previous simulation results for each method will also be cited.

In Section 2.1, two confidence intervals based on minimum variance unbiased estimators (MVUE) will be discussed. Section 2.2 provides details of interval estimations based on maximum likelihood estimators. In Section 2.3, step-by-step procedure for obtaining likelihood ratio based intervals will be discussed. Computer algorithms will be provided in the Appendix. Lastly, Section 2.4 presents the bootstrap-based interval.

2.1 Intervals Based on MVUE

The MVUE based intervals rely on the assumption that the distribution of MVUE $\hat{\kappa}$ is approximately normal. This approach will provide symmetric confidence intervals which are not expected for the confidence interval for the mean of a skewed distribution. The general form of 100(1-α)% MVUE-based confidence interval is

$$\hat{\kappa} \pm z_{\alpha/2}\sqrt{\hat{\nu}(\hat{\kappa})}$$

where $z_{\alpha/2}$ is the upper quantile of order $\alpha/2$ of the standard normal distribution.

2.1.1 Aitchison and Pennington's Estimators

Given the expressions (1.11) and (1.14), one can easily derive the confidence interval for the mean κ. Using the above expression, the corresponding 100(1-α)% MVUE-based confidence interval using Aitchison and Pennington's estimators is

$$\hat{\kappa} \pm z_{\alpha/2}\sqrt{\hat{\nu}_{pen}(\hat{\kappa})} \quad .$$

The only challenging part is to compute the Bessel function $G_{n_1}(t)$. Thus, this section will focus on deriving the value of $G_{n_1}(t)$. Recall that $G_{n_1}(t)$ is of the form

$$G_{n_1}(t) = 1 + \frac{n_1 - 1}{n_1}t + \sum_{j=2}^{\infty} \frac{(n_1-1)^{2j-1}t^j}{n_1^j(n_1+1)(n_1+3)\cdots(n_1+2j-3)j!} \quad .$$

Let ω be the logarithm of the last term of $G_{n_1}(t)$. Then, for a given j, ω can be expressed as

$$\begin{aligned} \omega &= (2j-1)\log(n_1-1) + j\log(t/n_1) - \log\Gamma(j+1) \\ &\quad -[\log(n_1+1) + \log(n_1+3) + \cdots + \log(n_1+2j-3)] \quad . \end{aligned}$$

By using the derived expression for ω, a simple iterative approach can solve $G_{n_1}(t)$.

Algorithm:

1. Set $j = 2$.

2. Set a tolerance level (tol). The value of tol depends on the precision desired. The smaller the tol, the higher the precision.

3. Compute the first 2 terms ($1 + \frac{n_1-1}{n_1}$t) and call it g.

4. Compute ω.

5. If $e^\omega > tol$, then $g = g + e^\omega$ and $j = j + 1$. Repeat steps 4 and 5. Else, stop. Then $g = G_{n_1}(t)$.

An R code version of this algorithm is presented in Appendix A. For the rest of this dissertation, this method will be referred as **MVUE1**.

MVUE1 is said to be the worst method (Fletcher, 2008) especially when there is a high level of skewness in the data. According to a simulation study performed by Fletcher, an increase in sample size does not help improve the performance of this method. This method provides a very imbalanced upper error rate (UER) and lower error rate (LER).

2.1.2 Using Asymptotic Variance Estimate of $\hat{\kappa}$

Given a large n and a δ that is substantially less than 1, Aitchison and Brown (1957) derived an approximation to the variance of the mean estimator $\hat{\kappa}$ expressed as

$$\nu_\infty(\hat{\kappa}) = \frac{e^{2\mu+\sigma^2}}{n}\left[\delta(1-\delta) + \frac{(1-\delta)(2\sigma^2+\sigma^4)}{2}\right]. \tag{2.1}$$

Since δ, μ and σ^2 are usually unknown, Owen and DeRouen (1980) suggested to use the unbiased estimates from the sample which are $\hat{\delta}$, $\hat{\mu}$ and $\hat{\sigma}^2$, respectively. In a simulation study that they carried out, results indicated that expression (2.1) performed well for $n \geq 15$ in terms of mean squared error (MSE) with the use of unbiased estimators. Thus,

$$\hat{\nu}_\infty(\hat{\kappa}) = \frac{e^{2\hat{\mu}+s^2}}{n}\left[\hat{\delta}(1-\hat{\delta}) + \frac{(1-\hat{\delta})(2s^2+s^4)}{2}\right], \tag{2.2}$$

Therefore, the corresponding 100(1-α)% MVUE-based confidence interval with large sample approximation of the variance is

$$\hat{\kappa} \pm z_{\alpha/2}\sqrt{\hat{\nu}_\infty(\hat{\kappa})} \quad .$$

This approach does not only rely on the normality assumption of $\hat{\kappa}$, but also on the accuracy of variance approximation. It was shown in previous simulation study that this method provides intervals with good coverage when skewness is very small. However, as skewness increases, the accuracy deteriorates rapidly. This method will be referred as **MVUE2**.

2.2 Intervals Based on MLE

Intervals calculated based on MLE also assume normality approximation of the MLE $\hat{\theta}$ and involve a two-step procedure. First, to calculate the interval for $\theta = \log(\kappa)$. Second, to transform back the endpoints to their original scale. In general, the 100(1-α)% MLE-based confidence interval is of the form

$$exp[\hat{\theta} \pm z_{\alpha/2}\sqrt{\hat{\nu}(\hat{\theta})}]$$

where $\hat{\theta}$ is the estimated mean of the log scale, $\hat{\nu}(\hat{\theta})$ is the variance estimate of $\hat{\theta}$.

2.2.1 Bias-Corrected Approach

By maximizing the likelihood function of the Δ-distribution (1.8), one can obtain $\hat{\delta}$, $\hat{\mu}$ and $\frac{n_1-1}{n_1}s^2$ as the maximum likelihood estimators of δ, μ and σ^2, respectively. It can also be shown that $\frac{n_1-1}{n_1}s^2$ is a biased estimator of σ^2. Because of this, Zhou and Tu (2000) proposed s^2 as the estimator for σ^2 to remove the bias. Therefore, θ can be estimated as

$$\hat{\theta} = \log(1-\hat{\delta}) + \hat{\mu} + \frac{s^2}{2} \quad . \tag{2.3}$$

Using delta method, they derived a consistent variance estimator of $\hat{\theta}$ given as

$$\hat{\nu}_{bc}(\hat{\theta}) = \frac{n_0}{nn_1} + \frac{s^2}{n_1} + \frac{s^4}{2n_1} \quad . \tag{2.4}$$

Through asymptotic normality of $\hat{\theta}$, the corresponding 100(1-α)% MLE-based confidence interval with bias correction is

$$\left(e^{\hat{\theta}-z_{\alpha/2}\sqrt{\hat{\nu}_{bc}(\hat{\theta})}}, e^{\hat{\theta}+z_{\alpha/2}\sqrt{\hat{\nu}_{bc}(\hat{\theta})}}\right) \quad .$$

This method will be referred as **MLE-BC**. According to a simulation study carried out by Zhou and Tu (2000), this method performs best when skewness is small. Also, MLE-BC has a satisfactory performance for moderate to large samples but tends to provide poor coverage when sample size is small.

2.2.2 Truncated Binomial Approach

Fletcher (2008) agreed that a bias-corrected estimator of σ^2 must be used. However, Δ-distribution should not be used in situations where n_1 is 0 or 1. As a result, he assumed n_1 to follow a truncated binomial distribution. Given the situation, he derived an approximately unbiased variance estimator of $\hat{\theta}$ and expressed it as

$$\hat{\nu}_{tb}(\hat{\theta}) = \frac{(\hat{d}-\hat{c})(1-\hat{c}\hat{d}) - n_1(1-\hat{c})^2}{n_1(1-\hat{c}\hat{d})^2} + \frac{s^2}{n_1} + \frac{s^4}{2(n_1+1)} \tag{2.5}$$

where $\hat{c} = (n_0/n)^{n-1}$ and $\hat{d} = 1 + (n-1)(n_1/n)$.

Because of the restriction in n_1, $\hat{\delta}$ is not the MLE of δ anymore and the "true" MLE can only be obtained by numerical search (Finney, 1949). However, the difference between the "true" MLE and $\hat{\delta}$ is too small. Thus, expression (2.3) is still valid to use (Fletcher, 2008).

Therefore, the corresponding 100(1-α)% MLE-based confidence interval with truncated binomial adjustment is

$$\left(e^{\hat{\theta}-z_{\alpha/2}\sqrt{\hat{\nu}_{tb}(\hat{\theta})}},\, e^{\hat{\theta}+z_{\alpha/2}\sqrt{\hat{\nu}_{tb}(\hat{\theta})}}\right) .$$

This method will be referred as **MLE-TB**. This method is said to perform good enough (Fletcher, 2008) for moderate to large samples but tends to provide poor coverage when sample size is small. When sample size is small and has low level of skewness, MLE-TB tends to provide a very high upper confidence limit.

2.3 Intervals Based on Likelihood Ratio

In this section, two likelihood ratio-based methods will be presented. The procedure is much more complicated because of the presence of two nuisance parameters. The profile likelihood of θ is needed in order to compute the likelihood ratio statistic.

Let ψ be the vector of parameters with dimension equal to three. Then ψ can be partitioned into (θ, τ) where the scalar θ is the parameter of interest and the vector τ is the vector of nuisance parameters (μ, η). From expression (1.7) the log-likelihood function of Δ-distribution can be expressed as

$$\begin{aligned} l(\psi; \boldsymbol{Y}) &= l(\theta, \tau; \boldsymbol{Y}) \\ &= n_0 \log[1-\exp(\theta-\mu-\eta/2)] + n_1(\theta-\mu-\eta/2) \\ &\quad -n_1(\log(2\pi)+\log(\eta))/2 - (T+n_1\mu^2)/2\eta + S\mu/\eta \end{aligned} \tag{2.6}$$

where δ is re-expressed as $1-\exp(\theta-\mu-\eta/2)$, $\eta = [(n_1-1)/n_1]\sigma^2$, $T = \sum_{i=1}^{n_1} \log Y_i^2$ and $S = \sum_{i=1}^{n_1} \log Y_i$. The derivation of the interval in this section is mainly based on the log-likelihood ratio statistic given as

$$W(\theta) = 2[l(\hat{\theta}, \hat{\tau}; \boldsymbol{Y}) - l(\theta, \hat{\tau}_\theta; \boldsymbol{Y})] \tag{2.7}$$

where ($\hat{\theta}$, $\hat{\tau}$) are ML estimates of (θ, τ) and $\hat{\tau}_\theta$ is the ML estimate of τ for a fixed θ. Unfortunately, there is no explicit expression for $\hat{\tau}_\theta$ thus, an iterative approach must be performed. It is known that $W(\theta)$ approximately follows χ^2_1. Therefore, the 100(1-α)% confidence limits are the two values of θ that will produce $W(\theta)$ equal to 100(1-α)th percentile of χ^2_1 distribution.

Note that the process involved in deriving the confidence interval for κ is similar in Section 2.2 in the sense that confidence interval for θ will be obtained first and then transformed back to original scale.

2.3.1 Signed Likelihood Ratio Approach

Davison and Hinkley (1997) suggested a square root transformation of the log-likelihood ratio statistic $W(\theta)$ (2.7) which can be written as

$$r(\theta) = \text{sgn}(\hat{\theta} - \theta)\sqrt{W(\theta)} \quad . \tag{2.8}$$

The statistic $r(\theta)$, more popularly known as the signed log-likelihood ratio, approximately follows a standard normal distribution with error of order $O(n^{-1/2})$ (Davison and Hinkley, 1997). Let r($\hat{\theta}_{1-\alpha/2}$) $= z_{\alpha/2}$ then, the corresponding 100(1-α)% confidence interval for θ based on r is

$$(\hat{\theta}_{\alpha/2}, \hat{\theta}_{1-\alpha/2}) \quad .$$

As mentioned earlier, this interval is not easy to derive mainly because of the nuisance parameters. From equation (2.7), $\hat{\tau}_\theta = (\hat{\mu}_\theta, \hat{\eta}_\theta)$ can be derived by simultaneously solving the two equations below:

$$\frac{n_0 e^{\theta-\hat{\mu}_\theta-\hat{\eta}_\theta/2}}{1-e^{\theta-\hat{\mu}_\theta-\hat{\eta}_\theta/2}} - n_1 + \frac{S - n_1\hat{\mu}_\theta}{\hat{\eta}_\theta} = 0$$

$$\frac{n_0 e^{\theta-\hat{\mu}_\theta-\hat{\eta}_\theta/2}}{1-e^{\theta-\hat{\mu}_\theta-\hat{\eta}_\theta/2}} - \frac{n_1}{2} - \frac{n_1}{2\hat{\eta}_\theta} + \frac{T - 2S\hat{\mu}_\theta + n_1\hat{\mu}_\theta^2}{2\hat{\eta}_\theta^2} = 0$$

with the constraints defined as $\hat{\eta}_\theta > 0$ and $\theta - \hat{\mu}_\theta - \hat{\eta}_\theta/2 < 0$. The complete algorithm for deriving the confidence interval for κ using this approach can be found in Appendix B.

Fortunately, this method is popular enough and certain softwares packages are designed for this approach. So, complexity of this approach should not be a problem in real practice. Some called this approach as profile-likelihood based confidence interval. In R, the package *Bhat* has function called **plkhci** that computes this type of interval. For industries that are regulated by some agencies and require SAS as their software, the book *SAS/IML User's Guide, Version 8* published by SAS Institute provides sample program for this approach.

According to Fletcher (2008) and Zhou and Tu (2000), this procedure outperforms all previously presented procedures except when the sample size is small. This method will be referred as **SLR**.

2.3.2 Adjusted Signed Likelihood Ratio Approach

Barndorff-Nielsen (1986) introduced a new statistic $r^*(\theta)$, an adjustment to $r(\theta)$, to help improve the approximation to normality. The statistic $r^*(\theta)$ is defined as

$$r^*(\theta) = r(\theta) + r(\theta)^{-1}\log\left[\frac{u(\theta)}{r(\theta)}\right]$$

where $u(\theta)$ is a certain quantity. It was shown by Barndorff-Nielsen that $r^*(\theta)$ approximately follows a standard normal distribution with error of order $O(n^{-3/2})$ (third order accurate). Tian and Wu (2006) derived the quantity $u(\theta)$ for Δ-distribution and is given as follows

$$u(\theta) = \frac{|l_{;t}(\hat{\psi};\mathbf{Y}) - l_{;t}(\hat{\psi}_\theta;\mathbf{Y})l_{\tau;t}(\theta,\hat{\tau}_\theta;\mathbf{Y})|}{|l_{\psi;t}(\hat{\psi};\mathbf{Y})|}\left[\frac{|j_{\psi\psi}(\hat{\psi})|}{|j_{\tau\tau}(\theta,\hat{\lambda}_\theta|)}\right]^{\frac{1}{2}} \quad ,$$

where

$$\begin{aligned} |l_{;t}(\hat{\psi};\mathbf{Y}) - l_{;t}(\hat{\psi}_\theta;\mathbf{Y})l_{\tau;t}(\theta,\hat{\tau}_\theta;\mathbf{Y})| \;=\; & (\hat{a}_\theta + 1 + \tfrac{\hat{\mu}_\theta}{\hat{\eta}_\theta})\{\tfrac{\hat{\mu}_\theta - \hat{\mu}}{2\hat{\eta}_\theta^2\hat{\eta}}\} \\ & +(\tfrac{-1}{2\hat{\eta}_\theta^3})\{\log[\tfrac{n_1}{n_0\hat{a}_\theta}] - \tfrac{\log[\hat{\eta}/\hat{\eta}_\theta]}{2} - \tfrac{\hat{\mu}^2}{2\hat{\eta}} + \tfrac{\hat{\mu}_\theta^2}{2\hat{\eta}_\theta}\} \\ & +(\tfrac{1}{\hat{\eta}_\theta})[\tfrac{\hat{a}_\theta}{2} + \tfrac{1}{2} + \tfrac{1}{2\hat{\eta}_\theta} + \tfrac{\hat{\mu}_\theta^2}{2\hat{\eta}_\theta^2}][\tfrac{1}{2\hat{\eta}} - \tfrac{1}{2\hat{\eta}_\theta}] \quad , \end{aligned}$$

$$|l_{\psi;t}(\hat{\psi};\mathbf{Y})| = \frac{-n}{2n_0\hat{\eta}^3} \quad ,$$

and

$$\hat{a}_\theta = \frac{e^{\theta - \hat{\mu}_\theta - \hat{\eta}_\theta/2}}{1 - e^{\theta - \hat{\mu}_\theta - \hat{\eta}_\theta/2}} \quad .$$

The matrix $j_{\psi\psi}(\hat{\psi})$ is the observed information matrix and $j_{\tau\tau}(\theta,\hat{\lambda}_\theta)$ is the observed nuisance information matrix:

$$|j_{\psi\psi}(\hat{\psi})| = \frac{-nn_1^3}{2n_0\hat{\eta}^3}$$

and

$$\begin{aligned} |j_{\tau\tau}(\theta,\hat{\lambda}_\theta)| \;=\; & (\tfrac{-n_0\hat{b}_\theta - n_1}{\hat{\eta}_\theta})(\tfrac{-n_0\hat{b}_\theta}{4} + \tfrac{n_1}{2\hat{\eta}_\theta^2} - \tfrac{T - 2S\hat{\mu}_\theta + n_1\hat{\mu}_\theta^2}{\hat{\eta}_\theta^3}) \\ & -(\tfrac{-n_0\hat{b}_\theta}{2} + \tfrac{n_1\hat{\mu}_\theta}{\hat{\eta}_\theta^2} - \tfrac{S}{\hat{\eta}_\theta^2})^2 \quad , \end{aligned}$$

where

$$\hat{b}_\theta = \frac{e^{\theta - \hat{\mu}_\theta - \hat{\eta}_\theta/2}}{(1 - e^{\theta - \hat{\mu}_\theta - \hat{\eta}_\theta/2})^2} \quad .$$

Now, let

$$\begin{array}{l}\theta^*_{LO} \rightarrow P(r^*(\theta^*_{LO}) \leq -z_{\alpha/2}) = \alpha/2 \\ \theta^*_{HI} \rightarrow P(r^*(\theta^*_{HI}) \leq z_{\alpha/2}) = 1 - \alpha 2\end{array} \ .$$

Then, the (1-α)100% confidence interval for κ is

$$(e^{\theta^*_{LO}}, e^{\theta^*_{HI}}) \ . \tag{2.9}$$

The complete algorithm to derive the interval using this approach can be found in Appendix C.

Unlike the profile likelihood approach, this approach is relatively new and no software package is currently offering this type of procedure. Although the extra complexity present in this procedure might provide a struggle for the users, it is said to be the best procedure among all existing ones (Tian and Wu, 2006) since it can provide nearly exact coverage probability with balance UER and LER.

2.4 Percentile-t Bootstrap of MLE-BC

The method presented here is the percentile-t bootstrap version of MLE-BC approach. The idea is to use the sampling technique to approximate the sampling distribution of a pivotal statistic to avoid asymptotic normal approximation of the MLE. This method was first introduced by Angus (1994) for the confidence interval of the lognormal mean. Then, Zhou and Tu (2000) extended this approach for the mean of Δ - distribution.

Consider the pivotal statistic T defined as

$$T = \frac{\hat{\theta} - \theta}{\sqrt{\hat{\nu}(\hat{\theta})}}$$

Then, from (2.3) and (2.4), T can be expressed as

$$T = \frac{(\log(1-\hat{\delta}) + \hat{\mu} + s^2/2) - (\log(1-\delta) + \mu + \sigma^2/2)}{\sqrt{n_0/nn_1 + s^2/n_1 + s^4/2n_1}} \tag{2.10}$$

With the idea of re-expressing the T statistic as a function of statistics following a certain distribution, then it can be shown that (2.10) has the same distribution as the following statistic:

$$T(\delta, \sigma^2) = \frac{\sqrt{\frac{n_1}{\sigma^2}}(\log(\frac{n_1}{n(1-\delta)}) + Z + \frac{\sqrt{n_1\sigma^2}}{2}\left\{\frac{X^2}{n_1} - 1\right\}}{\sqrt{\frac{n_0}{n\sigma^2} + \frac{X^2}{n_1}\left\{1 + \frac{\sigma^2 X^2}{2n_1}\right\}}} \tag{2.11}$$

where Z has standard normal distribution and X^2 has chi-square distributions with $n_1 - 1$ degrees of freedom, respectively. Since Zhou and Tu defined s^2 as unbiased estimator of σ^2, we believe it is more appropriate to use the fact that

$$\frac{(n-1)s^2}{\sigma^2} = X^2 \quad \sim \quad \chi^2_{n_1-1}$$

Therefore, the modified $T(\delta, \sigma)$ is

$$T_m(\delta, \sigma^2) = \frac{\sqrt{\frac{n_1}{\sigma^2}}(\log(\frac{n_1}{n(1-\delta)}) + Z + \frac{\sqrt{n_1\sigma^2}}{2}\left\{\frac{X^2}{n_1-1} - 1\right\}}{\sqrt{\frac{n_0}{n\sigma^2} + \frac{X^2}{n_1-1}\left\{1 + \frac{\sigma^2 X^2}{2(n_1-1)}\right\}}} \tag{2.12}$$

Please refer to Appendix D for the algorithm of deriving the confidence interval based on this approach. Also, proof of (2.12) is presented in Appendix E.

This method will be referred as **BOOTSTRAP-BC**. This method provides good accuracy when sample size is small and the level of skewness is relatively high. This method also performs well for moderate to large samples.

Bootstrap is very easy to implement in several software packages. The function **bootstrap** in S-plus and **boot** in R can do this procedure easily. For SAS, a downloadable macro in SAS website (http://support.sas.com/kb/24/982.html) is provided to handle bootstrap analyses.

Chapter 3

PROPOSED INTERVAL ESTIMATION

Two alternative approaches for interval estimation of the mean of Δ-distribution are proposed in this chapter. The first proposal is based on Al-Khouli's (1999) study on the robustness of the mean and variance estimators of Δ-distribution. Suggested robust point estimators are expected to produce robust confidence intervals. In addition, Al-Khouli's approach will be extended to the two MLE-based methods that were presented in Chapter 2. The second proposed method is a percentile-t bootstrap of the MLE-TB approach.

3.1 Al-Khouli's Idea

Al-Khouli's work is basically driven by the notion of invariance property of robustness. That is, a function of robust estimators is robust. For example, test statistics using robust estimators may lead to robust tests and robust confidence intervals.

From expressions (1.11) and (1.14), κ and ν are functions of δ, μ, and σ^2. The focus was to robustify μ and σ^2 since there is no practical reason to robustify δ. It is a fact that the classical estimators $\hat{\mu}$ and s^2 of μ and σ^2, respectively are sensitive to infrequent, very large observations. Hence, the estimators tend to overestimate and most likely the overestimation will be inherited by Δ-distribution-based estimates. In addition, the magnitude of overestimation can be very large, especially when exponentiated. Because of these, Al-Khouli suggested that by directly substituting $\hat{\mu}$ and s^2 with robust M-estimators, robust estimators of κ and ν can be obtained. M-estimators, that generalize the idea of maximum likelihood estimation, offer great advantages in performance, flexibility and convenience.

Al-Khouli investigated several combinations of M-estimators for μ and σ^2 and used them in the calculation of mean and variance of Δ-distribution. Some combinations are (Median, MAD), (T_H, MAD) and (T_H, S_{bi}) where MAD is the median absolute deviation, T_H is one-step Huber M-estimator of location and S_{bi} is a biweight A-estimator of scale (Lax, 1985).

3.1.1 Median Absolute Deviation

Median absolute deviation (MAD) is the median absolute deviation from the median. It is defined as

$$\text{MAD} = \text{median}_i(|X_i - \text{median}_j(X_j)|) \quad .$$

MAD is one of the most commonly used robust estimator of scale. It is known to be more resilient to outliers compared to the sample variance. It also exists for some distributions which may not have a variance.

3.1.2 Huber M-Estimator of Location

M-estimator of location is the solution to the problem of minimizing the deviation of the data from the location estimate. Let T_n be the M-estimator of location then

T_n is the value that will minimize

$$\sum_{i=1}^{n} \rho(y_i - T_n)$$

or equivalently, T_n is the value that satisfies

$$\sum_{i=1}^{n} \psi(y_i - T_n) = 0 \quad ,$$

where ρ is an arbitrary function (commonly known as the objective function) and $\psi(y;t)$ is the first derivative of ρ. The sample mean and the sample median are two of the most popular M-estimators of location. To take into account the scale, we define

$$u_i = \frac{y_i - T_n}{cS_n}$$

where S_n is an auxiliary estimator of scale and c is some tuning constant. The only drawback is that it presents a complicated problem which is the simultaneous estimation of T_n and S_n. Huber (1981) said that this extra complication is not necessary and suggested to choose a fixed auxiliary estimator, say S_0, beforehand. Thus, M-estimator is the value that satisfies

$$\sum_{i=1}^{n} \psi\left(\frac{y_i - T_n}{cS_0}\right) = 0 \quad .$$

Sometimes a solution that minimizes $\sum \rho$ cannot be found. A good approach to handle this situation is to use the Newton-Raphson algorithm. Let

$$u_i^{(j)} = \frac{y_i - T_n^{(j)}}{cS_0}$$

and

$$T_n^{(j+1)} = T_n^{j)} + cS_0 \frac{\sum_{i=1}^{n} \psi[u_i^{(j)}]}{\sum_{i=1}^{n} \psi'[u_i^{(j)}]} \tag{3.1}$$

where ψ' is the second derivative of the objective function ρ. Then, the M-estimator is the limit of $T_n^{(j)}$.

For Huber$M$-estimator, the objective function is defined as

$$\rho_H(u) = \begin{cases} u^2/2 & \text{for } |u| \le d \\ k|u| - k^2/2 & \text{for } |u| > d \end{cases} . \tag{3.2}$$

Consequently,

$$\psi_H(u) = \begin{cases} u & \text{for } |u| \le d \\ k\,\mathrm{sgn}(u) & \text{for } |u| > d \end{cases} \tag{3.3}$$

and

$$\psi'_H(u) = \begin{cases} 1 & \text{for } |u| \le d \\ 0 & \text{for } |u| > d \end{cases} . \tag{3.4}$$

Thus, using equation (3.1), we can now obtain a fully-iterated Huber M-estimator. Initial estimate $T_n^{(0)}$ is set to be the sample median and the fixed auxiliary estimate S_0 is set to be the median absolute deviation (MAD). It is recommended to use the rescaled or normalized MAD (NMAD) when using Huber estimate. NMAD is equal to 1.48×MAD which only means that the tuning constant c is equal to 1.48.

A simpler approach is the so-called one-step M-estimator. From expression (3.1), the one-step Huber M-estimator is defined to be

$$T_H = T_n^0 + cS_0 \frac{\sum_{i=1}^n \psi_H[u_i^{(0)}]}{\sum_{i=1}^n \psi'_H[u_i^{(0)}]} . \tag{3.5}$$

The fully iterated and the one-step M-estimators are said to be almost equally robust (Andrews, *et al.*, 1972). The function **hubers** from the package MASS in R is designed for calculation of Huber M-estimators.

3.1.3 Biweight A-Estimator of Scale

The term A-estimator of scale, first introduced by Lax (1985), refers to the finite sample approximation of the variance of M-estimator. A-estimator of scale is defined as

$$S_T = \frac{\sqrt{n}cMAD[\sum_{i=1}^{n}\psi^2(u_i)]^{1/2}}{|\sum_{i=1}^{n}\psi'(u_i)|}$$

where

$$u_i = \frac{y_i - M}{cMAD}$$

and M is the median.

For biweight estimators, ρ_{bi}, ψ_{bi} and ψ'_{bi} are defined as

$$\rho_{bi}(u) = \begin{cases} (1-u^2)^3/6 & \text{for } |u| \le 1 \\ 1/6 & \text{for } |u| > 1 \end{cases} \tag{3.6}$$

$$\psi_{bi}(u) = \begin{cases} u(1-u^2)^2 & \text{for } |u| \le 1 \\ 0 & \text{for } |u| > 1 \end{cases} \tag{3.7}$$

$$\psi'_{bi}(u) = \begin{cases} (1-u^2)(1-5u^2) & \text{for } |u| \le 1 \\ 0 & \text{for } |u| > 1 \end{cases} \quad . \tag{3.8}$$

Given the equations (3.6)-(3.8), the biweight A-estimator can be expressed as

$$S_{bi} = \frac{\sqrt{n}[\sum_{|u_i<1|}(y_i - M)^2(1-u_i^2)^4]^{1/2}}{|\sum_{|u_i<1|}(1-u_i^2)(1-5u_i^2)|} \quad . \tag{3.9}$$

Biweight A-estimator outperforms Huber A-estimator especially in the presence of extreme contamination. For biweight estimators, the reasonable value of the tuning constant is $6 \le c \le 9$ (Al-Khouli, 1999).

3.1.4 Proposed Interval Estimation Using Al-Khouli's Estimator

Al-Khouli considered two simulation models. The first model was designed to contaminate the positive part of the data by generating some observations from another lognormal with a higher value of σ. That is, some observations were generated from standard lognormal $LN(0,1)$ and some observations were generated from $LN(0,\sigma^2)$. The second model was designed to introduce contaminants from similarly-shaped distributions. Some observations were generated from $LN(\mu,\sigma^2)$ and some observations were generated from other skewed distributions, such as gamma and Weibull with the same mean and variance as in $LN(\mu,\sigma^2)$.

To assess the robustness of proposed estimators $\hat{\kappa}_M$ and $\hat{\nu}_M(\hat{\kappa}_M)$, relative efficiencies were obtained as

$$\text{Relative efficiency of } \hat{\kappa}_M = \frac{MSE(\hat{\kappa})}{MSE(\hat{\kappa}_M)}$$

and

$$\text{Relative efficiency of } \hat{\nu}_M(\hat{\kappa}_M) = \frac{MSE(\hat{\nu}_{pen}(\hat{\kappa}))}{MSE(\hat{\nu}_M(\hat{\kappa}_M))} \quad ,$$

where $\hat{\kappa}$ and $\hat{\nu}_{pen}(\hat{\kappa})$ are estimators of the mean and variance of Δ-distribution, respectively. In addition, relative biases have been compared and defined as

$$\text{Relative bias of } \hat{\kappa} = \frac{\text{Bias}(\hat{\kappa})}{\kappa} = \frac{\text{E}(\hat{\kappa}) - \kappa}{\kappa} \quad ,$$

$$\text{Relative bias of } \hat{\kappa_M} = \frac{\text{Bias}(\hat{\kappa}_M)}{\kappa} = \frac{\text{E}(\hat{\kappa}_M) - \kappa}{\kappa} \quad ,$$

$$\text{Relative bias of } \hat{\nu}_{pen}(\hat{\kappa}) = \frac{\text{Bias}(\hat{\nu}_{pen}(\hat{\kappa}))}{\nu(\hat{\kappa})} = \frac{\text{E}(\hat{\nu}_{pen}(\hat{\kappa})) - \nu(\hat{\kappa})}{\nu(\hat{\kappa})}$$

and

$$\text{Relative bias of } \hat{\nu}_M(\hat{\kappa}) = \frac{\text{Bias}(\hat{\nu}_M(\hat{\kappa}_M))}{\nu(\hat{\kappa})} = \frac{\text{E}(\hat{\nu}_M(\hat{\kappa}_M)) - \nu(\hat{\kappa})}{\nu(\hat{\kappa})} \quad .$$

Direct substitution of more robust estimators of $\hat{mu}$ and s^2 enhanced the performance of $\hat{\kappa}$ and $\hat{\nu}_{pen}(\hat{\kappa})$ and reduced the bias to a lesser extent (Al-Khouli, 1999). The pair (T_H, S_{bi}), the highlight of that study, is claimed to provide accurate results in terms of relative efficiency and relative bias for small samples and is significantly robust when contaminants are present. For T_H, the tuning constant c was set to 1.48 and d was set to 1.28. For S_{bi}, the tuning constant was set to 9.0.

The simulation study using the pair (T_H, S_{bi}) can be summarized as follows:

- If there is no contaminant, direct substitution of (T_H, S_{bi}) provides very efficient estimator of κ. The efficiency increases as n decreases or as δ increases. If contaminants are present, efficiency is directly related to the probability and size of the contamination and inversely related to the sample size.

- In terms of bias, proposed estimator $\hat{\kappa}_M$ is slightly biased when there is no contaminant, but the bias decreases as n_1 decreases. If contaminants are present, $\hat{\kappa}_M$ is the least biased.

- Direct substitution of (T_H, S_{bi}) also provides highly efficient variance estimator when compared to $\hat{\nu}_{pen}(\hat{\kappa})$. If contaminants are present, great resistance was observed.

- Proposed variance estimator $\hat{\nu}_M(\hat{\kappa}_M)$ is practically unbiased when there is no contaminant, but the bias decreases as n_1 decreases. If contaminants are present, $\hat{\nu}_M(\hat{\kappa}_M)$ is the least biased.

Given the results of Al-Khouli's simulation, we would like to propose to use the pair (T_H, S_{bi}) as a substitute in $\hat{\mu}$ and s^2 for interval estimation of κ using MVUE method. This method will be referred as **R-MVUE1**.

Let

$$\hat{\kappa}_M = \begin{cases} \frac{n_1}{n} e^{T_H} G_{n_1}\left(\frac{S_{bi}}{2}\right) & \text{if } n_1 > 1 \\ \frac{x_1}{n} & \text{if } n_1 = 1 \\ 0 & \text{if } n_1 = 0 \end{cases} \tag{3.10}$$

$$\hat{\nu}_M(\hat{\kappa}_M) = \begin{cases} \frac{n_1}{n} e^{2T_H} \left\{ \frac{n_1}{n} G_{n_1}\left(\frac{S_{bi}}{2}\right) - \frac{n_1-1}{n-1} G_{n_1}\left(\frac{n_1-2}{n_1-1} S_{bi}\right) \right\} & \text{if } n_1 > 1 \\ (\frac{x_1}{n})^2 & \text{if } n_1 = 1 \\ 0 & \text{if } n_1 = 0 \end{cases} . \tag{3.11}$$

Then, the corresponding 100(1-α)% confidence interval using R-MVUE1 method is

$$\hat{\kappa}_M \pm z_{\alpha/2}\sqrt{\hat{\nu}_M(\hat{\kappa}_M)} \quad .$$

3.1.5 Extension to MLE-Based Intervals

By examining MLE-based interval estimators, it can be noticed that they are also functions of $\hat{\mu}$ and s^2. Most likely, in the same case of MVUE interval estimators, the overestimation observed in $\bar{x}$ and s^2 when infrequent large observations are present will also be inherited by interval estimates based on MLEs. To demonstrate this, consider the following scenarios presented in Table 3.1 which are generated from Δ(0.4, -0.5, 1). From Table 3.1, it is very clear that MLE-based interval estimators are very sensitive to the value of s^2. It was shown that as s^2 increases, the estimate $\hat{\theta}$ also increases. In effect, a large point estimate and a large sample variance produce a very large upper confidence limit (especially, when exponentiated). Consequently, undesired interval width is also obtained. Note that the true mean for these situations is 0.6.

We will try to avoid the complication mentioned above by extending Al-Khouli's approach to MLE-based intervals. That is, we will directly substitute $\bar{x}$ and s^2 by estimators that are robust.

Let

Table 3.1: Sensitivity of MLE-Based Intervals to s^2

				Upper Bound of 95% CI	
n	n_1	s^2	θ	MLE-BC	MLE-TB
25	17	1.8763	-0.0164	2.503	2.532
25	10	3.9362	0.1256	9.343	9.952
25	15	9.6377	3.2639	1062.230	1172.794
25	12	12.8359	3.8419	9814.051	11824.500
25	15	15.3491	6.0627	126427.700	148883.100

$$\hat{\theta}_M = \log(1-\hat{\delta}) + T_H + S_{bi}/2 \quad , \tag{3.12}$$

$$\hat{\nu}_{tb_M}(\hat{\theta}_M) = \frac{(\hat{d}-\hat{c})(1-\hat{c}\hat{d}) - n_1(1-\hat{c})^2}{n_1(1-\hat{c}\hat{d})^2} + \frac{S_{bi}}{n_1} + \frac{S_{bi}^2}{2(n_1+1)} \quad , \tag{3.13}$$

$$\hat{\nu}_{bc_M}(\hat{\theta}_M) = \frac{n_0}{nn_1} + \frac{S_{bi}}{n_1} + \frac{S_{bi}^2}{2n_1} \quad . \tag{3.14}$$

Then, the corresponding 100(1-α)% confidence interval based on MLE-TB with Al-Khouli's estimators is

$$(e^{\hat{\theta}_M - z_{\alpha/2}\sqrt{\hat{\nu}_{tb_M}(\hat{\theta}_M)}}, e^{\hat{\theta}_M + z_{\alpha/2}\sqrt{\hat{\nu}_{tb_M}(\hat{\theta}_M)}}) \quad .$$

This method will be referred as **R-TB**.

In the same manner, the corresponding 100(1-α)% confidence interval based on MLE-BC with Al-Khouli's estimators is

$$(e^{\hat{\theta}_M - z_{\alpha/2}\sqrt{\hat{\nu}_{bc_M}(\hat{\theta}_M)}}, e^{\hat{\theta}_M + z_{\alpha/2}\sqrt{\hat{\nu}_{bc_M}(\hat{\theta}_M)}}) \quad .$$

This method will be referred as **R-BC**.

3.2 Percentile-t Bootstrap of MLE-TB

Recall that two MLE-based methods are available but their performance was not compared. To have an idea, we performed a simulation study to compare the two methods and results can be found in Table 3.2. For each scenario, 10000 random samples were generated. From Table 3.2, it can be noticed that MLE-TB provides nearly the same coverage probabilities with MLE-BC for all scenarios considered especially when sample size is relatively large. Although both methods do not perform well when sample size is small, it can be seen in Figure 3.1 that MLE-TB is relatively better because it usually generates intervals that have smaller width than Method-BC.

Previous studies indicate that using the percentile-t bootstrap of MLE-BC improve its performance. Since MLE-TB performs better than MLE-BC, we expect that the bootstrap version of MLE-TB will provide better confidence intervals. Thus, we would like to propose the percentile-t bootstrap of MLE-TB and will be referred as **BOOTSTRAP-TB**

Table 3.2: Comparison of MLE-Based Intervals

				MLE-TB		MLE-BC	
n	δ	μ	σ^2	CP	Length	CP	Length
15	0.4	-0.075	0.15	0.936	0.6042	0.938	0.6073
25				0.945	0.4623	0.945	0.4626
40				0.946	0.3637	0.946	0.3639
50				0.946	0.3246	0.946	0.3247
15	0.4	-1.0	2.0	0.904	13.9277	0.909	22.1939
25				0.922	2.1184	0.924	2.1790
40				0.930	1.3398	0.933	1.3576
50				0.938	1.1187	0.940	1.1297

Again, the idea is to re-express the pivotal statistic T as a function of statistics

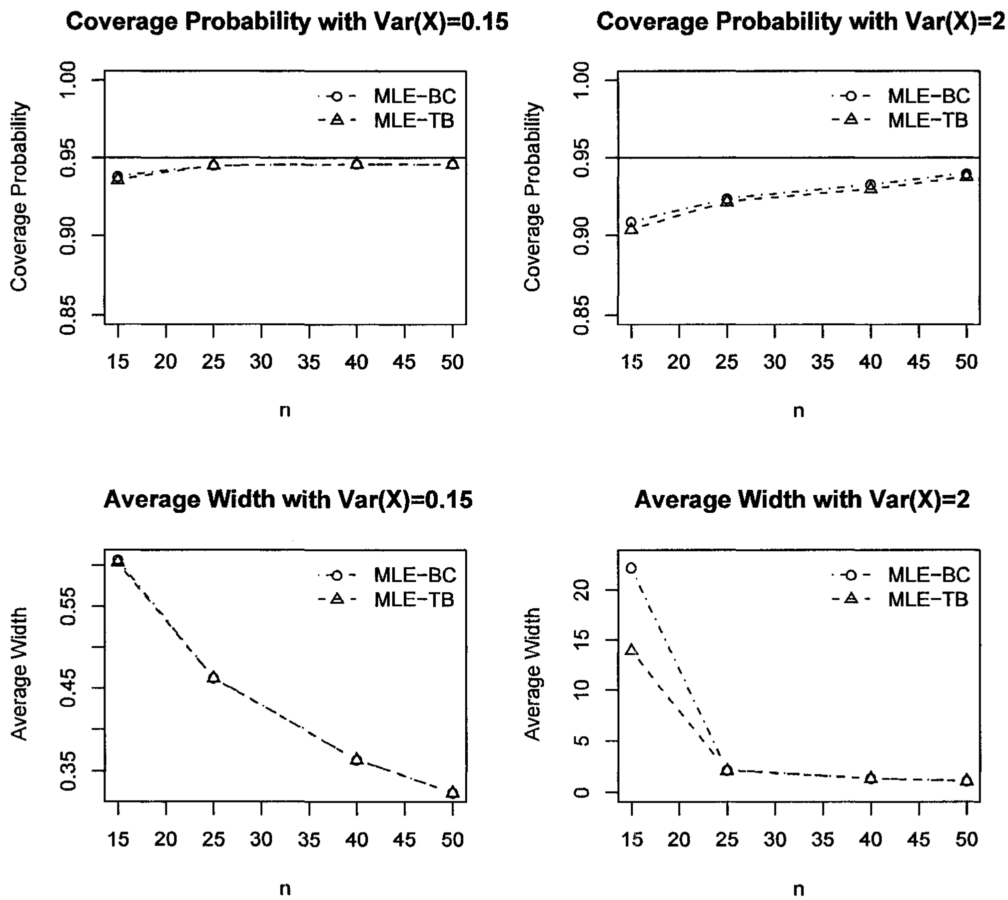

Figure 3.1: Comparison of MLE-Based Intervals

following certain distribution. Let T_{tb} be defined as

$$T_{tb} = \frac{\hat{\theta} - \theta}{\hat{\nu}_{tb}(\hat{\theta})} \quad .$$

Then, using the expressions (2.3) and (2.5), T can be written as

$$T_{tb} = \frac{(\log(1-\hat{\delta}) + \hat{\mu} + \frac{s^2}{2}) - (\log(1-\delta) + \mu + \frac{\sigma^2}{2})}{\left\{\frac{(\hat{d}-\hat{c})(1-\hat{c}\hat{d}) - n_1(1-\hat{c})^2}{n_1(1-\hat{c}\hat{d})^2} + \frac{s^2}{n_1} + \frac{s^4}{2(n_1+1)}\right\}^{\frac{1}{2}}} \quad . \tag{3.15}$$

It can be shown that the following T-statistic has the same distribution of the T-statistic presented above.

$$T_{tb}(\mu, \sigma^2) = \frac{\sqrt{n_1/\sigma^2}\log\frac{n_1}{n(1-\hat{\delta})} \; + \; Z \; + \; \sqrt{n_1\sigma^2}/2\left\{\frac{X^2}{n_1-1} - 1\right\}}{\sqrt{\frac{(\hat{d}-\hat{c})(1-\hat{c}\hat{d}) - n_1(1-\hat{c})^2}{\sigma^2(1-\hat{c}\hat{d})^2} + \frac{X^2}{n_1-1} + \frac{\sigma^2}{2(n_1+1)}\left\{\frac{X^2}{n_1-1}\right\}^2}} \tag{3.16}$$

Proof:

Rearranging the terms in numerator of (3.15), we have

$$T_{tb}(\mu, \sigma^2) = \frac{(\log(1-\hat{\delta}) - \log(1-\delta)) \; + \; (\hat{\mu} - \mu) \; + \; (s^2/2 - \sigma^2/2)}{\sqrt{\frac{(\hat{d}-\hat{c})(1-\hat{c}\hat{d}) - n_1(1-\hat{c})^2}{n_1(1-\hat{c}\hat{d})^2} \; + \; \frac{s^2}{n_1} \; + \; \frac{s^4}{2(n_1+1)}}} \quad .$$

Multiplying the numerator and denominator by $\sqrt{n_1}/\sigma$,

$$\begin{aligned} T_{tb}(\mu, \sigma^2) \; &= \frac{\sqrt{n_1}/\sigma\left(\log\frac{(1-\hat{\delta})}{(1-\delta)}\right) + \sqrt{n_1}/\sigma(\hat{\mu}-\mu) + \sqrt{n_1}/\sigma(s^2/2-\sigma^2/2)}{\sqrt{\frac{n_1}{\sigma^2}\frac{(\hat{d}-\hat{c})(1-\hat{c}\hat{d}) - n_1(1-\hat{c})^2}{n_1(1-\hat{c}\hat{d})^2} + \frac{n_1}{\sigma^2}\frac{s^2}{n_1} + \frac{n_1}{\sigma^2}\frac{s^4}{2(n_1+1)}}} \\ &= \frac{\sqrt{n_1}/\sigma\left(\log\frac{(1-\hat{\delta})}{(1-\delta)}\right) + \sqrt{n_1}/\sigma(\hat{\mu}-\mu) + \sigma\sqrt{n_1}/2(s^2/\sigma^2-\sigma^2/\sigma^2)}{\sqrt{\frac{(\hat{d}-\hat{c})(1-\hat{c}\hat{d}) - n_1(1-\hat{c})^2}{\sigma^2(1-\hat{c}\hat{d})^2} + \frac{s^2}{\sigma^2} + \frac{\sigma^2 n_1}{2(n_1+1)}\frac{s^4}{\sigma^4}}} \quad . \end{aligned}$$

Finally, we have

$$T_{tb}(\mu, \sigma^2) = \frac{\sqrt{n_1/\sigma^2}\log\frac{n_1}{n(1-\hat{\delta})} \; + \; Z \; + \; \sqrt{n_1\sigma^2}/2\left\{\frac{X^2}{n_1-1} - 1\right\}}{\sqrt{\frac{(\hat{d}-\hat{c})(1-\hat{c}\hat{d}) - n_1(1-\hat{c})^2}{\sigma^2(1-\hat{c}\hat{d})^2} \; + \; \frac{X^2}{n_1-1} \; + \; \frac{\sigma^2}{2(n_1+1)}\left\{\frac{X^2}{n_1-1}\right\}^2}} \quad ,$$

where

$$Z = \frac{(\hat{\mu} - \mu)}{\sigma/\sqrt{n_1}} \quad \sim \quad N(0,1)$$

and

$$X^2 = \frac{(n_1 - 1)s^2}{\sigma^2} \quad \sim \quad \chi^2_{n_1-1} \quad .$$

Using the same algorithm presented in Appendix D, the corresponding 100(1-α)% bootstrap percentile interval based on MLE-TB approach is

$$(e^{\hat{\theta} - t^{*(1-\alpha/2)}\sqrt{\hat{\nu}_{tb}(\hat{\theta})}}, e^{\hat{\theta} - t^{*(\alpha/2)}\sqrt{\hat{\nu}_{tb}(\hat{\theta})}}) \quad .$$

Chapter 4

SIMULATION STUDY AND RESULTS

Two simulation models were performed to investigate the robustness of the existing methods and the proposed methods. The first model is designed to check the performance of the methods when contaminants from more skewed lognormal distribution are present in the data. This model will be referred as **Model A**. The second model is designed to check the performance of the methods when lognormality assumption of the positive values is not met. Gamma, Weibull and Birnbaum-Saunders distributions have been considered as alternative sampling distributions to lognormal. This model will be referred as **Model B**. A brief introduction to Birnbaum-Saunders distribution is presented in Appendix F.

4.1 Judgment Criteria

Robustness of the different methods will be assessed using the following criteria:

- Coverage Probability (CP): proportion of intervals such that the true parameter falls within the interval

- Coverage Error (CE): absolute difference between coverage probability and nominal probability
- Lower Error Rate (LER): proportion of intervals such that the true parameter falls below the interval
- Upper Error Rate (UER): proportion of intervals such that the true parameter falls above the interval
- Average Width (Width): average of the widths of simulated intervals

Throughout the simulation study, we set the nominal probability to 0.95. Hence, desired results are CP=0.95, CE=0, LER=0.025, and UER=0.025. Also, average width should be as small as possible.

4.2 Model A: Framework and Results

4.2.1 Simulation Framework

As mentioned earlier, the general idea of this model is to contaminate the positive portion of the data by a fixed number of observations, u, from a more skewed lognormal distribution. Basically, for each random sample of size n, a number of zero observations n_0 is generated from $BIN(n, \delta)$. Then, u observations will be generated from $LN(0, \sigma^2)$. Fixed value of u represents different percentage of contamination for different sample sizes. Two contaminants (u=2) represent 13.33% contamination in a sample of 15 while it represents only 4% in a sample of 50. Last, $n - n_0 - u$ observations will be generated from $LN(0, 1)$. This process will be repeated 10,000 times for each combination of the parameters u, n, δ and

σ. Values of the parameters considered are similar to the parameters used by other authors. Complete simulation framework is presented below.

Simulation framework:

- Number of samples: 10,000
- Number of bootstrap samples for each sample: 2,000
- Sample size (n): 15, 25, 50
- Proportion of zeros (δ): 0.2, 0.4
- Number of outliers (u): 0, 1, 2, 3
- Generate n_0 number of zero observations from $BIN(n, \delta)$
- Generate $(n_1 - u)$ observations from $LN(0, 1)$
- Generate u observations from $LN(0, \sigma^2)$
- Variance of contaminants σ^2: 2, 4, 5

4.2.2 Simulation Results

For the sake of brevity, only selected tables are presented in this section. For the complete simulation results, please refer to Appendix G.

Here are the highlights of this simulation:

- In general, there is a noticeable improvement in the coverage probability of all methods as sample size increases. In addition, precision is acquired since all methods provide shorter intervals when samples are relatively large. However, methods that have imbalanced error rates remain to be imbalanced. On the other hand, the coverage probability is inversely related to the proportion of zeros. The width of the intervals tends to be narrower as proportion of zeros gets larger. The stability of error rates seems to be unaffected by δ.

- The performance of MVUE1 and MVUE2 are relatively the same, similarly with MLE-TB and MLE-BC, SLR and ASLR, R-TB and R-BC, and BOOTSTRAP-TB and BOOTSTRAP-BC.

- When there is no contamination, we have proven previous claims that intervals based on likelihood ratio provide a very satisfactory coverage probability and balance error rates. Bootstrap methods tend to provide very wide intervals when sample size is small and proportion of zeros is high but provide good coverage when sample size is large. On the other hand, MVUE1 and MVUE2 provide the poorest coverage. R-MVUE1 enhanced the performance of MVUE1 but imbalanced error rates are still observed. MLE-TB and MLE-BC have a satisfactory coverage but have imbalanced error rates. R-TB and R-BC are not the best method in this situation but they still have satisfactory performance. R-TB and R-BC can be good alternatives for the existing methods.

- The effect of contaminants can be greatly observed in cases where sample size is small. Existing methods tend to produce very wide intervals, especially when δ is large. But the effect of contaminant seems to reduce gradually as sample size increases. R-TB and R-BC showed great resistance to contaminants since they continuously provide high coverage probability while maintaining narrow interval width.

- The variance of the contaminants has a negative effect in the performance of the methods. Error rates tend to be more imbalanced as the variance of the contaminants gets larger. Tables provide evidence that likelihood methods (MLE-TB, MLE-BC, SLR and ASLR) including bootstrap methods have the tendency to produce very wide intervals as the variance of the contaminants increases, especially when sample size is small. R-TB and R-BC showed stability in terms of error rates and resistance in terms of the average width, regardless of how big the variance of the contaminants is.

- The number of the contaminants also has a negative effect in the performance of the methods. This negative effect is worsen by the increase in δ. The effect cannot be easily seen in terms of coverage probability and error rates but is very visible in terms of interval width. The more contaminants are present in the data, the wider the interval is produced. Intervals using methods R-TB and R-BC demonstrated great resilience.

In summary, direct substitution of $\hat{\mu}$ and s^2 by more robust estimators will enhance the performance especially, when contaminants are present. Intervals based on likelihood ratio provide best coverage when there are no contaminants. However, due to the complexity of these methods, R-TB and R-BC are good alternative methods since they provide satisfactory performance, convenience and flexibility. When contaminants are present, R-TB and R-BC showed resilience.

Table 4.1: 95% CI under $\Delta(0.2, 0, 1)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8600	0.0900	0.1394	0.0006	1.74709
MVUE2		0.8851	0.0649	0.1146	0.0003	2.17949
MLE-TB		0.9297	0.0203	0.0574	0.0129	2.42906
MLE-BC		0.9317	0.0183	0.0565	0.0118	2.47146
SLR		0.9386	0.0114	0.0199	0.0415	3.32812
ASLR		0.9390	0.0110	0.0198	0.0412	3.41266
BOOTSTRAP-BC		0.9293	0.0207	0.0303	0.0404	3.79864
R-MVUE1		0.8818	0.0682	0.1177	0.0005	1.62291
R-TB		0.9523	0.0023	0.0354	0.0123	2.09746
R-BC		0.9541	0.0041	0.0347	0.0112	2.12640
BOOTSTRAP-TB		0.9317	0.0183	0.0285	0.0398	3.87770
MVUE1	25	0.8903	0.0597	0.1087	0.0010	1.40987
MVUE2		0.9081	0.0419	0.0916	0.0003	1.61093
MLE-TB		0.9398	0.0102	0.0472	0.0130	1.70407
MLE-BC		0.9412	0.0088	0.0466	0.0122	1.71902
SLR		0.9453	0.0047	0.0219	0.0328	1.98898
ASLR		0.9452	0.0048	0.0219	0.0327	2.01101
BOOTSTRAP-BC		0.9409	0.0091	0.0286	0.0305	2.07930
R-MVUE1		0.9142	0.0358	0.0849	0.0009	1.34789
R-TB		0.9649	0.0149	0.0240	0.0111	1.58930
R-BC		0.9664	0.0164	0.0233	0.0103	1.60189
BOOTSTRAP-TB		0.9422	0.0078	0.0276	0.0302	2.09479
MVUE1	50	0.9138	0.0362	0.0837	0.0025	1.02457
MVUE2		0.9241	0.0259	0.0748	0.0011	1.09548
MLE-TB		0.9426	0.0074	0.0428	0.0146	1.12372
MLE-BC		0.9435	0.0065	0.0424	0.0141	1.12817
SLR		0.9457	0.0043	0.0222	0.0321	1.20191
ASLR		0.9480	0.0020	0.0218	0.0302	1.25987
BOOTSTRAP-BC		0.9453	0.0047	0.0261	0.0286	1.22485
R-MVUE1		0.9430	0.0070	0.0550	0.0020	1.00734
R-TB		0.9692	0.0192	0.0197	0.0111	1.09877
R-BC		0.9695	0.0195	0.0195	0.0110	1.10298
BOOTSTRAP-TB		0.9462	0.0038	0.0253	0.0285	1.22928

Table 4.2: 95% CI under $\Delta(0.4, 0, 1)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8464	0.1036	0.1527	0.0009	1.57092
MVUE2		0.8755	0.0745	0.1243	0.0002	2.09023
MLE-TB		0.9247	0.0253	0.0613	0.0140	2.52830
MLE-BC		0.9273	0.0227	0.0594	0.0133	2.60791
SLR		0.9312	0.0188	0.0219	0.0469	3.87526
ASLR		0.9363	0.0137	0.0216	0.0421	4.21340
BOOTSTRAP-BC		0.9192	0.0308	0.0303	0.0505	17.09941
R-MVUE1		0.8521	0.0979	0.1470	0.0009	1.43048
R-TB		0.9435	0.0065	0.0433	0.0132	1.98127
R-BC		0.9461	0.0039	0.0410	0.0129	2.01932
BOOTSTRAP-TB		0.9269	0.0231	0.0280	0.0451	34.33750
MVUE1	25	0.8818	0.0682	0.1168	0.0014	1.26860
MVUE2		0.9002	0.0498	0.0993	0.0005	1.49899
MLE-TB		0.9349	0.0151	0.0482	0.0169	1.63971
MLE-BC		0.9359	0.0141	0.0478	0.0163	1.65912
SLR		0.9406	0.0094	0.0235	0.0359	2.01803
ASLR		0.9408	0.0092	0.0235	0.0357	2.11366
BOOTSTRAP-BC		0.9346	0.0154	0.0303	0.0351	2.14000
R-MVUE1		0.8966	0.0534	0.1019	0.0015	1.20306
R-TB		0.9574	0.0074	0.0274	0.0152	1.48778
R-BC		0.9520	0.3580	0.0262	0.0146	1.50241
BOOTSTRAP-TB		0.9365	0.0135	0.0293	0.0342	2.16690
MVUE1	50	0.9043	0.0457	0.0942	0.0015	0.93086
MVUE2		0.9159	0.0341	0.0835	0.0006	1.01301
MLE-TB		0.9419	0.0081	0.0414	0.0167	1.05424
MLE-BC		0.9426	0.0074	0.0410	0.0164	1.05948
SLR		0.9431	0.0069	0.0226	0.0343	1.14476
ASLR		0.9452	0.0048	0.0218	0.0330	1.19023
BOOTSTRAP-BC		0.9401	0.0099	0.0287	0.0312	1.16677
R-MVUE1		0.9305	0.0195	0.0682	0.0013	0.90785
R-TB		0.9641	0.0141	0.0225	0.0134	1.01768
R-BC		0.9650	0.0150	0.0222	0.0128	1.02244
BOOTSTRAP-TB		0.9410	0.0090	0.028	0.0310	1.17190

Table 4.3: 95% CI under $\Delta(0.2, 0, 1)$ with One Contaminant from $\Delta(0.2, 0, 2)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8761	0.0739	0.1228	0.0011	1.91216
MVUE2		0.8996	0.0504	0.1002	0.0002	2.45595
MLE-TB		0.9343	0.0157	0.0484	0.0173	2.79614
MLE-BC		0.9365	0.0135	0.0479	0.0156	2.85288
SLR		0.9400	0.0100	0.0260	0.0340	3.94047
ASLR		0.9402	0.0098	0.0297	0.0301	4.21432
BOOTSTRAP-BC		0.9250	0.0250	0.0419	0.0331	5.33726
R-MVUE1		0.8860	0.0640	0.1131	0.0009	1.68541
R-TB		0.9532	0.0032	0.0328	0.0140	2.20370
R-BC		0.9552	0.0052	0.0318	0.0130	2.23532
BOOTSTRAP-TB		0.9279	0.0221	0.0391	0.0330	5.55965
MVUE1	25	0.9025	0.0475	0.0959	0.0016	1.48896
MVUE2		0.9178	0.0322	0.0818	0.0004	1.71604
MLE-TB		0.9401	0.0099	0.0430	0.0169	1.82305
MLE-BC		0.9413	0.0087	0.0424	0.0163	1.83989
SLR		0.9438	0.0062	0.0280	0.0282	2.14637
ASLR		0.9452	0.0048	0.0267	0.0281	2.92111
BOOTSTRAP-BC		0.9356	0.0144	0.0380	0.0264	2.25720
R-MVUE1		0.9220	0.0280	0.0763	0.0017	1.38471
R-TB		0.9643	0.0143	0.0219	0.0138	1.63892
R-BC		0.9653	0.0153	0.0215	0.0132	1.65215
BOOTSTRAP-TB		0.9367	0.0133	0.0371	0.0262	2.27533
MVUE1	50	0.9195	0.0305	0.0772	0.0033	1.05604
MVUE2		0.9301	0.0199	0.0689	0.0010	1.13192
MLE-TB		0.9446	0.0054	0.0377	0.0177	1.16227
MLE-BC		0.9458	0.0042	0.0374	0.0168	1.16699
SLR		0.9434	0.0066	0.0277	0.0289	1.24566
ASLR		0.9430	0.0070	0.0294	0.0276	1.22443
BOOTSTRAP-BC		0.9401	0.0099	0.0329	0.0270	1.27128
R-MVUE1		0.9464	0.0036	0.0509	0.0027	1.02187
R-TB		0.9677	0.0177	0.0184	0.0139	1.11603
R-BC		0.9683	0.0183	0.0183	0.0134	1.12034
BOOTSTRAP-TB		0.9409	0.0091	0.0321	0.0270	1.27600

Table 4.4: 95% CI under $\Delta(0.4, 0, 1)$ with One Contaminant from $\Delta(0.4, 0, 2)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8574	0.0926	0.1413	0.0013	1.72491
MVUE2		0.8897	0.0603	0.1101	0.0002	2.42330
MLE-TB		0.9287	0.0213	0.0512	0.0201	3.18327
MLE-BC		0.9324	0.0176	0.0495	0.0181	3.38871
SLR		0.9329	0.0171	0.0309	0.0362	4.68898
ASLR		0.9358	0.0142	0.0298	0.0344	5.46399
BOOTSTRAP-BC		0.9182	0.0318	0.0412	0.0406	$5.127e^{05}$
R-MVUE1		0.8560	0.0940	0.1427	0.0013	1.48680
R-TB		0.9389	0.0111	0.0429	0.0182	2.10212
R-BC		0.9421	0.0079	0.0408	0.0171	2.14499
BOOTSTRAP-TB		0.9268	0.0232	0.0385	0.0347	$1.536e^{07}$
MVUE1	25	0.8880	0.0620	0.1105	0.0015	1.36565
MVUE2		0.9089	0.0411	0.0907	0.0004	1.64745
MLE-TB		0.9351	0.0149	0.0427	0.0222	1.82275
MLE-BC		0.9368	0.0132	0.0419	0.0213	1.84685
SLR		0.9351	0.0149	0.0314	0.0335	2.30442
ASLR		0.9366	0.0134	0.0312	0.0324	2.45681
BOOTSTRAP-BC		0.9272	0.0228	0.0398	0.0330	2.48681
R-MVUE1		0.8982	0.0518	0.1003	0.0015	1.23697
R-TB		0.9508	0.0008	0.0308	0.0184	1.54265
R-BC		0.9520	0.0020	0.0301	0.0179	1.55839
BOOTSTRAP-TB		0.9289	0.0211	0.0385	0.0326	2.52799
MVUE1	50	0.9115	0.0385	0.0852	0.0033	0.96741
MVUE2		0.9251	0.0249	0.0733	0.0016	1.05763
MLE-TB		0.9430	0.0070	0.0359	0.0211	1.10286
MLE-BC		0.9437	0.0063	0.0354	0.0209	1.10858
SLR		0.9446	0.0054	0.0276	0.0278	1.20270
ASLR		0.9453	0.0047	0.0271	0.0276	1.34123
BOOTSTRAP-BC		0.9393	0.0107	0.0338	0.0269	1.22798
R-MVUE1		0.9318	0.0182	0.0655	0.0027	0.92280
R-TB		0.9636	0.0136	0.0204	0.0160	1.03703
R-BC		0.9640	0.0140	0.0203	0.0157	1.04196
BOOTSTRAP-TB		0.9401	0.0099	0.0331	0.0268	1.23361

Table 4.5: 95% CI under $\Delta(0.2, 0, 1)$ with Three Contaminants from $\Delta(0.2, 0, 5)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9443	0.0057	0.0544	0.0013	4.59939
MVUE2		0.9611	0.0111	0.0387	0.0002	10.29274
MLE-TB		0.8992	0.0508	0.0181	0.0827	35.23319
MLE-BC		0.9090	0.0410	0.0177	0.0733	41.87446
SLR		0.8946	0.0554	0.0938	0.0116	10.42776
ASLR		0.9012	0.0488	0.0724	0.0264	12.56445
BOOTSTRAP-BC		0.7873	0.1627	0.2032	0.0095	$1.096e^{06}$
R-MVUE1		0.8840	0.0660	0.1124	0.0036	2.14716
R-TB		0.9197	0.0303	0.0316	0.0487	3.08255
R-BC		0.9237	0.0263	0.0305	0.0458	3.14035
BOOTSTRAP-TB		0.7994	0.1506	0.1912	0.0094	$2.197e^{06}$
MVUE1	25	0.9579	0.0079	0.0391	0.0030	2.65414
MVUE2		0.9678	0.0178	0.0320	0.0002	3.57911
MLE-TB		0.8986	0.0514	0.0144	0.0870	4.27480
MLE-BC		0.9029	0.0471	0.0143	0.0828	4.35374
SLR		0.9112	0.0388	0.0789	0.0099	5.09820
ASLR		0.9115	0.0385	0.0775	0.0110	5.64008
BOOTSTRAP-BC		0.8212	0.1288	0.1691	0.0097	7.53086
R-MVUE1		0.9276	0.0224	0.0673	0.0051	1.59939
R-TB		0.9445	0.0055	0.0200	0.0355	1.95283
R-BC		0.9460	0.0040	0.0198	0.0342	1.97082
BOOTSTRAP-TB		0.8271	0.1229	0.1632	0.0097	7.71685
MVUE1	50	0.9515	0.0015	0.0360	0.0125	1.41403
MVUE2		0.9648	0.0148	0.0321	0.0031	1.56214
MLE-TB		0.9101	0.0399	0.0168	0.0731	1.62624
MLE-BC		0.9118	0.0382	0.0166	0.0716	1.63469
SLR		0.9046	0.0454	0.0823	0.0131	1.78585
ASLR		0.9099	0.0401	0.0788	0.0113	1.81277
BOOTSTRAP-BC		0.8685	0.0815	0.1191	0.0124	1.84591
R-MVUE1		0.9461	0.0039	0.0467	0.0072	1.09288
R-TB		0.9536	0.0036	0.0164	0.0300	1.20219
R-BC		0.9540	0.0040	0.0164	0.0296	1.20711
BOOTSTRAP-TB		0.8710	0.0790	0.1170	0.0120	1.85494

Table 4.6: 95% CI under $\Delta(0.4, 0, 1)$ with Three Contaminants from $\Delta(0.4, 0, 5)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9296	0.0204	0.0693	0.0011	5.20679
MVUE2		0.9532	0.0032	0.0465	0.0003	75.43320
MLE-TB		0.9042	0.0458	0.0199	0.0759	$8.103e^{04}$
MLE-BC		0.9143	0.0357	0.0192	0.0665	$2.031e^{05}$
SLR		0.9103	0.0397	0.0528	0.0369	$1.424e^{03}$
ASLR		0.9201	0.0299	0.0489	0.0310	$1.855e^{03}$
BOOTSTRAP-BC		0.7711	0.1789	0.2155	0.0134	$2.956e^{31}$
R-MVUE1		0.8587	0.0913	0.1387	0.0026	2.04217
R-TB		0.9054	0.0446	0.0393	0.0553	3.55522
R-BC		0.9096	0.0404	0.0375	0.0529	3.68290
BOOTSTRAP-TB		0.7900	0.1600	0.1987	0.0113	$3.135e^{34}$
MVUE1	25	0.9453	0.0047	0.0527	0.0020	2.77390
MVUE2		0.9619	0.0119	0.0379	0.0002	4.77709
MLE-TB		0.9004	0.0496	0.0158	0.0838	8.63029
MLE-BC		0.9075	0.0425	0.0155	0.0770	9.24612
SLR		0.9048	0.0452	0.0819	0.0133	11.12154
ASLR		0.9069	0.0431	0.0799	0.0132	12.35543
BOOTSTRAP-BC		0.8100	0.1400	0.1803	0.0097	$1.743e^{03}$
R-MVUE1		0.8952	0.0548	0.1006	0.0042	1.47845
R-TB		0.9301	0.0199	0.0283	0.0416	1.95787
R-BC		0.9318	0.0182	0.0277	0.0405	1.98297
BOOTSTRAP-TB		0.8171	0.1329	0.1734	0.0095	$2.865e^{03}$
MVUE1	50	0.9531	0.0031	0.0396	0.0073	1.39781
MVUE2		0.9659	0.0159	0.0324	0.0017	1.62027
MLE-TB		0.9043	0.0457	0.0151	0.0806	1.74102
MLE-BC		0.9068	0.0432	0.0146	0.0786	1.75438
SLR		0.9094	0.0406	0.0848	0.0058	2.10814
ASLR		0.9102	0.0398	0.0832	0.0066	2.19312
BOOTSTRAP-BC		0.8590	0.0910	0.1305	0.0105	2.10814
R-MVUE1		0.9339	0.0161	0.0600	0.0061	1.00789
R-TB		0.9462	0.0038	0.0190	0.0348	1.14814
R-BC		0.9471	0.0029	0.0189	0.0340	1.15413
BOOTSTRAP-TB		0.8613	0.0887	0.1282	0.0105	2.12362

4.3 Model B: Framework and Results

4.3.1 Simulation Framework

One major problem in dealing with lognormal distribution is the difficulty to detect any deviation from lognormality when sample size is less than 40, especially when contaminants came from similar skewed distribution, such as gamma and Weibull distributions (Myers and Pepin, 1990). This situation presents the importance of having robust estimators. The simulation model presented in this section is designed to check the performance of the interval estimators of the mean of Δ-distribution when contaminants from similar skewed distribution are present in the data.

The mechanism involved in this simulation model is similar to Model A. A fixed percentage p of positive values are being contaminated by data from similar skewed distribution. For each random sample of size n, a number of zero observations n_0 is generated from $BIN(n, \delta)$. Then, $(1-p)(n-n_0)$ observations will be generated from $LN(\mu, \sigma^2)$. Last, $p(n-n_0)$ observations will be generated from gamma, Weibull and Birnbaum-Saunders distributions with the same mean and variance as the lognormal distribution. This process will be repeated 10,000 times for each combination of the parameters p, n, δ and σ. Complete simulation framework is presented below.

Simulation framework:

- Number of samples: 10,000
- Number of bootstrap samples for each sample: 2,000
- Percent of contamination (p): 20%, 60%, 100%

- Sample size (n): 15, 25, 50
- Proportion of zeros (δ): 0.2, 0.4
- σ^2: 0.15, 0.50, 1.0, 2.0
- μ: - $\sigma^2/2$
- Distribution of the contaminants: gamma (G), Weibull (W) and Birnbaum-Saunders (BS)

Figure 4.1 shows the density function of the three distributions that are being considered along with the lognormal distribution. It can be seen that Birnbaum-Saunders distribution is the closest distribution to lognormal distribution, especially when σ^2 is very small.

The coverage performance of these methods is not affected by the choice of μ. Assuming the values of δ and σ are fixed, the error rates would not change for different values of μ (Fletcher, 2008). For convenience, we have set $\mu = -\sigma^2/2$. By doing this, the true mean κ is always 0.6 or 0.8 when δ is 0.4 or 0.2, respectively.

4.3.2 Simulation Results

Similar to Model A, only selected tables are presented in this section. For the complete result of Model B simulation, please refer to Appendix H.

Results of this simulation study can be summarized as follows:

- Evidently, the performance of all methods is very dependent on sample size n, proportion of zeros δ, variance σ^2(skewness) of the positive part of the data and the percent of contamination p.

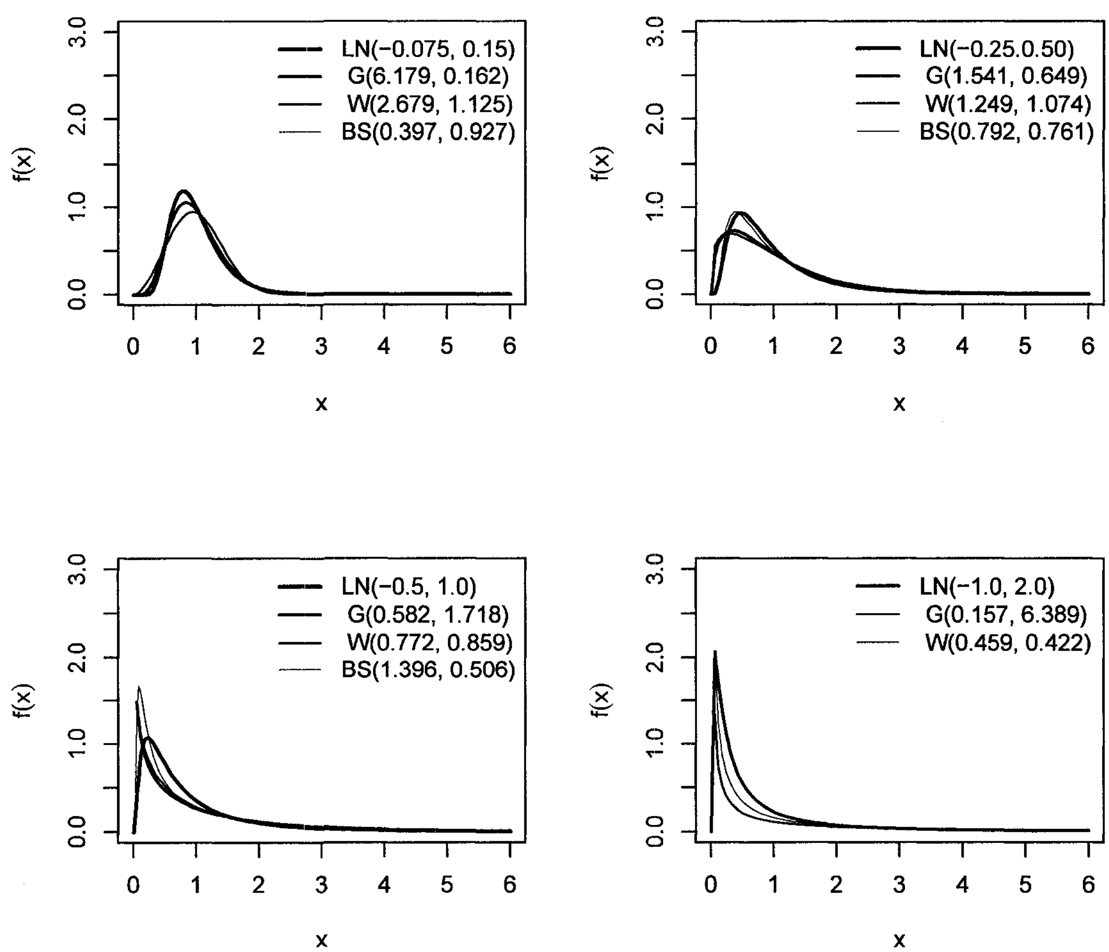

Figure 4.1: Density Function of Different Skewed Distributions

- In general, increase in sample size has a positive effect while increase in δ or σ (skewness) or percent of contaminants p has a negative effect in the performance of the methods.

- Coverage properties of MVUE1 and MVUE2 are relatively the same, same case with MLE-TB and MLE-BC, SLR and ASLR, R-TB and R-BC, and BOOTSTRAP-TB and BOOTSTRAP-BC.

- When there is no contamination, we have shown that intervals based on likelihood ratio (SLR and ASLR) are the best in several scenarios. We also have proven that MVUE-based intervals are worst, especially when level of skewness is high. Performance of proposed methods R-TB and R-BC are satisfactory, especially when there is moderate skewness. R-MVUE1 enhanced the performance of MVUE1 but not enough to outperform other methods. Bootstrap intervals do not perform well when sample size is small and skewness is high.

- In the presence of little to moderate contamination, coverage performance of existing methods depreciates. Existing methods tend to produce very wide intervals particularly when sample size is small and level of skewness is moderately high. Also, imbalanced error rates are observed in some scenarios. These negative effects of the contaminants are worsen by the increased in δ but lessen when sample size is increased. R-TB and R-BC also experienced imbalanced error rates in some situations. But these methods maintained the width of the intervals to be narrow.

- In the presence of extreme contamination, all methods experienced great deterioration in their coverage properties. Coverage probabilities are low, error rates are imbalanced and average interval width are too high. Proposed methods also experienced the same thing with the exception that they maintained the interval width to be small. Increase in sample size worsens the performance mainly because more extreme observations are present in the data.

To conclude, this simulation study has demonstrated the lack of resistance to contaminants of existing methods. Existing methods have good coverage properties when there are no contaminants but showed sensitivity in the presence of contaminants. They also tend to produce very wide intervals. This can be attributed to the fact that gamma and Weibull distributions have high probability of generating observations close to zero.

On the other hand, proposed methods, particularly R-TB and R-BC, have satisfactory performance in several scenarios. They showed resilience in the presence of contaminants by keeping the average width to be small while maintaining satisfactory coverage probability and less imbalanced error rates. However, poor performance is observed when skewness is too low ($\sigma^2 = 0.15$) and when skewness is too high ($\sigma^2 = 2.0$). Fortunately, these methods can be refined by adjusting the tuning constants of the robust estimators T_H and S_{bi}.

Table 4.7: 95% CI under $\Delta(0.4, -0.075, 0.15)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9294	0.0206	0.0467	0.0239	0.58030
MVUE2		0.9301	0.0199	0.0465	0.0234	0.58115
MLE-TB		0.9386	0.0114	0.0130	0.0484	0.60492
MLE-BC		0.9412	0.0088	0.0108	0.0480	0.60799
SLR		0.9423	0.0077	0.0398	0.0179	0.63287
ASLR		0.9423	0.0077	0.0398	0.0179	0.64223
BOOTSTRAP-BC		0.9422	0.0078	0.0244	0.0334	0.59589
R-MVUE1		0.9647	0.0147	0.021	0.0143	0.74519
R-TB		0.9471	0.0029	0.0045	0.0484	0.82983
R-BC		0.9491	0.0009	0.0033	0.0476	0.83613
BOOTSTRAP-TB		0.9611	0.0111	0.0242	0.0147	0.61868
MVUE1	25	0.9353	0.0147	0.0412	0.0235	0.45088
MVUE2		0.9352	0.0148	0.0416	0.0232	0.45125
MLE-TB		0.9417	0.0083	0.0160	0.0423	0.46252
MLE-BC		0.9419	0.0081	0.0158	0.0423	0.46288
SLR		0.9445	0.0055	0.0275	0.0280	0.50047
ASLR		0.9563	0.0063	0.0207	0.023	0.52680
BOOTSTRAP-BC		0.9440	0.0060	0.0268	0.0292	0.45745
R-MVUE1		0.9675	0.0175	0.0120	0.0205	0.59950
R-TB		0.9446	0.0054	0.0022	0.0532	0.64290
R-BC		0.9456	0.0044	0.0022	0.0522	0.64480
BOOTSTRAP-TB		0.9455	0.0045	0.0266	0.0279	0.45871
MVUE1	50	0.9433	0.0067	0.0366	0.0201	0.32065
MVUE2		0.9436	0.0064	0.0366	0.0198	0.32079
MLE-TB		0.9461	0.0039	0.0183	0.0356	0.32469
MLE-BC		0.9463	0.0037	0.0182	0.0355	0.32481
SLR		0.9479	0.0021	0.0241	0.0280	0.32142
ASLR		0.9565	0.0065	0.0206	0.0229	0.35774
BOOTSTRAP-BC		0.9485	0.0015	0.0243	0.0272	0.32267
R-MVUE1		0.9616	0.0116	0.0027	0.0357	0.43719
R-TB		0.9208	0.0292	0.0003	0.0789	0.45361
R-BC		0.9211	0.0289	0.0003	0.0786	0.45427
BOOTSTRAP-TB		0.9485	0.0015	0.0243	0.0272	0.03228

Table 4.8: 95% CI under $\Delta(0.4, -0.25, 0.50)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8917	0.0583	0.1007	0.0076	0.74759
MVUE2		0.9048	0.0452	0.0901	0.0051	0.83126
MLE-TB		0.9311	0.0189	0.0369	0.0320	0.90045
MLE-BC		0.9332	0.0168	0.0353	0.0315	0.91161
SLR		0.9301	0.0199	0.0365	0.0334	1.09458
ASLR		0.9308	0.0192	0.0361	0.0331	1.09923
BOOTSTRAP-BC		0.9189	0.0311	0.0348	0.0463	1.07788
R-MVUE1		0.9310	0.0190	0.0645	0.0045	0.85700
R-TB		0.9501	0.0001	0.0166	0.0333	1.06357
R-BC		0.9519	0.0019	0.0156	0.0325	1.07770
BOOTSTRAP-TB		0.9331	0.0169	0.0337	0.0332	1.14302
MVUE1	25	0.9180	0.0320	0.0753	0.0067	0.59633
MVUE2		0.9260	0.0240	0.0698	0.0042	0.63543
MLE-TB		0.9416	0.0084	0.0315	0.0269	0.66423
MLE-BC		0.9422	0.0078	0.0313	0.0265	0.66750
SLR		0.9409	0.0091	0.0255	0.0336	0.71463
ASLR		0.9424	0.0076	0.0252	0.0324	0.77812
BOOTSTRAP-BC		0.9366	0.0134	0.0302	0.0332	0.71378
R-MVUE1		0.9612	0.0112	0.0329	0.0059	0.71243
R-TB		0.9583	0.0083	0.0075	0.0342	0.82071
R-BC		0.9592	0.0092	0.0072	0.0336	0.82611
BOOTSTRAP-TB		0.9375	0.0125	0.0300	0.0325	0.71762
MVUE1	50	0.9294	0.0206	0.0613	0.0093	0.42794
MVUE2		0.9343	0.0157	0.0582	0.0075	0.44183
MLE-TB		0.9425	0.0075	0.0311	0.0264	0.45119
MLE-BC		0.9429	0.0071	0.0310	0.0261	0.45219
SLR		0.9429	0.0071	0.0259	0.0312	0.46462
ASLR		0.9433	0.0067	0.0256	0.0311	0.47005
BOOTSTRAP-BC		0.9418	0.0082	0.0283	0.0299	0.46477
R-MVUE1		0.9717	0.0217	0.0140	0.0143	0.52895
R-TB		0.9507	0.0007	0.0034	0.0459	0.57029
R-BC		0.9511	0.0011	0.0033	0.0456	0.57212
BOOTSTRAP-TB		0.9423	0.0077	0.028	0.0297	0.46568

Table 4.9: 95% CI under $\Delta(0.2, -0.50, 1.0)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8629	0.0871	0.1362	0.0009	1.06775
MVUE2		0.8871	0.0629	0.1127	0.0002	1.33332
MLE-TB		0.9275	0.0225	0.0592	0.0133	1.48771
MLE-BC		0.9301	0.0199	0.0576	0.0123	1.51397
SLR		0.9416	0.0084	0.0194	0.0390	2.03450
ASLR		0.9417	0.0083	0.0194	0.0389	2.10455
BOOTSTRAP-BC		0.9305	0.0195	0.0307	0.0388	2.38564
R-MVUE1		0.8845	0.0655	0.1149	0.0006	0.98938
R-TB		0.9543	0.0043	0.0339	0.0118	1.27903
R-BC		0.9565	0.0065	0.0325	0.0110	1.29669
BOOTSTRAP-TB		0.9328	0.0172	0.029	0.0382	2.43879
MVUE1	25	0.8923	0.0577	0.1069	0.0008	0.85569
MVUE2		0.9070	0.0430	0.0928	0.0002	0.97830
MLE-TB		0.9366	0.0134	0.0507	0.0127	1.03529
MLE-BC		0.9380	0.0120	0.0500	0.0120	1.04441
SLR		0.9450	0.0050	0.0199	0.0351	1.20917
ASLR		0.9448	0.0052	0.0198	0.0354	1.24112
BOOTSTRAP-BC		0.9386	0.0114	0.0280	0.0334	1.26606
R-MVUE1		0.9168	0.0332	0.0823	0.0009	0.81820
R-TB		0.9633	0.0133	0.0262	0.0105	0.96515
R-BC		0.9645	0.0145	0.0257	0.0098	0.97281
BOOTSTRAP-TB		0.9409	0.0091	0.0261	0.0330	1.27560
MVUE1	50	0.9156	0.0344	0.0818	0.0026	0.62292
MVUE2		0.9266	0.0234	0.0721	0.0013	0.66610
MLE-TB		0.9447	0.0053	0.0390	0.0163	0.68329
MLE-BC		0.9457	0.0043	0.0385	0.0158	0.68600
SLR		0.9466	0.0034	0.0232	0.0302	0.73090
ASLR		0.9470	0.0030	0.0230	0.0300	0.94129
BOOTSTRAP-BC		0.9443	0.0057	0.0284	0.0273	0.74542
R-MVUE1		0.9456	0.0044	0.0521	0.0023	0.61192
R-TB		0.9677	0.0177	0.0195	0.0128	0.66748
R-BC		0.9680	0.0180	0.0193	0.0127	0.67004
BOOTSTRAP-TB		0.9451	0.0049	0.0278	0.0271	0.74811

Table 4.10: 95% CI under $\Delta(0.4, -1.0, 2.0)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.7526	0.1974	0.2470	0.0004	1.29084
MVUE2		0.8244	0.1256	0.1755	0.0001	2.87386
MLE-TB		0.9084	0.0416	0.0859	0.0057	12.57300
MLE-BC		0.9114	0.0386	0.0836	0.0050	16.23939
SLR		0.9242	0.0258	0.0195	0.0563	6.78239
ASLR		0.9240	0.0260	0.0195	0.0565	7.82845
BOOTSTRAP-BC		0.9140	0.0360	0.0329	0.0531	$2.666e^{09}$
R-MVUE1		0.6125	0.3375	0.3871	0.0004	0.74603
R-TB		0.8565	0.0935	0.1396	0.0039	1.23186
R-BC		0.8625	0.0875	0.1339	0.0036	1.26737
BOOTSTRAP-TB		0.9234	0.0266	0.0275	0.0491	$1.639e^{11}$
MVUE1	25	0.8083	0.1417	0.1917	0.0000	1.07922
MVUE2		0.8575	0.0925	0.1425	0.0000	1.65082
MLE-TB		0.9273	0.0227	0.0659	0.0068	2.17149
MLE-BC		0.9309	0.0191	0.0636	0.0055	2.23606
SLR		0.9421	0.0079	0.0204	0.0375	3.43070
ASLR		0.9425	0.0075	0.0203	0.0372	3.61121
BOOTSTRAP-BC		0.9337	0.0163	0.0310	0.0353	6.08449
R-MVUE1		0.6404	0.3096	0.3595	0.0001	0.63564
R-TB		0.8636	0.0864	0.1339	0.0025	0.88140
R-BC		0.8671	0.0829	0.1305	0.0024	0.89419
BOOTSTRAP-TB		0.9370	0.0130	0.0286	0.0344	6.48733
MVUE1	50	0.8658	0.0842	0.1342	0.0000	0.82101
MVUE2		0.8938	0.0562	0.1062	0.0000	1.01260
MLE-TB		0.9389	0.0111	0.0516	0.0095	1.12333
MLE-BC		0.9404	0.0096	0.0503	0.0093	1.13436
SLR		0.9447	0.0053	0.0210	0.0343	1.35901
ASLR		0.9448	0.0052	0.0210	0.0342	1.41969
BOOTSTRAP-BC		0.9402	0.0098	0.0287	0.0311	1.46648
R-MVUE1		0.6307	0.3193	0.3693	0.0000	0.48963
R-TB		0.8282	0.1218	0.1709	0.0009	0.58353
R-BC		0.8315	0.1185	0.1677	0.0008	0.58737
BOOTSTRAP-TB		0.9423	0.0077	0.0273	0.0304	1.48070

Table 4.11: 95% CI under $\Delta(0.2, -0.50, 1.0)$ with 20% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8937	0.0563	0.1053	0.0010	2.05316
MVUE2		0.9182	0.0318	0.0816	0.0002	1.29490
MLE-TB		0.9473	0.0027	0.0395	0.0132	$7.749e^{04}$
MLE-BC		0.9493	0.0007	0.0386	0.0121	$1.076e^{05}$
SLR		0.9404	0.0096	0.0315	0.0281	$4.208e^{03}$
ASLR		0.9409	0.0091	0.0311	0.0280	$7.077e^{03}$
BOOTSTRAP-BC		0.9207	0.0293	0.0547	0.0246	$6.871e^{09}$
R-MVUE1		0.8253	0.1247	0.1739	0.0008	0.97338
R-TB		0.9340	0.0160	0.0556	0.0104	1.30033
R-BC		0.9361	0.0139	0.0541	0.0098	1.32027
BOOTSTRAP-TB		0.9266	0.0234	0.0494	0.0240	$1.375e^{10}$
MVUE1	25	0.9404	0.0096	0.0585	0.0011	1.68779
MVUE2		0.9519	0.0019	0.0479	0.0002	44.02819
MLE-TB		0.9544	0.0044	0.0220	0.0236	$1.106e^{04}$
MLE-BC		0.9568	0.0068	0.0217	0.0215	$1.385e^{04}$
SLR		0.9312	0.0188	0.0544	0.0144	28.79604
ASLR		0.9313	0.0187	0.0543	0.0144	48.99329
BOOTSTRAP-BC		0.9143	0.0357	0.0732	0.0125	$2.450e^{07}$
R-MVUE1		0.8638	0.0862	0.1351	0.0011	0.81202
R-TB		0.9432	0.0068	0.0486	0.0082	0.97709
R-BC		0.9440	0.0060	0.0479	0.0081	0.98556
BOOTSTRAP-TB		0.9182	0.0318	0.0694	0.0124	$3.524e^{07}$
MVUE1	50	0.9698	0.0198	0.0285	0.0017	1.10585
MVUE2		0.9754	0.0254	0.0241	0.0005	1.38310
MLE-TB		0.9274	0.0226	0.0112	0.0614	1.64618
MLE-BC		0.9288	0.0212	0.0112	0.0600	1.66340
SLR		0.8989	0.0511	0.0937	0.0074	1.57094
ASLR		0.9023	0.0477	0.0901	0.0076	1.61209
BOOTSTRAP-BC		0.8827	0.0673	0.1103	0.0070	2.45766
R-MVUE1		0.8880	0.0620	0.1112	0.0008	0.61230
R-TB		0.9473	0.0027	0.0451	0.0076	0.67593
R-BC		0.9488	0.0012	0.0440	0.0072	0.67876
BOOTSTRAP-TB		0.8848	0.0652	0.1082	0.0070	2.49673

Table 4.12: 95% CI under Δ(0.4, -0.50, 1.0) with 20% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8699	0.0801	0.1295	0.0006	1.75946
MVUE2		0.9022	0.0478	0.0976	0.0002	$1.558e^{04}$
MLE-TB		0.9412	0.0088	0.0449	0.0139	$9.091e^{09}$
MLE-BC		0.9439	0.0061	0.0435	0.0126	$2.418e^{10}$
SLR		0.9337	0.0163	0.0338	0.0325	$4.241e^{03}$
ASLR		0.9351	0.0149	0.033	0.0319	$8.432e^{03}$
BOOTSTRAP-BC		0.9136	0.0364	0.0535	0.0329	$3.254e^{37}$
R-MVUE1		0.8037	0.1463	0.1958	0.0005	0.85513
R-TB		0.9221	0.0279	0.0656	0.0123	1.23463
R-BC		0.9257	0.0243	0.0631	0.0112	1.26114
BOOTSTRAP-TB		0.9262	0.0238	0.0462	0.0276	$1.878e^{41}$
MVUE1	25	0.9132	0.0368	0.0864	0.0004	1.43523
MVUE2		0.9330	0.0170	0.0670	0.0000	1.99497
MLE-TB		0.9498	0.0002	0.0302	0.0200	$4.085e^{03}$
MLE-BC		0.9523	0.0023	0.0294	0.0183	$5.368e^{03}$
SLR		0.9350	0.0150	0.0436	0.0214	28.77077
ASLR		0.9372	0.0128	0.0415	0.0213	43.25099
BOOTSTRAP-BC		0.9170	0.0330	0.0621	0.0209	$1.142e^{08}$
R-MVUE1		0.8474	0.1026	0.1522	0.0004	0.72064
R-TB		0.9438	0.0062	0.0462	0.0100	0.91732
R-BC		0.9455	0.0045	0.0449	0.0096	0.92748
BOOTSTRAP-TB		0.9218	0.0282	0.0584	0.0198	$1.943e^{08}$
MVUE1	50	0.9544	0.0044	0.0438	0.0018	1.00494
MVUE2		0.9632	0.0132	0.0364	0.0004	1.51417
MLE-TB		0.9356	0.0144	0.0176	0.0468	2.81503
MLE-BC		0.9382	0.0118	0.0172	0.0446	2.90130
SLR		0.9132	0.0368	0.0746	0.0122	1.68637
ASLR		0.9184	0.0316	0.0698	0.0118	1.74233
BOOTSTRAP-BC		0.8964	0.0536	0.0919	0.0117	16.73328
R-MVUE1		0.8807	0.0693	0.1178	0.0015	0.54744
R-TB		0.9458	0.0042	0.0437	0.0105	0.62334
R-BC		0.9469	0.0031	0.0428	0.0103	0.62657
BOOTSTRAP-TB		0.8984	0.0516	0.0901	0.0115	17.86930

Table 4.13: 95% CI under Δ(0.2, -0.50, 1.0) with 60% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9458	0.0042	0.0534	0.0008	5.51535
MVUE2		0.9670	0.0170	0.0329	0.0001	$7.527e^{05}$
MLE-TB		0.9730	0.0230	0.0115	0.0155	$6.430e^{12}$
MLE-BC		0.9761	0.0261	0.0110	0.0129	$1.773e^{13}$
SLR		0.9273	0.0227	0.0667	0.0060	$8.397e^{05}$
ASLR		0.9294	0.0206	0.0650	0.0056	$1.443e^{07}$
BOOTSTRAP-BC		0.8850	0.0650	0.1104	0.0046	$1.296e^{29}$
R-MVUE1		0.7548	0.1952	0.2445	0.0007	0.97100
R-TB		0.9177	0.0323	0.0744	0.0079	1.40886
R-BC		0.9211	0.0289	0.0717	0.0072	1.43568
BOOTSTRAP-TB		0.8962	0.0538	0.0993	0.0045	$2.988e^{30}$
MVUE1	25	0.9788	0.0288	0.0206	0.0006	3.39512
MVUE2		0.9855	0.0355	0.0142	0.0003	19.33135
MLE-TB		0.9477	0.0023	0.0051	0.0472	362.91010
MLE-BC		0.9523	0.0023	0.0049	0.0428	413.68710
SLR		0.9124	0.0376	0.0712	0.0164	67.99203
ASLR		0.9161	0.0339	0.0689	0.0150	102.77451
BOOTSTRAP-BC		0.8317	0.1183	0.1661	0.0022	$6.953e^{04}$
R-MVUE1		0.7870	0.1630	0.2124	0.0006	0.82842
R-TB		0.9288	0.0212	0.0649	0.0063	1.05590
R-BC		0.9308	0.0192	0.0632	0.0060	1.06712
BOOTSTRAP-TB		0.8411	0.1089	0.1567	0.0022	$8.743e^{04}$
MVUE1	50	0.9950	0.0450	0.0043	0.0007	2.31911
MVUE2		0.9962	0.0462	0.0037	0.0001	3.47120
MLE-TB		0.8355	0.1145	0.0016	0.1629	4.75905
MLE-BC		0.8402	0.1098	0.0015	0.1583	4.83018
SLR		0.9105	0.0395	0.0705	0.0190	4.16914
ASLR		0.9143	0.0357	0.0684	0.0173	5.22093
BOOTSTRAP-BC		0.6988	0.2512	0.3003	0.0009	8.58443
R-MVUE1		0.8050	0.1450	0.1948	0.0002	0.62928
R-TB		0.9201	0.0299	0.0766	0.0033	0.71625
R-BC		0.9209	0.0291	0.0759	0.0032	0.71984
BOOTSTRAP-TB		0.7048	0.2452	0.2943	0.0009	8.76664

Table 4.14: 95% CI under Δ(0.4, -0.50, 1.0) with 60% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9075	0.0425	0.0917	0.0008	2.87989
MVUE2		0.9404	0.0096	0.0594	0.0002	$1.109e^{03}$
MLE-TB		0.9650	0.0150	0.0235	0.0115	$5.036e^{08}$
MLE-BC		0.9679	0.0179	0.0222	0.0099	$2.897e^{09}$
SLR		0.9326	0.0174	0.0506	0.0168	82.13606
ASLR		0.9340	0.0160	0.0494	0.0166	184.88675
BOOTSTRAP-BC		0.8982	0.0518	0.0864	0.0154	$2.191e^{69}$
R-MVUE1		0.7418	0.2082	0.2574	0.0008	0.83830
R-TB		0.9192	0.0308	0.0728	0.0080	1.33714
R-BC		0.9239	0.0261	0.0686	0.0075	1.37355
BOOTSTRAP-TB		0.9141	0.0359	0.0730	0.0129	$2.201e^{74}$
MVUE1	25	0.9543	0.0043	0.0453	0.0004	3.14177
MVUE2		0.9715	0.0215	0.0282	0.0003	$1.474e^{03}$
MLE-TB		0.9625	0.0125	0.0097	0.0278	$5.99549e^{07}$
MLE-BC		0.9662	0.0162	0.0092	0.0246	$1.147e^{08}$
SLR		0.9201	0.0299	0.0739	0.0060	$3.965e^{03}$
ASLR		0.9255	0.0245	0.0697	0.0048	$7.213e^{03}$
BOOTSTRAP-BC		0.8683	0.0817	0.1263	0.0054	$1.26688e^{18}$
R-MVUE1		0.7800	0.1700	0.2196	0.0004	0.71564
R-TB		0.9278	0.0222	0.0647	0.0075	0.97851
R-BC		0.9297	0.0203	0.0632	0.0071	0.99220
BOOTSTRAP-TB		0.8788	0.0712	0.1160	0.0052	$6.644e^{18}$
MVUE1	50	0.9885	0.0385	0.0111	0.0004	2.06767
MVUE2		0.9919	0.0419	0.0081	0.0000	5.05279
MLE-TB		0.8940	0.0560	0.0026	0.1034	24.60908
MLE-BC		0.8988	0.0512	0.0026	0.0986	26.11563
SLR		0.9122	0.0378	0.0859	0.0019	31.24387
ASLR		0.9184	0.0316	0.0798	0.0018	35.92321
BOOTSTRAP-BC		0.7675	0.1825	0.2308	0.0017	216.94330
R-MVUE1		0.8081	0.1419	0.1916	0.0003	0.55319
R-TB		0.9302	0.0198	0.0656	0.0042	0.65507
R-BC		0.9317	0.0183	0.0642	0.0041	0.65925
BOOTSTRAP-TB		0.7744	0.1756	0.2239	0.0017	235.86040

Table 4.15: 95% CI under Δ(0.2, -0.50, 1.0) with 100% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9627	0.0127	0.0368	0.0005	7.83179
MVUE2		0.9810	0.0310	0.0187	0.0003	$8.717e^{05}$
MLE-TB		0.9808	0.0308	0.0052	0.0140	$1.033e^{12}$
MLE-BC		0.9837	0.0337	0.005	0.0113	$2.050e^{12}$
SLR		0.9105	0.0395	0.0859	0.0036	12.30083
ASLR		0.9156	0.0344	0.0821	0.0023	39.49083
BOOTSTRAP-BC		0.8505	0.0995	0.1471	0.0024	$3.529e^{22}$
R-MVUE1		0.6976	0.2524	0.3019	0.0005	0.97286
R-TB		0.9204	0.0296	0.0738	0.0058	1.54470
R-BC		0.9241	0.0259	0.0706	0.0053	1.58006
BOOTSTRAP-TB			0.9500			
MVUE1	25	0.9889	0.0389	0.0109	0.0002	5.34512
MVUE2		0.9938	0.0438	0.0062	0.0000	85.62328
MLE-TB		0.9406	0.0094	0.0018	0.0576	$7.351e^{03}$
MLE-BC		0.9487	0.0013	0.0018	0.0495	$8.866e^{03}$
SLR		0.9264	0.0236	0.0629	0.0107	10.79133
ASLR		0.9302	0.0198	0.0548	0.0150	24.45512
BOOTSTRAP-BC		0.7662	0.1838	0.2333	0.0005	$8.339e^{06}$
R-MVUE1		0.7382	0.2118	0.2616	0.0002	0.84037
R-TB		0.9279	0.0221	0.0700	0.0021	1.14022
R-BC		0.9311	0.0189	0.0668	0.0021	1.15453
BOOTSTRAP-TB		0.7798	0.1702	0.2197	0.0005	$1.235e^{07}$
MVUE1	50	0.9978	0.0478	0.0018	0.0004	3.73823
MVUE2		0.9988	0.0488	0.0012	0.0000	6.95627
MLE-TB		0.7471	0.2029	0.0003	0.2526	13.58948
MLE-BC		0.7560	0.1940	0.0003	0.2437	13.91790
SLR		0.8956	0.0544	0.0892	0.0152	7.58515
ASLR		0.9021	0.0479	0.0812	0.0167	10.29454
BOOTSTRAP-BC		0.5558	0.3942	0.4442	0.0000	34.51173
R-MVUE1		0.7401	0.2099	0.2598	0.0001	0.65500
R-TB		0.9122	0.0378	0.0867	0.0011	0.77384
R-BC		0.9141	0.0359	0.0848	0.0011	0.77838
BOOTSTRAP-TB		0.5633	0.3867	0.4367	0.0000	35.68331

Table 4.16: 95% CI under $\Delta(0.2, -0.075, 0.15)$ with 20% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9326	0.0174	0.0350	0.0324	0.54756
MVUE2		0.9337	0.0163	0.0355	0.0308	0.55182
MLE-TB		0.9349	0.0151	0.0138	0.0513	0.56373
MLE-BC		0.9355	0.0145	0.0135	0.0510	0.56477
SLR		0.9205	0.0295	0.0597	0.0198	0.59404
ASLR		0.9216	0.0284	0.0587	0.0197	0.60002
BOOTSTRAP-BC		0.9329	0.0171	0.0399	0.0272	0.58186
R-MVUE1		0.9791	0.0291	0.0067	0.0142	0.77045
R-TB		0.9546	0.0046	0.0016	0.0438	0.83807
R-BC		0.9554	0.0054	0.0015	0.0431	0.84214
BOOTSTRAP-TB		0.9355	0.0145	0.0396	0.0249	0.58428
MVUE1	25	0.9416	0.0084	0.0336	0.0248	0.42906
MVUE2		0.9429	0.0071	0.0333	0.0238	0.43112
MLE-TB		0.9454	0.0046	0.0149	0.0397	0.43644
MLE-BC		0.9455	0.0045	0.0148	0.0397	0.43689
SLR		0.9483	0.0017	0.0245	0.0272	0.44529
ASLR		0.9486	0.0014	0.0244	0.0270	0.44989
BOOTSTRAP-BC		0.9425	0.0075	0.0301	0.0274	0.44229
R-MVUE1		0.9781	0.0281	0.0023	0.0196	0.61816
R-TB		0.9461	0.0039	0.0003	0.0536	0.65211
R-BC		0.9467	0.0033	0.0003	0.0530	0.65401
BOOTSTRAP-TB		0.9427	0.0073	0.0299	0.0274	0.44271
MVUE1	50	0.9501	0.0001	0.0269	0.0230	0.30427
MVUE2		0.9505	0.0005	0.0268	0.0227	0.30495
MLE-TB		0.9490	0.0010	0.0156	0.0354	0.30672
MLE-BC		0.9490	0.0010	0.0156	0.0354	0.30687
SLR		0.9523	0.0023	0.0254	0.0223	0.30957
ASLR		0.9523	0.0023	0.0254	0.0223	0.31043
BOOTSTRAP-BC		0.9517	0.0017	0.0264	0.0219	0.30821
R-MVUE1		0.9497	0.0003	0.0007	0.0496	0.44930
R-TB		0.9005	0.0495	0.0001	0.0994	0.46197
R-BC		0.9011	0.0489	0.0001	0.0988	0.46265
BOOTSTRAP-TB		0.9519	0.0019	0.0262	0.0219	0.30835

Table 4.17: 95% CI under $\Delta(0.4, -0.50, 1.0)$ with 20% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8605	0.0895	0.1383	0.0012	1.25963
MVUE2		0.8940	0.0560	0.1053	0.0007	21.11044
MLE-TB		0.9364	0.0136	0.0492	0.0144	$2.406e^{04}$
MLE-BC		0.9391	0.0109	0.0473	0.0136	$4.277e^{04}$
SLR		0.9377	0.0123	0.0270	0.0353	38.62434
ASLR		0.9395	0.0105	0.0263	0.0342	64.96651
BOOTSTRAP-BC		0.9203	0.0297	0.0426	0.0371	$1.272e^{17}$
R-MVUE1		0.8189	0.1311	0.1801	0.0010	0.86218
R-TB		0.9318	0.0182	0.0552	0.0130	1.23578
R-BC		0.9351	0.0149	0.0527	0.0122	1.26199
BOOTSTRAP-TB		0.9297	0.0203	0.0382	0.0321	$1.094e^{18}$
MVUE1	25	0.9059	0.0441	0.0930	0.0011	1.01036
MVUE2		0.9258	0.0242	0.0739	0.0003	1.56871
MLE-TB		0.9502	0.0002	0.0343	0.0155	3.34635
MLE-BC		0.9517	0.0017	0.0335	0.0148	3.64552
SLR		0.9464	0.0036	0.0300	0.0236	2.19849
ASLR		0.9467	0.0033	0.0299	0.0234	2.63293
BOOTSTRAP-BC		0.9347	0.0153	0.0432	0.0221	$1.716e^{03}$
R-MVUE1		0.8679	0.0821	0.1310	0.0011	0.07273
R-TB		0.9513	0.0013	0.0374	0.0113	0.92152
R-BC		0.9533	0.0033	0.0359	0.0108	0.93154
BOOTSTRAP-TB		0.9376	0.0124	0.0407	0.0217	$2.665e^{03}$
MVUE1	50	0.9412	0.0088	0.0570	0.0018	0.75815
MVUE2		0.9502	0.0002	0.0493	0.0005	0.89897
MLE-TB		0.9507	0.0007	0.0232	0.0261	0.99926
MLE-BC		0.9517	0.0017	0.0227	0.0256	1.00856
SLR		0.9435	0.0065	0.0409	0.0156	1.11133
ASLR		0.9451	0.0049	0.0399	0.0150	1.20045
BOOTSTRAP-BC		0.9313	0.0187	0.0535	0.0152	1.34829
R-MVUE1		0.9026	0.0474	0.0959	0.0015	0.55173
R-TB		0.9592	0.0092	0.0314	0.0094	0.62610
R-BC		0.9601	0.0101	0.0308	0.0091	0.62928
BOOTSTRAP-TB		0.9332	0.0168	0.0515	0.0152	1.36709

Table 4.18: 95% CI under $\Delta(0.2, -0.25, 0.50)$ with 60% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9399	0.0101	0.0557	0.0044	1.02176
MVUE2		0.9522	0.0022	0.0449	0.0029	1.26322
MLE-TB		0.9610	0.0110	0.0191	0.0199	1.46888
MLE-BC		0.9625	0.0125	0.0182	0.0193	1.50194
SLR		0.9662	0.0162	0.0194	0.0144	1.86118
ASLR		0.9685	0.0185	0.0188	0.0127	1.94223
BOOTSTRAP-BC		0.9517	0.0017	0.0336	0.0147	6.26217
R-MVUE1		0.9491	0.0009	0.0481	0.0028	0.97944
R-TB		0.9686	0.0186	0.0110	0.0204	1.20204
R-BC		0.9698	0.0198	0.01090	0.0193	1.21562
BOOTSTRAP-TB		0.9540	0.0040	0.0317	0.0143	6.96626
MVUE1	25	0.9668	0.0168	0.0297	0.0035	0.83843
MVUE2		0.9731	0.0231	0.0247	0.0022	0.94494
MLE-TB		0.9709	0.0209	0.0093	0.0198	0.99861
MLE-BC		0.9715	0.0215	0.0092	0.0193	1.00673
SLR		0.9663	0.0163	0.0267	0.0070	1.15792
ASLR		0.9674	0.0174	0.0260	0.0066	1.17112
BOOTSTRAP-BC		0.9571	0.0071	0.0359	0.0070	1.21684
R-MVUE1		0.9749	0.0249	0.0223	0.0028	0.81576
R-TB		0.9723	0.0223	0.0043	0.0234	0.93128
R-BC		0.9729	0.0229	0.0043	0.0028	0.93745
BOOTSTRAP-TB		0.9582	0.0082	0.0348	0.0070	1.22632
MVUE1	50	0.9773	0.0273	0.0148	0.0079	0.61221
MVUE2		0.9820	0.0320	0.0134	0.0046	0.64818
MLE-TB		0.9617	0.0117	0.0060	0.0323	0.66289
MLE-BC		0.9623	0.0123	0.0059	0.0318	0.66520
SLR		0.9522	0.0022	0.0428	0.0050	0.70333
ASLR		0.9603	0.0103	0.0361	0.0036	0.74221
BOOTSTRAP-BC		0.9445	0.0055	0.0513	0.0042	0.71406
R-MVUE1		0.9806	0.0306	0.0087	0.0107	0.60546
R-TB		0.9648	0.0148	0.0025	0.0327	0.64936
R-BC		0.9653	0.0153	0.0025	0.0322	0.65147
BOOTSTRAP-TB		0.9454	0.0046	0.0504	0.0042	0.71635

Table 4.19: 95% CI under $\Delta(0.4, -1.0, 2.0)$ with 100% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8665	0.0835	0.1335	0.0000	32.53336
MVUE2		0.9438	0.0062	0.0562	0.0000	$2.455e^{21}$
MLE-TB		0.9740	0.0240	0.0206	0.0054	$2.860e^{47}$
MLE-BC		0.9774	0.0274	0.0193	0.0033	$4.744e^{49}$
SLR		0.9299	0.0201	0.0602	0.0099	$1.052e^{19}$
ASLR		0.9316	0.0184	0.0591	0.0093	$2.032e^{20}$
BOOTSTRAP-BC		0.8094	0.1406	0.1813	0.0093	$1.733e^{89}$
R-MVUE1		0.3058	0.6442	0.6942	0.0000	0.52952
R-TB		0.7488	0.2012	0.2500	0.0012	1.35872
R-BC		0.7609	0.1891	0.2381	0.0010	1.43404
BOOTSTRAP-TB		0.8383	0.1117	0.1532	0.0085	$2.301e^{98}$
MVUE1	25	0.9449	0.0051	0.0551	0.0000	29.27150
MVUE2		0.9775	0.0275	0.0245	0.0000	$9.781e^{05}$
MLE-TB		0.9550	0.0050	0.0068	0.0382	$2.671e^{11}$
MLE-BC		0.9640	0.0140	0.0065	0.0295	$4.850e^{11}$
SLR		0.9205	0.0295	0.0654	0.0141	$8.312e^{06}$
ASLR		0.9266	0.0234	0.0600	0.0134	$1.214e^{07}$
BOOTSTRAP-BC		0.7218	0.2282	0.2766	0.0016	$6.895e^{25}$
R-MVUE1		0.2743	0.6757	0.7257	0.0000	0.46702
R-TB		0.7007	0.2493	0.2993	0.0000	0.89125
R-BC		0.7106	0.2394	0.2984	0.0000	0.91445
BOOTSTRAP-TB		0.7418	0.2082	0.2566	0.0016	$6.003e^{26}$
MVUE1	50	0.9934	0.0434	0.0066	0.0000	19.72354
MVUE2		0.9970	0.0470	0.0003	0.0000	$1.075e^{04}$
MLE-TB		0.7710	0.1790	0.0006	0.2284	$1.803e^{07}$
MLE-BC		0.7849	0.1651	0.0005	0.2146	$2.181e^{07}$
SLR		0.5825	0.3675	0.4171	0.0004	18.49578
ASLR		0.5921	0.3579	0.4072	0.0007	34.3201
BOOTSTRAP-BC		0.5127	0.4373	0.4872	0.0001	$3.254e^{10}$
R-MVUE1		0.1906	0.7594	0.8094	0.0000	0.38271
R-TB		0.5358	0.4142	0.4641	0.0001	0.55072
R-BC		0.5428	0.4072	0.4571	0.0001	0.55669
BOOTSTRAP-TB		0.5228	0.4272	0.4771	0.0001	$4.741e^{10}$

Table 4.20: 95% CI under $\Delta(0.2, -0.075, 0.15)$ with 20% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9346	0.0154	0.0319	0.0335	0.58119
MVUE2		0.9372	0.0128	0.0318	0.0310	0.58933
MLE-TB		0.9275	0.0225	0.0134	0.0591	0.60276
MLE-BC		0.9277	0.0223	0.0134	0.0589	0.60410
SLR		0.9322	0.0178	0.0493	0.0185	0.63877
ASLR		0.9325	0.0175	0.0490	0.0185	0.64411
BOOTSTRAP-BC		0.9265	0.0235	0.0489	0.0246	0.62354
R-MVUE1		0.9774	0.0274	0.0065	0.0161	0.81064
R-TB		0.9446	0.0054	0.0015	0.0539	0.88973
R-BC		0.9459	0.0041	0.0015	0.0526	0.89451
BOOTSTRAP-TB		0.9284	0.0216	0.0484	0.0232	0.62620
MVUE1	25	0.9339	0.0161	0.0322	0.0339	0.45290
MVUE2		0.9348	0.0152	0.0324	0.0328	0.45659
MLE-TB		0.9298	0.0202	0.0166	0.0536	0.46251
MLE-BC		0.9301	0.0199	0.0166	0.0533	0.46307
SLR		0.9358	0.0142	0.0384	0.0258	0.47389
ASLR		0.9356	0.0144	0.0386	0.0258	0.47866
BOOTSTRAP-BC		0.9292	0.0208	0.0456	0.0252	0.46928
R-MVUE1		0.9683	0.0183	0.0023	0.0294	0.65087
R-TB		0.9255	0.0245	0.0005	0.0740	0.69043
R-BC		0.9262	0.0238	0.0005	0.0733	0.69265
BOOTSTRAP-TB		0.9295	0.0205	0.0454	0.0251	0.46979
MVUE1	50	0.9476	0.0024	0.0188	0.0336	0.32275
MVUE2		0.0946	0.8551	0.0186	0.0328	0.32409
MLE-TB		0.9381	0.0119	0.0118	0.0501	0.32611
MLE-BC		0.9383	0.0117	0.0117	0.0500	0.32630
SLR		0.9453	0.0047	0.0395	0.0152	0.32981
ASLR		0.9459	0.0041	0.0391	0.0150	0.33481
BOOTSTRAP-BC		0.9414	0.0086	0.0433	0.0153	0.32794
R-MVUE1		0.9334	0.0166	0.0003	0.0663	0.32794
R-TB		0.8694	0.0806	0.0000	0.1306	0.48899
R-BC		0.8703	0.0797	0.0000	0.1297	0.48978
BOOTSTRAP-TB		0.9417	0.0083	0.0430	0.0153	0.32813

Table 4.21: 95% CI under Δ(0.4, -0.25, 0.50) with 60% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9029	0.0471	0.0924	0.0047	0.83396
MVUE2		0.9181	0.0319	0.0788	0.0031	0.95975
MLE-TB		0.9353	0.0147	0.0319	0.0328	1.05515
MLE-BC		0.9380	0.0120	0.0306	0.0314	1.07093
SLR		0.9361	0.0139	0.0363	0.0276	1.35864
ASLR		0.9363	0.0137	0.0361	0.0276	1.37221
BOOTSTRAP-BC		0.9227	0.0273	0.0427	0.0346	1.36054
R-MVUE1		0.9340	0.0160	0.0629	0.0031	0.91515
R-TB		0.9502	0.0002	0.0156	0.0342	1.17051
R-BC		0.9530	0.0030	0.0148	0.0322	1.18781
BOOTSTRAP-TB		0.9330	0.0170	0.0403	0.0267	1.44351
MVUE1	25	0.9252	0.0248	0.0695	0.0053	0.65519
MVUE2		0.9350	0.0150	0.0613	0.0037	0.71164
MLE-TB		0.9443	0.0057	0.0257	0.0300	0.74908
MLE-BC		0.9451	0.0049	0.0254	0.0295	0.75372
SLR		0.9440	0.0060	0.0314	0.0246	0.82612
ASLR		0.9443	0.0057	0.0311	0.0246	0.83223
BOOTSTRAP-BC		0.9382	0.0118	0.0379	0.0239	0.82986
R-MVUE1		0.9624	0.0124	0.0330	0.0046	0.75401
R-TB		0.9567	0.0067	0.0055	0.0378	0.88486
R-BC		0.9583	0.0083	0.0053	0.0364	0.89146
BOOTSTRAP-TB		0.9399	0.0101	0.0374	0.0227	0.83506
MVUE1	50	0.9413	0.0087	0.0467	0.0120	0.47513
MVUE2		0.9489	0.0011	0.0426	0.0085	0.49551
MLE-TB		0.9447	0.0053	0.0204	0.0349	0.50761
MLE-BC		0.9453	0.0047	0.0204	0.0343	0.50902
SLR		0.9450	0.0050	0.0351	0.0199	0.52844
ASLR		0.9451	0.0049	0.0350	0.0199	0.53965
BOOTSTRAP-BC		0.9399	0.0101	0.0410	0.0191	0.53023
R-MVUE1		0.9725	0.0225	0.0113	0.0162	0.56536
R-TB		0.9403	0.0097	0.0027	0.0570	0.61572
R-BC		0.9414	0.0086	0.0027	0.0559	0.61794
BOOTSTRAP-TB		0.9410	0.0090	0.0402	0.0188	0.53152

Table 4.22: 95% CI under $\Delta(0.4, -0.50, 1.0)$ with 100% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.7817	0.1683	0.2179	0.0004	0.86380
MVUE2		0.8266	0.1234	0.1732	0.0002	1.16901
MLE-TB		0.9065	0.0435	0.0888	0.0047	1.42294
MLE-BC		0.9093	0.0407	0.0863	0.0044	1.47194
SLR		0.9306	0.0194	0.0089	0.0605	2.38383
ASLR		0.9311	0.0189	0.0088	0.0601	2.69034
BOOTSTRAP-BC		0.9223	0.0277	0.0130	0.0647	37.60716
R-MVUE1		0.7694	0.1806	0.2303	0.0003	0.77195
R-TB		0.9178	0.0322	0.0776	0.0046	1.09574
R-BC		0.9209	0.0291	0.0748	0.0043	1.11815
BOOTSTRAP-TB		0.9309	0.0191	0.0118	0.0573	416.33140
MVUE1	25	0.8047	0.1453	0.1950	0.0003	0.70895
MVUE2		0.8395	0.1105	0.1604	0.0001	0.85258
MLE-TB		0.9157	0.0343	0.0799	0.0044	0.93886
MLE-BC		0.9173	0.0327	0.0783	0.0044	0.95083
SLR		0.9373	0.0127	0.0062	0.0565	1.17731
ASLR		0.9373	0.0127	0.0062	0.0565	1.19022
BOOTSTRAP-BC		0.9378	0.0122	0.0085	0.0537	1.24420
R-MVUE1		0.8047	0.1453	0.1952	0.0001	0.65497
R-TB		0.9359	0.0141	0.0609	0.0032	0.82626
R-BC		0.9379	0.0121	0.0589	0.0032	0.83509
BOOTSTRAP-TB		0.9396	0.0104	0.0079	0.0525	1.25911
MVUE1	50	0.8134	0.1366	0.1866	0.0000	0.51964
MVUE2		0.8394	0.1106	0.1606	0.0000	0.57099
MLE-TB		0.9102	0.0398	0.0875	0.0023	0.59641
MLE-BC		0.9110	0.0390	0.0867	0.0023	0.59965
SLR		0.9303	0.0197	0.0035	0.0662	0.65336
ASLR		0.9310	0.0190	0.0035	0.0655	0.66606
BOOTSTRAP-BC		0.9330	0.0170	0.0054	0.0616	0.66863
R-MVUE1		0.8199	0.1301	0.1801	0.0000	0.49363
R-TB		0.9296	0.0204	0.0690	0.0014	0.55894
R-BC		0.9310	0.0190	0.0676	0.0014	0.56175
BOOTSTRAP-TB		0.9338	0.0162	0.0050	0.0612	0.67182

Chapter 5

CONCLUSIONS AND FUTURE RESEARCH

Existing interval estimators for the mean of a delta distribution have shown very good performance when model assumptions are met. In particular, intervals based on likelihood ratio (SLR, ASLR) have demonstrated great stability in their coverage properties when there are no contaminants in the data. However, these methods are not robust. They seem to be sensitive to small departures from the assumed distribution. On the other hand, present methods that are functions of classic estimators $\bar{x}$ and σ^2 can be protected from the effect of contaminants by simple direct substitution of more robust estimators, such as T_H and S_{bi}. This simple process has shown positive results.

To a certain extent, proposed methods, especially R-TB and R-BC, possess the three features of a robust estimator that Huber (1981) has defined. First, these methods have shown reasonably good performance at the assumed delta distribution. Second, coverage properties of these methods were slightly affected when there are small to moderate contaminants in the data. Last, they have demonstrated resilience when there are large deviations from the model. This last feature was very evident in terms of interval width.

However, proposed methods, using the tuning constants 1.28 and 9 respectively for T_H and S_{bi}, have shown inferior performance on situations where level of skewness is too small or too high. Fortunately, the performance of these proposed methods is not limited to the results presented in this study. They can still be refined on scenarios where they did not show good coverage properties by adjusting the tuning constants. Overall, proposed methods are very good alternatives to existing methods, especially when contamination in the data is anticipated.

We have chosen M-estimators, particularly T_H and S_{bi}, as substitutes of the classic ones mainly because they offer great flexibility, high performance and convenience. However, the users can still choose other robust estimators. In order to select the proper estimators, the user must have a good understanding of the data in terms of level of skewness, number of zero observations, sample size and type and amount of contamination they are anticipating. These information will also allow them to choose the proper tuning constants.

For future research, aside from the presence of zero observations, data could contain left censored values. Performance of the methods presented in this paper can be studied by setting censored values to zero. Also, their coverage properties can be compared to the classic interval estimators of lognormal distribution with censored values.

More powerful test for normality of log-transformed data should be developed. It is important to detect small deviation from lognormality especially for small sample sizes.

Appendix A

R Code for the Bessel Function $G_{n_1}(t)$

```
`Gn1` <- function(n1, tee, tol=0.00000001)
{ n2 <- n1 - 1
  ans <- 1 + n2/n1*tee
  g <- function(i, n1, tee, n=n2)
     (2*i-1)*log(n) + i*log(tee/n1) - lgamma(i+1) - sum(log(n1+2*(2:i)-3))
  i <- 2
  repeat {
     if( abs(current <- exp(g(i,n1,tee,n2))) < tol )break
     ans <- ans + current
     i <- i+ 1
  }
  ans
}
```

Appendix B

Algorithm for SLR

Algorithm for deriving the upper confidence bound:

1. Set a tolerance level (*tol*), δ for numerical differentiation, m for maximum iteration, j=0 and θ_0 as initial guess for θ.

2. Compute the first likelihood (L1) based on maximum likelihood estimators.

3. For the given θ_0, derive the ML estimates for two nuisance parameters (μ_θ and η_θ). The function **constroptim** in R can easily do this.

4. Compute the second likelihood (L2) based on θ_0, $\hat{\mu}_{\theta_0}$ and $\hat{\eta}_{\theta_0}$.

5. Compute $r(\theta_0)$.

6. Repeat 3-5 for $r^*(\theta_0 + \delta)$ and $r^*(\theta_0 - \delta)$.

7. Then, compute the modified Newton-Raphson algorithm:

$$\theta_1 = \theta_0 + \frac{z_{\alpha/2} - r(\theta_0)}{[r(\theta_0 + \delta) - r(\theta_0 - \delta)]/2\delta} \quad .$$

8. $j= j + 1$. If $|\theta_1 - \theta_0| < tol$ or $j= m$, then $\theta_{HI} = \theta_0$. Else $\theta_0 = \theta_1$. Repeat steps 3-8.

The same algorithm can be used to determine the lower bound of the interval by using $-z_{\alpha/2}$ in step 7.

Therefore, the (1-α)100% confidence interval for κ is

$$\left(e^{\theta_{LO}}, e^{\theta_{HI}}\right) \quad . \tag{B.1}$$

Appendix C

Algorithm for ASLR

Algorithm for deriving the upper confidence bound:

1. Set a tolerance level (*tol*), δ for numerical differentiation, m for maximum iteration, j=0 and θ_0 as initial guess for θ.

2. Compute the first likelihood (L1) based on maximum likelihood estimators.

3. For the given θ_0, derive the two nuisance parameters (μ_θ and σ_θ) that will maximize the second likelihood (L2). There are no explicit expressions for these so, a numerical search must be performed.

4. Compute the second likelihood (L2) based on θ_0, μ_{θ_0} and σ_{θ_0}.

5. Compute the quantity $u(\theta)$ based on θ_0, μ_{θ_0} and σ_{θ_0}.

6. Compute $r^*(\theta_0)$.

7. Repeat 3-6 for $r^*(\theta_0 + \delta)$ and $r^*(\theta_0 - \delta)$.

8. Then, compute the modified Newton-Raphson algorithm:

$$\theta_1 = \theta_0 + \frac{z_{\alpha/2} - r^*(\theta_0)}{[r^*(\theta_0 + \delta) - r^*(\theta_0 - \delta)]/2\delta} \quad .$$

9. $j = j + 1$. If $|\theta_1 - \theta_0| < tol$ or $j = m$, then $\theta_{HI} = \theta_0$. Else $\theta_0 = \theta_1$. Repeat steps 3-9.

The same algorithm can be used to determine the lower bound of the interval by using $-z_{\alpha/2}$ in step 8.

Appendix D

Algorithm for Bootstrap-BC

Algorithm:

1. Generate $n1^*$, numbers of nonzero values from a binomial distribution $BIN(n, \hat{\delta})$.

2. Generate Z^* and $\chi^{2*}_{n_1-1}$.

3. Compute $T^*_m(\hat{\delta}, s^2)$ where $\hat{\delta} = n_0/n$.

4. Repeat steps 1-3 B times where B is the desired number of bootstraps.

5. Arrange T^*_m in ascending order. Let $t^{*(\alpha)}$ be the α^{th} percentile of the bootstraps.

Then, the corresponding 100(1-α)% bootstrap percentile interval is

$$\left(e^{\hat{\theta}-t^{*(1-\alpha/2)}\sqrt{\hat{\nu}_{bc}(\hat{\theta})}}, e^{\hat{\theta}-t^{*(\alpha/2)}\sqrt{\hat{\nu}_{bc}(\hat{\theta})}}\right) .$$

Appendix E

Derivation of the the Pivotal Statistic $T_m(\delta, \sigma)$

Consider the pivotal statistic T defined as

$$T = \frac{\hat{\theta} - \theta}{\sqrt{\hat{\nu}(\hat{\theta})}} \quad .$$

Using (2.3) and (2.4), T can be expressed as

$$T = \frac{(\log(1-\hat{\delta}) + \hat{\mu} + \frac{s^2}{2}) - (\log(1-\delta) + \mu + \frac{\sigma^2}{2})}{\sqrt{\frac{n_0}{nn_1} + \frac{s^2}{n_1} + \frac{s^4}{2n_1}}} \quad .$$

Rearranging the terms in numerator, we have

$$T = \frac{(\log(1-\hat{\delta}) - \log(1-\delta)) + (\hat{\mu} - \mu) + (\frac{s^2}{2} - \frac{\sigma^2}{2})}{\sqrt{\frac{n_0}{nn_1} + \frac{s^2}{n_1} + \frac{s^4}{2n_1}}} \quad .$$

Multiplying the numerator and denominator by $\sqrt{n_1}/\sigma$,

$$\begin{aligned} T &= \frac{\sqrt{n_1}/\sigma \log \frac{(1-\hat{\delta})}{(1-\delta)} + \sqrt{n_1}/\sigma(\hat{\mu}-\mu) + \sqrt{n_1}/\sigma\left(s^2/2-\sigma^2/2\right)}{\sqrt{\frac{n_1}{\sigma^2}\frac{n_0}{nn_1} + \frac{n_1}{\sigma^2}\frac{s^2}{n_1} + \frac{n_1}{\sigma^2}\frac{s^4}{2n_1}}} \\ &= \frac{\sqrt{n_1}/\sigma \log \frac{(1-\hat{\delta})}{(1-\delta)} + \sqrt{n_1}/\sigma(\hat{\mu}-\mu) + \sigma\sqrt{n_1}/2\left(s^2/\sigma^2-\sigma^2/\sigma^2\right)}{\sqrt{\frac{n_0}{n\sigma^2} + \frac{s^2}{\sigma^2} + \frac{s^4}{2\sigma^2}}} \quad . \end{aligned}$$

Let

$$Z = \frac{(\hat{\mu} - \mu)}{\sigma/\sqrt{n_1}} \quad \sim \quad N(0,1)$$

and

$$X^2 = \frac{(n_1 - 1)s^2}{\sigma^2} \quad \sim \quad \chi^2_{n_1-1} \quad .$$

Then, we finally have

$$T = \frac{\sqrt{n_1/\sigma^2}\log\frac{n_1}{n(1-\hat{\delta})} + Z + \sqrt{n_1\sigma^2}/2\left\{\frac{X^2}{n_1-1} - 1\right\}}{\sqrt{\frac{n_0}{n\sigma^2} + \frac{X^2}{n_1-1}\left\{1 + \frac{\sigma^2 X^2}{2(n_1-1)}\right\}}} = T_m(\delta, \sigma^2) \quad .$$

Appendix F

Introduction to Birnbaum-Saunders Distribution

The Birnbaum-Saunders distribution, also known as the fatigue life distribution, is a two-parameter distribution proposed by Birnbaum and Saunders (1969) for fatigue failure caused under cyclic loading. It is denoted as $BS(\alpha,\beta)$ where α and β are the shape and scale parameters, respectively.

Suppose $T_1, ..., T_n$ follow $BS(\alpha,\beta)$ then the *cumulative distribution function* (CDF) of T is given as

$$F_T(t;\alpha,\beta) = \Phi\left[\frac{1}{\alpha}\left\{\left(\frac{t}{\beta}\right)^{1/2} - \left(\frac{\beta}{t}\right)^{1/2}\right\}\right] \ , \quad 0 < t < \infty, \quad \alpha,\ \beta > 0$$

where $\Phi(\cdot)$ is the standard normal CDF. Correspondingly, the *probability density function* of T can be written as

$$\begin{aligned} f_T(t;\alpha,\beta) &= \frac{1}{2\sqrt{2\pi}\alpha\beta}\left[\left(\frac{\beta}{t}\right)^{1/2} + \left(\frac{\beta}{t}\right)^{3/2}\right] \\ &\quad \times \exp\left[-\frac{1}{2\alpha^2}\left(\frac{t}{\beta} + \frac{\beta}{t} - 2\right)\right] \ , \quad t > 0, \quad \alpha,\ \beta > 0 \ . \end{aligned}$$

It can be seen in Figure F.1 that the density function of Birnbaum-Saunders distribution is unimodal. Its shape is nearly symmetric and short tailed when β is large, and highly skewed and long tailed when β is small.

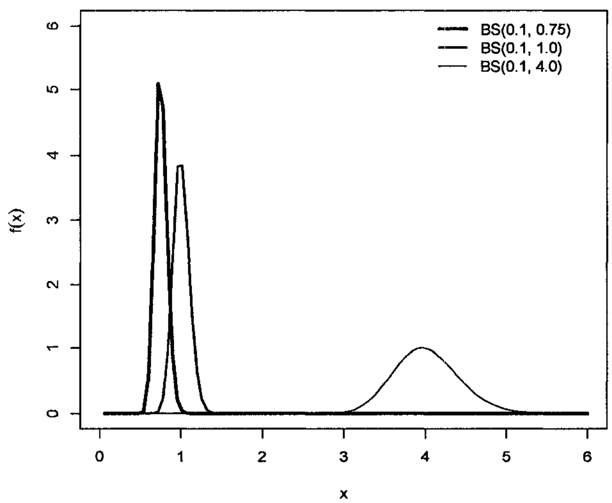

Figure F.1: Density Function of Birnbaum-Saunders

The expected value and variance of T can be easily obtained as

$$E(T) = \beta\left(1 + \frac{1}{2}\alpha^2\right)$$

$$Var(T) = (\alpha\beta)^2\left(1 + \frac{5}{4}\alpha^2\right) \quad .$$

The package **bs** in R is designed for this distribution.

Appendix G

Model A Simulation Results

Table G.1: 95% CI under $\Delta(0.2, 0, 1)$ with Two Contaminants from $\Delta(0.2, 0, 2)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8901	0.0599	0.1088	0.0011	2.07341
MVUE2		0.9136	0.0364	0.0862	0.0002	2.74044
MLE-TB		0.9354	0.0146	0.0437	0.0209	3.18936
MLE-BC		0.9376	0.0124	0.0431	0.0193	3.26299
SLR		0.9357	0.0143	0.0343	0.0300	4.54126
ASLR		0.9366	0.0134	0.0339	0.0295	5.12909
BOOTSTRAP-BC		0.9183	0.0317	0.0528	0.0289	7.77848
R-MVUE1		0.8872	0.0628	0.1117	0.0011	1.74250
R-TB		0.9515	0.0015	0.0321	0.0164	2.30506
R-BC		0.9540	0.0040	0.0307	0.0153	2.33940
BOOTSTRAP-TB		0.9213	0.0287	0.0501	0.0286	8.27485
MVUE1	25	0.9153	0.0347	0.0833	0.0014	1.57760
MVUE2		0.9296	0.0204	0.0700	0.0004	1.83802
MLE-TB		0.9435	0.0065	0.0370	0.0195	1.96350
MLE-BC		0.9446	0.0054	0.0368	0.0186	1.98276
SLR		0.9406	0.0094	0.0341	0.0253	2.33972
ASLR		0.9410	0.0090	0.0340	0.0250	2.44081
BOOTSTRAP-BC		0.9330	0.0170	0.0444	0.0226	2.47652
R-MVUE1		0.9280	0.0220	0.0703	0.0017	1.41817
R-TB		0.9630	0.0130	0.0225	0.0145	1.68662
R-BC		0.9640	0.0140	0.0217	0.0143	1.70054
BOOTSTRAP-TB		0.9345	0.0155	0.0431	0.0224	2.49792
MVUE1	50	0.9262	0.0238	0.0701	0.0037	1.08409
MVUE2		0.9365	0.0135	0.0625	0.0010	1.16460
MLE-TB		0.9453	0.0047	0.0339	0.0208	1.19692
MLE-BC		0.9458	0.0042	0.0336	0.0206	1.20190
SLR		0.9430	0.0070	0.0309	0.0261	1.28517
ASLR		0.9439	0.0061	0.0305	0.0256	1.31113
BOOTSTRAP-BC		0.9383	0.0117	0.0375	0.0242	1.31282
R-MVUE1		0.9484	0.0016	0.0487	0.0029	1.03279
R-TB		0.9663	0.0163	0.0180	0.0157	1.12922
R-BC		0.9669	0.0169	0.0177	0.0154	1.13363
BOOTSTRAP-TB		0.9394	0.0106	0.0366	0.0240	1.31781

Table G.2: 95% CI under $\Delta(0.4,\ 0,\ 1)$ with Two Contaminants from $\Delta(0.4,\ 0,\ 2)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8710	0.0790	0.1281	0.0009	1.89961
MVUE2		0.9019	0.0481	0.0979	0.0002	2.82933
MLE-TB		0.9309	0.0191	0.0446	0.0245	4.47900
MLE-BC		0.9338	0.0162	0.0435	0.0227	5.51278
SLR		0.9293	0.0207	0.0380	0.0327	5.59772
ASLR		0.9314	0.0186	0.0366	0.0320	5.62099
BOOTSTRAP-BC		0.9126	0.0374	0.0522	0.0352	$3.807e^{10}$
R-MVUE1		0.8580	0.0920	0.1409	0.0011	1.54759
R-TB		0.9372	0.0128	0.0411	0.0217	2.23437
R-BC		0.9413	0.0087	0.0387	0.0200	2.28290
BOOTSTRAP-TB		0.9206	0.0294	0.0483	0.0311	$6.014e^{12}$
MVUE1	25	0.8970	0.0530	0.1013	0.0017	1.44845
MVUE2		0.9167	0.0333	0.083	0.0003	1.78036
MLE-TB		0.9384	0.0116	0.0376	0.0240	1.99363
MLE-BC		0.9396	0.0104	0.0370	0.0234	2.02270
SLR		0.9373	0.0127	0.0354	0.0273	2.59973
ASLR		0.9381	0.0119	0.0347	0.0272	2.73242
BOOTSTRAP-BC		0.9271	0.0229	0.0465	0.0264	2.86512
R-MVUE1		0.9000	0.0500	0.0982	0.0018	1.26441
R-TB		0.9523	0.0023	0.0291	0.0186	1.58876
R-BC		0.9537	0.0037	0.0284	0.0179	1.60549
BOOTSTRAP-TB		0.9303	0.0197	0.0436	0.0261	2.91266
MVUE1	50	0.9192	0.0308	0.0775	0.0033	1.00028
MVUE2		0.9323	0.0177	0.0661	0.0016	1.09827
MLE-TB		0.9434	0.0066	0.0324	0.0242	1.14739
MLE-BC		0.9446	0.0054	0.0322	0.0232	1.15357
SLR		0.9437	0.0063	0.0318	0.0245	1.25634
ASLR		0.9444	0.0056	0.0315	0.0241	1.27333
BOOTSTRAP-BC		0.9367	0.0133	0.0395	0.0238	1.28507
R-MVUE1		0.9328	0.0172	0.0643	0.0029	0.93506
R-TB		0.9622	0.0122	0.0202	0.0176	1.05290
R-BC		0.9636	0.0136	0.0195	0.0169	1.05798
BOOTSTRAP-TB		0.9375	0.0125	0.0388	0.0237	1.29122

Table G.3: 95% CI under Δ(0.2, 0, 1) with Three Contaminants from Δ(0.2, 0, 2)

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9007	0.0493	0.0982	0.0011	2.24456
MVUE2		0.9229	0.0271	0.0768	0.0003	3.05747
MLE-TB		0.9374	0.0126	0.0383	0.0243	3.64943
MLE-BC		0.9400	0.0100	0.0377	0.0223	3.74504
SLR		0.9316	0.0184	0.0425	0.0259	5.19990
ASLR		0.9320	0.0180	0.0417	0.0263	5.80902
BOOTSTRAP-BC		0.9113	0.0387	0.0647	0.0240	11.30261
R-MVUE1		0.8902	0.0598	0.1084	0.0014	1.79958
R-TB		0.9515	0.0015	0.0298	0.0187	2.40846
R-BC		0.9533	0.0033	0.0290	0.0177	2.44566
BOOTSTRAP-TB		0.9152	0.0348	0.0610	0.0238	12.33373
MVUE1	25	0.9133	0.0367	0.0848	0.0019	1.63775
MVUE2		0.9300	0.0200	0.0697	0.0003	1.92117
MLE-TB		0.9423	0.0077	0.0333	0.0244	2.05870
MLE-BC		0.9438	0.0062	0.0329	0.0233	2.07957
SLR		0.9351	0.0149	0.0424	0.0225	2.46868
ASLR		0.9364	0.0136	0.0409	0.0227	2.51123
BOOTSTRAP-BC		0.9214	0.0286	0.0571	0.0215	2.61998
R-MVUE1		0.9212	0.0288	0.0768	0.0020	1.43501
R-TB		0.9625	0.0125	0.0207	0.0168	1.71214
R-BC		0.9636	0.0136	0.0203	0.0161	1.72647
BOOTSTRAP-TB		0.9239	0.0261	0.0548	0.0213	2.64330
MVUE1	50	0.9300	0.0200	0.0656	0.0044	1.11274
MVUE2		0.9409	0.0091	0.0578	0.0013	1.19810
MLE-TB		0.9443	0.0057	0.0309	0.0248	1.23249
MLE-BC		0.9455	0.0045	0.0306	0.0239	1.23773
SLR		0.9406	0.0094	0.0359	0.0235	1.32581
ASLR		0.9421	0.0079	0.0348	0.0231	1.32981
BOOTSTRAP-BC		0.9359	0.0141	0.0422	0.0219	1.35561
R-MVUE1		0.9496	0.0004	0.0473	0.0031	1.04361
R-TB		0.9656	0.0156	0.0171	0.0173	1.14232
R-BC		0.9662	0.0162	0.0169	0.0169	1.14681
BOOTSTRAP-TB		0.9369	0.0131	0.0413	0.0218	1.36089

Table G.4: 95% CI under $\Delta(0.4, 0, 1)$ with Three Contaminants from $\Delta(0.4, 0, 2)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8830	0.0670	0.1158	0.0012	2.10932
MVUE2		0.9144	0.0356	0.0854	0.0002	3.34694
MLE-TB		0.9364	0.0136	0.0369	0.0267	5.05264
MLE-BC		0.9401	0.0099	0.0358	0.0241	5.42665
SLR		0.9399	0.0101	0.0356	0.0245	6.64611
ASLR		0.9411	0.0089	0.0348	0.0241	7.10234
BOOTSTRAP-BC		0.9054	0.0446	0.0663	0.0283	$5.641e^{03}$
R-MVUE1		0.8643	0.0857	0.1343	0.0014	1.61873
R-TB		0.9423	0.0077	0.0354	0.0223	2.38760
R-BC		0.9451	0.0049	0.0341	0.0208	2.42200
BOOTSTRAP-TB		0.9151	0.0349	0.0600	0.0249	$1.868e^{04}$
MVUE1	25	0.9065	0.0435	0.0919	0.0016	1.53993
MVUE2		0.9267	0.0233	0.0729	0.0004	1.92949
MLE-TB		0.9396	0.0104	0.0331	0.0273	2.18904
MLE-BC		0.9421	0.0079	0.0321	0.0258	2.22426
SLR		0.9426	0.0074	0.0319	0.0255	2.89147
ASLR		0.9433	0.0067	0.0314	0.0253	3.00432
BOOTSTRAP-BC		0.9208	0.0292	0.0557	0.0235	3.36191
R-MVUE1		0.9024	0.0476	0.0957	0.0019	1.12958
R-TB		0.9513	0.0013	0.0283	0.0204	1.64110
R-BC		0.9529	0.0029	0.0273	0.0198	1.65895
BOOTSTRAP-TB		0.9239	0.0261	0.0532	0.0229	3.43463
MVUE1	50	0.9260	0.0240	0.0703	0.0037	1.03294
MVUE2		0.9379	0.0121	0.0603	0.0018	1.13878
MLE-TB		0.9431	0.0069	0.0297	0.0272	1.19185
MLE-BC		0.9442	0.0058	0.0292	0.0266	1.19849
SLR		0.9451	0.0049	0.0290	0.0259	1.31000
ASLR		0.9450	0.0050	0.0288	0.0262	1.31452
BOOTSTRAP-BC		0.9329	0.0171	0.0449	0.0222	1.34221
R-MVUE1		0.9340	0.0160	0.0626	0.0034	0.94761
R-TB		0.9615	0.0115	0.0189	0.0196	1.06920
R-BC		0.9621	0.0121	0.0188	0.0191	1.07444
BOOTSTRAP-TB		0.9343	0.0157	0.0437	0.0220	1.34886

Table G.5: 95% CI under $\Delta(0.2,\ 0,\ 1)$ with One Contaminant from $\Delta(0.2,\ 0,\ 4)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8940	0.0560	0.1052	0.0008	2.27252
MVUE2		0.9152	0.0348	0.0845	0.0003	3.17961
MLE-TB		0.9302	0.0198	0.0421	0.0277	3.92646
MLE-BC		0.9338	0.0162	0.0414	0.0248	4.04272
SLR		0.9341	0.0159	0.0377	0.0282	5.10150
ASLR		0.9342	0.0158	0.0370	0.0288	5.42887
BOOTSTRAP-BC		0.9069	0.0431	0.0667	0.0264	17.95314
R-MVUE1		0.8815	0.0685	0.1170	0.0015	1.75254
R-TB		0.9463	0.0037	0.0339	0.0198	2.32185
R-BC		0.9484	0.0016	0.0332	0.0184	2.35661
BOOTSTRAP-TB		0.9112	0.0388	0.0628	0.0260	20.17997
MVUE1	25	0.9128	0.0372	0.0851	0.0021	1.64928
MVUE2		0.9257	0.0243	0.0735	0.0008	1.95221
MLE-TB		0.9332	0.0168	0.0391	0.0277	2.10747
MLE-BC		0.9356	0.0144	0.0377	0.0267	2.13009
SLR		0.9336	0.0164	0.0402	0.0262	2.55068
ASLR		0.9349	0.0151	0.0395	0.0256	2.81123
BOOTSTRAP-BC		0.9216	0.0284	0.0540	0.0244	2.75446
R-MVUE1		0.9166	0.0334	0.0811	0.0023	1.40902
R-TB		0.9576	0.0076	0.0252	0.0172	1.67651
R-BC		0.9587	0.0087	0.0245	0.0168	1.69038
BOOTSTRAP-TB		0.9236	0.0264	0.0522	0.0242	2.78184
MVUE1	50	0.9308	0.0192	0.0655	0.0037	1.11870
MVUE2		0.9421	0.0079	0.0569	0.0010	1.20599
MLE-TB		0.9431	0.0069	0.0306	0.0263	1.24139
MLE-BC		0.9438	0.0062	0.0303	0.0259	1.24672
SLR		0.9441	0.0059	0.0340	0.0219	1.33673
ASLR		0.9445	0.0055	0.0337	0.0218	1.34012
BOOTSTRAP-BC		0.9334	0.0166	0.0461	0.0205	1.36771
R-MVUE1		0.9463	0.0037	0.0515	0.0022	1.03395
R-TB		0.9672	0.0172	0.0175	0.0153	1.13082
R-BC		0.9677	0.0177	0.0171	0.0152	1.13524
BOOTSTRAP-TB		0.9340	0.0160	0.0456	0.0204	1.37312

Table G.6: 95% CI under $\Delta(0.4, 0, 1)$ with One Contaminant from $\Delta(0.4, 0, 4)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8812	0.0688	0.1181	0.0007	2.23368
MVUE2		0.9082	0.0418	0.0917	0.0001	4.26017
MLE-TB		0.9281	0.0219	0.0411	0.0308	14.71453
MLE-BC		0.9332	0.0168	0.0399	0.0269	18.83575
SLR		0.9294	0.0206	0.0384	0.0322	15.31242
ASLR		0.9389	0.0111	0.0302	0.0309	21.03450
BOOTSTRAP-BC		0.8926	0.0574	0.0768	0.0306	$1.695e^{07}$
R-MVUE1		0.8520	0.0980	0.1467	0.0013	1.59104
R-TB		0.9314	0.0186	0.0432	0.0254	2.32903
R-BC		0.9359	0.0141	0.0406	0.0235	2.38213
BOOTSTRAP-TB		0.9015	0.0485	0.0716	0.0269	$7.589e^{07}$
MVUE1	25	0.9047	0.0453	0.0939	0.0014	1.57971
MVUE2		0.9234	0.0266	0.0763	0.0003	2.05296
MLE-TB		0.9355	0.0145	0.0342	0.0303	2.50165
MLE-BC		0.9379	0.0121	0.0339	0.0282	2.57020
SLR		0.9383	0.0117	0.0311	0.0306	2.99120
ASLR		0.9380	0.0120	0.0311	0.0309	3.01108
BOOTSTRAP-BC		0.9137	0.0363	0.0616	0.0247	59.62850
R-MVUE1		0.8952	0.0548	0.1033	0.0015	1.27335
R-TB		0.9523	0.0023	0.0276	0.0201	1.60634
R-BC		0.9543	0.0043	0.0266	0.0191	1.62356
BOOTSTRAP-TB		0.9161	0.0339	0.0596	0.0243	89.32289
MVUE1	50	0.9256	0.0244	0.0713	0.0031	1.03427
MVUE2		0.9370	0.0130	0.0623	0.0007	1.14208
MLE-TB		0.9389	0.0111	0.0315	0.0296	1.19649
MLE-BC		0.9400	0.0100	0.0315	0.0285	1.20323
SLR		0.9366	0.0134	0.0391	0.0243	1.31740
ASLR		0.9399	0.0101	0.0372	0.0229	1.33056
BOOTSTRAP-BC		0.9285	0.0215	0.0487	0.0228	1.35145
R-MVUE1		0.9322	0.0178	0.0648	0.0030	0.93476
R-TB		0.9611	0.0111	0.0204	0.0185	1.05223
R-BC		0.9624	0.0124	0.0199	0.0177	1.05730
BOOTSTRAP-TB		0.9290	0.0210	0.0482	0.0228	1.35827

Table G.7: 95% CI under $\Delta(0.2, 0, 1)$ with Two Contaminants from $\Delta(0.2, 0, 4)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9136	0.0364	0.0853	0.0011	2.94639
MVUE2		0.9333	0.0167	0.0666	0.0001	4.92713
MLE-TB		0.9236	0.0264	0.0318	0.0446	9.18541
MLE-BC		0.9278	0.0222	0.0313	0.0409	10.08110
SLR		0.9301	0.0199	0.0301	0.0398	12.32134
ASLR		0.9311	0.0189	0.0297	0.0392	14.22120
BOOTSTRAP-BC		0.8692	0.0808	0.1114	0.0194	$3.571e^{04}$
R-MVUE1		0.8865	0.0635	0.1117	0.0018	1.88626
R-TB		0.9355	0.0145	0.0335	0.0310	2.57159
R-BC		0.9384	0.0116	0.0323	0.0293	2.61356
BOOTSTRAP-TB		0.8760	0.0740	0.1051	0.0189	$6.636e^{04}$
MVUE1	25	0.9334	0.0166	0.0638	0.0028	1.93795
MVUE2		0.9478	0.0022	0.0518	0.0004	2.37238
MLE-TB		0.9271	0.0229	0.0263	0.0466	2.61083
MLE-BC		0.9294	0.0206	0.0261	0.0445	2.64321
SLR		0.9286	0.0214	0.0581	0.0133	3.25048
ASLR		0.9302	0.0198	0.0399	0.0299	3.40003
BOOTSTRAP-BC		0.8907	0.0593	0.0926	0.0167	3.61255
R-MVUE1		0.9224	0.0276	0.0738	0.0038	1.48168
R-TB		0.9517	0.0017	0.0223	0.0260	1.77937
R-BC		0.9536	0.0036	0.0218	0.0246	1.79470
BOOTSTRAP-TB		0.8947	0.0553	0.0888	0.0165	3.65571
MVUE1	50	0.9448	0.0052	0.0495	0.0057	1.21089
MVUE2		0.9562	0.0062	0.0426	0.0012	1.31570
MLE-TB		0.9369	0.0131	0.0233	0.0398	1.35897
MLE-BC		0.9381	0.0119	0.0228	0.0391	1.36521
SLR		0.9388	0.0112	0.0383	0.0229	1.47268
ASLR		0.9401	0.0099	0.0352	0.0247	1.50387
BOOTSTRAP-BC		0.9176	0.0324	0.0668	0.0156	1.51214
R-MVUE1		0.9475	0.0025	0.0489	0.0036	1.05538
R-TB		0.9636	0.0136	0.0167	0.0197	1.15698
R-BC		0.9644	0.0144	0.0162	0.0194	1.16159
BOOTSTRAP-TB		0.9186	0.0314	0.0658	0.0156	1.51858

Table G.8: 95% CI under $\Delta(0.4, 0, 1)$ with Two Contaminants from $\Delta(0.4, 0, 4)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8992	0.0508	0.0999	0.0009	3.00619
MVUE2		0.9251	0.0249	0.0744	0.0005	228.55450
MLE-TB		0.9239	0.0261	0.0320	0.0441	$5.399e^{06}$
MLE-BC		0.9291	0.0209	0.0307	0.0402	$2.255e^{07}$
SLR		0.9193	0.0307	0.0575	0.0232	8.49549
ASLR		0.9203	0.0297	0.0566	0.0231	11.14889
BOOTSTRAP-BC		0.8577	0.0923	0.1198	0.0225	$2.457e^{45}$
R-MVUE1		0.8530	0.0970	0.1449	0.0021	1.73858
R-TB		0.9215	0.0285	0.0427	0.0358	2.76573
R-BC		0.9247	0.0253	0.0411	0.0342	2.85792
BOOTSTRAP-TB		0.8697	0.0803	0.1104	0.0199	$2.909e^{52}$
MVUE1	25	0.9184	0.0316	0.0795	0.0021	1.86766
MVUE2		0.9376	0.0124	0.0622	0.0002	2.57118
MLE-TB		0.9275	0.0225	0.0271	0.0454	3.82969
MLE-BC		0.9316	0.0184	0.0266	0.0418	4.22169
SLR		0.9325	0.0175	0.0409	0.0266	5.31256
ASLR		0.9338	0.0162	0.0399	0.0263	5.91231
BOOTSTRAP-BC		0.8882	0.0618	0.0927	0.0191	$1.353e^{06}$
R-MVUE1		0.8948	0.0552	0.1025	0.0027	1.34475
R-TB		0.9446	0.0054	0.0284	0.0270	1.72303
R-BC		0.9471	0.0029	0.0271	0.0258	1.74269
BOOTSTRAP-TB		0.8929	0.0571	0.0882	0.0189	$2.629e^{06}$
MVUE1	50	0.9422	0.0078	0.0536	0.0042	1.14989
MVUE2		0.9529	0.0029	0.0460	0.0011	1.28980
MLE-TB		0.9365	0.0135	0.0222	0.0413	1.36155
MLE-BC		0.9376	0.0124	0.0219	0.0405	1.37013
SLR		0.9370	0.0130	0.0459	0.0171	1.52114
ASLR		0.9389	0.0111	0.0438	0.0173	1.60023
BOOTSTRAP-BC		0.9138	0.0362	0.0709	0.0153	1.57160
R-MVUE1		0.9376	0.0124	0.0589	0.0035	0.96373
R-TB		0.9612	0.0112	0.0177	0.0211	1.08977
R-BC		0.9621	0.0121	0.0176	0.0203	1.09518
BOOTSTRAP-TB		0.9151	0.0349	0.0696	0.0153	1.58067

Table G.9: 95% CI under $\Delta(0.2, 0, 1)$ with Three Contaminants from $\Delta(0.2, 0, 4)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9303	0.0197	0.0679	0.0018	3.59020
MVUE2		0.9481	0.0019	0.0516	0.0003	6.44832
MLE-TB		0.9160	0.0340	0.0242	0.0598	11.36842
MLE-BC		0.9229	0.0271	0.0236	0.0535	12.16892
SLR		0.9256	0.0244	0.0481	0.0263	9.04731
ASLR		0.9250	0.0250	0.0455	0.0295	10.90879
BOOTSTRAP-BC		0.8316	0.1184	0.1541	0.0143	82.65387
R-MVUE1		0.8838	0.0662	0.1128	0.0034	2.03203
R-TB		0.9267	0.0233	0.0343	0.0390	2.85909
R-BC		0.9302	0.0198	0.0330	0.0368	2.90992
BOOTSTRAP-TB		0.8401	0.1099	0.1459	0.0140	$1.185e^{03}$
MVUE1	25	0.9405	0.0095	0.0570	0.0025	2.22314
MVUE2		0.9544	0.0044	0.0452	0.0004	2.82151
MLE-TB		0.9178	0.0322	0.0201	0.0621	3.18303
MLE-BC		0.9222	0.0278	0.0200	0.0578	3.22881
SLR		0.9300	0.0200	0.0404	0.0296	3.96780
ASLR		0.9308	0.0192	0.0385	0.0307	4.12667
BOOTSTRAP-BC		0.8630	0.0870	0.1259	0.0111	4.76204
R-MVUE1		0.9192	0.0308	0.0766	0.0042	1.54771
R-TB		0.9484	0.0016	0.0204	0.0312	1.87713
R-BC		0.9505	0.0005	0.0198	0.0297	1.89400
BOOTSTRAP-TB		0.8683	0.0817	0.1208	0.0109	4.83595
MVUE1	50	0.9473	0.0027	0.0433	0.0094	1.31398
MVUE2		0.9590	0.0090	0.0381	0.0029	1.43953
MLE-TB		0.9259	0.0241	0.0185	0.0556	1.49246
MLE-BC		0.9273	0.0227	0.0182	0.0545	1.49977
SLR		0.9289	0.0211	0.0586	0.0125	1.62801
ASLR		0.9303	0.0197	0.0557	0.0140	1.72357
BOOTSTRAP-BC		0.8938	0.0562	0.0943	0.0119	1.67666
R-MVUE1		0.9477	0.0023	0.0461	0.0062	1.08148
R-TB		0.9600	0.0100	0.0169	0.0231	1.18850
R-BC		0.9608	0.0108	0.0166	0.0226	1.19332
BOOTSTRAP-TB		0.8955	0.0545	0.0927	0.0118	1.68433

Table G.10: 95% CI under $\Delta(0.4, 0, 1)$ with Three Contaminants from $\Delta(0.4, 0, 4)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9131	0.0369	0.0863	0.0006	3.96044
MVUE2		0.9388	0.0112	0.0611	0.0001	501.24540
MLE-TB		0.9126	0.0374	0.0245	0.0629	$1.332e^{09}$
MLE-BC		0.9203	0.0297	0.0238	0.0559	$1.005e^{10}$
SLR		0.9199	0.0301	0.0496	0.0305	$1.036e^{03}$
ASLR		0.9242	0.0258	0.0457	0.0301	$1.213e^{03}$
BOOTSTRAP-BC		0.8148	0.1352	0.1691	0.0161	$5.078e^{80}$
R-MVUE1		0.8543	0.0957	0.1441	0.0016	1.92672
R-TB		0.9090	0.0410	0.0413	0.0497	3.20780
R-BC		0.9131	0.0369	0.0392	0.0477	3.31219
BOOTSTRAP-TB		0.8287	0.1213	0.1577	0.0136	$5.340e^{93}$
MVUE1	25	0.9337	0.0163	0.0653	0.0010	2.25513
MVUE2		0.9485	0.0015	0.0514	0.0001	3.41614
MLE-TB		0.9168	0.0332	0.0222	0.0610	4.86000
MLE-BC		0.9209	0.0291	0.0218	0.0573	5.05514
SLR		0.9261	0.0239	0.0481	0.0258	5.81032
ASLR		0.9259	0.0241	0.0475	0.0266	6.21064
BOOTSTRAP-BC		0.8543	0.0957	0.1311	0.0146	58.54946
R-MVUE1		0.8951	0.0549	0.1028	0.0021	1.40934
R-TB		0.9376	0.0124	0.0278	0.0346	1.84136
R-BC		0.9391	0.0109	0.0270	0.0339	1.86388
BOOTSTRAP-TB		0.8613	0.0887	0.1244	0.0143	72.60327
MVUE1	50	0.9484	0.0016	0.0457	0.0059	1.26361
MVUE2		0.9597	0.0097	0.0387	0.0016	1.44039
MLE-TB		0.9221	0.0279	0.0171	0.0608	1.53383
MLE-BC		0.9244	0.0256	0.0170	0.0586	1.54457
SLR		0.9262	0.0238	0.0604	0.0134	1.64093
ASLR		0.9305	0.0195	0.0582	0.0113	1.73205
BOOTSTRAP-BC		0.8937	0.0563	0.0943	0.0120	1.81407
R-MVUE1		0.9346	0.0154	0.0604	0.0050	0.98746
R-TB		0.9537	0.0037	0.0182	0.0281	1.12170
R-BC		0.9543	0.0043	0.0179	0.0278	1.12744
BOOTSTRAP-TB		0.8945	0.0555	0.0936	0.0119	1.82611

Table G.11: 95% CI under $\Delta(0.2, 0, 1)$ with One Contaminant from $\Delta(0.2, 0, 5)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8946	0.0554	0.1044	0.0010	2.47617
MVUE2		0.9173	0.0327	0.0825	0.0002	3.65544
MLE-TB		0.9327	0.0173	0.0381	0.0292	4.80510
MLE-BC		0.9363	0.0137	0.0375	0.0262	4.97955
SLR		0.9358	0.0142	0.0373	0.0269	5.67785
ASLR		0.9412	0.0088	0.0352	0.0256	6.12098
BOOTSTRAP-BC		0.8921	0.0579	0.0829	0.0250	24.62005
R-MVUE1		0.8787	0.0713	0.1203	0.0010	1.76820
R-TB		0.9445	0.0055	0.0367	0.0188	2.35226
R-BC		0.9467	0.0033	0.0359	0.0174	2.38790
BOOTSTRAP-TB		0.8989	0.0511	0.0770	0.0241	27.68234
MVUE1	25	0.9202	0.0298	0.0780	0.0018	1.77622
MVUE2		0.9359	0.0141	0.0638	0.0003	2.15350
MLE-TB		0.9331	0.0169	0.0330	0.0339	2.36788
MLE-BC		0.9361	0.0139	0.0322	0.0317	2.39676
SLR		0.9346	0.0154	0.0442	0.0212	2.84675
ASLR		0.9405	0.0095	0.0397	0.0198	3.12328
BOOTSTRAP-BC		0.9132	0.0368	0.0674	0.0194	3.29132
R-MVUE1		0.9210	0.0290	0.0768	0.0220	1.42246
R-TB		0.9598	0.0098	0.0231	0.0171	1.69585
R-BC		0.9610	0.0110	0.0222	0.0168	1.71000
BOOTSTRAP-TB		0.9167	0.0333	0.0641	0.0192	3.33467
MVUE1	50	0.9337	0.0163	0.0614	0.0049	1.14759
MVUE2		0.9455	0.0045	0.0528	0.0017	1.24082
MLE-TB		0.9393	0.0107	0.0295	0.0312	1.27898
MLE-BC		0.9401	0.0099	0.0294	0.0305	1.28462
SLR		0.9432	0.0068	0.0357	0.0211	1.38057
ASLR		0.9431	0.0069	0.0348	0.0221	1.40212
BOOTSTRAP-BC		0.9277	0.0223	0.0542	0.0181	1.41370
R-MVUE1		0.9431	0.0069	0.0538	0.0031	1.03520
R-TB		0.9654	0.0154	0.0199	0.0147	1.13243
R-BC		0.9659	0.0159	0.0196	0.0145	1.13686
BOOTSTRAP-TB		0.9290	0.0210	0.0531	0.0179	1.41940

Table G.12: 95% CI under $\Delta(0.4, 0, 1)$ with One Contaminant from $\Delta(0.4, 0, 5)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8846	0.0654	0.1147	0.0007	2.54581
MVUE2		0.9129	0.0371	0.0868	0.0003	6.22632
MLE-TB		0.9270	0.0230	0.0387	0.0343	74.36465
MLE-BC		0.9329	0.0171	0.0376	0.0295	143.08290
SLR		0.9279	0.0221	0.0517	0.0204	6.92399
ASLR		0.9285	0.0215	0.0445	0.0270	8.12907
BOOTSTRAP-BC		0.8765	0.0735	0.0931	0.0304	$8.434e^{18}$
R-MVUE1		0.8527	0.0973	0.1463	0.0010	1.62681
R-TB		0.9289	0.0211	0.0438	0.0273	2.40751
R-BC		0.9322	0.0178	0.0425	0.0253	2.46499
BOOTSTRAP-TB		0.8873	0.0627	0.0866	0.0261	$4.206e^{22}$
MVUE1	25	0.9124	0.0376	0.0864	0.0012	1.67380
MVUE2		0.9293	0.0207	0.0703	0.0004	2.21068
MLE-TB		0.9353	0.0147	0.0319	0.0328	2.66404
MLE-BC		0.9376	0.0124	0.0313	0.0311	2.73016
SLR		0.9342	0.0158	0.0421	0.0237	3.39832
ASLR		0.9405	0.0095	0.0385	0.0210	3.65871
BOOTSTRAP-BC		0.9066	0.0434	0.0706	0.0228	298.99710
R-MVUE1		0.8997	0.0503	0.0982	0.0021	1.28362
R-TB		0.9490	0.0010	0.0299	0.0211	1.62212
R-BC		0.9512	0.0012	0.0288	0.0200	1.63964
BOOTSTRAP-TB		0.9106	0.0394	0.0668	0.0226	$1.326e^{03}$
MVUE1	50	0.9275	0.0225	0.0692	0.0033	1.06746
MVUE2		0.9361	0.0139	0.0624	0.0015	1.18564
MLE-TB		0.9406	0.0094	0.0302	0.0292	1.24609
MLE-BC		0.9419	0.0081	0.0297	0.0284	1.25343
SLR		0.9402	0.0098	0.0354	0.0244	1.38007
ASLR		0.9409	0.0091	0.0348	0.0243	1.38023
BOOTSTRAP-BC		0.9256	0.0244	0.0520	0.0224	1.42081
R-MVUE1		0.9316	0.0184	0.0654	0.0030	0.93761
R-TB		0.9610	0.0110	0.0218	0.0172	1.05611
R-BC		0.9614	0.0114	0.0217	0.0169	1.06122
BOOTSTRAP-TB		0.9270	0.0230	0.0509	0.0221	1.42846

Table G.13: 95% CI under $\Delta(0.2,\ 0,\ 1)$ with Two Contaminants from $\Delta(0.2,\ 0,\ 5)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9203	0.0297	0.0782	0.0015	3.49991
MVUE2		0.9410	0.0090	0.0587	0.0003	6.63591
MLE-TB		0.9126	0.0374	0.029	0.0584	16.41386
MLE-BC		0.9182	0.0318	0.0289	0.0529	18.47251
SLR		0.9123	0.0377	0.0465	0.0412	12.28038
ASLR		0.9166	0.0334	0.1470	0.0181	13.99605
BOOTSTRAP-BC		0.8349	0.1151	0.1470	0.0181	$1.815e^{04}$
R-MVUE1		0.8797	0.0703	0.1176	0.0027	1.95586
R-TB		0.9249	0.0251	0.0361	0.0390	2.69696
R-BC		0.9282	0.0218	0.0350	0.0368	2.74251
BOOTSTRAP-TB		0.8432	0.1068	0.1389	0.0179	$2.916e^{04}$
MVUE1	25	0.9422	0.0078	0.0559	0.0019	2.16407
MVUE2		0.9540	0.0040	0.0456	0.0004	2.75027
MLE-TB		0.9203	0.0297	0.0243	0.0554	3.11476
MLE-BC		0.9242	0.0258	0.0237	0.0521	3.16026
SLR		0.9291	0.0209	0.0502	0.0207	3.87647
ASLR		0.9360	0.0140	0.0412	0.0228	4.00451
BOOTSTRAP-BC		0.8681	0.0819	0.1177	0.0142	4.71917
R-MVUE1		0.9200	0.0300	0.0765	0.0035	1.50075
R-TB		0.9448	0.0052	0.0261	0.0291	1.80932
R-BC		0.9468	0.0032	0.0250	0.0282	1.82515
BOOTSTRAP-TB		0.8729	0.0771	0.1129	0.0142	4.79607
MVUE1	50	0.9446	0.0054	0.0477	0.0077	1.27653
MVUE2		0.9564	0.0064	0.0413	0.0023	1.39572
MLE-TB		0.9262	0.0238	0.0234	0.0504	1.44606
MLE-BC		0.9277	0.0223	0.0231	0.0492	1.45303
SLR		0.9300	0.0200	0.0390	0.0310	1.57515
ASLR		0.9345	0.0155	0.0345	0.0310	1.59004
BOOTSTRAP-BC		0.9042	0.0458	0.081	0.0148	1.62131
R-MVUE1		0.9452	0.0048	0.0505	0.0043	1.06211
R-TB		0.9618	0.0118	0.017	0.0212	1.16511
R-BC		0.9631	0.0131	0.0163	0.0206	1.16978
BOOTSTRAP-TB		0.9056	0.0444	0.0798	0.0146	1.62862

Table G.14: 95% CI under $\Delta(0.4, 0, 1)$ with Two Contaminants from $\Delta(0.4, 0, 5)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9077	0.0423	0.0918	0.0005	3.60727
MVUE2		0.9338	0.0162	0.0661	0.0001	2.32673
MLE-TB		0.9172	0.0328	0.0294	0.0534	$4.068e^{04}$
MLE-BC		0.9233	0.0267	0.0285	0.0482	$1.926e^{05}$
SLR		0.9208	0.0292	0.0431	0.0361	14.32051
ASLR		0.9284	0.0216	0.0412	0.0304	18.32961
BOOTSTRAP-BC		0.8247	0.1253	0.1541	0.0212	$5.399e^{39}$
R-MVUE1		0.8461	0.1039	0.1523	0.0016	1.80829
R-TB		0.9122	0.0378	0.0474	0.0404	2.90277
R-BC		0.9163	0.0337	0.0453	0.0384	2.99111
BOOTSTRAP-TB		0.8398	0.1102	0.1417	0.0185	$3.368e^{48}$
MVUE1	25	0.9263	0.0237	0.0729	0.0008	2.16593
MVUE2		0.9438	0.0062	0.0560	0.0002	3.22109
MLE-TB		0.9173	0.0327	0.0253	0.0574	4.42595
MLE-BC		0.9213	0.0287	0.0249	0.0538	4.59502
SLR		0.9301	0.0199	0.0398	0.0301	4.89621
ASLR		0.9322	0.0178	0.0389	0.0289	5.01590
BOOTSTRAP-BC		0.8582	0.0918	0.1244	0.0174	142.22803
R-MVUE1		0.8957	0.0543	0.1025	0.0018	1.37327
R-TB		0.9389	0.0111	0.0310	0.0301	1.77534
R-BC		0.9417	0.0083	0.0296	0.0287	1.79627
BOOTSTRAP-TB		0.8657	0.0843	0.1175	0.0168	214.75381
MVUE1	50	0.9464	0.0036	0.0484	0.0052	1.24206
MVUE2		0.9562	0.0062	0.0422	0.0016	1.41582
MLE-TB		0.9263	0.0237	0.0192	0.0545	1.50934
MLE-BC		0.9285	0.0215	0.0190	0.0525	1.51999
SLR		0.9310	0.0190	0.0472	0.0218	1.71036
ASLR		0.9308	0.0192	0.0470	0.0222	1.75221
BOOTSTRAP-BC		0.8962	0.0538	0.0913	0.0125	1.79280
R-MVUE1		0.9362	0.0138	0.0593	0.0045	0.97379
R-TB		0.9587	0.0087	0.0173	0.0240	1.10313
R-BC		0.9595	0.0095	0.0172	0.0233	1.10868
BOOTSTRAP-TB		0.8985	0.0515	0.0893	0.0122	1.80500

Appendix H

Model B Simulation Results

Table H.1: 95% CI under Δ(0.2, -0.075, 0.15)

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9309	0.0191	0.0394	0.0297	0.53447
MVUE2		0.9308	0.0192	0.0406	0.0286	0.53624
MLE-TB		0.9353	0.0147	0.0142	0.0505	0.54718
MLE-BC		0.9357	0.0143	0.0141	0.0502	0.54803
SLR		0.9374	0.0126	0.0402	0.0224	0.57352
ASLR		0.9391	0.0109	0.0385	0.0224	0.64086
BOOTSTRAP-BC		0.9283	0.0217	0.0377	0.034	0.56203
R-MVUE1		0.9821	0.0321	0.0063	0.0116	0.76345
R-TB		0.9564	0.0064	0.0011	0.0425	0.82804
R-BC		0.9579	0.0079	0.0011	0.0410	0.83193
BOOTSTRAP-TB		0.9313	0.0187	0.0374	0.0313	0.56432
MVUE1	25	0.9403	0.0097	0.0309	0.0288	0.41780
MVUE2		0.9406	0.0094	0.0314	0.0280	0.41867
MLE-TB		0.9430	0.0070	0.0152	0.0418	0.42351
MLE-BC		0.9434	0.0066	0.0151	0.0415	0.42387
SLR		0.9474	0.0026	0.0267	0.0259	0.43108
ASLR		0.9473	0.0027	0.0265	0.0262	0.43766
BOOTSTRAP-BC		0.9411	0.0089	0.0323	0.0266	0.42837
R-MVUE1		0.9739	0.0239	0.0027	0.0234	0.61417
R-TB		0.9414	0.0086	0.0006	0.0580	0.64669
R-BC		0.9423	0.0077	0.0006	0.0571	0.64852
BOOTSTRAP-TB		0.9415	0.0085	0.0320	0.0265	0.42870
MVUE1	50	0.9426	0.0074	0.0322	0.0252	0.29643
MVUE2		0.9425	0.0075	0.0324	0.0251	0.29671
MLE-TB		0.9440	0.0060	0.0187	0.0373	0.29837
MLE-BC		0.9440	0.0060	0.0187	0.0373	0.29849
SLR		0.9457	0.0043	0.0265	0.0278	0.30071
ASLR		0.9479	0.0021	0.0258	0.0263	0.31121
BOOTSTRAP-BC		0.9434	0.0066	0.0284	0.0282	0.29959
R-MVUE1		0.9436	0.0064	0.0005	0.0559	0.44471
R-TB		0.8927	0.0573	0.0001	0.1072	0.45682
R-BC		0.8934	0.0566	0.0001	0.1065	0.45747
BOOTSTRAP-TB		0.9435	0.0065	0.0283	0.0282	0.29971

Table H.2: 95% CI under $\Delta(0.2, -0.25, 0.15)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9077	0.0423	0.0862	0.0061	0.78272
MVUE2		0.9198	0.0302	0.0760	0.0042	0.85555
MLE-TB		0.9383	0.0117	0.0376	0.0241	0.89527
MLE-BC		0.9400	0.0100	0.0369	0.0231	0.90213
SLR		0.9434	0.0066	0.0196	0.0370	1.03095
ASLR		0.9437	0.0063	0.0194	0.0369	1.09322
BOOTSTRAP-BC		0.9292	0.0208	0.0333	0.0375	1.04214
R-MVUE1		0.9620	0.0120	0.0339	0.0041	0.94320
R-TB		0.9655	0.0155	0.0086	0.0259	1.11725
R-BC		0.9672	0.0172	0.0085	0.0243	1.12787
BOOTSTRAP-TB		0.9312	0.0188	0.0322	0.0366	1.05014
MVUE1	25	0.9245	0.0255	0.0677	0.0078	0.61969
MVUE2		0.9331	0.0169	0.0617	0.0052	0.65375
MLE-TB		0.9443	0.0057	0.0342	0.0215	0.67030
MLE-BC		0.9449	0.0051	0.0339	0.0212	0.67310
SLR		0.9459	0.0041	0.0232	0.0309	0.72054
ASLR		0.9476	0.0024	0.0226	0.0298	0.73210
BOOTSTRAP-BC		0.9388	0.0112	0.0294	0.0318	0.72305
R-MVUE1		0.9777	0.0277	0.0150	0.0073	0.77458
R-TB		0.9632	0.0132	0.0049	0.0319	0.86311
R-BC		0.9639	0.0139	0.0048	0.0313	0.86794
BOOTSTRAP-TB		0.9394	0.0106	0.0290	0.0316	0.72552
MVUE1	50	0.9340	0.0160	0.0570	0.0090	0.44255
MVUE2		0.9391	0.0109	0.0538	0.0071	0.45460
MLE-TB		0.9467	0.0033	0.0311	0.0222	0.46002
MLE-BC		0.9472	0.0028	0.0309	0.0219	0.46093
SLR		0.9474	0.0026	0.0232	0.0294	0.47538
ASLR		0.9497	0.0003	0.0226	0.0277	0.48122
BOOTSTRAP-BC		0.9442	0.0058	0.0269	0.0289	0.47574
R-MVUE1		0.9764	0.0264	0.0056	0.0180	0.56923
R-TB		0.9462	0.0038	0.0015	0.0523	0.60255
R-BC		0.9470	0.0030	0.0015	0.0515	0.60421
BOOTSTRAP-TB		0.9443	0.0057	0.0268	0.0289	0.47659

Table H.3: 95% CI under $\Delta(0.4, -0.50, 1.0)$

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8374	0.1126	0.1609	0.0017	0.93498
MVUE2		0.8694	0.0806	0.1303	0.0003	1.23334
MLE-TB		0.9250	0.0250	0.0595	0.0155	1.48299
MLE-BC		0.9283	0.0217	0.0570	0.0147	1.52945
SLR		0.9341	0.0159	0.0219	0.0440	2.36347
ASLR		0.9354	0.0146	0.0215	0.0431	3.40321
BOOTSTRAP-BC		0.9191	0.0309	0.0305	0.0504	66.19126
R-MVUE1		0.8477	0.1023	0.1513	0.0010	0.85989
R-TB		0.9412	0.0088	0.0442	0.0146	1.18842
R-BC		0.9441	0.0059	0.0421	0.0138	1.21110
BOOTSTRAP-TB		0.9296	0.0204	0.0282	0.0422	828.23502
MVUE1	25	0.8751	0.0749	0.1235	0.0014	0.76848
MVUE2		0.8977	0.0523	0.1018	0.0005	0.90940
MLE-TB		0.9352	0.0148	0.0496	0.0152	0.99546
MLE-BC		0.9365	0.0135	0.0490	0.0145	1.00732
SLR		0.9384	0.0116	0.0222	0.0394	1.23027
ASLR		0.9409	0.0091	0.0220	0.0371	1.27009
BOOTSTRAP-BC		0.9334	0.0166	0.0286	0.0380	1.29646
R-MVUE1		0.8945	0.0555	0.1037	0.0018	0.72831
R-TB		0.9575	0.0075	0.0289	0.0136	0.90180
R-BC		0.9593	0.0093	0.0274	0.0133	0.91072
BOOTSTRAP-TB		0.9355	0.0145	0.0273	0.0372	1.31170
MVUE1	50	0.9121	0.0379	0.0862	0.0017	0.56644
MVUE2		0.9250	0.0250	0.0740	0.0010	0.61677
MLE-TB		0.9464	0.0036	0.0376	0.0160	0.64203
MLE-BC		0.9473	0.0027	0.0370	0.0157	0.64524
SLR		0.9491	0.0009	0.0212	0.0297	0.69749
ASLR		0.9496	0.0004	0.0212	0.0292	0.70453
BOOTSTRAP-BC		0.9479	0.0021	0.0249	0.0272	0.71066
R-MVUE1		0.9337	0.0163	0.0648	0.0015	0.55098
R-TB		0.9677	0.0177	0.0194	0.0129	0.61794
R-BC		0.9684	0.0184	0.0189	0.0127	0.62084
BOOTSTRAP-TB		0.9485	0.0015	0.0245	0.0270	0.71379

Table H.4: 95% CI under Δ(0.2, -1.0, 2.0)

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.7840	0.1660	0.0216	0.0000	1.52381
MVUE2		0.8399	0.1101	0.1601	0.0000	2.63428
MLE-TB		0.9156	0.0344	0.0788	0.0056	4.02632
MLE-BC		0.9199	0.0301	0.0761	0.0040	4.25997
SLR		0.9328	0.0172	0.0185	0.0487	5.82871
ASLR		0.9329	0.0171	0.0184	0.0487	6.00301
BOOTSTRAP-BC		0.9256	0.0244	0.0328	0.0416	659.90111
R-MVUE1		0.6206	0.3294	0.3794	0.0000	0.87582
R-TB		0.8525	0.0975	0.1448	0.0027	1.29500
R-BC		0.8575	0.0925	0.1401	0.0024	1.32066
BOOTSTRAP-TB		0.9293	0.0207	0.0298	0.0409	859.42668
MVUE1	25	0.8326	0.1174	0.1674	0.0000	1.25650
MVUE2		0.8698	0.0802	0.1302	0.0000	1.72549
MLE-TB		0.9286	0.0214	0.0643	0.0071	2.03541
MLE-BC		0.9311	0.0189	0.0626	0.0063	2.07118
SLR		0.9423	0.0077	0.0195	0.0382	2.83925
ASLR		0.9426	0.0074	0.0195	0.0379	2.93212
BOOTSTRAP-BC		0.9386	0.0114	0.0302	0.0312	3.37893
R-MVUE1		0.6318	0.3182	0.3682	0.0000	0.73483
R-TB		0.8456	0.1044	0.1530	0.0014	0.94483
R-BC		0.8477	0.1023	0.1510	0.0013	0.95514
BOOTSTRAP-TB		0.9412	0.0088	0.0283	0.0305	3.44182
MVUE1	50	0.8716	0.0784	0.1284	0.0000	0.93975
MVUE2		0.8935	0.0565	0.1065	0.0000	1.10364
MLE-TB		0.9383	0.0117	0.0505	0.0112	1.18442
MLE-BC		0.9395	0.0105	0.0496	0.0109	1.19280
SLR		0.9452	0.0048	0.0222	0.0326	1.36010
ASLR		0.9458	0.0042	0.022	0.0322	1.39544
BOOTSTRAP-BC		0.9431	0.0069	0.0284	0.0285	1.44169
R-MVUE1		0.5827	0.3673	0.4173	0.0000	0.55422
R-TB		0.7695	0.1805	0.2302	0.0003	0.63366
R-BC		0.7723	0.1777	0.2274	0.0003	0.63691
BOOTSTRAP-TB		0.9439	0.0061	0.0278	0.0283	1.45168

Table H.5: 95% CI under $\Delta(0.2, -0.075, 0.15)$ with 20% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9315	0.0185	0.0367	0.0318	0.53877
MVUE2		0.9319	0.0181	0.0381	0.0300	0.54130
MLE-TB		0.9345	0.0155	0.0140	0.0515	0.55255
MLE-BC		0.9348	0.0152	0.0140	0.0512	0.55345
SLR		0.9274	0.0226	0.0525	0.0201	0.58040
ASLR		0.9298	0.0202	0.0511	0.0191	0.58546
BOOTSTRAP-BC		0.9329	0.0171	0.0381	0.0290	0.56797
R-MVUE1		0.9777	0.0277	0.0079	0.0144	0.76686
R-TB		0.9555	0.0055	0.0019	0.0426	0.83294
R-BC		0.9570	0.0070	0.0019	0.0411	0.83692
BOOTSTRAP-TB		0.9359	0.0141	0.0379	0.0262	0.57032
MVUE1	25	0.9394	0.0106	0.0328	0.0278	0.42137
MVUE2		0.9397	0.0103	0.0330	0.0273	0.42253
MLE-TB		0.9391	0.0109	0.0165	0.0444	0.42755
MLE-BC		0.9393	0.0107	0.0165	0.0442	0.42793
SLR		0.9456	0.0044	0.0269	0.0275	0.43578
ASLR		0.9457	0.0043	0.0270	0.0273	0.44226
BOOTSTRAP-BC		0.9406	0.0094	0.0320	0.0274	0.43247
R-MVUE1		0.9740	0.0240	0.0037	0.0223	0.61641
R-TB		0.9398	0.0102	0.0008	0.0594	0.64966
R-BC		0.9406	0.0094	0.0008	0.0586	0.65153
BOOTSTRAP-TB		0.9407	0.0093	0.0319	0.0274	0.43283
MVUE1	50	0.9465	0.0035	0.0298	0.0237	0.29891
MVUE2		0.9465	0.0035	0.0300	0.0235	0.29932
MLE-TB		0.9466	0.0034	0.0180	0.0354	0.30102
MLE-BC		0.9467	0.0033	0.0180	0.0353	0.30115
SLR		0.9483	0.0017	0.0263	0.0254	0.30352
ASLR		0.9485	0.0015	0.0261	0.0254	0.30944
BOOTSTRAP-BC		0.9462	0.0038	0.0277	0.0261	0.30236
R-MVUE1		0.9480	0.0020	0.0008	0.0512	0.44689
R-TB		0.8992	0.0508	0.0003	0.1005	0.45929
R-BC		0.8995	0.0505	0.0003	0.1002	0.45995
BOOTSTRAP-TB		0.9463	0.0037	0.0276	0.0261	0.30248

Table H.6: 95% CI under Δ(0.4, -0.075, 0.15) with 20% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9280	0.0220	0.0507	0.0213	0.58130
MVUE2		0.9276	0.0224	0.0514	0.0210	0.58275
MLE-TB		0.9378	0.0122	0.0155	0.0467	0.60690
MLE-BC		0.9399	0.0101	0.0136	0.0465	0.61002
SLR		0.9297	0.0203	0.0603	0.0100	0.62267
ASLR		0.9311	0.0189	0.0597	0.0092	0.62844
BOOTSTRAP-BC		0.9400	0.0100	0.0245	0.0355	0.59815
R-MVUE1		0.9629	0.0129	0.0241	0.0130	0.74470
R-TB		0.9468	0.0032	0.0062	0.0470	0.82996
R-BC		0.9489	0.0011	0.0051	0.0460	0.83628
BOOTSTRAP-TB		0.9601	0.0101	0.0242	0.0157	0.62148
MVUE1	25	0.9348	0.0152	0.0426	0.0226	0.45401
MVUE2		0.9354	0.0146	0.0423	0.0223	0.45484
MLE-TB		0.9409	0.0091	0.0152	0.0439	0.46643
MLE-BC		0.9410	0.0090	0.0152	0.0438	0.46684
SLR		0.9465	0.0035	0.0284	0.0251	0.45985
ASLR		0.9471	0.0029	0.0280	0.0249	0.46002
BOOTSTRAP-BC		0.9469	0.0031	0.0264	0.0267	0.46154
R-MVUE1		0.9703	0.0203	0.0104	0.0193	0.60277
R-TB		0.9398	0.0102	0.0015	0.0587	0.64757
R-BC		0.9410	0.0090	0.0014	0.0576	0.64954
BOOTSTRAP-TB		0.9496	0.0004	0.0263	0.0241	0.46293
MVUE1	50	0.9466	0.0034	0.0312	0.0222	0.32241
MVUE2		0.9465	0.0035	0.0313	0.0222	0.32267
MLE-TB		0.9450	0.0050	0.0141	0.0409	0.32660
MLE-BC		0.9450	0.0050	0.0141	0.0409	0.32673
SLR		0.9506	0.0006	0.0266	0.0228	0.32356
ASLR		0.9531	0.0031	0.0260	0.0209	0.32885
BOOTSTRAP-BC		0.9512	0.0012	0.0269	0.0219	0.32469
R-MVUE1		0.9567	0.0067	0.0023	0.0410	0.43966
R-TB		0.9127	0.0373	0.0001	0.0872	0.45637
R-BC		0.9132	0.0368	0.0001	0.0867	0.45705
BOOTSTRAP-TB		0.9515	0.0015	0.0267	0.0218	0.32481

Table H.7: 95% CI under $\Delta(0.2, -0.25, 0.50)$ with 20% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9197	0.0303	0.0742	0.0061	0.84306
MVUE2		0.9297	0.0203	0.0662	0.0041	0.94740
MLE-TB		0.9486	0.0014	0.0297	0.0217	1.00690
MLE-BC		0.9493	0.0007	0.0297	0.0210	1.01706
SLR		0.9543	0.0043	0.0178	0.0280	0.95427
ASLR		0.9552	0.0052	0.0173	0.0275	0.96221
BOOTSTRAP-BC		0.9392	0.0108	0.0322	0.0286	1.28754
R-MVUE1		0.9598	0.0098	0.0365	0.0037	0.95753
R-TB		0.9658	0.0158	0.0107	0.0235	1.14527
R-BC		0.9678	0.0178	0.0102	0.0220	1.15674
BOOTSTRAP-TB		0.9415	0.0085	0.0307	0.0278	1.30409
MVUE1	25	0.9346	0.0154	0.0581	0.0073	0.66732
MVUE2		0.9422	0.0078	0.0525	0.0053	0.71408
MLE-TB		0.9494	0.0006	0.0295	0.0211	0.73608
MLE-BC		0.9500	0.0000	0.0292	0.0208	0.73980
SLR		0.9505	0.0005	0.0238	0.0257	0.80362
ASLR		0.9523	0.0023	0.0229	0.0248	0.82663
BOOTSTRAP-BC		0.9430	0.0070	0.0308	0.0262	0.81217
R-MVUE1		0.9748	0.0248	0.0184	0.0068	0.78277
R-TB		0.9634	0.0134	0.0052	0.0314	0.87758
R-BC		0.9648	0.0148	0.0052	0.0300	0.88273
BOOTSTRAP-TB		0.9441	0.0059	0.0297	0.0262	0.81559
MVUE1	50	0.9505	0.0005	0.0406	0.0089	0.48365
MVUE2		0.9551	0.0051	0.0378	0.0071	0.50038
MLE-TB		0.9568	0.0068	0.0190	0.0242	0.50748
MLE-BC		0.9574	0.0074	0.0189	0.0237	0.50868
SLR		0.9559	0.0059	0.0270	0.0171	0.52779
ASLR		0.9560	0.0060	0.0269	0.0171	0.53114
BOOTSTRAP-BC		0.9523	0.0023	0.0317	0.0160	0.53005
R-MVUE1		0.9784	0.0284	0.0067	0.0149	0.58098
R-TB		0.9528	0.0028	0.0015	0.0457	0.61720
R-BC		0.9536	0.0036	0.0015	0.0449	0.61899
BOOTSTRAP-TB		0.9527	0.0027	0.0314	0.0159	0.53119

Table H.8: 95% CI under $\Delta(0.4, -0.25, 0.50)$ with 20% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8941	0.0559	0.1010	0.0049	0.78617
MVUE2		0.9089	0.0411	0.0878	0.0033	0.91607
MLE-TB		0.9381	0.0119	0.0355	0.0264	1.06630
MLE-BC		0.9405	0.0095	0.0340	0.0255	1.09523
SLR		0.9361	0.0139	0.0329	0.0310	0.93607
ASLR		0.9363	0.0137	0.0327	0.0310	0.95114
BOOTSTRAP-BC		0.9298	0.0202	0.0298	0.0404	190.83406
R-MVUE1		0.9251	0.0249	0.0718	0.0031	0.85606
R-TB		0.9524	0.0024	0.0204	0.0272	1.07324
R-BC		0.9551	0.0051	0.0186	0.0263	1.08816
BOOTSTRAP-TB		0.939	0.0110	0.0290	0.0320	927.47971
MVUE1	25	0.9275	0.0225	0.0666	0.0059	0.62973
MVUE2		0.9356	0.0144	0.0602	0.0042	0.68413
MLE-TB		0.9517	0.0017	0.0236	0.0247	0.72175
MLE-BC		0.9528	0.0028	0.0228	0.0244	0.72635
SLR		0.9521	0.0021	0.0238	0.0241	0.80122
ASLR		0.9552	0.0052	0.0221	0.0227	0.82390
BOOTSTRAP-BC		0.9473	0.0027	0.0293	0.0234	0.81164
R-MVUE1		0.9615	0.0115	0.0332	0.0053	0.71495
R-TB		0.9624	0.0124	0.0064	0.0312	0.83025
R-BC		0.9635	0.0135	0.0062	0.0303	0.83602
BOOTSTRAP-TB		0.9491	0.0009	0.0286	0.0223	0.81753
MVUE1	50	0.9464	0.0036	0.0455	0.0081	0.45956
MVUE2		0.9519	0.0019	0.0419	0.0062	0.47896
MLE-TB		0.9534	0.0034	0.0204	0.0262	0.49068
MLE-BC		0.9536	0.0036	0.0204	0.0260	0.49202
SLR		0.9528	0.0028	0.0272	0.0200	0.51047
ASLR		0.9533	0.0033	0.0270	0.0197	0.51332
BOOTSTRAP-BC		0.9496	0.0004	0.0311	0.0193	0.51215
R-MVUE1		0.9736	0.0236	0.0146	0.0118	0.53593
R-TB		0.9537	0.0037	0.0035	0.0428	0.58018
R-BC		0.9542	0.0042	0.0035	0.0423	0.58213
BOOTSTRAP-TB		0.9503	0.0003	0.0305	0.0192	0.51339

Table H.9: 95% CI under Δ(0.2, -1.0, 2.0) with 20% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9325	0.0175	0.0675	0.0000	$1.021e^{16}$
MVUE2		0.9581	0.0081	0.0419	0.0000	$8.496e^{79}$
MLE-TB		0.8414	0.1086	0.0176	0.1410	$1.062e^{140}$
MLE-BC		0.8728	0.0772	0.017	0.1102	$2.408e^{142}$
SLR		0.7113	0.2387	0.2674	0.0213	$1.160e^{74}$
ASLR		0.7324	0.2176	0.2512	0.0164	$4.123e^{78}$
BOOTSTRAP-BC		0.5964	0.3536	0.3952	0.0084	$4.362e^{256}$
R-MVUE1		0.3460	0.6040	0.6539	0.0001	0.64380
R-TB		0.6121	0.3379	0.3873	0.0006	1.06122
R-BC		0.6183	0.3317	0.3811	0.0006	1.08730
BOOTSTRAP-TB		0.6172	0.3328	0.3747	0.0081	$2.528e^{266}$
MVUE1	25	0.9742	0.0242	0.0258	0.0000	$6.080e^{12}$
MVUE2		0.9845	0.0345	0.0155	0.0000	$2.844e^{47}$
MLE-TB		0.6156	0.3344	0.0060	0.3784	$1.486e^{78}$
MLE-BC		0.6352	0.3148	0.0056	0.3592	$1.164e^{79}$
SLR		0.5243	0.4257	0.4696	0.0061	$1.209e^{23}$
ASLR		0.5511	0.3989	0.4432	0.0057	$9.556e^{23}$
BOOTSTRAP-BC		0.4208	0.5292	0.5775	0.0017	$5.116e^{114}$
R-MVUE1		0.2740	0.6760	0.7260	0.0000	0.52245
R-TB		0.4947	0.4553	0.5051	0.0002	0.72204
R-BC		0.4999	0.4501	0.4999	0.0002	0.73150
BOOTSTRAP-TB		0.4292	0.5208	0.5691	0.0017	$7.969e^{116}$
MVUE1	50	0.9972	0.0472	0.0028	0.0000	$2.090e^{10}$
MVUE2		0.9983	0.0483	0.0017	0.0000	$4.221e^{22}$
MLE-TB		0.2881	0.6619	0.0006	0.7113	$5.990e^{32}$
MLE-BC		0.2940	0.6560	0.0006	0.7054	$8.600e^{32}$
SLR		0.2534	0.6966	0.7458	0.0008	$1.427e^{12}$
ASLR		0.2703	0.6797	0.7289	0.0008	$5.231e^{13}$
BOOTSTRAP-BC		0.1942	0.7558	0.8057	0.0001	$1.374e^{39}$
R-MVUE1		0.1440	0.8060	0.8560	0.0000	0.38848
R-TB		0.2887	0.6613	0.7113	0.0000	0.46151
R-BC		0.2921	0.6579	0.7079	0.0000	0.46426
BOOTSTRAP-TB		0.1976	0.7524	0.8023	0.0001	$2.787e^{39}$

Table H.10: 95% CI under Δ(0.4, -1.0, 2.0) with 20% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8895	0.0605	0.1104	0.0001	$7.003e^{09}$
MVUE2		0.9326	0.0174	0.0673	0.0001	$3.221e^{22}$
MLE-TB		0.9148	0.0352	0.0327	0.0525	$3.667e^{52}$
MLE-BC		0.9353	0.0147	0.0313	0.0334	$9.782e^{58}$
SLR		0.7773	0.1727	0.1830	0.0397	$1.055e^{15}$
ASLR		0.8020	0.1480	0.1794	0.0186	$2.445e^{16}$
BOOTSTRAP-BC		0.6534	0.2966	0.328	0.0186	$7.231e^{123}$
R-MVUE1		0.3725	0.5775	0.6274	0.0001	0.55497
R-TB		0.6523	0.2977	0.3449	0.0028	1.05930
R-BC		0.6607	0.2893	0.3365	0.0028	1.10447
BOOTSTRAP-TB		0.6797	0.2703	0.3040	0.0163	$1.299e^{199}$
MVUE1	25	0.9506	0.0006	0.0494	0.0000	$9.287e^{16}$
MVUE2		0.9695	0.0195	0.0305	0.0000	$7.712e^{95}$
MLE-TB		0.7506	0.1994	0.0122	0.2372	$3.067e^{168}$
MLE-BC		0.7779	0.1721	0.0119	0.2102	$2.046e^{171}$
SLR		0.6241	0.3259	0.3606	0.0153	$6.241e^{89}$
ASLR		0.6502	0.2998	0.3398	0.0100	$2.443e^{95}$
BOOTSTRAP-BC		0.5205	0.4295	0.4734	0.0061	$6.643e^{269}$
R-MVUE1		0.3245	0.6255	0.6755	0.0000	0.45886
R-TB		0.5799	0.3701	0.4193	0.0008	0.69593
R-BC		0.5864	0.3636	0.4128	0.0008	0.70841
BOOTSTRAP-TB		0.5338	0.4162	0.4601	0.0061	$2.472e^{277}$
MVUE1	50	0.9913	0.0413	0.0087	0.0000	$2.435e^{10}$
MVUE2		0.9945	0.0445	0.0055	0.0000	$2.090e^{28}$
MLE-TB		0.4199	0.5301	0.0025	0.5776	$5.902e^{43}$
MLE-BC		0.4306	0.5194	0.0025	0.5669	$1.251e^{44}$
SLR		0.3669	0.5831	0.6293	0.0038	$1.183e^{14}$
ASLR		0.3825	0.5675	0.6142	0.0033	$8.212e^{14}$
BOOTSTRAP-BC		0.2926	0.6574	0.7062	0.0012	$1.358e^{56}$
R-MVUE1		0.2179	0.7321	0.7821	0.0000	0.34291
R-TB		0.4073	0.5427	0.5925	0.0002	0.42952
R-BC		0.4099	0.5401	0.5899	0.0002	0.43289
BOOTSTRAP-TB		0.2971	0.6529	0.7017	0.0012	$5.083e^{56}$

Table H.11: 95% CI under Δ(0.2, -0.075, 0.15) with 60% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9333	0.0167	0.0361	0.0306	0.54708
MVUE2		0.9349	0.0151	0.0357	0.0294	0.55116
MLE-TB		0.9376	0.0124	0.0129	0.0495	0.56291
MLE-BC		0.9381	0.0119	0.0128	0.0491	0.56392
SLR		0.9245	0.0255	0.0559	0.0196	0.59159
ASLR		0.9246	0.0254	0.0558	0.0196	0.61311
BOOTSTRAP-BC		0.9347	0.0153	0.0372	0.0281	0.57985
R-MVUE1		0.9779	0.0279	0.0072	0.0149	0.77426
R-TB		0.9553	0.0053	0.0010	0.0437	0.84336
R-BC		0.9561	0.0061	0.0010	0.0429	0.84752
BOOTSTRAP-TB		0.9381	0.0119	0.0369	0.0250	0.58225
MVUE1	25	0.9467	0.0033	0.0288	0.0245	0.42671
MVUE2		0.9474	0.0026	0.0289	0.0237	0.42847
MLE-TB		0.9450	0.0050	0.0146	0.0404	0.43363
MLE-BC		0.9451	0.0049	0.0146	0.0403	0.43405
SLR		0.9531	0.0031	0.0239	0.0230	0.44232
ASLR		0.9540	0.0040	0.0235	0.0225	0.45045
BOOTSTRAP-BC		0.9458	0.0042	0.0307	0.0235	0.43905
R-MVUE1		0.9783	0.0283	0.0024	0.0193	0.62144
R-TB		0.9465	0.0035	0.0003	0.0532	0.65588
R-BC		0.9471	0.0029	0.0003	0.0526	0.65782
BOOTSTRAP-TB		0.9461	0.0039	0.0304	0.0235	0.43945
MVUE1	50	0.9524	0.0024	0.0254	0.0222	0.30420
MVUE2		0.9524	0.0024	0.0255	0.0221	0.30484
MLE-TB		0.9526	0.0026	0.0151	0.0323	0.30661
MLE-BC		0.9528	0.0028	0.0151	0.0321	0.30676
SLR		0.9549	0.0049	0.0234	0.0217	0.30939
ASLR		0.9556	0.0056	0.0231	0.0213	0.31256
BOOTSTRAP-BC		0.9528	0.0028	0.0255	0.0217	0.30795
R-MVUE1		0.9505	0.0005	0.0004	0.0491	0.45231
R-TB		0.9047	0.0453	0.0001	0.0952	0.46529
R-BC		0.9051	0.0449	0.0001	0.0948	0.46598
BOOTSTRAP-TB		0.9528	0.0028	0.0255	0.0217	0.30809

Table H.12: 95% CI under $\Delta(0.4, -0.075, 0.15)$ with 60% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9229	0.0271	0.0513	0.0258	0.58774
MVUE2		0.9228	0.0272	0.0515	0.0257	0.59073
MLE-TB		0.9363	0.0137	0.0123	0.0514	0.61559
MLE-BC		0.9392	0.0108	0.0095	0.0513	0.61875
SLR		0.9206	0.0294	0.0699	0.0095	0.63316
ASLR		0.9254	0.0246	0.0666	0.0080	0.63855
BOOTSTRAP-BC		0.9413	0.0087	0.0261	0.0326	0.60880
R-MVUE1		0.9619	0.0119	0.0226	0.0155	0.75210
R-TB		0.9440	0.0060	0.0045	0.0515	0.84087
R-BC		0.9460	0.0040	0.0036	0.0504	0.84734
BOOTSTRAP-TB		0.9612	0.0112	0.0257	0.0131	0.63210
MVUE1	25	0.9405	0.0095	0.0379	0.0216	0.45777
MVUE2		0.9410	0.0090	0.0375	0.0215	0.45932
MLE-TB		0.9474	0.0026	0.0123	0.0403	0.47129
MLE-BC		0.9474	0.0026	0.0123	0.0403	0.47174
SLR		0.9504	0.0004	0.0278	0.0218	0.46553
ASLR		0.9518	0.0018	0.0268	0.0214	0.47102
BOOTSTRAP-BC		0.9524	0.0024	0.0252	0.0224	0.46684
R-MVUE1		0.9727	0.0227	0.0093	0.0180	0.60565
R-TB		0.9459	0.0041	0.0019	0.0522	0.65202
R-BC		0.9467	0.0033	0.0017	0.0516	0.65407
BOOTSTRAP-TB		0.9554	0.0054	0.0251	0.0195	0.46829
MVUE1	50	0.9485	0.0015	0.0333	0.0182	0.32540
MVUE2		0.9487	0.0013	0.0331	0.0182	0.32590
MLE-TB		0.9525	0.0025	0.0143	0.0332	0.32996
MLE-BC		0.9526	0.0026	0.0143	0.0331	0.33009
SLR		0.9543	0.0043	0.0221	0.0236	0.32718
ASLR		0.9545	0.0045	0.0219	0.0236	0.33010
BOOTSTRAP-BC		0.9541	0.0041	0.0231	0.0228	0.32801
R-MVUE1		0.9639	0.0139	0.0027	0.0334	0.44238
R-TB		0.9231	0.0269	0.0003	0.0766	0.45971
R-BC		0.9236	0.0264	0.0003	0.0761	0.46042
BOOTSTRAP-TB		0.9543	0.0043	0.0230	0.0227	0.32814

Table H.13: 95% CI under Δ(0.2, -0.25, 0.50) with 60% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9316	0.0184	0.0630	0.0054	0.95446
MVUE2		0.9446	0.0054	0.0522	0.0032	1.12746
MLE-TB		0.9587	0.0087	0.0216	0.0197	1.23520
MLE-BC		0.9594	0.0094	0.0214	0.0192	1.25310
SLR		0.9630	0.0130	0.0191	0.0179	1.57694
ASLR		0.9654	0.0154	0.0184	0.0162	1.60541
BOOTSTRAP-BC		0.9489	0.0011	0.0329	0.0182	1.98906
R-MVUE1		0.9533	0.0033	0.0432	0.0035	0.97693
R-TB		0.9699	0.0199	0.0094	0.0207	1.19148
R-BC		0.9712	0.0212	0.0093	0.0195	1.20457
BOOTSTRAP-TB		0.9513	0.0013	0.0308	0.0179	2.04596
MVUE1	25	0.9537	0.0037	0.0404	0.0059	0.76840
MVUE2		0.9617	0.0117	0.0342	0.0041	0.84599
MLE-TB		0.9631	0.0131	0.0144	0.0225	0.88279
MLE-BC		0.9645	0.0145	0.0141	0.0214	0.88877
SLR		0.9609	0.0109	0.0274	0.0117	0.99474
ASLR		0.9616	0.0116	0.0270	0.0114	1.00004
BOOTSTRAP-BC		0.9528	0.0028	0.0359	0.0113	1.02567
R-MVUE1		0.9731	0.0231	0.0212	0.0057	0.81056
R-TB		0.9685	0.0185	0.0042	0.0273	0.92113
R-BC		0.9693	0.0193	0.0041	0.0266	0.92707
BOOTSTRAP-TB		0.9533	0.0033	0.0354	0.0113	1.03200
MVUE1	50	0.9704	0.0204	0.0221	0.0075	0.56336
MVUE2		0.9755	0.0255	0.0190	0.0055	0.59097
MLE-TB		0.9653	0.0153	0.0087	0.0260	0.60213
MLE-BC		0.9655	0.0155	0.0087	0.0258	0.60398
SLR		0.9601	0.0101	0.0327	0.0072	0.63386
ASLR		0.9641	0.0141	0.0302	0.0057	0.64111
BOOTSTRAP-BC		0.9540	0.0040	0.0395	0.0065	0.64064
R-MVUE1		0.9817	0.0317	0.0072	0.0111	0..60183
R-TB		0.9609	0.0109	0.0020	0.0371	0.64399
R-BC		0.9615	0.0115	0.0020	0.0365	0.64603
BOOTSTRAP-TB		0.9546	0.0046	0.0389	0.0065	0.64243

Table H.14: 95% CI under Δ(0.4, -0.25, 0.50) with 60% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9148	0.0352	0.0803	0.0049	0.88249
MVUE2		0.9302	0.0198	0.0668	0.0030	1.09846
MLE-TB		0.9528	0.0028	0.0211	0.0261	1.36304
MLE-BC		0.9548	0.0048	0.0197	0.0255	1.42359
SLR		0.9520	0.0020	0.0303	0.0176	1.81410
ASLR		0.9519	0.0019	0.0303	0.0177	1.84223
BOOTSTRAP-BC		0.9461	0.0039	0.0317	0.0222	$8.324e^{03}$
R-MVUE1		0.9261	0.0239	0.0712	0.0027	0.88282
R-TB		0.9573	0.0073	0.0164	0.0263	1.13776
R-BC		0.9589	0.0089	0.0150	0.0261	1.15525
BOOTSTRAP-TB		0.9534	0.0034	0.0300	0.0165	$3.902e^{04}$
MVUE1	25	0.9397	0.0103	0.0544	0.0059	0.71118
MVUE2		0.9506	0.0006	0.0454	0.004	0.80445
MLE-TB		0.9619	0.0119	0.0155	0.0226	0.86631
MLE-BC		0.9622	0.0122	0.0155	0.0223	0.87436
SLR		0.9607	0.0107	0.0253	0.0140	1.01209
ASLR		0.9618	0.0118	0.0249	0.0133	1.04110
BOOTSTRAP-BC		0.9559	0.0059	0.0302	0.0139	1.07794
R-MVUE1		0.9573	0.0073	0.0376	0.0051	0.73161
R-TB		0.9670	0.0170	0.0075	0.0255	0.86391
R-BC		0.9677	0.0177	0.0071	0.0252	0.87060
BOOTSTRAP-TB		0.9568	0.0068	0.0298	0.0134	1.09056
MVUE1	50	0.9599	0.0099	0.0337	0.0064	0.52155
MVUE2		0.9661	0.0161	0.0295	0.0044	0.55384
MLE-TB		0.9622	0.0122	0.0105	0.0273	0.57159
MLE-BC		0.9626	0.0126	0.0105	0.0269	0.57374
SLR		0.9595	0.0095	0.0304	0.0101	0.60677
ASLR		0.9603	0.0103	0.0298	0.0099	0.61994
BOOTSTRAP-BC		0.9539	0.0039	0.0374	0.0087	0.61306
R-MVUE1		0.9735	0.0235	0.0175	0.0090	0.55085
R-TB		0.9578	0.0078	0.0044	0.0378	0.60226
R-BC		0.9586	0.0086	0.0042	0.0372	0.60452
BOOTSTRAP-TB		0.9551	0.0051	0.0363	0.0086	0.61514

Table H.15: 95% CI under Δ(0.2, -1.0, 2.0) with 60% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9885	0.0385	0.0115	0.0000	$5.458e^{12}$
MVUE2		0.9960	0.0460	0.0040	0.0000	$1.765e^{95}$
MLE-TB		0.7250	0.2250	0.0013	0.2737	$1.121e^{178}$
MLE-BC		0.7881	0.1619	0.0012	0.2107	$1.584e^{182}$
SLR		0.4299	0.5201	0.5584	0.0117	$1.993e^{41}$
ASLR		0.4721	0.4779	0.5221	0.0058	$1.657e^{45}$
BOOTSTRAP-BC		0.2500	0.7000	0.7493	0.0007	$2.556e^{212}$
R-MVUE1		0.1171	0.8329	0.8829	0.0000	0.38289
R-TB		0.4916	0.4584	0.5081	0.0003	1.05478
R-BC		0.5042	0.4458	0.4956	0.0002	1.10656
BOOTSTRAP-TB		0.2746	0.6754	0.7247	0.0007	$8.324e^{221}$
MVUE1	25	0.9991	0.0491	0.0009	0.0000	$2.239e^{20}$
MVUE2		0.9997	0.0497	0.0003	0.0000	$1.394e^{87}$
MLE-TB		0.2902	0.6598	0.0000	0.7098	$5.576e^{141}$
MLE-BC		0.3182	0.6318	0.0000	0.6818	$1.969e^{143}$
SLR		0.1911	0.7589	0.8089	0.0000	$2.327e^{69}$
ASLR		0.2243	0.7257	0.7757	0.0000	$1.244e^{71}$
BOOTSTRAP-BC		0.0922	0.8578	0.9078	0.0000	$2.886e^{209}$
R-MVUE1		0.0645	0.8855	0.9355	0.0000	0.31364
R-TB		0.3061	0.6439	0.6939	0.0000	0.61052
R-BC		0.3160	0.6340	0.6840	0.0000	0.62416
BOOTSTRAP-TB		0.0977	0.8523	0.9023	0.0000	$3.218e^{14}$
MVUE1	50	1.0000	0.0500	0.0000	0.0000	$1.017e^{16}$
MVUE2		1.0000	0.0500	0.0000	0.0000	$5.209e^{38}$
MLE-TB		0.0202	0.9298	0.0000	0.9798	$2.969e^{55}$
MLE-BC		0.0212	0.9288	0.0000	0.9788	$5.040e^{55}$
SLR		0.1709	0.7791	0.8291	0.0000	$2.445e^{21}$
ASLR		0.2002	0.7498	0.7998	0.0000	$1.221e^{22}$
BOOTSTRAP-BC		0.0060	0.9440	0.9940	0.0000	$1.031e^{67}$
R-MVUE1		0.0116	0.9384	0.9884	0.0000	0.23425
R-TB		0.0759	0.8741	0.9241	0.0000	0.33824
R-BC		0.0777	0.8723	0.9223	0.0000	0.34135
BOOTSTRAP-TB		0.0062	0.9438	0.9938	0.0000	$2.519e^{67}$

Table H.16: 95% CI under $\Delta(0.4, -1.0, 2.0)$ with 60% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9577	0.0077	0.0423	0.0000	$2.121e^{15}$
MVUE2		0.9861	0.0361	0.0139	0.0000	$3.212e^{55}$
MLE-TB		0.8952	0.0548	0.0053	0.0995	$8.059e^{77}$
MLE-BC		0.9422	0.0078	0.0050	0.0528	$1.665e^{82}$
SLR		0.5527	0.3973	0.4194	0.0279	$1.620e^{29}$
ASLR		0.5733	0.3767	0.4105	0.0162	$4.221e^{34}$
BOOTSTRAP-BC		0.3741	0.5759	0.6233	0.0026	$8.345e^{156}$
R-MVUE1		0.1618	0.7882	0.8381	0.0001	0.33958
R-TB		0.5724	0.3776	0.4266	0.0010	1.21422
R-BC		0.5872	0.3628	0.4118	0.0010	1.36905
BOOTSTRAP-TB		0.4126	0.5374	0.5850	0.0024	$2.334e^{187}$
MVUE1	25	0.9952	0.0452	0.0048	0.0000	$3.435e^{15}$
MVUE2		0.9986	0.0486	0.0016	0.0000	$9.830e^{90}$
MLE-TB		0.5239	0.4261	0.0005	0.4756	$5.926e^{54}$
MLE-BC		0.5730	0.3770	0.0005	0.4265	$2.090e^{56}$
SLR		0.3272	0.6228	0.6710	0.0018	$4.553e^{18}$
ASLR		0.3611	0.5889	0.6371	0.0018	$9.448e^{18}$
BOOTSTRAP-BC		0.1827	0.7673	0.8172	0.0001	$6.332e^{185}$
R-MVUE1		0.0993	0.8507	0.9007	0.0000	0.26798
R-TB		0.4256	0.5244	0.5742	0.0002	0.63079
R-BC		0.4357	0.5143	0.5641	0.0002	0.65324
BOOTSTRAP-TB		0.1964	0.7536	0.8035	0.0001	$2.441e^{200}$
MVUE1	50	1.0000	0.0500	0.0000	0.0000	$1.711e^{16}$
MVUE2		1.0000	0.0500	0.0000	0.0000	$4.916e^{46}$
MLE-TB		0.0846	0.8654	0.0000	0.9154	$2.169e^{72}$
MLE-BC		0.0903	0.8597	0.0000	0.9097	$7.608e^{72}$
SLR		0.0646	0.8854	0.9354	0.0000	$2.573e^{51}$
ASLR		0.0844	0.8656	0.9156	0.0000	$7.112e^{51}$
BOOTSTRAP-BC		0.0261	0.9239	0.9739	0.0000	$3.845e^{92}$
R-MVUE1		0.0313	0.9187	0.9687	0.0000	0.20190
R-TB		0.1643	0.7857	0.8357	0.0000	0.32573
R-BC		0.1688	0.7812	0.8312	0.0000	0.33000
BOOTSTRAP-TB		0.0272	0.9228	0.9728	0.0000	$3.049e^{93}$

Table H.17: 95% CI under Δ(0.2, -0.075, 0.15) with 100% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9344	0.0156	0.0361	0.0295	0.55327
MVUE2		0.9358	0.0142	0.0363	0.0279	0.55836
MLE-TB		0.9426	0.0074	0.0113	0.0461	0.57050
MLE-BC		0.9432	0.0068	0.0111	0.0457	0.57160
SLR		0.9401	0.0099	0.0401	0.0198	0.57345
ASLR		0.9411	0.0089	0.0395	0.0194	0.58023
BOOTSTRAP-BC		0.9392	0.0108	0.0342	0.0266	0.58837
R-MVUE1		0.9808	0.0308	0.0071	0.0121	0.77888
R-TB		0.9597	0.0097	0.0009	0.0394	0.84994
R-BC		0.9609	0.0109	0.0009	0.0382	0.85424
BOOTSTRAP-TB		0.9427	0.0073	0.0341	0.0232	0.59083
MVUE1	25	0.9472	0.0028	0.0278	0.0250	0.43361
MVUE2		0.9475	0.0025	0.0280	0.0245	0.43603
MLE-TB		0.9495	0.0005	0.0117	0.0388	0.44142
MLE-BC		0.9500	0.0000	0.0117	0.0383	0.44189
SLR		0.9558	0.0058	0.0243	0.0199	0.45104
ASLR		0.9598	0.0098	0.0230	0.0172	0.46213
BOOTSTRAP-BC		0.9504	0.0004	0.0297	0.0199	0.44730
R-MVUE1		0.9765	0.0265	0.0024	0.0211	0.62754
R-TB		0.9456	0.0044	0.0005	0.0539	0.66334
R-BC		0.9459	0.0041	0.0005	0.0536	0.66536
BOOTSTRAP-TB		0.9504	0.0004	0.0297	0.0199	0.44773
MVUE1	50	0.9541	0.0041	0.0257	0.0202	0.30910
MVUE2		0.9548	0.0048	0.0256	0.0196	0.30996
MLE-TB		0.9551	0.0051	0.0138	0.0311	0.31182
MLE-BC		0.9552	0.0052	0.0137	0.0311	0.31198
SLR		0.9591	0.0091	0.0223	0.0186	0.31487
ASLR		0.9602	0.0102	0.022	0.0178	0.31702
BOOTSTRAP-BC		0.9573	0.0073	0.0244	0.0183	0.31321
R-MVUE1		0.9525	0.0025	0.0003	0.0472	0.45715
R-TB		0.9025	0.0475	0.0000	0.0975	0.47067
R-BC		0.9036	0.0464	0.0000	0.0964	0.47139
BOOTSTRAP-TB		0.9574	0.0074	0.0243	0.0183	0.31336

Table H.18: 95% CI under Δ(0.4, -0.075, 0.15) with 100% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9326	0.0174	0.0450	0.0224	0.59099
MVUE2		0.9327	0.0173	0.0454	0.0219	0.59555
MLE-TB		0.9417	0.0083	0.0115	0.0468	0.62121
MLE-BC		0.9441	0.0059	0.0093	0.0466	0.62455
SLR		0.9326	0.0174	0.0501	0.0173	0.63935
ASLR		0.9385	0.0115	0.0480	0.0135	0.64023
BOOTSTRAP-BC		0.9483	0.0017	0.0242	0.0275	0.61608
R-MVUE1		0.9671	0.0171	0.0209	0.0120	0.75279
R-TB		0.9500	0.0000	0.0037	0.0463	0.84399
R-BC		0.9513	0.0013	0.0027	0.0460	0.85066
BOOTSTRAP-TB		0.9643	0.0143	0.0241	0.0116	0.63981
MVUE1	25	0.9409	0.0091	0.0397	0.0194	0.46243
MVUE2		0.9412	0.0088	0.0396	0.0192	0.46467
MLE-TB		0.9454	0.0046	0.0143	0.0403	0.47687
MLE-BC		0.9457	0.0043	0.0142	0.0401	0.47737
SLR		0.9500	0.0000	0.0255	0.0245	0.47212
ASLR		0.9507	0.0007	0.0252	0.0241	0.48504
BOOTSTRAP-BC		0.9515	0.0015	0.0240	0.0245	0.47333
R-MVUE1		0.9732	0.0232	0.0106	0.0171	0.61119
R-TB		0.9418	0.0082	0.0022	0.0560	0.65907
R-BC		0.9427	0.0073	0.0022	0.0551	0.66120
BOOTSTRAP-TB		0.9537	0.0037	0.0240	0.0223	0.47477
MVUE1	50	0.9479	0.0021	0.0309	0.0212	0.32985
MVUE2		0.9484	0.0016	0.0305	0.0211	0.33065
MLE-TB		0.9467	0.0033	0.0143	0.0390	0.33483
MLE-BC		0.9467	0.0033	0.0143	0.039	0.33498
SLR		0.9527	0.0027	0.0264	0.0209	0.33246
ASLR		0.9549	0.0049	0.0263	0.0193	0.34102
BOOTSTRAP-BC		0.9544	0.0044	0.0263	0.0193	0.33311
R-MVUE1		0.9597	0.0097	0.0021	0.0382	0.44757
R-TB		0.9214	0.0286	0.0003	0.0783	0.46564
R-BC		0.9216	0.0284	0.0003	0.0781	0.46639
BOOTSTRAP-TB		0.9544	0.0044	0.0263	0.0193	0.33326

Table H.19: 95% CI under $\Delta(0.2, -0.25, 0.50)$ with 100% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9440	0.0060	0.0507	0.0053	1.06349
MVUE2		0.9559	0.0059	0.0410	0.0031	1.31166
MLE-TB		0.9636	0.0136	0.0154	0.0210	1.47876
MLE-BC		0.9651	0.0151	0.0150	0.0199	1.50619
SLR		0.9678	0.0178	0.0207	0.0115	1.98567
ASLR		0.9699	0.0199	0.0196	0.0105	2.00132
BOOTSTRAP-BC		0.9525	0.0025	0.0351	0.0124	3.20082
R-MVUE1		0.9481	0.0019	0.0484	0.0035	1.00418
R-TB		0.9692	0.0192	0.0097	0.0211	1.24952
R-BC		0.9701	0.0201	0.0094	0.0205	1.26449
BOOTSTRAP-TB		0.9551	0.0051	0.0329	0.0120	3.35981
MVUE1	25	0.9685	0.0185	0.0276	0.0039	0.86808
MVUE2		0.9751	0.0251	0.0222	0.0027	0.98151
MLE-TB		0.9717	0.0217	0.0091	0.0192	1.03665
MLE-BC		0.9723	0.0223	0.0089	0.0188	1.04522
SLR		0.9665	0.0165	0.0273	0.0062	1.20261
ASLR		0.9672	0.0172	0.0268	0.0060	1.24908
BOOTSTRAP-BC		0.9578	0.0078	0.0363	0.0059	1.25730
R-MVUE1		0.9734	0.0234	0.0228	0.0038	0.83712
R-TB		0.9728	0.0228	0.0045	0.0227	0.96458
R-BC		0.9735	0.0235	0.0045	0.0220	0.97135
BOOTSTRAP-TB		0.9595	0.0095	0.0347	0.0058	1.26668
MVUE1	50	0.9797	0.0297	0.0121	0.0082	0.64069
MVUE2		0.9843	0.0343	0.0104	0.0053	0.68078
MLE-TB		0.9652	0.0152	0.0046	0.0302	0.69695
MLE-BC		0.9655	0.0155	0.0046	0.0299	0.69950
SLR		0.9574	0.0074	0.0394	0.0032	0.74166
ASLR		0.9600	0.0100	0.0375	0.0025	0.74856
BOOTSTRAP-BC		0.9469	0.0031	0.0502	0.0029	0.75405
R-MVUE1		0.9827	0.0327	0.0074	0.0099	0.62682
R-TB		0.9680	0.0180	0.0017	0.0303	0.67599
R-BC		0.9685	0.0185	0.0017	0.0298	0.67832
BOOTSTRAP-TB		0.9482	0.0018	0.0490	0.0028	0.75657

Table H.20: 95% CI under $\Delta(0.4, -0.25, 0.50)$ with 100% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9200	0.0300	0.0756	0.0044	0.96061
MVUE2		0.9422	0.0078	0.0551	0.0270	1.28543
MLE-TB		0.9556	0.0056	0.0187	0.0257	2.05635
MLE-BC		0.9581	0.0081	0.0175	0.0244	2.31768
SLR		0.9572	0.0072	0.0294	0.0134	2.34002
ASLR		0.9599	0.0099	0.0288	0.0113	2.50881
BOOTSTRAP-BC		0.9496	0.0004	0.0337	0.0167	$6.285e^{06}$
R-MVUE1		0.9153	0.0347	0.0817	0.0003	0.89289
R-TB		0.9566	0.0066	0.0171	0.0263	1.17910
R-BC		0.9597	0.0097	0.0154	0.0249	1.19866
BOOTSTRAP-TB		0.9548	0.0048	0.0321	0.0131	$1.804e^{07}$
MVUE1	25	0.9526	0.0026	0.0429	0.0045	0.78733
MVUE2		0.9631	0.0131	0.0388	0.0031	0.91991
MLE-TB		0.9646	0.0146	0.0121	0.0233	1.00753
MLE-BC		0.9652	0.0152	0.012	0.0228	1.01917
SLR		0.9618	0.0118	0.0274	0.0108	1.23765
ASLR		0.9670	0.0170	0.0268	0.0062	1.28444
BOOTSTRAP-BC		0.9535	0.0035	0.0358	0.0107	1.34535
R-MVUE1		0.9547	0.0047	0.0411	0.0042	0.75257
R-TB		0.9660	0.0160	0.0083	0.0257	0.90372
R-BC		0.9668	0.0168	0.0081	0.0251	0.91141
BOOTSTRAP-TB		0.9552	0.0052	0.0345	0.0103	1.36493
MVUE1	50	0.9725	0.0225	0.0219	0.0056	0.58710
MVUE2		0.9782	0.0282	0.0187	0.0031	0.63365
MLE-TB		0.9662	0.0162	0.0063	0.0275	0.65774
MLE-BC		0.9669	0.0169	0.0063	0.0268	0.66073
SLR		0.9618	0.0118	0.0333	0.0049	0.70880
ASLR		0.9619	0.0119	0.0332	0.0049	0.71453
BOOTSTRAP-BC		0.9530	0.0030	0.0419	0.0051	0.72043
R-MVUE1		0.9775	0.0275	0.0167	0.0058	0.57112
R-TB		0.9677	0.0177	0.0032	0.0291	0.63068
R-BC		0.9680	0.0180	0.0032	0.0288	0.63329
BOOTSTRAP-TB		0.9540	0.0040	0.0410	0.0050	0.72337

Table H.21: 95% CI under $\Delta(0.4, -0.50, 1.0)$ with 100% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9225	0.0275	0.0771	0.0004	4.53345
MVUE2		0.9600	0.0100	0.0400	0.0000	$1.404e^{06}$
MLE-TB		0.9753	0.0253	0.0143	0.0104	$3.020e^{15}$
MLE-BC		0.9779	0.0279	0.0133	0.0088	$7.084e^{16}$
SLR		0.9257	0.0243	0.0634	0.0109	$1.122e^{03}$
ASLR		0.9255	0.0245	0.0634	0.0111	$2.155e^{03}$
BOOTSTRAP-BC		0.8791	0.0709	0.1128	0.0081	$7.503e^{184}$
R-MVUE1		0.6743	0.2757	0.3252	0.0005	0.81435
R-TB		0.9092	0.0408	0.0846	0.0062	1.44682
R-BC		0.9150	0.0350	0.0790	0.0060	1.49514
BOOTSTRAP-TB		0.8972	0.0528	0.0960	0.0068	$9.617e^{212}$
MVUE1	25	0.9685	0.0185	0.0314	0.0001	4.58606
MVUE2		0.9837	0.0337	0.0162	0.0001	73.12168
MLE-TB		0.9643	0.0143	0.0046	0.0311	$2.174e^{06}$
MLE-BC		0.9701	0.0201	0.0043	0.0256	$3.170e^{06}$
SLR		0.9674	0.0174	0.0281	0.0045	19.36891
ASLR		0.9703	0.0203	0.0269	0.0028	33.22098
BOOTSTRAP-BC		0.8146	0.1354	0.1838	0.0016	$5.474e^{22}$
R-MVUE1		0.7116	0.2384	0.2883	0.0001	0.70799
R-TB		0.9273	0.0227	0.0691	0.0036	1.04865
R-BC		0.9297	0.0203	0.0668	0.0035	1.06655
BOOTSTRAP-TB		0.8295	0.1205	0.1690	0.0015	$3.262e^{24}$
MVUE1	50	0.9953	0.0453	0.0044	0.0003	3.36076
MVUE2		0.9976	0.0476	0.0024	0.0000	1.05643
MLE-TB		0.8430	0.1070	0.0007	0.1563	55.08054
MLE-BC		0.8511	0.0989	0.0006	0.1483	58.63700
SLR		0.8216	0.1284	0.1782	0.0002	74.77800
ASLR		0.8276	0.1224	0.1722	0.0002	84.93762
BOOTSTRAP-BC		0.6671	0.2829	0.3328	0.0001	$1.006e^{03}$
R-MVUE1		0.7528	0.1972	0.2470	0.0002	0.56764
R-TB		0.9233	0.0267	0.0740	0.0027	0.70440
R-BC		0.9261	0.0239	0.0712	0.0027	0.70977
BOOTSTRAP-TB		0.6745	0.2755	0.3254	0.0001	$1.142e^{03}$

Table H.22: 95% CI under Δ(0.2, -1.0, 2.0) with 100% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9891	0.0391	0.0109	0.0000	$4.837e^{16}$
MVUE2		0.9983	0.0483	0.0017	0.0000	$2.531e^{18}$
MLE-TB		0.6994	0.2506	0.0006	0.3000	$2.300e^{25}$
MLE-BC		0.7782	0.1718	0.0006	0.2212	$9.002e^{25}$
SLR		0.6101	0.3399	0.3012	0.0887	$1.214e^{15}$
ASLR		0.6302	0.3198	0.2906	0.0792	$2.123e^{17}$
BOOTSTRAP-BC		0.1692	0.7808	0.8308	0.0000	$3.123e^{56}$
R-MVUE1		0.0441	0.9059	0.9559	0.0000	0.22927
R-TB		0.4612	0.4888	0.5386	0.0002	1.12659
R-BC		0.4842	0.4658	0.5156	0.0002	1.20950
BOOTSTRAP-TB		0.1920	0.7580	0.808	0.0000	$1.920e^{64}$
MVUE1	25	0.9997	0.0497	0.0003	0.0000	$1.158e^{20}$
MVUE2		1.0000	0.0500	0.0000	0.0000	$9.859e^{76}$
MLE-TB		0.2060	0.7440	0.0000	0.7940	$6.611e^{120}$
MLE-BC		0.2302	0.7198	0.0000	0.7698	$5.849e^{121}$
SLR		0.1855	0.7645	0.8145	0.0000	$2.830e^{97}$
ASLR		0.1906	0.7594	0.8094	0.0000	$3.443e^{100}$
BOOTSTRAP-BC		0.0427	0.9073	0.9573	0.0000	$2.088e^{164}$
R-MVUE1		0.0147	0.9353	0.9853	0.0000	0.19062
R-TB		0.2676	0.6824	0.7324	0.0000	0.60259
R-BC		0.2808	0.6692	0.7192	0.0000	0.62308
BOOTSTRAP-TB		0.0471	0.9029	0.9529	0.0000	$1.353e^{166}$
MVUE1	50	1.0000	0.0500	0.0000	0.0000	$8.139e^{13}$
MVUE2		1.0000	0.0500	0.0000	0.0000	$1.846e^{35}$
MLE-TB		0.0047	0.9453	0.0000	0.9953	$2.676e^{51}$
MLE-BC		0.0054	0.9446	0.0000	0.9946	$4.610e^{51}$
SLR		0.0031	0.9469	0.9969	0.0000	$3.512e^{41}$
ASLR		0.0104	0.9396	0.9896	0.0000	$4.199e^{41}$
BOOTSTRAP-BC		0.0004	0.9496	0.9996	0.0000	$1.481e^{61}$
R-MVUE1		0.0001	0.9499	0.9990	0.0000	0.15702
R-TB		0.0429	0.9071	0.9571	0.0000	0.31108
R-BC		0.0454	0.9046	0.9546	0.0000	0.31520
BOOTSTRAP-TB		0.0004	0.9496	0.9996	0.0000	$3.746e^{61}$

Table H.23: 95% CI under Δ(0.4, -1.0, 2.0) with 100% Contamination from Gamma Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9518	0.0018	0.0482	0.0000	$3.638e^{17}$
MVUE2		0.9910	0.0410	0.0090	0.0000	$1.235e^{22}$
MLE-TB		0.9028	0.0472	0.0027	0.0945	$9.160e^{44}$
MLE-BC		0.9462	0.0038	0.0021	0.0517	$4.601e^{48}$
SLR		0.5253	0.4247	0.4348	0.0399	$2.110e^{28}$
ASLR		0.5402	0.4098	0.4205	0.0393	$4.122e^{28}$
BOOTSTRAP-BC		0.2916	0.6584	0.7078	0.0006	$3.887e^{112}$
R-MVUE1		0.0713	0.8787	0.9287	0.0000	0.20385
R-TB		0.5562	0.3938	0.4432	0.0006	1.45372
R-BC		0.5803	0.3697	0.4191	0.0006	1.70225
BOOTSTRAP-TB		0.3313	0.6187	0.6682	0.0005	$2.966e^{123}$
MVUE1	25	0.9955	0.0455	0.0045	0.0000	$2.371e^{21}$
MVUE2		0.9996	0.0496	0.0004	0.0000	$5.723e^{24}$
MLE-TB		0.4771	0.4729	0.0000	0.5229	$9.374e^{33}$
MLE-BC		0.4663	0.4837	0.0000	0.5337	$7.989e^{36}$
SLR		0.4517	0.4983	0.5483	0.0000	$2.389e^{28}$
ASLR		0.4542	0.4958	0.5458	0.0000	$6.865e^{28}$
BOOTSTRAP-BC		0.1086	0.8414	0.8914	0.0000	$1.616e^{85}$
R-MVUE1		0.0287	0.9213	0.9713	0.0000	0.15889
R-TB		0.4019	0.5481	0.5980	0.0001	0.65202
R-BC		0.4192	0.5308	0.5807	0.0001	0.68798
BOOTSTRAP-TB		0.1205	0.8295	0.8795	0.0000	$2.849e^{122}$
MVUE1	50	0.9990	0.0490	0.0001	0.0000	$3.305e^{22}$
MVUE2		1.0000	0.0500	0.0000	0.0000	$1.968e^{74}$
MLE-TB		0.0407	0.9093	0.0000	0.9593	$6.522e^{111}$
MLE-BC		0.0438	0.9062	0.0000	0.9562	$2.941e^{112}$
SLR		0.0303	0.9197	0.0000	0.9697	$1.993e^{76}$
ASLR		0.0411	0.9089	0.0000	0.9589	$2.0387e^{78}$
BOOTSTRAP-BC		0.0062	0.9438	0.9938	0.0000	$6.001e^{140}$
R-MVUE1		0.0063	0.9437	0.9937	0.0000	0.13126
R-TB		0.1244	0.8256	0.8756	0.0000	0.30995
R-BC		0.1297	0.8203	0.8703	0.0000	0.31596
BOOTSTRAP-TB		0.0069	0.9431	0.9931	0.0000	$6.335e^{141}$

Table H.24: 95% CI under $\Delta(0.4, -0.075, 0.15)$ with 20% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9285	0.0215	0.0484	0.0231	0.58563
MVUE2		0.9286	0.0214	0.0481	0.0233	0.58911
MLE-TB		0.9381	0.0119	0.0125	0.0494	0.61417
MLE-BC		0.9406	0.0094	0.0102	0.0492	0.61748
SLR		0.9295	0.0205	0.0521	0.0184	0.61922
ASLR		0.9304	0.0196	0.0515	0.0181	0.62114
BOOTSTRAP-BC		0.9440	0.0060	0.0238	0.0322	0.61065
R-MVUE1		0.9639	0.0139	0.0226	0.0135	0.74636
R-TB		0.9456	0.0044	0.0049	0.0495	0.83321
R-BC		0.9473	0.0027	0.0036	0.0491	0.83966
BOOTSTRAP-TB		0.9620	0.0120	0.0238	0.0142	0.63445
MVUE1	25	0.9334	0.0166	0.0430	0.0236	0.45808
MVUE2		0.9343	0.0157	0.0429	0.0228	0.45963
MLE-TB		0.9411	0.0089	0.0127	0.0462	0.47149
MLE-BC		0.9411	0.0089	0.0127	0.0462	0.47194
SLR		0.9424	0.0076	0.0316	0.0260	0.46611
ASLR		0.9442	0.0058	0.0302	0.0256	0.47412
BOOTSTRAP-BC		0.9433	0.0067	0.0292	0.0275	0.46757
R-MVUE1		0.9703	0.0203	0.0094	0.0203	0.60378
R-TB		0.9362	0.0138	0.0017	0.0621	0.64888
R-BC		0.9370	0.0130	0.0016	0.0614	0.65086
BOOTSTRAP-TB		0.9472	0.0028	0.0292	0.0236	0.46902
MVUE1	50	0.9467	0.0033	0.0318	0.0215	0.32672
MVUE2		0.9472	0.0028	0.0314	0.0214	0.32729
MLE-TB		0.9501	0.0001	0.0148	0.0351	0.33137
MLE-BC		0.9591	0.0091	0.0148	0.0351	0.33151
SLR		0.9515	0.0015	0.0255	0.0230	0.32870
ASLR		0.9542	0.0042	0.0247	0.0211	0.34211
BOOTSTRAP-BC		0.9524	0.0024	0.0260	0.0216	0.32967
R-MVUE1		0.9621	0.0121	0.0034	0.0345	0.44080
R-TB		0.9195	0.0305	0.0006	0.0799	0.45780
R-BC		0.9200	0.0300	0.0006	0.0794	0.45850
BOOTSTRAP-TB		0.9524	0.0024	0.0260	0.0216	0.32980

Table H.25: 95% CI under Δ(0.2, -0.25, 0.50) with 20% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9178	0.0322	0.0761	0.0061	0.86213
MVUE2		0.9290	0.0210	0.0675	0.0035	0.98799
MLE-TB		0.9453	0.0047	0.0330	0.0217	1.07965
MLE-BC		0.9464	0.0036	0.0327	0.0209	1.09450
SLR		0.9530	0.0030	0.0197	0.0273	1.28597
ASLR		0.9555	0.0055	0.0182	0.0263	1.65590
BOOTSTRAP-BC		0.9404	0.0096	0.0312	0.0284	2.36619
R-MVUE1		0.9542	0.0042	0.0422	0.0036	0.95402
R-TB		0.9663	0.0163	0.0110	0.0227	1.14242
R-BC		0.9679	0.0179	0.0105	0.0217	1.15392
BOOTSTRAP-TB		0.9425	0.0075	0.0301	0.0274	2.57345
MVUE1	25	0.9353	0.0147	0.0585	0.0062	0.69638
MVUE2		0.9418	0.0082	0.0538	0.0044	0.75493
MLE-TB		0.9521	0.0021	0.0265	0.0214	0.78395
MLE-BC		0.9530	0.0030	0.0262	0.0208	0.78857
SLR		0.9518	0.0018	0.0249	0.0233	0.87030
ASLR		0.9524	0.0024	0.0245	0.0231	0.88043
BOOTSTRAP-BC		0.9446	0.0054	0.0322	0.0232	0.89158
R-MVUE1		0.9715	0.0215	0.0225	0.0060	0.78241
R-TB		0.9661	0.0161	0.0059	0.0280	0.87876
R-BC		0.9670	0.0170	0.0055	0.0275	0.88398
BOOTSTRAP-TB		0.9455	0.0045	0.0315	0.0230	0.89636
MVUE1	50	0.9524	0.0024	0.0388	0.0088	0.49940
MVUE2		0.9565	0.0065	0.0369	0.0066	0.51838
MLE-TB		0.9559	0.0059	0.0198	0.0243	0.52634
MLE-BC		0.9567	0.0067	0.0196	0.0237	0.52768
SLR		0.9533	0.0033	0.0297	0.0170	0.54904
ASLR		0.9534	0.0034	0.0298	0.0168	0.55244
BOOTSTRAP-BC		0.9479	0.0021	0.0351	0.0170	0.55218
R-MVUE1		0.9769	0.0269	0.0089	0.0142	0.58081
R-TB		0.9546	0.0046	0.0023	0.0431	0.61736
R-BC		0.9551	0.0051	0.0022	0.0427	0.61916
BOOTSTRAP-TB		0.9481	0.0019	0.0349	0.0170	0.55345

Table H.26: 95% CI under $\Delta(0.4, -0.25, 0.50)$ with 20% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9067	0.0433	0.0876	0.0057	0.80238
MVUE2		0.9221	0.0279	0.0741	0.0038	0.94525
MLE-TB		0.9420	0.0080	0.0299	0.0281	1.09827
MLE-BC		0.9444	0.0056	0.0287	0.0269	1.12560
SLR		0.9428	0.0072	0.0314	0.0258	1.38087
ASLR		0.9432	0.0068	0.0311	0.0257	1.39545
BOOTSTRAP-BC		0.9339	0.0161	0.0311	0.0350	15.26922
R-MVUE1		0.9325	0.0175	0.064	0.0035	0.86189
R-TB		0.9551	0.0051	0.0157	0.0292	1.08339
R-BC		0.9581	0.0081	0.0142	0.0277	1.09830
BOOTSTRAP-TB		0.9451	0.0049	0.0303	0.0246	22.77766
MVUE1	25	0.9254	0.0246	0.0691	0.0055	0.64918
MVUE2		0.9357	0.0143	0.0605	0.0038	0.71670
MLE-TB		0.9504	0.0004	0.0248	0.0248	0.76695
MLE-BC		0.9511	0.0011	0.0247	0.0242	0.77305
SLR		0.9486	0.0014	0.0259	0.0255	0.86481
ASLR		0.9491	0.0009	0.0257	0.0252	0.88347
BOOTSTRAP-BC		0.9438	0.0062	0.0315	0.0247	0.95239
R-MVUE1		0.9585	0.0085	0.0370	0.0045	0.71495
R-TB		0.9635	0.0135	0.0077	0.0288	0.83154
R-BC		0.9643	0.0143	0.0075	0.0282	0.83739
BOOTSTRAP-TB		0.9455	0.0045	0.0311	0.0234	0.96591
MVUE1	50	0.9501	0.0001	0.0418	0.0081	0.47282
MVUE2		0.957	0.0070	0.0373	0.0057	0.49485
MLE-TB		0.9580	0.0080	0.0175	0.0245	0.50778
MLE-BC		0.9585	0.0085	0.0174	0.0241	0.50929
SLR		0.9574	0.0074	0.0257	0.0169	0.53056
ASLR		0.9580	0.0080	0.0252	0.0168	0.53324
BOOTSTRAP-BC		0.9536	0.0036	0.0301	0.0163	0.53331
R-MVUE1		0.9748	0.0248	0.0144	0.0108	0.53670
R-TB		0.9581	0.0081	0.0035	0.0384	0.58174
R-BC		0.9583	0.0083	0.0035	0.0382	0.58373
BOOTSTRAP-TB		0.9541	0.0041	0.0297	0.0162	0.53471

Table H.27: 95% CI under $\Delta(0.2, -0.50, 1.0)$ with 20% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8853	0.0647	0.1136	0.0011	1.39149
MVUE2		0.9096	0.0404	0.0903	0.0001	3.25236
MLE-TB		0.9460	0.0040	0.0419	0.0121	77.81260
MLE-BC		0.9478	0.0022	0.0410	0.0112	101.92120
SLR		0.9438	0.0062	0.0270	0.0292	34.38883
ASLR		0.9447	0.0053	0.0263	0.0290	52.88554
BOOTSTRAP-BC		0.9280	0.0220	0.0448	0.0272	$3.719e^{05}$
R-MVUE1		0.8479	0.1021	0.1510	0.0011	0.98567
R-TB		0.9445	0.0055	0.0452	0.0103	1.30700
R-BC		0.9466	0.0034	0.0435	0.0099	1.32658
BOOTSTRAP-TB		0.9321	0.0179	0.0412	0.0267	$7.019e^{05}$
MVUE1	25	0.9263	0.0237	0.0734	0.0003	1.19774
MVUE2		0.9392	0.0108	0.0607	0.0001	1.68815
MLE-TB		0.9525	0.0025	0.0291	0.0184	2.44413
MLE-BC		0.9540	0.0040	0.0287	0.0173	2.52221
SLR		0.9440	0.0060	0.0369	0.0191	2.17297
ASLR		0.9445	0.0055	0.0365	0.0190	2.21540
BOOTSTRAP-BC		0.9291	0.0209	0.0536	0.0173	10.57274
R-MVUE1		0.8830	0.0670	0.1166	0.0004	0.82158
R-TB		0.9501	0.0001	0.0395	0.0104	0.98631
R-BC		0.9528	0.0028	0.0371	0.0101	0.99479
BOOTSTRAP-TB		0.9327	0.0173	0.0505	0.0168	11.48220
MVUE1	50	0.9564	0.0064	0.0419	0.0017	0.85275
MVUE2		0.9633	0.0133	0.0362	0.0005	0.96007
MLE-TB		0.9487	0.0013	0.0178	0.0335	1.01315
MLE-BC		0.9504	0.0004	0.0174	0.0322	1.01910
SLR		0.9340	0.0160	0.0522	0.0138	1.11862
ASLR		0.9349	0.0151	0.0514	0.0137	1.12427
BOOTSTRAP-BC		0.9234	0.0266	0.0637	0.0129	1.18226
R-MVUE1		0.9178	0.0322	0.0810	0.0012	0.61992
R-TB		0.9582	0.0082	0.0324	0.0094	0.68288
R-BC		0.9589	0.0089	0.0319	0.0092	0.68569
BOOTSTRAP-TB		0.9252	0.0248	0.0619	0.0129	1.18914

Table H.28: 95% CI under $\Delta(0.2, -1.0, 2.0)$ with 20% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8533	0.0967	0.1466	0.0001	15.07276
MVUE2		0.9029	0.0471	0.0971	0.0000	$4.996e^{06}$
MLE-TB		0.9484	0.0016	0.0441	0.0075	$1.198e^{13}$
MLE-BC		0.9520	0.0020	0.0428	0.0052	$2.683e^{13}$
SLR		0.9135	0.0365	0.0559	0.0306	$4.113e^{05}$
ASLR		0.9146	0.0354	0.0551	0.0303	$1.665e^{06}$
BOOTSTRAP-BC		0.8901	0.0599	0.0882	0.0217	$2.921e^{30}$
R-MVUE1		0.5325	0.4175	0.4674	0.0001	0.82150
R-TB		0.8122	0.1378	0.1860	0.0018	1.29642
R-BC		0.8178	0.1322	0.1808	0.0014	1.32574
BOOTSTRAP-TB		0.8996	0.0504	0.0790	0.0214	$3.207e^{31}$
MVUE1	25	0.9091	0.0409	0.0909	0.0000	7.03598
MVUE2		0.9363	0.0137	0.0637	0.0000	$1.405e^{03}$
MLE-TB		0.9396	0.0104	0.0256	0.0348	$7.328e^{05}$
MLE-BC		0.9448	0.0052	0.0247	0.0305	$9.051e^{05}$
SLR		0.8894	0.0606	0.0945	0.0161	74.22672
ASLR		0.9023	0.0477	0.0899	0.0078	89.43087
BOOTSTRAP-BC		0.8621	0.0879	0.1255	0.0124	$1.558e^{09}$
R-MVUE1		0.5033	0.4467	0.4967	0.0000	0.68895
R-TB		0.7657	0.1843	0.2337	0.0006	0.92458
R-BC		0.7696	0.1804	0.2299	0.0005	0.93590
BOOTSTRAP-TB		0.9704	0.0204	0.1176	0.0120	$2.341e^{09}$
MVUE1	50	0.9642	0.0142	0.0357	0.0001	3.02025
MVUE2		0.9732	0.0232	0.0268	0.0000	8.99818
MLE-TB		0.8946	0.0554	0.0113	0.0941	43.55634
MLE-BC		0.8982	0.0518	0.0110	0.0908	45.42271
SLR		0.8350	0.1150	0.1573	0.0077	4.67471
ASLR		0.8409	0.1091	0.1523	0.0068	9.89670
BOOTSTRAP-BC		0.8065	0.1435	0.1874	0.0061	271.42030
R-MVUE1		0.4297	0.5203	0.5703	0.0000	0.52697
R-TB		0.6537	0.2963	0.3460	0.0003	0.61628
R-BC		0.6574	0.2926	0.3423	0.0003	0.61977
BOOTSTRAP-TB		0.8112	0.1388	0.1829	0.0059	288.78180

Table H.29: 95% CI under Δ(0.4, -1.0, 2.0) with 20% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8023	0.1477	0.1977	0.0000	12.90831
MVUE2		0.8723	0.0777	0.1277	0.0000	$8.007e^{09}$
MLE-TB		0.9326	0.0174	0.0612	0.0062	$1.187e^{21}$
MLE-BC		0.9354	0.0146	0.0595	0.0051	$6.855e^{21}$
SLR		0.9120	0.0380	0.0438	0.0442	$9.557e^{11}$
ASLR		0.9187	0.0313	0.0398	0.0415	$1.442e^{12}$
BOOTSTRAP-BC		0.8862	0.0638	0.0803	0.0335	$1.878e^{71}$
R-MVUE1		0.5329	0.4171	0.4670	0.0001	0.69548
R-TB		0.8129	0.1371	0.1840	0.0031	1.23939
R-BC		0.8188	0.1312	0.1782	0.0030	1.28079
BOOTSTRAP-TB		0.9014	0.0486	0.0687	0.0299	$6.884e^{82}$
MVUE1	25	0.8724	0.0776	0.1276	0.0000	0.59261
MVUE2		0.9142	0.0358	0.0858	0.0000	$5.616e^{03}$
MLE-TB		0.9382	0.0118	0.0383	0.0235	$2.729e^{07}$
MLE-BC		0.9432	0.0068	0.0372	0.0196	$4.094e^{07}$
SLR		0.9054	0.0446	0.0715	0.0231	$7.135e^{04}$
ASLR		0.9112	0.0388	0.0675	0.0213	$1.445e^{05}$
BOOTSTRAP-BC		0.8786	0.0714	0.1031	0.0183	$1.630e^{14}$
R-MVUE1		0.5307	0.4193	0.4693	0.0000	0.59410
R-TB		0.8076	0.1424	0.1910	0.0014	0.86730
R-BC		0.8135	0.1365	0.1853	0.0012	0.88161
BOOTSTRAP-TB		0.8879	0.0621	0.0940	0.0181	$6.419e^{14}$
MVUE1	50	0.9439	0.0061	0.0561	0.0000	2.49688
MVUE2		0.9599	0.0099	0.0401	0.0000	8.99446
MLE-TB		0.9203	0.0297	0.0162	0.0635	56.55374
MLE-BC		0.9246	0.0254	0.0158	0.0596	60.23218
SLR		0.8627	0.0873	0.1270	0.0103	14.85099
ASLR		0.8661	0.0839	0.1242	0.0097	24.77459
BOOTSTRAP-BC		0.8327	0.1173	0.1594	0.0079	616.63738
R-MVUE1		0.4951	0.4549	0.5049	0.0000	0.46188
R-TB		0.7358	0.2142	0.2638	0.0004	0.56749
R-BC		0.7396	0.2104	0.2601	0.0003	0.57168
BOOTSTRAP-TB		0.8379	0.1121	0.1542	0.0079	683.69052

Table H.30: 95% CI under $\Delta(0.2, -0.075, 0.15)$ with 60% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9381	0.0119	0.0303	0.0316	0.56998
MVUE2		0.9404	0.0096	0.0299	0.0297	0.57866
MLE-TB		0.9383	0.0117	0.0106	0.0511	0.59227
MLE-BC		0.9391	0.0109	0.0102	0.0507	0.59367
SLR		0.9343	0.0157	0.0502	0.0155	0.62752
ASLR		0.9345	0.0155	0.0500	0.0155	0.63108
BOOTSTRAP-BC		0.9439	0.0061	0.0366	0.0195	0.61608
R-MVUE1		0.9767	0.0267	0.0086	0.0147	0.78040
R-TB		0.9549	0.0049	0.0011	0.0440	0.85304
R-BC		0.9556	0.0056	0.0011	0.0433	0.85743
BOOTSTRAP-TB		0.9459	0.0041	0.0365	0.0176	0.61882
MVUE1	25	0.9544	0.0044	0.0228	0.0228	0.44919
MVUE2		0.9556	0.0056	0.0223	0.0221	0.45337
MLE-TB		0.9543	0.0043	0.0098	0.0359	0.45940
MLE-BC		0.9545	0.0045	0.0098	0.0357	0.45999
SLR		0.9607	0.0107	0.0236	0.0157	0.47116
ASLR		0.9623	0.0123	0.0228	0.0149	0.48443
BOOTSTRAP-BC		0.9564	0.0064	0.0280	0.0156	0.46734
R-MVUE1		0.9789	0.0289	0.0023	0.0188	0.62869
R-TB		0.9491	0.0009	0.0003	0.0506	0.66525
R-BC		0.9502	0.0002	0.0003	0.0495	0.66730
BOOTSTRAP-TB		0.9565	0.0065	0.0279	0.0156	0.46788
MVUE1	50	0.9581	0.0081	0.0198	0.0221	0.32092
MVUE2		0.9587	0.0087	0.0194	0.0219	0.32243
MLE-TB		0.9552	0.0052	0.0111	0.0337	0.32450
MLE-BC		0.9553	0.0053	0.0111	0.0336	0.32470
SLR		0.9601	0.0101	0.0251	0.0148	0.32832
ASLR		0.9604	0.0104	0.0248	0.0148	0.33439
BOOTSTRAP-BC		0.9585	0.0085	0.0266	0.0149	0.32656
R-MVUE1		0.9537	0.0037	0.0005	0.0458	0.45798
R-TB		0.9072	0.0428	0.0001	0.0927	0.47179
R-BC		0.9076	0.0424	0.0001	0.0923	0.47252
BOOTSTRAP-TB		0.9586	0.0086	0.0265	0.0149	0.32676

Table H.31: 95% CI under $\Delta(0.4, -0.075, 0.15)$ with 60% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9306	0.0194	0.0456	0.0238	0.60365
MVUE2		0.9319	0.0181	0.0438	0.0243	0.61270
MLE-TB		0.9409	0.0091	0.0127	0.0464	0.64085
MLE-BC		0.9442	0.0058	0.00980	0.046	0.64469
SLR		0.9108	0.0392	0.0795	0.0097	0.67069
ASLR		0.9185	0.0315	0.0721	0.0094	0.67442
BOOTSTRAP-BC		0.9494	0.0006	0.0235	0.0271	0.64452
R-MVUE1		0.9618	0.0118	0.0236	0.0146	0.75436
R-TB		0.9483	0.0017	0.0054	0.0463	0.84766
R-BC		0.9502	0.0002	0.0041	0.0457	0.85456
BOOTSTRAP-TB		0.9644	0.0144	0.0233	0.0123	0.66969
MVUE1	25	0.9449	0.0051	0.0358	0.0193	0.47419
MVUE2		0.9453	0.0047	0.0357	0.0190	0.78690
MLE-TB		0.9503	0.0003	0.0108	0.0389	0.49208
MLE-BC		0.9503	0.0003	0.0108	0.0389	0.49274
SLR		0.9563	0.0063	0.0251	0.0186	0.49066
ASLR		0.9563	0.0063	0.0251	0.0186	0.50023
BOOTSTRAP-BC		0.9577	0.0077	0.0230	0.0193	0.49122
R-MVUE1		0.9733	0.0233	0.0107	0.0160	0.60980
R-TB		0.9472	0.0028	0.0014	0.0514	0.65822
R-BC		0.9477	0.0023	0.0014	0.0509	0.66038
BOOTSTRAP-TB		0.9601	0.0101	0.0228	0.0171	0.49283
MVUE1	50	0.9543	0.0043	0.0248	0.0209	0.33800
MVUE2		0.9549	0.0049	0.0242	0.0209	0.33948
MLE-TB		0.9535	0.0035	0.0096	0.0369	0.34396
MLE-BC		0.9536	0.0036	0.0095	0.0369	0.34416
SLR		0.9592	0.0092	0.0260	0.0148	0.34241
ASLR		0.9595	0.0095	0.0258	0.0147	0.35660
BOOTSTRAP-BC		0.9598	0.0098	0.0267	0.0135	0.34293
R-MVUE1		0.9623	0.0123	0.0028	0.0349	0.44693
R-TB		0.9240	0.0260	0.0006	0.0754	0.46519
R-BC		0.9241	0.0259	0.00006	0.0753	0.46595
BOOTSTRAP-TB		0.9598	0.0098	0.0267	0.0135	0.34312

Table H.32: 95% CI under Δ(0.4, -0.25, 0.50) with 60% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9163	0.0337	0.0789	0.0048	0.92831
MVUE2		0.9355	0.0145	0.0611	0.0035	1.31973
MLE-TB		0.9514	0.0014	0.0228	0.0258	4.94774
MLE-BC		0.9528	0.0028	0.0222	0.0250	6.47923
SLR		0.9500	0.0000	0.0319	0.0181	2.15668
ASLR		0.9543	0.0043	0.0289	0.0168	3.44232
BOOTSTRAP-BC		0.9407	0.0093	0.0351	0.0242	$2.290e^{06}$
R-MVUE1		0.9172	0.0328	0.0801	0.0027	0.87619
R-TB		0.9555	0.0055	0.0180	0.0265	1.13630
R-BC		0.9578	0.0078	0.0170	0.0252	1.15413
BOOTSTRAP-TB		0.9487	0.0013	0.0333	0.0180	$6.886e^{06}$
MVUE1	25	0.9507	0.0007	0.0441	0.0052	0.76442
MVUE2		0.9602	0.0102	0.0367	0.0031	0.89915
MLE-TB		0.9621	0.0121	0.0140	0.0239	1.01856
MLE-BC		0.9628	0.0128	0.0138	0.0234	1.03468
SLR		0.9584	0.0084	0.0282	0.0134	1.20043
ASLR		0.9613	0.0113	0.0254	0.0133	1.25887
BOOTSTRAP-BC		0.9513	0.0013	0.0353	0.0134	2.55663
R-MVUE1		0.9520	0.0020	0.0437	0.0043	0.73510
R-TB		0.9634	0.0134	0.0099	0.0267	0.87231
R-BC		0.9643	0.0143	0.0094	0.0263	0.87926
BOOTSTRAP-TB		0.9540	0.0040	0.0329	0.0131	2.81412
MVUE1	50	0.9667	0.0167	0.0269	0.0064	0.56461
MVUE2		0.9715	0.0215	0.0235	0.0050	0.60709
MLE-TB		0.9628	0.0128	0.0081	0.0291	0.62972
MLE-BC		0.9638	0.0138	0.0081	0.0281	0.63246
SLR		0.9576	0.0076	0.0359	0.0065	0.67646
ASLR		0.9581	0.0081	0.0356	0.0063	0.68119
BOOTSTRAP-BC		0.9503	0.0003	0.0435	0.0062	0.68794
R-MVUE1		0.9752	0.0252	0.0172	0.0076	0.55458
R-TB		0.9623	0.0123	0.0043	0.0334	0.60800
R-BC		0.9628	0.0128	0.0042	0.0330	0.61035
BOOTSTRAP-TB		0.9514	0.0014	0.0424	0.0062	0.69067

Table H.33: 95% CI under $\Delta(0.2, -0.50, 1.0)$ with 60% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9261	0.0239	0.0732	0.0007	2.12016
MVUE2		0.9492	0.0008	0.0506	0.0002	10.63763
MLE-TB		0.9693	0.0193	0.0199	0.0108	424.13580
MLE-BC		0.9710	0.0210	0.0193	0.0097	543.57810
SLR		0.9526	0.0026	0.0354	0.0120	65.01213
ASLR		0.9529	0.0029	0.0351	0.0120	84.22134
BOOTSTRAP-BC		0.9272	0.0228	0.0629	0.0099	$3.993e^{06}$
R-MVUE1		0.8076	0.1424	0.1918	0.0006	0.99459
R-TB		0.9368	0.0132	0.0555	0.0077	1.40013
R-BC		0.9392	0.0108	0.0531	0.0077	1.42488
BOOTSTRAP-TB		0.9337	0.0163	0.0566	0.0097	$7.029e^{06}$
MVUE1	25	0.9645	0.0145	0.0349	0.0006	1.84743
MVUE2		0.9750	0.0250	0.0249	0.0001	3.23650
MLE-TB		0.9649	0.0149	0.0101	0.0250	6.66616
MLE-BC		0.9678	0.0178	0.0099	0.0223	7.01342
SLR		0.9259	0.0241	0.0682	0.0059	4.42041
ASLR		0.9266	0.0234	0.0679	0.0055	4.90243
BOOTSTRAP-BC		0.9003	0.0497	0.0947	0.0050	59.50236
R-MVUE1		0.8386	0.1114	0.1607	0.0007	0.83815
R-TB		0.9501	0.0001	0.0446	0.0053	1.04850
R-BC		0.9516	0.0016	0.0434	0.0050	1.05899
BOOTSTRAP-TB		0.9059	0.0441	0.0891	0.0050	67.18158
MVUE1	50	0.9893	0.0393	0.0097	0.0010	1.35775
MVUE2		0.9924	0.0424	0.0075	0.0001	1.67186
MLE-TB		0.9277	0.0223	0.0028	0.0695	1.87065
MLE-BC		0.9303	0.0197	0.0027	0.0670	1.88673
SLR		0.8757	0.0743	0.1229	0.0014	2.15876
ASLR		0.8801	0.0699	0.1188	0.0011	2.31198
BOOTSTRAP-BC		0.8514	0.0986	0.1473	0.0013	2.46338
R-MVUE1		0.8742	0.0758	0.1255	0.0003	0.63982
R-TB		0.9523	0.0023	0.0445	0.0032	0.72086
R-BC		0.9535	0.0035	0.0436	0.0029	0.72429
BOOTSTRAP-TB		0.8544	0.0956	0.1444	0.0012	2.48732

Table H.34: 95% CI under $\Delta(0.4, \text{-}0.50, 1.0)$ with 60% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8943	0.0557	0.1050	0.0007	1.79748
MVUE2		0.9298	0.0202	0.0700	0.0002	404.44730
MLE-TB		0.9589	0.0089	0.0283	0.0128	$9.027e^{06}$
MLE-BC		0.9618	0.0118	0.0270	0.0112	$1.916e^{07}$
SLR		0.9426	0.0074	0.0372	0.0202	67.44914
ASLR		0.9453	0.0047	0.0349	0.0198	82.87555
BOOTSTRAP-BC		0.9210	0.0290	0.0597	0.0193	$4.700e^{23}$
R-MVUE1		0.7736	0.1764	0.2258	0.0006	0.86008
R-TB		0.9274	0.0226	0.0620	0.0106	1.32640
R-BC		0.9302	0.0198	0.0598	0.0100	1.35994
BOOTSTRAP-TB		0.9292	0.0208	0.0533	0.0175	$4.552e^{25}$
MVUE1	25	0.9369	0.0131	0.0623	0.0008	1.54106
MVUE2		0.9573	0.0073	0.0426	0.0001	4.67308
MLE-TB		0.9653	0.0153	0.0157	0.0190	61.36730
MLE-BC		0.9681	0.0181	0.0154	0.0165	72.78730
SLR		0.9408	0.0092	0.0483	0.0109	4.51780
ASLR		0.9409	0.0091	0.0482	0.0109	5.08766
BOOTSTRAP-BC		0.9146	0.0354	0.0755	0.0099	$3.854e^{04}$
R-MVUE1		0.8173	0.1327	0.1818	0.0009	0.73003
R-TB		0.9458	0.0042	0.0481	0.0061	0.97142
R-BC		0.9478	0.0022	0.0463	0.0059	0.98393
BOOTSTRAP-TB		0.9215	0.0285	0.0689	0.0096	$5.591e^{04}$
MVUE1	50	0.9790	0.0290	0.0200	0.0010	1.18045
MVUE2		0.9861	0.0361	0.0138	0.0001	1.65129
MLE-TB		0.9472	0.0028	0.0053	0.0475	2.36494
MLE-BC		0.9500	0.0000	0.0052	0.0448	2.41498
SLR		0.9107	0.0393	0.0861	0.0032	2.20080
ASLR		0.9115	0.0385	0.0857	0.0028	2.31129
BOOTSTRAP-BC		0.8838	0.0662	0.1132	0.0030	6.57151
R-MVUE1		0.8643	0.0857	0.1349	0.0008	0.56483
R-TB		0.9555	0.0055	0.0399	0.0046	0.66008
R-BC		0.9561	0.0061	0.0394	0.0045	0.66404
BOOTSTRAP-TB		0.8901	0.0599	0.1069	0.0030	6.87174

Table H.35: 95% CI under Δ(0.2, -1.0, 2.0) with 60% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9104	0.0396	0.0896	0.0000	31.15367
MVUE2		0.9533	0.0033	0.0467	0.0000	$3.726e^{07}$
MLE-TB		0.9712	0.0212	0.0180	0.0108	$2.579e^{14}$
MLE-BC		0.9773	0.0273	0.0167	0.0060	$5.460e^{14}$
SLR		0.8745	0.0755	0.1100	0.0155	148.10510
ASLR		0.8821	0.0679	0.1095	0.0084	212.44309
BOOTSTRAP-BC		0.8154	0.1346	0.1778	0.0068	$3.523e^{27}$
R-MVUE1		0.3902	0.5598	0.6098	0.0000	0.73165
R-TB		0.7622	0.1878	0.2372	0.0006	1.33694
R-BC		0.7712	0.1788	0.2284	0.0004	1.37529
BOOTSTRAP-TB		0.8365	0.1135	0.1568	0.0067	$2.004e^{28}$
MVUE1	25	0.9670	0.0170	0.0330	0.0000	16.34843
MVUE2		0.9830	0.0330	0.0170	0.0000	$1.485e^{05}$
MLE-TB		0.9289	0.0211	0.0049	0.0662	$1.263e^{10}$
MLE-BC		0.9376	0.0124	0.0049	0.0575	$1.862e^{10}$
SLR		0.7876	0.1624	0.2081	0.0043	$1.465e^{04}$
ASLR		0.8001	0.1499	0.1965	0.0034	$3.112e^{04}$
BOOTSTRAP-BC		0.7262	0.2238	0.2711	0.0027	$2.969e^{16}$
R-MVUE1		0.3542	0.5958	0.6458	0.0000	0.63127
R-TB		0.6955	0.2545	0.3041	0.0004	0.94085
R-BC		0.7026	0.2474	0.2970	0.0004	0.95516
BOOTSTRAP-TB		0.7404	0.2096	0.2569	0.0027	$6.178e^{16}$
MVUE1	50	0.9944	0.0444	0.0056	0.0000	9.26077
MVUE2		0.9965	0.0465	0.0035	0.0000	44.57924
MLE-TB		0.7246	0.2254	0.0012	0.2742	322.88860
MLE-BC		0.7337	0.2163	0.0012	0.2651	340.04550
SLR		0.5780	0.3720	0.4215	0.0005	133.01430
ASLR		0.5823	0.3677	0.4174	0.0003	256.07810
BOOTSTRAP-BC		0.5287	0.4213	0.4709	0.0004	$3.052e^{03}$
R-MVUE1		0.2480	0.7020	0.7520	0.0000	0.49230
R-TB		0.5144	0.4356	0.4856	0.0000	0.61132
R-BC		0.5181	0.4319	0.4819	0.0000	0.61551
BOOTSTRAP-TB		0.5367	0.4133	0.4629	0.0004	$3.310e^{03}$

Table H.36: 95% CI under $\Delta(0.4, -1.0, 2.0)$ with 60% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8586	0.0914	0.1414	0.0000	12.80546
MVUE2		0.9280	0.0220	0.0720	0.0000	$7.731e^{12}$
MLE-TB		0.9663	0.0163	0.0279	0.0058	$1.026e^{28}$
MLE-BC		0.9708	0.0208	0.0255	0.0037	$1.432e^{29}$
SLR		0.8924	0.0576	0.0823	0.0253	$1.323e^{10}$
ASLR		0.9002	0.0498	0.0820	0.0178	$6.543e^{10}$
BOOTSTRAP-BC		0.8401	0.1099	0.1482	0.0117	$7.949e^{161}$
R-MVUE1		0.4068	0.5432	0.5932	0.0000	0.61340
R-TB		0.7849	0.1651	0.2135	0.0016	1.31518
R-BC		0.7940	0.1560	0.2044	0.0016	1.37343
BOOTSTRAP-TB		0.8646	0.0854	0.1244	0.0110	$1.450e^{198}$
MVUE1	25	0.9370	0.0130	0.0630	0.0000	297.26320
MVUE2		0.9672	0.0172	0.0328	0.0000	$9.616e^{11}$
MLE-TB		0.9575	0.0075	0.0102	0.0323	$9.835e^{21}$
MLE-BC		0.9647	0.0147	0.0093	0.0260	$2.478e^{22}$
SLR		0.8438	0.1062	0.1494	0.0068	$1.372e^{07}$
ASLR		0.8612	0.0888	0.1365	0.0023	$2.443e^{07}$
BOOTSTRAP-BC		0.7844	0.1656	0.2121	0.0035	$2.274e^{36}$
R-MVUE1		0.4004	0.5496	0.5996	0.0000	0.53638
R-TB		0.7536	0.1964	0.2460	0.0004	0.89209
R-BC		0.7641	0.1859	0.2355	0.0004	0.91102
BOOTSTRAP-TB		0.8012	0.1488	0.1953	0.0035	$2.326e^{37}$
MVUE1	50	0.9863	0.0363	0.0137	0.0000	18.73297
MVUE2		0.9926	0.0426	0.0074	0.0000	$4.126e^{04}$
MLE-TB		0.8246	0.1254	0.0015	0.1739	$1.376e^{08}$
MLE-BC		0.8344	0.1156	0.0015	0.1641	$1.716e^{08}$
SLR		0.6827	0.2673	0.3164	0.0009	$1.213e^{04}$
ASLR		0.7002	0.2498	0.2994	0.0004	$3.199e^{04}$
BOOTSTRAP-BC		0.6227	0.3273	0.3767	0.0006	$3.261e^{11}$
R-MVUE1		0.3111	0.6389	0.6889	0.0000	0.42121
R-TB		0.6206	0.3294	0.3793	0.0001	0.55834
R-BC		0.6260	0.3240	0.3739	0.0001	0.56346
BOOTSTRAP-TB		0.6325	0.3175	0.3669	0.0006	$5.082e^{11}$

Table H.37: 95% CI under Δ(0.2, -0.075, 0.15) with 100% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9474	0.0026	0.0240	0.0286	0.59712
MVUE2		0.9505	0.0005	0.0224	0.0271	0.61173
MLE-TB		0.9465	0.0035	0.0067	0.0468	0.62769
MLE-BC		0.9470	0.0030	0.0067	0.0463	0.62960
SLR		0.9516	0.0016	0.0392	0.0092	0.67199
ASLR		0.9552	0.0052	0.0358	0.0090	0.67554
BOOTSTRAP-BC		0.9533	0.0033	0.0341	0.0126	0.66142
R-MVUE1		0.9796	0.0296	0.0054	0.0150	0.79065
R-TB		0.9571	0.0071	0.0009	0.0420	0.86877
R-BC		0.9579	0.0079	0.0009	0.0412	0.87350
BOOTSTRAP-TB		0.9553	0.0053	0.0336	0.0111	0.66459
MVUE1	25	0.9550	0.0050	0.0196	0.0254	0.46949
MVUE2		0.9564	0.0064	0.0192	0.0244	0.47610
MLE-TB		0.9514	0.0014	0.0083	0.0403	0.48298
MLE-BC		0.9515	0.0015	0.0083	0.0402	0.48376
SLR		0.9625	0.0125	0.0259	0.0116	0.49793
ASLR		0.9626	0.0126	0.0259	0.0115	0.50002
BOOTSTRAP-BC		0.9573	0.0073	0.0311	0.0116	0.49385
R-MVUE1		0.9764	0.0264	0.0022	0.0214	0.63784
R-TB		0.9469	0.0031	0.0003	0.0528	0.67707
R-BC		0.9476	0.0024	0.0003	0.0521	0.69270
BOOTSTRAP-TB		0.9577	0.0077	0.0308	0.0115	0.49454
MVUE1	50	0.9657	0.0157	0.0115	0.0228	0.33616
MVUE2		0.9665	0.0165	0.0111	0.0224	0.33849
MLE-TB		0.9588	0.0088	0.0057	0.0355	0.34082
MLE-BC		0.9591	0.0091	0.0057	0.0352	0.34107
SLR		0.9653	0.0153	0.0268	0.0079	0.34561
ASLR		0.9666	0.0166	0.026	0.0074	0.35012
BOOTSTRAP-BC		0.9626	0.0126	0.0299	0.0075	0.34370
R-MVUE1		0.9551	0.0051	0.0003	0.0446	0.46660
R-TB		0.9098	0.0402	0.0000	0.0902	0.48148
R-BC		0.9104	0.0396	0.0000	0.0896	0.48227
BOOTSTRAP-TB		0.9627	0.0127	0.0298	0.0075	0.34394

Table H.38: 95% CI under $\Delta(0.4, -0.075, 0.15)$ with 100% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9365	0.0135	0.0446	0.0189	0.62099
MVUE2		0.9403	0.0097	0.0406	0.0191	0.63686
MLE-TB		0.9480	0.0020	0.0097	0.0423	0.66898
MLE-BC		0.9499	0.0001	0.00800	0.0421	0.67350
SLR		0.9329	0.0171	0.0561	0.0110	0.71633
ASLR		0.9372	0.0128	0.0531	0.0097	0.72531
BOOTSTRAP-BC		0.9534	0.0034	0.0229	0.0237	0.69457
R-MVUE1		0.9631	0.0131	0.0243	0.0126	0.76062
R-TB		0.9540	0.0040	0.0036	0.0424	0.86047
R-BC		0.9561	0.0061	0.0028	0.0411	0.86777
BOOTSTRAP-TB		0.9685	0.0185	0.0226	0.0089	0.72516
MVUE1	25	0.9474	0.0026	0.0318	0.0208	0.48864
MVUE2		0.9494	0.0006	0.0302	0.0204	0.49571
MLE-TB		0.9523	0.0023	0.0076	0.0401	0.51028
MLE-BC		0.9528	0.0028	0.0076	0.0396	0.51113
SLR		0.9584	0.0084	0.0285	0.0131	0.51263
ASLR		0.9590	0.0090	0.0283	0.0127	0.51433
BOOTSTRAP-BC		0.9585	0.0085	0.0276	0.0139	0.51227
R-MVUE1		0.9724	0.0224	0.0092	0.0184	0.61637
R-TB		0.9446	0.0054	0.0017	0.0537	0.66764
R-BC		0.9456	0.0044	0.0015	0.0529	0.66996
BOOTSTRAP-TB		0.9607	0.0107	0.0273	0.0120	0.51403
MVUE1	50	0.9548	0.0048	0.0241	0.0211	0.34953
MVUE2		0.9555	0.0055	0.0236	0.0209	0.35203
MLE-TB		0.9519	0.0019	0.0101	0.0380	0.35692
MLE-BC		0.9520	0.0020	0.0101	0.0379	0.35718
SLR		0.9575	0.0075	0.0286	0.0139	0.35667
ASLR		0.9582	0.0082	0.0281	0.0137	0.35989
BOOTSTRAP-BC		0.9576	0.0076	0.0295	0.0129	0.35688
R-MVUE1		0.9633	0.0133	0.0024	0.0343	0.45256
R-TB		0.9216	0.0284	0.0009	0.0775	0.47207
R-BC		0.9223	0.0277	0.0009	0.0768	0.47288
BOOTSTRAP-TB		0.9576	0.0076	0.0295	0.0129	0.35712

Table H.39: 95% CI under Δ(0.2, -0.25, 0.50) with 100% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9556	0.0056	0.0407	0.0037	1.19405
MVUE2		0.9674	0.0174	0.0302	0.0024	1.61621
MLE-TB		0.9691	0.0191	0.0109	0.0200	2.24218
MLE-BC		0.9705	0.0205	0.0108	0.0187	2.35398
SLR		0.9717	0.0217	0.0202	0.0081	2.55277
ASLR		0.9725	0.0225	0.0194	0.0081	3.21129
BOOTSTRAP-BC		0.9543	0.0043	0.0381	0.0076	371.89440
R-MVUE1		0.9454	0.0046	0.0518	0.0028	1.00868
R-TB		0.9698	0.0198	0.0105	0.0197	1.27262
R-BC		0.9713	0.0213	0.0100	0.0187	1.28873
BOOTSTRAP-TB		0.9576	0.0076	0.0351	0.0073	631.50950
MVUE1	25	0.9738	0.0238	0.0220	0.0042	0.98446
MVUE2		0.9796	0.0296	0.0184	0.002	1.15674
MLE-TB		0.9695	0.0195	0.0081	0.0224	1.25518
MLE-BC		0.9708	0.0208	0.0078	0.0214	1.26874
SLR		0.9618	0.0118	0.0332	0.0050	1.47850
ASLR		0.9617	0.0117	0.0331	0.0050	1.52322
BOOTSTRAP-BC		0.9511	0.0011	0.0444	0.0045	1.70634
R-MVUE1		0.9675	0.0175	0.0285	0.0040	0.84413
R-TB		0.9739	0.0239	0.0056	0.0205	0.98141
R-BC		0.9746	0.0246	0.0054	0.0200	0.98865
BOOTSTRAP-TB		0.9537	0.0037	0.0421	0.0042	1.72928
MVUE1	50	0.9848	0.0348	0.0082	0.007	0.72713
MVUE2		0.9890	0.0390	0.0074	0.0036	0.78431
MLE-TB		0.9581	0.0081	0.0032	0.0387	0.80801
MLE-BC		0.9591	0.0091	0.0032	0.0377	0.81147
SLR		0.9418	0.0082	0.0560	0.0022	0.87061
ASLR		0.9421	0.0079	0.0558	0.0021	0.89342
BOOTSTRAP-BC		0.9306	0.0194	0.0674	0.0020	0.89119
R-MVUE1		0.9804	0.0304	0.0117	0.0079	0.63478
R-TB		0.9690	0.0190	0.0025	0.0285	0.68777
R-BC		0.9695	0.0195	0.0025	0.0280	0.69025
BOOTSTRAP-TB		0.9318	0.0182	0.0662	0.0020	0.89473

Table H.40: 95% CI under $\Delta(0.4, -0.25, 0.50)$ with 100% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9246	0.0254	0.0716	0.0038	1.04325
MVUE2		0.9446	0.0054	0.0524	0.0003	1.64278
MLE-TB		0.9606	0.0106	0.0185	0.0209	23.07833
MLE-BC		0.9632	0.0132	0.0172	0.0196	38.43748
SLR		0.9699	0.0199	0.0224	0.0077	12.97473
ASLR		0.9713	0.0213	0.0214	0.0073	21.09553
BOOTSTRAP-BC		0.9506	0.0006	0.0328	0.0166	$2.230e^{12}$
R-MVUE1		0.9075	0.0425	0.0895	0.0030	0.89169
R-TB		0.9605	0.0105	0.0190	0.0205	1.19708
R-BC		0.9624	0.0124	0.0180	0.0196	1.21821
BOOTSTRAP-TB		0.9583	0.0083	0.0303	0.0114	$1.497e^{13}$
MVUE1	25	0.9575	0.0075	0.0383	0.0042	0.87446
MVUE2		0.9697	0.0197	0.0281	0.0022	1.08351
MLE-TB		0.9653	0.0153	0.0103	0.0244	1.26755
MLE-BC		0.9660	0.0160	0.0103	0.0237	1.29217
SLR		0.9613	0.0113	0.0313	0.0074	1.58305
ASLR		0.9625	0.0125	0.0302	0.0073	1.67770
BOOTSTRAP-BC		0.9518	0.0018	0.0411	0.0071	3.42401
R-MVUE1		0.9440	0.0060	0.0523	0.0037	0.75411
R-TB		0.9664	0.0164	0.0092	0.0244	0.91574
R-BC		0.9672	0.0172	0.0090	0.0238	0.92401
BOOTSTRAP-TB		0.9544	0.0044	0.0387	0.0069	3.73672
MVUE1	50	0.9820	0.0320	0.0134	0.0046	0.65835
MVUE2		0.9864	0.0364	0.0109	0.0027	0.72689
MLE-TB		0.9609	0.0109	0.0041	0.0350	0.76390
MLE-BC		0.9612	0.0112	0.0041	0.0347	0.76823
SLR		0.9490	0.0010	0.0475	0.0035	0.83728
ASLR		0.9492	0.0008	0.0473	0.0035	0.84211
BOOTSTRAP-BC		0.9380	0.0120	0.0589	0.0031	0.87023
R-MVUE1		0.9754	0.0254	0.0192	0.0054	0.57495
R-TB		0.9682	0.0182	0.0043	0.0275	0.63825
R-BC		0.9688	0.0188	0.0043	0.0269	0.64101
BOOTSTRAP-TB		0.9392	0.0108	0.0577	0.0031	0.87510

Table H.41: 95% CI under Δ(0.2, -0.50, 1.0) with 100% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9459	0.0041	0.0537	0.0004	2.87053
MVUE2		0.9676	0.0176	0.0323	0.0001	106.56600
MLE-TB		0.9781	0.0281	0.0120	0.0099	$7.593e^{04}$
MLE-BC		0.9801	0.0301	0.0113	0.0086	$1.112e^{05}$
SLR		0.9703	0.0203	0.0232	0.0065	19.63076
ASLR		0.9719	0.0219	0.0223	0.0058	41.88102
BOOTSTRAP-BC		0.9106	0.0394	0.0847	0.0047	$2.795e^{17}$
R-MVUE1		0.7613	0.1887	0.2384	0.0003	0.99904
R-TB		0.9387	0.0113	0.0556	0.0057	1.50414
R-BC		0.9423	0.0077	0.0527	0.0050	1.53514
BOOTSTRAP-TB		0.9212	0.0288	0.0741	0.0047	$1.860e^{18}$
MVUE1	25	0.9743	0.0243	0.0252	0.0005	2.35186
MVUE2		0.9843	0.0343	0.0156	0.0001	4.48586
MLE-TB		0.9699	0.0199	0.0039	0.0262	8.59921
MLE-BC		0.9730	0.0230	0.0038	0.0232	8.9993
SLR		0.9668	0.0168	0.0318	0.0014	6.64328
ASLR		0.9672	0.0172	0.0315	0.0013	7.02443
BOOTSTRAP-BC		0.8808	0.0692	0.1182	0.001	64.90590
R-MVUE1		0.8000	0.1500	0.1994	0.0006	0.85676
R-TB		0.9457	0.0043	0.0501	0.0042	1.11866
R-BC		0.9484	0.0016	0.0476	0.0040	1.13141
BOOTSTRAP-TB		0.8887	0.0613	0.1104	0.0009	72.21372
MVUE1	50	0.9942	0.0442	0.0053	0.0005	1.88887
MVUE2		0.9964	0.0464	0.0035	0.0001	2.50539
MLE-TB		0.8937	0.0563	0.0014	0.1049	2.94796
MLE-BC		0.8985	0.0515	0.0014	0.1001	2.97879
SLR		0.9085	0.0415	0.0908	0.0007	3.51815
ASLR		0.9102	0.0398	0.0891	0.0007	3.63111
BOOTSTRAP-BC		0.7662	0.1838	0.2332	0.0006	4.22451
R-MVUE1		0.8266	0.1234	0.1731	0.0003	0.66494
R-TB		0.9410	0.0090	0.0573	0.0017	0.76890
R-BC		0.9421	0.0079	0.0562	0.0017	0.77305
BOOTSTRAP-TB		0.7706	0.1794	0.2289	0.0005	4.27724

Table H.42: 95% CI under Δ(0.4, -0.50, 1.0) with 100% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8999	0.0501	0.0998	0.0003	2.13622
MVUE2		0.9410	0.0090	0.0589	0.0001	$6.293e^{03}$
MLE-TB		0.9706	0.0206	0.0209	0.0085	$7.454e^{10}$
MLE-BC		0.9725	0.0225	0.0200	0.0075	$3.393e^{11}$
SLR		0.9250	0.0250	0.0502	0.0248	94.79544
ASLR		0.9297	0.0203	0.0476	0.0227	$1.342e^{03}$
BOOTSTRAP-BC		0.9181	0.0319	0.0693	0.0126	$5.605e^{49}$
R-MVUE1		0.7280	0.2220	0.2717	0.0003	0.84550
R-TB		0.9272	0.0228	0.0664	0.0064	1.41486
R-BC		0.9329	0.0171	0.0613	0.0058	1.45713
BOOTSTRAP-TB		0.9309	0.0191	0.0589	0.0102	$2.140e^{53}$
MVUE1	25	0.9524	0.0024	0.0473	0.0003	2.07173
MVUE2		0.9723	0.0223	0.0276	0.0001	7.37650
MLE-TB		0.9677	0.0177	0.0092	0.0231	73.25448
MLE-BC		0.9716	0.0216	0.0087	0.0197	85.48152
SLR		0.9268	0.0232	0.0678	0.0054	14.01258
ASLR		0.9302	0.0198	0.0635	0.0063	24.76770
BOOTSTRAP-BC		0.8930	0.0570	0.1024	0.0046	$3.486e^{04}$
R-MVUE1		0.7821	0.1679	0.2175	0.0004	0.73371
R-TB		0.9439	0.0061	0.0492	0.0069	1.03554
R-BC		0.9466	0.0034	0.0469	0.0065	1.05129
BOOTSTRAP-TB		0.9024	0.0476	0.0932	0.0044	$5.273e^{04}$
MVUE1	50	0.9888	0.0388	0.0108	0.0004	1.65564
MVUE2		0.9928	0.0428	0.0072	0.0000	2.61606
MLE-TB		0.9294	0.0206	0.0016	0.0690	4.06234
MLE-BC		0.9335	0.0165	0.0016	0.0649	4.16860
SLR		0.9311	0.0189	0.0502	0.0187	7.32143
ASLR		0.9309	0.0191	0.0500	0.0191	8.00341
BOOTSTRAP-BC		0.8206	0.1294	0.1786	0.0008	12.65663
R-MVUE1		0.8181	0.1319	0.1817	0.0002	0.57709
R-TB		0.9473	0.0027	0.0502	0.0025	0.69750
R-BC		0.9482	0.0018	0.0493	0.0025	0.70232
BOOTSTRAP-TB		0.8279	0.1221	0.1713	0.0008	13.51227

Table H.43: 95% CI under $\Delta(0.2, -1.0, 2.0)$ with 100% Contamination from Weibull Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9221	0.0279	0.0779	0.0000	87.32188
MVUE2		0.9666	0.0166	0.0334	0.0000	$2.632e^{10}$
MLE-TB		0.9757	0.0257	0.0111	0.0132	$8.439e^{19}$
MLE-BC		0.9815	0.0315	0.0108	0.0077	$2.183e^{20}$
SLR		0.8912	0.0588	0.0987	0.0101	$3.432e^{09}$
ASLR		0.9012	0.0488	0.0899	0.0089	$6.445e^{11}$
BOOTSTRAP-BC		0.7700	0.1800	0.2268	0.0032	$1.884e^{42}$
R-MVUE1		0.2905	0.6595	0.7095	0.0000	0.64730
R-TB		0.7223	0.2277	0.2776	0.0001	1.38249
R-BC		0.7345	0.2155	0.2654	0.0001	1.43098
BOOTSTRAP-TB		0.7967	0.1533	0.2002	0.0031	$2.638e^{44}$
MVUE1	25	0.9773	0.0273	0.0227	0.0000	44.88749
MVUE2		0.9894	0.0394	0.1060	0.0000	$1.641e^{06}$
MLE-TB		0.9028	0.0472	0.0026	0.0946	$1.016e^{11}$
MLE-BC		0.9190	0.0310	0.0026	0.0784	$1.406e^{11}$
SLR		0.8521	0.0979	0.1289	0.0190	$1.886e^{08}$
ASLR		0.8612	0.0888	0.1201	0.0187	$2.043e^{08}$
BOOTSTRAP-BC		0.6375	0.3125	0.3619	0.0006	$1.028e^{17}$
R-MVUE1		0.2428	0.7072	0.7572	0.0000	0.56872
R-TB		0.6371	0.3129	0.3629	0.0000	0.95000
R-BC		0.6443	0.3057	0.3557	0.0000	0.96728
BOOTSTRAP-TB		0.6544	0.2956	0.3450	0.0006	$1.838e^{17}$
MVUE1	50	0.9984	0.0484	0.0016	0.0000	17.62258
MVUE2		0.9996	0.0496	0.0004	0.0000	133.2322
MLE-TB		0.6221	0.3279	0.0001	0.3778	$1.595e^{03}$
MLE-BC		0.6338	0.3162	0.0001	0.3661	$1.690e^{03}$
SLR		0.4503	0.4997	0.5497	0.0000	19.64866
ASLR		0.4589	0.4911	0.5411	0.0000	28.30342
BOOTSTRAP-BC		0.3873	0.5627	0.6127	0.0000	$1.787e^{04}$
R-MVUE1		0.1427	0.8073	0.8573	0.0000	0.45361
R-TB		0.4053	0.5447	0.5947	0.0000	0.60106
R-BC		0.4093	0.5407	0.5907	0.0000	0.60584
BOOTSTRAP-TB		0.3968	0.5532	0.6032	0.0000	$1.945e^{04}$

Table H.44: 95% CI under Δ(0.4, -0.075, 0.15) with 20% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9189	0.0311	0.0550	0.0261	0.61320
MVUE2		0.9212	0.0288	0.0541	0.0247	0.62138
MLE-TB		0.9285	0.0215	0.0158	0.0557	0.64927
MLE-BC		0.9311	0.0189	0.0134	0.0555	0.65308
SLR		0.9289	0.0211	0.0610	0.0101	0.67767
ASLR		0.9311	0.0189	0.0589	0.0100	0.68212
BOOTSTRAP-BC		0.9322	0.0178	0.0332	0.0346	0.64840
R-MVUE1		0.9573	0.0073	0.0274	0.0153	0.77641
R-TB		0.9374	0.0126	0.0057	0.0569	0.87643
R-BC		0.9399	0.0101	0.0043	0.0558	0.88381
BOOTSTRAP-TB		0.9510	0.0010	0.033	0.0160	0.67350
MVUE1	25	0.9387	0.0113	0.0377	0.0236	0.47833
MVUE2		0.9398	0.0102	0.0372	0.0230	0.48201
MLE-TB		0.9375	0.0125	0.0134	0.0491	0.49504
MLE-BC		0.9379	0.0121	0.0131	0.0490	0.49563
SLR		0.9425	0.0075	0.0339	0.0236	0.49233
ASLR		0.9428	0.0072	0.0336	0.0236	0.50004
BOOTSTRAP-BC		0.9417	0.0083	0.0337	0.0246	0.49243
R-MVUE1		0.9708	0.0208	0.0091	0.0201	0.62942
R-TB		0.9340	0.0160	0.0011	0.0649	0.68134
R-BC		0.9348	0.0152	0.0010	0.0642	0.68368
BOOTSTRAP-TB		0.9448	0.0052	0.0331	0.0221	0.49389
MVUE1	50	0.9426	0.0074	0.0279	0.0295	0.34094
MVUE2		0.9432	0.0068	0.0277	0.0291	0.34225
MLE-TB		0.9357	0.0143	0.0136	0.0507	0.34672
MLE-BC		0.9358	0.0142	0.0136	0.0506	0.34691
SLR		0.9408	0.0092	0.0385	0.0207	0.34494
ASLR		0.9412	0.0088	0.0382	0.0206	0.35002
BOOTSTRAP-BC		0.9406	0.0094	0.0404	0.0190	0.34525
R-MVUE1		0.9488	0.0012	0.0037	0.0475	0.46103
R-TB		0.9018	0.0482	0.0010	0.0972	0.48069
R-BC		0.9025	0.0475	0.0010	0.0965	0.48151
BOOTSTRAP-TB		0.9406	0.0094	0.0404	0.0190	0.34543

Table H.45: 95% CI under $\Delta(0.2, -0.25, 0.50)$ with 20% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9036	0.0464	0.0889	0.0075	0.81079
MVUE2		0.9164	0.0336	0.0789	0.0047	0.89288
MLE-TB		0.9313	0.0187	0.0430	0.0257	0.93668
MLE-BC		0.9324	0.0176	0.0426	0.0250	0.94436
SLR		0.9397	0.0103	0.0206	0.0397	1.09010
ASLR		0.9402	0.0098	0.0205	0.0393	1.11056
BOOTSTRAP-BC		0.9238	0.0262	0.0352	0.0410	1.10502
R-MVUE1		0.9628	0.0128	0.0366	0.0045	0.96280
R-TB		0.9619	0.0119	0.0102	0.0279	1.14819
R-BC		0.9635	0.0135	0.0100	0.0265	1.15951
BOOTSTRAP-TB		0.9254	0.0246	0.0343	0.0403	1.11395
MVUE1	25	0.9261	0.0239	0.0679	0.0060	0.64187
MVUE2		0.9331	0.0169	0.0626	0.0043	0.68062
MLE-TB		0.9442	0.0058	0.0334	0.0224	0.69910
MLE-BC		0.9449	0.0051	0.0330	0.0221	0.70225
SLR		0.9463	0.0037	0.0238	0.0299	0.75525
ASLR		0.9463	0.0037	0.0237	0.0300	0.77041
BOOTSTRAP-BC		0.9385	0.0115	0.0308	0.0307	0.75963
R-MVUE1		0.9783	0.0283	0.0158	0.0059	0.78780
R-TB		0.9646	0.0146	0.0050	0.0304	0.88470
R-BC		0.9652	0.0152	0.0048	0.0300	0.88985
BOOTSTRAP-TB		0.9392	0.0108	0.0302	0.0306	0.76242
MVUE1	50	0.9388	0.0112	0.0488	0.0124	0.46360
MVUE2		0.9429	0.0071	0.0460	0.0111	0.47759
MLE-TB		0.9449	0.0051	0.0260	0.0291	0.48680
MLE-BC		0.9455	0.0045	0.0257	0.0288	0.48471
SLR		0.9434	0.0066	0.0322	0.0244	0.50113
ASLR		0.9433	0.0067	0.0323	0.0244	0.51004
BOOTSTRAP-BC		0.9422	0.0078	0.0350	0.0228	0.50231
R-MVUE1		0.9741	0.0241	0.0045	0.0214	0.97410
R-TB		0.9430	0.0070	0.0016	0.0554	0.62165
R-BC		0.9441	0.0059	0.0016	0.0543	0.62344
BOOTSTRAP-TB		0.9428	0.0072	0.0345	0.0227	0.50327

Table H.46: 95% CI under Δ(0.4, -0.25, 0.50) with 20% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8954	0.0546	0.0981	0.0065	0.77212
MVUE2		0.9097	0.0403	0.0867	0.0036	0.86754
MLE-TB		0.9334	0.0166	0.0374	0.0292	0.94359
MLE-BC		0.9361	0.0139	0.0356	0.0283	0.95607
SLR		0.9326	0.0174	0.0337	0.0337	1.15899
ASLR		0.9344	0.0156	0.0330	0.0326	1.16221
BOOTSTRAP-BC		0.9243	0.0257	0.0330	0.0427	1.15562
R-MVUE1		0.9328	0.0172	0.0636	0.0036	0.87294
R-TB		0.9527	0.0027	0.0166	0.0307	1.09429
R-BC		0.9555	0.0055	0.0151	0.0294	1.10946
BOOTSTRAP-TB		0.9355	0.0145	0.0323	0.0322	1.25358
MVUE1	25	0.9170	0.0330	0.0766	0.0064	0.61213
MVUE2		0.9250	0.0250	0.0702	0.0048	0.65615
MLE-TB		0.9419	0.0081	0.0309	0.0272	0.68741
MLE-BC		0.9424	0.0076	0.0307	0.0269	0.69106
SLR		0.9420	0.0080	0.0261	0.0319	0.74517
ASLR		0.9422	0.0078	0.0260	0.0318	0.75111
BOOTSTRAP-BC		0.9373	0.0127	0.0316	0.0311	0.74584
R-MVUE1		0.9619	0.0119	0.0326	0.0055	0.72262
R-TB		0.9565	0.0065	0.0074	0.0361	0.83760
R-BC		0.9580	0.0080	0.0071	0.0349	0.84334
BOOTSTRAP-TB		0.9396	0.0104	0.0310	0.0294	0.75014
MVUE1	50	0.9389	0.0111	0.0534	0.0077	0.44175
MVUE2		0.9435	0.0065	0.0504	0.0061	0.45758
MLE-TB		0.9500	0.0000	0.0244	0.0256	0.46775
MLE-BC		0.9502	0.0002	0.0243	0.0255	0.46887
SLR		0.9501	0.0001	0.0254	0.0245	0.48342
ASLR		0.9519	0.0019	0.0245	0.0236	0.49021
BOOTSTRAP-BC		0.9464	0.0036	0.0301	0.0235	0.48400
R-MVUE1		0.9772	0.0272	0.0113	0.0115	0.53955
R-TB		0.9498	0.0002	0.0027	0.0475	0.58366
R-BC		0.9503	0.0003	0.0025	0.0472	0.58560
BOOTSTRAP-TB		0.9470	0.0030	0.0296	0.0234	0.48500

Table H.47: 95% CI under $\Delta(0.2, -0.50, 1.0)$ with 20% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8524	0.0976	0.1469	0.0007	1.04305
MVUE2		0.8812	0.0688	0.1186	0.0002	1.30368
MLE-TB		0.9290	0.0210	0.0631	0.0079	1.45366
MLE-BC		0.9311	0.0189	0.0614	0.0075	1.47929
SLR		0.9347	0.0153	0.0124	0.0529	2.00806
ASLR		0.9356	0.0144	0.0122	0.0522	2.12543
BOOTSTRAP-BC		0.9371	0.0129	0.0222	0.0407	2.28461
R-MVUE1		0.8717	0.0783	0.1277	0.0006	0.96610
R-TB		0.9556	0.0056	0.0371	0.0073	1.25300
R-BC		0.9569	0.0069	0.0361	0.0070	1.27051
BOOTSTRAP-TB		0.9397	0.0103	0.0204	0.0399	2.34836
MVUE1	25	0.8741	0.0759	0.1254	0.0005	0.84674
MVUE2		0.8937	0.0563	0.1063	0.0000	0.97134
MLE-TB		0.9333	0.0167	0.0575	0.0092	1.02940
MLE-BC		0.9348	0.0152	0.0565	0.0087	1.03866
SLR		0.9427	0.0073	0.0146	0.0427	1.20582
ASLR		0.9430	0.0070	0.0146	0.0424	1.23967
BOOTSTRAP-BC		0.9408	0.0092	0.0204	0.0388	1.26471
R-MVUE1		0.8980	0.0520	0.1015	0.0005	0.80385
R-TB		0.9577	0.0077	0.0347	0.0076	0.95166
R-BC		0.9592	0.0092	0.0336	0.0072	0.95935
BOOTSTRAP-TB		0.9420	0.0080	0.0194	0.0386	1.27443
MVUE1	50	0.8998	0.0502	0.0980	0.0022	0.61803
MVUE2		0.9107	0.0393	0.0885	0.0008	0.66241
MLE-TB		0.9404	0.0096	0.0500	0.0096	0.68012
MLE-BC		0.9411	0.0089	0.0495	0.0094	0.68288
SLR		0.9464	0.0036	0.0156	0.0380	0.72888
ASLR		0.9471	0.0029	0.0151	0.0378	0.73885
BOOTSTRAP-BC		0.9457	0.0043	0.0194	0.0349	0.74404
R-MVUE1		0.9268	0.0232	0.0713	0.0019	0.60166
R-TB		0.9648	0.0148	0.0281	0.0071	0.65776
R-BC		0.9652	0.0152	0.0277	0.0071	0.66032
BOOTSTRAP-TB		0.9465	0.0035	0.0190	0.0345	0.74681

Table H.48: 95% CI under Δ(0.4, -0.50, 1.0) with 20% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8273	0.1227	0.1717	0.0010	0.93402
MVUE2		0.8621	0.0879	0.1376	0.0003	1.24646
MLE-TB		0.9223	0.0277	0.0660	0.0117	1.50564
MLE-BC		0.9254	0.0246	0.0636	0.0110	1.55306
SLR		0.9334	0.0166	0.0191	0.0475	2.45404
ASLR		0.9345	0.0155	0.0186	0.0469	2.51131
BOOTSTRAP-BC		0.9199	0.0301	0.0281	0.0520	6.60950
R-MVUE1		0.8338	0.1162	0.1654	0.0008	0.84707
R-TB		0.9384	0.0116	0.0501	0.0115	1.17929
R-BC		0.9418	0.0082	0.0473	0.0109	1.20232
BOOTSTRAP-TB		0.9284	0.0216	0.0259	0.0457	22.20403
MVUE1	25	0.8693	0.0807	0.1305	0.0002	0.75831
MVUE2		0.8901	0.0599	0.1098	0.0001	0.90162
MLE-TB		0.9353	0.0147	0.0533	0.0114	0.98934
MLE-BC		0.9367	0.0133	0.0524	0.0109	1.00142
SLR		0.9433	0.0067	0.0171	0.0396	1.23091
ASLR		0.9434	0.0066	0.0171	0.0395	1.25332
BOOTSTRAP-BC		0.9378	0.0122	0.0236	0.0386	1.30085
R-MVUE1		0.8833	0.0667	0.1165	0.0002	0.71237
R-TB		0.9571	0.0071	0.0333	0.0096	0.88538
R-BC		0.9584	0.0084	0.0323	0.0093	0.89428
BOOTSTRAP-TB		0.9398	0.0102	0.0222	0.0380	1.31701
MVUE1	50	0.8930	0.0570	0.1047	0.0023	0.55940
MVUE2		0.9068	0.0432	0.0924	0.0008	0.61043
MLE-TB		0.9400	0.0100	0.0475	0.0125	0.63594
MLE-BC		0.9406	0.0094	0.0469	0.0125	0.63918
SLR		0.9440	0.0060	0.0176	0.0384	0.69225
ASLR		0.9443	0.0057	0.0174	0.0383	0.70554
BOOTSTRAP-BC		0.9424	0.0076	0.0221	0.0355	0.70596
R-MVUE1		0.9118	0.0382	0.0859	0.0023	0.54096
R-TB		0.9604	0.0104	0.0293	0.0103	0.60805
R-BC		0.9614	0.0114	0.0284	0.0102	0.61095
BOOTSTRAP-TB		0.9434	0.0066	0.0216	0.0350	0.70913

Table H.49: 95% CI under $\Delta(0.2, -0.075, 0.15)$ with 60% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9357	0.0143	0.0291	0.0352	0.67656
MVUE2		0.9428	0.0072	0.0280	0.0292	0.70040
MLE-TB		0.9170	0.0330	0.0108	0.0722	0.72008
MLE-BC		0.9180	0.0320	0.0107	0.0713	0.72271
SLR		0.9407	0.0093	0.0446	0.0147	0.77747
ASLR		0.9411	0.0089	0.0442	0.0147	0.78002
BOOTSTRAP-BC		0.9162	0.0338	0.0671	0.0167	0.76336
R-MVUE1		0.9754	0.0254	0.0061	0.0185	0.91322
R-TB		0.9264	0.0236	0.0009	0.0727	1.02795
R-BC		0.9285	0.0215	0.0009	0.0706	1.03492
BOOTSTRAP-TB		0.9186	0.0314	0.0656	0.0158	0.76711
MVUE1	25	0.9408	0.0092	0.0222	0.0370	0.52713
MVUE2		0.9449	0.0051	0.0215	0.0336	0.53807
MLE-TB		0.9162	0.0338	0.0111	0.0727	0.54660
MLE-BC		0.9168	0.0332	0.0110	0.0722	0.54770
SLR		0.9291	0.0209	0.0576	0.0133	0.56750
ASLR		0.9298	0.0202	0.0570	0.0132	0.56998
BOOTSTRAP-BC		0.9183	0.0317	0.0682	0.0135	0.56207
R-MVUE1		0.9642	0.0142	0.0022	0.0336	0.73441
R-TB		0.8972	0.0528	0.0003	0.1025	0.79153
R-BC		0.8985	0.0515	0.0003	0.1012	0.79472
BOOTSTRAP-TB		0.9196	0.0304	0.0671	0.0133	0.56304
MVUE1	50	0.9310	0.0190	0.0112	0.0578	0.37587
MVUE2		0.9342	0.0158	0.0109	0.0549	0.37979
MLE-TB		0.9041	0.0459	0.0050	0.0909	0.38269
MLE-BC		0.9043	0.0457	0.0050	0.0907	0.38306
SLR		0.9136	0.0364	0.0802	0.0062	0.38945
ASLR		0.9137	0.0363	0.0801	0.0062	0.39266
BOOTSTRAP-BC		0.9061	0.0439	0.0880	0.0059	0.38714
R-MVUE1		0.8917	0.0583	0.0002	0.1081	0.53700
R-TB		0.7964	0.1536	0.0000	0.2036	0.55855
R-BC		0.7978	0.1522	0.0000	0.2022	0.55968
BOOTSTRAP-TB		0.9067	0.0433	0.0874	0.0059	0.38748

Table H.50: 95% CI under $\Delta(0.4, -0.075, 0.15)$ with 60% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9312	0.0188	0.0463	0.0225	0.68684
MVUE2		0.9376	0.0124	0.0427	0.0197	0.71321
MLE-TB		0.9233	0.0267	0.0126	0.0641	0.75152
MLE-BC		0.9259	0.0241	0.0107	0.0634	0.75691
SLR		0.9254	0.0246	0.0647	0.0099	0.80860
ASLR		0.9266	0.0234	0.0635	0.0099	0.81100
BOOTSTRAP-BC		0.9293	0.0207	0.0461	0.0246	0.77652
R-MVUE1		0.9643	0.0143	0.0213	0.0144	0.85628
R-TB		0.9302	0.0198	0.0037	0.0661	0.99805
R-BC		0.9320	0.0180	0.0029	0.0651	1.00799
BOOTSTRAP-TB		0.9415	0.0085	0.0455	0.0130	0.80694
MVUE1	25	0.9394	0.0106	0.0298	0.0308	0.53840
MVUE2		0.9437	0.0063	0.0287	0.0276	0.55057
MLE-TB		0.9261	0.0239	0.0098	0.0641	0.56796
MLE-BC		0.9266	0.0234	0.0098	0.0636	0.56917
SLR		0.9369	0.0131	0.0491	0.0140	0.57715
ASLR		0.9376	0.0124	0.0487	0.0137	0.59221
BOOTSTRAP-BC		0.9325	0.0175	0.0532	0.0143	0.57429
R-MVUE1		0.9654	0.0154	0.0075	0.0271	0.69717
R-TB		0.9164	0.0336	0.0019	0.0817	0.77015
R-BC		0.9175	0.0325	0.0018	0.0807	0.77360
BOOTSTRAP-TB		0.9345	0.0155	0.0529	0.0126	0.57637
MVUE1	50	0.9426	0.0074	0.0180	0.0394	0.38456
MVUE2		0.9454	0.0046	0.0173	0.0373	0.38888
MLE-TB		0.9181	0.0319	0.0088	0.0731	0.39477
MLE-BC		0.9186	0.0314	0.0088	0.0726	0.39515
SLR		0.9294	0.0206	0.0593	0.0113	0.39657
ASLR		0.9299	0.0201	0.0590	0.0111	0.40054
BOOTSTRAP-BC		0.9243	0.0257	0.0648	0.0109	0.39592
R-MVUE1		0.9366	0.0134	0.0023	0.0611	0.51350
R-TB		0.8600	0.0900	0.0003	0.1397	0.54123
R-BC		0.8612	0.0888	0.0002	0.1386	0.54243
BOOTSTRAP-TB		0.9249	0.0251	0.0642	0.0109	0.39626

Table H.51: 95% CI under Δ(0.2, -0.25, 0.50) with 60% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9197	0.0303	0.0733	0.0070	0.88348
MVUE2		0.9323	0.0177	0.0640	0.0037	0.99033
MLE-TB		0.9388	0.0112	0.0315	0.0297	1.04592
MLE-BC		0.9407	0.0093	0.0309	0.0284	1.05584
SLR		0.9451	0.0049	0.0279	0.0270	1.24166
ASLR		0.9451	0.0049	0.0279	0.0270	1.25339
BOOTSTRAP-BC		0.9268	0.0232	0.0457	0.0275	1.27368
R-MVUE1		0.9616	0.0116	0.0343	0.0041	1.01430
R-TB		0.9605	0.0105	0.0075	0.3200	1.22927
R-BC		0.9619	0.0119	0.0075	0.0306	1.24239
BOOTSTRAP-TB		0.9286	0.0214	0.0447	0.0267	1.28500
MVUE1	25	0.9332	0.0168	0.0588	0.0080	0.69808
MVUE2		0.9420	0.0080	0.0534	0.0046	0.74812
MLE-TB		0.9444	0.0056	0.0265	0.0291	0.77110
MLE-BC		0.9450	0.0050	0.0263	0.0287	0.77508
SLR		0.9447	0.0053	0.0325	0.0228	0.84248
ASLR		0.9449	0.0051	0.0323	0.0228	0.87654
BOOTSTRAP-BC		0.9372	0.0128	0.0401	0.0227	0.85274
R-MVUE1		0.9749	0.0249	0.0177	0.0074	0.83189
R-TB		0.9593	0.0093	0.0038	0.0369	0.94105
R-BC		0.9606	0.0106	0.0036	0.0358	0.94694
BOOTSTRAP-TB		0.9379	0.0121	0.0394	0.0227	0.85636
MVUE1	50	0.9506	0.0006	0.0353	0.0141	0.50495
MVUE2		0.9573	0.0073	0.0317	0.0110	0.52305
MLE-TB		0.9461	0.0039	0.0176	0.0363	0.53059
MLE-BC		0.9465	0.0035	0.0175	0.0360	0.53189
SLR		0.9425	0.0075	0.0426	0.0149	0.55242
ASLR		0.9430	0.0070	0.0422	0.0148	0.56884
BOOTSTRAP-BC		0.9364	0.0136	0.0495	0.0141	0.55514
R-MVUE1		0.9720	0.0220	0.0046	0.0234	0.61706
R-TB		0.9319	0.0181	0.0009	0.0672	0.65856
R-BC		0.9329	0.0171	0.0009	0.0662	0.66058
BOOTSTRAP-TB		0.9374	0.0126	0.0485	0.0141	0.55637

Table H.52: 95% CI under $\Delta(0.2, -0.50, 1.0)$ with 60% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8231	0.1269	0.1763	0.0006	1.02303
MVUE2		0.8585	0.0915	0.1411	0.0004	1.29148
MLE-TB		0.9190	0.0310	0.0742	0.0068	1.44599
MLE-BC		0.9215	0.0285	0.0723	0.0062	1.47213
SLR		0.9390	0.0110	0.0103	0.0507	2.02315
ASLR		0.9396	0.0104	0.0101	0.0503	2.19883
BOOTSTRAP-BC		0.9352	0.0148	0.0176	0.0472	2.27106
R-MVUE1		0.8347	0.1153	0.1648	0.0005	0.93417
R-TB		0.9421	0.0079	0.0517	0.0062	1.22334
R-BC		0.9436	0.0064	0.0506	0.0058	1.24097
BOOTSTRAP-TB		0.9374	0.0126	0.016	0.0466	2.30973
MVUE1	25	0.8455	0.1045	0.1543	0.0002	0.81860
MVUE2		0.8708	0.0792	0.1291	0.0001	0.94325
MLE-TB		0.9257	0.0243	0.0694	0.0049	1.00116
MLE-BC		0.9274	0.0226	0.0680	0.0046	1.01038
SLR		0.9423	0.0077	0.0100	0.0477	1.17758
ASLR		0.9432	0.0068	0.0098	0.0470	1.21001
BOOTSTRAP-BC		0.9430	0.0070	0.0129	0.0441	1.23708
R-MVUE1		0.8632	0.0868	0.1366	0.0002	0.76955
R-TB		0.9492	0.0008	0.0470	0.0038	0.91584
R-BC		0.9506	0.0006	0.0457	0.0037	0.92343
BOOTSTRAP-TB		0.9439	0.0061	0.0122	0.0439	1.24679
MVUE1	50	0.8621	0.0879	0.1374	0.0005	0.60282
MVUE2		0.8763	0.0737	0.1235	0.0002	0.64831
MLE-TB		0.9234	0.0266	0.0722	0.0044	0.66652
MLE-BC		0.9244	0.0256	0.0715	0.0041	0.66932
SLR		0.9383	0.0117	0.0075	0.0542	0.71628
ASLR		0.9399	0.0101	0.0073	0.0528	0.72912
BOOTSTRAP-BC		0.9425	0.0075	0.0092	0.0483	0.73199
R-MVUE1		0.8798	0.0702	0.1197	0.0005	0.57880
R-TB		0.9488	0.0012	0.0487	0.0025	0.63488
R-BC		0.9498	0.0002	0.0479	0.0023	0.63742
BOOTSTRAP-TB		0.9431	0.0069	0.0088	0.0481	0.73481

Table H.53: 95% CI under $\Delta(0.4, -0.50, 1.0)$ with 60% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.8127	0.1373	0.1871	0.0002	0.90617
MVUE2		0.8520	0.0980	0.1479	0.0001	1.21862
MLE-TB		0.9213	0.0287	0.0723	0.0064	1.47315
MLE-BC		0.9235	0.0265	0.0706	0.0059	1.52009
SLR		0.9376	0.0124	0.0123	0.0501	2.43961
ASLR		0.9375	0.0125	0.0123	0.0502	2.51662
BOOTSTRAP-BC		0.9276	0.0224	0.0183	0.0541	6.84062
R-MVUE1		0.8062	0.1438	0.1936	0.0002	0.81385
R-TB		0.9350	0.0150	0.0588	0.0062	1.14633
R-BC		0.9385	0.0115	0.0559	0.0056	1.16928
BOOTSTRAP-TB		0.9378	0.0122	0.0159	0.0463	41.21129
MVUE1	25	0.8350	0.1150	0.1647	0.0003	0.73430
MVUE2		0.8634	0.0866	0.1365	0.0001	0.87898
MLE-TB		0.9298	0.0202	0.0652	0.0050	0.96652
MLE-BC		0.9317	0.0183	0.0634	0.0049	0.97861
SLR		0.9434	0.0066	0.0095	0.0471	1.20760
ASLR		0.9448	0.0052	0.0091	0.0461	1.24330
BOOTSTRAP-BC		0.9410	0.0090	0.0138	0.0452	1.27872
R-MVUE1		0.8466	0.1034	0.1531	0.0003	0.68316
R-TB		0.9512	0.0012	0.0448	0.0040	0.85634
R-BC		0.9533	0.0033	0.0428	0.0039	0.86524
BOOTSTRAP-TB		0.9430	0.0070	0.0127	0.0443	1.29401
MVUE1	50	0.8617	0.0883	0.1378	0.0005	0.54246
MVUE2		0.8808	0.0692	0.1191	0.0001	0.59436
MLE-TB		0.9346	0.0154	0.0599	0.0055	0.62016
MLE-BC		0.9352	0.0148	0.0594	0.0054	0.62344
SLR		0.9458	0.0042	0.0078	0.0464	0.67768
ASLR		0.9472	0.0028	0.0076	0.0452	0.68221
BOOTSTRAP-BC		0.9451	0.0049	0.0112	0.0437	0.69191
R-MVUE1		0.8807	0.0693	0.1188	0.0005	0.51889
R-TB		0.9559	0.0059	0.0401	0.0040	0.58555
R-BC		0.9566	0.0066	0.0395	0.0039	0.58843
BOOTSTRAP-TB		0.9456	0.0044	0.0107	0.0437	0.69515

Table H.54: 95% CI under $\Delta(0.2, -0.075, 0.15)$ with 100% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9397	0.0103	0.0295	0.0308	0.76859
MVUE2		0.9486	0.0014	0.0280	0.0234	0.81176
MLE-TB		0.9080	0.0420	0.0126	0.0794	0.83972
MLE-BC		0.9100	0.0400	0.0123	0.0777	0.84404
SLR		0.9323	0.0177	0.0547	0.0130	0.93235
ASLR		0.9345	0.0155	0.0528	0.0127	0.95332
BOOTSTRAP-BC		0.9040	0.0460	0.0807	0.0153	0.91790
R-MVUE1		0.9751	0.0251	0.0079	0.0170	1.00395
R-TB		0.9156	0.0344	0.0021	0.0823	1.15703
R-BC		0.9176	0.0324	0.0021	0.0803	1.16637
BOOTSTRAP-TB		0.9065	0.0435	0.0791	0.0144	0.92353
MVUE1	25	0.9284	0.0216	0.0186	0.0530	0.60554
MVUE2		0.9375	0.0125	0.0179	0.0446	0.62566
MLE-TB		0.8939	0.0561	0.0081	0.0980	0.63760
MLE-BC		0.8948	0.0552	0.0081	0.0971	0.63941
SLR		0.9041	0.0459	0.0871	0.0088	0.67052
ASLR		0.9048	0.0452	0.0870	0.0082	0.67452
BOOTSTRAP-BC		0.8886	0.0614	0.1022	0.0092	0.66641
R-MVUE1		0.9485	0.0015	0.0021	0.0494	0.82004
R-TB		0.8670	0.0830	0.0004	0.1326	0.89755
R-BC		0.8691	0.0809	0.0004	0.1305	0.90184
BOOTSTRAP-TB		0.8897	0.0603	0.1011	0.0092	0.66797
MVUE1	50	0.9091	0.0409	0.0056	0.0853	0.43299
MVUE2		0.9157	0.0343	0.0054	0.0789	0.44024
MLE-TB		0.8588	0.0912	0.0023	0.1389	0.44427
MLE-BC		0.8594	0.0906	0.0023	0.1383	0.44487
SLR		0.8649	0.0851	0.1325	0.0026	0.45478
ASLR		0.8700	0.0800	0.1279	0.0021	0.45995
BOOTSTRAP-BC		0.8528	0.0972	0.1450	0.0022	0.45292
R-MVUE1		0.8552	0.0948	0.0001	0.1447	0.60200
R-TB		0.7274	0.2226	0.0000	0.2726	0.63125
R-BC		0.7295	0.2205	0.0000	0.2705	0.63275
BOOTSTRAP-TB		0.8534	0.0966	0.1444	0.0022	0.45348

Table H.55: 95% CI under $\Delta(0.4, -0.075, 0.15)$ with 100% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9305	0.0195	0.0445	0.0250	0.76249
MVUE2		0.9395	0.0105	0.0405	0.0200	0.81117
MLE-TB		0.9131	0.0369	0.0137	0.0732	0.86199
MLE-BC		0.9149	0.0351	0.0126	0.0725	0.86944
SLR		0.9219	0.0281	0.0644	0.0137	0.96298
ASLR		0.9225	0.0275	0.0639	0.0136	0.96884
BOOTSTRAP-BC		0.9135	0.0365	0.0658	0.0207	0.93022
R-MVUE1		0.9606	0.0106	0.0244	0.015	0.93119
R-TB		0.9165	0.0335	0.0070	0.0765	1.11570
R-BC		0.9194	0.0306	0.0058	0.0748	1.12827
BOOTSTRAP-TB		0.9227	0.0273	0.0645	0.0128	0.96958
MVUE1	25	0.9365	0.0135	0.0278	0.0357	0.60172
MVUE2		0.9436	0.0064	0.0264	0.0300	0.62432
MLE-TB		0.9041	0.0459	0.0094	0.0865	0.64680
MLE-BC		0.9050	0.0450	0.0094	0.0856	0.64880
SLR		0.9163	0.0337	0.0711	0.0126	0.67080
ASLR		0.9165	0.0335	0.071	0.0125	0.67932
BOOTSTRAP-BC		0.9086	0.0414	0.0791	0.0123	0.66706
R-MVUE1		0.9616	0.0116	0.0077	0.0307	0.76686
R-TB		0.8912	0.0588	0.0014	0.1074	0.86341
R-BC		0.8929	0.0571	0.0014	0.1057	0.86810
BOOTSTRAP-TB		0.9100	0.0400	0.0785	0.0115	0.66983
MVUE1	50	0.9356	0.0144	0.0128	0.0516	0.42956
MVUE2		0.9403	0.0097	0.0123	0.0474	0.43764
MLE-TB		0.8894	0.0606	0.0060	0.1046	0.44517
MLE-BC		0.8897	0.0603	0.0060	0.1043	0.44578
SLR		0.9033	0.0467	0.0901	0.0066	0.45139
ASLR		0.9045	0.0455	0.0893	0.0062	0.47121
BOOTSTRAP-BC		0.8945	0.0555	0.0990	0.0065	0.45070
R-MVUE1		0.9228	0.0272	0.0019	0.0753	0.56539
R-TB		0.8209	0.1291	0.0006	0.1785	0.60205
R-BC		0.8220	0.1280	0.0006	0.1774	0.60366
BOOTSTRAP-TB		0.8952	0.0548	0.0984	0.0064	0.45127

Table H.56: 95% CI under $\Delta(0.2, -0.25, 0.50)$ with 100% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9231	0.0269	0.0716	0.0053	0.94617
MVUE2		0.9350	0.0150	0.0621	0.0029	1.07941
MLE-TB		0.9417	0.0083	0.0295	0.0288	1.14873
MLE-BC		0.9432	0.0068	0.0292	0.0276	1.16116
SLR		0.9459	0.0041	0.0298	0.0243	1.40200
ASLR		0.9463	0.0037	0.0297	0.0240	1.43886
BOOTSTRAP-BC		0.9281	0.0219	0.0459	0.0260	1.45069
R-MVUE1		0.9582	0.0082	0.0386	0.0032	1.05313
R-TB		0.9606	0.0106	0.0087	0.0307	1.29568
R-BC		0.9627	0.0127	0.0084	0.0289	1.31050
BOOTSTRAP-TB		0.9308	0.0192	0.0441	0.0251	1.46609
MVUE1	25	0.9421	0.0079	0.0490	0.0089	0.75291
MVUE2		0.9516	0.0016	0.0432	0.0052	0.81497
MLE-TB		0.9401	0.0099	0.0226	0.0373	0.84282
MLE-BC		0.9414	0.0086	0.0221	0.0365	0.84768
SLR		0.9396	0.0104	0.0428	0.0176	0.93022
ASLR		0.9401	0.0099	0.0417	0.0182	0.95287
BOOTSTRAP-BC		0.9279	0.0221	0.0539	0.0182	0.94554
R-MVUE1		0.9753	0.0253	0.0164	0.0083	0.86898
R-TB		0.9490	0.0010	0.0048	0.0462	0.99179
R-BC		0.9505	0.0005	0.0045	0.0450	0.99837
BOOTSTRAP-TB		0.9290	0.0210	0.0530	0.0180	0.94999
MVUE1	50	0.9528	0.0028	0.0275	0.0197	0.54641
MVUE2		0.9585	0.0085	0.0259	0.0156	0.56906
MLE-TB		0.9334	0.0166	0.0152	0.0514	0.57824
MLE-BC		0.9341	0.0159	0.0150	0.0509	0.58816
SLR		0.9271	0.0229	0.0606	0.0123	0.60490
ASLR		0.9292	0.0208	0.0505	0.02033	0.62128
BOOTSTRAP-BC		0.9192	0.0308	0.0690	0.0118	0.60919
R-MVUE1		0.9677	0.0177	0.0044	0.0279	0.64700
R-TB		0.9154	0.0346	0.0015	0.0831	0.69396
R-BC		0.9167	0.0333	0.0015	0.0818	0.69623
BOOTSTRAP-TB		0.9196	0.0304	0.0686	0.0118	0.61069

Table H.57: 95% CI under Δ(0.4, -0.25, 0.50) with 100% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.9048	0.0452	0.0889	0.0063	0.88113
MVUE2		0.9200	0.0300	0.0765	0.0035	1.03450
MLE-TB		0.9294	0.0206	0.0342	0.0364	1.14910
MLE-BC		0.9319	0.0181	0.0330	0.0351	1.16882
SLR		0.9326	0.0174	0.0399	0.0275	1.52750
ASLR		0.9355	0.0145	0.0372	0.0273	1.63445
BOOTSTRAP-BC		0.9183	0.0317	0.0471	0.0346	1.66961
R-MVUE1		0.9312	0.0188	0.0648	0.0040	0.94515
R-TB		0.9450	0.0050	0.0179	0.0371	1.22812
R-BC		0.9474	0.0026	0.0169	0.0357	1.24735
BOOTSTRAP-TB		0.9278	0.0222	0.0459	0.0263	2.03697
MVUE1	25	0.9313	0.0187	0.0616	0.0071	0.70556
MVUE2		0.9439	0.0061	0.0526	0.0035	0.77772
MLE-TB		0.9393	0.0107	0.0213	0.0394	0.82306
MLE-BC		0.9416	0.0084	0.0208	0.0376	0.82897
SLR		0.9391	0.0109	0.0419	0.0190	0.92530
ASLR		0.9395	0.0105	0.0417	0.0188	0.93620
BOOTSTRAP-BC		0.9271	0.0229	0.0534	0.0195	0.93595
R-MVUE1		0.9632	0.0132	0.0302	0.0066	0.78754
R-TB		0.9457	0.0043	0.0067	0.0476	0.93554
R-BC		0.9476	0.0024	0.0064	0.0460	0.94305
BOOTSTRAP-TB		0.9287	0.0213	0.0524	0.0189	0.94159
MVUE1	50	0.9538	0.0038	0.0331	0.0131	0.51340
MVUE2		0.9614	0.0114	0.0294	0.0092	0.53944
MLE-TB		0.9430	0.0070	0.0130	0.0440	0.55400
MLE-BC		0.9433	0.0067	0.0130	0.0437	0.55575
SLR		0.9407	0.0093	0.0468	0.0125	0.58124
ASLR		0.9409	0.0091	0.0466	0.0125	0.58901
BOOTSTRAP-BC		0.9309	0.0191	0.0568	0.0123	0.58410
R-MVUE1		0.9736	0.0236	0.0096	0.0168	0.59352
R-TB		0.9312	0.0188	0.0024	0.0664	0.65091
R-BC		0.9335	0.0165	0.0023	0.0642	0.65343
BOOTSTRAP-TB		0.9315	0.0185	0.0562	0.0123	0.58573

Table H.58: 95% CI under Δ(0.2, -0.50, 1.0) with 100% Contamination from Birnbaum-Saunders Distribution

Method	Sample Size	CP	CE	LER	UER	Width
MVUE1	15	0.7839	0.1661	0.2158	0.0003	0.97083
MVUE2		0.8253	0.1247	0.1746	0.0001	1.23380
MLE-TB		0.9082	0.0418	0.0897	0.0021	1.38661
MLE-BC		0.9103	0.0397	0.0878	0.0019	1.41255
SLR		0.9390	0.0110	0.0051	0.0559	1.95694
ASLR		0.9395	0.0105	0.0050	0.0555	2.01329
BOOTSTRAP-BC		0.9378	0.0122	0.0087	0.0535	2.23974
R-MVUE1		0.7831	0.1669	0.2166	0.0003	0.88042
R-TB		0.9285	0.0215	0.0695	0.0020	1.16286
R-BC		0.9319	0.0181	0.0664	0.0017	1.18009
BOOTSTRAP-TB		0.9404	0.0096	0.0076	0.0520	2.29072
MVUE1	25	0.8082	0.1418	0.1918	0.0000	0.78815
MVUE2		0.8350	0.1150	0.1650	0.0000	0.91176
MLE-TB		0.9087	0.0413	0.0894	0.0019	0.96923
MLE-BC		0.9102	0.0398	0.088	0.0018	0.97835
SLR		0.9365	0.0135	0.0041	0.0594	1.14340
ASLR		0.9364	0.0136	0.0041	0.0595	1.15772
BOOTSTRAP-BC		0.9379	0.0121	0.0067	0.0554	1.20359
R-MVUE1		0.8082	0.1418	0.1918	0.0000	0.73546
R-TB		0.9334	0.0166	0.0648	0.0018	0.87940
R-BC		0.9353	0.0147	0.0632	0.0015	0.88684
BOOTSTRAP-TB		0.9387	0.0113	0.0065	0.0548	1.21321
MVUE1	50	0.8030	0.1470	0.1970	0.0000	0.57890
MVUE2		0.8243	0.1257	0.1757	0.0000	0.62400
MLE-TB		0.8936	0.0564	0.1048	0.0016	0.64207
MLE-BC		0.8956	0.0544	0.1029	0.0015	0.64483
SLR		0.9236	0.0264	0.0026	0.0738	0.69125
ASLR		0.9243	0.0257	0.0025	0.0732	0.70333
BOOTSTRAP-BC		0.9283	0.0217	0.0034	0.0683	0.70726
R-MVUE1		0.8124	0.1376	0.1876	0.0000	0.55167
R-TB		0.9156	0.0344	0.0835	0.0009	0.60664
R-BC		0.9167	0.0333	0.0824	0.0009	0.60911
BOOTSTRAP-TB		0.9287	0.0213	0.0034	0.0679	0.71004

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