Western Michigan University
ScholarWorks at WMU

Honors Theses

Lee Honors College

12-9-2005

Change Bell Ringing and Mathematics: A High School Student Excursion into Graph Theory and Group Theory

Kristi Carlson
Western Michigan University, sissyfromcab@hotmail.com

Follow this and additional works at: https://scholarworks.wmich.edu/honors_theses

Part of the Curriculum and Instruction Commons, Mathematics Commons, and the Science and Mathematics Education Commons

Recommended Citation

Carlson, Kristi, "Change Bell Ringing and Mathematics: A High School Student Excursion into Graph Theory and Group Theory" (2005). *Honors Theses*. 660.
https://scholarworks.wmich.edu/honors_theses/660

This Honors Thesis-Open Access is brought to you for free and open access by the Lee Honors College at ScholarWorks at WMU. It has been accepted for inclusion in Honors Theses by an authorized administrator of ScholarWorks at WMU. For more information, please contact wmu-scholarworks@wmich.edu.

WESTERN MICHIGAN UNIVERSITY

The Carl and Winifred Lee Honors College

THE CARL AND WINIFRED LEE HONORS COLLEGE

CERTIFICATE OF ORAL EXAMINATION

Kristi Carlson, having been admitted to the Carl and Winifred Lee Honors College in Fall 2002 successfully presented the Lee Honors College Thesis on December 9, 2005.

The title of the paper is:

"Change Bell Ringing and Mathematics: A High School Student Excursion into Graph Theory and Group Theory"

Dr. Arthur White, Mathematics

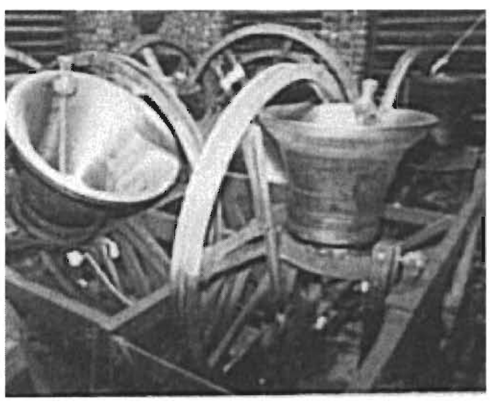
Dr. Tabitha Mingus, Mathematics

Dr. Allison Kelaher-Young, Teaching, Learning, and Leadership

1903 W. Michigan Ave.
PHONE: (269) 387-3230 FAX: (269) 387-3903

Change Bell Ringing And Mathematics:

A High School Student Excursion into Graph Theory and Group Theory



Kristi Carlson
Honors Thesis

Monday	Tuesday	Wednesday	Thursday	Friday
1. Intro to Change Ringing	2. Methods in Change Ringing	3. Principles in Change Ringing	4. Calls in Change Ringing	5. *The Nine Tailors*
6. Intro to Graphs	7. Graphs	8. Hamiltonian Graphs and Terminology	9. Graphs from Change Ringing	10. Extents from Change Ringing
11. Intro to Groups	12. Permutation Groups	13. Permutation Groups in Change Ringing	14. Even and Odd Permutations	15. Extents from Groups
16. Change Ringing in ABEL	17. How Many Extents?	18.Extents of Minor	19. Extents of Minor	

Introduction

This unit was created as a way to introduce higher level mathematics concepts to advanced high school students. If the concepts of graph theory and group theory were presented traditionally to students in high school, most would be overwhelmed, but by presenting them in the context of change bell ringing, students will be able to work with the concepts and hopefully understand them. All five of the National Council of Teachers of Mathematics Process Standards are found in this unit, though none of the Content Standards are explicitly addressed, because the content of this unit is beyond what would be studied in the traditional high school curriculum.

The first process standard found is problem solving. Throughout the unit, students will be introduced to a problem and then by solving it, will learn new mathematical content. For instance, in their introduction to graph theory, students are given the Königsberg Bridge Problem to solve. While students work within the context of the problem, they explore the problem that led to the creation of graph theory. As students try to find a solution, they find that there is no solution possible, and they must explain their reasoning that there is no solution possible. This leads right into the next process standard, which is reasoning & proof. Throughout the entire unit, students will be expected to explain their reasoning for their answers. It may be something as simple as citing a definition to say that a particular example is not a graph in the Graphs lesson or may be more complex, like explaining why there are only 4 extents of Plain Bob Doubles and what restricts the number of different extents that are possible.

Communication will be an essential process standard in this unit. For most of the unit, students will be working within small groups. Within the small groups students will need to explain their reasoning for their answers to one another so that each can understand what the others are thinking. Also, once students learn the correct mathematical terminology, they will be expected to use it in their discussions. Representations will be another process standard that is used extensively in this unit. Students will work out an extent of Plain Bob Doubles by hand in one of the early lessons and quickly find that this is bulky and not very fun to do. Once they are introduced to the concepts in graph theory and group theory, students will be able to use representations present in graph theory and group theory to show the mathematical ideas present in the structure of change ringing; students will find these representations much easier to work with than the straight change ringing context.

The final process standard that is found is connections. This is found in many areas throughout this unit. The students will have to make connections between how they can use graph theory and group theory to both represent what they see happening in change ringing. They will also have to see the connections between change bell ringing and the mathematics present in both graph theory and group theory.

I would like to thank my thesis committee:

Dr. Arthur White

Dr. Tabitha Mingus

Dr. Alison Young

And my friends and family for all of their support throughout this process.

(And my computer for not deleting everything I had in the middle of the semester, like it threatened to do!)

Lesson Title: Introduction to Change Ringing
Lesson length: 1 hour

Goal: Students will understand the basics of what change ringing is.
Materials: Computers, handouts with websites, projector, handbells, signs numbered 1-8

Terms to know: change ringing, change, row, tower, bell, handbell, extent, cross, stand, rounds

Lesson:

- Short explanation of change ringing
  - Bells are hung on a full wheel, takes about 2 seconds to ring bell once around before it can ring again
  - Because of this, can't ring tunes, so ring changes instead
    - Changes are different permutations
  - Rules of changes
    - The first and last changes are rounds
    - No changes can be repeated
    - No bell can change more than one position between changes
    - *No bell stays in the same position for more than 2 changes*
    - *The working bells do the same work*
    - *Each lead is palindromic*
  - Explain # of possible changes on different numbers of bells (n!)
    - 3 – 6; 4 – 24; 5 – 120; 6 – 720; 7 – 5,040; 8 – 40,320; 9 – 362,880; 10 – 3,628,800; 11 – 39,916,800; 12 – 479,001,600
  - Try to ring quarters (≥1250 changes) or peals (≥5000 changes)
- Split students into groups of 3-4 and give 1 handout per group
- Have students look at websites to find out more information about change ringing, encourage to not just look at the first few sites
- Have students ring on bodies
  - Give 8 students numbered sign and appropriate handbell
  - Have students cross in pairs (rule #1) – What will happen if cross again?
  - Have first/last stand and others cross (rule #2)
  - Continue alternating rules until back to rounds
- If time permits, show video of ringing on a projector

Websites to look at:
http://www.nagcr.org/ - Homepage of the North American Guild of Change Ringers

http://www.cccbr.org.uk/ - Homepage of the Central Council of Change Bell Ringers

http://web.mit.edu/bellringers/www/ - Homepage for MIT's Guild of Bellringers

http://www.kzoo.edu/ringers/index.htm - Homepage of the Kalamazoo College Guild of Change Ringers

http://www.ringingworld.co.uk/ - Homepage of *The Ringing World*

http://www.whitechapelbellfoundry.co.uk/ - Homepage of Whitechapel Bell Foundry

http://www.campanophile.co.uk/ - Homepage of Campanophile

http://www.cs.ubc.ca/spider/kvdoel/bells/bells.html - The Bells Applet: Change Ringing on the Web

http://www.changeringers.karoo.net/ - Homepage of Change Ringers Web Dictionary

http://www.ringing.org/main/pages/intro - Homepage of ringing.org

1

2

3

4

5

6

7

8

Lesson Title: Methods in Change Ringing
Lesson length: 1 hour

Goal: Students will understand how methods are constructed and construct their own method.
Materials: Overhead with all minimus methods and of treble hunt/dodge, graph paper to write out methods/plain hunt

Terms to know: Method, rounds, cross(x), stand (12)

Lesson:

- Review rules for methods
  - The first and last changes are rounds
  - No changes can be repeated
  - No bell can change more than one position between changes
  - *No bell stays in the same position for more than 2 changes*
  - *The working bells do the same work*
  - *Each lead is palindromic*
- Methods are treble dominated
  - Treble can hunt (1, 2, 3, 4, 5, 6, 6, 5, 4, 3, 2, 1)
  - Treble can treble dodge (1, 2, 1, 2, 3, 4, 3, 4, 5, 6, 5, 6, 6, 5, 6, 5, 4, 3, 4, 3, 2, 1, 2, 1)
- Examples of changes between rows – allowed or not?
  - 123456 -> 124356
  - 342165 -> 342561
  - 542316 -> 453216
  - 135246 -> 142536
- Remember the hunting we did on handbells yesterday? That was plain hunting.
  - Rule # 1: Every pair crosses.
  - Rule # 2: First and last bells stand and others cross.
  - Start with rule # 1 and alternate until back to rounds.
  - Write out plain hunt on 6.
- Split students into groups of 3-4 and have groups try to come up with a method (minimus) like plain hunt but that contains all 24 changes (most likely Plain Bob).
  - Compare different methods found and tell method names.
- If extra time, compare methods with one another
  - What is similar between multiple/all methods? What is different?

Treble Hunt

1-----
-1----
--1---
---1--
----1-
-----1
-----1
----1-
---1--
--1---
-1----
1-----

Treble Dodge

1-----
-1----
1-----
-1----
--1---
---1--
--1---
---1--
----1-
-----1
----1-
-----1
-----1
----1-
-----1
----1-
---1--
--1---
---1--
--1---
-1----
1-----
-1----
1-----

Minimus Methods

Plain Bob	Reverse Bob	Double Bob	Canterbury	Reverse Canterbury	Double Canterbury
1234	1234	1234	1234	1234	1234
2143	2143	2143	2143	2134	2134
2413	2413	2413	2413	2314	2314
4231	4231	4231	2431	3241	2341
4321	2431	2431	4231	3421	3241
3412	4213	4213	4213	4312	3214
3142	4123	4123	4123	4132	3124
1324	1432	1432	1432	1432	1324
1342	1342	1423	1342	1423	1342
3124	3124	4132	3124	4123	3142
3214	3214	4312	3214	4213	3412
2341	2341	3421	3241	2431	3421
2431	3241	4321	2341	2341	4321
4213	2314	3412	2314	3214	4312
4123	2134	3142	2134	3124	4132
1432	1243	1324	1243	1324	1432
1423	1423	1342	1423	1342	1423
4132	4132	3124	4132	3142	4123
4312	4312	3214	4312	3412	4213
3421	3421	2341	4321	4321	4231
3241	4321	3241	4231	4231	2431
2314	3412	2314	2413	2413	2413
2134	3142	2134	2143	2143	2143
1243	1324	1243	1243	1243	1243
1234	1234	1234	1234	1234	1234

Single Court	Reverse Court	Double Court	St. Nicholas	Reverse St. Nicholas
1234	1234	1234	1234	1234
2134	2143	2134	2134	2143
2314	2413	2314	2314	2413
3241	2431	2341	3241	2431
3421	2341	2431	2341	4231
4312	2314	2413	3214	4213
4132	2134	2143	3124	4123
1432	1243	1243	1324	1432
1342	1423	1423	1342	1423
3142	4132	4123	3142	4132
3412	4312	4213	3412	4312
4321	4321	4231	4321	4321
4231	4231	4321	3421	3421
2413	4213	4312	4312	3412
2143	4123	4132	4132	3142
1243	1432	1432	1432	1324
1423	1342	1342	1423	1342
4123	3124	3142	4123	3124
4213	3214	3412	4213	3214
2431	3241	3421	2431	3241
2341	3421	3241	4231	2341
3214	3412	3214	2413	2314
3124	3142	3124	2143	2134
1324	1324	1324	1243	1243
1234	1234	1234	1234	1234

Lesson Title: Principles in Change Ringing
Lesson length: 1 hour

Goal: Students will understand the difference between methods and principles and write out a plain course of Stedman Doubles.
Materials: Overhead of Stedman Doubles, graph paper to write out plain course

Terms to know: Principle

Lesson:

- Explain difference between method and principle
  - Methods – Treble hunts or treble dodges, other bells work alike
  - Principle – All bells work alike, including treble
- Explain rule for Stedman Doubles
  - 3.1.5.3.1.3.1.3.5.1.3.1.
- Have students split into groups of 3-4 and write out the plain course of Stedman Doubles
  - Notice how many rows it has (60), is there a way to get the other 60?
  - That's where calls come in
- Have students compare what happens in principle vs. method
  - All bells work
  - Not symmetric
  - Same number of leads as number of bells

Stedman Doubles

12345	53412	24153	31524	45231
21354	35421	42135	13542	54213
23145	34512	41253	15324	52431
32415	43152	14523	51234	25341
23451	34125	41532	15243	52314
24315	31452	45123	12534	53241
42351	13425	54132	21543	35214
43215	14352	51423	25134	32541
34251	41325	15432	52143	23514
43521	14235	51342	25413	32154
45312	12453	53124	24531	31245
54321	21435	35142	42513	13254
53412	24153	31524	45231	12345

Lesson Title: Calls in Change Ringing
Lesson length: 1 hour

Goal: Students will understand what calls do and be able to use them to write out an extent of Plain Bob Doubles.
Materials: Overhead with Plain Bob Doubles calls written out, overhead of extent of Plain Bob Doubles, graph paper to write out extent

Terms to know: Bob, Single, Extent

Lesson:

- Well, we've written out methods and principles, but what if we want to get all the possible changes?
- We have calls that change the order of bells (at lead ends usually)
  - Bobs change the order of 3 bells in a 3-cycle
  - Singles change the order of a pair of bells
- Show overhead with calls and explain what happens
- Split students into groups of 3-4 and have try to write out an extent of Plain Bob Doubles
- If none of the groups are successful, show an extent on overhead

Plain Bob Doubles **Bob**

3514
315 4
13 54
1 354
1534
5143

If about to make 2^{nd}'s, run out.
If about to dodge 3-4 down, run in.
If about to dodge 3-4 up, make the bob.
If making long 5^{th}'s, stay unaffected.

Plain Bob Doubles **Single**

35142
31524
13254
13245
31425
34152

If about to make 2^{nd}'s, stay unaffected.
If about to dodge 3-4 down, make 3^{rd}'s.
If about to dodge 3-4 up, run out.
If about to make long 5^{th}'s, run in.

Extent of Plain Bob Doubles

12345	15234	14352 -	15423
21435	51324	41532	51243
24153	53142	45123	52134
42513	35412	54213	25314
45231	34521	52431	23541
54321	43251	25341	32451
53412	42315	23514	34215
35142	24135	32154	43125
31524	21453	31245	41352
13254	12543	13425	14532
13524	12453	13245	15432 -
31254	21543	31425	51342
32145	25134	34152	53124
23415	52314	43512	35214
24351	53241	45321	32541
42531	35421	54231	23451
45213	34512	52413	24315
54123	43152	25143	42135
51432	41325	21534	41253
15342	14235	12354	14523
13542 -	14325	12534	14253
31452	41235	21354	41523
34125	42153	23145	45132
43215	24513	32415	54312
42351	25431	34251	53421
24531	52341	43521	35241
25413	53214	45312	32514
52143	35124	54132	23154
51234	31542	51423	21345
15324	13452	15243	12435
15234	14352 -	15423	12345

Lesson Title: *The Nine Tailors*
Lesson length: 1 hour

Goal: Students will integrate the mathematical concept of change ringing with literature.
Materials: Handouts with quote, paper for student journals

Terms to know:

Lesson:

- This is a paragraph from the novel *The Nine Tailors* by Dorothy Sayers.
    - The novel is a mystery about change ringing and a mysterious death.
    - Let's read this popcorn style.
    - "The art of change-ringing is peculiar to the English, and, like most English peculiarities, unintelligible to the rest of the world. To the musical Belgian, for example, it appears that the proper thing to do with a carefully-tuned ring of bells is to play a tune upon it. By the English campanologist, the playing of tunes is considered to be a childish game, only fit for foreigners; the proper use of bells is to work out mathematical permutations and combinations. When he speaks of the music of his bells, he does not mean musicians music—still less what the ordinary man calls music. To the ordinary man, in fact, the pealing of bells is monotonous jangle and a nuisance, tolerable only when mitigated by a remote distance and sentimental association. The change-ringer does, indeed, distinguish musical differences between one method of producing his permutations and another; he avers, for instance, that where the hinder bells run 7,5,6 or 5,6,7, or 5,7,6, the music is always prettier, and can detect or approve, where they occur, the consecutive fifths of Titums and the cascading thirds of the Queen's change. But what he really means is, that by the English method ringing with rope and wheel, each several bell gives forth her fullest and noblest note. His passion—and it is a passion—finds its satisfaction in mathematical completeness and mechanical perfection, and as his bell weaves her way rhythmically up from lead to hinder place and down again, he is filled with solemn intoxication that comes of intricate ritual faultlessly performed. To any disinterested spectator, peeping in upon the rehearsal, there might have been something a little absurd about the eight absorbed faces; the eight tense bodies poised in a spell-bound circle on the edges of eight dining room chairs; the eight upraised right hands, decorously wagging the handbells upward and downward; but to the performers, everything was serious and important as an afternoon with the Australians at the Lord's." (22)
    - What do you think of this passage? What stands out to you? What part do you like best? Based on what you know about change ringing right now, what do you think of the description?
- Write a journal entry for 10 minutes.
- In large group, talk about what wrote in journal entries.

From *The Nine Tailors* by Dorothy Sayers

"The art of change-ringing is peculiar to the English, and, like most English peculiarities, unintelligible to the rest of the world. To the musical Belgian, for example, it appears that the proper thing to do with a carefully-tuned ring of bells is to play a tune upon it. By the English campanologist, the playing of tunes is considered to be a childish game, only fit for foreigners; the proper use of bells is to work out mathematical permutations and combinations. When he speaks of the music of his bells, he does not mean musicians music—still less what the ordinary man calls music. To the ordinary man, in fact, the pealing of bells is monotonous jangle and a nuisance, tolerable only when mitigated by a remote distance and sentimental association. The change-ringer does, indeed, distinguish musical differences between one method of producing his permutations and another; he avers, for instance, that where the hinder bells run 7,5,6 or 5,6,7, or 5,7,6, the music is always prettier, and can detect or approve, where they occur, the consecutive fifths of Titums and the cascading thirds of the Queen's change. But what he really means is, that by the English method ringing with rope and wheel, each several bell gives forth her fullest and noblest note. His passion—and it is a passion—finds its satisfaction in mathematical completeness and mechanical perfection, and as his bell weaves her way rhythmically up from lead to hinder place and down again, he is filled with solemn intoxication that comes of intricate ritual faultlessly performed. To any disinterested spectator, peeping in upon the rehearsal, there might have been something a little absurd about the eight absorbed faces; the eight tense bodies poised in a spell-bound circle on the edges of eight dining room chairs; the eight upraised right hands, decorously wagging the handbells upward and downward; but to the performers, everything was serious and important as an afternoon with the Australians at the Lord's."

Lesson Title: Intro to Graphs (Königsberg Bridge Problem)
Lesson length: 1 hour

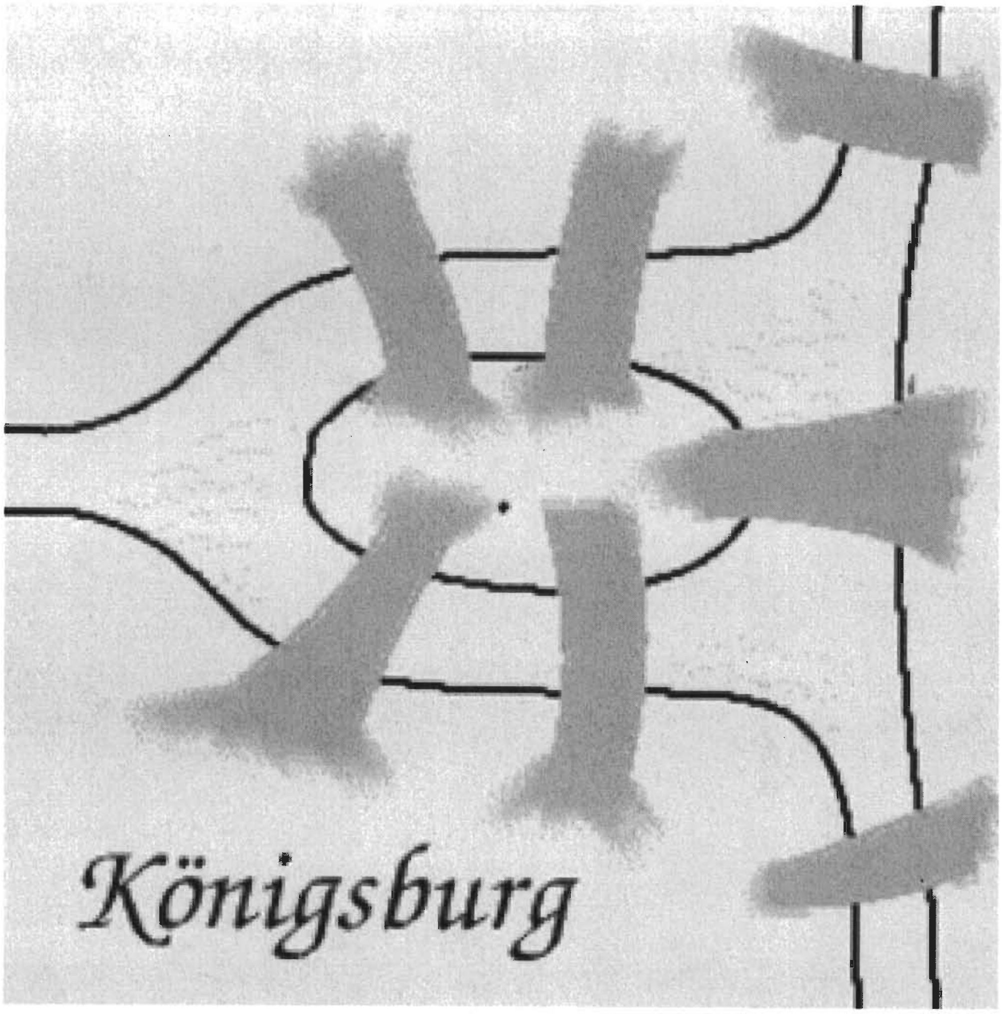
Goal: Students will explore the basic concepts of graph theory/discrete mathematics.
Materials: Worksheet

Terms to know: Graph, vertex, edge, walk, path, trail, circuit, Eulerian circuit, Hamiltonian cycle, degree of a vertex

Lesson:

- Can you see the map on the worksheet I handed you? You can see that there are seven bridges. Well, the people of Königsberg wanted to know if it was possible to walk around the city and cross each bridge exactly once. What do you think?
- Split up into groups of 3-4 people
- Try in groups to find a path
- Back in whole group, ask if anyone found a path
- Discuss in groups if anyone figured out any problems
- Back in whole group, discuss problems
- Look at Euler's solution
- How does it relate to Königsberg Bridge Problem?
- Discuss in groups, then in whole group
- If extra time:
    - Draw another graph on board and ask class to see if it has an Euler path or circuit
- Can anyone tell me why we can't find a way to cross each bridge in Königsberg exactly once?
- In the real world, the people of Königsberg built another bridge so that the city looked like this:
    - Is it possible now to begin and end at the same point and cross each bridge exactly once?
    - What about just crossing each bridge once?

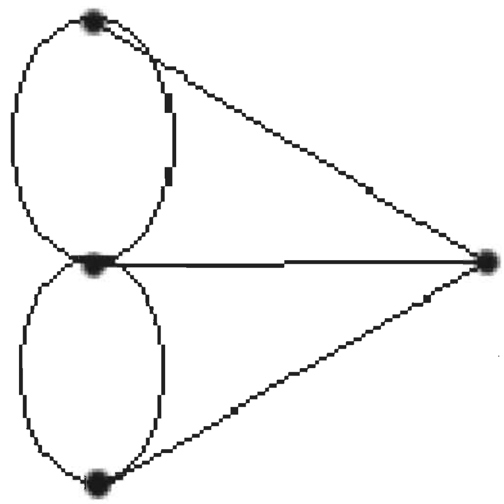
Königsberg Bridge Problem



In Königsberg, Germany, a river ran through the city such that in its center was an island, and after passing the island, the river broke into two parts. Seven bridges were built so that the people of the city could get from one part to another.
The people wondered whether or not one could walk around the city in a way that would involve crossing each bridge exactly once.

Try it – Can you find a way to cross each bridge exactly once and end at the point you began?
What if you relax the conditions and only have to cross each bridge exactly once, but not begin and end at the same point?

Solution



Euler labored and exposed the truth. First he divided the vertices into odd and even based on the parity of the number of paths directly connected to that vertex. He then noted about any graph the following four rules:

1. There will always be an even number of odd vertices.
2. If there are no odd vertices, then there is an Euler circuit starting at every vertex.
3. If there are two odd vertices, then there is no Euler circuit, but at least one Euler path starting at one odd vertex and ending at the other.
4. If there are four or more odd vertices, then there are no Euler paths.

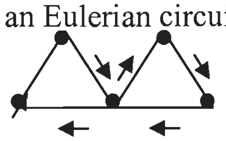
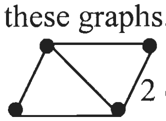
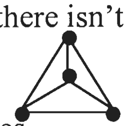
(Note, an Euler circuit starts and ends at the same point, as well as crossing each bridge only once, while an Euler path only needs to cross each bridge exactly once.)

Lesson Title: Graphs
Lesson length: 1 hour

Goal: Students will understand what the basic parts of a graph are.
Materials: worksheet

Terms to know: Graph, vertex, edge, degree of a vertex, walk, path, trail, circuit, Eulerian trail, Eulerian circuit, cycle, Hamiltonian cycle

Lesson:

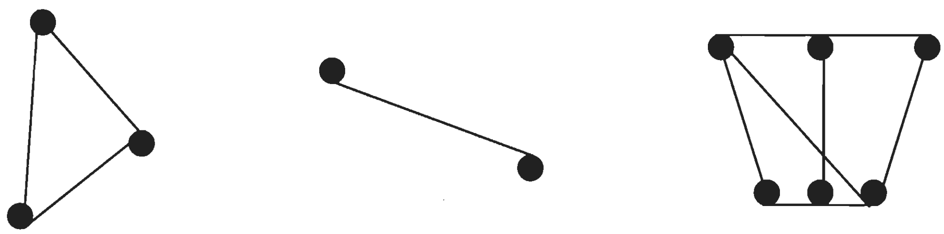
- Let's look at some more examples of graphs and see if we can put names to similar features.
- Definitions
  - Graph: a set of vertices and edges
  - Vertex: the "dot"
  - Edge: the "line(s)" between vertices
  - Degree (of vertex): the number of edges coming out of a vertex
- Split into groups of 3-4 and work on worksheet one problem at a time, then come back together to discuss as a whole class before moving on to the next question.
  - Are these graphs? Why or why not?
    - Yes, yes, no, yes – no vertices on line, others have vertices and/or edges
  - What are the degrees of the vertices in these graphs?
    - Triangle 2,2,2; line 1,1; trapezoid top - 3,3,2 bottom - 2,3,3
  - Find a circuit in these graphs. If there isn't one, explain why.
    - Answers will vary.
  - Find an Eulerian circuit in these graphs. If there isn't one, explain why.
    -   2 odd vertices  all odd vertices
- New Definitions*
  - Walk: an alternating sequence of vertices and edges where each edge joins the two vertices next to it $\{v_0, e_0, v_1, e_1, \ldots\}$
  - Path: a walk that does not repeat any vertices
  - Trail: a walk that does not repeat any edges
  - Circuit: a trail that begins and ends at the same vertex
  - Eulerian trail: a trail that contains all edges
  - Eulerian circuit: a circuit that contains all edges

* Definitions from: Chartrand, Gary. Introductory Graph Theory. Dover Publications, Inc: New York City, NY, 1977.

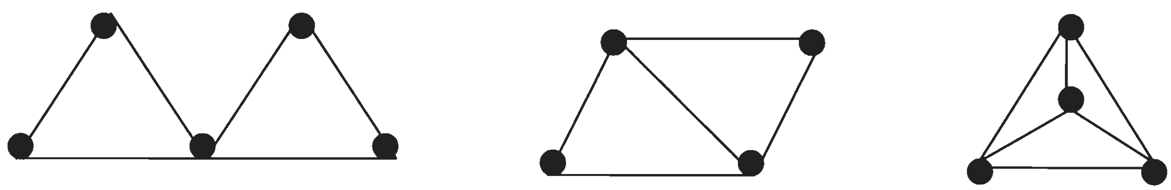
Graphs Handout

Are these graphs? Why or why not?

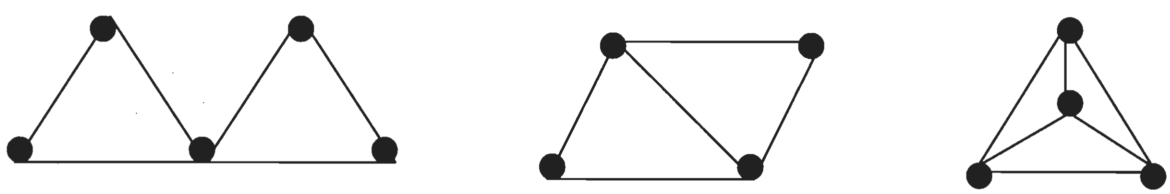
What are the degrees of the vertices in these graphs?



Find a circuit in these graphs. If there isn't one, explain why.



Find an Eulerian circuit in these graphs. If there isn't one, explain why.



Lesson Title: Hamiltonian Graphs and Terminology
Lesson length: 1 hour

Goal: Students will learn the proper terminology for graph theory and understand what a Hamiltonian graph is.
Materials: Handout

Terms to know: Graph, vertex, edge, degree of a vertex, walk, path, trail, circuit, Eulerian trail, Eulerian circuit, cycle, Hamiltonian cycle

Lesson:

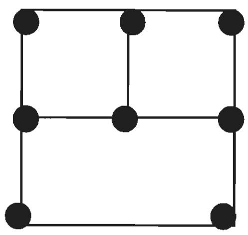
- Review what a graph is
    - Graph: a set of vertices and edges
    - Vertex: the "dot"
    - Edge: the "line(s)" between vertices
    - Degree (of vertex): the number of edges coming out of a vertex
- Split into groups of 3-4
- Introduce Salesman problem
    - Students should find that the first graph is Hamiltonian and the second graph is not.
- Definitions*
    - Walk: an alternating sequence of vertices and edges where each edge joins the two vertices next to it $\{v_0, e_0, v_1, e_1, \ldots\}$
    - Path: a walk that does not repeat any vertices
    - Trail: a walk that does not repeat any edges
    - Circuit: a trail that begins and ends at the same vertex
    - Eulerian trail: a trail that contains all edges
    - Eulerian circuit: a circuit that contains all edges
    - Cycle: a circuit that begins and ends at the same vertex, but does not repeat any other vertices
    - Hamiltonian cycle: a cycle that contains every vertex

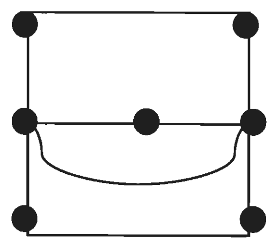
* Definitions from: Chartrand, Gary. <u>Introductory Graph Theory</u>. Dover Publications, Inc: New York City, NY, 1977.

The Salesman Problem

Suppose a salesman's territory includes several cities with highways connecting certain pairs of these cities. His job requires him to visit each city personally. Is it possible for him to schedule a round trip by car enabling him to visit each specified city exactly once?

Try it with these two "maps."





Problem from: Chartrand, Gary. Introductory Graph Theory. Dover Publications, Inc: New York City, NY, 1977.

Lesson Title: Graphs of Change Ringing
Lesson length: 1 hour

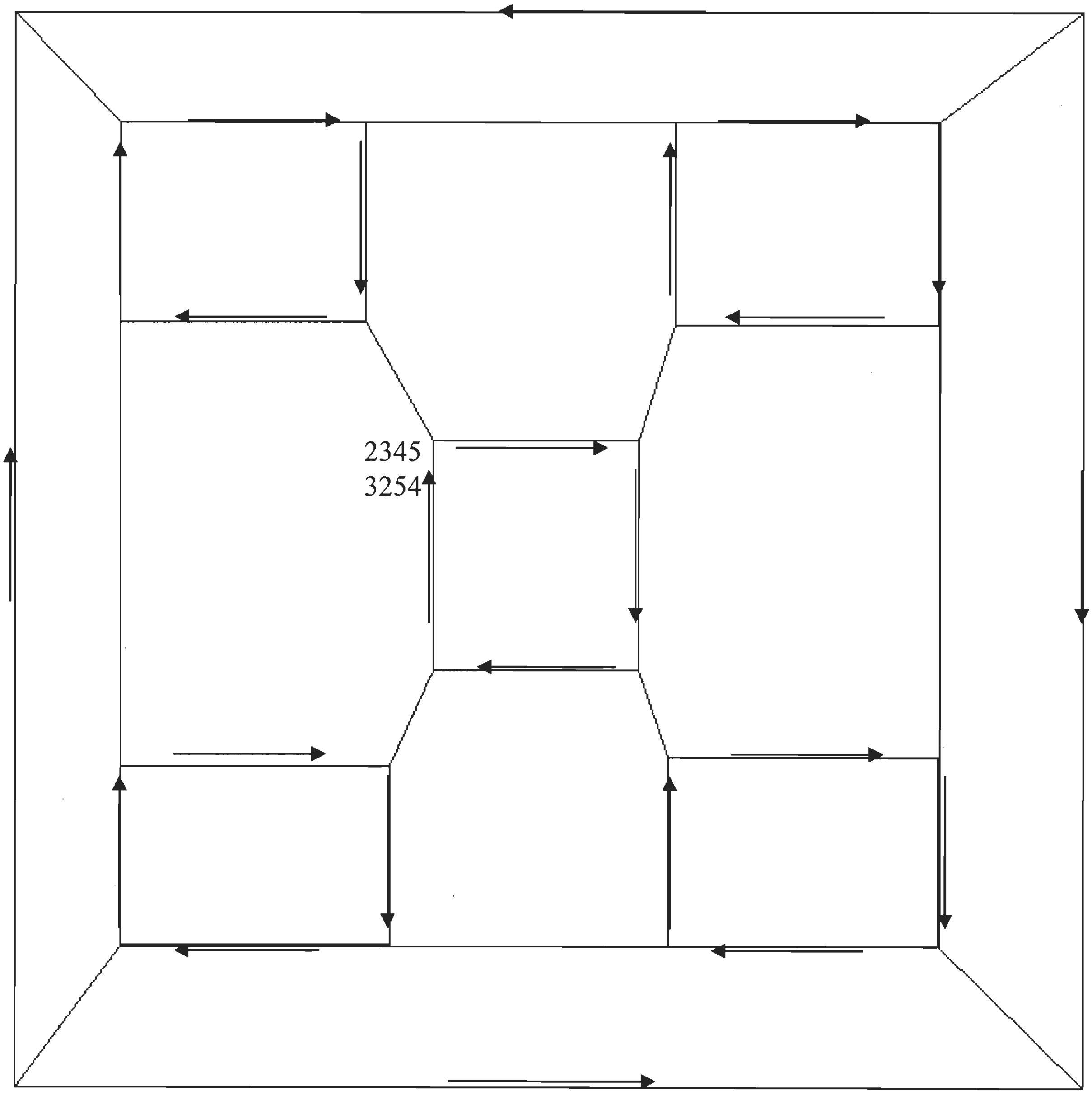
Goal: Students will use a provided graph to graph Plain Bob Doubles.
Materials: Worksheet of graph of Plain Bob Doubles, overhead of finished graph

Terms to know: Vertex, lead end, bob lead, plain lead

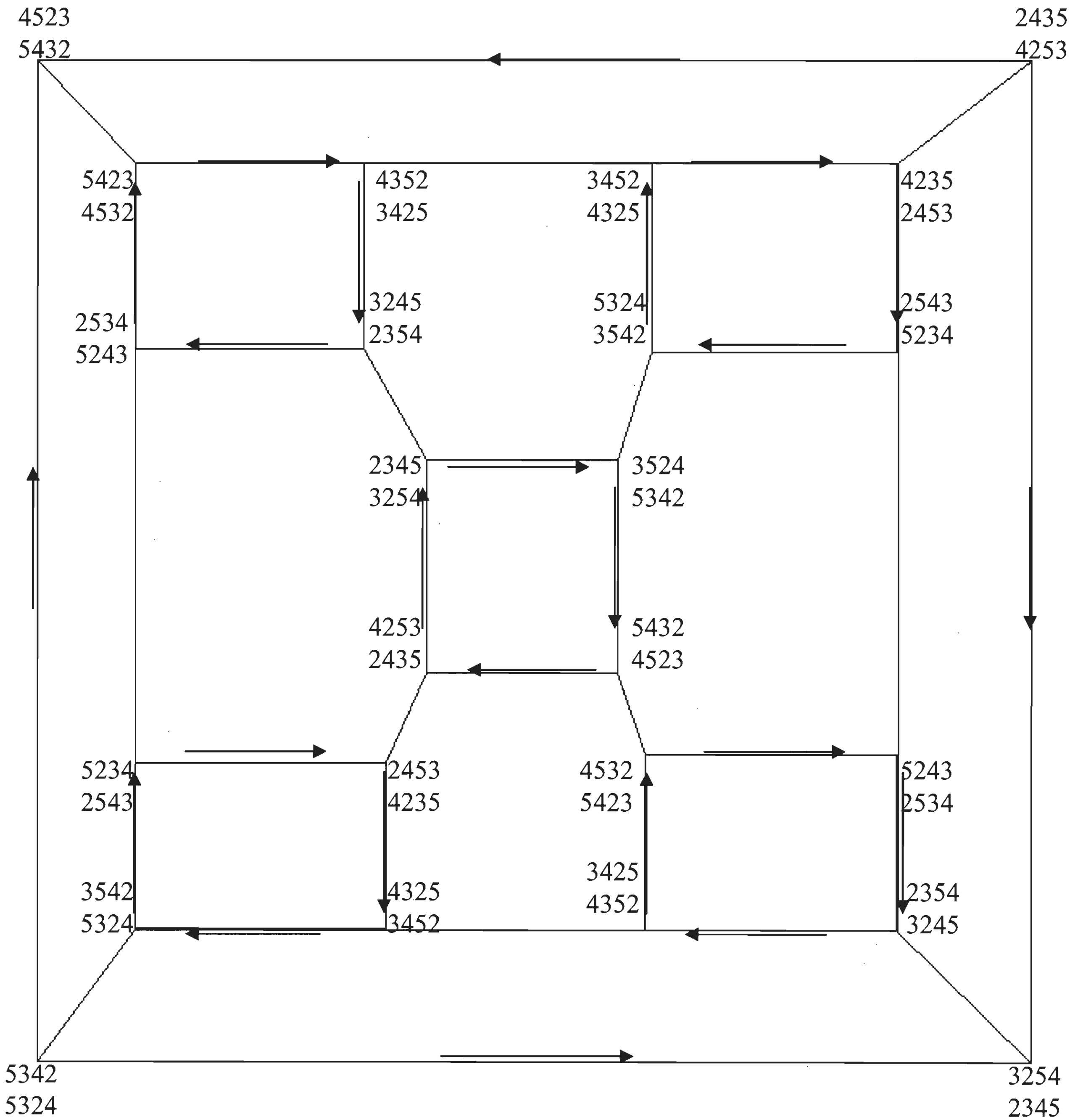
Lesson:

- Show graph of plain hunt on 3
- Explain how to graph a method
    - Start with rounds at indicated vertex
    - Find lead end of plain lead
    - Continue finding resulting plain lead ends until complete cycle
    - Pick another lead end and apply bob to find new lead end
    - Repeat finding plain leads until complete cycle
    - Again pick another lead end and apply bob to find new lead end
    - Repeat until all vertices have lead ends
- Split into groups of 3-4
- Have groups work together to fill out graph

Plain Bob Doubles



Plain Bob Doubles



Lesson Title: Extents from Graphs
Lesson length: 1 hour

Goal: Students will use the graph of Plain Bob Doubles created in a previous lesson to find an extent of Plain Bob Doubles.
Materials: Graph of Plain Bob Doubles from previous lesson

Terms to know: Walk, path, trail, circuit, Eulerian trail, Eulerian circuit, cycle, Hamiltonian cycle, extent, "legally" ringable

Lesson:

- Let's quickly review the rules for things to be "legally" ringable
    - **The first and last changes are rounds**
    - No changes can be repeated
    - No bell can change more than one position between changes
    - *No bell stays in the same position for more than 2 changes*
    - *The working bells do the same work*
    - *Each lead is palindromic*
        - Will any of these be automatically satisfied in our graph? Which do we need to worry about to make any route we find "legally" ringable?
- Using your graph, find a walk and write it down.
    - Is your walk "legally" ringable? Why or why not?
- Using your graph, find a path and write it down.
    - Is your path "legally" ringable? Why or why not?
- Using your graph, find a trail and write it down.
    - Is your trail "legally" ringable? Why or why not?
- Using your graph, find a circuit and write it down.
    - Is your circuit "legally" ringable? Why or why not?
- Split into groups of 3-4.
- As a group, find an Eulerian trail in your graphs and write it down.
    - Is your Eulerian trail "legally" ringable? Why or why not?
- As a group, find an Eulerian circuit in your graphs and write it down.
    - Is your Eulerian circuit "legally" ringable? Why or why not?
- As a group, find a cycle in your graphs and write it down.
    - Is your cycle "legally" ringable? Why or why not?
- As a group, find a Hamiltonian cycle in your graphs and write it down.
    - Is your Hamiltonian cycle "legally" ringable? Why or why not?
    - What is significant about the Hamiltonian cycle? (It is the extent.)
    - Is this the same extent you found by hand?

- At each stage, if not "legally" ringable, can you find one that is?
- Also, what other possible solutions are there? Are the solutions limited in number?
- If extra time, what kind of restrictions does being "legally" ringable place on your solutions?

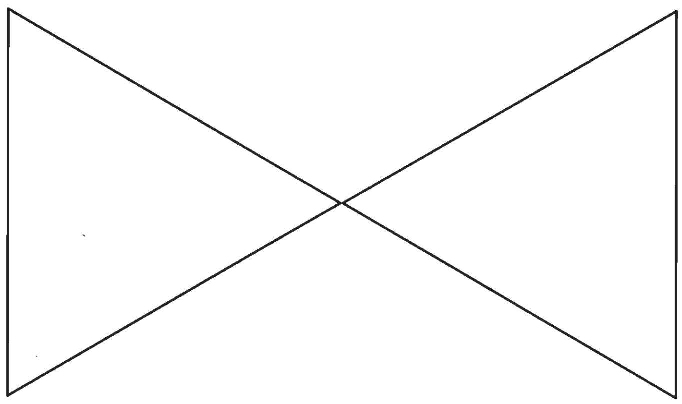
Lesson Title: Intro to Groups
Lesson length: 1 hour

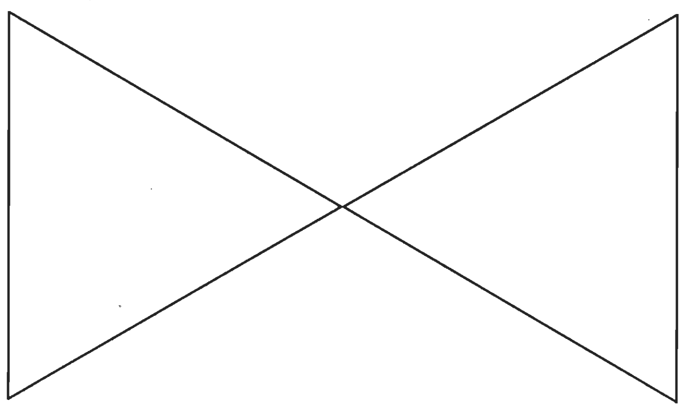
Goal: Students will work with different shapes to discover the basic properties of groups.
Materials: Handouts (1 shape per group), overheads of group tables and of one shape

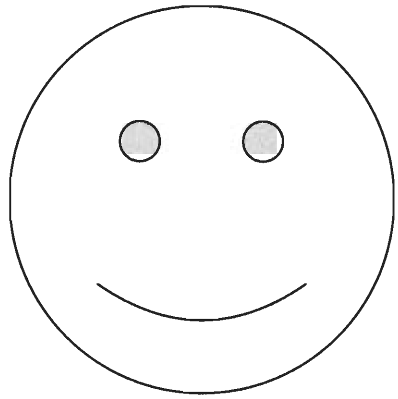
Terms to know: reflection, rotation, group, group table, identity, associativity, commutative, inverse, closure

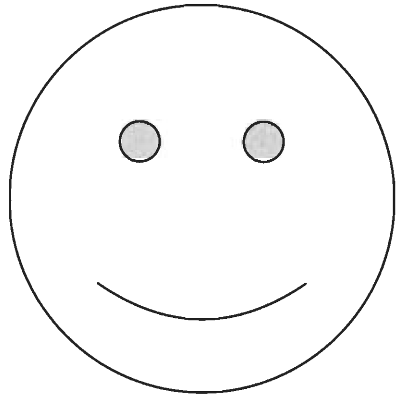
Lesson:

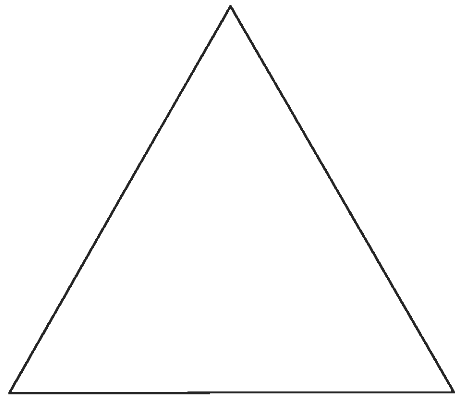
- Show one shape on overhead and have students tell how to move it (reflect/rotate) so that it looks the same
- Combine reflections/rotations that students find to try to make another reflection/rotation
- Split into groups of 3-4
- Have groups work with one shape and try to find all reflections and rotations
- Have groups try to do combinations of reflections/rotations to see if the result is a different reflection/rotation
- Have groups try to make a group table if time permits or just show on overhead
- Have students try to define what a group is
    - Give definition
    - A group is a nonempty set G with a binary operation * that satisfies:
        - Closure: If $a \in G$ and $b \in G$, then $a*b \in G$
        - Associativity: $a*(b*c) = (a*b)*c$ for all $a, b, c \in G$
        - Identity: There exists $e \in G$ such that $e*a = a*e = a$ for all $a \in G$
        - Inverse: For each $a \in G$, there exists $d \in G$, such that $a*d = e = d*a$
            - Note: d is typically written as a^{-1}
        - *Commutativity: (Abelian groups)* $a*b = b*a$ *for all* $a, b \in G$
    - Do all of our shapes rotations and reflections make a group?
- If time permits: Do you notice anything in common between the bowtie and the rectangle?
    - Hint: Look at their group tables.
    - If we look at their groups tables that we created they are identical.
    - They are isomorphic, i.e., they have the same structure.
        - Also isomorphic to $Z_2 \times Z_2$
- Which groups are abelian?

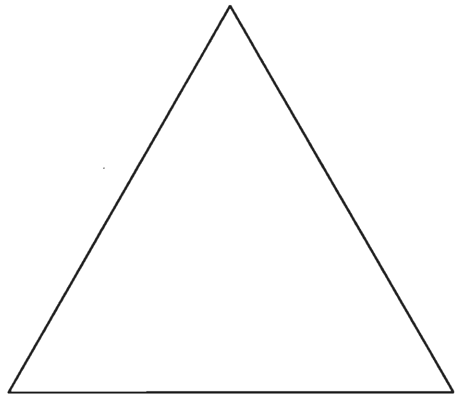












L

L

Square

*	R_0	R_{90}	R_{180}	R_{270}	$R_{y=0}$	$R_{y=x}$	$R_{x=0}$	$R_{y=-x}$
R_0	R_0	R_{90}	R_{180}	R_{270}	$R_{y=0}$	$R_{y=x}$	$R_{x=0}$	$R_{y=-x}$
R_{90}	R_{90}	R_{180}	R_{270}	R_0	$R_{y=x}$	$R_{x=0}$	$R_{y=-x}$	$R_{y=0}$
R_{180}	R_{180}	R_{270}	R_0	R_{90}	$R_{x=0}$	$R_{y=-x}$	$R_{y=0}$	$R_{y=x}$
R_{270}	R_{270}	R_0	R_{90}	R_{180}	$R_{y=-x}$	$R_{y=0}$	$R_{y=x}$	$R_{x=0}$
$R_{y=0}$	$R_{y=0}$	$R_{y=-x}$	$R_{x=0}$	$R_{y=x}$	R_0	R_{270}	R_{180}	R_{90}
$R_{y=x}$	$R_{y=x}$	$R_{y=0}$	$R_{y=-x}$	$R_{x=0}$	R_{90}	R_0	R_{270}	R_{180}
$R_{x=0}$	$R_{x=0}$	$R_{y=x}$	$R_{y=0}$	$R_{y=-x}$	R_{180}	R_{90}	R_0	R_{270}
$R_{y=-x}$	$R_{y=-x}$	$R_{x=0}$	$R_{y=x}$	$R_{y=0}$	R_{270}	R_{180}	R_{90}	R_0

Rectangle

*	R_0	R_{180}	$R_{y=0}$	$R_{x=0}$
R_0	R_0	R_{180}	$R_{y=0}$	$R_{x=0}$
R_{180}	R_{180}	R_0	$R_{x=0}$	$R_{y=0}$
$R_{y=0}$	$R_{y=0}$	$R_{x=0}$	R_0	R_{180}
$R_{x=0}$	$R_{x=0}$	$R_{y=0}$	R_{180}	R_0

Bowtie

*	R_0	R_{180}	$R_{y=0}$	$R_{x=0}$
R_0	R_0	R_{180}	$R_{y=0}$	$R_{x=0}$
R_{180}	R_{180}	R_0	$R_{x=0}$	$R_{y=0}$
$R_{y=0}$	$R_{y=0}$	$R_{x=0}$	R_0	R_{180}
$R_{x=0}$	$R_{x=0}$	$R_{y=0}$	R_{180}	R_0

Triangle

*	R_0	R_{120}	R_{240}	R_1	R_2	R_3
R_0	R_0	R_{120}	R_{240}	R_1	R_2	R_3
R_{120}	R_{120}	R_{240}	R_0	R_3	R_1	R_2
R_{240}	R_{240}	R_0	R_{120}	R_2	R_3	R_1
R_1	R_1	R_2	R_3	R_0	R_{120}	R_{240}
R_2	R_2	R_3	R_1	R_{240}	R_0	R_{120}
R_3	R_3	R_1	R_2	R_{120}	R_{240}	R_0

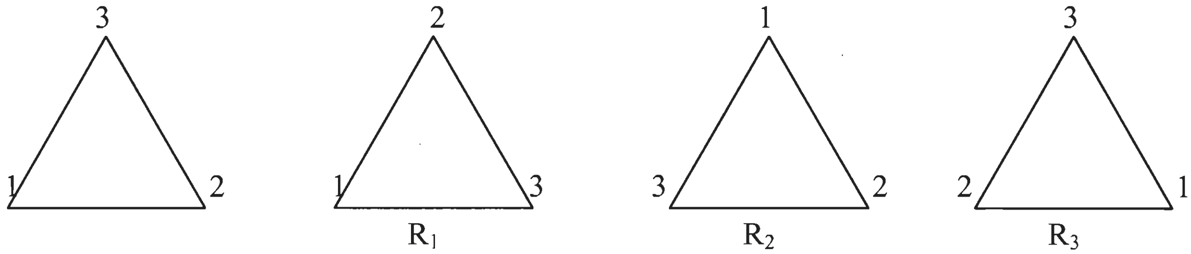
Smile

*	R_0	$R_{x=0}$
R_0	R_0	$R_{x=0}$
$R_{x=0}$	$R_{x=0}$	R_0

L

*	R_0
R_0	R_0

Triangle



Lesson Title: Permutation Groups
Lesson length: 1 hour

Goal: Students will understand what a permutation group is and how to computer permutations.
Materials: Overhead to write examples on

Terms to know: permutation, group

Lesson:

- Does anyone remember the definition from previous math classes?
    - Permutation:
- There are two different ways to represent a permutation.
    - (123) – "cycle" notation
    - (231) – "array" notation
    - Note: (213) = (12)(3) = (12)
    - Practice having students use both notations.
        - (54321)
        - (54321)
        - (35712648)
        - (146)(25)
        - Note: Both are useful, but for our purposes, we will use the "cycle" notation.
- We can compose two permutations.
    - Ex. (132) o (132) = (123)
    - Practice having students do compose functions.
        - (145326) o (64253) = (146523)
        - (135246) o (142536) = (16)(2)(3)(4)(5)
        - (2543) o (15423) = (1453)(2)
        - (18563742) o (16384572) = (135467)(28)
    - Do you think permutations might make a group?
        - Let's check the rules.
            - Closure: If a ϵ G and b ϵ G, then a*b ϵ G
            - Associativity: a*(b*c) = (a*b)*c for all a, b, c ϵ G
            - Identity: There exists e ϵ G such that e*a = a*e = a for all a ϵ G
            - Inverse: For each a ϵ G, there exists d ϵ G, such that a*d = e = d*a
        - Work through all rules. So it is a group.
        - Do you think permutations are a commutative group?
            - Commutativity: a*b = b*a for all a, b ϵ G
        - Work through an example.
            - (13) o (12) = (123)
            - (12) o (13) = (132)
        - So permutations aren't a commutative group.

Lesson Title: Permutation Groups in Change Ringing
Lesson length: 1 hour

Goal: Students will understand how to use permutation groups to represent change ringing.
Materials: Overhead of Plain Bob Doubles, handouts of Plain Bob Doubles for students to write changes on

Terms to know:

Lesson:

- We can represent the changes in change ringing with the permutation notation we learned yesterday.
    - Ex. 123456 -> 124356 (34)
    - Note: Make sure to note the place changes, not the specific bell changes.
    - So which one of these is right?
    - 135246 -> 132564 (34)(56) or (25)(46)?
- Why don't you try some examples in pairs?
    - 123456 -> 213456 (12)
    - 123456 -> 214365 (12)(34)(56)
    - 142536 -> 415263 (12)(34)(56)
    - 654321 -> 645312 (23)(56)
- What did you notice about the second and third examples that you tried?
    - Did you notice anything in general about the changes?
        - All changes are between adjacent places.
- Let's look at a plain course of Plain Bob Doubles.
    - Can you figure out the changes (in terms of permutations)?

Plain Bob Doubles

12345	13524	15432	14253
21435	31254	51342	41523
24153	32145	53124	45132
42513	23415	35214	54312
45231	24351	32541	53421
54321	42531	23451	35241
53412	45213	24315	32514
35142	54123	42135	23154
31524	51432	41253	21345
13254	15342	14523	12435
13524	15432	14253	12345

Lesson Title: Even and Odd Permutations
Lesson length: 1 hour

Goal: Students will understand the difference between even and odd permutations.
Materials:

Terms to know: permutation, even, odd, in-course change (even), out-of-course change (odd)

Lesson:

- We can look at two different kinds of permutations.
  - Even permutations have an even number of changes from rounds.
  - Odd permutations have an odd number of changes from rounds.
- Let's look at a few examples.
  - 132465 - (23)(56), even
  - 214365 – (12)(34)(56) odd
- Let's look at Plain Hunt on 4 and see if we can figure out the even and odd changes.
  - We'll call the change (12)(34) – a and (23) – b.
  - 1234 e even
    2143 a even
    2413 ab odd
    4231 aba odd
    4321 $(ab)^2$ even
    3412 $(ab)^2$ a even
    3142 $(ab)^3$ odd
    <u>1324</u> $(ab)^3$ a odd
    1234 $(ab)^4$ =e even
- Split into groups of 3-4.
- Let's look at some other minimus methods and see if we find any patterns in even and odd changes.
- What patterns did you find?
- Note: All even permutations are known as the Alternating group and is a subgroup of the permutation group. ($A_4 \leq S_4$)

Minimus Methods

Plain Bob	Reverse Bob	Double Bob	Canterbury	Reverse Canterbury	Double Canterbury
1234	1234	1234	1234	1234	1234
2143	2143	2143	2143	2134	2134
2413	2413	2413	2413	2314	2314
4231	4231	4231	2431	3241	2341
4321	2431	2431	4231	3421	3241
3412	4213	4213	4213	4312	3214
3142	4123	4123	4123	4132	3124
1324	1432	1432	1432	1432	1324
1342	1342	1423	1342	1423	1342
3124	3124	4132	3124	4123	3142
3214	3214	4312	3214	4213	3412
2341	2341	3421	3241	2431	3421
2431	3241	4321	2341	2341	4321
4213	2314	3412	2314	3214	4312
4123	2134	3142	2134	3124	4132
1432	1243	1324	1243	1324	1432
1423	1423	1342	1423	1342	1423
4132	4132	3124	4132	3142	4123
4312	4312	3214	4312	3412	4213
3421	3421	2341	4321	4321	4231
3241	4321	3241	4231	4231	2431
2314	3412	2314	2413	2413	2413
2134	3142	2134	2143	2143	2143
1243	1324	1243	1243	1243	1243
1234	1234	1234	1234	1234	1234

Single Court	Reverse Court	Double Court	St. Nicholas	Reverse St. Nicholas
1234	1234	1234	1234	1234
2134	2143	2134	2134	2143
2314	2413	2314	2314	2413
3241	2431	2341	3241	2431
3421	2341	2431	2341	4231
4312	2314	2413	3214	4213
4132	2134	2143	3124	4123
1432	1243	1243	1324	1432
1342	1423	1423	1342	1423
3142	4132	4123	3142	4132
3412	4312	4213	3412	4312
4321	4321	4231	4321	4321
4231	4231	4321	3421	3421
2413	4213	4312	4312	3412
2143	4123	4132	4132	3142
1243	1432	1432	1432	1324
1423	1342	1342	1423	1342
4123	3124	3142	4123	3124
4213	3214	3412	4213	3214
2431	3241	3421	2431	3241
2341	3421	3241	4231	2341
3214	3412	3214	2413	2314
3124	3142	3124	2143	2134
1324	1324	1324	1243	1243
1234	1234	1234	1234	1234

Lesson Title: Extents in Groups
Lesson length: 1 hour

Goal: Students will use group theory to find an extent of Plain Bob Doubles.
Materials:

Terms to know: extent, lead end

Lesson:

- Do you remember how a bob or single changes the lead end in Plain Bob Doubles?
  - Bob (23)
    35142
    31524
    <u>13254</u>
    12354
    21534
    25143
  - Single (45)
    35142
    31524
    <u>13254</u>
    13245
    31425
    34152
  - The real change is at the lead end; what is the permutation for these changes?
- Split into groups of 3-4.
- Have groups work together to find an extent of Plain Bob Doubles using group theory.

Lesson Title: Change Ringing in Abel
Lesson length: 1 hour

Goal: Students will attempt to ring the extents found in earlier lessons in the Abel ringing simulator.
Materials: Computers with Abel software

Terms to know: Abel Ringing Simulator

Lesson:

- So, we've learned all this information about change ringing, who would like to actually try it?
    - Since it takes about 6 months to learn to ring a real bell, we're going to use a computer simulator.
- Split into groups and have students watch a plain course of Plain Bob Doubles
- Have students try ring a bell in a plain course of Plain Bob Doubles
- Have students try to ring the one of the extents of Plain Bob Doubles that was found in previous lessons
    - Have one student ring a bell, while another acts as the conductor and puts in the calls at the appropriate time

Lesson Title: How Many Extents?
Lesson length: 1 hour

Goal: Students will work with Plain Bob Doubles to find all the possible extents.
Materials: Overheads of different extents

Terms to know: Plain Bob Doubles, extent

Lesson:

- So far, we've found extents of Plain Bob Doubles in 3 different ways – by hand, using graph theory, and using group theory. Some of you may have found up to 3 different extents.
- Using only bobs, are there only 3 different extents of Plain Bob Doubles? Are there more? How many more? Is there a pattern to find them?
- Split into groups of 3-4.
- Have each group try to find as many extents of Plain Bob Doubles as possible.
    - If some groups finish before others, challenge to find other extents.
    - Also, challenge to find a pattern that makes finding an extent easy.
    - Is there a reason that there are only n extents?
- Come back to whole group.
- How many extents did you find?
    - 4 – 2 as observation, 3 as observation, 4 as observation, 5 as observation
- Have students show and compare different extents.
- Did anyone have any ideas as to why there were only four extents? What limits the number of extents?

Extent of Plain Bob Doubles – 2 Observation ☺

12345
21435
24153
42513
45231
54321
53412
35142
31524
13254
13524
31254
32145
23415
24351
42531
45213
54123
51432
15342
13542 -
31452
34125
43215
42351
24531
25413
52143
51234
15324
15234

15234
51324
53142
35412
34521
43251
42315
24135
21453
12543
12453
21543
25134
52314
53241
35421
34512
43152
41325
14235
14325
41235
42153
24513
25431
52341
53214
35124
31542
13452
14352 -

14352 -
41532
45123
54213
52431
25341
23514
32154
31245
13425
13245
31425
34152
43512
45321
54231
52413
25143
21534
12354
12534
21354
23145
32415
34251
43521
45312
54132
51423
15243
15423

15423
51243
52134
25314
23541
32451
34215
43125
41352
14532
15432 -
51342
53124
35214
32541
23451
24315
42135
41253
14523
14253
41523
45132
54312
53421
35241
32514
23154
21345
12435
12345

Extent of Plain Bob Doubles – 3 Observation

12345	15423	12534	13452
21435	51243	21354	31542
24153	52134	23145	35124
42513	25314	32415	53214
45231	23541	34251	52341
54321	32451	43521	25431
53412	34215	45312	24513
35142	43125	54132	42153
31524	41352	51423	41235
13254	14532	15243 -	14325
13524	14352	12543	14235
31254	41532	21453	41325
32145	45123	24135	43152
23415	54213	42315	34512
24351	52431	43251	35421
42531	25341	34521	53241
45213	23514	35412	52314
54123	32154	53142	25134
51432	31245	51324	21543
15342	13425	15234	12453
15432	13245	15324	14253 -
51342	31425	51234	41523
53124	34152	52143	45132
35214	43512	25413	54312
32541	45321	24531	53421
23451	54231	42351	35241
24315	52413	43215	32514
42135	25143	34125	23154
41253	21534	31452	21345
14523	12354	13542	12435
15423 -	12534	13452	12345

Extent of Plain Bob Doubles – 4 Observation

12345	14532	12453	13524
21435	41352	21543	31254
24153	43125	25134	32145
42513	34215	52314	23415
45231	32451	53241	24351
54321	23541	35421	42531
53412	25314	34512	45213
35142	52134	43152	54123
31524	51243	41325	51432
13254	15423	14235	15342
12354 –	15243	14325	15432
21534	51423	41235	51342
25143	54132	42153	53124
52413	45312	24513	35214
54231	43521	25431	32541
45321	34251	52341	23451
43512	32415	53214	24315
34152	23145	35124	42135
31425	21354	31542	41253
13245	12534	13452	14523
13425	15234 –	13542	14253
31245	51324	31452	41523
32154	53142	34125	45132
23514	35412	43215	54312
25341	34521	42351	53421
52431	43251	24531	35241
54213	42315	25413	32514
45123	24135	52143	23154
41532	21453	51234	21345
14352	12543	15324	12435
14532	12453	13524 –	12345

Extent of Plain Bob Doubles – 5 Observation

12345	14253	15324	14532
21435	41523	51234	41352
24153	45132	52143	43125
42513	54312	25413	34215
45231	53421	24531	32451
54321	35241	42351	23541
53412	32514	43215	25314
35142	23154	34125	52134
31524	21345	31452	51243
13254	12435 -	13542	15423
13524	14235	13452	15243
31254	41325	31542	51423
32145	43152	35124	54132
23415	34512	53214	45312
24351	35421	52341	43521
42531	53241	25431	34251
45213	52314	24513	32415
54123	25134	42153	23145
51432	21543	41235	21354
15342	12453	14325	12534
15432	12543	13425 -	12354
51342	21453	31245	21534
53124	24135	32154	25143
35214	42315	23514	52413
32541	43251	25341	54231
23451	34521	52431	45321
24315	35412	54213	43512
42135	53142	45123	34152
41253	51324	41532	31425
14523	15234	14352	13245
14253	15324	14532	12345 -

Lesson Title: Extents of Minor
Lesson length: 2 x 1 hour

Goal: Students will use the graph theory and group theory that they have learned to find an extent of Minor (in a method of their choosing).
Materials: Copies of *Diagrams* books for each group, graph paper for students to write out extent

Terms to know: Minor, extent

Lesson:

- We've worked a lot with the method of Plain Bob Doubles. Now I want to see if you can apply what you've learned to a minor (6 bell) method.
- Split up into groups of 3-4.
- Give each group copies of *Diagrams*.
- As a group I want you to pick a minor method to work with. Please pick something other than Plain Bob Minor, because it will be very similar to what you have worked with in Plain Bob Doubles. Also, I want each group to pick out a different method.
    - Help groups pick out methods. (Try to stick to treble hunting methods?)
- Does anyone remember how many changes are in an extent of minor?
        - 720 = 6*5*4*3*2*1
- Ok, now that you've picked out a method, I want you to work with your group to figure out an extent of this method.
- Don't worry if you don't finish today, you can work on this tomorrow too.
- If groups finish early, encourage them to pick out another (treble dodging) method to find an extent of.
    - Do they notice any similarities or differences in finding extents with different work for the treble?

Bibliography

Chartrand, Gary. *Introductory Graph Theory*. Toronto, Ontario, Canada: Dover, 1977.

Hungerford, Thomas, W. *Abstract Algebra: An Introduction*. Brooks/Cole, 1997.

Sayers, Dorothy L. *The Nine Tailors*. San Diego, CA: Harcourt, Inc, 1934.

Snowdon, Jasper and William Snowdon. *Diagrams*. Burton Latimer, Kettering, Northants, England: MPG Books Ltd., 1998.

Extra Resources for Students and Teachers

Change Bell Ringing

Coleman, Steve. *The Bellringer's Bedside Companion*. Trowbridge, Whitshire, Great Britian: Redwood Books, 1994.

Cook, W. T. and C.A. Wratten. J. Sanderson (Ed). *Change Ringing: The History of and English Art, Volume 3: The Eighteenth Century, A Regional Survey*. Morpeth, Northumberland, England: The Central Council of Church Bell Ringers, 1994.

Copson, Pam. *One Per Learner*. Wellesbourne, Warwick, England, 1992.

Copson, Pam. *The Follow-On Book for Bell Ringers*. Wellesbourne, Warwick, England, 1988.

Eisel, J.C. and C.A. Wratten. J. Sanderson (Ed). *Change Ringing: The History of and English Art, Volume 2: The Eighteenth Century, an Overview*. Morpeth, Northumberland, England: The Central Council of Church Bell Ringers, 1992.

J. Sanderson (Ed). *Change Ringing: The History of and English Art, Volume 1*. Morpeth, Northumberland, England: The Central Council of Church Bell Ringers, 1987.

Smith, Robert B. *Standard Methods*. Burton Latimer, Kettering, Northants, England: MPG Books Ltd., 1980.

The Towers and Belfries Committee (Ed). *The Towers and Bells Handbook*. Brackley, Northhamptonshire: Smart and Company (Printers) Ltd., 1973.

Graph Theory

Wilson, Robin J. *Introduction to Graph Theory, 3rd Edition*. Pitman: Bath, 1985.

Group Theory

Pinter, Charles C. *A Book of Abstract Algebra, 2nd Edition*. McGraw-Hill: New York City, NY, 1990.

Graph Theory, Group Theory, and Change Bell Ringing

White, Arthur T. Graphs of Groups on Surfaces. North-Holland: Amsterdam, Netherlands, 2001.