Slug Tests in Unconfined Aquifers

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SLUG TESTS IN UNCONFINED AQUIFERS

by

Rozkar Ismael

A thesis submitted to the Graduate College in partial fulfillment of the requirements for the degree of Master of Science in Geosciences
Western Michigan University
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SLUG TESTS IN UNCONFINED AQUIFERS

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Western Michigan University, 2016

This research presents a hydraulic conductivity (K) analysis of unconfined aquifers using slug tests. Slug tests are used to determine in situ aquifer hydraulic conductivity more quickly and economically than by a pump test. This study examines how to best conduct a slug test using a physical slug. Different common slug test analysis methods are compared, including Bouwer and Rice (1976), Hvorslev (1951), Dagan (1978) and Kansas Geological Survey (KGS, 1994). Questions that motivated this study include: Which methods are better for performing and analyzing slug tests? Does the size of the physical slug affect the results? Do large initial water level displacements produce better results than smaller displacements?

Slug tests were performed at two sites: 1- Asylum Lake in Kalamazoo, Michigan, in a well 0.33 ft in diameter and 97 ft deep. 2- Another unconfined aquifer in Portage, Michigan, in wells 0.167 ft in diameter and relatively shallow depth of 16 ft. Both sites were slug tested using two different sizes of physical slug rods. The smaller slug was 5 ft in length with 0.12 ft diameter. The larger slug was 6.91 ft long and 0.125 ft in diameter.

The four-way analysis of variance (ANOVA) examination of 39 slug tests on wells in these two aquifers confirmed that log K depends highly significantly upon slug in/out, and as expected, the aquifer tested. What was not expected was the average log K for tests in 5-cm diameter wells using slug in was highly significantly larger than average log K for tests using slug out; these means also differed significantly by slug size. Importantly, mean log K’s produced by the Hvorslev (1951) method are highly significantly larger than mean log K’s produced by the other three methods above. The size of the initial water-level deflection, Yo, does not affect the results for log K. The use of a larger or smaller slug does not significantly affect the log K values. Finally, assuming that a large K value better represents the true value, closer to a pump test K value, Hvorslev (1951) gives the largest K values.
I appreciate my thesis supervisor and committee chair, Dr. Duane R. Hampton, who has guided me and helped me in all fields of the hydrogeology; he helped me improve my master thesis writing. I would like to thank the other committee members, Dr. Mohamed Sultan and Dr. Alan Kehew, and other professors who taught me graduate courses, namely Dr. Daniel P Cassidy and Dr. William A. Sauck. I am grateful for for Dr. Steven Kohler, who performed the statistical analysis using R software. Also, I will not forget professors who taught me in my undergraduate classes in University of Salahaddin in Erbil, Kurdistan.

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Rozkar Ismael
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1 Introduction

1.1 Background

Slug test methods are one of the most commonly used methods to determine in situ aquifer hydraulic conductivity (K) or transmissivity, it is more cost effective and faster to acquire compared to other tests such as pump tests. Slug tests usually yield smaller values for aquifer hydraulic conductivity compared to those estimated from pumping tests. Pump tests values are generally considered to better represent aquifer behavior (Butler and Healey, 1998). A slug test induces a rapid change in water level in a slugged well, and then one observes how fast the water level returns to equilibrium. The local horizontal hydraulic conductivity of the aquifer material surrounding the slugged well is related to the rate of water-level equilibration, which is determined. The water level equilibrates faster in higher transmissivity aquifers. The hydraulic conductivity of the screened interval is paramount.

Overall, this study attempts to answer the following questions: Which methods are better for performing and analyzing slug tests in unconfined aquifers? Does the size of the physical slug affect the results? Do large initial water level displacements produce better results than smaller displacements? This study does not systematically address the question of “Is there a difference in hydraulic conductivity (K) values obtained from slug tests and those from pumping tests?” Dennis (1987) found that slug test values are typically smaller than pump test values; if a well is developed adequately, then those values become more similar.

Slug testing is used to obtain hydraulic conductivity (K) in-situ in both confined and unconfined aquifers. Also, it is used for determining K at multiple locations at one site, including different depths, to help evaluate groundwater velocity, contaminant migration, and remedial plan effectiveness. While there is guidance on how to perform slug tests to get the best possible results (e.g., Butler, 1998; Fetter, 2001; Weight, 2008), this study’s objective is to compare the results (1) performed with different-size slugs, (2) using both slug in and slug out, and (3) analyzed using different methods such as Bouwer and Rice (1976), Hvorslev (1951), Dagan (1978), and Kansas Geological Survey (1994) (KGS) to determine the best practices for slug tests including analysis. This study
primarily relied upon physical rather than pneumatic (pressure or vacuum) slugs. That choice was made because for wells in unconfined aquifers where the water table is below the top of the screen, which is quite common, using a physical slug or a pneumatic slug below a packer is the only option for conducting a slug test. This comparison was done to help decide which of these slug sizes, modes (slug in or out) and analysis methods would be best for estimating unconfined aquifer parameters.

The most commonly-used slug test methods for unconfined aquifers are: (a) Bouwer and Rice (1976), (b) Hvorslev (1951), (c) Dagan (1978) and (d) Kansas Geological Survey (KGS) (1994). Slug test data are affected by factors such as well skin and aquifer anisotropy (Chirlin, 1990). The Bouwer and Rice (1976) method is most popular than other methods because it was designed specifically for tests of unconfined aquifers, the most common kind of slug test performed. The Hvorslev (1951) method was thoroughly critiqued by Chirlin (1989) and Hyder et al. (1994). In the Dagan (1978) method, the water table was included into the hydraulic conductivity (K) calculations (Widdowson et al., 1990). It is working best for partially penetrating well screens. But the simplicity of the assumptions could lead to errors in an aquifer’s hydraulic conductivity (K) estimates (Hyder and Butler, 1994). The Kansas Geological Survey (1994) model (see Hyder et al., 1994, Hyder and Butler, 1995, and Liu and Butler, 1995), should be most correct because it is the only one based on a transient equation and includes the effects of specific storage (Ss), aquifer anisotropy ratio (Kz/Kr), and well skin on (K) values. With the different assumptions and aquifer parameters which characterize these slug test analysis methods, their hydraulic conductivity results would be expected to vary, and they do.

1.2 Objectives and Scope

The goal of this research is to evaluate the performance of slug tests on unconfined aquifers analyzed using different methods. Those methods are Bouwer and Rice (1976), Hvorslev (1951), Dagan (1978) and Kansas Geological Survey (1994) (KGS). Which one or ones of these methods are most suited for unconfined aquifers? Does using a larger physical slug affect the results? Do large initial water level displacements produce better results than smaller displacements because a larger aquifer volume is affected? Does it matter whether the slug is inserted or pulled out of the well?
Thesis Organization

This thesis contains sections which explain slug tests on unconfined aquifers, where and how they were performed and the results obtained therefrom. Different methods are used to determine and analyze hydraulic conductivity (K) values.

Chapter one discusses the general background of this research introducing slug tests, and the use of different methods to perform the test and to analyze the results. This chapter also states the objective and scope of this research.

Chapter two explains the basics of slug tests and how slug tests work as well as aquifer concepts. Topics include confined and unconfined aquifers, and general assumptions of slug test methods. The chapter also describes previous work about slug tests.

Chapter three describes the four common methods used to perform slug tests and analyze their results. These are: Bouwer and Rice (1976), Hvorslev (1951), Dagan (1978) and Kansas Geological Survey (1994). This chapter presents equations and assumptions for these methods. The chapter also discusses steps to obtain the values of hydraulic conductivity of the aquifer for each of these four methods.

Chapter four shows slugged wells in two sites: Asylum Lake Preserve in Kalamazoo and Portage aquifer near to Austin Lake, Portage, MI. This chapter presents location of wells and equipment used to achieve slug test in both sites. Finally, the software used to analyze the data of slug test.

Chapter four shows well slug tested at two sites: Asylum Lake Preserve in Kalamazoo, Michigan, and an aquifer near Austin Lake, Portage, Michigan. This chapter presents the well locations and equipment used to conduct slug tests at both sites. Finally, the software used to analyze the slug test data is reviewed.

Chapter five discusses the results of slug tests performed at both sites. This chapter compares hydraulic conductivity values produced from these four methods, from tests in two aquifers, using larger and smaller slugs and changing the water level using both slug in and slug out. In addition, the results of all slug tests are statistically analyzed to see which of the above factors in their differences are significant. Finally, the effect of the size of the slug’s initial water-level deflection, Yo, on K values is examined.
Chapter six concludes with observations and outcomes from this research, and provides recommendations for future research.
2 Background and Literature Review

2.1 Introduction

One of the most useful methods to determine the hydraulic conductivity of an aquifer is slug testing. Slug tests have been used for many years as an inexpensive and quick method to estimate the hydraulic properties of aquifers Figure 2-1 (Fetter, 2001). This works by measuring the change of head in a well after instantaneous change in the head at that well. A solid object or slug is inserted into or removed from the well, causing a sudden change (increase or decrease) in the water level in the well. As the water level returns to the static level, the head is measured as a function of time. This data is used to calculate the hydraulic conductivity (K). Pneumatic pressure is another way that is used to change the water level in the well by pulling the water up using vacuum or pushing it down by pressure using a pump Figure 2-2(Todd, 2005). Hydrogeologists should know something about the different properties of the aquifers.

Figure 2-1 Schematic of a typical well installation.
2.2 Aquifer Concepts

An aquifer is a geologic unit that contains sufficient saturated permeable material to yield significant quantities of water to wells or to the surface such as springs or base flow (Weight, 2008). The permeability of aquifers ranges from $10^{-1}$ to over $10^3$ m/day. Examples of rock units known to be aquifers are: unconsolidated sands and gravels, limestones and dolomites, sandstones, basalt flows, and fractured plutonic and metamorphic rocks (Fetter, 2001). Confined and unconfined aquifers are types of aquifers Figure 2-3.

2.2.1 Confined Aquifers

A confined (or artesian) aquifer is an aquifer that has low-permeability formations or confining beds both above and below the aquifer (Pinder and Celia, 2006). The water in this kind of aquifer must be under pressure (Fetter, 2001). Recharge to these aquifers can occur either in a recharge area where the aquifer outcrops, or by slow leakage through leaky confining layers from adjacent aquifers under higher pressure.

2.2.2 Unconfined Aquifers

An unconfined aquifer has a water table and is typically close to the land surface. The water table is not an interface separating unsaturated from saturated zones. The
unsaturated-saturated interface can be several centimeters up to a couple of meters above the water table depending on the height of the capillary fringe (Lee, 1999). Recharge to these aquifers occurs either from downward seepage through the unsaturated zone, through lateral ground-water flow, or by upward seepage from underlying layers (Fetter, 2001).

![Aquifers and wells](Image)

Figure 2-3 Aquifers and wells (Environment Canada, USGS).

### 2.3 Pump Tests

A pumping test or aquifer test is the most common and the best test for obtaining hydraulic information from aquifers. In a pump test, a well is pumped and the rate of decline of the water level in nearby observation wells is noted. Transmissivity of the aquifer is yielded from analysis of the drawdown data. To conduct a pump test, a pumping well and one or more observation wells are needed (Fetter, 2001).

#### 2.3.1 General assumptions related to pumping tests

- The observation wells and the pumping well are screened throughout the entire thickness of the aquifer, and only in that aquifer.
- There is no recharge during the pump test.
- The ground water zone is infinite in areal extent.
- The zone is homogeneous, isotropic, and of uniform thickness over the area influenced by the test.
- Prior to pumping, the water table or potentiometric surface is horizontal, or nearly so, over the area to be influenced.
- No other pumping affects the area of the pump test.
- The well water removed from storage is discharged instantaneously with decline of head; the well diameter is infinitesimal.

2.3.2 General assumptions related to slug tests

- The groundwater zone has a large areal extent around the tested well.
- The zone is homogeneous and of uniform thickness over the area influenced by the test.
- Prior to the test, the water table or potentiometric surface is (nearly) horizontal over the well’s area of influence.
- The head in the well is changed instantaneously at time \( t_0 = 0 \).
- The inertia of the water column in the well and the linear and non-linear well losses are negligible.
- Well installation and development process are assumed to have not changed the hydraulic characteristics of the formation.
- The well diameter is finite; hence storage in the well cannot be neglected.
- No phases other than water (such as gasoline) are assumed to be present in the well or groundwater.
- Water is assumed to flow horizontally into or out of the well screen.

2.3.3 Differences between pump tests and slug tests

Butler and Healey (1998) highlighted the differences between pump tests and slug tests, which generally produce different values of \( K \):
- Infrequent-in-space zones of relatively high hydraulic conductivity will have a very limited impact on parameter estimates obtained from pump tests. Thus, it is difficult to explain the difference between slug tests and pump test parameters based on the existence of infrequent high-\( K \) conduits. But if these zones have a
significant effect on the drawdown from a particular pump test, this should be clearly revealed on a semi-log drawdown versus time plot.

- Pump tests are not affected by low-K skins (the annular volume around the well screen which has a different permeability than the surrounding aquifer) if the Cooper-Jacob semi-log distance-drawdown method or observation wells at a distance from the pumping well are used. However, slug tests are extremely sensitive to altered, near-well conditions. Low-K skins produce slug test estimates that may be orders of magnitude lower than the average hydraulic conductivity of the formation in the vicinity of the well screen.

- Failure to account for vertical anisotropy in the analysis of slug test data can result in underestimating the K by up to a factor of 3. Vertical anisotropy doesn’t have an effect on pump tests if the Cooper-Jacob semi-log method and/or observation wells at a distance from the pumping well are used.

- In a pump test, the full thickness of the aquifer will eventually contribute flow to the discharge wells, regardless of whether that well partially or fully penetrates the unit. Consequently, a reasonable estimate of the aquifer thickness is required to convert the transmissivity measured from a pump test into an average K for the aquifer. For a partially-penetrating well, Weight (2008, p. 486--492) shows that effective thickness will increase with time through a pump test.

- Effective screen length, which is required for the analysis of a slug test, may be difficult to estimate practically. Use of the length of the gravel pack or the nominal screen length may introduce considerable error into the slug-test K estimate if the well has not been thoroughly developed. Uncertainty about this quantity will have no effect on estimates of hydraulic conductivity from pump tests if the Cooper-Jacob semi-log method and/or observation wells at a distance from the pumping well are used.

2.4 Importance of Slug Tests

There are many advantages of slug tests to become an alternative method of other methods used to measure hydraulic conductivity and other parameters of aquifers:
• Simplicity: it is a simple method that can be used to measure the change of water level over time.

• Cost-effectiveness: slug tests are much less expensive than pump tests.

• Time duration: slug tests are much quicker than a pump test.

• In situ parameter measurement: other than a pump test, it is the best method for estimating the aquifer’s parameters in situ over a small extent.

• No addition or removal of water is required: a physical or pneumatic slug is used for slug tests. Hence no treatment or disposal of contaminated water is needed.

• Broad applicability: Slug tests can be used to determine K in a wide range of wells (i.e., production and monitoring wells). Also, slug tests are useful in tight layers or thin layers in which a pump test cannot be performed.

2.5 Literature Review

2.5.1 Herman Bouwer and R. C. Rice

In 1976, Herman Bouwer and R. C. Rice developed one of the most useful methods for analyzing slug tests in wells in unconfined aquifers. This method depends on a mathematical model, which assumes: (a) that the storage effects can be neglected; (b) that Theim’s steady-state equation can be used; and (c) the aquifer is isotropic and homogeneous. Overall, there is similarity the work done by Bouwer and Rice’s (1976) and Hvorslev (1951), who conducted empirical experiments exploring the effective radius with fully penetrating wells in confined aquifers.

2.5.2 Antoine Saucier, Clément Frappier, and Robert Chapuis

Antoine Saucier, Clément Frappier, and Robert Chapuis (2010) published a paper titled, “Sinusoidal Oscillations Radiating from a Cylindrical Source in Thermal Conduction or Groundwater Flow: Closed-Form Solution.” Authors explained the
phenomena of groundwater flow and thermal conduction using the following differential equation:

\[
\frac{\partial T}{\partial t} = \frac{k}{C} \nabla^2 T
\]

Where \( T \) is the temperature, \( C \) is the volumetric heat capacity, \( k \) is the thermal conductivity, and \( t \) is the time. In fully saturated conditions, groundwater flows are described by this differential equation:

\[
\frac{\partial h}{\partial t} = \frac{K}{s_s} \nabla^2 h
\]

Where \( h \) is the hydraulic head, \( s_s \) is the specific storativity, \( K \) is the saturated hydraulic conductivity, and \( t \) is the time. The ratio \( \frac{K}{s_s} \) is called the hydraulic diffusivity.

According to the authors, this type of differential equation can be utilized in several thermal conduction and groundwater flow applications. In one application, the authors used a sinusoidal signal starting from a vertical cylindrical well to analyzing the equations. In another application, the authors calculated the hydraulic properties of an aquifer around a well. Generally, they showed that the hydraulic properties could be obtained using the aquifer’s hydraulic conductivity results from slug and pump tests. In their results, Saucier, Frappier, and Chapuis (2010) indicated that the slug test values represent such a small scale because the results were mostly controlled by the first 10 to 30 cm around the well filter pack.

2.5.3 Mark A. Widdowson, Fred J. Molz, and Joel G. Melville

In the paper “An Analysis Technique for Multilevel and Penetrating Slug Test Data,” Widdowson et al. (1990), developed an analysis technique for multileveled and partial penetrating slug test data. Specifically, their research applied slug tests for multilevel and partially penetrating wells to sites of potential contamination to obtain vertically averaged hydraulic conductivity values, or values at discrete vertical intervals, without the withdrawal and subsequent disposal of contaminated groundwater. The multilevel slug tests provide data under suitable conditions to determine the local hydraulic conductivity at a specific depth. In contrast, Widdowson et al. (1990) noted that
slug test methods, such as those developed by Hvorslev (1951), Cooper et al. (1967) and Dagan (1978), ignored vertical flow, and that the position of the screen relative to the nearest aquifer boundary is not considered directly as a variable. They considered that the Bouwer-Rice method (Bouwer and Rice, 1976) could apply to the double packer flow problem under certain restrictive conditions; however, the method fails (> 25% error) if L (the water-filled screen length) is much smaller than H (the vertical distance from the top of the aquifer to the bottom of the screen). Additional inconsistencies occur when this method is applied to confined aquifers.

The model of Widdowson et al. (1990), is a finite element flow model (EFLOW), that includes radial and vertical, anisotropic, axisymmetric flow to or from the test interval. It is largely based on Dagan (1978). In a heterogeneous, anisotropic aquifer, the equation that governed for non-steady, saturated groundwater flow in cylindrical coordinates was:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left[ r k_r \frac{\partial h}{\partial r} \right] + k_z \frac{\partial^2 h}{\partial z^2} = s_s \frac{\partial h}{\partial t}
\]

Where \( h \) is hydraulic head, \( S_s \) is specific storage, \( K_r \) is hydraulic conductivity aligned with the radial coordinate, \( r \), \( K_z \) is hydraulic conductivity in the vertical direction, \( z \), and \( t \) is time. If \( K_r \) is assumed to be constant in the domain at any elevation along the \( z \)-axis, then the equation can be written as:

\[
k_r(z) \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial h}{\partial r} \right) \right] + k_z \frac{\partial^2 h}{\partial z^2} = s_s \frac{\partial h}{\partial t}
\]

Dagan (1978) suggested that storativity has a limited role in slug tests because of the small volumes of water involved, i.e., the storage term is negligible (Widdowson et al., 1990). Widdowson et al. (1990) stated that the main advantage of the EFLOW method over Cooper and Hvorslev-type models was that match points from type curves and estimation of an effective radius (Re) are not required. Moreover, they showed that the EFLOW method was applicable to a complete range of multilevel slug test geometries, and did not have the numerical difficulties with small \( L \) (screen length) values associated with Bouwer and Rice (1976) and Dagan (1978) methods.
2.5.4 Vitaly Zlotnik

In 1994, Vitaly Zlotnik published a paper titled, “Interpretation of Slug and Packer Tests in Anisotropic Aquifers.” He noted that the Bouwer-Rice method (Bouwer and Rice, 1976) used by many practitioners disregards any anisotropy of the tested formation in slug and packer test interpretations. Consequently, researchers such as Dawson and Istok (1991) have recommended that the Bouwer-Rice method be used only for isotropic conditions, whereas other methods such as those developed by Widdowson, Molz, and Melville (1990) are more appropriate for anisotropic conditions. However, Zlotnik (1994) contended that slug or packer test results obtained in isotropic conditions could be extended to anisotropic cases when the anisotropy is quantified. Zlotnik (1994) tried to develop an extension technique of the Bouwer-Rice method that can be utilized in anisotropic conditions.

The local values of hydraulic conductivity in aquifers are determined using data obtained from slug or packer tests. Zlotnik’s (1994) extension technique utilized a transformation of the radial dimension by the well radius, dividing the true well radius by the square root of the anisotropy ratio. He used this application to correct the Bouwer-Rice method in anisotropic cases. It yielded accurate results when compared with other models such as Widdowson et al. (1990).

2.5.5 Kenneth Belitz and Weston Dripps

Belitz and Dripps (1999) used different methods such as those developed by Hvorslev (1951), Cooper et al. (1967), Bouwer and Rice (1976), and the KGS (Hyder et al., 1994) method to estimate hydraulic conductivity in the sub-basin of the Sleepers River. They noted it can be difficult to discern the effect of anisotropy and the presence of well skin when the slugged well is the only source of water level data. Thus they posited that observation well data is necessary to estimate other factors.

Belitz and Dripps (1999) noted that while there is much research concerning tests in fully penetrating wells in isotropic, confined aquifers, there is less research on partially penetrating wells. They cited the KGS (1994) semi-analytical method to interpret cross-well slug tests in both confined and unconfined aquifers. Since this method accounts for
elastic storativity and anisotropy in the aquifer, it also could be used to assess errors introduced into aquifer parameter estimates using the single-well methods of Cooper et al. (1967) and Bouwer and Rice (1976).

Belitz and Dripps (1999) used the KGS method for analysis of cross-well slug tests and compared them to results from Cooper et al. (1967) and Bouwer and Rice (1976). The values derived from the Bouwer and Rice method were lower than the other methods, and they suggested that the source of errors can arise from imprecise application of the method’s graphical procedure. Belitz and Dripps (1999) suggested that given the underestimate of hydraulic conductivity in a hypothetical test without considering the effects of a wellbore skin, it is useful to examine a hypothetical aquifer with a skin. When the Bouwer-Rice method was used to interpret the hypothetical test, it resulted in an estimate only two-thirds of the specified conductivity, which is lower than the KGS value obtained by testing the aquifer system without a skin. Belitz and Dripps (1999) concluded that the Bouwer and Rice method consistently provides field conductivity values that are lower than those indicated by the cross-well slug test.

2.5.6 Y. Jeffrey Yang and Todd M. Gates

In their paper titled, “Wellbore Skin Effect in Slug-Test Data Analysis for Low-permeability Geologic Materials,” Yang and Gates (1997) used numerical simulation and field tests to analyze slug tests results affected by wellbore skin for a well at progressive stages of development. According to Yang and Gates (1997), the zone around a well which was disturbed by drilling and smearing, should be simulated by a flow model. They used well development to remove the small particles. Based on numerical modeling results following Hyder and Butler’s (1995) research for wellbore skin, Yang and Gates (1997) noted that the Bouwer and Rice (1976) method introduces error more than two orders of magnitude in the hydraulic conductivity estimates for low-permeability materials. Also, they noted that there is no agreement on which part of data can give hydraulic conductivity estimates with little or no influence from effect of the wellbore skin. They concluded that the wellbore skin significantly affects water level recovery in low-permeability formations.
2.5.7 M. Bayani Cardenas and Vitally A. Zlotnik

Cardenas and Zlotnik (2003) conducted tests using constant-head hydraulic injection for in situ estimation of hydraulic conductivity ($K$) of sandy streambeds. Slug test methods were the most accurate means for determining the $K$ distribution of sandy streambeds among different methods used such as constant-head and falling-head permeameter tests, seepage meter, grain size analysis, and slug tests. They assumed the streambed is isotropic ($Kr = Kz = K$). Therefore, $K$ can be estimated from:

$$K = \frac{Q}{2\pi L P_y}$$

The Bouwer and Rice (1976) approximate method is most commonly used for the shape factor:

$$P = \frac{1.1}{\ln((l+L)/r_w)} + \frac{A + B \ln[b - (l + L)/r_w]}{L/r_w}$$

Where $A$ and $B$ are dimensionless coefficients.

They tested assumption of linearity of relationship between $Q$ and $y$ and verified the accuracy of the constant-head injection test (CHIT) compared with other techniques. Cardenas and Zlotnik (2003) found that CHIT allows for rapid $K$ estimation on a three-dimensional grid of sample points.

2.5.8 David L. Brown, T. N. Narasimhan, and Z. Demir

Brown et al. (1995) evaluated the Bouwer and Rice (1976) method, which is widely used for analyzing slug test data to estimate hydraulic conductivity ($K$). This method is specifically intended to be applicable to unconfined aquifers. The authors investigate the limits of accuracy of the $K$ estimates obtained with this method. By using a numerical model for transient flow, they evaluated the method from two perspectives. First, they applied the method to synthetic slug test data to study the error in estimated $K$ values. Second, they analyzed the logical basis of the method. Parametric studies helped evaluate the role of the effective radius parameter, screen length, specific storage, and
well radius on the estimated values of K. They studied the difference between unconfined and confined systems via conditions on the upper boundary of the flow domain.

For the cases studied, the Bouwer and Rice (1976) method was found to give good estimates of K values, with errors ranging from 10% to 100%. They found that the estimates of K were consistently superior to those obtained with Hvorslev's (1951) basic time lag method. Brown et al. (1995) noted that in general, the Bouwer-Rice method tends to underestimate K, the greatest errors occurring in the presence of a damaged zone around the well or when the top of the well screen is close to the water table. There is no difference manifested between confined and unconfined conditions when the top of the screen is far removed from the upper boundary of the system. They suggested that it is reasonable to infer from the simulated results that when the well screen is close to the upper boundary, the results of the Bouwer-Rice method agree more closely with a "confined" idealization than an "unconfined" idealization. In effect, this method treats the aquifer system as an equivalent radial flow permeameter with an effective radius, R(e), which is a function of the flow geometry. Their transient simulations suggest that R(e) varies with time and specific storage. Therefore, the effective radius may be reasonably viewed as a time-averaged mean value. The fact that the method provides reasonable estimates of hydraulic conductivity suggests that the empirical, electric analog experiments of Bouwer and Rice (1976) have yielded shape factors that are better than the shape factors implicit in the Hvorslev (1951) method.
3 Analyzing Slug Tests in Unconfined Aquifers

3.1 Bouwer and Rice (1976) Method

A very useful method to analyze slug tests in unconfined aquifer comes from Bouwer and Rice (1976). This method can be used for both fully or partially penetrating wells. The original model of Bouwer and Rice was developed for unconfined aquifers, but it can be used for confined aquifers if the top of well screen is below the bottom of the confining layer. The Bouwer and Rice method depends on mathematical model. The two assumptions of this model are: that the elastic storage mechanisms effects can be neglected and the water table level does not change; also during the course of a test, the saturated thickness does not change. Although the original model of Bouwer and Rice was defined for isotropic conditions, Zlotink (1994) extended the method to the general anisotropic case. Furthermore, the idea of this model is similar to Hvorslev (1951) who did empirical measurements of the effective radius with a fully penetrating well in a confined aquifer. The model uses the following equations (Todd, 2005; Butler, 1998).

\[
\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + \frac{K_z}{K_r} \frac{\partial^2 h}{\partial z^2} = 0
\]

\[H(0) = H_0\]

\[h(r, 0, t) = 0, r_w < r < R_e, t > 0\]

\[\frac{\partial h(r, B, t)}{\partial z} = 0, r_w < r < R_e, t > 0\]

\[h(R_e, z, t) = 0, 0 \leq z \leq B, t > 0\]

\[h(r_w, z, t) = H(t), d \leq z \leq H, t > 0\]

\[2\pi r_w K_r \int_d^H \frac{\partial h(r_w, z, t)}{\partial r} dz = \pi r_c^2 \frac{dH(t)}{dt}, t > 0\]

\[\frac{\partial h(r_w, z, t)}{\partial r} = 0, 0 \leq z < d, H < z \leq B, t > 0\]

The analytical solution to this model defined by above equations can be written as (Butler, 1998):
\[
\ln \left( \frac{H(t)}{H_0} \right) = -\frac{2K_r bt}{r_c^2 \ln \left( \frac{R_e}{r_w^*} \right)}
\]

Where: \( r_w^* = r_w \left( \frac{K_z}{K_r} \right)^{1/2} \)

To estimate the hydraulic conductivity of the aquifer, this method uses the plot of drawdown versus time on semi-logarithmic paper, with drawdown on the logarithmic scale, and a straight line fit to those data. The slope of the straight line fit to the response data is used to calculate the hydraulic conductivity. Here are the steps to calculate \( K \) using either the Bouwer and Rice (1976) method or the Hvorslev (1951) method:

1. Plot data points for time versus the logarithm of drawdown since the test began. For Hvorslev (1951), normalize the drawdowns (divide by the initial, maximum value).
2. Fit a straight line to the data plot visually or by an automated regression routine.
3. Calculate the slope of the fitted line. For Hvorslev (1951), use the basic time lag \( (T_0) \), the time at which normalized drawdown is 0.37; the slope becomes \(-1/T_0\).
4. Estimate the effective radius for Bouwer and Rice (1976). Assume the anisotropy ratio equals one.
5. Compute the hydraulic conductivity using the following similar equations: for Hvorslev (1951)

\[
K_r = \frac{r_c^2 \ln \left( \frac{L_e}{r_w} \right)}{2L_e T_0}
\]

Or Bouwer and Rice (1976)

\[
K_r = \frac{r_c^2 \ln \left( \frac{R_e}{r_w^*} \right) 1 \ln \left[ \frac{H_0}{H_t} \right]}{2L_e t}
\]

Where

\( K_r \) = the hydraulic conductivity of the aquifer adjacent the slugged well (L/T).
$r_c$ = the radius of the well casing (L)

$\rho_w$ = the radius of the gravel envelope or zone disturbed by drilling (L)

$R_e$ = the effective radial distance over which head is dissipated (L)

$b$ or $L_e$ = the length of screen through which water can enter (L)

$H_0$ = the maximum drawdown which occurs at time $t = 0$ (L)

$H_t$ = the drawdown at time $t = t$ (L)

$t$ = the time since $H = H_0$ (T)

d = aquifer thickness contributing water to the well (L)

Transmissivity can be calculated by multiplying the above equation by the thickness of the aquifer as the following equation:

$$T = \frac{d \cdot r_c^2 \ln\left(\frac{R_e}{\rho_w}\right)}{2L_e} \frac{1}{t \ln\left(\frac{H_0}{H_t}\right)}$$

Bouwer and Rice (1976) used empirical equations to estimate $\ln\left(\frac{R_e}{\rho_w}\right)$ values. They obtained the following coefficients by fitting data from many experiments:

$$\ln\left(\frac{R_e}{\rho_w}\right) = \left[\frac{1.1}{\ln\left(\frac{H}{\rho_w}\right)} + \frac{A + B \left(\ln\left(\frac{D - H}{\rho_w}\right)\right)}{L_e/\rho_w}\right]^{-1}$$

Where: $A$, $B$ are empirical coefficients, (dimensionless).

Bouwer and Rice (1976) used the following equation when the bottom of well was at an impermeable layer, or $D = H$.

$$\ln\left(\frac{R_e}{\rho_w}\right) = \left[\frac{1.1}{\ln\left(\frac{H}{\rho_w}\right)} + \frac{C}{L_e/\rho_w}\right]^{-1}$$

Where: $C$ is an empirical coefficient, (dimensionless).

$\ln\left(\frac{R_e}{\rho_w}\right)$ was calculated using an electrical resistance network for various combinations of well length ($L_w$), well radius ($\rho_w$), aquifer thickness ($D$), and screen length ($L_e$). Results of the network simulation are given as curves of three parameters $A$, $B$, and $C$. Analytic expression for values of these parameters was formulated by regression analysis. The resulting expressions are:

$$A = 1.4720 + 3.537 \times 10^{-2} (L_e/\rho_w) - 8.148 \times 10^{-5} (L_e/\rho_w)^2 + 1.028 \times 10^{-7} (L_e/\rho_w)^3 - 6.484 \times 10^{-11} (L_e/\rho_w)^4 + 1.573 \times 10^{-14} (L_e/\rho_w)^5$$
\[
B = 0.2372 + 5.151 \times 10^{-3} \left( \frac{L_e}{r_w} \right) - 2.682 \times 10^{-6} \left( \frac{L_e}{r_w} \right)^2 - 3.491 \times 10^{-10} \left( \frac{L_e}{r_w} \right)^3 + 4.738 \times 10^{-13} \left( \frac{L_e}{r_w} \right)^4
\]
\[
C = 0.7920 + 3.993 \times 10^{-2} \left( \frac{L_e}{r_w} \right) - 5.743 \times 10^{-5} \left( \frac{L_e}{r_w} \right)^2 + 3.858 \times 10^{-8} \left( \frac{L_e}{r_w} \right)^3 - 9.659 \times 10^{-12} \left( \frac{L_e}{r_w} \right)^4
\]
(Butler, 1998).

Figure 3-1 shows the curves for A, B and C.

Figure 3-1 Curves of coefficients A, B, and C versus log(L_e/r_w).

During the slug test the saturated thickness of the aquifer is the whole saturated thickness regardless of capillary fringe, so the water table position does not change. Flow above the water level is also neglected. The well losses are negligible, and the aquifer is assumed to be homogeneous and isotropic. After suddenly removing the slug from the well, the rise of the water level in the well is measured over time. In the Bouwer and Rice (1976) equation for K, the \( \ln \left( \frac{R_e}{r_w} \right) \) term is evaluated using one of the equations above. Then, fitting a straight line to a plot of \( \ln H_t \) versus time will allow the choice of two points from that straight line (\( t = 0, H_o \); and \( t, H_t \)) to use in the other \( \ln \) term in the equation, \( \ln \left( \frac{H_o}{H_t} \right) / t \), which represents the slope of the straight line. Then, \( K \) can be calculated.
In the paper “The Bouwer and Rice Slug Test – An Update,” Bouwer (1989a) shows that sometimes when the drawdown data were plotted as $\ln H_t$ versus $t$, the graph had two different straight lines in early time, called a double straight line effect (see also Butler, 1998) as shown in Figure 3-2. Bouwer (1989a) assumed that a coarse gravel pack around the well produced the double straight line effect, with the earlier, steeper straight line resulting from water moving rapidly through the highly-permeable gravel pack into the well, and the later, less steep line resulting from water moving less rapidly through the aquifer into the gravel pack and then into the well. Bouwer suggested that the well radius, $r_w$, is the radial distance from the outer surface of the gravel pack to the center of the well when a double straight line effect is evident.

![Figure 3-2 The double straight-line effect.](image)

### 3.2 Hvorslev (1951) Method

In 1951, Mikael Juul Hvorslev developed one of the most popular methods of analyzing slug test data. This method can be applied to both confined and unconfined...
aquifers (Weight, 2008). The Hvorslev method is based on the following three assumptions (Todd, 2005):

1. Although the hydraulic head in the aquifer still varies with time, the specific storage is assumed to be very small that its effects can be neglected. So, any change in head in the well is immediately propagated throughout the flow system, its designated as a quasi-steady-state representation of the slug-induced flow.

2. The slug does not need to be introduced in an instantaneous fashion. However, all experts agree that the slug should be inserted or removed as instantaneously as possible.

3. Lateral constant-head boundaries are at a finite distance $R_e$ of the test well.

4. The mathematical model of Hvorslev 1951 in fully penetrating wells is defined as follows:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = 0$$

$$H(0) = H_0$$

$$h(R_e, t) = 0, t > 0$$

$$h(R_w, t) = H(t), t > 0$$

$$2\pi r_w K_r B \frac{\partial h(r_w, t)}{\partial r} = \pi r_c^2 \frac{dH(t)}{dt}, t > 0$$

Where: $R_e$ is effective radius of the slug test, (L).

The analytical solution to this mathematical model can be written as:

$$\ln \left( \frac{H(t)}{H_0} \right) = -\frac{2K_r B t}{r_c^2 \ln(R_e/R_w)}$$

To estimate the hydraulic conductivity component in Hvorslev method, there are four steps (Butler, 1998):

1. Plot the logarithm of the normalized drawdown versus time since the test began.
2. Fit a straight line to the data visually or by an automated regression routine.
3. Calculate the slope of the fitted line. A common method to calculate the slope is to read from the fitted line the time at which a normalized head of 0.368 (the natural logarithm of which is -1) occurs. This time, designated as $T_0$, is termed the basic time.
lag by Hvorslev (1951). Since the logarithm of the normalized head and the time are both zero at the beginning of the test, the slope calculated in this manner is just $\log_{10}$ of 0.368 over $T_0$, which becomes $-1/T_0$.

4. Calculate the hydraulic conductivity from the Hvorslev (1951) equation:

$$K_r = \frac{r_c^2 \ln(R_e/r_w)}{2BT_0}$$

Where: $T_0 =$ time at which a normalized head of 0.368 occurs, (T) in Figure 3-3.

There are three issues with above equation (Todd, 2005):

1. The assumption of a straight line fit to the semilog plot of the test data is appropriate as long as the effect of the elastic storage mechanisms can be neglected. It is common, however, for storage mechanisms to have some effect on the response data. In many cases the test data will display a distinct concave-upward curvature when it plotted in a semilog format. In 1989, Gary R. Chirlin provided a theoretical explanation of this behavior, showing that the concave-upward curvature is primarily a function of the dimensionless storage parameter ($\alpha$).

2. This equation requires an estimate of $R_e$, which is the effective radius of the slug test, $R_e$, an empirical parameter that is a function of $\alpha$. Typically values of the well screen length, $L_e$, or 200 times the effective radius of the well screen are used for $B$ and $R_e$. If the well screen length $L_e$ is less than 8 times $r_w$, the above equation should not be used (Fetter, 2001).

3. The slug does not have to be introduced nearly instantaneously in the Hvorslev method as in the Cooper et al. and other slug test methods. A quasi-steady-state represents the slug-induced flow, eliminating the need for nearly instantaneous initiation, with the assumption that the slug introduction is completed prior to the collection of response data. The Hvorslev method can be used to analyze the response data that have been affected by non-instantaneous slug introduction. However, all experts agree that to get better data and a clean test, the slug should be inserted or removed as instantaneously as possible.
Figure 3-3 Basic time lag (63 seconds) is when water level change is 37% of initial.

### 3.3 Dagan (1978) Method

A mathematical solution by Dagan (1978) is useful for the analysis of slug tests in partially penetrating wells in unconfined aquifers. Cole and Zlotnik (1994) and Widdowson et al. (1990) outline extensions of the approach to confined aquifers. This method is similar to the Bouwer and Rice method, with only one difference that a constant-head boundary is not assumed at a finite radial distance from the test well. Instead of, the hydrologic boundaries in the lateral plane are assumed to be at an infinite distance from the well (Butler, 1998).

\[
\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + \frac{K_z}{K_r} \frac{\partial^2 h}{\partial z^2} = 0
\]

\[H(0) = H_0\]

\[h(r, 0, t) = 0, \quad r_w < r < \infty, \quad t > 0\]

\[\frac{\partial h(r, B, t)}{\partial z} = 0, \quad r_w < r < \infty, \quad t > 0\]
\( h(\infty, z, t) = 0, \ 0 \leq z \leq B, \quad t > 0 \)

\( h(r_w, z, t) = H(t), \ d \leq z \leq H, \quad t > 0 \)

\[ 2\pi r_w K_r \int_d^H \frac{\partial h(r_w, z, t)}{\partial r} \, dz = \pi r_c^2 \frac{dH(t)}{dt}, \quad t > 0 \]

\[ \frac{\partial h(r_w, z, t)}{\partial r} = 0, \ 0 \leq z < d, \quad H < z \leq B, \quad t > 0 \]

The steps for applying this method are as follows (Butler, 1998):

1. Plot the logarithm of the normalized response data versus the time since the test began.
2. Fit a straight line to the plot data by an automated regression routine or by visual inspection.
3. Calculate the slope of the fitted line.
4. Estimate the parameter \( \Psi \) for the particular well formation configuration obtained. In most cases, the anisotropy ratio is one.
5. Using \( \Psi \), the normalized distance from the water table \((d + b)/b\), and the normalized length of the well screen \((b/B)\), select a value of \( P \), the dimensionless flow parameter.
6. Estimate the radial component of hydraulic conductivity using this equation:

\[ K_r = \frac{r_c^2 \left( \frac{1}{P} \right)}{2bT_0} \]

Where \( P \) is a dimensionless flow or shape parameter.

### 3.4 KGS (1994) Method

The KGS (Kansas Geological Survey) model using for unconfined aquifer, it is based on a mathematical model defined as follows (Butler, 1998):

\[ \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + \frac{K_z}{K_r} \frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K_r} \frac{dh}{dt} \]

\( h(r, z, 0) = 0, \ r_w < r < \infty, \quad 0 \leq z \leq B \)

\( H(0) = H_0 \)

\( h(\infty, z, t) = 0, \ t > 0, \quad 0 \leq z \leq B \)
\[
\frac{\partial h(r,0,t)}{\partial z} = \frac{\partial h(r,B,t)}{\partial z} = 0, \quad r_w < r < \infty, \quad t > 0
\]

\[
\frac{1}{b} \int_d^{d+b} h(r_w,z,t)dz = H(t), \quad t > 0
\]

\[
2\pi r_w K_r b \frac{\partial h(r_w,z,t)}{\partial r} = \pi r_c^2 \frac{dH(t)}{dt} \square(z), \quad t > 0
\]

Where: \(\square(z)\) boxcar function = 0, \(z<d\), \(z>d+b\),

=1, elsewhere.

The analytical solution to this model could be defined as (Butler, 1998)

\[
\frac{H(t)}{H_0} = f(\beta, \alpha, \psi, \frac{d}{b}, \frac{b}{B})
\]

Where

\[
\beta = K_r B t / r_c^2 \quad \text{dimensionless time parameter.}
\]

\[
\alpha = (r_w^2 S_e B) / r_c^2 \quad \text{dimensionless storage parameter.}
\]

\[
\Psi = \frac{\sqrt{K_z / K_r}}{r_w} \quad \text{dimensionless proportion of vertical to radial flow.}
\]

KGS accounts for storage properties of the media in the analysis, and requires that the slug be introduced in a near-instantaneous fashion relative to the aquifer response.

The KGS method consists of the following six steps:

1. Plot normalized drawdown versus the logarithm of time since the test began.
2. Estimate the aquifer and well characteristics such as \(\psi\), \(d/b\), and \(b/B\). Usually \(K_z/K_r\) parameter is assumed = 1 as a first guess.
3. The \(\alpha\) type curves are shifted along the \(x\)-axis until one of them matches the field data. Figure 3-4 shows type curves for normalized head vs. \(\log \beta\) for different values of \(\psi\) and \(\alpha\).
4. β is set to 1.0, and the real time (t_{1.0}) corresponding to β=1.0 is read from the x-axis; 
   \( \alpha_{cal} \) is obtained from the type curve most closely matching the field data.

5. An estimate of radial hydraulic conductivity will be 
   \[ K_r = \frac{r_c^2}{bt_{1.0}} \]

6. An estimate of specific storage is 
   \[ S_s = \frac{\alpha_{cal} r_c^2}{r_w b} \]

Figure 3-4 Type curves for normalized head vs. log β for different values of \( \psi \) and \( \alpha \).
4 Case Study: Location of Wells

4.1 Wells near Austin Lake, Portage, MI

The Portage aquifer is located over 100 m from Austin Lake in Portage, Michigan, at a private residence shown in. The circle in the Figure 4-1 is the approximate location of the well site which is located at latitude 42° 10' 11.7264" N and at longitude 85° 34’ 9.0084" W. The well site was chosen to be in a relatively low-elevation area where the water table is shallow, and it’s close to power outlets. The names of the five wells and the distances between them are shown in Figure 4-2.

Figure 4-1 Location of the Portage aquifer in Portage, Michigan.
4.2 Wells of Asylum Lake, Kalamazoo, MI

The wells in Asylum Lake Preserve in Kalamazoo, Michigan, are located at latitude 42° 15' 51.0372” North and longitude 85° 38' 39.4872" West. The circle at the center of Figure 4-3 is the approximate location of the wells. The geology of the area consists of glacial deposits. The aquifer in this site is an unconfined water table aquifer, in which the water table is approximately 60 ft below the surface Figure 4-4.
Figure 4-3 Location of wells in Asylum Lake Preserve, Kalamazoo, Michigan.

Figure 4-4 Well AL-164.
4.3 Slug Test Data Acquisition

A Hermit 2000 data logger and In-Situ transducers were used in the field to collect slug test data. Later the data stored in the device were transferred to a computer.

4.3.1 HERMIT 2000

The HERMIT 2000 was once the most popular data logger due to its one button step test capability and ability to store data from 8 channels at once Figure 4-5. This was used during the slug test experiments in the field. The LCD HERMIT 2000 is able to record as many as 20 different tests in memory, each with its own unique setup and data, before having to dump the data between tests to free up memory. Tests are recorded sequentially from test 0 to test 19. The first step for running a test is to define the basic test conditions such as: the test number, how many input channels will be used, at what rate to sample the input channels, and what type of data will be collected on each input channel.

Figure 4-5 Hermit 2000 data logger (from Alfaifi, 2015).

4.3.2 Transducers

Pressure transducers convert a physical quantity (which is pressure) into an electrical signal. The semiconductor strain-gauge transducer is the most common type of transducer used for slug tests (Butler, 1998). Before starting the slug tests, the number of input channels is programmed in the data logger. The transducers are connected to the
channels that will be recorded during the slug test. Every transducer needs to have its quadratic calibration parameters, scale, linearity, and offset, correctly entered into the data logger Figure 4-6 to convert the electrical signal to a pressure to a drawdown.

![Figure 4-6 In-Situ pressure transducers and their calibration coefficients.](image)

### 4.3.3 Slug Rods

We used two different sizes of slug rods during slug tests. The larger slug is 6.91 ft long and 0.125 ft in diameter, made from a solid PVC rod with some parts routed out to accommodate the transducer and its vented cable. The smaller slug is a 1 inch diameter PVC pipe filled with sand and capped. It is 5 ft long, 0.087 ft in diameter Figure 4-7.

![Figure 4-7 Larger slug and smaller slug.](image)
4.4 Slug Test Analysis

The software used to analyze the slug test data is AQTESOLV (see Figure 4-8 through 4-12), which is designed for the analysis of aquifer tests (pump tests, slug tests and others).

Figure 4-8 AQTESOLV program.
Figure 4-9 Hydraulic conductivity obtained from Bouwer and Rice method.

Figure 4-10 Hydraulic conductivity obtained from Hvorslev method.
Figure 4-11 Hydraulic conductivity obtained from Dagan method.

Figure 4-12 Hydraulic conductivity obtained from KGS method.
5 Results and Discussion

5.1 Hydraulic Conductivity

The average hydraulic conductivity values were calculated for each slugged well, using test data obtained at each of the two sites with both the larger slug and the smaller slug. The hydraulic conductivity means were obtained using four different analysis methods: Bouwer and Rice (1976), Dagan (1978), Hvorslev (1951), and Kansas Geological Survey method assuming no well skin (Hyder et al. 1994). Table 5-1 Hydraulic conductivity (K) values using small slug in Portage, lists the results of 14 slug tests conducted using a smaller slug in wells in the Portage aquifer. Table 5-2 Hydraulic conductivity (K) values using larger slug in Portage, presents the K values from 15 tests in the Portage aquifer using the larger slug. The 4 tests reported in Table 5-3 Hydraulic conductivity (K) values using smaller slug in AL-164, are for the smaller slug in AL-164. Table 5-4 Hydraulic conductivity (K) values using larger slug in AL-164, gives results from 6 tests using the larger slug in Asylum Lake in Kalamazoo. Table 5-5 Hydraulic conductivity (K) values in the Portage using slug in, shows 11 tests in the Portage aquifer using slug in. The 18 tests reported in Table 5-6 Hydraulic conductivity (K) values in the Portage using slug out, are for slug out in the Portage aquifer. Table 5-7 Hydraulic conductivity (K) values in AL-164 using slug in, represents 5 tests using slug in AL-164. The 5 tests showed in Table 5-8 Hydraulic conductivity (K) values in AL-164 using slug out, are slug out of AL-164 at Asylum Lake in Kalamazoo. Table 5-9 Means of hydraulic conductivity (K) values from groups of slug tests, summarizes the results from Tables 5-1 through 5-8.

<table>
<thead>
<tr>
<th>Well Slugged</th>
<th>Slug Size</th>
<th>B &amp; R (K) ft/day</th>
<th>Dagan (K) ft/day</th>
<th>Hvorslev (K) ft/day</th>
<th>Kansas Geological Survey (Kr) ft/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>S test 1</td>
<td>SS</td>
<td>25.4</td>
<td>21.9</td>
<td>29.7</td>
<td>26.0</td>
</tr>
<tr>
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<td>SS</td>
<td>26.0</td>
<td>23.8</td>
<td>37.3</td>
<td>39.7</td>
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<td>36.2</td>
<td>65.4</td>
<td>36.1</td>
</tr>
<tr>
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<td>SS</td>
<td>49.2</td>
<td>40.9</td>
<td>55.9</td>
<td>48.0</td>
</tr>
<tr>
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<td>38.0</td>
<td>57.6</td>
<td>54.5</td>
</tr>
<tr>
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<td>SS</td>
<td>46.8</td>
<td>39.4</td>
<td>59.5</td>
<td>58.5</td>
</tr>
<tr>
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<td>SS</td>
<td>29.4</td>
<td>24.4</td>
<td>39.8</td>
<td>26.4</td>
</tr>
<tr>
<td>S test 4</td>
<td>SS</td>
<td>31.5</td>
<td>29.1</td>
<td>45.3</td>
<td>43.4</td>
</tr>
</tbody>
</table>
Table 5-1 Hydraulic conductivity (K) values using small slug in Portage.

<table>
<thead>
<tr>
<th>Well Slugged</th>
<th>Slug Size</th>
<th>B &amp;R (K) ft/day</th>
<th>Dagan (K) ft/day</th>
<th>(Hv) (K) ft/day</th>
<th>(KGS) (Kr) ft/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>S test 1</td>
<td>SS</td>
<td>31.7</td>
<td>26.4</td>
<td>45.9</td>
<td>28.1</td>
</tr>
<tr>
<td>S test 2</td>
<td>SS</td>
<td>32.2</td>
<td>26.5</td>
<td>46.7</td>
<td>32.5</td>
</tr>
<tr>
<td>S test 4</td>
<td>SS</td>
<td>32.1</td>
<td>25.8</td>
<td>44.0</td>
<td>32.0</td>
</tr>
<tr>
<td>S test 5</td>
<td>SS</td>
<td>33.1</td>
<td>30.0</td>
<td>47.6</td>
<td>33.1</td>
</tr>
<tr>
<td>S test 11</td>
<td>SS</td>
<td>32.3</td>
<td>26.4</td>
<td>38.8</td>
<td>33.8</td>
</tr>
<tr>
<td>S test 14</td>
<td>SS</td>
<td>31.8</td>
<td>27.5</td>
<td>40.2</td>
<td>35.1</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>35.2</td>
<td>29.7</td>
<td>46.7</td>
<td>37.7</td>
</tr>
</tbody>
</table>

Table 5-2 Hydraulic conductivity (K) values using larger slug in Portage.

<table>
<thead>
<tr>
<th>Well Slugged</th>
<th>Slug Size</th>
<th>B &amp;R (K) ft/day</th>
<th>Dagan (K) ft/day</th>
<th>(Hv) (K) ft/day</th>
<th>(KGS) (Kr) ft/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>S test 5</td>
<td>BS</td>
<td>49.5</td>
<td>44.6</td>
<td>56.3</td>
<td>61.0</td>
</tr>
<tr>
<td>S test 7</td>
<td>BS</td>
<td>38.1</td>
<td>42.0</td>
<td>50.0</td>
<td>44.0</td>
</tr>
<tr>
<td>S test 15</td>
<td>BS</td>
<td>55.8</td>
<td>51.5</td>
<td>78.5</td>
<td>72.5</td>
</tr>
<tr>
<td>S test 16</td>
<td>BS</td>
<td>58.6</td>
<td>47.3</td>
<td>80.5</td>
<td>48.0</td>
</tr>
<tr>
<td>S test 18</td>
<td>BS</td>
<td>53.4</td>
<td>47.6</td>
<td>60.2</td>
<td>48.1</td>
</tr>
<tr>
<td>S test 6</td>
<td>BS</td>
<td>22.8</td>
<td>25.7</td>
<td>32.8</td>
<td>28.0</td>
</tr>
<tr>
<td>S test 8</td>
<td>BS</td>
<td>23.2</td>
<td>25.9</td>
<td>33.2</td>
<td>33.4</td>
</tr>
<tr>
<td>S test 9</td>
<td>BS</td>
<td>23.2</td>
<td>28.0</td>
<td>34.6</td>
<td>50.6</td>
</tr>
<tr>
<td>S test 10</td>
<td>BS</td>
<td>23.2</td>
<td>27.8</td>
<td>35.0</td>
<td>44.1</td>
</tr>
<tr>
<td>S test 2</td>
<td>BS</td>
<td>32.3</td>
<td>28.1</td>
<td>45.4</td>
<td>28.1</td>
</tr>
<tr>
<td>S test 7</td>
<td>BS</td>
<td>31.6</td>
<td>26.6</td>
<td>44.0</td>
<td>30.9</td>
</tr>
<tr>
<td>S test 8</td>
<td>BS</td>
<td>32.6</td>
<td>28.5</td>
<td>47.3</td>
<td>33.0</td>
</tr>
<tr>
<td>S test 9</td>
<td>BS</td>
<td>24.8</td>
<td>21.6</td>
<td>31.8</td>
<td>25.5</td>
</tr>
<tr>
<td>S test 17</td>
<td>BS</td>
<td>31.0</td>
<td>25.9</td>
<td>32.1</td>
<td>25.1</td>
</tr>
<tr>
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<td>BS</td>
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<td>68.9</td>
<td>100.2</td>
<td>61.3</td>
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<tr>
<td>Average</td>
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<td>38.7</td>
<td>36.0</td>
<td>50.8</td>
<td>42.3</td>
</tr>
</tbody>
</table>

Table 5-3 Hydraulic conductivity (K) values using smaller slug in AL-164.

<table>
<thead>
<tr>
<th>Well Slugged</th>
<th>Slug Size</th>
<th>B &amp;R (K) ft/day</th>
<th>Dagan (K) ft/day</th>
<th>(Hv) (K) ft/day</th>
<th>(KGS) (Kr) ft/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL-164 test 13</td>
<td>SS</td>
<td>9.7</td>
<td>6.6</td>
<td>12.6</td>
<td>12.0</td>
</tr>
<tr>
<td>AL-164 test 14</td>
<td>SS</td>
<td>9.8</td>
<td>6.7</td>
<td>12.7</td>
<td>9.8</td>
</tr>
<tr>
<td>AL-164 test 15</td>
<td>SS</td>
<td>8.9</td>
<td>6.0</td>
<td>11.7</td>
<td>12.5</td>
</tr>
<tr>
<td>AL-164 test 16</td>
<td>SS</td>
<td>9.9</td>
<td>6.7</td>
<td>12.5</td>
<td>11.4</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>9.6</td>
<td>6.5</td>
<td>12.4</td>
<td>11.4</td>
</tr>
<tr>
<td>Well Slugged</td>
<td>Slug Size</td>
<td>B &amp; R (K) ft/day</td>
<td>Dagan (K) ft/day</td>
<td>(Hv) (K) ft/day</td>
<td>(KGS) (Kr) ft/day</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>------------------</td>
</tr>
<tr>
<td>AL-164 test 7</td>
<td>BS</td>
<td>9.1</td>
<td>6.1</td>
<td>11.6</td>
<td>12.1</td>
</tr>
<tr>
<td>AL-164 test 8</td>
<td>BS</td>
<td>9.1</td>
<td>6.2</td>
<td>11.6</td>
<td>9.8</td>
</tr>
<tr>
<td>AL-164 test 9</td>
<td>BS</td>
<td>9.4</td>
<td>6.6</td>
<td>12.5</td>
<td>13.4</td>
</tr>
<tr>
<td>AL-164 test 10</td>
<td>BS</td>
<td>9.3</td>
<td>6.3</td>
<td>12.5</td>
<td>9.5</td>
</tr>
<tr>
<td>AL-164 test 11</td>
<td>BS</td>
<td>9.3</td>
<td>6.3</td>
<td>12.5</td>
<td>15.6</td>
</tr>
<tr>
<td>AL-164 test 12</td>
<td>BS</td>
<td>9.3</td>
<td>6.3</td>
<td>12.1</td>
<td>12.0</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>6</td>
<td>9.2</td>
<td>6.3</td>
<td>12.1</td>
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</table>

Table 5-4 Hydraulic conductivity (K) values using larger slug in AL-164.

<table>
<thead>
<tr>
<th>Well Slugged</th>
<th>Slug In</th>
<th>B &amp; R (K) ft/day</th>
<th>Dagan (K) ft/day</th>
<th>(Hv) (K) ft/day</th>
<th>(KGS) (Kr) ft/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>S test 1</td>
<td>In</td>
<td>25.4</td>
<td>21.9</td>
<td>29.7</td>
<td>26.0</td>
</tr>
<tr>
<td>S test 3</td>
<td>In</td>
<td>26.0</td>
<td>23.8</td>
<td>37.3</td>
<td>39.7</td>
</tr>
<tr>
<td>S test 5</td>
<td>In</td>
<td>49.5</td>
<td>44.6</td>
<td>56.3</td>
<td>61.0</td>
</tr>
<tr>
<td>S test 7</td>
<td>In</td>
<td>38.1</td>
<td>42.0</td>
<td>50.0</td>
<td>44.0</td>
</tr>
<tr>
<td>S test 3</td>
<td>In</td>
<td>45.0</td>
<td>36.2</td>
<td>65.4</td>
<td>36.1</td>
</tr>
<tr>
<td>S test 10</td>
<td>In</td>
<td>49.2</td>
<td>41.0</td>
<td>55.9</td>
<td>48.0</td>
</tr>
<tr>
<td>S test 12</td>
<td>In</td>
<td>47.1</td>
<td>38.0</td>
<td>57.6</td>
<td>54.5</td>
</tr>
<tr>
<td>S test 13</td>
<td>In</td>
<td>46.8</td>
<td>39.4</td>
<td>59.2</td>
<td>58.5</td>
</tr>
<tr>
<td>S test 15</td>
<td>In</td>
<td>55.8</td>
<td>51.5</td>
<td>78.5</td>
<td>72.5</td>
</tr>
<tr>
<td>S test 16</td>
<td>In</td>
<td>58.6</td>
<td>47.3</td>
<td>80.5</td>
<td>48.0</td>
</tr>
<tr>
<td>S test 18</td>
<td>In</td>
<td>53.4</td>
<td>47.6</td>
<td>60.2</td>
<td>48.1</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>11</td>
<td>45.0</td>
<td>39.4</td>
<td>57.3</td>
</tr>
</tbody>
</table>

Table 5-5 Hydraulic conductivity (K) values in the Portage using slug in.

<table>
<thead>
<tr>
<th>Well Slugged</th>
<th>Slug Out</th>
<th>B &amp; R (K) ft/day</th>
<th>Dagan (K) ft/day</th>
<th>(Hv) (K) ft/day</th>
<th>(KGS) (Kr) ft/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>S test 2</td>
<td>Out</td>
<td>29.4</td>
<td>24.4</td>
<td>39.8</td>
<td>26.4</td>
</tr>
<tr>
<td>S test 4</td>
<td>Out</td>
<td>31.5</td>
<td>29.1</td>
<td>45.3</td>
<td>43.4</td>
</tr>
<tr>
<td>S test 6</td>
<td>Out</td>
<td>22.8</td>
<td>25.7</td>
<td>32.8</td>
<td>29.0</td>
</tr>
<tr>
<td>S test 8</td>
<td>Out</td>
<td>23.2</td>
<td>25.9</td>
<td>33.2</td>
<td>33.4</td>
</tr>
<tr>
<td>S test 9</td>
<td>Out</td>
<td>23.2</td>
<td>28.0</td>
<td>34.6</td>
<td>50.6</td>
</tr>
<tr>
<td>S test 10</td>
<td>Out</td>
<td>23.2</td>
<td>27.8</td>
<td>35.0</td>
<td>44.1</td>
</tr>
<tr>
<td>S test 1</td>
<td>Out</td>
<td>31.7</td>
<td>26.4</td>
<td>45.9</td>
<td>28.1</td>
</tr>
<tr>
<td>S test 2</td>
<td>Out</td>
<td>32.3</td>
<td>28.1</td>
<td>45.4</td>
<td>28.1</td>
</tr>
</tbody>
</table>

38
<table>
<thead>
<tr>
<th>Well Slugged</th>
<th>Slug</th>
<th>B &amp; R (K) ft/day</th>
<th>Dagan (K) ft/day</th>
<th>(Hv) (K) ft/day</th>
<th>(KGS) (Kr) ft/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL-164 test 13</td>
<td>In</td>
<td>12.6</td>
<td>2.6</td>
<td>9.7</td>
<td>12.0</td>
</tr>
<tr>
<td>AL-164 test 15</td>
<td>In</td>
<td>11.7</td>
<td>6.0</td>
<td>8.9</td>
<td>12.5</td>
</tr>
<tr>
<td>AL-164 test 7</td>
<td>In</td>
<td>11.6</td>
<td>6.1</td>
<td>9.1</td>
<td>12.1</td>
</tr>
<tr>
<td>AL-164 test 9</td>
<td>In</td>
<td>12.5</td>
<td>6.6</td>
<td>9.4</td>
<td>13.4</td>
</tr>
<tr>
<td>AL-164 test 11</td>
<td>In</td>
<td>12.5</td>
<td>6.3</td>
<td>9.3</td>
<td>15.6</td>
</tr>
<tr>
<td>Average</td>
<td>5</td>
<td>12.2</td>
<td>6.3</td>
<td>9.3</td>
<td>13.1</td>
</tr>
</tbody>
</table>

Table 5-7 Hydraulic conductivity (K) values in AL-164 using slug in.

<table>
<thead>
<tr>
<th>Well Slugged</th>
<th>Slug</th>
<th>B &amp; R (K) ft/day</th>
<th>Dagan (K) ft/day</th>
<th>(Hv) (K) ft/day</th>
<th>(KGS) (Kr) ft/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL-164 test 14</td>
<td>Out</td>
<td>12.7</td>
<td>6.7</td>
<td>9.8</td>
<td>9.8</td>
</tr>
<tr>
<td>AL-164 test 16</td>
<td>Out</td>
<td>12.5</td>
<td>6.70</td>
<td>9.9</td>
<td>11.4</td>
</tr>
<tr>
<td>AL-164 test 8</td>
<td>Out</td>
<td>11.6</td>
<td>6.2</td>
<td>9.1</td>
<td>9.8</td>
</tr>
<tr>
<td>AL-164 test 10</td>
<td>Out</td>
<td>12.50</td>
<td>6.3</td>
<td>9.3</td>
<td>9.5</td>
</tr>
<tr>
<td>AL-164 test 12</td>
<td>Out</td>
<td>12.1</td>
<td>6.3</td>
<td>9.3</td>
<td>12.0</td>
</tr>
<tr>
<td>Average</td>
<td>5</td>
<td>12.3</td>
<td>6.5</td>
<td>9.5</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Table 5-8 Hydraulic conductivity (K) values in AL-164 using slug out.
<table>
<thead>
<tr>
<th>Test Type</th>
<th>B&amp;R K(ft/d)</th>
<th>Dagan K(ft/d)</th>
<th>Hvorslev K(ft/d)</th>
<th>KGS K(ft/d)</th>
<th>Tests Done</th>
<th>all methods K(ft/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>all Portage tests</td>
<td>37.0</td>
<td>33.0</td>
<td>48.8</td>
<td>40.1</td>
<td>29</td>
<td>37.6</td>
</tr>
<tr>
<td>all Kalamazoo tests</td>
<td>9.4</td>
<td>6.4</td>
<td>12.2</td>
<td>11.8</td>
<td>10</td>
<td>10.0</td>
</tr>
<tr>
<td>all smaller slug Portage</td>
<td>35.2</td>
<td>29.7</td>
<td>46.7</td>
<td>37.7</td>
<td>14</td>
<td>32.7</td>
</tr>
<tr>
<td>all larger slug Portage</td>
<td>38.7</td>
<td>36.0</td>
<td>50.8</td>
<td>42.3</td>
<td>15</td>
<td>36.6</td>
</tr>
<tr>
<td>all small slug Kalamazoo</td>
<td>9.6</td>
<td>6.5</td>
<td>12.4</td>
<td>11.4</td>
<td>4</td>
<td>8.8</td>
</tr>
<tr>
<td>all larger slug Kalamazoo</td>
<td>9.2</td>
<td>6.3</td>
<td>12.1</td>
<td>12.1</td>
<td>6</td>
<td>9.2</td>
</tr>
<tr>
<td>all slug in Portage</td>
<td>45.0</td>
<td>39.4</td>
<td>57.3</td>
<td>48.8</td>
<td>11</td>
<td>40.3</td>
</tr>
<tr>
<td>all Slug out Portage</td>
<td>32.2</td>
<td>29.1</td>
<td>43.6</td>
<td>34.7</td>
<td>18</td>
<td>31.5</td>
</tr>
<tr>
<td>all slug in Kalamazoo</td>
<td>9.3</td>
<td>6.3</td>
<td>12.2</td>
<td>13.1</td>
<td>5</td>
<td>9.2</td>
</tr>
<tr>
<td>all slug out Kalamazoo</td>
<td>9.5</td>
<td>6.5</td>
<td>12.3</td>
<td>10.5</td>
<td>5</td>
<td>8.7</td>
</tr>
<tr>
<td>All tests</td>
<td>23.5</td>
<td>19.9</td>
<td>30.8</td>
<td>26.2</td>
<td>39</td>
<td>23.2</td>
</tr>
</tbody>
</table>

Table 5-9 Means of hydraulic conductivity (K) values from groups of slug tests.

The hydraulic conductivity means obtained using four different analysis methods, Bouwer and Rice (1976), Dagan (1978), Hvorslev (1951), and Kansas Geological Survey (1994) without skin, were then compared Figure 5-1.

![Average K(ft/day) Slug in vs. Slug out](image)

Figure 5-1 Comparison of K means from different methods using slug in and out.
The comparison shows that the hydraulic conductivity means are significantly larger using slug in than the K values obtained using the slug out (Figure 5-1 and Table 5-9). Table 9 shows that are particularly true in the Portage aquifer, in which all the wells have a diameter of 5 cm (2 inches). Well AL-164 in the unconfined aquifer at Asylum Lake in Kalamazoo is 10 cm in diameter (4 inches); there was very little difference between slug in and slug out in that well, which is much wider than both slugs. Why did slug in result in higher K values than slug out? From the point of view of physics, there should be no difference. The difference may be due to the speed of slug insertion and removal. This hypothesis could be tested by varying and recording the speed.

Several of these K means are different, as is evident from the figures and tables. It would be prudent to use the power of statistics to test whether these differences are significant. That analysis follows.

5.2 Statistical Analysis of Results

Statistical tools have been developed to compare the means of different populations to determine whether their differences are significant. The t test is used to compare two different populations to see if they are statistically similar. Analysis of variance (ANOVA) is used to compare three or more populations for similarity. The main point of comparison is the population means, though the method looks at the variances around those means. The method assumes the populations are Gaussian, or normally distributed. Since hydraulic conductivity is known to be log-normally distributed, all ANOVA analyses were done upon log-transformed K values so the populations would become Gaussian and satisfy the method’s assumptions.

Results of the 39 slug tests were analyzed with a 4-factorial mixed model analysis of variance using package phia in R statistical software, version 3.0.2 (R core team, 2014). The response variable was log hydraulic conductivity (K). Treatment factors were SlugSize (smaller, larger), SlugInOut (slug in versus slug out), Well (Asylum Lake well AL-164, Portage aquifer well S), and Test (four methods to calculate K). Because several observations were made at both sites, well (or aquifer) was included as a random factor in
the model. K was log transformed to meet the assumption of the linear model. F represents the F statistic used in ANOVA. Values of probability (P) of main and interaction effects, which represent the probabilities of obtaining the observed results by chance alone, were obtained using package phiacar with Kenward-Roger degrees of freedom (DF) in Table 5-10. The degrees of freedom are the number of variables minus one.

The results of the four-way ANOVA are that log K depends upon only three of the four treatment factors: highly significantly upon slug in/out, the analysis method used, and the aquifer tested. Average log K for the Portage aquifer (well S) was significantly larger than average log K for the AL-164 (Figure 5-2; Location effect: \( F_{1272.8} = 48.88, P << 0.001 \)). This difference was expected; aquifers usually have different K values. Also, mean log K’s produced by the Hvorslev (1951) method are highly significantly larger than mean log K’s produced by the other three methods, Bouwer and Rice (1976), Dagan (1978), and KGS (1994) (Figure 5-2; Test effect: \( F_{36.81} = 1.41, P << 0.001 \)). This is consistent with the results found by Alfaifi (2015). Average log K for tests using the slug in was highly significantly larger than average log K for tests using the slug out, which was not expected (Figure 5-3; Slug in/out effect: \( F_{46.188} = 1.77, P << 0.001 \)). There were also interactions among two of the treatment factors, SlugInOut and SlugSize, the latter of which otherwise had no statistical impact on K values Table 5-10. The difference in average log K between slug in and slug out was significantly greater for the larger slug than for the smaller slug (see Figure 5-3; Slug size: Slug in/out effect: \( F_{7.39} = 0.284, P = 0.0075 < 0.01 \)). No other interaction effects were significant (P > 0.5).

<table>
<thead>
<tr>
<th>Response: log K</th>
<th>Effect or treatment</th>
<th>DF</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr (&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well (aquifer)</td>
<td></td>
<td>1</td>
<td>48.88</td>
<td>48.88</td>
<td>1272.8123</td>
<td>&lt; 2.2E-16</td>
</tr>
<tr>
<td>Slug Size (larger or smaller)</td>
<td></td>
<td>1</td>
<td>0.04</td>
<td>0.04</td>
<td>1.0507</td>
<td>0.307299</td>
</tr>
<tr>
<td>Test (4 analysis methods)</td>
<td></td>
<td>3</td>
<td>4.241</td>
<td>1.414</td>
<td>36.8092</td>
<td>&lt; 2.2E-16</td>
</tr>
<tr>
<td>Slug In/Out</td>
<td></td>
<td>1</td>
<td>1.774</td>
<td>1.774</td>
<td>46.1882</td>
<td>3.74E-10</td>
</tr>
<tr>
<td>Interactions of effects</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slug Size: Test</td>
<td></td>
<td>3</td>
<td>0.061</td>
<td>0.02</td>
<td>0.5312</td>
<td>0.661656</td>
</tr>
<tr>
<td>Slug Size:Slug In/Out</td>
<td></td>
<td>1</td>
<td>0.284</td>
<td>0.284</td>
<td>7.3869</td>
<td>0.007488</td>
</tr>
<tr>
<td>Test:Slug In/Out</td>
<td></td>
<td>3</td>
<td>0.103</td>
<td>0.034</td>
<td>0.8948</td>
<td>0.445914</td>
</tr>
</tbody>
</table>
Table 5-10 Statistical analysis of results.

<table>
<thead>
<tr>
<th>Slug Size:Test:Slug In/Out</th>
<th>3</th>
<th>0.016</th>
<th>0.005</th>
<th>0.1389</th>
<th>0.936563</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residuals</td>
<td>127</td>
<td>4.877</td>
<td>0.038</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Significance codes</td>
<td>** Significant (P &lt; 0.01)</td>
<td>*** Highly Significant (P &lt; 0.001)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Log K differed substantially between the wells. This effect of well is held constant by the statistical model, allowing us to focus on the other factors. Taking the effect of the well into consideration, there is a significant difference in log K among the 4 tests. Analysis of orthogonal contrasts indicates that the Hvorslev method (1951) mean K value is significantly larger than the means of the other three tests, and the Dagan method (1978) mean K is significantly smaller than the KGS (1994) mean K. Bouwer and Rice (1976) mean K is not significantly different from the mean K’s of Dagan and KGS. There
is a significant interaction between slug size and slug in/out. This occurred because the difference in log K between slug in and slug out was much greater for the large slug than the smaller slug Figure 5-3.

![Graph showing Log K means by slug size and slug in/out. Error bars are ±1 SE.](image)

Figure 5-3 Log K means by slug size and slug in/out. Error bars are ±1 SE.

The size of the slug’s initial water level deflection, \( Y_o \), doesn’t affect the results for hydraulic conductivity, \( K \), values Figure 5-4. This disagreed with the hypothesis advanced by Dan Greene, a noted slug test expert with FTCH consultants in Grand Rapids, Michigan.
Figure 5-4 Slug’s initial water level deflection $Y_0$. 

$R^2 = 0.2315$
6 Conclusions and Future Research

6.1 Conclusions

Aquifer parameters can be estimated by many different methods. The slug test method is one of the most important techniques to estimate hydraulic conductivity (K) in situ in unconfined aquifers. Slug tests move the static water level suddenly up or down and record the changes in water level inside the well while it returns to its static condition. There are several methods to analyze slug test data in different aquifers. In unconfined aquifers, Bower and Rice (1976), Hvorslev (1951), Dagan (1978) and Kansas Geological Survey (KGS) (1994) are the most commonly-used methods. All methods rely on mathematical models to determine hydraulic conductivity from unconfined aquifers depending on the effective radius. But they don’t incorporate in their equations the effects of a slug rod’s size or the size of the initial water level deflection on the effective radius.

The goal of this study was to answer questions about how to slug test wells: Which methods are better for performing and analyzing slug tests? Does the size of the slug affect the results, or do large initial water level displacements produce better results than small displacements? Knowing these answers could help conduct slug tests that produce K values closer to pump test values (i.e., larger).

The four-way ANOVA examination of 39 slug tests on wells in two aquifers confirmed that log hydraulic conductivity (K) depends highly significantly upon slug in/out and the analysis method used, the aquifer tested, and significantly upon slug size of slug in/out. Average log K for the Portage aquifer (well S) was highly significantly larger than average log K for the Asylum Lake well AL-164. Average log K for tests in 5-cm diameter wells using slug in was highly significantly larger than average log K for tests using slug out, which was not expected. Also, mean log K’s produced by the Hvorslev (1951) method are highly significantly larger than mean log K’s produced by the other three methods, Bouwer and Rice (1976), Dagan (1978), and KGS (1994).

The size of the slug’s initial deflection, Yo, did not affect the results for hydraulic conductivity values in these tests. The size of the slug rod had only a weak, statistically insignificant effect on K values, mostly on slug in compared to slug out; Alfaifi (2015) found a significant difference between K values based on slug size. Finally, assuming
that a large K value better represents the true value, closer to a pump test K value, then Hvorslev (1951) gives the largest K values.

Larger K values typical of pump tests are generally known to be superior to smaller values from slug tests, largely due to inadequate development of wells that are slugged (Butler and Healy, 1998). Butler (1998) says that “the hydraulic-conductivity estimate obtained from a slug test should virtually always be viewed as a lower bound on the hydraulic conductivity of the formation in the vicinity of the well.” That is why larger K values are considered to be inherently better or more potentially true than smaller values.

An ideal test of that hypothesis was carried out by Dennis (1987), who demonstrated that by developing a slugged well between rounds of slug tests, resulting K values increased until they achieved parity with the results of a pump test performed at the end of the experiments at that well.

Nevertheless, the ideal of slug testing and pump testing the same well is difficult to achieve, because when a pump is in a pumping well, the well cannot be slug tested. The pump and discharge pipe and electrical wiring all must be removed to make slug tests possible.

Another challenge in equating the K obtained from slug testing with the results obtained from pump testing is that pump tests produce values of transmissivity, which can be converted to K values by dividing by the aquifer thickness. Since most water-table wells are only partially penetrating, the issue becomes what thickness should be used to calculate K, the aquifer thickness or the screen length or some value in between? Weight (2008) showed that the thickness to use in that calculation increases during the pump test, eventually reaching the full saturated thickness of the aquifer being tested. For the ideal comparison of pump and slug test results, the pumped and slugged well should fully penetrate the water-table aquifer so the conversion from transmissivity to hydraulic conductivity is straightforward.

6.2 Future Research

Based on the results of this study, further studies are warranted on several related topics. First, the reasons for obtaining larger hydraulic conductivity values when inserting
the slug than when pulling out the slug from wells need to be better understood. Field experiments varying the speed of slug insertion and removal should be compared to see if that variable is significant, but still would not explain why that would be significant. Perhaps numerical modeling of slug testing using a detailed finite element code could help illuminate a cause. Second, more studies and analyses using Hvorslev (1951) and Kansas Geological Survey (KGS) (1994) methods should be conducted to see if those methods consistently produce larger K values than the Bouwer and Rice (1976) and Dagan (1978) methods in wells that fully penetrate the unconfined aquifer. Percent penetration may be a significant variable. Third, more experiments should be conducted using different size slugs, especially pneumatic (pressure and vacuum) slugs, to more clearly determine the effects of slug size and the initial water level deflection, $Y_o$, on K values. These slug tests should be performed in wells that are very close to several observation wells which can be monitored to determine the radius of influence of the slug tests as a function of slug size, slug in and out, and initial water-level deflection. Finally, fully-penetrating wells in water table aquifers should be both slug tested and pump tested to understand which factors are responsible for pump test K values being larger than slug test results.

If slug testing procedures and analysis could produce results which consistently correlated with and predicted pump test results, then slug testing could be used more confidently in place of pump tests, saving considerable time, money and effort.
References


http://water.usgs.gov/edu/earthgwaquifer.html
Appendix

Well S test 1 (06-02-2015)

Well S test 2
Well S test 3
Well S test 4

Well S test 5
Well S test 7

Well S test 8
Well S test 9
Well S test 10

Well S test 1 (07-30-2015)
Well S test 2
Well S test 2 (09-07-2015)

Well S test 3
Well S test 4
Well S test 5

- Obs. Wells: 1 Well S
- Aquifer Model: Uncoupled
- Solution: Bravais-Rice
- Parameters:
  - $K = 33.11$ ft/day
  - $J = 2.031$ ft
  - $y = 1.016$ ft

Well S test 7

- Obs. Wells: 1 Well S
- Aquifer Model: Uncoupled
- Solution: Bravais-Rice
- Parameters:
  - $K = 29.97$ ft/day
  - $J = 2.018$ ft
  - $y = 1.016$ ft
Well S test 8
Well S test 9

Well S test 10 (10-23-2015)
Well S test 11
Well S test 12

Well S test 13
Well S test 14
Well S test 15

Well S test 16
Well S test 17
Well S test 18

Well S test 19
Well - 164 test 7 (01-27-2016)
Well – 164 test 8

Well – 164 test 9
Well – 164 test 10
Well – 164 test 11

Well – 164 test 12
Well – 164 test 13
Well – 164 test 14

Well – 164 test 15
Well – 164 test 16