Application of Wavelet Transform in Structural Health Monitoring

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APPLICATION OF WAVELET TRANSFORM IN STRUCTURAL HEALTH MONITORING

by

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Structural Health Monitoring (SHM) is a process of implementing a damage detection strategy in structures to evaluate the condition of existing structures to ensure the safety of users in the future. The changes in the material, geometric and/or structural properties affect structural responses, which can be captured and analyzed for condition assessment. Various vibration-based damage detection algorithms have been developed in the past decades. Among them, the Continuous Wavelet Transform (CWT) gained popularity as an efficient method of signal processing to build a framework to identify model properties and detect damage in structures. In this study, the application of wavelet transform for identification of modal properties and damage detection is presented and numerically verified through verification examples. Then the method is used in two case studies: a single-span steel girder bridge in Holland, Michigan and a cable-stayed bridge in mainland China. The results are compared with those obtained from the Fast Fourier Transform (FFT) method. It is shown that CWT used for output-only identification and damage detection yields good agreement with results from FFT, while CWT can provide time-domain information and FFT cannot.
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CHAPTER 1

1 INTRODUCTION

1.1 Structural Health Monitoring and Damage Detection

The process of using an on-structure sensing system to monitor its performance and evaluate the health state is defined as Structural Health Monitoring (SHM) (Chang and Thambiratnam 2011). The changes of material and geometric properties of structures including boundary condition, changes in loading conditions, deterioration with age, etc. affect the structure performance and can cause damage. While some damage is visible, others may be difficult to spot. The damage causes a decrease in stiffness of the structure and hence may affect its vibration characteristics. The vibration based SHM process involves the observation of a system over time using periodically sampled dynamic response measurements from an array of sensors. In addition, SHM uses the extraction of damage-sensitive features, such as vibration characteristics and the statistical analysis to determine the current state of system health.

Sensors are placed on the structure to allow communication between sensors as well as the location of the damage. As the density of sensors increases on a structure, the quality and resolution of damage information also increases. However, sensor installation costs, sensor power consumption, and data processing capacity act as limiting factors for sensor density. Sensors measure structure quantities such as strain, displacement, and acceleration as well as environmental conditions like temperature, wind, and moisture. The output of this process periodically updates information regarding the ability of the structure to perform its intended function and degradation results from operational environments for long term serviceability.
SHM is used to provide rapid, real and reliable information regarding the functions of the structure. A complete SHM approach consists of five basic steps which involve:

(1) Detection – Identification of damage occurrence in the structure, if any
(2) Localization – Identification of single or multiple damage locations
(3) Assessment – Quantification of the level of damage
(4) Prognosis – Evaluation of structural performance and its useful remaining life
(5) Remediation – Determination, implementation, and evaluation of effective remediation and repair efforts

This concept is widely used in various types of engineering structures to reduce monetary losses and guarantee and ensure the safety of users.

The study presented in this thesis is focused on SHM of bridge structures. Bridges continuously suffer from destructive effects due to material aging, corrosion of reinforced bars in concrete structures, corrosion of steel members in steel structures. In addition, failures due to traffic volume, overloading and the overall deterioration of structural components also affect the health condition of structures. These factors result in the loss of load carrying capacity of bridges and lead to the unsatisfactory condition of bridge structures. Carrying out periodical bridge inspections and assessing its condition is a crucial challenge. Even though visual inspections and nondestructive testing methods have been used for a long time period, it does not pave the path to perform periodical evaluation of bridge structures. Therefore, a continuous monitoring system is the best solution to resolve the aforementioned issues to ensure safety of the traveling public. The most common objectives of bridge health monitoring are to obtain quantitative data about structural behavior in order to confirm design assumptions for new bridges and to evaluate the
current condition of existing bridges and allow engineers to make informed decision about maintenance plan or repair actions.

1.2 Problem Statement

The health condition of bridges must be periodically inspected and monitored due to the age deterioration and environmental effects. There are many methods to evaluate the conditions of bridges. Visual inspections are the primary inspection method where inspectors visit the field and access the structure’s condition. This method is sufficient for non-critical structures, but it does not always provide reliable results of the actual health condition. Since this method is highly depending on the experience of the inspector, the consistency of inspections is questionable. Even though well experienced inspectors are able to spot external issues of the structures, the internal delamination, cracks, loss of stiffness etc. are difficult to identify. Therefore, more accurate, effective and efficient structural monitoring systems and evaluation techniques are essential.

Identification of alteration in mode shape and natural frequency in damaged structural elements in comparison with pre-damaged state of elements is one of the popular methods in damage detection. These changes of structural modal properties are often small, and measurements are polluted by noise, which causes the method to be inefficient in detecting proper location of damage. Therefore, methods of inspection need robust methodology to identify damage location and so forth.

Most vibration based SHM techniques require the knowledge of pre-damaged status. Many researchers have used shifts in resonance frequencies to identify damage. The Damage Location
Assurance Criterion (DLAC) method is one of the common frequency shift methods used in SHM. This determines the degree of correlation between the shifts of experimented natural frequencies to the frequencies of numerically modeled damage cases. However, frequency shift methods have significant practical limitations for real world applications (Doebling et al. 1998). The forward problem method is another damage detection method used with respect to measured frequencies. The damage is represented in a mathematical model, and the measured frequencies are compared with predicted values in order to identify the damage. Cawley and Adams (1979) created a mathematical formulation to detect damage in composite materials. Mode shape comparison techniques provide a descriptive behavior of physical properties of a structure. Modal Assurance Criteria (MAC) is another mode shape based damage detection method used in SHM. A numerical model of the structure is used as a reference model and includes various cases of damage. The change in the flexibility matrix of a structure is also used in damage detection and localization. A comparison study of damaged status and undamaged status is used with the flexibility matrix using experimental data. This method does not need a reference model, which reduces the computing time and uncertainties. Further, Fast Fourier Transformation (FFT) is one of the widely used techniques in modal analysis which allows for the detection of the characteristic frequencies of structures. However, Fourier analysis does not pave the path to study the nature of the time series in the time frequency domain. Therefore, under the analysis of the Fourier Transform, the time information along the time series is lost. Hence, it is difficult to distinguish transient relations and identify when the structural changes have exactly occurred. Further, these techniques are only appropriate for the time series with stable statistical properties i.e. stationary time series (Conraria and Soares 2011). As an alternative, the wavelet transform has been proposed. In this study, CWT is used for the analysis.
Research Objectives and Scope

The main goal of this research is to apply CWT for SHM of bridges to identify modal properties and detect damage. The vibration based SHM is subjected to a prior environmental noise removal technique and identifies the damages and modal properties of two real world applications. MATLAB simulations are used to develop algorithms and subjected to acquired acceleration data analysis for two case studies. The analyzed results are compared with FFT for further verification. The advantages of the CWT method are identified and compared to the widely used Fourier analysis system in SHM. The following objectives are used to achieve the desired goals

1) Review the state-of-the-art and practice for the application of Continuous Wavelet Transform in SHM.

2) Code CWT algorithms using MATLAB and verify them with numerical examples.

3) Identify a suitable environmental noise removal method and apply for real world applications.

4) Apply the CWT algorithm to obtain modal frequencies and damping ratios for a single span bridge in Holland, Michigan and compare the results with FFT.

5) Apply the CWT algorithm in the damage detection of a cable-stayed bridge in mainland China and compare the results with FFT.
1.3 Thesis Overview


Chapter 1: Introduction

The definition, importance and basic steps of structural health monitoring process are presented in this chapter. The problem statement and the research objectives are presented. The organization of the thesis is outlined.

Chapter 2: Literature Review

This chapter includes the basic methods of SHM including visual inspections, non-contact testing methods and vibration based testing methods. Next, literature of wavelet transforms methods used in SHM in different fields is reviewed.

Chapter 3: Environmental Noise Removal Method

The effect of noise and importance of noise removal method is discussed with basic filter types. The mathematical background, behaviors, advantages, design procedure, practical behavior and real world application of a specific filter type is further discussed.
Chapter 4: CWT Theory and Numerical Algorithms

The mathematical background, usage, and the importance of the CWT method are presented. The advantages of CWT method Vs. FFT is then presented. Moreover, the Random Decrement Technique (RDT) method used to convert the ambient vibration measurement to free decay function is discussed. Finally, the numerical algorithm of damping ratio identification using the CWT method is presented.

Chapter 5: Validation of the Numerical Algorithms

The results validation of the numerical algorithms is presented using two numerically built signals with known frequencies and damping ratio. Further, damage is being detected using two signals with reduced stiffness in order to see the changes of scalograms corresponding to the prescribed damage.

Chapter 6: Identification of Modal Properties in Holland Bridge

A detail overview of the bridge structure is presented with plan views and location. The sensor configuration during the data acquisition process and the sample frequencies are described. The results obtained using CWT and FFT algorithms are presented with a comparative study for identification of the frequency of the structure. Further, the damping ratio of the structure is identified using the CWT method.
Chapter 7: Damage Detection and Frequency Comparison of the Cable-stayed Bridge

A cable-stayed bridge in mainland China is used as a case study of using CWT to detect damage. With the acceleration data collected on-site, the CWT scalograms are generated for different stages of the bridge within a period of seven months. These scalograms are then compared to identify the damage in the structure. The results are compared with FFT.

Chapter 8: Summary, Conclusion and Future Work

This chapter summarizes the main points of the research and draws conclusions about CWT application in SHM. The recommendations for future research work are presented.
CHAPTER 2

2 LITERATURE REVIEW

2.1 Structural Health Monitoring Methods

2.1.1 Visual Inspection

The first and foremost basic structural health monitoring system is visual inspection. That is, visually inspect the cracks, delaminated areas, deformations of structural members in order to clarify the structural healthiness of the structure. Even though this is helpful as an initial inspection method, this has several drawbacks and inconsistency of results depending on the experience of the inspectors. Hence, there is a special need of using consistent reliable methods in order to overcome these drawbacks.

2.1.2 Non-contact Monitoring

Contacting sensors have to be mounted directly onto the measuring points of structures and be connected to a neighboring stationary reference point. For practical infrastructures, it is difficult to find an ideal stationary reference point near the measuring point, and it is also inconvenient for the connection between sensors and stationary reference points. Therefore contacting sensors troublesome in installation and expensive in maintenance too (Malesa et al. 2010). Non-contacting sensors can be located outside the infrastructures without being connected to a stationary reference point directly which is quite convenient for SHM compared to contact technologies for health monitoring.

The development of innovative non-contact systems for vibration measurement is convenient to use compared to contact technologies and has recently drawn the attention of several researchers
for non-contact technologies. Non-contact sensors include Laser Doppler Vibrometer (Cunha and Caetano 1999), Global Positioning Systems (Nickitopoulou et al. 2006,) and vision-based systems using digital image processing techniques etc. In addition, the microwave interferometry has recently emerged as an innovative technology, suitable to the non-contact vibration monitoring of large structures.

2.1.3 Vibration Based Structural Health Monitoring

Vibration Based Structural Health Monitoring (VBSHM) has drawn significant attention in health monitoring techniques. The basic characteristic of this method is that the characteristic changes in structures such as mass, stiffness and damping will affect to the global vibration response of the monitored structure. Thus, unknown properties of the structure can be identified by studying the changes of the measured vibration behavior. When the changes in structural properties inversely affect to the performance of structure, it will be defined as damage (Guan and Karbhari 2008). The process of identifying those structural changes is referred as vibration based damage identification or vibration based damage detection. Compared to other monitoring system, this method has the advantage of monitoring the global nature of the vibration characteristics. The capability of identifying the modal parameters such as natural frequencies and mode shapes, pave the path to monitor not only a single individual structural component but the entire structure. Hence, large civil engineering structures such as bridges can be effectively monitored with limited number of sensors and equipment. In this report, vibration based structural health monitoring for two bridge structures are performed.

Structural dynamic response measurements are achieved with an instrumentation system which includes sensors, transmission and storage of dynamic response data. Sensor type, sampling
frequencies need to be customized according to the application. Acceleration, velocity and
displacement are the most common dynamic response measurements. The instrumentation
system must be designed to manage a data collection if continuous monitoring of a structure is
needed with large amount of data.

Dynamic response of the structure is utilized in order to find the corresponding modal parameters
which contain important characteristics of the structural dynamic response. Those are easy to use
in further analysis and storage compared to actual raw data.

2.2 Data Acquisition Methods

2.2.1 Wired Sensor Networks

Wired sensor networks are consists of cables to carry different electrical signals from one end to
the other. The speed of operation is high compared to wireless networks but installation is
cumbersome and requires more time. In wired sensor network, the mobility is limited as it
operates in an area covered by a connected system. Channel interference is less as one wired
network will not affect the other.

2.2.2 Wireless Sensor Networks

A wireless sensor network consists of bunch of multi-functional sensor nodes having sensing,
computational and communication capabilities for responses in structures. It consists of four
basic components which are sensor unit, Analog Digital Converter (ADC), Central Processing
Unit (CPU), and a power unit. Sensor nodes sense or capture the physical data at the area of
interest. The sensed data by sensors is digitalized by ADC and sent to controllers for further
processing. Wireless sensor nodes are usually a small electronic device which can only be
equipped with limited power. This network can set up without any fixed infrastructure and ideal for non-reachable places such as mountains, deep forests, sea, and rural area. Implementation is comparatively cheaper than wired sensors.

2.3 Wavelet Transform for Structural Health Monitoring

Wavelet Transform (WT) has achieved the ability to overcome many of the limitations in Fourier analysis. Hence, WT is widely used not only in civil engineering field but in many fields including mechanical systems and aerospace as a signal processing tool in structural health monitoring showing its generality. WT is based on dilated scales and shifted windows which has the ability to perform a good time frequency resolution of a data signal contributed to widespread applications in engineering. A time domain signal is converted into WT in terms of the projection of the original signal on to a family of functions that are normalized dilations and translations of wavelet transform. A function $\psi(t)$ is defined as the mother wavelet and it dilates (scaled) and translates (shifted) as daughter wavelets. Scaling in WT means stretching or compressing it in the time domain. Smaller scales represent more compressed wavelets while larger scales produce more stretched wavelets. Major applications in WT were focused on feature extraction and pattern recognition.

Wavelet based in depth analysis of the status and estimation of a system’s remaining useful life was performed by Farrar et al. (2004). Patsias and Staszewski (2002) presented the possibility of damage detection using WT from optically observed mode shapes. Kumara et al. (1999) and Sohn et al. (2003) showed delamination detection of composite structures using CWT. Damage detection is performed by analyzing the structural response collected from piezoelectric sensors and observing the signal energy levels in the wavelet scalogram. Qi et al. (1997) showed that
Wavelet Multi-resolution Analysis (WMRA) not only has the ability of detecting the damage but also the level of damage from energy computed from decomposed signals. Daywood et al. (2002) successfully achieved WMRA de-noising of signals which were contaminated with noise produced by thermal effects of the structure. Yan and Yam (2004) detected small structural damage using wavelet packets and energy spectrum. Further, damage location and severity of damage was identified in a composite structure by Yam et al. (2003). These researches highlights that they can detect unobservable damages in composite structures using WT.

WT has widely used in SHM for large scale bridge structures (Reda et al. 2006). The research carried for a bridge which is hung from concrete arched girders in Calgary, Canada showed that peak acceleration response indicated in scalogram with high intensities showing high frequency component at that time. Liew and Wang (1998) represented that non propagating crack identification using WT in structural systems such as simply supported beams is more efficient compared to eigenvalue analysis. Daouka et al. (2003) determined the damage location and the size of the crack using CWT in a beam using vibration modes. The size of the crack related to the wavelet coefficients. Similarly, Gentlie and Messina (2003) showed detection of damage location and crack size from both clean and noisy data signals. Melhem and Kim (2003) suggested damage detection in a structural beam and an asphalt pavement using wavelet ridges. It has proved that damage occurrence can be detectable using CWT scalograms.

Moreover, damaged part in machineries can be detected using wavelet coefficients of the wavelet transformed signal. Giurgiuțiu et al. (2001) reviewed different damage identification in helicopter components and showed that WT is one of the efficient method for early damage detection in mechanical structures. Similarly, fatigue crack growth in rotating machineries were detected by
Rubini and Menegheiti (2001) using WT. These research backgrounds highlight the suitability of using WT in SHM in various fields.
CHAPTER 3

3 ENVIRONMENTAL NOISE REMOVAL METHOD

3.1 Introduction

Noise is irrelevant and meaningless data which naturally add to data sets during the data collection process. In any real signal analysis, noise is an unpleasant phenomenon. Removing the effects of noise is an important goal in most types of data analysis. Most existing data filtering, cleaning and pre-processing methods focus on removing noise that will negatively affect and result to the low level data errors with imperfect data collection, hence, many researchers have developed many techniques to overcome this phenomenon. In developing a noise reduction technique, keeping the original structure of the data signal is one of the main concerns. Especially researches use different types of filters and smoothing functions in order to improve the quality of collected data.

3.2 Basic Filter Types

Filter is a device which removes irrelevant features and components of a data signal in signal processing. It removes some of the frequencies which has interfered in the original data signal and reduce the effects of background noises. Filters do frequency selection or separation. As discussed, noises are the main issue that engineers face while doing signal analysis. Before starting the analysis process, it is necessary to remove unwanted noise signals from the raw data because such noise signals distort the original data of the signal. Therefore, it is necessary to identify such signals and filter out noise signal from the original wave as much as possible. It is possible to provide band widths and filter any particular part of a raw data signal depending on
frequencies and gains. The process of design of filters starts with the expected characteristics of the filter. There are four basic types of filters as shown in Figure 3-1, which are also called ideal filters. It shows ideal situation or characteristics of each type of filters. However, actual filter behaviors deviate from the ideal filter characteristics.

**Low-pass filters**

Low pass filters allow passing the entire signal which is lower than the specified cutoff frequency, $f_c$ while impeding all other frequencies which exceed the $f_c$ value. (Figure 3-1, a).

**High-pass filters**

High pass filters only allow passing high frequency signals above its cutoff frequency $f_c$, and blocking all which are below. (Figure 3-1, b).

**Bandpass filter**

Band pass filters allow passing the signal falling between a certain frequency range while blocking all other frequencies below and above the specified frequency band. (Figure 3-1, c).

**Bandstop filter**

Band stop filter allow passing the entire signal falling outside the specified band and blocking rest of the frequencies which falls within the range (band). (Figure 3-1, d).
Figure 3-1. Basic filter types. (a) Low-pass. (b) High-pass (HP). (c) Bandpass (BP). (d) Bandstop (BS).

These characteristic curves show a clear boundary between the passband and the stopband. Passband to stopband change of the characteristic curve is abrupt and the rolloff slope is infinitely steep. This type of response would be ideal because it allows to completely separate signals at different frequencies from one another. Unfortunately, such a filter characteristic curve is not physically realizable. It is required to design filters based on approximations that will meet the requirement of the application. Deciding on the best approximation involves making a compromise between various properties of the filter's transfer function.

The order of the filter is one of the important approximations due to several reasons. It is directly related to the number of components in the filter, which affect to its cost, physical size, and the
complexity of the design task. Higher order filters are more expensive, take up more space, and are more difficult to design. The primary advantage of a higher order filter is that, it will have a steeper rolloff slope than a similar lower order filter.

The rolloff rate is another important approximation. It is usually expressed as the amount of attenuation in dB for given ratio of frequencies. Some filters have steeper attenuation slopes near the cutoff frequency than others of the same order. If a filter is intended to reject a signal very close in frequency to a signal that must be passed, a sharp cutoff characteristic is desirable. Ideally, a filter has a unit gain in the passband and gain of zero in the stopband.

In practical situations, a finite transition band which is the gap in between the passband and the stopband, always exists. In the transition band, the gain of the filter changes gradually from one in the passband to zero in the stopband. Practical filters might have passband ripple, and the stopband attenuation of the filter cannot be infinite. In many applications, it is possible the gain in the passband to vary slightly from unity. This variation in the passband is the passband ripple, or the difference between the actual gain and the desired gain of unity. In practice stopband attenuation cannot be infinite.

Further, in real situations, it is required to satisfy Nyquist sampling theorem (Sedra and Smith, 2004), when selecting sampling frequency. According to the theorem, sampling frequency must be twice or greater than message signal frequency (data signal frequency) to minimize sampling errors. When the sampling frequency is greater than two times of the message signal frequency, it is also called over sampling. Filters can also be used along with the sampler to avoid interference from extremely high frequency signals. Designing a filter to attenuate a desired frequency band without distortion is somewhat challengeable.
Final aim of performing such kind of research is to use them for real world applications. It is possible to simulate, test, and take results in virtual software platform like MATLAB. But, designing a hardware circuits unit is a special necessity when it is introducing as a real product and use to provide signals directly to a structural monitoring centers without doing manual simulations. In such situations, it is required to design hardware circuits for noise filtering as well as to the frequency level detection. Hence, cost and quality factors are required to take into consideration and necessary to take decisions about design approximation such as gain, cut off frequencies, filter order and etc. for a better functioning real product.

3.3 Butterworth Filter

Butterworth filter is used in applications where the maximum passband flatness is required. Narrow transition band and roll off characteristic outside the passband are added benefits of the Butterworth filter. Hence, it is very good at simulating the passband of an ideal filter. It has no ripples and less distortion whereas in some filter types such as Chebeshev (Sedra and Smith, 2004) in the passband or stopband. Therefore, it is sometimes called maximally flat filter. Since the transition band width is less compared to other filter types the total loss of the actual signal is less even if another filter is being designed after the cutoff frequency. The values of the elements of Butterworth filter are more practical and less critical than many other filter types (Valkenburg, 1982). When order increases, the filter closes to the behavior of an ideal filter. The ideal filters have characteristics of maximum flatness, maximum pass band gain and maximum stop band attenuation. However, it is impractical to use higher order Butterworth filters as none
of the real situations behave as same as an ideal filter. Butterworth bandpass filter was used in this analysis.

**3.3.1 Filter Order Calculation**

Most of research activities do on a virtual environment at its initial stages. At some situations, it is possible to check the results by varying the order of the filter in a simulation software environment. The order of the filter can be decided by considering the results or output, but, it is not necessarily to do assumptions on filter order, since the order of the filter can directly be obtained from the filter specifications.

It is possible to use the lowpass to bandpass transformation in reverse, and convert the bandpass specification into equivalent lowpass specifications. The banspass specifications are given as two frequencies $\omega_1$ and $\omega_2$ which define the bandwidth $b_w$, are identical to those shown in Figure 3-2.
Assume that the low frequency and high frequency attenuation specifications are identical and specified by $\alpha_{\text{min}}$. The two frequencies at which the attenuation is $\alpha_{\text{min}}$ are identified as $\omega_3$ and $\omega_4$. According to the Figure 3-2 at attenuation frequencies $\omega_3$ and $\omega_4$, the frequencies below $\omega_3$ and frequencies above $\omega_4$ will be totally rolled off according to the designed filter. Also $\omega_3$ and $\omega_4$ must be satisfied the equation 3-1. Note that $\alpha_{\text{min}}$ and $\alpha_{\text{max}}$ are identical in Figure 3-2 and transform only the frequencies and not attenuation.

$$\omega_3 \omega_4 = \omega_0^2$$

3-1
Also,

$$\omega_1 \omega_2 = \omega_0^2 \quad 3-2$$

Equation 3-3 and 3-4 will be used in order to transform the frequencies in Butterworth bandpass to lowpass filter. $\Omega_p$ and $\Omega_s$ is the corresponding equivalent passband and stopband frequencies in the normalized low pass specifications.

$$\Omega_p = \frac{-\omega_2^2 + \omega_0^2}{\omega_2 (\omega_2 - \omega_1)} \quad 3-3$$

$$\Omega_s = \frac{\omega_4 - \omega_3}{\omega_2 - \omega_1} \quad 3-4$$

Considering the analysis of lowpass filter, the order of the filter $n$ is given by in terms of $\Omega_s$ and $\Omega_p$, then, the order number can be determined using Equation 3-5.

$$n = \log_{10} \left[ \left( \frac{10^{\alpha_{\text{min}, \text{dB}/10}} - 1}{10^{\alpha_{\text{max}, \text{dB}/10}} - 1} \right) \frac{\Omega_s}{\Omega_p} \right] 2 \log_{10} \frac{\Omega_s}{\Omega_p} \quad 3-5$$

### 3.4 Applications in the Case Study

According to the analyzed environmental data in the cable-stayed bridge in China, the passband width of data signal was identified after running a basic FFT (Fast Fourier Transformation)
analysis for wind velocity, wind angle, and environmental temperature for each day. Since all
the considerable frequencies affected from environment was below 0.2 Hz, finally the frequency
range was selected as 0.2 Hz-3Hz for the analyzed Chinese bridge. According to the frequency
requirements of the analyzed data, Butterworth bandpass filter characteristic curve was
developed accordingly using MATLAB. However, -3 dB standard gain and -30 dB gains were
used to design the characteristic curve at passband and stopband frequencies respectively. Hence,
all the gains with less than -30 dB was considered as noise frequencies data and removed from
the original signal. The order of the curve was developed according to the gains and frequency
range using Equation 3-5. According to the calculation, the respective frequencies were
calculated as \( \omega_0 = 0.77, \omega_1 = 0.2, \omega_2 = 3, \omega_3 = 0.1 \omega_4 = 5.8 \) and order of the Butterworth filter was
finally selected as 5.
CHAPTER 4

4 CWT THEORY AND NUMERICAL ALGORITHMS

4.1 Continuous Wavelet Transform (CWT)

Wavelet Transform is one of the most important techniques in signal processing to build a framework in the identification of modal properties. As a time frequency analysis tool, wavelet transform has the advantages of dealing with non-stationary, transient, and non-linear signals. The other very popular signal processing tool, Fourier analysis does not pave the path to study the nature of the time series in the time frequency domain. Under the analysis of Fourier Transform, the time information along the time series is lost. Hence, it is difficult to distinguish transient relations and identify when the structural changes have exactly occurred. Further, these techniques are only appropriate for the time series with stable statistical properties, i.e. stationary time series (Conraria and Soares 2011). As an alternative, the wavelet transform has been proposed. Wavelet analysis allows studying the spectral characteristics of a time series as a function of time. It clearly illuminates the changes of different periodic components along the time series. One major advantage of wavelet transform is the ability to carry out natural local analysis of time series while the wavelet stretches into a long function to measure the low frequency movements, and it compresses into a short function in order to measure the high frequency movements. The Continuous Wavelet Transform is widely used as an analysis tool which provides highly redundant information in time frequency domain. This It has the ability to recover the original time series from its transform (Conraria and Soares 2011).

In practice, there are two types of wavelet analysis called Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT). Both are extendable from a one-dimensional to
multidimensional case and implemented in a digital way. The main advantage of CWT is the ability to analyze a signal at arbitrary scales and locations. Even though, CWT is highly redundant compared to DWT, it is very time consuming. DWT uses orthogonal functions for signal decomposition and its analysis is non redundant and very fast. However, DWT analyses signals at dyadic scales and integer translations. The general CWT formula used to decompose a function \( x(t) \) into frequency-time domain is defined in the following term.

\[
CWT\{x(t)\} = W_x(\tau, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t - \tau}{s}\right) dt
\]

Where \( \psi^* \) and \( \tau \) are the complex conjugate of \( \psi \) and the translation respectively. \( W_x(\tau, s) \) Measures the similitude between the function \( x(t) \), and the time \( \tau \) and the scale \( s \). The amplitude of wavelet coefficient, \( |W_x(\tau, s)| \) can construct the wavelet amplitude map which illustrates the amplitude of the features in the original signal and variation with time. Once the amplitude map is obtained, local maxima indicate the natural frequencies of the system. The complex Morlet wavelet is commonly used for the CWT method as a mother wavelet.
4.2 Random Decrement Method

Random decrement free decay functions are extracted from ambient vibration measurements of structures which is subsequently useful to determine the damping parameters accurately without performing expensive dynamic tests (Kolling et al. ND). Random decrement technique provides the path to extract functions similar to free decay functions from vibration measurements. This technique was developed by Cole in the late 1960s in order to analyze the space structures exposed to ambient loads (Kolling et al. ND.). The basic concept of this method is to decompose the acceleration response $y(t)$ on the structure into fee vibration component and forced vibration component. Free vibration response consists of impulse or step response while the forced vibration component contains to random loads such as wind load.

The main goal of this technique is to obtain large number of time segments from the original signal with the same initial value. By averaging over those signals tends to be disappeared the random part of the response and remains the response to initial condition of the system. The resulting function will be named as the RD function which is similar to the free decay function of the structure which contains the same damping coefficients and frequencies. The main parameters of the RDT method is trigger value ($a$) and the length of a time segment ($l$ or $\tau$). According to Kolling et al. (ND) a large number of time segments are needed in order to achieve a good averaging. When extracting the damping ratio, curve fitting is applied. Figure 4-1 illustrates the principal of random decrement technique.
The random decrement signal (free response) \( x(\tau) \) of the measured signal is defined by

\[
x(\tau) = \frac{1}{N} \sum_{m=1}^{N} y(t_m + \tau) y(t_m) = a
\]

Where \( N \) is the total number of time sample averaged, \( \tau \) is the time length of the free response; \( t_m \)'s can be taken as those of zero crossing time or any reasonable trigger value depending on the signal. Since zero crossing fixed value contains noise in noisy signals, it is better to consider any fixed value above the standard deviation of the signal.

### 4.3 Identification of Damping Ratio Using CWT

The identification of damping in multi degree of freedom (MDOF) systems is comparatively more difficult than estimating mass and stiffness in structures. The damping ratio identification needs dynamic test analysis while mass and stiffness can be obtained using static testing procedures. Therefore, damping analysis requires complex procedures to acquire more accurate results of frequency response function and modal data. One of the signal processing tools to
extract damping ratios with high level of efficiency and accuracy is Continuous Wavelet Transform (CWT). This method has powerful advantage of filtering data, which resolve the significant errors due to the influence of environmental noise. In this report, CWT based on Morlet-wavelet function was used to identify the modal damping ratios of linear vibrations. The CWT was used to decompose a function \( x(t) \) into frequency-time domain as defined in the

\[
\text{4-1. Once the wavelet map is obtained, local maxima indicate the natural frequencies of the system. Then, the wavelet envelop, } |W_x(\tau, \omega)| \text{ is extracted at natural frequencies to obtain damping ratio using following equations.}
\]

\[
\delta_i = \frac{1}{m} \ln \left( \frac{|W_{x_n}(t, \omega_{di})|}{|W_{x_n}(t + mT_i, \omega_{di})|} \right) \tag{4-3}
\]

\[
\zeta_i = \frac{\delta_i}{\sqrt{4\pi^2 + \delta_i^2}} \tag{4-4}
\]

Where \( \delta_i, m \) are logarithmic decrement and any positive integer respectively and \( T_i = \frac{2\pi}{\omega_{di}} \).

### 4.4 Identification of Damage Using Scalograms

The natural frequencies are usually obtained according to the each peak gives from the time frequency, and wavelet coefficient power spectrum in CWT analysis. According to the damage occurrence, the frequencies in each mode at damaged status will decrease compared to the undamaged system. According to the Melhem and Kim, 2003, it is found that all the natural frequencies decrease as the number of fatigues cycles increase in the experimented pre-stressed concrete beam. In other words, frequencies decrease as the crack length increases.
Further, According to the contour maps of CWT analysis of scalograms, clear ridge patterns can be identified and compared between the undamaged and damages status. Figure 4-2 shows the salograms extracted by Liao et al., (2015) for four story steel frame at healthy and damaged state. Clear ridges have identified at damaged status compared to undamaged status.

![Figure 4-2. Scalogram of four-story steel frame](Liao et al. 2015) (With permission from SPIE)

Another experimental results obtained by Melhem and Kim in 2003 indicates that difference in ridge pattern when damage is being occurred as shown in Figure 4-3. According to Melhem and Kim, number of ridges has increased when the load is increased and number of cracks increased in an experiment performed on a Portland cement concrete pavement. Subsequently, results obtained for a prestressed beam is shown in Figure 4-4. It is noticed that when the fatigue loads are applied and the crack grows, the magnitude of different ridges decreases.
Figure 4-3. Wavelet transform scalograms for different loads on a concrete pavement. (a) before damage. (b) after 120 k load cycles. (c) after 180 k load cycles. (d) after 260 k load cycles

(Melhem and Kim, 2003) (With permission from ASCE)
Figure 4-4. Scalogram of a prestressed beam for different loads. (a) before damage. (b) after 200 k load cycles. (c) after 350 k load cycles. (d) after 2,000 k load cycles
(Melhem and Kim, 2003) (With permission from ASCE)
CHAPTER 5

5 VALIDATION OF THE NUMERICAL ALGORITHMS

In this study, CWT and FFT numerical algorithms were coded using MATLAB. In order to verify the correctness of the algorithms, two signals with known frequencies and damping ratio were numerically built. The MATLAB codes were used to identify the frequencies and damping ratios of the two signals and the results were then compared with the known values.

5.1 Frequency Analysis Using FFT and CWT

Figure 5-1 represents the numerically built signal in the time domain. The parameters of the signal are given in Table 5-1. The signal was analyzed both in FFT and CWT methods. In order to find the corresponding frequencies of the signal, local maxima of power frequency spectrum was identified. Figure 5-2 shows the corresponding frequencies obtained using FFT and those were 50 Hz, 150 Hz, and 300 Hz respectively.

CWT was performed using Morlet wavelet in order to identify the natural frequencies with local maxima. Figure 5-4 indicates the three dimensional wavelet amplitude map with three peaks along axes of time, frequency and amplitude. Further, two dimensional wavelet plot represents by Figure 5-3 with corresponding three frequencies used to generate the signal.

<table>
<thead>
<tr>
<th>Table 5-1. Parameters of the Test Signal 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling frequency (fs)</td>
</tr>
<tr>
<td>Time step (t)</td>
</tr>
<tr>
<td>Implemented frequency 1</td>
</tr>
<tr>
<td>Implemented frequency 2</td>
</tr>
<tr>
<td>Implemented frequency 3</td>
</tr>
</tbody>
</table>
Figure 5-1. Test signal 1
Figure 5-2. FFT power frequency spectrum-test signal 1
Figure 5-3. CWT power frequency spectrum-test signal 1

Figure 5-4. Wavelet amplitude map-test signal 1
5.2 Damping Identification Using CWT

Damping ratio was identified for the second numerical test signal generated using MATLAB with known parameters. RDT method was used to convert the ambient vibration signal to a simply decay function to identify the damping ratio of the signal. Moreover, random white noise was added as an input force to verify the noise removal method and its effectiveness.

Test signal 2 was generated according to the parameters given in Table 5-2. Natural frequency was set to 0.2 Hz while 2% damping was introduced to the signal. Analyzed signal is shown in Figure 5-5 with the decaying function as presented in Figure 5-6.

The local maxima indicated both in FFT and CWT analysis was in a good compatibility and it introduces the natural frequency obtained using the analysis as 0.2 Hz. The results are shown in Figure 5-7 and Figure 5-8 respectively. Further, Figure 5-8 (a) shows the frequency obtained for RDT signal and Figure 5-8 (b) shows the frequency obtained from the original signal, showing a very consistent values with each other. Figure 5-9 indicates the 3D envelope of the wavelet map along time, frequency and amplitude axes.

In order to calculate the damping ratio using analyzed results, the wavelet envelope \(|W_x(t, \omega_d)|\) was extracted from the wavelet amplitude map by a slice parallel to the time axis through 0.2 Hz frequency in frequency axis. According to 4-3 and 4-4 formulas in chapter 4, the damping ratio can be calculated and results are shown in Table 5-3. The result showed 1.9% damping ratio which is close enough with 2% and had a good agreement with the expected value.
Table 5-2. Parameters of the Test Signal 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency ($f$)</td>
<td>0.2 Hz</td>
</tr>
<tr>
<td>Eigen frequency ($\omega_0$)</td>
<td>$2\pi f$ rad/sec</td>
</tr>
<tr>
<td>Damping Ratio ($\zeta$)</td>
<td>0.02</td>
</tr>
<tr>
<td>Damped eigenfrequency ($\omega_d$)</td>
<td>$\omega_0\sqrt{1 - \zeta^2}$ rad/sec</td>
</tr>
<tr>
<td>Modal mass (M)</td>
<td>1 kN</td>
</tr>
<tr>
<td>Modal stiffness (K)</td>
<td>$\omega_0^2 M$</td>
</tr>
<tr>
<td>Modal damping (C)</td>
<td>$2\zeta M \omega_0$</td>
</tr>
<tr>
<td>Force amplitude (F0)</td>
<td>0.1 kN</td>
</tr>
<tr>
<td>Initial displacement (X0)</td>
<td>$F_0/K$ m</td>
</tr>
<tr>
<td>Initial speed (V0)</td>
<td>0</td>
</tr>
<tr>
<td>White noise (F)</td>
<td>$F_0 [\text{randn}(1,N)]$ kN</td>
</tr>
</tbody>
</table>

Figure 5-5. Test signal 2

Figure 5-6. Decayed function of test signal 2
Figure 5-7. FFT power frequency spectrum-test signal 2

Figure 5-8. CWT power frequency spectrum-test signal 2.  
(a) For RDT signal. (b) For original signal
Figure 5.9. Wavelet amplitude map-test signal 2

Figure 5.10. Wavelet envelope extracted from wavelet map

Table 5.3. Results of Damping Identification-Test Signal 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency (f)</td>
<td>0.2 Hz</td>
</tr>
<tr>
<td>Ti =1/f</td>
<td>0.5 Sec</td>
</tr>
<tr>
<td>m</td>
<td>22</td>
</tr>
<tr>
<td>t1</td>
<td>7.224 Sec</td>
</tr>
<tr>
<td>t2</td>
<td>119.2 Sec</td>
</tr>
<tr>
<td>$</td>
<td>W_x(t, \omega_{di})</td>
</tr>
<tr>
<td>$</td>
<td>W_x(t + mT_i, \omega_{di})</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>0.177</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.019</td>
</tr>
</tbody>
</table>
5.3 Damage Detection Using Cwt Scalograms

Damage detection can be performed using CWT scalograms by observing and comparing the ridge patterns from before and after the occurrence of damage. Generally, increase in number of clear ridges is caused by increase in number of cracks. Further, when crack grows the magnitude of different ridges decreases. Low scale components which correspond to high frequency components of the signal do not change much. However, the low frequency components decrease significantly as the crack grows (Melhem and Kim, 2003). Moreover, all the natural frequencies decrease as the crack grows.

In order to verify above mentioned characteristics, a numerical signal was analyzed with different frequencies to observe the changes in ridge patterns with respect to the change in frequency values. Decrease in frequency can be considered as decrease in stiffness of a structure since \( \omega = \sqrt{\frac{k}{m}} \).

Ridges in scalograms were analyzed according to the change in frequency values. A numerically built signal using MATLAB as described in Appendix C was used for the analysis. The frequency value of the signal was changed and the ridge patterns in the scalograms were compared. Figure 5-11 indicates the scalograms obtained using CWT for 0.2 Hz, 0.1 Hz, and 0.02 Hz. According to the Ovanesova and Suárez (2004), wavelet coefficients with the lighter color correspond to the larger coefficients and darker color to small coefficients. They observed larger coefficients with lighter color nearest to the cracked point, where the stiffness reduced in their experiment. Similarly, in the numerical example, the clarity of ridges increased (i.e.
increase in wavelet coefficients) when the frequency was decreased from 0.2 Hz to 0.05 Hz. In addition, the length of the clear ridges was also increased with a good agreement to the results obtained by Ovanesova and Sua´rez (2004),

(a)
Figure 5-0-11. Scalograms for reduced stiffness (frequency), (a) 0.2 Hz. (b) 0.1 Hz. (c) 0.02 Hz.
CHAPTER 6

6 IDENTIFICATION OF MODAL PROPERTIES IN HOLLAND BRIDGE

6.1 Introduction

Ambient vibrations due to traffic load in a bridge situated in Holland, Michigan were collected using a wireless sensor network in May 2011 (Teng, 2012). Acceleration along the girders with two configurations was acquired with 3 leaf nodes which were installed at the bottom flange of I-beams. Three different sampling frequencies were used for different sensors and those were 280 Hz, 100 Hz, and 50Hz.

The collected acceleration data was used to analyze the modal properties of the bridge (i.e. frequency and damping ratio) using CWT method. Finally, results obtained using CWT was compared with FFT results.

6.2 Bridge Description and Background

The selected bridge was constructed in 1972 with following details as shown in Table 6-1. Further, Figure 6-1 represents the Holland Bridge schematic cross section to identify the girder configuration.

<table>
<thead>
<tr>
<th>Table 6-1. Geometric Properties and Other Details of the Bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span type</td>
</tr>
<tr>
<td>Span length</td>
</tr>
<tr>
<td>Width</td>
</tr>
<tr>
<td>Number of supports</td>
</tr>
<tr>
<td>Number of I-beams</td>
</tr>
</tbody>
</table>
6.3 Sensor Configuration

Two types of configurations used for data collection as shown in Figure 6-2. Table 6-2 presents parameters for each data sets used in both configurations. Configuration 1 is useful to identify the behavior of each girder to daily traffic load while configuration 2 is helpful to identify the mode shapes. Further, configuration 1 provides the maximum vibrations of the bridge since the sensors have located at the mid span of the girders. Therefore, configuration 1 was used to identify the modal frequencies and the damping ratios of three individual girders. However, sensor 131 could not be gained the response due to an installation error.

In configuration two, three sensors were installed at near two supports and one at the mid span of middle girder. This will help to identify the variance of results due to the amplitude vibration response. Hence, the deflection of the girder at support is low whereas the middle of the span deflection is comparatively high. Unexpectedly, the sensor 131 was shut down while set 7 data was being collected.
Figure 6-2. Sensor configurations, (a) Configuration 1. (b) Configuration 2

Table 6-2. Parameters of Data Set

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Configuration</th>
<th>Sampling Frequency (Hz)</th>
<th>Sensor ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>1</td>
<td>280</td>
<td>65, 128</td>
</tr>
<tr>
<td>Set 3</td>
<td>1</td>
<td>100</td>
<td>65, 128</td>
</tr>
<tr>
<td>Set 4</td>
<td>2</td>
<td>50</td>
<td>65, 128</td>
</tr>
<tr>
<td>Set 5</td>
<td>2</td>
<td>280</td>
<td>65, 128, 131</td>
</tr>
<tr>
<td>Set 6</td>
<td>2</td>
<td>100</td>
<td>65, 128, 131</td>
</tr>
<tr>
<td>Set 7</td>
<td>2</td>
<td>50</td>
<td>65, 128</td>
</tr>
</tbody>
</table>
6.4 Results

6.4.1 Frequency Identification Using Continuous Wavelet Transform Method

The collected acceleration data in Holland Bridge was analyzed using both FFT and CWT for comparison. Butterworth filter described in chapter 3 was used to eliminate the effects of environmental noise. The filtered signal was then used to convert to a decay function using random decrement method. A sample of raw acceleration data signal, filtered signal and the decayed function is shown in Figure 6-3. Random decrement method was used with small time sub segments of 12 seconds.
FFT plot for data set 1 is shown in Figure 6-4 for sensor 65 and 128 in first configuration. The first five modal frequencies were obtained according to the local maxima and those were between 3.5 Hz to 17.5 Hz. 2D and 3D amplitude maps extracted from CWT method is presented in Figure 6-6, Figure 6-7, and Figure 6-8. X, Y, Z axes in 3D plots represents time, frequency and wavelet amplitude respectively. The advantage in CWT compared to FFT is clear with 3D plots with time axes. Time axes in CWT illustrates the exact time which a certain modal frequency was occurred while FFT provides only the frequency value. Clear peaks can be
obtained in CWT for those frequencies with large amplitude. However, both 3D and 2D plots have to be observed to identify the minor peaks.

In CWT method, initially a scalogram was produced as indicated in Figure 6-5 and it was then converted to frequency using MATLAB functions.

![Single-Sided Amplitude Spectrum](image)

**Figure 6-4. FFT plot-configuration 1-set 1-sensor 65 and 128**
Figure 6-5. 3D scalogram

Figure 6-6. CWT plot-configuration1-set 1-sensor 65
Figure 6-7. 3D amplitude map-configuration1-set 1-sensor 65

Figure 6-8. Frequency vs. wavelet amplitude map from 3D plot-configuration1-set 1-sensor 65
Appendix A shows the local maxima indicating first five modal frequencies obtained in both configurations using FFT and CWT. The summary of modal frequencies is presented in Table 6-3 and Table 6-4 for configuration 1 and 2 respectively.

In order to identify the compatibility of FFT and CWT methods, percentage difference of modal frequencies were calculated and majority of difference were less than 10% and shows a good agreement with each other. Further, CWT could find some of the frequencies which could not be able to detect from FFT plots.

According to the amplitude map of FFT and CWT plots, larger amplitudes were observed in sensor 65 which was placed on the middle span of the structure in the second configuration. This indicates both FFT and CWT power amplitude is sensitive to vibration response of the structure.

In conclusion, all the modal frequencies were in good agreement and all the sampling frequencies indicated consistency between results obtained using both FFT and CWT methods. Moreover, middle span of the bridge is more sensitive to both methods and sensor placement is highly recommended at mid span in order to identify clear peaks.
### Table 6-3. Summary of FFT and CWT Modal Frequency Values for Configuration 1

<table>
<thead>
<tr>
<th>Sensor Number</th>
<th>Set 1 (280 Hz)</th>
<th>Set 3 (100 Hz)</th>
<th>Set 4 (50 Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (Hz)</td>
<td>% Diff</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td></td>
<td>FFT</td>
<td>CWT</td>
<td>FFT</td>
</tr>
<tr>
<td>65</td>
<td>3.5</td>
<td>3.53</td>
<td>0.9 %</td>
</tr>
<tr>
<td>6.67</td>
<td>6.69</td>
<td>0.3 %</td>
<td>6.67</td>
</tr>
<tr>
<td>12.42</td>
<td>12.64</td>
<td>1.8 %</td>
<td>12.25</td>
</tr>
<tr>
<td>17.5</td>
<td>16.85</td>
<td>3.7 %</td>
<td>17.75</td>
</tr>
<tr>
<td>128</td>
<td>3.5</td>
<td>3.45</td>
<td>1.4 %</td>
</tr>
<tr>
<td>6.75</td>
<td>6.59</td>
<td>2.4 %</td>
<td>6.67</td>
</tr>
<tr>
<td>10.83</td>
<td>10.58</td>
<td>2.3 %</td>
<td>11.83</td>
</tr>
<tr>
<td>12.42</td>
<td>12.64</td>
<td>1.7 %</td>
<td>12.33</td>
</tr>
<tr>
<td>17.58</td>
<td>16.25</td>
<td>7.6 %</td>
<td>17.67</td>
</tr>
</tbody>
</table>

### Table 6-4. Summary of FFT and CWT Modal Frequency Values for Configuration 2

<table>
<thead>
<tr>
<th>Sensor Number</th>
<th>Set 5 (280 Hz)</th>
<th>Set 6 (100 Hz)</th>
<th>Set 7 (50 Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (Hz)</td>
<td>% Diff</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td></td>
<td>FFT</td>
<td>CWT</td>
<td>FFT</td>
</tr>
<tr>
<td>65</td>
<td>3.58</td>
<td>3.67</td>
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</tr>
<tr>
<td>6.67</td>
<td>6.59</td>
<td>1.2 %</td>
<td>6.67</td>
</tr>
<tr>
<td>10.67</td>
<td>10.58</td>
<td>0.8 %</td>
<td>10.5</td>
</tr>
<tr>
<td>12.33</td>
<td>12.64</td>
<td>2.5 %</td>
<td>12.33</td>
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<tr>
<td>15.08</td>
<td>15.17</td>
<td>0.6 %</td>
<td>15</td>
</tr>
<tr>
<td>128</td>
<td>3.58</td>
<td>3.45</td>
<td>3.6 %</td>
</tr>
<tr>
<td>-</td>
<td>10.83</td>
<td>-</td>
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</tr>
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<td>12.64</td>
<td>-</td>
<td>12.25</td>
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<td>-</td>
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<td>5.1 %</td>
<td>17.75</td>
</tr>
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<td>131</td>
<td>3.25</td>
<td>3.7</td>
<td>13.8 %</td>
</tr>
<tr>
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<td>6.89</td>
<td>3.3 %</td>
<td>6.58</td>
</tr>
<tr>
<td>12.42</td>
<td>12.3</td>
<td>1 %</td>
<td>12.25</td>
</tr>
<tr>
<td>15.75</td>
<td>16.85</td>
<td>5.1 %</td>
<td>17.83</td>
</tr>
</tbody>
</table>
6.4.2 Damping Ratio Identification Using Continuous Wavelet Transform Method

Damping ratios were calculated using the CWT method for configuration 1 and 2. The amplitude spectrum, obtained from a window parallel to the time axis at the first five frequencies in time and frequency and amplitude map, was used to determine the damping ratio for the corresponding frequencies. According to the equation 4-4 illustrated in chapter 4, the corresponding values of damping ratio were calculated. All the variable values used for calculations are shown in Table 6-7, Table 6-8, and Table 6-9 for sensor 65 in configuration 1 for set 1 (280 Hz), set 3 (100 Hz), and set 4 (50 Hz) respectively. Figure 6-9, Figure 6-10, Figure 6-11 show the corresponding envelopes for the first five frequencies. Similarly, Table 6-10, Table 6-11 and Table 6-12 display the variable values in Equation 4-3 for sensor 128 in configuration 2 for set 5 (280 Hz), set 6 (100Hz), and set 7 (50 Hz) respectively. Figure 6-12, Figure 6-13, and Figure 6-14 show the corresponding 2D envelopes obtained from the CWT method. Similarly, Table 6-13 and Figure 6-15 represent variable values and the envelope extracted from sensor 131 in configuration 2 for set 6 (100 Hz).

Those figures show the consistency of damping ratio values of same sensor with different sampling rates in both the first and second configurations. Further, it shows a good agreement between the values with different locations of the placed sensors in the structure. This concludes that majority of damping ratio values were between 2.8 % - 1.5 % in configuration 1 and 2.8% - 1.2 % in configuration 2. Table 6-5 and Table 6-6 represent the summary of damping ratio values corresponding to the first five frequencies in configuration 1 and 2 respectively.
Table 6-5. Summary of Frequency and Damping Ratio for Configuration 1 Using CWT Method

<table>
<thead>
<tr>
<th>Sensor Number</th>
<th>Set 1 (280 Hz)</th>
<th>Set 3 (100 Hz)</th>
<th>Set 4 (50 Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (Hz)</td>
<td>Damping Ratio</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>65</td>
<td>3.53</td>
<td>2.8 %</td>
<td>3.69</td>
</tr>
<tr>
<td></td>
<td>4.46</td>
<td>2.2 %</td>
<td>4.28</td>
</tr>
<tr>
<td></td>
<td>6.69</td>
<td>1.6 %</td>
<td>6.25</td>
</tr>
<tr>
<td></td>
<td>12.64</td>
<td>2.3 %</td>
<td>13.54</td>
</tr>
<tr>
<td></td>
<td>16.85</td>
<td>1.5 %</td>
<td>16.25</td>
</tr>
<tr>
<td>128</td>
<td>3.45</td>
<td>2.4 %</td>
<td>3.61</td>
</tr>
<tr>
<td></td>
<td>6.59</td>
<td>1.7 %</td>
<td>6.77</td>
</tr>
<tr>
<td></td>
<td>10.58</td>
<td>1.9 %</td>
<td>10.83</td>
</tr>
<tr>
<td></td>
<td>12.64</td>
<td>1.7 %</td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>16.25</td>
<td>1.5 %</td>
<td>16.25</td>
</tr>
</tbody>
</table>

Table 6-6. Summary of Frequency and Damping Ratio for Configuration 2 Using CWT Method

<table>
<thead>
<tr>
<th>Sensor Number</th>
<th>Set 5 (280 Hz)</th>
<th>Set 6 (100 Hz)</th>
<th>Set 7 (50 Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (Hz)</td>
<td>Damping Ratio</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>65</td>
<td>3.67</td>
<td>2.2 %</td>
<td>3.61</td>
</tr>
<tr>
<td></td>
<td>6.59</td>
<td>1.9 %</td>
<td>6.77</td>
</tr>
<tr>
<td></td>
<td>10.58</td>
<td>1.9 %</td>
<td>10.83</td>
</tr>
<tr>
<td></td>
<td>12.64</td>
<td>1.6 %</td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>15.17</td>
<td>1.4 %</td>
<td>14.77</td>
</tr>
<tr>
<td>128</td>
<td>3.45</td>
<td>2.5 %</td>
<td>3.61</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>10.83</td>
</tr>
<tr>
<td></td>
<td>10.83</td>
<td>1.9%</td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>12.64</td>
<td>1.7 %</td>
<td>14.77</td>
</tr>
<tr>
<td></td>
<td>16.85</td>
<td>1.4 %</td>
<td>16.25</td>
</tr>
<tr>
<td>131</td>
<td>3.7</td>
<td>2.8 %</td>
<td>3.61</td>
</tr>
<tr>
<td></td>
<td>6.89</td>
<td>1.9 %</td>
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</tr>
<tr>
<td></td>
<td>12.3</td>
<td>1.5 %</td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>15.17</td>
<td>1.3 %</td>
<td>14.77</td>
</tr>
<tr>
<td></td>
<td>16.85</td>
<td>1.3 %</td>
<td>16.25</td>
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Figure 6-9. Set 1-sensor 65

Table 6-7. Set 1-Sensor 65

<table>
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<th>Parameter</th>
<th>Value</th>
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<td>Natural Frequency (f) (Hz)</td>
<td>3.53</td>
</tr>
<tr>
<td></td>
<td>4.46</td>
</tr>
<tr>
<td></td>
<td>6.69</td>
</tr>
<tr>
<td></td>
<td>12.64</td>
</tr>
<tr>
<td></td>
<td>16.85</td>
</tr>
<tr>
<td>Ti =1/f (Sec)</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
</tr>
<tr>
<td>m</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td>t1 (Sec)</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>1.41</td>
</tr>
<tr>
<td></td>
<td>1.65</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>t2 (Sec)</td>
<td>5.38</td>
</tr>
<tr>
<td></td>
<td>3.06</td>
</tr>
<tr>
<td></td>
<td>5.38</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
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<td></td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>W_x(t, ω_d)</td>
</tr>
<tr>
<td></td>
<td>1.38</td>
</tr>
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<td>3</td>
</tr>
<tr>
<td></td>
<td>1.98</td>
</tr>
<tr>
<td></td>
<td>W_x(t + mTi, ω_d)</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>δ_i</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td>ĺ_i</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>0.023</td>
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<tr>
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<td>0.015</td>
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Table 6-8. Set 3-Sensor 65

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<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency (f) (Hz)</td>
<td>3.69</td>
<td>4.28</td>
<td>6.25</td>
<td>13.54</td>
<td>16.25</td>
</tr>
<tr>
<td>Ti = 1/f (Sec)</td>
<td>0.27</td>
<td>0.23</td>
<td>0.16</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>m</td>
<td>5</td>
<td>10</td>
<td>11</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>t1 (Sec)</td>
<td>1.18</td>
<td>1.06</td>
<td>1.18</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td>t2 (Sec)</td>
<td>2.5</td>
<td>3.5</td>
<td>3</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>W_ξ(t, ω_d(t))</td>
<td></td>
<td>0.7</td>
<td>1.2</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>W_ξ(t + mT_i, ω_d(t))</td>
<td></td>
<td>0.3</td>
<td>0.3</td>
<td>0.04</td>
</tr>
<tr>
<td>δ_i</td>
<td>0.17</td>
<td>0.13</td>
<td>0.1</td>
<td>0.17</td>
<td>0.11</td>
</tr>
<tr>
<td>ζ_i</td>
<td>0.028</td>
<td>0.021</td>
<td>0.015</td>
<td>0.027</td>
<td>0.018</td>
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Figure 6-10. Set 3-sensor 65
### Table 6-9. Set 4-Sensor 65

<table>
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<tr>
<td>Natural Frequency (f) (Hz)</td>
<td>3.87 4.28 6.25 13.54 16.25</td>
</tr>
<tr>
<td>Ti =1/f (Sec)</td>
<td>0.26 0.23 0.16 0.07 0.06</td>
</tr>
<tr>
<td>m</td>
<td>10 4 26 17 20</td>
</tr>
<tr>
<td>t1 (Sec)</td>
<td>0.12 0.7 0.46 0.08 0.3</td>
</tr>
<tr>
<td>t2 (Sec)</td>
<td>2.6 1.6 4.6 1.3 1.5</td>
</tr>
<tr>
<td></td>
<td>W_x(t, ωd1)</td>
</tr>
<tr>
<td></td>
<td>W_x(t + mT, ωd1)</td>
</tr>
<tr>
<td>δi</td>
<td>0.17 0.14 0.12 0.16 0.11</td>
</tr>
<tr>
<td>ζi</td>
<td>0.027 0.023 0.019 0.026 0.178</td>
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**Wavelet Amplitude Map**

![Wavelet Amplitude Map](image)

Figure 6-11. Set 4-sensor 65
Table 6-10. Set 5-Sensor 128

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<td>m</td>
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</tr>
<tr>
<td>t1 (Sec)</td>
<td>1</td>
</tr>
<tr>
<td>t2 (Sec)</td>
<td>5.3</td>
</tr>
<tr>
<td>$</td>
<td>W_c(t, \omega_d)</td>
</tr>
<tr>
<td>$</td>
<td>W_c(t + mTi, \omega_d)</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>0.16</td>
</tr>
<tr>
<td>$\zeta_i$</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>10.83</td>
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<td></td>
<td>0.9</td>
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<td>30</td>
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<td>0.017</td>
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Figure 6-13. Set 6-sensor 128

Table 6-11. Set 6-Sensor 128

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<th>Value</th>
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</tr>
<tr>
<td></td>
<td>10.83</td>
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<tr>
<td></td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>14.77</td>
</tr>
<tr>
<td></td>
<td>16.25</td>
</tr>
<tr>
<td>Ti = 1/f (Sec)</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
</tr>
<tr>
<td>m</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
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<td>47</td>
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<tr>
<td></td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>26</td>
</tr>
<tr>
<td>t1 (Sec)</td>
<td>0.12</td>
</tr>
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<td>0.4</td>
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<td>t2 (Sec)</td>
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<td>2</td>
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<td></td>
<td>2</td>
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<td>W_x(t, \omega_d)</td>
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<tr>
<td></td>
<td>W_x(t + mT_i, \omega_d)</td>
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<tr>
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</tr>
<tr>
<td></td>
<td>0.08</td>
</tr>
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<td></td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
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<td>\zeta_i</td>
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Table 6-12. Set 7-Sensor 128

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<td>6.25</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>13.54</td>
</tr>
<tr>
<td></td>
<td>16.25</td>
</tr>
<tr>
<td>$T_i = 1/f$ (Sec)</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>0.16</td>
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<td>0.06</td>
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<td>$m$</td>
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<tr>
<td></td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>61</td>
</tr>
<tr>
<td>$t_1$ (Sec)</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td>$t_2$ (Sec)</td>
<td>6.9</td>
</tr>
<tr>
<td></td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>4.7</td>
</tr>
<tr>
<td></td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>3.8</td>
</tr>
<tr>
<td>$</td>
<td>W_x(t, \omega_{at})</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>1.02</td>
</tr>
<tr>
<td>$</td>
<td>W_x(t + mT_i, \omega_{at})</td>
</tr>
<tr>
<td></td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
</tr>
<tr>
<td>$\zeta_i$</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>0.012</td>
</tr>
</tbody>
</table>
Wavelet Amplitude Map

- Mode 1 - 3.61 Hz
- Mode 2 - 6.25 Hz
- Mode 3 - 12.5 Hz
- Mode 4 - 14.77 Hz
- Mode 5 - 16.25 Hz

Figure 6-15. Set 6-Sensor 131

Table 6-13. Set 6-Sensor 131

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency (f) (Hz)</td>
<td>3.61</td>
</tr>
<tr>
<td></td>
<td>6.25</td>
</tr>
<tr>
<td></td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>14.77</td>
</tr>
<tr>
<td></td>
<td>16.25</td>
</tr>
<tr>
<td>Ti = 1/f (Sec)</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
</tr>
<tr>
<td>m</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>47</td>
</tr>
<tr>
<td>t1 (Sec)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td>t2 (Sec)</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>W_x(t, ω_m)</td>
</tr>
<tr>
<td></td>
<td>W_x(t + mTi, ω_m)</td>
</tr>
<tr>
<td>δi</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
</tr>
<tr>
<td>ζi</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>0.013</td>
</tr>
</tbody>
</table>
6.5 Conclusion

The following conclusions were made during the analysis for the single span Holland Bridge.

- The majority of percentage difference was below 10% and modal frequencies were in very good agreement between FFT and CWT analysis. Therefore, CWT is a very good tool to identify modal properties with added benefits.
- Damping ratios showed a very good consistency in both configurations, and logarithmic decrement method is highly recommended to calculate the damping ratios.
- It is highly recommended to place sensors in the middle span in order to identify clear peaks during the analysis.
- The CWT method provides the time component apart from the frequency values, illustrating when a certain frequency has occurred.
- CWT needs more computational time compared to FFT.
- The randomly decayed signal takes less computational time in CWT analysis compared to the original signal.
CHAPTER 7

7 DAMAGE DETECTION AND FREQUENCY COMPARISON OF THE CABLE-STAYED BRIDGE

7.1 Introduction

Vibration based nondestructive method was used to detect the damage of concrete bridge girders in a cable-stayed bridge situated in China. The acceleration response has been collected for a full-scale bridge benchmark problem by the Center of Structural Monitoring and Control at the Harbin Institute of Technology (HIT), China. A damage detection algorithm based on Morlet wavelet transform was performed in order to detect the damage using acquired acceleration data. The acquired data was pre-processed using Butterworth filtering to reduce the effect of environmental factors. Initial FFT analysis was performed in order to see the frequency range which can make a considerable effect to the original data. Accelerations were monitored from January to July 2008 and the data was made public by the Center of Structural Monitoring and Control at HIT. The data collected on January 1, January 17, February 3, March 19, March 30, April 9, May 31, June 7, June 16, and July 31, 2008 was selected to represent the benchmark time history of the bridge from healthy status to damaged status. Data acquisition was performed for 24 hours with a sampling rate 100 Hz. The analysis of ridges obtained in continuous wavelet transform scalograms were used to distinguish the healthy state and the damaged state of the bridge from January to July. Meantime, the change of natural frequencies in the first five modes was analyzed in CWT method from healthy to damaged status.
7.2 Bridge Description

The bridge shown in Figure 7-1 is one of the first cable-stayed bridges constructed in mainland China. The bridge was instrumented by a comprehensive structural health monitoring system which monitors vehicle loads, cable stresses, girder strains, acceleration, wind, temperature, and humidity.

![Image of the bridge](Image)

**Figure 7-1. Investigated bridge in mainland China**

(Li et al. 2013) (With permission from Li)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of spans</td>
<td>3</td>
</tr>
<tr>
<td>Main span length</td>
<td>260 m</td>
</tr>
<tr>
<td>Side span-1 length</td>
<td>25.15 m</td>
</tr>
<tr>
<td>Side span-2 length</td>
<td>99.85 m</td>
</tr>
<tr>
<td>Length of the bridge</td>
<td>510 m</td>
</tr>
<tr>
<td>Width of the bridge</td>
<td>11 m</td>
</tr>
<tr>
<td>Tower height</td>
<td>60.5 m</td>
</tr>
</tbody>
</table>

Table 7-1. Geometric Properties and Other Details of the Chinese Bridge
The bridge provides 2x1m passage for foot traffic while providing 9m for vehicular traffic. The concrete bridge tower consists of two transverse beams and tower was constructed using sliding formwork technology. The main girder was assembled with 74 precast concrete girder segments which are formed continuously by cast-in-place joints that connect the girder ends and form transversely reinforced diaphragms. It consists, a total of 88 pairs of cables containing steel wires 5mm in diameter and the minimum and maximum number of wires within a cable is 69 and 199 respectively. The design cable tension forces under dead load range from 559.4-1,706.8 kN (approximately 450MPa stress) while live load design stress is 160 MPa. The bridge construction started in 1983 and it was opened to traffic in December 1987.

![Figure 7-2. Sensor configuration of Chinese bridge](Li et al. 2013) (With permission from Li)

### 7.3 Application of CWT in Damage Detection

Acceleration response collected along January to July was analyzed using CWT and frequency response of healthy status and damaged status were initially compared according to the damaged date detected by researchers at the HIT. Thereafter, wavelet based ridges were analyzed in scalograms obtained from the data acquired from 2nd accelerometer, in order to detect and confirm the damaged state. The eleventh hour was selected to analyze the structure along January to July as same as HIT researchers used.
Butterworth filtering and random decrement method was used as signal pre-processing tools.

In Figure 7-3 raw acceleration data signal, filtered acceleration data signal and corresponding decayed signal is presented as per the January 17th at healthy state. 20 sub segments were used in order to decay the non-stationary signal using random decrement method. Figure 7-4 represents corresponding 3D, 2D scalograms with frequency spectrum obtained using CWT. Frequency values for first seven modes were between 0.41 Hz – 1.45 Hz and it had a god agreement with the values obtained by Harbin researchers using FFT which was around 0.42 Hz – 1.44 Hz. 2D scalogram presented in Figure 7-4 indicates four clear ridges of scale ranges between 14 and 53. According to Melhem, H and Kim, H, when the damage is occurred, decrease in natural frequencies and increases of clear ridges are expected. This was verified using a numerical example in chapter 5 with different frequencies. (i.e. low frequencies correspond to low stiffness).

Figure 7-7 shows the scalograms from January to July. It indicates an abrupt increase of number of clear ridges in July. Initially at the healthy status, there were around 4 clear ridges and it continued till June 16th indicating the bridge is still at the healthy state. Whereas, according to the vibration response on July 31st, around 15 clear ridges were visualized other than the initial 4 ridges. An instant increase of number of ridges may conclude that bridge has changed the status from healthy to damage by July 31st. The damage might have occurred before July 31st and unfortunately acceleration data in between June 16th to July 31st has not monitored. Until June 16th, bridge was in healthy state. In the 23rd hour in July 31st, many ridges were observed and this is believed to be the reason of increment of the damage.
Raw acceleration signal, filtered signal, and decayed signal is shown in Figure 7-6 as per July 31st. More fluctuations are visualized in the signal compared to the Figure 7-3 at healthy state. Moreover, the decrease in frequencies at first seven modes is indicated in Figure 7-7 with corresponding 3D and 2D scalograms. The scalograms also indicate a lot of fluctuations than in January 17th. According to the Li et al. 2013, HIT researchers also identified decrease in frequency values at first seven modes using FFT in damaged state compared to healthy state. Further, they observed a lot of fluctuations in the data signal at damaged status compared to the healthy stage. Therefore, the results are in good agreement with each other.

Frequencies obtain using FFT and CWT methods were compared with the frequencies obtained from the finite element model prepared by Harbin researchers. The percentage difference was then calculated for both healthy and damaged stages. However, the majority of percentage difference was below 5% and showed a good agreement. Table 7-2 summaries all the frequency values and calculated percentage differences for both stages.
Figure 7-3. Vibration-time history sample signal-January 17th-11th hour-sensor 2
Figure 7-4. Scalograms and frequency values at healthy status-January 17th - 11th hour-sensor 2
Continuous Transform, absolute coefficients.

1/1/11\textsuperscript{th} hour-sensor 2

Continuous Transform, absolute coefficients.

1/17/11\textsuperscript{th} hour-sensor 2
Figure 7-5. 2D Scalograms from January to July

Figure 7-6. Vibration-time history sample signal-July 31st-11th hour-sensor 2
Figure 7-7. Scalograms and frequency values at damaged status- July 31st-11th hour-sensor 2
Table 7-2. Summary of Frequency Comparison

<table>
<thead>
<tr>
<th>Vertical mode of bridge deck</th>
<th>January 17th (Healthy status)</th>
<th>July 31st (Damaged status)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Identified frequencies by Harbin researches using FFT</td>
<td>Computed frequency</td>
</tr>
<tr>
<td>V1</td>
<td>0.417</td>
<td>0.418</td>
</tr>
<tr>
<td>V2</td>
<td>0.587</td>
<td>0.595</td>
</tr>
<tr>
<td>V3</td>
<td>0.891</td>
<td>0.879</td>
</tr>
<tr>
<td>V4</td>
<td>1.038</td>
<td>1.046</td>
</tr>
<tr>
<td>V5</td>
<td>1.101</td>
<td>1.090</td>
</tr>
<tr>
<td>V6</td>
<td>1.271</td>
<td>1.214</td>
</tr>
<tr>
<td>V7</td>
<td>1.478</td>
<td>1.441</td>
</tr>
</tbody>
</table>

7.4 Conclusion

The scalograms were obtained using the CWT method from January to July 2008, and damage detection was performed according to the changes occurred in ridge patterns in the scalogram. Signal preprocessing tools were used to remove the impact of environmental effects.

The following conclusions were made during the analysis for the cable-stayed bridge in China:

- From January to June 16th the bridge was in healthy status, and on July 31st the analysis of the collected acceleration data showed that the status of bridge was changed, indicating the possible damage of the structure.
- Further, frequencies of the first seven modes were decreased by July 31st indicating damage status in bridge girders.
• The frequencies obtained using CWT and FFT had a good agreement, and the majority percentage difference was below 5%.

• The CWT scalograms is a powerful tool in damage detection. In the scalograms, the discontinuities of the signal can be clearly identified with a lower scale range compared to higher scale range. The number of clear ridges and the length of ridges increased when the status of the bridge change from healthy to damage state.
CHAPTER 8

8 SUMMARY, CONCLUSION AND FUTURE WORK

8.1 Summary

The study presented herein applied CWT in evaluating the modal parameter (damping ratio and frequency) and damage detection in bridges structures. Nowadays, vibration based structural health monitoring is commonly used in bridge engineering to overcome the drawbacks of conventional monitoring systems. In vibration based SHM, the acceleration data was acquired using either wired or wireless sensors for analysis. In this study, the algorithms were coded using MATLAB, and numerical examples were built in order to verify the algorithms before using those for real case study data. A comparison study was done between results obtained in FFT and CWT, and the results showed that they were in very good agreement. According to the analysis, following conclusion are made:

• CWT is a very effective tool, which is able to provide reliable results both in modal parameters identification and damage detection.

• CWT analysis is time consuming compared to FFT, but it provides a detailed view of when a certain incident has occurred in the structure.

• CWT analysis combined with the random decrement method for ambient signals reduces the computational time.

• Wireless sensors have properly captured the dynamic response of the structures.
• Damage detection can be performed using CWT without constructing numerical models for the pre-damaged structure system.

• In general, the damage is easier to detect with a lower level of noise and more severe damage.

• The wavelet approach is less model-dependent as it only requires measured data for the analysis.

8.2 Future Work

There are many possible future research directions concerning structural health monitoring using CWT starting from the work presented in this thesis. This section provides the author’s recommendations for future research in this research area.

• Perform mode shape analysis using the CWT method and Finite Element analysis of structures.

• Perform detailed level 1 wavelet decomposition to identify the locations of the damage.

• For the Chinese cable-stayed bridge, quantify the detected damage and evaluate the performance of the damaged structure, so that its useful remaining life can be estimated.

• Determine, implement, and evaluate effective remediation and repair efforts for the damaged structure.
• Implement further environmental noise removal methods, such as nonlinear principal component analysis, in order to see the effectiveness of signal preprocessing and to perform a comparison study.
REFERENCES


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35. Saripalli, L., and Zimmerman, D.C. *Structural Health Monitoring Using the Hilbert-Huang Transform and Beating*. University of Houston, Houston, Texas, USA.


41. Teng, C. (2012). *STRUCTURAL HEALTH MONITORING OF A BRIDGE STRUCTURE USING WIRELESS SENSOR NETWORK*, Western Michigan University, Kalamazoo, Michigan, USA.


APPENDIX A

FFT and CWT Plots for Configuration 1 and 2 - Holland Bridge

(a) Single-Sided Amplitude Spectrum

(b) Single-Sided Amplitude Spectrum
Figure 8-0-1. FFT Plots-configuration 1, (a) Set 1, (b) Set 3, (c) Set 4
Figure 8-0-2. FFT Plots - configuration 2, (a) Set 5, (b) Set 6, (c) Set 7
Figure 8-0-3. CWT Plots-configuration 1
Figure 8-0-4. CWT Plots-configuration 2
MATLAB Code for Numerical Example-Frequency Identification

```matlab
figure(1)
fs = 1000;
t = (0 : 1/fs : 1)';
frq = [ 50, 150, 300 ];
y= sum( cos(2*pi*t*frq), 2 );
plot(t,y)
title('Signal')
xlabel('Time (sec)')
ylabel('Acceleration (m/sec^2)')

%%FFT

figure(3);
N=length(t);
L=N;
f = (0:L/2);
Y = fft(y);
P2 = abs(Y/L);
P1 = P2(1:L/2+1);
P1(2:end-1) = 2*P1(2:end-1);
plot(f,P1)
title('Single-Sided Amplitude Spectrum of x(t)')
xlabel('f (Hz) in bins')
ylabel('|P1(f)|')

figure(4)
x = linspace(0,fs/2,length(P1));
plot(x,P1)
title('Single-Sided Amplitude Spectrum')
xlabel('f (Hz)')
ylabel('|P1(f)|')

%%CWT
figure(5)
subplot(2,1,1)
wnname = 'morl';
scales = 1:0.5:256;
subplot(2,2,3)
cw1=cwt(y,scales,wnname,'plot');
% (plots the continuous wavelet transform coefficients, using default coloration 'absglb')
title('Continuous Transform, absolute coefficients.')
ylabel('Scale')
subplot(2,1,2)
[cw1,sc] = cwt(y,scales,wnname,'scalCNT');
%( displays a contour representation of the scalogram)
title('Scalogram')
```

APPENDIX B
ylabel('Scale')

%-----------------------------------------------------------%
%%%Time vs Scale vs Amplitude%%%
%-----------------------------------------------------------%

figure(6)
cw2 = cwt(y,scales,wname,'3Dlvl'); % (display 3D plot)
colormap winter;

%-----------------------------------------------------------%
%%%Scales and Frequencies%%%
%-----------------------------------------------------------%

figure(7)
%fs=280;
cfs = abs(cw2).^2;
pfreq = scal2frq(scales,wname,1/fs); % (returns the pseudo-frequencies corresponding to the scales given and the wavelet function 'wname' and the sampling period fs.)

plot(pfreq,cfs);

axistight;
gridon;
xlabel('Approximate Frequency (Hz)');
ylabel('Power of Coefficients');
title('CWT with Power vs Frequency');

figure(8)
cw3 = cwt(y,pfreq,wname,'3Dlvl'); % (display 3D plot)
colormap summer;
axistight;
gridon;
xlabel('Time');
ylabel('Freq');
zlabel('Coef');
title('CWT with Coef, Time vs Freq');

X=t;
Y=pfreq;
Z=cfs;

figure(9)
surf(X,Y,Z); shading('interp');
xlabel('Time (Sec)'); ylabel('Frequency (z)'); zlabel('Wavelet Amplitude')
title('Wavelet Amplitude Map');

figure(10)
contour(X, Y, Z)
xlabel('Time'); ylabel('Frequency')

figure(11)
y=16.25;
z=interp2(X,Y,Z,X,y);
plot(X,z);
xlabel('Time (Sec)');
ylabel('Power of Coefficients');
title('Amplitude Spectrum at Selected Natural Frequency');
APPENDIX C

MATLAB Code for Numerical Example-Damping Ratio Identification and Damage Detection

%% INITIALISATION
% The time series from a SDOF is computed using the central difference method, and a white noise is used as an input force.

% modal parameters
w0 = 2*pi*0.2; % eigen-frequency (rad/s) -> 0.2 Hz is the natural frequency
ta = 0.02; % damping ratio
wd = w0.*sqrt(1-ta.^2); % damped eigen frequency (rad/s)
M = 1; % mass
K = w0^2.*M; % stiffness
C = 2*ta.*M.*w0; % damping

N = 3000; % number of time step
t = linspace(0,1800,N); % time
dt = median(diff(t)); % time step
fs = 1/dt;
F0 = 0.1; % amplitude of force
w = w0;
% initial conditions
v0 = 0; % no initial speed
x0 = F0/K; % initial displacement

% A white noise is used as an input
F = F0.*randn(1,N);
% output response
y = CentDiff(F,M,K,C,dt,x0,v0);

% Visualization of the data
clf;close all;
figure
subplot(211)
plot(t,y)
xlabel('Time (sec)')
ylabel('Acceleration (m/sec^2)')

%% Band Pass Filtering
figure(2)
order = 5;
%fs = ;
fcutlow = 0.01;
fcuthigh = 0.8;
[b,a] = butter(order,[fcutlow/(fs/2) fcuthigh/(fs/2)], 'bandpass');
yy = filter(b,a,y);
```matlab
plot(t,yy)
title('Bandpass Filtered Signal')

newDT = 1/fs
newY = yy;

% triggering value
ys = max(abs(yy))/5;
% subsegment duration
Ts = round(t(end)/10);

figure
% RDT function
[IRF,newT] = RDT(newY,ys,Ts,newDT);
plot(newT,IRF,'b');
xlabel('time (s)')
ylabel('normalized displacement')

figure(3);
N=length(newT);
L=N;
f = (0:L/2);
Y = fft(IRF);
P2 = abs(Y/L);
P1 = P2(1:L/2+1);
P1(2:end-1) = 2*P1(2:end-1);
plot(f,P1)
title('Single-Sided Amplitude Spectrum')
xlabel('f (Hz) in bins')
ylabel('|P1(f)|')

figure(4)
x = linspace(0,fs/2,length(P1));
plot(x,P1)
title('Single-Sided Amplitude Spectrum')
xlabel('f (Hz)')
ylabel('|P1(f)|')

figure(51)
% subplot(2,1,1)
wname = 'morl';
scales = 1:0.5:256;
% subplot(2,2,3)

figure(52)
cw1=cwt(IRF,scales,wname,'plot');
% (plots the continuous wavelet transform coefficients, using default coloration 'absglb')
title('Continuous Transform, absolute coefficients.')
ylabel('Scale')
% subplot(2,1,2)
```

101
figure(53)
[cw1,sc] = cwt(IRF,scales,wname,'scalCNT');  \%( displays a contour representation of the scalogram\%)
title('Scalogram')
ylabel('Scale')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%Time vs Scale vs Amplitude%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(6)
cw2 = cwt(IRF,scales,wname,'3Dlvl'); \%(display 3D plot\%)
colormap winter;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%Scales and Frequencies%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(7)
%fs=280;
cfs = abs(cw2).^2;
pfreq = scal2frq(scales,wname,1/fs); \%(returns the pseudo-frequencies corresponding to the scales given and % the wavelet function 'wname' and the sampling period fs.)%
plot(pfreq,cfs);
axis tight;
grid on;
xlabel('Approximate Frequency (Hz)');
ylabel('Power of Coefficients');
title('CWT with Power vs Frequency');

figure(8)
cw3 = cwt(IRF,pfreq,wname,'3Dlvl'); \%(display 3D plot\%)
colormap summer;
axis tight;
grid on;
xlabel('Time (Sec)');
ylabel('Frequency (Hz)');
zlabel('Coefficient');
title('CWT with Coefficient, Time vs Frequency');

X=newT;
Y=pfreq;
Z=cfs;

figure(9)
surf(X,Y,Z); shading('interp');
xlabel('Time (sec)'); ylabel('Frequency (Hz)'); zlabel('Wavelet Amplitude');
title('Wavelet Amplitude Map');

figure(10)
contour(X, Y, Z)
xlabel('Time'); ylabel('Frequency')

figure(11)
y=0.2;
z=interp2(X,Y,Z,X,y);
plot(X,z);
xlabel('Time (Sec)');
ylabel('Power of Coefficients');
title('Amplitude Spectrum at Selected Natural Frequency');

figure(12)
logamp=log(z);
plot(X,logamp)
APPENDIX D

MATLAB Code for Holland Bridge

```matlab
figure(1)
Data=xlsread('Set1.xlsx')
t=Data(:,1)
y=Data(:,3)

plot(t,y)
xlabel('Time (sec)')
ylabel('Acceleration (m/sec^2)')
title('Original Signal')

%% Band Pass Filtering

figure(2)
order = 5;
fs=280;
fcutlow = 3;
fcuthigh = 20;
[b,a] = butter(order,[fcutlow/(fs/2) fcuthigh/(fs/2)], 'bandpass');
yy = filter(b,a,y);
plot(t,yy)
title('Bandpass Filtered Signal')

%% Random Decrement Technique (RDT)

newDT = 1/fs
newY = yy;

% triggering value
ys = max(abs(yy))/5;
% subsegment duration
Ts = round(t(end)/25);

figure
% RDT function
[IRF,newT] = RDT(newY,ys,Ts,newDT);
plot(newT,IRF,'b');
xlabel('time (s)')
ylabel('Normalized Acceleration')

figure(12)
subplot(311)
plot(t,y)
xlabel('Time (sec)')
ylabel('Acceleration (m/sec^2)')
title('Raw Signal')

subplot(312)
plot(t,yy)
title('Filtered Signal')
```

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```matlab
xlabel('Time (sec)')
ylabel('Acceleration (m/sec^2)')

subplot(313)
plot(newT,IRF,'b');
title('Decay Signal')
xlabel('Time (sec)')
ylabel('Normalized Acceleration (m/sec^2)')

figure(3);
N=length(newT);
L=N;
f = (0:L/2);
Y = fft(IRF);
P2 = abs(Y/L);
P1 = P2(1:L/2+1);
P1(2:end-1) = 2*P1(2:end-1);
plot(f,P1)
title('Single-Sided Amplitude Spectrum of x(t)')
xlabel('f (Hz) in bins')
ylabel('|P1(f)|')

figure(4)
x = linspace(0,fs/2,length(P1));
plot(x,P1)
title('Single-Sided Amplitude Spectrum')
xlabel('f (Hz)')
ylabel('|P1(f)|')

figure(5)
subplot(2,1,1)
wname = 'morl';
scales = 1:0.5:256;
subplot(2,2,3)
cw1=cwt(IRF,scales,wname,'plot');
   title('Continuous Transform, absolute coefficients.')
ylabel('Scale')
subplot(2,1,2)
[cw1,sc] = cwt(IRF,scales,wname,'scalCNT');
   title('Scalogram')
ylabel('Scale')

figure(6)
cw2 = cwt(IRF,scales,wname,'3Dlvl');
   title('3Dscalogram')
colormap winter;
```
figure(7)
fs=280;
cfs = abs(cw2).^2;
pfreq = scal2frq(scales,wname,1/fs);  %(returns the pseudo-frequencies corresponding to the scales given and the wavelet function 'wname' and the sampling period fs.)%
plot(pfreq,cfs);
axistight;
gridon;
xlabel('Approximate Frequency (Hz)');
ylabel('Power of Coefficients');
title('CWT with Power vs Frequency');

figure(8)
cw3 = cwt(IRF,pfreq,wname,'3Dlvl');  %(display 3D plot)%
colormap summer;
axistight;
gridon;
xlabel('Time');
ylabel('Freq');
zlabel('Coef');
title('CWT with Coef, Time vs Freq');

X=newT;
Y=pfreq;
Z=cfs;

figure(9)
surf(X,Y,Z); shading('interp');
xlabel('Time'); ylabel('Frequency');zlabel('Wavelet Amplitude')

figure(10)
contour(X, Y, Z)
xlabel('Time'); ylabel('Frequency')

figure(11)
y=0.41;
z=interp2(X,Y,Z,X,y);
plot(X,z);
xlabel('Time (Sec)');
ylabel('Power of Coefficients');
title('Amplitude Spectrum at Selected Natural Frequency');
MATLAB Code for Chinese Bridge

figure(1)
load ('d_08_7_31_11')
t=Data(:,1)
y=Data(:,3)
plot(t,y)
xlabel('Time (sec)')
ylabel('Acceleration (m/sec^2)')
title('Original Signal')

%% Band Pass Filtering
figure(2)
order    = 5;
fs=100;
fcutlow  = 0.2;
fcuthigh = 3;
[b,a]    = butter(order,[fcutlow/(fs/2) fcuthigh/(fs/2)], 'bandpass');
yy        = filter(b,a,y);
plot(t,yy)
title('Bandpass Filtered Signal')

%% Random Decrement Technique (RDT)
newDT = 1/fs
newY = yy;

% triggering value
ys = max(abs(yy))/5;
% subsegment duration
Ts = round(t(end)/10^12);
figure
% RDT function
[IRF,newT] = RDT(newY,ys,Ts,newDT);
plot(newT,IRF, 'b');
xlabel('time (s)')
ylabel('Normalized Acceleration')

figure(12)
subplot(311)
plot(t,y)
xlabel('Time (sec)')
ylabel('Acceleration (m/sec^2)')
title('Raw Signal')

subplot(312)
plot(t,yy)
title('Filtered Signal')
xlabel('Time (sec)')
ylabel('Acceleration (m/sec^2)')
subplot(313)
plot(newT,IRF,'b');
title('Decay Signal')
xlabel('Time (sec)')
ylabel('Normalized Acceleration (m/sec^2)')

figure(3);
N=length(newT);
L=N;
f = (0:L/2);
Y = fft(IRF);
P2 = abs(Y/L);
P1 = P2(1:L/2+1);
P1(2:end-1) = 2*P1(2:end-1);
plot(f,P1)
title('Single-Sided Amplitude Spectrum of x(t)')
xlabel('f (Hz) in bins')
ylabel('|P1(f)|')

figure(4)
x = linspace(0,fs/2,length(P1));
plot(x,P1)
title('Single-Sided Amplitude Spectrum')
xlabel('f (Hz)')
ylabel('|P1(f)|')

figure(51)
%subplot(311)
wname = 'morl';
scales = 1:0.5:256;
figure(52)
%subplot(312)
cw1=cwt(IRF,scales,wname,'plot');
% (plots the continuous wavelet transform coefficients, using default coloration 'absglb')

%title('Continuous Transform, absolute coefficients.')
ylabel('Scale')
figure(53)
%subplot(313)
[cw1,sc] = cwt(IRF,scales,wname,'scalCNT');
%( displays a contour representation of the scalogram)
%title('Scalogram')
ylabel('Scale')

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% %Time vs Scale vs Amplitude %
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure(6)
cw2 = cwt(IRF,scales,wname,'3Dlvl');
%(display 3D plot)
colormap winter;
figure(7)
%fs=280;
cfs = abs(cw2).^2;
pfreq = scal2frq(scales,wname,1/fs); % (returns the pseudo-frequencies
% corresponding to the scales given and
% the wavelet function 'wname' and the sampling period fs.)%
plot(pfreq,cfs);
axis tight;
grid on;
xlabel('Approximate Frequency (Hz)');
ylabel('Power of Coefficients');
title('CWT with Power vs Frequency');

figure(8)
cw3 = cwt(IRF,pfreq,wname,'3Dlvl'); % (display 3D plot)%
colormap summer;
axis tight;
grid on;
xlabel('Time');
ylabel('Freq');
zlabel('Coef');
title('CWT with Coef, Time vs Freq');

X=newT;
Y=pfreq;
Z=cfs;

figure(9)
surf(X,Y,Z); shading('interp');
xlabel('Time'); ylabel('Frequency'); zlabel('Wavelet Amplitude')

figure(10)
contour(X, Y, Z)
xlabel('Time'); ylabel('Frequency')

figure(11)
y=0.41;
z=interp2(X,Y,Z,X,y);
plot(X,z);
xlabel('Time (Sec)');
ylabel('Power of Coefficients');
title('Amplitude Spectrum at Selected Natural Frequency');
APPENDIX F

Letters of Permission

11/04/2016

Dear Sir/Madam,

I'm a Masters student working under Dr. Yufeng Hu for my thesis; Application of Wavelet Transform in Structural Health Monitoring.

I would like to request your permission to include an excerpt from the following item in my thesis

Damage Detection in Concrete by Fourier and Wavelet Analyses (DOI: 10.1061/ASCE0733-9399(2003)129:5(571))

I'm wrapping up my thesis and I need 2 figures, Figure 3 and Figure 8, of course with citing the resources. This two figures will help the readers to get a clear idea of the features of scalograms. The source will receive full credit in the manuscript.

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Yashodhya Kankanamge
y.liyanakankanamge@wmich.edu

Graduate Assistant
Department of Civil &Construction Engineering
Western Michigan University
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Dear Sir/Madam,

I'm a Masters student working under Dr. Yufeng Hu for my thesis; Application of Wavelet Transform in Structural Health Monitoring.

I would like to request your permission to include an excerpt from the following item in my thesis

Title: Sequential Damage Detection based on the Continuous Wavelet Transform
Authors: Yizheng Liaoa and Konstantinos Balafasb and Ram Rajagopala and Anne S. Kiremidjianb
(Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems 2015,
© 2015 SPIE · CCC code: 0277-786X/15/$18 · doi: 10.1117/12.2084495)

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Thank you for your time and attention to this matter.

Sincerely,

Yashodhya Kankanamge
y.liyanakankanamge@wmich.edu

Graduate Assistant
Department of Civil &Construction Engineering
Western Michigan University
Dear Ms. Kankanamge,

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Hi Yashodhya:

I think SPIE holds the copyright of this paper. You may contact them. I am fine with it as long as you have a citation within your paper.

Thanks,
Yizheng

Best wishes,
Yizheng
Dr. Li,

I hope this email finds you well. My student Yashodhya Kankanamge is doing her master's thesis on Application of Continuous Wavelet Transform in Structural Health Monitoring. She used the acceleration data from your website for the cable bridge. We actually contacted you back in this summer and asked for your help for data downloading. We really appreciate that you provided the password to us.

Now Yashodhya is wrapping up her thesis. In her thesis, she wants to use the two figures in your following paper:


She wants to use figure 1 and the top part of figure 3 in the paper, of course with citing the resource. This two figures will help the readers get an idea of the general view of the bridge and the location of the accelerometers. I wonder if she has your permission to use them in her thesis. Please let us know. Either way we are very thankful for your help in the study. The data your group provided is very important in her research.

Also, we want to let you know, since her thesis will be available through ProQuest, by agreeing to the use of the item in her thesis, you give ProQuest Information and Learning (PQIL) the right to supply copies of this material on demand as part of her master thesis. Please attach any other terms and conditions for the proposed use of this item.

Thank you for your time and attention to this matter.

Best regards,
Yufeng Hu

--

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Phone: (269) 276-3310
Fax: (269) 276-3211
Dear Dr. Hu,

Yashodhya can use the figures, no problem. I hope that she cite the paper.

Regards

Hui

lihui@hit.edu.cn