An Equivalent Plate Model with Orthotropic Material Properties for Adjacent Box-Beam Bridge Superstructure

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AN EQUIVALENT PLATE MODEL WITH ORTHOTROPIC MATERIAL PROPERTIES FOR ADJACENT BOX-BEAM BRIDGE SUPERSTRUCTURE

by

Timothy Alexander Schnell

A thesis submitted to the Graduate College in partial fulfillment of the requirements for the degree of Master of Science in Engineering (Civil)
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Reflective cracking over the longitudinal joints of adjacent box-beam bridges is a well-documented and recurrent problem; it is a concern because it leads to poor durability performance. These cracks develop during early stages of bridge construction and have not been eliminated by the design revisions implemented so far.

This study is aimed at developing a rational load demand analysis model for calculating the moment acting at the longitudinal joints in order to determine the impact of post-tensioning on mitigating reflective cracking.

The scope of the study includes calculating moment demand at the shear keys using a rational analysis model based on composite materials and mechanics of materials, and determining the post-tensioning required to eliminate tensile stresses developed at the shear keys due to static loading. Additionally, concrete shrinkage will be modeled in ABAQUS to allow consideration of volume change loading.

The results of this study show that volume change loads cause the initiation of reflective cracking and current post-tensioning practice is insufficient to mitigate reflective cracking. Moment demand at longitudinal joints and corresponding required transverse post-tensioning is also presented for several static loading scenarios.
# TABLE OF CONTENTS

LIST OF TABLES .......................................................................................................................................... viii
LIST OF FIGURES ........................................................................................................................................ ix

CHAPTER

1. INTRODUCTION .................................................................................................................................. 1
   1.1: Objective and Scope....................................................................................................................... 2
   1.2: Outline of Thesis .......................................................................................................................... 2

2. RESEARCH METHODOLOGY ........................................................................................................... 3

3. STATE-OF-THE-ART LITERATURE REVIEW ................................................................................ 5
   3.1: Introduction and Objective........................................................................................................... 5
   3.2: Bridge Performance .................................................................................................................... 5
   3.3: Common Details ......................................................................................................................... 7
      3.3.1: Box-beam geometry............................................................................................................... 7
      3.3.2: Shear key geometry ............................................................................................................. 10
      3.3.3: Shear key material ............................................................................................................. 13
      3.3.4: Transverse post-tensioning ................................................................................................. 15
      3.3.5: Reinforced cast-in place concrete slab .............................................................................. 19
      3.3.6: Bearing pad details.............................................................................................................. 20
# Table of Contents-Continued

## CHAPTER

3.4: Shrinkage ........................................................................................................................ 20

3.5: Temperature Effects ....................................................................................................... 21

3.6: Modeling and Analysis .................................................................................................. 22

3.6.1: Grillage analysis ..................................................................................................... 23

3.6.2: Plate analysis using equivalent properties .............................................................. 24

3.6.3: Numerical analysis and finite element modeling .................................................... 25

3.7: Summary ........................................................................................................................ 26

4. FIELD INSPECTION ............................................................................................................ 28

4.1: Introduction .................................................................................................................... 28

4.2: Bridge Description ......................................................................................................... 28

4.2.1: Bridge location ........................................................................................................ 28

4.2.2: Bridge geometry and construction .......................................................................... 28

4.3: Bridge Inspection Summary ........................................................................................... 30

4.4: Conclusions of Field Investigation ................................................................................. 33

5. MODELING CONCRETE SHRINKAGE ............................................................................ 34

5.1: Introduction .................................................................................................................... 34

5.2: ABAQUS Modeling ....................................................................................................... 34
Table of Contents-Continued

CHAPTER

5.2.1: Units and coordinate system ................................................................. 35
5.2.2: Parts ...................................................................................................... 35
5.2.3: Meshing ................................................................................................ 37
5.2.4: Assembly ............................................................................................... 37
5.2.5: Material ................................................................................................ 38
5.2.6: Steps ..................................................................................................... 39
5.2.7: Boundary conditions ............................................................................. 40
5.2.8: Loading ................................................................................................ 40
5.3: User Subroutine ....................................................................................... 40
5.3.1: Subroutine USDFLD ........................................................................ 40
5.3.2: Subroutine UEXPAN .......................................................................... 40
5.4: Model Results ......................................................................................... 41

6. ANALYTICAL ANALYSIS USING EQUIVALENT MATERIAL PROPERTIES .... 46
6.1: Introduction ............................................................................................. 46
6.2: Procedure ................................................................................................. 46
6.3: Representative Volume Element ............................................................. 48
6.4: Laminate .................................................................................................. 48
# Table of Contents-Continued

## CHAPTER

6.5: Micromechanics ............................................................................................................. 49

6.6: Macromechanics ......................................................................................................... 51

   6.6.1: Force-strain relationship ..................................................................................... 51

   6.6.2: Plane stress matrix ............................................................................................. 52

   6.6.3: The [ABD] stiffness matrix and its submatrices ..................................................... 53

6.7: Equivalent Material Properties ................................................................................... 54

6.8: CSiBridge Model ........................................................................................................ 56

6.9: Selection of CSiBridge ............................................................................................... 56

6.10: Model Validation ..................................................................................................... 57

   6.10.1: Isotropic plate behavior ....................................................................................... 57

   6.10.2: Orthotropic plate behavior .................................................................................. 59

   6.10.3: Equivalent material properties validation ............................................................ 64

   6.10.4: CSiBridge model verification results ................................................................. 68

6.11: Discussion .................................................................................................................. 70

7. Calculation of Moment Demand at the Shear Keys ...................................................... 71

   7.1: Introduction .............................................................................................................. 71

   7.2: Model Geometry ..................................................................................................... 71
Table of Contents-Continued

CHAPTER

7.2.1: Box beam geometry ................................................................................................ 71

7.2.2: Bridge width and span ............................................................................................ 71

7.3: Loading .......................................................................................................................... 72

7.3.1: Dead load ................................................................................................................ 72

7.3.2: Live load ................................................................................................................. 73

7.4: Results and Discussion ................................................................................................... 75

7.4.1: Effect of bridge width ............................................................................................. 76

7.4.2: Effects of bridge span ............................................................................................ 79

7.4.3: Effect of beam width ............................................................................................... 81

7.4.4: Effect of beam depth ............................................................................................... 82

7.4.5: Effect of loading ..................................................................................................... 82

7.5: Transverse Post-tensioning Force .................................................................................. 83

7.5.1: Post-tensioning force calculation procedure ........................................................... 84

7.5.2: Two-stage post-tensioning results .......................................................................... 85

7.5.3: Comparison with Hanna et al. (2009) equation ...................................................... 87

7.5.4: Comparisons with state post-tensioning practice .................................................... 88

8. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS ........................................ 91
# Table of Contents-Continued

## CHAPTER

8.1: Summary and Conclusions ................................................................. 91

8.2: Recommendations ............................................................................. 92

REFERENCES ............................................................................................... 93

## APPENDICES

A. Equivalent Material Property Calculations for Load Analysis of Adjacent Box-beam Bridge Superstructures ................................................................. 100

B. Plate Analysis Results ........................................................................... 113

C. Moment Demand Results ...................................................................... 116

D. 2-Stage Post-tensioning Requirement Results ........................................ 120
LIST OF TABLES

1. State DOT Post-tensioning Practice................................................................. 16
2. Grout Properties and Requirements ................................................................. 30
3. Young’s Modulus and the Corresponding Field Variable .................................. 39
4. Resulting Equivalent Material Properties ...................................................... 61
5. Plate Analysis Results for 6 in. Thick Plate ..................................................... 63
6. Material Properties ......................................................................................... 66
7. Multiple Presence Factors ............................................................................. 75
8. Post-Tensioning Force Requirements for 36 ft Wide Bridge ............................. 86
9. Post-Tensioning Requirements for 60 ft Wide Bridge ...................................... 86
10. Post-Tensioning Requirements for 84 ft Wide Bridge ..................................... 87
11. State DOT Post-tensioning Practice Considering Per-Foot Force ................... 90
# LIST OF FIGURES

1. Research methodology ................................................................................................................ 3
2. Longitudinal Reflective Cracking and Efflorescence (Pictures Taken by: Timothy Schnell).... 6
3. Typical Side-by-Side Box-Beam Bridge Cross-Section ............................................................. 7
4. Example Box Beam Cross-Section ............................................................................................ 8
5. Box Beam Diaphragm ................................................................................................................. 9
6. Common Shear Key Configurations, Partial- and Full- depth ................................................. 10
7. Top, Mid-Depth, and Bottom Shear Key Configurations Discussed by Kim et al. (2008) ...... 12
8. Japanese Box-Girder Shear Key Configurations ...................................................................... 13
9. South Korean Box-Beam Shear Key Geometry ....................................................................... 13
10. Wide-Joint Connection Detail Based on Detail Provided in Hanna et al. (2011) ................... 18
11. Narrow-Joint Connection Detail Based on Detail Provided in Hanna et al. (2011) ............... 18
12. Causes of Cracking in Bridge Decks ...................................................................................... 21
13. Joint Movement Caused by Daily Thermal Loading .............................................................. 22
14. Bridge Location (Source: Google Maps) ................................................................................ 28
15. B-26-40 Details .......................................................................................................................... 29
16. Overview of B-26-40 ................................................................................................................... 31
17. Crack Width Measurement (Crack Along the Vertical Face of the Slab Over West Abutment) ............................................................................................................................. 32
List of Figures - Continued

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.</td>
<td>Geometry of Crack 1 Over West Abutment</td>
<td>32</td>
</tr>
<tr>
<td>19.</td>
<td>Crack 2 Along Vertical Face of Slab</td>
<td>33</td>
</tr>
<tr>
<td>20.</td>
<td>ABAQUS Model Tree</td>
<td>35</td>
</tr>
<tr>
<td>21.</td>
<td>Box Beam Part</td>
<td>36</td>
</tr>
<tr>
<td>22.</td>
<td>Shear Key Part</td>
<td>36</td>
</tr>
<tr>
<td>23.</td>
<td>Deck Part</td>
<td>36</td>
</tr>
<tr>
<td>24.</td>
<td>Final Bridge Assembly</td>
<td>37</td>
</tr>
<tr>
<td>25.</td>
<td>Edit Material Dialog Box</td>
<td>38</td>
</tr>
<tr>
<td>26.</td>
<td>Stress (S11) Due to Concrete Shrinkage Over the Abutment</td>
<td>42</td>
</tr>
<tr>
<td>27.</td>
<td>Shrinkage Model Results</td>
<td>44</td>
</tr>
<tr>
<td>28.</td>
<td>Post-Tensioning Model Results</td>
<td>45</td>
</tr>
<tr>
<td>29.</td>
<td>Flowchart Depicting Major Steps in Equivalent Material Property Calculation</td>
<td>47</td>
</tr>
<tr>
<td>30.</td>
<td>RVE as Proposed by Attanayake and Aktan (2009)</td>
<td>48</td>
</tr>
<tr>
<td>31.</td>
<td>Representative Volume Element Definition</td>
<td>49</td>
</tr>
<tr>
<td>32.</td>
<td>Layer Divisions of the RVE</td>
<td>49</td>
</tr>
<tr>
<td>33.</td>
<td>Distances Between Midplan Layers</td>
<td>54</td>
</tr>
<tr>
<td>34.</td>
<td>Material Property Data Dialog Box - CSiBridge</td>
<td>57</td>
</tr>
<tr>
<td>35.</td>
<td>Square, Isotropic Plate Model in SAP2000</td>
<td>59</td>
</tr>
<tr>
<td>36.</td>
<td>SAP2000 Orthotropic Plate Model and Results</td>
<td>61</td>
</tr>
<tr>
<td>37.</td>
<td>Box-Beam Partitioning</td>
<td>65</td>
</tr>
<tr>
<td>38.</td>
<td>Meshed ABAQUS Assembly</td>
<td>66</td>
</tr>
</tbody>
</table>
List of Figures - Continued

39. Model Loading .................................................................................................................. 67
40. Model Stress Results ....................................................................................................... 67
41. Model Deflection Results ............................................................................................... 68
42. CSiBridge Model Verification Results ........................................................................... 69
43. Barrier Loading ............................................................................................................... 72
44. CSiBridge Loading Dialog Boxes .................................................................................. 74
45. Two Lane Loading in CSiBridge .................................................................................... 75
46. Maximum Positive Transverse Moment ....................................................................... 77
47. Maximum Negative Transverse Moment ....................................................................... 79
48. Span Effect on Positive Transverse Moment ............................................................... 80
49. Span Effect on Negative Transverse Moment ............................................................... 80
50. Beam Width Effect on Positive Transverse Moment .................................................... 81
51. Beam Width Effect on Negative Transverse Moment .................................................. 81
52. Loading and Resulting Positive Transverse Moment .................................................. 83
53. Loading and Resulting Negative Transverse Moment ................................................ 83
54. Example Bridge Transverse Moment Results (kip-ft/ft) ............................................... 84
1. INTRODUCTION

Bridge systems are a key component of a nation’s civil infrastructure and play a significant role in a country’s economics and competitiveness in a global market. The health of a bridge has significant impacts on transport systems, particularly in large metropolitan areas. In the United States there are more than 609,000 bridges, with almost a quarter being defined as structurally deficient or functionally obsolete. While the number of deficient bridges has declined over the last several years, with the structurally deficient category dropping from eleven percent (11%) in 2012 to nine and a half percent (9.5%) in 2015, the concern associated with the significant number of deficient bridges remains.

Accelerated Bridge Construction (ABC) has been at the forefront in the effort to repair bridges in the United States. ABC has many advantages coming from the reduction in construction time including reduced traffic delays and associated costs. The use of prefabricated bridge elements and systems (PBES) is one of the methods employed to reach the objectives of ABC (Culmo, 2009). PBES can include deck, beam, and pier elements as well as many other miscellaneous elements. One such beam element is the box-beam; box-beams are placed adjacent to each other to make the bridge superstructure.

Side-by-side or adjacent box beam bridges have been a popular choice for medium and short span bridges throughout the United States and represent a significant portion of the bridges built every year. The popularity of adjacent box-beam bridges stems from their favorable span-to-depth ratios, ease of construction, and the high quality that is associated with precast members (Fu et al., 2011; Russell, 2009; PCI, 2009; Culmo. 2009; Grace and Jenson, 2008; Aktan et al., 2005; Miller et al., 1999; El-Remaly et al., 1996). The use of precast members also makes adjacent box-beam bridges a popular choice for ABC projects. However, the issue of longitudinal reflective cracking and ensuing bridge deterioration continues to plague these bridges.

The use of box beams to build bridges first started in the 1950’s and evolved from open channel designs; shear keys allowed load transfer between beams and the box shape provided the torsional rigidity required to sustain this method of load transfer (Miller et al., 1999). In the years since its introduction, numerous changes to box-beam details have occurred in an effort to
improve the durability of these bridges, but the longitudinal connections continue to remain vulnerable to degradation (Attanayake and Aktan, 2014; Attanayake and Aktan, 2009).

1.1: Objective and Scope
This study is aimed at developing a rational load demand analysis model using equivalent material properties for calculating the moment acting at the longitudinal joints of side-by-side box-beam bridge superstructures in order to determine the impact post-tensioning has in mitigating reflective cracking. Currently no rational process exists for determining the moment demand at the shear key locations.

The scope of the study includes (i) developing rational analysis models using basics of composite materials and mechanics of materials, (ii) validating the applicability of modeling concept for side-by-side box-beam bridges, (iii) developing moment demands at shear keys for commonly used girder sizes and bridge geometries, and (iv) modeling reflective cracking in order to determine the effect of post-tensioning on preventing reflective cracks. In addition, a design example will be included to present the application of the concept.

1.2: Outline of Thesis
Chapter 1: this chapter presents the objective of the research and the scope of work.
Chapter 2: this chapter presents the research methodology followed throughout the study.
Chapter 3: this chapter presents an in-depth literature review pertaining to the performance and construction of adjacent box-beam bridges.
Chapter 4: this chapter presents the result of a field inspection of a newly constructed adjacent box-beam bridge in Wisconsin. Earlier age reflective cracking is identified
Chapter 5: this chapter discusses the development of a concrete shrinkage subroutine and modeling shrinkage of the cast-in-place deck on an adjacent box-beam bridge
Chapter 6: this chapter presents the development of the load demand analysis model using equivalent material properties. Additionally, this chapter discusses the verification of the model.
Chapter 7: this chapter presents the results of the model and shows the transverse moment demand at the longitudinal joints. The analysis procedure and required transverse post-tensioning is discussed.
Chapter 8: this chapter provides a conclusion that summarizes this study and provides recommendation for future study.
2. RESEARCH METHODOLOGY

In an effort to accomplish this study’s objectives a multifaceted research methodology is developed. The research methodology is presented as a flowchart in Figure 1. In order to accomplish the objectives of this study three primary tasks need to be completed.

![Flowchart of Research Methodology](image)

**Task-1** is to develop a load demand model and identify the causes of longitudinal reflective cracking on box-beam bridges. Modeling adjacent box-beam bridges, though various means, has been a major component in the research of reflective deck cracking. Literature on the various modeling techniques including grillage, equivalent stiffness, equivalent material properties, and finite element will be reviewed. Simplifying the complex geometry of the adjacent box-beam superstructure to an orthotropic plate is desired. Additionally, this task requires developing a thorough understanding of adjacent box-beam bridges including common bridge details, best construction practices, and commonly used materials. The in-depth literature review will also consider common elements and details of adjacent box-beam bridges, including beam and shear key geometries, materials used in construction, and current post-tensioning practices. The literature review will be presented in Chapter 3.
Task-2 is the validation of the developed load analysis model. In order to accomplish this validation the results from the developed analysis model will be compared with results from a refined finite element analysis. Simultaneously, an effort will be made to model the factors that cause reflective cracking. These factors will have been identified through the literature review and an inspection of a newly constructed box-beam bridge.

Task 3 is the computation of the transverse moment along the longitudinal joints. A structural analysis program will be utilized to solve for the transverse moment at the longitudinal joint locations on an orthotropic plate representing the adjacent box-beam bridge superstructure.

Finally, the computed moment demand will be used to determine the required transverse post-tensioning to prevent tensile forces from forming in the deck under static loading. The results of the model considering factors affecting reflective cracking will be considered as as the impact of post-tensioning on mitigating reflective cracking.
3. STATE-OF-THE-ART LITERATURE REVIEW

3.1: Introduction and Objective

The following section presents an in-depth review of literature. Existing literature dealing with box-beam bridge performance, detailing, materials, and structural modeling concepts is reviewed. Additionally, considerations are given to existing analysis methods and tools for analyzing adjacent box-beam bridge structures.

The objective of this literature review is to identify information directly related to adjacent box-beam bridges. Particular interest has been given to the structural system and components of the modern box-beam bridge as well as box-beam bridge performance and design modifications that have occurred since its introduction. An investigation into modeling precast orthotropic bridge superstructure systems and identifying available analysis methods is also covered.

3.2: Bridge Performance

Longitudinal reflective deck cracking at shear key locations is a well-known and documented problem with adjacent box-beam bridges (Attanayake and Aktan, 2013; Attanayake and Aktan, 2011; Grace and Jenson, 2008; Sharpe, 2007; Harries, 2006; Aktan et al., 2005; Huckelbridge et al., 1995; Lall et al., 1998; Miller et al., 1999; El-Remaily et al., 1996). Reflective cracking allows water ingress, which becomes a durability concern in states that use deicing salts. The water that penetrates into the beams is often laced with chloride ions that initiate the corrosion of steel reinforcements and prestressing tendons (Huffman, 2012). Efflorescence forming below the shear key locations is evidence of water penetration. Because shear keys and transverse post-tensioning serve to develop moment and shear stiffness across the entirety of the cross-section, developing issues could lead to serious structural damage. Figure 2 shows some adjacent box-beam bridges that have been affected by longitudinal reflective cracking and water ingress.
National Cooperative Highway Research Program (NCHRP) Synthesis Report 393 was developed to address the need for construction practices that assure the durability of the system; the report documents the best practices implemented by various highway agencies (Russell,
A report was also published by the Precast/Prestressed Concrete Institute describing current practices for the design and construction of side-by-side box-beam bridges (PCI, 2011). The reports included data collected from two surveys that were focused on determining the extent of the reflective cracking problem and current construction practices. Both Russell (2009) and PCI (2011) provided recommendations for design, fabrication, and construction to mitigate reflective cracking.

Michigan has been one of the few states to implement the changes suggested by Russell (2009) and PCI (2011). Attanayake and Aktan (2013) described the evolution of box-beam bridge design in Michigan along with documenting the performance of in-service bridges. Even the most up-to-date specifications implemented in Michigan could not prevent reflective cracking from occurring. Grace and Jensen (2008) suggested increasing the post-tensioning force magnitude to prevent cracking, however, it has since been shown that increasing the force magnitude alone will not stop deck cracking (Ulku et al., 2010).

### 3.3: Common Details

The majority of adjacent box-beam bridges share similar geometries consisting of several rectangular box-beams placed side-by-side. Adjacent box-beam bridge designs vary slightly depending on usage and location. Figure 3 shows a standard bridge cross-section.

![Figure 3. Typical Side-by-Side Box-Beam Bridge Cross-Section](image)

#### 3.3.1: Box-beam geometry

Typical box beams consist of a rectangular section with either rectangular or circular voids. The exact details of the beam vary around the world and even from state to state. Many of these variations are due to fabricator preference. An example of a standard section used in the United
States is shown in Figure 4. The void sections are formed using Styrofoam blocks that are placed after pouring the bottom slab of the beam.

Diaphragms provide cross-section stability and along with post-tensioning provide additional transverse stiffness to the bridge superstructure. Diaphragms are located at various points along the beam. The number of diaphragms varies from bridge to bridge and is typically related to the overall span length. The majority of bridges have between two to five diaphragms. Presently diaphragms also serve as a location for post-tensioning to be applied to the bridge.
Figure 5. Box Beam Diaphragm
3.3.2: Shear key geometry

One of the most significant components of side-by-side box-beam bridges is the shear key. Shear key failure compromises the ability of the bridge to distribute the load amongst the beams and allows water ingress which leads to corrosion problems. Shear keys are commonly defined as partial- or full-depth, where depth is in reference to the depth of the keyway. Some states, such as Michigan, use a full-depth grouted partial depth shear key (Attanayake and Aktan, 2008). Figure 6 shows the layout of partial- and full-depth shear keys.

![Figure 6. Common Shear Key Configurations, Partial- and Full-depth](image)

The use of partial-depth shear keys was common prior to the year 1992 (Lall et al., 1998). However, partial depth-shear keys do not fully connect adjacent beams, allowing a ‘hinging’ action and the development of transverse moments; these moments in turn produce tensile
stresses that lead to longitudinal cracking. In a study by Lall et al. (1998) it was found that the use of full-depth shear keys along with transverse post-tensioning reduced the percentage of bridges that suffer from longitudinal cracking. Miller et al. (1999) also noted that the use of full-depth shear keys may prevent the ‘hinging’ action that leads to keyway joints opening. El-Esnawi (1994) also suggested a shear key design that would involve moving the keyway from the upper third of the longitudinal joint to mid-height. El-Esnawi’s research included an experimental testing program and modeling the proposed detail in a finite element program. Results indicated that this change in shear key configuration could nearly triple the static load capacity of the shear key, all other variables held constant.

Kim et al. (2008) conducted a study that involved load testing and finite element analysis. Kim et al. concluded that shear keys should be formed with cast-in-place concrete and transverse post-tensioning should be used. Additional, it was found that the location of the shear key can have an effect on the deflection of side-by-side box beams. This was done by moving the location of the shear key to different depths along the side of the box beam. Four models were run, each one with the keyway at a different depth. The models consisted of shear keys placed near the top of the beam, mid-depth, and bottom, as well as a model with no shear key. It was found that the shear keys located near the top and bottom of the beam suffered more deflection than the shear key with the keyway placed at mid-depth. Figure 7 shows the top, mid-depth, and bottom shear key configurations. The deflections with the shear key placed at the bottom of the beam were 5% larger when compared with the results from the mid-depth shear key model; the model with no shear key had 40% larger deflections (Kim et al., 2008)
Box beams from Japan vary from those in the United States in that the longitudinal shear keys are detailed differently. Yamane et al. (1994) reported on various types of precast prestressed bridges in Japan and described two different types of box beams used. The first type referred to was described as a pretensioned voided box girder. The pretensioned voided box girder lacks a keyway slot and common practice involves filling the full-depth shear key with cast-in-place concrete, followed by application of post-tensioning and covering the section with a two to three inch wearing surface (Yamane et al., 1994). The second box beam discussed was the post-tensioned concrete box girder; this system resembles a box-beam with T-girder like flanges added to the top flange of the box-girder. This second type of box-girder bridge is constructed in a way that resembles a spread box-beam bridge, with cast-in-place diaphragms added along the span of the bridge. Figure 8 shows the geometries of the two box-girder configurations used in Japan.
South Korean box beams are similar to the pretensioned voided box girder in Japan, but have a keyway slot along their depth. Kim et al. (2008) used the South Korean geometry in the study previously discussed. Figure 9 shows the geometry used in South Korea, reproduced based on drawings in Kim et al. (2008).

3.3.3: Shear key material
Shear keys are formed by filling the keyways between box beams with grout, mortar, or cast-in-place concrete. The grouting material is responsible for transferring the flexural and shear stresses between the beams. The importance of the mechanical properties of the shear key
material has led to numerous studies being conducted on the performance of various shear key materials.

### 3.3.3.1: Concrete, grout, and mortar

The use of mortar to form shear keys is common throughout the United States. A forensic investigation by Attanayake and Aktan (2013) showed that there was a loss of bond between the shear key mortar and the beam. Attanayake and Aktan further noted that when typical cement mortar is used, cracking develops at the grout-beam interface within two to three days of grouting. They further reported that reflective deck cracking appeared within fifteen days of deck pouring. These results indicate that there is a limited bond that develops between the shear key material and the beam.

The performance of shear keys with cementitious grout and fiber reinforced cementitious grout was evaluated by Sang (2010), he found that higher bond strength was obtained by using epoxy grout and fiber reinforced cementitious grout. High Performance Fiber Reinforced Concrete (HPFRC) was also evaluated by Hoomes et al. (2014), several mixes and test methods where used and documented. Hoomes et al. (2014) recommended the use of bond strength tests to determine the expected structural response of the connection. Sharpe (2007) noted that the beam-shear key interface bond was the weakest failure mode related to keyways.

Several studies have assessed commercial grout for shear keys, due to the interest shown in them by various DOTs. One study was conducted by Gulyas et al. (1995) who evaluated non-shrink grout through various tests. Gulyas et al. (1995) found that the bond strength was lower than the seven-day tensile strength of the grout used in the test. The use of magnesium ammonium phosphate (Mg-NH₄-PO₄) increased the bond strength by 214%, when compared to the non-shrink grout. The approach for testing the grouted shear key presented by Gulyas et al. (1995) consisted of a composite direct tension test and was found to be an effective method for evaluating the performance properties of various shear key layouts and compositions. Nottingham (1995) followed up on Gulyas’ work and commented that an impermeable high quality grout, with low shrinkage, high bond, high early strength and low temperature curing was needed. The optimal grout should also have user friendly characteristics, such as good workability.
3.3.3.2: Ultra-High Performance Concrete (UHPC)

A relatively new option is Ultra-High Performance Concrete (UHPC). Developed in the mid- to late-90’s, UHPC first started being implemented by the U.S. highway system in 2006; UHPC has low water-to-cement ratios, around 0.25, and contains steel fiber reinforcement (Graybeal, 2011; Russell and Graybeal, 2013; Ubbing, 2014). According to Russell and Graybeal (2013) UHPC exhibits compressive strengths above 21.7 ksi and a tensile strength above 0.72 ksi.

Several bridges have been constructed using UHPC either for use as a shear key or to connect deck panels, however widespread use has not been seen. UHPC has been used in several states and has been specified for joints in precast construction. One example is the Highway 6 Bridge over Keg Creek in Iowa constructed in 2011; Phares et al. (2013) reported on the project. The UHPC was used for the longitudinal connections; Phares et al. (2013) noted that seven months after project completion, cracking occurred at the UHPC connection and girder flange interface. While UHPC’s ability to develop high early strength is a desirable attribute for shear keys it displays considerable shrinkage and without adequate development of bond strength it becomes a less desirable alternative for shear keys (French et al., 2011; Freyne et al., 2012). Life cycle cost is another one of several issues that UHPC suffers from, Piotrowski and Schmidt (2012) reported that a life cycle cost analysis on UHPC revealed it was 23% more expensive than normal concrete. Graybeal (2009) noted another reason for contractor reluctance to use UHPC is the absence of detailed design codes.

The primary reason for considering UHPC for shear keys would be to resist the tensile forces that result in longitudinal cracking. In addition, post-tensioning of bridge decks is time consuming particularly when being used on Accelerated Bridge Construction (ABC) projects, and a high strength material with good bonding properties could have negated the post-tensioning requirement. However, Hartwell (2011), through testing, found that UHPC joints underwent early debonding, even below service load conditions. Hartwell (2011) suggested further investigation into the improvement of the direct tensile bond strength of UHPC connections.

3.3.4: Transverse post-tensioning

Grace et al. (2008) mentioned that the service life of an adjacent box-beam bridge has a direct correlation with its ability to distribute loads between the beams. The implications of this relationship demonstrate just how important both post-tensioning and shear key structure is for
the adjacent box beam bridge. According to Section 4.6.2.2.1, the AASHTO Bridge Design Manual requires a minimum stress magnitude of 0.25 ksi (AASHTO LRFD, 2012). However, transverse post-tensioning force and location vary by state. New York State DOT, for example, requires that post-tensioning be placed at three locations for spans less than 50 feet and five locations for spans of greater length (NYSDOT, 2014). Attanayake and Aktan (2008, 2009) discussed Michigan’s post-tensioning; Michigan recommends post-tensioning occur at the mid-height of the beam or at the 1/3 points along the height for beams less than and greater than 27 inch in depth, respectively. Russel (2009) presented some of the average post-tensioning forces used by various states and recommend by different authors; figure 15 in Russel (2009) shows these ranges of average transverse force. Russel calculated the post-tensioning force per-foot of the bridge by dividing the force specified in each states respective bridge design manual by the maximum distance between the specified post-tensioning locations.

Table 1 was developed by reviewing various state DOT bridge design guides. The table highlights the location, along the span, at which post-tensioning is applied and the post-tensioning force specified.

### Table 1. State DOT Post-tensioning Practice

<table>
<thead>
<tr>
<th>State DOT</th>
<th>Post-Tensioning Location*</th>
<th>Post-Tensioning Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ends</td>
<td>¼ Pt.</td>
</tr>
<tr>
<td>Connecticut</td>
<td>X</td>
<td>75 &lt; S</td>
</tr>
<tr>
<td>Delaware</td>
<td>X</td>
<td>120 &lt; S ≤ 160</td>
</tr>
<tr>
<td>Illinois</td>
<td>X</td>
<td>N = (S/25 – 1) ≥ 1</td>
</tr>
<tr>
<td>Massachusetts</td>
<td>X</td>
<td>50 &lt; S</td>
</tr>
<tr>
<td>Michigan</td>
<td>X</td>
<td>50 &lt; S ≤ 100</td>
</tr>
<tr>
<td>New Hampshire</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>New York</td>
<td>X</td>
<td>50 ≤ S</td>
</tr>
<tr>
<td>Ohio</td>
<td>X</td>
<td>75 &lt; S</td>
</tr>
<tr>
<td>Oregon</td>
<td>X</td>
<td>120 &lt; S</td>
</tr>
<tr>
<td>Pennsylvania</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>Virginia</td>
<td>X</td>
<td>50 &lt; S</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>X</td>
<td>120 &lt; S ≤ 160</td>
</tr>
</tbody>
</table>

*Location may depend on Span and Skew*
Hanna et al. (2009) proposed an equation for calculating the required post-tensioning magnitude. The method used to develop this equation was grillage analysis. Similar to the method developed by El-Remaily et al. (1996), post-tensioned transverse diaphragms were assumed to be the primary mechanism for transferring loading across the bridge. Longitudinal beam elements represented the box girders and transverse beam elements represented the diaphragms. The equation developed provides the force per unit length of the bridge required for post-tensioning. Thus, by using the spacing between ducts, span, width and skew the required force per diaphragm can be determined. The equation proposed by Hanna et al. (2009) is given below.

\[
P = \left(\frac{0.9W}{D} - 1.0\right) K_L K_S \leq \left(\frac{0.2W}{D} + 8.0\right) K_L K_S
\]

Where,

- \(D\) = box depth (in.)
- \(W\) = bridge width (in.)
- \(K_L\) = correction factor for span-to-depth ratio
  \[= 1.0 + 0.003\left(\frac{L}{D} - 30\right)\]
- \(K_S\) = skew correction factor
  \[= 1.0 + 0.002\theta\]
- \(L\) = bridge span (in.)
- \(\theta\) = skew angle (deg.)

In a later article, Hanna et al. (2011) considered two alternatives to post-tensioned joints. The first alternative was a wide-joint system. The wide-joint system uses transverse reinforcement traveling through the top and bottom flange along with spiral reinforcement placed into recesses along the shear key. Figure 10 shows the proposed wide-joint system. The other alternative was a narrow-joint system from the Illinois Department of Transportation (IDOT). The narrow-joint system consists of tie rods placed through the top and bottom flange of the box-beam. Figure 11 shows the IDOT narrow-joint system.
Hanna found that the proposed connections improved the capacity of the bridge system when compared with the current post-tensioning system. The performance was evaluated by comparing the results of three-dimensional (3-D) computer models and the capacity of specimens developed. The 3-D computer models were composed of shell elements and were used to compute the required tensioning force. The specimens consisted of two beams with the respective joint connection, simulated HL93 loading was applied to the specimens for between 10,000 cycles to 3 million cycles. Throughout the duration of the applied loading the joint was monitored for cracking and water leakage. Grout used in the shear key locations was allowed to reach a compressive strength of 6000 psi prior to loading.

The specimen representing the current IDOT post-tensioning design failed after 10,000 cycles. Failure of the wide-joint connection occurred at the shear key location with cracking occurring at the grout beam interface. The wide-joint connection failed after a load of 162 kips was applied, or an equivalent uniform load of 20.3 kip/ft. The narrow-joint connection did not fail during the first 2 million cycles. Narrow-joint failure occurred when the applied load was 156 kips, or at an
equivalent uniform load of 19.5 kip/ft. Results of a cost-analysis indicated that this proposed connection would be more economical than post-tensioning.

Another analysis on transverse post-tensioning, done by Grace et al. (2012), using finite element analysis considered the effects of traffic loading and temperature. Recommendation based on this study include increasing the number diaphragms and included suggestions for post-tensioning force based on bridge width.

Aktan et al. (2009) showed that post-tensioning did not compress the shear keys in regions between the diaphragms. Additionally, Attanayake and Aktan (2013) showed that the cracking occurred in the cast-in-place slab before the bridge had been opened to traffic. Therefore Attanayake and Aktan proposed a two stage post-tensioning sequence that would compress the cast-in-place concrete slab. The effectiveness of this two stage post-tensioning sequence was further demonstrated by Ulku et al. (2010), however consideration was not given to the effects of creep and shrinkage. A related earlier study by Ulku et al (2009) demonstrated that compression as a result of post-tensioning was limited to 1.35 times the width of the precast panel in the direction parallel to post-tensioning. When considering the previous studies Attanayake and Aktan (2013) recommended limiting the spacing between post-tensioning locations to the width of the fascia beam, however they did not perform an in-depth evaluation into the effects of this change.

3.3.5: Reinforced cast-in place concrete slab

A survey performed by the PCI subcommittee (1995) on adjacent member bridges presented a number of lessons learned and suggestions for future construction. One of these suggestions was the use of a cast-in-place deck. A number of states now use cast-in-place concrete slabs including Illinois, New York, Wisconsin, and Michigan (Macioce et al., 2007; Russel, 2009; Attanayake and Aktan, 2013). However, reflective cracking is still present on bridges with cast-in-place decks. Attanayake and Aktan (2013) documented the development of longitudinal reflective cracking prior to any live loading on the bridges in Michigan with a 6-inch cast-in-place deck. The early age of the cracking indicates that reflective cracks are due to concrete shrinkage, temperature gradient, or prestress losses (Attanayake and Aktan, 2013).

The effect of concrete shrinkage in cast-in-place decks was also described by Sharpe (2007). Sharpe (2007) used finite element analysis to model concrete deck shrinkage and the effects of temperature loads. Sharpe (2007) found that concrete shrinkage occurred near the support
regions of the bridge and recommended considering methods to reduce shrinkage effects. Additionally, Sharpe’s (2007) research indicated that daily temperature fluctuation can be large enough to cause stress reversal and cause cracking in cast-in-place concrete decks.

3.3.6: Bearing pad details

Miller et al. (1999) noted that longitudinal cracking initiated over the abutments and then spread toward the mid-span. This observation indicates that bearing pad layout may be one of the key factors leading to reflective cracking. Lall et al. (1998) recommended the use of full-width bearing pads. Lall et al. (1998) continued stating that New York State standards required the bearing pad be at least half as wide as the beam; however, this raised the concern that such a narrow bearing pad would allow the beam to rotate about its longitudinal axes. Sang (2010) used Finite Element to model the two alternative layouts of bearing pads presented by Miller et al. (1999) and recommended placing the bearing pads underneath the shear keys. Sang (2010) found that placing the bearing pads underneath the keyway reduced the tensile stress developed near the bottom of the shear key and helped prevent differential movement between the adjacent beams. Russell (2009) recommended using a three-point bearing system to minimize rocking of girders.

3.4: Shrinkage

The combination of shrinkage, restraint, and thermal stresses are considered the major causes of longitudinal cracking (Brown et al., 2001). Krauss and Rogalla (1996) identified shrinkage as one of the factors contributing to concrete bridge deck cracking, noting that the girders provide significant restraint against the decks natural shrinkage and thermal movements, resulting in the development of stresses in the deck. Krauss and Rogalla (1996) continued stating that the concrete composition, material properties, early age elastic modulus, creep, temperature during placement, and aggregate type along with heat generated during hydration and drying shrinkage all have an influence on longitudinal bridge deck cracking. Through a survey of state transportation agencies, Krauss and Rogalla (1996) developed recommendations to minimize the effects of shrinkage including lowering water-cement ratio to between 0.41 and 0.49, reducing peak and placement concrete temperatures, and using aggregates with low shrinkage values. Brown et al. (2001) also considered shrinkage, along with thermal stresses and restraint, to be a cause of bridge deck cracking, as shown in Figure 12. As concrete cures, its volume changes.
slightly, mainly due to shrinkage. The overall shrinkage is due to four different types of shrinkage including plastic shrinkage, drying shrinkage, autogenous shrinkage, and thermal shrinkage. Plastic shrinkage is a process that occurs as water is evaporated during concrete curing and is often the ‘first’ shrinkage experience. The effects of plastic shrinkage can be minimized through various curing techniques. Drying shrinkage occurs in conjunction with curing, but continues to occur after the concrete has cured; the effects of drying shrinkage are considered to be long-term in nature. Autogenous shrinkage is also experienced during the curing process and is related to the hydration and crystal structure formation. Thermal shrinkage also occurs after curing and is related to concrete expansion and contraction after undergoing hydration.

3.5: Temperature Effects

Loads due to changes in temperature can cause significant deformations of bridge superstructures and is considered another important factor affecting deck cracking. Thermal stresses and loads result from daily temperature cycles in ambient air temperature and radiation from the sun. These daily cycles impose a thermal gradient through the depth of the cross-section. Coupled with restraint the thermal gradient can lead to tensile stresses and eventually cracking (Brown et al., 2001; Krauss and Rogalla, 1996). The thermal gradient throughout the structure can be affected by properties of the concrete such as density, thermal conductivity, and specific heat as well as parameters such as bridge orientation and wind speed (Taysi and Abid, 2014).

Miller et al. (1999) stated that the early cracking at shear key locations, even before completion of the construction phase, could be explained by the tensile forces developed at shear key locations due to joint movement caused by daily temperature variation. Miller et al. (1998), through instrumentation of a bridge, found that during the day the beams tend to camber upward, causing transverse strains exceeding 300 με. Figure 13 shows how this camber can be
exaggerated when girder axis are not parallel, the figure is drawn based on a figure presented by Miller et al. (1999). One of the major causes of thermal cracking was beam axis not being parallel. Through further testing, using full sized box girders, Miller et al. (1999) found that moving the keyway to the neutral axis of the beam and not grouting to the top of the keyway made them more resilient to cracking; however this approach did not fully prevent cracking and the comment was made that during field implementation the empty space at the top of the keyway would fill with water and debris. Miller et al. (1998) concluded by recommending that the keyway be moved to the neutral axis, the shear key be fully grouted with epoxy, and that post-tensioning be used. Additionally, the recommendation was made to consider different methods of sealing the shear key and preventing salt laden water from entering the keyway.

**Figure 13. Joint Movement Caused by Daily Thermal Loading**

Thermal stresses can also be developed as a result of the hydration process. This process consists of a simultaneous rise in temperature and increase in modulus of elasticity. Saadeghvaziri and Hadidi (2002) noted that if concrete is restrained, as is often the case when decks are poured, the cooling that follows heat of hydration can induce tensile stresses in the bridge deck.

**3.6: Modeling and Analysis**

Modeling adjacent box-beam bridges, though various means, has been a major component in the research of reflective deck cracking and the behavior in the shear key region. El-Remaily et al. (1996) used grillage analysis to calculate transverse post-tensioning magnitudes and El-Esnawi
and Hucklebridge (1996) used finite element analysis to study stresses near the shear key for different shear key configurations. More recently Sang (2010) considered the use of both grillage analysis and finite element analysis to evaluate potential modifications to shear keys in order to reduce stress levels and Ubbing (2014) used finite element analysis to consider the effects of incorporating UHPC grout and transverse dowel bars in shear keys. Modeling is an important tool in the analysis of bridge behavior, and if done correctly can yield good results. The following section will consider various modeling techniques that have been used to analyze various factors influencing reflective cracking and the behavior of adjacent box-beam bridges.

3.6.1: Grillage analysis

Grillage analysis is a common method used to model bridge superstructure systems. The Grillage Analogy (GA) method was first used, with the assistance of computer software, by Lightfoot and Sawko (1959). The GA approach is based on bridge decks being continuous in two dimensions, resulting in the distribution of shear forces and moments in two directions. The GA method simplifies the bridge deck/superstructure onto a ‘skeleton structure’ or grid system constructed of one-dimensional beams running in the longitudinal and transverse directions (Sadeghi and Fathali, 2012). Each of these beams, or grillage members, represents the corresponding stiffness in the longitudinal and transverse directions.

Hambly (1991) suggested that grillage analysis of side-by-side box beam superstructures should be performed in such a way that equal spacing exists between the members in the grid system. Additionally, Hambly (1991) noted that shear keys are to be modeled as hinges, which would be consistent with the behavior of a side-by-side box beam bridge that was not subjected to transverse post-tensioning.

El-Remairy et al. (1996) used the grillage method to analyze different box beam types and developed charts for the design of transverse post-tension. The models consisted of longitudinal beam elements, located at the longitudinal joint locations (shear keys) and transverse beam elements representing the transverse stiffness properties of the superstructure. Spacing between the longitudinal grillage elements was equal to the beam width and the transverse grillage elements were placed at the diaphragm locations. Required transverse post-tensioning was calculated by cross checking the resulting stresses with the allowable limits. El-Remairy et al. (1996) found that the required post-tensioning was nearly constant along the length of the span and suggested that equal post-tensioning force be applied at all diaphragm locations.
Sang (2010) conducted a grillage analysis on a simply supported adjacent box beam bridge in order to determine the load effects at the shear key locations. The bridge modeled consisted of 12 box beams, was 45 feet wide and had an 80 foot span. The model consisted of several longitudinal beam elements, located at the center lines of the beams and transverse beam elements represented the transverse stiffness properties of the superstructure. Spacing between bordering longitudinal and transverse beam members was kept nearly identical, following the recommendations of Hambly (1991). Sang modeled the bridge under HS25 truck loading and PennDOT P-82 truck loading. Sang (2010) found that the use of a full depth shear key was able to reduce tensile stresses when compared to partial depth shear keys, located at the top and mid-height of the beam.

While the GA method is useful for approximating the behavior of a superstructure, it was demonstrated by Gordon and May (2004) that grillage analysis does not accurately represent plate behavior. In their analysis they compared the results of three different slabs using finite element and theoretical solutions; these different slabs included a square plate supported on all sides, a rectangular plate supported on two sides, and a skewed plate supported on two sides. Gordon and May (2004) found that the grillage analysis results had significant differences when compared to the finite element and theoretical solutions. Similarly Sadeghi and Fathali (2012) found that significant differences, ranging from 17 to 21%, existed between the grillage analysis and the experimental results. Sadeghi and Fathali (2012) data showed that as the loading increased, so did the difference between the experimental and grillage results; they concluded that the theoretical GA results are satisfactory when applied loads are low and when the bridge system being modeled is not complex.

3.6.2: Plate analysis using equivalent properties

The behavior of adjacent box-beam bridges resembles that of an orthotropic plate (Attanayake et al., 2011; Attanayake, 2006). The modeling of similar structures as orthotropic plates has been done in several industries (Biancolini and Brutti, 2003; Peng et al., 2007; Biancolini, 2005; Zhou, 2002). Researchers have used both equivalent material properties and equivalent stiffness properties to model complex structures as plates. Determining the Equivalent Material Properties to model complex anisotropic structures as a homogeneous material is common in many industries.
Lok and Cheng (2000) compared the elastic stiffness properties of several truss-core sandwich panels. To do this they idealized the sandwich panel as an equivalent two-dimensional orthotropic plate. During their study Lok and Cheng (2000) compared the closed-form solution with finite-element results; a strong correlation between the two was found, but the effects of shear could be seen better in the finite element analysis. When comparing the closed-form solution with finite element modeling for deflection of several sandwich plates an error between 1.37% and 12.31% was found. They concluded by presenting the stiffness constants for a variety of sandwich panels.

Cai et al. (2009) studied the development of equivalent material properties using finite element modeling. Cai et al.’s model focused on determining the equivalent material properties for fiber-reinforced polymer (FRP) composite bridge deck panels. After selecting a representative volume element they applied unit forces in the finite element program ANSYS and recorded displacements. The geometry and the displacements were used to compute the equivalent material properties of the FRP deck panel. Cai et al. (2009) found that a complicated sandwich panel can successfully be simplified to an equivalent orthotropic plate.

Wondwosen (2014) created a micromechanics based model for spherically voided slab systems. Wondwosen (2014) accomplished this by defining a representative volume element and then using ANSYS to determine the equivalent stiffness properties. These equivalent stiffness properties included the extensional stiffness matrix [A], the coupling stiffness matrix [B], and the bending stiffness matrix [D]. Wondwosen (2014) verified the micromechanical homogenization procedure and then used a finite element based micromechanical analysis to compare the behavior of spherically voided slabs with solid slabs with equal depth.

3.6.3: Numerical analysis and finite element modeling

Numerical analysis and the use of the Finite Element Method (FEM) has been gaining popularity for the last several decades. FEM software has been developed by a number of manufactures, including Simula, which has produced ABAQUS.

Ulku et al. (2010) discussed the development of a FE model using ABAQUS software. Ulku described the advantages of ABAQUS and noted important steps required to create a model. Additionally, Ulku et al. considered the effects of temperature gradient. The results of Ulku’s (2010) research showed the benefits of multi-stage post-tensioning and also demonstrated that transverse post-tensioning does not provide sufficient clamping stresses along the longitudinal
joints. Figure 5 in Ulku et al. (2010) shows the distribution of the stresses on the shear key region due to post-tensioning. From the figure, the limited distribution of the post-tensioning force can be seen. The distribution is approximately 1.35 times the width of the beam. Sang (2010) developed a number of two dimensional finite element models. These models consisted of point-loads representing the transverse post-tensioning. When considering the entire superstructure, Sang (2010) defined the various elements to represent the thickness of the beam in that area, for instance areas that have a void/flanges were considered to be 11 inches thick and the web and diaphragm areas were considered to be 33 inches thick. The results of the superstructure analysis showed the current post-tensioning layout was insufficient to meet the compressive stress magnitude specified in AASHTO LRFD. Models of beam cross-sections revealed that a full depth shear key had the greatest effect in reducing tensile stress in the shear key and that a partial depth shear key, with the keyway placed the mid-height of the beam did not outperform the full-depth shear key. Steinberg et al. (2013) also discussed the development of a finite element model for adjacent box beams highlighting the development of defining of various beam parts to best reflect the expected behavior. The finite element model developed by Steinberg et al. (2013) and further developed by Ubbing (2014) was used in the evaluation of UHPC for shear key material for a bridge in Fayette County, Ohio. The evaluation of the shear key using UHPC was done with a model consisting of two beams; Ubbing (2014) concluded that UHPC could be used in the shear key. Ubbing (2014) also noted that the performance of the shear key could have been better analyzed if the entire bridge had been modeled in ABAQUS, as opposed to the two beam model. Dong et al. (2007) used finite element analysis to evaluate the stress in the shear key. Dong used fictitious materials and analyzed different shear key geometries. Dong et al. (2007) made the suggestions that future research consider nonlinear analysis.

3.7: Summary

A thorough review of literature was conducted covering box-beam geometry, transverse post-tensioning, shear key configurations, and various materials used in the construction of box-beam bridges. The geometry of box-beams in the United States, Japan, and Korea was discussed as well as several shear key configurations. The review showed that despite many changes implement to prevent reflective cracking, reflective crack development is persistent.
The use of full-depth grouted shear keys is preferable, as it opposes any ‘hinging’ that may lead to the keyway opening (El-Esnawi, 1994). Full-depth shear keys have been implemented by many state DOTs as well as Japan and Korea. Several materials for shear key grouting were considered. Non-shrink properties are highly desirable as well as high bond strength. Aktan et al. (2009) and Ulku et al. (2010) showed that much of the area in-between post-tensioning locations are not compressed and the compression zone is limited to 1.35 times the width of a beam. Attanayake and Aktan (2013) proposed two-stage post-tensioning, in order to compress the concrete deck. Two-stage post-tensioning has the potential to reduce deck stress under service loads and will be considered in this study.

The use of plate analysis using equivalent material properties was reviewed and was shown to have the ability to model orthotropic plates, such as the box-beam superstructure. Further development of a model using this methodology could simplify the analysis procedure and provide a way to analyze the load response of such a structure.

Longitudinal reflective cracking occurs early, even prior to the bridge being opened to traffic. The combined effects of shrinkage, restraint and temperature loading has been shown to be a leading cause in the cracking (Saadeghvaziri and Hadidi, 2002). Attanayake and Aktan (2013) inspected a bridge under construction and found cracking had occurred prior to barrier or live loading further indicating that cracking resulted from the effects of heat of hydration, shrinkage, or thermal exposure.
4. FIELD INSPECTION

4.1: Introduction
This chapter discusses the field inspection of a newly constructed adjacent box-beam bridge. The scope of this inspection was limited to the development of longitudinal reflective cracking. The inspection data was collected visually and recorded using digital photography.

4.2: Bridge Description
Wisconsin bridge B-26-40 was selected for inspection. B-26-40 is a new construction bridge replacing an existing bridge that carries Wisconsin-47 over Lost Creek.

4.2.1: Bridge location
B-26-40 is located in the town of Sherman in Iron County, Wisconsin (Figure 14), just southeast of the unincorporated community of Manitowish. The bridge carries W-47 over Lost Creek. B-26-40 is aligned in the east-west direction.

Wisconsin-47 has an average daily traffic volume of less than 1,100 vehicles per day. After demolition of the existing bridge, the new bridge was constructed at the same alignment. The existing bridge, was demolished in September 2016. The entire structure was replaced.

4.2.2: Bridge geometry and construction
Bridge B-26-40 is constructed of nine 4-ft wide × 17 inch deep prestressed box-beams. The new bridge has a single 35 ft span with no skew and a width of 37.5 ft. The bridge is meant to accommodate two 12 ft wide traffic lanes and two 5 ft shoulders. The bridge superstructure is transversely post-tensioned over the abutments, quarter-points and mid-span. Post-tensioning
force magnitude is specified as 86.7 kips per duct (about 12.38 kip/ft). Figure 15 shows the cross-section and post-tensioning details for bridge B-26-40.

![Figure 15. B-26-40 Details](image)

The beams are overlain by a 6-in cast-in-place concrete deck slab; the deck overhangs the exterior girders by 5-inches. The plans specify a minimum 28-day compressive strength of 4,000 psi for the concrete deck slab. Full-depth shear keys are used. Full post-tension force was applied after the grout had cured 2 days, having attained a compressive strength of 3,000 psi. The shear key grout mix requirements are provided in Table 2. Girders are positioned at different elevations to form a road crown.
Table 2. Grout Properties and Requirements

<table>
<thead>
<tr>
<th>Property</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength</td>
<td></td>
</tr>
<tr>
<td>3 days</td>
<td>≥ 5000 psi</td>
</tr>
<tr>
<td>7 days</td>
<td>≥ 6000 psi</td>
</tr>
<tr>
<td>Slump/flow</td>
<td>Pourable/flowable</td>
</tr>
<tr>
<td>Early age expansion at final set</td>
<td>Min: 0% Max: 4%</td>
</tr>
<tr>
<td>Expansion of hardened grout</td>
<td>Min: 0.02% Max: 1%</td>
</tr>
<tr>
<td>Shrinkage</td>
<td>0% from max expansion</td>
</tr>
<tr>
<td>Air content</td>
<td>8% +/- 1.5%</td>
</tr>
</tbody>
</table>

4.3: Bridge Inspection Summary

Cast-in-place deck slab was poured on the 6th of October. The bridge was inspected during the second week of October 2016. During the two-day inspection period, October 10th to 11th, temperatures ranged from the mid 50’s to high 60’s. Rain occurred on the second morning of the inspection. The slab was inspected prior to pouring the approach; this allowed inspection of the deck slab over the abutments and observing the cross-section of the slab. As specified in the plans, the deck was moist cured for seven days. This was done by placing wet burlap over the entire deck and covering the burlap with a large plastic sheet and a tarp. Figure 16 shows an overall view of the bridge.

Two cracks were identified over the West abutment. Both cracks had an average width of less than 0.005 inches. The first crack was located above the shear key between girders 5 and 6. Figure 18 documents the geometry of the crack. As shown the crack has reached the full depth of the deck; additionally the crack has propagated approximately 8 inches along the deck.
Figure 16. Overview of B-26-40
Figure 17. Crack Width Measurement (Crack Along the Vertical Face of the Slab Over West Abutment)

(a) crack to full depth of deck

(b) crack propagation along deck

Figure 18. Geometry of Crack 1 Over West Abutment
The second crack occurred above girder 8. Crack 2 appeared slightly smaller, widthwise, than the first crack. This crack also had reached the full depth of the deck and had propagated approximately 2 inches along the deck. Figure 19 shows crack 2.

![Figure 19. Crack 2 Along Vertical Face of Slab](image)

### 4.4: Conclusions of Field Investigation

Bridge B-26-40 was inspected prior to opening and construction of the approaches. Cracks were observed just five days after pouring the deck. During this time, the deck slab was subjected to only barrier loads and volume change loads such as heat of hydration, shrinkage, and temperature gradient due to ambient weather. The early appearance of these cracks indicates that the cracking is initiated primarily due to the aforementioned loads and propagated to full length due to subsequent loading.
5. MODELING CONCRETE SHRINKAGE

5.1: Introduction

This chapter discusses the development and results of a finite element analysis considering the effects of shrinkage on a cast-in-place concrete deck. As was shown in both the literature review and field investigation the early appearance of reflective cracking indicates that cracking is initiated primarily due to heat of hydration, shrinkage, and temperature gradient due to ambient weather. This analysis serves to further investigate the causes of reflective cracking and the location these cracks will occur. The analysis will demonstrate that the stresses from these loads, considered separate for self-weight and live loading, have the potential to initiate reflective cracking over the longitudinal joints. This chapter will also discuss the results of a model considering post-tensioning.

5.2: ABAQUS Modeling

A three dimensional, non-linear, finite element model is created using ABAQUS 6.14 CAE. The input file is modified using Notepad++, and Intel Composer XE 2013 SP1 is used to compile a FORTRAN subroutine.

Although ABAQUS has a wide array of abilities and features, only those used in developing the model will be discussed. Some of the benefits of using ABAQUS include an easy to use graphical user interface via CAE, extensive processing abilities, and ability to incorporate user defined subroutines.

ABAQUS CAE modeling is accomplished using a model tree (Figure 20). Each limb of the tree falls under one of the modules required to define a model. Following these modules in order allows the creation of a working model. The modules that are important for this model are the part, property, assembly, step, interactions, mesh, and job modules.
5.2.1: Units and coordinate system

A consistent use of units is required because ABAQUS does not have a built in unit system. Everything entered by the user must have consistent units, including part geometries, material properties, and loading (Kasera, 2014). The analysis parameters, in this model, are input in the units of inches, pounds, and days. Results from the analysis will have the same units.

The coordinate system in ABAQUS is defined in terms of x-, y-, and z-axis. The model was created so that the z-axis was in line with the span of the bridge, the x-axis is the width of the bridge and the y-axis is the depth/height of the bridge.

5.2.2: Parts

Each of the parts in the ABAQUS model were drawn in AutoCAD Civil3D and imported into ABAQUS as solid, elastic parts using the import part option. The shear key geometry and sides of the box girder were simplified to reduce solving time. Each of the 27-inch deep and 36-inch wide box beams included voids along their length. In AutoCAD, prior to importing the part into ABAQUS, center and end diaphragms were added to the beam and joined. Figure 21 shows the box beam part, after import and partitioning in ABAQUS.
The shear keys were drawn to match the height of the box beam and are 3-inches in width. Once imported into ABAQUS as a three-dimensional, deformable, solid part, the shear key part was partitioned. Shear key contact surfaces were tied to the adjoining box beam; beams were assigned as the master surface and shear keys as the slave surface. Figure 22 shows the shear key part, after import and partitioning in ABAQUS.

A 6-inch cast-in-place deck was considered. The deck drawn was divided into six, one-inch thick layers drawn to the full span and width of the bridge. The deck was also assigned a tie constraint. Because the deck would be undergoing shrinkage the deck was assigned as the master surface and the beams as the slave surface. Figure 23 shows the deck part, after drawing and partitioning in ABAQUS.
5.2.3: Meshing
The entire ABAQUS model was meshed using C3D8R elements. The ABAQUS Analysis User’s Guide defines C3D8R elements as “8-node linear brick, reduced integration” elements (ABAQUS, 2013). Reduced integration reduces the number of integration points, from eight, to one located at the center of the element. Using the reduced integration elements significantly reduced the analysis time.
Prior to meshing the parts needed to be partitioned and seeded. Partitioning is used to separate sections of a part and can be utilized to improve meshing quality. Seeds are markers used in ABAQUS to define the density of the mesh, particularly along the edges of the part. After several failed analysis attempts a mesh with seed approximately 10 inches apart was applied to the model.

5.2.4: Assembly
The assembly is created by adding and defining the location of instances of the parts. The assembly represents the final geometry of the structure. In order to create the assembly for this model, eight box-beam instances and seven shear key instances were added and placed adjacent to each other to match the layout of a typical box-beam bridge. Finally six deck layers were added to the model, each one being a single instance of the six deck layers defined. Figure 24 shows the final assembly of the adjacent box-beam bridge used in the model.

Figure 24. Final Bridge Assembly
5.2.5: Material

A total of eight different material properties were defined. Six properties would be assigned to the various ‘layers’ of the deck, with only slight changes in the subroutine between them. The other two were the properties assigned to the box-beam and the shear key.

The edit material dialog box allows various material properties to be defined (Figure 25). Material definitions can include numerous characteristics including the modulus of elasticity, density, Poisson’s ratio and various other properties. The following section will discuss the properties of each of these materials.

![Figure 25. Edit Material Dialog Box](image)

5.2.5.1: Box-beam material properties

The concrete box-beam was created using a linear elastic material. The density, modulus of elasticity, and Poisson’s ratio was defined for this material. The properties for the concrete box beam are listed below:

- Density: 0.086 lbs/in$^3$
- Modulus of Elasticity: 5,850,000 psi
- Poisson’s Ratio: 0.2

5.2.5.2: Shear key properties

Similar to the concrete box-beam, the shear key was created using a linear elastic material. The properties for the shear key are listed below:

- Density: 0.086 lbs/in$^3$
- Modulus of Elasticity: 3,600,000 psi
- Poisson’s Ratio: 0.2
5.2.5.3: Deck properties
The cast-in-place deck was created by defining six different material properties. However the only difference between the six properties is the equation describing the total shrinkage, which was included in the UEXPAN subroutine.

General properties were assigned to all layers of the cast-in-place deck. These include density, depvar, and a user defined field. The deck material was assigned a density of 0.086 lbs/in$^3$. Depvar is used to define the size of the STATEV array in a user subroutine. Ten solution-dependent state variables are specified using the Depvar property. The User Defined Field material property notifies ABAQUS that one of the other material properties relies on a changing field variable. In order to use this option a USDFLD subroutine must be defined.

Mechanical properties were also defined for the deck material. The defined properties are elasticity and expansion. The elasticity included definitions of Young’s modulus and Poisson’s ratio. In order to model the increase in the concrete strength that is associated with curing concrete field variables were added to the elastic material property. Table 3 shows the Young’s modulus along with the assigned field variable. The USDFLD subroutine defines the field variable and sets it equal to the time that has passed since the beginning of the analysis.

| Table 3. Young’s Modulus and the Corresponding Field Variable |
|-----------------|-----------------|
| Young’s Modulus (ksi) | Field Variable (Time, days) |
| 3,360            | 1               |
| 3,600            | 3               |
| 3,750            | 7               |
| 4,200            | 14              |
| 4,250            | 28              |

Selecting the expansion behavior provides the user with the option to either define a coefficient of thermal expansion or specify the use of a user subroutine UEXPAN. Selecting this option allows the incorporation of a subroutine that defines the shrinkage of concrete with time.

5.2.6: Steps
Two different models were considered, the first considering the effects of concrete shrinkage and the second considering the effects of post-tensioning. Steps are defined in ABAQUS in order to differentiate various loading and boundary conditions. Both models consisted of a single step
analysis. The shrinkage analysis consisted of a single 28 day step. The maximum increment during this step is one day. The minimum increment is $1 \times 10^{-7}$ days.

The post-tensioning analysis consisted of a single ‘one’ day step; during this step the post-tensioning force was linearly increased until the specified post-tensioning force had been applied.

5.2.7: Boundary conditions
Both models had the same boundary conditions applied. The bridge assembly was simply supported. Pinned and roller conditions were applied to the ends of the bridge span, both over a 1-foot section of the end of the beam.

5.2.8: Loading
Only the post-tensioning model had defined loads. Concentrated loads were applied at the mid-depth of the beam, over the abutments and the mid-span. The post-tensioning force was 120 kips, applied to both sides of the bridge.

5.3: User Subroutine
One of the many advantages of using ABAQUS is the ability to include subroutines that can simulate complex properties and conditions. Subroutines used by ABAQUS must be in the FORTRAN language and be saved with either the ‘.f’ or ‘.for’ file extension. Multiple subroutines can be included in a single file. In this study two subroutines were used simultaneously to model the concrete deck curing and shrinkage. USDFLD and UEXPAN are the subroutines implemented.

5.3.1: Subroutine USDFLD
The USDFLD subroutine serves two purposes. First it is used to obtain instantaneous values of strain throughout the analysis. Second it is used to change field variables with time. The instantaneous strain is called using the GETVRM command and placed as a state variable in the STATEV array for every element during each increment of the analysis.

5.3.2: Subroutine UEXPAN
The ABAQUS User Subroutine Reference Manual states that the UEXPAN subroutine is used to define incremental thermal strains. The strains can be a function of temperature, field variables and/or state variables. By excluding thermal effects the subroutine can be modified to model
other forms of expansion and shrinkage (Kasera, 2014). The UEXPAN subroutine is run for each integration point within the model.

Shrinkage strain was determined using ACI-209’s shrinkage model (ACI, 2008). Incremental shrinkage can be determined through a slight modification of the equation as presented by Kasera (2014). The equation to determine time dependent shrinkage is shown below:

\[ \Delta \varepsilon_{cr} = v_u \varepsilon_i \left( \frac{t_n}{35 + t_n} - \frac{t_{n-1}}{35 + t_{n-1}} \right) \]

Where:
- \( \Delta \varepsilon_{cr} \): creep strain increment
- \( v_u \): ultimate creep coefficient
- \( \varepsilon_i \): instantaneous elastic strain
- \( t_n \): time since loading at current increment
- \( t_{n-1} \): time since loading at previous increment

Non-uniform shrinkage occurs through the depth of the concrete deck. In order to model this effect various levels of shrinkage were defined for each inch-thick layer of the deck. ACI 209 allows correction factors to be used in the shrinkage model, one of these correction factors includes thickness factor, where the thickness of the concrete structure being modeled determines the extent of the shrinkage. These factors were used to simulate increased shrinkage near the surface of the deck.

5.4: Model Results

The analysis results for the shrinkage model are presented in Figure 26 and Figure 27. As shown, maximum tensile stresses are located over the abutments, with peak stresses occurring above the shear key locations. Reviewing the stresses through the depth of the cast-in-place deck shows stresses higher near the top of the deck. This indicates that cracking, due to shrinkage, would initiate over the shear keys at the abutments on the exposed concrete surface.
Result for the shrinkage model show $S_{11}$ ($S_{xx}$) stresses. Because cracking occurs perpendicular to the maximum stresses, peak stress in the S11 direction over shear keys would likely cause the longitudinal cracking pattern seen on bridge decks. Peak tensile stresses due to concrete shrinkage are in the 310 to 318 psi range.

The results of the post-tensioning model are presented in Figure 28. It should be noted that compressive stresses are isolated around the post-tensioning location. The distribution of the compressive stress is similar to that found by Ulku et al. (2010). Ulku et al. (2010) found that the compressive stress distribution due to post-tensioning was limited to approximately 1.35 times the beam width. Ulku et al. (2010) also showed through a finite element analysis that peak compressive stresses were in the 110 to 114 psi range. The results from the post-tensioning model indicate that post-tensioning compressive stresses are in the range of 8 to 10 psi at the exposed surface of the deck. Considering the stresses at the bottom surface of the deck (Figure 28– B) higher stresses are seen, in the range of 90 to 108 psi. Further reviewing the stresses through the vertical face of the cast-in-place slab, a decrease in stress can be seen, decreasing from the top of the box-girders to the surface of the deck.
As shown in the results from the post-tensioning model, the resulting maximum compression near the surface of the beams was in the range of 90 to 108 psi. Less compression occurred near the exposed surface of the deck, between 8 to 10 psi. Comparing the results to the shrinkage model tensile stresses, the tensile stress ranged from 310 to 318 psi. It can be seen that insufficient compressive force is provided by post-tensioning to overcome the tensile stresses due to concrete shrinkage.

The compression provided by the post-tensioning is not sufficient to compress the deck after shrinkage occurs. If the desire is to mitigate cracking using post-tensioning, an increase in post-tensioning force will be required. Additionally, as shown in the model results and previously discussed by Ulku et al. (2010), the compression zone is limited to the area directly around the concrete diaphragm. To fully compress the deck, the addition of post-tensioning locations, potentially through the top and bottom flange, should be considered.

The developed shrinkage model serves as a platform for future research into the effect of volume change loading. This model has the potential to be expanded to include the effects of creep, temperature gradient, as well as other thermal loads.
Figure 27. Shrinkage Model Results

(a) Overview
(b) Stresses over abutment
(c) Stresses through vertical face of deck
Figure 28. Post-Tensioning Model Results

(a) overview
(b) model with deck removed
(c) stresses through vertical face of deck
6. ANALYtical ANALYSIS USING EQUIVALENT MATERIAL PROPERTIES

6.1: Introduction
Analyzing complex structures using either analytical or finite element methods is often time consuming and cumbersome. As shown through an extensive literature review, models can be developed that idealize the completed adjacent box-beam superstructure as an orthotropic plate subject to flexure and shear deformations.

6.2: Procedure
This chapter demonstrates the calculation of equivalent material properties for an adjacent box-beam bridge superstructure using mechanics of composite materials and plate theory. The equivalent material properties are used with structural analysis software to determine the moment demand at the longitudinal connections. The relationships and equations used have been presented by A.T. Nettles (1994) in “Basic Mechanics of Laminated Composite Plates” and L. Kollar (2003) in “Mechanics of Composite Structures.” The procedure will follow a similar route to the one suggested by Kollar (2003).
A flowchart presenting the steps that must be accomplished to determine the equivalent material properties are shown in Figure 29.
Step-1 is the selection of the representative volume element (RVE). Selecting an RVE that can accurately represent the behavior of the structure is extremely important. For structures that have a sandwich like construction, the RVE will then need to be divided into several distinct layers or ply.

Step-2 is the consideration of micromechanics. Micromechanics is utilized to determine the properties of each of the individual lamina of the RVE. Once the RVE is broken down into several lamina the rule of mixtures is used to determine the longitudinal and transverse Young’s modulus, shear modulus, and Poisson’s ratio. The properties of the matrix and fiber in each of the lamina must be known to accomplish this step.

Step-3 is the application of macromechanics. By considering macromechanics and the relationships that exist in orthotropic plates the key sub-matrices of the stiffness matrix can be developed. These sub-matrices include the extensional or in plane stiffness matrix [A], the coupling stiffness matrix [B], and the bending or flexural stiffness matrix [D].

Once the stiffness matrix has been developed Cramer’s rule can be applied to compute the equivalent material properties for an orthotropic plate.
6.3: Representative Volume Element

Since we are modeling a complex structure as a laminate plate, the structure must be simplified in a way that will allow layers to be defined. Therefore a representative unit or volume will be selected from which the major engineering constraints will be taken. A representative volume element (RVE) consisting of two halves of adjacent box beams, including the shear key between them, was selected (Figure 30). This RVE was first proposed by Attanayake (2006), and is advantageous in that it allows the behavior of the shear key to be captured. Additionally, by considering the entire unit, as opposed to the shear key itself, the goal of calculating the load demand for a structure capable of transferring load between adjacent beams can be accomplished.

![Figure 30. RVE as Proposed by Attanayake and Aktan (2009)](image)

6.4: Laminate

The RVE must be divided into layers in order to consider the superstructure a laminate plate. The geometry has been defined as shown in Figure 31. Figure 32 shows how the defined RVE was divided into four distinct layers. Layer A consists of the cast-in-place deck, layer B the top flange of the box beam. Layer C and D consist of the box beam web and bottom flange respectively.
Micromechanics aims to determine the properties of heterogeneous materials using the properties of its constituents. The computational process relies on the concept that a well-defined RVE exists and can accurately be used to approximate the behavior of the composite material. Using micromechanics the four engineering constants for a given material can be determined. These constants include:

- Longitudinal Young’s modulus ($E_1$)
- Transverse Young’s modulus ($E_2$)
- Longitudinal shear modulus ($G_{12}$)
- Longitudinal Poisson’s ratio ($\nu_{12}$)
The rule of mixtures, as discussed by Kollar (2003) in “Mechanics of Composite Structures” will be used to determine the four engineering constants for each of the previously defined layers (Figure 32). The rule of mixtures describes a relationship that exists between a matrix and contained ‘fibers.’ The constants can be calculated using the following equations, which rely on volume fractions.

The volume fractions relate the fraction of the matrix and fiber that exist in the composite:

\[ V_m := \frac{A_m}{A_m + A_f} \]

\[ V_f := \frac{A_f}{A_m + A_f} \]

Where:

- \( A_m \) is the area of the matrix.
- \( A_f \) is the area of the fiber.

The longitudinal Young’s modulus can then be calculated as:

\[ E_1 := V_f E_f + V_m E_m \]

The transverse Young’s modulus:

\[ E_2 := \left( \frac{\sqrt{V_f}}{E_{b2}} + \frac{1 - \sqrt{V_f}}{E_m} \right)^{-1} \]

Where:

\[ E_{b2} := \sqrt{V_f} E_f + (1 - \sqrt{V_f}) E_m \]

The longitudinal shear modulus:

\[ G_{12} := \left( \frac{\sqrt{V_f}}{G_{b2}} - \frac{1 - \sqrt{V_f}}{G_m} \right)^{-1} \]

Where:
\[ G_{b2} := \sqrt{V_f G_f} + \left(1 - \sqrt{V_f}\right) G_m \]

The longitudinal Poisson’s ratio:

\[ \nu_{12} := V_f \nu_f + V_m \nu_m \]

The Young’s modulus and Poisson’s ratio were known for the deck, box beam and shear key prior to performing the micromechanical analysis to determine each layers properties. Layer A consists of the cast-in-place deck, because this layer is isotropic micromechanics relationships were not required to determine the layer’s properties. Layer B consists of the top flange of the box beam; it is composed of both shear key material and box beam material. Using the rule of mixtures the box beam was considered the matrix and the shear key was considered the fiber. Layer C is the web of the box beam. This layer is composed of both shear key and box beam material and also contains the voided section of the box beam. The voided section requires that the engineering constants for the entire layer be modified by the ratio of non-voided area to total area. Equivalent properties for Layer D, the bottom flange of the box beam, were determined using the same method applied to layer B. Example calculations for the micromechanical process are included in Appendix A.

6.6: Macromechanics

Once the engineering constants have been computed, macromechanics can be used to determine the components of the [A], [B], and [D] matrices that make up the stiffness matrix. The stiffness matrix is used to calculate the equivalent material properties.

6.6.1: Force-strain relationship

An understanding of the force-strain relationship that exists is vital for the development of the stiffness matrix. The force-strain relationship is described by the following matrix:
Where, $N_x$, $N_y$, and $N_{xy}$ represent the in-plane axial forces and $M_x$, $M_y$, and $M_{xy}$ represent the bending and twist moments. Further $\varepsilon_x$, $\varepsilon_y$, and $\gamma_{xy}$ are unit strains and $\kappa_x$, $\kappa_y$, and $\kappa_{xy}$ are unit curvatures. The coefficients, $A$, $B$, and $D$, define the components of the $[A]$, $[B]$, and $[D]$ sub-matrices that make up the stiffness matrix $[ABD]$. The sub-matrices are referred to as the extensional or in plane stiffness matrix $[A]$, the coupling stiffness matrix $[B]$, and the bending or flexural stiffness matrix $[D]$. As is typical with a stiffness matrix, all coefficients are defined per unit length. The geometry of the RVE causes the torsion created by $M_{xy}$ to be distributed to the flanges, resulting in the twisting moment $M_z$.

### 6.6.2: Plane stress matrix

In order to determine the $[A]$, $[B]$, and $[D]$ matrices the plane stress matrix $[Q]$ must first be determined for each ply. According to Nettles (1994) the $[Q]$ matrix may be defined as:

$$
Q := \begin{pmatrix}
Q_{11} & Q_{12} & Q_{13} \\
Q_{21} & Q_{22} & Q_{23} \\
Q_{31} & Q_{32} & Q_{33}
\end{pmatrix} = \begin{pmatrix}
E_1 & \frac{\nu_{12}E_2}{1-\nu_{12}^2\nu_{21}} & 0 \\
\frac{\nu_{21}E_1}{1-\nu_{12}^2\nu_{21}} & E_2 & 0 \\
0 & 0 & G_{12}
\end{pmatrix}
$$

The components, $Q_{ij}$’s, are called the reduced stiffnesses. For orthotropic plates the $[Q]$ matrix may be simplified to:
Where:

\[
Q := \begin{pmatrix}
\frac{E_1}{D_Q} & \frac{\nu_{12}E_2}{D_Q} & 0 \\
\frac{\nu_{12}E_2}{D_Q} & \frac{E_2}{D_Q} & 0 \\
0 & 0 & G_{12}
\end{pmatrix}
\]

\[
D_Q := 1 - \frac{E_2}{E_1} \cdot \nu_{12}^2
\]

### 6.6.3: The [ABD] stiffness matrix and its submatrices

After each layer’s [Q] matrix has been developed, the submatrices can be determined in reference to the geometric midplane of the RVE. Because middle strains and curvatures are not a function of the height of the laminate, \( z \), and each ply has a constant stiffness matrix the height components can be separated as constants from the stiffness matrices. The [A], [B], and [D] matrices can therefore be defined as:

\[
A_{ij} = \sum_{k=1}^{n} [Q_{ij}]_k (h_k - h_{k-1})
\]

\[
B_{ij} = \frac{1}{2} \sum_{k=1}^{n} [Q_{ij}]_k (h_k^2 - h_{k-1}^2)
\]

\[
D_{ij} = \frac{1}{3} \sum_{k=1}^{n} [Q_{ij}]_k (h_k^3 - h_{k-1}^3)
\]

Where, given the distances between the midplane and layers of the defined RVE (Figure 33), the coefficients of the submatrices can be calculated as:
Figure 33. Distances Between Midplan Layers

The submatrices can then be combined as the [ABD] matrix:

\[
A_{ij} := (z_4 - z_3)Q_a + (z_3 - z_2)Q_b + (z_2 - z_1)Q_c + (z_1 - z_0)Q_d
\]

\[
B_{ij} := \frac{1}{2} \left( (z_4^2 - z_3^2)Q_a + (z_3^2 - z_2^2)Q_b + (z_2^2 - z_1^2)Q_c + (z_1^2 - z_0^2)Q_d \right)
\]

\[
D_{ij} := \frac{1}{3} \left( (z_4^3 - z_3^3)Q_a + (z_3^3 - z_2^3)Q_b + (z_2^3 - z_1^3)Q_c + (z_1^3 - z_0^3)Q_d \right)
\]

6.7: Equivalent Material Properties

Determining the equivalent material properties requires the use of the original [ABD] matrix, along with its corresponding coefficients and the height, H, of the RVE. Nettles (1994) considers the derivation of the equations used to determine the material properties of a laminate plate, the derivation uses Cramer’s rule to simplify the force-strain relationship. The engineering constants can be calculated as follows.
Young’s Modulus:

\[ E_1 := \begin{vmatrix} A_{22} & A_{23} & B_{12} & B_{22} & B_{23} \\ A_{23} & A_{33} & B_{13} & B_{23} & B_{33} \\ B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\ B_{22} & B_{23} & D_{12} & D_{22} & D_{23} \\ B_{23} & B_{33} & D_{13} & D_{23} & D_{33} \end{vmatrix} \left( \frac{1}{H} \right) \]

Shear Modulus:

\[ G_{12} := \begin{vmatrix} A_{11} & A_{13} & B_{11} & B_{12} & B_{13} \\ A_{13} & A_{33} & B_{13} & B_{23} & B_{33} \\ B_{11} & B_{13} & D_{11} & D_{12} & D_{13} \\ B_{12} & B_{23} & D_{12} & D_{22} & D_{23} \\ B_{13} & B_{33} & D_{13} & D_{23} & D_{33} \end{vmatrix} \left( \frac{1}{H} \right) \]

\[ G_{23} := \left[ \frac{2(1 + \nu_{23})}{E_2} \right]^{-1} \]

Poisson’s Ratio:
6.8: CSiBridge Model

CSiBridge is a specialized analysis and design software developed by Computers and Structures, Incorporated (CSI). CSiBridge evolved from CSI’s other major software package, SAP2000. The user-friendly interface presented by the program provides an object-orientated modeling approach. CSiBridge analyzes the structure by converting the object-based model into a mathematical finite element model (CSI website). This finite element meshing process is done automatically; with the user having little control over defining meshing volumes.

6.9: Selection of CSiBridge

CSiBridge was selected for use with the developed analytical model for a number of reasons. To begin with, CSiBridge allows the user to easily define an orthotropic material and input material property data (Figure 34), the program then incorporates the material property data into its structural analysis engine. CSI’s large online Technical Knowledge Base provides documentation and test problems explaining how this is accurately accomplished. Another reason CSiBridge was selected includes its ability to perform a number of load cases and combinations; additionally a large library of predefined moving loads, including those in AASHTO LRFD, are available.
6.10: Model Validation

Before continuing, it is necessary to validate both the tools used to calculate the load demand and the use of equivalent material properties. In order to accomplish this validation, the following section will be broken down into several smaller cases that will be used to demonstrate the reliability, or any lack thereof, of the model. This breakdown will accomplish the following objectives:

1. Show the shell elements used in CSi’s software suite can be used to model isotropic plate behavior
2. Demonstrate that shell elements used in CSi’s software suite can incorporate equivalent material properties to model orthotropic plate behavior
3. Validate the use of equivalent material properties through a comparison of an orthotropic plate model with refined finite element analysis
4. Establish, that under similar boundary conditions, CSiBridge can produce similar results

6.10.1: Isotropic plate behavior

Validation that the shell elements used in CSi’s software could be used to accurately model an isotropic plate involved the comparison of results between a model created in CSi’s SAP2000
software and the theoretical results. Verification was done using SAP2000, as opposed to
CSiBridge, because the user interface allows for easier modification of the boundary conditions.
It was determined that this was acceptable because both programs run using the same software
ingine. The theoretical results were obtained using Navier’s Method as presented in Ugural
Navier’s Method gives the deflected shape \( w \) of a uniformly loaded \( p_0 \) rectangular, isotropic
plate simply supported on four edges as:

\[
\frac{w}{D} = \frac{16p_0}{D\pi^6} \sum_{m=1,3,5,\ldots} \sum_{n=1,3,5,\ldots} \frac{\sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{2n\pi y}{b} \right)}{mn \left( \frac{m^2 \pi^4}{a^4} + \frac{4n^2 \pi^4}{b^4} \right)}
\]

And the moment about the x-axis \( M_x \) as:

\[
M_x = \frac{16p_0}{\pi^4} \sum_{m=1,3,5,\ldots} \sum_{n=1,3,5,\ldots} \frac{(m/a)^2 + \nu(n/b)^2}{mn \left( \frac{m^2 \pi^4}{a^4} + \frac{4n^2 \pi^4}{b^4} \right)} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{2n\pi y}{b} \right)
\]

Where:

\[
D = \frac{Eh^3}{12(1-\nu^2)}
\]

\( a, b = \) length, width
\( E \) is the modulus of elasticity, \( \nu \) is Poisson’s ratio, and \( h \) is the plate thickness.

The SAP2000 model consisted of a square, isotropic plate simply supported on four edges. The
plate was meshed into 100 shell elements. The model was \( a \) 10-feet long, \( b \) 10-feet wide, \( h \)
and 1-foot thick. A uniform load of \( p_0 \) 10 kips per square foot was applied. The modulus of
elasticity \( E \) was 519119.5 kips per square foot (3600 ksi) and Poisson’s ratio \( \nu \) was 0.3.
Figure 35 shows the SAP2000 model.
The results of the SAP2000 isotropic plate model showed the maximum moment $M_{x,\text{max}} = M_{y,\text{max}} = 45.18$ kip-ft and maximum deflection $w_{\text{max}} = 0.0091$ feet. Solving for the theoretical values, Navier’s equations reduce to:

$M_{x,\text{max}} = M_{y,\text{max}} = 0.0479p_0a^2 = 47.9$ kip-ft

and

$w_{\text{max}} = 0.0454\frac{p_0a^4}{Eh^3} = 0.00874$ ft

Comparing the results of the SAP2000 model with the theoretical solution an error of 5.6% for the moment and 4.1% for the deflection can be seen. During a comparison of Navier’s Method with exact solutions, Ventsel and Krauthammer (2001) stated that an error between 2.5% and 11.5% could be expected. The comparison of SAP2000 to theoretical solutions shows that reasonably accurate results can be obtained using CSi’s plate elements.

**6.10.2: Orthotropic plate behavior**

Validation of CSi’s orthotropic plate behavior was handled using a similar approach as taken for isotropic plates. A square orthotropic plate model, simply supported on four sides was constructed in SAP2000 and compared with the theoretical results. The theoretical results were obtained by using Navier’s method for simply supported orthotropic plates as presented in Ugural (1981) and Ventsel and Krauthammer (2001). PTC MathCAD was used to expedite the evaluation of these equations.
Navier’s method, when applied to orthotropic plates, gives the deflected shape \( w \) of a uniformly loaded \( (p_0) \) rectangular plate, simply supported on four sides as:

\[
w := \left( \frac{16 p_0}{\pi^3} \right) \pi \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{(-1)^{n+m}}{(2n-1)(2m-1)} \left[ \frac{(2n-1)+(2m-1)}{2} \right]^{-1} \left[ \frac{(2n-1)}{a^2} + 2D_d \left[ \frac{(2m-1)^2}{a^2 b^2} \right] + D_y \left( \frac{2n-1}{b^4} \right) \right]
\]

Where:

\[
D_x := \left( \frac{E_x}{1 - v_x v_y} \right) \left( \frac{h^3}{12} \right)
\]

\[
D_y := \left( \frac{E_y}{1 - v_x v_y} \right) \left( \frac{h^3}{12} \right)
\]

\[
D_{xy} := \left( \frac{E_x v_y}{1 - v_x v_y} \right) \left( \frac{h^3}{12} \right)
\]

\[
D_{yx} := \left( \frac{E_y v_x}{1 - v_x v_y} \right) \left( \frac{h^3}{12} \right)
\]

\[
D_s := G h^3 \left( \frac{1}{12} \right)
\]

\[
H_d := D_{xy} + 2D_s
\]

\( E_x \) and \( E_y \) are the modulus of elasticity in the x- and y-directions, respectively. \( G \) is the shear modulus and \( h \) is the height of the plate.

The SAP2000 model consisted of a square orthotropic plate simply supported on all four edges. The model was \( (a) \) 19-feet long, \( (b) \) 19-feet wide, and \( (h) \) 0.5-feet thick. A uniform load \( (p_0) \) of 1 kip per square foot was applied to the model. Using the method developed earlier in this chapter, the equivalent material properties for an MDOT 27-inch deep \( \times \) 36-inch wide box beam with a 6-inch cast-in-place deck was calculated. The box beam and deck were assumed to have an original modulus of elasticity of 5000 ksi, and the shear key material to have a modulus of 3600 ksi. The Poisson’s Ratio’s for all three materials was 0.2. The resulting equivalent material properties for the beam, used in the SAP2000 model, are shown in Table 4.
<table>
<thead>
<tr>
<th>Young’s Modulus (ksi)</th>
<th>Poisson’s Ratio</th>
<th>Shear Modulus (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1 = 2985$</td>
<td>$\nu_{12} = 0.2$</td>
<td>$G_{12} = 1269$</td>
</tr>
<tr>
<td>$E_2 = 2081$</td>
<td>$\nu_{23} = 0.2$</td>
<td>$G_{23} = 867$</td>
</tr>
<tr>
<td>$E_3 = 2081$</td>
<td>$\nu_{21} = 0.139$</td>
<td></td>
</tr>
</tbody>
</table>

The SAP2000 model was finely meshed into 1,600 shell elements and executed the analysis. Figure 36 shows the SAP2000 model and the resulting deflection.

Figure 36. SAP2000 Orthotropic Plate Model and Results

The results of the SAP2000 model showed a maximum deflection of 0.122 feet. Solving Navier’s equation, for an orthotropic plate, results in a theoretical maximum deflection of 0.121 feet. A comparison reveals a 1% error. The same results were obtained using a thick shell element in SAP2000. Thick shell elements were considered because they account for shear behavior and transverse shear deformation. Although Navier’s classical solution does not fully account for transverse shear in thick plates, it does provide satisfactory results for thin plates by
considering plate thickness as a function of rigidity and including it in the general equation (Ventsel and Krauthammer, 2001).

SAP2000’s orthotropic plate modeling capabilities were further investigated by performing a sensitivity analysis and comparing the results with theoretical solutions. This analysis involved reducing each of the equivalent material property values by 25%, 50% and 75% and recording the resulting deflection.

Table 5 show the results of the analysis for the plate described above.
Table 5. Plate Analysis Results for 6 in. Thick Plate

<table>
<thead>
<tr>
<th>% reduction</th>
<th>Modulus of Elasticity</th>
<th>Poisson Ratio</th>
<th>Shear Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deflection (ft)</td>
<td>Deflection (ft)</td>
<td>Deflection (ft)</td>
</tr>
<tr>
<td></td>
<td>E1</td>
<td>SAP2000</td>
<td>MathCAD</td>
</tr>
<tr>
<td>0</td>
<td>2985</td>
<td>0.122</td>
<td>0.121</td>
</tr>
<tr>
<td>25</td>
<td>2238.75</td>
<td>0.136</td>
<td>0.134</td>
</tr>
<tr>
<td>50</td>
<td>1492.5</td>
<td>0.152</td>
<td>0.149</td>
</tr>
<tr>
<td>75</td>
<td>746.25</td>
<td>0.173</td>
<td>0.168</td>
</tr>
<tr>
<td>0</td>
<td>2081</td>
<td>0.122</td>
<td>0.121</td>
</tr>
<tr>
<td>25</td>
<td>1560.75</td>
<td>0.126</td>
<td>0.129</td>
</tr>
<tr>
<td>50</td>
<td>1040.5</td>
<td>0.13</td>
<td>0.135</td>
</tr>
<tr>
<td>75</td>
<td>520.25</td>
<td>0.126</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>E3</td>
<td>SAP2000</td>
<td>MathCAD</td>
</tr>
<tr>
<td>0</td>
<td>2081</td>
<td>0.122</td>
<td>0.121</td>
</tr>
<tr>
<td>25</td>
<td>1560.75</td>
<td>0.122</td>
<td>0.121</td>
</tr>
<tr>
<td>50</td>
<td>1040.5</td>
<td>0.122</td>
<td>0.121</td>
</tr>
<tr>
<td>75</td>
<td>520.25</td>
<td>0.122</td>
<td>0.121</td>
</tr>
</tbody>
</table>

(Note: Thin plate does not consider E3) (Note: Thin plate calc does not consider V23)
The results of the sensitivity analysis indicate that $E_1$ and $G_{12}$ have the largest effect on the resulting deflection. Additionally we can see that error between SAP2000’s results and the theoretical solutions ranged from 1% to 10%. Similar analyses were performed for a 1-foot thick plate and a 1.5-ft thick plate. As the plate thickness increased, so did the error between Navier’s solution and SAP2000 results. Error ranged from 7%-24% and 7%-19% for the 1-foot and 1.5-foot thick plates, respectively. Tables including the results for the thicker plates have been included in the Appendix B.

6.10.3: Equivalent material properties validation

The previous two sections demonstrated that CSi’s software is capable of producing satisfactory results, when compared with theoretical values, given equivalent material properties. However, the ability to accurately model a complex structure using equivalent material properties was not shown. The objective of this section is to validate the plate analysis using equivalent material properties. In order to accomplish this objective a detailed finite element model will be created using ABAQUS and compared with the results of a plate analysis. The ABAQUS model will consider the geometry of the girders, deck, and shear key. The SAP2000 model will consist of an orthotropic plate model that uses equivalent material properties calculated from the geometry used in the ABAQUS model.

6.10.3.1: Bridge geometry

Validation of the equivalent material property method will involve comparing the results of an orthotropic plate with a refined finite element model. Both the plate and the finite element model will be based off of the same fictitious bridge. This idealized/prototype adjacent box-beam bridge will have a 50-ft (600-inch) span and 25.75-ft (609-inch) width. The superstructure will consist of eight 27-inch deep by 36-inch wide box-beams with a 3-inch wide, full-depth shear key. A 6-inch cast-in-place concrete deck will also be in place.

6.10.3.2: SAP2000 model

CSi’s SAP2000 will be used to model the plate with equivalent material properties. The span and width of the plate will match the dimensions of the bridge. Close consideration of CSi’s shell element formulation led to the use of thick shell elements. Additionally, based off the programs calculation of stresses, the thickness of the plate was defined as 33-inches (equivalent to the depth of the beam plus the cast-in-place deck). The plate was partitioned into smaller shell
elements for addition refinement. The equivalent material properties calculated based on the geometry and materials used, and incorporated into the plate material property definitions.

6.10.3.3: ABAQUS parts and assembly

The same bridge model discussed in the shrinkage modeling section is used for the analysis. Parts were drawn in AutoCAD Civil3D and imported into ABAQUS as three-dimensional solid parts. 27-inch deep by 36-inch wide box beams with simplified sides are utilized, this simplification was considered acceptable because the RVE also considers simplified sides. Shear keys have the same depth as the box-beam and a width of 3-inches.

The only major difference between part geometries, when compared with the previous analysis, is the cast-in-place deck. In the shrinkage analysis model the cast-in-place slab divided into six layers, this model will consider a single layer deck with a thickness of 6 inches.

Partitioning is an important part of the modeling process. It allows parts to be divided into separate sections and can be used to improve the mesh quality. The beam was partitioned to allow high quality meshing of the flanges, web and diaphragm locations. Figure 37 shows the partitioning applied to the box-beam part.

![Figure 37. Box-Beam Partitioning](image)

6.10.3.4: Material properties

The same material properties were defined for both the equivalent material properties calculation and the ABAQUS model. The properties defined include Young’s Modulus of Elasticity and Poisson’s ratio for the beam, deck, and shear key materials. The values used were chosen based on previous experiences as well as on numerous sources. Table 6 shows the properties of the materials defined in ABAQUS. The box beams and cast-in-place deck were constructed of concrete and the shear key of grout.
### Table 6. Material Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity (ksi)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>5,000</td>
<td>0.2</td>
</tr>
<tr>
<td>Grout</td>
<td>3,600</td>
<td>0.2</td>
</tr>
</tbody>
</table>

6.10.3.5: **Meshing and assembly**

The entire ABAQUS model was meshed using C3D8R elements. The ABAQUS Analysis User’s Guide defines C3D8R elements as “8-node linear brick, reduced integration” elements (ABAQUS, 2013). Reduced integration reduces the number of integration points from eight to one located at the center of the element. The edges of all parts were seeded to produce elements with a 6-inch side.

The final assembly consists of eight 27-inch deep girders placed side-by-side, with shear keys in-between and a 6-inch cast-in-place deck on top. The final meshed assembly is shown in Figure 38.

![Figure 38. Meshed ABAQUS Assembly](image)

6.10.3.6: **Loading and boundary conditions**

The same load and boundary conditions were applied to both the plate and the ABAQUS model. A distributed load of 1 kip per square foot was applied to the surface of both models. Both models were also assigned boundary conditions to represent pinned connections on both ends. SAP2000 applies joint restraints at the centerline of the shell element by default; in order to match this effect the location of the neutral axis on the ABAQUS model was determined and assigned a pin restraint. Figure 39 show the applied loading and boundary conditions of both models.
6.10.3.7: ABAQUS model results

This section will compare the results from the ABAQUS finite element model and the SAP2000 model. Prior to comparing the results, reaction forces at the pinned connections were checked to ensure correct loading. Both ABAQUS and SAP2000 had correct reaction forces for the applied loading. Figure 40 displays the results from both the ABAQUS and SAP2000 models, shown as stress contours. Note that although axis conventions are not the same in ABAQUS and SAP2000, the same stress component was considered. A review of the results reveals that maximum compressive stress occurs on the deck surface at the mid-span, as expected. ABAQUS results indicate a stress value of 1.50 ksi and SAP2000 results a stress of 1.68 ksi.
Similarly Figure 41 displays the results of both the ABAQUS and SAP2000 models as displacement contours. The magnitude of the displacement, in the direction of gravity, was considered for both models under the defined loading. Review of the results found that maximum displacements occurred in the same location in both models, on the outer edge of the mid-span. Deflected shapes of both models are similar and are consistent with plate bending. Resulting maximum displacements were 0.98-inch in the ABAQUS model and 1.18-inch in the SAP2000 model.

![ABAQUS model](image1.png)  ![SAP2000 model](image2.png)

**Figure 41. Model Deflection Results**

Comparing the results of the two models shows a difference of 0.18 ksi (12%) and 0.20-inches (20%) for maximum stress and displacement, respectively. A further review of results showed additional similarities between both the ABAQUS and SAP2000 models, as well as expected plate behavior. The minor differences between ABAQUS and SAP2000 were determined to be acceptable given that SAP2000 provided more conservative results in all areas considered. The results of the ABAQUS and SAP2000 models highlight that incorporating equivalent material properties into a plate model can produce a usable result.

### 6.10.4: CSiBridge model verification results

Previous sections considered the use of SAP2000 models. The objective of this section is to demonstrate that ability of CSiBridge to produce results that are comparable with both the SAP2000 and ABAQUS models. The CSiBridge model was constructed so as to match the geometry of the idealized bridge used in the previous section. Equivalent material property definitions were the same as the earlier SAP2000 model, based off the MDOT 27-inch deep × 36-inch wide box girder. The applied load was identical to the previous models.
CSiBridge’s boundary conditions move the joint restraint to the bottom of the shell element. Additional changes in boundary conditions, includes the CSiBridge options to define abutment, bearing pad, and foundation spring definitions; were the shell elements are connected to the bearing pads which have surface contact with the abutment. The abutment was defined as a concrete beam with a modulus of elasticity of 5000 ksi and a Poisson’s ratio of 0.2. The CSiBridge defaults for bearing pads and foundation springs was selected. The default bearing restraints act as a pin-connection that prevents displacement along the z-axis (elevation/height) and the y-axis (transverse direction) and limits the displacement along the x-axis (span of the bridge). Default foundation springs act as fixed supports for the abutment.

The results of the CSiBridge model are presented in Figure 42. Maximum compressive stress indicated at the mid-span was 1.65 ksi. Maximum deflection was 1.324-inches. Stress and deflection were comparable to the SAP2000 model results. All other definitions held constant, the slight changes in values are most likely due to the change in boundary conditions.

Figure 42. CsiBridge Model Verification Results
6.11: Discussion

This chapter demonstrated a method to determine both the equivalent material properties and equivalent section stiffness. This method is based on mechanics of composite structures and laminate plates. It was then shown that, given a set of equivalent material properties, a plate could be defined in the structural analysis program CSiBridge and produce satisfactory results when compared to the theoretical results for both isotropic and orthotropic plates. Additionally, by comparison with a refined finite element analysis, it was established that a plate model incorporating equivalent material properties, could generate similar results.
7. CALCULATION OF MOMENT DEMAND AT THE SHEAR KEYS

7.1: Introduction
This chapter discusses the analysis of different adjacent box-beam bridge geometries utilizing equivalent material properties and CSiBridge. The procedure discussed in the previous chapter is implemented to calculate the equivalent material properties, which is then input into CSiBridge using the material property data dialog box.

The analysis parameters include, box beam width and depth, and bridge superstructure width and length. Dead and live loads were considered. The analysis results are presented as graphs and the observed moment demand variation with respect to the analysis parameters is discussed. Based on the moment demands and the AASHTO LRFD service and strength requirements, transverse post-tensioning requirements are calculated and compared with post-tensioning recommendations from literature and specifications from various state DOTs.

7.2: Model Geometry
In order to better understand the variables affecting the moments acting at the longitudinal joints several bridge geometries are considered. As shown through the earlier literature review, bridge width and beam depth appear to have the largest effect on the required transverse post-tensioning force (Hanna, 2008; Hanna et al., 2009; Grace et al., 2012). In order to consider this four beam depths and three bridge widths are considered during the analysis.

7.2.1: Box beam geometry
The four box beam depths selected for analysis are 17-inches, 27-inches, 39-inches, and 48-inches. The effect of beam width is considered and both 36-inch wide and 48-inch wide beams are used during the analysis. The only exception to this is the 17-inch deep beam, which only has a width of 36-inch specified, and the 48-inch deep beam which only has 48-inch wide specifications. MDOT box beam geometries were considered.

7.2.2: Bridge width and span
The three different bridge widths considered are 36-ft, 60-ft, and 84-ft. These widths were selected in an effort to cover the broad range of widths that are seen on existing adjacent box beam bridges. Additionally, each of these three widths can use either 36-inch wide or 48-inch wide box girders, assuming a negligible effect from the shear key width.
An adjacent box-beam bridge’s span has a large influence on the girder depth used in the design. Commonly a beam will be used on spans between 30 to 50 times it depth. Therefore, the analysis will consider spans based off of the span-to-depth ratios of 30 and 50 for each respective beam depth. That is to say for a span-to-depth ratio of 30, for each bridge width, a span of 42.5-ft, 67.5-ft, 97.5-ft, and 120-ft for 17-inch, 27-inch, 39-inch, and 48-inch deep beams will be considered, respectively. Similarly for a span-to-depth ratio of 50, spans of 70.8-ft, 112.5-ft, and 162.5-ft for 17-inch, 27-inch, and 39-inch deep beams will be considered, respectively. A span to depth ratio of 50 will not be considered for the 48-inch deep beam because 200-ft exceeds the spans typically seen for adjacent box-beam bridges.

7.3: Loading

This analysis is performed considering several loading conditions. These conditions include the loading due to cast-in-place concrete deck or asphalt wearing surface, and HL-93 live loading. Multiple lane loading conditions are checked to determine critical loading. The following sections discuss loading in more detail.

7.3.1: Dead load

The dead load consists of the various overlays and the concrete barrier. Self-weight of the beams is not considered because it is evenly distributed to each beam and does not affect the transverse moment. The concrete New Jersey barrier produces a negative moment and is located along the outer edge of the exterior beam. The weight of the concrete New Jersey barrier was assumed to be 0.3667 kips/ft$^2$. The positioning of the barrier load is shown in Figure 43. The load is applied to a width of 1.5 ft, starting at the outer edge of both exterior beams, and along the full span of the bridge.

![Figure 43. Barrier Loading](image-url)
An analysis of dead loading is performed that considered only the weight of the fresh concrete. Research performed by Ulku et al. (2010) and Attanayake and Aktan (2009) was the leading reason for this analysis. Determining the moment due to the self-weight of the concrete allows calculation of the required post-tension to eliminate tensile stresses at the shear key. The barrier weight is then used during the live load analysis, at which point the deck is considered to be hardened. This method allows two stage post-tension to be considered. The application of the live, barrier and deck weight allows the calculation of the moment and the required post-tension to eliminate tensile stresses due to applied loads. During the analysis of the self-weight of concrete, the plate models thickness was set equal to the beam height and the equivalent material properties calculation did not consider the influence of the deck. Similarly an analysis for the self-weight of asphalt was considered.

7.3.2: Live load

Live load application consisted of HL-93 loading according to AASHTO LRFD specifications. HL-93 includes a design truck load plus a design lane load. The design truck loading is identical to the AASHTO HS20 truck loading configuration, which consists of three axle loads of 8 kips, 32 kips, and 32 kips. The distance between the 8 kip load and the first 32 kip load is 14 ft; the distance between the two 32 kip loads can vary between 14 ft to 30 ft. CSiBridge accounts for the variation between the two 32 kip loads and selects the distance that maximizes the load effect.

In addition to the truck load, a land load of 0.64 kips/ft is distributed across the land width, assumed to cover 10 ft transversely. Loading information can be input into CSiBridge by means of several dialog boxes, two of which were utilized. The Vehicle Data and Vertical Loading dialog boxes are displayed in Figure 44.
For each of the three bridge width used, different numbers of lanes were used. Lanes placed on the bridge were 2, 4 and 6 for the 36-ft, 60-ft, and 84-ft width bridges, respectively. Figure 45 shows the lanes applied to the 36-ft wide bridge. In all models the multiple presence factor was considered, as discussed in AASHTO LRFD specifications. Table 7 shows the multiple presence factors in reference to the number of lanes used. During analysis CSiBridge considers different lane loading combinations to maximize the load effect.
Table 7. Multiple Presence Factors

<table>
<thead>
<tr>
<th>Number of Lane Loaded</th>
<th>Multiple Presence Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.20</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>0.85</td>
</tr>
<tr>
<td>&gt; 3</td>
<td>0.65</td>
</tr>
</tbody>
</table>

One of the primary reasons for selecting CSiBridge is its ability to analyze moving traffic loads, consider variations in distance between axials, and vary lane loading to develop the various bridge response envelopes. For each model the effects of live loading were considered. Additionally, AASHTO LRFD load combinations for Service I and Strength I were used to consider the effect of the dead and live loading. The AASHTO LRFD load combination and load factors for Service I and Strength I are shown in Table 3.4.1-1 of the AASHTO LRFD manual.

7.4: Results and Discussion

The following section will present a discussion of the results. The effect of various factors, including bridge width, bridge span, and beam width, on the transverse moment will be discussed. Required transverse post-tensioning will then be determined using the calculated transverse moments. The transverse moment results are presented, in tabular form, in Appendix C.
7.4.1: Effect of bridge width

To analyze the influence of bridge width on the resulting transverse moment, three different bridge widths were used with each beam depth considered. Figure 46 shows the maximum positive transverse moment that occurs along a shear key. The maximum positive transverse moment results in compression at the mid-span of the bridge. Within each graph in Figure 46 the span of each respective beam remains constant. As expected, an increase in transverse moment occurs as the width of the bridge increases. The increase in transverse moment as bridge width increases is consistent for all beam widths and depths.
Figure 47 shows the maximum negative moment. The negative moment results in tensile stresses at the top of the deck, primarily over the supports. Similar to the maximum positive moment, as the bridge width increases the maximum negative transverse moment increases. A review of the magnitudes and location of maximum positive and negative moments reveals that the maximum positive transverse moment occurs near the mid-span of the centermost shear key. The maximum negative transverse moment occurs over the abutments along the shear key for the exterior girders. A close review of the resulting transverse moments also shows that negative moments have a larger magnitude, when compared to positive moments, on narrower bridges. As the bridge width increases, the positive moment increases more rapidly than the negative moment and consistently excides the magnitude of the negative transverse moment on wider bridges.
(a) beam width: 36-in; span ratio: 30

(b) beam width: 36-in; span ratio: 50

(c) beam width: 48-in; span ratio: 30
7.4.2: Effects of bridge span

To analyze the influence of bridge span on the resulting transverse moment, two different spans were used for each beam considered. Figure 48 provides a comparison of resulting transverse moments for the two spans considered for the 27-inch deep and 39-inch deep beams. As seen in Figure 48, for all three bridge widths considered, an increase from a span ratio of 30 to a span ratio of 50 resulted in an increase in maximum transverse movement. The same effect was seen for all other beam depths and widths considered. An increase in maximum negative transverse moment is also seen with an increase in bridge span. The comparison between span ratios of 30 and 50, for the 27-inch and 39-inch deep beams, for the three bridge widths considered are producing a negative transverse moment are shown in Figure 49.
Figure 48. Span Effect on Positive Transverse Moment

Figure 49. Span Effect on Negative Transverse Moment
7.4.3: Effect of beam width

To analyze the effect of beam width on the resulting transverse moment, 36-inch wide and 48-inch wide beams were considered. Figure 50 provides a comparison of resulting maximum positive transverse moment for the two beam widths considered. From an analysis of the results it is shown that the wider girder typically has a lower transverse moment. This result is typical for both positive and negative transverse moment. However, at narrower bridge widths, the positive transverse moment will be greater on the 48-in wide beam than the 36-in wide beam. Figure 51 shows a comparison between two different beam widths, for two different beam depths, of the maximum negative transverse moment.

Figure 50. Beam Width Effect on Positive Transverse Moment

Figure 51. Beam Width Effect on Negative Transverse Moment
7.4.4: Effect of beam depth

Increases in beam depth result in higher transverse moments. However, the increase in transverse moment is relatively small compared to the increase in transverse stiffness provided by the deeper beam. Figure 46 and Figure 47, presented earlier, demonstrate this increase in transverse moment with the increase in depth. However, once the required transverse post-tensioning is calculated, process presented in the following section, a decrease in required post-tensioning with the increase in beam depth can be seen.

7.4.5: Effect of loading

Dead and live loading configurations are discussed in a previous section. The section discussed dead and live loading scenarios that were considered to calculate the moment demand due to the self-weight of the cast-in-place deck and then due to live and other dead loads acting on the composite section. The loading scenarios are loads due to self-weight of an asphalt deck, self-weight of 6 inch cast-in-place concrete deck, and Live loading which includes barrier loads. These loading scenarios allow the required post-tension to be calculated when two-stage post-tensioning is considered. As is expected, increases in loading result in higher transverse moment demand at the longitudinal joints. The various loading scenarios and their effect on the maximum positive transverse moment are shown in Figure 52. The loading scenarios and their effect on the maximum negative transverse moment are shown in Figure 53. Both figures present the result for a bridge with 39 inch deep beams and a span ratio of 30. Similar results are seen for all other bridge and beam geometries.
7.5: **Transverse Post-tensioning Force**

Using equivalent material properties and CSiBridge it was possible to determine the transverse moments acting at the shear keys. Using these moments the transverse post-tensioning can be computed using working stress analysis. This section demonstrates the calculation of the required post-tensioning force and then discusses and compares the results with other methods and recommendations for post-tensioning. The example presented considers single stage post-tensioning. Two-stage post-tensioning has a similar procedure and will also be described.
7.5.1: **Post-tensioning force calculation procedure**

The prototype bridge considered for the example has a 50-ft span and 25.75-ft width, with a superstructure consisting of eight 27-inch deep by 36-inch wide box beams. A 6-inch cast-in-place concrete deck is also considered. Following the procedure outlined earlier the equivalent material properties are calculated and input into CSiBridge. The results of the analysis (Figure 54) show that the maximum transverse moment, occurring at the center span of the middle shear key, is 22.998 kip-ft/ft.

![Figure 54. Example Bridge Transverse Moment Results (kip-ft/ft)](image)

The following information will be used in the working stress analysis:

- **Bridge Span**: 50 ft
- **Bridge Width**: 25.75 ft
- **Box-beam depth**: 27 in.
- **Box-beam width**: 36 in.
- **Concrete strength**, $f'_c$: 6,000 psi
- **Grout strength**, $f'_c$: 3,600 psi

Moment demand is calculated per foot, therefore a one foot section will be considered.

- **Area**, $A = 324 \text{ in}^2$ \hspace{1cm} (12 in. $\times$ beam depth)
- **Moment of Inertia**, $I = 1,968 \text{ in}^4$ \hspace{1cm} $[(12 \text{ in.} \times \text{beam depth}^3)/12]$

The maximum positive and negative moment load cases are checked. The maximum positive moment occurs at the midspan and maximum negative over the abutment. The allowable compressive strength is reduced due to effective prestress. Because the grout is also compressed
the maximum allowable compressive load is \(0.6 \times f'_c = 0.6 \times 3,600 = 2,160\) psi. Additionally, tension is not allowed. The basic formulation for working stress analysis is as follows.

\[
\begin{align*}
    f_{bot} &= \left( \frac{M}{I} \right) + \frac{P}{A} \geq 0 \text{ psi} \\
    f_{top} &= \left( \frac{M}{I} \right) + \frac{P}{A} \leq 2160 \text{ psi}
\end{align*}
\]

Where:
- \(M\) Moment
- \(I\) Moment of Inertia
- \(y\) Distance from the neutral axis
- \(P\) Post-tensioning force
- \(A\) Area

Solving for the required Post-tensioning:

\[
\begin{align*}
    P &\geq 61,328 \text{ psi} \\
    \left[ 2160 - \left( \frac{27}{2} \frac{M-12000}{I} \right) \right] A &= 6.385 \times 10^5 \\
    P &\leq 638,500 \text{ psi}
\end{align*}
\]

The required post-tensioning is 61.3 kips/ft.

The preceding example considered only single stage post-tensioning. Ulku et al. (2010) showed that two-stage post-tension greatly reduced tensile stresses in the deck. Two-stage post-tensioning slightly modifies the procedure. The same working stress analysis is performed twice; the first time considering the moment from the dead-weight of the deck, and the second time considering the live loads. The resulting post-tensioning force, calculated from the dead-weight of the deck, is applied as the first stage of post-tensioning. Once the deck has cured the post-tensioning force is increased to the second calculated value which considers the remaining dead and live loads.

7.5.2: Two-stage post-tensioning results

Using working stress analysis the required post-tensioning for all bridge configurations considered is calculated. CSiBridge results include transverse moment along the entire span at each longitudinal joint; this allows the maximum positive and negative transverse moments to be
identified. Because the transverse moment demand varies along the bridge span, different post-tensioning requirements can be calculated between the abutments and the mid-span.

The maximum positive and negative transverse moments consistently occurred at the mid-span and over the supports, respectively. The maximum positive transverse moment resulted in compression of the deck and the negative moment resulted in tension of the deck. Considering these two maximum transverse moments, the post-tensioning force required at the mid-span and over the supports is calculated, for both first- and second-stage post-tensioning.

The following three tables show some of the results for required post-tensioning. Additional tables showing the results for the remaining bridge configuration considered are in Appendix D. Table 8, Table 9, and Table 10 show the post-tensioning force requirements for a 36 ft wide, 60 ft wide, and an 84 ft wide bridges respectively. The three tables present the results for bridges constructed of 39-inch deep box-beams with a span ratio of 30 (total span of 97.5 ft).

Table 8. Post-Tensioning Force Requirements for 36 ft Wide Bridge

<table>
<thead>
<tr>
<th></th>
<th>Over support</th>
<th>Mid-span</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior to Deck Placement (first stage)</td>
<td>6.5</td>
<td>7.3</td>
</tr>
<tr>
<td>After Deck Cures (second stage)</td>
<td>27.5</td>
<td>11.4</td>
</tr>
<tr>
<td>Total Post-tension</td>
<td>34.6</td>
<td>18.7</td>
</tr>
</tbody>
</table>

Table 9. Post-Tensioning Requirements for 60 ft Wide Bridge

<table>
<thead>
<tr>
<th></th>
<th>Over support</th>
<th>Mid-span</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior to Deck Placement (first stage)</td>
<td>9.3</td>
<td>15.6</td>
</tr>
<tr>
<td>After Deck Cures (second stage)</td>
<td>38.5</td>
<td>53.6</td>
</tr>
<tr>
<td>Total Post-tension</td>
<td>47.8</td>
<td>69.2</td>
</tr>
</tbody>
</table>
The post-tensioning magnitudes are affected by the beam used and bridge geometry. One of the largest factors is the bridge width; as the width of the bridge increases so does the required post-tensioning. Similarly higher spans require increases in post-tensioning.

7.5.3: Comparison with Hanna et al. (2009) equation

As discussed in the literature review, a variety of recommendations on the required transverse post-tensioning force exist. One such example exists in the form of an equation developed by Hanna et al. (2009). The prototype bridge, used in the previous sections, will once again be used as a calculation example. The equation is provided below.

\[ P = \left( \frac{0.9W}{D} - 1.0 \right) K_L K_S \leq \left( \frac{0.2W}{D} + 8.0 \right) K_L K_S \]

Where,

\[ D = \text{box depth (in.)} = 27\text{-inches} \]
\[ W = \text{bridge width (in.)} = 309\text{-inches} \]
\[ L = \text{bridge span (in.)} = 600\text{-inches} \]
\[ K_L = 1.0 + 0.003 \left( \frac{L}{D} - 30 \right) = 0.977 \]
\[ \theta = \text{skew angle (deg.)} = 0 \]
\[ K_S = 1.0 + 0.002\theta = 0 \]

\[ P = \left( \frac{0.9 \times 309}{27} - 1.0 \right) (0.977)(1.0) \leq \left( \frac{0.2 \times 309}{27} + 8.0 \right) (0.96)(1.0) = 9.083 \text{kips/ft} \]

Comparing the results of the proposed method, 61.3 kips/ft, with the results of Hanna’s equation, 9.08 kips/ft, it is seen that Hanna’s equation produces a considerably lower post-tensioning force per foot. Similar results are found when comparing other bridge geometries as well. The development of the equation using a grillage analysis with a wide spacing between transverse members may be a possible explanation for the difference in results. According to Hanna (2008)
the transverse member spacing correlated with the diaphragm locations, resulting in distances up to 30 feet between transverse members. The large spacing between transverse members may not accurately represent the transverse stiffness characteristic to adjacent box-beams. Transverse moment can only be calculated at the locations of the transverse members and the results can be skewed by the occurrence of negative and positive moments within the region the transverse member represents. The developed procedure overcomes this obstacle by allowing the transverse moment to be determined at any point along the span.

Hambly (1991) recommended that the spacing of transverse members be similar to the spacing of the longitudinal members when performing a grillage analysis. Sang (2010) followed Hambly’s recommendation when modeling an adjacent box-beam bridge with a span of 80 ft and skew of 0° constructed of twelve 33 inch deep by 48 inch wide box-beams. Through a grillage analysis, with a longitudinal and transverse member spacing of 4 ft, Sang calculated a maximum moment of 225 kip-ft (2701 kip-in) at the mid-span transverse member. This is about 56 kip-ft/ft. Using working stress analysis the required post-tensioning for the bridge, based upon Sang’s transverse moment, would be 122.6 kips/ft. Applying Hanna’s equation to this same bridge geometry results in a recommended post-tensioning force of 14.6 kips/ft.

7.5.4: Comparisons with state post-tensioning practice

Russel (2009) performed a survey of states that use adjacent box-beam bridges, this survey revealed that 81% of the states did not perform design calculations to determine the number of transverse ties required; additionally, post-tensioning forces were just taken from the respective states bridge design manual. A review of current state practices reveals a wide range of suggested post-tensioning forces. The highlights of the state DOT practices are included in the literature review. The same table included in the literature review was modified (Table 11) to include a column with the per-foot post-tensioning force. The per-foot post-tensioning force was calculated by dividing the spacing between post-tensioning locations by the DOT specified post-tensioning force.

All of the states shown in the table specify lower post-tensioning forces than that calculated in this study. Additionally, many of the forces specified are lower than those calculated for the first-stage post-tensioning (Section 7.5.2: and Appendix D).

As is shown, very little relationship exists between different states post-tensioning specifications. Michigan, New York, and Wisconsin specify some of the highest transverse post-tensioning
forces of the various state DOTs. Attanayake and Aktan (2013) performed field inspections of 15 in-service bridges in Michigan and found longitudinal reflective cracking on the bridges inspected. The approximate per-foot forces specified by Michigan and New York are close to the values that can be obtained from Hanna et al.’s equation; the continued occurrence of reflective cracking indicates that the current post-tensioning specifications are not sufficient.

The calculation of post-tensioning per foot by dividing the specified post-tensioning by the duct spacing is useful for comparison, however is not realistic when considering the force distribution described by Ulku et al. (2010). In order to use this method it must be assumed that the entire area between post-tensioning locations is compressed and this is not true. If the distribution described by Ulku et al. (2010) is considered much higher ‘per foot’ forces can be seen. If, for example, a 48-in wide box-beam in Michigan is considered the per foot force within the distribution area would be 120/(1.35*4) = 22.2 kips/ft. Considering the force that is being applied within this distribution area the results obtained from the analysis are reasonable. Further, in order to provide continuous compression of the deck a smaller post-tensioning spacing needs to be considered.
Table 11. State DOT Post-tensioning Practice Considering Per-Foot Force

<table>
<thead>
<tr>
<th>State DOT</th>
<th>Post-tensioning Location*</th>
<th>Specified Post-tensioning Force</th>
<th>Post-tensioning force (kip/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ends</td>
<td>¼ Pt.</td>
<td>1/3 Pt.</td>
</tr>
<tr>
<td>Connecticut</td>
<td>X</td>
<td>75 &lt; S</td>
<td>50 &lt; S ≤ 75</td>
</tr>
<tr>
<td>Delaware</td>
<td>X</td>
<td>120 &lt; S ≤ 160</td>
<td>80 &lt; S ≤ 120</td>
</tr>
<tr>
<td>Illinois</td>
<td>X</td>
<td>N = (S/25 – 1 ) ≥ 1</td>
<td>-</td>
</tr>
<tr>
<td>Massachusetts</td>
<td>X</td>
<td>50 &lt; S</td>
<td>-</td>
</tr>
<tr>
<td>Michigan</td>
<td>X</td>
<td>50 &lt; S ≤ 100</td>
<td>-</td>
</tr>
<tr>
<td>New Hampshire</td>
<td>-</td>
<td>-</td>
<td>40 &lt; S</td>
</tr>
<tr>
<td>New York</td>
<td>X</td>
<td>50 ≤ S</td>
<td>-</td>
</tr>
<tr>
<td>Ohio</td>
<td>X</td>
<td>75 &lt; S</td>
<td>50 &lt; S ≤ 75</td>
</tr>
<tr>
<td>Oregon</td>
<td>X</td>
<td>120 &lt; S</td>
<td>80 &lt; S ≤ 120</td>
</tr>
<tr>
<td>Pennsylvania</td>
<td>X</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Virginia</td>
<td>X</td>
<td>50 &lt; S</td>
<td>-</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>X</td>
<td>120 &lt; S ≤ 160</td>
<td>80 &lt; S ≤ 120</td>
</tr>
</tbody>
</table>

¹Not specified in state bridge design manual
8. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

8.1: Summary and Conclusions
The objective of this study was to develop a rational load demand analysis model using equivalent material properties. The model would then be used to calculate the moment acting at the longitudinal joints of adjacent box-beam bridges. A literature review was conducted to understand the causes of reflective cracking and the construction of adjacent box-beam bridges. A newly constructed bridge was inspected. During the inspection cracks were found over the abutments. The lack of live loading indicates that these cracks were due to volume change loads. Concrete shrinkage was modeled using the finite element program ABAQUS. This was accomplished by incorporating a user subroutine and the shrinkage equation developed in ACI 209. This model confirmed that cracking would occur over the shear key locations near the supports. Additionally, a model of post-tensioning showed the force distribution was insufficient to fully compress the deck along the span and that current levels of post-tensioning are insufficient to prevent cracking due to concrete shrinkage.

A load demand analysis model was developed and used in conjunction with CSiBridge to determine the transverse moment. The model results were validated through a comparison with the results of Navier's method and a finite element analysis. It was demonstrated that using equivalent material properties with CSiBridge can produce reliable results and that the transverse moments in the shear key location could be calculated.

The developed model is used to determine the moment demands at shear keys for many commonly used girder sizes and bridge geometries. The moment demands were then used to calculate the required post-tensioning; the calculated post-tensioning forces are compared with those suggested in literature and by various state DOTs. Additionally, recommendations were given for post-tensioning forces when considering multi-stage post-tensioning. These recommendations included the required post-tensioning over the supports and at the mid-span. The results from the load demand model also showed that tensile stress would occur over the supports and compressive stresses would occur over the mid-span. Thus cracks would not expand to full length under live load. The propagation of cracks across the entire length of the
bridge indicate that other factors, such as volume change loads, are responsible for the initiation and propagation of reflective cracks.

8.2: Recommendations

A review of current literature and DOT specifications revealed that many inconsistencies exist between state transverse post-tensioning practices. Their specified transverse post-tensioning forces range from 30 kips to 120 kips per tensioning location. The analysis model presented in this thesis shows that these forces should be reviewed and potentially increased. This is especially vital at locations over the supports where longitudinal cracking has been documented to begin.

While the developed model is useful in determining the transverse moment, it did not incorporate the potential loads that were identified as the causes of cracking. These loads including heat of hydration, shrinkage, and temperature gradient due to ambient weather need to be considered. Understanding the effects of these loads and accounting for them is essential to prevent the continued development of reflective cracking.

Additionally, as discussed in the literature review and post-tensioning calculation section, alternatives to current post-tensioning practices need to be considered. While post-tensioning cannot prevent the initiation of reflective cracking, multi-stage post-tensioning could potentially prevent continued crack propagation along the deck. Similarly a smaller spacing needs to be considered between post-tensioning locations in order to maintain compression throughout the slab.

Recommendations for future study are as follows:

- Consider the effects of heat of hydration, shrinkage, and temperature gradient simultaneously to develop design recommendations to address the resulting stresses.
- Analysis of smaller spacing between post-tensioning locations. Post-tensioning through the top and bottom flange spaced as described by Ulku (2010).
- Further study of multi-stage post-tensioning and its ability to compress the deck and prevent propagation of cracking.
REFERENCES


Precast/Prestressed Concrete Institute (PCI). (2003) Precast prestressed bridge design manual, PCI, Chicago, IL.
Precast/Prestressed Concrete Institute (PCI). (2009) The State-of-the-Art of Precast, Prestressed Adjacent Member Bridges, PCI, Chicago, IL.


APPENDIX A

Equivalent Material Property Calculations for Load Analysis of Adjacent Box-beam Bridge Superstructures
This section presents an example calculation of equivalent material properties. The equivalent material properties will be calculated for a bridge constructed of MDOT 36-inch wide × 17-inch deep box-beams. The figure below shows the geometry of the box-beam.

A 6-inch cast-in-place deck will also be considered. To simplify the calculation the geometry of the key way is not considered in the RVE.

**Section data for cast in place deck:**
- Height of deck (in): \( H_d := 6 \)
- Youngs Modulus (ksi): \( E_d := 5000 \)
- Poisson's Ratio: \( \nu_d := 0.2 \)

**Section data for box beam:**
- Height of box beam (in): \( H_b := 17 \)
- Height of top flange (in): \( H_{bf1} := 5 \)
- Height of bottom flange (in): \( H_{bf2} := 5 \)
- Width of box beam (in): \( W_b := 36 \)
- Width of webs (in): \( W_w := 5 \)
- Size of bevel (in): \( B_s := 3 \)

**Youngs Modulus (ksi):**
- \( E_c := 5000 \)
- Poisson's Ratio: \( \nu_c := 0.2 \)
Section data for shear key:

<table>
<thead>
<tr>
<th>Width of shear key (in)</th>
<th>Youngs Modulus (ksi)</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_s := 3$</td>
<td>$E_s := 3600$</td>
<td>$\nu_s := 0.2$</td>
</tr>
</tbody>
</table>

Mid plane axis location, from the bottom of the section (in):

$$Y_m := \frac{H_d + H_b}{2} = 11.5$$

Area of the box beam (in$^2$):

$$A_c := H_b \cdot W_b - \left[ \left( H_b - H_{bf1} - H_{bf2} \right) \left( W_b - 2 \cdot W_w \right) \right] - 2 \cdot B_s^2 = 448$$

Step-1 Representative Volume Element

The same RVE described in this study will be used. The geometry of the RVE is adjusted based on the box-beam selected. The section was divided into four layers as shown in the figure below. Layer A consists of the cast-in-place deck, layer B the top flange of the box beam. Layer C and D consist of the box-beam web and bottom flange, respectively.
Step-2 Micromechanics

Micromechanics is used to determine the properties of the individual lamina of the RVE.

Layer A - Cast in place deck:

This layer is isotropic and will not require micromechanical calculations.

Longitudinal Young Modulus (ksi):

\[ E_{1a} := E_d = 5000 \]

Transverse Young Modulus (ksi):

\[ E_{2a} := E_d = 5000 \]

Longitudinal Shear Modulus (ksi):

\[ G_{12a} := \frac{E_d}{2(1 + \nu_d)} = 2083 \]

Longitudinal Poisson's Ratio:

\[ \nu_{12a} := \nu_d = 0.2 \]

Layer B - Top flange of box beam:

For micromechanical calculations, the top flange of the box beam will act as the matrix and the shear key will act as the fiber. If the section does not include a shear key, this layer will be isotropic.

Areas for both the matrix and fibers (in²):

\[ A_{mB} := H_{bf1} \cdot W_b = 180 \quad A_{fB} := H_{bf1} \cdot W_s = 15 \]

Volume Fractions:

\[ V_{mB} := \frac{A_{mB}}{A_{mB} + A_{fB}} = 0.923 \quad V_{fB} := \frac{A_{fB}}{A_{mB} + A_{fB}} = 0.077 \]

Longitudinal Young Modulus (ksi):

\[ E_{1b} := V_{fB} \cdot E_s + V_{mB} \cdot E_c = 4892 \quad A_{mB} := H_{bf1} \cdot W_b = 180 \]
Transverse Young Modulus (ksi):

\[ E_{b2B} := \sqrt{V_{fB}} E_s + \left(1 - \sqrt{V_{fB}} \right) E_c = 4612 \]

\[ A_{fB} := H_{bf1} W_s = 15 \]

\[ E_{2b} := \begin{cases} \frac{\sqrt{V_{fB}} + 1 - \sqrt{V_{fB}}}{E_{b2B} + E_c} & \text{if } W_s > 0 \\ E_{1b} & \text{otherwise} \end{cases} \]

Longitudinal Shear Modulus (ksi):

\[ G_{12mB} := \frac{E_c}{2(1 + \nu_c)} = 2083 \]

\[ G_{12fB} := \frac{E_s}{2(1 + \nu_s)} = 1500 \]

\[ G_{b2B} := \sqrt{V_{fB}} G_{12fB} + \left(1 - \sqrt{V_{fB}} \right) G_{12mB} = 1922 \]

\[ G_{12b} := \begin{cases} \frac{\sqrt{V_{fB}} + 1 - \sqrt{V_{fB}}}{G_{b2B} + G_{12mB}} & \text{if } W_s > 0 \\ G_{12mB} & \text{otherwise} \end{cases} \]

Longitudinal Poisson's Ratio:

\[ \nu_{12b} := V_{fB} \cdot \nu_s + V_{mB} \cdot \nu_c = 0.2 \]

Layer C - Web of box beam and shear key:

Areas for both the matrix and fibers (in²):

\[ A_{mC} := A_c - (H_{bf1} + H_{bf2}) \cdot W_b = 88 \]

\[ A_{fC} := (H_b - H_{bf1} - H_{bf2}) \cdot W_s = 21 \]

Volume Fractions:

\[ V_{mC} := \frac{A_{mC}}{A_{mC} + A_{fC}} = 0.807 \]

\[ V_{fC} := \frac{A_{fC}}{A_{mC} + A_{fC}} = 0.193 \]

Longitudinal Young Modulus (ksi):

\[ E_{w1C} := V_{fC} E_s + V_{mC} E_c = 4730 \]

Transverse Young Modulus (ksi):

\[ E_{b2C} := \sqrt{V_{fC}} E_s + \left(1 - \sqrt{V_{fC}} \right) E_c = 4385 \]
The voids in this layer cause the Transverse Young's Modulus to become zero. The Longitudinal Young's Modulus, Longitudinal Shear Modulus, and Poisson's Ratio will be directly proportional to the volume of the material compared to the void area.

**Transverse Young Modulus (ksi):**

\[ E_{w2C} := \begin{cases} 
\left( \frac{\sqrt{V_{fC} E_c}}{E_{b2C} E_c} \frac{1 - \sqrt{V_{fC}}}{E_{c}} \right)^{-1} & \text{if } W_s > 0 \quad = 4710 \\
E_{w1C} & \text{otherwise}
\end{cases} \]

**Longitudinal Shear Modulus (ksi):**

\[ G_{12mC} := \frac{E_c}{2(1 + \nu_c)} = 2083 \quad \text{and} \quad G_{12fC} := \frac{E_s}{2(1 + \nu_s)} = 1500 \]

\[ G_{b2C} := \sqrt{V_{fC} G_{12fC} + \left( 1 - \sqrt{V_{fC}} \right) G_{12mC}} = 1827 \]

\[ G_{w12C} := \begin{cases} 
\left( \frac{\sqrt{V_{fC} G_{b2C}}}{G_{b2C} G_{12mC}} \frac{1 - \sqrt{V_{fC}}}{G_{12mC}} \right)^{-1} & \text{if } W_s > 0 \quad = 1963 \\
G_{12mC} & \text{otherwise}
\end{cases} \]

**Poisson's Ratio:**

\[ \nu_{w12C} := V_{fC} \nu_s + V_{mC} \nu_c = 0.2 \]

The voids in this layer cause the Transverse Young's Modulus to become zero. The Longitudinal Young's Modulus, Longitudinal Shear Modulus, and Poisson's Ratio will be directly proportional to the volume of the material compared to the void area.

**Volume Fractions for the web compared to the voids:**

\[ V_{\text{web}} := \frac{A_c - (H_{bf1} + H_{bf2}) W_b + (H_b - H_{bf1} - H_{bf2}) W_s}{(H_b - H_{bf1} - H_{bf2}) W_b + W_s} = 0.399 \]

**Longitudinal Young Modulus (ksi):**

\[ E_{1c} := V_{\text{web}} E_{w1C} = 1889 \]

**Transverse Young Modulus (ksi):**

\[ E_{2c} := 0 \]

**Longitudinal Shear Modulus (ksi):**

\[ G_{12c} := V_{\text{web}} G_{w12C} = 784 \]
Layer D - Bottom flange of box beam:

For micromechanical calculations, the bottom flange of the box beam will act as the matrix and the shear key will act as the fiber. If the section does not include a shear key, this layer will be isotropic.

Areas for both the matrix and fibers (in\(^2\)):

\[
A_{mD} := H_{bf} W_b = 180 \quad \text{and} \quad A_{fD} := H_{bf} W_s = 15
\]

Volume Fractions:

\[
V_{mD} := \frac{A_{mD}}{A_{mD} + A_{fD}} = 0.923 \quad \text{and} \quad V_{fD} := \frac{A_{fD}}{A_{mD} + A_{fD}} = 0.077
\]

Longitudinal Young Modulus (ksi):

\[
E_{1d} := V_{fD} E_s + V_{mD} E_c = 4892
\]

Transverse Young Modulus (ksi):

\[
E_{b2D} := \sqrt{V_{fD}} E_s + \left(1 - \sqrt{V_{fD}}\right) E_c = 4612
\]

\[
E_{2d} := \begin{cases} 
\frac{\sqrt{V_{fD}}}{E_{b2D}} + \frac{1 - \sqrt{V_{fD}}}{E_c} & \text{if } W_s > 0 \\
E_{1b} & \text{otherwise}
\end{cases}
\]

Longitudinal Shear Modulus (ksi):

\[
G_{12mD} := \frac{E_c}{2\left(1 + \nu_c\right)} = 2083.333 \quad \text{and} \quad G_{12fD} := \frac{E_s}{2\left(1 + \nu_s\right)} = 1500
\]

\[
G_{b2D} := \sqrt{V_{fD}} G_{12fD} + \left(1 - \sqrt{V_{fD}}\right) G_{12mD} = 1922
\]

\[
G_{12d} := \begin{cases} 
\frac{\sqrt{V_{fD}}}{G_{b2D}} + \frac{1 - \sqrt{V_{fD}}}{G_{12mD}} & \text{if } W_s > 0 \\
G_{12mD} & \text{otherwise}
\end{cases}
\]

Longitudinal Poisson Ratio:

\[
\nu_{12d} := V_{fD} \nu_s + V_{mD} \nu_c = 0.2
\]
Step-3 Macromechanics

Macromechanics is used to calculate the complete stiffness matrix and its sub-matrices, the [A], [B], and [D] matrices. The matrices are calculated using the plane stress matrix [Q]. The plane stress matrix must be computed for each layer of the RVE.

Calculate Layer Stiffness Matrices:

Each layer will have a plane stress stiffness matrix Q:

\[
Q := \begin{pmatrix}
Q_{11} & Q_{12} & Q_{13} \\
Q_{21} & Q_{22} & Q_{23} \\
Q_{31} & Q_{32} & Q_{33}
\end{pmatrix}
\]

For isotropic and orthotropic materials, Q is calculated using the following equation:

\[
Q := \begin{pmatrix}
\frac{E_1}{DQ} & \frac{\nu_{12}E_2}{DQ} & 0 \\
\frac{\nu_{12}E_2}{DQ} & \frac{E_2}{DQ} & 0 \\
0 & 0 & G_{12}
\end{pmatrix}
\]

Where: \( D_Q := 1 - \frac{E_2}{E_1} \cdot \nu_{12}^2 \)

Layer (A) plane stress stiffness matrix \([Q_a]\) (psi):

\[
D_{Qa} := 1 - \frac{E_{2a}}{E_{1a}} \cdot \nu_{12a}^2 = 0.96
\]

\[
Q_a := \begin{pmatrix}
\frac{E_{1a} \cdot 1000}{D_{Qa}} & \frac{\nu_{12a} E_{2a} \cdot 1000}{D_{Qa}} & 0 \\
\frac{\nu_{12a} E_{2a} \cdot 1000}{D_{Qa}} & \frac{E_{2a} \cdot 1000}{D_{Qa}} & 0 \\
0 & 0 & G_{12a} \cdot 1000
\end{pmatrix} = \begin{pmatrix}
0.521 & 0.104 & 0 \\
0.104 & 0.521 & 0 \\
0 & 0 & 0.208
\end{pmatrix} \cdot 10^7
\]

Layer (B) plane stress stiffness matrix \([Q_b]\) (psi):

\[
D_{Qb} := 1 - \frac{E_{2b}}{E_{1b}} \cdot \nu_{12b}^2 = 0.96
\]

107
Layer Geometry:

RVE dimensions (in):

\[ z_0 := -Y_m = -11.5 \]
\[ z_1 := -Y_m + H_{bf2} = -6.5 \]
\[ z_2 := -Y_m + H_b - H_{bf1} = 0.5 \]
\[ z_3 := -Y_m + H_b = 5.5 \]
\[ z_4 := -Y_m + H_b + H_d = 11.5 \]
Determine the coefficients of the sub-matrices

In-plane stiffness matrix [A] (lb / in):

\[
A := \begin{pmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{pmatrix}
\]

\[
A := (z_4 - z_3) \cdot Q_a + (z_3 - z_2) \cdot Q_b + (z_2 - z_1) \cdot Q_c + (z_1 - z_0) \cdot Q_d = \begin{pmatrix}
9.543 & 1.643 & 0 \\
1.643 & 8.214 & 0 \\
0 & 0 & 3.834
\end{pmatrix} \cdot 10^7
\]

Coupling stiffness matrix [B] (lb - in):

\[
B := \begin{pmatrix}
B_{11} & B_{12} & B_{13} \\
B_{21} & B_{22} & B_{23} \\
B_{31} & B_{32} & B_{33}
\end{pmatrix}
\]

\[
B := \frac{1}{2} \left[ (z_4^2 - z_3^2) \cdot Q_a + (z_3^2 - z_2^2) \cdot Q_b + (z_2^2 - z_1^2) \cdot Q_c + (z_1^2 - z_0^2) \cdot Q_d \right]
\]

\[
B = \begin{pmatrix}
7.309 & 2.259 & 0 \\
2.259 & 11.295 & 0 \\
0 & 0 & 2.872
\end{pmatrix} \cdot 10^7
\]

Flexural stiffness matrix [D] (lb - in):

\[
D := \begin{pmatrix}
D_{11} & D_{12} & D_{13} \\
D_{21} & D_{22} & D_{23} \\
D_{31} & D_{32} & D_{33}
\end{pmatrix}
\]

\[
D := \frac{1}{3L} \left[ (z_4^3 - z_3^3) \cdot Q_a + (z_3^3 - z_2^3) \cdot Q_b + (z_2^3 - z_1^3) \cdot Q_c + (z_1^3 - z_0^3) \cdot Q_d \right]
\]

\[
D = \begin{pmatrix}
492.384 & 94.955 & 0 \\
94.955 & 474.773 & 0 \\
0 & 0 & 197.091
\end{pmatrix} \cdot 10^7
\]
Assemble stiffness matrix [ABD]:

\[
[ABD] := \begin{pmatrix}
A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\
A_{21} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} \\
A_{31} & A_{32} & A_{33} & B_{31} & B_{32} & B_{33} \\
B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\
B_{21} & B_{22} & B_{23} & D_{21} & D_{22} & D_{23} \\
B_{31} & B_{32} & B_{33} & D_{31} & D_{32} & D_{33}
\end{pmatrix}
\]

\[
[ABD] = \begin{pmatrix}
9.543 & 1.643 & 0 & 7.309 & 2.259 & 0 \\
1.643 & 8.214 & 0 & 2.259 & 11.295 & 0 \\
0 & 0 & 3.834 & 0 & 0 & 2.872 \\
7.309 & 2.259 & 0 & 492.384 & 94.955 & 0 \\
2.259 & 11.295 & 0 & 94.955 & 474.773 & 0 \\
0 & 0 & 2.872 & 0 & 0 & 197.091
\end{pmatrix} \cdot 10^7
\]

Calculate Equivalent Material Properties

Cramer’s rule is now employed to solve for the equivalent material properties.

Calculate the engineering constants:

Young's Modulus (ksi):

\[
E_1 := \frac{|ABD|}{\begin{pmatrix}
A_{22} & A_{23} & B_{12} & B_{22} & B_{23} \\
A_{23} & A_{33} & B_{13} & B_{23} & B_{33} \\
B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\
B_{22} & B_{23} & D_{21} & D_{22} & D_{23} \\
B_{23} & B_{33} & D_{31} & D_{32} & D_{33}
\end{pmatrix}} \cdot \begin{pmatrix}
\frac{1}{H_b + H_d} \\
1
\end{pmatrix} = 3998
\]
Shear Modulus (ksi):

\[
E_2 := \begin{vmatrix} A_{11} & A_{13} & B_{11} & B_{12} & B_{13} \\
A_{13} & A_{33} & B_{13} & B_{23} & B_{33} \\
B_{11} & B_{13} & D_{11} & D_{12} & D_{13} \\
B_{12} & B_{23} & D_{12} & D_{22} & D_{23} \\
B_{13} & B_{33} & D_{13} & D_{23} & D_{33} \end{vmatrix} \left( \frac{1}{H_b} + \frac{1}{H_d} \right) = 3367
\]

\[
G_{12} := \begin{vmatrix} A_{11} & A_{12} & B_{11} & B_{12} & B_{13} \\
A_{12} & A_{22} & B_{12} & B_{22} & B_{23} \\
B_{11} & B_{12} & D_{11} & D_{12} & D_{13} \\
B_{12} & B_{22} & D_{12} & D_{22} & D_{23} \\
B_{13} & B_{23} & D_{13} & D_{23} & D_{33} \end{vmatrix} \left( \frac{1}{H_b} + \frac{1}{H_d} \right) = 1649
\]

Poisson's Ratio:

\[
\nu_{12} := \begin{vmatrix} A_{12} & A_{23} & B_{12} & B_{22} & B_{23} \\
A_{13} & A_{33} & B_{13} & B_{23} & B_{33} \\
B_{11} & B_{13} & D_{11} & D_{12} & D_{13} \\
B_{12} & B_{23} & D_{12} & D_{22} & D_{23} \\
B_{13} & B_{33} & D_{13} & D_{23} & D_{33} \end{vmatrix} = 0.2
\]
Summary of the engineering constants:

<table>
<thead>
<tr>
<th>Young's Modulus (ksi)</th>
<th>Poisson's Ratio</th>
<th>Shear Modulus (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1 = 3998$</td>
<td>$\nu_{12} = 0.2$</td>
<td>$G_{12} = 1649$</td>
</tr>
<tr>
<td>$E_2 = 3367$</td>
<td>$\nu_{23} := \nu_{12} = 0.2$</td>
<td>$G_{23} := \left[ \frac{2(1 + \nu_{23})}{E_2} \right]^{-1} = 1403$</td>
</tr>
<tr>
<td>$E_3 := E_2 = 3367$</td>
<td>$\nu_{21} := \frac{E_2}{E_1} \cdot \nu_{12} = 0.168$</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX B

Plate Analysis Results
Plate analysis results for 1-ft thick plate

<table>
<thead>
<tr>
<th>% reduction</th>
<th>Modulus of Elasticity</th>
<th>Poisson Ratio</th>
<th>Shear Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E1 SAP2000 MathCAD Error</td>
<td>V12 SAP2000 MathCAD Error</td>
<td>G12 SAP2000 MathCAD Error</td>
</tr>
<tr>
<td>0</td>
<td>2985 0.016 0.015 7%</td>
<td>0.2 0.016 0.019 16%</td>
<td>1269 0.016 0.019 16%</td>
</tr>
<tr>
<td>25</td>
<td>2238.75 0.018 0.017 6%</td>
<td>0.15 0.016 0.019 16%</td>
<td>951.75 0.017 0.021 19%</td>
</tr>
<tr>
<td>50</td>
<td>1492.5 0.02 0.019 5%</td>
<td>0.1 0.017 0.019 11%</td>
<td>634.5 0.02 0.024 17%</td>
</tr>
<tr>
<td>75</td>
<td>746.25 0.023 0.021 10%</td>
<td>0.05 0.0176 0.019 7%</td>
<td>317.25 0.023 0.028 18%</td>
</tr>
<tr>
<td>0</td>
<td>2081 0.016 0.019 16%</td>
<td>0.2 0.016 0.019 16%</td>
<td>867 0.016 0.015 7%</td>
</tr>
<tr>
<td>25</td>
<td>1560.75 0.0162 0.02 19%</td>
<td>0.15 0.016 0.019 16%</td>
<td>650.25 0.016 0.015 7%</td>
</tr>
<tr>
<td>50</td>
<td>1040.5 0.017 0.02 15%</td>
<td>0.1 0.016 0.02 20%</td>
<td>433.5 0.016 0.015 7%</td>
</tr>
<tr>
<td>75</td>
<td>520.25 0.0163 0.021 22%</td>
<td>0.05 0.016 0.021 24%</td>
<td>216.75 0.016 0.015 7%</td>
</tr>
<tr>
<td></td>
<td>E3 SAP2000 MathCAD</td>
<td>V23 SAP2000 MathCAD Error</td>
<td>G23</td>
</tr>
<tr>
<td>0</td>
<td>2081 0.016 0.019 16%</td>
<td>0.2 0.016 0.015 7%</td>
<td>867 0.016 0.015 7%</td>
</tr>
<tr>
<td>25</td>
<td>1560.75 0.016 0.019 16%</td>
<td>0.15 0.016 0.015 7%</td>
<td>650.25 0.016 0.015 7%</td>
</tr>
<tr>
<td>50</td>
<td>1040.5 0.016 0.019 16%</td>
<td>0.1 0.016 0.015 7%</td>
<td>433.5 0.016 0.015 7%</td>
</tr>
<tr>
<td>75</td>
<td>520.25 0.016 0.019 16%</td>
<td>0.05 0.016 0.015 7%</td>
<td>216.75 0.016 0.015 7%</td>
</tr>
</tbody>
</table>
Plate analysis results for 1.5-ft thick plate

<table>
<thead>
<tr>
<th>% reduction</th>
<th>Modulus of Elasticity</th>
<th>Poisson Ratio</th>
<th>Shear Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deflection (ft)</td>
<td>Deflection (ft)</td>
<td>Deflection (ft)</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>2238.75</td>
<td>0.0054</td>
<td>0.0049</td>
</tr>
<tr>
<td>50</td>
<td>1492.5</td>
<td>0.0060</td>
<td>0.0055</td>
</tr>
<tr>
<td>75</td>
<td>746.25</td>
<td>0.0069</td>
<td>0.0062</td>
</tr>
<tr>
<td>0</td>
<td>2081</td>
<td>0.0048</td>
<td>0.0045</td>
</tr>
<tr>
<td>25</td>
<td>1560.75</td>
<td>0.0051</td>
<td>0.0056</td>
</tr>
<tr>
<td>50</td>
<td>1040.5</td>
<td>0.0060</td>
<td>0.0059</td>
</tr>
<tr>
<td>75</td>
<td>520.25</td>
<td>0.0051</td>
<td>0.0061</td>
</tr>
<tr>
<td>0</td>
<td>2081</td>
<td>0.0048</td>
<td>0.0045</td>
</tr>
<tr>
<td>25</td>
<td>1560.75</td>
<td>0.0051</td>
<td>0.0059</td>
</tr>
<tr>
<td>50</td>
<td>1040.5</td>
<td>0.0060</td>
<td>0.0059</td>
</tr>
<tr>
<td>75</td>
<td>520.25</td>
<td>0.0051</td>
<td>0.0061</td>
</tr>
</tbody>
</table>
APPENDIX C

Moment Demand Results
Moment demand results are presented in the following tables. The result of three load cases are presented; the Asphalt load case represents the self-weight of an asphalt wearing surface, Deck considers the self-weight of a 6-inch cast-in-place concrete deck, and Traffic Loading considers the dead loads and the HL93 traffic loading. The maximum positive (Pos.) transverse movement and maximum negative (Neg.) transverse moment are presented for all load cases. All results are presented in units of kip-ft/ft.

### 36-inch wide × 17-inch deep, span ratio: 30

<table>
<thead>
<tr>
<th>Bridge Width (ft)</th>
<th>Asphalt</th>
<th>Deck</th>
<th>HL-93 Traffic Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>1.4382</td>
<td>-0.5267</td>
<td>2.1573</td>
</tr>
<tr>
<td>60</td>
<td>1.9804</td>
<td>-0.5687</td>
<td>2.96</td>
</tr>
<tr>
<td>84</td>
<td>2.1459</td>
<td>-0.5713</td>
<td>3.2189</td>
</tr>
</tbody>
</table>

### 36-inch wide × 17-inch deep, span ratio: 50

<table>
<thead>
<tr>
<th>Bridge Width (ft)</th>
<th>Asphalt</th>
<th>Deck</th>
<th>HL-93 Traffic Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.2342</td>
<td>-1.4373</td>
<td>3.3513</td>
</tr>
<tr>
<td>60</td>
<td>4.1234</td>
<td>-1.8016</td>
<td>6.1851</td>
</tr>
<tr>
<td>84</td>
<td>5.257</td>
<td>-1.886</td>
<td>7.886</td>
</tr>
</tbody>
</table>

### 36-inch wide × 27-inch deep, span ratio: 30

<table>
<thead>
<tr>
<th>Bridge Width (ft)</th>
<th>Asphalt</th>
<th>Deck</th>
<th>HL-93 Traffic Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.1882</td>
<td>-1.2746</td>
<td>3.2823</td>
</tr>
<tr>
<td>60</td>
<td>3.885</td>
<td>-1.5749</td>
<td>5.8275</td>
</tr>
<tr>
<td>84</td>
<td>4.8107</td>
<td>-1.3915</td>
<td>7.216</td>
</tr>
<tr>
<td>Bridge Width (ft)</td>
<td>Asphalt Deck</td>
<td>HL-93 Traffic Loading</td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>--------------</td>
<td>-----------------------</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>2.1583</td>
<td>-0.9854</td>
<td>3.2375</td>
</tr>
<tr>
<td>60</td>
<td>3.8496</td>
<td>-1.3198</td>
<td>5.7744</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bridge Width (ft)</th>
<th>Asphalt Deck</th>
<th>HL-93 Traffic Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.673</td>
<td>-3.5895</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bridge Width (ft)</th>
<th>Asphalt Deck</th>
<th>HL-93 Traffic Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.6473</td>
<td>-2.3223</td>
</tr>
<tr>
<td>60</td>
<td>6.0788</td>
<td>-3.65338</td>
</tr>
<tr>
<td>84</td>
<td>9.2182</td>
<td>-4.2867</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bridge Width (ft)</th>
<th>Asphalt Deck</th>
<th>HL-93 Traffic Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.6514</td>
<td>-2.3631</td>
</tr>
<tr>
<td>60</td>
<td>5.6579</td>
<td>-3.3598</td>
</tr>
</tbody>
</table>
48-inch deep × 39-inch deep, span ratio: 30

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>5.6039</td>
<td>-2.8268</td>
<td>8.4058</td>
<td>-4.2403</td>
<td>35.7119</td>
<td>-22.9786</td>
</tr>
<tr>
<td>84</td>
<td>8.0297</td>
<td>-3.232</td>
<td>12.0445</td>
<td>-4.848</td>
<td>47.1328</td>
<td>-25.1328</td>
</tr>
</tbody>
</table>

36-inch wide × 39-inch deep, span ratio: 50

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>7.214</td>
<td>-7.6264</td>
<td>10.8211</td>
<td>-11.4395</td>
<td>42.5611</td>
<td>-46.4557</td>
</tr>
</tbody>
</table>

48-inch deep × 39-inch deep, span ratio: 50

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.8533</td>
<td>-4.7411</td>
<td>4.2799</td>
<td>-7.1117</td>
<td>11.6799</td>
<td>-25.4996</td>
</tr>
<tr>
<td>60</td>
<td>7.1771</td>
<td>-6.636</td>
<td>10.7591</td>
<td>-9.954</td>
<td>40.8528</td>
<td>-41.367</td>
</tr>
<tr>
<td>84</td>
<td>12.2001</td>
<td>-8.5013</td>
<td>18.3001</td>
<td>-12.7519</td>
<td>60.5306</td>
<td>-49.2617</td>
</tr>
</tbody>
</table>

48-inch deep × 48-inch deep, span ratio: 30

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>84</td>
<td>9.7509</td>
<td>-4.9615</td>
<td>13.7224</td>
<td>-7.0257</td>
<td>53.4867</td>
<td>-34.3909</td>
</tr>
</tbody>
</table>
APPENDIX D

2-Stage Post-tensioning Requirement Results
2-stage post-tensioning requirements are presented in the following tables. First stage post-tensioning was determined using the moment demand from the weight of the cast-in-place deck. Total post-tensioning requirement was determined using the moment demand from the live load analysis. Second stage post-tensioning is the difference between the first and total post-tensioning requirements. Post-tensioning was calculated using the working stress analysis presented in this thesis. All post-tensioning results are in units of kip/ft.

2-Stage Post-tensioning Requirements for 36-inch wide Box-Beams over Abutments

<table>
<thead>
<tr>
<th>Beam Depth (in)</th>
<th>17</th>
<th>27</th>
<th>39</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Stage P.T.</td>
<td>2nd Stage P.T.</td>
<td>Total P.T.</td>
<td>1st Stage P.T.</td>
</tr>
<tr>
<td>Bridge Width (ft)</td>
<td>1st Stage P.T.</td>
<td>2nd Stage P.T.</td>
<td>Total P.T.</td>
</tr>
<tr>
<td>30</td>
<td>36</td>
<td>3.3</td>
<td>20.3</td>
</tr>
<tr>
<td>60</td>
<td>3.6</td>
<td>25.8</td>
<td>29.4</td>
</tr>
<tr>
<td>84</td>
<td>3.6</td>
<td>26.3</td>
<td>29.9</td>
</tr>
<tr>
<td>50</td>
<td>36</td>
<td>9.1</td>
<td>30.9</td>
</tr>
<tr>
<td>60</td>
<td>11.4</td>
<td>59.8</td>
<td>71.2</td>
</tr>
<tr>
<td>84</td>
<td>11.9</td>
<td>62</td>
<td>73.9</td>
</tr>
</tbody>
</table>
### 2-Stage Post-tensioning Requirement for 48-inch wide Box-beams over Abutments

<table>
<thead>
<tr>
<th>Span Ratio</th>
<th>30</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Depth (in)</td>
<td>27</td>
<td>39</td>
</tr>
<tr>
<td>Bridge Width (ft)</td>
<td>1&lt;sup&gt;st&lt;/sup&gt; Stage P.T.</td>
<td>2&lt;sup&gt;nd&lt;/sup&gt; Stage P.T.</td>
</tr>
<tr>
<td>36</td>
<td>5.0</td>
<td>23.9</td>
</tr>
<tr>
<td>60</td>
<td>7.8</td>
<td>34.6</td>
</tr>
<tr>
<td>84</td>
<td>8.9</td>
<td>37.5</td>
</tr>
<tr>
<td>36</td>
<td>13.1</td>
<td>34.0</td>
</tr>
<tr>
<td>60</td>
<td>18.4</td>
<td>58.0</td>
</tr>
<tr>
<td>84</td>
<td>23.5</td>
<td>67.4</td>
</tr>
</tbody>
</table>

### 2-Stage Post-tensioning Requirement for 36-inch wide Box-beam at Mid-Span

<table>
<thead>
<tr>
<th>Span Ratio</th>
<th>30</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Depth (in)</td>
<td>17</td>
<td>27</td>
</tr>
<tr>
<td>Bridge Width (ft)</td>
<td>1&lt;sup&gt;st&lt;/sup&gt; Stage P.T.</td>
<td>2&lt;sup&gt;nd&lt;/sup&gt; Stage P.T.</td>
</tr>
<tr>
<td>36</td>
<td>8.7</td>
<td>15.0</td>
</tr>
<tr>
<td>60</td>
<td>15.5</td>
<td>65.3</td>
</tr>
<tr>
<td>84</td>
<td>19.2</td>
<td>76.6</td>
</tr>
<tr>
<td>36</td>
<td>10.7</td>
<td>16.2</td>
</tr>
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<td>80.1</td>
</tr>
<tr>
<td>84</td>
<td>37.1</td>
<td>104.8</td>
</tr>
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## 2-Stage Post-tensioning Requirement for 48-inch wide Box-beam at Mid-Span

<table>
<thead>
<tr>
<th>Span Ratio</th>
<th>Beam Depth (in)</th>
<th>27</th>
<th>39</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Bridge Width (ft)</td>
<td>1st Stage P.T.</td>
<td>2nd Stage P.T.</td>
<td>Total P.T.</td>
</tr>
<tr>
<td>36</td>
<td>8.6</td>
<td>17.4</td>
<td>26.0</td>
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<td>60</td>
<td>15.4</td>
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<td>19.6</td>
<td>71.3</td>
<td>90.9</td>
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</table>

<table>
<thead>
<tr>
<th>Span Ratio</th>
<th>Beam Depth (in)</th>
<th>27</th>
<th>39</th>
<th>48</th>
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</thead>
<tbody>
<tr>
<td>50</td>
<td>Bridge Width (ft)</td>
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<td>2nd Stage P.T.</td>
<td>Total P.T.</td>
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<td>19.0</td>
<td>29.6</td>
<td>7.9</td>
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<tr>
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<td>100.3</td>
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