Size Imperfections in One-Dimensional Periodic Optical Arrays

Yan Zhong

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SIZE IMPERFECTIONS IN ONE-DIMENSIONAL PERIODIC OPTICAL ARRAYS

by

Yan Zhong

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Faculty of The Graduate College
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SIZE IMPERFECTIONS IN ONE-DIMENSIONAL PERIODIC OPTICAL ARRAYS

Yan Zhong, M.A.
Western Michigan University, 1993

The concepts of band theory for electrons can also be employed to describe the behavior of electromagnetic waves propagating in periodic dielectric structures. These periodic structures can produce photonic band gaps in which the propagation of electromagnetic waves is strictly prohibited. The introduction of impurities in such system gives rise to donor and acceptor gap modes in electron system and to energy gap modes in photonic system.

In my thesis, the impurity modes in one-dimensional, periodic, dielectric system will be studied. These modes are introduced in the 1-d dielectric structure by altering the thickness of one slab in an otherwise periodic array. Narrow resonant modes are then found in the forbidden gaps. We study these modes as function of impurity slab thickness and dielectric constant. The calculations presented in this thesis are based on numerically studying the transmittance of impurity modes through one-dimensional, periodic arrays of dielectric slabs.
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Yan Zhong
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Size imperfections in one-dimensional periodic optical arrays

Zhong, Yan, M.A.

Western Michigan University, 1993
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CHAPTER I

INTRODUCTION

We have already known from Solid State Physics that, electrons in periodic structures are arranged in energy bands separated by forbidden regions in energy for which no wave-like electron orbits will exist. Such forbidden regions are called band gaps.\(^1\) Recently, It has been shown that the concepts of band theory for electrons can also be employed to describe the behavior of electromagnetic waves propagating in a periodic dielectric structure in any number of spatial dimensions (1, 2, or 3), and that these structures can be of technological importance.\(^2,3\) These optical periodic structures can produce optic band gaps, frequency regions in which no waves at any of the values of the wave vector in the Brillouin Zone can propagate. In this thesis, first, we will use slabs, characterized by a dielectric constant \(\varepsilon_b\), to form a one-dimensional periodic optical dielectric structure. When electromagnetic waves propagate through this structure, they will exhibit band gaps. Secondly, we will put some dielectric impurity into this structure, just like in producing doped semiconductor and electronic devices, we dope donor or acceptor state

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into the silicon materials. To achieve this theoretically, we simply replace one slab of the periodic array by an impurity slab of dielectric constant $\varepsilon_e$, but different thickness. We then observe the variation of the optical band, and the possible presence of an impurity level in the gap. We will see, (it is the same as in semiconductors,) this impurity will generate states in the band gaps. After this, we change the thickness of the impurity slabs, and study the relation between the frequency of the impurity mode and the impurity slab thickness. In the following introduction, we will: (a) discuss the technological interest in periodic dielectric structures; (b) discuss the interest in the simple impurity problem; and (c) give a brief review in words of the methods we shall use to solve our problem.

It has been recognized for many years that spontaneous emission between two excited states of an atom occurs as a result of interactions between the atom and space even in the absence of an externally applied emission field. Changes in the environment that an atom finds itself in can, however, modify the density of electromagnetic states available to the atom to emit radiation into and modify the rate of spontaneous emission. This is important as it is found that spontaneous emission plays a fundamental role in limiting the performance of semiconductor lasers, hetero-
junction bipolar transistors, and solar cells. Unwanted spontaneous emissions can act as a source of energy losses. Hence, inhibition of spontaneous emission can greatly enhance the performance of many electronic devices.

The starting research on the inhibition of spontaneous emission was done by Purcell as early as in 1946, when he found that the spontaneous emission rate for a two state atomic system is increased if the atom is surrounded by a cavity tuned to the atomic transition frequency, \( v \). If on the contrary, the cavity is mistuned and \( v \) lies below the fundamental frequency of the cavity, spontaneous emission is inhibited. This arises because at this time, light of frequency of the spontaneous emission has no electromagnetic modes in space to radiate into. In 1985, Gabrielse and Dehmelt reported the first observation of inhibited spontaneous emission. They presented convincing evidence that the radiative decay of the cyclotron motion of a single electron is significantly inhibited when the electron is in thermal equilibrium at nearly 4K and is located within a microwave cavity formed by the electrodes of a Penning trap. The Penning trap is used to confine the electron in a region of space which has a certain natural cavity oscillator frequency. They observed damping times as much as 5 times longer than the free space value and attributed the increased lifetime to cavity resonance.
effects from the surrounding electrons. Basically, this experiment had proved that spontaneous emission can be suppressed by employing structures which have natural oscillation frequencies. Subsequently, Hulet, Hilfer and Kleppner did another experiment of inhibited spontaneous emission of a atom in a Rydberg state. They used a cavity consisting of two plates that are separated by six disk-shaped quarter spacers to eliminate the vacuum modes at the transition frequency. They observed that the spontaneous emission "switched off" abruptly as the transition wavelength was varied across the cavity's cutoff wavelength and the lifetime was measured to increase by a factor of at least 20. All these methods are no less important in the solid state where spontaneous emission in the form of electron-hole radiation recombination plays a decisive role in energy loss of solid state devices. The common goal of these methods is to find a means of opening a optical band gap or forbidden region in the electromagnetic frequency spectrum, so as to prohibit the propagation of the electromagnetic waves away from a spontaneously emitting atom which is placed in the structure.

In 1987, Eli Yablonovitch gave a new way to open direct band gaps in the electromagnetic dispersion relation that can inhibit spontaneous emission. He pointed out that if a three-dimensional periodic dielectric structure has an
electromagnetic band gap which overlaps the electronic band edge, then spontaneous emission can be rigorously forbidden. We shall be interested in this new method in this thesis.

The performance of semiconductor lasers, heterojunction bipolar transistors, and solar cells are all limited by spontaneous emission. Each of these devices is limited by spontaneous emission in a characteristically different way which we shall indicate below:

1. In semiconductor lasers, the inhibition of spontaneous emission at frequencies other than the lasing frequency prevent energy losses into these other non-lasing frequencies.

2. In bipolar junction transistors, the current gain in the transistor can be enhanced if the electron-hole recombination rate is minimized. This can be accomplished by the suppression of radiation associated with this recombination.

3. In solar cells, the suppression of certain spontaneous emission mechanisms leads to higher efficiency in the conversion of solar energy by again suppressing electromagnetic radiations emitted by the solar cell.

These three problems 1 to 3, mentioned above, then were the motivation for Yablonovitch to study inhibition of spontaneous emission through his approach based on periodic
dielectric media.

It has been mentioned before, that a periodic dielectric structure will produce photonic band gaps to electromagnetic waves. These band gaps are bands of frequencies in which electromagnetic waves are forbidden to propagate. Similarly in electronic semiconductors, the three dimensional, periodic potential from the positive ions in the crystals create band gaps in the propagation of electron waves.

In electronic semiconductors, it has been observed also that, even if a small amount of atomic impurities exist in the semiconductor material, impurity modes can be created in the band gaps which will have a great effect on the electrical, magnetic and optical properties of semiconductors. In fact, many of the properties utilized in semiconductor devices are produced by suitably doping the material to some appropriate impurity contribution. For example, transistors are created by doping doper or acceptor impurities into the silicon material to control the conductivity type (n or p material).

In 1991, it was pointed by Eli Yablonovitch⁹ that the perfect three-dimensional translational symmetry of a dielectric structure can be lifted in either one of the two ways: (1) by adding extra dielectric material to one of the unit cells, this gives rise to donor modes which have their
origin at the bottom of the conduction band, just like a donor atom in semiconductor; or (2) by removing some dielectric material from one of the unit cells, this gives rise to acceptor modes which have their origin at the top of the valence band. Such defects resemble donor or acceptor atoms in semiconductors. In a recent experiment, Eli Yablonovitch and T. J. Gmitter have chosen a photonic crystal, which consist of a continuous three-dimensional web of dielectric material, made up of interconnecting ribs to study the impurity problem. The Wigner-Seitz unit cell of the photonic crystal is the standard fcc rhombic dodecahedron with "air atoms" created by drill holes, about 45 millimeters in diameter, centered on the top three faces. When this crystal was probed with microwave radiation, it prevented frequencies between 13 and 16 gigahertz. Donor defects were chosen to consist of a single dielectric sphere centered in an air atom. Likewise, by breaking one of the interconnecting ribs, it is easy to create acceptor modes. From the experiment, Eli Yablonovitch and T. J. Gmitter found that the frequencies of either donor or acceptor modes is a function of donor or acceptor defect volume. The larger the defect volume, the deeper the impurity modes.

In another series of experiment 1991, S.L. McCall, etc., have calculated and measured the properties of x-
band microwaves propagating in a 2d array of low-loss high-dielectric-constant cylinders. They found that by removing a rod from the system it gives rise to an impurity mode whose frequency falls in the second band gap. Dr. McGurn and Dr. Maradudin have also sketched out a theory of the defect modes that can occur in a periodic, two-dimensional, dielectric structures when a substitutional or interstitial line defect is introduced to them. They examined the particular case of defect states of E-polarization. A theory of defect modes of H-polarization can be constructed along similar lines.

Impurity modes can form high Q resonator states in the band gaps. So the disturbed periodic dielectric structure can be used to produce high-Q resonators. It is now possible to make high-Q electromagnetic cavities of about 1 cubic wavelength, for short wavelengths at which metallic cavities are useless. These new dielectric cavities can cover the range from mm to uv wavelengths.

In this thesis, we will theoretically construct a one-dimensional periodic dielectric structure, show the optical band gaps in this kind of structures, as well as study the impurity modes of the optical wave propagation by changing thickness of impurity slabs in the structure. The one-dimensional structure we study in this thesis illustrates all of the aspects of the 2-d and 3-d systems discussed
above.
CHAPTER II

ONE-DIMENSIONAL OPTICAL BAND STRUCTURE AND IMPURITY MODES IN THE STRUCTURE

In this chapter, first, we shall treat the optics of the simplest layered structure, which is a single homogeneous and isotropic layer sandwiched between two semi-infinite media. The bounding media we selected are simply vacuum with dielectric constant \( \varepsilon = 1 \). Specifically, we shall investigate transmittance, reflectance, as well as phase shifts associated with the reflection and transmission of electromagnetic radiation incident normal to the surface of the layer. Then, we duplicate this kind of structure periodically in space for 10-20 times, and study the optical band structure we get. We do this by studying the relation between transmittance and frequency of the optical wave. Finally, we will study the ways of creating impurity modes in this kind of structure. In particular, we do research on the impurity mode introduced by changing the thickness of one of the dielectric layers.

Optics of a Single Homogeneous and Isotropic Layer

Referring to Figure 1, we consider the reflection and transmission of electromagnetic radiation at a thin die-
lectric layer between two semi-infinite media. We assume that the dielectric constant of the thin dielectric layer is $\varepsilon_b$, the bounding media are just vacuum with $\varepsilon=1$, so that the whole structure for a slab of thickness $d$ can be described by:

\[
\begin{array}{ccc}
\text{Vacuum} & \varepsilon_b & \text{Vacuum} \\
E_1(x) & E_2(x) & E_3(x) \\
\end{array}
\]

I II III

Figure 1. The Structure of a Single Dielectric Slab.

\[
\varepsilon(x) = \begin{cases} 
1 & x \leq x_0 \\
\varepsilon_b & x_0 < x \leq x_0 + d \\
1 & x_0 + d < x 
\end{cases} 
\quad (2.1)
\]

For an electromagnetic wave propagating in the $-x$ direction, the wave function of the electric field can be represented by:
where $A$ is the amplitude of the wave, $k$ is the wave vector in the $x$ direction and $w$ is frequency of the wave. If we further assume that a plane wave is incident from the right, i.e. the direction of incidence is along $-x$ axis, a general solution of the wave $E(x)$ through a single isotropic layer can be written as:

$$E(x) = A e^{i(-kx - wt)}$$  \hfill (2.2)

$$E(x) = \begin{cases} 
A_1 e^{i(k_0 x - wt)} + A_2 e^{i(-k_0 x - wt)} & x \leq x_0 \\
A_2 e^{i(k_0 x - wt)} + A_3 e^{i(-k_0 x - wt)} & x_0 < x < x_0 + d \\
A_3 e^{i(k_0 x - wt)} + A_4 e^{i(-k_0 x - wt)} & x_0 + d < x 
\end{cases} \hfill (2.3)$$

Where $A_1 = 0$ in our single slab transmission problem.

The complex amplitudes $A_1$, $B_1$, $A_2$, $B_2$, $A_3$ and $B_3$ are constants, $k_0$ and $k$ are the $x$ components of the wave vectors in two different areas with dielectric constant $\varepsilon = 1$ and $\varepsilon = \varepsilon_b \neq 1$.

$$k_0 = \frac{\nu}{c} \hfill (2.4)$$

$$k = \frac{\nu}{c} \sqrt{\varepsilon_b} = k_0 \sqrt{\varepsilon_b} \hfill (2.5)$$

The constant $B_3$ is the amplitude of the incident wave. $A_3$ and $B_1$ are amplitudes of the reflected and transmitted waves.
respectively.

According to the boundary conditions at the film surface, both the electric field and its normal derivative at x=x₀ and x=x₀+d should be continuous, that is:

$$
\begin{align*}
A_1 e^{i(k_0x_0-wt)} + B_1 e^{i(-k_0x_0-wt)} &= A_2 e^{i(k_0x_0-wt)} + B_2 e^{i(-k_0x_0-wt)} \\
-ik_0 A_1 e^{i(k_0x_0-wt)} - ik_0 B_2 e^{i(-k_0x_0-wt)} &= ik_0 B_1 e^{i(k_0x_0-wt)} - ik_0 A_2 e^{i(-k_0x_0-wt)}
\end{align*}
$$

(2.6)

at x=x₀.

Solving for the relation between A₂, B₂ and A₁, B₁, we have:

$$
\begin{bmatrix}
A_2 \\ B_2
\end{bmatrix} = -\frac{1}{2k} \begin{bmatrix}
-(k+k_0) e^{-i(k-k_0)x_0} - (k-k_0) e^{-i(k+k_0)x_0} \\
-(k-k_0) e^{i(k+k_0)x_0} - (k+k_0) e^{i(k-k_0)x_0}
\end{bmatrix} \begin{bmatrix}
A_1 \\ B_1
\end{bmatrix}
$$

(2.7)

At x=x₀+d, our boundary conditions give:

$$
\begin{align*}
A_2 e^{i(k_0(x_0+d)-wt)} + B_2 e^{i(-k_0(x_0+d)-wt)} &= A_1 e^{i(k_0x_0-d)-wt} + B_1 e^{i(-k_0x_0-d)-wt} \\
-ik_0 A_2 e^{i(k_0(x_0+d)-wt)} - ik_0 B_1 e^{i(-k_0(x_0+d)-wt)} &= ik_0 B_2 e^{i(k_0(x_0+d)-wt)} - ik_0 A_1 e^{i(-k_0(x_0+d)-wt)}
\end{align*}
$$

(2.8)

Similarly, we have:

$$
\begin{bmatrix}
A_3 \\ B_3
\end{bmatrix} = -\frac{1}{2k_0} \begin{bmatrix}
-(k+k_0) e^{i(k-k_0)(x_0+d)} - (k-k_0) e^{i(k+k_0)(x_0+d)} \\
-(k-k_0) e^{-i(k+k_0)(x_0+d)} - (k+k_0) e^{-i(k-k_0)(x_0+d)}
\end{bmatrix} \begin{bmatrix}
A_2 \\ B_2
\end{bmatrix}
$$

(2.9)

Combining equation (2.7), (2.9), we find the relation between A₃, B₃, and A₁, B₁:
\[
\begin{bmatrix}
A_3 \\
B_3 
\end{bmatrix} = \frac{1}{4kk_0} \begin{bmatrix}
(k+k_0)^2 e^{i(k-k_0)d} - (k-k_0)^2 e^{-i(k+k_0)d} \\
-(k^2-k_0^2) e^{i[(k+k_0)d^2+k_0]} - (k^2-k_0^2) e^{-i[(k-k_0)d^2+k_0]} 
\end{bmatrix}
\begin{bmatrix}
A_1 \\
B_1 
\end{bmatrix}
\]

Assuming that the phase of electrical wave is \( \Phi \),
\[
\Phi = kkd = \sqrt{\varepsilon B}k_0d = \sqrt{\varepsilon B}D
\]

\[
D = k_0d = \frac{\nu}{c} \]

then the relation between \( A_3 \), \( B_3 \), \( A_1 \) and \( B_1 \) can be rewritten as:
\[
\begin{bmatrix}
A_3 \\
B_3 
\end{bmatrix} = \frac{1}{4\sqrt{\varepsilon B}(1-\varepsilon_B)} \begin{bmatrix}
(\sqrt{\varepsilon_B}+1)^2 e^{i(\sqrt{\varepsilon_B}-1)D} - (1-\sqrt{\varepsilon_B})^2 e^{-i(\sqrt{\varepsilon_B}+1)D} \\
(\varepsilon_B-1) e^{i(2\varepsilon_B(1-\sqrt{\varepsilon_B})D}) - (\varepsilon_B-1) e^{i[2\varepsilon_B(1-\sqrt{\varepsilon_B})D]} 
\end{bmatrix}
\begin{bmatrix}
A_1 \\
B_1 
\end{bmatrix}
\]

The transmittance is defined as the fraction of energy transmitted of the slab from the dielectric structure and is given by:
Because we have already assumed that the plane wave was incident from area \( III \) \((x>x_0+d)\), it should pass through area \( II \) \((x_0<x<x_0+d)\) to arrive area \( I \) \((x<x_0)\). There would be no reflected wave vector existing in area \( I \), only the transmitted wave is present. Therefore, we can set \( A_1=0 \), and if we select \( B_1=1 \), then the transmittance becomes:

\[
T = \left| \frac{B_1}{E_1} \right|^2
\]

(2.15)

By changing the frequency \( \omega \) of the incident wave, we get different \( D \), obtaining the relation between transmittance \( T \) and \( D \) for a single dielectric layer. The calculations of matrix and transmittance for different \( D \) are completed on VAX machine with FORTRAN scientific programs. Figure 2 gives a particular example with the dielectric constant of the slab \( \varepsilon_b=6 \). We can see that, for a single dielectric layer, the transmittance \( T \) is a continuous function of \( D \).

Optics of One-dimensional Periodic Dielectric Structure

The one-dimensional periodic dielectric structure is
Figure 2. Plot of Transmittance $T$ Versus $\omega d/c$ for a Single Dielectric Slab When $\varepsilon_r=6.0$.

formed by periodic array of slabs. The slabs can be treated as a series of single slab transmissions. (See Figure 3)

As in the single slab problem, a plane wave is incident from the right on the periodic structure along the $-x$ axis. Passing through the periodic dielectric structure, it finally arrives in area I. We still select $B_z=1$, then the transmittance becomes:

$$T = \left| \frac{1}{B_z} \right|^2$$

(2.16)
Figure 3. One-Dimensional Periodic Dielectric Structure Formed by Periodic Array of Slabs

To achieve this kind of structure theoretically, in my FORTRAN computer program, I simply use a Do loop to let one slab dielectric structure shown in Figure 1 be repeated for n times. Doing this, a periodic dielectric structure shown in Figure 3 is obtained. The transmittance is solved numerically on computer by calculating a matrix product using the product of single slab matrices. From the data we get by changing the frequency of the incident wave, a T versus D plot can again be obtained. I have calculated the transmittance versus D relation for dielectric constant $\varepsilon_b = 4.0, 5.0, 6.0, 8.0, 10.0$. Seen from the graphs we have got (Figure 4-8), it is very clear that T is no longer a continuous function of w. Instead, band gaps are bound to exist between particular frequency ranges. This means that the electromagnetic waves with frequencies in these ranges (band gaps) can not propagate through the one-dimensional periodic dielectric structure being considered.
In Figure 4, which has the dielectric constant of the slabs equal to 4.0, we find the lowest two band gap edges are opened in frequency which ranges $0.84 < \omega d/c < 1.23$ and $1.91 < \omega d/c < 2.30$. Figure 5 shows the band structures with dielectric constant $\varepsilon_b = 5.0$, and the lowest two band gaps are opened in frequencies with $0.75 < \omega d/c < 1.15$ and $1.73 < \omega d/c < 2.18$. For $\varepsilon_b = 6.0$, 8.0, 10.0, the gaps are opened at $0.69 < \omega d/c < 1.08$ and $1.59 < \omega d/c < 2.07$ (Figure 6), $0.60 < \omega d/c < 0.98$.
Figure 5. Plot of Transmittance $T$ Versus $\omega d/c$ for Periodic Dielectric Slabs When $\varepsilon_r=5.0$.

and $1.39<\omega d/c<1.90$ (Figure 7), $0.54<\omega d/c<0.90$ and $1.26<\omega d/c<1.76$ (Figure 8) respectively.

Impurity Modes in One-dimensional Periodic Optical Band Structure

When the perfect periodic structure of the one-dimensional dielectric array of slabs is disrupted, impurity modes can exist in the band gaps. The impurity mode in the structures being considered here can be produced in
Figure 6. Plot of Transmittance $T$ Versus $\omega d/c$ for Periodic Dielectric Slabs When $\varepsilon_b=6.0$.

two ways:

1. By introducing another kind of dielectric material into the structure. For example, in the one dimensional periodic dielectric structure with $\varepsilon_b=4.0$, after every $n$ slabs with $\varepsilon=\varepsilon_b=4.0$, we replace one slab by an impurity slab with $\varepsilon=\varepsilon_i=2.0$, and calculate the transmittance (See Figure 9). Since for $n>5$, the impurity mode does not change with increasing $n$, this indicates that this is the correct fre-
Figure 7. Plot of Transmittance $T$ Versus $\omega d/c$ for Periodic Dielectric Slabs When $\varepsilon_r = 8.0$.

Frequency for a single impurity. In our case, we select $n=6$. From Figure 9, we see that, most of the spectrum is unaffected except that two sharp impurity peaks exist within the first two forbidden gaps respectively. They are called impurity modes. In the first gap, the impurity peak is located at $\omega d/c = 0.89$. In the second gap, the impurity peak is located at $\omega d/c = 2.03$. This means that the electrical waves with frequencies that make $\omega d/c = 0.89$ and 2.03 are
bound to the dielectric impurities in this structure.

2. By changing the thickness of one of the dielectric slab periodically. In my research, with the background dielectric constant $\varepsilon_b=4.0, 5.0, 6.0, 8.0, 10.0$ respectively, after every 6 slabs, I change the thickness of one of the slab to $x_d$, where $0.0\leq x\leq 1.0$, and again study the relation between $T$ and $D$. (See Figure 10)

Figure 11 and Figure 12 show the situations for $\varepsilon_b=4.0$, 

Figure 8. Plot of Transmittance $T$ Versus $wd/c$ for Periodic Dielectric Slabs When $\varepsilon_b=10.0$. 

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Figure 9. Plot of Transmittance $T$ Versus $\omega d/c$ for Periodic Dielectric Slabs With Impurity in it. Here, After Every 6 Slabs With $\varepsilon_b=4.0$, We Replace One Slab With $\varepsilon_b=2.0$.

$x=0.3$, and $\varepsilon_b=6.0$, $x=0.3$ respectively. In Figure 11, in the first forbidden gap, the impurity mode is created at the location where $\omega d/c=0.89$, at which the transmittance is 0.23. In the second gap, the impurity mode is created at the position where $\omega d/c=2.13$, at which $T=0.52$. In the case described in Figure 12, there is an impurity mode at $\omega d/c=0.76$ in the first band gap. Its transmittance $T$ is 0.998.
This means that the electromagnetic wave with this particular frequency is transmitted through the structure almost with no energy loss, and another impurity mode exists at $wd/c=1.96$ in the second gap, at which $T$ is 0.0018.

By changing the thickness of the disordered slab for $\varepsilon_\infty=4.0, 5.0, 6.0, 8.0,$ and $10.0$ cases, (i.e., increasing $x$ from 0.0 to 1.0), we obtain the graphs Figure 13 to Figure 17, these plots show the impurity frequency as a function of the thickness of the disordered slab. In fact, they display the relations between $wd/c$ and $x$. When the thickness of the disordered slab is relatively small, the impurity level is deep (near upper edge). A thicker disordered slab will result in a shallow impurity level (near the lower gap.
edge). If the thickness of the impurity slab is larger than a threshold value, the impurity levels fall within the continuum of levels below the top of the lower permitting bands. For $\varepsilon_b=4.0$, the threshold thickness is 0.55d for impurity in the first band gap, and about 0.90d for that in the second gap (See Figure 13). For $\varepsilon_b=5.0$, the threshold thick-
Figure 12. Plot of Transmittance $T$ Versus $\omega d/c$ for Periodic Dielectric Slabs With Size Imperfection. After Every 6 Slabs With Thickness $d$, Replace One Slab With Thickness $x d$. Here, $\varepsilon_b = 6.0$, $x = 0.3$.

Transmittance is about $0.61d$ for impurity in the first gap, and $0.91d$ for that in the second gap. (See Figure 14). For $\varepsilon_b = 6.0, 8.0$, and $10.0$ cases, the threshold thickness are $0.63d$, $0.75d$, and $0.75d$ in the first gap, and $0.91d$, $0.97d$, and $0.97d$ in the second gap respectively (See Figure 15–Figure 17). Table 1 lists the gap edges, the highest $D (= wd/c)$ of
Figure 13. Plot of Impurity Mode $w_d/c$ Versus Impurity Thickness Coefficient $x$ When $\varepsilon_b=4.0$.

the impurity modes, and the threshold thickness for the first two forbidden gaps.
Figure 14. Plot of Impurity Mode $\frac{w_d}{c}$ Versus Impurity Thickness Coefficient $x$ When $\varepsilon_r=5.0$. 

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Figure 15. Plot of Impurity Mode $\omega d/c$ Versus Impurity Thickness Coefficient $x$ When $a_b=6.0$. 

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Figure 16. Plot of Impurity Mode \( \frac{w}{d} \) Versus Impurity Thickness Coefficient \( x \) When \( \varepsilon_b=8.0 \).
Figure 17. Plot of Impurity Mode $\frac{w_d}{c}$ Versus Impurity Thickness Coefficient $x$ When $\varepsilon_b = 10.0$. 

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Table 1

Items Related to the First Two Forbidden Gaps in the One-dimensional Optical Band Structure With Size Imperfections

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>1st Band Gap</th>
<th>2nd Band Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gap Edge</td>
<td>$D_h$</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>4.0</td>
<td>0.84</td>
<td>1.23</td>
</tr>
<tr>
<td>5.0</td>
<td>0.75</td>
<td>1.15</td>
</tr>
<tr>
<td>6.0</td>
<td>0.69</td>
<td>1.08</td>
</tr>
<tr>
<td>8.0</td>
<td>0.60</td>
<td>0.98</td>
</tr>
<tr>
<td>10.0</td>
<td>0.54</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Annotation:

$D_h$: Highest $D$ of the impurity mode.

$X_T$: Threshold value of $x$.  

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CHAPTER III

CONCLUSION

According to the band theory in solid state physics, electrons in periodic crystals are arranged in energy bands separated by band gaps. Similarly, this band theory can also be employed to explain the propagation of electromagnetic waves in periodic dielectric structures in any number of spatial dimensions. Electromagnetic waves with frequencies falling in the photonic band gaps are totally absent from the system. Similarly, as with donor or acceptor impurities in n-type or p-type semiconductor materials, impurity modes can also be introduced into the optical band structures by replacing some of the dielectric material with impurity material of a different dielectric constant or size.

In Chapter II, we have discussed the propagation of electromagnetic waves through one single dielectric slab, one-dimensional periodic structure consisting of an array of parallel dielectric slabs, and also through the periodical system with a single impurity. The transmission function versus the frequencies of electromagnetic waves was calculated in all these cases. From these graphs, we see that, for the periodic arrangement of the slabs, the
transmittance is no longer a continuous function of frequency, as in the single slab case, but instead, there are band gaps in the dispersion curves. The lower-upper edges of the first two band gaps are presented in frequencies $0.84 < \omega d/c < 1.23$ and $1.91 < \omega d/c < 2.30$ for slabs with dielectric constant $\varepsilon_b = 4.0$, $0.75 < \omega d/c < 1.15$ and $1.73 < \omega d/c < 2.18$ for slabs with dielectric constant $\varepsilon_b = 5.0$. $0.69 < \omega d/c < 1.08$ and $1.59 < \omega d/c < 2.07$, $0.60 < \omega d/c < 0.98$ and $1.39 < \omega d/c < 1.90$, $0.54 < \omega d/c < 0.90$ and $1.26 < \omega d/c < 1.76$ for slabs with dielectric constant $\varepsilon_b = 6.0, 8.0, 10.0$ respectively. In addition, the introduction of size impurities in these periodic structures is found to introduce additional impurity modes in the band gaps.

By changing the thickness of one dielectric slab in the periodic 1-d system, narrow impurity modes are introduced in the band gaps. After every 6 slabs with thickness equal to $d$, we change one slab's thickness to $xd$ for $\varepsilon_b = 4.0$, $5.0$, $6.0$, $8.0$, and $10.0$, and study the impurity mode presented in the gaps as function of $x$. For each case, the frequency at which the impurity mode occurs is found to decrease as $x$ increase from 0.0 to 1.0. And also, there exists a threshold $x_T$ such that when $x$ is larger than $x_T$, the impurity mode will disappear into the lower continuum spectrum of the pass band.

The properties of optical band structure and the
localized impurity mode in band gaps can be very useful in the improvement of many electronic devices. By opening a photonic band gap or forbidden region in certain electromagnetic frequency spectrum, the spontaneous emission of atoms will be effectively inhibited if the frequency of the emission happens to fall into this range. By creating some kind of impurity disorder into the perfect periodic dielectric structure, impurity modes can be introduced into the optical band gap at some particular frequencies. This kind of dielectric structure can be used as narrow band filters, isolators, antennas, or as high quality resonators. A number of application examples are already illustrated in a recent edition of Scientific American.\textsuperscript{11}

All the impurity modes studied in our work are linear impurities. The future work in this area can be done in the area of electromagnetic wave propagation in one-dimensional periodic dielectric structures with non-linear impurities. In such material the dielectric constant of the impurity slab is a function of the intensity of the electromagnetic wave being propagating.
REFERENCES


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