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Two High School Teachers’ Initial Use of Geometers Sketchpad: Issues of Implementation

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TWO HIGH SCHOOL TEACHERS’ INITIAL USE OF GEOMETER’S SKETCHPAD: ISSUES OF IMPLEMENTATION

by

Kathryn G. Shafer

A Dissertation
Submitted to the Faculty of The Graduate College in partial fulfillment of the requirements for the Degree of Doctor of Philosophy Department of Mathematics

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Kathryn G. Shafer
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CHAPTER I: INTRODUCTION

Statement of the Problem

The initial reform effort from the National Council of the Teachers of Mathematics (NCTM) expressed in the Standards (NCTM, 1989) document advocated increased emphasis on geometry at all levels. At the high school level, teachers are encouraged to "conduct increasingly independent exploration, which allows the student to develop a deeper understanding of important geometry ideas such as transformations and symmetry" (p. 309). The Standards (NCTM, 1989) emphasized that exploration is crucial to the activity of discovering patterns and conjectures.

The high school geometry curriculum is defined, for the purpose of this document, as the study of geometric figures with both a pure and applied approach. Main themes are similarity and congruence, which provide the context for the study of proof, as well as work with two- and three-dimensional shapes. Proof is approached from the standpoint of a logical argument that may be represented in a two-column or in a paragraph format. Research on high school geometry identifies areas of concern as well as a new direction for geometry instruction that addresses these concerns. Studies on high school geometry students find that both the curriculum and the teacher's expectations related to it are at a much higher level than students' abilities in the areas of level of thinking and vocabulary. The van Hiele research (Fuys, Geddes, Lovett, & Tischler, 1988) developed levels of reasoning, ranging from visual recognition at level 1 to an understanding of different axiomatic systems at level 5. Typically, students enter a high school geometry course at a van Hiele level of 1 or 2 while the curriculum assumes students are at level 3 which involves the logical ordering of properties of figures and the understanding of relationships between figures (Burger & Shaughnessy, 1986; Senk, 1985). A characteristic of teachers and students operating at different van Hiele levels is a gap between the teacher's formal
geometric language and the student's informal language (Burger & Shaughnessy, 1986; Chazan, 1993; Fuys et. al., 1988) with an even more pronounced difference found for second language learners (de Villiers, 1998). Another concern is that typically students define learning mathematics as the memorization of mathematical rules and procedures. Students who have been successful in mathematics as a result of their rote memorization skills are often at a loss when asked to think independently. One approach to resolving the discrepancies of language, level of thinking, and student's learning style is via instruction that encourages exploration and conjecture through the use of a dynamic geometry system (Chazan, 1993; Chazan & Yerushalmy, 1998; Choi-koh, 1999; de Villiers, 1998).

In The Professional Standards for School Mathematics (PSSM) (NCTM, 2000), the NCTM outlined the role and benefits of using technology in mathematics education. The technology principle from PSSM states, “Electronic technologies, computers, and calculators, are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they communicate efficiently and accurately” (p. 24). In addition, the PSSM specifically references the benefits of the dynamic geometry environment for student learning of geometric concepts. Within this type of program, the student can drag a node (vertex) around and the shape on the screen changes, giving the student immediate feedback regarding potentially invariant properties and specific measurements (provided the figure was constructed correctly – see literature review for a full discussion). Technology also provides a focal point as students discuss with one another and with their teacher the objects on the screen and the effects of the various dynamic transformations that the technology allows. Although technology use does not guarantee a change in the teacher's geometry instruction, Chazan and Yerushalmy (1998) maintain that it best supports the type of student thinking called for by the NCTM. Ideally, insight gained from the use of the dynamic geometry system Geometer’s Sketchpad (Jackiw, 1991) will aid the teacher in assessment and future instructional planning with respect to the geometry curriculum.
A verbalized, but frequently unrealized, goal of many teachers is teaching content through student exploration (Senger, 1999). Past instructional practice and issues of teacher efficacy are among the many roadblocks for teachers who desire to change their instruction from a procedural focus to one of exploration (Smith, 1996). In some cases, a lack of content knowledge has been cited as a cause for a reliance on direct instruction (Ball & Bass, 2000; Shulman, 1987). Swafford, Jones, and Thornton (1997) indicate that increasing teacher knowledge is one way to improve instructional practice in geometry. “The common belief is that the more the teacher knows about the subject and the way students learn, the more effective that individual will be in nurturing mathematical understanding” (p. 407). This common belief is substantiated by the work of Ma (1999). In addition to a high level of content knowledge, effective instruction requires teacher knowledge and skill in many dimensions including selecting tasks, creating and maintaining the learning environment, employing a variety of instructional strategies, and facilitating discourse in the classroom. Effective instruction with technology places new demands on teachers. This study identifies the challenge that a change in the direction called for by NCTM creates for teachers.

A particular challenge for high school teachers is to integrate technology in their teaching as a way of encouraging students to explore and develop conjectures while continuing to help them understand the need for proof, or counter examples of conjectures (PSSM, p. 310).

Teachers have the additional burden of needing to know how the chosen technology works and knowing how to use the technology as a tool for instruction.

As is the case with any tool, the level and efficacy of the use of technology to support teaching is dependent on numerous factors. These factors may include instructional time, state and local standards, the use of standardized tests, institutional practice (tradition), beliefs of the teacher, cultural expectations (beliefs of fellow teachers and other school personnel), availability of hardware or software, and knowledge regarding specific technology tools and mathematics content. Thus, for many educators, using technology effectively often involves a difficult change...
in a coordination of the goals of instruction, the teacher’s role in the classroom, classroom management and planning, and training in technology and its use with students. The confluence of these factors, many beyond the realm of teacher control, can conspire to inhibit the use of technology in the teaching of high school geometry.

High school geometry teachers gain certification to teach through the completion of undergraduate coursework. Many teachers graduated long before much of the current technology was available while others may have attended an undergraduate or graduate program that did not include the use of technology. In either case, these practicing teachers need to know how to use technology in order to implement it in their teaching practice.

General factors that foster technology acceptance and effective use by teachers include adequate information, leadership, and a sense of obligation for its use (PSSM). Beyond this, a key provision is support from a technology specialist or colleague who is experienced with technology and can help the teacher learn the program and release time to experiment with the technology to plan for instruction (Saye, 1998). In a study specific to the implementation issue of Geometer’s Sketchpad (GSP), McDougall (1997) states, “The in-service teachers should be given a mentor or coach to help them accept a temporary loss of control in the classroom as part of the change process.” This study provided the in-service teacher with a mentor (the researcher) to aid in the integration of GPS activities into their geometry curriculum. The researcher, at the same time, investigated how the teachers accepted and integrated the program into their instruction. This prompts the following research questions:

1. How do teachers implement Geometer’s Sketchpad in their planning and teaching?
2. How does the role of a mentor facilitate the professional development of the classroom teacher with the implementation of Geometer’s Sketchpad?
Purpose of the Study

This study examines two issues that are central to mathematics reform. First, as previously stated, the goal of the current reform movement is to shift teachers from the role of dispensers of knowledge to that of facilitators (Chazan, 1993; Chazan & Yerushalmy, 1998; Cuoco, Goldenberg, & Mark, 1996; de Villiers, 1998; Glass & Deckert, 2001; Hannafin, Burruss, & Little, 2001; McDougall, 1997). This study followed two teachers as they implemented GSP and documents how they integrate the program into their classroom instruction (e.g., do they use direct teaching or assume the role of facilitator while using GSP). The study examined changes that took place in the areas of goals of instruction, method of instruction, role of the teacher, role of the student, and the role and source of curricular materials. Second, teacher educators are looking for vehicles that will promote this pedagogical shift (Lampert & Ball, 1999). While many in-service teachers attend short-term classes and workshops, Lampert and Ball suggest these settings rarely generate the type of change in the role of the teacher that is desired by the NCTM.

In-service workshops are usually required by school districts and not highly valued as sources of knowledge about improving instruction … Prospective and practicing teachers learn about constructivist theories of learning, communities of learners, and authentic tasks, but often the courses and workshops in which they hear about these ideas are taught in ways that do not make use of the very same ideas. With little or no firsthand experience with learning of the kind that reformers advocate, neither beginning nor experienced teachers have adequate images of what these ideas mean, what it might mean to draw on them in practice, and the complications they raise for teaching and learning (p. 39).

Therefore, a second goal of this study was to examine the effect the professional development model, which employs the use of a mentor, had on these two teacher’s instructional decisions regarding the use of GSP.

The use of a mentor when implementing GSP is recommended based on the results of the dissertation work of McDougall (1997). In Mathematics teachers’ needs in dynamic geometric computer environments: In search of control, McDougall identifies three control issues teachers have to face:
• Management control - student discipline,
• Personal control – expectations of the student, and assessment issues, and
• Professional control – the teacher no longer has all the answers.

While McDougall found students thrive in the dynamic geometry environment, he recommends teachers be given a mentor to help with these control issues and the process of change. Fullan (2001) describes this as the "primacy of personal contact" where teachers need to participate in one-to-one and group opportunities, to receive and give help, and more simply to converse about the meaning of change" (emphasis in original, p. 124).

The work of Hannifin, Burrus, and Little (2001) also supports this study since “few investigations have examined the teachers’ or students’ attitudes about using dynamic geometry tools in environments less structured than their normal classroom” (p. 135). In their study, the teacher used GSP as a form of practice, which severely limits the potential of the program. With the use of a mentor (in this study) it was anticipated that the teachers would implement GSP for exploration and discovery, thus validating the role of a mentor as a form of professional development.

Framework of the Study

This study has three interrelated goals which attempt to answer the research questions: 1) introduce two high school geometry teachers to the Geometer’s Sketchpad program; 2) mentor them on how to implement it into their instruction; and, 3) at the same time, document how they integrate GSP into their instructional practice. The design of the study is based upon current research models of professional development, research on teacher change, and finally research that links the use of a dynamic geometry environment to an increase in student achievement in geometry that is verified by the van Hiele model. Studies on instruction and student learning may focus on the interaction between the teacher and her students, which can be characterized as one interaction level. This study resides on the level of researcher to teacher. Since student learning is not the direct focus of this study, the researcher must be sensitive to the teacher’s goals for
student learning by respecting her position as the instructional decision maker. These levels are linked through the professional development process. Thus, the theoretical framework for the study incorporates the following to support these different aspects:

- Professional Development Research
- Cognitively Guided Instruction Model for Research and Curriculum Development
- Teacher Knowledge and Beliefs Research
- Van Hiele Theory
- Teacher Change Model

Currently, teacher professional development (TPD) appears to have a relatively weak effect on improving or changing a teacher’s instructional practice. While policy mandates that set academic standards, govern the use of reform curricula, or require high-stakes testing may serve to instigate teacher change, change is primarily dependent on the teacher. “Curricular change, like all important changes in education, ultimately relies on teacher understanding, skill, and will [beliefs]” (Sykes, 1999). To elaborate on these three points, teacher understanding refers to teacher content knowledge which many researchers cite as crucial to curricular change (Ball & Bass, 2000; Cooney, 1994; Fennema & Franke, 1992; Munby, Russell, & Martin, 2001; Shulman, 1987; Swafford et al., 1997). Teacher skill refers to the way in which teachers craft their lessons and is a distinctive trait of the expert (Ball & Bass, 2000; Ball & Cohen, 1999; Ball, Lubienski, & Mewborn, 2001; Lampert & Ball, 1999). Finally, teacher beliefs refer to the teacher’s beliefs regarding instruction and learning, but more importantly what they believe mathematics to be. For example, teachers who believe mathematics to be the study of procedures and facts will likely stress the memorization of procedures and facts in their instruction. The literature is replete with research regarding beliefs and how they shape what takes place in a classroom (Artzt & Armour-Thomas, 2002; Fullan, 2001; Saye, 1998; Senger, 1999; Smith, 1996; Thompson, 1984).

Carpenter and Fennema (1991) and their work with Cognitively Guided Instruction (CGI) portray the links between teacher knowledge, teacher beliefs, and instructional decision-making (see Figure 1).
In addition to the issues of teacher knowledge and teacher beliefs, the effective TPD must be based on current learning theories with a goal of bringing about higher standards of performance for students (Hawley & Valli, 1999). Hawley and Valli developed guidelines for a new generation of professional development based on a review of current TPD efforts. These guidelines were chosen as a framework for this study due to their comprehensive nature. Further discussion of these principles and how they specifically support this study is found in the literature review.

Finally, the study draws on the current information regarding teacher change (Artzt & Armour-Thomas, 2002; Etchberger & Shaw, 1992; Fullan, 2001; McDougall, 1997; Munby et al., 2001; Saye, 1998; Senger, 1999; Shaw & Jakubowski, 1991; Smith, 1996; Swafford et al., 1997; Tobin & Dawson, 1991; Wilson & Goldenberg, 1998). Senger (1999) identifies stages that elementary mathematics teachers go through as they progress from experimental practice to principled practice while they adopt a constructivist teaching pedagogy based on recursive cycles of experiment and reflection (Figure 2). This model is used as a framework for the discussion of teacher change since it affords the researcher a comprehensive breakdown of the steps involved in

Figure 1. CGI Model (Carpenter & Fennema, 1991, p. 8)
change specific to the setting of mathematics. A discussion of teacher change and a comparison of different change models are found in the literature review.

Figure 2. Recursive Change Model (Senger, 1999, p. 211)
CHAPTER II: LITERATURE REVIEW

This chapter brings together research on a number of different topics with major divisions being: using technology to teach geometry; professional development; teacher knowledge, beliefs, and change; and mentoring. The literature review also examined articles and studies on the use of dynamic geometry systems. This review explores how the use of a dynamic geometry system impacts the curriculum, student achievement, proof, and the role of the teacher. Since this study involved working with two teachers in an in-service setting, the literature review also includes studies of leading models of professional development. The research on teacher knowledge, teacher beliefs, and teacher change provides a background for a discussion of how teachers in this study plan for and implement a new piece of technology into their existing teaching practice. The final section of the literature review examines the various uses of mentoring in educational settings.

Geometry

In 1989 the NCTM published the Standards that called for significant changes in the way mathematics is taught. In teaching geometry, the Standards called for decreased emphasis on the presentation of geometry as a complete deductive system and a decrease in two-column proofs. The Standards favored an increase in open exploration and conjecturing and increased emphasis on topics in transformational geometry. In a continuation of the call for change in the geometry curriculum, the PSSM (NCTM, 2000) recognized the impact current dynamic geometry programs could have as an addition to the standard geometry tools of compass and protractor. Dynamic geometric systems are identified interchangeably as either a dynamic geometry environment (DGE) or a dynamic geometric system (DGS).
Dynamic Geometry Systems

The two major DGS packages currently used in the United States are The Geometer’s Sketchpad (Jackiw, 1991) and Cabri Geometry (Cabri Geometry II, 1996). Use of such software provides an interactive geometry environment that allows the user to alter geometric objects by moving components such as vertices and edges to different locations on the screen. When the original objects on the screen are modified, the results of the constructions and any measurements on the screen are immediately updated.

There are two options for creating a geometric object. One is to draw it freehand. For example, a student draws a “square” on the computer screen by creating points and connecting them with segments. When they “drag” one of the vertices of the square, the drawn figure becomes distorted and no longer appears to have the features of a square.

The other format is to construct the object based on its defining properties. If the student constructs a square using the construction menu tools, the results of “dragging” a vertex or an edge will result in moving the entire square on the screen or, depending on the procedure used, will enlarge, shrink, or rotate the square. As long as the square is constructed the defining properties of the square will remain invariant. This supports the realization that the construction process can be used to discover many properties which hold for that particular object.

The next section will outline the van Hiele theory. This theory provides a framework for the discussion of student achievement and treatment of proof in the high school curriculum.

The van Hiele Levels

The van Hiele theory originated as a combination of the doctoral dissertation of Diana van Hiele-Geldof and the work of her husband, Pierre van Hiele, at the University of Utrecht, Netherlands, in 1957. P. van Hiele looked at the difficulties students have in geometry education while D. van Hiele’s dissertation documented a teaching experiment that used a prescriptive approach to teaching geometry. Her approach focused on ordering the content and learning
activities of the students based on the level of thinking of the student. These levels of thinking appeared to be fairly discrete levels of understanding that could be quantified and qualified into five distinct categories or what are commonly called the five van Hiele levels. The activities followed a format that moved the student from their everyday language, through exploration, to the more formal language of geometry. de Villiers (1998) highlights the van Hiele levels and the application of this theory to the teaching of geometry within a DGS. The first four levels are designated as follows:

Level 1: Recognition
Students will visually recognize figures by their global appearance. They recognize triangles, squares, parallelograms, and so forth by their shape, but they do not explicitly identify the properties of these figures.

Level 2: Analysis
Students start analyzing the properties of figures and learn the appropriate technical terminology for describing them, but they do not interrelate figures or properties of figures.

Level 3: Ordering
Students logically order the properties of figures by short chains of deductions and understand the interrelationships between figures (for example, class inclusions).

Level 4: Deduction
Students start developing longer sequences of statements and begin to understand the significance of deduction, the role of axioms, theorems, and proof (p. 11).

According to de Villiers, the van Hiele theory can be used to help explain why the traditional geometry curriculum fails. Often, the curriculum that is presented is found to be at a higher level than that of the student (Senk, 1985). In many cases, the student cannot understand the teacher, nor can the teacher understand why the students have so much trouble. Fuys, Geddes, Lovett, and Tischler (1988) wrote, in the NCTM monograph on the van Hiele theory, that a high school text may assume a van Hiele level of 3, whereas Senk (1985) found that the beginning van Hiele level of most high school geometry students is a level 1 or 2. She also confirmed that the high school geometry curriculum is at a level 3 and that there is a strong correlation between a van Hiele level of at least a 3 and success in writing traditional two column proofs.
The Geometry Curriculum

Traditional geometry courses tend to fall into three categories: attempts at replicas of a Euclidean treatment of geometry, complete with two column proofs; a Euclidean approach without proof with an emphasis on formulas; and inductive geometry. In these approaches, proof takes on either a ritualistic role and is treated as a skill, or is ignored completely (Goldenberg, Cuoco, & Mark, 1998), while geometric constructions are sometimes taught as skills to be committed to memory (Schoenfeld, 1988).

Many have responded to the current situation with a plea that geometry assume a new focus and that proof and constructions play a different role. In discussions concerning how mathematicians do mathematics, we see a different role for argumentation and constructions. More specifically, with the use of a DGS, the problem posed can be changed from “problems to prove” to “problems to explore.” In the DGS, students construct a diagram, manipulate and measure it, and make a conjecture. This is characteristic behavior of what Cuoco, Goldenberg, and Mark (1996) identify as habits of mind and what the Standards (NCTM, 1989) calls mathematical power.

Chazan and Yerushalmy (1998) highlight the use of a DGS over paper and pencil drawing because of the way that the DGS focuses the students’ attention to the procedure used in the construction. In the paper and pencil construction, the process is usually lost and the diagram becomes the focus. Chazan and Yerushalmy highlight how geometry construction programs can help:

The program separates the particular diagram, which results from a construction procedure, from the procedure itself ... the geometry construction programs provide the users with the ability to:

- Draw starting shapes (points, lines, segments, polygons).
- Make compass and straightedge constructions (perpendicular and parallel lines, circles centered on one point and through another, etc.).
- Measure aspects of those drawings and constructions (lengths, areas, angles, etc.) and display those measurements (in charts and/or graphs).
• View a representation of the procedure they have created.
• Generate other diagrams that satisfy the same construction (by repeating the construction on a new starting shape or by dragging one of the vertices of the starting shape[s]) (p. 73).

The DGS enables students to experiment with invariant properties and provides the support for them to make and test conjectures. In the DGS, students can visualize geometric objects in ways not possible with pencil and paper representations. Transformations, as well as coordinate geometry, can enter the discussion thus extending the study of geometry to an approach that connects it to other areas of mathematics such as algebra or discrete mathematics (NCTM, 2000). Assuming the student has engaging and well-designed tasks to explore, the role of proof changes dramatically to one of explanation (Chazan & Yerushalmy, 1998; de Villiers, 1998; Goldenberg et al., 1998; Goldenberg & Cuoco, 1998).

Proof

For a discussion of proof we look to Laborde (2000) for a working definition. “A proof in mathematics is a specific kind of discourse meant both for validating the truth of a statement and for convincing others of the validity of this assertion” (p. 155). de Villiers (1999) outlines six different roles proof can take on:

• Verification – concerned with truth of a statement.
• Explanation – providing insight into why it is true.
• Systematization – the organization of various results into a deductive system of axioms, major concepts, and theorems.
• Discovery – the discovery or invention of new results.
• Communication – the transmission of mathematical knowledge.
• Intellectual challenge – the self-realization/fulfillment derived from constructing a proof (p. 4).

In a discussion of the proof process, de Villiers cites the role of conviction and how this drives mathematicians to develop a proof. “In real mathematical research, personal conviction usually depends on a combination of quasi-empirical verification and the existence of a logical (but not necessarily rigorous proof)” (de Villiers, 1999, p. 4). He goes on to say that, logically,
we require a formal proof, but psychologically, we are led to it from some sort of experimental exploration or intuition.

Student intuition can be developed through exploration in the DGS by the use of measurement on a large number of cases. In this setting, we suspect that a conjecture is true through an examination of the data. This does not explain why it is true. A proof that explains why a conjecture is true is realized through an examination of the process used to construct the geometric object.

Chazan (1993) goes into great detail regarding the role of evidence in convincing the student of the validity of a deductively proven theorem. He found that students viewed a deductive proof as a single case and did not appreciate the generic aspect of it. In the DGS, the role of proof is encountered more often as explanation. Here the student is led to make a conjecture that may be true and are more likely to want to understand why it is true in general. While some feel that the role of proof may be diminished in the high school curriculum by the use of a DGS, research thus far suggests that the discussion of why a conjecture is true leads to a more secure role of proof (and counterexample) in the high school curriculum (Chazan & Yerushalmy, 1998; de Villiers, 1998; Laborde, 2000; Mariotti, 2000).

The PSSM (NCTM, 2000) states, “the use of dynamic geometry software enables students to examine many cases, thus extending their ability to formulate and explore conjectures” (p. 309-310). In the high school geometry section of the PSSM, a hypothetical example is given concerning the ratio of the area of a midpoint triangle to the original triangle. The students are shown how to make conjectures that may lead to the need for proofs as well as conjectures that are found to be false due to the discovery of a counterexample.

An advantage of visualizing many examples at once is summarized in Lester’s dissertation (cited in Glass & Deckert, 2001), “the Geometer’s Sketchpad program allows students to manipulate diagrams easily, thereby freeing them to focus on the various relationships and patterns that are displayed on the screen and in the requested measurements” (p. 225). The
themes of exploration and knowledge construction are linked to gains in student proof writing abilities and achievement (Chazan & Yerushalmy, 1998; de Villiers, 1999; PSSM, 2000; Glass & Deckert, 2001;).

**Student Achievement**

A few studies have specifically looked at the achievement of students within a DGS with regard to van Hiele level. An example of published results is a case study design used by Choi-koh (1996, 1999) that examined the progress of individual students with specific lessons on GSP. In one situation, a student progressed from a level 1 to a level 4 over the course of nine one-on-one sessions. At the end of the sessions the student was able to verify a deductive proof through a visual examination of the properties of the shape.

Results from a doctoral dissertation by R. A. July (2001) indicate that GSP is a viable tool to teach students about three-dimensional objects. Results of her study indicate that the students' geometric thinking and spatial ability improved significantly, especially for students in the lower van Hiele levels.

**Reinvention**

One common theme in a variety of research studies and articles on geometry education is the reconstructive process (Fawcett, 1938; Fuys et al., 1988). A reconstruction or reinvention approach allows the student to construct his or her own mathematical knowledge and forms the foundation for the Dutch geometry curriculum (Gravemeijer, 1998).

According to the reinvention principle, a learning route has to be mapped out, along which the student can find the result by him- or herself. Of course, the students cannot be expected to reinvent, in a short period of time, mathematics that took outstanding mathematicians centuries to invent. Therefore, the developer has to develop a set of instructional activities that give the students the opportunity to experience such a reinvention process in a condensed form (p. 53-54).
Through the use of a DGS, students follow a cycle of exploration, conjecture, and validation in various contexts. These socially accepted results then form the basis for their own axiom system (de Villiers, 1998; Mariotti, 2000).

The Role of Definitions

Definitions play a crucial role in the study of geometry. Goldenberg and Cuoco (1998) discuss definition in the DGS and then place it into the context of mathematics in general.

... Dynamic geometry offers an interesting arena within which to watch students construct or reconstruct definitions of categories of geometric objects because it allows the students to transgress their own tacit category boundaries without intending to do so, creating a kind of disequilibrium, which they must somehow resolve... The making of definitions is central to mathematics. To learn the importance and purpose of careful definition, students must be afforded explicit opportunities to participate in such definition-making themselves (de Villiers, 1994). Calling explicit attention in class to issues like the ones just described may help students both to examine and to refine their definitions of these specific mathematical objects and even to think more deeply about the very nature of definition (p. 357).

Jones (2000) found that, by working with GSP activities that exposed the definitions of the different quadrilaterals, middle school students were better able to explain their classification hierarchy, an ability characteristic of van Hiele level three. At the collegiate level, pre-service teachers used GSP to explore the necessary and sufficient requirements for the definition of a rhombus. This work increased their content knowledge on the importance of concise yet rigorous definitions and introduced them to how GSP can be used to aid students in understanding definitions (Govender, 2002).

The Role of the Teacher

The issues of teacher knowledge, beliefs, and control dominate research on the use of technology in the classroom. First, using technology for exploration requires a shift by the teacher to the role of facilitator and guide, a shift many researchers cite as difficult. The teacher is responsible for “creating experimental environments where collaborative learning and student
exploration are encouraged" (Chazan, 1993, p. 72). When this experimental environment is achieved, the teacher must be able to follow many different student solution paths. In this environment, some students will progress well beyond the current task. In either case the teacher must possess a deep understanding of mathematical content in order to either confirm or redirect the student explorations. A lack of content knowledge in the field of mathematics is cited as a roadblock for teachers using technology in the classroom. (Ball & Cohen, 1999; Chazan & Yerushalmy, 1998; de Villiers, 1998; Goldenberg et al., 1998; Gravemeijer, 1998; Koedinger, 1998). Strong content knowledge of mathematics is crucial, but so is a good understanding of the technology, both hardware and software, being used to investigate the mathematics. Thus, working with technology itself brings about another set of skills with which the teacher must be able to deal. For example, when the computer reports an error, this can be intimidating to students and even more so to their teachers (Koedinger, 1998).

Teacher beliefs and personality type influence how technology is used in the classroom. Saye (1998) studied characteristics of teachers who use technology for reinforcement versus exploration. He found that the driving force behind the way the teacher used technology was determined by his or her beliefs and educational goals. Teachers with the goal of efficiency and control use technology to save time and increase performance. He labels these teachers as “accidental tourists” since they avoid the unfamiliar. Teachers that desire enrichment and empowerment use technology to engage students with the content and require them to be active learners. Saye labels these teachers as “voyageurs” and describes them as risk takers who are stimulated by novelty and experimentation. Results of his study suggest that if we want teachers to use technology as a tool for exploration, we need to impact their belief system, moving them from a control-based to a constructivist-based pedagogy.

In another look at the issue of control, we find teachers’ beliefs and goals as the underlying motivator. In a study on student learning in a DGE, Hannafin and Berry (1998) make the following aside regarding the teachers that participated in the study. “Teachers agreed to
granting this freedom (unguided student exploration) before the session began, but they later related they were uncomfortable with allowing some students to ‘struggle’ through their own naive and misguided assumptions” (p. 7). A study on open-ended learning environments by Hannafin, Burns, and Little, (2001) examined the teachers’ role. “The importance of the teachers’ role in facilitating student inquiry in open-ended learning environments is unquestioned: teachers’ ability or willingness to adapt their teaching style to facilitate such inquiry is, however another matter” (p. 133). This group cites three basic modes of though teachers assume in their attitude toward the use of technology and instruction: basics first; structured problem solving; or guided generation. The first two modes stress direct instruction and the avoidance of errors (accidental tourist) whereas the last mode allows the teacher to help the student through misconceptions (voyageur). Findings of their study suggest that experienced teachers have fairly well established views of how learning should take place and that it may be important to learn more about changing teachers attitudes since “teachers are the ultimate gatekeepers for the kind of activities that occur in their classrooms” (p. 133).

If teachers are the gatekeepers of the classroom, then they need to be seen as pivotal characters in the quest for educational reform. Many practicing teachers were not taught in undergraduate or even graduate coursework with or about new technology like the current DGS packages. This raises the issue of how teachers come to know technology that is new to them, which underscores the second research questions. Ideally, practicing teachers should be provided with training to update their skills with new technologies prior to the local purchase of new software or hardware.

Unfortunately, a number of factors including budget constraints, scheduling conflicts, and lack of a qualified teacher educator cause this form of training to be late or missing altogether. In general, very few professional development opportunities are organized to support and influence teacher learning (Little, 1999).
Teachers report that they find little satisfaction in episodic 'in-service training,' but are otherwise often thrust on their own resources in what Michael Huberman (1995) has termed a 'lone wolf scenario (p. 234).'

The following discussion looks at general guidelines for teacher professional development and how this impacts the role of the classroom teacher. The current state of professional development underscores the need for the examination of new approaches to in-service teacher training which is addressed in the second research question.

Professional Development: A New Approach

The need for a new approach to teacher professional development (TPD) is emphasized by Sykes (1999) as he discusses the connection between professional development and goals of student learning. "Teachers’ professional development appears weakly coupled to effects on students … Whether it is university course work or the workshops provided by consultants, in-service planned by districts, school-wide improvement plans, or teachers’ voluntary activity, too often the relationship to improvement in student learning is remote to nonexistent" (p. 161).

Hawley and Valli (1999) suggest new formats for professional development need to be attempted since conventional efforts have been demonstrated to be ineffective and wasteful (Ball & Cohen, 1999; Ball, Lubienski, & Mewborn, 2001; Heaton, 2000; Richardson & Placier, 2001).

Fullan (2001) cautions that not only are the formats ineffective, but the presence of too many disconnected projects push teachers to go through the motions for a few months until asked to change yet again. Fullan cites the biggest problem facing schools as the fragmentation and overload caused by uncoordinated reform projects. He warns “change will always fail until we find some way of developing infrastructure and processes that engage teachers in developing new understandings” (p. 37). He extends his caution by stressing that such understanding must include “deep meaning about new approaches to teaching and learning” (p. 38). While it is important to look at research on student learning to inform and justify what is done within
professional development studies, we must look at research on professional development itself to help define a best practice for a new generation of professional development opportunities.

Through an examination of a variety of national research reviews, Hawley and Valli (1999) found a remarkable amount of congruence in what is recommended. They synthesized their findings into the eight “Principles of Effective Professional Development” (Table 1, column 1). An important theme that is seen throughout their principles is the high level involvement by the classroom teacher as the link between external support and the students. Also noted is the cyclic activity of needs assessment, planning, and evaluation as important for both teacher learning and curriculum development.

In addition to identifying the need for new approaches to TPD, Sykes (1991) offers advice similar to Hawley and Valli. First, greater attention must be paid to the “core relationship between what teachers learn and what students learn, building outward from this essential connection” (p. 175). Student learning should form the basis for the planning and evaluation cycle of the TPD. Specific characteristics of professional development identified by Sykes are listed in Table 1, column 2.

The final column of Table 1 includes the professional development guidelines Richardson and Placier (2001) developed as a result of their own in-depth review of studies on staff development. An examination of Table 1 highlights the completeness of the principles developed by Hawley and Valli and their inclusion of the guidelines developed by Sykes, as well as those developed by Richardson and Placier.

An interesting note is that many of the authors who write about successful professional development (including both Sykes and Richardson and Placier) cite the Cognitively Guided Instruction (CGI) research model as a highly successful model of professional development (Lampert & Ball, 1999; Munby, Russell, & Martin, 2001; Swafford, Jones, & Thornton, 1997).
Table 1. Comparison of Professional Development Models

<table>
<thead>
<tr>
<th>Hawley and Valli (p. 138)</th>
<th>Sykes (p. 161)</th>
<th>Richardson and Placier (p. 917-918)</th>
</tr>
</thead>
</table>
| **P1: Goals and Student Performance**  
Driven, fundamentally, by analysis of differences between 1) goals for learning and 2) student performance. | Use the teacher-student learning connection as a criterion for the selection and design of the TPD. | |
| **P2: Teacher Involvement**  
Involves learners (such as teachers) in the identification of their learning needs, and when possible, the development of the learning opportunity and/or process to be used. | Include attention to student learning in the TPD associated with the implementation of curricular and instructional innovations. | |
| **P3: School Based**  
Is primarily school based and integral to school operations. | Embed TPD in the specific content of the student curriculum. | The program should be school-wide and context specific. |
| **P4: Collaborative Problem Solving**  
Provides learning opportunities that relate to individual needs but for the most part are organized around collaborative problem solving. | | The process should encourage collegiality. School principals should be supportive of the process and encouraging of change. |
| **P5: Continuous and Supported**  
Is continuous and ongoing, involving follow-up support for further learning, including support from those external to the school. | | The program should be long-term with adequate support and follow-up. |
| **P6: Information Rich**  
Incorporates evaluation of multiple sources of information in implementing the lessons learned through professional development. | Integrate examination of student learning, using multiple sources of evidence, into the TPD. Reference both formative and summative evaluation of TPD into student learning. | |
| **P7: Theoretical Understanding**  
Provides opportunities to develop a theoretical understanding of the knowledge and skills to be learned. | | The program content should incorporate current knowledge obtained through well-designed research. |
| **P8: Part of a Comprehensive Change Process**  
Is integrated with a comprehensive change process that deals with the full range of impediments to and facilitators of student learning. | | The program should include adequate funds for materials, outside speakers, and substitute teachers so that teachers can observe each other. |

The CGI research model is based on the classroom teachers’ conceptions and experience with the mathematical content. A teacher’s beliefs and content knowledge directly affect that teacher’s instructional decisions and are key points in the CGI research model (Carpenter & Fennema,
1991), (see Figure 1, Chapter I). This leads to a discussion of teacher change within the context of both teacher knowledge and then teacher beliefs.

**Teacher Knowledge**

Shulman (1987) identifies teacher knowledge as a primary influence on instructional practice. He further identifies three key areas of teacher knowledge as content knowledge (CK), pedagogical knowledge (PK), and pedagogical content knowledge (PCK). Frank and Fennema (1992) define these areas as follows:

1. The content of mathematics (CK) component includes teacher knowledge of the concepts, procedures, and problem-solving processes within the domain in which they teach, as well as related content domains. It includes knowledge of the concepts underlying the procedures, the interrelatedness of these concepts, and how these concepts and procedures are used in various types of problem solving. Crucial also to teacher knowledge of content is the manner in which the knowledge is organized, indicating teacher knowledge of the relationships between mathematical ideas.

2. Pedagogical knowledge (PK) includes teachers' knowledge of teaching procedures such as effective strategies for planning, classroom routines, behavior management techniques, classroom organizational procedures, and motivational techniques.

3. Learners' cognitions (PCK) includes knowledge of how students think and learn and, in particular, how this occurs within specific mathematics content. This includes knowledge of how students acquire the knowledge of the mathematics being addressed, as well as understanding the processes the students will use and the difficulties and successes that are likely to occur (p. 162).

Mathematical content knowledge has been shown to influence teaching (Leinhardt, 1989; Munby et al., 2001). In a well-known study, Ma (1999) investigated the mathematical content knowledge of elementary teachers in the United States and in China. Her work illustrates the effect subject matter knowledge, or lack thereof, has on teachers’ explanations of key topics in the elementary school curriculum.

Swafford, Jones, and Thornton (1997) conducted a study that examined the role of teacher knowledge in a geometry setting. This study looked for changes in the geometry instruction of middle school mathematics teachers as a result of a summer institute on both geometry concepts and the van Hiele model of reasoning. As a result of an increase in content knowledge, the teachers increased the time in their curriculum that was devoted to geometry. The
teachers also felt the quality of their instruction improved and they were more willing to try new ideas and new instructional approaches. Many of the teachers reported they were more likely to engage in risk-taking that enhanced student learning and were more confident in their abilities to provoke and respond to higher levels of geometric thinking as a result of the summer institute. While the summer course increased the teacher’s knowledge of geometry, it also included work on the van Hiele model of reasoning which helped the teachers within the area of pedagogical content knowledge. While the study cannot attribute the changes directly to the increase in content knowledge (due to the confounding of the two treatments), the participants in the study cite this as a major reason for the changes in their teaching.

Fennema and Frank (1992) examine the issue of teacher knowledge and its impact on student learning. They explain that teacher knowledge covers a large area of interrelated but not often isolated components. They note that a teacher’s specific mathematical content knowledge is often masked in studies that compare an expert to a novice teacher or in studies that combine content knowledge with pedagogical content knowledge, like the Swafford study. Fennema and Frank also point out that mathematical knowledge alone does not account for quality instruction. Teachers have to take their knowledge of content and somehow transform it, using what they know of students and student learning, so their students can interact with the material and learn from it.

Pedagogical content knowledge involves knowledge about how ideas might be represented to students and how students learn what they find difficult. A key aspect to understanding PCK is that it has more to do with how the mathematics is learned. Three examples from research are often cited as model examples of an integration of the three knowledge areas CK, PK, and PCK. Lamperts’ (1999) writings from an experience teaching fifth grade highlight the on-the-job activity of the teacher as she presents the topic of exponents. She asks questions that provoke an exploration of patterns and leads the discussion through the argumentation necessary for the class to agree on what they find. A similar example is that of
Schoenfeld (1982, 1992) teaching a college level class on problem solving. His deep understanding of the mathematics enables him to ask the right questions as he guides and models the problem solving process. The CGI research (Carpenter & Fennema, 1991; Fennema & Franke, 1992) provides the final example of an integration of the domains of teacher knowledge. In CGI training, the teacher gains an in-depth understanding of the many different models of addition and subtraction along with the patterns students may use to solve the problems. The teachers are also made aware of the need to build on the students’ prior knowledge. As a result, the teacher is better able to gear questions to the level appropriate to the students’ current knowledge and, at the same time, push them to the next level. In addition to educating the teacher in the areas of teacher knowledge, the CGI research assumes a belief system that is based on a constructivist teaching philosophy. While qualitative studies on teacher beliefs are relatively new they are addressing the role that beliefs play in shaping the teachers instructional behavior.

Teacher Beliefs

Artzt and Amour-Thomas (2002) define beliefs as “an integrated system of personalized assumptions about the nature of the subject, the students, learning, and teaching” (p. 22). Beliefs are shaped by the teacher’s prior experience as a student, as well as by his or her level of mathematical knowledge (Ball et al., 2001). A teacher’s beliefs have a profound impact on classroom practice (Thompson, 1984).

The belief that a teacher may hold regarding the nature of mathematics generally falls into three philosophical areas. Ernest (cited in Thompson, 1992) distinguishes each view as follows:

- [Firstly], there is a dynamic, problem-driven view of mathematics as a continually expanding field of human creation and invention, in which patterns are generated and then distilled into knowledge. This mathematics is a process of [inquiry] and coming to know, adding to the sum of knowledge. Mathematics is not a finished product, for its results remain open to revision (the problem-solving view).
- Secondly, there is the view of mathematics as a static but unified body of knowledge, a crystalline realm of interconnecting structures and truths, bound together by filaments of
logic and meaning. Thus mathematics is a monolith, a static immutable product. Mathematics is discovered, not created (the Platonist view).

• Thirdly, there is the view that mathematics, like a bag of tools, is made up of an accumulation of facts, rules, and skills to be used by the trained artisan skillfully in the pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts (the instrumentalists view) (p. 132).

Many teachers view mathematics as an organized set of rules and procedures that is to be learned and, in some cases, memorized. Others see mathematics as a process of problem solving and reflection. The teachers’ conception of what mathematics is drives their beliefs regarding the role of the student; curriculum and instruction; local, state and national standards; high-stakes testing; and the role of the teacher. Thompson (1992) reports, “Differences in teachers’ conceptions of mathematics appear to be related to differences in their views about mathematics teaching” (p. 135). The teacher who believes that mathematics is a set of rules and procedures to be learned sees the teacher as the dispenser of knowledge and the student as the recipient. The favored mode of instruction for this type of teacher is often the lecture. Learning mathematics involves watching the teacher provide a clear step-by-step demonstration and evidence of that learning is reproducing that knowledge on a test. The teacher and the text are seen as the mathematical authority. By contrast, the teacher who believes that mathematics involves active participation with the construction of mathematical ideas identifies the role of the student as a problem solver and expects all students to be actively engaged in meaningful tasks. The role of the teacher becomes that of facilitator and classroom instruction becomes focused on exploration. The student is expected to lead and engage in discussions with the teacher and classmates. The teacher and the text are not seen as the sole authorities and validation is frequently found through debate and agreement within the group. The teacher’s role is different in that control of learning is turned over to the students.

The PSSM (NCTM, 2000) promotes shifting the role of the teacher from one who dispenses knowledge to one who listens, questions, and probes for student understanding. While some teachers admit to being unwilling to implement a reform style of teaching, other teachers...
indicate they believe in mathematics reform but are unable to make the necessary changes in their practice. What may be causing some of the difficulty is the perception teachers have regarding teacher efficacy, or "what good teaching looks like." Smith (1996) identifies many areas in which the teacher has difficulty implementing a reform style of teaching. First of all, traditional teaching provides benchmarks for instructors to evaluate their practice. A shift to reform teaching changes these benchmarks and the teacher becomes unsure of how to evaluate his or her own teaching. Second, a reform teaching style requires a deep knowledge of the subject, which many teachers do not possess. Lastly, the reform style of teaching calls for teachers to share control and authority with their students. Many teachers are not ready or willing to take such a risk, or genuinely do not see their students as a source of authority in the classroom.

In terms of integrating technology into one's teaching, the same problems exist. Teachers unfamiliar with the technology do not possess the necessary benchmarks or expectations of either themselves or of their students in regards to the use of the technology. Teachers with limited mathematical knowledge may remain immersed in the text and may not look to integrate activities that generate risk, either for them or their students (Saye, 1998). If technology is used, it generally fills the role of drill and practice as opposed to exploration and discovery (Hannafin & Barry, 1998).

Change in teaching with technology will require changing teachers' beliefs in two areas: their concept of the nature of mathematics and their efficacy beliefs. Smith (1996) identifies guidelines that can be used to facilitate this change process:

- Choosing the problem – Choose problems that quickly engage the student in work with significant mathematics.
- Predicting student responses – Know the standard representations and solution methods in particular topic areas. Know some of the approaches students will take and the language they will most likely use.
- Generating and directing discourse – Lead discussion without being the authority. Being open to different strategies and solution methods suggests a different view, that mathematics is dynamic, growing, and created by people.
- Judicious telling – There are times when the teacher needs to model terminology, representations, counterexamples, and mathematical activity. Selective telling of this sort
is quite different from the general telling model where the teacher is the sole knowledge provider and telling is the central teaching task. (p. 400).

The researcher, in the role of a mentor, will follow these guidelines while working with the teachers in planning sessions to promote a new set of teacher beliefs.

### Teacher Change

Change in the educational setting is usually initiated by district administrators as a reaction to “do something” about a problem. Fullan (2001) categorizes this as a top-down model of organizational change since a person, or persons, above the level in which the change is sought, make the decision. Fullan categorized stages of the organizational change process as 1) initiation, 2) implementation, and 3) institutionalization. Many times change that is sought via a top-down model is not successfully implemented and is abandoned (Fullan, 2001) or is not implemented as it was intended because the burden of change rests with the individual teacher (Cohen, 1990; Goldenberg et al., 1998; Wilson & Goldenberg, 1998). Fullan indicates innovations that affect mathematics teachers usually involve three aspects of change:

- Materials are new – either curriculum or technology.
- New teaching approach – new strategy and activities.
- Altered beliefs – personal theory of learning mathematics.

Teachers work through these aspects of change in a process that parallels the organizational change process. These stages – initiation, implementation, and institutionalization – will be discussed in relation to the models of individual instructional change displayed in Table 2.

The three individual change models listed in the table originate from a variety of sources. Etchberger and Shaw (1991) restate a change model that is attributed to Tobin (Table 2, column 2). The NCTM publication **Bold Ventures (Volume 3)** (Raizen & Britton, 1996) provides a case study of an Urban Mathematics Collaborative (UMC) that successfully moved teachers to a more constructivist teaching style (Table 2, column 3). The final model originates in the work of
Senger (1999) as she conducted a case study on three teachers in the content area of elementary school mathematics over the course of a year (Table 2, column 4).

Table 2. Models of Instructional Change

<table>
<thead>
<tr>
<th>Fullan Etchberger and Shaw</th>
<th>Bold Ventures – UMC project</th>
<th>Senger</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initiation</strong></td>
<td>Perturbation</td>
<td>Disequilibrium Exposure</td>
</tr>
<tr>
<td><strong>Implementation</strong></td>
<td>Commitment to Change Vision</td>
<td>Existence Proof Modeling Support</td>
</tr>
<tr>
<td></td>
<td>Vision</td>
<td>Modeling Support</td>
</tr>
<tr>
<td></td>
<td>Projection into that Vision</td>
<td>Experimentation</td>
</tr>
<tr>
<td><strong>Institutionalization</strong></td>
<td>Reality</td>
<td>Reflection</td>
</tr>
</tbody>
</table>

Each individual change model listed in Table 2 begins in the initiation stage with a sense of uneasiness caused by exposure to something new. In Senger’s study, it was a shift to a constructivist teaching philosophy. Once the teacher becomes aware of a new direction for his or her teaching, each model indicates implementation involves creating an image or a vision of what the changed practice should look like, followed by a period of exploration. The UMC project points out the critical role networking and mentoring played for the teachers in the midst of change. Teachers were brought together to give them knowledge and support, but also so they could share ideas and problems with each other. Fullan supports the aspect of collegiality and cites a main reason for failed change is the isolation of teachers as they are attempting something new. In this study the “something new” is the GSP program with the existence proof that it can be done in the classroom being offered by the mentor. The mentor also alleviates the isolation factor by working alongside the teachers.

Fullan describes institutionalization, or the final stage of change, as a step that Senger identifies by a change in the teacher’s verbalization regarding instruction. Throughout the
experimental stage, the teacher's verbalization includes characteristics of tentativeness, constant repetition, and often, disbelief. At the final stage, teachers exhibit *principled discourse* "which was defined in the study as language reflecting commitment of some type where the teacher could base and defend ideas as integral parts of a belief system" (Senger, p. 209).

Inherent in the initiation stage of individual change is the reflective action of the teacher as they experience a perturbation or disequilibrum and ask themselves a "what if" question. This process of personal reflection is integral to the remainder of the individual change process and is often used to instigate change in the educational environment.

**Reflective Practitioner**

Reflection in this context is defined as thinking about teaching. According to Artzt and Amour-Thomas (2002) it involves "the thoughts teachers have before, during, and after the actual enactment of a lesson" (p. 7). Most well known for his work defining the reflective practitioner, Schön (1983) describes reflection as a cyclic process of 1) problem identification, 2) reframing, and 3) resolution. For example, after teaching a lesson many teachers will look back and perform an analysis of what took place and, based on this analysis, may alter the lesson the next time they teach it. This results in a new analysis of the modified lesson. Reflective activity on the part of the mentored teachers took place in this study but the role of teacher reflection was secondary to the role played by the introduction and use of GSP.

**Mentoring**

Mentors are used in the educational arena for three general purposes, to train potential teachers, to support novice teachers in their first years of practice, and to improve instruction with practicing teachers through novice/expert mentoring. A review of the research indicates a lack of subject specific mentoring projects.
Typically, teacher education programs pair the pre-service teacher with a practicing teacher, or mentor, during what is often called student teaching. During the student teaching experience, the novice watches the practicing teacher and then slowly begins to assume responsibility for instruction with the support of the practicing teacher. Mewborn (2000) cites the use of a mentor as particularly important in promoting reflective thinking among pre-service teachers. A study from Norway (Gudmundsdottir, Nilssen, & Wangsmocappelen, 1998) documents the progress of a pre-service elementary teacher in the teaching of multiplication under the guidance of a mentor. Student teaching as a form of mentoring is commonplace and has been, in some places, extended to yearlong programs for participants in alternative certification programs. An example of this is found in Georgia with the Teacher Recruitment and Internship Project for Success, an intern-mentor alternative certification program geared to recruiting teachers for the urban environment.

Mentoring has also found its place in schools with the purpose of supporting new teachers. Darling-Hammond (1998) recommends that the future teacher's career path include the development of authentic-learning experiences with mentor teachers. Darling-Hammond reports that while the purpose of mentoring to new teachers is to keep them in the profession and increase their knowledge base, the majority of the new teachers' concerns and mentored activities center on classroom management issues.

A final form of mentoring is school improvement where the institution pairs an experienced teacher with a novice teacher with the goal of increased academic performance for students. Much of the research in this area includes general models for implementing mentoring, or peer teaching programs (Bunyan & Eckmier, 1995). Results specific to mathematics education include a study by Paine and Wang (2001) on the learning of a novice first grade teacher in China where the mentor provided a clear and concise approach to the content, as well as modeled reflective thinking. While studies on mentoring within the field of mathematics are rare, a study outside of mathematics on the role of the mentor makes specific recommendations that are
followed in this study. Halai (1998) examined the role of the mentor with a group of mid-career teachers in order to better understand how mentoring practices influence teacher learning. Halai found that the mentees’ perception of the mentor was problematic if the mentor was perceived to be in the role of problem solver, or evaluator of performance. By contrast, the teachers did show professional growth when the mentor and mentee relationship was established on mutual trust.

A final study that highlights the use of mentors focuses on the integration of technology into science and mathematics courses. Friedman, Holahan, and Jurkat (2000) trained both pre-service teachers and potential mentors on the use of technology. Results indicate that the pre-service teachers were better prepared to implement technology into their teaching, and the practicing teachers, that were to become mentors, changed how they taught mathematics and science by more often adopting the role of the facilitator in their own classrooms.

Summary

Teaching is a complex activity. The process of adjusting one’s teaching to respond to the demands of reform in mathematics places additional demands on this already complex activity. Research within the content area of geometry calls for the integration of dynamic geometry systems to increase the opportunity for student exploration. The research base effectively makes the case that teachers should embrace this new technology. This supports one of the questions that this research study will examine: How do teachers implement GSP into their teaching practice? The nature of this question provoked an examination of the research on teacher knowledge, beliefs, and teacher change. This research base indicates that what a teacher knows and believes about mathematics has a strong impact on how they teach. The research question proposed is also dependent on teachers making a change in their practice to include teaching with technology. How teachers manage this process can be examined amidst the backdrop of teacher change models provided by researchers like Senger. Finally, this study is a professional development project. Research on professional development points to a need for change within
this area. This supports the second research question regarding the use of a mentor to facilitate
the teachers' change process.
CHAPTER III: METHODOLOGY

This study uses a teacher development experiment nested in a case study design to investigate how two teachers implement Geometer's Sketchpad when a mentor supports them through the phases of learning, planning, and teaching. This study is modeled after a pilot study, which was conducted to test the feasibility of the researcher working as a mentor in a public school setting to elicit change in the teachers' practice [see Appendix A for a description of the pilot]. The teacher from the pilot study briefly participated in this study as a resource on GSP. The following describes the design specifics, phases of the study, data collection tools, and the coding and analysis procedure for this study.

Design of Study

This study is a teacher development experiment with the researcher working with two teachers in a mentoring role in their classrooms. Currently there are two alternate approaches to working with teachers to elicit change in their teaching. One approach is to assume that changes in beliefs and/or knowledge will lead to specific changes in practice. This assumption indicates that short-term workshops or in-service training, outside the classroom, would be an avenue to promote a change in beliefs, or knowledge, that will then impact classroom instruction. The discussion of teacher professional development (literature review) indicates that such a model is usually ineffective.

Another approach, based on the work of Cobb, Wood, and Yackel (1990), uses the teacher's classroom for both the research setting and the arena for change. These researchers conducted a study on how second-grade students perform on problem-centered activities. Their methodology included the regular classroom teacher as a member of the research team.
the focus of the study was student understanding, these researchers discovered student performance on the classroom activities provided the evidence the second-grade teacher needed to both provoke and support change in her beliefs and habits regarding traditional instruction.

Our experiences of interacting with the project teacher who participated in the classroom teaching experiment profoundly influenced the way in which we inducted the other teachers into the project… Our continued interaction with the teachers throughout the year reflects our belief that classrooms are a learning environment (p. 139).

Cobb, Wood, and Yackel advocate teacher professional development outside of the classroom, where new ideas are presented, followed by work within the classroom where the teacher is confronted with possible problems in his or her own teaching.

In doing so we encourage the teachers to make aspects of their current practice problematic. In effect we asked the teachers to reconsider what they thought they knew. Only then would they have both an initial awareness of other possibilities and reason and motivation to pursue these possibilities by developing a new form of practice (p. 143).

Further support for this design is found in the writing of Simon (2000). The Teacher Development Experiment (TDE) employs a methodology based on the constructivist teaching experiment. The researcher can study development by fostering the development as part of a continuous cycle of analysis and intervention. The TDE builds on the fact that learning can be seen as both a psychological process of the individual and a social process of the group.

On combining case study methodology within the TDE Simon states, “The TDE methodology also integrates a case study approach into its approach of the teaching experiment in order to collect and coordinate individual and group data on teacher development” (p. 336). Simon points out, “The case studies of individual teachers that are included in the TDE make use of traditional case study methodology (Stake, 1995) but with a particular modification based on the TDE developmental stance” (p. 348).

This study includes data collected from observations, lesson plans and post-lesson reflections, surveys, interviews, participant journal prompts, field notes, a collection of student work, as well as a mentor log. Selected teaching episodes were video taped for the purpose of validating the real time coding as well as for future analysis.
Setting

Harrison High School (all names have been changed) is located in the Midwest. The town has two high schools, which serve a community of about 50,000. Attendance for the 2002-2003 school year was approximately 1,750 predominately middle class students. Twenty percent of the students were on a free lunch program and 10% were on reduced lunch. The student population is approximately 74% Caucasian, 16% African American, 7% Hispanic, 2% Asian, and 1% Multiracial. The school is a typical suburban 9-12 high school.

Harrison High School is a pilot site for the state level end-of-course assessment similar to the New York regent exams. Assessments were piloted in 9th and 11th grade English classes as well as Geometry, Algebra I, and Algebra II in spring 2003. Due to the nature of the state test, the geometry curriculum at Harrison is strongly affected by the state geometry standards.

The mathematics department consists of 14 teachers. The teachers are not required to follow similar lesson plans and technology use is classified by the department chair as limited to hand held calculators with computer software used primarily for diagnostic testing and drill in the algebra classes. The school has a number of computer labs teachers can use if they reserve the lab ahead of time.

Participants

The two teachers in this study, Alice and June, have worked together at Harrison for the past two years. They collaborate often on pacing and content issues concerning their geometry preparation. Alice and June would consider themselves to be friends as well as colleagues. The two teachers were exposed to GSP for about an hour at an NCTM sponsored geometry institute during 2003. During that hour they could not recall learning much more than how to draw a line segment. Due to their exposure to GSP at this conference, Alice and June were recommended to the researcher as potential participants for this study by their district curriculum coordinator.
Alice had been teaching for four years. She came to the teaching profession after approximately 20 years as a secretary. She completed a master’s degree in May of 2003 in secondary education. Alice is well liked by many students.

June had taught mathematics for 21 years, the last four at Harrison High School. She also has a master’s degree in secondary education. Many of June’s colleagues and students regard her as a good mathematics teacher.

Alice and June were well suited to this research study since they worked together on coverage and pacing issues of the school’s geometry curriculum. These two teachers had already negotiated a social norm for their planning time. The aspect of a researcher working with them on curriculum planning became a natural extension of what the teachers were already doing. This helped to focus the research on the intervention and not on the social negotiation of their relationship. By following two teachers who worked as a team, the researcher was also afforded a glimpse into the social aspect of change as the teachers learned from, and supported, each other. Raizen and Britton (1996) stress the importance of teachers supporting each other as an avenue of successful change in the Urban Mathematics Collaborative.

The final aspect these specific teachers brought to the study is that they were at two different stages in their teaching careers. June was a veteran teacher and Alice was only in her second year of high school teaching.

The final participant in the study was the researcher. The researcher participated in the study as a mentor to the teachers. This action on the part of the researcher promoted a split between the roles of observer and active participant. Due to the involvement of the researcher the following section will address the researcher’s past experience with the use of Geometer’s Sketchpad.
Researcher Experience

The researcher began her involvement with the use of GSP in her own public high school teaching from 1993 to 1996. During this time she preferred to have students develop ‘sketches’ and created the following social norm for class work. Students would be given a worksheet that included a set of directions to guide them in the use of GSP to construct a given figure. Students would work in pairs on one computer. The students would typically be asked to measure some aspect of the figure, either segments or angles, manipulate the figure, and then come up with a conjecture, such as “the sum of the remote interior angles of a triangle equals the exterior angle.” In some cases the sketch (file) would be printed and turned in, but if animation were required the sketches would be collected on disk. The role of the teacher was to facilitate the activity by asking questions, and to answer questions regarding the use of the program itself. The role of the program was to aid the student in the development of an empirical understanding of geometric theorems.

After leaving the public school setting the researcher gained further experience with GSP while teaching an undergraduate modern geometry course in 2001. In the setting of hyperbolic geometry (the Poincaré Disc) the researcher again challenged students to construct figures, manipulate them, and develop conjectures. GSP provided the students a tool that helped them investigate properties of the geometry as well as convince them of the truth of certain theorems. The social norm that was established included a time of investigation, which involved discourse between students and student to teacher, followed by whole class discussion.

The researcher claims past experience with the use of GSP puts her in a favorable position to act as the mentor for the study. Simon (2000) cautions that any researcher who has not conducted a teaching experiment independently, “should engage in exploratory teaching in order to become thoroughly acquainted with the [teachers’] ways and means of operating with respect to whatever particular domain of mathematical concepts are of interest” (p. 354). This caution prompted the researcher to conduct a pilot study. In the fall of 2002, during a three-
month time span, the researcher worked as a mentor with a public school teacher in his geometry classroom. This experience, while limited, afforded the researcher experience in the role of a mentor as well as research facilitator (a detailed discussion of the pilot study is contained in Appendix A).

Intervention

The intervention for this study was twofold. For the first intervention Alice and June met with the researcher in June of 2003 with the goal of learning how to use the program with content from their first semester geometry course. The meetings consisted of three half-day sessions where each teacher brought in her personal laptop computer with GSP version 4 installed. They were provided with supplemental material to work on at the session and were encouraged to work on it at home between sessions. The researcher had carefully selected the labs for the sessions in order to both teach GSP as well as showcase the role it could play as a tool for student geometric exploration and conjecture.

The second form of intervention was a type of teaching experiment where the researcher acted as a mentor, or specialist, by facilitating the act of selecting and teaching GSP activities for the teachers in their classrooms. During September and October of 2003, the researcher met with the teachers for three after-school meetings to select GSP lab activities. During the planning meetings the researcher attended to the teacher change guidelines developed by Saye (1996):

- The researcher helped the teachers select GSP labs that engaged the students with the program as well as significant mathematics.
- The researcher predicted student responses and solution paths for the proposed GSP labs.
- The researcher suggested possible teaching trajectories for the teachers to follow.

Over the two months the researcher was also available to teach labs for the teachers and the following routine was established. Since Alice taught three geometry sections and June taught two sections the researcher was asked by the teachers to teach the first sections as the teachers observed. When the teachers felt comfortable they would resume the role of lead teacher.
with the remaining sections and the researcher would become an observer and an assistant to the
teacher and students. By teaching the GSP labs the researcher provided Alice and June with a
vision or a mental image of what instruction with GSP could look like. When the teachers
resumed the role of instructor in the computer lab the researcher assumed a position of support as
they entered a phase of experimental change in their teaching practice.

The Geometer’s Sketchpad can be a highly effective tool that many instructors use to
teach the concepts of geometry in an exploratory fashion. An in-depth discussion on this can be
found in the literature review. The use of a mentor is unique to this study and is based on the
recommendation of McDougall (1997).

Study Description and Data Collection

This study examined the following research questions:

1. How do teachers implement Geometer’s Sketchpad in their planning and teaching?
2. How does the role of a mentor facilitate the professional development of the classroom
teacher with the implementation of Geometer’s Sketchpad?

This study addressed the research questions in four distinct phases as outlined in Table 3, with the
intervention placed in phases two and three. While the phases of the study followed a linear path
from May to January, the analysis of data and the planning done by the researcher followed an
iterative process that is typical of a teaching experiment (see Coding and Analysis section for
more discussion on this).

Table 3. Four Phases of the Study

<table>
<thead>
<tr>
<th>Phase</th>
<th>Phase Code</th>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Baseline</td>
<td>Observations and Surveys</td>
<td>May - June</td>
</tr>
<tr>
<td>2.</td>
<td>Learn</td>
<td>Introduction to GSP</td>
<td>June</td>
</tr>
<tr>
<td>3.</td>
<td>Plan/Teach</td>
<td>Collaborative Planning and Teaching with GSP</td>
<td>September - October</td>
</tr>
<tr>
<td>4.</td>
<td>Follow-up</td>
<td>Exit Interview and Surveys</td>
<td>January</td>
</tr>
</tbody>
</table>

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The Baseline phase involved three observations of June's and four observations of Alice's classroom instruction in order to characterize their teaching style, methods of instruction, and goals of instruction. Data collected from the teachers included a lesson plan and a post-lesson reflection for each of the teaching episodes. The researcher completed a classroom observation form called the Classroom Lesson Record (Appendix B), which is described in the next section of this chapter, and videotaped each teaching episode for back up purposes. The teachers also completed a van Hiele test (CDASSG project, 1983) and a Belief Survey (Appendix C). At the conclusion of the Baseline phase the researcher compiled a narrative analysis of each teachers' teaching style, goals of instruction, and other related observations.

During the Learn phase the teachers met with the researcher for three half-day sessions that involved two specific tasks. As a first task, the teachers worked selected activities from the book Exploring Geometry with Geometer's Sketchpad (Bennett, 1999) published by Key Curriculum Press in order to both become familiar with GSP and to investigate a source for curriculum supplements. The teachers then identified the topic of transformations for more in-depth work with the researcher on GSP. Data that were collected during this phase included a mentor log of the activities, audiotape of the second and third sessions, as well as researcher field notes.

On the final half-day session the teachers met with Jacob, the teacher that was involved with the pilot study, for a half-hour, question-and-answer session. This session was audio recorded and transcribed for later analysis.

The Plan/Teach phase involved the planning and implementation of GSP activities into the teachers' curriculum. The teachers' goal for the fall semester was to use the Geometer's Sketchpad to replace hand-constructed transformations. The original goal of the researcher was to space out the use of GSP with one computer lab session per week, which proved to be a successful pace in the pilot study. However, the teachers chose to concentrate the majority of the
GSP labs within a three-week time span, with daily trips to the computer lab in order to cover the topics of transformations.

The teachers met with the researcher in September to plan out introductory activities and then in late September and early October to plan out the chapter unit on transformations. The result was a total of four introductory labs that were implemented in September and nine labs on transformations that were implemented in October. The researcher maintained field notes as each of the GSP labs were implemented. The researcher also conducted informal interviews with the teachers on an as-needed basis. These field notes and interviews were transcribed for later analysis.

Each of the planning meetings was audio taped and transcribed. For each of the GSP lessons taught, a lesson plan (Appendix D) and post-lesson reflection (Appendix D) were collected from each teacher. For each teacher, three GSP teaching episodes were videotaped for later analysis. In addition to the lesson plans and post-lesson reflections the teachers were given specific journal prompts (Appendix D) to complete on a weekly basis. Once the teachers were completing post-lesson reflections they felt the journal prompts were repetitive and asked to only complete one or the other. The researcher agreed to the completion of only the post-lesson reflections at this point in the study.

The Follow-up phase of the study involved a repeat of the surveys and an exit interview (Appendix D). In the interview the teachers were asked to look back over the duration of the study and reflect on how they felt about using GSP and recall any changes in their instruction when using GSP. Specific questions focused on the areas of the role of the teacher, role of the student, goals of instruction, the role and source of curricular materials, and the typical instructional method. They were also asked to reflect on the role of the mentor as a professional development model. Prior to the final interview the teachers were asked to repeat the belief survey and the van Hiele test. Where pre-post differences were found to teachers responses on the belief survey, the teachers were asked to comment on their responses and offer an explanation.
as to what may have been responsible for bringing about the change. Pre-post differences found on the van Hiele test were very minor and were therefore not addressed in the final interview.

In each phase the researcher monitored the activity of the teachers, but she was not physically present during all parts of the study. For example, in the summer the teachers took the materials home to further explore the software and curriculum. In the fall the teachers taught a few of the GSP lessons without the researcher present due to a time conflict. In such cases the teachers were asked to document their activity and their reactions for later analysis by the researcher.

Data Collection Tools and Rationale

Data were collected from a variety of sources. Table 4 lists the phases of the study with the corresponding activity, data collection tool, and date of activity. A discussion of the instruments follows and each tool is described in terms of how it addresses the two research questions. All of the data collection tools were selected with the purpose of supporting the first research question while selected data collection tools were re-examined to support the second research question.

As the teachers accepted and used the GSP program there was the potential for distinct changes in many aspects of their geometry instruction. One way to document teacher change is through an examination of content knowledge and professed beliefs. Thus, two instruments were used for a pre/post comparison. The first was a van Hiele test developed by Usiskin (CDASSG project, 1983). The second item was the adapted Belief Survey from McDougall’s (1997) doctoral dissertation.
Table 4. Study Time Line with Data Collection Tools

<table>
<thead>
<tr>
<th>Phase</th>
<th>Activity</th>
<th>Data Tool</th>
<th>Date of Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td>Teaching Episode Observation</td>
<td>Lesson Plan, Observation and Video, Post-Lesson Reflection, Mentor Log</td>
<td>May 20-27, 2003</td>
</tr>
<tr>
<td></td>
<td>Surveys</td>
<td>Belief Survey, Van Hiele Test</td>
<td>Early June</td>
</tr>
<tr>
<td>Phase 2</td>
<td>Learn GSP</td>
<td>Audio Recording, Journal Prompts, Mentor Log</td>
<td>June 11, 13, and 19</td>
</tr>
<tr>
<td>Phase 3</td>
<td>Curriculum Planning</td>
<td>Audio Recording, Journal Prompts, Mentor Log, Planned GSP Labs</td>
<td>September 11, September 29, and October 2</td>
</tr>
<tr>
<td></td>
<td>Teaching Episodes</td>
<td>Lesson Plan, Observation and Video - 3 sessions, Post-Lesson Reflection, Student Work, Mentor Log</td>
<td>September - October</td>
</tr>
<tr>
<td>Phase 4</td>
<td>Surveys</td>
<td>Belief Survey, Van Hiele Test</td>
<td>Early January 2004</td>
</tr>
<tr>
<td></td>
<td>Exit Interview</td>
<td>Interview Protocol, Analysis of Survey</td>
<td>Alice: January 20, June: January 22</td>
</tr>
<tr>
<td></td>
<td>Teaching Episode Observation</td>
<td>Observation and Video: Alice only</td>
<td>Alice only: February 3 and 5</td>
</tr>
</tbody>
</table>

To further document teacher change it was important to observe the teachers’ instruction over a period of time. Tools that would potentially expose beliefs and knowledge, as well as changes in instruction, included lesson plans, classroom observations, student work, and post-lesson reflections. In Phase 1, the lesson plans, and post-lesson reflections, were gathered from the teachers but they were not given a specific protocol to follow. At that point in the study, the goal was to collect data that could be used to describe the teachers’ instruction prior to working with the researcher on GSP. The lesson plan and the post-lesson reflection became more specific in Phase 3 to assure that appropriate amounts of data were collected. For example, the post-lesson reflection is a one-page handout that includes the following prompts that focus on GSP:
• Were the goal(s) of the lesson achieved and if so how do you know?
• Describe how GSP was implemented in the lesson.
• Describe what your students did with GSP.
• Compare this lesson to a non-GSP lesson on the same topic.

In Phases 1 and 3 the teachers were observed and coded (when possible) in real time. A backup video was made for the purpose of validating the coding of the lesson as well as to preserve the classroom lessons for further analysis. The Class Lesson Record (CLR) was developed, and is used in Minneapolis, Minnesota (Math/Science Matters (M/SM) Class Lesson Record – Section V only). This instrument requires coding at five minute intervals in the following categories: Group Size, Lesson Phase, Level of Student Thinking, Level of Student Engagement, Student Learning Behaviors, Processes (Mathematics/Science), and Instructional Strategies. The instrument also calls for written notes regarding the teachers’ questions and a description of the activity, again at five-minute intervals.

The CLR was adapted for documenting the observations in Phase 3 to incorporate the use of GSP in the following ways. First, if students were working on computers individually or in pairs, the observation focused on a small group of students in order to capture a sense of the discourse that was taking place as well as to document the role of technology. The Processes coding section indicates that if a mathematics class was being observed the process codes would include Communication, Connections, Problem Solving, Reasoning and Proof, and Representations. For Phase 3 the researcher allowed use of the science codes in the Process category. These additional science codes included Observation, Comparison, Organization, Relationships, Inference, and Application. A final modification was the addition of three new discourse codes (Table 5), which applied to both the teacher and the student. The prefix for the code is T for teacher and S for student. The suffix for the code indicates Q for question, A for answer, and C for comment.
Table 5. Discourse Codes

<table>
<thead>
<tr>
<th>Code</th>
<th>Category</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/S Tech Q/A/C</td>
<td>Technology</td>
<td>How do I put a circle here?</td>
</tr>
<tr>
<td>T/S Voc Q/A/C</td>
<td>Vocabulary</td>
<td>How do I know if a triangle is acute?</td>
</tr>
<tr>
<td>T/S Dis Q/A/C</td>
<td>Discovery</td>
<td>These angles are always congruent!</td>
</tr>
</tbody>
</table>

An important stage of the teacher change model is when a perturbation is introduced. It was anticipated that beginning in the Learn stage the teachers might begin to see the potential of GSP with their students. In order to document this process the sessions were audio taped and sections were transcribed for coding. The teachers were also asked to journal for each occasion that they experimented with the program (especially on their own time). In order to assure that the teachers used the journal, specific prompts were given. Journal prompts specific to GSP are:

Specific Journal Prompts Phase 2:
1. Describe what you learned about GSP in this meeting.
2. Describe how you might implement GSP into your teaching.
3. Describe how this new GSP knowledge may change your teaching.

The intervention continued into the third phase of the study. Additional journal prompts concerning GSP are:

Additional Journal Prompts Phase 3:
1. Describe the lesson planning session.
2. Did you notice any changes in your instruction due to this week’s GSP activity (describe)?
3. Have you used additional GSP activities this week? If so please include the post-lesson reflection prompts in your journal.

A final tool that was used to gather evidence of teacher change was the exit interview. Here the teachers were asked to reflect on how their teaching with GSP may have impacted their instruction in regard to the teacher’s role, student’s role, goals of instruction, the role and source of curricular materials, and the typical instructional method.

An immediate analysis of the exit interviews prompted the researcher to request additional classroom observations of Alice’s teaching. Although the observations were not part of the original study plan, Alice agreed to the observation and video recording of two non-GSP
teaching episodes in early February 2004. The purpose of the observations was to investigate a verbalized change in Alice's beliefs regarding her geometry instruction.

Figure 3 was created to indicate how the data collection instruments relate to each other in the various stages of the study with regard to the first research question: How do teachers implement Geometer's Sketchpad in their planning and teaching?
For example, the same (or similar) tools are used between phases 1 and 4 as well as between phases 1 and 2, and phases 2 and 3. This consistency in data collection aided the researcher in focusing on possible changes due to using GSP.

The second research question: How does the role of the mentor facilitate the professional development of the classroom teacher with the implementation of Geometer's Sketchpad? is specific to the role of the researcher as a mentor. Figure 4 outlines the data collection instruments that were used to collect data relevant to this question.

In Phase 2 the teachers encountered the journal prompt, “Describe the researcher’s role in the meeting.” Phase 3 has a similar prompt with, “Describe what the researcher is doing this week.” Each day that a GSP lesson was conducted the teachers completed a post-lesson reflection. In the post-lesson reflection the teachers were prompted with the question “Describe the role of the researcher in the lesson.” It was anticipated this series of questions would prompt data that could provide an analysis of the role the researcher was playing. A final set of questions was encountered in the exit interview regarding the mentoring relationship and the mentoring form of professional development.
The mentor log was maintained throughout the study with attention being paid to three areas. First, mentor action, or what the mentor was doing during the learning session, planning meeting, or teaching episode. Second, the mentor log documents the GSP activities that were selected and comments on issues of implementation. Finally the mentor log contains suggested action to be taken by the mentor prior to the next session. For example, the mentor developed a few of the GSP labs and this activity was documented in the mentor log.

Data Coding and Analysis

Data analysis began with the researcher transcribing the audio taped summer learning sessions, the fall planning meetings, and the exit interviews. Prior to initial coding of the data, the researcher reviewed procedural guidelines for coding qualitative data (Miles & Huberman 1994), which prompted a review of the research questions, and a review of codes that were hypothesized in the research proposal. From this activity the researcher developed a list of initial codes to be used with each section of transcribed data. The researcher added and combined codes as necessary. The unit of analysis in the transcribed text ranged from a clause, or whole sentence, to a group of sentences. The units of analysis in the videotapes were distinct discussions on a specific topic, example, or math problem. These would range from a few sentences to a full page of text.

Phase 1 data included videotape of the teachers’ classes, a classroom lesson record which recorded activity at five minute intervals, lesson plans and reflections, a belief survey, and the van Hiele test. The researcher viewed the videotapes and reviewed the classroom lesson record and found distinct patterns within the areas of goals of instruction, role of the teacher, role of the student, method of instruction, and curriculum. While the Classroom Lesson Record reflected these patterns, the researcher decided that actual sections of discourse between the teacher and students provided better support for the analysis that was being made. The decision then was to select one videotaped lesson per teacher that would provide the bulk of the material to be
described or transcribed. The May 21st first- and fourth-hour lessons were selected because both teachers were covering almost identical material and these particular lessons provided examples of what was seen across the other observed lessons and could be categorized as typical lessons. The belief survey was analyzed in light of what the teachers demonstrated (through their teaching practice) as their beliefs in the classroom. The van Hiele test was graded according to its protocol. Information from this test was also analyzed with regard to what the teachers demonstrated in the classroom.

Phase 2 data were of two types, the mentor log and the transcribed audiotape from the learning lessons. The researcher did not audiotape the first day of the summer session because she did not anticipate the teachers’ discussion to project into issues of teaching with Geometer’s Sketchpad. The researcher made general notations of what was discussed that day and the following two days were audio taped to preserve what was discussed. The data from the mentor log and the transcribed learning sessions were coded from a list of preexisting codes. The codes were modified and the sections were coded a second time. The sections of text were then separated and grouped by code. Original codes were combined which resulted in four distinct groups of coded text.

Phase 3 data included transcripts of three planning meetings, the teachers’ lesson plans and post-lesson reflections, the videotapes of three Geometer’s Sketchpad lessons per teacher, the Classroom Lesson Record for these recorded lessons, and the mentor log. Again the researcher developed a list of codes (including the codes from Phase 2 data) to use on each type of data. The data were reviewed and the list of codes adjusted to include patterns that had emerged in the new data. The researcher coded each set of data at two distinct time periods to determine the stability of the codes being used as well as code-recode reliability. This resulted in the modification of a few codes. Again, each section of text was separated and grouped by code. Examples of student work that were collected in phase 3 were used by the mentor during the study but were not used in the final analysis of the two teachers.
The final set of data from Phase 4 included a repeat of the van Hiele test, the belief survey, and the exit interview. The van Hiele test was again graded according to its protocol. The pre- and post-belief surveys were compared and the researcher highlighted statements in which the teachers changed their level of agreement (or disagreement). These highlighted statements then became part of the discussion in the exit interview. The exit interview was coded using codes that were developed in the previous phases of the study. The interviews were coded at two distinct periods in time and the text grouped by code. A mathematics education researcher familiar with the project validated the researcher’s findings.

Limitations of the Study

This study focuses on describing the changes, if any, a teacher makes when integrating the use of a dynamic geometry program into their existing curriculum. The intent is to examine and describe the change process. While research supports such a shift in the high school curriculum, the issues of student achievement or student use of the software take a secondary role, thus the findings can only be applied to teacher change and not student learning or achievement.

A second limitation is the fact that change in a teacher’s practice may take years, while this study looks at change over a six-month time-span. It is assumed that the mentoring relationship may accelerate a teacher’s inclusion of GSP activities into the existing classroom practice, thus provoking issues of change that can be observed within a narrower timeframe. However, it is likely that only a longitudinal study would be able to document change over a period of years, which extends beyond the resources available for this study.

This study does not intend to imply that the findings with regard to the participants of this study can be generalized to all teachers. The teachers in this study are unique. However, the teachers are exemplars of practicing teachers and the descriptions of their change process may be
similar to those of other teachers given similar circumstances and similar professional development experiences.

A final limitation is the use of the researcher as a mentor. The quality of the research depends directly on the knowledge, skill, and interactive abilities of the researcher, because the researcher was one of the data collection instruments and one of the treatments in the study. Care was taken in the reporting of the results to clearly document action taken by the mentor. Care was also taken to use data sources from the participants and use the mentor log only to support findings. As previously stated a mathematics education researcher familiar with the project validated the researcher's findings. This was done, in part, to detect instances of researcher bias.

The following chapters will discuss the results of this study. Chapter IV will discuss the Baseline and Learning phases. Chapter V will discuss results from the planning meetings from the Plan/Teach phase. The implementation of GSP as well as the Follow-up Phase is reported in Chapter VI. In Chapter VII the main results will be summarized, and related to current research, followed by recommendations regarding future research.
CHAPTER IV: BASELINE TEACHING OBSERVATIONS AND LEARNING GSP

This chapter provides a pre-intervention description of the two mentored teachers involved in the study through the analysis of observed class episodes, a belief survey, and van Hiele test. Following the pre-intervention description is the discussion of the Phase 2 summer sessions where the two mentored teachers were introduced to the basics of the Geometer’s Sketchpad program.

The first research question required the analysis of the teachers’ typical teaching style, beliefs, and knowledge prior to the introduction of the summer intervention, which was the mentor introducing the teachers how to use Geometer’s Sketchpad. [The analysis is considered to be a “snapshot” of the teachers’ pre-intervention behavior since it was drawn from a limited number of observations during a single month.] The snapshots of each teacher are important because the teachers’ knowledge and beliefs regarding the teaching and learning of mathematics determined, to a large extent, how they, in turn, implemented Geometer’s Sketchpad into their teaching.

The summer session data is included in this chapter as well. The main result of this phase of the study was the revelation of certain forces that the researcher identified as influencing the teachers as they began to make decisions regarding how they would implement Geometer’s Sketchpad.

The participants involved in the intervention were Alice and June. Alice had taught high school geometry for two years and came to teaching mathematics after a 20-year career as a secretary. June had taught high school mathematics 21 years. The researcher was also a participant in this study beginning in the summer sessions.
A Snapshot of Alice

Classroom Observation

In May of 2003, Alice was observed teaching geometry on four separate occasions. Each class period lasted 90 minutes and was comprised of up to five distinct activities. The five activities (described in detail below) include the warm-up, reading comic strips, grading and review of homework, the discussion of new material, and group time to work on the new homework assignment. Since only one of the four observed classes included the group homework time, this section is not included in the following analysis. Alice explained that she usually had group time but it was the end of the year and she was trying to finish up the material she needed to cover. While the majority of the transcribed selections originate in period 1 on May 21st, other class periods may be referenced. The researcher acknowledges that while it is impossible to thoroughly describe Alice’s teaching from four classroom observations, the following sections will illustrate a typical class period, providing a snapshot of Alice’s teaching practice.

Warm-Up

Each class period began with a prompt to the students to use their warm-up page to complete the problems listed on the overhead. A warm-up consisted of two problems asking the students to categorize triangles by both sides and angles. The two problems had three side measurements and two angle measurements for each triangle. Alice read the directions to the class and the students began to work with a low level of talking. At one point Student 1 (S1) challenged Alice on the angle classification of the triangles and said the problems could not be answered. Alice reviewed the definition of obtuse and acute triangles and S1 proceeded to complete the problems. Students were given about five minutes to complete the two problems and then asked to “trade [papers] to the right.” Alice then worked out the problems on the overhead, giving instructions in a rather quick pace. The following is the explanation that was
given on the acute classification for the first triangle, with the comments again originating with
S1. This example illustrates Alice’s desire to make math easier for her students by teaching a
short cut.

A: You could add up the two, subtract it from 180 and see if it is greater than 90 but the
shortcut could be, if we know we have to have an angle greater than 90, could that other
angle, if I even just add 90 to that I get 170, well the other angle is 37 so it is going to be
greater than 180 so I know it’s not obtuse, so it’s acute.

S: That’s the easy way? It seems kind of hard to me.

A: Or you could add and subtract from 180 and see what it is.

S1: That’s what I did, a lot easier that way...

The classroom atmosphere during warm-ups in three of the four class periods was usually
quiet but this particular class had a group of very talkative boys which may have been due to the
class make up of thirteen boys and only three girls. When questioned on the large number of
boys Alice confirmed that no one was absent and the gender balance for that particular class was
very lopsided.

Grading of the warm-up finished within the next five minutes and the students returned
papers to the owners. The warm-up sheets were routinely turned in on the last class meeting of
each week so scores could be recorded in the grade book (observed on Thursday, May 22). The
purpose of the warm-ups was to review material that would be on the GeoCap test. The GeoCap
test refers to the state’s end of the course assessment for geometry. Alice stated at two different
times “this is a review for the GeoCap.” On the following day the warm-up consisted of
identifying types of parallelograms and again the caution was made that they needed to know this
for the GeoCap. Out of the four classes observed, the warm-up usually lasted ten minutes but on
May 27th it lasted for almost 20 minutes.

Comics

On three out of the four days that Alice was observed, she displayed three to four
cartoons on the overhead as a transition from the warm up activity to the homework activity.
Over the years Alice had collected comics that had to do with mathematics, made overhead
copies of them and in some cases had colored them. Alice would explain some of the vocabulary in the comics that the students might not understand and then made sure that they understood the punch line. During the class on May 21st Alice and a group of boys talked freely on the content of the comics. It was a relaxed time and the students seemed to enjoy it as was evidenced by all students actively reading the comics.

Homework

Following the five-minute comic activity the students were prompted to grade homework. Students in the third-hour class had their homework checked for a completion grade while they completed the warm-up. The first-hour class followed an alternate grading routine that was observed again in one other class period. The first-hour students were again asked to “trade [papers] to the right” to grade their peers’ homework papers. Alice selected ten problems from the homework to be graded for a total of 10 points. The method for doing this was to write the page number, problem number, and solution on the overhead. In this particular class the boys challenged Alice on three of the answers that they recalled from their own papers or the ones they were grading. The first student was told that they would go over his particular solution during the homework review and Alice wrote down the student’s answer for future reference. The second challenge came from three different students as follows.

S1: What if it said 30, that is close?
A: No that would be wrong.
S2: What if it said 38?
A: 36 is the only answer.
S3: What if it is 36.1?
A: No it has to be 36.

A final dispute on the homework concerning when and how to round solutions provides an example of Alice answering a direct question incorrectly.

A: 2 is not the same as 2 1/3 ... if the decimal carries out to the hundredths or the thousandths, then round that part.
S1: So the whole number and the first decimal point should be the same, right?
A: beg your pardon?
S1: The whole number and the first decimal point should be the same, right?
A: Yes, pretty much ... I will try to get one on the [chapter] test that you will know very distinctly, you won't have that problem.

After the homework was graded, S1 joked with Alice on the fact that his homework grade was low, due to the problems she selected to grade. While on the surface, he appeared to be a lazy student who was trying to waste time; in fact, this student remained highly engaged during the entire lesson, often pointing out errors and making very intriguing comments as will be shown in later sections. Alice was not bothered by S1's remarks and began to record the homework grades. Her routine was to call on the students, they would give her their score out of ten and she would record it in the grade book. This particular assignment resulted in scores for about half the class from zero to two points, three students with three or four points, and five students with six through nine points. This indicates that at the most, only half of the first period students completed the homework assignment.

Alice proceeded to display an overhead sheet that contained all of the solutions for the homework. As students graded their own papers she repeatedly removed the answer sheet and worked out selected problems on a clean overhead sheet. This checking and review of the homework problems lasted from 30 to 40 minutes in each of the observed classes. Alice would go back and forth between asking specific students and the whole class if they had certain homework problems correct or finished. She would then say "ok I will do that one" and she worked through the problem on the overhead. In some cases, certain problems were indicated as ones to know because they would be on the test.

Of particular interest during the homework review was the stress Alice placed on the problems that would be on the GeoCap test and how students would recognize them. On this day the homework covered the following topics; side splitting theorem, geometric mean, and two-step problems where they used the Pythagorean theorem as well as the geometric mean. The following excerpt illustrates the discussion used to explain the first important homework problem.
A: My question to you would be did you get [problem] three right because that might happen to be on the test ... I got 7.5 ... For three the ratio is 2 to 5, for three it's 2 to 5, 2 to 5 equals, stay in the numerator, keep consistent with the smaller triangle, 3 to, and then your gonna find CE. The key is to stay consistent to the way you set it up. I have, in the numerator I have the smaller triangle and the denominator is the larger triangle. There's another way that you could write it too.

After going over three or four more problems it became apparent to Alice that not very many students had done the homework so she asked the class “At this point who has the homework so I know who I am talking to.” Three students raised their hands. After discussing a few more solutions a student was asked to open his book and follow along.

The next section of explanation began as follows. “Geometric mean – ok who has this section done? One, two, ... only two of you did this ... that is not a good percentage out of this class.” The following example illustrates the discussion of a homework problem on the geometric mean. Due to the number of responses observed, more students were engaged in the discussion than the two that completed the homework.

A: What did you get for f) Tiffany? ... ok I'll go over it ... one of them is SQ ... I'll go over it. Right triangle, the altitude to the hypotenuse, remember it goes through the 90 degrees and goes over to the hypotenuse, that's why it's called the altitude to the hypotenuse.
A: For f) they say RS is the geometric mean.
S2: What is the geometric mean?
A: The geometric mean, actually in simple terms it is the geometric mean because it is; a over g equals g over b, remember that a over g equals g over b, so that means you have two triangles that are sharing the same side, of the geometric mean. So RS is the geometric mean, if I go to RS right here, what two triangles would share that side?
Choral response: A variety of answers.
A: Ok how about if I turn it that way maybe, can you see it that way?
S3: Is it a over b or a over h?
A: a over g, see it is on the first page of that section.
S2: The geometric mean is the line that is shared by two triangles?
A: The side that is shared with two triangles.
A: If RS is the geometric mean then I substitute it in for g in this proportion, so my proportion would be RS is here and then I know RS is going to be in the numerator, but I need to find the other two sides then?
S1: You need to find a?
A: Ok you said there are two triangles right, RS, this is the side, and where are the two triangles here?
S1: And b?
A: For the two that I need to fill in here, the way I usually tell my students is, well which ones do you see?
Students: SQ
A: SQ? not SQ because SQ is for the entire triangle, just the big triangle.
Students: No PQ is the entire triangle.
A: Ok yes so SQ and PS, so SQ and PS [are the extremes]
S3: Can we do any of the other ones like p) or g)?
A: Uh g) we’ll do g). Ok so g) is, and you have two questions on the GeoCap on this; you really need to know this, and how to set up your proportion.

Alice solved two more homework problems on the geometric mean before going on to a section where the students needed to use the geometric mean and the Pythagorean theorem on a two-step problem. A student noticed that when Alice drew in the altitude it did not go through the corner of the right triangle tic mark. Alice pointed out it only had to go through the 90-degree box, not the corner of it. The following example illustrates Alice reinforcing the way to recognize when a geometric mean problem was being presented. This excerpt illustrates her belief that procedural knowledge is important.

A: Remember when on the test whenever you see a right triangle with a line going through that 90-degree box over to the hypotenuse you are going to have to do something with geometric means, and this is geometric means. So keep that in mind, when you see that line going through the 90 degree box, geometric means, geometric means is a over g equals g over b.

The next problem that was worked out in class involved the repeated use of the geometric mean and a mistake was made in the set up of the first proportion. S1 caught the mistake and asked for an explanation since the equation on the overhead did not match the solution that was written. The mistake was uncovered and Alice was impressed that S1 was paying close attention and had caught the error.

The last homework problem involved a sketch of a person using a book to site the height of a tower. By using right triangles and known distances the solution was found. S1 challenged the situation saying “you couldn’t do that in real life.” Alice smiled and said that yes it really does work, took out a book and sited the height of the clock. She invited the skeptic boys to come up and see that it really could be done. A similar scenario played out in the following third-period class. The students enjoyed acting out the problem and Alice seemed pleased to show that geometry really is useful.
New Material

Upon completion of the homework review the class transitioned to taking notes on the new material. Students were encouraged to take out a blue sheet of paper that contained formulas they had learned. They were cautioned that they would have to use a state-produced formula sheet for the GeoCap test but they should still keep on adding to their blue formula sheet. Students were asked to volunteer for reading and many of the boys and one girl volunteered for this. It was routine for Alice to offer candy as a reward for student reading. Alice later explained that her decision to spend class time having students read out-loud was a result of a school corporation focus on reading across the curriculum.

During the remainder of the class period the students took turns reading from the text on the topics of 45-45-90 and 30-60-90 triangles and the theorems for their respective side measurements as well as the next section of the book on the topic of tangent of an angle. In general, after a section was read by a student Alice would go over the book's example in detail.

Alice began the discussion of the isosceles right triangle with an illustration from home and the measurement of fabric. She related that her daughter had purchased fabric and wanted to know if it was a square so she could make a poodle skirt. Alice asked the class how they could fold the material in order to know if it was a square. (The abbreviation “S?” is used for an unidentified student who responded.)

A: I want to know if the sides were the same length. So what would you do?

Students provide many responses.

S?: Fold it in half.

A: Fold it in half but fold it in half which way? On the …

S?: The long way.

A: The long way, which is the …

Female student: The hypotenuse.

A: The hypotenuse, very good, yes the diagonal, fold it on the diagonal. And so she folded it on the diagonal and low and behold it was a perfect square…. So I use my math at home.

The folding of a square was demonstrated on the overhead with the result being the diagram of an isosceles right triangle. The theorem was then written on the overhead.
Isosceles Right Triangle
Leg = x
Hyp = x \sqrt{2}

These formulas were then used to solve the example problems in the book. Alice prompted the students that if a leg was given what would the hypotenuse be and, if the hypotenuse was given what would the leg be.

A: [Calls on student 5] If my leg equals 5 what is my hypotenuse, of an isosceles right triangle, using that formula, remember now my leg is x [points to the theorem on the overhead] so what is my hypotenuse? [Pause] What is my hypotenuse?
S5: What is your hypotenuse?
A: Yes what is the length of my hypotenuse, if a leg is x, what would I do for the hypotenuse?
S5: Put 5 in for x.
A: Yes, 5 times the square root of 2. Yes that’s what you do…

Alice followed the example with an explanation of the isosceles right triangle theorem proof that is based on the Pythagorean theorem. She stated that you could use the Pythagorean theorem to solve these types of problems and then stopped part way into simplifying the equation and stated, “it gets really complicated, but it’s just easier to remember the [new] theorem and use your blue sheet.”

The next example showed the students how to rationalize the denominator of a fraction. A caution was given regarding never leaving a square root in the denominator of a fraction. An example was modeled by Alice, which demonstrated rationalizing the denominator in a problem that used the isosceles right triangle theorem. The class was informed that they would need to know this skill for Algebra II.

The reading continued with the 30-60-90 triangle and again the side length theorem was listed on the overhead as:

30-60-90 Triangle
Short Leg = x
Long Leg = x \sqrt{3}
Hyp = 2x
Alice worked out problems that involved finding the measurements of two sides given a third side. Students were called on and prompted to answer by Alice pointing to the overhead theorem in a manner similar to what is recorded above for the isosceles right triangle.

The students continued taking turns reading the section on tangent. They were asked to take out their calculators and to make sure the setting was on degrees. The students were asked to punch in the tangent of 26 degrees and helped each other until each one had the correct answer. Alice then completed a word problem that involved finding the height of a flagpole, using the angle of elevation and the distance to the flagpole. Alice focused on how to recognize that a problem calls for the tangent function and how to plug numbers into the function correctly.

Following this example, the discussion moved to the inverse tangent. Alice completed an example and the students followed along with finding the inverse tangent of 5/12. S1 caused an interesting interruption as Alice tried to finish the book material for the day.

A: There is one more section ... example three ... so get your books out, we still have five minutes ...
S1: Wait, hold on, what did that inverse thing do exactly? Mrs., what did the inverse thing do?
A: It gives you information.
S1: On what?
[A female student goes over to S1 and quietly explains it.]
S1: Oh SHHHHHHHHHH, that's neat.
A: It gives the measure of angle D.
S1: It gives you the measure of the smallest angle?

Much of the final discussion focused on when to leave the square root in the solution so the answer was “exact” like leaving the square root 3 in the solution. At this point S1 was overheard to say, “Where did they get the 3 at?” Unfortunately he did not pursue this line of questioning. Alice then cautioned the class that she could not loan out calculators and that if a student did not have one he would have to use the trig table. This prompted a hunt through the book for a trig table, which was not found. One student suggested getting it off the Internet.

With the conclusion of the discussion of tangent Alice passed out a worksheet as part of the
homework assignment and cautioned not to forget the book problems. The final activity of the class period was to offer the candy jar to the students who had volunteered to read that day.

Discussion

The following discussion will focus on the five areas as outlined in the methodology: goals of instruction, role of the teacher, role of the student, method of instruction, and curriculum.

Goals of Instruction

Alice appears to have the following goals for her instruction.

- Cover the material in the book
- Prepare the students for the GeoCap test
  - Recognize visual cues that indicate an appropriate theorem or formula
  - Memorize formulas and theorems
  - Apply formulas and theorems – accurate answers
- Exposure to the use of mathematics outside the classroom
- Student reading

An analysis of the observed class periods indicates that Alice made sure that she covered the material in the text and gave many examples of each topic. The number of topics and the frequency of examples that are worked out in detail, in each phase of her teaching, provide support for this goal. In addition to coverage of the text, Alice repeatedly points out what types of questions are going to be on the GeoCap, which indicates that student preparation for this test is a definite goal of hers.

In regard to the use of theorems and formulas the following pattern was observed in Alice’s teaching. She promoted the memorization of visual cues that indicated which formula or theorem to use. This was especially noticeable in the review of the geometric mean homework. Following this, the students were drilled on the formula or theorem, as seen in the sections on the geometric mean and the use of the special triangles theorems. This was followed by stress on correctly substituting values into the selected formula or equation in order to produce the correct answer.
To illustrate that mathematics can be used outside of the classroom Alice recalled a sewing problem that also produced an introduction to the discussion of the isosceles right triangle. A final goal for Alice was that her students read out loud in class. Students were given the opportunity to volunteer to read and many took her up on this. The students that read were rewarded for their effort with a candy treat at the end of class.

**Role of the Teacher**

The following points describe the role of the teacher for Alice.

- Directly model the material to the students
- Make mathematics class fun
- The teacher is the authority in the classroom
- Give short cuts to make math easier

Alice closely directed every activity with the exception of group work. She used the overhead projector to complete each example that is gone over in class. Very rarely did she ask students to explain what they had done or would do on a given problem. An interesting observation was made regarding student reading. While students were asked to read the text and examples, when it came time for an explanation this became the job of the teacher.

Alice appeared to enjoy teaching geometry, and maintained an informal atmosphere in the classroom. Alice smiled and laughed often during the class period. She used math comic strips to create a fun atmosphere.

While the classroom atmosphere was informal, Alice maintained a position of authority. Her form of direct teaching indicated that she did not choose to share control of the instructional process with her students. It is not clear from the observations why Alice did not allow students to model solutions, answer other student’s questions or why she only used low-level prompts when presenting new material. She may have thought her students were not capable of modeling solutions or answer each other’s questions. An alternate explanation is that she believed these activities have to be performed by the teacher as part of “teaching.” It was observed that Alice
always had an answer; even if she was incorrect she had an answer. Since the role of the researcher was that of observer, not evaluator of Alice’s teaching, the researcher did not intervene on those occasions where Alice’s incorrect answers, or mistakes, went unchallenged by any student and were left as facts. On the one hand, in the role of the observer, the researcher was very aware that an evaluation of Alice’s teaching was not a goal of the research study and evaluative comments could potentially strain the mentor-teacher relationship that was forming. On the other hand, the researcher was in a position of supporting Alice and made a suggestion to her with this attitude. For example, the researcher illustrated that the angle of elevation should be drawn from a person’s head up to the object they are sighting, as opposed to drawing it from their feet. This suggestion was incorporated into the appropriate problem in the following class period.

A final observation that was made regarding Alice was her attempts to teach short cuts. She sincerely wanted her students to be able to succeed in mathematics and would stress short cuts that she had learned to use. This was illustrated in the classifying an acute triangle warm-up problem.

**Role of the Student**

The students in Alice’s classes appeared in the observations to be very passive with an exception of a few students in each class that would carry on open dialogues with her. Some students were allowed to sit the entire period and do nothing. Since students were not interviewed it is unknown if they were listening to the lesson or not. A few students were highly engaged and received a lot of attention from Alice. In each class observed, the most talkative students were males. Students in Alice’s classes assumed the following roles:

- Bystander
- Passively involved
- Aggressively involved
- Source of knowledge
A few students, labeled the bystanders, were routinely observed just sitting at their desks and, at times, were sleeping. They were quiet and did not disturb class and for the most part were allowed to continue in this pattern. The second type of student observed was the passively involved student who was actively watching and writing down information but would not volunteer information. On a rare occasion one of these students would be called on to answer a question. A few of these students would be among the ones that would volunteer to read out loud.

The final group of students was highly engaged in the classroom discourse. These students followed each example and were quick to both ask good questions and point out mistakes that were being made. Unique examples of this are the questions and comments made by S1. He obviously did not do his homework but remained very engaged during the entire class period. He also attempted to push the discussion beyond a procedural understanding and a computational level. He stopped class to ask what the "inverse thing is" and was overheard contemplating the origin of the square root of 3 for the longest leg of a 30-60-90 triangle.

While students were rarely put into positions of leadership or treated as a source of knowledge by Alice, observations of the classes indicate that at times students would just turn and help a classmate. This was seen when the female student went over to S1 and explained the inverse tangent function.

Method of Instruction and Curriculum

The method of instruction for Alice was primarily lecture. While there was a lot of informal talking going on, the instruction would be labeled as direct teaching with a focus on mathematical procedures.

The curriculum for the geometry classes consists primarily of the University of Chicago School Mathematics Project Geometry (1998) text and their supplemental worksheets. No other curriculum sources were observed in the limited observations in May.
Teacher Beliefs and Knowledge

An analysis of the beliefs survey (Appendix C) indicates that in some areas Alice’s beliefs align well with her observed behavior. Table 6 displays three of the survey items that were observed to be in agreement with the snapshot that was presented earlier.

Table 6. Alice: Teacher Beliefs

<table>
<thead>
<tr>
<th>Your Views about Mathematics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2. A lot of things in math must simply be accepted as true and remembered; there aren’t really explanations for them.</td>
<td>Agree</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Learning Mathematics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7. For students to get better at math, they need to practice a lot.</td>
<td>Strongly Agree</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teaching Mathematics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18. Teachers should not necessarily answer student’s questions but should let them puzzle things out themselves.</td>
<td>Disagree</td>
</tr>
</tbody>
</table>

These items were ranked consistent with what was observed in class. Alice did not spend time developing mathematical ideas; she merely presented them as facts the students had to remember for the test. The review of homework was observed to be extremely repetitious as students would see the same problem type over and over again, which illustrates the role of practice in the learning of mathematics. In the teaching of mathematics Alice disagreed with the survey statement regarding answering student questions and letting them puzzle things out for themselves. This was also consistent with the classroom observations in which she answered every question put to her. The researcher inferred from this behavior that Alice felt she had to answer every question, even if she did not know the correct response.

On a few of the survey items a mismatch was found between Alice’s stated belief and her classroom practice. In the Learning of Mathematics section Alice agreed with statement 13) 

Average mathematics students, with little guidance, should be able to discover the basic ideas of mathematics for themselves. The use of discovery learning was not evident in the observed class periods. Furthermore, the reliance on direct teaching and a lecture format potentially deprived the
students of the chance to discover anything for themselves. The instructional method used implied that Alice believed that students would only know what she told them.

In the survey section on Teaching Mathematics there were two statements that appeared to be inconsistent with Alice's classroom practice. First, she disagreed with statement 15) If a student asks a question in math, the teacher should know the answer. There is a difference between knowing the answer and not choosing to share the answer and not knowing the answer. As was stated above, Alice always answered student questions and was never observed stating that she did not know.

The final discrepancy was found on statement 21) Teachers should spend most of their class period explaining how to work specific problems. To this item Alice disagreed. By looking at the time allocations for the classes observed, the majority of the class period was used to do just this, work specific problems.

In an analysis of any type the researcher must attend to the task of defending their analysis. An alternate explanation could be found with the items that were classified as discrepancies in the area of Teaching Mathematics if the observed teaching episodes were not a true representation of what takes place in Alice's classroom. Due to this early finding, instances where Alice's beliefs regarding her practice were not aligned with her teaching practice were explored in later phases of this study.

Knowledge of Geometry

In early June of 2003 Alice completed the van Hiele Geometry Test. This version of the van Hiele Geometry Test was developed to assess high school students' geometric knowledge. The researcher admits to going outside of this intended audience. However, the van Hiele Geometry Test has a high ceiling, due to the abstract nature of level 5, and one cannot assume that high school geometry teachers are at the highest van Hiele level. There is an absence of documentation regarding practicing high school teacher's van Hiele levels but Swafford, Jones,
and Thornton (1997) state, "Research conducted in the 1980's documented that both practicing and pre-service elementary teachers exhibited low levels of geometrical knowledge (Fuys, Geddes, & Tischler, 1985; Hershkowitz & Vinner, 1984; Mason & Schell, 1988; Mayberry, 1983)" (p. 469). Thus the van Hiele Geometry Test was selected to determine a baseline level for Alice's knowledge of geometry. Her score placed her at level four. The grading criteria place the score at the level where two out of five items were missed.

Alice was observed teaching at a procedural level when introducing and reviewing the material. This may be due to the goal that students only need to know procedures because that is what is on the test (GeoCap). By contrast, she may have believed they would not understand (or care) for an explanation, or she may have felt she just did not have time to spend on this.

A few examples that may shed some light on the depth of her understanding were observed in her explanation of the geometric mean and also when she attempted to develop the proof for the isosceles right triangle theorem and stated that it gets complicated. It appeared that, in these cases, when asked by students to go beyond the procedural definitions, she was unsure of the mathematics herself. Thus, in general, it is unclear if she is at a procedural level or just chooses not to go beyond it because of her students.

Summary

Alice was very conscious of the end-of-year state assessment. Her goals of instruction focused on problem identification, memorization of theorems and formulas, and the accurate completion of procedures to obtain the correct solution. Teaching math meant covering the material by direct instruction. Importance was placed on the demonstration and repetition of similar problems within each problem type. The teacher's role was to disseminate information and the student was there to receive it. The other teacher in the study, June, provides a contrast to Alice.
A Snapshot of June

Classroom Observation

June was observed teaching geometry on three different days in May of 2003. The class activities were a warm up (in two of three observations), a review of the homework, and a presentation of the new material. Students were not observed working in groups, or independently, on class work or homework. June pointed out she usually gave time in class for students to begin their homework, but it was the end of the year and she had to complete the chapter on triangles.

June's style of teaching was centered on students answering her prompts in each of these class activities. The examples that are transcribed are taken from the May 21st observation, with other class periods referenced as necessary. This class period covered a review of the homework on the geometric mean and the sidesplitting theorem, and also new material on the two special right triangle theorems. June paced her comments on the material so the students could easily follow her and she usually displayed adequate wait time when asking a question or prompting a solution.

Warm-Up

The class period began with a warm-up set of problems displayed on the overhead projector. June used the warm-up activity time to settle the students down and to take attendance. The problems were selected to provide a review for the GeoCap.

J: Work on the two problems up there, (pause) shhhhh, work on the warm up guys. This is one of the ones similar to a possible GeoCap exam so take it seriously. [Groans are heard from many students.]

The students are called together after a few minutes to go over the two problems, which were not evaluated for a grade. The problems involved categorizing two triangles by their side and angle measurements. (When it was difficult to identify which student was speaking the code S? was used. When a group of students respond the code SS was used.) Notice how June began the
warm-up activity by soliciting an approach instead of a solution. She took class time to follow a student’s thought pattern as he attempted to draw the triangle. When a different student determined the triangle was acute there was a class discussion on triangle vocabulary terms.

J: Ok, what do we have to do on these?
SA: Draw it.
J: Draw it?
SA: Maybe, I don’t know.
J: Ok, I always harp on you to draw a picture don’t I? So we can draw it if you want to.
SA: I don’t know, is there? (Pause) If I draw it, how do you want me to draw it? SB, why don’t you come up and draw it for us?
SB: Where at?
J: Anywhere, how about right there [indicates space on the overhead].
SB: How am I going to draw the 80 and the 37?
J: Can you approximate it? Ok, you are having trouble then doing it that way aren’t you, drawing without any tools?

[Class discussion ensues, involving many students, on how to draw the 80-degree angle and the need for a protractor.]
SB: Like that?
J: Ok, so now you got your 80-degree angle, ok now you’ve got your 80-degree angle, now what? Can we draw a representation of it without drawing it exact?
SA: Sure.
J: Ok, label this as your 80-degrees. Where would you draw your 37-degrees [angle]?
SB: Over here [Points to the end of a segment.]
J: Ok.
SB: It would be about right there.
J: Ok.
SB: That’s not right, no that’s not right, pick someone else.

[SB has drawn a fairly accurate acute triangle on the overhead.]
J: I see your 37. Now where are we running into trouble? Have a seat, thanks. Why are we running into trouble?
S?: We only have two angles.
J: We have two angles, do we know what the third one measures?
SC: Yes, we can figure it out.
J: How?
SC: Minus, add those two together and subtract from 180.
J: You add those two together and you get?
SS: 117
J: From 180 is?
S?: Sixty-three.
J: This would have to be a 63-degree angle.
SA: That is pretty close [referring to the triangle drawn by SB.]
J: Ok, so what’s that tell us about the triangle?
S?: It’s acute.
J: What, it’s acute, so it’s acute.
J: What about the measurements of the segments?
SC: It’s isos, it’s scalene.
J: How do you know they are scalene?
SC: Because none of the sides are equal.
J: None of the sides are equal.
SA: What is scalene?
J: Ok, so she is saying I have to use scalene over here, and acute up here.
J: Did we use the picture?
SS: Yes ... no, not really.
J: Could we have done the same without the picture?
SA: That’s what I was referring to at the start of the problem.
S?: What makes it scalene and acute?
J: Let’s run through these terms because some of you are having trouble with them.

June began a discussion of the triangle vocabulary with a focus on their textbook’s definition of isosceles and equilateral.

SA: Then why doesn’t isosceles and equilateral mean the same thing then?
J: It’s like a square and a rectangle. A square is a rectangle; a rectangle is not necessarily a square.
SA: So if you have two it could be either isosceles or equilateral?
J: If you have exactly two it is?
SC: Isosceles.
J: If you have three its?
SC: Isosceles or equilateral.
J: Isosceles and equilateral.

Discussion of the first problem concluded with a review of acute, obtuse, and right triangles.

June walked the students through the solutions for the second problem and the class quickly transitioned into a review of the homework assignment.

Homework

The homework activity began by June reading the answers to the assigned problems.

Students who were not finished or had incorrect solutions were encouraged to write in the correct answer with an ink pen so they could go back and use this worksheet to study for the chapter test.

June reviewed the theorems on the overhead with a diagram for reference.

J: I want you to look at page 761. I have a feeling that when we left here last time, especially the back page [of the worksheet] was going to be kinda tough ... I had a feeling that this was going to be a real tough one for us. Ok right triangle and altitude theorem. Let’s put that into, I kinda tried to do this last time so lets try to work on this theorem again, rather than putting this into the letters that they use, lets use some words here. Number 1: The altitude to the hypotenuse is the geometric mean of? If it is the geometric mean, remember we said if we set up a proportion, whatever is the geometric mean we are going to put in these two positions. So the altitude to the hypotenuse, I am
going to write altitude here, ‘alt’ for altitude, this only works in a right triangle, the altitude to, and it has to be the altitude to the hypotenuse, the altitude to the hypotenuse is the geometric mean of what ever these two are [pointing to segments] the segment to which it divides the hypotenuse, so this is the hypotenuse segment, that’s one hypotenuse segment, the second hypotenuse segment goes here, the two segments of the hypotenuse are your extremes, the altitude to the hypotenuse are your means [referring to one diagram but two written proportions], both times you put the same thing in here. That is the first part of that theorem.

June continued in this manner to explain the second proportion that could be used with the leg as the geometric mean of the hypotenuse and the leg adjacent to the hypotenuse. She concluded this review with two proportions written with abbreviations that indicate the correct pieces of the right triangles that need to be used in the two proportions. The following discussion involved deciding which proportion should be used for the problems that were on the worksheet.

J: Now I am going to leave that up there and I want you to have it on your desk to use as notes as we go over the problems. Now do you agree that if you get them set up in the proportions you can solve them? Cross-multiplying is not the problem.

June set up the diagram and given information for problem number five from the worksheet.

J: Now look at that and see what we have, which one of those words up there do we have here? If this is your right triangle, your big right triangle, what is [segment] MN?
S?: The hypotenuse.
J: It’s the hypotenuse, and MP is? a segment of the hypotenuse right. So lets look here, this one uses hypotenuse segment and the other hypotenuse segment, we don’t have that do we? I mean we can figure it, and we may need to, but with what’s given, that’s not what we have now. We have the segment of the hypotenuse, the hypotenuse, and we are finding a what? Everybody? What are we finding?
SS: A leg.
J: A leg, is there one up here that uses a leg, a hypotenuse, and the segment of a hypotenuse?

The proportion for problem number five was set up and quickly solved. The homework review followed this process for three more problems. June and the students debated back and forth which formula to use by examining which pieces of information were given and needed. An interesting line of questioning ensued when a student pointed out that parts of the hypotenuse are also legs of their respective right triangles. June continued to explain, using the vocabulary of the two proportions, how to decide what to call the pieces and how this helps determine which proportion to use.
The last few homework problems were what June called “two steppers” since they involved the use of the geometric mean proportion twice to find the missing segment’s measurement. After completing one of these problems a student pointed out that the Pythagorean theorem could have been used to solve the second part. June laughed and agreed with the student that, yes it would have been an easier way, and encouraged the students to use Pythagorean theorem when the opportunity presents itself.

New Material

The new material for this day consisted of the theorems for the isosceles right triangle and the 30-60-90-triangle. June usually assigned reading from the text, as part of the homework, with the dual purpose of incorporating reading into the curriculum and introducing the students to the new topics. While reading, the students were encouraged to place a yellow sticky note in their book any place where they did not understand the text and to ask questions about it during class time. Since she forgot to assign the reading for that day she had the class take ten minutes to read a section in the textbook on the special right triangle. After the silent reading from the textbook, June began the following review of radicals:

J: Ok, some of you, a place where you might get bogged down on this is seeing how they’re simplifying radicals. So I am going to take a moment to go over, to review with you how to simplify radicals. Look up here for a minute, \[\sqrt{72}\] is written on the overhead] up here, we had some bell work this year, that the topic was to simplify radicals, remember we said find a perfect square that will divide into 72 evenly. Think of a perfect square.

SS: Nine ... Eight
J: Nine times?
SC: Eight.
J: Now do any of those have a perfect square factor?
SS: Eight.
J: Eight has a perfect square factor of a ...
SS: Four.
J: Four which leaves me a two. So here’s my eight, here’s my nine. Square root of nine is?
SS: Three.
J: Square root of four is?
SS: Two.
J: Square root of two, you can't do so we are going to leave it there. That gives me six times the square root of two, six radical two. Now we could have also instead of saying nine times eight, what's another way I could have done that?

SS: Thirty-six times two.

J: Thirty-six times two, which is six radical two isn't it? So as long as you are taking out perfect squares and taking the square root of them, you'll be ok. Ok now another thing... is this kind of simplifying. We can't leave a square root, a radical, in the denominator, got to get rid of that [the fraction four over radical three is written on the overhead]. Does anybody remember how we do that? In your bell work earlier this year? I want to multiply both the numerator and the denominator by something?

SC: The square root of three.

J: By what, square root three? Because if I multiply both the numerator and the denominator by the square root of three, the square root of three over the square root of three is what number?

SS: One.

J: And anything times one is equal to itself. So when I do that, four radical three over, what's the square root three times square root three?

SS: Three.

J: Which is the square root of nine, which is three, so that gives me four root three over three.

June continued the review of simplifying the denominator with four more examples. The amount of prompting done by June decreased with each new example.

In the discussion that follows June used the general information from an isosceles right triangle and the Pythagorean theorem to develop the formulas hyp = x and leg = xSQRT(2).

J: Now I want you to look at this... this is the triangle that's on page 765, anyone have a question mark on your yellow sticky note?... How did they get c² = x² + x²? How did they get this?

SD: Pythagorean theorem.

J: Pythagorean theorem.

S?: What?

J: A right triangle. We are on page 765, x is the length of one leg, x is the length of the other leg, it's an isosceles triangle, the legs are the same size.

S?: Why did they leave them both x?

J: Why did they leave them both what?

SS: Both x's?

J: Because they are identical, they are the same size. Inaudible

J: That's why they are the same. It's an isosceles triangle, that's what we are talking about here. Now how did they go from x² + x² to 2x²?

SS: Combine like terms / Combine two x's.

J: Combine like terms, Ok, how did they go from c² = 2x² to this step? Oh they did not put that step in, that's an additional one I put in. What did I do? What did I do to both sides?

SS: Square root them.

J: How did I go from here [points to overhead] to here [points to next line on the overhead], what did I take out? Took the square root of x² because x² is a perfect square.

SA: What if x was like three?
J: What if the x was a three, then it would be three root two. What that is trying to show you by using a variable for your sides, length of the side, this relationship between the leg and the length of the hypotenuse of an isosceles right triangle is always the same. If I know a leg of an isosceles right triangle measures six then the hypotenuse is six root two.

To finish this problem the students were instructed to use a calculator to find an approximation for the solution. After completing this, one student asked why you don’t just use the Pythagorean theorem in the problem, as opposed to using the new formula. June stated that he could use the Pythagorean theorem but that the proof provides a short cut.

J: And SD we are not going to ask you to take it back to this each time, now that I have shown you that it works, it’s one of those things like proofs we did first semester. Once we proved that, um, you know parallel lines cut by a transversal and alternate interior angles are the same, we didn’t go back and understand it and prove it each time, we used it.

S?: So the hypotenuse equals whatever one of the legs is times the square root of two?
J: That’s it, if it is a 45-45-90 triangle.
S?: So that is just an equation that someone made up?
J: It always works, this is the explanation [pointing to the overhead].
S?: The square root of two is just something someone like just made up?
J: It comes from here. Whatever the length is, this says c is whatever the leg is times the square root of two. It is always going to work because we justified each step ... so now we can accept that this works.

It became obvious to June that although she had illustrated an algebraic proof, this particular student was not making the connection to the resulting formula. She tried to emphasize the role of proof in the introduction to the following discussion of the 30-60-90 triangle theorem.

J: Page 767, we are going to go through the proof because these are proofs you can understand. These are ones you can get. We are going to go through the proof one step at a time. If every step I say, you see that there is a reason for it, then that ending conclusion, we can accept, and then you don’t have to go back and have to rethink the proof each time. We can again, the results we can accept Ok, they give you triangle ABC, a 30-60-90 triangle.

The class discussion followed the text’s proof of the theorem for the 30-60-90 triangle with students supplying the reasons for each step of the proof. June also discussed how the 30-60-90 triangle fits in the equilateral triangle, which helps explain two out of three of the side lengths.

The final activity for the day consisted of June going through a few examples on the worksheet that was assigned as homework. She provided the hint of placing the side length pattern on the appropriate sides of a sketched triangle. This would provide the information
needed to help set up the necessary equation. Student responses to these examples were positive
“Is that all we have to do?”

Discussion

The following discussion will focus on the same five areas as outlined in the
methodology: goals of instruction, role of the teacher, role of the student, method of instruction,
and curriculum.

Goals of Instruction

The following goals of instruction are identified in the observations of June’s classes:

• Cover material in the text
• GeoCap exam preparation
• Student reading
• Teach for understanding

June reviewed and taught material that would be seen on the GeoCap exam. She also had
a goal of covering a set amount of material in the text. These two goals were observed in the
references to the exam in the warm-up problems and in the push to finish the chapter before the
end of the school year. An additional instructional goal was to stress reading as part of the
mathematics curriculum. June assigned reading for homework or gave time in class for reading
as described in the snapshot. The students were encouraged to place a sticky note on any section
they had questions on and then to ask those questions in class.

The final goal that was identified was to teach for understanding. June was very attentive
to overarching constructs like proof and the importance of geometric vocabulary and definitions.
This was seen in the conversation with the student who could not see where the isosceles right
triangle theorem came from when the proof had just been completed in class. Another example
would be the presentation of the geometric mean theorems. June took the time to rewrite the
proportions using right triangle vocabulary instead of meaningless variables. June also made connections between what students already knew and the new material.

June was observed asking a lot of questions. A frequency count of teacher questions, or prompts, indicates that during the fourth-hour class described in the snapshot June asked 34 questions during the warm-up activity; 44 questions during the homework review; and 94 questions during the discussion of the new material. An analysis of these prompts and questions reveals that over half were low level and required a response that was the result of either computation or inspection of a diagram. Less than half the questions were of the type observed in the warm-up discussion where June was eliciting the solution path taken by the students or guiding classroom discourse.

Role of the Teacher

The three observations of June’s teaching support the following roles:

- Mixed teacher/student-centered mode of direct instruction
- Teach for understanding: Expose how and why geometry works
- Sequence lessons into developmental chunks
- Acknowledged student’s knowledge – limited group of engaged students

June exhibited the behaviors that Artzt and Amour-Thomas (2002) categorize as teacher centered. June’s goal was to teach for understanding yet her behavior conformed to a belief that the role of the teacher is to transmit mathematical content, demonstrate procedures for solving problems, and explain the process of solving sample problems. June would set up a problem and then prompt students toward a solution. June took time to thoroughly explain the geometric mean theorem to her students with the belief that they could learn the content by listening to her and remembering what they were told.

Less frequently, June displayed behaviors that are considered student-centered. During the warm-up activity and again during the new material presentation June was observed guiding
class discussions. She solicited the solution path of the student as problems were worked out. Unfortunately not all of the students in the class were engaged in these discussions.

       June assumed that it is the teacher's role to explain how and why geometry works. Her approach to the isosceles right triangle theorem was to work, with the students, through a proof using the Pythagorean theorem. She began the discussion of the 30-60-90-triangle theorem by discussing how this triangle fits in an equilateral triangle. From there the students were prompted to recall what they knew about this figure. Related geometric properties were referenced to reinforce the concept that the short leg is opposite the 30-degree angle. For June, teaching mathematics did not center on presenting rules to be memorized but involved promoting understanding how the different concepts of geometry are connected.

       June was observed on two occasions breaking down the new material into developmental chunks. The first example was seen in the review of radicals and rationalizing the denominator that was provided prior to the introduction of the special triangle theorems. A second example was observed prior to the introduction of the trigonometric functions a week later. June spent time discussing the vocabulary terms adjacent leg and opposite leg, for an indicated angle. This provided the necessary background information for the students to be able to select and then make the substitutions into the correct trigonometric function.

       A final characteristic of June's role was that she did not present herself as the only source of knowledge in the classroom. When a student pointed out a different solution strategy for the two-step homework problems, June agreed with him. June asked questions and prompted responses from the class. Having students supply the solution strategy of the warm-up problem indicated that she valued their knowledge.

Role of the Student

Student action and behavior represented in June's classroom fell into the following categories:
• Actively involved
• Passively involved
• Mentally absent
• Source of knowledge – limited group of engaged students

Several students, in each class observed, appeared to be actively involved in the lesson. This group of students offered solutions and responses to June’s prompts as well as asked questions and made comments with the goal of working toward understanding. A few students were passively involved in the lesson. These students remained quietly engaged by watching and listening to the activity at the overhead. Another fairly large group of students can be categorized as mentally absent. These students doodled, wrote notes, or slept. The Classroom Lesson Record shows the following researcher notes taken about 35 minutes into the class period.

2:10 – 4 students are sleeping at this time, many students are doodling. (Engagement in the class is categorized as low).
2:15 – “Stand up and Stretch” – 4 had to wake up / “Calm down and get out sticky notes, read 13.5” (forgot to assign reading) – 2 students still sleeping. Got books out for students without books (CLR, 6-21).

As discussed in the section Role of the Teacher, students were considered a source of knowledge. Unfortunately, this role only applied to about a third of the students in the class.

Method of Instruction and Curriculum

June’s method of instruction included short presentations on new material, teacher directed work through sample problems, and short classroom discussions. This type of instruction could be classified as traditional in comparison to what is recommended in the NCTM reform initiatives. This form of instruction was effective with the students that could learn in this setting but students that chose a passive role were basically ignored.

The curriculum for the geometry classes consisted primarily of the University of Chicago School Mathematics Project Geometry (1998) text and their supplemental worksheets. No other sources were observed in the limited observations in May.
Teacher Beliefs and Knowledge

The teacher belief survey consists of statements in three areas: views about mathematics, learning mathematics, and teaching mathematics. June’s responses in the belief survey appeared to be closely aligned with her behavior as seen in the classroom observations from May. Sample responses from each of the three areas will be discussed.

In the first area June disagreed with statement 2) A lot of things in mathematics must simply be accepted as true and remembered; there aren’t really explanations for them. Support for this belief was seen as June repeatedly provided explanations and support for the geometric theorems that were covered in class. June strongly agreed with statement 5) There is more than one right way to get an answer in mathematics; and statement 6) Mathematics is not just a bag of tricks. June’s teaching acknowledged different solution strategies and did not stress tricks or short cuts. An alternate solution strategy was supplied by a student on what June called “two-steppers” and June agreed that it was a good use of the Pythagorean theorem.

In the second area of the belief survey June disagreed with the statement 9) If students get into arguments about ideas and procedures in math class, it can interfere with their learning of mathematics. While students were not seen arguing, June was observed to encourage discussion regarding student understanding, which at times involved different approaches to a procedure. June’s teaching also was observed to be consistent with her agreement on statement 12) How you get an answer is as important as whether the answer is right or wrong. June devoted the majority of the class time to the discussion and development of solution strategies.

In the final area of the belief survey, June disagreed with statement 15) If a student asks a question in math, the teacher should know the answer. June was not observed to be unable to answer a question that was posed to her by a student, but her introduction of the researcher and the video camera to her students was done in a manner that indicated she was involved in a project to help her learn something new and she was excited about that.
Two additional statements from this section were found to be consistent with the classroom observations. June disagreed with 17) Students should never leave math class feeling confused or stuck, and agreed with 18) Teachers should not necessarily answer student’s questions but should let them puzzle things out themselves. June revealed that the students had probably left the geometry class a bit confused when the assignment on the geometric mean was given. This indicated that she acknowledged that some assignments and topics would push the student into areas they cannot grasp in one class period. The final comment regarding not answering students’ questions and letting them puzzle things out for themselves forms the basis of June’s teaching style. When students would ask questions she would not directly answer the question but would ask the student what they thought or would back up and conduct a whole class review. She would, through prompts, find out what knowledge the student brought to the situation and then prompt them, or the class, to think of how they could answer the question. This was seen in the warm up activity on classifying triangles by side and angle measurements.

Knowledge of Geometry

In early June 2003, June completed the van Hiele Geometry Test. The grading criteria placed her score at level five. June’s teaching displayed characteristics from each of the van Hiele levels. For example, students were seen analyzing the properties of figures and learning the appropriate terminology in the class discussion on the geometric mean theorems, which indicates a van Hiele level two. Level four work was seen when June tried to point out to a student that by using the Pythagorean theorem she really had proven the isosceles right triangle theorem.

June displayed knowledge of student’s mathematical thinking in the different topics by breaking down the material into appropriately sequenced topics. For example, when introducing the special triangle theorems she began with a review of radicals. She was aware of what the students would encounter and broke off this chunk prior to handling the new theorems. In fact, a close examination of the transcript from the 21st indicates June had been reviewing radicals in
what she called "bell work" throughout the year, which is an indication of the attention she had paid to this particular topic and its role in her geometry curriculum.

The pre-intervention observations indicate that June made informed decisions regarding how content is sequenced for both the chapter, and the class period, based on her knowledge of student thinking. In the 2002-2003 school year, the pace of the blocked geometry classes was usually two new sections per day. June only taught one section on May 21st and chose to wait and teach all of the trigonometric functions, the next two sections of the text, on a later day. The trigonometry lesson began with a discussion of the appropriate vocabulary and formula for each function. The rest of the class time was devoted to prompting students on how to decide which function to use and how to use it correctly. The thoughtful sequencing of topics, both in the chapter and in the class period, indicates that June displayed a high level of knowledge regarding how students learn geometry.

Summary

June's teaching can be categorized as traditional. June was aware of end-of-year testing but directly prepping students for this test did not appear to be a primary goal in her teaching. June spent the majority of the class time engaged with students that were participating. The focus of the discussions was geometric vocabulary and relationships. She selected appropriate examples and prompted students to help supply a solution strategy. June displayed a knowledge of what students might have trouble with and had a pedagogical approach that let student thinking drive both the class discussions and her decisions regarding the pacing of topics.

Comparison of Alice and June

Alice and June displayed a few similarities within their goals of instruction and the roles that the students take in their classrooms. However, subtle differences were found within their goals of instruction, role of the teacher, role of the student, and method of instruction. A
The shared goals of instruction include covering the material in the textbook, preparation for the GeoCap, and the integration of student reading in the mathematics curriculum. Students took on very similar roles for each of the two teachers. Each teacher communicated with a select cohort within her classroom who aggressively sought information. Within each teacher’s classroom there were students who were watching and listening but did not engage in the ongoing discourse. Also observed for both teachers was a group of disengaged students with a few of June’s students sleeping through most of the class.

A few subtle differences in the goals of instruction for these two teachers were found. Alice tried to make math fun with the use of comics and meaningful by using a sewing example. Another goal was to help the students memorize visual cues and theorems. June did not refer to

<table>
<thead>
<tr>
<th>Domains</th>
<th>Alice</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goals of Instruction</td>
<td>Cover the material in the book</td>
<td>Cover material in the text</td>
</tr>
<tr>
<td></td>
<td>GeoCap preparation</td>
<td>GeoCap preparation</td>
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<tr>
<td></td>
<td>Be able to recognize visual cues</td>
<td>Teach for understanding</td>
</tr>
<tr>
<td></td>
<td>Memorization of formulas and theorems</td>
<td>Student reading</td>
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<td></td>
<td>Correct application of formulas and theorems</td>
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<td></td>
<td>Exposure to the use of mathematics outside the classroom</td>
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<tr>
<td></td>
<td>Student reading</td>
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<tr>
<td>Role of the Teacher</td>
<td>Directly model the material to the students</td>
<td>Mixed teacher/student-centered mode of direct instruction</td>
</tr>
<tr>
<td></td>
<td>Make mathematics class fun</td>
<td>Teach for understanding</td>
</tr>
<tr>
<td></td>
<td>The teacher is the authority in the classroom</td>
<td>Acknowledge student knowledge – limited group</td>
</tr>
<tr>
<td></td>
<td>Give short cuts to make math easier</td>
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<tr>
<td>Role of the Student</td>
<td>Bystander</td>
<td>Mentally absent</td>
</tr>
<tr>
<td></td>
<td>Passively involved</td>
<td>Passively involved</td>
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<td>Actively involved</td>
<td>Actively involved</td>
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<tr>
<td></td>
<td>Source of knowledge</td>
<td>Source of knowledge – limited group of involved students</td>
</tr>
<tr>
<td>Method of Instruction</td>
<td>Direct Instruction</td>
<td>Short presentations</td>
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<tr>
<td></td>
<td>Work through sample problems</td>
<td>Work through sample problems</td>
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<tr>
<td></td>
<td></td>
<td>Discussion – limited group of involved students</td>
</tr>
<tr>
<td>Curriculum Source</td>
<td>Textbook</td>
<td>Textbook</td>
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</table>

The shared goals of instruction include covering the material in the textbook, preparation for the GeoCap, and the integration of student reading in the mathematics curriculum. Students took on very similar roles for each of the two teachers. Each teacher communicated with a select cohort within her classroom who aggressively sought information. Within each teacher’s classroom there were students who were watching and listening but did not engage in the ongoing discourse. Also observed for both teachers was a group of disengaged students with a few of June’s students sleeping through most of the class.

A few subtle differences in the goals of instruction for these two teachers were found. Alice tried to make math fun with the use of comics and meaningful by using a sewing example. Another goal was to help the students memorize visual cues and theorems. June did not refer to...
uses of mathematics outside the classroom and did not display a goal of having fun. She did have a goal of teaching for understanding and tried to accomplish this through in-depth explanations and class discussions.

In the role of the teacher both Alice and June can be categorized as traditional but Alice used only a direct instruction teaching method. She was the authority in the classroom and provided all of the instruction and modeling of problems. Alice attempted to make math fun and easy for her students and stressed memorization and short cuts. June used short lectures as well as discussions that focused on vocabulary and connections within geometry. June attempted to make geometry make sense for her students through detailed explanations and teacher guided sample problems.

Subtle differences were found between the role students were allowed to take regarding authority and knowledge. Alice did not directly put students in positions of authority while June actively elicited information from students (who choose to participate) and acknowledged their mathematical knowledge. Alice did not ask students to supply their solution strategies where June did. Alice answered student questions herself while June reflected student questions back to the class thus acknowledging their ability to answer each other’s questions.

Many of the differences noted in the method of instruction use by the two teachers have been presented through the discussion above. Alice used direct teaching with little student involvement while June made an effort to include students in the instruction process.

These similarities and subtle differences form the backdrop, or the setting, for the rest of this study, which involves each teacher interacting with the researcher, each other, and their students, as they were introduced to GSP during the summer of 2003 and implemented GSP into their teaching in the fall of 2003. The following section of this chapter will describe and interpret the teachers’ activity during the summer of 2003.
Summer Sessions

The summer sessions took place on three half days in June 2003. The teachers met with the researcher (who will now be referred to as the mentor) at the local library. Alice and June were able to set up their laptop computers, which had GSP version 4 installed on them. The format for the training sessions was to work on Geometer’s Sketchpad activities selected by the mentor and let the teachers discuss their concerns and ask questions as they came up. The teachers were encouraged to work on specific chapters at home between meetings.

I modeled the labs that had been done [in the pilot study] from the publisher and also labs that had been hand made. For each of the three days I had a loose agenda that we followed, along with breaks where we would just talk (Mentor Log, 7-2).

The teachers’ questions and concerns would provide topics for discussion as well as guide the mentor in the planning of the following session. Many activities were selected from the following chapters of Exploring Geometry with the Geometer’s Sketchpad (2002).

- Chapter 1: Lines and Angles
- Chapter 2: Transformations, Symmetry, and Tessellations
- Chapter 3: Triangles

In addition to the published GSP activities about half of the labs used in this study had been developed by either Jacob (pilot teacher) or the mentor. A specific list of GSP topics and labs that were used in the summer sessions is presented in Table 8. On the last day of the summer sessions Jacob was able to join the group for a 30-minute question and answer time. This gave the teachers a chance to revisit their concerns and receive information from someone other than the mentor. Jacob was not present for actual work with GSP.
An interesting observation from the first session concerns the motivation of the participants. The mentor planned the sessions with the goal of providing instruction on GSP to show both how the program works and how it can be used as a tool for exploration, conjecture, and communication. The teachers approached the sessions by reflecting on their role as teachers, using this new technology.

Sessions covered an introduction to the program. As much as I wanted to focus on the software, the teachers wanted to focus on the curriculum (Mentor Log, 7-2).

The mentor shifted her goals to include more discussion on the issues that were brought up while at the same time maintain a focus on her original goals. Data gathered from the summer session were coded and the following themes emerged.

- GSP Technological Knowledge
- Goals of Instruction
These themes provide an insight into the forces that were acting on these two teachers as they were contemplating incorporating GSP into their teaching. Each area will be discussed in detail.

**GSP Technological Knowledge**

The first day began with the mentor explaining the basics of GSP, the tool bar and the measurement menu. Questions and discussion focused on what the program could do like drawing a triangle, measuring the angles, and then calculating the sum of the angles. This prompted a discussion on dragging and making conjectures. The sample labs selected by the mentor for the first session included finding the sum of the angles of a triangle, the construction of a square, and the construction of the midpoint quadrilateral (for sources see Table 8). After constructing the midpoint quadrilateral a discussion took place on if the quadrilateral would always be a parallelogram, which highlighted the role of GSP in making and confirming conjectures.

At the conclusion of the first session the teachers were encouraged to complete as many of the labs as possible, in chapter one of the workbook, prior to the next meeting. This involved reading the directions that were printed on each page and possibly consulting the GSP reference manual, or help command, if necessary. The teachers were encouraged to keep a log of their activity and responses to their work. Alice did not have time to work at home. June worked at home but her records only show technical questions.

The second session began with the mentor answering many technical questions from June. She had completed most of the labs in chapter one of Exploring Geometry with the Geometer's Sketchpad at home and had specific questions on animation and the precision of GSP measurements and rounding error.
J: How do we get a segment to revolve around a point?
M: The animated point has to have a path. [The mentor explained how animation worked and pointed out the Parent/Child relationship with objects in GSP.]
J: So we have to be really careful with animation and selecting.
J: Is there a way to drag this to exactly 63 [degrees]?
M: Precision is set here and it is better to work in inches [because they are a larger unit of measurement]. Then take it to tenths. Changes in preferences have to be done first [before drawing]. Put it into tenths and it will then go to tenths.
J: Angles ... I want tenths. Because it will look better – more exact.
M: Sometimes you will add two segments and it will not add up correctly so inches will work better (Lines 6-8).

Alice asked questions regarding how students could record their work, which prompted a discussion of the text box, special values like pi, and a return to the rounding problem.

A: Can they explain what they did?
M: This is what we put in a text box.
A: They don’t have a pi key.
M: Go into value, I know it has pi in there, and it should have pi in there [pointing].
A: I did not check it [her calculation]. Ok times two, 1.8, so we picked up a rounding error. That is a concern for kids (Lines 22-27).

Other concerns the teachers had that day regarding the technical aspect of GSP focused on scripts and hiding objects. The teachers then shifted to teacher mode and asked questions about printing GSP files, editing text, and if GSP would work with a word processing program.

J: Did he [Jacob] just print this from GSP?
M: Yes. We just printed it from GSP.
J: Did he put this into a text box or is it easier in WORD?
M: He did it in a text box (Lines 65-68).
[Discussion of the symbols for angles, segments, etc.]

At the third summer session, June asked very few technical questions and was observed to be proficient with using the different tools and menu items. Alice was observed to have difficulty switching back and forth between the tools and was much slower on the labs.

June took to the program well. Alice still was having problems on the third day. June worked many of the labs at home and came back with questions. Alice did not appear to have worked at home (Mentor Log, 7-2).

By the end of the three sessions the teachers had been exposed to the basics of the program, which involved the tools and the main menu items. The teachers were also exposed to
the way GSP could be used as a tool in their curriculum, which will be discussed in the following section.

**Goals of Instruction**

June and Alice indicated at the beginning of the research study that their main goal was to use GSP to replace hand constructions in a chapter on transformations. This indicated the program would be used as a glorified drawing program that would decrease both the time spent on the chapter and errors the students made in hand constructions. It therefore became a goal of the mentor to expose the role that GSP can play in proof and conjecture. In the first session the mentor had the goal of teaching the program as well as appropriate use (as recommended in NCTM documents). The following comments are found in the Mentor Log from June 11th.

And proving and illustrating – different …
Discussion on proving versus empirical evidence took place …
Then a discussion of dragging and the dynamic environment took place …
Discussion of how to prove that the midpoint quadrilateral is always a parallelogram due to the midline theorem took place (Mentor Log, 6-11).

Following the examples worked on the first day, June commented, “This [GSP] really confirms definitions,” which indicated June might use GSP for more than just transformations.

In the second summer session, the teachers’ discussion focused on using GSP for more than the topic of transformations. As they experienced the introductory labs in chapter one they expressed the desire to introduce GSP thorough a few of the chapter one labs prior to transformations.

In the last session, Jacob shared a unique use of GSP. When his students had free time at the end of a class he had them return to a previous chapter and illustrate a theorem, using GSP. He discussed a theorem about a tangent to a circle and how some of the students ran into trouble. “[The] converse proof was tricky since they assumed what they were proving.” The teachers listened to this idea but did not comment on it.
Alice and June made comments in the summer sessions that indicated they wanted to use GSP in their teaching but these comments did not indicate what their specific goals would be. What was apparent from their comments was how the issues of curriculum coverage and the time that they had to accomplish this would affect their decisions and goals of instruction.

**Curriculum and Time**

Issues of what to cover are very closely linked to how much time it would take in the classroom. An issue that quickly surfaced in the first session was the amount of time each teacher currently spent going over homework. The teachers commented they would not have time to go over homework for as long as they currently do and still have time to use the computer lab. This prompted a discussion on different ways to review and grade homework.

Comments on the first day addressed the large amount of time spent going over homework and how that could be changed to accommodate the labs. Good discussion on this (Mentor Log, 7-2).

A second issue that came up regarding time and curriculum was the GeoCap test. The teachers discussed if the test would be done on computer (like the Algebra test) or be done with pencil and paper, since this would determine how the teachers would teach some of the topics like constructions. For example, if students have to use the computer for the GeoCap then it becomes unnecessary to teach constructions by hand, which saves time for other topics that are on the test. Alice asked if the basic constructions should be taught by hand before the students do them on the GSP program. Jacob suggested the students learn the constructions by hand first and then do the same constructions on GSP.

The question of time and how often the student should go to the lab came up in each session.

J: Do you think it works out well if we only get in there once a week (6-11: Line 4)?
A: I want to demo in the class first. We only have 45 minutes (6-13: Line 155).
J: How often did you [Jacob] go in the lab?
Jacob: Once a week for half a lab [half of a block schedule class period] (6-19: Lines 17-18).
This question is important because the teachers perceived GSP as an addition to the curriculum, and it prompted a discussion on what the teachers would gain by using GSP. The mentor commented in the second session, “You are adding to the curriculum but you really aren’t because they will know it (meaning the teachers won’t have to re-teach it in the classroom).”

The final concern regarding curriculum focused on assessment. Alice asked if Jacob assessed his students on GSP. The mentor answered, “There were questions that he asked [in] the labs that showed up on the test.” Jacob’s focus was not on the mechanics of the program, it was on the concepts they should have learned in the labs.

The issues the teachers brought up regarding curriculum were if the GeoCap would be administered on a computer or by pencil and paper (and what this meant for their teaching), if they should test their students directly on GSP, as well as how they could create more time in their curriculum to add GSP use to it.

Further analysis of the summer sessions indicated that the issue of curriculum coverage and time had a direct affect on the teachers’ desire to participate in a few of the summer session labs. Jacob, the teacher in the pilot study, created many of the labs the mentor chose for the second session. This was done in response to the teachers’ questions concerning how to create a lab of their own. The teachers reacted well to labs that were related to areas of geometry that they currently “covered” but Alice was reluctant to complete a lab that was on a topic they didn’t cover. Examples of this were seen in both the Euler line lab and the Apothem lab. The mentor attempted to justify the labs by pointing out that the labs did include some very common geometry topics. The mentor then modified the labs by pulling out topics that might fit the teachers’ curriculum as illustrated below.

M: Let’s do this lab because it is going to cover a lot of ground, centroid, circumcenter, and incenter.
A: We don’t get to that. It is in chapter 14.
M: But they do altitudes and medians.
[Alice begins looking up, in the text, how far they go with these topics.]
J: Circumcenter?
M: You may not do those points but you do altitudes, medians. The angle bisector does not fall on the Euler line. You would take this line out when you print the lab [instructions]. Let's go through it.

[Altitudes, medians, and perpendicular bisectors are constructed for a generic triangle.] (6-13: Lines 94-104).

M: This is a lot of material and lots of review.
A: We don't do all of this.
M: Let's do angle bisectors before we leave (6-13: Lines 157-159).

Following this exchange on the Euler lab, the mentor was more cautious when introducing the last lab for the day.

M: This last one — do you do apothems?
A: We don't do that.
M: What I want to do is just make the hexagon. He assumes that you can make a [regular] hexagon.

[The mentor leads them through the construction of a hexagon using rotations and a reflection] (6-13: Lines 201-206).

The reluctance by Alice to work on certain labs could be explained by her desire to only focus on examples of labs she may actually use in her classroom the following year. An alternate explanation would be that she did not want to spend time on geometry topics that she may have been less familiar with.

Implementation and Logistics

The idea of using GSP prompted questions on how to physically use it in the classroom or the computer lab. The first question concerned using the program to demonstrate to a whole class versus taking the students to the computer lab.

M: Also, you can work on a computer in the classroom.
J: I have to switch rooms all day long (6-11: Lines 8-9).

A: Is this important that the kids do it or can you do it in the classroom as a demo?
M: You can do it either way (6-13: Lines 198-199).

More discussion took place about how soon they could use GSP in the classroom and also the use of in class demonstrations [with a projection screen] versus lab time (Mentor Log, 6-13).

A: Demo in class?
Jacob: Yes, I used it as a class demo only, 3-4 times.
A: In lab demo first?
Jacob: Mostly at the beginning of the year but later, not [as much] (6-19: Lines 6-9).

For June, using the program in her own classroom became a concern since she was not in the same room all day. Alice’s discussion with Jacob about how he used the computer exposed that at different times he used it in different ways, depending on the time of year and what the topic was.

Another logistical concern that came up was how to seat the students in the computer lab. This issue revolved around having the students work alone or in pairs at the computer.

Discussion immediately turned to the use of GSP in the beginning of the year and how soon they would use it. We also discussed how to pair up students. I suggested matching them up close to ability (Mentor Log, 6-11).

A: Did they work in partners [in the pilot]? 
M: No, they were alone (6-13: Lines 187-188).

During the second summer session the teachers became concerned as to how often the mentor would be in their classrooms and what her role would be. Also in this session, Alice confided her fears about using GSP.

J: Are you going to teach for us? 
M: Now if you had the advantage of having someone teach your classes, what would you do? 
A: Yes, I would go around and help kids (Lines 175-178).

A: How often do you plan on being in our classes? 
M: Count on you each getting a day per week. 
[Discussion of my schedule took place] (Lines 179-182).

A: One of my greatest fears is that there is only one of me. 
J: Before they ask you they need to ask the person next to them. 
[Discussion of cooperative groups took place] (Lines 227-229).

The mentor made it clear she would definitely be there to support the teachers and make recommendations but they would be responsible for making final decisions regarding the curriculum and how involved the mentor was in teaching any of their classes. The reality of having the mentor teaching their classes would be dependent on the level of trust they had in her, as well as their own comfort levels with instructing the GSP labs. June broached this issue by asking if the mentor would teach for them. The mentor turned it around and asked the teachers -
what would they do if they were given that opportunity. This made the issue of having the mentor teach in their classrooms less of a requirement but an option they could take advantage of.

Throughout the sessions Alice voiced some concerns that were affecting her decision to implement the program. She wanted to know if the students liked the program (6-13) and if it helped with motivation and understanding (6-19). The mentor assured her that yes the students really do like the program while Jacob discussed with Alice that he found GSP helped with both student motivation and student understanding.

The last entry in the Mentor Log dated July 2nd summarized the teacher’s feelings about using the program, as perceived by the mentor.

Both teachers were very positive about using the program with Alice’s dialogue very mentor dependent. She wanted assurance that I would be selecting their labs in the fall. I indicated that I would help but that it would be their decision (Mentor Log, 7-2).

Summary

In the summer sessions the teachers were introduced to the basics of GSP by working through labs that were selected by the mentor. June put additional time in at home and by the third session she displayed a basic level of proficiency with the GSP tools and menus. Alice did not work at home and by the third session was still having difficulty manipulating the toolbox items.

An analysis of the work sessions exposed major areas of concern the mentor identified as forces that act on the teachers’ decision-making process. Learning the mechanics of the program almost became secondary to figuring out how the teachers would coordinate the use of the program within their existing curriculum, class routine, and goals of instruction. Figure 5 identifies these forces with questions that capture the main concerns as voiced by Alice and June.
The actions taken by the mentor included selecting and teaching GSP labs as well as entering into the teachers’ discussions regarding their concerns with using the program. By listening to the comments and watching the progress at each session the mentor was able to then plan for the following session. This action by the mentor was consistent with the change models discussed in Chapter II. The mentor provided the vision of something new or a possible perturbation in the teachers’ geometry instruction. Through the use of the pilot teacher the mentor provided an existence proof of the new vision. Finally, the mentor assured the teachers of her support with the selection of and teaching with GSP in the fall.

Chapter V will examine how Alice and June, with the support of the mentor, planned for the use of GSP in their teaching practice. The themes that are identified during the summer learning sessions can be found in Chapter V but the categories used will change to better reflect the data from the activities of planning for the use of GSP.
CHAPTER V: PLANNING FOR INSTRUCTION WITH GSP

The first research question in this study refers to how teachers plan for and implement GSP. This chapter will explore how the teachers planned for its use through an examination of three, after school, planning meetings.

First Planning Meeting

The first planning meeting took place after school on September 11, 2003. The meeting with June and Alice lasted almost two hours. The purpose of the meeting was to plan introductory GSP labs to be used prior to class work on transformations. It was apparent that the teachers had spent time after the summer learning session with the Exploring Geometry with Geometer's Sketchpad workbook since they suggested three possible labs from this book during the meeting. The analysis of the meeting will begin with the activity and motivation of the mentor. Following a discussion of the mentor's role the analysis will look at themes that emerged during this meeting.

Mentor: Role and Activity

The mentor encountered two types of suggested labs. The labs originated in the Exploring Geometry with Geometer's Sketchpad workbook and were labs she had either used with students or had not used with students prior to the study. In either case the mentor focused the discussion on an assessment of the lab, modifications that may be necessary, and the need to "trouble shoot" each lab before use with students. The mentor also projected possible student responses as well as possible teacher prompts or instructional hints. This action by the mentor follows Smith's (1996) guidelines as discussed in the literature review.
The first lab discussed was how to introduce the GSP program to the students. This ‘lab’ was a hands-on demonstration the mentor suggested and did not include a handout. The mentor explained a section of the lab as follows:

M: Right, I basically go through, you have the arrow tool, you have the point tool, you have the compass tool, and you have the line tool, which opens out to the segments, rays, and then lines.
A: Uh, uh.
M: Just going through what all these do. Going though the text. It is an A, having them just play with it. Now, that can be scripted out onto a worksheet, where it says …
A: But you don’t know of one that is done [as a worksheet from the book], or anything (Lines 81-86)?

The teachers decided they would have the mentor use this demonstration lab as an introductory session with their classes. See Appendix E for the oral script of this lab activity.

The second lab was one the teachers suggested. It covered points, segments, rays, and lines and was found in Exploring Geometry with Geometer’s Sketchpad on pages 3-6. This is an example of a lab the mentor had not previously used with students. The assessment and modification process demonstrated by the mentor to evaluate this lab for classroom use is characteristic of how the remaining labs were also evaluated, and if necessary, modified.

The mentor began the analysis of this lab by reading through the technical aspects of the lab. On page 4, step 16, it became clear the students would be using the animation feature and creating an animation button in their sketch.

16. In the Edit menu, drag to the Action Buttons submenu and choose Animation.
You’ll get a dialogue box you can use to specify animation settings. To choose the default settings, click OK. You’ve created an Animation action button in your sketch (p. 40).

The mentor gave a technical overview of the animation being used in the lab and pointed out the advanced nature of the animation feature. The teachers concluded that the animation section was not important to their goal of introducing the program features and stated that if this lab were chosen, page 4 would be skipped.
Alice supported using the material on the following pages since it covered rays and lines.

The mentor pulled the discussion back to evaluate the question on the top of the page that would be skipped.

M: Um, this second question could be something they would have to write out. ‘How did the length of a segment compare to the distance between its endpoints?’ And how are they going to say, it’s the same thing? This is just two ways to get this measurement, distance versus length (Lines 192-195).

The mentor continued through the lab by reading and evaluating both the GSP tasks as well as the questions being asked in order to model how GSP labs need to be assessed prior to their use.

M: This starts with a new sketch and it has them go to the ray tool, a ray through point A, a ray with end point A that passes through point B. Could it also be called ray BA? That’s a good question (Lines 222-224).

M: Know its length is good; know why you can’t measure length of a ray, that’s a good question (Lines 229-230).

M: Put a point on the ray, why can’t you construct a midpoint of a ray? That’s a good question (Lines 232-233).

M: Press and hold the ray then drag right to choose the line tool and (inaudible). List all the similarities and differences you can between segments, rays, and lines.
A: Yea, that’s a good idea.
M: That’s a good question (Lines 235-238).

The last task of the lab involved creating a line without using the line tool. This prompted a discussion of how you could accomplish this.

M: If you constructed a line without using a line, if you used the ray.
J: And then two rays …
M: Two rays could make a line.
A: Is that what you are doing?
M: Construct a line without using the line tool, you have to have two overlapping rays going the opposite way (Lines 250-255).

After figuring out how to complete the task the mentor went on to explain what the students might do to help lay a pedagogical foundation for the teachers.

M: So, ok where were we, it depends on how they construct it. If they overlap the rays like this it will work.
A: Oh, okay.
M: If they attach them end-to-end it won’t work. So that depending on how they do [it] it could work (Lines 269-273).
The discussion on this particular lab concluded with the decision to use it in class but to skip most of page 4 which was animating and then tracing a point.

The third lab that was suggested by the teachers was found in Exploring Geometry with Geometer's Sketchpad on page 67. This lab covered the triangle inequality theorem as well as the theorem involving longest (shortest) sides being opposite the largest (smallest) angles. This lab was a lab the mentor had used with students and one she recommended. This lab also provided an excellent example of the need to trouble shoot GSP labs for potential problems. The content of the lab was good, but there were a number of typographical errors (this was uncommon for the labs in this workbook). The assessment of this lab involved careful reading to know how to fix the typing errors. Once this was done, the mentor drew the teachers' attention to the third question, “summarize your findings as a conjecture about the sum of the lengths of any two sides of a triangle” and hinted the students may not be able to come up with an answer. She indicated the teachers might need to be ready to step in and provide a hint.

M: They may not be able to come up with this. So how much of a hint do you want to give?
A: On this one here, ahem.
M: They may not conjecture (Lines 352-354).

The teachers felt the students would be able to answer the question since this was a topic they had recently gone over in class. A discussion of Alice’s teaching, later in this chapter, revealed many of the students did need help summarizing the triangle inequality theorem. The teachers made notes of what had to be corrected and decided to use the lab with their students since it met the goals of introducing GSP features and fit well with what they would have just covered in class.

The next lab selected for discussion was found on page 25 of Exploring Geometry with Geometer’s Sketchpad. The mentor had not used the lab titled Distance from a Point to a Line, with students. This lab was not discussed in detail because the teachers were unsure of using it since they had not covered the topic yet in class. The mentor read through the lab, endorsed its
use, and pointed out that topics could be presented in GSP before they were introduced by the text.

M: So, LOOKS VERY GOOD – this point to a line.
A: Because we cover distance.
J: Hum, hum, but this comes up later.
A: Does it?
J: Where we want to do the perpendicular.
A: Is this for the perpendicular? Or should I …
J: Uh, uh, isn’t it?
M: Hum, yea, because …
J: The shortest distance …
M: The shortest distance is what they are trying to get through, but that’s not – you don’t have to go through this stuff in class before you go to the computer lab.
J: Hum, hum, but it is far enough – where is it in the book? If it’s far enough down the road that we want to save it for another time (Lines 400-412).

This illustrates one of the teachers’ concerns regarding if topics will first be presented in classroom or the computer lab. Other possible topics for labs suggested were slope of a line and properties of parallel and perpendicular lines. The discussion of these two topics did not develop into viable labs during this meeting.

The four labs that had been discussed, including the distance from a point to a line, were selected by both the teachers to use with their students and were used during the following two weeks of class. The decision to use a lab depended on two aspects. First, the topic had to be closely aligned between the time it appeared in the geometry curriculum and the time it would be completed in the computer lab. Second, the lab had to showcase basic features of GSP students would need to use in the chapter on transformations. Discussion of implementation of these labs will follow the analysis of the planning meetings.

Teacher Concerns with using GSP

The mentor was only one of the participants in the planning meeting. The teachers were also involved and the discussion covered much more than the assessment and modification of potential GSP labs. Analysis of the transcripts reveals two major areas of concern the teachers
faced when planning for the use of GSP labs in their classrooms. The first area is scheduling and time management. This first section covers the following related concerns:

- Mentor schedule/High school schedule
- Computer lab schedule
- Curriculum schedule
- Curriculum: past topic/future topic
- Lab length
- Reading and the State Standards

The second major area of concern for the teachers was if and how they would copy the worksheets. While this may seem trivial, the school corporation gives each teacher a limited amount of copy paper, thus the teachers are forced to evaluate how they will use their limited supply. The teachers debated over different solutions ranging from a classroom set of copies (for each pair of students), to printed answer sheets, to each student getting their own copy of the labs.

Scheduling

In order to work with the mentor, the teachers had to coordinate their class schedules with the mentor's availability. This prompted much discussion about what days of the week and what times the teachers would be using the computer lab. The high school was on a block schedule, which alternated classes between schedule A days and schedule B days. The typical student would have a total of eight classes that would meet every other day. For the first semester of 2003-2004 Alice taught geometry on schedule A days during second and fourth hour and fourth hour on Schedule B days. June taught geometry on schedule B days, third and fourth hours. This discussion of schedules revealed that Alice had planned to use the computer lab during the second half of the class block while June had planned on using the computer lab during the first half of the class block. The teachers both taught geometry during fourth hour on schedule B days so this meant they could share the computer lab (and the mentor) during that class.

The second form of scheduling that took place was the signing up for the computer lab. This was an issue because other teachers could use the lab and often signed up to use it well in
advance. This prompted the following dilemma: Should the teachers sign up for future dates without knowing if and how they would use the lab that day or should they wait to sign up, taking the chance the lab would still be open? The following section of transcript captures the lab-scheduling problem:

J: That’s going to be the hard part about this whole process is getting computer time schedules when we want them.
A: It would take a day or two (inaudible).
J: We really need to be deciding when we want them for a good share of the semester and that’s going to be tough to do. We have so many programs [other subject areas] in just that lab.
M: In 222?
J: It’s going to get to the point where business wants to use the same lab.
A: Yes, but I think if your name is down on the list you should get it first.
J: Right, that’s what I mean, we need to …
M: Oh.
J: Would we be well off to just schedule one day a week?
M: You can always release it.
J: Right.
M: And I will, I will guarantee you will be able to find something to do.
J: At least once a week …
M: Yes, you will.
J: Uh, uh, because we are only talking one 45-minute period, half a period, half a class period. Is there a day that works best for your schedule? During both hours that we marked down (Lines 443-462)?

The discussion that followed involved coordination of the mentor’s schedule as well as coordination between the two teachers and their curricular goals for the next few weeks. By looking at their curricular goals the teachers were made aware of how much time they had to spend on chapter three, which covered angles and lines. Teaching involves planning at many levels. After looking at when they might schedule the computer lab the teachers backed up and examined their lesson planning books. The teachers were told by the math department how much of the text had to be covered in the first semester and were held accountable for this since many of the students would change geometry instructors at the semester break. Thus, scheduling the use of GSP had to take into account the curriculum schedule the high school geometry teachers had been given.
One of the deciding factors to determine if a lab was going to be discussed, or used, during the first planning meeting was how closely the topic coincided with the content of the first three chapters. The teachers were willing to plan a lab that would cover a past topic. The teachers wanted to wait to plan labs on future topics so that their use coincided with when that topic was introduced in the text.

A: Of course a lot from your first chapter. We haven’t got into angles yet.
J: I don’t know if we want to go into …
A: We haven’t had angles (Lines 138-142).
A: Because slope of equations of a line, we haven’t really covered that (Line 433).
J: So which day did you request the lab?
A: I haven’t yet, yea because that was out there a bit and I want to make sure (Lines 1128-1129).

June sums up her thoughts on how to coordinate the topics used in the lab as follows.

J: I think we would try to get in like closer to the time though and do this closer to the time, than to do it now (Lines 427-428).

Closely related to the decision of when to place a lab into the schedule based on the curriculum, the teachers also evaluated how long the lab would take to complete. For each of the four labs discussed in the planning meeting the teachers asked how long it would take or they expressed a time related issue.

A: How long do you think that would take [Introduction lab] (Line 88)?
M: To get through all of these and then to get through measures …
A: By the time we get in there and sit down …
M: Get people logged in …
J: You won’t have more than a half-hour (Lines 131-134).

J: Do you think? Will we get through the, all the toolbars and everything and do that one (Line 378)?
J: You said it was a pretty quick one, right [Distance from a point to a line] (Line 1095)?

Two final issues, which affected scheduling, were the school corporation’s focus on reading across the curriculum and the State Geometry Standards. Since reading was a school goal the teachers had to make sure their class activities included reading.
A: And another thing we are faced with is that we have to incorporate reading into our curriculum and that’s why we have been so busy trying to scramble and we have to justify this [using Geometer’s Sketchpad].

J: How many reading strategies for this (Lines 480-483)?

The state standards on basic constructions prompted a discussion on how to teach the basic constructions with GSP.

A: But we have to go back to the standards.

M: Right.

A: Because we have to take time out of class to do the constructions.

M: Right.

A: We do have to construct angles, and angle bisectors.

J: So you mean we could be modeling it on Sketchpad and they could be doing it with the compass, is that what you are saying (1136-1143)?

The mentor went through a possible scenario for angle construction on GSP along with paper and pencil and offered to make up a lab for this, but the teachers did not reach a consensus on how constructions would be taught and if there would be time to include GSP with this topic. After much discussion it boiled down to the GeoCap test.

A: Distinguish between constructing and drawing, I mean GSP is fine but then we are torn, we are driven by the State Standards.

M: Yea.

A: But they think they want the computer but they [are] not gonna incorporate the Geometers’ Sketchpad, the computer but, for the test.

J: I was going to say what did they have to construct on the GeoCap last year?

A: Something new, angle bisector or something, it had to do something to do with an angle, I think it was a bisector (Lines 1197-1204).

The next largest section of the transcripts focused on if the teachers would copy the lab worksheets and how they would accomplish this.

Worksheet Copies

The issue of how to copy the worksheets prompted a discussion that centered on how, and where, the students would answer the printed questions. If the students did not write on the worksheets they could be reused with other classes. The question then became, where would they write their answers? The teachers discussed a number of options including one that had the students working on a word processing program, but this resulted in ramifications that had to be
worked through. The mentor tried to stay in the background as the teachers debated a solution to
the copy problem.

A: I was wondering if [we could] make one working with Geometer’s Sketchpad and the
computer next to them will be blank and the other one can type it in, the question, as they
are talking back and forth, I was wondering...

J: Oh so use the word processor or something?

A: To type their answers in ...

J: I don’t know, so are you thinking of them attaching as an email or something.

A: Print it off and hand it in.

J: They could just send it as an attachment.

A: Yea.

J: Do they want it though printed out anyways so you want them to keep it in their
notebook?

A: Yea.

M: You want a hard copy to grade? Or do you want to have to open a file and grade it?

J: I think it would be easier to grade a hard copy.

A: But not everybody will have a hard copy (pause) but I will not take a grade on it.

M: In general what you’re doing now is setting your first class procedure, how they are going
to do stuff.

J: I think we are going to want a hard copy.

M: So do you want them answering the questions on the word processor?

A: Um, I don’t think they will have enough time to do that.

M: It is going to take more time, I can tell you that right now, it’s going to take more time.

A: They will take it out with them. Should we retype the questions on another sheet of
paper? Or what, will we have time, or are they going to play with it and then have the
other person write it down, just have one recorder and one on the computer, how we split
up the responsibility, they need to be writing at the same time or we are going to have to
give them a paper to walk out with.


The idea of having a copy of the lab to walk out with changed the focus to producing a sheet of
paper that only contained the questions and not the rest of the GSP information.

J: We could give them a paper with just the questions typed on it and it could just, you
could get six on a page, you know just give them a little short thing.

A: Yea, but when they come to it here they need to go to their paper and write it in, don’t
you think? So just leave it on here and straight copy it.

J: Well you probably want them to see where in the process they need to be writing don’t
you? So I think it’s got to stay on here.

A: Yea, yea stay on here just copy these as they are and don’t . . . the questions there, the
questions on the other sheet, is that what you are talking, or just tell them to just take a
sheet.

J: To really make sense they really need to have the question written.

A: With their answer . . .

J: I mean if we want them to save them and put them in their notebooks, and have it make
any sense to them later, they need the questions.

A: Yea (inaudible).

A: So do we want to make up an answer sheet then (Lines 848-861)?
It is not really clear if the teachers are making a hand out with just the questions or copying the worksheet, and if each student will get their own copy.

M: Do we want them to each have a copy?
?: The problem, each have a copy.
M: This one they are going to want to write in because they are going to see the little boxes and they are going to want to write in that one [page 25 lab].
M: I mean this would make a good, for them to have.
J: Well, except that they wouldn’t really have to have the whole process.
M: No it just needs ...
J: The chart.
M: They would just need that, one two three four, the chart. By the time you would copy these questions, give them space to write, if you are thinking about giving them space to write. You are ...
A: It might as well be the whole paper (Lines 863-874).

The discussion still had not resolved the issue of how many copies would be made and if each student would have to write out their own answers. June asked if the group should answer the questions and Alice responded that she would have them answer as pairs. At this point the mentor stepped in and pointed out the importance of the worksheets for review and projected how the questions could be answered.

M: It depends on how much you’re having them going back and use this as a, as a resource. Um, what I would do? The way I would have them answer the questions, the questions should be apparent in the answer ok, it should be, and that’s something that I teach them. I teach them that you don’t write the answer, ‘it’s 200’ you know. The question should start out by, the answer should start out by saying, building from the question. You know, the length … measurement because there are two different formats to write.
A: And incorporate their reading and writing.
M: That was something that we really worked on, ‘is it possible for the sum of the two side lengths of the triangle to be equal to the third side?’ They like ‘explain’ so the answer should say, “no it is not possible for the sum of two sides of the triangle, two side lengths of the triangle to be equal to the third. If it was, there would not be, the lines would be on top of each other. I don’t know if they need to have the question. I think it would be beneficial for them to think about, how can I answer this question in a complete sentence. Um.
A: Because, I know this part, they would answer with YES or NO, and then the ‘explain’ would bring it out.
M: Right and modeling, and this might be something that we say ‘ok we are going to model this because’, I just say ‘this is something that my students that, have had a problem in the past, how would you answer this question? And I need something more than just yes or no’ (Lines 884-904).
June finally asks why they cannot all be writing it down. At about the same time Alice realizes that the students may need to refer back to these worksheets.

J: But if they are talking about it, and that one person is writing, why can’t they all write it at the same time.
A: Um hum.
J: I mean would that work?
A: Maybe they need to reflect back on, as we get deeper in to Geometer’s Sketchpad, ‘well remember when we did that one worksheet and …’
J: Um hum.
A: And they have it and could turn back to that in their binder (Lines 917-924).

The teachers finally came to the conclusion that the worksheets would be copied as they were printed and that each student would get a copy so they could fill in the answers and have them for review purposes. This procedure was also followed for future labs that were copied from the Exploring Geometry with Geometer’s Sketchpad workbook. The teachers were able to save paper on a few of the labs that were planned in the second and third planning meetings since the students were asked to complete their answers within the GSP file they were printing out.

Alice and June

Major concerns for both of the teachers were identified as scheduling and copying the worksheets. By looking at what each individual teacher was saying or asking, it became apparent Alice was apprehensive about teaching the GSP labs by herself. Throughout the meeting Alice asked the mentor if she would be there, to teach these first few labs for her. The mentor assumed Alice was not comfortable teaching with GSP due to a lack of experience using the program with students. She may also have been uncomfortable because of her level of GSP technical knowledge.

A: Now will you be here to do any of those demonstrations (Line 553)?
A: I am glad you guys are going first [June’s classes meet before hers] (Line 646).
M: I will be here so we can have me do it, you can do it, we can team do it [teach].
A: You will be here on Friday too?
M: Yea.
A: The whole day?
M: Yea, I’ll be here two and four (Lines 653-657).
A: So you are going to teach Tuesday?
M: Monday second period I am doing that one and then I am going to watch and be here fourth period. Tuesday I am coming in and doing third with June and then fourth hour (Lines 762-765).

A: Ok, I will see if I feel comfortable [teaching the third class] (Line 774).

A: So Friday you will be here.
M: Friday I will be here, I will be here, utilize me as you want to (Lines 782-783).

By comparison, June appeared to look forward to teaching with GSP. June did not comment on any potential discomfort in teaching with GSP and was not upset that the mentor would not always be able to help with her third-hour classes due to a scheduling conflict. June was given the choice of having the mentor there for the first demo class or the second lab on the triangle inequality. She opted to teach the second lab herself.

The first planning meeting exposed one major difference between the way Alice and June would come to use GSP in their teaching. As noted earlier, Alice wanted to come into the lab after spending the first half of her class time in her room. June wanted to schedule her lab time for the beginning of the class block and use GSP to introduce topics. During one of the extended discussions on scheduling the lab June asked Alice how she was planning to use GSP.

J: So are you thinking of using the lab as reinforcement, or as exploratory?
A: I think reinforcement because I want to be sure I get the material in (Lines 475-476).

A few lines later Alice sheds some light on why she thinks she has to begin class in her classroom.

A: And again, because I wanted to make sure I covered the homework and had enough time, because then the parents would say “you’re not covering the homework and you’re not teaching” (Lines 486-487).

A different approach was taken by June. Late in the first planning meeting June planed out her coverage of chapter three and included the use of a lab on parallel lines. June’s placement of the lab indicates that GSP is being used to introduce the topic. The fact she would conduct the lab without the mentor present indicates her confidence in teaching with GSP.
J: Well if I work Friday, if we did lesson 3.5 on Friday and did the lab for parallel lines on Friday and they came back on Monday and talk about parallel lines and perpendicular lines from the book ... (Lines 1089-1092).

While these lines make up a fraction of the transcript for the planning meeting they are very telling of what Alice and June feel about teaching with GSP. For Alice, GSP can be used for reinforcement, if there is time left after “teaching” the material. June sees GSP as an avenue of exploration that will be followed with classroom discussion and exercises.

The first planning meeting concluded with the selection of four introductory labs scheduled for use by both teachers, with the mentor assisting with teaching, and the hint of a fifth lab to be used by June on her own. The second planning meeting took place after this preliminary round of labs was taught and picked up with the planning of chapter four on transformations.

The Second Planning Meeting

The second planning meeting took place after school on September 29th and lasted more than one hour. At the beginning of the study the teachers had indicated the desire to use GSP to help them efficiently cover chapter four on transformations. The problem they were trying to remedy was a discrepancy between the State Standards and the textbook (USCMP Geometry). The authors of the textbook introduced transformations in chapter four and integrated this approach throughout the remaining chapters. The teachers did not feel the State Standards were asking for a transformational approach to geometry. Thus the teachers’ goals were split between covering the state standards (for the GeoCap) and covering enough of what was in chapter four so the rest of the year made sense to the students. In addition to this, the teachers’ long-range curricular goals called for them to decrease the time spent on chapter four from four weeks to only two weeks to accommodate the coverage of other topics in the State Standards.

The second planning meeting revisited many of the themes from the first meeting. Scheduling was still a main concern and a number of GSP labs were assessed and modified for use. A new theme that surfaced was how to assign homework from the text as well as how to
administer the traditional end-of-chapter assessment. The main theme that emerged from the meeting was how to cover the State Standards for transformations as well as efficiently cover enough of chapter four so the students would have the background necessary for later chapters of their textbook. The role of the mentor exposes this question early on in the meeting.

**Role of the Mentor: Standards versus Textbook**

In this meeting, the mentor continued to help the teachers assess and modify GSP labs. The first lab suggested by Alice was Tour One, which comes from the teacher workshop booklet published by GSP. The mentor pointed out many problems with this lab that would confuse the students. Following this discussion, neither teacher offered up any other labs for discussion. This prompted the mentor to shift roles and assume the role of questioner in order to help the teachers focus on what they wanted to accomplish with GSP. The discussion centered on how much to include within the topic of transformations. For example, did the teachers want to expose the properties of the transformations, did they want to relate them back to reflections, and/or did they want to look at real world applications? For translations, the mentor asked if they wanted to focus on a vector approach and/or develop translations as a reflection over parallel lines. For rotations the question was similar. Did they just want to use the rotation menu and/or did they want to approach rotations as a reflection over intersecting lines? The issue for the teachers was to cover what they had to for the State Standards and what they needed to from the textbook for future work. It was apparent that this was a tough line of questioning and it appeared the teachers would not be relating transformations and rotations back to reflections.

M: Well I have a question for you, the way the book approaches translations is a reflection over parallel lines. The way Sketchpad [transformation menu] is going to approach it is a vector. And I am going to give you that starting vector and this is the magnitude and this is the direction you are going to translate in, which is pretty universal.

A: Um, hum.

M: So you, you need to decide how you want to ...

A: We get to vectors at the end of the chapter.

M: How do you want to lead into um translations?

J: Um, hum.
M: And how much do you need them to know that the distance of the translation is twice the distance between the parallel lines (Lines 330-338)?

J: I can’t think that there is anything, from after we are out of chapter four. I can’t think of anything that there is anything that uses what is there.

A: The properties?

J: And once again it is not in our standards (Lines 349-350).

Due to the sentiment June expressed regarding learning just the properties of the transformations the mentor began to suggest possible labs. The first set of labs, from Exploring Geometry with Geometer’s Sketchpad, covered reflections (page 38) and translations (page 39) in the coordinate plane. The mentor pointed out that by reflecting a triangle in the coordinate plane and coming up with general results for reflecting over the x- or y-axis, the students had completed what was in sections 4.1 and 4.2 of their text.

M: Here we have reflections in the coordinate plane, which I did this one, and then I did some of the homework from their book actually.

A: From their book?

M: Yes, from your book, on Sketchpad. It looks like section 4.2 problem 14, it’s what would the new vertices be? Right here, what would the new vertices be? So I did that on Sketchpad, I graphed, I plotted the point, and I had it reflected down … (Lines 71-76).

The mentor went on to describe and assess the rest of the lab on page 38, which involved snapping points to the grid and how to label coordinates. A similar discussion took place regarding translating a triangle in the coordinate plane (lab on page 39). Here the mentor observed that the last question may be confusing for students due to the repeated use of the variables lower case a and b.

M: So in that case I would do, I would do page 39, for translation in the coordinate plane because that is where you are going to have a vector that you can then move around and you are going to, you are going to translate the triangle, and you have to come up with, you know if I just had, as the last question, if I had coordinates (a, b), what would the coordinates of my new image point be, given the translation a, little a, little b, oh I don’t like the repeated use of a and b (Lines 353-358).

The teachers agreed these two labs on reflection and translation in the coordinate plane did meet their goals and selected them for use with their students.
Since the properties of each of the transformations was one topic where both the state standards and the text matched, the mentor suggested a set of three short labs that would cover what the teachers identified as the ABCD properties of transformations.

A: Yea, because we go a-angle, b-betweeness, c-co-linearity ...
J: And d-distance ...
A: abed (Lines 486-488).

The mentor had previously created this series of labs, which involved students drawing a triangle, reflecting (or translating or rotating) it, and answering questions regarding what measurements were preserved, as well as if the orientation changed. The mentor offered to modify the labs by adding betweeness and collinearity, which the teachers agreed to (See Appendix E for these three labs).

Another area of agreement between the text and the standards was the ability to apply transformations in problem solving. The third section of chapter four used golf and billiards as applications of reflection. The Exploring Geometry with Geometer’s Sketchpad workbook contained a lab on lasers (page 42). The mentor projected that students would have trouble with the laser lab because the student would have to create a sketch of the room to be used based on the measurements of their own bedroom. The mentor suggested standardizing the room or creating a problem based on a pool table, set at a specific place on the coordinate plane. It was determined the mentor would design a lab based on golf, billiards, or lasers for the teachers to use and this would be discussed at the next planning meeting.

Other topics that received discussion for possible labs were slope and tessellations. Slope had come up in the first meeting but a lab had not been identified for use with the students. The mentor suggested just having the students experiment with two lines and make observations regarding their slopes as the lines were dragged around the screen. The teachers wanted a prewritten lab and discussion turned to the labs on pages 20-24 in Exploring Geometry with Geometer’s Sketchpad. Here the mentor brought up the issue of time and what the teachers’ goals for slope were.
M: How much time do you want to take? … So how much time do you want to spend on slope?
A: I guess not that much because the book does it, we just have to refresh their memory.
M: Right.
A: I am going to do it as warm-ups in here, are you getting …
J: And I have already done it in warm-ups. We have hit them once.
M: I would suggest a review work sheet on slope … I think I would do that in a handout as opposed to doing all this [labs from the workbook] (Lines 116-131).

It was decided that the mentor would develop a review worksheet for slope.

A final topic that created discussion of possible labs was tesselations. The teachers explained that it was the last section of chapter four. Possible labs were found in Exploring Geometry with Geometer’s Sketchpad. The mentor liked them and suggested trouble shooting them for use. As the teachers looked at the labs (pages 55-59) they began to recall that the topic of tiling was covered again later in the book and that these topics were not on the state standards.

J: Yea, that’s beyond here.
A: That’s in another chapter (Lines 613-614).

A: Tessellations with regular polygons, we will get that later on (Line 620).

A: Isn’t it on the standards? Where’s the standard book at?
J: I don’t think so
A: [Reading from the state standards] Slides, flips, turns, expansions, and contractions … know that the images formed by slides flips and turns are congruent to the original shape (Lines 625-633).

Following this discussion the teachers indicated that they would not include GSP examples of tesselations in chapter four.

In the first planning meeting the mentor’s role was to model the analysis and trouble shooting of suggested labs. In addition to this, the mentor’s role included projecting possible student responses as well as instructional paths to follow. In the second planning meeting the mentor took on an additional duty of trying to help the teachers decide on their goals for teaching transformations. The teachers were not suggesting possible labs from the workbook and appeared to be at a loss due to their goal to not deviate from the textbook. The mentor took this as an indicator that she needed to help the teachers examine their goals for instruction. The mentor became more proactive in suggesting labs and offering to create labs for use in the classroom. As

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the discussion progressed from reflections to translations and then finally to rotations the mentor again forced the issue of only covering the properties of rotations and not how rotations relate to reflections.

M: Reflections over two lines that are intersecting, but you don’t want to do this. How do you want to lead into [rotations]? Do you want to lead into it, that it is a reflection over intersecting lines, or do you want to just do rotations and just say ok this is what happens when we do a rotation. Let’s measure this angle as it is rotated over here. What do you know about ABC and A’, B’, and C’?

A: Hum (Lines 443-448).

M: What would be your goal for rotation … (Line 542)?

M: I just want to know what they need, what they need to know how to, you know (Line 551).

This discussion of how much to include for transformations was continued in the third planning meeting. The remaining discussion of the second planning meeting covered the topics of scheduling, which involve the issues of time, reading, and the standards; and the new topic of homework and assessment.

Scheduling

Scheduling covers many areas. The teachers had to schedule the computer lab as well as coordinate the curriculum between their text and the planned labs. As the teachers redesigned chapter four, by looking for GSP labs to supplement the textbook, they tried to schedule the labs on an appropriate day to keep the progression of topics coordinated with the book as much as possible. They also worked hard to determine if enough time was allotted for students to complete each lab. In addition, the topics of reading and the State Standards - issues from the first meeting - resurfaced here.

The teachers identified student reading as reading from the textbook and were faced with how much of this type of reading could be assigned since the transformations were going to be done on computer as opposed to by hand, as the book would explain.
J: You know reading in this section, this chapter is going to be a pain when we are not doing everything, having them do all the reflections everything the way the book’s doing it.

A: Um hum.

J: I think I may give up on reading on that chapter, and come back to it again in chapter five (Lines 12-7).

A: On section 4.1 we could at least read that, we have already talked about image and pre-image, um transformations, the notes for reflection.

J: Yea, it really won’t hurt them to read it ahead of time but (Lines 28-30).

A: You know what, on the day of test three we can have them read 4.1 (Line 49).

A: So when we get to 4.2 we may, skip some of the reading, except for like on page 192 where they actually show the Geometer’s Sketchpad.

J: Um hum.

A: So we need, can talk about it there.

J: I am not thinking the reading in [section] two is that bad, do you think (Lines 280-284)?

The teachers came to the conclusion, for the first few sections of the chapter, that the students should be able to follow the text, so they planned to assign the reading as they had done in the past.

These two teachers were also very aware of the State Standards. On many occasions throughout the meeting they consulted the State Standards document to decide what might be on the GeoCap exam. A few examples of the teachers referring to the standards have already been mentioned as the teachers debated what would be covered. A final example from the transcripts revisits applications of transformations.

At one point the mentor suggested the Burning Tent problem as an application of reflection and was met with pointed opposition from June. The burning tent problem involves finding the shortest path from where a person is standing, to a river to fetch water, back to the burning tent to douse the flames.

J: That is REALLY not in our standards (Line 171).

It became obvious that the teachers and the mentor may have interpreted the term ‘application’ differently. Apparently the phrase ‘apply the transformation’ did not mean applications of transformations. Apply the transformation simply meant to complete the given transformation on
the given figure. This brings up a question regarding the teachers’ acceptance of the other applications of reflections. One explanation would be that the golf and billiards problems were in the textbook and the burning tent problem was not. June summarized the concern regarding the standards and the textbook as follows.

A: I don’t think there was anything more in there [standards] was there?
J: I don’t think so but I am checking just to make sure. But we just about need to do a lot of it really, just to continue using our textbook (Lines 220-222).

As much as the teachers wanted to skip through chapter four they found they were forced to complete much of it since transformations would show up in many of the later chapters.

**Homework and Assessment**

Following a discussion on the issue of reading, the teachers also identified ramifications for assigning homework and testing. June commented on homework as follows.

J: I don’t think that it’s going to be real easy to, to do it on the computer and then come back and, I think it is going to be impossible to assign homework out of the book in a lot of cases, you know I am not really sure that we need to do everything according to the book (Lines 553-556).

Later, Alice voiced the same concern regarding testing.

A: I don’t think that we are going to do much testing out of this chapter (Line 647).

At several times throughout the meeting the mentor tried to promote the use of writing prompts for both homework and assessment purposes. June agreed that they could do this but no one took the idea further to develop this type of homework or assessment for the chapter. While the issues of homework and assessment did not receive much discussion, nor were any conclusions reached, they were concerns the teachers had to deal with as they planned to incorporate GSP into their curriculum.

The second planning meeting found the teachers continuing to struggle with scheduling and the selection of labs with the added task of deciding how much of the textbook to cover. These issues continued in the third planning meeting.
The Third Planning Meeting

Introduction

Three days after the second planning meeting, the teachers and mentor met after school to finalize the plans for chapter four. Alice set the tone of the meeting by looking through the chapter, section-by-section, asking what labs would be used. The role of the mentor was consistent with the past meetings. In this meeting the mentor assessed and modified potential labs, made suggestions regarding alternate homework and assessment activities, and made suggestions on potential student responses and teaching strategies.

The themes that surfaced during the meeting were consistent with the past meetings. The selection and scheduling of the GSP labs took up over half of the meeting time. The topics of reading, state standards, length of labs, homework, the chapter test, and copying worksheets remained concerns for both teachers. A new issue that emerged was working on the school's computer network. The teachers had to deal with storing, saving, and retrieving one of the potential labs.

As a result of this meeting, nine GSP labs were selected to be implemented in the following weeks. The discussion that follows will highlight the Laser Lab and the reflections over two parallel lines lab. The remaining labs will be reviewed briefly.

The Laser Lab

The discussion that took place for the Laser Lab is representative of what took place for many of the other labs. The textbook section 4.3 covered golf and billiards as applications of reflections. The mentor had created and e-mailed a lab to the teachers that involved constructing, in a rectangular room, the path from a laser to a target. The exercise progressed from one bounce, off an indicated wall, up to having the laser bounce off of four walls (given a specific order) before hitting the target (see Appendix E). The lab did not use the coordinate plane. The lab was contained in a GSP file where the directions, example, and exercises would appear on the screen.
when the file was opened. A new task for the teachers would be to use their network to store, save, and retrieve files.

M: And we have to figure how to get it on the system, I would send, e-mail her (school Computer Specialist) the file and say we need this, like you should have a teacher folder somewhere that they can access, to get a common file.
J: So we have our folder.
M: You should have a common file that the kids …
J: Well we do on …
A: That the kids can’t get into it can they?
J: There is a student common drive.
M: There’s a student common drive and you should have a place on that where you can store something and you can designate it, if you go into preferences you can designate it as a read only file.
J: So we want it on the student common drive as a read only.
M: As a read only, so they can’t move the points around, because my only fear is that if they move the target and they move the laser it could reflect it where you can’t see it.
J: Ok.
A: But as a “read only” [file] they can go in and use it?
M: They can use it.
J: They just can’t save it (back to the same file).
A: Ok (Lines 504-523).

It appeared June had worked with the different network drives and understood how the system worked and Alice had not.

In the following section the mentor suggests a potential homework problem with graph paper and lasers. Alice found a similar problem in the text. This discussion exposes the controversy of using the computer versus the hand constructions and how the teachers would modify class reading and homework to accommodate the computer approach.

M: But what might be cool would be to give them then a piece of graph paper, and say here is your room, what are the coordinates that this should hit to get to the target.
J: Um.
M: And give them, you know here’s the laser and here’s the target.
A: Um hum.
M: Tell me what coordinates it should hit on the wall, so then they would be reviewing section 4.2
A: Um hum.
M: And combining it with this, so it would give them something to do outside the class (Lines 434-444).
A: Um, the only one that uses coordinates is that number 12.
M: Right or you could assign number 12, because I was looking in the book to see ok where would you want to go from here, you know with it, you are going to want to hold them accountable.
J: Yeah.
M: And say ok you are going to have a homework problem where you are going to have to
do this and we are going to actually put this on a grid.
A: Can we maybe just assign that whole page to them?
M: Is that part of the reading, have they already done that? Are you going to assign that?
A: Um, usually, I don’t know, we may probably read it before we go.
J: If we assign all this though we have to show them how to do reflections with ...
A: Um hum.
J: A protractor and either a ruler or a compass.
A: Well let’s see because they are not really not showing it here really, but yeah how to do
it.
M: Are they just having them eyeballing it?
A: No.
J: No.
M: Oh.
J: No, they’re doing it exact.
M: Ok
J: It would be a lot easier I think to do it with coordinates.
M: So if you gave them graph paper.
J: Um hum.
M: If you gave them coordinates then they could do it.
J: Um hum.
M: Because you are not worried about them doing it exact by hand, that’s not the point.
J: No, right.
M: You could even modify number 12 by moving either the cue ball, what is it, I can’t see it,
what does it say? Ok so they are doing off of three walls.
J: Um hum
M: So you could modify that problem and say ok just bank it off of two walls, and you
probably for uniformity would want to tell them what two walls, bank it off of one, bank
it off of two.
J: Um hum.
A: Would that be enough? Do you think?
M: It wouldn’t be very much.
J: Well, we could do one with graph paper with one, two, and three.
M: Have them show all the work, show all the work out.
J: Um hum.
M: I mean one problem really done in detail is probably better than ten problems done just
kinda ugh, graph it out three times (Lines 449-491).

While Alice appears to be unsure of a direction, June agrees that combining a laser problem with
the coordinate plane would solve the problem of the accuracy of hand constructions and the issue
of using the protractors.

A closely related concern to homework was giving quizzes and tests. In the following
section the teachers debate the merit of giving the standard mid-chapter quiz.

A: That means according to this book they’re about time for a quiz already, but I don’t think
we’ll be giving those quizzes.
J: I don’t think I am going to worry about quizzes during this chapter.
A: Because I won’t take the time out of class to do them, actually.
J: I would rather they have the hands-on demo on the computer I think.
A: So 4.3 labs then for me in one day. Because I think that will take one day yea definitely (Lines 492-498).

For now, the teachers planned to skip the quiz and devote the time to the computer activities.

In the discussion of the Laser Lab the teachers wanted to know how long the lab might take to complete. The mentor addressed this concern by pointing out that if a student did not finish the four-wall laser shot they should discover the required pattern of reflecting the target by completing the one, two, and three-wall laser shots. The rest of the Laser Lab discussion focused on what days the lab would be taught on and the mentor’s schedule.

Reflections over Two Parallel Lines

The topic of reflections took place in the first two sections of chapter four. The teachers had previously agreed on two GSP labs that would coordinate well with these two sections. The first was Reflections in the Coordinate Plane from Exploring Geometry with Geometer’s Sketchpad, page 38. The second was a mentor-developed lab on the properties of reflections (see Appendix E). As the discussion of the textbook section 4.4 began the teachers revisited their debate on if they should cover translations as the composition of reflections over parallel lines or skip to section 4.6, which was translations with vectors. The issue was how to balance the needs of the students as they continued with the text as well as coverage necessary for the state standards.

M: Well it’s a reflection over two parallel lines, so, see we talked about how much of this do you want them to know about the distance between the parallel lines and the distance there.
J: Our book does a lot with that.
A: Um hum.
M: Ok.
J: Whether, but the standards don’t.
A: Yeah.
M: So what we talked about doing?
J: I think and our book really doesn’t after this chapter, do they?
A: No.
J: I don’t think they do (Lines 608-619).

The teachers retraced how many days they had scheduled in the lab and confirmed they planned to do two labs on translations: One in the coordinate plane and one over the properties of translations. After confirming this they come back to the need to cover section 4.5 and composition of reflections over parallel lines. The mentor pointed out that their book does use this approach in the following chapter, at least for the rotation transformation, which is section 4.6 in the text.

J: And we don’t need it to be real accurate.
A: Um hum, because that’s 4.5.
J: Because the State Standards.
M: This is 4.5.
A: Yeah, um hum.
M: I looked at your next chapter and it does piggyback on these, so the notation, I don’t know if they even use the rotation [notation] a lot, they use the double reflection.
A: Um hum.
M: So knowing that relationship I think is something that I think that you would want to spend one day on, and you can almost do it as an exploration (Lines 667-676).

A lab in Exploring Geometry with Geometer’s Sketchpad, pages 44 and 45, had not been previously discussed so the mentor took the time to read though it and project how the teachers might approach it with their students.

M: So those would be more open-ended type discovery things where you go in and say ok we’re going to try this and just write out the questions on the worksheet, you’ve got two questions, you know this is, what your set up, and this, what you find, spend ten minutes trying to figure this out, in ten minutes we are going to stop and we’re going to see if anybody’s got it (Lines 684-688).

The teachers agree this lab was one they would like to use and they moved on to the discussion of a lab on rotations as the composition of reflections over intersecting lines (Exploring Geometry with Geometer’s Sketchpad, pages 46 and 47), which they also selected for use.

**Glides and the Chapter Plan**

Alice began the discussion of section 4.7 by asking what the workbook might have for a “walk.” This section of the textbook is labeled Isometries and includes the glide-reflection.
A: How about a walk, are we covering that at all?
M: You mean glide?
A: They don’t have, they call it a walk in here, where is all that?
J: 224, right there.
A: Ok.
M: They call it a glide, they should be, they should be able to do it by then.
A: They don’t have anything doing that [in the workbook]?
M: Here is, here is your glide.
A: What page?

The mentor had not used the lab on glide-reflection with students but suggested that if the students pay attention to the directions in the margin they should be able to complete the lab.

June agreed with stressing the importance of reading the margin hints.

J: They need to get good at reading those, because ...
M: Right.
J: They’re there for a reason.
M: Right, yeah they really do help (Lines 863-866).

The teachers quickly agreed to include this lab on glide-reflections and this was the last section of the book discussed.

At the end of the planning meeting the teachers spent 10 minutes reviewing with the mentor the exact schedule for the GSP labs for the following 12 school days, which came to six days per geometry class, due to the block schedule. The discussion also involved the availability of the mentor. The results are listed below.

- Day One – *Reflections in the Coordinate Plane* and Properties of Reflections
- Day Two – Laser Lab
- Day Three – *Reflections over Two Parallel Lines*
- Day Four – *Translations in the Coordinate Plane* and Properties of Translations
- Day Five – *Reflections over Two Intersecting Lines* and Properties of Rotations
- Day Six – *Glide-Reflections*

A review of the topics of the planned labs reveals the teachers did not stray far from their text.

They succeeded in changing the method that the students would learn transformations from hand constructions to using GSP but did not cut any of the material from chapter four.
Summary of the Planning Meetings

Teachers have to schedule what they are doing for each class session, topic, chapter, and semester. Planning to use GSP in the classroom involved an examination of many different aspects of scheduling. When and how to schedule the computer lab may create problems when teachers are competing for its use. Scheduling the use of GSP requires the coordination of lab topics with the text. Scheduling labs also requires an idea of how much class time they will take. Teachers have to make every minute count and planning a successful GSP activity means having enough time to finish while at the same time not wasting valuable class time. Finally the teachers in the study had to respond to both local and state requirements. Scheduling discussions took up the largest amount of time in each of the meetings.

For the teachers to implement GSP into their curriculum they had to examine how they would modify the daily homework, reading, and assessment activities to accommodate the new approach. At many times, the solution they suggested was to abandon the textbook reading, homework, and publisher created assessments in favor of just spending time in the lab. The teachers did not attempt to create new assignments or assessments as the mentor suggested. In the end, the textbook became the organizational tool, which helped determine and order which GSP labs were chosen. As will be revealed in the discussion on teaching with GSP, the teachers returned to the textbook for reading and homework assignments as well as publisher created quizzes and chapter tests.

The planning meetings reveal some very pointed differences between Alice and June. First, Alice was wary about teaching with GSP where June was observed to be confident. Second, Alice scheduled the computer lab for the second half of the class period so that she could cover the topic in the classroom first and then use GSP to reinforce the concept. By contrast, June scheduled the computer lab for the beginning of class and planned to use GSP to introduce new topics.
The next chapter will explore how each of these two teachers, with the assistance of the mentor, implemented GSP into their curriculum from mid September to the end of October 2003.
CHAPTER VI: TEACHING WITH GEOMETER'S SKETCHPAD

This chapter contains discussions on three related topics. The first section incorporates data from Phases 3 and 4 of this study to analyze how the teachers implemented GSP into their teaching practice. The second section focuses on a possible change in beliefs for Alice due to the use of GSP.

Teaching with GSP: The Mentor’s Role

Following the first planning meeting, the teachers were ready to begin implementing the selected GSP labs into their teaching practice. The mentor’s intent in the study was to model the GSP labs and have the teachers watch the lessons and notice specific strategies they might adapt for their own instructional purposes. The mentor provided the teachers with the existence proof that this form of instruction could work with their students. Specific strategies the mentor modeled included:

- Model new GSP tools and menus through the chosen GSP lab.
- Use of student(s) on the demonstration computer.
- Direct student reading of the worksheet instructions.
- Model complete answers to the printed questions.
- Guide the students in their conjectures.
- Answer or assist in geometry questions.
- Answer or assist in technical questions or problems.

Each of the GSP labs used different features of the program that the students needed to learn how to use. By having students operate the demonstration computer that was projected to the front of the class, the remaining students could watch their peers, working at their speed, go through the necessary menu items. This offered the students proof that this form of technology was accessible to them.

The routine established with the students was to work through the first part of the GSP lab as a class. The first part of each of the labs was used to set up a polygon (usually a triangle)
on the screen, measure its coordinates, angles, or segments and, using one of the transformations, create its image. Once the students had the appropriate figure to manipulate the mentor instructed them to complete the lab questions with their partners. The mentor continuously monitored student progress by walking around the room and noticing what was on their screen or their worksheet. When a majority of the students had worked to a point of frustration, the mentor would pull the class back together and ask if anyone could volunteer a suggestion. This discussion would help guide the remainder of the students in what they needed to be looking at in order to answer the question(s). For many of the students this was a new type of learning environment and they needed to be supported as they worked with GSP. During the first few GSP labs the mentor would model responses with complete sentences. In later GSP labs the mentor would read what the students had written and ask them questions to help them rethink and possibly revise their responses.

An example of a geometry question that surfaced early in the transformation labs was the definition of orientation. This was a term neither teacher had discussed in class prior to the lab, so the mentor brought the class together and explained what was meant by the orientation of a polygon. An example of a technical question that often came up was why the measurement command would not work. This was usually caused by the student selecting (highlighting) something on their screen in addition to the object to be measured. Technical problems usually arose when the student would draw lines or segments instead of using the construction menu.

Since the researcher in this study participated in the role of mentor, this information is self-reported. In order to shed light on the actions of the mentor a pre-service high school mathematics teacher, who was observing Alice as part of her undergraduate program, was asked to document one of the GSP classes taught by the mentor. Due to the resources available for the study she was not trained in observation techniques. The following report was submitted:

I had the opportunity to observe Prof. Shafer instructing a geometry class at Harrison High School. The class was studying the topic of translations and Prof. Shafer guided the class through it with the computer program called Geometer’s Sketchpad.
Using this technology, the students were able to create shapes and manipulate them in various ways to assist in their understanding. Prof. Shafer’s work with the students helped them greatly and allowed for a successful lesson.

In the class I observed, Prof. Shafer had constructed a worksheet to introduce the students to reflections on the computer. One side of the paper was a lab from the workbook Geometer’s Sketchpad [Reflections in the Coordinate Plane] and the other was a lab for the students to work through on their own [Reflection Lab, see Appendix E]. Prof. Shafer guided the students through the first page helping them find the keyboard controls and instructing them on the construction of triangles. The students were grouped into partners and helped each other along through the activities and would switch roles every once in a while [who was controlling the mouse]. One student’s computer screen was projected onto the pull down screen at the front of the classroom. I think this was very beneficial for the rest of the class because they were able to have an example they could work from if they were lost or falling behind. Also, Prof. Shafer was able to guide the student in the front of the class through the steps of the assignment and know at what pace to continue on. It also helped her to see any problems the students might come across as they worked.

Prof. Shafer continually walked around to assist the students with their work and answer any questions they had. The students were quick to volunteer for reading aloud and answer questions. Prof. Shafer actively engaged the students by allowing them to work along with the “example” student at the front of the class. This was very beneficial to the students and kept them engaged in the lesson the entire time. Even the students who were not working on the computer were still working alongside their partner. Overall, I think the students really enjoyed working on the computers and getting a chance to learn through a different method than they were used to. Using this technology helped them to interact with the lesson and gain a hands-on understanding of geometry. Prof. Shafer did an excellent job keeping the students involved with the lesson and guiding them through the activities. I think the students definitely benefited from the time Prof. Shafer spent with them in their geometry class (Pre-service Teacher Geometry Observation, October 10, 2003).

This undergraduate student had completed the majority of her observation hours in Alice’s pre-algebra and geometry classes when this observation took place and was aware of Alice’s typical instructional method.

Once the teachers observed the mentor teach a specific GSP lesson they would resume instruction in their remaining geometry section(s). At this time the mentor became an assistant to the teachers, observed the classes, and helped students when necessary. Other duties assumed by the mentor throughout the study were to assist the teachers with the maintenance of the computer labs and assist with the purchase of the software for the high school.
Teaching with GSP

In the months of September and October of 2003, Alice incorporated 13 GSP labs into her teaching and June used a total of 14 GSP labs. The labs are listed in Table 9 along with the date they were taught and the source of the lab. Two dates are listed for most of the labs due to the block schedule at the high school. The analysis of teaching with GSP will focus on the goals of instruction, role of the teacher, and role of the student. Sources of data are the mentor log, the teachers’ Lesson Plans and Post-Lesson Reflections (PLR), the Classroom Lesson Records (CLR), and the exit interviews. The videotaped lessons were studied for evidence to support or refute the findings from the other data sources. Review of the taped lessons revealed teacher behaviors that corroborate the findings from the other data sources. Thus, those sections of the tapes that provided additional supporting evidence were transcribed. Following an analysis of the individual teachers’ practice with GSP, similarities and differences are discussed. This is followed by a discussion of teacher beliefs.

Table 9. Fall GSP labs, dates of use, and source

<table>
<thead>
<tr>
<th>Date(s)</th>
<th>GSP Lab</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sept 15 &amp; 16</td>
<td>Introduction to GSP (script only)</td>
<td>Mentor Developed</td>
</tr>
<tr>
<td>Sept 15 &amp; 16</td>
<td>Introducing Points, Segments, Rays, and Lines</td>
<td>*Bennett - pages 3, 5, and 6</td>
</tr>
<tr>
<td>Sept 18 &amp; 19</td>
<td>Triangle Inequalities</td>
<td>Bennett - page 67</td>
</tr>
<tr>
<td>Sept 18 &amp; 19</td>
<td>Distance From a Point to a Line</td>
<td>Bennett - page 25</td>
</tr>
<tr>
<td>Oct 2</td>
<td>Properties of Parallel Lines</td>
<td>Bennett - pages 17-18</td>
</tr>
<tr>
<td>Oct 10 &amp; 13</td>
<td>Reflections in the Coordinate Plane</td>
<td>Bennett - page 38</td>
</tr>
<tr>
<td>Oct 10 &amp; 13</td>
<td>Reflection Lab: Properties</td>
<td>Mentor Developed</td>
</tr>
<tr>
<td>Oct 14 &amp; 15</td>
<td>Laser Lab</td>
<td>Mentor Developed</td>
</tr>
<tr>
<td>Oct 16 &amp; 17</td>
<td>Reflections over Two Parallel Lines</td>
<td>Bennett - pages 44-45</td>
</tr>
<tr>
<td>Oct 20 &amp; 21</td>
<td>Translations in the Coordinate Plane</td>
<td>Bennett - page 39</td>
</tr>
<tr>
<td>Oct 20 &amp; 21</td>
<td>Translations Lab: Properties</td>
<td>Mentor Developed</td>
</tr>
<tr>
<td>Oct 22 &amp; 23</td>
<td>Reflection over Two Intersecting Lines</td>
<td>Bennett - pages 46-47</td>
</tr>
<tr>
<td>Oct 22 &amp; 23</td>
<td>Rotation Lab: Properties</td>
<td>Mentor Developed</td>
</tr>
<tr>
<td>Oct 27 &amp; 28</td>
<td>Glide Reflections</td>
<td>Bennett - pages 48-49</td>
</tr>
</tbody>
</table>

*Exploring Geometry with Geometer’s Sketchpad by Bennett.
Teaching with GSP: June

The following discussion examines the goals of instruction, the role of the teacher, and the role of the student for June’s teaching practice as she implemented GSP activities into her curriculum. A review of the method of instruction and the curriculum sources will be omitted from this analysis since they will become obvious in the discussion.

Goals of Instruction

June possesses two main goals while teaching geometry with GSP. First, cover a set amount of material as dictated by the school corporation. June also maintained the goal of student understanding. She identified GSP as a tool for exploration and discovery to help foster student understanding. An examination of her lesson plans exposed the first goal of covering the material. Table 10 indicates her stated goals for each of the lessons. Second, June wanted to get through the topic of transformations efficiently and quickly, because it was not stressed on the GeoCap exam. The Post-Lesson Reflection question 5 asked the teachers to compare the GSP lesson they had just taught to a non-GSP lesson. This question exposed the result that June thought GSP provided a quick way to cover the material (see Table 11).

June possessed a second goal of fostering an understanding of geometry in her students. The second question on the Post-Lesson Reflection asked the teachers how they implemented GSP (see Table 12). The table entries indicated that after the 10-2 lesson, which was both review and new material, GSP was used to introduce a new topic and the students were to explore and investigate results.

A final table is used to highlight the intended goals of instruction while using GSP. The Post-Lesson Reflection question 1 (see Table 13) asked if the goals of the lesson were achieved, and if so, how the teacher knew this. The issue of time arises on 10-2 and 10-22 with the students either not finishing or finishing quickly. The issue of understanding arises on 10-14, 10-22, and
on 10-27, where the students successfully carry over what they accomplished in the computer lab to the classroom.

Table 10. June: Goals from Lesson Plans

<table>
<thead>
<tr>
<th>Date</th>
<th>Goal of Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-18</td>
<td>To continue familiarizing students with the tools of Geometer's Sketchpad</td>
</tr>
<tr>
<td></td>
<td>To find the shortest distance from a point to a line.</td>
</tr>
<tr>
<td>10-10</td>
<td>Draw figures by applying the definition of reflection image.</td>
</tr>
<tr>
<td></td>
<td>Draw reflection images of segments, angles, and polygons over a given line.</td>
</tr>
<tr>
<td></td>
<td>Apply properties of reflections to make conclusions, using one or more of the</td>
</tr>
<tr>
<td></td>
<td>following justifications:</td>
</tr>
<tr>
<td></td>
<td>Reflections preserve distance.</td>
</tr>
<tr>
<td></td>
<td>Reflections preserve angle measure.</td>
</tr>
<tr>
<td></td>
<td>Reflections switch orientation.</td>
</tr>
<tr>
<td></td>
<td>Figure Reflection Theorem.</td>
</tr>
<tr>
<td></td>
<td>Find coordinates of reflection images of points over the coordinate axes.</td>
</tr>
<tr>
<td>10-14</td>
<td>Use reflections to find a path from an object to a particular point.</td>
</tr>
<tr>
<td>10-20</td>
<td>Draw translation images of figures.</td>
</tr>
<tr>
<td></td>
<td>Find coordinates of translation images of points over the coordinate axis.</td>
</tr>
<tr>
<td>10-22</td>
<td>Draw or identify images of figures under composites of two reflections.</td>
</tr>
<tr>
<td></td>
<td>Apply properties of reflections to obtain properties of rotations.</td>
</tr>
<tr>
<td></td>
<td>Apply the Two-Reflection Theorem for Rotations.</td>
</tr>
<tr>
<td>10-27</td>
<td>Draw glide-reflection images of figures.</td>
</tr>
<tr>
<td></td>
<td>Apply properties of reflections to obtain properties of other isometries.</td>
</tr>
<tr>
<td></td>
<td>Determine the isometry, which maps one figure onto another.</td>
</tr>
</tbody>
</table>

Table 11. June: Post-Lesson Reflection Question 5

<table>
<thead>
<tr>
<th>Date</th>
<th>Compare this lesson to a non-GSP lesson on the same topic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-10</td>
<td>Students were able to see the results of reflections much more quickly using GSP</td>
</tr>
<tr>
<td></td>
<td>than in previous years when reflections were completed with the use of</td>
</tr>
<tr>
<td></td>
<td>protractors and compasses.</td>
</tr>
<tr>
<td>10-14</td>
<td>Students were actively involved in performing the reflections. In past years,</td>
</tr>
<tr>
<td></td>
<td>students got lost in doing the fine details of the reflections, losing sight of</td>
</tr>
<tr>
<td></td>
<td>the big picture.</td>
</tr>
<tr>
<td>10-20</td>
<td>Since past non-GSP lessons have always been taught with completing the</td>
</tr>
<tr>
<td></td>
<td>translations with protractor and compass, this lesson has been extremely</td>
</tr>
<tr>
<td></td>
<td>time-consuming in the past. Students were able to visualize the results of a</td>
</tr>
<tr>
<td></td>
<td>translation in a much more efficient manner by using GSP.</td>
</tr>
<tr>
<td>10-22</td>
<td>By using GSP, students were able to quickly see the result of reflecting a</td>
</tr>
<tr>
<td></td>
<td>figure over two intersecting lines and equate the resulting figure as also a</td>
</tr>
<tr>
<td></td>
<td>rotation about a point very quickly. In past years, students have labored over</td>
</tr>
<tr>
<td></td>
<td>completing rotations with the use of protractor and compass or ruler, thereby</td>
</tr>
<tr>
<td></td>
<td>losing sight of the desired goals because they mainly focused on the tedious</td>
</tr>
<tr>
<td></td>
<td>constructions to obtain the rotation.</td>
</tr>
<tr>
<td>10-27</td>
<td>The discussion of this lesson went much more quickly than in previous years</td>
</tr>
<tr>
<td></td>
<td>without the use of GSP. Students proceeded through the lab quickly and stayed</td>
</tr>
<tr>
<td></td>
<td>on task.</td>
</tr>
</tbody>
</table>
Table 12. June: Post-Lesson Reflection Question 2

<table>
<thead>
<tr>
<th>Date</th>
<th>Describe how GSP was implemented in the lesson.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-2</td>
<td>The material for this lesson was presented to the class before going to the lab and the GSP lab was to help reinforce the presented material and to investigate upcoming special angles formed by parallel lines. Students were to complete the lab independently.</td>
</tr>
<tr>
<td>10-10</td>
<td>Geometer's Sketchpad was used as an introduction/exploration to reflections in the coordinate plane.</td>
</tr>
<tr>
<td>10-14</td>
<td>GSP was used as an introduction/exploration with class discussion following.</td>
</tr>
<tr>
<td>10-20</td>
<td>Students explored the results of reflecting a figure over two parallel lines.</td>
</tr>
<tr>
<td>10-22</td>
<td>Students used GSP as an investigation of rotations.</td>
</tr>
<tr>
<td>10-27</td>
<td>GSP was used as an introduction/exploration with class discussion following.</td>
</tr>
</tbody>
</table>

Table 13. June: Post-Lesson Reflection Question 1

<table>
<thead>
<tr>
<th>Date</th>
<th>Were the goal(s) of the lesson achieved and if so how do you know this?</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-2</td>
<td>Time ran out before an assessment could be performed. Most students did not finish the lab.</td>
</tr>
<tr>
<td>10-14</td>
<td>Yes. Students related very well to the reading/discussion in the classroom of Lesson 4-3 after completing the Laser Lab. As students were working on their worksheets, many were heard saying things such as the miniature golf hole is the same as the laser target, the golf ball as the same is laser, etc.</td>
</tr>
<tr>
<td>10-20</td>
<td>Students had difficulty finding the coordinates of translation images of points over the coordinate axis. They need additional work in naming the coordinates of points, even in identifying the x- and y-axis and the x- and y-coordinates of a point. Translating a figure was too complicated for students. Next year try translating a point first, and then try with a figure.</td>
</tr>
<tr>
<td>10-22</td>
<td>Although the goals were not measured directly, students seemed to pick up very quickly on the classroom discussion and be able to work the worksheet problems with little assistance.</td>
</tr>
<tr>
<td>10-27</td>
<td>Yes, students were able to relate the work in the lab with the reading material in the textbook, and they equated the steps in the book with the lab procedure steps.</td>
</tr>
</tbody>
</table>

A problem with student understanding, and a potential remedy, are found in the entry on 10-20. On this lab, many of the students had problems with, as June stated, identifying the x- and y-coordinates. The wording on the lab caused some of the confusion. The mentor made the following remarks regarding this particular class session.

Today is two labs: Translations on a grid and the translation lab. June taught the translation on a grid first, with students modeling it up front. The students were slow with the questions. The ones I read had a few mistakes like switching the x- and y-axis. They became really lost when we got to the last question on B = (a, b) and a point being (x, y) and where the new image point will be. At this point June noticed that they were having lots of trouble and we discussed how it exposed a lack of understanding on their part with coordinates. I expressed the benefit of knowing this weakness and then being able to know, as the instructor, that she needed to go back over it in the classroom. I suggested that they stop that lab and go to the other side and try to
finish that one in the remaining time. That would be the translation properties lab that I created (Mentor Log, 10-20).

The data sources above indicate June used GSP for the introduction and exploration of geometric principles to promote student understanding. An excerpt from the lesson taught on 10-10 gives an illustration of what this looked like in June’s practice. The topic for this lab was reflecting points over the x- and y-axis in the coordinate plane. The students had not discussed this topic prior to the GSP lab session.

J: Once you measure the coordinates, take a look at those coordinates, take a look at the A coordinates and the A' coordinates, look at the B coordinates and the B' coordinates. Think about it [said to a student], look at the C coordinates and the C' coordinates, think about it for a minute and then we are going to give you a chance to write. Number 7, now [reading] ‘drag the vertices to different points on the grid.’ Look for a relationship between the point’s coordinates and the reflected images across the y-axis. So just like I asked you to do, after you drag it, when you look at A coordinates, what other coordinates are you going to look at? You are going to look at A [pause] and A’.

[Reading] ‘Question One: Describe any relationship you observe between the coordinates of the vertices of your original triangle and the coordinates of their reflected images across the y-axis.’ What do you notice about that A and A’? Write it down. What I want you to do is I want you to decide what you are going to write, go ahead and write it down, and then I want you to share it with the person sitting beside you, after you have written it down, see if you agree.

Let’s use some mathematical terms. I hear some of you saying first number, second number. It’s ok but can you think about what coordinate it is? (20:42-23:20).

June let the class work for a few minutes while she helped students individually. She called them back together and asked for volunteers to read their answer to question one. A female student responded that the new x-coordinate is negative. Another female student pointed out that the x-coordinates are opposites. June asks who is right? Since the first girl made a nasty remark (directed at the girl that was correct) June backed up and asked her if she was correct for the diagram she had created, which she was. June then suggested she move point A to quadrant II and make a new observation. At this time the student agreed that the x-coordinate of A’ is the opposite of the x-coordinate of point A. The students were urged to finish the lab on their own and they did.

In the reflection lab June gave the students the necessary hints to discover the relationship between the reflected points. As the students became more knowledgeable of GSP June gave
them fewer prompts, which supports her goal of student understanding and her belief that students can construct knowledge. The mentor observed this on the Laser Lab lesson on 10-14.

June let the students experiment with possible solutions before stepping in and discussing what they might want to try.

I got there to see how the Laser Lab would go. June started them and they did room one but over wall A so they redid it over wall B (before I got there). We had them do room 2, which was a two bank shot, without help and then gave them the strategy. This worked well and they (many) followed it. Both of us helped students. Some kept reflecting the original target and not the image. I had students that were done help others because it is such a large class. They saved the file to their class’s “inbox” on the high school’s computer network (Mentor Log, 10-14).

A minor goal of instruction was observed when June stressed both reading and writing in the computer lab. The GSP lab worksheets contained detailed directions. June would ask students to read these out loud and then someone would interpret what was read as they applied the directions to the program. An additional goal was observed as June expected the students to make conjectures regarding what they had observed in the GSP lab. June’s stated goals for instruction with GSP were consistent with what was observed in the computer lab.

In the snapshot analysis of June’s teaching, prior to the intervention, her goals of instruction were summarized as follows:

- GeoCap exam preparation
- Cover material in the text
- Student reading
- Teach for understanding

A comparison of goals of instruction between GSP and non-GSP teaching shows consistency in the goals of covering the material, student reading, and teaching for understanding. A comparison of June’s instructional goals also exposes two differences. First the GeoCap exam was not mentioned in the computer lab. The second difference was the focus of the tasks. In the classroom the observed focus was on fostering student understanding through direct instruction. In the computer lab, with GSP, the focus was on student understanding through exploration and making conjectures on new material like the properties of a translation.
Role of the Teacher

June assumed the role of facilitator when in the computer lab. She found that what worked best for her students was guided discovery. Her routine was to work through the first half of the GSP lab in order to create the figure(s) as a class and then have the students work with a partner to complete the questions. June directed the class to take turns reading and completing the GSP worksheet instructions together since they were still learning the GSP tools and menus. For example, if a student did not follow what was done on the demonstration computer, she would pause the class and students who had followed the instructions were asked to explain what they did.

June came to this routine after taking her classes to the lab on 10-2 with the assumption that she could let them work on the computers with only the worksheet instructions to guide them. The students did not complete the worksheet on parallel lines cut by a transversal due to a lack of knowledge of GSP and their inability to read and follow the instructions on the worksheet. June found that while GSP is a great tool, it has to be taught along with the mathematics. What this means is that students need to be shown how to manipulate within the menus and toolbars of GSP in order to use it correctly. Without this instruction the students will not be able to use GSP effectively and the mathematics in the lab becomes lost as they try to get through the requested construction.

Factors that inhibited independent student work were the size of the classes and characteristics of students in those classes. The mentor observed that June’s classes were both large, with one class of over 30 students, and each class contained a few students with behavior problems as well as students with special needs. While teaching these classes herself, the mentor experienced defiant behavior as well as very dependent behavior from a few specific students.

An observation of a typical GSP lesson found June working with the class to create the appropriate figure as well as prompting them on the questions they needed to think about. The CLR for 10-10 revealed the following codes for the teacher activity:
On this particular day, June modeled the GSP lab herself on the front computer. This physically took her away from the class, resulting in management problems. She commented in the PLR that she would try a different strategy for the next GSP lab.

Next time I will try having a student-team work at the demo computer rather than me doing so, enabling me to better keep students on task (PLR 10-10).

After allowing the student-team to work on the demo computer for the Laser Lab, June made the following observation:

Having a student team working on the demo computer was extremely helpful. This allowed me to keep track of student progress and still allowed students to see what steps they should be completing in their lab (PLR 10-14).

An examination of June’s lesson plans partially support the guided discovery role that was observed in the classroom (see Table 14). An inspection of the entry on 10-14 indicated June guided the class through the first Laser Lab. What this meant was that she guided them through the room with “the one bounce problem” and then let them work on the rest of the lab. A whole class discussion took place only after the students had attempted the two-wall bounce. The entry on 10-27 also indicated she guided them on the lab up to the point where the students were to make a conjecture at Q2 (question 2). Other entries in her lesson plans indicate she was modeling and leading the students through the labs but do not indicate what that “leading” looked like. The researcher attributes this to the fact that, in general, June was guiding the lesson as opposed to just sending them into the lab and letting them work on the worksheets themselves.

June indicated, in the exit interview, that she perceived her role when using GSP as more of a facilitator (with less directed instruction) than when she was teaching in the regular
classroom. She also indicated that with a better-behaved group of students she might have been able to loosen the control a bit.

Table 14. June: Role of the Teacher

<table>
<thead>
<tr>
<th>Date</th>
<th>Teacher Goal in Lesson Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-18</td>
<td>Teacher will model the worksheet (&quot;Distance from a Point to a Line&quot;) with students, stressing answering questions with complete sentences and reflecting the question in the answer.</td>
</tr>
<tr>
<td>10-10</td>
<td>Teacher will walk students through Reflections in the Coordinate Plane lab, p. 38 in <em>Exploring Geometry with the Geometer’s Sketchpad</em>. Stress importance of reading instructions in left-hand margin of lab directions. Stress answering questions with complete sentences, which reflect the question.</td>
</tr>
<tr>
<td>10-14</td>
<td>Teacher will walk students through the first room of the Laser Lab.</td>
</tr>
<tr>
<td>10-20</td>
<td>Teacher will lead students through Translations in the Coordinate Plane lab, p. 39.</td>
</tr>
<tr>
<td>10-22</td>
<td>Teacher will lead students through Reflections over Two Intersecting Lines lab, p. 46-47.</td>
</tr>
<tr>
<td>10-27</td>
<td>Teacher will lead students through Glide Reflections lab, p. 48-49 through Q2.</td>
</tr>
</tbody>
</table>

Another aspect observed in the computer lab was June’s shared approach to knowledge. She constantly asked students how to complete a GSP task or asked, “What do I do next?” When something did not work on the demo computer June would ask the class to identify the mistake that was made.

The description of role of the teacher presented in the snapshot of June was found to be consistent with what was observed when she was working in the computer lab.

- Mixed teacher/student centered mode of direct instruction
- Teach for understanding: Expose how and why geometry works
- Sequence lessons into developmental chunks
- Shared approach to authority and knowledge

**Role of the Student**

In the computer lab the students in June’s classes took on roles similar to what was observed in the snapshot analysis. Some students were actively involved, others were passively involved (watching), and a few students were not involved with the lessons. Over the duration of
the chapter on transformations this changed. In fact, a particular boy who had been repeatedly removed from class for behavior reasons was observed to be present and participating on 10-16.

To be able to use the computers at Harrison High School students had to have the appropriate hole punched in their ID cards. Prior to the time when GSP would be used, June tried to make sure her students had the appropriate permission to use the computer. Students had to bring in a signed permission slip and have their ID cards punched by the student services office.

By the third day in the computer lab all of her students were on the computer.

Unfortunately, as previously stated, the classes were large and at times students were found to be off task. The mentor made the following observation on 10-10.

- Students are not able to log in and coming really late to class delayed the start time.
- Teacher up front made it so students were not monitored.
- Students in the back row did not follow directions as I observed them to be behind much of the time.
- Many students were bored and playing solitaire.
- Class time went over 45 minutes. And students were just not able to measure things independently. In other words, they did not recall how to measure things in GSP (Mentor Log 10-10).

As previously mentioned, June had identified that she needed to put students on the front computer so she was able to better monitor the class. The result of moving physically in the room is found in her PLR comments on the Laser Lab.

Students seemed to really get into this lab … their progress and cooperation were much better today than in any other lab (PLR 10-14).

An alternate reason the students were successful and cooperative could be that the Laser Lab did not include a handout that had to be read and followed (or that hitting a target with a laser might be fun).

The lab activity for 10-10 was Reflections in the Coordinate Plane. What was observed in this class is representative of what took place in the computer lab during other days. Some of the students had fallen behind and others had worked ahead and were bored. Thus the student
engagement in the class varied from medium to high. The CLR reveals the following student activity:

- R - reading
- L - listening
- DS - discussing
- TS - technical comment from a student
- V - vocabulary comment or question
- T - computer activity by student
- W - writing

The codes in the Process and Thinking categories from the CLR are more revealing. Process codes included Observation, Comparison, and Inference. Thinking codes were Knowledge Representation and Knowledge Construction. For example, on 10-10, after the appropriate figure was created and correctly reflected the students were comparing the coordinates of the points in order to come up with a generalization. This pattern was observed in each of the labs as the students were asked to drag a constructed feature around the screen and make a conjecture that generalized the situation. The mentor log for 9-16 includes an observation of this type of comparison activity.

I finally got to observe the intro lab and pages 3, 5, and 6 and observed the following activities: reading, interpreting the written text to apply it to GSP, using GSP to compare and contrast. Examples are length versus distance, segment versus ray versus line (Mentor Log 9-16).

One main difference was found in the role of the student between the snapshot analysis and the GSP lessons. During the GSP labs the students engaged in the labs were working at a higher level of thinking than what was observed in the non-GSP classroom. Another result of working in the computer lab was an increase in student discussion. In the snapshot analysis the students maintained a two-way dialogue between themselves and the teacher with very little student-to-student discussion. A brief example of the typical pattern of student talk was found on 10-16. A female student remarked to the pair of students next to her that her line was bigger than the page. The male student beside her pointed out she had a line instead of a line segment. This
example also indicates students are considered a source of knowledge and shared authority, which is consistent with what was observed in the non-GSP lessons.

Epilogue

In January of 2004, June was asked to reflect back on the study. Her responses revealed her overall goal of efficiently covering the material on transformations in the textbook in order to have time for other topics.

J: It just really, I think [GSP] helped them not to work with the precision, not to have to worry about how precise they were, because so many times they have just gotten lost, and it doesn't, it doesn't look right because they'll do one little part of the construction wrong, and it just blows it (Lines 156-160).

J: I think, one of the biggest advantages of it was that it allowed us to get almost an extra chapter accomplished during the second nine, or the first semester, we were able to get all the way through chapter seven, which allowed us to complete the majority of the proofs that are in the book (Lines 50-53).

June commented that the students were more actively involved in exploration with the GSP labs and were able to carry over the new knowledge into the classroom discussions.

J: Well they were more, they were definitely more involved, too many times my kids are sitting there listening and, and not actively participating and this definitely gave them a way to actively participate (Lines 102-194).

J: Well as far as during the lab time the students were, a little more on their own so it was a little more of an exploration type activity (Lines 77-78).

J: I thought that was really quite effective, a lot of times then when we'd get back and we'd talk about the particular topic that we were doing in the lab that day or do more of an application type thing with it, the kids would say 'oh yeah that's just like in the lab we did, you know this happened' and so I thought especially again the ones that were really involved in the lab seemed to really be able to draw that comparison (Lines 94-98).

When asked if June had used GSP after October 27th she replied that she had not. Follow up questioning at the end of the 2004 school year also indicate June had not gone back into the computer lab. The response given was she did not have time. This may indicate she did not feel she had time in the curriculum or she did not have time to plan for the use of GSP. June indicated she would use GSP the following year if it were included in the new textbooks.
J: I’m hoping that when we adopt new textbooks that we look for something that has Sketchpad built into the textbook series (Lines 46-47).

Due to the time frame of the study it was not possible to follow June into the following school year to see if she used GSP more or less than in the 2003-2004 school year. The following section of this chapter will examine Alice’s teaching with GSP.

Teaching with GSP: Alice

The following discussion will examine Alice’s teaching practice while using GSP in the areas of goals of instruction, role of the teacher, and role of the student. The analysis will relate Alice’s non-GSP teaching with her GSP teaching.

Goals of Instruction

Alice appeared to have three main goals of instruction when using GSP. She used the GSP lab activities to reinforce what had already been taught in the classroom. Alice used GSP to help save time in the coverage of transformations. Alice also expected GSP to motivate her students to work harder in the classroom. These goals indicate Alice saw GSP as an extra activity that could be added to the curriculum versus the real geometry work completed in the textbook and in class. Alice continued to make student reading a goal and by the end of the study she discovered the relationship between GSP and the goal of student understanding at a conceptual level.

In the preceding analysis of June’s teaching, the researcher was able to use information from the Lesson Plans and the Post-Lesson Reflections to show distinct plans or reactions to the different labs. When the same analysis was performed on Alice’s data it was discovered that in some areas the same phrases were used repeatedly. Thus, the phrase “GSP was used to reinforce the classroom instruction” was found in the PLR on 10-13, 10-15 and again on 10-28 as part of
the response for question 2. A response that was found in the additional comments section of the PLR for 10-28 states:

This was a short lab, just reinforcement. It took the students only 20 minutes to complete, which gave me more time in classroom. I tend to like using the Geometers Sketchpad as reinforcement (PLR 10-28).

The goal of using GSP for reinforcement is supported by the fact Alice pre-taught almost all of the topics before bringing the students to the computer lab. This will be discussed further in the section Role of the Teacher.

A second goal Alice had for the use of GSP was to quickly cover the topic of transformations. An examination of her responses to PLR question 5 supports this observation (see Table 15).

Table 15. Alice: Post-Lesson Reflection Question 5

<table>
<thead>
<tr>
<th>Date</th>
<th>Compare to a non-GSP lesson on the same topic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-19</td>
<td>A non-GSP lesson would have been time consuming. With the GSP, students were able to quickly draw a conjecture and see the findings more visibly.</td>
</tr>
<tr>
<td>10-15</td>
<td>A non-GSP lesson on the same topic would involve more copies printed so the students could have a specific model of golf and billiards to do their reflections off of. It would have taken more time in class to instruct how to do it. The students would need the proper tools, protractor, to do the hand constructions of the golf and billiards.</td>
</tr>
<tr>
<td>10-17</td>
<td>This is a quite lengthy topic for the students to hand construct on a hard copy. It is also confusing for them if they are not careful in their color-coding on the hand copy. They did take less time to achieve the goal using the GSP activity since they could see it quickly being a transformation.</td>
</tr>
<tr>
<td>10-21</td>
<td>It was easier for the students to understand how a vector works and how it translates. It was less time consuming and the students seemed to enjoy.</td>
</tr>
</tbody>
</table>

In an interview with Alice on 9-19 she commented on the quickness of using GSP.

A: I thought it was really well, it was much faster than doing it in pencil because what they were doing, I was thinking, they would never have been able to do on pencil and paper, that comparing the lengths, make this side longer, what is the angle, it would take them forever, so I think they appreciate the quickness of being able to draw their conclusions and conjectures with it (Lines 39-43).

Another major goal Alice had for using GSP was that it would motivate her students to become more interested in geometry and work harder in the classroom. This is first identified in the interview on 9-19.
A: And what I am hoping, and although we will not get in there as much as I would like to, that it will put interest in them into geometry when we are in the classroom, and I think they will be able to relate better (Lines 52-54).

The Mentor Log entry on 9-19 included the following reference to student motivation.

We discussed the class afterward and she was upset they were doing the computer assignments and not the book stuff. I asked if they normally do the book stuff and she said "no" so I said that this is an improvement and Alice agreed that it was but that she wanted it all (Mentor Log 9-19).

Interestingly enough, GSP did prove to have this motivating affect on her students. Alice documented this in her PLR after the Laser Lab.

An observation was made by a student in the schedule A4 class that after the homework, textbook homework, was graded, he made the comment that the students who received 10 points (which is an excellent total) were the students who came in during success period for help. This is a class that would have 50% of their homework with zeros and I have seen an increase in the interest to complete the homework and to be motivated since the use of The Geometer's Sketchpad in this classroom (PLR 10-15).

The motivation factor was also recorded in the Mentor Log.

Alice suggested we do a survey on GSP. She was thrilled that students had come in for help and quiz scores were all high. She noticed the students completed a larger percentage of homework. She noted this was a big difference from the past weeks for the same bunch of students (Mentor Log 10-16).

The mentor did not conduct a survey of the students' interest in GSP since the students were not the focus of the study.

Student reading was an observed goal in the snapshot of Alice's non-GSP teaching. This goal carried over to the computer lab where she had students read the directions for the GSP labs to the class or to their partner.

As mentioned earlier, Alice treated GSP as an extra activity that was an addition to the curriculum. This attitude was shared by the students and was documented in the PLR for 10-23. On this day Alice allowed her second-hour schedule A class to talk her out of taking them to the lab. Alice documented that the fourth-hour class that participated in the GSP lab had a much better understanding of the concepts in the book than the students who remained in the classroom. The topic for the lesson was reflections over intersecting lines.
PLR Question 1: Were the goals of the lesson achieved, and if so how do you know this?
For A2: The A2 class before we went to the lab was complaining about grades and of what was taken in class, whether they wanted to go to the lab or not or work on the homework, and the students chose to stay in the classroom and not go to the lab. So the A2 class did not go to the lab.
The goals were not met for this class because the next class period, when we went over the homework they did not understand the angle measurement of the non-obtuse angle of the intersecting lines and how it relates to the rotation.
The A4 class went to the lab and did the reflecting over intersecting lines. The A4 class had a better understanding of rotations and the angle measurement, and the two-reflection theorem for rotations; they understand the theorem, and I feel that the goals were met in the A4.

PLR Question 5: Compare to a non-GSP lesson on this topic.
Since the A2 did not go to the lab this gave me a good comparison of the A2 to A4 with labs and without labs. When reviewing the homework the A2 had more questions, did not understand the textbook's problems and scored lower on the homework. A4 reviewing of the homework, since they went to the lab, required less time, reviewing homework - understood it, and they were confident, and pleased with themselves of what they had accomplished with the GSP.

Alice offered the final lab on properties of rotations as a bonus assignment that further illustrates the non-essential role of GSP.

PLR Additional comments on this lesson on 10-23:
The GSP definitely made a difference in the comparison of the two classes. Students seemed to enjoy bonus when it is offered and completed. Usually when bonus is offered only three or four students turn it in, but the majority of the students turned in the bonus that was offered using the GSP.

As a result of not taking one class to the computer lab it appears Alice discovered GSP could help students understand geometry concepts. In her additional comments on the following lab on Glide Reflections, Alice refers to student understanding.

Students seem to understand the concepts if they can discover it first and they understand it (PLR10-28).

It appears that on the last scheduled day of using GSP Alice acknowledges the role GSP can play in student understanding.
Role of the Teacher

Two forms of instruction were observed when Alice taught with GSP. She engaged in direct instruction to model the labs and reminded students of what she had previously taught in the classroom. Alice then switched gears to monitor students working either individually or with partners as they finished their worksheets.

In the classroom Alice used direct instruction. This continued into the lessons where GSP was included as she pre-taught each of the topics prior to bringing her students to the computer lab. A review of her Lesson Plans indicated that for each lab she planned to pre-teach the topic (see Table 16). Alice had completed Lesson Plans and Post-Lesson Reflections, but was reluctant to give the researcher a hand written copy. In order to gain access to this data the researcher suggested Alice tape-record the Lesson Plans and Post-Lesson Reflections and they would be transcribed for her to view. This resulted in Alice using past tense in some of the lesson plans.

Table 16. Alice: Lesson Plans

<table>
<thead>
<tr>
<th>Date</th>
<th>Overview of Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-13</td>
<td>In the classroom, students will check their answers for the assignment on Reflecting Points. Review of homework will be a lead in for the lab. Second half of period will be in the lab.</td>
</tr>
<tr>
<td>10-15</td>
<td>The class then read aloud the golf and billiards section in the textbook. The teacher instructed examples of miniature golf and billiards to the class on a hardcopy before they went into the lab.</td>
</tr>
<tr>
<td>10-17</td>
<td>When they are finished with the quiz they were to read from the textbook composing reflections over parallel lines. Five minutes before entering the lab the teacher instructed what a translation is and collected make up labs from the students before they entered the lab.</td>
</tr>
<tr>
<td>10-23</td>
<td>Introduction to reflections over intersecting lines, and then students will go the lab and work on the lab sheet of reflecting over two intersecting lines.</td>
</tr>
<tr>
<td>10-28</td>
<td>Discuss the new section isometrics and glides, and then students will go to the lab second half.</td>
</tr>
</tbody>
</table>

Further evidence of pre-teaching the topics was viewed in the videotape of 10-13. At the beginning of the class Alice reminded her students they already should know what the lab was asking.
A: Ok what you are going to do in this activity is what happens to the coordinates of points when you reflect them across the x- and y-axis, which is very similar to one homework problem that we did and we’ve already talked about, so you could already do a conjecture if I asked you on this one already (10-13, 2:58-3:14).

Once in the computer lab Alice would direct the first half of the lab by reading and modeling on the demo computer. Once the students were having success she would monitor their work by walking around the class and would help individual students as necessary.

An example of directly teaching the lab was recorded in the Mentor Log on 9-19. Notice that in this lesson Alice models the lab and then provides her students with the solution that the lab is leading them to find on their own. She tells them that the shortest distance between the line and the point is when you have a ninety-degree angle or perpendicular lines.

Alice basically went though the lab and demonstrated how the program would work, starting with putting a line on the screen and then a line segment to a point on the line and then showing them [what buttons to click] to measure the segment and the distance to the line stressing the complete use of sentences for solutions, for example stating that the shortest distance to the line is obtained when you have a 90 degree angle, or that the lines are perpendicular. The students were then advised to put a text box on with their name on the screen and they printed out, they all sent it to the printer, which worked very successfully (Mentor Log 9-19).

On 10-13 Alice directly taught the first part of the lab on Reflections in the Coordinate Plane and then let them finish this and continue on the second lab on their own. This is documented in the Mentor Log.

They were then told to turn it over and do the backside. This was impressive. They all worked on it without a leading walk through. Some had questions but it went really well. They answered the questions on screen and printed it out. Most all finished – Good Class (Mentor Log 10-13).

Alice maintained her position as an authority in the computer lab, even when she was incorrect. While leading the instruction on the Reflections in the Coordinate Plane, Alice had her students measure the angles of the plotted triangle. They were supposed to measure the coordinates of the triangle’s vertices. When questioned by a student on this (twice) Alice went on to measure the coordinates but told the class to keep the angle measurements there because they
would use them later (which they did not). The mentor commented on this in the mentor log entry for that day.

Then they measured angles instead of coordinates. Alice never told them they really did not need angles (Mentor Log 10-13).

Alice was consistent in her roles for the teacher between GSP and non-GSP lessons. She assumed she had to directly teach the material to her students. She was the authority in the classroom. She also liked to make math easier and fun and saw using GSP as a way to do this.

Role of the Student

Activities that defined the role of the student as recorded in the snapshot of Alice’s teaching included the following:

- Bystander
- Passive recipient of information
- Active seeker of information
- Source of knowledge

This is consistent with what was observed during the GSP labs in September and October of 2004. Unfortunately, a large number of Alice’s students did not have the appropriate hole punched on their ID cards and were not allowed to work on a computer when the class went to the computer lab. The students who were not allowed to participate sat at desks or tables along the side of the computer lab and could be considered bystanders. Most of these students eventually brought in the appropriate permission slip, but a few students went the entire time without getting on the computer. The PLR for 10-28, which was the last lab on Glide Reflections, indicates these students were not able to participate in this lab because they did not have permission to use technology.

The students were to do the GSP silently, individually and it was offered to them as bonus points. A total of 9 bonus points could be given on the rubric. Most students chose to do this activity. Out of 67 students only 8 students chose not to do the activity. Some of those students [were] because they did not have their ID punched for computer use (PLR 10-28).
A few of the students who were able to work on the computer did not complete the worksheets. At one point Alice insisted that even though they were working in pairs, each student had to turn in a worksheet with the questions answered. These passive recipients made sure they had written something down but were not too concerned with the accuracy of their answers. The mentor noted that in the really small class of 12, all the students were actively involved in the labs. This is the class referred to earlier as improving their quiz scores.

Alice’s students were observed working on the computer with varying levels of success. In an early lab the mentor noted the students having success working independently with the topic of the triangle inequality.

Students proceeded to work on the triangle lab independently and many of them completed it and turned it in. There were some excellent discussions between students and both the teachers who were in the room concerning what would happen when you add the two sides of the triangle and then try to make that equal to the third side by manipulating the one point. Students were able to collapse it down on the line and it needed some discussion to figure out what that really meant in their diagram, and what that meant for them to write down. And at that point you could see the light bulb go on and know that they really did at that point finally understand the triangle inequality because you only get one line when those distances are the same. It was a, I think a very good experience for the kids; I think that they really enjoyed being in the lab (Mentor Log, 9-19).

Later in the study Alice made the following observation of her students’ activities in the additional comments section of the PLR for 10-17 Reflections over Parallel Lines.

The teacher has noticed that the students are using the discovery method to develop their answers, they are increasing their writing skills and they seem to enjoy using their geometry terms that they are able to understand. The students are also communicating with each other and talking geometry as they are working with the lab (PLR 10-17).

The activities noted are confirmed in the CLR’s for Alice’s teaching. The students were active in reading, writing, and discussing the topics with appropriate vocabulary. The students were discovering new concepts even though they had already been taught the concept in the classroom. This was noted above in the example of the triangle inequality where the students had been told the property in the classroom but by encountering it in the computer lab they actually discovered what the property means.
Alice taught three geometry classes that ranged in size from 12 to about 20 students. She began modeling GSP by using the front demonstration computer herself. After a few labs she tried putting students at the demonstration computer and found it allowed her to move around the computer lab to assist and monitor students. She also found in her larger class that it worked better if they worked in pairs. The Lesson Plan and the Additional Comments from the PLR for 10-13 document this aspect of the role of the student.

A student at the overhead computer will model reflections in the coordinate plane lab while the teacher gives verbal directions (Lesson Plan 10-13).

Note-It is better to use partners in lab, especially a large class. Smaller classes like to use the GSP individually and seem to work better when they fill up two rows (12 students). It is a benefit if the class turns in lab homework before entering the lab instead of using up lab time. There are fewer discipline problems when in lab versus the classroom time (PLR 10-13).

Since Alice directly taught and modeled the GSP labs the students were not overtly treated as sources of knowledge and authority but many students took on this role when working in the computer lab. For example, while teaching the lab Reflections in the Coordinate Plane, Alice read all of the steps on the worksheet and explained each set of GSP instructions. She did not call on students for either the reading or to explain the GSP instructions but when she mistakenly instructed the students to measure the triangle’s angles instead of the coordinates of the vertices many students asked if they should be measuring the coordinates. Students who had measured the coordinates were told to delete them until Alice finally realized this was the correct step. This illustrates her students could both read and help others follow the directions but they were not asked to do so during this lesson.

Epilogue

Alice was interviewed in January of 2004. During the interview she confirmed she had taken her classes to the computer lab after the conclusion of the planned GSP labs. This new GSP activity was conducted in December on the triangle congruence postulates. Alice explained...
her textbook contained a GSP exploration activity in the introduction to the chapter on triangle congruence. She took her classes to the lab, prior to teaching them anything, and found they discovered which situations would, and would not, prove triangle congruence. When asked why she treated this topic differently than transformations she replied she was following the book and they were not supposed to know anything yet.

A: The first 15, 20 minutes of class, I didn’t use my whole half a period in the computer lab in there, it was the warm up, the introduction, meet me down there and it just took a few minutes to do (Lines 177-179).

In the sections on transformations the textbook did not use GSP in the warm-up sections. This example stressed how dependent Alice was on the textbook. When questioned about future use with GSP, Alice stated she did not have time because GSP did not meet the State Geometry Standards.

A: I think that it is a useful tool but with what the State is requiring us on the standards it doesn’t meet that need. It can help them connect it and did help them to see the overall picture, in that one, the first section of the textbook. It helped them to see the overall picture, but it can be time consuming and it would require, because State Standards requires them to construct and not on a drawer (Lines 28-33).

A: The University of Chicago it went well, because it uses the reflection, and the next textbook that we’re looking at, it may not go well, um, and it’s just sad to say that there just isn’t. I feel there just isn’t enough time in the classroom to go to the computer lab and to meet the standards. I guess we are so standard driven that it’s taken everything out of the classroom (Lines 99-103).

A follow up question at the end of the school year confirmed Alice did not return to the computer lab after the exploratory lesson in December.

Comparison of Alice and June

Alice and June taught with GSP in different manners with different goals, but for the most part, their GSP teaching was consistent with their non-GSP teaching. The main difference was found in the way they used GSP, which was a reflection of their goals of instruction. June used GSP during the first half of the geometry class period to introduce a topic and for student exploration. Alice used GSP in the second half of the geometry class period to reinforce a topic.
that had already been taught during the first half of the class period. The most striking example of this was the GSP lab on Reflections over Parallel Lines. Question 1 on the worksheet asked the students to project what single transformation will take the original figure to the second reflected image. They were instructed to skip the question and come back if they did not know the answer. At the conclusion of the lab the students are told to go back and answer Question 1. Alice prompted them the answer to Question 1 in the beginning of the class and even wrote “Translation” on the chalkboard (CLR 10-17). June prompted her students for the answer after 20 minutes into the lab session (CLR 10-16).

Alice and June shared the goal of quickly covering the transformation chapter. Another similarity was that both teachers said GSP helped their students learn the chapter on transformations but neither one of them felt they had time to use it for other geometry topics.

Follow up questioning in the exit interview asked the teachers to reflect on specific answers to the belief survey that was conducted in June of 2003 and again in early January 2004. June was consistent with her answers and any differences in ranking were explained by the way in which she had interpreted the statements. By contrast, Alice stated she had changed a few of her beliefs about teaching mathematics from Phase 1 of the study to the conclusion of the study. This prompted the mentor to ask permission to return to Alice’s non-GSP geometry classes to conduct observations as well as videotape, in order to confirm the new beliefs. The following section of this chapter discusses the findings with regard to Alice’s shift in beliefs.

A Change in Beliefs for Alice

The question of change in teacher beliefs was not a specifically stated research question for this study, but it was an overarching goal of the study to use GSP as a vehicle to promote teacher beliefs that align with the current mathematics reform recommendations. The following section will explore the impact working with GSP had on Alice’s belief system. The researcher
followed Alice back into the regular geometry classroom to verify if her newly stated beliefs were realized in her teaching practice.

At the beginning of the study, Alice completed a belief survey. Many of the responses were found to be consistent with her teaching practice and were documented in the Snapshot of Alice. A few of the responses were not consistent with what was observed in the classroom in May 2003 and this was documented as well. At the end of the study, Alice repeated the belief survey and was asked in the exit interview to comment on items that received responses that had moved two or more categories on the response scale (which contained five responses from strongly agree to strongly disagree). Each item is listed with Alice’s pre- and post- responses, followed by a brief discussion. The following item was listed second in the section on views about mathematics:

2) A lot of things in math must simply be accepted as true and remembered: there aren’t really explanations for them. Agree to Strongly Disagree.

A: Hum, I remember, hum, gosh, I would say why did I change that? I would say that they need, to say then need to explore and find out on their own now.

M: Ok, maybe say that louder.

A: Ok because why did I change that, for them not to accept it is that, I have found a way that they actually can explore and find out on their own with the Geometers’ Sketchpad, without the Geometers’ Sketchpad basically they have to accept some things.

M: Ok.

A: In the book, with Geometers they can go down there and explore it and say ok now I know why they say that, much quicker, where it would be time consuming to do it with pencil and paper.

M: Ok.

A: I think that is probably why I changed that (Lines 263-275).

Alice changed her response to strongly disagree but when she was questioned on this item she had to stop and think why she changed her response. Alice indicated that she had found a tool to use that would help students explore math but that without GSP they would just have to accept things. She also indicated that with GSP the understanding could come quickly. This implies an underlying belief that if understanding cannot be attained quickly the students will just have to accept what is being presented. The fourth item in this section was:

4) To be good at mathematics, you need to have a kind of “mathematical mind.” Agree to Disagree.
A: Maybe not necessarily a mathematical mind but the thought process has to be there, in
genral, to do it, but you don’t have to be good at mathematics to, you don’t have to be
good at understanding mathematics to be good in mathematics.
M: Ok.
A: Just if you could have that thought process.
M: Ok are you agreeing with what you said last summer?
A: You need to, I say I disagree, with this.
M: But now you disagree, you don’t need a mathematical mind?
A: Yeah.
M: Just a mathematical thought process?
A: Um hum, um hum (Lines 279-289).

The researcher had a hard time differentiating between Alice’s definitions of a mathematical
mind and a mathematical thought process. By focusing more on the change in the response it
seems Alice now felt she might have broadened her view of what it takes to be good at
mathematics. This set of statements indicate Alice’s beliefs are complex and further questioning
would have been necessary to confirm and explain much of what Alice was saying, including
what it means to be successful in mathematics. The intent of this section was to expose the fact
that by working with GSP Alice did experience changes in her beliefs about mathematics.

The next two items were from the section on learning mathematics. Item number seven
had to do with practice and item number ten with mastery.

7) For students to get better at math, they need to practice a lot. Strongly Agree to
Neutral.
M: So you changed there?
A: Yeah, drill and kill.
M: Um hum.
A: I think sometimes I have done that, because I drill them too much and I kill them.
M: Hum.
A: I think I am trying to think more of the problems that I am assigning, not the quantity of it
but the quality of it (Lines 195-300).

10) In learning math, students must master topics and skills at one level before
going on. Agree to Disagree.
A: Um, hum, because in mathematics we review so much that I feel that if they do not catch
it the first time around that they may the second or third, or fourth, but the key is to keep
reviewing that so, so I have changed, (laughing) for the good (Lines 308-310).

Alice reflected that she had changed her thinking in these two areas and indicated these were
positive changes in her instruction.

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The last area contained items on teaching mathematics. The researcher asked for comments on the following items in the order presented here (item 19 then item 18).

19) The most important issue is not whether the answer to any math problem is correct, but whether students can explain their answers. Strongly disagree to Agree.
M: Can you explain what moved you in this direction?
A: Whether they can explain their answers, because anyone, I feel that you can guess at the right answer, and come up with it, but if you can’t explain it to me, how you got that answer then you haven’t got it, so let’s say it was multiple choice they could guess at it, and get the correct answer.
M: Um hum.
A: But they couldn’t explain how to do it, so I am more interested in your explanation on how you did it (Lines 318-325).

18) Teachers should not necessarily answer student questions but should let them puzzle things out themselves. Disagree to Agree.
A: Um hum, yea definitely can’t believe that I said that, (laughing), I can’t believe I said that.
M: You don’t, you don’t think this was an honest …
A: Well no, I probably did, I don’t think I, yeah how I have changed, but um, yeah because I don’t answer every [one of] the students questions, I always come back with them and say ‘well what do you think?’ and I try to have them try to work it out.
M: Ok.
A: Instead of answering it direct, because they are not learning anything if I answer it directly (Lines 330-338).

The last two items indicate a major shift in how Alice sees her method of instruction.

She stated she was stressing student understanding and she deflected student questions back to the students in order to explore their thinking. This pedagogical shift came as a revelation to Alice in the interview and she commented on the change in the following excerpt.

A: I guess, you know, looking over that I’ve gone more into the thought process.
M: Can you think of why that might be?
A: Well that is what I am wondering, why, and maybe it was the Geometer’s Sketchpad, because I haven’t had any other training or any other schooling this …
M: Between these times there was no other factor that could?
A: No, I went to one other training, but that doesn’t, it maybe had a little influence because it was standards, so there is, that was standard driven, to me, so that is why I am standard driven now, but into the thought process and stuff, I think I saw the value in the students exploring and finding things out for themselves, with the Geometer’s Sketchpad, the value in it (Lines 340-349).

Beliefs are difficult to put into words and can be difficult to defend. In general, Alice appears to have made a shift in her beliefs on mathematics teaching and learning by placing more emphasis on student understanding and less emphasis on drill. What is interesting is the
researcher found the responses to belief survey items 2, 7, and 18 (from June 2003) to be consistent with what she observed in the baseline classroom observations. An additional question for the researcher to pursue presented itself at this time. Were Alice’s new beliefs consistent with Alice’s current practice in 2004?

The researcher asked permission to observe and videotape Alice teaching geometry in the following weeks. The researcher did not explain exactly what she was looking for; just that she felt a need to have more data at the end of the study for comparison purposes.

The researcher observed Alice teach her fourth-hour geometry class on February 2nd and 5th in 2004. The researcher did not find evidence to support the change in beliefs Alice expressed in the exit interview. Alice conducted the classes in a manner similar to what was observed in May 2003. She worked problems from the warm-up and from the homework herself. When the students were working problems from the reading discussion, Alice gave them the answers. The following comments were recorded on the CLR for February 2nd:

- “I’ll get you started” (1:35)
- Teacher modeled all solutions (1:45)
- Let’s do number ten; Teacher demo set up and solved area of triangle. So far only the teacher has solved each problem. She asks, “Do I need to work it?” (1:55)
- On number 22 – teacher worked it out (2:00)
- In both cases the teacher talked the students through the problems
- Teacher gave student the shape and prompted the solution
- Teacher found the area of the graphed trapezoid
- Teacher stated, “I know the sides are parallel” (2:25)

On February 5th, the warm-up and homework review was followed by a lecture on the circumference of a circle formula and then arc length. Alice had the students take notes on a yellow paper with a pre-printed format. On the left-hand side of the paper was a column of three blocked areas for diagrams. Matching these three blocked areas were blocked areas on the right side of the paper with lines for writing text or equations. The students were to copy what was on the overhead into the indicated block on the paper. This technique of directly teaching the procedure of finding the circumference focused on substituting the correct values into the
formula. Student codes for this 25-minute time frame were Watch, Copy, and Listen. The researcher commented to Alice about this more formalized lecture time with the special pre-printed notepaper. Her reply was that this was what her students wanted. She stated that the students wanted good notes to follow to help them with the homework.

It is possible Alice had made a shift in her belief system but was unable to put those new beliefs into practice due to a stronger set of beliefs about what she thinks her students want as well as what she thinks teaching mathematics looks like.

Mentoring

The study participants were able to gain knowledge on how to implement the program by working with the mentor. When technological or pedagogical questions came up they were promptly addressed. On the one hand, the researcher does not want to imply that Alice and June left the study as experts in the use of GSP. On the other hand, Alice and June indicated the mentor had helped them with learning the program and learning how to use it with their students.

In the exit interviews June was able to reflect on the study and provided the following insight.

J: It was very nice to have, have you there to, to help guide us through sometimes, or to give us advice on how, you know, we might approach a particular lab or, or concept (Lines 114-116).

J: I thought it was, it was good when I would sometimes watch you, do a lesson and then I would teach it the next class, with the next group. I am trying to think if we ever did it in reverse, if I ever taught and then you did, I can’t remember that we did. But yeah that was, that was just really helpful, to see somebody more experienced working with Sketchpad to teach it in the first place (Lines 148-152).

J: I didn’t have any, I don’t think I had any concerns with it [GSP]. I don’t know, I don’t know if the mentoring was the reason that I didn’t (Lines 300-301).

In the exit interview it was difficult to prompt Alice to reflect on the role of the mentor (and the study in general) but she did make the following comment.

A: That you actually spent more time with us, it wasn’t just thrown at us and said here do it, we did not have to read the manual on how to do it, but you were there to help us (Lines 92-94).
The discussion of the role of the mentor and then mentoring as a tool for professional development will continue in Chapter VII. This chapter concludes with a review of each of the teachers as they implemented GSP into their teaching practice.

Review

This chapter began with the implementation of GSP for Alice and June. The following sections will review and summarize the learning, planning, and implementation phases of GSP for June and Alice beginning with the summer sessions.

Review of June and GSP

When June worked with the program in the summer of 2003, she approached the program from the viewpoint of a teacher and asked many questions regarding how to use it with her students. She worked through the selected labs either with the mentor, or at home, and came back with many questions. When something did not work out she had questions as to why it did not work. June was committed to learning the program and put time in at home completing suggested activities.

June continued the learning process while using GSP with her students. If June was demonstrating a step in a lab and it did not work correctly, she would back up and explain her mistake or ask the class what went wrong. She had a good sense of humor about it and made learning with GSP fun. June caught on quickly to the menu items and displayed a high level of confidence in her ability to teach with GSP.

June planned to use GSP as a tool for discovery. She scheduled her classes into the computer lab for the first half of the block period and introduced the topic for the day through a lab activity. Problems June encountered when planning for the use of GSP included advanced scheduling of the lab, deciding what GSP labs to use to support the textbook, coordination of the topics in the textbook to the State Standards, deciding how to reproduce the labs, as well as
modifying the traditional textbook reading, homework and assessment activities. June felt pressure to cover the material in the textbook but also recognized the benefit of using GSP to meet this goal (while working with transformations).

Teaching with Geometer's Sketchpad, for June, looked very similar to teaching in her classroom in the traditional setting. June worked through the GSP labs with the students by asking questions along the way. June stressed geometric vocabulary and often answered the students' questions by saying, "I don't know, what do you think?" in an attempt to get them to try to go further on their own. When the GSP lab got to a point where the students had to make a conjecture she helped them focus on what they were comparing or evaluating, encouraged them to write down their conjecture, and then share it with a partner sitting next to them. In the cognate domains of role of the teacher and goals of instruction, we find a slight change in June's teaching between the classroom and the computer lab. June's role as a facilitator increased while using GSP as the goals of instruction expanded to include student discovery of geometric principles. Many of the students became more involved with the subject of geometry at a higher level than what was observed in the classroom. Most of the students were engaged on the computer and actively discussing either the program or the assignment. The method of instruction also changed since students were first guided through the given lab and then given time to make conjectures on their own or with partners.

June reconciled many of her scheduling concerns by using much of the textbook for reading, and homework assignments, and she used publisher produced assessment activities. She found that by skipping certain problems, the textbook could still be used as the primary curriculum source.

June finished the study with positive feelings as to the benefits of GSP for her students. She did comment that it might have worked better if each class would have gone only once a week over the semester as opposed to the concentrated use in September and October. Interestingly though when asked (in January) if June had used or would use the program again in
the 2003-2004 school year she answered “no.” When the researcher asked June if she would use the program the following school year she indicated that it depended on what textbook was chosen and if the textbook would have GSP activities already in it. This was a most interesting result since June had demonstrated confidence in using the program with her classes and indicated that the program had helped many students blossom in geometry. One possible explanation for this would be to examine the teachers’ initial purpose for using GSP with their students. June was looking for an efficient way to cover both the State Standards and the textbook chapter on transformations. This goal had been accomplished. The students did cover transformations and June felt they had a better understanding of it due to the use of GSP. After the chapter on transformations was completed June stated she simply did not have time to take them to the computer lab since they had to get through proofs by the end of the semester. This same reason was given for the second semester work. She did not have time.

Review of Alice and GSP

Alice was introduced to GSP in the summer of 2003. She approached the program from the viewpoint of a teacher. While learning the program Alice expressed concerns regarding how to assess her students on GSP, if they would like using the program, and logistical concerns like when to use the computer lab versus a demonstration in the classroom. She displayed uncertainty in her confidence in using the program with her students and only wanted to complete labs on topics she currently covered in the curriculum.

Alice planned to use GSP as a tool for reinforcement. She scheduled the computer lab for the second half of the blocked period. The first half of the class time was spent teaching the topic that would then be reinforced in the computer lab by an appropriate GSP lab. The difficulties Alice encountered in planning for the use of GSP are similar to what has already been described for June. The main problem was a discrepancy between the State Standards and the textbook on the topic of transformations and the desire to quickly cover transformations. In the
third planning meeting Alice used the chapter on transformations to organize the selection of GSP labs. At first Alice thought she might have to skip most of the textbook reading, homework, and assessments due to the GSP approach but after careful examination found she was able to use the text for most of the chapter.

Alice used direct teaching in the classroom and carried this method of instruction over into the computer lab. She led the students through the activities of the lab, either by whole class demonstrations or by working with individual students. Alice treated GSP as an extra activity that could be done after she taught the geometry topic in the classroom. Interestingly, Alice’s students picked up on this attitude and one class asked to skip the lab on rotations as a composition of reflections over intersecting lines. Alice documented that the class that completed the lab had met the goals of the lesson and did not have questions on the homework while the class that chose not to complete the lab did not understand the relationship between intersecting lines and the rotation transformation. Thus, by the end of the study, Alice began to see the merit of using GSP for developing student understanding.

The role of the student was similar between non-GSP and GSP classes with a few students not participating in the class activities as well as others that were very engaged. Alice approached GSP as a fun way to do geometry and hoped it would motivate her students to complete their textbook assignments. This goal was realized, especially in the schedule A fourth hour class. Differences that were noted in the role of the student included an increase in student-to-student discussions and an increase in the mathematical level of those discussions as students were making discoveries (or confirming what they had been told) about geometric principles.

About six weeks following the transformation unit and the conclusion of working with the mentor, Alice took her classes to the computer lab to complete a discovery activity on triangle congruence postulates. At this time she did not pre-teach the topic and used GSP to introduce the unit as was scripted in her textbook. While this was only a small part of one class period, Alice reflected on the impact it had on her students. They could really see why, for example, AAA
would not prove two triangles congruent. When asked why she treated GSP differently this time, Alice responded that she was just following the textbook and it was an introduction to the chapter as opposed to one of the lessons she had to teach them. After this use of GSP in December, Alice also stated she did not have time to use GSP for the rest of the year.

This concludes the discussion of specific results found in this study for these teachers. The following chapter will attempt to more globally answer the two research questions as well as link the specific findings of this study to the current literature as discussed in Chapter II.
CHAPTER VII: DISCUSSION AND INTERPRETATION OF FINDINGS

This chapter will revisit the research questions posed in Chapter I. This will be followed by a discussion of limitations, implications, and suggestions for further research.

Research Question One

The first research question states: How do teachers implement Geometer's Sketchpad in their planning and teaching? This discussion will focus on Alice, June and Jacob, the pilot study teacher. Each of these three teachers illustrates a different way in which Geometer's Sketchpad was implemented. Alice used GSP as a tool for reinforcement and motivation. GSP was used as a tool for exploration within one topic by June and Jacob used GSP as a tool for exploration and communication regardless of topic. When comparing the three teachers' beliefs regarding the nature of mathematics, goals for instruction, and level of content knowledge, the researcher noted distinct differences.

At the outset of the study Alice and June were surveyed and their teaching practices were observed to determine their beliefs and goals for teaching geometry. Alice maintained an Instrumentalist view of mathematics (Thompson, 1992) and viewed the role of teacher as the dispenser of knowledge. She drilled her students on the memorization of theorems and formulas through lectures and step-by-step demonstrations. This belief system drove the manner in which she used the program as a tool to reinforce or practice what she had already taught. This is consistent with Saye’s (1998) category of the accidental tourist. Teachers of this type have a goal of efficiency and control and use technology to save time and to increase performance. They also tend to avoid the unfamiliar. This was a characteristic often noted in the study as Alice avoided any GSP lab that fell into a topic area with which she was unfamiliar.
Alice can also be identified as being in the “basics first” category of technology instruction as described by Hannafin, Burns, and Little (2001). This model stresses direct instruction and the avoidance of errors when using technology. Alice had a strong belief system regarding what teaching geometry looks like and did not treat the use of GSP as real teaching. She used GSP to reinforce her teaching with the goal of increasing student motivation. According to Smith (1996), teachers with a strong traditional view of teaching have difficulty adjusting their practice due to the absence of a new set of benchmarks with which to evaluate their practice. This was observed in Alice’s teaching at the end of the study where she had begun to verbalize a change in selected beliefs regarding teaching but was unable to break from the pattern of classroom instruction she had established for herself.

Survey analysis and observations of June’s teaching indicated she held a process view of mathematics (Thompson, 1992). Although she used direct teaching in the regular classroom her goal was to help students make sense of and to understand the origin and application of geometric theorems and formulas. June used GSP as a tool for discovery. She fits into Saye’s (1998) voyageur category. Voyageurs desire empowerment and enrichment and use technology to engage the student with content. He identifies this type of teacher as a risk taker. This was evident in the study when June decided to teach a GSP lab on parallel and intersecting lines, early in October, without the aid of the mentor. Using the category system defined by Hannafin, Burns, and Little (2001), June could be identified as having an attitude of “guided generation.” She allowed the students to work through mistakes to come to an understanding of a geometric principle.

The impact teachers’ beliefs have on their instructional goals and teaching practice is well documented in the work of Carpenter and Fennema (1991). In their model for research and development, emphasis is placed on teacher knowledge as a factor that influences teacher decisions. The following discussion will review the influence of teacher knowledge in this study.
Results of the baseline phase of this study indicate that June possessed a stronger geometry background as well as a deeper level of pedagogical knowledge than Alice. Baseline van Hiele test results indicate Alice was at level 4 but classroom observations found her working primarily at a procedural level, possibly due to the fact that she was put “on the spot” in front of her students. When attempts were made to venture beyond this she was generally not able to finish the problems. This was evidenced by her attempted proof of the isosceles right triangle theorem. By contrast, June’s instruction moved between the different van Hiele levels as she guided the development of geometric theorems.

A similar discrepancy in the level of knowledge regarding GSP between Alice and June was noted while they were instructing with GSP. Alice was not comfortable with the program. Alice wanted the mentor to model the lessons and to be present during her initial GSP teaching experiences. During instruction with GSP she was tentative in her tone and on a few occasions asked the mentor if she was correct or to clarify an instruction. She was not flexible in her use of the program. An example of this was when she insisted students use the distance between two points method of measuring a line segment when measuring the segment would provide the same information. By contrast, June was comfortable with the program and was able in one class session to extend the lesson by developing an additional sequence of commands for the students to follow.

Underlying factors that supported the implementation of GSP in the pilot study include Jacob’s content knowledge, goals of instruction, and his beliefs regarding mathematics. He possessed a high degree of content knowledge. His goals of instruction included deeper coverage of geometry topics than what was observed in the main study. Also, Jacob viewed learning mathematics as a process where communication of students’ ideas is a key issue.

A model of GSP use that characterizes the pilot study will incorporate the categories of technology use coined by Saye (1998). June and Jacob can be identified as voyageurs or as risk takers who are stimulated by novelty and experimentation. Figure 6 illustrates the factors that
support the voyageur role. Beliefs, Content Knowledge, and Goals of Instruction serve as the foundation. Traits identified with June and Jacob lead up from the foundation to support the category of voyageur.

Figure 6. GSP use as a Voyageur

The way in which Alice used technology is consistent with what Saye (1998) identifies as the accidental tourist. Alice used GSP to save time, to increase student performance, and to reinforce prior learning. Alice displayed a level of content knowledge that was at times inferior to some of her students. Alice viewed mathematics as the study of procedures and indicated that her goals of instruction were at the procedural level. Figure 7 incorporates Figure 6 with the addition of traits specific to Alice pointing downward leading to the accidental tourist category.

The model of GSP use indicates that in general a teacher that views math as a process, has a high level of content knowledge, and maintains student understanding as a goal of instruction is better prepared to use GSP as a tool for discovery. In general, a teacher that views math as procedures, has a low or medium level of content knowledge, and maintains a procedural goal of instruction for his or her students would tend to use GSP for routine practice and drill.

This model is consistent with what is described in the literature (Hannafin, Burns, & Little, 2001; Saye, 1998). When a teacher holds a procedural view of mathematics his or her instruction is usually at the procedural level. When a teacher holds a process view of
mathematics his or her goals for instruction will focus on student understanding. Content knowledge supports instructional beliefs and goals. Teaching for understanding requires a higher level of content knowledge.

![Diagram](image)

Figure 7. Model of GSP Use

Research Question Two

The second research question states: How does the role of a mentor facilitate the professional development of the classroom teacher with the implementation of Geometer's Sketchpad? Senger's (1999) model of recursive change describes the various paths a teacher may take when they are involved in changing an aspect of their instruction. This change model will be used to describe how Alice and June came to use GSP and the role the researcher played in the implementation process. The model will then be used to discuss how Jacob came to use GSP. The methodology of the pilot study will be reviewed since the professional development model used in the pilot study facilitated the movement of Jacob to the final stage of the change model.
Alice and June became aware of GSP, which is identified in the model as stage one or new knowledge when they attended an NCTM regional workshop on geometry and GSP was demonstrated. This awareness was fostered through discussions with the researcher who offered to mentor the teachers in how to use GSP with their students. The mentor encouraged the teachers to investigate GSP thus inhibiting them from rejecting it at the first stage of the change model.

In the summer of 2003 the researcher worked with the teachers providing them with experience using GSP. During the summer months the teachers began to experience stage two of the change model, which is creating mental images of new possibilities. In the summer months the teachers also began to verbalize how they might change their instruction to include GSP, which is found in the third stage of Senger’s change model.

The teachers continued to verbalize about using GSP with their students, which Senger identifies as experimental change in verbalization in the fall of 2003. During this time the researcher assisted the teachers in planning and teaching a number of GSP labs. This action prevented the teachers from rejecting GSP until they had experimented with it in their own classrooms, which is the other activity of stage three in the model. Unfortunately, once this support ended the teachers rejected the use of GSP for the remainder of the school year. Factors that contributed to them rejecting GSP will be discussed (see Limitations) and suggest that a modification be made to Senger’s change model to support teachers in being convinced of the merits of GSP. The modification that is suggested is a purposeful examination of student understanding when GSP is used. Results of the pilot study indicate when Jacob reflected on his students’ understanding, which was communicated through their work with GSP, he became convinced of the importance of using GSP and moved to the final stage of the change model, change in teaching practice and change in verbalization.
Figure 8 coordinates the Senger change model with the phases of this study. It also includes the addition of *teacher reflection on student understanding* as part of the third stage of change.

![Modified Senger Recursive Change Model](image)

Figure 8. Modified Senger Recursive Change Model

In the pilot study the sequence of learning, planning, and instruction with GSP were completely controlled by the mentor and were not delineated into separate phases as was done in the main study. In level one of the pilot study, Jacob supplied the mentor with a list of topics to
choose from and the mentor would select a lab to teach. The mentor would teach the lab to three different geometry classes on Tuesday of each week and Jacob would instruct the Wednesday sections. Jacob then assessed the results of each of the labs. Figure 9 illustrates the cycle of activity that took place each week. Jacob learned how the program would work, how to plan for its use, and how to implement the program in classroom instruction through an apprenticeship with the researcher in the role of a mentor or coach.

Figure 9. Pilot Study: Level One

This level of the pilot study can be identified as having characteristics of both the initiating and implementing stages of change since Jacob was exposed to something new and was provided with a vision of what this something new could look like.

Level two of the pilot study was identified as a time of transition where Jacob became more proactive in topic and task selection but remained the observer while the mentor taught. Jacob initiated an informal assessment of lab activities and resulting student work. Results of this assessment process were used as a springboard for classroom discussions. Jacob found GSP provided his students a setting where they could experiment and then communicate their findings. The assessment of student writing gave Jacob valuable feedback that he used to plan future instruction. Figure 10 is expanded to include the shift in responsibilities between the mentor and
Jacob as well as the increased use of GSP lab results in the regular classroom. This level of the pilot study is a continuation of the implementation stage of change where Jacob participated in experimental change in his teaching practice.

Figure 10. Pilot Study: Level Two

The third and final level of the pilot study was identified when Jacob assumed full responsibility for classroom instruction and the mentor became the observer of the classes. At this time Jacob was also sharing and defending his new beliefs regarding the use of GSP as an instructional tool to his colleagues. Both of these activities are characteristic of the final stage of personal change. Characteristics of the pilot study's third level are consistent with the institutionalization of a change in practice. The following discussion will review the differences in the methodology used by the mentor in the two studies as well as differences in the study participants.

In the main study, the learning of GSP, planning to use GSP, and the teaching of GSP were separated into distinct phases. Also, the amount of control held by the mentor in the planning phase was modified in order to grant the classroom teachers more control of this aspect. Figure 11 illustrates the phases of the study.
A comparison of Figures 10 and 11 indicates a difference in the nature of planning, teaching, and assessing the GSP activities with one being cyclical and the other linear. The linear nature of the main study developed in response to the teachers' goal of teaching an entire chapter with GSP. Alice and June were overwhelmed with the up front tasks of planning and implementing instruction with GSP on a daily basis. Thus, the tasks of assessment and reflection on the instructional cycle, which Jacob engaged in during the pilot study, did not receive adequate attention by the teachers.

The lack of teacher reflection on student understanding with GSP implementation was a primary difference between the two studies. This component accounted for the progress that Jacob showed and hence supports the addition of teacher reflection on student understanding as part of the third stage of the Senge change model.

Limitations

There are two major types of limitations in this research study that can be attributed to decisions in the execution of the methodology. The first involves a number of issues surrounding
Alice and June as participants in the study. The second involves decisions made by the researcher in the selection of Alice and June as well as granting them too much decision making control in the intervention phases of the study.

Alice and June did not have a strong and sustainable goal for using Geometer’s Sketchpad in their teaching practice. The teachers’ goal was to quickly cover the unit on transformations by using GSP as a tool for accurate constructions. Once this goal was met the teachers were not motivated to continue using GSP.

A second factor that limited the study was that Alice and June were not reflective teachers. A comparison of results from the pilot study and the main study highlights this issue. Jacob was reflective in his teaching practice. His use of formative assessment to inform decisions about future instruction is an indicator of his reflective nature. Jacob watched the mentor teach GSP labs for half the block class period, in three of his classes, one day per week. This gave him time to review and to reflect on what had occurred in the lab. The following day he taught the GSP labs to the remaining two geometry classes. He focused on the results of the GSP labs and found that use of the program allowed his students to explore, but more importantly provided a way for them to communicate about what they were thinking. Their responses to questions on various GSP labs provided a starting point for discussion in the following non-GSP class session.

In the case of Alice and June, there was no observed or documented evidence of the use of student assessment to prompt further student progress or to inform subsequent instructional decisions.

Another factor that influenced Alice and June was the atmosphere in which they worked. A marked difference existed in the extended support provided by the respective mathematics departments and department chairs. Jacob’s mathematics department was very excited and positive about the opportunity Jacob was involved in. The department chair assisted the mentor in logistics like parking and a key to the math office as well as showing genuine interest in the study. The school administrators were apprised of the pilot study and reported on this in a
general faculty meeting. The mentor was made aware of this while having lunch with members of the counseling office at Jacob’s school.

An atmosphere of support was not found once the study began at Harrison High School. At this school, mathematics department dynamics were not supportive of the older “veteran” teachers, which included Alice and June. The mentor observed that the older teachers were given the low-level classes while the younger teachers were assigned the higher-level classes. The department chair was a young man and did not enter into any discussions with the mentor on the GSP study that was taking place.

Alice and June followed the state geometry standards but their interpretation of these standards was narrowed due to the state-level assessment. The state was piloting an end of course assessment (GeoCap). Since results were reported to the corporation for each building, scores for individual teacher’s classes could be determined. The ramifications for Alice and June were that they must cover what was listed in the state geometry standards but particular items on the GeoCap test dictated their interpretation of these standards. Another example of Alice and June narrowing the curriculum is seen in their interpretation of reading in the mathematics class. Reading across the curriculum was a school-wide mandate; both Alice and June interpreted this to mean reading from their textbook and not other sources like a GSP lab.

Another issue that played a limiting role was the level of content knowledge displayed by Alice in particular. Results of the baseline phase of this study indicate June possessed a stronger geometry background as well as a deeper level of pedagogical knowledge than Alice. Classroom observations found Alice working primarily at a procedural level.

A final issue is the level of textbook dependence found with Alice and June. The issue of textbook dependence is related to content knowledge. In general, the teacher’s level of content knowledge has been linked to how well the teacher can use or modify the text (Ball, Lubienski, & Mewborn, 2001). Alice and June appeared to be at a loss during the second planning meeting when trying to decide how to negotiate their need to cover topics for the GeoCap and cover
enough of the chapter on transformations so the students could follow the remainder of their text. They debated on possible alternatives to homework and assessments but, in the end, returned to the textbook.

Limitations existed due to decision made by the researcher. Due to the initial support displayed by the department chair at Harrison High School the researcher was unaware of the culture in the department and that veteran and older teachers were not in positions of leadership or respected within the math department. Due to the involvement of volunteer subjects the researcher felt obligated to grant them the “final word” in the implementation phases of the study. In the learning phase the teachers were adverse to the risk of exploring unfamiliar geometric context using GSP, which prevented them from following the trajectory that their students would experience when using GSP as an exploratory tool.

Implications

The results of this study suggest topics that carry implications for mathematics education. In order of importance these topics are: forms of technological knowledge, a mentor model of professional development, the link between student learning and teacher change, and change in teachers beliefs.

Forms of Technological Knowledge

Results of this study indicate there are three forms of technological knowledge necessary to teach with Geometer’s Sketchpad. The first form is technical knowledge (TK) and can be compared to content knowledge. Teachers need to possess an in-depth knowledge about how the given technology works. An example of this would be knowing how to turn on the “snap to grid” feature and know this feature will force all plotted points to coordinate values that can only be integer values.
The second form of technological knowledge is pedagogical knowledge of technology and can be compared to pedagogical knowledge. This form of knowledge is illustrated by what the mentor modeled in the study for the participants. Pedagogical knowledge of technology (PKT) includes decisions regarding goals of instruction, task selection, and pacing decisions. PKT is closely related to PK, which was defined in Chapter II to be the teachers' knowledge of teaching procedures such as effective strategies for planning, classroom routines, behavior management techniques, classroom organizational procedures, and motivational techniques. A discussion of these issues surfaced in the summer planning meetings when the teachers asked questions regarding how to group students, how long the labs would take, if they should pair students up at machines, and if their students would like the program.

In phase 3 of the study the mentor modeled three techniques that fall in the area of PKT to aid in balancing the instruction between the geometry lesson and the new features of GSP. First, the mentor used students on the demo computer. Second, the mentor kept the class together during the set up portion of the GSP lab. Third, the mentor prompted students to answer the questions on the geometric concepts the particular lab would reveal. When the students appeared to be pushed to a point of frustration the mentor would give them a hint or ask a leading question. If most of the class experienced frustration the mentor would pull the students together for a whole class discussion. When Jacob, Alice and June experimented with GSP instruction they adopted features of this teaching trajectory into their practice. The main difference noted was that Alice pre-taught the concepts prior to using the GSP labs while Jacob and June used the GSP labs to introduce new topics.

The final form of technological knowledge is pedagogical content knowledge of Technology (PCKT). Pedagogical content knowledge was defined in Chapter II as knowledge of how students think and learn and in particular how this occurs within specific mathematics content areas. PCKT is defined here as how students think about and learn mathematics with technology. This form of knowledge surfaced in two places in the study. As the teachers began
instruction with GSP the mentor was called upon to sort out cases where "something wasn't right," which usually resulted from a student not reading and following directions carefully. The action taken by the mentor usually involved conducting a drag test on a segment or point in the sketch to determine if it was constructed correctly, or reading what was in the sketch to determine what errant geometric relationships were governing the sketch. Being able to find and to correct an error in a student's sketch is one form of advanced knowledge that usually only comes from the experience of using the program with students. Koedinger (1998) suggests that while errant computer constructions can be intimidating to students they are often a source of frustration for teachers and could potentially inhibit the implementation of GSP. The researcher suggests that practicing the diagnosis of errant sketches could be incorporated in an initial GSP training phase by bringing in samples of student work and having teachers examine the sketches to identify an error in either the student's construction process or interpretation of directions.

Knowing the difficulties that are likely to occur with technology is one aspect of PCKT. Another aspect of PCKT is knowledge of how students can learn with technology. Early in the study the mentor attempted to impact this form of teacher knowledge by having Alice and June experience GSP labs as their students would. The mentor's goal in the summer sessions was to have the teachers gain knowledge of the technology as well as experience GSP as a tool for exploration. The resistance of the teachers to participate as "students" inhibited the development of their PCKT. The researcher suggests that in future studies classroom teachers need to spend more time as "students" in a GSP training phase in order to gain experience that will influence their use of GSP as a pedagogical tool.

Planning, and teaching with GSP involves an advanced knowledge or understanding that only comes from experience. McDougall (1997) points out that when teachers first begin working with GSP they experience a loss of control. This loss of control can be easily identified in these areas as the teacher struggles to attain the advanced level of knowledge (TK) necessary to effectively use GSP (PKT). By working with a mentor the teachers in the study did not report a
loss of control in their teaching. They did not report a loss of control of class management (PKT) and Alice thought her students behaved better in the computer lab. They did not report a loss of control of their curricular expectations of their students (PKT). Nor did they report a loss of control of their professional role.

**A Mentor Model for Professional Development**

Use of a mentor as a professional development model is an avenue that warrants continued investigation. The pilot study indicates the potential for success of the model. While time constraints prevented the researcher from working with a group of teachers at this school, by working closely with one teacher the whole math department was affected. In the pilot study, Jacob learned GSP and how to implement it through an apprenticeship. Lave and Wenger (1991) suggest that through the process of apprenticeship people learn although there is little instruction. The learning process follows a trajectory of watching and practice as the apprentice moves from activity on the periphery to full participation. In a summary of research on teacher learning and professional development Wilson and Berne (1999) refer to a number of studies that stress working within the teacher's classroom and offering them a coach.

An examination of the design of this study indicates it served two purposes. The study was a professional development opportunity for two teachers, but it was also a research opportunity for the mentor. A professional development opportunity of this type can also provide the opportunity to gather classroom-based information that can, in turn, be used to improve the research base on the given topic. An example of a yearlong mentoring relationship is the basis for the book *Mathematical Power: Lessons from a Classroom* by Parker (1993). Through the published results of Parker's work with a fifth-grade teacher and her students, the research community gained an in-depth examination of the many issues teachers face as they realign their beliefs and instruction with the vision provided by the NCTM reform initiative.
The Link between Student Learning and Teacher Change

As a result of this study the Senger (1999) teacher change model was modified to include teacher reflection on student learning. This modification is supported by Syke’s (1991) recommendations for TPD, which include: (1) integrate examination of student learning, using multiple sources of evidence, into the TPD; and (2) reference both formative and summative evaluation of TPD into student learning. Hawley and Valli (1999) echo this recommendation for TPD by stressing the importance of the relationship between what teachers learn and what students learn. They state that student learning should form the basis for the planning and evaluation of the cycle of the TPD. This was exhibited in the pilot study, as Jacob was naturally reflective. This type of activity needs to be purposefully instigated for teachers who are not naturally reflective such as the participants in the main study. Reflection on student learning is one way that TPD could achieve the “deep meaning about new approaches to teaching and learning” Fullan (2001, p. 38) proffered.

Change in Beliefs

Results of the study indicate Alice changed selected beliefs about her role in teaching mathematics from a “drill-and-kill” procedural approach to a focus on student understanding. Throughout the study Alice progressed through the first three change stages as outlined by Senger (1999). To begin, Alice became aware of technology that afforded a new approach to teaching geometry. Using GSP for discovery was modeled by both the mentor and by June, which provided a mental image of new possibilities. At stage three, experimental change in practice and/or verbalization, Alice experimented by first using new technology with her old teaching approach. This is one avenue Fullan (1999) cautions against since it will usually not foster the desired change.

Toward the end of the study, Alice found her class that used GSP for a lab on rotations outperformed a class that did not use GSP on that topic. When interviewed she noted that use of
the program helped her students understand the principles of geometry, when she had time to use it. According to Senger (1999) in the stage following experimental change in practice and/or verbalization the teacher will either reject or become convinced of change in their practice. At the end of the study Alice was still in the experimental phase of both her teaching and her verbalized beliefs. This is evident by her returning to use GSP for an introduction to the triangle congruence postulates. This is also supported by the tentative nature of her exit interview and a tone of disbelief in her statements. Alice was surprised at her responses to the belief survey items that had changed instead of being able to strongly defend them.

In January 2004, Alice stated her teaching focused on student understanding. Since this would be a change from what was observed in May of 2003, the researcher completed additional classroom observations. The stated changes in belief were not observed in Alice’s teaching practice. Thompson (1984) discussed this phenomenon as she documented discrepancies between what a teacher says and what she actually does. Thompson reports a teacher’s beliefs can be discrepant with their instructional practice. This may indicate the “absence of an integrated conceptual system operating to modify her actions” (p. 403). An alternate explanation to the misalignment between what Alice was stating and her practice was Alice held very strong ideas of what teaching mathematics looked like and she was unable to reconcile her role as the teacher with her newly verbalized focus on student understanding.

Suggestions for Research

This study verifies the relationship between teacher beliefs and teacher knowledge and the way in which the teacher implements technology in the classroom. This emphasizes the importance that should be placed on the teacher’s belief system and his or her content knowledge in both pre-service and in-service training.

This study also reveals the implications of textbook dependence, which highlights the importance of well-written textbooks and training with continued support in their use. The
teachers in this study were faced with the pressures of state assessments and standards, which had a narrowing affect on their curriculum, and in the case of Alice tacitly supported her procedural teaching. Finally, this study exposes the issue of logistics. When teachers cannot schedule their classes into a computer lab they cannot use GSP. When they are forced to schedule computer lab use for future dates, not knowing if or how they will use that time, teachers become frustrated and give up.

During the teaching phase of the study the mentor became concerned with a lack of focus on formative assessment of student work by the teachers. Thus the researcher recommends the following adjustments be made in the study design:

- Space out the use of GSP to no more than one class session per week so that pre- and post-GSP lesson meetings/discussions can take place.
- Meet with the prospective teacher(s) before each GSP class session to discuss goals and concerns.
- Meet with the prospective teacher(s) after each GSP class session to review student results.

An additional change that could be made in the study design is hinted at by the action Alice took when she allowed one of her classes to skip a GSP lab. The result of this action on her part allowed her to focus on a difference in her students’ knowledge on rotations. This assessment activity helped Alice see the merit of using GSP. This prompts the following possible change to the study design:

- When appropriate in the study, have the teacher use GSP with only one geometry class or cohort of students. Then compare results between the GSP lesson and the non-GSP lesson.

This focus on formative assessment is supported by the work of Cobb, Wood, and Yackel (1990) and was cited in the methodology for this study (see pages 38-39). The difference is Cobb et al. focused on student understanding, whereas this study shifted students to the background while it focused on the teachers. The examination of student work, to determine student understanding, provides the teacher with the existence proof of what can be gained which supports their efforts to make a change in their practice and should be included in future studies of this type.
Implementation of these suggested changes in a future study may involve the mentor requiring certain activities from teachers. Results of this study favor the approach that was taken in the pilot study. When the mentor spaced out use of GSP and assumed control for the planning and teaching, Jacob was able to focus first on assessment. While Jacob spontaneously reflected on his students’ work, this may need to be a second aspect that is prompted by the mentor. Not all teachers reflect on their teaching practice. By prompting the participants to reflect on their teaching, the mentor may be able to help the teacher reveal, and if necessary confront, their beliefs regarding mathematics, that shape their goals for instruction.

Actions to promote reflective behavior in the participants were not incorporated into the design of the study. The researcher believed that such inclusion would be a confounding variable. It is now regarded as a crucial factor.

One final change that could be made to a future study is to incorporate the use of cases (Berne & Wilson, 1999) to provide the existence proof that GSP helps students learn geometry. Research on professional development has found that groups that worked together developed a set of “canonical stories” that served as a reminder of certain cases. Stories that are known on a national level include Cohen’s (1990) “Mrs. O” and Ball’s (1993) “Shea numbers.” Wilson and Berne (1999) state, “while such cases are meant to be disseminated and shared, the knowledge that appears in these professional development communities suggests that ‘case knowledge’ has validity (it naturally arises in these communities of learning teachers) and potential as a research tool or site for measuring teacher knowledge” (p. 179).

In closing, based on the results of this study the researcher makes seven general recommendations to improve the methodology of the next iteration of a study of this type:

1. Approach the school first to help ensure a supportive atmosphere for the study to take place. Then approach teachers that are considered leaders within the school.

2. Provide an out-of-school GSP learning opportunity that features a longer more intensive instruction time with a stipend or some sort of incentive for teacher participation.
3. Develop the use of cases of individual high school students and their GSP activities (using videotape, GSP files, and other written work) to be used as exemplars with the teachers.

4. Use a mentor to model GSP instruction in the teacher's classroom following an apprenticeship model. Allow the teachers time to concentrate on formative assessment prior to the burden of planning and instruction with GSP.

5. Incorporate reflective activities where teachers reflect on their students' work with GSP to build new cases.

6. Monitor the use of GSP use so it is not pigeonholed into one particular content area within the geometry curriculum.

7. Provide long-term support and follow-up. Ideally this could take up to two years.
Appendix A

Pilot Study
Introduction

In the fall term of the 2002-2003 school year, a research project was conducted to explore the feasibility and results that might be obtained from a long-term mentoring relationship between the researcher and a local high school geometry teacher, with a focus on the implementation of DGS activities. A high school was contacted in May of 2002 and a teacher named “Jacob” agreed to participate. The high school is located in a medium-sized town in the midwest. The school has a fairly large Hispanic population (10%). Jacob has taught at the school since 1968 and has a master’s degree from the University of Notre Dame. The school had purchased an updated (version 4) Geometer’s Sketchpad program, but was not using it in any of their classes. Jacob admitted to having little experience with the program but was interested in participating in the project.

The project consisted of the researcher working in the school almost every Tuesday from September to November of 2002. On Tuesday the schedule was two regular and two honors geometry classes. The researcher and classroom teacher worked in the lab for half of the block time or approximately 40 minutes each week. Jacob would then repeat the lessons with his Wednesday classes the following day by himself.

Topics: Selection and Instruction Modeling

At the start of the project, Jacob would e-mail a list of topics the researcher could then choose from. The researcher would select a lab (hand out) from a published source, create a lab, or outline a demonstration (no handout) to be done with the students. In a typical session the researcher would complete a demonstration and a lab activity. The topics of the labs are listed below:

Sept 17: Copy a segment; introducing angles
Sept 24: Midpoint formula; slopes of parallel and perpendicular lines
Oct 1: GSP Tours 1 and 2 (parallelogram in a quadrilateral)
Oct 8: Perspective drawings; rotation of a point demo; rotation 1 lab
Oct 15: Perpendicular bisector theorem; rotation 2 lab, introduction to translations
Oct 22: Burning Tent problem (reflections); vertical angle theorem

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Nov 5: Triangle (angle) theorems
Nov 12: Triangle Congruence theorems
Nov 19: Triangles: medians and perpendicular bisectors.

While the researcher conducted the lessons in the lab, Jacob took on a number of roles. In the beginning he worked through the labs on a computer, observed the instruction, and observed students. Soon he began to work with students. He easily adapted to the role of facilitator and asked questions like “How do you know that?” and “Why did that do that?” or “What is the relationship here?” As the semester progressed he took on a more decisive role in selecting, and then designing, the activities to be studied in the lab. The following e-mail communications illustrate the progress that was made in this area.

Sept 19
... Looking ahead: 1) equations of parallel and perpendicular lines 2) perspective drawings 3) rotations 4) also, midpoint formula will be among the things we will do, hopefully by mid-October, so would be fine to intro in the lab next week ... your choice.

Oct 8
... I would like to do the lab on page 28. Then we can do at least the 1st of your rotation labs .... I like them. BEFORE doing the rotation lab, it would be good to simply have them rotate a segment AB around the center O, measure AO, A’O, BO, B’O, angle AOA’ and BOB’ so they understand what a rotation does.

Nov 27
Planning to do the orthocenter and incenter constructions for triangles. Page 77 first and do it through Explore more #1, then page 75, 76 but not the Explore more. I am comfortable doing these if you want to observe OR even more comfortable observing you ... your choice. ... I thought we would do the Euler line next.

The e-mails show a progression in ownership of the curricular content of the labs. Taking over the Tuesday teaching took much longer than the researcher expected (see the e-mail of Nov 27th). While Jacob would teach the labs himself each Wednesday, he took the opportunity on Tuesdays to observe instruction of the lessons. Jacob enjoyed watching the researcher teach. He was able to watch how someone else might interact with the software and have the students interact with the software. He felt it helped him skip some of the learning curve one would normally experience while experimenting with a new instructional tool. It should be noted that

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once a lab was introduced, the two of us worked as a team, helping the students in the lab.

Comments from the exit interview address this issue:

The thing that happened here was by having you in the classroom with me, that was just a tremendous experience for me, and easy way for me to learn GSP, which I was not terribly familiar with, I had minimum exposure to it, but for me to learn that and also, obviously, for me to see how you interacted and expected the students to use, to use sketchpad ...

The format was that the initial session, the initial class session, on any particular lab, that you conducted that and I observed and then I went ahead and did the others on my own. Yeah that was just a wonderful process because modeling how to work with the students as well as learning the actual function of sketchpad was a very, very comfortable, and efficient way to learn it (lines 68-78).

Concern

While the results of the project were positive, making a commitment to going to the lab once a week was a cause of concern for Jacob. He felt there needed to be a tight sequencing of topics between the days in and out of the lab. Since the project design restricted lab use to every Tuesday, topics had to be selected that had not been introduced yet. This turned out to have a positive result on the student’s learning of these specific topics. Jacob noted in October that since his students had been initially exposed to a topic in the lab, they knew it better. He commented that in classroom discussions he often found them recalling a lab that had been done.

Jacob stated he wasn’t sure at the beginning of the year how the GSP work would mesh with his curriculum but that it is working out well and that he needs to spend more time working in the integration of ideas between the lab and the book. For example, we had started rotations and he has not gotten to that yet --(next week) but he stated they would probably know it better because of the labs that we are doing (Oct 8 field notes).

This was commented on in the exit interview. “Well, one of my concerns I guess before we started, was the need to tightly sequence the things we were introducing with what we were doing in the classroom and I found out as we went that that really didn’t need to be an issue or concern at all (lines 117-120).” The following section notes the two areas Jacob cited as changes in his practice as well as an analysis of changes.
Change in Practice

Communication Goal

Jacob stated the use of the Geometer’s Sketchpad program helped him fulfill not only geometry goals but also helped the students in communication skills.

One emphasis at our school right now is to work with students on the communication process, in all classes, not just their language arts classes and Sketchpad is one way to do this. Often the labs are printed out and the students have to read and follow the directions. Then, often times, we have them write responses to those so we are doing much more communicating (lines 29-35).

An example of Jacob working with the students on communication was noted when he took a variety of student responses to a construction (Parallelograms, October 1) and placed them on one handout (and overhead). In the following class session the students read through the different student responses and had a discussion regarding what they had described when constructing the midpoint quadrilateral of a general quadrilateral (you get a parallelogram). More importantly, they discussed how to word their response so it would be correct and understandable and yet not long.

Change in Instruction

Jacob reported he had definitely made changes in his instruction due to observing students in the lab. From the exit interview we find the following summary:

I think in terms of getting the students, having them use the, take a hands on approach towards discovery had lead me to see how much better it is when kids are actively involved with the, the initial step in learning. So, yes, for me at least, there had been less of a presentation of ‘here it is’ presented by me and trying to get them active, even in the business of class discussion, to get them more involved in the actual discovery or learning process. I know that has taken place (lines 52-59).

Jacob also pointed out the power of Sketchpad in looking at multiple cases of a situation.

All of these things we had done in pasts years, but we didn’t do it with GSP, we had to construct or own figures. GSP enables us to do so much. It is just so much more efficient, students, rather than looking at just one example, that they have created [paper and pencil], they can create as many examples as they want and make comparisons (lines 110-112).
While Jacob noticed the students were learning more they were also motivated. In a recent lab on polygons, Jacob stated, “students would scream with delight when they got it, when they finally saw the relationship between the two [measurements], (e-mail communication 2/25/03).

Analysis of Change in Instruction

Senger traces the change process from a beginning level of an awareness of something new, through the experimental change in teaching practice and verbalization, to the final stage of ‘being convinced.’ It is believed Jacob traversed this model and was well into the stage of ‘experimental change in teaching practice.’ In November, the researcher experienced a sign that Jacob was ‘convinced’ of his new belief in a most unusual manner. On November 5th, two math teachers (at different times, while in the hall) both spoke highly of the geometry project. One asked if he could get copies of the labs that had been done in the computer lab and the other asked about the duration of the project at their school. Since these two teachers were not involved with the project, their interest in the project had to have come from Jacob. The comments these teachers made were a result of conversations Jacob had with them. Jacob had been engaging in conversations regarding the use of GSP that had persuaded his peers of its merits. This type of verbalization is characteristic of the final stage of change. To engage in persuasive conversations is an indication that a person has begun to integrate the new knowledge into their existing belief system.

A major change in Jacob’s practice is also noted by an examination of his classes following the conclusion of the researcher’s time in his school. While the project formally ended in November of 2002, Jacob continued to take his classes to the lab at the same pace as the fall semester. Labs he conducted include the Euler line and labs on similar polygons. A current e-mail from him indicates the following activity:
Apr 3, 03
Labs we have done since we spoke last: Measuring arcs in circles, central angles, inscribed angles. Also, tangent to a circle, lengths of tangent segments to a circle (I added angle formed by tan segments and a TOUGH bonus for constructing a tangent to a circle from an exterior point).
Labs are unavailable most of April and May due to MAPS testing 😞

This email illustrates a complete inclusion of GSP activities into Jacobs teaching. The fact that he cannot use the computer lab at the end of the year is noted with the unhappy face.

A final analysis of Jacob's motives with the use of GSP places him somewhere between the Accidental Tourist and the Voyageur (as most teachers are according to Saye) with goals of efficiently discovering geometric concepts.
Appendix B

Classroom Lesson Record
Section V (adapted pages 2 – 4 from original by adding bold items below)

Lesson Description (completed by recorder during the lesson)

Use the following codes to record selected or requested characteristics of the lesson during five-minute intervals of the class. Record this information in the boxes on the next page.

<table>
<thead>
<tr>
<th>Code</th>
<th>Group Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Class</td>
</tr>
<tr>
<td>I</td>
<td>Individual</td>
</tr>
<tr>
<td>LG</td>
<td>Large group (6 to C-1)</td>
</tr>
<tr>
<td>PT</td>
<td>Partners (2)</td>
</tr>
<tr>
<td>SG</td>
<td>Small Group (3-5)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Level of Student Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Receipt of Knowledge</td>
</tr>
<tr>
<td>2</td>
<td>Application of Procedural Knowledge</td>
</tr>
<tr>
<td>3</td>
<td>Knowledge Representation</td>
</tr>
<tr>
<td>4</td>
<td>Knowledge Construction</td>
</tr>
<tr>
<td>5</td>
<td>Other</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Level of Student Engagement</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>High engagement (80% or more)</td>
</tr>
<tr>
<td>M</td>
<td>Mixed engagement (40% to 60%)</td>
</tr>
<tr>
<td>L</td>
<td>Low engagement (20% or less)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Process (Math / Science)</th>
</tr>
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<tbody>
<tr>
<td>CM</td>
<td>Communication</td>
</tr>
<tr>
<td>CN</td>
<td>Connections</td>
</tr>
<tr>
<td>PS</td>
<td>Problem Solving</td>
</tr>
<tr>
<td>R&amp;P</td>
<td>Reasoning and Proof</td>
</tr>
<tr>
<td>REP</td>
<td>Representation</td>
</tr>
<tr>
<td>OB</td>
<td>Observation</td>
</tr>
<tr>
<td>CP</td>
<td>Comparison</td>
</tr>
<tr>
<td>OR</td>
<td>Organization</td>
</tr>
<tr>
<td>Re</td>
<td>Relationships</td>
</tr>
<tr>
<td>I</td>
<td>Inference</td>
</tr>
<tr>
<td>AP</td>
<td>Application</td>
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</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Instructional Strategies</th>
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<tbody>
<tr>
<td>A</td>
<td>Assessing</td>
</tr>
<tr>
<td>DM</td>
<td>Demonstrating</td>
</tr>
<tr>
<td>FGW</td>
<td>Facilitating group work</td>
</tr>
<tr>
<td>GD</td>
<td>Giving directions</td>
</tr>
<tr>
<td>IN</td>
<td>Interruption (describe)</td>
</tr>
<tr>
<td>L</td>
<td>Lecturing</td>
</tr>
<tr>
<td>M</td>
<td>Modeling</td>
</tr>
<tr>
<td>Q</td>
<td>Questioning</td>
</tr>
<tr>
<td>RS</td>
<td>Redirecting students</td>
</tr>
<tr>
<td>AQ</td>
<td>*Answering questions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Student Learning Behavior</th>
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</thead>
<tbody>
<tr>
<td>CL</td>
<td>Collaborative Learning</td>
</tr>
<tr>
<td>DS</td>
<td>Discussion</td>
</tr>
<tr>
<td>EX</td>
<td>Experiment</td>
</tr>
<tr>
<td>HOA</td>
<td>Hands-on activity</td>
</tr>
<tr>
<td>IN</td>
<td>Interruption (describe)</td>
</tr>
<tr>
<td>R</td>
<td>Reading (describe)</td>
</tr>
<tr>
<td>RD</td>
<td>Recording data</td>
</tr>
<tr>
<td>RQ</td>
<td>Responding to questions</td>
</tr>
<tr>
<td>PR</td>
<td>Presentation</td>
</tr>
<tr>
<td>ST</td>
<td>Stations/learning centers</td>
</tr>
<tr>
<td>T</td>
<td>Technology (describe)</td>
</tr>
<tr>
<td>TR</td>
<td>Transition</td>
</tr>
<tr>
<td>W</td>
<td>Writing (describe)</td>
</tr>
<tr>
<td>WL</td>
<td>Watching / Listening</td>
</tr>
<tr>
<td>WS</td>
<td>Worksheet (describe)</td>
</tr>
<tr>
<td>O</td>
<td>Other (describe)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Discourse</th>
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</thead>
<tbody>
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<td>Tech</td>
<td>*Technology</td>
</tr>
<tr>
<td>Voc</td>
<td>*Vocabulary</td>
</tr>
<tr>
<td>Dis</td>
<td>*Discovery</td>
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</table>

*Added Categories
Classroom Lesson Record Continued

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<thead>
<tr>
<th>Time (5 minute intervals)</th>
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<th>:</th>
<th>:</th>
<th>:</th>
<th>:</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Group Size:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student:</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Teacher:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lesson Phase:</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Process</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Thinking</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Engagement</td>
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</tbody>
</table>

Notes:

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Appendix C
Belief Survey
The Belief Survey was adapted from McDougall, D. E. (1997). Mathematics teachers' needs in dynamic geometric computer environments: In search of control. Unpublished Dissertation, University of Toronto. This survey was adjusting by deleting questions that referred to junior high or rewording them to fit the high school population.

<table>
<thead>
<tr>
<th>I. YOUR VIEWS ABOUT MATHEMATICS</th>
<th>Alice</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I can handle high school level math, but I don't have the kind of mind needed to do advanced mathematics.</td>
<td>D</td>
<td>N</td>
</tr>
<tr>
<td>2. A lot of things in math must simply be accepted as true and remembered; there aren't really explanations for them.</td>
<td>A</td>
<td>SD</td>
</tr>
<tr>
<td>3. Math helps you think better.</td>
<td>SA</td>
<td>SA</td>
</tr>
<tr>
<td>4. To be good at mathematics, you need to have a kind of &quot;mathematical mind.&quot;</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>5. There is more than one right way to get the right answer in mathematics.</td>
<td>SA</td>
<td>SA</td>
</tr>
<tr>
<td>6. Mathematics is not just a bag of tricks.</td>
<td>SA</td>
<td>A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>II. LEARNING MATHEMATICS</th>
<th>Pre</th>
<th>Post</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. For students to get better at math, they need to practice a lot.</td>
<td>SA</td>
<td>N</td>
<td>SA</td>
<td>SA</td>
</tr>
<tr>
<td>8. If high school students use calculators, they won't learn the math they need to know.</td>
<td>D</td>
<td>D</td>
<td>N</td>
<td>D</td>
</tr>
<tr>
<td>9. If students get into arguments about ideas and procedures in math class, it can interfere with their learning of mathematics.</td>
<td>D</td>
<td>SD</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>10. In learning math, students must master topics and skills at one level before going on.</td>
<td>A</td>
<td>D</td>
<td>N</td>
<td>D</td>
</tr>
<tr>
<td>11. Once students can reason abstractly, the use of models and other visual aids becomes less necessary.</td>
<td>D</td>
<td>SD</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>12. How you get an answer is as important as whether the answer is right or wrong.</td>
<td>SA</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>13. Average mathematics students, with a little guidance, should be able to discover the basic ideas of mathematics for themselves.</td>
<td>A</td>
<td>A</td>
<td>D</td>
<td>N</td>
</tr>
<tr>
<td>14. The teacher should consistently use activities, which require original thinking.</td>
<td>N</td>
<td>N</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>III. TEACHING MATHEMATICS</th>
<th>Pre</th>
<th>Post</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>15. If a student asks a question in math, the teacher should know the answer.</td>
<td>D</td>
<td>N</td>
<td>D</td>
<td>N</td>
</tr>
<tr>
<td>16. Being personally good at mathematical problem solving has little to do with being a good math teacher.</td>
<td>SD</td>
<td>SD</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>17. Students should never leave math class feeling confused or stuck.</td>
<td>D</td>
<td>N</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>18. Teachers should not necessarily answer student's questions but should let them puzzle things out themselves.</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>19. The most important issue is not whether the answer to any math problem is correct, but whether students can explain their answers.</td>
<td>D</td>
<td>SA</td>
<td>D</td>
<td>N</td>
</tr>
<tr>
<td>20. Teachers should follow the math textbook that is used in their school.</td>
<td>N</td>
<td>D</td>
<td>A</td>
<td>N</td>
</tr>
<tr>
<td>21. Teachers should spend most of their class period explaining how to work specific problems.</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>22. Students working in cooperative groups can learn just as well as from whole class instruction.</td>
<td>SD</td>
<td>SD</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>
Appendix D

Data Collection Tools
Phase 3: Lesson Plan

Date: ________________

Class Section: _______________________

Topic(s) of Lesson: ____________________________________________________________

Goal of Lesson: ________________________________________________________________

Curriculum sources: ____________________________________________________________

Give a general overview of the lesson:

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

Homework Assignment:

___________________________________________________________________________
Phase 3: Post-lesson Reflection

1. Were the goal(s) of the lesson achieved and if so how do you know this?

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

2. Describe how GSP was implemented in the lesson.

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

3. Describe what your students did with GSP.

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

4. Describe the role of the researcher in the lesson.

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

5. Compare this lesson to a non-GSP lesson on the same topic.

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

On the back – note anything else that may be relevant.
Personal Journal Prompts

Directions:

Please keep a journal of this project. Please write in it on days that we have met or as soon after as possible. Please journal at other times as necessary to keep a log of the activity of this project as it personally affects you.

Specific Journal Prompts Phase 2:

1. Describe what you learned about GSP in this meeting.
2. Describe how you might implement GSP into your teaching.
3. Describe how this new GSP knowledge may change your teaching.
4. Describe researcher’s role in the meeting.

Specific Journal Prompts Phase 3:

1. Describe the lesson planning session.
2. Did you notice any changes in your instruction due to this week’s GSP activity? Describe.
3. Have you used additional GSP activities this week? If so, please complete a post-lesson reflection for this activity.
4. Describe what the researcher is doing this week.
Exit Interview

1. Describe the study from your perspective.

2. How do you feel about GSP? Has it provoked any changes in your instruction?

3. Looking back over the entire study, reflect on any change GSP knowledge has made on your:
   a. Goals of instruction
   b. Role of the teacher
   c. Role of the student
   d. Mathematics curriculum
   e. Typical instruction method

   Possible follow up question for each would be “how do you know this?”

4. Reflect on the mentoring relationship.
   a. How are teachers usually supported during change?
   b. How was this study different?

5. Would you recommend that this study be repeated:
   a. In your school?
   b. In other schools?

6. In the first few observations you did (data from phase one) and now you spend more time doing (data from phase three). Describe the change in your instructional approach.
Appendix E

Mentor Created GSP Labs
Lesson # One:

Points: Click on the point tool and place some points on the screen.

Compass Tool: Click on the compass tool and put some circles on the screen. Notice the control point on the circle. Never use this in an assignment.

Arrow Tool: Practice selecting and deleting points and circles.

Stop and clear the screen by making a box around all of the items. This box will turn everything pink, which means it is selected.

Line Segment Tool: Draw a line segment on the screen.

Select and delete the segment. Now use the Edit menu to get it back.

Text tool: Click on each endpoint and then the segment. Click again and the labels go off.

Points are Capitals and segments are lower case lines. To change the letter of a point from what is given, double click on the letter and then retype it in the box.

Make a text box with the text tool: In the box - type your name, partner's name, and class time (for example).

Ray Tool: Practice drawing a few rays on the screen.

Line Tool: Practice drawing a few lines on the screen.

Clear your screen by highlighting and deleting everything.

Using the ray tool draw an angle on your screen.

Label the points using the Text tool.

Measure the angle. Discuss how we name an angle and what point goes in the middle (vertex).

Drag one of the endpoints around so you see the measurement change. Can you make a right angle? Can you make a straight angle?
Reflection Lab

1. Draw a triangle in the lower left side of your screen. Label it $\triangle ABC$.

2. Construct the interior of the triangle by selecting the three vertices and Construct Triangle Interior.

3. Above the triangle draw a line and label it "Mirror."

4. Select the line and mark it as a mirror. (Transform Mark Mirror)

5. Select the triangle interior, sides, and vertices and reflect them over the "Mirror" line. (Transform Reflect)

6. Measure $\overline{AB}$, $\angle CAB$, $\triangle ABC$'s perimeter, and area. Hint: for perimeter and area select the interior first.

7. Now measure $\overline{A'B'}$, $\angle C'A'B'$, $\triangle A'B'C$'s perimeter, and area.

Q1 Drag parts of the original triangle around. Make a conjecture regarding the four measurements you made above and how they behave under the transformation of reflection.

Q2 Define Orientation and describe the orientation of points for the original triangle and it's image.

8. Place a point X on segment AB and reflect the point over the Mirror.

Q3 Notice where the image $X'$ lies. Can you move it so it is not between points A and B? Can you move it off segment $A'B'$? What does this say about betweenness of points and collinearity of points under the reflection transformation.
WARM UP: Your goal is to hit the target with one banked shot off side wall A.

Solution:
1. Reflect the target over the side wall A and construct a segment from the Laser to the reflected target.
2. Find the intersection of the side wall and the path of the laser.
3. Complete the path of the Laser from the intersection point to the target.
Laser Lab – Continued

**ROOM TWO**

![Diagram of Room Two]

Task: Bank a shot off wall A, then wall B, to the target.

**ROOM THREE**

![Diagram of Room Three]

Task: Bank a shot off of walls A, B, and C, in that order, to the target.
Laser Lab – Continued

Task: Bank a shot off of walls A, B, C, and D, in that order, to the target.
Translation Lab

Before you begin go to Preferences (under Edit) then to TEXT and click in the box show labels automatically FOR ALL NEW POINTS.

Translation Lab: Put your names in a text box on the screen.

1. Draw a triangle in the lower left side of your screen. Label it Δ.ABC.

2. Construct the interior of the triangle by selecting the three vertices and Construct Triangle Interior.

3. Above and to the right of the triangle draw a line segment and label it MN.

4. Select the points M and N, in that order, and mark the vector. (Transform Mark Vector)

5. Select the triangle interior, sides, and vertices and translate them by marked vector. (Transform Translate). Label the points on the new triangle.


IN A TEXT BOX ON YOUR SCREEN ANSWER THE FOLLOWING:

Q1. Drag parts of the original triangle around. Make a conjecture regarding the four measurements you made above and how they behave under the transformation of translation.

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Rotation Lab

1. Draw a triangle in the upper left side of your screen. Label it \( \triangle ABC \).

2. Construct the interior of the triangle by selecting the three vertices and \( \text{Construct} \mid \text{Triangle Interior} \).

3. Below and to the right of the triangle plot a point and rename it "Center."

4. Select the Center and mark it as the center of rotation \( \text{Transform} \mid \text{Mark Center} \).

5. Select the triangle interior, sides, and vertices and rotate them all by 60° \( \text{Transform} \mid \text{Rotate} \mid \text{type in 60} \).

6. Measure \( AB, \angle CAB, \triangle ABC \)'s perimeter and area. Hint: for perimeter and area select the interior first.

7. Now measure \( A'B', \angle C'A'B', \triangle A'B'C \)'s perimeter and area.

Q1 Drag parts of the original triangle around. Make a conjecture regarding the four measurements you made above and how they behave under the transformation of rotation.

Q2 Describe the orientation of points for the original triangle and its image.

8. Place a point \( X \) on segment \( AB \) and rotate the point 60°.

Q3 Notice where the image \( X' \) lies. Can you move it so it is not between points \( A \) and \( B \)? Can you move it off segment \( A'B' \)? What does this say about betweeness of points and collinearity of points under the rotation transformation.

\[ m\angle BAC = 49.86^\circ \]
\[ AB = 2.16 \text{ cm} \]
\[ \text{Area} \ P_1 = 2.90 \text{ cm}^2 \]
\[ \text{Perimeter} \ P_1 = 8.36 \text{ cm} \]
Appendix F

HSIRB Approval Letter
Western Michigan University Department of Mathematics
Principle Investigator: Tabitha Mingus
Student Investigator: Kathryn Shafer
Title of Study: Geometry and Technology

You have been invited to participate in a research project entitled “Geometry and Technology.” This research project is intended to follow a practicing High School Teacher as they learn and implement the Geomètre’s Sketchpad Program (GSP). This project is Kathryn Shafer’s dissertation project.

You will be asked to participate in the following activities:

- Allow the researcher to observe you teach a geometry class at least three times before using GSP. A lesson plan and a post lesson reflection will be collected for each lesson.
- Meet with the researcher for a minimum of five half-day sessions in the summer of 2003 in order to learn GSP.
- Maintain a journal documenting thoughts and observations while learning and using GSP.
- Allow the researcher to observe you teach a geometry class using GSP at weekly intervals in the fall semester of 2003. This may involve team teaching or the researcher teaching in your class. A lesson plan and a post lesson reflection will be collected for each lesson. Copies of relevant student work may be collected.
- Meet with the researcher weekly in the fall of 2003 to discuss the use of GSP in the geometry class.
- Participate in a final reflective interview in December of 2003.

One potential inconvenience of participation would be the time involved in learning the program and meeting with the researcher. The researcher will minimize this by meeting with you at your school or at a location convenient for you.

One way in which you may benefit from this activity is having a chance to learn how to use the Geometer’s Sketchpad program. Participation in the project may have a positive impact on your students. Participation in this project may benefit other mathematics teachers in your school.

All of the information collected from you is confidential. That means that your name will not appear on any papers on which this information is recorded. The forms will all be coded, and Kathryn Shafer will keep a separate master list with the names of participants and the corresponding code numbers. Once the data are collected and analyzed, the master list will be destroyed. All other forms will be retained for at least three years in a locked file in the principal investigator’s office.
You may refuse to participate or quit at any time during the study without prejudice or penalty. If you have any questions or concerns about this study, you may contact either Kathryn Shafer at [redacted] or Tabitha Mingus at [redacted]. You may also contact the chair of Human Subjects Institutional Review Board at 616-387-8293 or the vice president for research at 616-387-8298 if questions or problems arise in the course of the study.

This consent document has been approved for use for one year by the Human Subjects Institutional Review Board as indicated by the stamped date and signature of the board chair in the upper right corner. Do not participate in this study if the stamped date is older than one year.

Your signature below indicates that you have read and/or had explained to you the purpose and requirements of the study and that you agree to participate.

_____________________________  _______________________
Signature                          Date

Consent obtained by:    ________________________
Initials of researcher                        Date

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