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Hypernuclear Photoproduction Via $(\gamma,K^*)$

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HYPERNUCLEAR PHOTOPRODUCTION
VIA (γ,K')

by

Kimberly Ann Hodgkinson

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HYPERNUCLEAR PHOTOPRODUCTION
VIA (γ,K⁺)

Kimberly Ann Hodgkinson, M.A.
Western Michigan University, 1987

The nuclear (γ,K⁺) reaction is studied using plane
wave impulse approximation (PWIA) and a distorted wave
impulse approximation (DWIA), which includes full Coulomb
and optical distortions. The model and calculations for
each model are presented. For \(^{12}\text{C}(\gamma,K^+)_{A}B\), the two
approximation methods are compared at 2 energies:
1.47 GeV/c and 1.84 GeV/c. \(^{208}\text{Pb}(\gamma,K^+)_{A}\) is also
examined. Distortion effects are found to greatly alter
the magnitude of the cross sections, but the shape of the
angular distributions remains the same. Conclusions
suggest further study.
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Kimberly Ann Hodgkinson
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Hyperfine photoproduction via \((\gamma, K^+)\)

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Western Michigan University, 1987
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CHAPTER I

INTRODUCTION

In nuclear physics, the study of hypernuclei is a primary problem. Since the major objective in nuclear physics is to understand and describe the structure of nuclei in terms of the particles that form them and the interactions among these particles, hypernuclear research helps reach this objective in two ways. First, in understanding the properties of hypernuclei, it will aid in determining the nature of the forces acting between baryons. Second, since the hyperon acts as a probe of its host nucleus, hypernuclei further the study of the properties of nuclei.

Hypernuclear research focuses its studies on $\Lambda$-nucleus interaction. The $\Lambda$ particle bound in a nucleus is an excellent probe of nuclear properties; it has a mass which exceeds the mass of a nucleon by less than 20%. The $\Lambda$-nucleus interaction is about half as strong as the $N$-nucleus interaction. Therefore, the $\Lambda$ particle can be expected to behave in the nucleus very much like a nucleon, except that the $\Lambda$ particle will have strangeness $S=-1$ so that it can be distinguished from other nucleons. One way to produce hypernuclei is
known as a strangeness-exchange reaction. An excellent example of strangeness exchange is the $K^- + n \rightarrow \pi^- + \Lambda$ process. In $(K^-, \pi^-)$ the strangeness of the $K$ meson ($S=-1$) is transferred to the $\Lambda$ baryon ($S=-1$). Since the $\Lambda$ baryon can be produced with low momentum, the $(K^-, \pi^-)$ process is rather simple to study due to the fact that the $\Lambda$ baryon can be put easily in the nucleus.

Unlike the $(K^-, \pi^-)$ process, the $\gamma + p \rightarrow K^+ + \Lambda$ process creates strangeness rather than transferring it. This reaction is called associated production; this occurs when strange particles are produced in conjugate pairs from an initial system of zero strangeness. In $(\gamma, K^+)$, the $K^+$ meson ($S=+1$) and the $\Lambda$ baryon ($S=-1$) are created from an initial zero strangeness system. If the $\Lambda$ baryon replaces a proton that is within the nucleus, the $(\gamma, K^+)$ process will change the initial nucleus into a nucleus with negative strangeness ($\Delta S = -1$).

As early calculations by R.H. Dalitz and A. Gal (1978) predicted, the $(K^-, \pi^-)$ reaction experimentally reveals only natural parity states, most evident in the spectrum are the substitutional states where the $\Lambda$ baryon just replaces a neutron in the nucleus.
without otherwise changing its wave function. These states are due to the low momentum transfer to the Λ baryon which leads to the low total angular momentum Λs=0 states. However, the (γ,K') reaction has one major advantage over the (K',π⁻) reaction: the photon has unit spin which implies that both natural and unnatural parity hypernuclear states can be excited, and this in turn will lead to the opportunity to determine the properties of hypernuclear spin-flip (Δs≠0) excitations. With the future addition of high-intensity, high-energy electron accelerators to the research world, the necessary tool will be available to study (γ,K'). Of particular interest is hypernuclear photoproduction, and in this paper it will be shown that the reaction

\[ Λ + γ \rightarrow K^- + Λ' \]  

(1)

in the total center-of-mass energy region from 0.70 GeV (threshold) to 2 GeV is a prime candidate for studying hypernuclear structure.

In Chapter II, the kinematics and the theoretical model behind the (γ,K') reaction are discussed. The basic theory behind the plane wave impulse approximation (PWIA) is presented in Chapter III, along with actual cross section calculations using the PWIA. Chapter IV presents the distorted wave impulse approximation (DWIA) in the
same manner as Chapter III discusses the PWIA. In addition, Chapter IV compares the results from the PWIA and the DWIA. Finally, Chapter V presents the conclusions about the two approximation methods and stresses the need to continue studying this subject.
CHAPTER II

THEORY

Kinematics

When the photon lab energy ($E'_\gamma$) is 0.7 GeV, as in Figure 1, the net momentum transfer is just under 3 fm$^{-1}$ and the change in net momentum transfer is less than 0.08 fm$^{-1}$ from 0°-30°. This minute change is due to the fact that the kaons are just beginning to be produced at this particular energy. As $E'_\gamma$ increases to 1.5 GeV, the change in momentum transfer from 0°-30° increases to approximately 2.1 fm$^{-1}$. Also, the kaon lab momentum is 0.825 Gev/c. This is a very unusual energy region for nuclear studies; most such studies involve $K^-$ with a low kinetic energy.

Lastly, note the increase in the net momentum transfer $\vec{Q}$ of all angles but 0° at $E'_\gamma = 1.5$ GeV. The decrease in $\vec{Q}$ at 0° occurs as $k_\perp$ increases. So as $k_\perp$ increases and $\vec{Q}$ decreases, the contribution of the nuclear form factor $F_n(Q^2)$ will in general increase. This form factor is an integral of the
Figure 1. Kinematics for $\gamma + p \rightarrow K^+ + A$. Momentum Transfer for the Nuclear Target at Rest in the Lab System as a Function of the Kaon Lab Kinetic Energy at Four Different Lab Angles.
transition density \( p(\vec{r}) \) times a phase factor over a nuclear volume and may be written in terms of \( \vec{Q} \). The differential cross section for (1) is proportional to \( F_n(\vec{Q}) \). In this endothermic reaction at all \( E_L \), relatively high Fourier components of \( F_n(\vec{Q}) \) are probed.

**Theoretical Model**

After looking at the kinematics of (1), it is evident that research on (1) is of definite interest. Numerous authors, such as Thom (1966), Adelseck, Bennhold, and Wright (1985) and Rosenthal, Halderson, and Tabakin (WMU-PGH) (1987), have published theoretical studies on \((\gamma,K)\) and they all use the same basic model for the amplitudes.

For this model the center-of-mass 4-vectors of the interacting particles are defined as follows:

- proton \((p_2)\): \((E_p,-k)\),
- photon \((p_1)\): \((k, k)\),
- kaon \((p_3)\): \((\omega, \vec{q})\),
- lambda \((p_4)\): \((E_\Lambda,-\vec{q})\).

The basic amplitude for the reaction is then

\[
F = F_1(\vec{\sigma} \cdot \vec{e}) + F_2(i\vec{\sigma} \cdot \vec{q} \cdot \vec{e} \cdot \vec{k}) + F_3(\vec{\sigma} \cdot \vec{k} \cdot \vec{e}) + F_4(\vec{\sigma} \cdot \vec{q} \cdot \vec{e}), \tag{2}
\]

where \( \vec{\sigma} \) is the Pauli spin vector, \( \vec{e} \) is the photon spin polarization vector, and the \( F_i \)'s are the invariant amplitudes.
These $F_i$'s are independent of the initial and final spin states. At forward angles $F_1$ and $F_2$ are most important, while the spin-flip hypernuclear excitations are proportional to $F_1 - F_2 \cos \theta$ and, at all angles, non-flip transitions are proportional to $F_2 \sin \theta$. In Figure 2 the $F_i$'s, which are discussed later, are shown evaluated for $\theta = 0^\circ$. Near threshold only $F_1$ is significant while the other $F_i$'s vanish, but the other amplitudes do become significant around $k_t = 1.4$ GeV/c. For this type of reaction, the $F_i$'s are related to the invariant amplitudes, $A_i$, of the Lorentz covariant CGLN form:

$$M = \sum_{i=1}^{4} A_i \bar{u}(p_2', s_2) M_i u(p_1', s_1), \quad (3)$$

where the matrices $M_i$ are given by

$$M_1 = -\gamma_5 \frac{\not{k}}{k},$$
$$M_2 = 2\gamma_3 (\not{\epsilon} \cdot p_1 k \cdot p_2 - \not{\epsilon} \cdot p_2 k \cdot p_2),$$
$$M_3 = \gamma_5 (\not{\epsilon} \cdot p_1 - \not{\epsilon} \cdot p_2),$$
$$M_4 = 2\gamma_3 (\not{\epsilon} \cdot k \cdot p_2 - \not{\epsilon} \cdot p_2).$$

The details of the CGLN forms for $F_i$ and $A_i$ can be found in Thom (1966).

This amplitude (2) is then used in finding the $(\gamma, K^*)$ cross section

$$\frac{d\sigma}{dn} = \frac{\alpha}{k} \left| \langle x_i | F | x_i \rangle \right|^2, \quad (4)$$

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Figure 2: Predicted Amplitudes for the Two Different Model Sets. The Solid Curves are for WM-UG, Dashed for OUD.
where \( \chi_i, \chi_f \) are initial and final Pauli spinors.

For unpolarized initial states, the cross section is

\[
\frac{d\sigma}{d\Omega}(\theta) = \frac{q}{k} \text{Re}\{ |F_1|^2 + |F_2|^2 - 2F_1^*F_2\cos\theta + F_1^*F_4 + F_2^*F_3 \\
+ \left[\left(\frac{1}{2}\right)|F_3|^2 + \left(\frac{1}{2}\right)|F_4|^2\right] \sin^2\theta + F_3^*F_4\cos\theta\}.
\]

(5)

and the \( \lambda \) polarization in the direction \( \vec{k} \times \vec{q} \) is

\[
\vec{P}_\lambda \frac{d\sigma}{d\Omega} = \frac{q}{k} \sin\theta \Im\{-2F_1^*F_2 - F_1^*F_3^*F_4 + F_3^*F_4 \sin^2\theta + (F_2^*F_3 - F_1^*F_4)\cos\theta\}.
\]

(6)

The model of the \( F_i \)'s which we use is derived from the Born terms of the diagrams in Figure 3. The vertex factors for the Born terms (including the vector kaon exchange term) may be found in Thom (1966). Only three of the 9 coupling constants \( (\epsilon, \mu, \mu_H) \) are known, and the 6 remaining can be put into pairs to form adjustable parameters. The \( K\Lambda \) coupling constant is multiplied by a pseudo-scalar \( \gamma_5 \) to account for different parity states. It is \( g_{\text{KNA}} \) that is the prime factor allowing for a good fit to the data. However, some disagreement has arisen as to the exact value for \( g_{\text{KNA}} \).

Various analyses by such groups as Thom (1966) and Dover and Walker (1982) have deduced values for \( g_{\text{KNA}} \). Most of these fits favor \( g_{\text{KNA}}/\sqrt{4\pi} \geq 3.7 \) for hadron scattering. Recently Adelseck, Berndhold, and Wright...
Figure 3. The Born Terms for $\gamma + p \rightarrow K^+ + \Lambda$. 

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(1985) extracted a value of $\frac{g_{\text{KNA}}}{\sqrt{4\pi}} = 1.03$. The details of the steps taken to produce this amplitude, which is known as the OU amplitude, are discussed in the above authors' articles. By including an additional t-channel exchange $K^\ast$ in OU's method, the WMU-PGH group improves the fit but does not alter the discrepancy between electromagnetic and hadronic determinations of $g_{\text{KNA}}$. The WMU-PGH amplitude produces a low value of $\frac{g_{\text{KNA}}}{\sqrt{4\pi}} = 0.9$.

The diagrams in Figure 3 are associated with the $F_i$'s and consist of the standard Born terms and $K^\ast$ (892 MeV). The Born terms can give a realistic picture of the data as the vector meson $K^\ast$ is included in the t-channel. Thom (1966) showed that for photokaon production no single resonance seems to dominate the reaction.

For $k_t = 1.2$ and 1.4 GeV/c, Figures 4 and 5 show the theoretical cross sections in fairly good agreement with the data. However, with this type of fit, the agreement of the forward angle data at 1.2 GeV/c will lead to an overestimation of the forward-angle cross sections at 1.4 GeV/c. Therefore, no one value of $g_{\text{KNA}}$ will provide a good fit at all energies.

Since $g_{\text{KNA}}$ cannot provide a good fit for all
Figure 4. Comparison of Fitted $\gamma + p \rightarrow K^* + \Lambda$ Cross Sections at $k_t = 1.2$ GeV/c.
Figure 5. Comparison of Fitted $\gamma + p \rightarrow K^* + \Lambda$
Cross Sections at $k_t = 1.4\text{GeV/c}$. 

- $\text{WMU-PGH}$
- $\text{OU}$
energies, this adds to the suspicions that some important physics must be missing from the model. Perhaps other channels interfere with (1) or the angle dependence may change due to the subnucleonic degrees of freedom, but these are both new areas of physics. Whether or not such degrees of freedom are necessary for a quantitative understanding must await experimental developments. For the rest of this paper, we treat the present model as adequate.
CHAPTER III

PLANE WAVE IMPULSE APPROXIMATION (PWIA)

In the PWIA, it is assumed that the projectile strikes only one target particle and that the incident and emitted particles have no other interactions with the nucleus except for the interaction among these particles which cause the reaction. Therefore, up to a normalization constant, both the photon and kaon may be represented as the following plane waves:

\[ \psi_{\gamma}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \quad \text{and} \quad \psi_{\pi}(\vec{r}) = e^{i\vec{q}\cdot\vec{r}}, \]

where the plane wave is produced from the wave equation

\[ \left( \frac{-\hbar^2}{2M} \nabla^2 + U_{\text{opt}} \right) \psi_{\pi}(\vec{r}) = 0, \quad (7) \]

in which the optical potential effective potential, is set equal to zero.

PWIA Model

The \( A(\gamma,K')A' \) cross section can be estimated using the PWIA. The cross section in the \( \gamma \)-nucleus center-of-mass system (ACM) is then

\[ \frac{d\sigma}{d\Omega}(\theta) = \frac{q_{\gamma}}{K_c} \left( \frac{2\pi}{\hbar c} \right)^4 \omega_\Lambda^2 \sum_{M_\Lambda M'\Lambda} |T_{\Lambda}|^2, \quad (8) \]
where $k_c$ and $q_c$ are the momentum transfer in the two-body center-of-mass system (2CM) for the photon and kaon, respectively, and $\tilde{\omega}_A$, the reduced kaon energy in the ACM, is

$$\tilde{\omega}_A = \left[\frac{k_w q_w E_N E_A}{s_A}\right]^{1/2}$$ (9)

and $\sqrt{s_A}$ is the total ACM energy. The $t$-matrix $T_{\Pi}$ in the ACM is summed over the initial ($M_j$) and final ($M_f$) nuclear spins and the photon polarization $\lambda$ and is defined as follows:

$$T_{\Pi} = (2\pi)^{-3/2} \frac{\gamma}{\omega_c} \left(\frac{\hbar c}{2\pi}\right)^2 \int d\vec{r} \ e^{i\vec{s} \cdot \vec{r}} F_\Pi F_{\phi_j}.$$ (10)

In this expression,

$$\gamma = \left[\frac{E^c E^c E^c E^c}{E^A E^A E^A E^A}\right]^{1/2},$$ (11)

which results from Lorentz transforming the 2CM $t$-matrix to an ACM $t$-matrix;

$$\bar{\omega}_c = \left[\frac{E^c E^c E^c E^c}{s_c}\right]^{1/2},$$ (12)

which is the reduced kaon energy in the 2CM and $\sqrt{s_c}$ is the total 2CM energy. Note that $\tilde{Q} = \tilde{k} (M_j/M_f) \tilde{Q}$ and this $\tilde{Q}$ is due to the fact that the nuclear center-of-mass is not fixed in this reaction. A recoil factor $(M_j/M_f)$
must multiply the kaon momentum.

The notation $I$ is used to represent the integrand in (10), so that $I$ obeys the equation

$$\sum_{L,M} |I|^2 = \sum_{p=1}^7 c_p.$$  

(13)

The seven coefficients comprising this sum have been evaluated by Halderson (1987) and are:

$$c_1 = 4\pi (F_1 - F_2 \cos \theta_c)^2 \sum_{L} 2\left(\frac{2J_c+1}{2J+1}\right) (-1)^{L+1} \sum_{l=0,2}^{L} \frac{2^L}{\sqrt{\pi}} \frac{\sqrt{2J_c+1}}{\sqrt{2J+1}} \frac{L}{I} \frac{L}{I'} \frac{L}{L'} \frac{L}{L} \frac{\mathcal{A}(l,I',I,J) P_L(\cos \theta_a)}{\mathcal{A}(l,I',I,J) P_L(\cos \theta_a)},$$

$$c_2 = 2 \text{Re}(4\pi (F_1 - F_2 \cos \theta_c) (F_2 + F_3)^2) \sum_{L} \left[\frac{2^L}{\sqrt{\pi}} \frac{\sqrt{2J_c+1}}{\sqrt{2J+1}} \frac{L}{I} \frac{L}{I'} \frac{L}{L'} \frac{L}{L} \frac{\mathcal{A}(l,I',I,J) P_L(\cos \theta_a)}{\mathcal{A}(l,I',I,J) P_L(\cos \theta_a)},

$$

$$c_3 = 2 \text{Re}\left\{ \frac{(4\pi)^2 (F_1 - F_2 \cos \theta_c) F_4^2 \sin \theta_c}{\mathcal{A}(l,I',I,J) P_L(\cos \theta_a)} \sum_{L} \left[\frac{2^L}{\sqrt{\pi}} \frac{\sqrt{2J_c+1}}{\sqrt{2J+1}} \frac{L}{I} \frac{L}{I'} \frac{L}{L'} \frac{L}{L} \mathcal{A}(l,I',I,J) P_L(\cos \theta_a)\right] \right\},$$

$$c_4 = 4\pi |F_2|^2 \sin^2 \theta_c \sum_{L} \left[\frac{2J_c+1}{(2L+1)^2} B(l)\right],$$

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\[ c_5 = 4\pi |F_2 + F_3|^2 \sin^2 \theta_e \sum_{L I} (-1)^I \left( \frac{2J_f + 1}{2J + 1} \right) c_{000}^{I L} c_{000}^{11L} \]
\[ \times \left( \frac{1}{I I' J'' L} \right) A(I, I', J) \ P_L(\cos \theta_e), \]
\[ c_6 = 2\text{Re} \left\{ (4\pi)^2 (F_2 + F_3) F_4 \sin^2 \theta_e \sum_{J' L'} \left( \frac{2J + 1}{3(2J + 1)} \right) (-1)^{J' L - 1} \right\} \]
\[ \times c_{000}^{I L} \left( \frac{1}{I I' J'' L} \right) A(I, I', J) \sum_{M} c_{-M-M_0}^{111} y_M^I(Q) y_M^J(Q_e), \]
\[ c_7 = (4\pi)^2 |F_4|^2 \sin^2 \theta_e \sum_{L} (-1)^{J' J} \left( \frac{2J + 1}{(2L+1)(2J+1)} \right) \left( \frac{1}{I I' J'' L} \right) \]
\[ \times c_{000}^{I L} c_{000}^{111} A(I, I', J) y_M^I(Q) y_M^J(Q_e). \]

All seven coefficients consist of combinations of the \( F_i \)'s previously discussed, various Clebsch-Gordan and 6-J coefficients, and angular dependent functions. Except for \( c_4 \), the other six coefficients include
\[ A(I, I', J) = \sum_{a, a', \bar{a}} \tau_{a a'}^{I I' J'} \tau_{a \bar{a}}^{I I', J}, \tag{14} \]
which is a bilinear combination of the overlap integral
\[ \tau_{a a'}^{I I'} = i^I \frac{1}{\sqrt{(2I+1)J_a}} \left[ \int R_a(r) J_1(Qr) R_{a'}(r) r^2 \, dr \right] \]
\[ \times \langle J | [a_a(\Lambda) a_{p}(\bar{\Lambda}) J_f] | J_i > \langle \alpha || [Y^I(u') J'] || \beta >, \tag{15} \]

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where \( \alpha = \left[ \gamma^1 \gamma^2 \gamma^2 \right]^{1/2} \) and \( \beta = \left[ \gamma^1 \gamma^2 \gamma^4 \right]^{1/2} \). Note that these terms are sensitive to the spin angular-momentum transfer. As for \( c_4 \), it depends only on the orbital angular-momentum transfer; consequently, it is multiplied by a different bilinear combination of overlap integrals.

\[
B(I) = \sum_{i \neq f} T^{I}_{e \sigma} T^{I}_{i \beta},
\]

(16)

in which the overlap integral is now

\[
T^{I}_{e \sigma} = i^{I} \sqrt{(2I+1)(2j_e+1)} \left( \int R_e(x) j_f(Qr) R_i(x) x^I dx \right) \\
\times \langle J_f | (\alpha(\Lambda) \alpha(p))^{I} | J_i \rangle \langle \alpha | | \gamma^I | \beta \rangle.
\]

(17)

Glancing at the above seven coefficients, it is clear that at forward angles, \( c_1 \) will dominate since no \( \sin \theta_c \) term is present. Also, for \( \Delta s=0 \), \( c_4 \) will be the only contribution to the cross section; whereas, \( \Delta s = 1 \) all other terms except \( c_4 \) contribute to the cross section. This is due to the fact that \( c_4 \) depends only on the orbital angular momentum.

**PWIA Calculations**

Using the PWIA, actual photoproduction differential
cross sections were calculated for the \( \left( \frac{P_{3/2}}{A} \frac{A_{1/2}}{2} \right) \) of \(^{12}C\) for \( J = 0^\ast, 1^\ast, 2^\ast, 3^\ast \). In Figures 6 and 7, angular distributions of this "substitutional" multiplet are shown for \( k_t = 1.47 \text{ GeV/c} \) and \( 1.84 \text{ GeV/c} \). Characteristic of this reaction, the stretched state \( J = 3^\ast \) is the most excited state. Because the large momentum transfer \( Q \) implies that the orbital momentum transfers \( \Delta l - QR \) will be large and that the reaction is mainly by spin-flip \( (\Delta s = 1) \), a maximum \( J \) transfer must occur; therefore, the highest \( J \)-state is the most strongly excited in this reaction.

Examination of the smaller \( J \)-states in Figures 6 and 7, shows that the \( 2^\ast \) state is of next importance to the \( 3^\ast \) state. As \( J \) becomes smaller, the states are harder to excite. This is evident in the above figures, where the 2 smallest states are compared to the \( 2^\ast \) state.

As mentioned above, the PWIA assumes that the only interaction is the one causing the reaction. However, when a particle approaches close enough to have a reaction, this model ignores the effects of nuclear and coulomb potentials, which leads to scattering and perhaps absorption of the particle. Consequently, a more refined method of calculating the cross section is needed.
Figure 6. Predicted Photoproduction Differential Cross Sections as $k_t = 1.47$ GeV/c for a Pure $\left( {^{1}_{3/2}^P} \right)$ Multiplet, Calculated with Only Coulomb Distortions Using $b_N = b_A = 1.7$ fm.
Figure 7. Predicted Photoproduction Differential Cross Sections as $k_z = 1.84$ GeV/c for a Pure $\{p_3/2, p_3/2\}$ Multiplet, Calculated with Only Coulomb Distortions Using $b_N = b_A = 1.7$ fm.
CHAPTER IV

DISTORTED WAVE IMPULSE APPROXIMATION (DWIA)

As seen in the PWIA, it is difficult to obtain an exact solution to meson-nucleus scattering because of the many-body aspects, so we need to go one step beyond the PWIA to the DWIA. The DWIA neglects the binding forces on the target particles during the collision and assumes that the projectile strikes only one target particle. Since this approximation neglects three-body and higher correlations, the process is related to two-particle scattering. In this paper, all the kaons are considered to be above inelastic thresholds.

Using the DWIA, the $A(\gamma,K^')A'$ cross sections can be calculated. However, first let us look at how the plane wave kaon from the PWIA is replaced by a distorted wave kaon in the DWIA.

DWIA Model

The DWIA treats the kaon as moving under the influence of a $K'$ nucleus optical potential, so that a plane wave can no longer represent the kaon-hypernucleus wave function. Since the exact kaon wave function is not known, the $K'$ optical potential $U_{opt}$ is used in the wave equation (7) to generate a distorted kaon wave.
function. The $K^*$ optical potential takes the form

$$U_{\text{opt}} = -4\pi A \left(\frac{\hbar c}{2}\right)^2 \frac{1}{2K^*} \left[ 1 + \frac{r}{r_n} \right] b \rho(r), \quad (18)$$

where $\rho(r)$ is the nuclear density and

$$b = b_0 + b_1 \langle \vec{t} \cdot \vec{t} \rangle, \quad (19)$$

where $\vec{t}$ is the nuclear isospin operator and $\vec{t}$ is the kaon isospin operator. The value of $b$ is obtained from the Martin (1975) $K^*-N$ amplitudes and the two coefficients of $b$ are shown in Figure 8 for $K^* = 1-2$ GeV/c.

Once a form for the kaon hypernucleus interaction is assumed, the $A(\gamma,K^*)A'$ cross section can be obtained in the ACM system from the following expression:

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{\rho_n}{2(2J+1)} \sum_{J,M,J',M'} \left| \langle J,M;K^*(q) | H_\lambda | J,M';\gamma(k,\lambda) \rangle \right|^2, \quad (20)$$

where the photoproduction operator which involves the kaon distorted wave is

$$H_\lambda = \vec{J} \cdot \vec{\lambda} \chi_\lambda^{(-)} V_\lambda, \quad (21)$$

$\vec{\lambda}$ is the photon field where $\lambda$ and $k$ are the photon polarization and momentum, respectively; $\chi_\lambda^{(-)}$ is the kaon distorted wave; $V_\lambda$ is the SU(3) ladder operator which changes an up to a strange quark, or a proton to a $\Lambda$, and $\rho_n$ is the density of states factor. In
Figure 8. (A) Isovector Kaon Optical Parameter $b_1$ for $^{12}$B. (B) Isoscalar Kaon Optical Parameter $b_0$ for $^{12}$B. Solid Curves are Real and Dashed are Imaginary Parts of $b_1$ and $b_0$. 

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this reaction, the cross section is averaged over the initial nuclear spins and the photon polarizations and summed over the final nuclear spins.

The above photoproduction operator also includes the photokaon transition current density

\[ \mathbf{J}_s \cdot \hat{\epsilon} = \frac{4\pi \sqrt{\kappa}}{\sqrt{E_n E_m}} F_c, \]  

(22)

where \( F_c \) is the \((\gamma,K')\) amplitude in the 2CM. However, since the DWIA calculates the \( A(\gamma,K')A^\prime \) cross section in the ACM, \( F_c \) must be calculated using \( \vec{k}, \vec{q}, \lambda \) so that \( \mathbf{J}_s \cdot \hat{\epsilon} \) is calculated in the ACM. In order for \( F_c \) to make the transition from the 2CM to the ACM, a series of approximations in the spirit of forward scattering approximations are made (Rosenthal, Halderson, Hodgkinson, Tabakin, 1987).

Assuming the kaons transverse 2CM and ACM momenta are equal, the approximations allow us to map the 2CM amplitudes into the ACM system

\[ F = F_0 \mathbf{\sigma} \cdot \hat{\epsilon} - F_2 \mathbf{\sigma} \cdot \hat{q}_e \cdot \hat{k} + F_3 \mathbf{\sigma} \cdot \hat{k} \times \hat{q}_e + (F_2 + F_3) \mathbf{\sigma} \cdot \hat{q}_e \cdot \hat{q} + F_4 \mathbf{\sigma} \cdot \hat{q}_e \cdot \hat{q} \cdot \hat{q}. \]  

\[ \rightarrow F_0 \mathbf{\sigma} \cdot \hat{\epsilon} - F_2 \frac{\mathbf{q}_{\lambda} \cdot \hat{q}_e \cdot (\mathbf{k} \times \hat{q})}{\mathbf{k} \cdot \mathbf{q}_e} + \frac{(F_2 + F_3)}{\mathbf{k} \cdot \mathbf{q}_e} \mathbf{\sigma} \cdot \hat{q}_e \cdot \hat{q} + \frac{F_4}{\mathbf{q}_e \cdot \mathbf{q}} \mathbf{\sigma} \cdot \hat{q}_e \cdot \hat{q} \cdot \hat{q}. \]  

(23)

Using \( \hat{q} = \hat{p} + \hat{k} - \hat{p}', \) in the operator \( \mathbf{\sigma} \cdot \hat{q} \cdot \hat{q} \cdot \hat{\epsilon}, \) the
transition current density in the ACM is then in the form

\[
\mathbf{J}_s \hat{e} = a_0 \hat{e} + a_7 (\bar{q} \cdot \vec{k}) \cdot \hat{e} + a_4 (\bar{q} \cdot \vec{p}_f) \cdot \hat{e} + a_2 (\bar{q} \cdot \vec{p}_f) \cdot \hat{e} + a_2 (\vec{q} \cdot \hat{e}) + a_0 (\vec{q} \cdot \hat{e}).
\]  

Now the transition current only depends linearly on the momenta \( \vec{p}_f, \vec{p}_f, \) and \( \vec{q} \). The terms of the CGLN amplitudes express the strength of each term in \( \mathbf{J}_s \) as

\[
\begin{bmatrix}
    a_0 \\
    a_7 \\
    a_4 \\
    a_2 \\
    a_5
\end{bmatrix} = \frac{4\pi\hbar}{\sqrt{E_A E_N}} \begin{bmatrix}
    F_1 \\
    -iF_2 / q_c \\
    -F_2 / q \\
    F_4 / q q_c \\
    \frac{(F_2 + F_3)}{q_c} + kF_4 / qq_c \\
    -F_4 / q q_c
\end{bmatrix}.
\]  

Since the \( a_i \)'s and \( F_i \)'s are evaluated at 0°, we can conclude that the momentum dependence and angular dependence of \( \mathbf{J}_s \) is contained in the operators and not the \( a_i \)'s.

The absolute values of the magnitude of the \( a_i \)'s as calculated from the WMU-PGH and OU models are shown in Figure 9. As the plot indicates, the strength of each operator in \( \mathbf{J}_s \) varies with the incident photon momentum for each model. It is quite evident from the plots that \( a_0, a_7, \) and \( a_4 \) are the important coefficients of \( \mathbf{J}_s \hat{e} \), since they depend on \( F_1 \) or \( F_2 \). States excited
Figure 9. Predicted Transition Strengths for the Two Amplitude Sets. Solid Curves are WMU-PGH and Dashed are OU.
by the \( \sigma \cdot \varepsilon \) are expected to dominate the spectrum because \( a_1 \) is the dominant component of \( \tilde{J}_3 \cdot \varepsilon \). On the other hand, \( a_8 \), which depends on \( F_2 \), also multiplies \( \sigma \cdot \varepsilon \cos \theta \) so that it acts like \( a_1 \). The phases are such that destructive interference for both WMU-PGH and OU amplitude sets is evident. In addition, \( a_7 \) excites \( A_s=0 \) states and therefore contributes to the richness of the spectrum. It should be noted that both amplitude sets are in close agreement for \( a_1, a_7, \) and \( a_8 \).

The other three coefficients, \( a_3, a_7, \) and \( a_9 \), are mostly dependent on \( F_3 \) and \( F_4 \) and contribute to the \( A_s=1 \) excitations. Since \( F_3 \) and \( F_4 \) are such small values, none of these components have a noticeable effect on \( \tilde{J}_3 \cdot \varepsilon \); therefore, the large disagreement in the models has no effect on the final \( \tilde{J}_3 \cdot \varepsilon \).

**DWIA Calculations**

In actual calculations of the photoproduction differential cross sections using the DWIA, the \((P_{12}^{-1} A_{12})^\dagger\) of \(^1\text{H}\) for \( J=0^+, 1^+, 2^+, 3^+ \) is again used. Figures 10 and 11 show that the DWIA retains the basic shape of the PWIA calculations (Figures 6 and 7), but the
Figure 10. Predicted Photoproduction Differential Cross Sections as $k_\perp = 1.47$ GeV/c for a Pure $^3P_{1/2} - ^3P_{3/2}$ Multiplet, Calculated with Full Kaon Distortions.
Figure 11. Predicted Photoproduction Differential Cross Sections as $k_t = 1.84$ GeV/c for a Pure ($p_{3/2}^1 \Lambda_{3/2}$) Multiplet, Calculated with Full Kaon Distortions.
magnitudes of the cross sections is decreased. Again, the highest J-state is the most excited state, a common feature of the \((\gamma,K')\) reaction.

In addition to the \(^{12}\text{C}\), or light nuclei, calculations, interest in heavy nuclei has arisen. Since \(K'\) mesons are assumed to be weakly absorbed by nuclei, a reasonable cross section may be obtained for the \((\gamma,K')\) reaction when a \(\Lambda\) particle is deeply bound in a heavy nucleus in the final hypernuclear state. In Figures 12 and 13 the angular distributions for the photoproduction differential cross sections for the \((\gamma,^9\text{Be})^\text{13}\text{Si}\) or \(\text{a}\ \text{13}\text{-}^\text{Si}\) are shown for the plane wave (CW) and the distorted wave (DW) calculations. The cross sections of the \(^{208}\text{Pb}(\gamma,K')^{208}\text{Tl}\_\Lambda\) reaction are suppressed by a factor of five at both the low and high energies. This shows a basic independence of distortion effects on energy.

In the DWIA calculation, the cross section is given in terms of phase shifts by making a partial wave analysis of the cross section. Each partial wave in the cross section corresponds to a definite value of the angular momentum \(L\). Since \(L=\text{kR}\), this value of \(L\) should represent the number of partial waves necessary for the cross sections to saturate. In Figure 14 the cross
Figure 12. Cross Section to an Assumed Pure $^3\mathbf{S}_1^2$ state at $k_L = 1.2$ GeV/c.
\[ \frac{\mathrm{d} \sigma}{\mathrm{d} \Omega} \left( 10^{-3} \, \mu \text{b/sr} \right) \]

\hspace{2cm} \theta (\text{degrees})

Figure 13. \[ ^{208}\text{Pb}(\gamma, K^+)_{^8\Lambda}^{208}\text{Tl} \] Cross Section to an Assumed Pure \( (p_{1/2} \lambda_{S_{1/2}}^{-1})^{1S_0} \) State at \( k_L = 1.84 \text{ GeV/c} \).
Figure 14. Comparison of Differential Cross Sections Using Coulomb Waves and Full Distorted Waves for a Pure $^{1}_{2}S_{1/2}$ State at $k_L = 1.84 \, \text{GeV/c}$.

$k_L = 1.84 \, \text{GeV/c}$
sections for $A = 12$ do not saturate until 25 partial waves are included. The kaon momentum is about 2.5 fm$^{-1}$ here in the ACM, which implies that the arguments from elastic scattering, which relate $kR - 8$ to the number of necessary partial waves, is obviously not true here. As can be seen from the value of $kR$, many more partial waves are needed, essentially because of the significant centrifugal barrier effects in the final state only. Therefore, 25 kaon partial waves for $A = 12$ and 45 for $A = 208$ are required for accuracy in the photokaon production calculation.

Comparison and Analysis of DWIA and PWIA

The PWIA and the DWIA both produce photoproduction differential cross sections. However, the PWIA does not include a kaon optical potential in the wave equation (7), which produces the kaon wave function. In Figures 6, 7, 10 and 11, the results of the two calculations of $(p_\gamma^p A p_\gamma^p)^J$ for $\frac{d\sigma}{d\Omega}$ at $k_\gamma = 1.47$ GeV/c and 1.84 GeV/c can be compared. The PWIA gives an accurate picture of the shape of the angular distribution of the cross sections, and it also gives the approximate location of the maximum cross section. Unfortunately the PWIA is not very accurate in predicting cross sections. This is
where the DWIA uses the PWIA as a guide. The DWIA produces the same shape curve as the PWIA and the maximum cross section is within $1^\circ$ of the PWIA; however, the DWIA cross sections are smaller in magnitude and, to the extent that the optical potential correctly summarizes the effects of multiple scattering, are more realistic.

Although the PWIA does not produce accurate cross sections, it is still useful for estimating magnitudes for the dominantly excited states. The reason is as follows: the ratio of the maximum DWIA cross section to the maximum PWIA cross section provides a practical factor which roughly converts a PWIA cross section to a DWIA cross section for a given energy and $J$-state. This conversion factor is referred to as the distortion factor $D = \frac{\text{max}(\sigma_{\text{DW}})}{\text{max}(\sigma_{\text{PW}})}$.

In Figures 6, 7, 10 and 11, the effects of kaon distortion at both energies yield about a 35% decrease in the predicted cross sections for the $3^+$ and $2^+$ states. Using actual cross sections generated by the two approximation methods for $3^+$, $D = 0.67$ at 1.47 GeV/c and $D=0.64$ at 1.84 GeV/c. For $2^+$, $D = 0.68$ and 0.65 at the low and high energies, respectively. Although the actual values of the cross sections are not the same at the two different energies, $D$ does not vary much as a function of energy.
In addition to weak energy dependence, $D$ reveals a small dependence on the $J$-state within a given multiplet. This is because for a given multiplet $D$ depends on the overlap of radial transition densities with kaon and photon wave functions. In calculating cross sections, only the operators $a_1$, $a_7$, and $a_9$ are important and these all have the same transition density $\rho$. Therefore, the overlap tends to be dominated only by a single $J$-transfer.

The angular distributions of the cross sections in Figures 6, 7, 10 and 11 are distinct from one another. This will make it easier for the experimenter to determine which $J$-state is being excited. By looking at these figures and Figure 15, it is clear that for relatively light nuclei the angular distribution of the cross sections is independent of nuclear structure and dependent only on $J$. Also, in comparing the calculations in Figure 16 of $(p^1_3\pi^\nu_3, d^1_3)$ of $^{12}\text{C}$ to those for Figure 7, it is not only evident that the stretched states dominate, but that the simplest nuclear structure yields the largest cross sections. Notice that even with distortions, the maximum $J$-states continue to dominate the spectrum.

As for the heavy nuclei, the $^{208}\text{Pb}(\gamma, K^-)^{208}\text{Tl}$ reaction
Figure 15. Energy Integrated Excitation Functions for Stretched Configurations.
Figure 16. Predicted Photoproduction Differential Cross Section as $k_L=1.84$ GeV/c for $(p_{2\theta}, d_{1\pi})$ Multiplet, Calculated with Full Kaon Distortions.
produces tiny cross sections without distortion factors. Once the optical potential is included, the cross sections become so minute (Figures 12 and 13) that they can barely, if at all, be detected. Even configuration mixing will not improve the cross sections; it will only further suppress the value of the cross section. Therefore, it is most unlikely that this reaction can be used to study a $\Lambda$ particle in nuclear matter.
CHAPTER V

CONCLUSIONS

The major emphasis of this paper is to study the characteristics of the nuclear $(\gamma, K^\pm)$ reaction in order to show that this reaction is an excellent candidate for exploring hypernuclear structure. Since the $\gamma + p \rightarrow K^+ + \Lambda$ reaction tends to preferentially excite the $\Delta s = 1$ states, it can provide a means to investigate these particular states over a range of momentum transfers.

A major conclusion of this study reveals a definite angular "signature" for each excited state of the $(\gamma, K^\pm)$. Since the $\Delta s \neq 0$ transitions can only weakly be excited in strangeness exchange reactions, it is difficult to study these states. Therefore, although experimenters can only allow an energy resolution of about 0.5 MeV, by varying the detection angle the experimenter will be able to see how strongly the states are excited by knowing the particular angular distributions. Even though little can be done to improve the energy resolution, good angular distributions will lead to spectroscopic factors for these states. Thus, $(\gamma, K^\pm)$ will
provide an important spectroscopic tool and gives us the possibility of extending our knowledge of hypernuclear structure.

As shown in earlier studies of \((\gamma,K^+)\), the excited hypernuclear multiplet tends to be dominated by a single state. Our conclusions show that a single state does dominate the spectrum and that particular state is the maximum \(J\)-state. However, other states are also significantly excited, especially away from forward angles.

Lastly, our studies reveal that the next generation of experiments should include research on \((\gamma,K^+)\) for light nuclei. Experiments on deeply bound states in heavier nuclei are not recommended since the cross sections are too small to be detected. The next generation of high-intensity, high-energy accelerators will provide the necessary tool for studying \((\gamma,K^+)\).
BIBLIOGRAPHY


