A Study of the Direct Capture Process of Protons by $^{27}\text{Al}$

Rozana Hussain

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A STUDY OF THE DIRECT CAPTURE PROCESS OF PROTONS BY $^{27}\text{Al}$

by

Rozana Hussain

A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
requirements for the
Degree of Master of Arts
Department of Physics

Western Michigan University
Kalamazoo, Michigan
April 1987
An experimental investigation of the $^{27}\text{Al}(p,\gamma)^{28}\text{Si}$ reaction was made for protons in the energy range 550- to 1785-keV and at angles of 0, 20, 55, 90 and 118 degrees. Four angular distributions at $E_p = 845-$, $1055-$, $1135-$, and $1618-$-keV were selected for detailed study. These angular distributions were fit with theoretical calculations employing a direct-capture model. These fits permitted the extraction of spectroscopic factors for the population of the first excited-state in $^{28}\text{Si}$ by proton transfer.
ACKNOWLEDGEMENTS

I wish to thank the faculty and staff of the Physics Department at Western Michigan University, Kalamazoo for all their help and a very special thanks to Professor Gerald Hardie for his help, suggestions and encouragement during the course of this work.

Rozana Hussain
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A study of the direct capture process of protons by aluminum-27

Hussain, Rozana, M.A.

WESTERN MICHIGAN UNIVERSITY, 1987
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CHAPTER I

INTRODUCTION

The purpose of the experiment is to study the process of direct capture\(^1\) of a proton by a nucleus. In this process an intermediate compound state is not formed but rather the proton is captured directly into its final orbit with the emission of a \(\gamma\)-ray. This process is depicted in Figure 1.

\[ \bullet \rightarrow 0 \]

\(\gamma\)-ray

proton nucleus

\[ 0 \]

final nucleus

Figure 1. Direct Capture of a Proton by a Nucleus

The advantage of this reaction is that the transition occurs via the well-known and simple electromagnetic interaction rather than the complicated and poorly understood nuclear interaction. Unlike the nuclear force, the electromagnetic interaction is sufficiently weak so that the simple first-order,
time-independent perturbation theory can be used to calculate cross sections.

In the theoretical calculation of the cross section the incoming proton is represented by a distorted plane wave and the target nucleus by a Woods-Saxon potential given by:

\[ V(r) = -\frac{V_0}{1 + e^{(r-R)/a}} \quad \text{with} \quad R = R_0 A^{1/3} \]

in which \( A \) is the mass number of the target nucleus. The bound state of the proton is also represented by a real Woods-Saxon potential with the same \( R \) and \( 'a' \) but \( V_0 \) chosen to produce the binding energy of the proton added. One important reason for understanding direct capture is that it is an important process in the interior of stars. However the main interest in this thesis is with the spectroscopic factors. Essentially the spectroscopic factor is a measure of the extent to which the final nucleus can be represented by the target nucleus and an orbiting proton. Although these can be obtained using other nuclear reactions it is expected that the simplicity of the theoretical calculations for direct capture (due to the knowledge of the electromagnetic interaction) will result in more accurate spectroscopic factors.
In the experiment reported in this thesis the direct capture of protons by $^{27}\text{Al}$ is reported. In Figure 2 the simplest shell model description of the ground state ($5/2^+$) of $^{27}\text{Al}$ is given. This figure also shows some of the experimentally-determined lower-lying states in $^{28}\text{Si}$, the final nucleus.

![Shell Model Description](image_url)

Figure 2. Shell Model Description of $^{27}\text{Al}$ and $^{28}\text{Si}$

In the present work capture to form the first excited state ($2^+$) in $^{28}\text{Si}$ was studied, as this is the strongest transition. This ($2^+$) state can be formed by placing the captured proton either in a $2S_1/2$ orbit or a $1d_{3/2}$ orbit (see Figure 2). There will be a spectroscopic factor associated with each possibility.
The differential cross section is written as

$$\frac{d\sigma}{d\Omega} = C^2 S \left\{ \frac{d\sigma}{d\Omega} \right\} (\text{capture into } 2S_{1/2} \text{ state}) + C^2 S \left\{ \frac{d\sigma}{d\Omega} \right\} (\text{capture into } 1d_{3/2} \text{ state})$$

in which $C^2 S$ represents the spectroscopic factor and the individual $d\sigma/d\Omega$ are calculated using the simple direct capture model described below. The procedure is then to theoretically calculate the individual $d\sigma/d\Omega$ appearing on the right-hand side of the above equation and then to adjust the $C^2 S$'s until one gets agreement with the measured $d\sigma/d\Omega$.

There are several basic difficulties with the experiment, one theoretical and one experimental. The theoretical difficulty is that the calculated differential cross sections depend sensitively on the parameters chosen for the Woods-Saxon potential. The values chosen in the present experiment ($R_o = 1.25\text{fm}$, $V_o = 55.5\text{MeV}$ and $a = 0.65\text{fm}$) were taken from a compilation of optical model parameters. These values were not adjusted to obtain better fits to the experimental data. The experimental difficulty results from the very low cross section for the direct capture process. There is a second process which occurs and which will contribute to
the measured cross section. This second process is resonance capture.

The resonances in the \(^{27}\text{Al}(p, \gamma)\) reaction are well-known\(^5\) and so one can attempt to measure the direct capture by going between two resonances. However, the "tails" of the resonances contribute to the cross section "between" resonances. Although the contributions from the "tails" of the resonances rapidly becomes small as one moves away from the resonance energy, the direct capture cross section itself is so small that it may not dominate over the contributions from the "tails" of nearby resonances.

The scope of the present work is first to obtain some angular distributions at proton energies where the contributions from the "tails" of resonances may be small compared to the direct reaction. Then an attempt will be made to fit these angular distributions with a direct capture calculation. If this is successful the spectroscopic factors will be presented.
CHAPTER II

EXPERIMENTAL ARRANGEMENT

The experiments were performed at the Argonne National Laboratory using the Dynamitron Accelerator. This device is capable of producing high currents of low-energy charged particles. The protons used in these experiments had energies ranging from 500 keV to 1800 keV while beam currents were as large as 300 μA. A typical experimental arrangement is shown in Figure 3. Target thicknesses ranging from 50-120 μg/cm² were used. The large beam currents would quickly destroy the Al target so a special arrangement was used. First the target was wobbled so that the particle-beam traced a circle on the target thus depositing the energy in the beam over a larger area. Secondly a stream of water was used to cool the target holder.

Two types of targets were used. The first consisted of evaporating Al onto a Ta cup. Ta was chosen because the high nuclear coulomb-barrier precluded the production of a background of γ-rays. However, Ta is not one of the better heat conductors and so the temperature of the Al target could get quite high. In a second arrangement Cu, which is an excellent heat...
Figure 3. Experimental Arrangement
conductor, was used. The disadvantage of Cu is that it has a rather low Z (low coulomb-barrier) and contains low-Z impurities, thus possibly giving rise to a background of γ-rays. To reduce this possibility an Au layer of about 10 mg/cm² was electroplated onto the inside of the Cu cup and then the Al was evaporated onto the Au. Au has a high Z and is an excellent heat conductor. The purpose of the Au is to sufficiently reduce the energy of the incident protons so they will not produce γ-rays by nuclear interactions with the Cu or its impurities.

A second problem was that carbon would build up on the target. Unless special precautions are taken carbon is always present in a vacuum system from pump oil or components made with hydrocarbons (e.g., O-rings). Carbon build-up on the target was avoided by precipitating it out on a Ni-plated Cu-tube kept at liquid-nitrogen temperature. Three lithium-drifted germanium (Ge(Li)) detectors were used to observe the γ-rays produced when the proton beam struck the Al target. The large active volume of these detectors, about 80cm³, gave reasonable efficiencies for the high-energy (around 12MeV) γ-rays under investigation. In three different experiments different sets of angles were chosen:
10°, 55°, 90° in May, 1984; 10°, 55°, 118° in June, 1984; and 0°, 90°, 118° in March, 1985.

The electronics arrangement is shown in Figure 4. As data for a single point were often collected over a 24-hour period, very stable electronics had to be used. The heights of the pulses from the detectors are proportional to the energies of the γ-rays. These heights are converted to numbers by the analogue-to-digital converters (ADC's) and stored on magnetic tape by the PDP11/45 computer. This computer permitted the data to be displayed as they were being collected, thus permitting decisions to be made while the experiment was in progress.

While an ADC processes a pulse, it will not accept another, thus there is a certain amount of "dead-time," the amount depending upon the counting rate. This dead-time, which must be known to correct the data, was determined as follows. The current integrator sends out voltage pulses, the number N of these pulses being proportional to the charge collected. In fact the charge Q is given by:

\[ Q = (\text{current scale of integrator}) \times (N) \times (10^{-2}) \]
Figure 4. Electronics Arrangement
One can preset \( N \), and the beam turned off after this preset number is reached. An ADC, on the other hand, gives a voltage pulse ("BUSY" signal) whenever it will not accept another pulse from the detector. This "BUSY" pulse from the ADC is placed in anti-coincidence with the voltage pulses from the integrator and the output pulse of the anti-coincidence unit is counted by a scaler (see Figure 4). If, for example, the ADC is never busy, all the \( N \) pulses will be registered by the scaler. If, on the other hand, the ADC is busy 50\% of the time, then the scaler will register \( N/2 \). Hence the preset number \( N \) and the value recorded by the scaler (stored on magnetic tape) is used to correct data for the dead-time.
CHAPTER III

DATA REDUCTION AND RESULTS

The formula used to calculate the differential cross section is

\[
\frac{d\sigma}{d\Omega} = \frac{Y}{\epsilon \Omega} \cdot \frac{Q \Delta_{\text{res}}}{4\pi \epsilon_{\text{res}} 10^{-30}}
\]

where:

- \( Y \) : yield, which is corrected for dead time
- \( \epsilon \Omega \) : "efficiency of detector"
- \( Q \) : total charge collected
- \( \Delta_{\text{res}} \) : target thickness (measured as width of 992keV resonance peak)
- \( \epsilon_{\text{res}} \) : 7.76 x 10^{-15} \text{eV/atom/cm}

(refer to appendix A for derivation of \( d\sigma/d\Omega \))

To calculate cross sections the thickness, \( \Delta_{\text{res}} \), of the Al target must be known. This was measured by going over the well-known \(^{27}\text{Al}(p, \gamma)\) resonance at \( E_p = 992\text{keV} \).

Since the intrinsic width of this resonance is 0.1keV the width observed (see Figure 5) is essentially due to the thickness of the target. Also because one
knows that resonance occurs at an energy of 992keV one can calibrate the generating voltmeter of the accelerator. For example, from Figure 5 it is seen that a generating voltmeter reading of 980keV is equivalent to a proton energy of 992keV. Periodically throughout the experiment this resonance curve was generated to check the possibility of deterioration of the Al target. Also any carbon build-up on the target would be detected as this would shift the location of the resonance.

The product of the efficiency of the detector and the solid angle subtended by the detector ($\varepsilon \Omega$ in the above formula) must also be known.

The formula used to calculate $\varepsilon \Omega$ is

$$
\varepsilon \Omega = \frac{2 \xi}{\pi \lambda^2} \left( \frac{A_t}{A_i + A_t} \right) \left( \frac{Ye}{QbrK} \right) \frac{1}{\omega \gamma}
$$

where:

- $\xi$ : stopping power of the target for protons
- $A_t$ : target mass number
- $A_i$ : incident particle mass number
- $\lambda$ : wave number for incident protons (G.M. system)
- $\omega \gamma$ : resonance strength

Y : yield, which is corrected for dead-time

Q : total charge collected

br : branching ratio

K : correction for angular distribution effects
Figure 5. $^{27}\text{Al} (p, \gamma)$ Resonance
The efficiency of the detector was calculated using the 992keV resonance because the strength, \( \omega_\gamma \), and the angular distribution is well-known.\(^6\) An example of the efficiency measured for one of the detectors is given in Figure 6. The efficiency is measured as a function of the gamma-ray energy where

\[
E_\gamma = Q_0 + \frac{m_1}{m_1 + m_2} \left( E_p - E_x \right)
\]

and

\[
Q_0 : \text{ reaction Q-value } = 11585\text{keV}
\]
\[
E_x : \text{ excitation energy}
\]
\[
E_p : \text{ proton energy}
\]
\[
m_1 : \text{ mass of } ^{27}\text{Al} = 27
\]
\[
m_2 : \text{ mass of proton} = 1
\]

so that

\[
E_\gamma = 11585 + \frac{27}{28} E_p - E_x
\]

A summary of the differential cross sections calculated from the formula given above is given in Table 1. The results of these differential cross sections are for proton energies ranging from 585keV to 1785 keV. Most of these cross sections are dominated by nearby resonances, but there may be regions between resonances at which the cross section is mainly due to direct capture.
Figure 6. Detector Efficiency
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<th>E (keV)</th>
<th>Run Date</th>
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<th>E -Δ</th>
<th>θ=0°</th>
<th>θ=20°</th>
<th>θ=55°</th>
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<th>θ=118°</th>
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<td>7.08±21.21</td>
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<td>713</td>
<td>6.74</td>
<td>21.16</td>
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<td>7.08±21.21</td>
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<td>714</td>
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<td>763</td>
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<td>7.22±21.24</td>
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<td>7.22±21.24</td>
<td>7.08±21.21</td>
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<td>811</td>
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<td>812</td>
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<td>0.00176±0.00031</td>
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<td>33.3±5.0</td>
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<td>6/84</td>
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<td>987</td>
<td>46.0±7.0</td>
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<td>0.0254±0.0040</td>
<td>0.0177±0.0031</td>
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TABLE 1 (continued)

SUMMARY OF $\frac{d\gamma}{d\Omega}$ (µb/sr)

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<td>(E_p)</td>
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<td>1427</td>
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<td>20°</td>
<td>1431</td>
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<td>11.9 ± 4.9</td>
<td>1425</td>
<td>11.9 ± 4.9</td>
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<td>55°</td>
<td>1431</td>
<td>5.84</td>
<td>14.2 ± 5.2</td>
<td>1427</td>
<td>14.2 ± 5.2</td>
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TABLE 1 (continued)

SUMMARY OF \(d\) (ub/sr)

<table>
<thead>
<tr>
<th>(\theta = 0^\circ)</th>
<th>(\theta = 20^\circ)</th>
<th>(\theta = 55^\circ)</th>
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<tr>
<td>(d_{118})</td>
<td>(d_{20})</td>
<td>(d_{55})</td>
</tr>
<tr>
<td>0.315 ± 0.044</td>
<td>0.300 ± 0.042</td>
<td>0.300 ± 0.042</td>
</tr>
<tr>
<td>0.394 ± 0.058</td>
<td>0.298 ± 0.056</td>
<td>0.153 ± 0.057</td>
</tr>
<tr>
<td>0.225 ± 0.118</td>
<td>0.104 ± 0.123</td>
<td>0.194 ± 0.02</td>
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<tr>
<td>1.04 ± 0.19</td>
<td>3.81 ± 2.5</td>
<td>2.34 ± 2.2</td>
</tr>
<tr>
<td>11.2 ± 2.0</td>
<td>11.1 ± 2.0</td>
<td>11.1 ± 2.0</td>
</tr>
<tr>
<td>0.194 ± 0.02</td>
<td>0.194 ± 0.02</td>
<td>0.194 ± 0.02</td>
</tr>
<tr>
<td>0.12 ± 0.22</td>
<td>0.12 ± 0.22</td>
<td>0.12 ± 0.22</td>
</tr>
<tr>
<td>0.194 ± 0.02</td>
<td>0.194 ± 0.02</td>
<td>0.194 ± 0.02</td>
</tr>
</tbody>
</table>

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To locate energies at which direct capture may be important, an estimate of the resonance contribution was made by summing incoherently the amplitudes for all the known resonances in this energy region. The result is shown in Figures 7(A) to 7(C). In these figures, the effect of the target thickness was included. The total cross sections as measured in the present experiment are also shown in the figures.

As can be seen from these figures, there are energy regions in which the measured cross sections are substantially greater than those estimated from the resonances. This difference could be due to constructive interference among resonances or between resonances and direct capture or due to a direct capture contribution which is much greater than the resonance contribution. Further information on this can be obtained by considering the angular distributions.

Angular distributions on resonances are rather flat whereas direct capture theory predicts cross sections which are substantially peaked. We have selected four energies for further investigation. The four energies are $E_p = 845\text{-}, 1055\text{-}, 1135\text{-} \text{ and } 1618\text{-} \text{keV}$. 

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Figure 7(A). Cross Sections
Figure 7(B). Cross Sections (continued)
Figure 7(C). Cross Sections (continued)
These energies were selected because the measured total cross section was substantially greater than that predicted from resonances alone and the angular distributions are not flat but rather peaked at $90^\circ$ and we have substantial data at these energies. The angular distributions at these four different energies are given in Figures 8(A) through 8(D). An attempt was made to fit the data with Legendre Polynomials using the equation

$$\frac{d\sigma}{d\Omega} = a_0 + a_1 P_1 (\cos \theta) + a_2 P_2 (\cos \theta)$$

where $a_0$, $a_1$ and $a_2$ are the angular distribution coefficients and $P_1 (\cos \theta)$ and $P_2 (\cos \theta)$ are the Legendre Polynomials. As seen from these figures, there is a good fit to the angular distributions, indicating that higher order polynomials are not needed.
Figure 8(A). Angular Distribution - Not Normalized
Figure 8(B). Angular Distribution - Not Normalized

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Figure 8(C). Angular Distribution - Not Normalized
Figure 8(D). Angular Distribution - Not Normalized

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In order to remove some uncertainties from one run to the next, the different runs were normalized to the March 1985 run at 90°. These normalized angular distributions are shown in Figures 9(A) through 9(D).

A summary of the angular distribution coefficients \( a_i \) is given in Table 2. Plots of these values of \( a_i \)'s and \( \frac{a_2}{a_0} \) are also given in Figures 10(A) through 10(C). From these results it is seen that the value for \( a_1 \) is essentially equal to zero so that the equation of the Legendre Polynomial fit to the angular distributions becomes

\[
\frac{dc}{dn} = a_0 + a_2 P_2 (\cos \theta)
\]

As can be seen from Figure 10(C) all of these angular distributions have significant negative \( \frac{a_2}{a_0} \)'s which seem to decrease with increasing energy. In the next section we investigate to what extent these angular distributions can be reproduced by a simple direct capture calculation.
Figure 9(A). Angular Distribution - Normalized

$E_p = 845$ keV
Figure 9(B). Angular Distribution - Normalized

$E_p = 1055$ keV

$\frac{d\sigma}{d\Omega} \, (10^{-2} \mu b / sr)$

$\Theta \, (\text{deg.})$

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Figure 9(C). Angular Distribution - Normalized

\[ E_p = 1135 \text{ keV} \]
Figure 9(D). Angular Distribution - Normalized

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### TABLE 2

**SUMMARY OF a₁'s**

<table>
<thead>
<tr>
<th>E_p (keV)</th>
<th>a₀</th>
<th>a₁</th>
<th>a₂</th>
<th>a₁/a₀</th>
<th>a₂/a₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>845</td>
<td>0.00309±0.00033</td>
<td>0.00139±0.00094</td>
<td>-0.00341±0.00089</td>
<td>0.450±0.308</td>
<td>-1.10±0.310</td>
</tr>
<tr>
<td>1055</td>
<td>0.01692±0.00173</td>
<td>0.00025±0.00446</td>
<td>-0.00810±0.00398</td>
<td>0.015±0.268</td>
<td>-0.479±0.240</td>
</tr>
<tr>
<td>1135</td>
<td>0.03156±0.00223</td>
<td>-0.00113±0.00599</td>
<td>-0.01485±0.00573</td>
<td>-0.036±0.191</td>
<td>-0.471±0.185</td>
</tr>
<tr>
<td>1618</td>
<td>0.14126±0.01427</td>
<td>-0.02022±0.03674</td>
<td>-0.03500±0.03054</td>
<td>-0.143±0.260</td>
<td>-0.248±0.219</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E_p (keV)</th>
<th>a₀</th>
<th>a₁</th>
<th>a₂</th>
<th>a₁/a₀</th>
<th>a₂/a₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>845</td>
<td>0.00292±0.00033</td>
<td>0.00049±0.00089</td>
<td>-0.00236±0.00085</td>
<td>-0.168±0.306</td>
<td>-0.808±0.305</td>
</tr>
<tr>
<td>1055</td>
<td>0.01731±0.00184</td>
<td>0.00250±0.00475</td>
<td>-0.01019±0.00461</td>
<td>-0.144±0.274</td>
<td>-0.589±0.274</td>
</tr>
<tr>
<td>1135</td>
<td>0.03349±0.00264</td>
<td>0.00260±0.00722</td>
<td>-0.01904±0.00791</td>
<td>-0.078±0.217</td>
<td>-0.569±0.241</td>
</tr>
<tr>
<td>1618</td>
<td>0.14558±0.01555</td>
<td>-0.00087±0.03897</td>
<td>-0.05284±0.03640</td>
<td>-0.006±0.269</td>
<td>-0.363±0.253</td>
</tr>
</tbody>
</table>
Figure 10(A). Angular Distribution Coefficients - Not Normalized

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Figure 10(B). Angular Distribution Coefficients - Normalized
Figure 10(B). Angular Distribution Coefficient $a_2/a_0$

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CHAPTER IV

COMPARISON WITH THEORETICAL CALCULATIONS

The calculation of the direct capture cross section is simple in principle but tedious to perform. For a given single-particle final state characterized by an orbital angular momentum $l$, the theoretical cross section $\sigma_{\text{TH}}(l)$ is calculated using the code HIKARI\textsuperscript{7} which uses a more realistic model than some of the earlier computer codes written to calculate $\sigma(l)$. In the present case, there are two possible angular momenta, $l=0$ and $l=2$, and the total cross section is then given by

$$\sigma_{\text{TH}} = C^2 S(l=0) \sigma_{\text{TH}}(l=0) + C^2 S(l=2) \sigma_{\text{TH}}(l=2)$$

in which the spectroscopic factors, $C^2 S$, must be calculated by some nuclear model or obtained experimentally. In the present experiment, the simple direct capture model was used to calculate $\sigma_{\text{TH}}$. To obtain the spectroscopic factors we let $\sigma_{\text{TH}} = \sigma_{\text{EXP}}$ and the spectroscopic factors were then adjusted to give a reasonable agreement between the calculated and experimental cross sections. The same real Woods-Saxon potential was used to represent both the target nucleus,
$^{27}\text{Al}$, and the bound state of the proton. The parameters used for the Woods-Saxon potential are

$$R_0 = 1.25 \text{ fm} \quad \text{and} \quad a = 0.65 \text{ fm}$$

and the bound state of the proton is chosen to reproduce the binding energy of the proton. The spectroscopic factors obtained are given in Table 3. Current literature values$^8$ of the spectroscopic factors are also given in the same table.

The fits to the four angular distributions using the adjusted values of the spectroscopic factors are shown in Figures 11(A) through 11(D). As seen from these figures, the fit is excellent at the lowest and highest energies but the cross sections are not high enough at the two intermediate energies. To keep the number of parameters varied small, no attempt was made to change the parameters in the calculation of the direct capture. However, some adjustment of these parameters might give a better overall fit to the experimental data. Also a more complicated optical model, including absorption and spin-orbit contributions might improve the fit. The discrepancy in the two middle angular distributions may be due to contributions from the "tails" of nearby isolated resonances.
<table>
<thead>
<tr>
<th>( C^2S(2^+ \text{state}) )</th>
<th>Present Work</th>
<th>Reference$^8$</th>
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<tbody>
<tr>
<td>( 2S_{1/2} )</td>
<td>0.14</td>
<td>0.19 + 0.43</td>
</tr>
<tr>
<td>( 1d_{3/2} )</td>
<td>0.90</td>
<td>0 + 0.91</td>
</tr>
</tbody>
</table>

Table 3

SPECTROSCOPIC FACTORS
Figure 11(A). Angular Distribution with $C^2 S$ Adjusted

$E_p = 845$ keV

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Figure 11(B). Angular Distribution with $C^2S$ Adjusted
Figure 11(C). Angular Distribution with $C^2S$ Adjusted
Figure 11(D). Angular Distribution with $C^2S$ Adjusted
CHAPTER V

SUMMARY AND CONCLUSIONS

Differential cross sections were measured for the $^{27}\text{Al}(p, \gamma)^{28}\text{Si}^*$ reaction for proton energies of 585keV to 1785keV. Four angular distributions, at energies $E_p = 845-, 1055-, 1135-$ and 1618-keV, were then selected for further theoretical analysis. An attempt was made to fit these angular distributions with the direct capture model, thus extracting the spectroscopic factors. From the results, it is seen that there is a good fit at the lowest and highest energies while the cross sections were low at the two intermediate energies. From this experiment, the spectroscopic factors are found to be $0.14(p + s)$ and $0.90(p + d)$. A more complete analysis which would include contributions from neighboring isolated resonances might account for these discrepancies. However, from this experiment, there is no firm evidence of any other process such as semi-direct capture as suggested by M. Kicinska-Habior et al.\textsuperscript{9}
REFERENCES


7. We thank Professor H. Weller of the Physics Department, Duke University for supplying us with the computer code HIKARI.


APPENDIX A

DERIVATION OF $\frac{d\sigma}{d\Omega}$

Suppose a beam of protons with $I$ particles per second incident on an area "Area" of the target, the target being of thickness $T$. If $Y$ represents the number of $\gamma$-rays emitted in a solid angle $d\Omega$ in time $dt$, then

$$Y = \frac{(\text{# of target centers}) \left( \frac{d\sigma}{d\Omega} \right) d\Omega}{\text{Area}} \times \left( \frac{\text{# of incident particles in time } dt}{\text{time } dt} \right)$$

where

$$\text{# of target centers} = \frac{\rho (\text{Area}) T N_Q}{A} = n (\text{Area})$$

$$\text{# of incident particles in time } dt = \frac{I dt}{e} = \frac{Q}{e}$$

and

$$\frac{d\sigma}{d\Omega} : \text{ cross section}$$

$$\rho : \text{ density of the target}$$

$$N_Q : \text{ Avogadro's number}$$

$$A : \text{ mass number}$$

$$Q : \text{ total charge collected}$$

$$n : \text{ number of atoms per unit area} = \frac{\rho N_Q T}{A}$$

Therefore the expression for $Y$ becomes

$$Y = n \left( \frac{d\sigma}{d\Omega} \right) d\Omega \frac{Q}{e}$$
If all the measurements were taken in cm, we would like to convert the units so that $\frac{d\sigma}{d\Omega}$ is in $\mu$b/$\text{sr}$ by multiplying the right hand side of the above equation by $10^{-30}$ since $1\mu$b = $10^{-30}$ cm$^2$. Let $\epsilon$ be the fraction of $\gamma$-rays that are detected that actually end up in the peak during analysis. We must then multiply the right hand side of the equation again by $\epsilon$ to obtain the correct value for the yield $Y$.

If we now define the "efficiency of the detector" as $\frac{\epsilon d\Omega}{4\pi}$ then

$$Y = n \left( \frac{d\sigma}{d\Omega} \right) 4\pi \times 10^{-30} \left( \frac{\epsilon d\Omega}{4\pi} \right) \frac{Q}{\epsilon}$$

Lastly, the number of atoms/area, $n$, can be obtained from the 992-keV resonance curve where

$$n = \frac{\Delta_{\text{res}}}{\epsilon_{\text{res}}}$$

and $\Delta_{\text{res}}$ is just the width of the resonance curve which is also the target thickness and $\epsilon_{\text{res}} = 7.76 \times 10^{-15}$ eV/atom/cm. The final equation for $Y$ is

$$Y = \left( 4\pi \times 10^{-30} \right) \frac{\Delta_{\text{res}}}{\epsilon_{\text{res}}} \frac{Q}{\epsilon} \frac{d\sigma}{d\Omega}$$
and therefore the differential cross section is then given by

\[
\frac{d\sigma}{d\Omega} = \frac{\frac{Y}{\epsilon \Omega}}{Q \Delta_{\text{res}} \frac{4\pi}{e} \epsilon_{\text{res}}^{\text{res}} 4\pi \times 10^{-30}} \quad \text{in \( \mu b/sr \).}
\]
BIBLIOGRAPHY


J. Kalifa, G. Rotbard, M. Vergnes and G. Ronsin, "Etude des niveaux de \(^{28}\text{Si} \) et \(^{32}\text{S} \) au moyen de la réaction \(^{3}\text{He},d\) à 8 MeV," *Journal de Physique 34*, 139 (1973).


