Numerical Studies of Transition for Flows around Multi-Element Airfoils

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NUMERICAL STUDIES OF TRANSITION FOR FLOWS AROUND MULTI-ELEMENT AIRFOILS

by

Fengjun Liu

A Dissertation
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
requirements for the
Degree of Doctor of Philosophy
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Western Michigan University
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The transition of flows around a multi-element airfoil has been numerically studied using RANS with a $k-\varepsilon$ two-equation transition model, LST and DNS. The transition model uses an effective eddy-viscosity by coupling an intermittence-like correction to a turbulence eddy-viscosity that can be obtained via solving a parent $k-\varepsilon$ turbulence model. The transition model is truly predictive in that it is able to predict transition onset locations without having to specify prior knowledge of the targeted transition process. The predicted transition onset locations for all the cases studied were compared with the measured data. The results suggest that a better understanding of the confluent wake/boundary layers occurring near the main-element or flap leading edge is important to consistent predictions of such complex flows.

To better understand the confluent wake/boundary layer flow, a linear analysis of the confluent wake/boundary has been performed using a global numerical solution method. The modes associated with the boundary layer and the wake, respectively, have been identified. The modes associated with the wake, including a symmetric mode and an antisymmetric mode, are stabilized by the reduced distance between the wall and the wake. On the other hand, the boundary-layer mode has been found being
amplified as the wake approaches the wall. The important effects of the wake modes on the disturbance growth of the boundary layer have been confirmed using a DNS approach. The initial conditions for the DNS were provided by the linear analysis. The mean flow velocity profile was obtained using a time-averaged Navier-Stokes equation solver. The DNS results show that the disturbances in the wake region grow rapidly and promote a growth of the disturbances in the boundary layer. The effects of different disturbance forcing, such as amplitude and frequency, on the numerical simulations have been discussed.
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Fengjun Liu
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CHAPTER I

INTRODUCTION

The overall performance of a multi-element airfoil plays an important role in the sizing, economics, and safety of any airplane configuration. Compared to the flow occurred in cruise, there are still many phenomena that are presently not well understood for the flows found in landing and takeoff modes. These include laminar-to-turbulence transition, confluent boundary layers, cove flow separations and large-scale flow separations. They often present very different and distinct challenges. In the following, the current understanding of flow transition process, its experimental observations and numerical simulations as are relevant to the present dissertation research are reviewed.

Boundary-Layer Transition Physics and Prediction

Transition is a complex process through which a laminar flow transitions into a turbulent state as flow disturbances develop. It is often observed in various kinds of fluid machines. The measured data (Bertelrud 1997, Dam et al. 1997a, 1997b) for a multi-element airfoil indicate that a significant portion of the flow around the multi-element airfoil can be transitional over a wide range of operating conditions. The locations of the transition onset vary with angles of attack and Reynolds numbers. The transition of the boundary layer has attracted much attention for more than a century.
Although many issues remain unsolved, it is generally believed that the natural flow transition goes through three main stages (see Figure 1). They are the receptivity stage in which disturbances enter the boundary layer, the linear stage in which the Tollmien-Schlichting (T-S) waves are amplified as they travel downstream, and the nonlinear breakdown where secondary instability develops. The flow physics in each stage has been extensively studied theoretically and experimentally and has been reviewed (Morkovin 1969, Kachanov 1994).

![Figure 1. A Flat-Plate Boundary Layer Transition Process (from Kachanov 1994).](image)

During the receptivity stage, in the region with a relatively low Reynolds number, viscous traveling instability waves are usually generated. Disturbances in the freestream, such as sound or vorticity, may enter the boundary layer to form other types of instabilities depending on the Reynolds number, wall curvature, sweep, and surface roughness. These instabilities may exist individually or simultaneously, establishing the initial conditions of disturbance amplitude, frequency, and phase for the breakdown of laminar flow. The amplitudes of the disturbances entering the
boundary layer amplify exponentially during the linear stage of the T-S wave development. When the amplitudes of the disturbances become large, generally about 1-2% of the freestream velocity, the flow enters the nonlinear breakdown stage where turbulent spots appear and transition begins. Two types of transition routes have been identified. Klebanoff et al. (1962) conducted an experiment where a vibrating ribbon was used to create two-dimensional T-S waves that propagate as they travel downstream. It was observed that the ribbon-induced disturbances develop into three-dimensional A-shaped structure (A-vortices), showing alternating regions of high and low shear in the spanwise direction denoted as the splitting in peaks and valleys, respectively. The roll up of the high shear region results in the occurrence of disturbance spikes. The resulting three-dimensional vortex structures are characterized by an aligned pattern with the same fundamental streamwise wavelength as the original T-S waves. This is known as the K-type (Klebanoff et al. 1962) or fundamental breakdown. Another type of breakdown is referred to as subharmonic breakdown, or H-type (Herbert 1983a, 1983b) or N-type (Kachanov and Levchenko 1984, Kachanov 1987). The subharmonic breakdown appears in a staggered pattern of the A-vortices in the spanwise direction, implying that the streamwise wavelength is double the value of the initial T-S wave. In contrast to the fundamental regime, where the breakdown occurs due to the accumulation of spikes, the broadening of the frequency spectrum leads to transition even for very low amplitudes of the fundamental T-S waves.
The evolutionary characteristics of the two types of secondary instability in boundary layers and channel flows have been examined through numerical simulations. Early work concentrates on the temporal approach that uses a single wavelength computation domain and permits periodicity in the streamwise direction. This approach results in a saving of computer resources and avoids the problems with the outflow boundary conditions. Simulations of transition into breakdown were performed for channel flows (Orszag and Kells 1980, Orszag and Patera 1983, Biringen 1987, 1990) and for the boundary layer flows (Laurien and Kleiser 1989).

For a spatial approach, the transitional flow development in the physical space is computed. The inflow conditions were normally taken as the superposition of a two-dimensional disturbance wave and an oblique wave pair. Saiki et al. (1993) performed comparisons of the fundamental and the subharmonic transition simulations for a plane channel flow. The Λ-shaped vortex structures for the fundamental breakdown and the subharmonic breakdown were clearly visualized. The Λ-shaped vortex was also captured in the numerical simulations conducted by Liu and Liu (1994) for the boundary layer over a flat plate. The simulation of the fundamental breakdown for a flat-plate boundary layer (Fasel and Konzelmann 1990, Rist and Fasel 1995) generally agreed with the experiments by Kachanov and Lavchenko (1984) up to the second spike stage. Numerical simulations (Rist and Fasel 1995) indicate that the spanwise disturbances are generated at a much more rapid rate than two-dimensional components and the spanwise wave number spectrum spreads rapidly. This is consistent with the findings of Klebanoff et al. (1962), which suggests
that the development of the three-dimensional disturbance dominates the breakdown process. The subharmonic breakdown for the boundary layer was computed by Fasel and Konzelmann (1990) and Joslin et al. (1993) and general agreement with the experiment of Kachanov and Levchenko (1984) was obtained. Another route of boundary layer transition is the oblique wave breakdown that can be initiated by a pair of oblique waves (Berlin et al. 1999) or with additional complex conjugates (Joslin et al. 1993). The nonlinear interactions of the oblique wave pair lead to breakdown. The flow structures observed in the late stage of such flow transition were investigated in detail by Berlin et al. (1999).

In the following section, the transition on a multi-element airfoil will be reviewed.

Flow Over a Multi-Element Airfoil

For the flow over a multi-element airfoil, the onset location of the transition process varies with, for example, angles of attack and Reynolds numbers. In addition to the boundary layer transition, free shear flow transition is also likely to occur in, for example, the flow over the slat. As the slat wake develops downstream, it may further interact with the transitional flow over the succeeding elements, such as the flow over the main element and the flap. The resulting shear layer is referred to as the confluent wake/boundary layer where the wake and the boundary layer have strong interactions as shown in Figure 2. These effects change mainly with the airfoil configuration, angle of attack, Reynolds number, and freestream Mach number. The cove flow is
always separated, as is the flow on the lower surface of slat and flap at low angles of attack. The large-scale flow separation often occurs on the upper surface of the flap.

In order to understand the physics of the flow over multi-element airfoils, extensive experimental work has been performed on realistic multi-element airfoil configurations and in-flight aircrafts. Bertelrud (1997) made detailed transition onset location measurements on each element for a multi-element airfoil for a wide range of operating conditions. The tested angles of attack vary from 4° to 23° for a freestream Mach number of 0.2 and two Reynolds numbers, $9 \times 10^6$ and $5 \times 10^6$, respectively. The resulting transition onset data have been used to validate computational codes. The effects of Reynolds number and freestream Mach number on the overall performance

Figure 2. Flow Over a Multi-Element Airfoil (Liou and Liu 2000).
of a multi-element airfoil have been studied by Lin and Dominik (1997). Detailed measurements of the mean flow and the turbulence quantities have been made by Nakayama et al. (1990) using pressure and hot-wire probes in the shear flow regions around a multi-element airfoil in two different configurations for a Reynolds number of $3 \times 10^6$ and a freestream Mach number of 0.2. Their results indicate that the slat wake seems to have an important effect on the development of the boundary layer on the main element, particularly on the transition from laminar to turbulent flow.

To obtain detailed full-scale flow measurements on a multi-element high-lift system at various flight conditions, in-flight pressure measurements (Yip et al. 1995) and in-flight boundary layer measurements (Dam et al. 1997a, 1997b) have been conducted on the NASA Langley Transport Systems Research Vehicle, a B737-100 aircraft with the wing comprised of a slat, a main element and a triple-slotted flap. The lift curve, wall pressure coefficient, skin friction, and flow visualization were obtained (Yip et al. 1995) over a range of Reynolds numbers from approximately $1.0 \times 10^6$ to $2.0 \times 10^6$, and freestream Mach numbers from approximately 0.16 to 0.40 for angles of attack up to near-stall conditions. Extended linear lift curves with slopes steeper than the wind-tunnel results (Yip et al. 1995) were found. Also, increasing the angle of attack primarily increased the slat and main element pressure loading, with little effect on the flap loading (Yip et al. 1995). Flow relaminarization was found to occur on the leading edge of the slat (Dam et al. 1997a), the main element and the flap (Dam et al. 1997b).
The advent of the computational fluid dynamics (CFD) techniques capable of dealing with a complex flow has helped improving the calculation of the flow over a multi-element airfoil. The rather complicated geometry and complex flow, however, have made the prediction of the multi-element flow field a significant challenge to CFD. A two-dimensional unstructured Navier-Stokes solver (Anderson et al. 1995) and a parallel three-dimensional Navier-Stokes solver (Mavriplis and Pirzadeh 1999) were utilized for computing the flow around multi-element airfoils. The calculations generally over-predicted the angle of attack where the maximum lift occurred. A general agreement between the computed velocity profiles and the measured ones was found, but the velocity defect of the slat wake was under-predicted on the flap (Anderson et al. 1995). An incompressible Navier-Stokes code, INS2D (Rogers and Kwak 1990, Rogers et al. 1991, Rogers 1995), has been developed. The code has been extensively used to predict the overall performance trends that occur with configuration changes (Dominik 1994), to evaluate the use of different turbulence models (Rogers et al. 1994) and to design the components of a multi-element airfoil (Lin and Dominik 1997). Comparisons (Rogers et al. 1994) of the Baldwin-Barth one equation model (Baldwin and Barth 1990), the Spalart-Allmaras (SA) one-equation model (Spalart and Allmaras 1992), the Durbin-Mansour one-equation model (Durbin et al. 1994) and the $k-\omega$ shear stress transport (SST) two-equation model (Menter 1993) showed that the correct trends in lift were predicted for changes in flap gap and in the Reynolds number by all the models but the calculations generally over-predicted the angle of attack where the maximum lift occurred.
It is generally found that poor boundary-layer velocity profile predictions upstream lead to poor trailing edge near-wake profile predictions and, subsequently, poor predictions in the far-wake region. Often, inadequate wake predictions cannot be attributed only to inadequate models but also, at least in part, to a lack of a detailed understanding of the transition process on the generating element. Rumsey et al. (1998) used the CFL3D code (Thomas et al. 1990, Thomas 1993) with the SA turbulence model and the SST turbulence model to compute 2-D multi-element airfoil flow fields using one-to-one point connectivity grids. Transition onset locations were specified based on measured data. The use of these measured onset locations improved the prediction for the boundary layer thickness, skin friction, and wake profile shape. They concluded that the predicted transition onset locations are very important to accurate and consistent computations of the multi-element airfoil flow field.

A transition prediction for a multi-element airfoil was performed (Kusunose and Cao 1994) via examining three possible transition mechanisms: the growth of unsteady T-S waves, the laminar boundary layer separation, and the turbulence contamination. The pressure distributions were obtained by using the INS2D code and the laminar boundary layer parameters by solving the momentum integral equation. The predicted transition onset locations were in good agreement with the experimental data except for those on the flap. Another transition model capable of calculating transition onset (Young et al. 1993, Warren and Hassan 1998, McDaniel et al. 1999) was incorporated (Czerwiec et al. 1999) into the CFL3D and then used to
compute a multi-element flow field for two angles of attack, 8° and 19°, at a freestream Mach number of 0.2 and a Reynolds number of 9×10⁶. The transition model is characterized by a combination of a non-turbulence fluctuation part determined by using an analysis of the Tollmien-Schlichting (T-S) wave instability, a turbulent part by a $k$-$\zeta$ (enstrophy) model (Robinson and Hassan 1998) and an intermittency correction factor (Dhawan and Narasimha 1958). In this transition model, the freestream turbulence intensity has to be specified for calculating the non-turbulent part. But there is no measured value of the freestream turbulence intensity in the wind tunnel experiment. All computations (Czerwiec et al. 1999) assumed the freestream turbulence intensity of 0.05. Turbulent spots were assumed to appear at the location with minimum skin friction and the intermittency correction factor was obtained as a part of the flow solution. Thus, the model does not need to specify either the onset or the extent of transition. The results show that the model slightly under-predicts the transition onset for the slat and the main element at the angle of attack of 19° and greatly over-predicts the transition location for the slat at 8°.

The inability of the models to consistently predict the transition onset locations on a multi-element airfoil may be partially attributed to the lack of an appropriate understanding of the associated complex flow physics. The confluent wake/boundary layer is one of the flow phenomena that could have considerable influences on the transition process of the boundary layer over a succeeding element, the velocity development downstream and the overall aerodynamic performance.
Bario et al. (1982) used simplified geometries to investigate the confluent wake/boundary layer flow. In that study, a tandem airfoil arrangement consisting of a small and symmetric airfoil was followed by a larger symmetric airfoil immediately downstream. Three viscous regions: the wall boundary layer, the internal half wake and the external half wake, were examined in detail. The position of zero shear stress was found to be not identical with that of the zero velocity gradient due to the strong interactions between the wake and the boundary layer. In addition, an extensive set of experimental studies for the confluent wake/boundary layer flow were conducted and reported by the group in Cambridge University (Zhou and Squire 1983, Zhou and Squire 1985, Moghadam and Squire 1989, Agoropoulos and Squire 1988). In these studies, the wake generated by a flat plate and symmetric airfoils were allowed to merge with the turbulent boundary layer on the floor of a wind tunnel. After the wake velocity defect has vanished, the confluent wake/boundary layer tends to develop a thick boundary layer in which the von Karman constant is very different from a fully developed turbulent boundary layer (Zhou and Squire 1983). The introduction of an adverse pressure gradient (Zhou and Squire 1985) resulted in a rapid merging between the wake and the boundary layer and the center of the wake moved away from the wall. A three-dimensional interaction of the confluent wake/boundary layer was generated using a swept airfoil and bump (Moghadam and Squire 1989). It was found that the wake decayed very slowly with its centerline moving away from the wall,
causing a very slow mixing of the wake and the boundary layer. Comparisons between the measurements and computations for the confluent wake/boundary layer were reported (Agoropoulos and Squire 1988, Johnston and Horton 1986). The effects of two types of airfoil trailing edges, a streamlined body and a bluff body, on the confluent wake/boundary-layer flows were conducted (Tulapurkara et al. 1990). The bluff body caused faster interaction of the confluent wake/boundary layer flow than the streamlined body due to the higher level of fluctuating quantities behind the bluff body.

Recently, Thomas et al. (2000) employed a realistic geometry to investigate the interactions of a slat wake with a turbulent boundary layer on the main element. The effects of the confluent wake/boundary layer flow on the lift generation of a multi-element airfoil system were discussed in their studies. It was found that even in the absence of main element separation, lift was still reduced as a consequence of the confluence of the slat wake/the main element boundary layer due to an associated outward streamline displacement effect. The detailed measurements, including velocity profiles downstream, wall pressure coefficient and skin friction form a comprehensive multi-element airfoil flow field database for CFD coding validation.

According to the experimental studies mentioned above, the development of the confluent wake/boundary layer flow can be divided into three stages as shown in Figure 3. In the initial stage, the wake and the boundary layer are separated by a potential core. The lower half wake and the upper half wake can be defined by a wake centerline. The second stage refers to the region where the lower half wake interacts
with the boundary layer and there is no potential core between these two layers. Finally, the confluent flow enters the third stage where a thicker turbulent boundary layer develops.

The linear stability theory (LST) can be applied to study the behaviors of small disturbances. The boundary layer stabilities and the wake stabilities are fundamental problems that have received much attention. The theoretical basis of the LST stems from the well-known Orr-Sommerfeld equation (Orr 1907a, 1907b, Sommerfeld 1908) and the Squire equation (Squire 1933) that is derived based on linearized disturbance equations for an incompressible flow.

![Figure 3. Development of the Confluent Wake/Boundary Layer Flow.](image)

Solving the Orr-Sommerfeld equation was a great challenge for mathematicians at the turn of the 20th century when these equations were published. Today, there are various mathematical and numerical methods to solve the Orr-Sommerfeld equation. A commonly used method has been the shooting method with orthonormalization (Osborne 1964, 1967, Keller 1968) before the global methods
(Bridges and Morris 1983, Bridges and Vaserstein 1986) were applied. In the shooting method, an initial guess is made for the eigenvalue and the integration is performed over the computational domain repeatedly until matching conditions are satisfied. A good initial guess, thus, is required to assure the convergence of the iteration. The eigenvalues and eigenfunctions of the Orr-Sommerfeld equation for a parallel Blasius boundary layer were calculated for a wide range of values of frequency and Reynolds number (Jordinson 1970). The global method solves for all of eigenvalues without requiring an initial guess. The modes of interests are found by examining the resulting eigenvalue spectrum. The use of the global method is time consuming, especially for the case of a fine grid. The problem can be overcome by using the local method (Bridges and Morris 1983, Bridges and Vaserstein 1986), in which the initial values for the eigenvalue are provided by the global method in a course grid. In addition, the local method can be used to find eigenfunctions as soon as the eigenvalue is obtained. The global and the local methods have been extensively used to study various boundary layer stabilities (Joslin 1990, Joslin 1992, Liou et al. 2000).

It is generally believed that the LST is very effective in studying flow transition. For example, the existence of T-S disturbance wave before transition for a wall-bounded boundary layer was first predicted using LST (Tollmien 1931, Schlichting 1932) and then verified by experiments (Schubauer and Skramstad 1947). Good agreement between the predicted and the measured neutral curves, which represent the boundary layer instability, was found. In addition, theoretical predictions of the growth of the amplified disturbances also agree well with experiments in the
initial stage of transition. The presence of an inflexion point in the velocity profile has a destabilizing influence on flow (Schlichting 1979). Lin (1955) showed that a negative streamwise pressure gradient stabilizes the boundary layer and vice versa. Although LST can not describe the whole flow transition process, it has been shown to be very useful in identifying the type of velocity profile that could lead to instability, finding the frequency to trigger the fastest growth of disturbance, and determining the possible parameters to delay transition.

Possibly just as widely studied as the boundary layer instability, the instability of wake has also been examined in many reports (Sato and Kariki 1961, Mattingly and Criminale 1972, Hultgren and Aggarwal 1987, Monkewitz and Nguyen 1987). Two amplifying modes are often identified, a symmetric mode and an antisymmetric mode. For a near wake (Mattingly and Criminale 1972), the symmetric mode was found to have a higher spatial growth rate and was observed in the experiment. The instability analysis of the wake embedded in a boundary layer was studied by Bajwa (1995) for the modeling of the presence of particles in bounded and unbounded shear flows. Absolute instability was found to occur for cases with large values of the centerline wake defect (Hultgren and Aggarwal 1987).

For a swept multi-element wing, multiple instabilities may co-exist in the flow. These include streamwise viscous traveling wave instability, stationary and traveling crossflow instabilities, Görtler instability, attachment-line instability, and streamline-curvature instability. The streamwise viscous traveling wave instability is associated with the chordwise component of the flow and is quite similar to that in
two-dimensional flows, where the T-S waves generally develop. This usually occurs in zero or positive pressure-gradient regions on each element. Because of the effect of sweep, the crossflow velocity profile has a maximum velocity in the boundary layer. The crossflow instability is initiated by the existence of an inflexion-point in the spanwise velocity profile. It usually occurs in the negative pressure-gradient region near the leading edge. It could appear in the rear region if the angle of sweep is sufficiently large. Two types of the crossflow instability may exist on the multi-element airfoil, depending on operating conditions. They are the stationary crossflow instability that usually appears in a quiet freestream, such as a cruise position and the traveling crossflow instability that is likely to occur in high freestream turbulence in, for example, landing or takeoff modes. The Görtler instability occurs in the shear flow over the slat surface, the main-element surface, and the flap surface. It is induced by centrifugal forces. The Görtler instability becomes less important in the curved region of swept elements for a sufficiently large angle of sweep. Disturbances produced in a fuselage/wing juncture corner may propagate along the leading edge of each element and cause the attachment-line stability. An increase of angle of attack may lead to the large negative pressure-gradient near the leading edge of each element. As a consequence, the boundary-layer flow accelerates away from the attachment-line and enters the region where crossflow instability is likely to occur. The streamline-curvature instability due to the curvature of external streamlines is more likely to exist in the region close to the leading edge on each element. The wake instability occurs in the wake region or near the leading edge of the main element or
the flap. The main-element wake or the slat wake may exhibit absolute instability for sufficiently large wake defect that tends to appear at a high angle of attack. The critical wake defect to trigger the absolute instability usually varies with Reynolds number.

The process of external disturbances in the freestream entering the boundary layer is called receptivity (Morkovin 1969). Boundary-layer receptivity to freestream disturbances has been reviewed (Saric et al. 2002) in theory, computation and experiment. For the flow over a real swept multi-element airfoil, receptivity may have different paths through which to introduce a disturbance into the boundary layer. These are involved in the vortical parts of the freestream disturbances (turbulence) and the irrotational parts of the freestream disturbances (sound). Goldstein (1983) showed that receptivity occurs in regions where the mean flow changes rapidly in the streamwise direction. Near the leading-edge, the boundary layer is thin and grows rapidly such that receptivity is likely to occur. The interaction of disturbance with a discontinuous surface tends to cause the localized receptivity that may occur in the main-element cove region. Saric et al. (2002) showed that the theory for receptivity based on the Orr-Sommerfeld equation is not successful because the streamwise dependence of the mean flow is neglected in the Orr-Sommerfeld equation. Goldstein (1983, 1985) developed an asymptotic approach that has been used for the leading-edge receptivity and the localized receptivity.

As the confluent wake/boundary layer develops downstream, the initial small disturbances amplify, particularly for the disturbance in the highly unstable wake.
When the amplitudes of the disturbances reach considerable values, the nonlinear effects become significant. To numerically investigate such development of confluent wake/boundary layer flows, direct numerical simulations (DNS) that solve the Navier-Stokes equations can be used. Aided by the advances of computer capacity, DNS has become an effective research tool by which significant insights into the transition or turbulence physics have been gained that cannot be easily attained by experiments (Moin and Mahesh 1998).

The important numerical issues DNS has to deal with are the scheme, the time advancement, and the boundary conditions. The scheme adopted determines the capability of DNS to capture various scales of flow perturbations. Spectral methods provide higher spatial resolution than differencing methods but tend to give rise to higher aliasing error associated with the nonlinearity of the governing equations. Detailed discussions on eliminating the aliasing error can be found in Hussaini and Zang (1987) and Blasdell et al. (1996). For the wall-bounded boundary layer flow simulation, the commonly used schemes are the spectral methods (Laurien and Kleiser 1989, Lundbladh et al. 1994, Berlin et al. 1999), the combination of the spectral method and the finite difference methods (Joslin et al. 1992, Rist and Fasel 1995, Saiki and Biringen 1997) and high-order finite difference method (Rai and Moin 1993, Liu and Liu 1994, Wu et al. 1999). Recently, high-order compact schemes have attracted much attention due to their spectral-like accuracy and demonstrated ability to resolve shocks (Adams and Shariff 1996, Tolstykh 1991, Tolstykh and Lipavskii 1998, Zhong 1998). Implicit time advancement allows large
time steps in computations using the Reynolds averaged Navier-Stokes equations. The requirement of time accuracy over a wide range of scales, however, does not permit very large time steps in DNS. Additional iterations are often needed at each time step to eliminate the linearization errors from implicit operations (Rai and Moin 1993). For explicit time advancement, the third or fourth order Runge-Kutta scheme has been used to obtain high-order time accuracy (Joslin et al. 1992, Tolstykh 1994, Zhong 1998).

In the simulations of the incompressible boundary layer over a flat plate, the inflow conditions (Joslin 1992, 1993, Liu and Liu 1992, 1994) are usually provided by the solutions of the Orr-Sommerfeld equation. Outflow boundary conditions are required to suppress all possible reflections into a computation domain. The buffer domain method (Streett and Macaraeg 1989) is an efficient nonreflecting outflow boundary treatment. The disturbances are to be smoothly reduced to zero via using a damping function in the buffer domain that is appended to the end of the physical domain of interest. An improved buffer domain method (Liu and Liu 1992, 1994) reduces the length of the buffer domain from at least three T-S wavelengths to less than one wavelength.

Goals of Dissertation Research

The objective of this dissertation research is to numerically study the transitional flows over a multi-element airfoil by ways of modeling and full simulations.
The first part of the research is to calculate the flow over a multi-element airfoil using a predictive transition model (Liou and Shih 1997) and a RANS code. The model has been successfully applied to predict the transition onsets of flat plate boundary layers. The model is coupled with the INS2D flow solver. The predictions of the flow field over a multi-element airfoil will be performed for angles of attack from 8° to 19° with two Reynolds numbers of $9 \times 10^6$ and $5 \times 10^6$, respectively. The computed results are compared with experimental data for the surface pressure, skin friction, transition onset location and velocity magnitude.

A linear analysis and a non-linear analysis of the confluent wake/boundary layer will then be performed using the LST and DNS to examine the evolution of disturbance in confluent wake/boundary layers. The studies focus on the confluent wake/boundary layers that may be found around two-dimensional multi-element airfoils. The flow model adopted is a superposition of a boundary and a wake located over the boundary layer. For such the simplified flow geometry, the effects of wall curvature and wing-sweep are not considered and the hydrodynamic instabilities that will be included are the convective wake and boundary-layer modes. The linear analysis and the nonlinear analysis are to deal with the 2D confluent wake/boundary layer flow, with particular attention to how disturbances both in the wake and in the boundary layer propagate downstream and how the wake affects the boundary-layer transition.
Significance of the Dissertation Research

As mentioned earlier, the aerodynamics characteristics of a multi-element airfoil system can significantly impact the overall design of aircraft in terms of sizing, performance and safety certification. Currently, the design and optimization of this system remain very challenging due to the complicated geometry and flow physics. In general, the capability to accurately predict the flow over multi-element airfoil configuration is critical to the design and optimization of aircraft. Reliable predictions of the performance of the multi-element airfoil using CFD requires consistently accurate modeling of the relevant viscous flow, in particular, the transitional flow that occurs over a wide range of operating conditions. As a result, the predictions of the flow field over a multi-element airfoil have been the subject of a considerable number of studies using a variety of turbulence models. The first part of this thesis is to predict the flow field over a multi-element airfoil using a transition model. The results will help determine if it is viable transition model for predicting the complex flow over multi-element airfoils. The linear and nonlinear analyses of the confluent wake/boundary layer will be helpful for a better understanding the flow physics and improving the mathematical models. With a more accurate model, a RANS solver can be routinely used to design and optimize a multi-element airfoil configuration.

The thesis is organized as follows. Chapter II will present the transitional flow modeling for the flow over a multi-element airfoil. Chapter III will describe a linear analysis for a model confluent wake/boundary layer. Chapters IV will present a
nonlinear analysis of the confluent wake/boundary layers using DNS. The summary of this thesis will be given in Chapter V.
In this chapter, the natural transitional flow fields over a multi-element airfoil are calculated using a $k - \varepsilon$ two-equation transition model (Liou and Shih 1997). The model uses an effective eddy-viscosity by coupling an intermittence-like correction to a turbulence eddy-viscosity that can be obtained via solving a parent $k - \varepsilon$ turbulence model (Shih and Lumley 1993). The intermittency function varies with the evolution of the disturbance kinetic energy, $k$, which is a solution variable for the present $k - \varepsilon$ modeling methodology. The freestream turbulence level affects the transition in the boundary layer and is explicitly included in the model formulation of the intermittency function. Transition onset and end locations are predicted rather than being input data. In this sense, the transition model is a truly predictive transition model. The model has been successfully applied to by-pass transitional flows over flat plates (Liou and Shih 1997). The predicted transition onset and the length of the transition region were in a very good agreement with the measurement for all the cases studied. The transition model and the results of its application to a multi-element airfoil are presented in the following sections.

In the following section, the configuration of the multi-element airfoil is described. The transition model and its parent turbulence model are also presented.
Finally, the calculated results and discussion for a wide range of operating conditions of the multi-element airfoil are presented.

Multi-Element Airfoil Configuration

The geometry used for the present study was the McDonnell Douglas 30P30N landing configuration where the gap, overhang, and deflection angle are designed to be 0.0295, -0.025, and 30° for the slat, and 0.0127, 0.025, 30° for the flap. The chord of the main element is used as a reference length. The 30P30N airfoil geometry and survey stations on it are shown in Figure 4. This airfoil has been tested extensively at NASA Langley Low Turbulence Pressure Tunnel (LTPT) (Stainback et al. 1986, Bertelrud 1998) to form the database that can be used as a means of calibrating CFD codes. It is one of the test cases in a CFD Challenge Workshop held at NASA Langley in 1993 and the measured data are used here for comparison. This configuration has static pressure taps mainly in the mid-span region of the three elements. The 350 hot films were located in a single row on the surface. The wake profiles were obtained using probes located 4 inches apart along vertical direction. The transition region was determined from a combination of the standard deviation, the skewness, and flatness of the hot film signal. The autocorrelation function was used to provide information on the Taylor microscale and the integral length scale. In the analysis the scales have been used to indicate spectral information. The scales were used as a rough guide to identify sensors that might exhibit a problem or a region of strong correlation and thus
to make sure that any noise or disturbance that might be present is not interpreted a

Figure 4. 30P30N Geometry and Survey Stations.

The $k$-$\varepsilon$ Turbulence Model

In the parent turbulence model used here (Shih and Lumley 1993), the
turbulent Reynolds stresses, $\tau_{ij}$ are modeled via the turbulent eddy viscosity, $\mu_t$. That
is

$$
\tau_{ij} = 2\mu_t S_{ij} - \frac{2}{3} k \delta_{ij}
$$

(2.1)

where $\mu$ denotes the molecular viscosity and $S_y$ denotes the mean strain rate tensor,
i.e.,

$$
S_{ij} = \frac{1}{2} (U_{ij} + U_{ji}) - \frac{1}{3} U_k \delta_{ij}
$$

(2.2)
In the framework of $k$-$\varepsilon$ turbulence eddy viscosity model, the eddy viscosity $\mu_t$ is further expressed as

$$
\mu_t = C_\mu f_\mu \rho \frac{k^2}{\varepsilon}
$$

where $C_\mu$ is taken as 0.09 for the standard $k$-$\varepsilon$ turbulence model, $f_\mu$ is the wall damping function for the eddy viscosity and $\rho$ is a local density. The turbulent kinetic energy $k$ and the dissipation rate $\varepsilon$ are found from the following model transport equations

$$
\rho k_j + \rho U_j k_j = [((\mu + \mu_t) k_j)_j - \rho u_i u_j U_{i,j} - \rho \varepsilon]
$$

(2.4)

$$
\rho \varepsilon_j + \rho U_j \varepsilon_j = [((\mu + \mu_t) \varepsilon_j)_j - C_1 \frac{\varepsilon}{k} \rho u_i u_j U_{i,j} - C_2 f_2 \frac{\varepsilon^2}{k} + \frac{\mu}{\rho} S_j S_j]
$$

(2.5)

In light of experiments and numerical simulations, the model coefficient $C_\mu$ in equation (2.3) varies with flow situation. In the $k$-$\varepsilon$ turbulence model developed by Shih and Lumley (1993), the constant $C_\mu$ (=0.09) was replaced by a variable $C_\mu$ formulation. The new formulation of $C_\mu$, with an explicit dependence on the mean strain rate, was developed based on the realizability constraints of the Reynolds stresses. It takes the following form

$$
C_\mu = \frac{1}{A_0 + A_1 U^{(*)} \frac{k}{\varepsilon}}
$$

(2.6)
where

\[ U^{(\omega)} = \sqrt{S_{ij}S_{ij} + \bar{\Omega}_{ij}\bar{\Omega}_{ij}} \]
\[ \bar{\Omega}_{ij} = \tilde{\Omega}_{ij} - 2\varepsilon_{ijk} \omega_k \]
\[ \tilde{\Omega}_{ij} = \Omega_{ij} - \varepsilon_{ijk} \omega_k \]

\( \Omega_{ij} \) is the mean rotation rate viewed in a rotating reference frame with the angular velocity \( \omega_k \). The parameter \( A_s \) is determined by

\[ A_s = \sqrt{6} \cos \phi \]
\[ \phi = \frac{1}{3} \arccos(\sqrt{6}W) \]
\[ W = \frac{S_{ij}S_{jk}S_{ki}}{S^3} \]
\[ \bar{S} = \sqrt{S_{ij}S_{ij}} \]

The damping function is defined by

\[ f_\mu = \left[ 1.0 - \exp\left( -(a_1 R_k + a_3 R_k^2 + a_5 R_k^4) \right) \right]^{0.5} \quad (2.7) \]

where

\[ a_1 = 1.7 \times 10^{-3}, \ a_3 = 10^{-9}, \ a_5 = 5 \times 10^{-10}, \ R_k = \rho \frac{\sqrt{k}y}{\mu} \]

The model coefficients in the damping function were chosen that by calibrating the model solutions against the turbulent boundary layer flow over a flat plate with zero pressure gradient. Other model coefficients in equations (2.4) and (2.5) are taken as
\[ C_1 = 1.44, C_2 = 1.92, \sigma_x = 1.3, \]
\[ f_2 = 1.0 - 0.22 \exp(-\frac{R_t^2}{36}) \]
\[ R_t = \rho \frac{k^2}{\varepsilon \mu} \]

The near wall boundary conditions are obtained by examining the Kolmogorov behavior of near-wall turbulence proposed by Shih and Lumley (1993). The expressions of \( k \) and \( \varepsilon \) at the wall are given as

\[ k = 0.25 u_r^2 \]  \hspace{1cm} (2.8)

\[ \varepsilon = 0.25 \rho \frac{u_r^4}{\mu} \]  \hspace{1cm} (2.9)

The \( k - \varepsilon \) turbulence model has been shown to work well for a wide range of flows (Liou and Shih 1995, Yang et al. 1995).

The Predictive Transition Model

The transitional model (Liou and Shih 1997) has been developed based on the experimental observation that the flows in transition are highly intermittent. Locally, the flow is turbulent as the turbulent spots convect through. In between the passing of the turbulent spots, the flow relaxes to a disturbed laminar state. In this case, a measure of the fraction of time that the boundary layer is turbulent is the intermittency factor. Normally, the intermittent nature of the transitional flow is not explicitly accounted for in the development of turbulence models. Turbulence models have been
in general designed for flows in a fully turbulent state. To be able to use a turbulence model in the intermittent transitional flow region, we incorporate a weighting factor, $\gamma$, in the turbulent eddy viscosity, $\mu_t$, obtained from the “parent” $k-\varepsilon$ two-equation turbulence model mentioned above. That is,

$$\mu_\tau = \gamma \mu_t$$  \hspace{1cm} (2.10)

where $\mu_\tau$ denotes transitional flow eddy-viscosity. The intermittency correction function, $\gamma$, resembles the physical flow intermittency factor. Similar to the intermittency factor that determines the fraction of time the flow is turbulent in the transition region, $\gamma$ determines the fraction of turbulent eddy-viscosity to be used in the transition region. Therefore, it is assumed that $\gamma$ varies monotonically through the transition region.

In this model, the intermittency correction function $\gamma$ is defined in terms of the streamwise variation of the peak disturbance energy. The disturbance kinetic energy increases as the flow evolves from laminar to transitional state. Its local peak level also increases. Therefore, the disturbance energy level is an appropriate parameter to characterize the flow transitional region. The intermittency correction function involves the peak disturbance energy level and the local freestream turbulence level.

$$\gamma = \gamma_0 + (1 - \gamma_0) f^{1.8}$$  \hspace{1cm} (2.11)
\[ f = \frac{k_n^+ - k_1^+}{k_2^+ - k_1^+} \quad (2.12) \]

where

\[ k_n^+ = k_n^+(x) = \max(k^+(x,y)) \big|_{x - k_e^+(x)} \quad (2.13) \]

\[ k_e^+(x) = \frac{k_e(x)}{u_r^2(x)} \quad (2.14) \]

\( \gamma_0 \) is a model constant. It is taken as 0.6 (Liou and Shih 1997), based on flat plate boundary layer results. \( k_e \) and \( u_r \) denote the local disturbance kinetic energy in the free stream and the local frictional velocity, respectively. The freestream turbulence intensity was quite low in the wind tunnel where the transition measurements were made. In the present application, \( k_e \) is set according to the turbulence level in the outer stream. The model coefficient \( k_1^+ \) relates to the threshold for the formation of turbulent spots near the wall. It has been shown (Shih and Lumley 1993) that turbulent energetic eddies reduce to Kolmogorov eddies near the wall, where \( k^+ = 0.25 \), and all the wall parameters are characterized by the Kolmogorov scales.

An estimate for the spot generation threshold level in terms of \( k^+ \) is thus set at 0.25. It should be noted that the wall boundary layer condition for \( k \) be determined by the Kolmogorov scaling of the near-wall turbulence in the parent \( k - \varepsilon \) two-equation model. Therefore, the selection of the value of \( k_1^2 \) is consistent with the parent model.
$k_2^+$ is determined by examining fully developed turbulent boundary layer and its value is set at 4.5. The term $k_2^+ - k_1^+$ in equation (2.12) is used to normalize the term $k_\ast^+ - k_1^+$, such that the transition progress variable, $f$, satisfies the equation,

$$0 \leq f \leq 1 \quad (2.15)$$

The effect of the local freestream turbulence is accounted for in equations (2.13) and (2.14). In equation (2.13), $k_\ast^+$ represents the difference between the peak value and the freestream value of the $k^+$ profile at a streamwise location, $x$. In this study, the $k^+$ profile is taken along the direction normal to the wall surface. Equation (2.13) allows the flow transition to proceed according to the development of the internal peak level of $k^+$.

Note that no measurement was made to determine the turbulent intensity and the decay of turbulent kinetic energy in the experiment where the present data were taken. This information is needed to determine the value of the disturbance kinetic energy and its dissipation rate at the inlet of the computational domain in the present approach. A turbulent intensity level of 0.05% was cited and used in the present analysis and the equivalent eddy-viscosity was set at the laminar level.

The present intermittency function technique uses the variation of the disturbance energy to computationally characterize the transition progress. Therefore, the calculated intermittency correction function does not necessarily correspond to the physical flow intermittency factor.
The INS2D Flow Solver

All computations are carried out using the flow solver INS2D (Rogers and Kwak 1990, Rogers et al. 1991, Rogers 1995) that solves the two-dimensional incompressible Navier-Stokes equations. The artificial compressibility was introduced to deal with compressible effects. The code uses a third-order flux splitting difference for the convective terms and the second-order central difference for the viscous terms. Time accuracy is obtained in the numerical solutions by sub-iterating the equations in pseudotime for each physical time step. To accelerate convergence to a steady state, the code employs a nonfactored implicit line-relaxation algorithm that allows very large time steps. In addition, implicit boundary conditions are imposed at all of the boundaries. At a solid wall, the velocity is specified to be zero and the pressure at the boundary is obtained by specifying that the pressure gradient normal to the wall be zero. The method of characteristics is used for the inflow and outflow boundaries.

The INS2D code has been successfully used to compute two-dimensional multi-element airfoil flow fields (Rogers 1993, Kusunose et al. 1994, Rogers et al. 1994, Lin and Dominik 1997). For turbulent flows, INS2D currently employs a number of turbulence models, including Baldwin-Barth one-equation model, Spalart-Allmaras one-equation model, $k$-$\omega$ SST two-equation model, and Durbin-Mansour one-equation model. In the present study, the $k$-$\varepsilon$ Turbulence Model (Shih and Lumley 1993) has been coded into the INS2D as a parent turbulence model to employ the predictive transition model (Liou and Shih 1997).
The Overset Grid Generation

Generating the necessary grids for multi-element airfoil flow calculations can be very time-consuming regardless of the grid topology one chooses to use. The INS2D code runs on multiple-zone grids that are connected in point-to-point match or Chimera overset manner. In this work, overset grids were used and the grids were generated using the OVERMAGG (Rogers et al. 1998) software. OVERMAGG is an automated script system specifically designed to perform overset grid generation for multi-element airfoils. The use of OVERMAGG has resulted in a significant time saving during this study.

The Chimera method allows a system of relatively simple grids, each describing a component of a complex aerodynamic configuration, to be combined into a composite grid to yield solutions for complex flow fields. Application of the Chimera method requires two steps. Step 1 is a description of how each mesh is to communicate flow field information with other meshes. Step 2 is an execution of a flow solver that uses the communication information generated in step 1. For overset meshes, any mesh can receive information from appropriate meshes through outer boundary points by interpolation. Any mesh points that are contained within a certain region surrounding solid walls have to be excluded from the computational domain of that mesh. The boundary between the excluded points and the rest points are referred as the hole boundary. It can also be specified by interpolation. The interpolation work is performed in step 1 by using the code PEGSUS (Suhs and Tramel 1991). PEGSUS
can produce a composite mesh file that consists of the concatenation of all meshes in the multi-mesh configuration and an interpolation file that is a table associated with all interpolated boundary points in the composite mesh with mesh points supplying the interpolated flow field values. In step 2, the composite mesh and interpolation files are read by a flow solver such as INS2D.

The computational grid used in the present study consists of six zones and a total of about 103,000 grid points. Three C grids were used around the slat, the main element, and the flap, with sizes 175x45, 381x133, and 221x85, respectively. Three H grids were used for the main element cove, the wake region of the flap, and the wind tunnel. Figures 5 and 6 show the three C grids that were used to model the main element, the slat, and the flap, respectively. Figure 7 shows the two H grids that were used to model the main element cove region and the wake region of the flap. This was done in an attempt to accurately model the backward facing step of the cove and the flap wake regions.

Comprehensive grid-independence studies have been performed and reported previously using similar overset grids for the same geometry (Rogers et al. 1994, Rogers 1995). These and other calculations for the same geometry indicate that the grid density can have certain effects on the velocity profiles and the wall pressure is not sensitive to the grid density. These results also show that further refinement of the grid from the current level does not change the results appreciably. To examine the effect of grid on the use of the transition model, a fine grid with a total points of 136,000 arranged 175x85, 381x165 and 221x 113 for the slat, the main element and
the flap, respectively, has been used in the computation for Reynolds number of 9×10^6 at angles of attack of 19° and 8°. Results of the computations on these two grids

Figure 5. Grid Around the Main Element of 30P30N.

Figure 6. Grids Around the Slat and Flap of 30P30N.
showed that there was not much grid dependence in transition onset locations, surface pressure coefficients and velocity profiles.

Figure 7. Grids for the Main Element Cove and Flap Wake Regions of 30P30N.

In the following, results will be presented for two Reynolds numbers of $9 \times 10^6$ and $5 \times 10^6$ and a range of angles of attack ($AOA$) for each Reynolds number.

Results and Discussion

Wall Pressure Coefficient and Skin Friction Coefficient

Figure 8 shows a comparison of the wall surface pressure coefficient, $C_p$, distribution over the slat, main element, and flap for $Re = 9 \times 10^6$ and $AOA = 19^\circ$. The calculated wall pressure distributions on each element agree well with the measurement. The wall surface pressure coefficients for $AOA = 8^\circ$ are presented in
Figure 9. The predicted pressure over the suction side of the slat is slightly lower than the measured data. The overall agreement of the predicted surface pressure with the

Figure 8. Wall Pressure Coefficient on 30P30N. $Re = 9 \times 10^6$, $AOA = 19^\circ$.

Figure 9. Wall Pressure Coefficient on 30P30N. $Re = 9 \times 10^6$, $AOA = 8^\circ$. 

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measurement is satisfactory for both cases, showing that the transitional model is capturing the outer "inviscid" flow fields.

The experimental measurements of the skin friction coefficient are available on only a few points on the upper surface of the main element. Figure 10 shows that the calculated and the measured skin friction coefficients over the main element for \(Re = 9 \times 10^6\) and \(AOA = 19^\circ\). The computed results agree very well with the available measured data. The calculated skin friction coefficient distributions were employed to evaluate the transition onset location, which will be discussed later.

![Skin Friction Coefficient on 30P30N. Re = 9x10^6, AOA = 19^\circ.](image)

**Figure 10.** Skin Friction Coefficient on 30P30N. \(Re = 9 \times 10^6\), \(AOA = 19^\circ\).

**Kinetic Energy Distribution**

The transition model uses the evolution of the internal peak of the calculated distribution kinetic energy and the variation of the disturbance in the outer stream to
characterize the development of a transitional flow. For a multi-element airfoil, such as the 30P30N airfoil, the wakes of the preceding elements convect downstream and further interact with the transitional flow over the succeeding elements. Figure 11 shows the calculated distributions of the disturbance kinetic energy, $k$, along the direction normal to the upper surface of the slat, main element, and flap, respectively, for $R_e = 9 \times 10^6$ and $AOA = 19^\circ$. The local high values in the outer stream represent the high disturbance energy associated with the slat and main wakes. The wakes generated by the preceding elements can also be clearly observed in Figure 12 that shows velocity magnitude contour on 30P30N for $R_e = 9 \times 10^6$ and $AOA = 19^\circ$. For this operating condition, the slat wake can be identified as far downstream as in the flap region with little diffusion. It can be argued that there are receptive mechanisms for the boundary layer on the main, or flap element to be perturbed by the high-intensity disturbances in the slat, or main wake. In this study, the effect of the wake generated by the preceding element(s) is taken into account by using the highest $k$ in the wake(s) as the $k_e$ in equation (2.13) for the succeeding element(s). Near the leading edge of the main element, where the surface normal intersects with the slat cove, $k_e$ can vary significantly due to the separated shear layer in the cove region. While not affecting the wall pressure, it causes low amplitude variation in the skin friction coefficient and greatly delays convergence locally. In this application, an average value of $k_e$ over the intersect region was used for both the main element and the flap.
Figure 11. Disturbance Kinetic Energy Profiles on 30P30N. \( R_e = 9 \times 10^6, AOA = 19^\circ \).

Figure 12. Velocity Magnitude Contour on 30P30N. \( R_e = 9 \times 10^6, AOA = 19^\circ \).
Transition Onset Location

The flow around the slat is complicated and is difficult to measure due to the physical size of the slat. The transition of the boundary layer from laminar to turbulent flow on the upper suction surface has long been recognized as being one of the critical flow features that are yet to be modeled correctly. In the experiment, the onset and the extent of a transition process were determined based on the combination of the skewness, flatness, and standard deviation of hot film data. In this study, the onset location of flow transition is assumed to occur at the minimum value of the calculated skin friction coefficient, $C_f$. A solution of the flow field was first obtained using the parent $k-\varepsilon$ turbulence model and then using the transition model. Comparison of the two skin coefficients, obtained by using the turbulence and the transition models, was used to help identify the minimum value of $C_f$ in the transition model calculation. Figure 13 shows the calculated $C_f$ over the slat for $Re = 9 \times 10^6$ and $AOA = 19^\circ$. The $C_f$ obtained by using the parent turbulence model is also included for comparison. The transition onset location on the slat is determined to be at $x/c = -0.087$ by examining the minimum value of $C_f$. It should be noted that unsteady vortex shedding may exist behind airfoil elements. In the present study the transition onset location was predicted using a converged steady-state solution and effects of unsteady vortex shedding on the transition onset prediction were not considered.
Case 1: $Re = 9 \times 10^6$

Figure 14 shows a comparison of the predicted and the measured transition onset on the suction side of the three elements at $AOA = 8^\circ, 10^\circ, 12^\circ, 16^\circ,$ and $19^\circ$.

Figure 13. Criterion to Determine Transition Onset. $Re = 9 \times 10^6, AOA = 19^\circ$.

Figure 14. Transition Onset on the Three Elements. $Re = 9 \times 10^6$. 

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The variations of the transition onset locations with $AOA$ on the suction surface of the individual element are shown in Figures 15, 16, and 17. The results by

![Diagram](image1)

Figure 15. Slat Transition Onset. $R_e = 9 \times 10^6$.

![Diagram](image2)

Figure 16. Main Transition Onset. $R_e = 9 \times 10^6$.  

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Figure 17. Flap Transition Onset. \( Re = 9 \times 10^6 \).

Czerwiec et al. (1999) were included for comparison. Figure 15 shows the slat results. The range of \( x/c \) shown corresponds to the slat chord length. The measured data indicated a slight change of transition location toward the leading edge of the slat as the AOA increases. It is apparent that the present model has correctly predicted this trend. Moreover, the predicted transition locations also agree well with the measured data. A large difference between the prediction (Czerwiec et al. 1999) and the measurements was found at 8°. Figures 16 and 17 shows comparison between the predicted and the measured transition locations for the main element and flap, respectively. Similar to Figure 15, the range of \( x/c \) shown correspond to their respective chord lengths. The predicted locations of transition onset on the main element and flap move slightly with the change of AOA, which agrees with the
measurement. The predicted onsets occur slightly upstream of the measured locations for both the main element and flap.

Another view of the transition locations is given in Figure 18, which shows the locations of the predicted and measured transition onset on the 30P30N airfoil for $\text{AOA} = 19^\circ$.

![Figure 18. Airfoil Transition Onset. $R_e = 9 \times 10^6$, $\text{AOA} = 19^\circ$.](image)

- $O$: Experiment, $\Delta$: Present.

**Case 2: $R_e = 5 \times 10^6$**

Figure 19 shows a comparison of the predicted and the measured transition onset locations on the suction side of the three elements for $R_e = 5 \times 10^6$ and $\text{AOA} = 8^\circ, 10^\circ, 12^\circ, 16^\circ$, and $19^\circ$. For this lower Reynolds number, the present model also predicts onset locations that are in general agreement with the measured data for all three elements.

Figures 20, 21, and 22 show the variation of the transition onset location with $\text{AOA}$ for the slat, main element, and flap, respectively. The range of $x/c$ shown corresponds to their respective chord lengths. For the slat results shown in Figure 20,
the present model predicts reasonably well the transition locations on the suction side except for the lowest $AOA$ of $8^\circ$. As shown in Figure 21, the main element transition
onset locations were more under-predicted than those for $Re = 9 \times 10^6$. The prediction deviation from the measured data was also found for the flap transition onset location shown in Figure 22.

Figure 21. Main Transition Onset. $Re = 5 \times 10^6$.

Figure 22. Flap Transition Onset. $Re = 5 \times 10^6$. 

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Note that the present transition model predicts the transition location, rather than using it as an input. Through the ranges of angle of attack and Reynolds number tested, the present model has predicted, in general, satisfactorily the trend of change of the transition location with angle of attack on all three elements of the 30P30N geometry. The absolute locations of transition onset were predicted well for the slat. The only exception occurs at $\text{AOA} = 8^\circ$ for the case of $R_e = 5\times10^6$. In contrast, the model did not predict the absolute transition onset locations for the main element and flap very well. This may be caused by the presence of the confluent wake/boundary layer flow over the main element and the flap.

**Velocity Profiles**

**Angle of Attack = 19°**

Figures 23-28 show the velocity profiles at different survey stations on the main element and on the flap for $R_e = 9\times10^6$ and $\text{AOA} = 19^\circ$. At $x/c = 0.1075$ (as shown in Figure 23), there is almost a uniform offset between the measured data and the predictions. This may have been caused by possible improper data calibration (Rumsey et al. 1998). The predicted slat wake is located near the measured wake immediately behind the slat trailing edge. The calculated values for the wake deficit agree reasonably well with the data at $x/c = 0.45$ (see Figure 24) and 0.85 (see Figure 25). The measured wake is wider at $x/c = 0.45$. The merging of the calculated slat wake and the main boundary layer is apparently yet to occur at $x/c = 0.85$. 

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Figure 23. Velocity Profile. \( x/c = 0.1075, \ R_e = 9\times10^6, \ \text{AOA} = 19^\circ \).

The boundary layer velocity profiles are predicted well by the present transition model, which results in a better prediction of the main wake immediately behind the main element trailing edge (see Figure 26).

Figure 24. Velocity Profile. \( x/c = 0.45, \ R_e = 9\times10^6, \ \text{AOA} = 19^\circ \).
Over the flap (Figures 27 and 28, $x/c = 1.0321$ and $x/c = 1.1125$), the predicted development of the slat wake is satisfactory. The predicted deficit of the
main element wake is higher than the measured values. The width of the calculated main element wake agrees reasonably well with the data.

Figure 27. Velocity Profile. $x/c = 1.0321, \, R_e = 9 \times 10^6, AOA = 19^\circ$.

Figure 28. Velocity Profile. $x/c = 1.1125, \, R_e = 9 \times 10^6, AOA = 19^\circ$. 

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Angle of Attack = 8°

Figures 29, 30, and 31 show the velocity profiles at different survey stations on the main element for $R_e = 9 \times 10^6$ and $AOA = 8^\circ$. The predicted main element surface boundary layer is more developed than what the data has shown at $x/c = 0.1075$ (see Figure 29). There is a good agreement between the measurement and the calculated velocity profiles at $x/c = 0.45$ (see Figure 30) and $0.85$ (see Figure 31), where both the prediction and the measurement show a confluent wake/boundary layer.

![Figure 29. Velocity Profile. $x/c = 0.1075$, $R_e = 9 \times 10^6$, $AOA = 8^\circ$.](image)

Overall, the predicted velocity profiles agree reasonably well with the measurement. One of the biggest differences was observed at the station of $x/c = 0.1075$ where the wake defect was predicted larger than the measurement.
Figure 30. Velocity Profile. $x/c = 0.45$, $Re = 9 \times 10^6$, $AOA = 8^\circ$.

Figure 31. Velocity Profile. $x/c = 0.85$, $Re = 9 \times 10^6$, $AOA = 8^\circ$. 

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Summary

The transitional flows over a two-dimensional multi-element airfoil have been computed using a predictive transition model. The model couples an intermittency correction function to a parent $k-e$ two-equation turbulence model through an effective eddy-viscosity for the transitional flow. Comparisons with available measured data for two Reynolds numbers and a range of angles of attack were made. These include the surface pressure, skin friction, and velocity profile. The predicted transition onset locations on all three elements were also compared with the experimental data. The comparisons show that the present transition model predicts the transition onset very well for the slat with the only exception for $AOA = 8^\circ$ at lower Reynolds number. The predictions for the main element and the flap, e.g., the velocity profiles show a significant difference in the development of the confluent wake/boundary layers. It appears that the possible interactions between the wakes and the boundary layer are not adequately modeled. A better understanding of the confluent wake/boundary layer flow is needed.

As discussed in Chapter I, the disturbances entering into the confluent wake/boundary layer are assumed small such that LST can be used. The flow model adopted will be a superposition of the Blasius boundary layer and a wake located over the boundary layer. In this flow model, three parameters, the velocity defect $a$, the wake height $h$, the wake width $w$, will be defined to mainly characterize the confluent wake/boundary layer flow. Without the presence of the wall curvature and the sweep,
hydrodynamic instabilities occurring in the 2D CWB flow could be the wake modes, the discrete viscous traveling wave mode, and the continuous mode. The unstable modes, including the wake modes and the discrete viscous traveling wave mode are of interest in the present study.

In the following chapter, the linear analysis of the confluent wake/boundary layer flow will be presented using LST, with focusing on the behaviors of the instabilities, including the boundary layer mode and wake modes as functions of the wake velocity defect, the distance between the wall and the wake, and disturbance frequencies.
CHAPTER III

LINEAR ANALYSIS OF THE CONFLUENT WAKE/BOUNDARY LAYER

This chapter is to deal with the spatial linear instability of confluent wake/boundary flows. The characteristics of small disturbances in the confluent near wake and boundary layer are investigated. The flow model considered consists of a wake located above the Blasius boundary layer. The base flow velocity profile adopted represents a reasonable approximation to the measured (Chin et al. 1993) and the calculated mean velocity profiles in the confluent near wake/boundary layer region over a multi-element airfoil described in Chapter II. The amplitudes of the disturbances are assumed small such that a linearized form of the Navier-Stokes equations can be used. A global numerical solution technique is applied in this study. This method is capable of predicting the entire eigenvalue spectrum, including the discrete and the continuous parts, which is important to the current studies. As will be shown in the results, multiple discrete modes associated with the boundary layer and wake flow can appear and a global approximation is more desirable a method to use than the traditional local shooting method. As a result, the study establishes a link, based on a linear analysis, between the modes for confluent wake and laminar boundary layer, which often proceeds with transitional and turbulent boundary layer in the flow over multi-element airfoils.
The Orr-Sommerfeld equation and the Squire equation will be derived in the following section. It is followed by a description of the base velocity profile, numerical scheme, results and discussion and summary.

Stability Equations

The two-dimensional incompressible confluent wake/boundary layer is described in the Cartesian coordinate system, \((x^*,y^*,z^*)\), representing the streamwise, the wall-normal and the spanwise directions, respectively. Let \(U^*(x^*,y^*,z^*), V^*(x^*,y^*,z^*)\) and \(W^*(x^*,y^*,z^*)\) be the base flow components in the \(x^*\), \(y^*\) and \(z^*\) directions. The pressure in the base flow is \(P^* = P^*(x^*,y^*,z^*)\). The instantaneous value of a flow variable, \(\phi^*\), may be decomposed into a base flow component and a fluctuation component that is a function of time and space. That is given by,

\[
\begin{align*}
\tilde{u}^*(x^*,y^*,z^*,t^*) &= u^*(x^*,y^*,z^*,t^*) + U^*(x^*,y^*,z^*) \\
\tilde{v}^*(x^*,y^*,z^*,t^*) &= v^*(x^*,y^*,z^*,t^*) + V^*(x^*,y^*,z^*) \\
\tilde{w}^*(x^*,y^*,z^*,t^*) &= w^*(x^*,y^*,z^*,t^*) + W^*(x^*,y^*,z^*) \\
\tilde{p}^*(x^*,y^*,z^*,t^*) &= p^*(x^*,y^*,z^*,t^*) + P^*(x^*,y^*,z^*)
\end{align*}
\]  

(3.1)

where \(u^*, v^*\) and \(w^*\) are the disturbance velocity components in the \(x^*\), \(y^*\) and \(z^*\) directions, respectively. Here, \(p^*\) denotes pressure.

The incompressible Navier-Stokes equations in terms of instantaneous variables are given by
\[ \frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho^*} \frac{\partial p^*}{\partial x^*} + \nabla^2 u^* \]  
(3.2)

\[ \frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho^*} \frac{\partial p^*}{\partial y^*} + \nabla^2 v^* \]  
(3.3)

\[ \frac{\partial w^*}{\partial t^*} + u^* \frac{\partial w^*}{\partial x^*} + v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho^*} \frac{\partial p^*}{\partial z^*} + \nabla^2 w^* \]  
(3.4)

\[ \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \]  
(3.5)

where \( \nabla^2 \) denotes the Laplacian operator, \( \partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \partial^2 / \partial z^2 \). The base flow itself satisfies the Navier-Stokes equations. They are

\[ \frac{U^*}{\partial x^*} + V^* \frac{\partial U^*}{\partial y^*} + W^* \frac{\partial U^*}{\partial z^*} = -\frac{1}{\rho^*} \frac{\partial p^*}{\partial x^*} + \nabla^2 U^* \]  
(3.6)

\[ \frac{U^*}{\partial x^*} + V^* \frac{\partial V^*}{\partial y^*} + W^* \frac{\partial V^*}{\partial z^*} = -\frac{1}{\rho^*} \frac{\partial p^*}{\partial y^*} + \nabla^2 V^* \]  
(3.7)

\[ \frac{U^*}{\partial x^*} + V^* \frac{\partial W^*}{\partial y^*} + W^* \frac{\partial W^*}{\partial z^*} = -\frac{1}{\rho^*} \frac{\partial p^*}{\partial z^*} + \nabla^2 W^* \]  
(3.8)

\[ \frac{\partial U^*}{\partial x^*} + \frac{\partial V^*}{\partial y^*} + \frac{\partial W^*}{\partial z^*} = 0 \]  
(3.9)

Substituting equation (3.1) into the equations (3.2)-(3.5), neglecting nonlinear terms, and combining the base flow equations (3.6)-(3.9) lead to

\[ \frac{\partial u^*}{\partial t^*} + U^* \frac{\partial u^*}{\partial x^*} + V^* \frac{\partial u^*}{\partial y^*} + W^* \frac{\partial u^*}{\partial z^*} + u^* \frac{\partial U^*}{\partial x^*} + v^* \frac{\partial U^*}{\partial y^*} + w^* \frac{\partial U^*}{\partial z^*} = \]

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\[ -\frac{1}{\rho} \frac{\partial \rho}{\partial x} + \nu^2 u \quad (3.10) \]
\[ \frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} + W \frac{\partial v}{\partial z} = \quad (3.11) \]
\[ \frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} + W \frac{\partial u}{\partial z} = \quad (3.12) \]
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3.13) \]

For simplicity, a parallel flow assumption, commonly used in hydrodynamic stability analyses, is applied. As a result, the streamwise base flow velocity, \( U^* \), depends only on \( y^* \) while the remaining two components, \( V^* \) and \( W^* \) are supposed to be zero elsewhere, or \( V^* = W^* = 0 \) (Schlichting 1979). Consequently, equations (3.10)-(3.12) are reduced to

\[ \frac{\partial u^*}{\partial t} + U^* \frac{\partial u^*}{\partial x} + V^* \frac{dU^*}{dy^*} + \frac{1}{\rho^*} \frac{\partial \rho^*}{\partial x} = \nu^2 u \quad (3.14) \]
\[ \frac{\partial v^*}{\partial t} + U^* \frac{\partial v^*}{\partial x} + \frac{1}{\rho^*} \frac{\partial \rho^*}{\partial y^*} = \nu^2 v \quad (3.15) \]
Differentiating equation (3.14) with respect to $y^*$ minus differentiating equation (3.15) with respect to $x^*$ leads to

$$\frac{\partial w^*}{\partial t^*} + U^* \frac{\partial w^*}{\partial x^*} + \frac{1}{\rho^*} \frac{\partial p^*}{\partial z^*} = \nu \nabla^2 w^* \quad (3.16)$$

Differentiating equation (3.15) with respect to $z^*$ minus differentiating equation (3.16) with respect to $y^*$ leads to

$$\nu \nabla^2 \left( \frac{\partial u^*}{\partial y^*} - \frac{\partial v^*}{\partial x^*} \right) = \frac{\partial}{\partial x^*} \left( \frac{\partial v^*}{\partial z^*} - \frac{\partial w^*}{\partial y^*} \right) + U^* \frac{\partial}{\partial x^*} \left( \frac{\partial u^*}{\partial y^*} - \frac{\partial v^*}{\partial x^*} \right) + v^* \frac{d^2 U^*}{dy^*} + \frac{d U^*}{dx^*} \frac{\partial v^*}{\partial y^*} \quad (3.17)$$

Differentiating equation (3.15) with respect to $z^*$ minus differentiating equation (3.16) with respect to $y^*$ leads to

$$\nu \nabla^2 \left( \frac{\partial v^*}{\partial y^*} - \frac{\partial w^*}{\partial z^*} \right) = \frac{\partial}{\partial y^*} \left( \frac{\partial v^*}{\partial z^*} - \frac{\partial w^*}{\partial y^*} \right) \quad (3.18)$$

Differentiating the continuity equation (3.13) with respect to $y^*$ implies

$$\frac{\partial}{\partial x^*} \left( \frac{\partial u^*}{\partial y^*} - \frac{\partial v^*}{\partial x^*} \right) = \left( \frac{\partial^2 v^*}{\partial x^*} + \frac{\partial^2 v^*}{\partial y^*} + \frac{\partial^2 w^*}{\partial z^*} \right) \quad (3.19)$$

By differentiating equation (3.17) with respect to $x^*$ minus differentiating equation (3.18) with respect to $z^*$ and combining equation (3.19), one may have

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For three-dimensional disturbances, the equation for normal vorticity, $\Omega^*$, referred to as the Squire equation, is required in addition to equation (3.20). The pressure-derivative terms in equations (3.14) and (3.16) are eliminated by differentiating equation (3.14) with respect to $z^*$ minus differentiating equation (3.16) with respect to $x^*$. As a consequence, the Squire equation can be obtained by defining a variable, normal vorticity $\Omega^* = \partial u^*/\partial z^* - \partial w^*/\partial x^*$. That is

$$
\left( \nu \nabla^2 - U^* \frac{\partial}{\partial x^*} - \frac{\partial}{\partial t^*} \right) \Omega^* - \frac{dU^*}{dy^*} \frac{\partial v^*}{\partial z^*} = 0
$$

(3.22)

with boundary conditions

$$
\Omega^* = 0 \text{ at } y^* = 0 \text{ and } \Omega^* \to 0 \text{ as } y^* \to \infty
$$

(3.23)

When disturbances are assumed to be traveling waves, separable solutions in a normal mode form are then obtained by

$$
\begin{bmatrix} v^* \\ \Omega^* \end{bmatrix} = \begin{bmatrix} \hat{v}^*(y^*) \\ \hat{\Omega}^*(y^*) \end{bmatrix} \exp[i(\alpha^* x^* + \beta^* z^* - \omega^* t^*)] + \text{complex conjugate}
$$

(3.24)
where \( \{ \psi^*(y^*), \Omega^*(y^*) \} \) are the complex eigenfunctions, \( \omega^* \) is the frequency, \( \beta^* \) is the spanwise wave number, \( \alpha^* \) is the streamwise wave number.

Substituting equation (3.24) into equation (3.20) to (3.23) yields the Orr-Sommerfeld equation and the Squire equation that are, respectively, given by

\[
\frac{d^4 \psi}{dy^4} + \left[ -2(\alpha^2 + \beta^2) - i R_e^* (\alpha U - \omega) \right] \frac{d^2 \psi}{dy^2} + \left[ (\alpha^2 + \beta^2)^2 + i R_e^* (\alpha^2 + \beta^2)(\alpha U - \omega) + i R_e^* \frac{d^2 U}{dy^2} \right] \psi = 0 \quad (3.25)
\]

\[
\frac{d^2 \hat{\Omega}}{dy^2} + \left[ -2(\alpha^2 + \beta^2) - i R_e^* (\alpha U - \omega) \right] \hat{\Omega} + \left[ -i R_e^* \beta \frac{dU}{dy} \right] \hat{\psi} = 0 \quad (3.26)
\]

with boundary conditions

\[
\hat{\psi}, \quad \frac{d\hat{\psi}}{dy}, \quad \hat{\Omega} = 0 \quad \text{at} \quad y = 0 \quad \text{and} \quad \hat{\psi}, \quad \frac{d\hat{\psi}}{dy}, \quad \hat{\Omega} \to 0 \quad \text{as} \quad y \to \infty \quad (3.27)
\]

Note that equations (3.25)-(3.27) has been nondimensionalized using the freestream velocity, \( u_\infty^* \) and the local displacement thickness, \( \delta^* \). Reynolds number, \( R_e^* \), is then defined by \( u_\infty^* \delta^*/\nu \). The same reference length for nondimensionalism was reported in Jordinson (1970). Schlichting (1979) used the boundary layer thickness for the reference length. Other nondimensional relations are: \( \alpha = \alpha^* \delta^*, \beta = \beta^* \delta^* \) and \( \omega = \omega^* \delta^*/u_\infty^* \).
For a temporal stability, \( \alpha \) is real, while \( \omega \) is complex, \( \omega = \omega_r + i\omega_i \). In this way, the stability analysis provides a dispersion relation that determines \( \omega \) for a given \( \alpha \). For \( \omega_i > 0 \), the disturbance grows exponentially in time. In the case of a spatial stability, \( \omega \) is constrained to be real, while \( \alpha \) is considered complex, \( \alpha = \alpha_r + i\alpha_i \). The imaginary part of \( \alpha \) is a measure of the growth of the disturbance in the streamwise direction and is negative for spatially growing waves. The streamwise wavelength is defined by \( \lambda_x = 2\pi/\alpha_r \). The transformation between the temporal stabilities and the spatial stabilities have been established and verified (Nayfeh and Padhye 1979) for the Blasius flow as well as three-dimensional incompressible flows past sweptback wings. The spatial stability of the confluent wake/boundary layer flow is of interest in the present study.

Base Flow Velocity Profiles

The flow model depicted in Figure 32 consists of two separate regions where flow shearing can occur. They are: the region near the wall where the boundary layer develops, and the region away from the wall where the wake generated by an upstream element is located. The Blasius velocity profile is used for the boundary layer. The wake profile is defined by

\[
U = 1.0 - a \exp[-0.5(y-h)^2]
\]  

(3.28)
where $a$ represents the wake velocity defect and $h$ the wake height, or the distance between the wake and the wall. Equation (3.28) closely mimics measured wake velocity profile (Mattingly and Criminale 1972) and is a reasonable analytical expression to use in the current study. The base flow velocity is obtained by a superposition of the Blasius velocity profile and equation (3.28). A similar superposition of the velocity profiles has also been used in the convective stability analysis (Bajwa 1995) of a wake embedded in the Blasius boundary layer. Care has been taken to ensure that no discontinuities exist in the second derivative of the resulting velocity distribution, $d^2U/dy^2$.

Figure 32. Flow Model.

Equations (3.25) – (3.28) define an eigenvalue problem. Solutions are sought as the disturbances develop spatially downstream.
Numerical Methods

Discretization

The equations (3.25) – (3.27) were integrated numerically using the
Chebyshev collocation method (Canuto et al. 1988) and a sixth-order accurate finite
difference formulae.

Chebyshev Collocation Method

The Chebyshev polynomials have been shown to be very effective in the
numerical approximation of non-periodic boundary layer value problems. They are
cosine functions after a change of independent variable and given by (Canuto et al.
1988)

\[ T_k(\tilde{y}) = \cos(k \cos^{-1}(\tilde{y})) \]  

(3.29)

where \( \tilde{y} \) denotes the Chebyshev-Gauss-Lobatto points on the interval \([-1, 1]\) and is
defined by

\[ \tilde{y}_k = \cos\left(\frac{\pi k}{N}\right), \quad k = 0, 1, 2, \ldots, N \]  

(3.30)

where \( N \) is the number of the Chebyshev-domain intervals. When a function \( f(\tilde{y}) \) is
represented by the Chebyshev polynomials at the Gauss-Lobatto points, it may take
the form of
\[ f(\bar{y}) = \sum_{k=0}^{N} \hat{f}_k T_k(\bar{y}) \]  

(3.31)

where \( \hat{f}_k \) denotes the polynomial coefficients. When the function is considered an unknown discretized variable, \( \hat{f}_k \) corresponds to the function values on the discretization points. Derivatives of the function at the collocation points can be represented in matrix form as

\[ \frac{d f(\bar{y}_k)}{d \bar{y}} = \bar{D}_{i,j} \hat{f}_k \]  

(3.32)

where repeated indices indicate summation. The entries \( \bar{D}_{i,j} \) is given by

\[ \bar{D}_{i,j} = \begin{cases} 
\frac{c_i(-1)^{i+j}}{c_j \bar{y}_i - \bar{y}_j} & i \neq j \\
\frac{-\bar{y}_j}{2(1 - \bar{y}_j^2)} & j = 1, 2, \ldots, N-1 \\
\frac{2N^2 + 1}{6} & i = j = 0 \\
\frac{-2N^2 + 1}{6} & i = j = N
\end{cases} \]  

(3.33)

where \( c_i \) and \( c_j = 1 \) for \( i, j, \ldots, N-1 \) and \( c_0 = c_N = 2 \). Higher order derivatives are simply multiple powers of \( \bar{D} \), or

\[ \bar{D}_p = \bar{D}^p \]  

(3.34)
where \( p \) is the derivative order.

A transformation (Joslin et al. 1992) between the Chebyshev domain \([-1, 1]\) and the physical domain \([0, y_{\text{max}}]\) was used here. That is

\[
y = \frac{y_{\text{max}} S_p (1 + \bar{y})}{2S_p + y_{\text{max}} (1 - \bar{y})} \quad (3.35)
\]

where \( y \in [0, y_{\text{max}}], \quad \bar{y} \in [-1, 1], \quad y_{\text{max}} \) is the normal distance from the wall to the far-field boundary, \( S_p \) is the grid stretching parameter applied in the wall-normal direction. It controls the grid stretching in the direction normal to the wall. The typical range for \( S_p \) is from 6 to 12. As a result of the grid stretching, the derivative matrix in the Chebyshev domain is transformed into the physical domain via the following expressions:

\[
D = m\overline{D} \\
D^2 = m^2 \overline{D}^2 + mm'\overline{D} \\
D^3 = m^3 \overline{D}^3 + 3m^2 m' \overline{D}^2 + (m(m')^2 + m^2 m')\overline{D} \\
D^4 = m^4 \overline{D}^4 + 6m^3 m' \overline{D}^3 + (4m^3 m'' + 7m^2 (m')^2)\overline{D}^2 + (m^3 m''' + 4m^2 m'' m' + m(m')^3)\overline{D} 
\]

where

\[
m = \frac{d\bar{y}}{dy} = \frac{1}{d\bar{y}} , \quad m' = \frac{dm}{d\bar{y}} , \quad m'' = \frac{d^2 m}{d\bar{y}^2} , \quad m''' = \frac{d^3 m}{d\bar{y}^3} \quad (3.37)
\]
Equations (3.32)-(3.37) were used to evaluate the various order derivatives in the Orr-Sommerfeld equation (3.25) and the Squire equation (3.26). The discretized forms of equations (3.25) and (3.26) can then be obtained.

**Finite Differencing Method**

A sixth-order accurate finite differencing scheme was applied to discretize the equations (3.25) and (3.26). The differencing formulae are easily derived using the Taylor series expansion and are given in Appendix A. Here a transformation grid stretching is used.

\[ y_i = y_1 + \theta(y_{\text{max}} - y_1) \]  

(3.38)

where

\[ \theta = \frac{\eta}{\xi} + \frac{1}{\phi \xi} \tanh \left[ \left( \xi \frac{i-1}{n-1} - \eta \right) (0.5 \ln \frac{1+\phi}{1-\phi}) \right] \]

\[ \xi = 1, \quad \eta = 1 \quad \text{for clustering near } y_1 \]

\[ \xi = 1, \quad \eta = 0 \quad \text{for clustering near } y_{\text{max}} \]

\[ \xi = 2, \quad \eta = 1 \quad \text{for clustering near } y_1 \text{ and } y_{\text{max}} \]

\( \phi \) is a clustering coefficient that ranges between 0 and 1. The higher value of \( \phi \), the more grid node density near the point at which to cluster. The grid node distribution in the physical domain allows for high node density in the high shear regions near the wall and in the wake.
The discretization of equations (3.25) and (3.26) using the Chebyshev and the finite difference methods results in a system of homogeneous equations nonlinear in the parameter, $\alpha$.

\[ D_4(\alpha)\{\hat{\nu},\hat{\Omega}\}^T = 0 \quad (3.39) \]

The matrix, $D_4(\alpha)$, is a lambda matrix of degree four (Lancaster, 1966) and can be expressed as a scalar polynomial with matrix coefficients,

\[ D_4(\alpha) = C_0\alpha^4 + C_1\alpha^3 + C_2\alpha^2 + C_3\alpha + C_4 \quad (3.40) \]

**The Global Method**

Note that the boundary conditions are independent of the streamwise wave number, $\alpha$, and therefore, are imposed only in the matrix $C_4$ and the corresponding rows of the remaining matrices are set to zero. This implementation creates a singularity of matrices $C_i, i = 0, 1, 2, 3$ and thus causing infinite eigenvalues. In order to make the leading matrix $C_0$ nonsingular, the following transformation is used (Bridges and Morris 1983).

\[ \lambda = \frac{(1 + \alpha)}{(1 - \alpha)} \quad (3.41) \]

Substituting equation (3.41) into equation (3.40) leads to

\[ \tilde{D}_4(\lambda) = \tilde{C}_0\lambda^4 + \tilde{C}_1\lambda^3 + \tilde{C}_2\lambda^2 + \tilde{C}_3\lambda + \tilde{C}_4 \quad (3.42) \]
where

\[ \tilde{C}_0 = C_0 + C_1 + C_2 + C_3 + C_4, \]
\[ \tilde{C}_1 = -4C_0 - 2C_1 + 2C_3 + 4C_4, \]
\[ \tilde{C}_2 = 6C_0 - 2C_2 + 6C_4, \]
\[ \tilde{C}_3 = -4C_0 + 2C_1 - 2C_3 + 4C_4, \]
\[ \tilde{C}_4 = C_0 - C_1 + C_2 - C_3 + C_4 \]  

The matrices \( \tilde{C}_i \) are square matrices of order \( 2N \) in which \( N \) represents the number of grid points in \( y \). A linear companion matrix method has been used to linearize the lambda matrix. The resulting general eigenvalue problem becomes

\[
\begin{bmatrix}
-\tilde{C}_1 & -\tilde{C}_2 & -\tilde{C}_3 & -\tilde{C}_4 \\
I & 0 & 0 & 0 \\
0 & I & 0 & 0 \\
0 & 0 & I & 0 \\
\end{bmatrix}
- \lambda
\begin{bmatrix}
\tilde{C}_0 & 0 & 0 & 0 \\
0 & I & 0 & 0 \\
0 & 0 & I & 0 \\
0 & 0 & 0 & I \\
\end{bmatrix}
\begin{bmatrix}
\lambda^3 \psi \\
\lambda^2 \psi \\
\lambda \psi \\
\psi \\
\end{bmatrix} = 0
\]  

(3.44)

where \( \psi \) represents \( \{\hat{v}, \hat{\Omega}\}^T \). Equation (3.44) can be further transformed to an algebraic eigenvalue problem seeking the eigenvalues of matrix \( A \),

\[
A = \begin{bmatrix}
-\tilde{C}_0^{-1} \tilde{C}_1 & -\tilde{C}_0^{-1} \tilde{C}_2 & \tilde{C}_0^{-1} \tilde{C}_3 & \tilde{C}_0^{-1} \tilde{C}_4 \\
I & 0 & 0 & 0 \\
0 & I & 0 & 0 \\
0 & 0 & I & 0 \\
\end{bmatrix}
\]  

(3.45)

where the matrix \( A \) has an order of \( 8N \) and \( I \) is the identity matrix of order \( 2N \).

The eigenvalues may be obtained by using \( QR \) or \( QZ \) algorithms.
The Local Method

The local method (Bridges and Morris 1983) is more efficient than the global method. But it requires sufficiently accurate initial estimate of the eigenvalue, which may be obtained from the global method. The iterative formula is given by

$$\alpha_{k+1} = \alpha_k - \frac{2f(\alpha_k)}{[f(\alpha_k)]^2 - f^{(1)}(\alpha_k)}, \quad k = 0, 1, 2, \ldots$$

(3.46)

where

$$f(\alpha_k) = T_r \left\{ D_4^{-1}(\alpha_k) D_4^{(1)}(\alpha_k) \right\}$$

(3.47)

$$f^{(1)}(\alpha_k) = T_r \left\{ D_4^{(-1)}(\alpha_k) D_4^{(2)}(\alpha_k) - [D_4^{(-1)}(\alpha_k) D_4^{(1)}]^2 \right\}$$

(3.48)

$T_r(A)$ denotes the trace of matrix $A$, $D_4^{-1}$ is the inverse of $D_4$, and $D_4^{(J)}$ denotes the $j$th derivative of $D_4$ with respect to $\alpha$. To compute eigenvectors the inverse iteration (Bridges and Morris 1983) is used

$$D_4(\alpha) \begin{bmatrix} \hat{\phi}^{k+1} \\ \hat{\Omega}^{k+1} \end{bmatrix} = \sigma \begin{bmatrix} \hat{\phi}^{k} \\ \hat{\Omega}^{k} \end{bmatrix}$$

(3.49)

where $\sigma$ is a normalizing factor. This procedure is very effective and convergence is usually obtained in two to three iterations using an initial guess, $\{\phi^0, \hat{\Omega}^0\}^T = \sigma \{1, 1, \ldots, 1\}^T$. 

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Results and Discussion

Code Validation

A global numerical solver has been developed. The code validation has been done via examining the Blasius boundary layer flow and the wake flow, respectively. The present numerical solver with the Chebyshev method was first validated by calculating the eigenvalue for a two-dimensional disturbance with Reynolds number $R_e^*$ of 998 and frequency $\omega$ of 0.1122. Jordinson (1970) found the eigenvalue of $0.3086 - i0.0057$. The present global solver gives the value of $0.3085049 - i0.0058112$ for grid node number $N = 36$, $S_p = 10$, $y_{max} = 75$. With this initial value, the present local solver gives the converged results shown in Table 1. The present converged value in Table 1 agrees well with that of Jordinson (1970).

For three-dimensional disturbance, the eigenvalue of $0.19861 - i0.00420$ for $R_e^* = 1000$, $\omega = 0.07$ and $\beta = 0.12$ was found by Bridges and Morris (1987). The present global solver predicted the eigenvalue of $0.19860 - i0.00419$ with grid node number of 41 and $S_p = 10$.

The validations for the present global solver with the finite differencing method are shown in Table 2, Figure 33, 34 and 35. Table 2 shows a comparison of the calculated eigenvalue with that of Jordinson (1970) for the Blasius boundary layer. The present values agree well with the data, particularly for cases with a high number of grid points. In addition, good agreement between the present eigenfunctions and
Jordinson's eigenfunctions is shown in Figure 33. The eigenfunctions were obtained using the local method.

Table 1

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>0.3085049-i0.0058112</td>
</tr>
<tr>
<td>41</td>
<td>0.3086042-i0.0056938</td>
</tr>
<tr>
<td>51</td>
<td>0.3085902-i0.0057057</td>
</tr>
<tr>
<td>61</td>
<td>0.3085909-i0.0057083</td>
</tr>
<tr>
<td>71</td>
<td>0.3085912-i0.0057082</td>
</tr>
<tr>
<td>81</td>
<td>0.3085911-i0.0057083</td>
</tr>
<tr>
<td>91</td>
<td>0.3085911-i0.0057083</td>
</tr>
</tbody>
</table>

To validate the current numerical solver for the wake flow, a measured wake profile (Mattingly and Criminale 1972) is placed at \( h = 20 \) and the boundary layer removed from the viscous wall. Figures 34 and 35 show a comparison of the dispersion relationship obtained by using the present numerical solver and that reported (Mattingly and Criminale 1972) for both the symmetric and the
antisymmetric modes, based on the cross-stream velocity disturbance, \( \hat{v} \). The results obtained by using the current global method with \( N = 300 \) agree well with the previous calculation, which is based on a local shooting method.

Table 2

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \phi )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>0.998</td>
<td>0.308444 - i0.005965</td>
</tr>
<tr>
<td>41</td>
<td>0.998</td>
<td>0.308599 - i0.005662</td>
</tr>
<tr>
<td>51</td>
<td>0.998</td>
<td>0.308607 - i0.005698</td>
</tr>
<tr>
<td>61</td>
<td>0.998</td>
<td>0.308597 - i0.005707</td>
</tr>
<tr>
<td>71</td>
<td>0.998</td>
<td>0.308592 - i0.005707</td>
</tr>
<tr>
<td>81</td>
<td>0.997</td>
<td>0.308591 - i0.005704</td>
</tr>
<tr>
<td>91</td>
<td>0.994</td>
<td>0.308590 - i0.005704</td>
</tr>
</tbody>
</table>

The results for the boundary layer flow and the wake flow show that the global numerical solver used in the current study is accurately capturing the boundary layer and the wake instabilities. In the following, the instability modes associated with the model confluent wake/boundary layer flow are examined for \( \beta = 0 \).
Figure 33. Eigenfunctions: $R_e = 998, \omega = 0.1122, \alpha = 0.3086 - i0.0057, \beta = 0$.

Figure 34. Dispersion Relation for Symmetric Wake Mode.
Figure 35. Dispersion Relation for Antisymmetric Wake Mode.

Mode Identification

The flow model used in the present analysis is shown in Figure 32. Recall Figures 34 and 35 where the velocity defect is 0.56 (Mattingly and Criminale 1972). For this velocity defect, the present calculated eigenvalues show that there is no difference with or without the presence of the wall as long as the wake is sufficiently away from the wall, such as $h = 20$. Figure 36 shows the wave-speed spectrum for the model confluent wake/boundary layer flow with $Re^* = 998$ and $\omega = 0.1122$. Here $C_r$ and $C_i$ denote the real and the imaginary part of the wave speed, respectively. The wake height, $h$, is 20, the width is 4.83, and the wake defect, $a$, is 0.6. For comparison, the wave speed spectrum for the Blasius boundary layer alone with the same $Re^*$ and $\omega$ is also included. In addition to the boundary layer mode, two discrete
unstable modes appear in the case of the confluent wake/boundary layer, which are identified as Wake Mode 1 and Wake Mode 2. They correspond to the symmetric and the antisymmetric modes found in the linear stability of the wake flows, respectively. The presence of the wake above the boundary layer at this height apparently has no significant effects on the continuous part of the eigenvalue spectrum. In the following, the discrete unstable modes in the confluent wake/boundary layer will be examined in details as they are linearly unstable and are likely to be physically observed.

![Eigenvalue Spectrum](image)

**Figure 36. Eigenvalue Spectrum: h = 20, \( \omega = 0.1122, \alpha = 0.6 \).**

**Boundary Layer Mode**

Figure 37 shows the neutral curve of the boundary layer mode with \( h = 7.14 \) and \( \alpha = 0.6 \). The neutral curve for the Blasius boundary layer (Jordinson 1970) is also included for comparison. The presence of the wake has resulted in a significant
decrease of the critical Reynolds number. The value is 445 for the present case, compared to 520 for the Blasius boundary layer. The neutral curve moves up slightly

![Graph showing neutral curves for boundary layer mode]

Figure 37. Neutral Curves for Boundary Layer Mode: $a = 0.6, h = 7.14$.

for all Reynolds numbers with more significant changes in the high frequency range near the critical point. As can be seen from Figure 38, which shows the variation of the spatial growth rate, $-\alpha_i$, with frequency for $R_e^* = 998$, the neutral frequencies increase as the wake is brought closer to the wall. Figure 38 also shows that the maximum growth rate of the unstable boundary layer mode increases with the reduced distance between the wake and the wall.

To examine the effect of $h$ on the boundary layer mode shape, the eigenfunctions for a frequency of $\omega = 0.1122$, which roughly corresponds to the most unstable frequency, are calculated and their real parts are presented in Figure 39. The
Figure 38. Growth Rates for Boundary Layer Mode: $R_e^* = 998$, $a = 0.6$.

Figure 39. Eigenfunctions for Boundary Layer Mode: $a = 0.6$. 

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velocity defect is taken as 0.6. The eigenfunctions have been normalized by their respective maximum amplitudes. The eigenfunction of the boundary layer mode decays faster away from the wall as the wakes become situated closer to the wall. The damping effect of the wake on the eigenfunction of the boundary layer mode is quite significant in the region above the wake. The velocity defect of the wake in the model problem is determined by the value of $a$, which also determines the shear strength for a constant wake width.

The effects of the velocity defect of the wake on the eigenfunctions are shown in Figure 40 where the wake is located very close to the wall. The increase of the velocity defect tends to result in the rapid decay of disturbances in the wake region but has little effect on the region very near the wall.

![Figure 40. Eigenfunctions for Boundary Layer Mode: $h = 7.14$.](image)
Figure 41 shows the change in the growth rate of the boundary layer mode for $R_e^* = 998$ and $\omega = 0.1122$ with the wake velocity defect and wake height. The growth rate for such a mode in the Blasius boundary layer, $-0.0057$ (Jordinson 1970), is indicated in the Figure. We notice in Figure 41 that the amplifying effect of the reduced wake height on the growth rate of the boundary layer mode, as is discussed above, becomes more pronounced as the velocity defect of the wake increases. For example, when the value of $h$ is about 7, there is an increase of 400% for the growth rate for $a = 0.8$, as compared to about 40% for $a = 0.2$. For the given Reynolds number and frequency, the damping effects of the wake on the boundary layer mode gradually vanishes when the wake height $h \geq 15$. It should be noted that the lowest value of the centerline velocity defect for a Gaussian wake profile to exhibit absolute instability is 0.946 (Hultgen and Aggarwal 1987).

Figure 41. Growth Rates for Boundary Layer Mode: $R_e^* = 998$, $\omega = 0.1122$. 

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Wake Modes

In addition to the boundary layer mode, the eigenvalue spectrum for the confluent wake/boundary layer, shown in Figure 36, also contains two discrete, unstable modes that are directly associated with the wake. These modes have been identified as Wake Modes 1 and 2. As is discussed above, the Mode 1 disturbances are symmetric with respect to the wake center, where the velocity defect is the maximum, and Mode 2 represents antisymmetric disturbances. Figures 42 and 43 shows the variations of the spatial growth rates of Mode 1 and Mode 2 with wake height $h$ and velocity defect $a$. The Reynolds number is 998 and $\omega = 0.1122$. The growth rates for Mode 1 and Mode 2 are both seen to decrease as $h$ decreases. This appears true for the different value of the velocity defect selected, from 0.2 to 0.8, with more prominent decreases for the high-defect cases. Note that the boundary layer mode, as shown in Figure 41, becomes more unstable as the wake is placed closer to the wall. Therefore, the reduced distance between the wall and the wake has an amplifying effect on the boundary layer mode, but a damping effect on the wake modes. A composite view of the growth rates of the various modes, as a function of $a$ and $h$, is given in Figure 44. For a range of value of $a$, the wake modes are more unstable than the boundary layer mode at all wake height. The wake modes become less unstable than the boundary layer mode for cases where wakes with small velocity defect are placed close to the wall, i.e., cases with low values of $a$ and $h$. For cases with higher values of $a$, for example, 0.6 and 0.8, the spatial growth rates of the wake

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modes remain higher than that of the boundary layer mode even at the lowest wake height.

Figure 42. Effect of $h$ and $a$ on Wake Mode 1: $R_e = 998$, $\omega = 0.1122$.

Figure 43. Effect of $h$ and $a$ on Wake Mode 2: $R_e = 998$, $\omega = 0.1122$.  

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Figures 42, 43, and 44 suggest that for low velocity defect, Mode 1 is more unstable than the antisymmetric Mode 2 for all the wake height. The trend is then reversed when the velocity defect increases. In Figure 45 the effect of the wake velocity defect on the growth rate of Modes 1 and 2 are shown for $h = 20$. The growth rate of Mode 1 is higher than that of Mode 2 for the low values of $a$ and increases nearly linearly with $a$. The growth rate of Mode 2 is smaller than that of Mode 1 for the lower $a$, but increases rapidly as the value of $a$ increases. Mode 2 becomes more unstable than Mode 1 for $a$ higher than about 0.5. In other words, there is a switching of the relative level of the spatial growth rate between Mode 1 and Mode 2 based on the present confluent wake/boundary layer model. Calculations using the measured velocity profile (Mattingly and Criminale 1972) in the very near wake region where the velocity defect is high also show a similar unstable mode switching behavior.
between the symmetric and the antisymmetric modes. These results seem to indicate that the antisymmetric mode, which generates "puffing" disturbances, is possible to be observed in the near wake immediately downstream of the wake-generating body.

The unstable frequency spectra for Mode 1 and Mode 2 are examined and shown in Figure 46 for $R_\infty^* = 998$ and for $a = 0.4, 0.6, \text{ and } 0.8$. The unstable mode switching appears to have occurred only at low frequency range for $a = 0.6 \text{ and } 0.8$. The frequencies for the most unstable waves for both Modes 1 and 2 decrease slightly as $a$ increases. The growth rate for the most unstable Mode 1 is higher than that of Mode 2 for all of the values of $a$ tested. The wake mode spectrum for another wake height $h = 7.0$ is shown in Figure 45 where the Mode 1 exhibits a more unstable behavior than Mode 2.

![Figure 45. Unstable Mode Switching: $R_\infty^* = 998, \omega = 0.1122$.](image_url)
Figure 46. Growth Rates of Modes 1 and 2: $R^* = 998$, $h = 20$.

Figures 47 and 48 show the real part of the eigenfunctions of Modes 1 and 2 for $R^* = 998$, $\omega = 0.1122$, and $a = 0.4$ for $h = 7, 20, 40$. Figures 49 and 50 show the real part of the eigenfunctions of Modes 1 and 2 for $R^* = 998$, $\omega = 0.1122$, and $h = 0.4$ for $a = 0.2, 0.4, 0.6, 0.8$. The eigenfunctions were normalized by their respective highest amplitudes. It is apparent that the shapes of the eigenfunctions of the wake modes are not strongly dependent upon the wake height and the wake depth.

Summary

A linear spatial viscous instability analysis for incompressible confluent wake/boundary layer flows has been performed. The global numerical solution tool is
validated by comparisons with available data. Two types of discrete unstable modes are identified, which are rooted, respectively, to the boundary layer and the wake parts.

Figure 47. Eigenfunctions of Mode 1: $R_e^* = 998$, $\omega = 0.1122$, $a = 0.4$.

Figure 48. Eigenfunctions of Mode 2: $R_e^* = 998$, $\omega = 0.1122$, $a = 0.4$. 

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Figure 49. Eigenfunctions of Mode 1: $R_e^* = 998$, $\omega = 0.1122$, $h = 7.14$.

Figure 50. Eigenfunctions of Mode 2: $R_e^* = 998$, $\omega = 0.1122$, $h = 7.14$. 

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of the flow model. It is found that the critical Reynolds number associated with the boundary layer mode is reduced by the presence of the wake. The presence of the wake above the boundary layer appears to have an amplifying effect on the growth rate of the boundary layer mode. It is also shown that such an amplifying effect intensifies with an increase of the wake velocity defect. The mode shape of the boundary layer mode also seems to diminish above the wake, indicating that the wake has a confining influence on the disturbances associated with the boundary layer mode. The results suggest that the wake may play an important role in the growth of the linear disturbances and the initiation of transition in the boundary layer. The unstable modes associated with the wake are stabilized by the reduced wake height. An unstable mode switching of the symmetric and the antisymmetric modes is found to occur for low frequency wake modes for cases with high velocity defects, indicating a possible appearance of antisymmetric “puffing” disturbance immediately downstream of the wake-generating body.

When the confluent wake/boundary layer develops downstream, the initial small disturbances grow from a linear wave to a nonlinear breakdown before transition occurs. The flow in this process is complex. To further examine the flow mechanisms at work in confluent wake/boundary layers, direct numerical simulation (DNS) that solves the Navier-Stokes equations will be used in the next Chapter. Spatial DNS computes spatially evolving disturbances and can provide needed quantitative information about transition process. Initial conditions required by DNS can be obtained using the linear results that have been described in this Chapter.
CHAPTER IV

NUMERICAL SIMULATION OF THE CONFLUENT WAKE/BOUNDARY LAYER FLOW

This chapter presents a nonlinear analysis of the confluent wake/boundary layer using a DNS code (Joslin et al. 1992) that solves the incompressible Navier-Stokes equations expressed in terms of disturbance components. The flow model used is the same as one in the linear analysis described in Chapter III. The disturbances entering the confluent wake/boundary layer flow are assumed to be induced by the wake instabilities and the viscous traveling wave instabilities. The initial conditions are obtained by the linear results. The growth of the T-S waves in the 2-D parallel and nonparallel Blasius boundary layer are first examined by comparing with the LST results. The behaviors of the linearly developing waves in the confluent wake/boundary layer are examined by also comparing with the LST results. The simulations of the nonparallel confluent wake/boundary layer are performed for different forcing at the inflow boundary. The mean-flow profile is provided using a RANS code named ALLSPD (Chen and Choi 1991, Chen and Shuen 1994).

Governing Equations

The flow model studied in this paper is shown in Figure 51. The flow may be described by the time-dependent incompressible Navier-Stokes equations in the

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Cartesian coordinate system. Let \((x, y, z)\) represent the coordinates for the streamwise, the normal-wall, and the spanwise directions, respectively. The corresponding instantaneous velocities are \(\tilde{\mathbf{u}} = (\tilde{u}, \tilde{v}, \tilde{w})\) and the pressure is \(\tilde{p}\). The

\[
\begin{align*}
\tilde{u}_t + (\tilde{u} \cdot \nabla) \tilde{u} &= -\nabla \tilde{p} + \frac{1}{R^*} \nabla^2 \tilde{u} \\
\nabla \cdot \tilde{u} &= 0
\end{align*}
\]

Figure 51. Sketch of the Confluent Wake/Boundary Layer Flow.

momentum equations are given by

and the continuity equation by

where subscript \(t\) denote partial derivatives of velocities with respect to time. The equations are nondimensionalized by the free stream velocity \(u_\infty\) and the inlet displacement thickness \(\delta^*\). A Reynolds number can then be defined as \(R^*_e = u_\infty \delta^*/\nu\).
Let \( \vec{u} = (u, v, w) \) and \( p \) represent the disturbance velocity and pressure. The instantaneous velocity \( \vec{u}' = (\vec{u}, \vec{v}, \vec{w}) \) and pressure \( \bar{p} \) may then take the following form,

\[
\vec{u}(x, y, z, t) = \bar{U}(x, y, z) + \vec{u}(x, y, z, t), \quad \bar{p} = P(x, y, z) + p(x, y, z, t)
\]  

(4.3)

where \( \bar{U} = (U, V, W) \) and \( P \) are mean flow components that satisfy the Navier-Stokes equation. They are given by the momentum equations

\[
(\bar{U} \cdot \nabla) \bar{U} = -\nabla P + \frac{1}{Re} \nabla^2 \bar{U}
\]  

(4.4)

and the continuity equation

\[
\nabla \cdot \bar{U} = 0
\]  

(4.5)

Subtracting the mean-flow equations (4.4) and (4.5) from the Navier-Stokes equations (4.1) and (4.2) leads to an unsteady, nonlinear disturbance equations given by

\[
\vec{u}' + (\bar{u} \cdot \nabla) \vec{u} + (\bar{U} \cdot \nabla) \vec{u} + (\bar{u} \cdot \nabla) \bar{U} = -\nabla \bar{p} + \frac{1}{Re} \nabla^2 \vec{u}
\]  

(4.6)

and the continuity equation by

\[
\nabla \cdot \vec{u} = 0
\]  

(4.7)
The boundary conditions are given by requiring that the disturbances vanish at the far field.

\[
\bar{u} = 0 \text{ at } y = 0 \quad \text{and} \quad \bar{u} \rightarrow 0 \text{ as } y \rightarrow \infty \tag{4.8}
\]

Inflow conditions depend on the type of instability disturbances and outflow conditions require that all waves exit without reflections. They are described in a later section.

**Numerical Schemes**

The mean-flow velocity field has been obtained by using the ALLSPD code. Discretization was performed using the Chebyshev collocation method in the wall-normal direction, fourth-order finite differencing for the pressure equation, compact differencing for the momentum equations in the streamwise direction, and Fourier series in the spanwise direction. Time-splitting procedure was used with an implicit Crank-Nicolson differencing for the normal diffusion terms and a third-order low storage Runge-Kutta method for all the remaining terms. The initial and the inlet conditions were provided by the linear analysis of the confluent wake/boundary layer. At the outflow boundary, the buffer domain method was used to suppress possible wave reflections. The detailed numerical procedure used to solve the disturbance equations (4.6)-(4.8) can be found in Joslin et al. (1992). The code has been used for swept-wing transition (Joslin and Streett 1994, Joslin 1995a), attachment line instabilities (Joslin 1995b, 1996a, 1996b) as well as subharmonic and oblique wave
breakdowns (Joslin et al. 1993). A brief description of the numerical procedure is given here.

In the streamwise direction, fourth-order central finite differencing scheme was used for the pressure equation and fourth-order compact differences for the momentum equations. For interior nodes, fourth-order central difference approximations of the first and second derivatives \( f'_n \) and \( f''_n \) are given by

\[
\begin{align*}
  f'_n &= \frac{1}{12 h_x^2} (-f_{n-2} - 8 f_{n-1} + 8 f_{n+1} - f_{n+2}) + O(h_x^4) \\
  f''_n &= \frac{1}{12 h_x^2} (-f_{n-2} + 16 f_{n-1} - 30 f_n + 16 f_{n+1} - f_{n+2}) + O(h_x^4)
\end{align*}
\]

where \( h_x \) is the step size. Fourth-order forward and backward differences were used for boundary and near-boundary nodes. The compact scheme has an advantage to achieve higher accuracy without involving the use of additional neighboring nodes. The first and second derivatives for a fourth-order compact scheme are approximated by

\[
\begin{align*}
  f'_{n-1} + 4 f'_n + f'_{n+1} &= 3(f_{n+1} - f_{n-1})/h_x \\
  f''_{n-1} + 10 f''_n + f''_{n+1} &= 12(f_{n+1} - 2f_n + f_{n-1})/h_x^2
\end{align*}
\]

These equations yield tridiagonal systems that require boundary conditions obtained by one-sided fourth-order finite differences.
In the wall-normal direction, derivatives in equations (4.6) and (4.7) were evaluated by Chebyshev series described in the last Chapter.

In the spanwise direction, the flow is assumed to be periodic and symmetric about \( z = 0 \). The independent variables can then be expanded with cosine and sine series. They are

\[
\{u, v, p\} = \sum_{k=0}^{N_z} \{\hat{u}_k, \hat{v}_k, \hat{p}_k\} \cos(2\pi k \lambda_z z)
\]

\[
w = \sum_{k=0}^{N_z} \hat{w}_k \sin(2\pi k \lambda_z z)
\]

where \( \lambda_z \) is a specified spanwise wave number. Equations (4.13) and (4.14) permit computations on half the domain, which significantly saves computer sources.

For the time marching, the implicit Crank-Nicolson differencing was used for normal diffusion and an explicit third-order Runge-Kutta method for all remaining terms. For the fractional Runge-Kutta stage, an omission of the pressure term in equation (4.6) yields

\[
\frac{\bar{u}^+ - \bar{u}^m}{h_i^m} = C_1^n H^n (\bar{u}) + \frac{C_2^n}{R_e^*} D^2 (\bar{u}^+ + \bar{u}^m)
\]

where

\[
H^n (\bar{u}) = L^n (\bar{u}) + C_3^n H^{m-1} (\bar{u})
\]
\[ L(\bar{u}) = -[(\bar{U} \cdot \nabla)\bar{u} + (\bar{u} \cdot \nabla)\bar{U} + (\bar{u} \cdot \bar{U})\bar{u}] + \frac{1}{Re} \nabla^2 \bar{u} \quad (4.17) \]

where \( m \) denotes the number of Runge-Kutta stages, \( h \) time-step size, \( \bar{u}^+ \) velocities at the intermediate Runge-Kutta stages. For a full Runge-Kutta stage, the momentum equations with the pressure are

\[ \frac{\bar{u}^{m+1} - \bar{u}^m}{h_t} = C_1^m H^m(\bar{u}) + \frac{C_2^m}{Re} D^2(\bar{u}^{m+1} + \bar{u}^m) - \nabla p^{m+1} \quad (4.18) \]

Subtracting equation (4.15) from equation (4.18) leads to

\[ \frac{\bar{u}^{m+1} - \bar{u}^+}{h_t} = \frac{C_2^m}{Re} D^2(\bar{u}^{m+1} - \bar{u}^+) - \nabla p^{m+1} = \nabla \phi^{m+1} \quad (4.19) \]

where \( \phi \) is an introduced pressure-like quantity. The pressure-like equation can be obtained by taking the divergence of equation (4.19) and imposing zero divergence at each Runge-Kutta stage. That is

\[ \nabla^2 \phi^{m+1} = \frac{1}{h_t} (\nabla \cdot \bar{u}^+) \quad (4.20) \]

The intermediate Runge-Kutta velocities \( \bar{u}^+ \) are determined by solving equation (4.15). The pressure-like correction \( \phi^{m+1} \) is found by solving equation (4.20). Then, the full Runge-Kutta stage velocities \( \bar{u}^{m+1} \) are obtained by solving equation (4.19). The Runge-Kutta coefficients and time steps are given by
\[
\begin{bmatrix}
C^1_1 & C^1_2 & C^1_3 \\
C^2_1 & C^2_2 & C^2_3 \\
C^3_1 & C^3_2 & C^3_3
\end{bmatrix}
= \begin{bmatrix} 1 & 1/2 & 0 \\ 9/4 & 1/2 & -4 \\ 32/15 & 1/2 & -153/32 \end{bmatrix}
\] (4.21)

and

\[
\begin{bmatrix}
h^1_t \\ h^2_t \\ h^3_t
\end{bmatrix}
= \begin{bmatrix} (1/3)h_t \\ (5/12)h_t \\ (1/4)h_t \end{bmatrix}
\] (4.22)

The eigenvector-decomposition method and the influence-matrix method were used to solve the pressure-like equation (4.20). At the end of each full Runge-Kutta time step, a nonzero tangential velocity component may arise at the computational boundary since equation (4.20) is an inviscid calculation and is well posed. A slip-velocity correction is given by

\[
\hat{u}_r = \bar{u}_{BC} + h^m_t \left[ (1 + \frac{h^m_t}{h^{m-1}_t}) \nabla \varphi^m_t - \frac{h^m_t}{h^{m-1}_t} \nabla \varphi^{m-1}_t \right]
\] (4.23)

where $\bar{u}_{BC} = 0$ for a rigid wall and $\bar{u}_{BC} = \bar{u}_0$ for an inflow condition or for a wall slot condition evaluated at the appropriate time in the Runge-Kutta stage.

### Outflow Conditions

The buffer domain method (Streett and Macaraeg 1989) was used for outflow conditions. The ellipticity of the Navier-Stokes equations results from the viscous and
the pressure terms. To deal with the first source of upstream influence from the outflow boundary, the streamwise viscous terms are smoothly reduced to zero via multiplying by an appropriate function in a buffer domain, which is appended to the end of the computational domain of interest. Additionally, the convective velocity in the nonlinear advection terms, which in the domain of interest is the sum of mean flow and disturbance velocities, is modified in the buffer main to involve the mean flow velocity only at outflow. This is also done via a smooth coefficient function. The forcing function of the pressure-like equation (4.20) is also attenuated to zero at outflow in the buffer domain. The attenuation function $s_j$ is given by

$$s_j = \frac{1}{2} \left( 1 + \tanh \left( 4 \left[ 1 - 2 \frac{(j - N_b)}{(N_x - N_b)} \right] \right) \right)$$

(4.24)

where $N_b$ denotes the beginning of the buffer domain and $N_x$ the outflow boundary location. Generally, it takes about three streamwise wavelengths (Joslin et al. 1992) as a buffer-domain length to allow the disturbances to exit the outflow boundary without reflections. The spanwise direction is assumed to be infinite such that periodicity condition can be imposed in that direction.

Inflow Conditions

The inlet mean-flow model is taken as the superposition of a boundary layer and a wake located over the boundary layer. It is assumed to mimic the flow leaving the trailing edge of a symmetrical airfoil over a flat plate. For a given Reynolds
number, frequency and spanwise wave number, a superposition of the eigenfunctions
associated with the boundary layer mode and the two wake modes obtained from the
linear analysis, was taken as the disturbances at the inlet. That is,

\[ u(x = 0, y, z, t) = A_{BL} \text{Re}\{u_{BL}(y)\exp[i(\beta z - \omega_{BL}t)]\} + \]
\[ A_{W1} \text{Re}\{u_{W1}(y)\exp[i(\beta z - \omega_{W1}t)]\} + \]
\[ A_{W2} \text{Re}\{u_{W2}(y)\exp[i(\beta z - \omega_{W2}t)]\} \]  

(4.25)

where \( A_{BL} \), \( A_{W1} \), and \( A_{W2} \) are the amplitudes of the perturbation waves of the
boundary layer mode, the symmetric mode (Wake Mode 1), and the antisymmetric
mode (Wake Mode 2), respectively. The corresponding eigenfunctions are \( u_{BL} \), \( u_{W1} \),
and \( u_{W2} \), which were normalized by the maximum streamwise disturbance velocity.
\( \omega_{BL} \), \( \omega_{W1} \) and \( \omega_{W2} \) are the forcing frequencies for the boundary layer mode, wake
modes 1 and 2, respectively. \( \beta \) is the spanwise wave number.

Results and Discussion

Boundary Layer

As is mentioned earlier, the DNS code has been used for boundary layer
transition studies. For comparison with the confluent wake/boundary layer cases,
simulations have been performed for a parallel and a nonparallel Blasius boundary
layer with a 2-D disturbance imposed at the inflow boundary layer. The prescribed
waves at the inflow boundary were obtained from the global code described in
Chapter III for Reynolds number $R_e = 998$, frequency $\omega = 0.1122$ and spanwise wave number $\beta = 0$. The 2-D unstable mode has a streamwise wave number $\alpha = (0.3086 - i0.0057)$ and a disturbance amplitude of 0.1%. The spatial DNS was performed on a grid of $481 \times 101 \times 5$, denoting the streamwise, the wall-normal, and the spanwise direction, respectively. An algebraic mapping in equation (3.35) was employed for grid stretching in the direction normal to the wall. The far-field boundary was located at $75\delta^*$, and the streamwise distance was $238.5\delta^*$, which is about 12 T-S wavelengths. Three of them were used for the buffer domain. For the time marching, the disturbance period was divided into 320 time steps, and time was advanced with a three-stage Runge-Kutta method. Computations were performed on SGI Octane. For such a grid size, the CPU time needed for 14 periods of disturbance forcing is 76.32 hours.

Figure 52 gives a comparison between the DNS and LST predictions after fourteen periods of forcing at the inflow boundary. $y = 0.44$ corresponds to the location where the $u$ component reaches its maximum. The computed streamwise variations of the amplitudes and the phases for the streamwise disturbance $u$ and the wall-normal disturbance $v$ agree well with the LST results in the entire physical domain. It can be also seen from Figure 52 that the leading waves have gone through the buffer domain without reflections.

Fourier analysis is capable of helping investigate the prominent frequencies/wave numbers in the flow field. For this case, the $u$ velocity was recorded...
for five periods of the T-S wave in time and the signals were Fourier transformed in time. Figure 53 depicts the profile of Fourier amplitude of $|u|$ at the forcing frequency.
near the end of physical domain. There is a good agreement with that obtained by LST. The streamwise variations of the Fourier amplitude $|u|$ at $y = 0.44$ are presented in Figure 54. $f$ in Figure 54 denotes the ratio of Fourier frequency over the inlet forcing frequency of 0.1122. There is a spike at the forcing frequency in the otherwise flat spectra. The wave amplitudes remain small in the entire physical domain.

![Fourier Spectra of u Components, $y = 0.44$. The Parallel Boundary Layer Case.](image)

For the same operation parameters, simulations have been performed for a nonparallel Blasius boundary layer. The similarity Blasius velocity profile was taken as the mean flow velocity profile. Figure 55 shows the streamwise variations of $u$ and $v$ components and Figure 56 shows Fourier spectra of $u$ components near the wall, $y = 0.44$. For the given Reynolds number and frequency, the disturbances forcing at the inflow boundary experience a linear growth in the first three wavelengths and then a
Figure 55. Velocity Components. $y = 0.44$. the Nonparallel Boundary Layer Case.

Figure 56. Fourier Spectra of $u$ Components. $y = 0.44$. the Nonparallel Boundary Layer Case.
decay due to the nonparallel effect of the base flow. This is consistent with the neutral curve, as shown in Figure 37, where the flow develops from an unstable region to a stable region with an increase of Reynolds number at a constant frequency.

**Confluent Wake/Boundary Layer (CWB)**

Five cases were studied for the CWB. Cases 1 and 2 are for a parallel CWB and 3-5 for a nonparallel CWB. Those cases will be defined in the following subsections.

**Parallel CWB**

**Case 1.** To examine the linear behavior of the CWB, Case 1 was designed with an amplitude of 0.01% and a frequency $\omega = 0.1122$ forcing at the inlet for all modes. The streamwise wave numbers for the imposed inflow disturbances associated with the boundary layer mode and the wake mode 1 and 2 are $(0.3162-0.007371)$, $(0.1613-0.007528)$ and $(0.1147-0.008252)$, respectively. A short computational domain of seven T-S wavelengths was used, including four wavelengths for the physical domain and three for the buffer domain. The superposition of the parallel Blasius mean-flow profile and the wake profile described in equation (3.28) was employed. The wake, located at $y = 7.0$, has a velocity defect of 0.4. Simulations have been performed on a grid $281 \times 101 \times 5$ for $Re^* = 998$. Figures 57 and 58 give the $u$ and $v$ components comparisons between the DNS and the LST results in the wake.
region and in the boundary layer, respectively. The DNS results agree well with the
LST results in the physical domain.

Figure 57. Instantaneous Disturbance Velocity Components. $y = 7$. Case 1.

Figure 58. Instantaneous Disturbance Velocity Components. $y = 0.44$. Case 1.
**Case 2.** Case 2 is still for the parallel CWB but has a longer computational domain and a more streamwise grid nodes than Case 1. The grid used is the same as that for the boundary layer cases, i.e., $481 \times 101 \times 5$. The streamwise wave numbers for the imposed inflow disturbances associated with the boundary layer mode and the wake mode 1 and 2 are the same as those in Case 1. The amplitudes of the inlet disturbances were all set at 0.1%.

Figures 59-62 show the $u$ profiles at four different streamwise locations. The LST results are also shown for comparison. At the first two stations, $x = 39.74$ and 79.48, there is a good agreement between the DNS and the LST. At $x = 119.22$, the DNS starts to deviates from the LST in wake region. The profile at $x = 178.85$ shows a significantly different behavior compared with the linear results. Recall that, without

![Figure 59. $u$ Component Profiles. $x = 39.74$. Case2.](image-url)
Figure 60.  $u$ Component Profiles. $x = 79.48$. Case 2.

Figure 61.  $u$ Component Profiles. $x = 119.22$. Case 2.
Figure 62. $u$ Component Profiles. $x = 178.85$. Case 2.

The presence of the wake, the disturbance has been shown in Figure 53 to follow the linear analysis. With the presence of the wake, both the shape and the amplitude of the disturbance diverge from the linear behavior, particularly in the wake region. This is consistent with the observation made based on the linear analysis that the wake has an amplifying effect on the boundary layer mode. Due to the higher growth rates of the wake modes, the disturbance first starts to grow appreciably in the region of the wake. This, in turn, promotes disturbance growth in the boundary layer.

Figure 63 shows the instantaneous disturbance velocity profiles at $x = 178.85$. The profiles are generally symmetric with respect to the wake centerline, indicating that the symmetrical mode dominates the growth of wake disturbance. This is
consistent with the linear analysis suggesting that the symmetrical mode have the larger growth rate than the antisymmetrical mode over a wide range of frequencies.

Figure 63. Instantaneous Disturbance Velocity Profiles. $x = 178.85$. Case 2.

Figures 64-67 shows the frequency spectra of $u$ at the same four streamwise stations as those in Figures 59-62. Here $f$ denotes a ratio of a frequency over the inlet forcing frequency of 0.1122. As shown in Figure 64, four spikes are found at $x = 39.74$. They correspond to the zero-frequency mode, the forcing mode and its harmonics, and the mode at $f = 5.5$. According to the linear analysis, the last mode appears at the frequency for which the growth rate of the symmetric wake mode is the maximum. This mode then is the mode that is mostly likely to extract energy, grow and show up early in the spectrum. Additional modes appear in the spectra at the stations downstream. At $x = 79.48$, modes with frequency between the forcing frequency and the most unstable wake mode have gained energy. At $x = 119.22$, the
spectrum shows that the disturbance has a continuous distribution of frequency, with peaks at the most unstable wake mode and its first harmonic, indicating a nonlinear

![Frequency Spectra of $u. y = 7, x = 39.74$. Case 2.](image1)

Figure 64. Frequency Spectra of $u. y = 7, x = 39.74$. Case 2.

![Frequency Spectra of $u. y = 7, x = 79.48$. Case 2.](image2)

Figure 65. Frequency Spectra of $u. y = 7, x = 79.48$. Case 2.
Figure 66. Frequency Spectra of $u$. $y = 7$, $x = 119.22$. Case 2.

Figure 67. Frequency Spectra of $u$. $y = 7$, $x = 178.85$. Case 2.
energy cascade. At the last station, $x = 178.85$, the spectrum shows a widening of wave frequency range with appreciable amplitudes for all frequencies.

Figures 68-71 show the frequency spectra of $u$ at the same four streamwise stations as those in Figures 64-67. The data is now taken at $y = 0.44$, which is located inside the boundary layer. At the first station, a spike at the forcing frequency is apparent. At $x = 79.48$, the spectrum is filled up to about the frequency for the most unstable wake mode. At the later stations, the amplifying effects of the wake on the boundary layer mode are more pronounced.

![Figure 68](image)

**Figure 68.** Frequency Spectra of $u$. $y = 0.44$, $x = 39.74$. Case 2.

It can be seen in Figure 64 that a zero-frequency mode, representing a mean-flow distortion, has appeared at the downstream distance $x = 39.74$, the end of the second T-S wavelength. Its amplitude is lower than that of the forced mode. The stationary mode grows rapidly with downstream distance. Figures 68-71 also shows a
Figure 69. Frequency Spectra of $u$. $y = 0.44$, $x = 79.48$. Case 2.

Figure 70. Frequency Spectra of $u$. $y = 0.44$, $x = 119.22$. Case 2.
Figure 71. Frequency Spectra of $u$, $y = 0.44$, $x = 178.85$. Case 2.

similar increase of the stationary mode. At $x = 178.85$, or the end of the eighth wavelength, the amplitude associated with the mean-flow distortion mode has been amplified nearly 40 times the amplitude of the forced waves at the inflow boundary. It is apparent that the large amount of base flow correction should be considered.

**Nonparallel CWB**

To include the nonparallel effect, the ALLSPD was used to obtain the laminar base flow profiles. The ALLSPD code solves the Navier-Stokes equations in terms of primitive variables. It employs preconditioning and eigenvalue rescaling technologies, being capable of providing efficient solution of flows over a wide range of Mach numbers. The code has been successfully used in studying a variety of viscous flows (Chen and Choi 1991, Shuen et al. 1993, Chen and Shuen 1994). In this study,
laminar boundary layers were first calculated and compared with the Blasius profiles. The computational domain size and inlet parameters were taken as the same as that of Case 2. The computations were performed on a grid of $481 \times 61$ with stretching near the wall described in equation (3.38). Figures 72 - 74 show comparisons between the Blasius and ALLSPD for $u$ components, and their first and second derivatives, respectively. Here $L$ denotes the flat-plate length. Figure 72 indicates that the ALLSPD captures the similarity behavior of the Blasius flow. There is a good agreement between the calculated and the Blasius solution. Further inspection of the calculated profiles, as shown in Figures 73 and 74, indicates that the calculated profiles deviate slightly from the Blasius solution near the wall in about the first quarter of the computational-domain length. This small deviation is considerably reduced when the flow develops downstream.

![Figure 72. Velocity Profiles at Downstream Locations.](image-url)
Figure 73. First Derivatives of $u$ at Downstream Locations.

Figure 74. Second Derivatives of $u$ at Downstream Locations.
To alleviate the numerical effects on the CWB calculations, the $U$ and $V$ profiles from 20% of the domain length downstream were used in the DNS. For comparison with previous cases, Reynolds number at the 20% of the domain length was set to 998 by adjusting the inlet flow Reynolds number. The grid used in 601 x 61. The numbers of grid nodes per wavelength remain the same as those for Case 2.

Figure 75 gives the $U$ profiles, uniformly spaced between 20% and 100% of the domain length. A spline-interpolation was used to map the velocity profiles from the current grid to the grid for DNS that has a node number of 101 in the normal direction. The length of the computational domain for the present DNS is the same as that for Case 2. The inlet profile has a wake defect of about 0.351. Based on this profile, the disturbances associated with the boundary layer mode, wake mode 1 and wake mode 2 were found using the global code.

![Figure 75. CWB Velocity Profiles at Uniform Downstream Locations.](image)
The disturbances associated with the boundary layer mode and wake modes were forced in all cases. In Case 3, the disturbance frequency and amplitude forcing at the inlet boundary for all modes were, respectively, 0.1122 and 0.1%. Case 4 was forced with the frequency of 0.561 for the wake mode 1, 0.1122 for the wake 2, respectively. The amplitudes are 0.01% for both wake modes. The setup for the boundary layer mode is the same as that for Case 3. The frequency of 0.561, five times that of 0.1122, corresponds closely to the frequency at which the maximum growth rate for the wake mode 1 occurs. Thus it is referred as the most unstable wake frequency. With the same frequencies forcing as those in Case 4, the disturbance amplitudes of 0.1% was forced for the wake modes in Case 5. The parameters set in these five cases are given in Table 3.

**Case 3.** Figures 76 shows the frequency spectra in the wake region at \( x = 178.85 \) near the end of the physical domain. The wake mode is not seen being amplified. Instead, the first harmonic of the forcing frequency of 0.1122 appears to have grown. The frequency spectra in the boundary layer are shown in Figure 77. There is only one spike that refers to the forcing mode. It can be also seen from Figures 76 and 77 that the disturbance amplitude is nearly one order of magnitude smaller than that in Case 2. Figures 78 and 79 shows the instantaneous disturbance velocities calculated after sixteen T-S wave periods at \( y = 7 \) and \( y = 0.44 \), respectively. The disturbances decay in the boundary layer, which is also demonstrated in Figure 77. In the wake region, \( y = 7 \), the disturbances experience only slight amplitude growth.
Table 3
Parameters Setting for the CWB Simulations Presented

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
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<td>$\omega_{W2}$</td>
<td>0.1122</td>
<td>0.1122</td>
<td>0.1122</td>
<td>0.1122</td>
<td>0.1122</td>
</tr>
<tr>
<td>$(\alpha_i)_{BL}$</td>
<td>-0.007371</td>
<td>-0.007371</td>
<td>0.025610</td>
<td>0.025610</td>
<td>0.025610</td>
</tr>
<tr>
<td>$(\alpha_i)_{W1}$</td>
<td>-0.007528</td>
<td>-0.007528</td>
<td>-0.006465</td>
<td>-0.048463</td>
<td>-0.048463</td>
</tr>
<tr>
<td>$(\alpha_i)_{W2}$</td>
<td>-0.008252</td>
<td>-0.008252</td>
<td>-0.003536</td>
<td>-0.003536</td>
<td>-0.003536</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

where $\alpha_i$ in Table 3 represents the growth rate at the inflow boundary.
Figure 76. Frequency Spectra of $u, y = 7$. Case 3.

Figure 77. Frequency Spectra of $u, y = 0.44$. Case 3.
Figure 78. Instantaneous Disturbance Velocity Components. $y = 7$. Case 3.

Figure 79. Instantaneous Disturbance Velocity Components. $y = 0.44$. Case 3.
Case 4. With the low amplitude (0.01%) and the high frequency (0.561) for the wake mode 1, Figure 80 shows the frequency spectra for v-component at $x = 178.85$, about the end of the eighth T-S wavelength, and $y = 7$, the initial wake centerline. Here $f$ in the Figure denotes the ratio of Fourier frequency over the fundamental frequency of 0.1122. The spectra have four sharp peaks, representing the boundary-layer mode, the most unstable wake mode and its first and second harmonics. The highest peak at the most unstable wake frequency has a band around it. Its first harmonic peak is higher than that at the fundamental frequency.

Figure 80. Frequency Spectra of $v$. $y = 7$. Case 4.

Figure 81 shows that the spectra of $u$-component also have four peaks at the different modes similar to that of $v$-component. We notice in Figures 80 and 81 that the fastest growth mode associated with the $v$ signals is stronger than that with the $u$ signals for the given amplitudes forcing. This is also demonstrated in Figures 82 and
83 showing the streamwise variations of frequency spectra at $y = 7$ for $v$ component and $u$ component, respectively. Near the end of the fifth T-S wavelength, $x = 100,$

Figure 81. Frequency Spectra of $u$. $y = 7$. Case 4.

Figure 82. Frequency Spectra of $v$ with Downstream Distance. $y = 7$. Case 4.
the most unstable wake mode rapidly increases while the fundamental mode seems to remain neutrally stable (see Figure 82). Note that the zero-frequency mode associated with the $u$ signals starts to rapidly grow at about the end of the sixth T-S wavelength, $x = 120$ (see Figure 83).

Figure 84 shows the instantaneous velocity components at $y = 0.44$. The disturbance decays until about $x = 110$ and then starts to grow, compared to Case 3 where the disturbance decays in the physical domain (see Figure 79). Streamwise variations of $u$ spectra shown in Figure 85 indicate that the disturbance decay is associated with the fundamental frequency and the disturbance growth is associated with the most unstable wake frequency. This demonstrates that the disturbance associated with wake modes affect significantly the development of the disturbances in the boundary layer. This interaction is apparent in Figure 86 showing the
streamwise variation of the disturbances at $y = 2.96$, which is near the edge of the initial boundary layer. The disturbance is amplified as the flow develops downstream.

Figure 84. Instantaneous Disturbance Velocity Components. $y = 0.44$. Case 4.

Figure 85. Frequency Spectra of $u$ with Downstream Distance. $y = 0.44$. Case 4.
Figure 86. Instantaneous Disturbance Velocity Components, \( y = 2.96 \). Case 4.

At the end of the physical domain, the amplitude of the \( u \) component is about 30 times that of the initial value.

Figure 87 shows the streamwise frequency spectra of \( u \) at \( y = 2.96 \). We notice in Figure 87 that the amplitude of the \( u \) component associated with the most unstable wake frequency rapidly increases after about \( x = 110 \) while the fundamental mode is damped as that at \( y = 0.44 \) (see Figure 85). Fourier amplitude of \( u \) at \( x = 178.85 \), as shown in Figure 88, shows that the highest peak is associated with the most unstable wake frequency, with a band around it and its first harmonic next to it. The behavior of how the wake influences on the boundary layer can also be seen in Figure 89 that shows the spanwise vortex contour. Rapid growth of the wake mode results in the
roll-up of the boundary layer and thus trigger the disturbance growth in the boundary layer.

Figure 87. Frequency Spectra of $u$ with Downstream Distance. $y = 2.96$. Case 4.

Figure 88. Frequency Spectra of $u$. $y = 2.96$. Case 4.
Case 5. With inlet amplitude forcing at 0.1% for the wake modes, Figures 90 and 91 give the streamwise frequency spectra of $v$ and $u$ components at $y = 7$, respectively. The mode at the most unstable wake frequency has the fastest growth rate for the $v$ component and the zero-frequency mode grows fastest for the $u$ component.
Figure 92 shows the spectrum of $u$ at $x = 178.85$ and $y = 2.96$. The spectrum is filled in a continuous manner with distinct peaks at the most unstable wake frequency and its harmonics. Recall in Case 4 (Figure 88) that only one harmonic of the most unstable mode is generated. It indicates that the increase of the amplitude forcing for the wake modes promotes the disturbance growth in the boundary layer. It is also demonstrated in the spanwise vortex contour shown in Figure 93. The effect of the wake on the boundary layer is brought farther upstream compared to Case 4 (see Figure 89).

![Diagram of frequency spectra](image)

Figure 91. Frequency Spectra of $u$ with Downstream Distance. $y = 7$. Case 5.

Summary

Direct numerical simulations have been used to study the development of forced spatially developing confluent wake/boundary layer flow. The initial condition
was provided via solving the Orr-Sommerfeld equation. Based on an assumption of the parallel confluent wake/boundary layer, the flow in a linear stage has been examined and the results agree well with the LST results. Further downstream the
parallel assumption results in a significant distortion to the mean flow. The nonparallel effects must be considered in the simulation of the confluent wake/boundary layer. The mean flow profile was obtained using ALLSPD. For a given fundamental disturbance frequency of 0.1122 and amplitude of 0.1% for the boundary layer mode and the wake modes, the disturbances exhibit decaying behavior in the boundary layer and neutrally stable behavior in the wake region. The linear results show that the maximum growth rate associated with the symmetric wake mode approximately appears at a frequency of 0.561 for a velocity defect of 0.351. The most unstable wake mode forced at the inflow boundary tends to have an important influence on the disturbance evolution in the confluent wake/boundary layer. When the amplitude of forcing for the symmetric wake mode is 0.01%, the most unstable wake mode grows rapidly and generates harmonic modes. The amplifications of the disturbances in boundary layer were found being caused by the growth of the disturbance in the wake. The amplifying effects of the wake on the boundary layer mode can be seen clearly at the edge of the boundary layer. With the larger amplitude forcing, 0.1%, the harmonics of the most unstable wake mode appears and the spectra become filled. This suggests strong interactions among waves and a cascade of energy from the forcing modes to a wide distribution of modes. This spreading of energy mimics that of an early stage of flow transition. It should be noted that the $u$ signals start to become stronger than the $v$ signals when the amplitude increases. In this case, the zero-frequency mode associated with the $u$ signals grows rapidly and its amplitude is larger than any other modes.
CHAPTER V

CONCLUSIONS

The flow transition around a multi-element airfoil has been studied using RANS with a predictive transition model, LST and DNS. The transition model uses an intermittency correction function in terms of the local peak of kinetic energy that was obtained by a flow solver INS2D. The flowfield over a multi-element airfoil configuration has been predicted for two Reynolds numbers with five different angles of attack. Comparisons between the predictions and the measured data with regard to transition onset locations, wall pressure coefficients, skin friction coefficients and velocity profiles have been made. The LST and DNS have been used to study the confluent wake/boundary layer that has an important effect on the boundary layer transition. For the linear analysis, a numerical global code was used to find the eigenvalues and eigenfunctions associated with the boundary layer mode and the wake modes, respectively. A DNS code (Joslin et al. 1992) was used for the nonlinear analysis of the confluent wake/boundary layer. The initial conditions were obtained using the LST results. The mean flow velocity profile was provided using the ALLSPD. Based on the computed results and comparison with experiments, the following conclusions are made.

The calculated wall pressure coefficients generally agree well with experiments. Discrepancies occur at the lower angle of attack, 8°, particularly on the
slat. For velocity profiles, the agreement between the computations and the experiments is fairly good in depicting the wake location and the wake defect. The transition onset locations on the slat were predicted very well for all angles of attack tested at the higher Reynolds number. At the lower Reynolds number, the predictions were good except for AOA = 8°. Compared to the slat, discrepancies between the computational and experimental transition onset locations on the main element and the flap were found. That may be attributed to the confluent wake/boundary layer flow occurring near the main-element and the flap leading edge where the velocity profiles were not well predicted. The flow mechanism of the confluent wake/boundary layer is significantly different from the boundary layer and is not adequately understood, especially the possible interactions between the wake and the boundary layer.

These results suggest that a better understanding of the confluent wake/boundary layer flow is needed. As a result, linear and nonlinear analyses of the confluent wake/boundary layer flow have been performed. For the LST and DNS, a simplified flow model for the confluent wake/boundary layer has been used. It is a superposition of the Blasius boundary layer and a wake over the boundary layer. The flow was assumed to be parallel. A global numerical code to solve all eigenvalues has been developed and validated by comparing with the Blasius boundary layer results and the 2-D wake results, respectively. The wake modes include the symmetric and the antisymmetric modes. Their switching behavior was seen to occur for low frequency with high velocity defects. The interactions between the boundary layer mode and the wake mode have been found. The unstable modes associated with the
wake are stabilized by the reduced distance between the wake and the wall. On the other hand, the presence of the wake has an amplifying effect on the growth rate of the boundary layer mode and such amplification is intensified with an increase of the wake velocity defect. In addition, the critical Reynolds number associated with the boundary layer mode is reduced by the presence of the wake. The mode shape of the boundary layer mode also seems to diminish above the wake, indicating that the wake has a confining influence on the disturbances associated with the boundary layer mode. The results suggest that the wake may play an important role in the growth of the linear disturbances and the initiation of transition in the boundary layer. That has been confirmed by DNS when the symmetric wake mode is forced at the most unstable frequency. In general, the disturbances in the wake region grow rapidly and, as a result, promote the disturbance growth in the boundary layer. Such a promotion is more intensive in the region near the edge of the boundary layer than in the near-wall region. For a given fundamental frequency of 0.1122 for the boundary layer mode and the antisymmetric wake mode, and the most unstable wake frequency of 0.561 for the symmetric wake mode, the effects of the wake on the boundary-layer development depend mainly on the amplitude forcing for the symmetric wake mode. When the amplitude is forced at 0.01%, the frequency spectra show that the fastest growth mode for the $v$ signals occurs at the most unstable wake frequency and that for the $u$ signals at zero frequency. For the mode with the fastest growth rate, the amplitude for the $v$ signals is larger than that for the $u$ signals over the whole physical domain. With the larger amplitude forcing at 0.1%, the most unstable wake mode grows rapidly with its
harmonics and filled the spectra appearing. It indicates the initial stage of a transition process. On the other hand, the zero-frequency mode associated with the $u$ signals, however, also grows rapidly and tends to have the larger amplitude than that with the $v$ signals at the end of the physical domain. The zero-frequency mode represents mean-flow distortion. It is implied that a larger amount of mean-flow correction will be needed for simulations with large amplitude forcing for the wake modes.

The following is the suggested future research.

The present simulations for the confluent wake/boundary layer flow show that the zero-frequency mode may be significant depending upon forcing conditions. In the future, we suggest that the full Navier-Stokes equations be applied to simulate confluent wake/boundary layer flows with large amplitude forcing to include the potential large mean flow variation in the simulations.

The effects of inflow forcing on the boundary layer transition process should be studied. The different inflow forcing may include changes in the spanwise wave number, frequency and amplitude. The height of wake may also influence the boundary layer transition process and should be investigated.

For the multi-element airfoil with the higher Reynolds numbers and higher angles of attack, the local Mach number increases considerably such that the effects of compressibility may become significant. The present study can be extended to a compressible confluent wake/boundary layer flow by using LST and DNS. The initial conditions for DNS will be provided by LST results.
The accurate numerical simulation of a realistic multi-element airfoil can offer new possibilities in studying the transitional flow in great detail. The complexity of the geometry requires a large number of grid points. As a result, enormous computer speed and memory requirements are needed due to the large computational domains and intensive computations that are involved. Parallel computation is an attractive approach.
Appendix A

Finite Difference Relations

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All stencils were obtained using a code developed by Liou et al. (2000) and verified.

Sixth-order central-difference approximations for the fourth derivative are
given by

\[
f_{j}^{(4)} = \frac{1}{240h_x^4}(7f_{j-4} - 96f_{j-3} + 676f_{j-2} - 1952f_{j-1} + 2730f_{j} - 1952f_{j+1} + 676f_{j+2} - 96f_{j+3} + 7f_{j+4}) \quad (A.1)
\]

\[
f_{j}^{(4)} = \frac{1}{240h_x^4}(-26f_{j} + 361f_{j-2} - 1112f_{j-3} + 1260f_{j-4} - 188f_{j-5} - 794f_{j-6} + 744f_{j} - 308f_{j+1} + 70f_{j+2} - 7f_{j+3}) \quad (A.2)
\]

\[
f_{j}^{(4)} = \frac{1}{240h_x^4}(101f_{j} + 58f_{j-2} - 1860f_{j-3} + 5272f_{j-4} - 7346f_{j-5} + 6204f_{j-6} - 3428f_{j+1} + 1240f_{j+2} - 267f_{j+3} + 26f_{j+4}) \quad (A.3)
\]

\[
f_{N-3}^{(4)} = \frac{1}{240h_x^4}(-7f_{N-9} + 70f_{N-8} - 308f_{N-7} + 744f_{N-6} - 794f_{N-5} - 188f_{N-4} + 1260f_{N-3} - 1122f_{N-2} + 361f_{N-1} - 26f_{N}) \quad (A.4)
\]

\[
f_{N-2}^{(4)} = \frac{1}{240h_x^4}(26f_{N-9} - 267f_{N-8} + 1240f_{N-7} - 3428f_{N-6} + 6204f_{N-5} - 7346f_{N-4} + 5272f_{N-3} - 1860f_{N-2} + 58f_{N-1} + 101f_{N}) \quad (A.5)
\]

Sixth-order central-difference approximations for the third derivative are
given by

\[
f_{j}^{(3)} = \frac{1}{240h_x^3}(-7f_{j-4} + 72f_{j-3} - 338f_{j-2} + 488f_{j-1} - 488f_{j+1} + 338f_{j+2} - 72f_{j+3} + 7f_{j+4}) \quad (A.6)
\]
Sixth-order central-difference approximations for the second derivative are given by

\[ f^{(3)}_{j+4} = \frac{1}{240h_x^2} \left( 9f_{j} - 86f_{j+1} + 882f_{j+2} - 1370f_{j+3} + 926f_{j+4} - 324f_{j+5} + 70f_{j+6} - 7f_{j+7} \right) \]  
(A.7)

\[ f^{(3)}_{j+3} = \frac{1}{240h_x^2} \left( -5f_{j} + 424f_{j+2} + 1638f_{j+3} - 2504f_{j+4} + 2060f_{j+5} - 1080f_{j+6} + 394f_{j+7} - 88f_{j+8} + 9f_{j+9} \right) \]  
(A.8)

\[ f^{(3)}_{N-3} = \frac{1}{240h_x^2} \left( -9f_{N} + 86f_{N-1} + 100f_{N-2} - 882f_{N-3} + 1370f_{N-4} - 926f_{N-5} + 324f_{N-6} - 70f_{N-7} + 7f_{N-8} \right) \]  
(A.9)

\[ f^{(3)}_{N-2} = \frac{1}{240h_x^2} \left( 5f_{N} + 424f_{N-1} - 1638f_{N-2} + 2504f_{N-3} - 2060f_{N-4} + 1080f_{N-5} - 394f_{N-6} + 88f_{N-7} - 9f_{N-8} \right) \]  
(A.10)

\[ f^{*}_{j+3} = \frac{1}{180h_x^2} \left( 2f_{j-3} - 27f_{j-2} + 270f_{j-1} - 490f_{j} + 270f_{j+1} - 27f_{j+2} + 2f_{j+3} \right) \]  
(A.11)

\[ f^{*}_{j+3} = \frac{1}{180h_x^2} \left( -11f_{j} + 214f_{j-2} - 378f_{j-3} + 130f_{j-4} + 85f_{j-5} - 54f_{j-6} + 16f_{j-7} - 2f_{j-8} \right) \]  
(A.12)

\[ f^{*}_{j+2} = \frac{1}{180h_x^2} \left( 126f_{j} - 70f_{j-2} - 486f_{j-3} + 855f_{j-4} - 670f_{j-5} + 324f_{j-6} - 90f_{j-7} + 11f_{j-8} \right) \]  
(A.13)

\[ f^{*}_{j+1} = \frac{1}{180h_x^2} \left( 938f_{j} - 4014f_{j-2} + 7911f_{j-3} - 9490f_{j-4} + 7380f_{j-5} - 3618f_{j-6} + 1019f_{j-7} - 126f_{j-8} \right) \]  
(A.14)
Sixth-order central-difference approximations for the first derivative are given by

\[ f_{N-2}^* = \frac{1}{180h_x^2} (-11f_N + 214f_{N-1} - 378f_{N-2} + 130f_{N-3} + 85f_{N-4} - 
54f_{N-5} + 16f_{N-6} - 2f_{N-7}) \]  
(A.15)

\[ f_{N-1}^* = \frac{1}{180h_x^2} (126f_N - 70f_{N-1} - 486f_{N-2} + 855f_{N-3} - 670f_{N-4} + 
324f_{N-5} - 90f_{N-6} + 11f_{N-7}) \]  
(A.16)

\[ f_N^* = \frac{1}{180h_x^2} (938f_N - 4014f_{N-1} + 7911f_{N-2} - 9490f_{N-3} + 7380f_{N-4} - 
3618f_{N-5} + 1019f_{N-6} - 126f_{N-7}) \]  
(A.17)

\[ f_j' = \frac{1}{180h_x} (-3f_{j-3} + 27f_{j-2} - 135f_{j-1} + 135f_{j+1} - 27f_{j+2} + 3f_{j+3}) \]  
(A.18)

\[ f_{j+3}' = \frac{1}{180h_x} (6f_1 - 72f_2 - 105f_3 + 240f_4 - 90f_5 + 24f_6 - 3f_7) \]  
(A.19)

\[ f_{j+2}' = \frac{1}{180h_x} (-30f_1 - 231f_2 + 450f_3 - 300f_4 + 150f_5 - 45f_6 + 6f_7) \]  
(A.20)

\[ f_{j+1}' = \frac{1}{180h_x} (-441f_1 + 1080f_2 - 1350f_3 + 1200f_4 - 675f_5 + 216f_6 - 30f_7) \]  
(A.21)

\[ f_{N-2}^* = \frac{1}{180h_x^2} (-6f_N + 72f_{N-1} + 105f_{N-2} - 240f_{N-3} + 90f_{N-4} - 
24f_{N-5} + 3f_{N-6}) \]  
(A.22)

\[ f_{N-1}^* = \frac{1}{180h_x^2} (30f_N + 231f_{N-1} - 450f_{N-2} + 300f_{N-3} - 150f_{N-4} + 
45f_{N-5} - 6f_{N-6}) \]  
(A.23)
\[ f'_N = \frac{1}{180\lambda_x} \left( 441f_N - 1080f_{N-1} + 1350f_{N-2} - 1200f_{N-3} + 675f_{N-4} - 216f_{N-5} + 30f_{N-6} \right) \] (A.24)
BIBLIOGRAPHY


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