New Graphical Approach on the Analysis of Experimental Data

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NEW GRAPHICAL APPROACH ON THE
ANALYSIS OF EXPERIMENTAL DATA

by
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A Dissertation
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Faculty of The Graduate College
in partial fulfillment of the
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Department of Statistics

Western Michigan University
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This study presents a new graphical method to identify significant effects in factorial experiments. The proposed methods are obtained for the different cases in which the design can be of full factorial or fractional factorial and the factor levels can be pure or mixed.

We focus on the different decomposition methods, for example orthogonal components system and orthogonal contrast method, to make use of the chisquare plot which requires that the sums of squares are of the same degrees of freedom. Examples and simulations illustrating the different cases of the procedure are presented.
ACKNOWLEDGMENTS

I wish to express my gratitude to my advisor J. C. Wang whose continual guidance and support made this possible. I would like to thank my committee Gerald Sievers and Michael Stoline and Steven Butt for serving on my committee and giving numerous valuable suggestions. I would like to thank my wife, Yasemin, without whose support, understanding this work could not have completed. At last but not a bit least, I would like to thank my family in Turkey and friends for their support and encouragement.

Suha Sari
TABLE OF CONTENTS

ACKNOWLEDGEMENTS ...................................................... ii

LIST OF TABLES ........................................................................ v

LIST OF FIGURES ........................................................................ vi

CHAPTER

I. INTRODUCTION ............................................................................. 1

1.1 Statement of the Problem ...................................................... 1

1.2 Our Proposed Method ......................................................... 3

II. THE USE OF A GRAPHICAL METHOD TO IDENTIFY

FACTORIAL EFFECTS ...................................................................... 5

2.1 Introduction ............................................................................. 5

2.2 Chisquare Plot for the Sums of Squares of
s Level Factorial Data ............................................................... 6

2.2.1 Chisquare Plot for the Sums of Squares of
s Level Full Factorial Data ..................................................... 6

2.2.2 Chisquare Plot for the Sums of Squares of
s Level Fractional Factorial Data .......................................... 14

2.3 Chisquare Plot for the Sums of Squares of
s^m × 2 Level Factorial Data .................................................... 24

2.3.1 Chisquare Plot for the Sums of Squares of
s^m × 2 Level Full Factorial Data ........................................... 24

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LIST OF TABLES

1  Textile Data with 13 Components ............................................. 12
2  Analysis of Variance for Strength of Textile Data ....................... 14
3  Analysis of Variance for Strength of Seat-Belt Data .................... 23
4  Analysis of Variance for Y of Simulated Data ............................ 32
5  Generating Three 2-level Factors from One 4-level Factor ............. 39
6  Analysis of Variances for Y of Example One .............................. 42
7  Some Coefficients of Orthogonal Contrasts ................................. 44
8  Analysis of Variance for Y of Example Two ............................... 58
9  Analysis of Variance for Y of Example Three ............................. 58
10 Description and Sums Squares of Textile Data ............................ 67
11 Textile Data ............................................................................. 68
12 Description and Aliasing Pattern of Seat-Belt Data ...................... 69
13 Seat-Belt Data, Strength Location .............................................. 70
14 Sums of Squares of Seat-Belt Data, Strength Location .................. 70
15 Sums of Squares of Simulated Data ............................................ 71
16 Example One Data ................................................................. 72
17 Example Two Data ................................................................... 73
18 Example Three After One Factor Decomposed Into 3 Components .... 74
# LIST OF FIGURES

1. Chi-square and Half-Normal Plots of Textile Data .................................. 13
2. Chi-square Plot of Seat-Belt Data .......................................................... 22
3. Chi-square Plot of Simulated Mixed Level Full Factorial Data .................. 31
4. Chi-square Plot of Simulated Mixed Level Fractional Factorial Data .......... 38
5. Chi-square Plot of Example One ............................................................. 41
6. Textile Data with Method 1, 2, and 3 ....................................................... 48
7. Example Two with Method 1 ...................................................................... 49
8. Example Two with Method 2 ..................................................................... 50
9. Example Two with Method 3 ..................................................................... 51
10. Chi-square Plot of Blood Glucose Data ..................................................... 53
11. Chi-square and Half-Normal Plots of Blood Glucose Data ......................... 54
12. Chi-square Plot of Example Two ............................................................... 56
13. Chi-square Plot of Example Three ........................................................... 57
14. Chi-square Plot of Example 3 ................................................................. 60
15. Example 3 with Method 1 ......................................................................... 61
16. Example 3 with Method 2 ......................................................................... 62
17. Example 3 with Method 3 ......................................................................... 63
CHAPTER I
INTRODUCTION
1.1 Statement of the Problem

This dissertation presents a new graphical method which employs chi-square plots to identify the significant effects of mixed and pure level full factorial and fractional factorial experiments.

In practice, Graphical Methods have been used extensively for many different statistical purposes including analysis of experimental data, residual analysis, identification of factorial effects, normality assumption, constant variance assumption and many others. For example, the use of normal quantile plots and half normal plots (Daniel 1959) in the identification of factorial effects had resulted in numerous successful applications in industries.

Quantile plots have enjoyed a long history of successful applications. For example, the use of normal quantile plots and its variant, half normal quantile plot, since its introduction by Daniel (1959) in the identification of factorial effects of experimental data and in residual analysis in many modelling tasks had resulted in numerous successful applications in industries. Other types of quantile plots have also provided the practitioners the tools no less important or useful than the normal quantile plots. Among these, chi-square plots have been used to check for multivariate normality assumption.

In this dissertation, we will explore a new graphical method which employs...
chisquare plots to identify significant effects of factorial experiments. This method will be used for the data arisen from factorial experiments in which factors are either qualitative or quantitative.

We obtain a graphical method for factorial experiments in which design is either full or fractional factorial and in which factors are either at pure level or mixed level. Moreover, we have developed the necessary algorithms (see Appendix B for written R/Splus programs to obtain chisquare plots) in order to reach results. We also run some simulations for different cases and different parameters of the design.

Before discussing any specific result of our approach, we give more details for some previously known results of graphical method in identifying factor effects. Normal quantile plots and half normal quantile (Daniel 1959) plots which are mostly used in practice are generally used for the data which is quantitative. This method is explained in statistics books (see Montgomery (1991) or Devore (2000) or Walpole and Myers and Myers (1998) or Wu and Hamada (2000)). The normal probability plots are used to check the normality or constant variance assumptions. Under the hypothesis that the effects in the factorial effect designs are zero, the normalized effects resemble a random sample drawn from a normal population with mean zero and variance $\sigma^2$ and will tend to fall along a straight line on the plot. On the other hand, effects which do not lie along the straight line have nonzero means and have significant impact on the data.
Our approach to the graphical method essentially has some improvements over these results. Our approach is more general in a sense that it can deal with experimental data in which factors can be either qualitative or quantitative.

1.2 Our Proposed Method

Our new graphical approach is based on making use of the chi-square plot that have been used to check for multivariate normality assumption in identifying significant factorial effects of experimental data.

Since our method of using chi-square plot involves sums of squares of the same degrees of freedom (to be discussed later), effects will have to be decomposed into some components each having the same degrees of freedom. Interaction effects generally have higher degrees of freedom than the main effects do except for pure two-level full factorial data in which all main effects and interactions have one degree of freedom each. Therefore, interaction effects are generally decomposed into some components to have the same degrees of freedom. In Section 2.2.1, Section 2.2.2, Section 2.3.1, Section 2.3.2 and Section 2.4 orthogonal components system (see, for example, Wu and Hamada, 2000) is used as a decomposition method. In Section 3.2, orthogonal contrast method (see, for example, Wu and Hamada, 2000) is used as a decomposition method.

In Chapter II, we define and propose a graphical method for four different cases in which the design is of full factorial or fractional factorial and the factor levels are of pure or mixed levels. Moreover, each case is explained in detail
and some examples are given in this chapter. Also, more examples are given in
Chapter IV.

In Section 2.2.1, we develop a graphical method for pure level full factorial
data and establish the all necessary steps to construct chisquare plot for this case.
Also, we describe how to read a chisquare plot to identify the significant effects. In
Section 2.2.2, method is explained in the same manner as done in Section 2.2.1 for
pure level fractional factorial data. In Section 2.3.1, Section 2.3.2 and Section 2.4,
we establish a graphical method for mixed level full or fractional factorial data.

In above five sections, same decomposition method, orthogonal components
system (see, for example, Wu and Hamada, 2000) is used.

In Chapter III, a different decomposition method, namely, orthogonal con­
trast method (see, for example, Wu and Hamada, 2000) is used.

In Chapter IV, we consider some more examples and simulation studies for
all four different cases and different decomposition methods as explained above in
order to understand the motivation of our proposed graphical method.

In Chapter V, we conclude the discussion with a few remarks on the graph­
ical method and give a few suggestions on what should be done in the subsequent
research.
CHAPTER II
THE USE OF A GRAPHICAL METHOD TO IDENTIFY FACTORIAL EFFECTS

2.1 Introduction

Graphical Methods have been used extensively in the analysis of experimental data. For example, the use of normal quantile plots and half normal plots (Daniel 1959) in the identification of factorial effects had resulted in numerous successful applications in industries.

In this chapter, we will explore a new method which employs chi-square plots (to be discussed later) to identify the significant effects of mixed and pure level full factorial and fractional factorial experiments. This method will be used for the data arisen from factorial experiments in which factors are either qualitative or quantitative.

Two variants of the method are proposed depending on whether the factorial experiment is of pure level or mixed level. The proposed method is not used to replace half normal plots but rather to supplement and enhance them.
2.2 Chisquare Plot for the Sums of Squares of $s$ Level Factorial Data

Quantile plots have enjoyed a long history of successful applications. For instance, normal quantile plot, since its introduction by Daniel (1959), has been used extensively in residual analysis in many modelling tasks. The use of normal quantile plot and its variant, half normal quantile plot, in identifying significant factorial effects of experimental data has led to numerous successful stories. Other types of quantile plots have also provided the practitioners the tools no less important or useful than the normal quantile plots. Among these, chisquare plots have been used to check for multivariate normality assumption. In this section, we will explore the use of chisquare plots in identifying significant factorial effects of experimental data.

Two variants of the method for $s$ level factorial data are proposed depending on whether the experiment is of full or fractional factorial.

2.2.1 Chisquare Plot for the Sums of Squares of $s$ Level Full Factorial Data

In an $s$-level factorial experiment, each main effect accounts for $s-1$ degrees of freedom in the respective sum of squares. Interaction involving $m$ factors has $(s - 1)^m$ degrees of freedom in its sum of squares. For example, in a 3-level factorial experiment, while a main effect accounts for two degrees of freedom in its sum of squares, the sum of squares of a two-way interaction has four degrees of freedom. Therefore, interaction effects generally have higher degrees of freedom.
than the main effects do except for pure two-level full factorial data in which all
main effects and interactions have one degree of freedom each.

Since our method of using chisquare plot involves using sum of squares of
the same degrees of freedom (to be discussed later), interactions will have to be
decomposed into components each having the same degrees of freedom as main
effects do.

As an example, consider a $3^4$ experiment with factors A, B, C, and D.
There are four main effects of two degrees of freedom each, six two-way interac-
tions of four degrees of freedom each, four three-way interactions of eight degrees
of freedom each, and a four-way interaction with 16 degrees of freedom. We can
decompose the interactions in a way so that there are 40 components of factorial
effects of two degrees of freedom each (including the main effects). The decom-
position is illustrated as follows.

Since two-way interactions have four degrees of freedom, they are decom-
posed into two components each having two degrees of freedom as main effects do.
Therefore, there are 12 components of six two-way interactions in a $3^4$ experiment:

$$AB, AB^2, AC, AC^2, AD, AD^2, BC, BC^2, BD, BD^2, CD, CD^2.$$  

Similarly, three-way interactions are decomposed into four components of
two degrees of freedom each. Since there are four three-way interactions in a $3^4$
experiment, there are total of 16 components of two degrees of freedom corre-
sponding to three-way interactions:
Finally, the four-way interaction, $A \times B \times C \times D$, of 16 degrees of freedom is decomposed into eight components of two degrees of freedom each:

\[
ABC, ABC^2, AB^2C, AB^2C^2, ABD, ABD^2, AB^2D, AB^2D^2, ACD, ACD^2, AC^2D, AC^2D^2, BCD, BCD^2, BC^2D, BC^2D^2.
\]

The procedure of obtaining components is explained with an example. Consider $A \times B$ interaction. Denote the levels of $A$ and $B$ by $z_1$ and $z_2$, respectively, each having possible values of 0, 1, and 2. One of two components of $A \times B$ interaction is $AB$ component between the response values in which $z_1$ and $z_2$ satisfy

\[
z_1 + z_2 \pmod{3}.
\]

The other is $AB^2$ component between the response values in which $z_1$ and $z_2$ satisfy

\[
z_1 + 2z_2 \pmod{3}.
\]

Since each of these components is of three levels, the corresponding sums of squares have two degrees of freedom.

Now consider the three-way interaction involving factors $A$, $B$, and $C$. Denote their respective levels by $z_1$, $z_2$, and $z_3$ in a similar fashion as above. The four components of the $A \times B \times C$ interaction are $ABC$, $ABC^2$, $AB^2C$, and
$AB^2C^2$ satisfying, respectively, the following

\[ z_1 + z_2 + z_3 \pmod{3}, \]
\[ z_1 + z_2 + 2z_3 \pmod{3}, \]
\[ z_1 + 2z_2 + z_3 \pmod{3}, \]
and
\[ z_1 + 2z_2 + 2z_3 \pmod{3}. \]

Again, each component is of three levels and consequently its corresponding sum of squares has two degrees of freedom.

The eight components of the four-way interaction can be obtained similarly. This system of decomposition is called orthogonal components system (see, for example, Wu and Hamada, 2000).

Moreover, all 40 components are orthogonal. This implies that the sums of squares corresponding to these components are independent. If the experimental error distribution is normal with variance $\sigma^2$, then each sum of squares is $\sigma^2 \times \chi^2_2$ distributed.

Under the null hypothesis that all components are insignificant, these 40 sums of squares constitute a random sample from $\sigma^2 \times \chi^2_2$ distribution. Therefore, when plotted against $\chi^2_2$ quantiles, they should lie roughly on a straight line. Consequently, any obvious departure from the straight line that is implicated by a component implies its significant impact on the response variable. This explains why chi-square plots can be employed to identify significant factorial components.
Now we shall start describing the construction of a chisquare plot of sums of squares and how it is used for the identification of factorial effects. Consider an $s^k$ full factorial experiment.

**Construction of the plot**

**Step 1.** (Decomposing total sum of squares)

Proceed the decomposition of the total sum of squares ($s^k - 1$ degrees of freedom) into $(s^k - 1)/(s - 1)$ sums of squares as described in the example above. Each sum of squares is of $s - 1$ degrees of freedom.

**Step 2.** (Sorting the sums of squares)

Sort the corresponding sums of squares in increasing order.

**Step 3.** (Calculating chisquare quantiles)

Compute the $(s^k - 1)/(s - 1)$ chisquare quantiles (of $s - 1$ degrees of freedom).

**Step 4.** (Plotting quantile-quantile pairs)

Plot the sorted sums of squares against the chisquare quantiles.

**Use of the plot**

On the plot, smaller sums of squares will lie roughly on a straight line and a few largest sums of squares will likely stand clear off the straight line. The components corresponding to these small sums of squares are considered to be insignificant while the largest sums of squares are having significant impact on the response.
An example is given below to demonstrate the use of chisquare plots in identifying significant factorial effects. Consider the Textile Data taken from article written by Barella and Sust (an unpublished report cited by Box and Draper, 1987, in "Empirical Model-Building and Response Surfaces", page 206) arisen from a 27 runs full factorial experiment with three 3-level quantitative factors. The total sum of squares has 26 degrees of freedom. It is decomposed into 13 two-degree-of-freedom sums of squares, namely,

\[ A, B, C, AB, AB^2, AC, AC^2, BC, BC^2, \]
\[ ABC, ABC^2, AB^2C, \text{ and } AB^2C^2. \]

The chisquare plot of the sums of squares is given below in Figure 1. The half-normal plot of the 26 single-degree-of freedom orthogonal contrasts (obtained from orthogonal polynomial contrasts decomposition) and Analysis of Variance table in Table 2 (under the assumption that three-way interaction is negligible) are also given for comparison.
Table 1: Textile Data with 13 Components

<table>
<thead>
<tr>
<th>Factors</th>
<th>log(Y)</th>
<th>Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B C</td>
<td></td>
<td>AB AB² AC AC² BC BC² ABC ABC² AB²C AB³C²</td>
</tr>
<tr>
<td>0 0 0</td>
<td>2.83</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1 0 0</td>
<td>3.15</td>
<td>1 1 1 1 0 0 1 1 1 1 1 1</td>
</tr>
<tr>
<td>2 0 0</td>
<td>3.56</td>
<td>2 2 2 2 0 0 2 2 2 2 2 2</td>
</tr>
<tr>
<td>0 1 0</td>
<td>2.53</td>
<td>1 2 0 0 1 1 1 1 1 2 2 2</td>
</tr>
<tr>
<td>1 1 0</td>
<td>3.01</td>
<td>2 0 1 1 1 1 1 2 2 0 0 0</td>
</tr>
<tr>
<td>2 1 0</td>
<td>3.19</td>
<td>0 1 2 2 1 1 0 0 1 1 1 1</td>
</tr>
<tr>
<td>0 2 0</td>
<td>2.23</td>
<td>2 1 0 0 2 2 2 2 2 2 2 2</td>
</tr>
<tr>
<td>1 2 0</td>
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<td>3.06</td>
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<td>0 0 1</td>
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</tr>
<tr>
<td>1 0 1</td>
<td>3.08</td>
<td>1 1 2 0 1 2 2 2 0 2 2 0</td>
</tr>
<tr>
<td>2 0 1</td>
<td>3.50</td>
<td>2 2 0 1 1 2 0 2 0 1 0 0</td>
</tr>
<tr>
<td>0 1 1</td>
<td>2.42</td>
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<tr>
<td>1 1 1</td>
<td>2.79</td>
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</tr>
<tr>
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<td>3.03</td>
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<td>0 2 1</td>
<td>2.60</td>
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<tr>
<td>1 2 1</td>
<td>2.52</td>
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</tr>
<tr>
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<td>2.95</td>
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<td>2.47</td>
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<td>1 0 2</td>
<td>2.80</td>
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</tr>
<tr>
<td>2 0 2</td>
<td>3.30</td>
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<tr>
<td>0 1 2</td>
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<td>2.64</td>
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<tr>
<td>2 1 2</td>
<td>2.75</td>
<td>0 1 1 0 2 0 2 1 1 2 1 2</td>
</tr>
<tr>
<td>0 2 2</td>
<td>1.95</td>
<td>2 1 2 1 1 0 1 0 0 2 0 0</td>
</tr>
<tr>
<td>1 2 2</td>
<td>2.34</td>
<td>0 2 0 2 1 0 2 1 1 0 0 0</td>
</tr>
<tr>
<td>2 2 2</td>
<td>2.56</td>
<td>1 0 1 0 1 0 0 2 2 2 2 1</td>
</tr>
</tbody>
</table>
As it can be seen from Figure 1, all except the respective sums of squares of $A$, $B$, $C$, and $AB^2$ components lie roughly on a straight line. Therefore, all main effects and the $A \times B$ interaction are judged to be significant. This result is consistent with that of either Half-Normal plot in the first figure of Figure 1 or the analysis of variance in Table 2.
Table 2: Analysis of Variance for Strength of Textile Data

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>2.36276</td>
<td>1.18138</td>
<td>310.81</td>
<td>0.000</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>1.35125</td>
<td>0.67563</td>
<td>177.75</td>
<td>0.000</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>0.53203</td>
<td>0.26601</td>
<td>69.99</td>
<td>0.000</td>
</tr>
<tr>
<td>A x B</td>
<td>4</td>
<td>0.07590</td>
<td>0.01898</td>
<td>4.99</td>
<td>0.026</td>
</tr>
<tr>
<td>A x C</td>
<td>4</td>
<td>0.02586</td>
<td>0.00646</td>
<td>1.70</td>
<td>0.242</td>
</tr>
<tr>
<td>B x C</td>
<td>4</td>
<td>0.00297</td>
<td>0.00074</td>
<td>0.20</td>
<td>0.934</td>
</tr>
<tr>
<td>Error</td>
<td>8</td>
<td>0.03041</td>
<td>0.00380</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>26</td>
<td>4.38119</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.2.2 Chisquare Plot for the Sums of Squares of s Level Fractional Factorial Data

In many applications, it is highly impractical to run a full factorial experiment when there are seven, say, or more factors due to the run size consideration. In fact a run size of 64, say, is considered to be large in many physical experimentations. A common alternative is to use fractional factorial designs to plan experiments.

In an $s^{k-p}$ fractional factorial experiment, the main effects and all the interactions can be decomposed into $(s^k - 1)/(s - 1)$ components of $s - 1$ degrees of freedom each, and consequently the factorial effects account for $s^k - 1$ degrees of freedom, far exceeding the actual available degrees of freedom, $s^{k-p} - 1$. This is because of confounding in the fractional factorial design. In a regular fractional factorial design, that is, one which can be derived from a defining relation subgroup, all components can be grouped into $(s^{k-p} - 1)/(s - 1)$ aliased groups of
For instance, the defining relation for a $3^4-1$ resolution IV design is

$$I = ABCD^2,$$

where $I$ is the column of zeros. Following the similar notation as in Section 2.2.1, the defining relation is defined algebraically

$$z_1 + z_2 + z_3 + 2z_4 \equiv 0 \pmod{3}. \quad (2.1)$$

The aliasing pattern are then calculated based on the defining relation. For instance, adding $2z_3 \pmod{3}$ on both side of Equation (2.1), we have

$$z_1 + z_2 + 2z_4 \equiv 2z_3 \pmod{3},$$

or equivalently

$$C = ABD^2. \quad (2.2)$$

Note that $C = C^2$ because of $z_3 + 2z_3 \equiv 0 \pmod{3}$ and that $C$ is used by convention. Thus, $C$ is aliased with $ABD^2$. On the other hand, adding $z_3$ on both sides of Equation (2.1), we have

$$C = ABC^2D^2. \quad (2.3)$$

By combining results from Equations (2.2) and (2.3), we conclude that

$$C = ABD^2 = ABC^2D^2.$$

Other aliasing groups can be calculated likewise. We have thus 13 groups, each of two degrees of freedom, of aliasing components.
Under the null hypothesis that all components are insignificant, these 13 sums of squares constitute a random sample from $\sigma^2 \times \chi^2_2$ distribution. Therefore, when plotted against $\chi^2_2$ quantiles, they should lie roughly on a straight line. Consequently, any obvious departure from the straight line that is implicated by a component implies its significant impact on the response variable. This explains why chisquare plots can be employed to identify significant factorial components.

After decomposition and finding aliasing groups, we have total of 13 orthogonal components of same level which is the sufficient condition for making use of the chisquare plot.

Now we shall start describing the construction of a chisquare plot of sums of squares and how it is used for the identification of factorial effects. Consider an $s^{k-p}$ fractional factorial experiment.

**Construction of the plot**

**Step 1.** (Decomposing total sum of squares)

Proceed the decomposition of the total sum of squares ($s^{k-p} - 1$ degrees of freedom) into same degrees of $s - 1$ of sums of squares as described in the example of Section 2.2.1.

**Step 2.** (Obtaining aliased effect groups)

Find ($s^{k-p} - 1$)/($s - 1$) aliased effect groups by use of defining relation of the experiment.
Step 3. (Sorting the sums of squares)

Sort the corresponding sums of squares in increasing order.

Step 4. (Calculating chi-square quantiles)

Compute the chi-square quantiles (of $s - 1$ degrees of freedom).

Step 5. (Plotting quantile-quantile pairs)

Plot the sorted sums of squares against the chi-square quantiles.

**Use of the plot**

On the plot, smaller sums of squares will lie roughly on a straight line and a few largest sums of squares will likely stand clear off the straight line. The aliased group corresponding to these small sums of squares are considered to be insignificant while the largest sums of squares are having significant impact on the response. To determine which effect from the significant aliased group is truly significant, *effect hierarchical ordering principle* (see for example Hamada and Wu 2000) will be used. This principle states that lower order factorial effects are more likely to be important than higher order factorial effects. For example, for a significant aliased group with one main effect and one two-way interaction effect and one three-way interaction effect, first priority should be given to the main effect. Moreover, we can assume that three-way and higher interactions are negligible (this is because of the hierarchical ordering principle and the fact that interpretations and justification of higher order interactions are more difficult).
Therefore, if no main effect or two-way interaction is aliased with other main effects or two-way interactions, results will be very obvious. Otherwise one needs to consider the significant main effects to decide which two-way interaction is significant. For this purpose, *effect heredity principle* (Hamada and Wu 1992) will be used. This principle states that in order for an interaction to be significant, at least one of its parent factors should be significant. For example, for a significant aliased group with two two-way interactions and one three-way interaction, one may consider eliminating the three-way interaction effect according to hierarchical ordering principle. Subsequent to this, the two-way interaction is considered to be significant only if at least one of the two main effects involved in the interaction is significant.

Seat-Belt Data taken from Wu and Hamada (2000) is given below as an example. The data arisen from a 3-replicated design one-third-fraction factorial experiment of the $3^4$ is with one qualitative and three quantitative factors. There are 81 experimental runs and hence the total sums of squares has 80 degrees of freedom. There are $3^{4-1} \times (3 - 1) = 54$ degrees of freedom for the experimental error. The remaining 26 degrees of freedom can be used for estimating factorial effects. After decomposition of interactions, there are total of 40 components so that there are not enough degrees of freedom for all components. Therefore we cannot have all components on the plot as separate effects. Moreover, some effects will be aliased with other main effects or interactions. To find out this, defining
relation which will be used to define aliasing groups must be identified for this example. Column D in the seat-belt data which is in Appendix is the element-wise sum, modulus 3, of other three columns. We can define this relationship as,

$$D = ABC,$$  \hspace{1cm} (2.4)

where A, B, C, D whose levels are $z_1, z_2, z_3, z_4$ respectively are the main factors.

We can redefine Equation (2.4) in terms of $z_1, z_2, z_3, z_4$

$$z_4 \equiv z_1 + z_2 + z_3 \pmod{3}$$

or equivalently,

$$z_1 + z_2 + z_3 + 2z_4 \equiv 0 \pmod{3} \hspace{1cm} (2.5)$$

or

$$I = ABCD^2$$  \hspace{1cm} (2.6)

In Equation (2.5), coefficient of $z_4$ is represented by the squared power of D in Equation (2.6). Equation (2.6) gives the defining relation of Seat-Belt data, where $I$ is the column of zeros, which is identity element in group theory. Defining relation will be used to identify the aliasing patterns of Seat-Belt data. For instance, to find out the effects that are aliased with B main effect, we add $2z_2$ to both sides of Equation (2.5),

$$z_1 + z_2 + 2z_2 + z_3 + 2z_4 + 0 + 2z_2 \equiv 0 + 2z_2 \pmod{3}$$

or equivalently,

$$z_1 + z_3 + 2z_4 \equiv 2z_2 \pmod{3} \hspace{1cm} (2.7)$$
In terms of the effects, Equation (2.7) is equivalent to

\[
B = ACD^2
\]  

(2.8)

Thus, B is aliased with \( ACD^2 \).

And also, we can add \( z_2 \) to both sides of Equation (2.5), we will have

\[
z_1 + z_2 + z_2 + z_3 \equiv z_2 + z_4 \quad \text{(mod 3)}
\]

and add \( 2z_4 \) to both sides

\[
z_1 + 2z_2 + z_3 + 2z_4 \equiv z_2 \quad \text{(mod 3)}
\]

(2.9)

In terms of the effects, Equation (2.9) is equivalent to

\[
B = AB^2CD^2
\]

(2.10)

Thus, B is also aliased with \( AB^2CD^2 \).

Combining Equations (2.8) and (2.10), we have shown that B has two aliases, \( ACD^2 \) and \( AB^2CD^2 \):

\[
B = ACD^2 = AB^2CD^2
\]

Similarly, we can find all other aliasing groups. For instance, aliasing relations for A, B, C, D main effects and AB and AC interaction effects are shown below.
A = BCD^2 = AB^2C^2D
B = ACD^2 = AB^2CD^2
C = ABD^2 = ABC^2D^2
D = ABC = ABCD
AB = CD^2 = ABC^2D
AC = BD^2 = AB^2D

To see all aliased groups, refer to Table 12 in Appendix.

Since there are 27 runs with 13 orthogonal columns for this design, there will be 13 sets of aliased effects. After we decompose effects and find the defining relation and have 13 aliasing effects groups each with two degrees of freedom, we calculate the sums of squares of all 13 aliasing effects groups. Then we plot calculated \( \chi^2 \) quantiles against sorted sums of squares. Results are as following,
As it can be seen from Figure 2, all except the respective sums of squares of $A$, $C$, $D$ main effects, and $AC=BD^2$ and $AB=CD^2$ interaction components lie roughly on a straight line. Under the assumption that three-way and higher interactions are negligible, we conclude that $A$, $C$, $D$ main effects, and $AC=BD^2$ and $AB=CD^2$ interaction effects are significant. Since $A$ and $C$ main effects are significant, then we can conclude that $AC$ interaction effect from the first aliased
group \(AC=BD^2\) is more likely to be significant. Likewise, since \(C\) and \(D\) main effects are significant, then more likely \(CD^2\) interaction effect from the second aliased group \(AB=CD^2\) is significant. Finally, we can summarize significant effects as \(A, C, D, A \times C\) and \(C \times D\) for seat-belt example.

Analysis of Variance table (under the assumption that three-way and higher interactions are negligible) are also given for comparison. When we compare

Table 3: Analysis of Variance for Strength of Seat-Belt Data

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>34621746</td>
<td>17310873</td>
<td>85.58</td>
<td>0.000</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>938539</td>
<td>469270</td>
<td>2.32</td>
<td>0.108</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>9549481</td>
<td>4774741</td>
<td>23.61</td>
<td>0.000</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>4492927</td>
<td>2246464</td>
<td>11.11</td>
<td>0.000</td>
</tr>
<tr>
<td>(AB=CD^2)</td>
<td>2</td>
<td>2727451</td>
<td>1363725</td>
<td>6.74</td>
<td>0.002</td>
</tr>
<tr>
<td>(AB^2)</td>
<td>2</td>
<td>570795</td>
<td>285397</td>
<td>1.41</td>
<td>0.253</td>
</tr>
<tr>
<td>(AC=BD^2)</td>
<td>2</td>
<td>2985591</td>
<td>1492796</td>
<td>7.38</td>
<td>0.001</td>
</tr>
<tr>
<td>(AC^2)</td>
<td>2</td>
<td>886587</td>
<td>443294</td>
<td>2.19</td>
<td>0.122</td>
</tr>
<tr>
<td>(BC=AD^2)</td>
<td>2</td>
<td>427214</td>
<td>213607</td>
<td>1.06</td>
<td>0.355</td>
</tr>
<tr>
<td>(BC^2)</td>
<td>2</td>
<td>21134</td>
<td>10567</td>
<td>0.05</td>
<td>0.949</td>
</tr>
<tr>
<td>(AD)</td>
<td>2</td>
<td>263016</td>
<td>131508</td>
<td>0.65</td>
<td>0.526</td>
</tr>
<tr>
<td>(BD)</td>
<td>2</td>
<td>205537</td>
<td>102768</td>
<td>0.51</td>
<td>0.605</td>
</tr>
<tr>
<td>(CD)</td>
<td>2</td>
<td>245439</td>
<td>122720</td>
<td>0.61</td>
<td>0.549</td>
</tr>
<tr>
<td>Error</td>
<td>54</td>
<td>10922599</td>
<td>202270</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>80</td>
<td>68998897</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

chisquare plot result with analysis of variance for strength in Table 3 which also identifies \(A, C, D\) main effects and \(AC=BD^2\) and \(AB=CD^2\) interaction effects as significant, we reach the same conclusion.
2.3 Chisquare Plot for the Sums of Squares of $s^m \times 2$ Level Factorial Data

In practice, many experiments are of mixed level by nature. For this case, main effects do not have the same degrees of freedom.

In next two sections, we consider the case that two different level groups occur in the full or fractional factorial experiments. One group has at least two factors at the same level, $s, s > 2$, and other has one factor at two levels.

In another section, we consider that more than two different level groups occur in the full factorial experiments. Moreover, we consider that there is one factor at each group.

In this section, two slightly different methods for $s^m \times 2$ mixed level factorial data are proposed depending on whether the design is of full or fractional factorial. Moreover, one method for more than two different levels factorial data is proposed in Section 2.4.

2.3.1 Chisquare Plot for the Sums of Squares of $s^m \times 2$ Level Full Factorial Data

In this section, we consider $s^m \times 2$, mixed level data, where $s > 2$ and $m \geq 2$.

Since the use of our method of chisquare plot involves using sums of squares of the same degrees of freedom, these two different levels, $s$ and $2$, will have to be considered separately so that the same degrees of freedom can be obtained.

Under the assumption, 2-level factor is significant, first the chisquare plot
for pure level full factorial data explained in section 2.2.1 can be used on the group with at least two factors to identify significant effects for this group. Sums of squares involving only the s-level factors can be re-expressed as sums of squares of \((s - 1)\) degrees of freedom according to the orthogonal component systems. Moreover, sums of squares of the interactions involving the 2-level factor and at least one s-level factor can be re-expressed as the sums of squares of \((s - 1)\) degrees of freedom also. We demonstrate this with an example. For example, in \(3^3 \times 2\) experimental design, there are total of \(\left(\frac{s^m - 1}{s - 1}\right) = \left(\frac{3^3 - 1}{3 - 1}\right) = 13\) orthogonal column for three-level main effects and their interaction components each having two degrees of freedom in the respective sum of squares. Furthermore, total of 13 interaction components involving the 2-level factor and at least one 3-level factor will also have two degrees of freedom in the respective sums of squares. Therefore, there will be 26 components each having two degrees of freedom in the respective sums of squares to obtain the chisquare plot.

Now we shall start describing the construction of a chisquare plot of sums of squares for all effects and how it is used for the identification of factorial effects.

**Construction of the plot**

**Step 1.** (Decomposing total sum of squares)

Proceed the decomposition of the total sum of squares into same degrees as following steps:

**Step 1.A** (Obtaining the sum of squares of 2-level factor)
Find the sum of squares of 2-level factor.

**Step 1.B** (Obtaining the sums of squares of \((s^m - 1)/(s - 1)\) components)

Find the sums of squares of \((s^m - 1)/(s - 1)\) components of s-level main effects and their interaction components each having \(s - 1\) degrees of freedom in the respective sum of squares by using orthogonal columns.

**Step 1.C** (Obtaining the sums of squares of interaction components of the 2-level factor and at least one s-level factor)

Find the sums of squares of interaction components involving the 2-level factor and at least one s-level factor by following steps:

**Step 1.C.i** (Creating 2s-level column)

Multiply s-level column by 2-level column to create 2s-level column.

**Step 1.C.ii** (Calculating sum of squares of 2s-level column)

Find sum of squares of 2s-level column.

**Step 1.C.iii** (Calculating sums of squares of s-level and 2-level factors)

Find sums of squares of s-level and 2-level factors.

**Step 1.C.iv** (Calculating sums of squares of interaction component)

Subtract sums of squares of s-level and 2-level factors from sum of squares of 2s-level column to find sum of squares of interaction component involving the 2-level factor and at least one s-level factor.
Step 2. (Sorting the sums of squares)

Sort the $s - 1$ degrees of freedom sums of squares in increasing order.

Step 3. (Calculating chisquare quantiles)

Compute the chisquare quantiles (of $s - 1$ degrees of freedom).

Step 4. (Plotting quantile-quantile pairs)

Plot the sorted sums of squares against the chisquare quantiles.

Step 5. (Identifying significant effects and estimating $\sigma^2$)

As discussed earlier in Section 2.2.1, identify the significant effects by the distance from straight line on the plot. Find an estimate of $\sigma^2$.

Step 6. (Converting sum of squares, $SS$, of 2-level main effect to $s - 1$ degrees of freedom sum of squares, $SS'$, as the group do)

Sum of squares of 2-level main effect is converted to $s - 1$ degrees of freedom sum of squares as following:

1. Calculate

   $$\frac{SS}{\sigma^2}$$

   for sum of square of 2-level effect , which is a $\chi_1^2$ quantile.

2. Calculate the probability:

   $p = P(\chi_1^2 \leq \frac{SS}{\sigma^2}).$

3. Obtain the corresponding $\chi_{s-1}^2$ quantile, $q$, so that

   $p = P(\chi_{s-1}^2 \leq q).$
4. Multiply the previous result by $\hat{\sigma}^2$:

$$SS' = \hat{\sigma}^2 q.$$ 

**Step 7.** (Plotting quantile-quantile pairs for all effects)

Plot the sorted sums of squares of all effects including the converted 2-level sum of squares against the chisquare quantiles.

**Use of the plot**

On the plot, significant sums of squares stand clear off the straight line which covers the most of the data. On the other hand, insignificant sums of squares lie roughly on the straight line. If the result of the chisquare plot contradicts with the assumption that we made beginning of construction of the chisquare plot that 2-level factor is significant, we need to return to Step 5 and revise the estimate of $\hat{\sigma}^2$ and follow other steps after that.

Simulated mixed level full factorial data is given below as an example. For this example, we simulate 54 runs $3^3 \times 2$ full factorial data. Three level factors are $A$, $B$, $C$ and two level factor is $D$. We use following model to produce 54 runs

$$Y = \mu + A_i + B_i + D + A_iB_i + B_iD + \varepsilon$$

where $\mu = 20$ and $\varepsilon \sim N(0, \sigma^2)$ with effects
Simulated data and sums of squares are in Appendix. Two-way interactions involving two three level factors having four degrees of freedom are decomposed into two components, for instance, $A \times B$ interaction is decomposed into $AB$, and $AB^2$. Three-way interactions involving one two level factor and two three level factors having four degrees of freedom are decomposed into two components, for instance, $A \times B \times D$ is decomposed into $ABD$, and $AB^2D$. Three-way interaction, $A \times B \times C$, involving three 3-level factors having eight degrees of freedom is decomposed into four components, $ABC$, $ABC^2$, $AB^3C$, and $AB^2C^2$. Similarly, four-way interaction, $A \times B \times C \times D$, involving one two level factor and three three level factors having eight degrees of freedom is decomposed into four components, $ABCD$, $ABC^2D$, $AB^3CD$, and $AB^2C^2D$.

Sums of squares of 13 components involving only the s-level factors are calculated by using 13 orthogonal columns. Sums of squares of 13 interactions involving the 2-level factor and at least one s-level factor are calculated by following Step 1.C.i through Step 1.C.iv. For instance, sum of squares of $AD$ effect is calculated as following. First we multiply column $A$ by column $D$ to create 6-level column and then calculate sums of squares for this 6-level column, column $A$ and column $D$. We subtract sums of squares of column $A$ and column $D$ from sum of squares of
6-level column to obtain sum of squares of $AD$ effect. By using 26 components having two degrees of freedom each, we plot calculated $\chi^2_2$ quantiles against sorted sums of squares in increasing order. From the first chisquare plot of Figure 3, we identify the significant effects as $A$ main effect and $AB^2D$, $AD$ and $CD$ interaction effects.

To estimate $\sigma^2$, we pool all sums of squares of effects except sums of squares of significant effects and 2-level, $\hat{\sigma}^2 = \frac{132.6115}{44} = 3.013898$. By using the formula $\frac{SS}{\sigma^2} = \frac{5.9566}{3.013898} = 1.976377$, we calculate the value of $\chi^2_1$ quantile for sum of squares of 2-level effect. We obtain the probability of $\chi^2_1$, $p = P(\chi^2_1 \leq 1.976377) = 0.8402274$ and find the corresponding $\chi^2_2$ quantile, $0.8402274 = P(\chi^2_2 \leq q)$, $q = 3.668007$.

Finally, to have the new sum of squares, we need to multiply $\chi^2_2$ quantile by $\hat{\sigma}^2$, $SS' = 3.013898 \times 3.668007 = 11.055$. Two-level factor effect is converted to two degrees of freedom sums of squares as the group with three 3-level main factors does.

Now there are 27 components with two degrees of freedom. Since these components are orthogonal, we just follow the steps for pure level full factorial data explained in Section 2.2.1. Therefore, we sort the sums of squares and obtain $\chi^2_2$ quantiles. Finally, plot calculated $\chi^2_2$ quantiles against sorted sums of squares, $SS_t$. From the second chisquare plot of the Figure 3, most effects are close to each other and just a few values are far away from others. By using this information and Figure 3, we can draw an imaginary straight line to identify the significant ef-
Figure 3: Chi-square Plot of Simulated Mixed Level Full Factorial Data

Effects. Effects which are on the line or very close to line will be judged insignificant and effects which stand clear off the line are judged significant. For the simulated-data, we conclude that A main effect and $AB^2$, $AD$ and $CD$ interaction effects are significant. $D$ main effect is not significant which is a contradiction to our assumption that 2-level effect is significant. We revise estimate of $\sigma^2$, $\sigma^2_{new} = \frac{138.5681}{45}$ and convert sum of squares of 2-level $D$, $SS'_{new} = 11.12363$ by using revised $\sigma^2$ and
plot again which is the third plot of Figure 3. We decide that $D$ is insignificant.

Analysis of Variance table (under the assumption that three-way and higher order interactions are negligible) are also given for comparison. When we com-

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>2</td>
<td>86.649</td>
<td>43.324</td>
<td>11.13</td>
<td>0.000</td>
</tr>
<tr>
<td>$B$</td>
<td>2</td>
<td>15.299</td>
<td>7.649</td>
<td>1.97</td>
<td>0.159</td>
</tr>
<tr>
<td>$C$</td>
<td>2</td>
<td>0.876</td>
<td>0.438</td>
<td>0.11</td>
<td>0.894</td>
</tr>
<tr>
<td>$D$</td>
<td>1</td>
<td>5.953</td>
<td>5.953</td>
<td>1.53</td>
<td>0.226</td>
</tr>
<tr>
<td>$A \times B$</td>
<td>4</td>
<td>49.078</td>
<td>12.269</td>
<td>3.15</td>
<td>0.029</td>
</tr>
<tr>
<td>$A \times C$</td>
<td>4</td>
<td>5.976</td>
<td>1.494</td>
<td>0.38</td>
<td>0.818</td>
</tr>
<tr>
<td>$A \times D$</td>
<td>2</td>
<td>29.842</td>
<td>14.921</td>
<td>3.83</td>
<td>0.034</td>
</tr>
<tr>
<td>$B \times C$</td>
<td>4</td>
<td>5.643</td>
<td>1.411</td>
<td>0.36</td>
<td>0.833</td>
</tr>
<tr>
<td>$B \times D$</td>
<td>2</td>
<td>1.121</td>
<td>0.561</td>
<td>0.14</td>
<td>0.866</td>
</tr>
<tr>
<td>$C \times D$</td>
<td>2</td>
<td>27.706</td>
<td>13.853</td>
<td>3.56</td>
<td>0.042</td>
</tr>
<tr>
<td>Error</td>
<td>28</td>
<td>108.967</td>
<td>3.892</td>
<td></td>
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</tr>
<tr>
<td>Total</td>
<td>53</td>
<td>337.111</td>
<td></td>
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</tbody>
</table>

pare chisquare plot result with analysis of variance for strength in Table 3 which identifies $A$ main effect and $A \times B$, $A \times D$ and $C \times D$ interaction effects as significant, we reach the same conclusion.

2.3.2 Chisquare Plot for the Sums of Squares of $s^m \times 2$ Level Fractional Factorial Data

In this section, we consider $s^{m-l} \times 2$, $s \geq 3$, mixed level fractional factorial data.

Under the assumption, 2-level factor is significant, first the chisquare plot for pure level fractional factorial data explained in section 2.2.2 can be used on
the group with at least two factors to identify significant effects for this group. Sums of squares involving only the s-level factors can be re-expressed as sums of squares of \((s - 1)\) degrees of freedom according to the orthogonal component systems. Moreover, sums of squares of the interactions involving the 2-level factor and at least one s-level factor can be re-expressed as the sums of squares of \((s - 1)\) degrees of freedom also.

Below are steps to obtain chisquare plot for mixed level fractional factorial data in lights of the fact that the use of chisquare plot requires same degrees of freedom for sums of squares of components.

We start with the group with at least two factors as we do in mixed level full factorial data. Moreover, we follow similar concept for fractional factorial data. Consider an \(s^{m-l} \times 2\) fractional factorial experiment.

**Construction of the plot**

**Step 1.** (Decomposing total sum of squares)

Find how many aliasing groups there are by using the formula \((s^{m-l} - 1)/(s - 1)\). Produce necessary number of orthogonal columns. Proceed the decomposition of the total sum of squares into same degrees as following steps:

**Step 1.A** (Obtaining the sum of squares of 2-level factor)

Find the sum of squares of 2-level factor.

**Step 1.B** (Obtaining the sums of squares of \((s^{m-l} - 1)/(s - 1)\) aliasing
groups)

Find the sums of squares of \((s^{n-l} - 1)/(s - 1)\) aliasing groups involving s-level main effects and their interaction components each having \(s - 1\) degrees of freedom in the respective sum of squares by using orthogonal columns.

**Step 1.C** (Obtaining the sums of squares of interaction components of the 2-level factor and at least one s-level column)

Find the sums of squares of interaction components involving the 2-level factor and at least one s-level aliasing group by following steps:

**Step 1.C.i** (Creating 2s-level column)

Multiply s-level column by 2-level column to create 2s-level column.

**Step 1.C.ii** (Calculating sum of squares of 2s-level column)

Find sum of squares of 2s-level column.

**Step 1.C.iii** (Calculating sums of squares of s-level aliasing group and 2-level factors)

Find sums of squares of s-level aliasing group and 2-level factor.

**Step 1.C.iv** (Calculating sums of squares of interaction component)

Subtract sums of squares of s-level aliasing group and 2-level factor from sum of squares of 2s-level column to find sum of squares of interaction component involving the 2-level factor and at least one
s-level aliasing group.

**Step 2.** (Obtaining aliased effect groups)

Since the design is fractional factorial, there are not enough degrees of freedom for all components. Effects will be aliasing with the other effects. To have all aliasing effects groups, defining relation will be used. Therefore, find \((s^{m-l} - 1)/(s - 1)\) aliased effect groups by use of defining relation of the experiment.

**Step 3.** (Sorting the sums of squares)

Sort the corresponding sums of squares except the sum of square of 2-level main component in increasing order.

**Step 4.** (Calculating chisquare quantiles)

Compute the chisquare quantiles (of \(s - 1\) degrees of freedom).

**Step 5.** (Plotting quantile-quantile pairs)

Plot the sorted sums of squares against the chisquare quantiles.

**Step 6.** (Identifying significant effects and estimating \(\sigma^2\))

As discussed earlier in Section 2.2.2, identify the significant effects by the distance from the straight line on the plot and under the assumption that three-way and higher interactions are negligible.

**Step 7.** (Converting sum of squares, \(SS\), of 2-level factor effect to the same degrees of freedom sums of squares, \(SS'\), as the group do)
Sum of squares of 2-level factor effect is converted to the same degrees of freedom sums of squares as the group which has at least two main factors and has the same degrees of freedom for each factor by using estimated $\sigma^2$.

Steps to obtain new sum of squares for 2-level factor are as followed,

1. Calculate

$$\frac{SS}{\sigma^2}$$

for sum of squares of 2-level effect, which is a $\chi_1^2$ quantile.

2. Calculate the probability:

$$p = P(\chi_1^2 \leq \frac{SS}{\sigma^2}).$$

3. Obtain the corresponding $\chi_{s-1}^2$ quantile, q, so that

$$p = P(\chi_{s-1}^2 \leq q).$$

4. Multiply the previous result by $\hat{\sigma}^2$:

$$SS' = \hat{\sigma}^2 q.$$

**Step 8.** (Plotting quantile-quantile pairs for all effects)

Plot the sorted sums of squares of all effects including 2-level factor effect against the chisquare quantiles.

**Use of the plot**

On the plot, smaller sums of squares will lie roughly on a straight line and a few largest sums of squares will likely stand clear off the straight line. The aliased
group corresponding to these small sums of squares are considered to be insignificant while the largest sums of squares are having significant impact on the response. To determine which effect from the significant aliased group is truly significant, *effect hierarchical ordering principle* and *effect heredity principle* as described in Section 2.2.2 will be used. Moreover, we can assume that three-way and higher order interactions are negligible (this is because of the hierarchical ordering principle and the fact that interpretations and justification of higher order interactions are more difficult). Therefore, if no main effects or two-way interactions is aliased with other main effects or two-way interactions, results will be very obvious. Otherwise one needs to consider the significant main effects to decide which two-way interaction is significant. If the result of the chisquare plot contradicts with the assumption that we made beginning of constructing of the chisquare plot that 2-level factor is significant, we need to return to Step 5 and revise $\sigma^2$ and follow other steps after that.

Simulated mixed level fractional factorial data is given below as an example.

For this example, we simulate 54 runs $3^{4-1} \times 2$ fractional factorial data. Three level factors are $A$, $B$, $C$, $D$ and two level factor is $E$. Chisquare plot is given in Figure 4. $A$, $B$ and $AE$ are identified as significant among total of 26 two degrees of freedom sums of squares in first part of Figure 4. Then, one-degree-of-freedom sum of square is converted to two-degree-of-freedom and plotted with others in
second part of Figure 4. $A$, $B$, $E$ and $AE$ are identified as significant. Since 2-level factor $E$ is significant, there is no contradiction to the assumption that 2-level factor is significant and there is no need for further process.

![Chi-square Plot of Simulated Mixed Level Fractional Factorial Data](image)

**Figure 4:** Chi-square Plot of Simulated Mixed Level Fractional Factorial Data

### 2.4 Chisquare Plot for the Other Types of Mixed Level Factorial Data

In this section, we illustrate the use of chisquare plots for other types of mixed level factorial data. We will proceed first with examples on some $4 \times 3 \times 2$
full factorial data. Some mixed level data of other types will also be discussed theoretically.

2.4.1 A 4 × 3 × 2 Full Factorial Example

Since the use of the chisquare plot involves using sums of squares of the same degrees of freedom, one of three different levels will have to be decomposed into some components by method of replacement (to be discussed later) according to Addelman (1962) so that the degrees of freedom of components of this factor can be the same as one of other two factors. For example, in 4 × 3 × 2 factorial data, 4-level factor with three degrees of freedom in its sum of squares can be decomposed into three two-level columns each having one degree of freedom so that there will be four two-level columns and one three-level factor in the model.

An example of using method of replacement (Addelman 1962) will be given below in Table 5. Generating one 4-level factors from three 2-level factor or vice versa is shown here.

Table 5: Generating Three 2-level Factors from One 4-level Factor.

<table>
<thead>
<tr>
<th>2-level factors</th>
<th>4-level factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3</td>
<td>0</td>
</tr>
<tr>
<td>0 0 1</td>
<td>1</td>
</tr>
<tr>
<td>0 1 0</td>
<td>1</td>
</tr>
<tr>
<td>1 0 0</td>
<td>2</td>
</tr>
<tr>
<td>1 1 1</td>
<td>3</td>
</tr>
</tbody>
</table>
Since these new components after method of replacement are orthogonal components of main effect, design is called orthogonal components system as described before.

One of three examples of $4 \times 3 \times 2$ full factorial data which is taken from Devore (2000) and other two are taken from Walpole and Myers and Myers (1998) is given below and other two examples is given in Chapter IV. In the first example, 4-level factor is first decomposed into three 2-level components so that there will be four 2-level components and one 3-level factor in the design. After decomposition, there are total of seven components each having one degree of freedom and total of eight components each having two degrees of freedom. Chisquare plot of any one of two groups, say for seven components each having one degree of freedom, will be used to identify significant effects and estimate of $\sigma^2$. Sums of squares of eight components each having two degrees of freedom will be converted to one degree of freedom sums of squares. All 15 components each having one degree of freedom now will be plotted to identify significant effects. Choosing the first group to be plotted makes no differences for final result.

Chisquare plots for this example can be seen in Figure 5 and Analysis of Variance table is in Table 6.

As it can be seen from chisquare plot and Analysis of Variance table, $A$, $B$ and $C$ main effects are identified as significant. The results are consistent.
2.4.2 Other Examples

We shall give some hypothetical examples.

Example 1. A $6 \times 4$ full factorial.

Let $A$ be the 6-level factor and $B$ be the 4-level factor. Factor $A$ can be decomposed into $A_1$ with 3-level, $A_2$ with 2-level, and their interaction. Factor $B$ can be decomposed into three 2-level columns, $B_1$, $B_2$, $B_3$. Then there
Table 6: Analysis of Variances for Y of Example One

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>2.1600</td>
<td>0.7200</td>
<td>3.71</td>
<td>0.022</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>2.0617</td>
<td>1.0308</td>
<td>5.31</td>
<td>0.011</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>1.2033</td>
<td>1.2033</td>
<td>6.20</td>
<td>0.019</td>
</tr>
<tr>
<td>A x B</td>
<td>6</td>
<td>1.1250</td>
<td>0.1875</td>
<td>0.97</td>
<td>0.465</td>
</tr>
<tr>
<td>A x C</td>
<td>3</td>
<td>0.7500</td>
<td>0.2500</td>
<td>1.29</td>
<td>0.297</td>
</tr>
<tr>
<td>B x C</td>
<td>2</td>
<td>1.0617</td>
<td>0.5308</td>
<td>2.73</td>
<td>0.081</td>
</tr>
<tr>
<td>Error</td>
<td>30</td>
<td>5.8250</td>
<td>0.1942</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>47</td>
<td>14.1867</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

are eight 2-degrees-of-freedom sums of squares:

\[ A_1, A_1A_2, A_1B_1, A_1B_2, A_1B_3, A_1A_2B_1, A_1A_2B_2, \text{ and } A_1A_2B_3. \]

On the other hand, there are seven 1-degree-of-freedom sums of squares:

\[ A_2, B_1, B_2, B_3, A_2B_1, A_2B_2, A_2B_3. \]

The use of chi-square plot is similar to that in Section 2.4.1.

**Example 2.** A $8 \times 3$ full factorial.

Let $A$ be the 8-level factor and $B$ be the 3-level factor. Factor $A$ can be decomposed into seven 2-level columns, say $A_i$, $i = 1, \ldots, 7$, each is of one degree of freedom. On the other hand, there are eight 2-degrees-of-freedom sums of squares: $B$, and $A_iB$, $i = 1, \ldots, 7$. Then the use of chi-square plot is similar to that in Section 2.4.1.
CHAPTER III
OTHER DECOMPOSITION METHODS

3.1 Introduction

In this section, other decomposition methods will be considered to have the chisquare plot to identify the significant effects of mixed and pure level full factorial and fractional factorial experiments. These decomposition methods will be based on the orthogonal contrasts (see, for example, Wu and Hamada, 2000). For example, for a level factor, there will be $a - 1$ orthogonal contrasts partitioning the sums of squares into $a - 1$ independent single-degree-of-freedom components.

Since our method of using chisquare plot involves using sum of squares of the same degrees of freedom (to be discussed later), some single-degree-of-freedom components of a factor effect will be combined to have the same degrees of freedom as main effects do.

Now we shall start describing the decomposition of sums of squares by using orthogonal contrast method.

3.2 Chisquare Plot for Orthogonal Contrasts Partitioning Sums of Squares of Fractional Factorial Data

In the previous section, orthogonal components system (see, for example, Wu and Hamada, 2000) was used as a decomposition method of sums of squares. In this section, for all four cases of experimental data mentioned in the previous section, we will use orthogonal contrasts (see, for example, Wu and Hamada, 2000)
to decompose the sums of squares into single-degree-of-freedom sum of square. Steps to obtain chisquare plot for each case will be the same as the steps in previous corresponding section. Therefore, no construction steps of plot will be presented here.

On the other hand, since the use of chisquare plot involves using sums of squares of the same degrees of freedom as main effects do, we will practise three methods (possible number of methods can be increased when there are more main effects in the model) to obtain same degrees of freedom sums of squares.

For this purpose, first, main effects will be decomposed into single-degree-of-freedom according to orthogonal contrast. For instance, a 3-level factor will be decomposed into linear and quadratic contrasts whose levels will be shown in Table 7.

Table 7: Some Coefficients of Orthogonal Contrasts.

<table>
<thead>
<tr>
<th>Level</th>
<th>Linear</th>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two level</td>
<td>-1,1</td>
<td></td>
</tr>
<tr>
<td>Three level</td>
<td>-1,0,1</td>
<td>1,-2,1</td>
</tr>
<tr>
<td>Quadratic</td>
<td>1,-2,1</td>
<td></td>
</tr>
<tr>
<td>Four level</td>
<td>-3,-1,1,3</td>
<td>1,-1,-1,1</td>
</tr>
<tr>
<td>Cubic</td>
<td>-1,3,-3,1</td>
<td>-1,3,-3,1</td>
</tr>
</tbody>
</table>

Then single-degree-of-freedom sums of squares of interaction and main effects will be calculated and many different methods of various combinations of single-degree-of-freedom will be used to have same degrees of freedom as main
effects do. Here are three examples (many methods of different combinations of contrasts will result in the same conclusion but here we just give the explanations for three methods).

**Method 1.** Adding necessary number of largest sums of squares to have the same degrees of freedom as main effects do.

**Method 2.** Adding some largest sums of squares and some smallest sums of squares together.

**Method 3.** Adding any sums of squares together to obtain the same degrees of freedom as main effects do.

Other steps to construct the chi-square plot for the sums of squares of pure and mixed level full and fractional factorial data will be the same as the steps that are described in the previous sections of this chapter.

As an example, Textile Data will be revisited. We will use this data set to demonstrate that, using the three different methods as mentioned in this section will give consistent result and coincide with the result by using orthogonal component system as discussed in the previous chapter.

Since the main effects have three levels, they are decomposed into two orthogonal contrasts which are linear and quadratic. For instance, A main effect is decomposed into $A_l$ and $A_q$. After decomposing all three main effects into
two components, two-way and higher interaction components will be found by multiplying corresponding main effects components. Therefore, total of 26 single-degree-of-freedom components will be obtained for Textile Data can be seen in Table 10 in Appendix A.

Next step is to calculate corresponding sums of squares of all 26 components. For this purpose, equation (3.1) (see, for example, Montgomery, 2001) will be used

$$SS_{c} = \frac{\left(\sum_{i=1}^{a} c_{i}y_{i}\right)^{2}}{\sum_{i=1}^{a} n_{i}c_{i}^{2}}$$

(3.1)

where $c_{i}$ is orthogonal contrast coefficient corresponding to $i$th level, and $a$ is the total response corresponding to $i$th level, and $y_{i}$ is the total response of $i$th level, and $n_{i}$ is the number of observations for $i$th level.

Since main effects have two degrees of freedom, sums of squares of some components from an interaction will be added together to have two degrees of freedom. For instance, in the Textile Data the four single-degree-of-freedom components of the $A \times B$ interaction are $A_{1}B_{1}$, $A_{1}B_{q}$, $A_{q}B_{1}$ and $A_{q}B_{q}$. We then can combine two single-degree-of-freedom in three different ways. Three methods described above will be used to calculate sums of squares each having two degrees of freedom.

Now, there are 13 two degrees of freedom of sums of squares (including the main effects) which is the requirement to have the chisquare plot. Steps described
in previous section will be followed to construct the plot for all three cases for this example. In Figure 6, \( A_{1q}B_q \) stands for the addition of two single-degree-of sums of squares:

\[
A_{1q}B_q = A_1B_q + A_qB_q,
\]

and consequently it is of two degrees of freedom.

The same procedures will be exercised to obtain the decomposition for all cases explained in previous sections. (see Chapter IV for examples). Moreover, steps for constructing the chisquare plot will be the same as in previous sections. See Figure 7, Figure 8 and Figure 9. The results of three methods are consistent with each other and also with the results in the previous chapter.

See Chapter IV for more chisquare plots with data decomposed by orthogonal contrast method.
Figure 6: Textile Data with Method 1, 2, and 3
Before the Inclusion of 2-degree-of-freedom Sum of Squares

After the inclusion of 2-degree-of-freedom Sum of Squares

Figure 7: Example Two with Method 1
Figure 8: Example Two with Method 2
Figure 9: Example Two with Method 3

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CHAPTER IV
EXAMPLES

4.1 Examples Using Orthogonal Components System

Blood Glucose data which is taken from the paper by Henkin (1986) has 18 runs and is a \((3^7 - 4 \times 2)\) fractional factorial design with one qualitative factor which has two levels and seven 3-level quantitative factors. The design is non-regular and effects have complex aliasing patterns. First, we run chisquare plot for seven 3-level factors and identify the significant effects. Then, we convert one degree of freedom sums of squares to two degrees of freedom sums of squares.
Figure 10: Chi-square Plot of Blood Glucose Data
Chisquare plot identifies $E$ and $F$ as significant, with $A$ on the borderline. This result is similar to that of Hamada and Wu (1992). Consequently, these authors fitted a model including only $E_q$, $F_q$, and $E_q F_q$ with an $R^2$ of 68.4%. In Hamada and Wu (2000), however, the same authors fitted a completely different model including only $B_t$, $B_t H_q$, and $B_q H_q$ with an $R^2$ of 85.5%. It appears that both methods (chisquare and half-normal plots) failed to identify the “right” terms.
completely. Nevertheless, the failure should largely be attributed to the complex aliasing patterns common to all nonregular designs. Moreover, any graphical method is used to aid a modelling task but not to replace it.

**Example:** Engineering Data come from a $4 \times 3 \times 2$ full factorial experiment. The 4-level factor is first decomposed into three 2-level components so that there will be four 2-level components and one three-level factor in the design. After decomposition, there are total of seven components each having one degree of freedom, and eight components each having two degrees of freedom. Chisquare plot of any one of two groups, say for seven components each having one degree of freedom, will be used to identify significant effects and the estimate of $\sigma^2$. Sums of squares of eight components each having two degrees of freedom will be converted to one degree of freedom sums of squares. All 15 components each having one degree of freedom now will be plotted to identify significant effects. (Starting from the chisquare plot of 2-degrees-of-freedom sums of squares and then converting 1-degree-of-freedom sums of squares to 2-degrees-of-freedom sums of squares gives essentially the same result in this example.) Chisquare plots for this examples can be seen in Figure 13 and Figure 12 and Analysis of Variance tables are given in Table 8 and Table 9.
Before the Inclusion of 2-degree-of-freedom Sum of Squares

After the Inclusion of 2-degree-of-freedom Sum of Squares

Figure 12: Chi-square Plot of Example Two
Before the Inclusion of 2-degree-of-freedom Sum of Squares

After the Inclusion of 2-degree-of-freedom Sum of Squares

Figure 13: Chi-square Plot of Example Three
Table 8: Analysis of Variance for Y of Example Two

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>145.46</td>
<td>48.49</td>
<td>1.19</td>
<td>0.390</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>159.25</td>
<td>79.63</td>
<td>1.95</td>
<td>0.220</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>2340.38</td>
<td>2340.38</td>
<td>57.45</td>
<td>0.000</td>
</tr>
<tr>
<td>A × B</td>
<td>6</td>
<td>478.42</td>
<td>79.74</td>
<td>1.96</td>
<td>0.217</td>
</tr>
<tr>
<td>A × C</td>
<td>3</td>
<td>240.46</td>
<td>80.15</td>
<td>1.97</td>
<td>0.220</td>
</tr>
<tr>
<td>B × C</td>
<td>2</td>
<td>795.25</td>
<td>397.63</td>
<td>9.76</td>
<td>0.013</td>
</tr>
<tr>
<td>Error</td>
<td>6</td>
<td>244.42</td>
<td>40.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23</td>
<td>4403.63</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9: Analysis of Variance for Y of Example Three

<table>
<thead>
<tr>
<th>Source</th>
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<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>393.417</td>
<td>131.139</td>
<td>22.37</td>
<td>0.00</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>12.896</td>
<td>6.448</td>
<td>1.10</td>
<td>0.338</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>100.042</td>
<td>100.042</td>
<td>17.06</td>
<td>0.000</td>
</tr>
<tr>
<td>A × B</td>
<td>6</td>
<td>71.021</td>
<td>11.837</td>
<td>2.02</td>
<td>0.073</td>
</tr>
<tr>
<td>A × C</td>
<td>3</td>
<td>1.542</td>
<td>0.514</td>
<td>0.09</td>
<td>0.967</td>
</tr>
<tr>
<td>B × C</td>
<td>2</td>
<td>1.646</td>
<td>0.823</td>
<td>0.14</td>
<td>0.869</td>
</tr>
<tr>
<td>Error</td>
<td>78</td>
<td>457.271</td>
<td>5.862</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>95</td>
<td>1037.833</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example analyzed in Chapter 2.4.1 will be reanalyzed here by a different approach. There are total of seven 1-degree-of-freedom sums of squares and total of eight 2-degree-of-freedom sums of squares after decomposition. First, chisquare plot will be run for first group of sums of squares each having one degree of freedom (group order to run chisquare does not affect the results) and significant

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effects will be identified. Then, chisquare plot for the other group will be run and significant effects will be identified. Estimate of $\sigma^2$ will be obtained by use of pooling insignificant sums of squares in two groups together. The estimate of $\sigma^2$ will be used to convert two degrees of freedom sums of squares to one degree of freedom sums of squares. The same process can be followed to convert one degree of freedom sums of squares to two degrees of freedom sums of squares. Finally, plot all 15 same degrees of freedom sums of squares for both cases (see Figure 14). Any effect appears in at least one of the two plots is to be considered in the modelling task.
Figure 14: Chi-square Plot of Example 3

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4.2 Examples Using Orthogonal Contrasts Method

Figure 15: Example 3 with Method 1
Figure 16: Example 3 with Method 2
Figure 17: Example 3 with Method 3
CHAPTER V
SUMMARY AND SUGGESTIONS FOR FUTURE RESEARCH

5.1 Summary

In Chapter II, we have introduced a new graphical method to identify significant factors in a factorial experiment. Our proposed methods for pure level full factorial or fractional factorial experiments supply efficient results for all possible regular designs. However, our proposed methods for mixed level full factorial or fractional factorial experiments have been studied for some special cases in this dissertation.

For all proposed methods, factors can be either qualitative or quantitative. Moreover, these methods are efficient whether the factor levels are equally spaced or not.

Each proposed method is studied with some decomposition techniques, for example orthogonal components system explained in Chapter II and orthogonal contrast method explained in Chapter III.

In Chapter IV, some more examples for different cases are illustrated. Also, some different approaches to the problems discussed earlier are presented to demonstrate the flexibility in using chisquare plots. All data sets and some other necessary information are given in Appendix A and Appendix B.
5.2 Suggestions for Future Research

Further studies for different mixed level, especially fractional factorial, experiment settings are needed.

Different decomposition methods can be studied in the future.

Advantages of Chisquare plot over Half-Normal plot such as

1. the former can be applied disregarding whether the factor levels are equally space or not;

2. and that the former can be used whether the factors are quantitative or not can be examined in more details with examples to give a broad sense of comparison.
BIBLIOGRAPHY


### APPENDIX A

Data and Sums of Squares

Textile Data

Table 10: Description and Sums Squares of Textile Data

<table>
<thead>
<tr>
<th>Factor</th>
<th>Description</th>
<th>Level 0</th>
<th>Level 1</th>
<th>Level 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>length of specimen</td>
<td>250 (mm)</td>
<td>300</td>
<td>350</td>
</tr>
<tr>
<td>B</td>
<td>amplitude of load cycle</td>
<td>8 (mm)</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>load</td>
<td>40 (mm)</td>
<td>45</td>
<td>50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>A</th>
<th>B</th>
<th>AB</th>
<th>AB²</th>
<th>C</th>
<th>AC</th>
<th>AC²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2.36276</td>
<td>1.35125</td>
<td>0.02936</td>
<td>0.04654</td>
<td>0.53203</td>
<td>0.01321</td>
<td>0.01265</td>
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<table>
<thead>
<tr>
<th></th>
<th></th>
<th>BC</th>
<th>BC²</th>
<th>ABC</th>
<th>AB²C²</th>
<th>AB²C</th>
<th>ABC²</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.00021</td>
<td>0.000276</td>
<td>0.0005</td>
<td>0.0143</td>
<td>0.0046</td>
<td>0.0108</td>
</tr>
</tbody>
</table>

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Table 11: Textile Data

<table>
<thead>
<tr>
<th>factor</th>
<th>cycles of failure</th>
<th>log(Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
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<tr>
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<td>1</td>
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</tr>
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<tr>
<td>2</td>
<td>1</td>
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<td>1</td>
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<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
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# Seat-Belt Data, Strength Location

Table 12: Description and Aliasing Pattern of Seat-Belt Data

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<th>Description</th>
<th>Level 0</th>
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<td>1400</td>
<td>1700</td>
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<td>B</td>
<td>die flat</td>
<td>10.0(mm)</td>
<td>10.2</td>
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<td>C</td>
<td>crimp length</td>
<td>18(mm)</td>
<td>23</td>
<td>27</td>
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<td>D</td>
<td>anchor lot</td>
<td>P74</td>
<td>P75</td>
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\[
\begin{align*}
A &= BCD^2 = AB^2 C^2 D, \\
B &= ACD^2 = AB^2 CD, \\
C &= ABD^2 = ABC^2 D, \\
D &= ABC = ABCD, \\
AB &= CD^2 C = ABC^2 D, \\
AB^2 &= AC^2 D = BC^2 D, \\
AC &= BD^2 = AB^2 D, \\
AC^2 &= AB^2 D = BC^2 D, \\
AD &= AB^2 C^2 = BCD, \\
AD^2 &= BC = AB^2 C^2 D, \\
BC^2 &= AB^2 D^2 = AC^2 D, \\
BD &= AB^2 C = ACD, \\
CD &= ABC^2 = ABD.
\end{align*}
\]
Table 13: Seat-Belt Data, Strength Location

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Table 14: Sums of Squares of Seat-Belt Data, Strength Location

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<tr>
<th>A</th>
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<th>C</th>
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54 runs $3^3 \times 2$ Full Factorial Design

3 level factors: A, B, C

2 level factor : D

Simulated data (in standard order of factors A, B, C, and D):

22.08 19.46 16.61 19.12 23.93 22.07
19.81 21.76 20.41 18.03 21.74 18.78
17.65 21.57 20.42 16.47 18.69 17.05
20.10 20.95 15.96 16.20 23.27 24.36
19.96 20.11 18.04 18.16 22.46 19.35
16.49 20.07 24.28 19.49 20.99 23.75
20.38 21.93 24.06 17.03 22.19 22.75

Table 15: Sums of Squares of Simulated Data

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<th>D</th>
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<th>B</th>
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Table 16: Example One Data

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APPENDIX B

R/Splus Codes

"chisqplot" <- function(data, dof=2, datax=T,
  dlab=as.expression(substitute(chi[a]^2*plain(" Quantiles"),
    list(a=dof))),
  xlab=if(datax) "Sample Quantiles" else dlab,
  ylab=if(datax) dlab else "Sample Quantiles",
  identifying=0, line=as.logical(identifying), pos=NULL,
  offset=0.5, labels=NULL, pch=20, lty=3, col=2, ...)

# Chisquare Quantile Plot

# Input :
#   Required argument
#   1. data --- numeric data vector
#   Defaulted arguments
#   1. dof ---- a numeric value (default is 2) giving
#      degrees of freedom for chi-square plot
#   2. datax---- a logical value (default is TRUE) to flag
#      that data should be placed on x axis or not
#   3. dlab --- data label string (default as seen above) giving axis label for data
#   4. xlab --- x axis label (title), default = 'Sample Quantiles' if datax is TRUE, otherwise dlab
#   5. ylab --- y axis label (title), default is dlab if datax is TRUE, otherwise 'Sample Quantiles'
#   6. identifying---- a numeric value (nonnegative 'integer#' default is 0) to flag the number of points to be identified as the largest effects
#   7. line ---- a logical value (default is FALSE if identifying is 0, and TRUE, otherwise) to flag the plotting of straight line which is supposed to capture small effects
#   8. labels--- character or expression vector of data labels (default is NULL)
#   9. pos --- positions to place the labels, default is NULL; if specified, it is a numeric vector of length 'identifying' in which 1=bottom, 2=left, 3=top, and 4=right
#   10. offset---offset value (default is 0.5) as a fraction of current character expansion (i.e. character point size) to place effects' labels
#   11. pch --- plotting character code (default is 20 circle)
#   12. lty ---- line type (default is 3, dotted line), should line is to be drawn
#   13. col --- color of the line (default is 2, depending on color scheme), should line is to be drawn
#   14. ... ---- other high-level graphical parameters such as 'main', 'sub', ..., etc.

# Effect: chisquare plot is produced
### Prepare data and set graphical elements ###

\[
n \leftarrow \text{length(data)} \quad \text{# number of effects}
\]

\[
\text{if(is.null(labels) \&\& is.null(names(data)))} \quad \text{# if labels and effects' names not given}
\]

\[
names(data) \leftarrow \text{as.character(1:n)} \quad \text{# name effects as 1,...,n}
\]

\[
p \leftarrow \text{ppoints(n)} \quad \text{# chisquare probabilities}
\]

\[
\text{quant} \leftarrow \text{qchisq(p, dof)} \quad \text{# chisquare quantiles}
\]

\[
id \leftarrow \text{order(data)} \quad \text{# sorting order}
\]

\[
data \leftarrow \text{data[id]} \quad \text{# sort the data}
\]

\[
\text{if(!is.null(labels))} \quad \text{# if data labels are given}
\]

\[
\text{labels} \leftarrow \text{labels[id]} \quad \text{# then sort the labels accordingly}
\]

\[
\text{what.to.identify} \leftarrow \text{NULL} \quad \text{# set null to-be-identified effects' IDs}
\]

\[
\text{if(identifying) \{ # if there are effects to be identified}
\]

\[
\text{what.to.identify} \leftarrow \text{seq(identifying)} + n - \text{identifying}
\]

\[
\text{if (is .null(pos))} \quad \text{# if text positions on plot for to-be}
\]

\[
\text{pos<-rep(4,identifying)} \quad \text{# -identified effects are not given}
\]

\[
\text{pos[identifying]<-1} \quad \text{# set all text positions on}
\]

\[
\text{pos[identifying]<-1} \quad \text{# the right, except for the largest}
\]

\[
\text{pos[identifying]<-1} \quad \text{# effect which is on the bottom}
\]

\[
\text{if (is.null(labels))} \quad \text{# if labels are not given,}
\]

\[
\text{labels<names(data)} \quad \text{# use effects' names}
\]

### Start the Plot ###

\[
\text{if(datax){ # if effects are to be placed on the x axis}
\]

\[
\text{plot(data, quant, xlab=xlab, ylab=ylab, pch=pch,...)}
\]

\[
\text{if(identifying)\{ # if any effects to be identified}
\]

\[
\text{for(i in what.to.identify)} \quad \text{# place the effects'}
\]

\[
\text{text(data[i],quant[i],labels[i],#labels one by one}
\]

\[
\text{pos=pos[i-n+identifying],offset=offset})}
\]

\[
\text{if(line){ #if a line is to be drawn to capture insignificant}
\]

\[
\text{if(identifying) # use least square fit of the insignificant}
\]

\[
\text{abline(lm(quant~data,subset=(seq(n))}
\]

\[
\text{[-what.to.identify]),lty=lty,col=col)}
\]

\[
\text{else}
\]

\[
\text{abline(lm(data~quant),lty=lty,col=col)}
\]

\[
}\}
\]

\[
\text{else { # otherwise, (effects are to be placed on y axis)}
\]

\[
\text{plot(quant, data, xlab=xlab, ylab=ylab, pch=pch, ...)}
\]

\[
\text{if(identifying)\{ # if any effects to be identified}
\]

\[
\text{for(i in what.to.identify)#place effects' labels one by one}
\]

\[
\text{text(quant[i],data[i],labels[i],#labels one by one}
\]

\[
\text{offset=offset,pos=pos[i-n+identifying])}
\]

\[
\text{if(line){# #if a line is to be drawn to capture insignificant}
\]

\[
\text{if(identifying)# use least square fit of insignificant}
\]

\[
\text{abline(lm(data~quant,subset=(seq(n))}
\]

\[
\text{[-what.to.identify]),lty=lty,col=col)}
\]

\[
\text{else}
\]

\[
\text{abline(lm(data~quant),lty=lty,col=col)}
\]

\[
}\}
\]
"chisqplot2" <- function (SS, ns, dof, n = length(SS), labels = NULL, both = F, sigma2 = NULL, pch = rep(c(20, 1), ns), pos = 1, offset = 0.5, ...) {
# Given effects' sums of squares of two distinct degrees of freedom, identify significant effects using two-stage chisquare plot

# Input:
# Required arguments
# 1. SS -- Sums of Squares in which first ns[1] elements are of dof[1] degrees of freedom, and remaining
# 2. ns -- vector of length two of the numbers of SS
# 3. dof -- vector of length two of the degrees of freedom
# Optional arguments
# 1. n -- total number of SS
# 2. labels -- labels of the SS's to be used in the plot, expressions are allowed
# 3. both -- logical value (default is FALSE) to flag if two separate chisquare plots should be produced for the two sets of SS's of distinct degrees of freedom in order to estimate error variance, and then produce two separate chisquare plots of the two distinct degrees of freedom in which the modified SS's are plotted in each plot
# 4. sigma2 -- error variance estimate (default is NULL, indicating that estimate should be obtained by inspecting chisquare plot (or plots))
# 5. pch -- plotting characters which must be of the same length as SS
# 6. pos -- a numeric value (default is 1) indicating text placement position for the labels of SS's
# 7. offset -- text placement offset (default is 0.5)
# 8. ... -- other high-level graphical parameters used in plot functions such as 'main', 'sub', etc

{if(sum(ns) != n)
  stop("Elements of 2nd argument should sum up to 4th argument.")
} if (is.null(names(SS))) names(SS) <- as.character(1:n)
if (is.null(labels)) labels <- names(SS)
Message <- function()
  cat("Identify significant Sums of Squares!!!\n\n")
ylabels <- function(d)
  as.expression(substitute(chi[a]^2*plain(' Quantiles'), list(a=d)))
xlab <- "Sums of Squares"
first <- 1:ns[1]
quant2 <- qchisq(ppoints(ns[2]), dof[2])
sorted.id <- order(SS[-first])
ss2 <- SS[-first][sorted.id]
lab2 <- labels[-first][sorted.id]
if (both && is.null(sigma2)) {
  quant1 <- qchisq(ppoints(ns[1]), dof[1])
sorted.id <- order(SS[first])
ss1 <- SS[first][sorted.id]
lab1 <- labels[first][sorted.id]
plot(ss1,quant1,xlab=xlab,ylab=ylabels(dof[1]),...)
Message()
x1 <- identify(ss1, quant1, labels=rep("",ns[1]))
n1 <- ns[1] - length(x1)
if(n1<ns[1]) for(i in 1:length(x1))
  text(ss1[x1[i]],quant1[x1[i]],label=lab1[x1[i]],
pos=pos,offset=offset)
plot(ss2,quant2,xlab=xlab,ylab=ylabels(dof[2]),...)
Message()
x2 <- identify(ss2, quant2, labels=rep("",ns[2]))
n2 <- ns[2] - length(x2)
if(n2<ns[2]) for(i in 1:length(x2))
  text(ss2[x2[i]],quant2[x2[i]],label=lab2[x2[i]],
pos=pos,offset=offset)
sigma2 <- ( sum(if(n1<ns[1])ss1[-x1] else ss1) +
  sum(if(n2<ns[2])ss2[-x2] else ss2) ) /
  sum( c(n1,n2)*dof )
}
if (both) {
  for (i in 1:2){
    ss <- SS
    for(j in others)
      ss[j] <- sigma2 * qchisq( pchisq(ss[j]/sigma2, dof[3-i]), dof[i])
    y <- order(ss)
    quant <- qchisq(ppoints(n), dof[i])
    plot(ss[y], quant, xlab=xlab, ylab=ylabels(dof[i]),...)
    mtext(side=3,line=0.5,text=paste("DOF =",dof[i]))
    Message()
    x <- identify(ss[y], quant, labels=rep("",n))
    if(length(x)) for(j in 1:length(x))
      text(ss[y][x[j]],quant[x[j]],labels=labels[y][x[j]],
pos=pos,offset=offset)
  }
  return(invisible())
}
panel.text <- function(d,leading="Before"){
  d <- paste(d,"degree-of-freedom",sep="-")
  paste(leading,"the Inclusion of",d,"Sum of Squares")
}
SS <- c(SS[first],ss2)
labels <- c(labels[first],lab2)
ylab <- ylabels(dof[2])
plot(ss2, quant2, xlab=xlab, ylab=ylab,...)
mtext(side=3,line=0.5,text=panel.text(dof[1]))
Message()

x <- identify(ss2, quant2, labels=rep("",ns[2]))
if(length(x)) for(i in 1:length(x))
  text(ss2[x[i]],quant2[x[i]],labels=lab2[x[i]],
pos=pos,offset=offset)
if(is.null(sigma2))
sigma2 <- mean( if(length(x)) ss2[-x] else ss2 ) / dof[2]
for (i in first)
  SS[i] <- sigma2 * qchisq( pchisq(SS[i]/sigma2,dof[1]), dof[2] )
y <- order(SS)
quant <- qchisq(ppoints(n), dof[2])
plot(SS[y], quant, xlab=xlab, ylab=ylab, pch=pch[y],...)
mtext(side=3,line=0.5,text=panel.text(dof[1],"After"))
Message()

x <- identify(SS[y], quant,labels=rep("",n))
if (length(x)) for(i in 1:length(x))
  text(SS[y][x[i]],quant[x[i]],labels=labels[y][x[i]],
pos=pos,offset=offset)
invisible()

"SS" <- function(y, x, gsum, n=length(y), levs=length(unique(x)))
# Given data, y, and a factor, x, also the grand total of the #
data, gsum, calculate the Sum of Squares of the factor effect#
#
{
  (sum(tapply(y, as.factor(x), sum)~2)
   * levs - gsum^2) / n
}

"three.levels" <-
# Given k, the number of factors, produce 3-level full factorial #
# design matrix together with all possible interactions broken #
# down to 2-degree-of-freedom each (all columns are of 3-level #
# and are orthogonal). The column names are suitable for parsing #
# to expression by using R function parse(text=...) #
#
{ if(k==1) return(matrix(0:2,3,1,dimnames=list(NULL, fac.names[k])))
x <- Recall(k-1, fac.names[-k],sep="*")
  x <- cbind(x,y, (x+y)^2.3, (x+2*y)r/.3)
dimnames(X) <- list(NULL,c(dimnames(x)[[2]],fac.names[k],
paste(dimnames(x)[[2]],fac.names[k],sep=sep),
paste(dimnames(x)[[2]],paste(fac.names[k],"^2",sep=""),
  sep=sep)))
}

"halfnorm" <-
# Generate data, datax=T,
function(data,datax=T, xlab=if(datax) "|effects|" else "Half-Normal Quantiles",

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ylab=if(datax) "Half-Normal Quantiles" else "|effects|", identifying=0, line=as.logical(identifying), pos=NULL, offset=0.5, labels=NULL, pch=20, lty=3, col=2,...

# Half Normal Quantile Plot
#
# Input :
#  Required argument
#  1. data --- numeric data vector
#  Defaulted arguments
#  1. datax --- a logical value (default is TRUE) to flag that data should be placed on x axis or not#
#  2. xlab --- x axis label (title), default is '|effects'| if datax is TRUE, otherwise 'Half-Normal Quantiles'
#  3. ylab --- y axis label (title), default is 'Half-Normal Quantiles' if datax is TRUE, otherwise '|effects|
#  4. identifying --- a numeric value (nonnegative 'integer', default is 0) to flag the number of points to be identified as the largest effects
#  5. line --- a logical value (default is FALSE if identifying is 0, and TRUE, otherwise) to flag the plotting of the straight line which is supposed to capture small effects
#  6. labels --- character or expression vector of data labels (default is NULL)
#  7. pos --- positions to place the labels, default is NULL; if specified, it is a numeric vector of length 'identifying' in which 1=bottom, 2=left, 3=top, and 4=right
#  8. offset --- offset value (default is 0.5) as a fraction of the current character expansion (i.e. character point size) to place the effects' labels
#  9. pch --- plotting character code (default is 20, a circle)
#  10. lty --- line type (default is 3, dotted line), should the line is to be drawn
#  11. col --- color of the line (default is 2, depending on the color scheme), should the line is to be drawn
#  12. ... --- other high-level graphical parameters such as 'main', 'sub', ..., etc.
#  Effect: half normal plot is produced

{### Prepare data and set graphical elements ###
  n <- length(data) # number of effects
  if(is.null(labels) & is.null(names(data)))# if labels and # effects' names not given
    names(data) <- as.character(1:n) # name effects as 1,...,n
  p <- ppoints(n)*0.5 + 0.5 # half-normal prob's expressed #in normal prob's

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quant <- qnorm(p)  # half-normal quantiles
data <- abs(data)  # absolute values of effects
id <- order(data)  # sorting order
data <- data[id]  # sort the data

if(!is.null(labels))  # if data labels are given
  labels <- labels[id]  # then sort the labels accordingly
what.to.identify<-NULL  # set null to-be-identified effects’ IDs
if(identifying) {  # if there are effects to be identified
  what.to.identify <- seq(identifying) + n - identifying
  if(is.null(pos)) {  # if text positions on plot for
    pos<-rep(4,identifying)#set all text positions on right, except
    pos[identifying]<-1  # for largest effect which is on bottom
  }
  if (is.null(labels)) #if labels are not given, use effects’ names
    labels<-names(data)
}

### Start the Plot ###
if(datax){  # if effects are to be placed on the x axis
  plot(data, quant, xlab=xlab, ylab=ylab, pch=pch,...)
  if(identifying){  # if any effects to be identified
    for(i in what.to.identify) # place effects’ labels one by one
      text(data[i],quant[i],labels[i],
           pos=pos[i-n+identifying],offset=offset)
  }
  if(line){  # if a line is to be drawn to
    if(identifying)  # capture insignificant effects
      abline(lm(quant~data,subset=(seq(n))
        [-what.to.identify]),lty=lty,col=col)
    else  
      abline(lm(quant~data),lty=lty,col=col)
  }
}
else {  # otherwise, (effects are to be placed on y axis)
  plot(quant, data, xlab=xlab, ylab=ylab, pch=pch, ...)
  if(identifying){  # if any effects to be identified
    for(i in what.to.identify) # place effects’ labels one by one
      text(quant[i], data[i], labels[i],
           offset=offset, pos=pos[i-n+identifying])
  }
  if(line){  # if a line is to be drawn to capture insignificant
    if(identifying)  # use least square fit of the insignificant
      abline(lm(data~quant,subset=(seq(n))
        [-what.to.identify]),lty=lty,col=col)
    else  
      abline(lm(data~quant),lty=lty,col=col)
  }
}

"Contr" <-
function(x, n=nrow(x), r=ncol(x), yates.order=F, 
presub="[", postsub=",", sepsub="", int.sep="*")

# Required argument
# 1. x ----- design matrix of a factorial design (can be a #
#    data frame)
# Optional Arguments
# 1. n ---- number of rows of the design matrix
# 2. r --- number of columns of the design matrix
# 3. yates.order ---- logical value (default is FALSE) #
#    indicating whether the output contrast columns #
#    should be arranged according to Yates' order or#
#    otherwise lexical order is assumed
# 4. presub, postsub, sepsub --- defaults as seen above to#
#    produce contrasts' names such as A[1], B[c], #
#    C[q] which can be called by R function 'parse' #
#    to produce expression # # to be used by #
#    text-producing parameters of high-level R #
#    graphical functions to produce pretty #
#    display-math texts (here, the three examples #
#    produce subscripts)
# 5. int.sep --- single character (defaulted "*"),"" (NULL#
#    character) is allowed, to indicate separator #
#    between two contrasts for interaction. Again, #
#    default is intended for pretty display-math #
#    texts in high-level R graphical functions #
# Output : A matrix of all contrast columns #
# Example: Contr(expand.grid(A=0:2,B=0:2,C=0:2)) #
# will produce a 27 by 26 matrix in which columns#
# are contrasts and interactions (This is a #
# 3-level full factorial in 3 factors.) #

{ # The next line gets column names
cnames <- if(is.null(colnames(x))) LETTERS[1:r]
else substring(colnames(x),1,1)
decompose <- function(a,aname,len.a=length(a))
    # get orthogonal contrasts and degrees
    # of freedom for a column
    { b <- contrasts(as.ordered(a))
      colnames(b) <- tolower(substring(colnames(b),2))
      cb <- ncol(b)
      ans <- matrix(0,len.a,cb)
      colnames(ans) <- paste(aname,presub,colnames(b),
                          postsub, sep=sepsub)
      for(i in 1:cb) ans[,i] <- b[as.character(a),i]
    }
interact <- function(first,second,nr,nsecond,int.join="*")
    # calculate interaction columns of 'first' and 'second'
    {

nf <- ncol(first)
ans <- matrix(0,nr, nf*nsecond)
for(i in 1:nsecond){
e <- i*nf
s <- e - nf + 1
ans[,s:e] <- first * second[,i]
}
colnames(ans) <- outer(colnames(first),colnames
(second), FUN="paste",sep=int.join)
ans
}
Yates.contr <- function(d,con,nr,nd=length(d),int.join="*")
# all contrasts arranged in Yates' order
#
if(nd==1) return(con[[1]])
first <- Recall(d[-nd],con[-nd],nr,nd-1)
last <- interact(first,con[[nd]],nr,d[nd],int.join)
cbind(first,con[[nd]],last)
}
dofs <- numeric(r) # degrees of freedom of columns (factors)
contrs <- vector("list", length=r) # list of contrasts
# of main effects
for(i in 1:r){ # for each column
y <- decompose(x[,i],cnames[i],n)
dofs[i] <- y[[l]] # degrees of freedom
contrs[[i]] <- y[[2]] # contrasts
}
sol <- Yates.contr(dofs,contrs,n,r,int.sep)
if(yates.order) return(sol)
sol[,order(nchar(colnames(sol)))]
}
"normalized.Contr" <- function(contrs,y,nc=ncol(contrs))
# => Calculate Contrasts <=#
#
# Input :  
# Required Arguments 
# 1. contrs --- contrasts matrix, 
# 2. y ---- response vector of length equals nrow(contrs) 
# Optional Arguments 
# 1. nc ---- number of contrasts 
#
# Output : A vector of named contrasts
# ........................................................................
{
x <- t(contrs) %*% contrs
x <- diag(sqrt(round(x,4)))
ans <- (t(contrs) %*% y) / x
ans[1:nc]
}
"normalized. Contr" <- function(contrs,y,nc=ncol(contrs))
{
x <- t(contrs) %*% contrs
x <- diag(sqrt(round(x,4)))
ans <- (t(contrs) %*% y) / x
ans[1:nc]
}

chisqplot(textile.ss,identify=4,
labels=parse(text=names(textile.ss)))
textile.contr <- normalized.Contr(Contr(textile[,1:3]),textile[,4])
halfnorm(textile.contr,identify=4,
labels=parse(text=names(textile.contr)))

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