Reform Mathematics

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Honors Thesis
Reform Mathematics

In 1989, the Board of Directors of the National Council of Teachers of Mathematics established the Commission on Standards for School Mathematics as one mean to help improve the quality of the education of mathematics. The Board of Directors gave the Commission two tasks:

"... (1) create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields and (2) create a set of standards to guide the revision of the school mathematical curricula and its associated evaluation toward that vision." (Standards, 1989).

Why would the National Council of Teachers of Mathematics want to adopt a set of standards? Normally, groups adopt standards in order to state goals, promote change, and ensure quality. Therefore the NCTM (National Council of Teachers of Mathematics) wanted to state these new goals. Schools, teachers, students, and parents, at that time, had no protection from poor education. In 1989, the Board of Directors and the Commission believed that if students are to be adequately prepared to live in the twenty-first century, new standards should be created in order to achieve this.

At this time in history, the economic shift can be attributed to the new goals. Low-cost calculators, computers, and other technologies were now more prevalent than in the past.
Also, employers no longer needed strong, hands-on workers as was needed in the industrial ages. Employees need to have problem solving capabilities, belief in the importance of mathematics, and the ability to work with others on projects. As a part of this, employers also realized that employees often changed careers, on average, three to four times during their lifetime. To accommodate this, employees needed to always learn new things. Thus the concept of "life long learner" was introduced. Employers also wanted to have the same opportunity for all. This, of course, spread into the education realm of politics (Standards, 1989).

As you can see, we live in a changing world. As a result parts of the education world needed to change with it, like mathematics.

In grades five through eight, an ideal mathematics curriculum would expand the students’ knowledge of numbers, computations, estimation, measurement, geometry, statistics, probability, patterns and functions, and the fundamental concepts of algebra. These are the exact standards that the Commission stated for grades five through eight. The textbooks prior to the Standards presented by NCTM that contain these standards only at the end of the textbook—the part of the book often left undone by teachers due to lack of time. Therefore, a need to connect the standards all together (like we will see in reform curriculum) was needed. One popular reform mathematics curriculum for grades five through eight is the Connected Mathematics Curriculum.

In a high school setting an ideal mathematics curriculum would shift the emphasis from a curriculum dominated by memorization of isolated facts and procedures and by proficiency with paper-and-pencil skills to one that emphasizes conceptual understanding, multiple representations and connections, mathematical modeling, and
mathematical problem solving. The goal is no longer to be able to do fifty problems a night with no clear goal, but rather to gain the knowledge of the big concepts and understand how they work. One popular reform mathematics curriculum for grades nine through twelve is the Contemporary Mathematics in Context curriculum (Standards, 1989).

**Evaluation Criteria**

My evaluation criteria are taken from *Evaluating Instructional Materials* by Jamal Bernhard, Melissa Mellissinos Lernhardt, and Rose Miranda-Decker. This evaluation was formed from several resources: NCTM's Standards documents (1989, 1991, and 1995), a variety of textbooks and articles about mathematical assessment (Romberg 1995, Romberg and Wilson 1995, Swan 1993) and their own experiences as math teachers and math education researchers. Jamal Bernhard and Melissa Mellissinos Lernhardt both teach at San Diego State University, San Diego, California and are interested in preservice elementary and middle school teacher preparation. Rose Miranda-Decker teaches mathematics to high school students in San Diego.

The evaluation is divided into four areas: content, technology and instructional tools, assessment, and teacher support. The content area covers the math subject, its organization, structure, and the activities in which the students engage. The technology and instructional tool area covers how the tools operate, their flexibility and appropriateness for the classroom, the value of learning opportunity, and the practicality. The assessment area covers issues like integrating assessment into the unit, using assessment as a learning tool, and determining the authenticity or value of assessment.
practices. The teacher support area covers the degree of support provided for teachers and the empowerment of teachers to make curricular and classroom decisions.

This evaluation is likely to take more time than other evaluations I looked at (including that of Tami S. Martin, Cheryl A. Hunt, John Lannin, William Leonard Jr., Gerald L. Marshall, and Arsalan Wares; NCTM’s 1982 Evaluation; and AAAS project 2061) because it is not simply a rating scale. This evaluation forces evaluators to look deeper into the criteria rather than giving a numerical value to represent whether or not the criteria was met. This way the evaluator writes out to what extent it was met, not was it or was it not met. Some information may be lost in the array of numbers. This way evaluators and others know WHY something received a mediocre score or evaluation.

Here is how I will evaluate the Connected Mathematics Project curriculum and the Contemporary Mathematics in Context curriculum:

- Criteria for evaluating content
  - The mathematics
    - Covers appropriate content
    - Challenges all students (low/high achievers, diversity)
    - Connects to other subject areas
  - Organization and structure
    - Develops concepts clearly
    - Incorporates technology and tools effectively
    - Conducive to school environment (budgets, time-wise, clientele)
    - Allows flexibility/tailor-able
  - Student activities
- Develops reasoning skills
- Develops communication skills (written and verbal are equally important)
- Develops problem-solving skills—explore math
- Meaningful situations
- Promotes equity (language suitable for all students?)

- Criteria for evaluating technology and instructional tools
  - Runs smoothly (free of bugs/viruses/glitches)
  - Sets up quickly
  - Stimulates interest
  - Promotes equity (multiple perspectives to situations?)
  - Reflects technology in society
  - Promotes communication (written and verbal equally important)
  - Incorporates assessment
  - Allows flexibility (tailor-able)
  - Conducive to school environment (budget conscience/time)

- Criteria for evaluating assessment
  - Mathematics and learning
    - Includes realistic/worthwhile math tasks
    - Allows for knowledge, thinking process (not just an answer will do)
    - Build on understanding, interests, experiences
    - Encourages learning
- **Equity**
  - Promotes equity/multiple ways of demonstration
  - Allows flexibility for grading unexpected answers

- **Assessment practices**
  - Engages students to interact, decide for themselves
  - Conducive to school environment (time issues/ special needs students)

- **Criteria for evaluating teacher support**
  - **Teacher decision-making**
    - Empowers teachers to improve discourse with students
    - Empowers teachers to make curricular decisions
  - **Teacher support**
    - Provides adequate instructional materials
    - Supplies adequate support materials
    - Makes suggestions for improvements/what to do next/etc.

**Introduction to the Connected Mathematics Project Curriculum**

The Connected Math Project was funded by the National Science Foundation. It is a three-year curriculum spanning from sixth grade to eighth grade. The Connected Math Project is devoted to developing student knowledge and understanding of mathematics that is rich in connections. These connections can be connections from one mathematics concept to another, connections between mathematics and its applications in other school subjects, connections between the learning activity and the special interests and lives of middle school students, and/or connects in general to the outside world.
When designing any subject curriculum it is virtually impossible to separate what the students learn and how the students actually learn the material at hand. This means the curriculum’s content goals, representing what is to be learned, must be linked the curriculum’s process goals, representing how the material is learned. The Connected Math Project’s content goals include number sense, geometry and spatial sense, measurement, algebra (reasoning, equations, expression, etc.), statistics, and probability. The process goals include, for example, determining the number of elements (counting), visualization, comparing, estimation, measuring, constructing models and making inferences about these models, reasoning, investigation, oral communication, written communication, and using technology.

The Connected Math Project is divided into three grade levels. At each grade level, the curriculum is organized into eight units with each unit developing a major concept or cluster of related concepts. Each unit is about four to six weeks in length (Getting to Know Connected Mathematics, 1996).

In this thesis, I will look strictly at the concept of linear relationships. Linear relationships can be found in the “Moving Straight Ahead” unit in seventh grade and the “Thinking with Mathematical Models” unit in eighth grade. When looking at the linear relationship concept in each unit I will evaluate it using the previously stated evaluation criteria. The fact that I created the evaluation criteria to be used for this study is an important one. This means that these criteria are important to me. Other students doing the same research may have different criteria.
In Getting to Know Connected Math, the authors state differences between traditional curricula and the Connected Math Project Curriculum. Traditional curricula has (Connected Math does not have):

- Emphasis on manipulating symbolic expressions such as multiplying and factoring polynomials
- Operations on algebraic fractions
- Formal solutions of linear systems in two or more variables
- Formal study of direct and inverse variations
- Radicals and simplifications of radicals
- Operations on polynomials other than linear polynomials
- Completion of the square and the quadratic formula

While the Connected Math Project has (Traditional curricula does not have):

- Emphasis on variables and the representations of the relationship between variables in words, numeric tables, graphs, and symbolic statements
- Focus, on the rate of change between two variables, not only linear
- Development of functional point of view and applications
- Emphasis on modeling
- Earlier introduction of exponential growth and decay
- Development of alternative strategies for answering questions about algebraic expressions and equations (Getting to Know Connected Mathematics, 1996).

Once I evaluate the units with the stated criteria I will determine to what extent I agree with the authors. The authors also state how well the Connected Math Project has done after the first few years of applications in schools. The authors sense that students
did not receive enough practice doing hand manipulations of problems like those of the traditional curricula. However, the students were eager to learn mathematics and use it outside of the classroom. Again, I will make my own conclusions after the evaluation.

**Connected Mathematics and Evaluation**

After reading through a few lessons in the Connected Mathematics Project and visiting their website, I filled out my evaluation.

**Content:** In the Connected Mathematics curriculum, a table is given on page 1i of the introduction to *Moving Straight Ahead*. All of the concepts to be accomplished and covered are presented on the table. There is also information on what material has been previously covered, as well as material to be covered. Therefore, after looking through the table I feel the curriculum covered an appropriate content.

Also given in the curriculum are suggestions to teachers as to how to alter lessons for diversity and special needs students. This helps make the content appropriate for all students.

At the beginning of the lesson in linear relationships, the textbook authors introduce the topic of patterns by drawing on prior knowledge from experiences and school. The textbook tries to include topics from other school subjects, however, most deal with real life situations.

The concept of patterns is supposed to lead to the concept of slope and linear relationships in this lesson. The lesson is set up so that students collect data, look at the table, and then graph the data. From this graph students can then easily make inferences about slope. This goal is clearly stated throughout the lesson.
The lesson’s experiments use tools like cups, water, meter sticks, and bouncing balls. These tools are used to collect data. It appears that the tools will be used effectively to collect valuable data.

The first lesson in the linear relationship unit is supposed to take two days. I can easily see how. Day one would be used for the experiment to collect data. Then day two would be used for discussion and problem solving. Depending on classroom time, teachers could break up the lesson any way they wish or need. Plus, the teacher could choose which experiment to run if time constraints are present. This way, the lesson is both conducive and flexible to various school environments.

In this unit, students are asked to make predictions using the data they collected. Therefore, the students have to understand the connections between the data and the results in order to make predictions. Also, students are given problems where they are to write out their reasoning behind a given graph or table. They also complete the experiments in groups where it is necessary to communicate their reasoning or questions they may have. Because of this, the curriculum is also sufficient in developing reasoning and communication skills. Also, students have to make their own connects between data, graphs, and results so the curriculum is sufficient in developing problem-solving skills.

The situations presented in the unit appear to be very carefully chosen. They are meaningful to the students because they reach outside the classroom. The bike problem is especially meaningful to middle school students with minimal amounts of money who ride bicycles instead of driving.
One of the best aspects of the curriculum is that vocabulary is left out of the lesson until absolutely necessary. It is included only after working to understand the concepts. Another good aspect is that suggestions are given to help English as Second Language students. This way they are not unfairly left behind.

**Technology and Tools:**

In this unit, the students used many tools and calculators. The tools used will work smoothly at first, are easily set up, and are flexible. However, the paper cups with a hole in the bottom may weaken and break. Perhaps the students should use plastic cups or an actual faucet. Another good aspect is that the items (minus the calculators) are low cost. This way the technology is conducive to more school environments.

While using the tools and calculators in this unit, I think students will see the real life situations arising from the lesson. Then they may start to think about what other situations would work in this lesson. These types of experiments then stimulate their interest in the topic at hand.

The various types of tools used in this unit yield multiple perspectives on situations. Students can see graphs; witness experiments taking place, and discuss observations.

In the last lessons of this unit, students start to use their graphing calculator. After using their calculators students can discuss what they did, how they did it, and how comfortable they feel using the calculators. Therefore, I feel this curriculum sufficiently promotes communication.

One problem I did find with this curriculum was that I did not feel it reflected technology in today’s society. Paper cups and meter sticks are tools, but not technology
today. Graphing calculators are a start, but it would have been even better had the authors included some type of computer software to use.

Assessment:

Assessment today is not only tests and quizzes. Assessment can be tests and quizzes, but it can also be evaluation in classroom discussions, projects, and even as informal as discussion taking place in group work and on homework.

All assessment in this unit deals with the concept at hand or urge students to think about future math concepts. While this may seem obvious, the important issue to look for is that the tasks are worthwhile, realistic, and encourage learning.

This curriculum is very focused on building connections between real life situations and the mathematical world. All questions are centered around a situation that interests students. Therefore, the tasks are worthwhile and realistic to the students. When students realize that mathematics are in everyday situations, they feel as though they are learning something and are encouraged to continue learning. Consequently, these assessments encourage learning as well.

One of the best qualities of the assessments presented in this unit is that almost all the questions are stated as “Why?”, “How does this happen…”, and “Why?”. A simple answer no longer suffices. Students are required to think further about topics and concepts and the assessment allows for thinking processes.

Another great aspect of the curriculum is that the assessment builds on students’ experiences and interests. Many of the questions are situated around sports, school, or money, which middle school students think about all the time. I feel these questions not
only build on their experiences and interests but also make students think about what else is centered around this topic or concept.

One of the hardest things about assessment is how to assess unexpected answers. However, with this curriculum, students are able to their understanding in a variety of ways. For example, students can draw graphs, make tables, or simply write out their thought process. When a student responds with a graph, table, or written response a teacher can see how they understand the topic even if they may not have the expected response.

These unexpected responses can lead to valuable classroom discussions on how students see the same thing in different ways. Students are allowed to decide for themselves how to solve problems and interpret responses. Also, when discussing the experiments in class, students can communicate their data and their thoughts and feelings on predictions from the data.

Most of the assessment appears to take place in class. Teachers can easily arrange experiments, discussions, and problem solving time around their own classroom environment. Whether the school is based on fifty minute or ninety-minute class periods, the curriculum will work.

**Teacher Support:**

This topic is probably the hardest for a future teacher to fill out. Teacher support is an important issue when choosing a new curriculum. How will I get help if I need it? Are suggestions given for various situations like English as Second Language students or special needs students? What about workshops? Not being a teacher hinders me in
filling this out sufficiently because I am not completely aware of all the questions that arise in implementing a new curriculum.

I believe that the Connected Mathematics Curriculum does empower teachers to improve discussions with students and make curricular decisions. The Connected Mathematics website gives an abundance of information about discussions to have with students and when these discussions should take place. After a few lessons, I feel that teacher would pick up how to discuss topics with students on their level and make decisions for themselves on when to have these discussions. Plus, every class is different. The discussion that works first hour may fail second hour. The teacher needs to be able to handle the curriculum to know how to work around these failures in discussions.

After the lesson there are a few summary pages where the textbook makes suggestions about extensions to the lesson plans and what to do next. The website also gives even more suggestions as to what to do when problems arise.

One problem I have with the curriculum is that there are no supplemental instructional materials given. For example, there are no overheads, worksheets, or even more situations to solve if students have difficulty with the textbook’s lesson. I realize that a teacher could easily make these himself or herself if necessary, however pre-made materials would be more helpful (Moving Straight Ahead, 1998, Thinking with Mathematical Models, 1998).

Contemporary Mathematics in Context

Contemporary Mathematics in Context is a four-year program developed by the Core-Plus Mathematics Project. The Core-Plus Mathematics Project builds on the theme
of "mathematics as sense-making". Students are presented with real life situations where students develop a deeper understanding of important mathematics that makes sense, and that enables them to make sense out of new situations.

The Core-Plus curriculum features interwoven strands of algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics. Each strand is developed within units connected by fundamental ideas such as symmetry, functions, matrices, and data analysis and curve fitting. When students encounter each strand each year, a more mathematically sophisticated point of view is presented. Student’s understanding of mathematics and its connections deepen through the years.

The curriculum is presented in student and teacher materials for a core curriculum for all students for three years. A flexible fourth year course that continues the preparation of students for college mathematics is also available. Each of the first three years in the Core-Plus Mathematics Project curriculum contains seven units and a capstone activity. Each unit then contains three to six multi-day lessons, with each unit having an approximate three to six week duration period.

The capstone activity is a themed project activity that spans the course of two weeks. This activity pulls students together to apply the important mathematical concepts and methods developed in that unit (Overview of the CPMP Curriculum, 2001).

Core-Plus Mathematics and Evaluation

After reading through all of the material given by the curriculum authors, including instructional materials, assessment materials, and the textbooks, I filled out my evaluation.
Content:

The unit begins well with meaningful examples from real life situations. In many instances the textbook related problems to other school subjects, including history and science. There are certain problems in the Reflecting section of the questions where the text introduced material beyond the students’ means. Generally speaking the content was appropriate most of the time, but could have been better.

I do think all students will relate to the situations provided by the textbook. However, I am concerned that problems will arise when instructing English as Second Language students. There were no suggestions given by the authors and I think their understanding may not be up to par with other students. Therefore, it is tough to say that the curriculum promotes equity in that sense. However, I do feel that equity is met in terms of multiple representations (i.e., graphs, tables, discussions, etc.)

There were several times when I needed to reread the concepts being introduced. If I have difficulties, seeing as how I have already been through Algebra 1 and am about to teach it, then I fear that many students will have difficulties.

When the textbook introduced experiments and tools they were introduced in order for students to visualize what they were learning. The instructional tools were incorporated effectively and efficiently.

The mathematical content and structure was relevant, workable, and flexible for many different school environments. The text even suggests how to divide the entire four-year curriculum up in order to accommodate block scheduling, every other day scheduling, and traditional classroom settings.
The content allows students to predict occurrences, communicate with their group, as well as in classroom discussions. Also, the curriculum has a great deal of “problem solving” problems. These types of problems have students actually solve mathematical situations (like those presented in traditional textbooks) instead of describing how they predict occurrences (the “idea” of reform mathematics).

**Technology and Instructional Tools:**

While no technological tool is completely free of bugs, it is essential that they be minimal, especially when a teacher only has fifty minutes to get their point across. The technology used in this unit on linear relationships is used only to solidify the student’s understanding of the topic. The entire lesson is never built solely on technology. The tools are always easily set up and take little time to prepare.

Bringing in technology by using calculators always simulates interest in students. By each student having their own calculator, while expensive, they will be more interested in what else they can do with their calculators, which they had not thought of before. The graphing calculators can be expensive for students to purchase, so not every student will be able to purchase one. This is one drawback on some of the lessons. Many schools cannot fit graphing calculators into their budgets. But, other tools could be used to alleviate the problem. Other tools used in the unit can be used repeatedly and can even be found at home if necessary to complete lessons at home. Therefore, I would have to say that the curriculum is sufficient at allowing flexibility in units and is conducive to various school environments.
Multiple perspectives are present. Students can visually see graphs, write in their journals, talk in groups, and use their calculators to represent their understanding. One aspect is not dominant over the others.

One of the best things about the technology in this curriculum is that it is always followed by some kind of discussion, whether in groups or as a whole class. Also, assessment is incorporated into the technology. Students in groups, or as a class, can talk about what technology they used, their results, and why. This way, teachers can also see how well students understand the technology.

I think the technology does a fair job at meeting the criteria of reflecting technology in society today. However, if the curriculum introduced computer programs, I feel the technology would better reflect technology in society today.

Assessment:

The presence of real life, meaningful situations expressing the concept of linear relationships, clearly fills the criteria of including realistic and worthwhile mathematical problems very well. Also by incorporating real life situations of high school students, the curriculum builds on the students' interests and own experiences. Quite a few of the questions were built around sports, which is one of the biggest aspects of high school life.

When students see the connection between their interests and real life experiences, they begin to see why the topic is important and then see the point in learning it. After this experience they are encouraged to learn more about it.

One of the best things assessment can do is make students explain why something happened. This curriculum does not just ask students to answer questions. It asks
"Why?", "How come...?", and other questions in order to make students explain what occurred in the situation. This allows for thinking processes to be present.

Students can show this understanding through multiple representations. Students can show their understanding through graphs, writings, and various forms of technology. Because of this, there will be a chance of unexpected answers. However, as long as the question does not state "In a graph, show..." any form of representation or understanding will suffice. Therefore, there is room to grade the unexpected answers.

Because they are given a choice as to how to answer questions, students can decide for themselves how to accomplish this task. Also by introducing material to be learned through group experiments, students are forced to interact.

Assessment can be accomplished in a variety of ways in this curriculum. Teachers can assess students in group discussion, whole class discussions, in experiments, and in homework problems. Therefore no matter the scheduling or in-class time, all assessment can be achieved in all types in the classroom.

Teacher Support:

I feel that the Core-Plus Mathematics Project does a fairly good job at empowering teachers to improve discussions and make curricular decisions. There are several examples given in *Implementing the Core-Plus Mathematics Curriculum* textbook as to how to explain different situations to students depending on the subject being instructed. Also, how to set up routines, make adjustments to lessons, and make adjustments when students are absent. Plus, the curriculum gives suggestions as to how to set up the four courses depending on the scheduling of classes in a school district. One
thing the curriculum does not do is simply state what to do, so teacher can make their
own decisions or create their own.

The Core-Plus Mathematic Project gives an entire workbook dedicated to teacher
support and support materials. In this workbook there are extra handouts in case students
have difficulties, overheads, and extra experiments that may better interest their students.
Also, the Implementation workbook gives ideas on how to teach things like social skills,
how to decide on group sizes, and how to deal with diversity.

Another great aspect of this curriculum is that the authors included a website
( ) in case problems arise, further professional development, and
information on workshops (Contemporary Mathematics in Context textbooks parts A and

Personal Reflections

As a student not too far out of the high school scene, I can honestly say I wish I had
gone through a math reform curriculum. I have observed a Connected Mathematics
class, as well as a Core-Plus Mathematics class. In both classrooms, students seemed
eager to learn and eager to explore the topic of that day. I felt that they actually liked
their math class. They saw the connections between real life and school, which is
difficult to do in middle school or high school.

I am so jealous of these students! I went to school and had “school life”, where I
answered fifty math questions a day without understanding where I would ever use this
outside of school, and I had a “real life” where I did not even think about school or
homework. I never saw a connection between the two.
It was not until college when I understood the reality of mathematics. I wish I could go back in time and relive my math class. I think I would remember and understand a lot more of the concepts introduced.

Unfortunately, I cannot go back in time. But I switched my career path because of everything I learned once I got to Western. I now want to teach reform mathematics in a middle school or high school setting. Then I can “go back” and teach mathematics to students so that they understand how important it is in real life.

As a future parent, I would send my children to a school that has a reform mathematics curriculum. I feel it is vitally important to have connections between life and school. If a school only has the Connected Math Project Curriculum or the Contemporary Mathematics in Context Curriculum, I would choose the Connected Math Project because I feel it is important to introduce the mathematic concepts in middle school as real life situations more so than in only high school. Once the students can see how easy it is to connect the topics in middle school, they could do this on their own in high school. However, I fear that if the school district waited until high school to introduce the reform curriculum, they may have already lost some of the students interests in math.

Overall, I think both curriculums are far better than traditional mathematics. The connections and sense making skills are key to understanding the importance of mathematics. However, I would say that the Connected Mathematics Curriculum seems a little more thought out than the Contemporary Mathematics in Context. The content and ideas are more clearly expressed, the courses are more equal for all students, and the curriculum is flexible for more school district in the Connected Mathematics Project.
References


