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Determination of Phenomenological Optical Model Parameters for Neutrons on $^{208}\text{Pb}$ from 4 to 150 MeV

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DETERMINATION OF PHENOMENOLOGICAL OPTICAL MODEL PARAMETERS FOR NEUTRONS ON $^{208}$Pb FROM 4 TO 150 MeV

by

Randy L. Schutt

A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
requirements for the
Degree of Master of Arts
Department of Physics

Western Michigan University
Kalamazoo, Michigan
December 1986
Phenomenological optical model parameters for neutrons incident on $^{208}$Pb have been determined for neutron energies from 4 to 150 MeV using the code SNOOPY-8. Neutron total cross sections from 4 to 150 MeV and angular distributions for elastic scattering from 13.5 to 40.0 MeV were utilized. Analyses of total and differential cross sections were performed at a number of energies to determine energy dependent expressions for the real strengths of the central potential, imaginary surface potential, and imaginary volume potential. These energy dependent expressions for $V$, $W_d$, and $W_s$ were found to provide a good fit to the total cross section from 4 to 150 MeV. The energy dependence of the real central potential strength is linear up to about 60 MeV and logarithmic above that energy. The imaginary surface potential strength is approximately parabolic with a peak at about 20 MeV and tends to zero at about 50 MeV. The imaginary volume potential strength, $W_s$, is zero below 11 MeV, described by a polynominal between 11 and 50 MeV, and linear above 50 MeV.
ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to Dr. Robert Shamu; without his constant advice and guidance this research project would not have been possible. I am sure the knowledge gained from Dr. Shamu will prove invaluable in the future.

I would also like to extend my appreciation to Dr. Dean Halderson and Dr. Larry Oppliger; their counseling and teaching abilities were extremely beneficial to me. I also thank Dr. Michi Soga and Mark Clark for their help in preparation of this thesis.

Lastly, I would like to thank my wife, Ann, for her support, enthusiasm and confidence that were vital to the conclusion of this project.

Randy L. Schutt
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CHAPTER I

INTRODUCTION

Since the nuclear force between two nucleons is so complicated, it is not possible to predict exactly even the properties of light nuclei. Therefore it is necessary to make certain assumptions which simplify the nuclear many-body problem without introducing too much error. A particular set of assumptions used to describe certain properties is called a model. In nuclear physics a variety of models are employed.

Originally attractive square-well potentials were used to model the nucleon-nucleus force. One of the early modifications to this potential was introduced by Ostrofsky, Breit and Johnson (1936). The modification consisted of the addition of an imaginary term to describe absorption. In 1940 Bethe showed that a complex potential well could be used to describe high energy scattering. Then, in 1949, (Fernbach, Gerber, and Taylor) successfully reproduced energy variations in high-energy neutron total cross sections with a complex potential. The optical model was born. Shortly thereafter Feshbach, Porter, and Weisskopf (1954) showed that at low neutron energies the optical model could be used to describe not only total cross sections, but also the components of the total cross section. They related the imaginary part of the potential to compound nucleus formation, and successfully reproduced extrema in the averaged total
cross section data. The optical model used by Feshbach, Porter, and Weisskopf (1954), with some refinements, is the one used today. The refinements include the use of form-factors which give the potential wells diffuse edges, the division of the imaginary potential into surface and volume components, and the addition of a spin-orbit term.

The optical model attempts to describe nuclear scattering by replacing the many-body problem with a two-body problem. A complex potential is used to describe the interaction between an incident particle, e.g., a nucleon, and the target nucleus. At high energies, for example, the real part of the potential describes elastic processes and the imaginary term represents all non-elastic processes. The optical model derives its name from its similarities to optical phenomena, namely the scattering of light from a translucent glass sphere. Nucleons incident on a nucleus behave in many ways like light incident on a glass sphere: elastic scattering is similar to reflection, and diffraction and absorption also take place. Diffraction of light occurs when light is incident on an object of approximately the same size as the incident light's wavelength. This is also the case for incident nucleon waves comparable in size to the target nucleus. In optics one uses a complex index of refraction to describe absorption, whereas in the nuclear optical model one describes absorption of the incident neutron with a complex potential.

Many features of the optical model are supported by results obtained in other means, thereby justifying the model. One such
feature is that the nuclear radius is proportional to the cube root of the mass number, which supports the assumption that the nuclear force is short range. Another feature is the close relationship of the real potential to the potentials used in the shell model to calculate nuclear energy levels.

It is possible to derive the optical model potential from a microscopic calculation. Although capable of reproducing the features of the data, such potentials rarely predict the details of the cross section. Phenomenological optical model potentials are constructed by fitting scattering data.

Figure 1 shows typical neutron total cross section data plotted versus incident neutron energy for $^{208}$Pb. In the present work the phenomenological optical model will be used to fit total and differential cross sections for neutrons on $^{208}$Pb from 2 to 150 MeV to determine the optical model potentials over this energy range. Previous analyses have been performed for neutrons on $^{208}$Pb over much of this energy range (Finlay, Annand, Cheema, Rapaport, and Dietrich, 1984; Marshak, Landsford, Tamura, and Wong, 1970).
CHAPTER II

PHENOMENOLOGICAL OPTICAL MODEL

General

In the present work the data we wish to study will be mainly neutron total cross section data. Neutron total cross sections are particularly convenient to work with as they are measurable and calculable in the optical model. The measurable quantities are the elastic cross section and the reaction cross section, whereas the calculable quantities are the shape elastic cross section and the absorption cross section. Formally, the neutron total cross section is defined as (Marmier and Sheldon, 1970):

\[ \sigma_T = \sigma_{SE} + \sigma_A = \sigma_E + \sigma_R \]

where

- \( \sigma_T \) = total cross section
- \( \sigma_{SE} \) = shape elastic cross section
- \( \sigma_A \) = absorption cross section
- \( \sigma_E \) = elastic cross section
- \( \sigma_R \) = reaction cross section

It should be noted that although the calculated and measured total cross section are the same, the elastic and reaction cross sections generally cannot be individually compared with theoretical predictions for these quantities. This is evident from the definitions of the elastic and reaction cross sections:

\[ \sigma_E = \sigma_{CE} + \sigma_{SE} \]
where \( \sigma_{CE} \) is the compound elastic cross section. The shape elastic cross section represents those particles which are immediately scattered elastically. The compound elastic cross section is due to those particles which are momentarily absorbed and then emitted with the same energy they had before absorption. The compound elastic cross section poses a problem as it is theoretically difficult to determine, but is part of the measurable elastic cross section. However, at higher energies the compound elastic scattering becomes negligible, so that the elastic cross section is approximately equal to the shape elastic cross section and the reaction cross section is approximately equal to the absorption cross section.

The phenomenological optical model calculates total cross sections which reproduce gross structure in the total cross section spectrum only. Therefore, at low energies, where isolated resonances are present, it is necessary to average total cross sections over energy intervals which are large compared to energy level spacings.

Parameters of the Optical Model Potential for Nucleons

The phenomenological optical model potential (OMP) used in the present work is defined as follows:

\[
U(r) = U_c(r) - V_f(x_v) - iW_gf(x_g) + 4iW_D \frac{d}{dx_w}f(x_w) + \frac{h}{m_0^2}(V_{SO} + iW_{SO})f(x_{SO}) \sigma \cdot \hat{l}
\]
where \( U_c(r) \) is the Coulomb potential of a uniformly charged sphere of radius \( R_c \) and is given by
\[
U_c(r) = \begin{cases} 
    zZe^2(1/r) & \text{for } r > R_c \\
    zZe^2(1/2R_c)(3 - r^2/R_c^2) & \text{for } r < R_c 
\end{cases}
\]
and \( R_c = r_c^{1/3}, z = \) projectile charge number and \( Z = \) target charge number. For neutrons \( U_c(r) = 0 \). The parameter \( V \) is the real central nuclear potential strength, \( W \) is the imaginary volume potential strength, \( W_D \) is the imaginary surface potential strength and \( V_{so} \) and \( W_{so} \) are the real and imaginary spin-orbit potential strengths, respectively. For the energies of interest all potentials are attractive except the Coulomb and imaginary spin-orbit potentials.

The \( f(x_i) \) are Woods-Saxon form factors given by \((1 + e^{x_i})^{-1}\) with
\[
x_v = (r - r_o^{1/3})(1/a_v), \quad x_w = (r - r_w^{1/3})(1/a_w) \\
x_{so} = (r - r_{so}^{1/3})(1/a_{so})
\]
and \( x_i \) are the respective potential radii and the \( a_i \) are the potential diffuseness parameters. The derivative form factor used with \( W_d \) is significant only at the nuclear surface (Hodgson, P.E., 1971). A standard Thomas-Fermi form factor is used with the spin-orbit potentials. The spin-orbit term also contains the scalar product of the Pauli spin vector \( \vec{\sigma} \) and the orbital angular momentum \( \vec{l} \), as well as a normalization factor \((\hbar/m_c)^2\) which is equal to \(2fm^2\).

**Relativistic Corrections**

At the energies studied in the present work, relativistic effects
are significant; therefore Schroedinger theory is not adequate. Two standard relativistic corrections (Elton, 1965) are applied: first, relativistic kinematics were used and secondly, all potential strengths were renormalized by a factor \( \gamma \) defined as

\[
\gamma = 2(T_0 + mc^2)(\frac{1}{T_0 + 2mc^2}) = 1 + (\frac{T_0}{T_0 + 2mc^2})
\]

where \( T_0 \) is the total center of mass energy and \( m \) is the mass of the incident nucleon. See Appendix A for details.

Fitting the Data

Values of the optical model potential parameters described above are determined by comparing calculated cross sections with experimental data. For a given set of parameters, the neutron total cross section, the reaction cross section, differential elastic cross section and differential cross sections for the analyzing power are calculable. These calculated cross sections are then compared with the appropriate data. It is customary (Hodgson, 1971) to measure the agreement between \( N \) calculated cross sections and the data with a quantity \( \chi^2 \), defined as follows

\[
\chi^2 = \sum_{i=1}^{N} \left[ \frac{\sigma_{\text{calc}} - \sigma_{\text{data}}}{\Delta \sigma_{\text{data}}} \right]^2
\]

where \( \sigma_{\text{calc}} \) is the calculated cross section, \( \sigma_{\text{data}} \) is the corresponding measured cross section and \( \Delta \sigma_{\text{data}} \) is the uncertainty in \( \sigma_{\text{data}} \). A \( \chi^2 \) per point \((\chi^2/N)\) of ten or below usually gives good
agreement between calculated and measured cross sections. Usually automatic search routines are used to systematically vary the parameters to arrive at minimum $\chi^2$ values. Physically reasonable parameters for which $\chi^2$ is a minimum are called "best-fit" parameters.

The Computer Program SNOOPY-8

The computer routine for optical model calculations used in this study was SNOOPY-8 (Schwandt, 1982). The relativistic corrections discussed previously are an integral part of this routine. Given an optical model potential, SNOOPY-8 can calculate all the cross sections listed in the previous section. This program has automatic search capabilities for potential parameters which give best fits to reaction and differential cross sections, and analyzing power data. Up to eight optical model potential parameters can be varied simultaneously. It also performs grid calculations which allow up to three parameters to be incremented in specified steps.

In the present analysis SNOOPY-8 was modified to allow sequential calculations of total and reaction cross sections as a function of energy. Separate programs were written to generate desired energy dependent parameters over specified energy ranges and increments.
CHAPTER III

THE LANE MODEL

Lane has proposed that neutron-nucleus, proton-nucleus, and (p,n) charge-exchange scattering are simply related. For example, if one assumes a linear energy dependence for the isoscalar potential,

\[ V_\text{o}(E) = V_\text{o} - b_\text{o} E \]

then the strengths of the real, central optical model potential for neutrons (−) or protons (+) may be written

\[ V(E) = V_\text{o} - b_\text{o}(E - \Delta_\text{o}) \mp V_\text{i} \epsilon \]

where \( E \) is the bombarding energy of the incident nucleon, i.e., the kinetic energy at infinity, \( \Delta_\text{o} \) is the Coulomb energy of the incident particle and \( V_\text{i} \epsilon \) is the asymmetry term where \( \epsilon = (N - Z)/A \). The above expression assumes that \( V_\text{i} \), the asymmetry coefficient, has a negligible energy dependence. For a uniform charge distribution of radius \( r_\text{c} = r_\text{c} \text{A}^{1/3} \) the Coulomb energy for incident protons may be approximated by the expression (Satchler, 1983).

\[ \Delta_\text{o} = (1.73/r_\text{c})(Z/A^{1/3}) \text{ MeV} \]

In the present work we define \( \Delta_\text{o} \) for protons on \(^{208}\text{Pb}\) to be equal to the Coulomb displacement energy, which for the pair \(^{209}\text{Pb} - ^{209}\text{Bi}\) is 18.82 MeV (Wilkinson, 1969). For neutrons \( \Delta_\text{o} = 0 \).
CHAPTER IV

DETERMINATION OF OPTICAL MODEL PARAMETERS

Data

The neutron data analyzed in the present study were total cross sections from 2 to 150 MeV incident neutron energy and differential cross sections for elastic scattering at several energies from 13.5 to 40.0 MeV. The total cross section data sources and their respective energy ranges are listed in Table I. The data sources for the differential cross sections are given in Table II.

Table I

<table>
<thead>
<tr>
<th>Energy Range of Data [MeV]</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8 - 13.9</td>
<td>Satkowiak, L.J., (1979)</td>
</tr>
<tr>
<td>2.5 - 15</td>
<td>Foster, D.G., &amp; Glasgow, D.W., (1971)</td>
</tr>
</tbody>
</table>
Table II
Angular Distribution Data

<table>
<thead>
<tr>
<th>Neutron Energy [MeV]</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.3, 40.0</td>
<td>DeVito, R.P. (1979)</td>
</tr>
</tbody>
</table>

Methods

General

Throughout this study the real central potential strength, $V$, the imaginary surface potential strength, $W_d$, and the imaginary volume potential strength, $W_s$, were the only optical model parameters varied. The remaining optical model parameters were the average geometry and average spin-orbit potential strengths of van Oers, Huang Haw, Ingemarsson, Fagerstrom, and Tibell (1974). These fixed parameters are listed in Table III.

The analysis of the angular distribution data was a standard $\chi^2$ minimization procedure, discussed earlier in Chapter III. Energy independent analyses were performed at each of the energies listed in...
Table III
Average Geometry and Spin-Orbit Parameters
of van Oers et al. (1974)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_o$</td>
<td>1.183 fm</td>
</tr>
<tr>
<td>$a_o$</td>
<td>0.724 fm</td>
</tr>
<tr>
<td>$r_w$</td>
<td>1.273 fm</td>
</tr>
<tr>
<td>$a_w$</td>
<td>0.699 fm</td>
</tr>
<tr>
<td>$r_{so}$</td>
<td>1.166 fm</td>
</tr>
<tr>
<td>$a_{so}$</td>
<td>0.677 fm</td>
</tr>
<tr>
<td>$V_{so}$</td>
<td>6.18 MeV</td>
</tr>
<tr>
<td>$W_{so}$</td>
<td>0.0 MeV</td>
</tr>
</tbody>
</table>

Table II. Here the total cross section was utilized only to determine the differential cross section at zero degrees.

The total cross section data were fitted independently of the angular distribution data at low energies and at high energies. Below 11 MeV, where $W_s$ was assumed to be zero, $V$ and $W_d$ were found. Above 50 MeV, where $W_d$ was assumed to be negligible, $V$ and $W_s$ were found. A qualitative justification for the procedure utilized is given below.

The maxima and minima observed in a plot of the total cross section versus incident neutron energy are a consequence of interference between that portion of the neutron wave which goes
around the nucleus and that part of the wave which goes through the
nucleus (Peterson, 1962). If it is assumed that the ray for the
transmitted wave is a straight line, then the scattering amplitude in
the forward direction can be approximated by (Bohr and Mottelson,
1969):

\[ f(0) = \frac{1}{2} \left( 1 + \frac{2}{x^2} (1 - \exp[ix][1 - ix]) \right) \]

where \( x = 2(K - k)R \) and \( R \) is the nuclear radius. The wave numbers
inside and outside the nucleus, \( K \) and \( k \), respectively, are given by

\[ K = \left[ \frac{2m}{\hbar^2} \left( E + |V| + i|W| \right) \right]^{1/2} \]
\[ k = \left[ \frac{2mE_n}{\hbar^2} \right]^{1/2} \]

Where \( m \) is the neutron mass and the other symbols have their usual
meaning. At low energies, \( k \ll K \), so that for constant \( W \), \( x \) depends on
\( E \) and \( V \) in a symmetric fashion. Since a maximum (or minimum) can
occur when \( V \) is fixed and \( E \) is varied, a maximum (or minimum) should
also occur if the converse is true. It is well known that the major
effect of \( W \) is to change the magnitude of the maximum or minimum
(Bohr and Mottelson, 1969). Therefore the energy and magnitude of a
total cross section maximum (or minimum) may be used to find the
strengths \( V \) and \( W \), respectively.

At higher energies, where \( k \approx K \), a modification of this method
can be used.

Intermediate energies are more complicated because there are
three parameters to be found, $V$, $W_d$, and $W_g$, and it is difficult to
determine that fraction of the imaginary strength that is volume or
surface.

**Total Cross Section Analysis**

**Low Energies**

In order to determine $V$ and $W_d$ at low energies by the above
method, the energies and magnitudes of maxima and minima in the total
cross section must be known. Average cross sections must be used at
low energies. For the first maximum, shown in Figure 2, data was
averaged from 2.3 to 5.5 MeV in 0.2 MeV bins. The energy and
magnitude of this maximum were determined by a parabolic fit to the
averaged data and found to be 3.93 MeV and 7.84 b, respectively.

The next extremum, given in Figure 3, is a minimum near 11 MeV.
The energy range of data used here was from 9.3 to 12.5 MeV. Since
no resonance structure was present in this energy range, averaging
data was not necessary. Again, by a least-squares parabolic fit, the
energy of the minimum was found to be 10.78 MeV and the magnitude of
the total cross section at this minimum was 5.13 barn.

At each extremum the energy and cross section given above were
then employed to determine $V$ and $W_d$ using the code SNOOPY-8. Initial
potential strengths were taken from Satkowiak, (1979). At each
energy total cross sections were computed by incrementing the
parameters $V$ and $W_d$ around these initial values in grid calculations.
Figure 2. Least-squares Parabolic Fit to Neutron Total Cross Section Data Near 3.9 MeV. The Data Sources are: X Satkowiak (1979), + Foster and Glasgow (1971).

Figure 3. Least-squares Parabolic Fit to Neutron Total Cross Section Data Near 10.78 MeV. The Data Sources are: + Shamu et al. (1980), X Satkowiak (1979), and * Ferguson and Shamu (1976).
Figure 4 is a graph of total cross section versus $V$ at 3.9 MeV for three different values of $W_d$. One sees by interpolation that the curve which would peak at $7.84$ b would have $V(\text{peak}) = 49.3$ MeV. The parameter $W_d$ is obtained from Figure 5, which is a plot of $W_b$ versus $V$ for total cross section equal to $7.84$ b. From this curve the $W_d$ value for $V = 49.3$ MeV is $2.28$ MeV. Figure 6 is the plot of total cross section versus $V$ at 10.78 MeV and figure 7 is the $W_d$ vs. $V$ plot. In similar fashion to the above, $V$ and $W_d$ are determined to be $47.3$ and $3.84$ MeV, respectively.

High Energies

A new procedure is needed at high energies since the total cross section varies slowly with incident neutron energy and therefore will
Figure 5. The Parameter $W_d$ vs. $V$ at 3.9 MeV for a Total Cross Section of 7.84 $\text{b}$. 

Figure 6. Neutron Total Cross Section vs. $V$ at 10.78 MeV for: $W_d = 2.84(+)$, $W_d = 3.84(*)$, and $W_d = 4.84(X)$. 

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Figure 7. The Parameter $W_4$ vs. $V$ at 10.78 MeV for a Total Cross Section of 5.126 b.

also vary slowly with $V$. Here the optical model calculations were compared directly with data over a range of energies. This procedure requires a knowledge of the energy dependences of the potential strengths $V$ and $W_5$. Initially these energy dependences were assumed to be the same as those determined by Nadasen, Schwandt, Singh, Jacobs, Bacher, Debevec, Kaitchuck, and Meek, (1981) from an optical model study of proton-208Pb scattering.

Above 50 MeV the dependence of $V$ on the energy of the incident nucleon is believed to be logarithmic. Figure 8 is a semilogarithmic plot of $Vr^2$ for protons versus incident proton energy over the energy range 30 to 160 MeV. The quantity $Vr^2$ is plotted here instead of $V$ alone because van Oers et al. (1974) and Nadasen et al. (1981) used
Figure 8. Semilogarithmic Plot of $V r^2$ vs. Proton Energy. The Data are: *van Oers et al. (1974), + Nadasen et al. (1981). The Solid Line is a Least-squares Fit to the Data.

different radius parameters. The straight line is a least-squares fit to the proton data. Figure 9 is a similar plot for $V$ only on a linear scale. The two values of $W$ used initially are shown in Figure 10.

In Figure 9, $V$ is plotted as a function of $E - A_0$, where $E$ is the kinetic energy of the incident nucleon and $A_0$ is the Coulomb energy discussed in Chapter III. There are two curves plotted in Figure 9, one for $V_p$, the potential felt by protons and the other for $V_n$, the potential felt by neutrons. For neutrons $A_0 = 0$ and for protons $A_0$ was assumed to be equal to the Coulomb displacement energy, 18.82 MeV (Wilkinson, 1969). According to the Lane model, if the real central potential is energy independent, then the neutron
Figure 9. The Real Potential Strengths $V_r$ and $V_p$ vs. $E - \Delta_c$, where $\Delta_c$ is the Coulomb Energy of the Incident Particle. See Text for Details.

Figure 10. The Parameter $W$ vs. Energy. The Points Plotted are: * Satkowski (1979), + Nadasen et al. (1981), x Present Work. The Solid Line is Described in the Text.
curve should be shifted downward from the proton curve by an amount $2eV_1$ on such a plot.

Experimental cross sections and three calculated curves over the peak near 80 MeV are shown in Figure 11. The short-dashed line was computed assuming $V_1 = 17$ MeV and $W_s$ was given by the linear relationship found from the two proton points given in Figure 10. For the solid curve, $V_1$ was adjusted to fit the position of this peak. For these calculations $V$ was taken to be the lower curve in Figure 9. For the long-dashed curve in Figure 11 $W_s$ was adjusted to fit the magnitude of the peak. Values of $W_s$ used here correspond to that portion of the curve shown in Figure 10 above 50 MeV.

**Intermediate Energies**

In this energy range potential strengths determined from the total cross section served primarily as input parameters for the angular distribution searches. Initial values of $V$ were determined by interpolating between an assumed linear dependence at low energies and the logarithmic dependence at high energies shown in Figure 9. Values for $W_d$ and $W_s$ were determined by using the results of previous analyses and also the lower energy and higher energy results obtained above.

Preliminary values for $W_d$ are given in Figure 12. Here, the points at 3.9 and 10.8 MeV are those determined in the low energy analysis discussed above. Previous work at WMU has shown that a plot of $W_d$ vs. $E$ is approximately parabolic, peaks at 20 MeV and is...
Figure 11. Total Cross Section vs. Energy Near 80 MeV. The Data (x) is Shamu et al. (1980). The Three Lines Represent Different Values of V and Wg. See Text.
Figure 12. The Parameter $W$ vs. Energy. The Points Plotted are: + Present Work, * Previous Work, X Symmetric Reflections about 20 MeV of Low Energy Points. See Text for Details.

Symmetric around 20 MeV (Satkowiak, 1979). The point at 20 MeV corresponds to the peak of this previously determined parabola. The smooth curve shown here, linear below 10 MeV, parabolic between 10 and 30 MeV, and linear above 30 MeV provides a reasonable fit to the total cross section data over this energy range.

The energy dependence for $W$ is shown in Figure 10. The points at 11.5 and 13.5 MeV were determined in previous analyses (Satkowiak, 1979). The points at 50 and 80 MeV were determined from the high energy analysis discussed above. The smooth curve between 11 and 50 MeV is the result of a polynomial least-squares fit. The curve above 50 MeV is linear, as mentioned earlier.
Angular Distributions

The input data used for fits to the angular distributions were the differential cross sections for elastic scattering listed in Table II and differential cross sections at zero degrees which were computed from Wick's limit. Wick's limit is given by

$$\sigma_n(0^\circ) = \left[ \frac{k\sigma_T}{2\pi} \right]^2$$

where \(\sigma_T\) is the total cross section and \(k\) is the wave number of the incident neutrons. In the present work relativistic values of \(k\) were employed. It has been demonstrated that equality holds in the above relationship for \(^{208}\text{Pb}\) at energies greater than 7 MeV (Bucher and Hollandsworth, 1975). At each energy the measured differential cross sections and the zero degree cross section were fitted by two parameter and three parameter searches. The best fit calculated cross sections for the three parameter searches are shown in Figure 13. It is seen that the fits are very good at all energies and angles. Values of the parameters determined in the searches are listed in Table IV. Also shown for each energy is the value of the reduced \(\chi^2\), \(\chi^2_R\).
Figure 13. The Differential Cross Section vs. $\theta$. The Data Sources are: □ Haouat et al. (1978), ○ Finlay et al. (1984), • DeVito (1979). The □ is the Wick's Limit Value for the 0° Cross Section.
TABLE IV
Best Fit Parameter Search Results

<table>
<thead>
<tr>
<th>Energy [MeV]</th>
<th>V</th>
<th>W_s</th>
<th>W_d</th>
<th>$\chi^2_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.5</td>
<td>46.24</td>
<td>1.220</td>
<td>3.640</td>
<td>8.1</td>
</tr>
<tr>
<td>20.0</td>
<td>44.17</td>
<td>1.380</td>
<td>4.270</td>
<td>14.5</td>
</tr>
<tr>
<td>22.0</td>
<td>43.96</td>
<td>1.760</td>
<td>4.110</td>
<td>8.6</td>
</tr>
<tr>
<td>30.3</td>
<td>40.95</td>
<td>4.290</td>
<td>2.640</td>
<td>7.2</td>
</tr>
<tr>
<td>40.0</td>
<td>37.91</td>
<td>5.520</td>
<td>2.300</td>
<td>6.6</td>
</tr>
<tr>
<td>40.0*</td>
<td>35.66</td>
<td>6.920</td>
<td>0.840</td>
<td>19.8</td>
</tr>
</tbody>
</table>

It should be noted that at most energies the value of $\chi^2_R$ is very good even though only three parameters were varied.

The fits to the zero degree cross sections were very good except at 40.0 MeV where $\chi^2$, was 20. An attempt was made to improve the zero degree fit by reducing the cross section error from 2%, the value used in all previous analyses, to 0.2%. The results of this fit are listed in the 40.0* row. It is seen that $\chi^2_R$ here, which has been computed using the original 2% uncertainty for $\sigma_n (0°)$, is considerably larger than the other 40.0 MeV analysis. Therefore, only the original analysis at 40.0 MeV was used.

Results and Discussion

Values of the real central potential determined at 3.9 and 10.78
MeV from the total cross section analyses and at 13.5, 20.0, 22.0, 30.3, and 40.0 MeV from the three parameter differential cross section analyses are presented in Figure 14. Also shown is a least-squares fit to these values which is given by $V_n = (50.61 \pm 0.13) - (0.316 \pm 0.006)E_n$.

Figure 14. Linear Plots of the Real Central Potentials $V_n$ and $V_n$ vs. Energy. The Points X are from van Oers et al. (1974). The Values * are the Present Results. The Solid Lines are Linear Least-squares Fits.

Values of $W_d$ deduced from the differential cross section searches are in Figure 15. At 20 MeV the results of the two parameter and three parameter searches are given. The three parameter search
points are those marked x. The two parameter searches are marked * and +, where the * is the search over $W_d$ and $W_s$, and the + is the search on $V$ and $W_d$. It is seen that the value of $W_d$ obtained at 20 MeV for the three parameter search is very consistent with values obtained at adjacent energies. The smooth curve of Figure 12 was renormalized by a factor of 0.8 to provide better agreement with the differential cross section values of $W_d$. The curve shown in Figure 15 is the renormalized curve.

The values found in the differential cross section analyses for $W_s$ are shown in Figure 16. Again, the three parameter search results are marked x, at 20 MeV the * is the search on $W_d$ and $W_s$ and the + is the search over $V$ and $W_s$. The smooth curve here is identical to the curve given in Figure 10. The equations for the smooth curves for $V$, $W_d$, and $W_s$ shown in Figures 14, 15, and 16, respectively, are given in Table V.

As mentioned in Chapter III, the coefficient of the asymmetry term, $V_1$, can be determined from $V_n$ and $V_p$ provided that the Coulomb energy $\Delta_c$ is known. In the present work this Coulomb energy is defined to be equal to the Coulomb displacement energy, 18.82 MeV. The values for $V_p$ from van Oers et al. (1974) over approximately the equivalent proton energy range, 20 to 60 MeV, are shown in Figure 14. Their datum at 26.3 MeV for all angles has been omitted since its
Figure 15. The Imaginary Surface Strength $W_d$ vs. Energy. Also plotted are the Results of Angular Distribution Searches. The Symbols (Parameters Varied) are: $X(V, W_d, W_s)$, $+(V, W_d)$, and $*(W_d, W_s)$.

Figure 16. The Imaginary Volume Strength $W_s$ vs. energy. Also plotted are Angular Distribution Search Results. The Symbols (Parameters Varied) are: $X(V, W_d, W_s)$, $+(V, W_s)$, $*(W_d, W_s)$. 

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### TABLE V
Optical Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Present Work</th>
<th>Energy Range [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>(50.61 - 0.316(E_n))</td>
<td>(E_n \leq 60.0)</td>
</tr>
<tr>
<td></td>
<td>(-18.96 \ln E_n + 109.28)</td>
<td>(E_n &gt; 60.0)</td>
</tr>
<tr>
<td>(W_s)</td>
<td>0.0</td>
<td>(E_n \leq 11.0)</td>
</tr>
<tr>
<td></td>
<td>(-13.373 + 5.186(E_n^{1.2}) - 0.354(E_n))</td>
<td>(11.0 &lt; E_n \leq 50.0)</td>
</tr>
<tr>
<td></td>
<td>0.0188 (E_n) + 4.647</td>
<td>(E_n &gt; 50.0)</td>
</tr>
<tr>
<td>(W_d)</td>
<td>(-0.181(E_n) + 1.341)</td>
<td>(E_n \leq 10.0)</td>
</tr>
<tr>
<td></td>
<td>0.438 + 0.361(E_n) - 0.009(E_n^2)</td>
<td>(10.0 &lt; E_n \leq 30.0)</td>
</tr>
<tr>
<td></td>
<td>(-0.181(E_n) + 8.56)</td>
<td>(30.0 &lt; E_n \leq 47.45)</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>(E_n &gt; 47.45)</td>
</tr>
</tbody>
</table>

\(\chi^2_R\), 351, was significantly larger than the other values. An unweighted least-squares fit to the \(V_p\) in Figure 14 gives \(V_p = (63.03 \pm 0.97) - (0.323 \pm 0.025) E_p\). Using the expression for \(V_n\) for the neutron energy range 3.9 to 40.0 MeV given above and the expression for \(V_p\), one finds \(V_1 = (15.0 \pm 2.6) - (0.017 \pm 0.061) E_n\). It is important to note that this result is consistent with the statement that \(V_1\) is independent of energy over this range. The value of \(V_1\) obtained here, \(15.0 \pm 2.6\) MeV, is consistent with the value of 18 MeV from analyses of the Sm and Nd isotopes (Shamu, Bernstein, Ramirez, and LaGrange, 1980).
The present analysis indicates that $W_d$ is a maximum around 20 MeV and tends to zero near 50 MeV. This result is in agreement with the assumption that surface absorption is associated primarily with nucleon inelastic scattering.

Figure 16 shows that $W_s$, the imaginary volume strength, is negligible below about 10 MeV and increases rapidly with energy above that value. Since $(n, xn)$ processes show a similar energy dependence, this behavior suggests that the observed strong volume absorption just above 10 MeV may be associated with these reaction mechanisms.

Recent proton analyses (Nadasen et al. 1980; Schwandt, Meyer, Jacobs, Bacher, Vigdor, Kaitchuck, and Donoghue, 1982) as well as the present high energy analysis indicates that the energy dependence of $V_n$ is logarithmic above about 50 MeV. A linear dependence for $V_n$, valid at low energies, can be expressed as

$$V_n = a + bE_n$$

Similarly, a logarithmic energy dependence, valid at higher energies, can be written

$$V_n = a + \beta \ln (E_n)$$

If both $a$ and $b$ are known and the slopes and values of $V_n$ for the above expressions are matched at a given transition energy $E$,

$$\beta = -bE$$

and

$$a = V_n(E) + \beta \ln (E)$$
The parameters $a$ and $b$ for the linear fit are given in Table V. This transition energy was varied to optimize the fit over the total cross section peak near 80 MeV. A transition energy of 60 MeV provided the best fit. The values obtained for $a$ and $b$ at $E = 60$ MeV are given in Table V.

In Figure 17 the experimental values of the total cross section over the energy range 1 to 150 MeV are in good agreement with the calculated total cross section computed using the energy dependent expressions in Table V. The size of the $+$ symbols is about 1%.

The calculated value of the reaction cross section at 14.0 MeV, 2.604 $b$, is in excellent agreement with the experimental value at that energy, 2.580 $b$.

The unique feature of this study is the close relationship shown between the optical model parameters for neutrons on $^{208}$Pb derived here and those previously derived (van Oers et al., 1974) for protons.

Direct comparisons to previous neutron analyses (Finlay et al., 1984; Marshak et al., 1970) are difficult since different geometries and spin-orbit potentials were used. Qualitatively, the results of Finlay et al., who analyzed much of the same data, are similar to the results reported here. It should be noted that the slope of the real central potential over the energy range 7 to 50 MeV reported by Finlay et al., $-0.31$, is in excellent agreement with the slope derived in this study. The analysis of Perey (Marshak et al., 1970) failed to fit the total cross section below 10 MeV.
Figure 17. The Neutron Total Cross Section for Neutrons on $^{208}$Pb vs. Energy. The Solid Line is the Optical Model Calculation Using the Energy Dependent Parameters Given in Table V. The Data Plotted are: O Fowler et al. (1962), X Satkowiak (1979), * Ferguson et al. (1976), and + Shamu et al. (1980).
Phenomenological optical model parameters for neutrons incident on $^{208}$Pb have been determined for neutron energies from 4 to 150 MeV using the code SNOOPY-8. Neutron total cross sections from 4 to 150 MeV and angular distributions from elastic scattering from 13.5 to 40.0 MeV were utilized. The central potential geometry and spin-orbit potential employed were the average geometry and average spin-orbit potential of van Oers et al. (1974) for protons incident on $^{208}$Pb.

Analyses of total and differential cross sections were performed at a number of energies between 4 and 40 MeV to determine the real central potential, the imaginary surface potential and the imaginary volume potential strength at each energy. Very good fits were found for each of the angular distributions analyzed. For each of these parameters polynomial least-squares fits were done to determine their energy dependence. These energy dependent expressions were extrapolated above 40 MeV using measured total cross sections as a guide. These energy dependent expressions for $V$, $W_d$, and $W_s$ were found to provide a good fit to the total cross section from 1 to 150 MeV.

The present study indicates that the energy dependence of the real central potential strength is linear up to about 60 MeV and logarithmic above that energy. The imaginary surface potential
strength is approximately parabolic with a peak at about 20 MeV and tends to zero at about 50 MeV. The imaginary volume potential strength, $W_s$, is zero below 11 MeV, is described by a polynomial between 11 and 50 MeV, and is linear above 50 MeV.

For neutron energies from 4 to 40 MeV, the parameter $V$ for neutrons was compared to the proton $V$ in the equivalent energy range, 20 to 60 MeV, to find the asymmetry coefficient, $V_1$. It was found that $V_1$ can be expressed as:

$$V_1 = (15.0 \pm 2.6) - (0.17 \pm 0.061) E_n$$

and thus has a negligible energy dependence over this range.
Appendix

Relativistic Corrections in SNOOPY-8
Relativistic Corrections in SNOOPY-8

The relativistic analog of the Schrödinger equation for an incident particle of mass $m$, moving in a central potential $V(r)$ is

$$\left[ V^2 + k^2 - V(r) \right] \psi(r) = 0$$

where $\sqrt{S} = E_1 + E_2 - T_C + m + M$ is the total energy in the center of mass frame. Here $k$ and $U$ are

$$k = \frac{(M/\sqrt{S}) T_C (T_C + 2m) = (M^2/S) T_{1\frac{1}{2}} (T_{1\frac{1}{2}} + 2m)}{U(r) = 2\varepsilon V(r)}$$

where $T_{1\frac{1}{2}}$ is the projectile kinetic energy in the laboratory frame, $U(r)$ is the total potential (nuclear and Coulomb) and

$$\varepsilon = \frac{[M(T_C + M)]}{\sqrt{S}}$$

is the reduced energy where the target is treated nonrelativistically.

The radial wave equation becomes

$$\left[ \frac{d^2}{dp^2} + \left[ 1 - \frac{V(p)}{E_C} - \frac{L(L+1)}{p^2} \right] \right] f_L(p) = 0$$

where $p = kr$

$$\gamma = \frac{2(T_C + m)}{T_C + 2m} = 1 + \frac{T_C}{T_C + 2m}$$

Also, $T_C$, the total center of mass energy is derived from the projectile energy in the lab $T_{1\frac{1}{2}}$ by

$$T_C^2 + 2T_C (m + M) = 2MT_{1\frac{1}{2}}$$

In SNOOPY-8, calculations are done with potentials which have all been multiplied by $\gamma$. 

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