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**THE PAULI OPERATOR CORRECTION IN THE
EFFECTIVE AN INTERACTION**

by

Jaleh Owliaei

**A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
requirement for the
Degree of Master of Arts
Department of Physics**

**Western Michigan University
Kalamazoo, Michigan
April 1985**

THE PAULI OPERATOR CORRECTION IN THE EFFECTIVE ΛN INTERACTION

Jaleh Owliaei, M.A.

Western Michigan University, 1985

Hypernuclear structure calculations performed for ${}^6_{\Lambda}\text{Li}$, ${}^7_{\Lambda}\text{Li}$, and ${}^{16}_{\Lambda}\text{O}$ have reproduced the $1_{\Lambda} = 1$ levels inferred from the recent (\bar{k}, π^-) reactions. However the $1_{\Lambda} = 0$, groundstate of ${}^{16}_{\Lambda}\text{O}$ is overbound by 6 MeV in these calculations, indicating a density dependence in the effective ΛN interaction in the interior of the heavier system. In this work corrections to the ${}^{16}_{\Lambda}\text{O}$ effective interaction are introduced in terms of a Brueckner-Goldstone G-matrix with a modified Pauli operator. The G-matrix of ${}^{16}_{\Lambda}\text{O}$ derived from that of ${}^6_{\Lambda}\text{Li}$ with the corrected Pauli operator results in an improvement of about one-half MeV for the groundstate of ${}^{16}_{\Lambda}\text{O}$. This makes the inclusion of the repulsive three-body forces, which has been neglected in the present calculations, more urgent. Since the number of ΛNN interactions in ${}^{16}_{\Lambda}\text{O}$ are ten times more than those in ${}^6_{\Lambda}\text{Li}$, these three-body interactions would improve the overbinding in ${}^{16}_{\Lambda}\text{O}$ with little effect on ${}^6_{\Lambda}\text{Li}$.

ACKNOWLEDGEMENTS

I would like to express my gratitude to professor Dean Halderson for suggesting the topic of this thesis, and for the constant encouragement and help that he has offered me. It is with pleasure that I also would like to thank the other members of my committee professors Michitoshi Soga and Alvin Rosenthal for their valuable advise.

Jaleh Owliaei

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CHAPTER I

INTRODUCTION

The central problem of nuclear physics is to understand and describe the structure of nuclei in terms of the particles forming them and the interactions among these particles. In this study some aspects of ${}_{\Lambda}^{16}\text{O}$ nucleus are examined. Thus it is assumed that the constituent baryons are protons, neutrons and a Λ -hyperon. An effective interaction among these particles is formulated in terms of the Brueckner-Goldstone theory of nuclear matter.

Hypernuclei - Why Study Them?

Nuclear force with its short range attraction ($r \approx 2$ fm), hard core repulsion ($r \approx .5$ fm), non central, spin dependent nature and other peculiarities appears to be more complex than other known fundamental forces. Complicated forces in one well investigated case, that of chemical forces between atoms, is well understood in terms of internal structure of atoms. This idea suggests that the complexity of nuclear force might be a manifestation of internal structure and fundamental forces acting within nucleons. Internal nuclear motions can give rise to the excited nucleons (Gottfried & Weisskopf, 1984)

One uses the term "baryon" for fermions that interact via the strong interaction. The existence of baryons other than nucleons was first inferred from the analysis of high energy cosmic rays.

They are short-lived particles and carry different charges, but all are an integer multiple of the charge of electron. Among these baryons was found a neutral massive particle which decays into proton and pion, but rather slowly. This particle was identified as lambda particle. Its lifetime is 2.6×10^{-10} sec. This is much longer than 10^{-23} sec, typical of strong interaction; it is rather close to 10^{-8} sec, the order of weak interaction lifetime. Gellmann and Nishijima speculated that Λ has a property that proton and neutron do not have, and the decay $\Lambda \rightarrow p + \pi$ violates conservation of this property, which is called "strangeness", S . Strangeness is conserved in strong interaction but could be changed by 1 in weak interaction. Λ is defined as having $S = -1$ (Dalitz, 1981; Soga, 1983).

Later eight of these particles were grouped into a baryon octet, based on their 3-quark structure. Particles with no s-quarks in their wave function have $S = 0$, and these are proton and neutron. The other particles in baryon octet have one or two s-quarks in their structure and have S different from zero. Particles with $S = -1$, which are Λ and Σ baryons are called Y-particles. The $S \neq 0$ particles are collectively called "hyperons" or "strange particles".

The charge equation is modified as:

$$Q = I_3 + \frac{1}{2} Y, \quad Y = B + S + C$$

where inclusion of S and C now gives correct results for neutral Λ -particle as well as the rest of baryons. Here B is the

"baryon" number which is 1 for all baryons, Q is the modified charge, I_3 is the "isospin projection", S is strangeness, C is "charm" and Y is called "hypercharge". This general charge equation which now encompasses all the states, introduces a new set of quantum numbers. These numbers, far from being an artifice (to save the charge equation), each has its own significance in subnuclear physics. Some properties of the baryon octet are summarized in the following diagrams: (Gottfried & Weisskopf, 1984; Soga, 1983, 1984).

S	Y	I	I_3				
			-1	$-1/2$	0	$1/2$	1
0	1	$1/2$		N		P	
-1	0	0			Λ		
-1	0	1	Σ^-		Σ^0		Σ^+
-2	-1	$1/2$		Ξ^-		Ξ^0	

Figure 1. Baryon octet

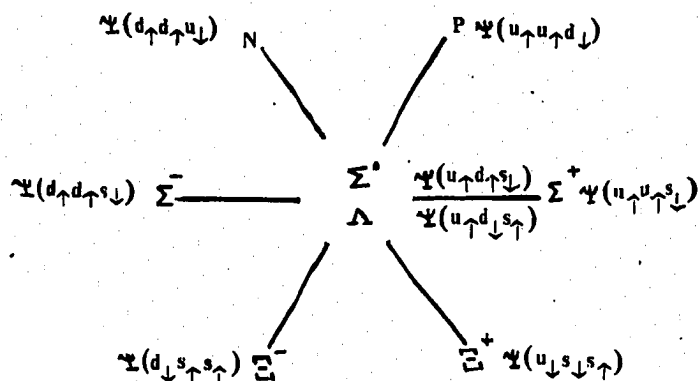


Figure 2. Quark structure of baryons

A hypernucleus is a nucleus in which one or more nucleons are replaced by hyperons. For example, Λ -hyperons are known to form particle stable bound states with all the stable nuclei as well as with some nuclei which are normally not particle stable. These Λ -nucleus systems are called Λ -hypernuclei, indicated by ${}^A_Z\Lambda$, in which Z is the symbol for the nuclear charge and A is the mass number.

Most of our knowledge about hyperon-nucleon interactions stems from the hyperon-nucleon binding energies. This is so because the short life time of hyperons make the hyperon-nucleon scattering experiments very difficult.

Since the path lengths traversed by hyperon before decay is so short relative to the mean path length for collision in liquid hydrogen, a direct determination of the hyperon-nucleon interaction from a study of hyperon-proton collision is very difficult... At the present the most information bearing on hyperon-nucleon interaction is that obtained from the binding of Λ hyperon to nuclei (Dalitz, 1965. p. 1).

Λ -hypernuclei are studied in more detail than the other hypernuclei, since the lifetime of Λ -hyperon is longer than the others, and also because it decays into two well measurable charged particles. Another reason is the absence of coulomb scattering interference for Λ -p interactions (Alexander Karshon, 1967). These considerations make the binding energy of Λ -particles an attractive topic of investigation.

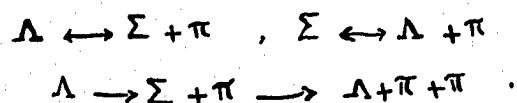
The Baryon-Baryon Interaction

The large binding energies of Λ -hyperons show that the strength of the Λ -nuclear interactions are comparable to NN

interactions. The absence of ΛN bound state though, together with the fact that the binding energy of the lightest Λ -hypernuclei ${}^3_{\Lambda}\text{H}$ is only slightly larger than that of deuteron indicates that Λ -binding energies although strong, are still substantially weaker than nucleon-nucleon interactions (Bodmer, 1967).

The most characteristic feature of hypernuclei stems from the distinguishability of the hyperon from nucleons, and consequently the absence of Pauli principle for hyperons. Besides this, one big difference between hyperon-nucleon and nucleon-nucleon interaction is the effective range of these forces. NN and YN forces are both transmitted through the meson field. The range of NN force is divided into three regions. The long ranges, $r \approx 2$ fm or more, are dominated by the exchange of single π -meson. This is the part of NN interaction which is best understood. Then there is the intermediate range of $r \approx 1$ fm, which is attributed to the σ -mesons that simulates contributions arising from exchange of pairs of π -mesons. The third region of $r \approx .5$ fm and less, is referred to as the "hard core" arising from heavy meson exchange (Barenger, 1969).

ΛN forces on the other hand, do not have the one pion exchange (OPE) tail, or the longer range of the NN forces. The N-N forces of the longest range is about .7 fm, and corresponds to the intermediate, imperfectly understood part of the NN interaction. As mentioned above one explains this region either by using two pion exchange or one boson exchange (OBE) models (Bodmer, 1967). The two pion exchange is made possible through the following decays:



Absorption of these pions by a neighbouring nucleon will give rise to the .7 fm ΛN interaction. Some typical two pion exchange (TPE) processes are shown in the following graphs (Dalitz, 1965, p. 62).

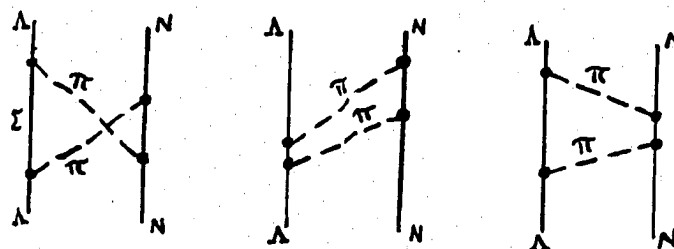


Figure 3. Some typical graphs contributing to the pionic component of the Λ -N interaction.

Other ΛN interactions also arise from exchange of k, ρ, ω and other mesons and resonant states which may exist (Dalitz, 1965, p. 76). (The k -meson exchange involves the transfer of strangeness between baryons and give rise to exchange potentials, having a range of $\approx .4$ fm).

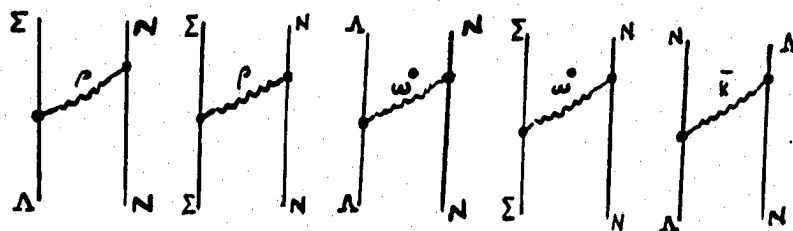


Figure 4. Typical ΛN graphs involving k, ρ , and ω exchange.

Finally, the above decays will lead to another possibility, the three body force, which is due to the absorption of pions by two

neighbouring nucleons (Dalitz, 1965, p. 3; Gal et al., 1970).

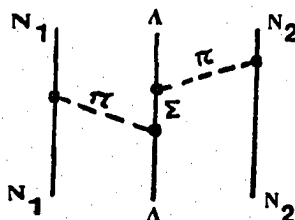


Figure 5. The pionic component of the Λ NN three body potential.

It will be seen in the present calculations that these three body potentials should not be overlooked and they could have appreciable contribution to the binding energies (de-Shalit, 1967, p.5; Gal et al., 1977).

The Overbinding Problem

Interest in Λ -particle studies is increased recently in attempts to explain and reproduce the important features of strangeness exchange (k^- , $\bar{\pi}$) reactions on a number of targets (Auerbach & Van Giai, 1979; Dalitz & Gal, 1978; Dalitz, 1981). In an article by D. Halderson (1984), some problems pertaining to the hypernuclear structure calculations are mentioned and a solution is offered. Namely, the fact that the hyperon wavefunction is different from that of nucleon, and also elimination of spurious center of mass excitations for hyperon orbitals other than $0S_{1/2}$ are taken into consideration (Dalitz & Gal, 1981; Halderson, 1984). Then calculations are made for ${}^6_{\Lambda}\text{Li}$, ${}^7_{\Lambda}\text{Li}$, and ${}^{16}_{\Lambda}\text{O}$.

In these calculations an interaction of the form:

$$v_{\Lambda N} = v(r)(1-s+sp_x)(1+a\vec{\sigma}_N \cdot \vec{\sigma})$$

is employed in which P_x is the space exchange operator and s and a are parameters chosen to fit data (Auerbach et al., 1983). The radial dependence $v(r)$ for the present calculation is:

$$v(r) = 4000 y(r/r_1) - 1358.05 y(r/r_2)$$

where $y(x) = e^{-x}/x$, and $r_1 = 0.25$ fm, and $r_2 = 0.4$ fm. These values provide self consistency in the Λ -single particle energy. A small symmetric spin-orbit odd interaction is added in the form (Halderson, 1984):

$$v_{\Lambda N}^{s0} = [-715.5 y(r/r_1) - 81.9 y(r/r_2)] \vec{l} \cdot \vec{s}$$

The $1_{\Lambda} = 1$ levels inferred from $(\vec{k}, \vec{\pi})$ are well reproduced, but the $1_{\Lambda} = 0$, the ground state of ${}^{16}_{\Lambda}O$ is overbound by 6 MeV, indicating a density dependence in the effective ΛN interaction in the interior of the heavier system.

In this thesis an effort is made to improve the overbinding problem of ${}^{16}_{\Lambda}O$ ground state. Corrections to the ${}^{16}_{\Lambda}O$ effective interaction are introduced by a proper Pauli operator. The role of Pauli operator in effective interaction is discussed in the next section.

CHAPTER II

GOLDSTONE-BRUECKNER THEORY

Effective Interactions

To solve the many-body problem of particles inside the nucleus, one may start by solving a non-relativistic Schrödinger equation for particles interacting through a two-body potential. Together with the explicit assumption of non-relativistic limit for the problem, it is also assumed that three-body and four-body potentials are negligible (Day, 1967). The mesonic degrees of freedom are included only as a resulting ΛN potential.

A criterion for a realistic two-body potential is the ability to reproduce the shell model properties of the nuclei. In shell model, the many-body problem of nucleons inside nuclei is solved by replacing the interaction of one particle with the rest, with the potential of single particle in an effective average central field due to the rest of the nucleons (de-Shalit & Talmi). Here though, we have to start with a realistic two-body force and hope to calculate the shell model wave functions and the effective potential.

There are some difficulties in nucleon-nucleon interaction however, mainly the hard core, which makes a self-consistent approach difficult. This interaction is also too strong to use ordinary perturbation techniques directly. However, the presence of many nucleons permits one to introduce an effective nucleon-

nucleon interaction which is well behaved and finite. One important effective interaction is the G-matrix, or reaction matrix central to Brueckner-Goldstone theory of nuclear matter (Ring & Schunck, 1980, chap. 4).

Goldstone Theorem

Goldstone theorem gives the perturbation expansion of the exact ground state function and the exact ground state energy of a many-fermion system (Baranger, 1969; Day 1967). The Hamiltonian is the sum of kinetic energies of all the particles plus the sum of two-body interactions between them:

$$H = \sum_i^A T_i + \sum_{i<j}^A v_{ij},$$

in which v_{ij} is the realistic two-body force. H is divided into a perturbed and an unperturbed part:

$$H = H_0 + H_1,$$

where $H_0 = \sum_i^A (T_i + U_i)$ and $H_1 = \sum_{i<j}^A v_{ij} - \sum_i^A U_i$.

Inclusion of the single particle potential U , in the unperturbed Hamiltonian permits one to make the perturbation H_1 very small. Still, the total Hamiltonian does not involve U and final result will not depend on U , but the energy which is an expansion in powers of H_1 will converge more rapidly for certain choices of U . This particular choice will be discussed in calculation section.

For the unperturbed part one can write:

$$(T_i + U_i) \Phi_p(r_i) = E_p \Phi_p(r_i)$$

where Φ_p are single particle wave functions and form a complete orthonormal set. The unperturbed ground state is the normalized Φ_0 represented by a Slater determinant:

$$\Phi_0 = (A!)^{-1/2} \mathcal{A}(\Phi(r_1) \dots \Phi(r_A))$$

where \mathcal{A} is antisymmetrizing operator. Then,

$$H_0 \Phi_0 = \epsilon_0 \Phi_0, \quad \epsilon_0 = \sum_{n=1}^A E_n$$

The exact ground state satisfies: $H\Psi = \epsilon\Psi$. The expansion for ϵ through third order in H_1 is:

$$\begin{aligned} \epsilon = & \epsilon_0 + \langle \Phi_0 | H_1 | \Phi_0 \rangle + \langle \Phi_0 | H_1 (\epsilon_0 - H_0)^{-1} P H_1 | \Phi_0 \rangle \\ & + \langle \Phi_0 | H_1 (\epsilon_0 - H_0)^{-1} P H_1 (\epsilon_0 - H_0)^{-1} P H_1 | \Phi_0 \rangle \\ & - \langle \Phi_0 | H_1 | \Phi_0 \rangle \langle \Phi_0 | H_1 (\epsilon_0 - H_0)^{-1} P H_1 | \Phi_0 \rangle \dots, \end{aligned}$$

where $P \equiv 1 - |\Phi_0\rangle\langle\Phi_0|$ is the projection operator which restricts the summation over intermediate states so that it contains only states above Fermi sea (Day, 1967). The matrix elements of H_1 would involve terms such as $\langle \Phi_1 \Phi_2 | v_{ij} | \Phi_3 \Phi_4 \rangle$. As it stands the matrix elements can get very large because of the hard core.

Brueckner Reaction Matrix

The same kind of problem which arises in the nucleon-nucleon interaction due to the short range repulsion in the nuclei also arises in nucleon-nucleon scattering in free space. Namely, in Goldstone expansion the hard core makes the matrix elements involving a potential infinite, and in the scattering problem, by

using the Born approximation in which one calculates the scattering amplitude to first order in v , one obtains inaccurately large values.

The treatment is also similar for the two cases. In scattering problem, one calculates the scattering amplitude to all orders in v , which amounts to solving the Lippmann-Schwinger equation, including multiple scattering (Joachain, 1975; Merzbacher, 1970; Soga, 1984). Then one gets correct results. In Goldstone expansion, instead of bare interaction, one introduces Brueckner's effective potential which is itself a sum of infinite scattering processes of two nucleons in nuclear matter, the bare interaction being just the Born term of this series (Ring & Schunck, 1980, Chap. 4). So it suffices to do a complete summation of all orders of the interaction for every pair of interacting particles.

Since the interaction is strong only at short ranges, this should be sufficient to ensure convergence and it should not be necessary to sum completely by all interactions between three nucleons interacting at close range, four nucleons etc... (Baranger, 1969, p. 542).

This effective interaction is called G-matrix, or reaction matrix. It is for two nucleons in nuclear matter, but analogous to scattering T-matrix for two nucleon in free space. It is defined in a similar manner, too. So in operator form G is defined as (Day, 1967):

$$G(\omega) = v + v (Q/e) v + v (Q/e) v (Q/e) v + \dots$$

$$G(\omega) = v + v (Q/e) [v + v (Q/e) v + \dots]$$

$$G(\omega) = v + v (Q/e) G(\omega)$$

and this equation is called the Bethe-Goldstone equation. Q and e are two particle operators defined as:

$$Q|pq\rangle = \begin{cases} |pq\rangle & \text{if } p>A \text{ and } q>A \\ 0 & \text{Otherwise} \end{cases}$$

$$e|pq\rangle = (\omega - E_p - E_q)|pq\rangle \quad |pq\rangle \text{ stands for } \Phi_p(r_1)\Phi_q(r_2)$$

The Pauli operator annihilates two particle states below Fermi sea. It is for two nucleons and is to be modified for Λ -hyperon. Operator e gives the starting energy ω minus the energy of the two particle state. It is important to note that G is a function of ω , and ω must be specified to avoid ambiguity in G .

The purpose of this kind of many-body technique is two fold: first, by resumming the series, one gets rid of the hard core problem; second, by elimination of the potential in favor of reaction matrix, one includes more of the many-body effects than if one had taken just the bare interaction.

CHAPTER III

CALCULATIONS

Choice of the Single Particle Potential

The perturbation H_1 , in the Goldstone expansion was divided into two parts:

$$H_1 = \sum_{i,j}^A v_{ij} - \sum_i^A U_i$$

i.e., a two-body and a one-body part. The two-body potential was replaced by the G-matrix in order to get rid of the hard core problem. Now one is to choose the single particle potential U .

We recall that the final result is independent of U , and inclusion of U in the unperturbed Hamiltonian H_0 is intended to ensure a rapid convergence, and to ease the numerical calculations. A convenient choice is harmonic oscillator potential. This fulfills the requirement of the Goldstone theorem; that the particle wavefunctions should form a complete orthonormal set. The main reason for this particular choice is, however, the fact that this H_0 nicely splits into a center of mass and a relative part (Baranger, 1969).

G-Matrix for ${}^{16}_\Lambda\text{O}$

As mentioned earlier, the effective ΛN interaction for ${}^{16}_\Lambda\text{O}$ should be modified. Since the interaction used for ${}^6_\Lambda\text{Li}$ gives reasonable results, the G-matrix for oxygen is calculated in terms

of that for ${}^6\text{Li}$ (Halderson, 1984). Both of these matrices should satisfy the Bethe-Goldstone equation: $G(\omega) = v + v(Q/e)G(\omega)$.

Let G_a and G_b be matrices satisfying the above equation, and let a and b be arbitrary operators.

$$G_a = v + vaG_a = v + vav + vavav + \dots$$

$$G_b = v + vbG_b = v + vbv + vbvbv + \dots$$

$$= v + v[a + (b-a)]v + v[a + (b-a)]v[a + (b-a)]v + \dots$$

So G_b can be expanded completely to include a large number of terms. These terms are now regrouped according to the number of $(b-a)$'s that occur:

$$\text{The 0th order: } v + vav + vavav + \dots = G_a$$

$$\begin{aligned} \text{The 1st order: } (v + vav + vavav + \dots)(b-a)(v + vav + vavav + \dots) \\ = G_a(b-a)G_a \end{aligned}$$

$$\begin{aligned} \text{The 2nd order: } (v + vav + vavav + \dots)(b-a)(v + vav + vavav + \dots)^* \\ (b-a)(v + vav + vavav + \dots) = G_a(b-a)G_a(b-a)G_a \end{aligned}$$

$$G_b = G_a + G_a(b-a)G_a + G_a(b-a)G_a(b-a)G_a + \dots$$

which is the iterative solution of the equation

$$G_b = G_a + G_a(b-a)G_b.$$

This equation may now be used with G_b as the corrected oxygen G-matrix, G_a the ${}^6\text{Li}$ G-matrix, and a and b as the corresponding Q/e operators (Baranger, 1969).

Treatment of the Pauli Operator and Energy Denominator

The form of operator Q which annihilates a two particle state below Fermi Sea, is governed by different Fermi levels for Λ particle and nucleons. The Fermi Sea, being the lowest filled states, of course differs for various nuclei. It is shown for ${}^6\text{Li}$ and ${}^{16}\text{O}$ in the following diagrams:

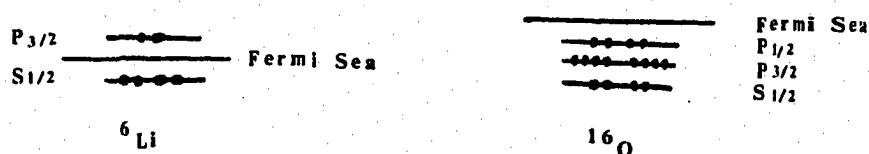


Figure 6. Fermi levels in ${}^6\text{Li}$ and ${}^{16}\text{O}$

Due to its distinguishability from nucleons, Λ -particle would not obey the Pauli principle and it resides in the $0S_{1/2}$ orbit. The Fermi levels for Λ and nucleons in ${}^6_\Lambda\text{Li}$ and ${}^{16}_\Lambda\text{O}$ are shown in the following diagrams:

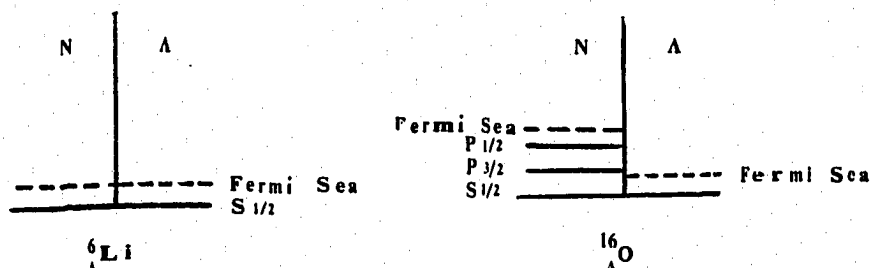


Figure 7. Fermi levels in ${}^6_\Lambda\text{Li}$ and ${}^{16}_\Lambda\text{O}$

Thus the projection operator Q_0 for ${}^6_\Lambda\text{Li}$ is defined as:

$$Q_0 = \begin{cases} 1 & \text{if } 2n_\Lambda + l_\Lambda > 0 \text{ and } 2n_N + l_N > 0 \\ 0 & \text{otherwise} \end{cases}$$

For ${}^{16}_\Lambda\text{O}$, however, the projection operator should annihilate Λ from $0S_{1/2}$, which is the same as Q_0 , and also should annihilate nucleons from $0P_{1/2}$, $0P_{3/2}$ and $0S_{1/2}$. Consequently Q for ${}^{16}_\Lambda\text{O}$ will be:

$$Q = \begin{cases} 1 & \text{if } 2n_N + l_N > 1 \text{ and } 2n_\Lambda + l_\Lambda > 0 \\ 0 & \text{otherwise} \end{cases}$$

The other quantity which has to be specified is the energy denominator. G is now written as:

$$G(\omega) = v + v_0 [Q/(\omega - H_0)] G(\omega)$$

H_0 is harmonic oscillator Hamiltonian and $H_0 = H_0(N) + H_0(\Lambda)$ with $\hbar\omega_\Lambda = 14.05$ MeV for Λ -particle and $\hbar\omega_N = 11.83$ MeV for nucleon. Due to the pairing interaction, the single particle energy of occupied orbits is lowered by an energy gap Δ (Cohen 1971; Soga, 1983). This gap is experimentally observed to be ≈ 30 MeV, therefore this amount is subtracted from H_0 in the denominator i.e., $\omega - (H_0 - \Delta)$ or equivalently added to ω : $\omega + \Delta - H_0$. This also lowers the single particle energy of unoccupied orbits by an amount comparable to that observed for the occupied orbits. ω is the starting energy $E_0(\Lambda) + E_0(N)$. In the present calculations the following experimental values are used (Dalitz & Gal, 1978, McCarthy & Davies, 1970):

TABLE 1

Experimental values for $E(N)$, $E(\Lambda)$, and the energy gap Δ .

$$\Delta = 30 \text{ Mev}$$

SHELL	$E(N)$	$E(\Lambda)$
$S_{1/2}$	-26 Mev	-13 Mev
$P_{3/2}$	-14 Mev	
$P_{1/2}$	-8 Mev	

Redefine $\omega = \omega + \Delta$, to include the energy gap. Thus, $\omega = 9, 3$, and -9 Mev corresponding to $0P_{1/2}$, $0P_{3/2}$, $0S_{1/2}$ orbits respectively.

$G_{Li}(\omega) = v + v[Q_0/(\omega - H_0)] G_{Li}(\omega)$, the G-matrix using ${}^6_{\Lambda}Li$ Pauli operator is formed first, then equation

$$G_0 = G_{Li} + G_{Li} [Q/(\omega - H_0) - Q_0/(\omega - H_0)] G_0$$

is employed to obtain the improved G matrix with the ${}^{16}_{\Lambda}O$ Pauli operator.

Binding Energy Calculations

Binding energy is the sum of potential and kinetic energies:

$$B = P.E. + K. E.$$

i) Kinetic energy:

From the total energy of the core nucleus and Λ -particle, the center of mass kinetic energy ($T_{c.m.}$) should be subtracted.

$$T = \sum_i P_i^2/2m + P_{\Lambda}^2/2m - T_{c.m.}$$

$$T_{c.m} = P^2/2M, \quad M = Am + m_\Lambda$$

where m and m_Λ are the masses of nucleon and Λ -particle respectively.

$$\begin{aligned} T_{c.m} &= 1/2M [p_\Lambda + \sum_i^A p_i]^2 \\ &= 1/2M \left[\sum_{ij}^A p_i \cdot p_j + 2 \sum_i^A p_i \cdot p_\Lambda + p_\Lambda^2 \right] \\ &= 1/2M \left[\sum_i^A p_i^2 + 2 \sum_{ij}^A p_i \cdot p_j + 2 \sum_i^A p_i \cdot p_\Lambda + p_\Lambda^2 \right] \\ T &= \sum_i^A (1/2m - 1/2M) p_i^2 + (1/2m_\Lambda - 1/2M) p_\Lambda^2 - 1/M \sum_{ij}^A p_i \cdot p_j \\ &\quad - 1/M \sum_i^A p_i \cdot p_\Lambda \end{aligned}$$

Now the total Hamiltonian is written as:

$$H = V_{\Lambda N} + V_{NN} + T$$

Substituting T in this formula and rearranging terms:

$$\begin{aligned} H &= V_{NN} + V_{\Lambda N} + \sum_i^A (1/2m - 1/2M) p_i^2 + (1/2m_\Lambda - 1/2M) p_\Lambda^2 \\ &\quad - 1/M \sum_{ij}^A p_i \cdot p_j - 1/M \sum_i^A p_i \cdot p_\Lambda \\ &= V_{NN} + \sum_i^A p_i^2/2m - 1/2M \sum_i^A p_i^2 - 1/M \sum_{ij}^A p_i \cdot p_j \end{aligned} \quad (3-1)$$

$$+ V_{\Lambda N} - 1/(m_\Lambda + Am) \sum_i^A p_i \cdot p_\Lambda \quad (3-2)$$

$$+ (1/2m_\Lambda - 1/2M) p_\Lambda^2 \quad (3-3)$$

$$\begin{aligned} (3-1) : \quad & V_{NN} + \sum_i^A p_i^2/2m - 1/2M \sum_i^A p_i^2 - 1/M \sum_{ij}^A p_i \cdot p_j \\ &= V_{NN} - 1/M \sum_{ij}^A p_i \cdot p_j + [m_\Lambda / (M_{core} + m_\Lambda)] \sum_i^A p_i^2/2m \end{aligned}$$

$$= V_{NN} - 1/M \sum_{i,j}^A p_i \cdot p_j + [m_\Lambda / (M_{\text{core}} + m_\Lambda)] (3/4 \hbar \omega_N)$$

$$(3-2) : \quad V_{\Lambda N} - 1/(m_\Lambda + A m) \sum_i p_i \cdot p_\Lambda \text{ is the } H_{\Lambda N} \text{ term.}$$

$$\begin{aligned} (3-3) : \quad & (1/2 m_\Lambda - 1/2 M) p_\Lambda^2 \\ &= (A m) p_\Lambda^2 / [(m_\Lambda + A m) 2 m] \\ &= [M_{\text{core}} / (M_{\text{core}} + m_\Lambda)] (3/4 \hbar \omega_\Lambda), \end{aligned}$$

since the Λ is assumed to reside in a $0S_{1/2}$ state.

Terms contributing to the Λ KE in this Hamiltonian are :

$$KE = [m_\Lambda / (m_{\text{core}} + m_\Lambda)] (3/4 \hbar \omega_N) + [M_{\text{core}} / (M_{\text{core}} + m_\Lambda)] (3/4 \hbar \omega)$$

ii) Potential energy :

Potential energy is found to be [See Appendix]:

$$PE = \sum_{j_a j_{m_a}} V_{j_a j_{m_a}}^J \frac{|J|}{|j|} - \sum_J V_{j_h j_{m_h}}^J (-)^{2J+2j_h} |J| \left\{ \begin{matrix} j_h & j & J \\ j_h & j & J_B \end{matrix} \right\}$$

in which the first term is due to the binding of Λ -particle to ^{16}O core and the second term corresponds to that of Λ with the hole that should be subtracted.

CHAPTER IV

RESULTS

$$KE = [m_A / (m_A + M_{core})] (3/4 \hbar \omega_N) + [M_{core} / (M_{core} + m_A)] (3/4 \hbar \omega_A)$$

with $m_A = 1115$ MeV , $M_{core} = (931 \times 15)$ MeV , $\hbar \omega_N = 11.83$ MeV and $\hbar \omega_A = 14.05$ MeV.

$$KE = 10.4144 \text{ MeV}$$

$$PE = \sum_{J, J_m, \alpha} V_{\alpha, \alpha}^J \frac{[J]}{[j]} - \sum_J V_{j_h j_l}^J (-)^{2j+2j_h} [J] \begin{Bmatrix} j_h & j & J \\ j_h & j & J_B \end{Bmatrix}$$

For $^{16}_\Lambda O$, $J_B = 1$ and :

$$\begin{aligned} PE = & 2 \langle OS_{1/2} OS_{1/2} (0) | V | OS_{1/2} OS_{1/2} \rangle^{1/2} \\ & + 2 \langle OS_{1/2} OS_{1/2} (1) | V | OS_{1/2} OS_{1/2} \rangle^{3/2} \\ & + 2 \langle OP_{3/2} OS_{1/2} (1) | V | OP_{3/2} OS_{1/2} \rangle^{3/2} \\ & + 2 \langle OP_{3/2} OS_{1/2} (2) | V | OP_{3/2} OS_{1/2} \rangle^{5/2} \\ & + 2 \langle OP_{1/2} OS_{1/2} (0) | V | OP_{1/2} OS_{1/2} \rangle^{1/2} \\ & + 2 \langle OP_{1/2} OS_{1/2} (1) | V | OP_{1/2} OS_{1/2} \rangle^{3/2} \\ & - \sum_J \langle OP_{1/2} OS_{1/2} (J) | V | OP_{1/2} OS_{1/2} \rangle [J] \begin{Bmatrix} 1/2 & 1/2 & J \\ 1/2 & 1/2 & 1 \end{Bmatrix} \quad (4-1) \end{aligned}$$

(4-1) is expanded as:

$$\begin{aligned} & \langle OP_{1/2} OS_{1/2} (0) | V | OP_{1/2} OS_{1/2} \rangle [0] \begin{Bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 1 \end{Bmatrix} \\ & \langle OP_{1/2} OS_{1/2} (1) | V | OP_{1/2} OS_{1/2} \rangle [1] \begin{Bmatrix} 1/2 & 1/2 & 1 \\ 1/2 & 1/2 & 1 \end{Bmatrix} \end{aligned}$$

A computer code is developed to calculate the above matrix

elements. Relevant matrix elements are summarized in the following table. To check the convergence, calculations are performed for two different basis sizes corresponding to $n \leq 6$ and $n \leq 8$ where

$$n = 2n_A + 1_A + 2n_N + 1_N$$

TABLE 2
CALCULATED MATRIX ELEMENTS

MATRIX ELEMENTS	ω	$n=6, n=8$ ${}^6\text{Li}_A$ G-Matrix	$n=6$ ${}^{16}\text{O}_A$ G-Matrix	$n=8$ ${}^{16}\text{O}_A$ G-Matrix
$\langle OS_{1/2} OS_{1/2} (0) V OS_{1/2} OS_{1/2} \rangle$	-9	-3.05709	-3.05709	-3.05709
$\langle OS_{1/2} OS_{1/2} (1) V OS_{1/2} OS_{1/2} \rangle$	-9	-2.11645	-2.11645	-2.11645
$\langle OP_{3/2} OS_{1/2} (1) V OP_{3/2} OS_{1/2} \rangle$	3	-1.51524	-1.46708	-1.46708
$\langle OP_{3/2} OS_{1/2} (2) V OP_{3/2} OS_{1/2} \rangle$	3	-1.24464	-1.21296	-1.21296
$\langle OP_{1/2} OS_{1/2} (0) V OP_{1/2} OS_{1/2} \rangle$	9	-1.06390	-1.03740	-1.03740
$\langle OP_{1/2} OS_{1/2} (1) V OP_{1/2} OS_{1/2} \rangle$	9	-1.31969	-1.27750	-1.27750

Binding energy of ${}^{16}_{\Lambda}0$ groundstate :

a- using G_{Li} -matrix

$$PE = -24.006535 \quad \text{MeV}$$

$$KE = +10.414392 \quad \text{MeV}$$

$$BE = -13.59093 \quad \text{MeV}$$

b- using G_0 -matrix

$$PE = -23.58493 \quad \text{MeV}$$

$$KE = +10.414392 \quad \text{MeV}$$

$$BE = -13.170538 \quad \text{MeV}$$

$$\Delta (BE) = 0.420455 \quad \text{MeV}$$

CHAPTER V

CONCLUSION

A modified ΛN interaction is considered for ${}^{16}_{\Lambda}O$ in terms of a Brueckner-Goldstone G -matrix with a corrected Pauli operator. Calculations were not sensitive to increase of n from 6 to 8 indicating that convergence is already achieved for $n = 6$. The contribution of Pauli operator correction is found to be about one half MeV. So, the overbinding problem of ${}^{16}_{\Lambda}O$ is improved but not corrected completely. This makes the inclusion of repulsive three-body forces more urgent. Since the number of ΛNN interactions in ${}^{16}_{\Lambda}O$ are ten times more than those in ${}^6_{\Lambda}Li$, these three-body interactions would improve the overbinding in ${}^{16}_{\Lambda}O$ without changing the results of ${}^6_{\Lambda}Li$ drastically.

APPENDIX

The potential energy for $^{16}_\Lambda\text{O}$ in which one nucleon is replaced by a Λ -particle involves particle-hole matrix elements of the form:

$$\begin{aligned}
 V &= \langle j_p j_h^{-1} | V | j_p' j_h^{-1} \rangle^{J_B} \\
 &= \sum \begin{pmatrix} j_p & j_h & J_B \\ m_p & -m_h & M_B \end{pmatrix} (-)^{j_h - m_h} \begin{pmatrix} j_p' & j_h' & J_B \\ m_p' & -m_h' & M_B \end{pmatrix} * \\
 &\quad \langle j_p m_p (j_h m_h)^{-1} | V | j_p' m_p' (j_h' m_h')^{-1} \rangle (-)^{j_h' - m_h'}
 \end{aligned}$$

C's are the Clebsh-Gordan coefficients ensuring that the total angular momentum J_B is a good quantum number, $(-)^{j_h - m_h}$ and $(-)^{j_h' - m_h'}$ ensure correct rotational transformation characteristic of the hole creation operators (Brink & Satchler, 1968/1979). The above matrix element can be written in its second quantized form:

$$\begin{aligned}
 &\langle j_p m_p (j_h m_h)^{-1} | V | j_p' m_p' (j_h' m_h')^{-1} \rangle \\
 &= 1/2 \sum_{\alpha\beta\gamma\delta} \langle 0 | a_h^\dagger a_p a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\delta a_p' a_h' | 0 \rangle v_{\alpha\beta\gamma\delta} \\
 &= 1/2 \sum_{\alpha\beta\gamma\delta} v_{\alpha\beta\gamma\delta} \langle 0 | a_p a_\alpha^\dagger a_\beta^\dagger (\delta_h^{h'} - a_h' a_h^\dagger) a_\delta a_\gamma a_p' | 0 \rangle \\
 &= 1/2 \sum_{\alpha\beta\gamma\delta} \langle 0 | a_p (a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\delta) a_p' | 0 \rangle \delta_h^{h'} \quad (A-1)
 \end{aligned}$$

$$\begin{aligned}
 &-1/2 \sum_{\alpha\beta\gamma\delta} v_{\alpha\beta\gamma\delta} \langle 0 | a_p a_\alpha^\dagger (\delta_h^{\beta'} - a_h' a_\beta^\dagger) \\
 &\quad * (\delta_h^\delta - a_\delta a_h^\dagger) a_\gamma a_p' | 0 \rangle \quad (A-2)
 \end{aligned}$$

The contribution of term (A-1) is:

$$\begin{aligned}
& \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \langle 0 | a_p (a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\delta) a_p^\dagger | 0 \rangle \delta_p^h \\
&= \sum \left[\begin{pmatrix} j_p & j_h & J_B \\ m_p & -m_h & M_B \end{pmatrix} \right]^2 \langle 0 | a_p V a_p^\dagger | 0 \rangle \delta_{j_p}^p \delta_{j_p'}^h \\
&= \frac{1}{2} \sum_{j_1 m_1} \sum_{j_2 m_2} \langle j_1 j_2 | V | j_1 j_2 \rangle - | j_2 j_1 \rangle \delta_p^p \delta_h^h \\
&\quad + \sum_{j_c m_c} \langle j_p m_p m_t j_c m_c m_t | V | j_p m_p m_t j_c m_c m_t \rangle \\
&\quad - | j_c - j_p \rangle \rangle \\
&= \sum_{j_1 m_1}^{core} \sum_{j_2 m_2} [J] \langle j_1 j_2 (J) m_1 m_2 | V | j_1 j_2 \rangle - (-)^{j_1+j_2-J} | j_2 j_1 \rangle \delta_p^p \delta_h^h \\
&\quad + \sum_{j_c m_c} \frac{[J]}{[j_p]} \langle j_p j_c (J) | V | j_p' j_c \rangle - (-)^{j_p'+j_c-J} | j_c j_p' \rangle \rangle
\end{aligned}$$

The first term contains interaction of the core with itself and does not contribute to Λ binding energy. The second term is the interaction of Λ -particle with the core, in which we neglect the exchange term corresponding to the exchange of Λ to N and vice versa. Then contribution from equation (A-1) becomes:

$$\sum_{j_c m_c} \frac{[J]}{[j_p]} \langle j_p j_c (J) | V | j_p' j_c \rangle \delta_h^h$$

The contribution of term (A-2) is:

$$\begin{aligned}
& - \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \langle 0 | a_p a_\alpha^\dagger (\delta_h^\beta - a_h^\dagger a_\beta^\dagger) (\delta_h^\delta - a_\delta a_h^\dagger) a_\gamma a_p^\dagger | 0 \rangle \\
&= - \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \langle 0 | a_p a_\alpha^\dagger a_\gamma a_p^\dagger | 0 \rangle \delta_h^\beta \delta_h^\delta
\end{aligned} \tag{A-3}$$

$$+ \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \langle 0 | a_p (\delta_h^\alpha - a_h^\dagger a_\alpha^\dagger) a_\beta^\dagger a_\gamma a_p^\dagger | 0 \rangle \delta_h^\delta \tag{A-4}$$

$$+ \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \langle 0 | a_p a_\alpha^\dagger a_\delta (\delta_h^\gamma - a_\gamma a_h^\dagger) a_p^\dagger | 0 \rangle \delta_h^\beta \tag{A-5}$$

$$- 1/2 \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \langle 0 | a_p a_\beta^\dagger a_\delta a_p^\dagger | 0 \rangle \delta_h^\alpha \delta_h^\gamma \quad (A-6)$$

$$(A-2) : (A-3) + (A-4) + (A-5) + (A-6)$$

$$\begin{aligned} &= -1/2 \sum_{\alpha\gamma} [V_{\alpha h' \gamma h} - V_{h' \alpha \gamma h} - V_{\alpha h h' \gamma} + V_{h' \alpha h \gamma}]^* \\ &\quad \langle 0 | (\delta_\alpha^p - a_\alpha^\dagger a_p) (\delta_\gamma^{p'} - a_\gamma^\dagger a_{p'}) | 0 \rangle \\ &= - \sum_{\alpha\gamma} [V_{\alpha h' \gamma h} - V_{\alpha h h' \gamma}] \langle 0 | a_\alpha^\dagger a_\gamma | 0 \rangle \delta_p^{p'} \\ &= - \langle j_p m_p j_h m_h | V [| j_p m_p j_h m_h \rangle - | j_h m_h j_p m_p \rangle] \\ &\quad - \sum_{j_c m_c m_c'} \langle j_c m_c j_h m_h | V [| j_c m_c j_h m_h \rangle - | j_h m_h j_c m_c \rangle] \delta_p^{p'} \end{aligned}$$

The second term is the interaction of hole with the core, and again will be neglected since it does not contribute to the Λ binding energy. The first term is the Λ hole interaction and should be subtracted from the Λ -core interaction found in (A-1).

The contribution from (A-2) is thus:

$$\begin{aligned} &- \langle j_p m_p j_h m_h | V [| j_p m_p j_h m_h \rangle - | j_h m_h j_p m_p \rangle] \\ &= - \sum \begin{pmatrix} j_p & j_h & J_B \\ m_p & -m_h & M_B \end{pmatrix} \begin{pmatrix} j_{p'} & j_{h'} & J_B \\ m_{p'} & -m_{h'} & M_B \end{pmatrix} \sum_{JM} \begin{pmatrix} j_{h'} & j_p & J \\ m_{h'} & m_p & M \end{pmatrix} \begin{pmatrix} j_h & j_{p'} & J \\ m_h & m_{p'} & M \end{pmatrix} \\ &\quad * (-)^{j_h - m_h + j_{h'} - m_{h'}} * \\ &\quad \langle j_h j_p(J) | V [| j_h j_{p'}(J) - (-)^{j_h + j_{p'} - J} | j_{p'} j_h(J) \rangle \end{aligned} \quad (A-7)$$

Again in the matrix element, (A-7), the exchange term is dropped.

Thus (A-2) is :

$$- \sum_J (A-7) \sum_{m's} [J_B][J] (-)^{j_h - j_p + j_{h'} - j_{p'} + j_p - j_{h'} + j_{p'} - j_h + j_h - m_h + j_{h'} - m_{h'}}$$

$$* \begin{pmatrix} j_p & j_h & J_B \\ m_p & -m_h & -M_B \end{pmatrix} \begin{pmatrix} j_p' & j_h' & J_B \\ m_p' & -m_h' & -M_B \end{pmatrix} \begin{pmatrix} j_h' & j_p & J \\ m_h' & m_p & -M \end{pmatrix} \begin{pmatrix} j_h & j_p' & J \\ m_h & m_p' & -M \end{pmatrix}$$

where Clebsch-Gordan coefficients are written in terms of 3-j symbols

(Messiah, 1966)

(A-2) :

$$- [J_B] \sum_J (A-7) [J] (-)^{j_p+j_h+j_h'+j_p'} \frac{1}{[J_B]} \left\{ \begin{matrix} j_p & j_h & J_B \\ j_p' & j_h' & J \end{matrix} \right\}$$

Here the 3-j symbols are written in terms of Racah coefficients

(Messiah, 1966). Therefore contribution from (A-2) becomes:

$$- \sum [J] (-)^{j_p+j_h+j_h'+j_p'} \left\{ \begin{matrix} j_p & j_h & J_B \\ j_p' & j_h' & J \end{matrix} \right\} \langle j_h' j_p(J) | V | j_h j_p'(J) \rangle$$

Thus contribution of (A-1) and (A-2) to the potential energy of Λ -particle are :

$$\sum_{j_c m_{j_c} J} \frac{[J]}{[j_p]} \langle j_p j_c(J) | V | j_p' j_c \rangle \delta_{h'}$$

$$- \sum_J [J] (-)^{j_p+j_h+j_h'+j_p'} \left\{ \begin{matrix} j_p' & j_h & J_B \\ j_p & j_h' & J \end{matrix} \right\} \langle j_h' j_p(J) | V | j_h j_p'(J) \rangle$$

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