
Gregory N. Vaughan
Western Michigan University

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A COMPARISON OF THE HUTCHINGS "LOW-STRESS", FULKERSON TENS METHOD, AND CONVENTIONAL ADDITION ALGORITHMS FOR SPEED, ACCURACY AND PREFERENCE WITH REGULAR EDUCATION STUDENTS

by

Gregory N. Vaughan

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Western Michigan University
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Gregory N. Vaughan, Ed.S.
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The differential calculation power (rate and accuracy) of three separate procedures for addition was investigated. The Hutchings' "low-stress, Fulkerson tens method, and conventional algorithms were compared on 5x7 array addition problems. The subjects were six male first semester third grade students approximately eight and one half years of age. A multielement design was used in which the three algorithms composed the multiple elements. During the last three daily sessions the students were given free choice of algorithms to use. The results indicated that the Hutchings "Low-stress" algorithm was superior to the Fulkerson tens method, which in turn was superior to the conventional algorithm in producing efficient and accurate calculations. When given a free choice, four students chose the Hutchings "Low-stress" algorithm, two students chose the conventional algorithm, and no students chose the Fulkerson tens method.
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Finally, I would like to thank my wife, Wendy, for taking the reliability checks and for her support throughout this endeavor.

Gregory N. Vaughan
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INTRODUCTION

The Problem

This study examined the effects of the Hutchings "Low-stress", Fulkerson, and conventional algorithms on percent accuracy, error rate, and correct rate using addition problems of a 5 x 7 array.

In recent years there has been a noted decline in math scores on standardized achievement tests. The Conference Board of the Mathematical Science National Advisory Committee on Mathematical Education (1975) found that Scholastic Aptitude Test (SAT) mean scores on the quantitative section declined from a high of 502 to a low of 472 from 1962 to 1975. A similar decline was found among high performers on the same test. The percentage of scores above 600 declined from 20.5 to 16.4. This general decline is occurring during a time in which good computational skills are more necessary than ever. In a chapter in Measurement In Education, Lloyd Hutchings (1976) noted:

A primary cause is the rate at which human knowledge is expanding. A majority of all the scholars and scientists who have ever lived are alive and producing now. Their work expands the mathematical disciplines on which science and technology are built, thereby increasing the number of "quantitative" professions and vocations and making existing professions and vocations more quantitative (pp. 218-220).

Math curricula in the United States have recently seen a shift in emphasis. Computational skills have given way to an emphasis on conceptual meanings and applications. Prior to this shift in emphasis a large portion
of classtime was dedicated to computational drill. However, classtime has not been expanded to include the extra load of learning conceptual meanings and applications. Thus, less time is being spent on computational drill (Alessi, 1974). Hutchings (1976) further supports this argument:

> The great increase in mathematical concepts and generalizations in the curriculum is fundamentally in conflict with the large amount of time and energy required for mastery of conventional algorithms. Moreover, increased conceptual requirements in no way reduce the requirements for computational skill. Understanding division is not the same as knowing how to divide quickly and accurately (pp. 218-220).

Special subjects, such as art, music, and gym, also provide competition for the time dedicated to computational drill. Thus, time that could have been used for computational drill is spent in other classes.

An obvious solution to the problem is to simply increase the amount of class time devoted to math instruction to allow students more practice on computational drill. However, Skinner (1968) postulated that "The figures and symbols of mathematics have become standard emotional stimuli. The glimpse of a column of figures...is likely to set off, not mathematical behavior, but a reaction of anxiety, guilt, or fear" (p. 18). This reaction may be due to the large number of mild punishers typically encountered while extensively practicing calculations. The punishers generally take the form of grading marks made by the teacher on the students' math worksheets that indicate errors or teacher comments that point out errors in student work. Unfortunately, because the conventional algorithm does not show a full record of the students' behavior used to produce the answer very little
information can be given back to the student on how to correct the mistake and the student is given the burden of teaching himself.

Another solution is to reduce the amount of dependence upon the conventional algorithm through the use of the calculator or the abacus. In Japan, students are instructed in the use of the abacus (Soroban) early in their education. The amount of drill required to master calculation exercises is thus reduced. As a result, the amount of teaching time dedicated to computational skills is reduced and more time is spent on conceptual meanings and applications (Alessi, 1974: Boyle, 1975).

As a third solution, investigators have developed alternative algorithms. Alternative algorithms appear to offer increased computational power without the use of mechanical or electronic devices and fit within the time constraints previously mentioned. The use of algorithms over the calculator or the abacus is suggested by Hutchings (1972) and Alessi (1974).

According to Rudolph (1976), the use of alternative algorithms is more in keeping with traditional American culture but also enjoys three other advantages: "a) algorithms can be accommodated more readily within the current mathematics instructional system used in the United States; b) algorithms do not involve the use of instruments more complicated than the traditional paper and pencil; and c) algorithms have historically retained their usefulness through the fact that their operation leaves a permanent record of the calculations performed" (p. 8).

The following section will present a description of the alternative algorithms currently presented in the literature.
Relevant Literature on Alternative Algorithms

In the absence of a full permanent record one would have to interview the student to identify error patterns (Lankford, 1974; Ashlock, 1976). The interview would require the student to state orally all of the steps he employed to solve the problem. The disadvantage to this system lies in the large amount of time required to perform the interviews.

The Hutchings "low-stress" algorithm for addition produces a full permanent record. Hutchings (1976) defined the algorithm by its operation:

The low stress addition algorithm uses a new notation, called half-space notation, to record individual steps. Half-space notation uses numerals of no more than a half-space in height to record the sum of two digits. With half-space notation, the units portion of the sum of two digits is written at the lower right of the bottom digit and the tens portion is written at the lower left of the bottom digit. When presented with single column addition problems, we add the first two digits...and record the sum in the new notation...The complete sum of each two-digit addition is recorded in half-space notation, but only the ones portion of each of the column sum is always the same as the ones portion of the last two-digit sum...The tens portion of the column sum is always the same as the number of tens recorded at the left of the column. These are simply counted...For a column in some multicolumn exercise, then, the last step - that is, counting the tens at the left of the columns - would be slightly changed. The counting itself is not changed in anyway, but the answer, the total number of tens, is no longer written in the tens place of the first column's sum but instead at the top of the next column at the left...Work continues in this manner until the exercise is completed. Note, however, that the column sum for the last column in a multicolumn example is recorded in exactly the same way as the sum of a single-column exercise (p. 220).
In the following example there are no ones, nine tens, and one
one-hundreds.

\[
\begin{array}{ccc}
2 & 8 & 8+5=13 \\
4 & & 3+7=10 \\
9 & 5 & 13+13 \text{ carry two tens to the next column} \\
6 & 7 & 2+2=4 \\
9 & 10 & 4+9=13 \\
\hline
19 & 0 & 3+6=9 \\
\end{array}
\]

1 one hundred is recorded at the left

In addition to providing a full record, Alessi (1979) has suggested
many other advantages to the low-stress addition algorithm. These
advantages include a reduced memory load, a reduced attention load, and an
increase in the efficiency of the algorithm with larger problems.

One of the advantages is that the memory load (number of math facts
to be remembered) is substantially reduced over the conventional algorithm.
In order to be proficient with the low-stress algorithm the student need
learn only the 100 basic addition facts. The basic facts involve only two
one digit addends...Complex facts (sums greater than 18) are often required
in the conventional algorithm. For example, in order to solve the following
problem with the conventional algorithm, complex facts would be used.

\[
\begin{array}{c}
9 \\
7 \\
5 \\
8 \\
+8 \\
\end{array}
\]

The student would have to recall 9+7=16, 16+5=21, 21+8=29, 29+8=37.
Using the low-stress method the student would use only the basic facts to solve the problem.

\[
\begin{array}{c}
9 \\
9+7=16 \\
6+5=11 \\
7 \\
16 \\
1+8=9 \\
5 \\
9+8=17 \\
11 \\
8 \\
9 \\
8 \\
\frac{17}{3}
\end{array}
\]

Another suggested advantage is the reduced attention load placed on the student. Because the sum of each binary operation is written down the student does not have to remember the previous sums obtained and add it to the next digit. Also, "The advantage offered by low-stress procedures is increased in proportion to the length of the column" (Hutchings, 1976). Thus, in addition to reduced attention and memory loads, low-stress procedures are as easy to use with very large addition problems as with smaller ones (Alessi, 1979). The conventional algorithm tends to become more difficult to use as the problem size increases. The low-stress algorithm's disadvantage lies in the fact that special spacing is needed to accommodate the half-step notation.

Fulkerson's (1963) "tens method" provides a partial record of each step used in the algorithmic process. Instead of indicating the tens of each binary operation with a one written to the lower left of a digit, a slash is used. The units portion is not recorded for each binary operation. The student is required to remember the last obtained units portion to be
added to the subsequent digit. For example:

\[
\begin{array}{c}
2 \\
28 \\
89 \\
+55 \\
172
\end{array}
\]

Like the low-stress algorithm, the tens method reduces the memory load required of students by using only the basic facts. Also, Fulkerson's method does not require the special spacing to accommodate the half-step notation used in the low-stress procedure and thus is readily adaptable to current workbooks and calculation drill sheets. However, it does not provide the full record needed for precise error analysis and correction.

VanHevel (1981) modified the Fulkerson tens method to produce a full record of each binary operation. The Fulkerson "full record" method retained the use of slash marks to indicate tens and used half-step notation to indicate the units portion of each binary operation. The Fulkerson "full record" method appears to share all of the advantages suggested by Alessi (1979). For example:

\[
\begin{array}{c}
21 \\
823 \\
03 \\
588 \\
511 \\
492 \\
903
\end{array}
\]

\[
1903
\]

O'Malley (1969) presented a procedure that added the tens obtained to the adjacent column immediately. Instead of using half-step notation or
a slash the ten is written in the appropriate column. For example:

\[\begin{align*}
  & 9\ 5 \\
  & 1\ 1 \\
  & 6\ 8 \\
  & 1 \\
  & 1\ 7\ 9 \\
  & 1\ 8\ 6 \\
  & 1 \\
  + & 3\ 7 \\
 \hline
  & 3\ 6\ 5 \\
\end{align*}\]

5+8=13  (write the 1 in the tens column)
3+9=12  (write the 1 in the tens column)
2+6=8   (proceed to the next digit)
8+7=15  (write the 1 in the tens column, the sum of the units is 5)

The same procedure is followed in the tens column.

9+1=10  (write the 1 in the hundreds column)
4+8=12  (write the 1 in the hundreds column)
2+1=3   (proceed to the next digit)
3+3=6   (the tens sum is 6)

The digits to the left of the tens column are summed and written in the hundreds place.

Memory and attention load requirements with O'Malley's procedure are the same as Fulkerson's tens method. Also, only a partial record is produced and special spacing is required.

Sanders (1971) presented an algorithm with all of the characteristics of the tens method except the production of a written record of calculations other than the answer. Instead of making a slash to indicate a ten, the student holds up a finger. For example:
7 7+5=12 (student holds up one finger)
5 2+8=10 (student holds up second finger)
8 0+3=3 (student continues to hold up two fingers)
3 3+9=12 (student holds up third finger)
\[ +9 \]
32 The sum of the units column is 2
The number of fingers held up is written in the tens portion of the answer.

The last alternative algorithm for addition to be presented was developed by Batarseh (1974) to reduce the amount of errors produced by the carrying process. Batarseh recommended that numerals be written underneath the adjacent column and underlined instead of carried to the top of the column. The underlined digit is then added to the sum of that column. This method does not reduce the memory or attention loads nor does it provide a written record of calculations other than the answer. For example:

\[
\begin{array}{c}
385 \\
+667 \\
\underline{12} \\
15 \\
1052
\end{array}
\]

Research on Alternative Algorithms

Increasing percent accuracy and rate of correct responding in arithmetic has historically been attempted by manipulating specific consequent events. Kirby and Shields (1972) used an adjusting fixed ratio schedule of positive reinforcement in the form of praise and feedback regarding the correctness of answers to increase the rate of correct responding. They also noted an increase in the percentage of time spent
attending. Copeland, Brown and Hall (1974) noted an increase in percent accuracy in addition in third graders when the building principal publically praised both improving students and highest performing students. McCarty, Griffin, Apolloni, and Shores (1977) increased arithmetic problem solving rates through group contingencies in a token economy based on money.

Time itself has also been used as an independent variable. Ayllon, Garber, and Pisor (1976) found that systematically reducing the amount of time allowed for completion of the math assignment in a token economy produced increases in the rate of correct responding ranging from 125% to 266%.

Little appears to have been done through manipulating antecedent events other than changing the algorithm. Lovitt and Curtiss (1968) increased the rate of correct responding and decreased the rate of incorrect responding through requiring the subject to verbalize the problem prior to making a written response.

The largest classes of antecedent events that may affect percent accuracy, rate of correct responding, and rate of incorrect responding have been the alternative algorithm, the abacus, and the calculator. Research on alternative algorithms has centered around a comparison of Hutchings' "low-stress" procedure to the conventional algorithm across various conditions. VanHevel (1981) was the only study that compared an alternative algorithm ("low-stress") to another alternative algorithm (Fulkerson "full-record"). Zoref (1976), Todd (1980), and Drew (1981) compared the calculator to the Hutchings' "Low-Stress" and conventional algorithms. No
comparison of the Sanders (1971), O'Malley (1969), Batareseh (1974), or Fulkerson (1963) methods have been reported in the literature. Reports of alternative algorithms have generally been of a descriptive nature only.

The earliest studies compared the "low-stress" procedure to the conventional algorithm (Alessi, 1974; Boyle, 1975; Dashiell, 1974; and Hutchings, 1972). Alessi (1974) compared "low-stress" to the conventional algorithm across varying conditions of token economy reinforcement and problem difficulty using a group factorial design. He found that higher scores for the number of columns attempted and computed correctly in a thirty minute period were obtained under the "low-stress" condition. It was also noted, "...that as the test forms increased in difficulty, the extent of superiority for the Hutchings algorithm decreased in a consistent fashion" (Alessi, 1974, pp. 76-77). However, this is the only study that found this difference.

Boyle (1975) also used a group factorial design to compare the "Low-stress" algorithm to the conventional under three conditions. Those conditions were: a) antecedent algorithm instruction; b) social reinforcement contingencies; and c) test and non-test conditions. The results of this study indicated that the "Low-stress" algorithm was superior to the conventional algorithm under all conditions of reinforcement and testing.

Within subject designs studying the Hutchings "Low-stress" method have recently been reported (Rudolph, 1976; Gillespie, 1976; Zoref, 1976; Buitendorp-Drew, 1980; Todd, 1980; Drew, 1981; Hadden, 1981; McCallum, 1981;
McGlinchey, 1981; and VanHevel, 1981). Each study found performance under the "Low-stress" algorithm to be superior to the conventional algorithm.

Rudolph (1976) compared the two algorithms in distracting and non-distracting settings with regular and special education students. He found that the superior performance when using the "Low-stress" algorithm did not appear to be dependent upon the level of distracting stimuli, the order of presentation, or whether the student was in special education or regular education.

Gillespie (1976) tested student preferences for either the Hutchings "Low-stress" algorithm or the conventional algorithm under increasing response efforts. Most of the students preferred the "Low-stress" methods up to 50% more effort. At 100% more effort, the preferences broke down and were affected only by reinforcement contingencies.

Zoref (1976) compared "Low-stress" and conventional algorithms with electronic calculators with low and high performers. Both 2x7 and 5x7 problem arrays were used. Overall, the results of this study indicated that the "Low-stress" algorithm was superior to the conventional algorithm and pocket calculator. These results were most noticeable with the 5x7 array. The rate of incorrect adding was found to be lowest with the "Low-stress" procedure irrespective of 2x7 and 5x7 arrays or low or high performers. The rate of correct adding was higher for high performers across all conditions. Low performers showed a higher correct rate with the calculator on 2x7 problems but the "Low-stress" correct rate was still higher than the conventional correct rate. In 5x7 problems the "Low-stress" correct rate

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was higher than the calculator or conventional algorithm.

Overall, the "Low-stress" algorithm brought the performance of low performers more in line with their high performing counterparts. A discrepancy of approximately 34 percentage points existed between low and high performers for the conventional algorithm (54% vs. 88%). A range of only 7.5 points existed between the two groups for the "Low-stress" algorithm (87.5% vs. 95.0%).

Buitendorp-Drew (1980) ran a systematic replication of the Gillespie (1976) study. Algorithm preferences were tested under increasing response efforts with and without reinforcement. Hutchings’ "Low-stress" algorithm and the conventional algorithm were the algorithms of choice. The "Low-stress" algorithm was preferred 81% of the time and 88% of the time at 50% more response effort with and without reinforcement. At 100% more response effort algorithm preferences broke down with reinforcement.

Todd (1980) compared the hand held calculator, Hutchings’ "Low-stress" algorithm, and the conventional algorithm for subtraction. Two levels of problem difficulty were also studied. The results indicated that the "Low-stress" algorithm was superior in producing low error rates but the conventional algorithm was the fastest, while the calculator was the slowest and least accurate at the lower level of problem difficulty. At the higher level of problem difficulty, the Hutchings "Low-stress" algorithm was the most accurate with the conventional being less accurate than "Low-stress" but more accurate than the calculator.

Drew (1980), in a systematic replication of the Zoref (1976) study,
compared the performances of high and low achieving students using the calculator, Hutchings' "Low-stress" algorithm and the conventional algorithm. The "Low-stress" algorithm was found to be superior to the conventional algorithm and calculator across both high and low achieving students. The calculator produced better results than the conventional algorithm irrespective of the achievement level of the students.

Hadden (1981) compared Hutchings' "Low-stress" algorithms for addition and subtraction to the conventional algorithms with low performing math students. He found that the "Low-stress" algorithm produced more efficient and accurate calculations for addition but did so only with one of four subjects for subtraction.

McCallum (1981) performed a component analysis of Hutchings' "Low-stress" addition algorithm by comparing it to the conventional algorithm with a written record and the conventional algorithm without a written record. The results showed a rank order of superiority with the "Low-stress" algorithm producing the best results as measured by accuracy and speed of calculations. The conventional algorithm with the written record produced the next best results and the conventional algorithm without the written record produced the lowest results.

McGlinchey (1981) compared the Hutchings "Low-stress" subtraction algorithm, conventional algorithm, and pocket calculator across two levels of problem difficulty. The conventional algorithm produced the fastest calculations for all subjects. No consistent accuracy data were obtained.

Van Hevel (1981) compared the Hutchings "Low-stress", Fulkerson Full
Record, and conventional algorithms for speed, accuracy, and preference. VanHevel noted "...that both the Hutchings "Low-stress" and Fulkerson "Full-record" algorithms were generally superior in producing stable, accurate, and efficient calculations" (p. 28). The study did not show any differences between the "Low-stress" and Full Record algorithms on the 5x7 problem arrays. A 17x12 problem array was used to test student preference. Four chose the Full Record, three chose the "Low-stress", and two students chose the conventional algorithm.

The Purpose of This Study

The Fulkerson Full Record algorithm is identical to the Hutchings "Low-stress" algorithm except that a slash mark is used to record tens instead of the digit 1 and no special spacing is required. The VanHevel (1981) results are in keeping with what a logical analysis would predict. The Fulkerson (1963) "tens method" is even one step farther removed from the "Low-stress" method because it does not use half-step notation to indicate the units portion of each binary operation. Thus, the attention load is increased over the "Low-stress" method. No comparison of the Fulkerson "Tens method" has been made to the Hutchings "Low-stress" method. The purpose of this study is to compare these two algorithms for speed and accuracy in columnar addition using problems of a 5x7 array.
METHOD

Special Considerations

This study's design was partially dictated by the following restrictions: a) the subjects would be drawn from one regular education classroom, b) the sessions would be held each day at the same time, c) the experimenter would run all sessions, d) all expenses incurred would be the responsibility of the experimenter, e) the results of the study would be made available to the school personnel and the parents of the subjects.

Subjects

The subjects of this study were six first semester third grade students, each approximately eight and one half years of age. All six students were males. Each student was enrolled full-time in a regular education classroom. The California Achievement Test was administered to each student in May of 1981. Four of the six students scored at approximately grade level, 2.9, for Math Computation. Two students received grade equivalent scores of .8 and 2.0.
Setting

The study took place in an elementary school building located in Kalamazoo County, Michigan. Sessions were held at 8:45 A.M. each school day. The sessions were held in the music room which was next door to the student classroom. The music room was the same size as their third grade classroom and contained only a piano and approximately twenty five writing chairs. The room was a quiet non-distracting environment.

Experimental Task

During each session each subject was required to compute 5x7 array addition problems using the Hutchings' "Low-stress", Fulkerson, and Conventional methods. The students were given two worksheets for each method. Each worksheet contained three 5x7 array addition problems. The digits used to construct the arrays were obtained from a random digits table. In accordance with a recommendation by Hutchings (1972), the identity element, zero, was not used. The subjects were given five minutes for each method and instructed to do as many problems as possible. Sessions were timed with a Casio F-81 Alarm Chronograph set in the stopwatch mode. No subjects ever completed all six problems.
Experimental Design

This study used a multielement design (Ullman & Sulzer-Azaroff, 1974). The three algorithms composed the multiple elements. Each subject came into contact with each independent variable each session. This design was used to provide as many data points as possible in the least amount of time and to allow for a day-by-day comparison within subjects as opposed to across subjects. The independent variables were presented in a random order each session. The order of presentation was determined through the use of a random digits table.

Independent Variables

The independent variables in this study were:

1) Hutchings' "Low-stress" addition algorithm versus the Fulkerson addition algorithm versus the conventional addition algorithm.

2) Hutchings' "Low-stress" algorithm was taught before teaching the Fulkerson method to five of the subjects. The Fulkerson method was taught before teaching the Hutchings' "Low-stress" algorithm to one of the subjects. While this individual student was being taught the Fulkerson method first the other five students had begun daily sessions of computing 5x7 array addition problems using the "Low-stress", Fulkerson, and conventional algorithms.
Dependent Variables

The dependent variables in this study were:

1) Percent correct: the number of correct columns divided by the total number of columns attempted. Columns were counted as correct only if both the units and carried portion of the same were computed correctly. However, any daily worksheet on which no columns were attempted was scored as 0%.

2) Rate incorrect: the total number of incorrect columns attempted divided by the session length, five minutes, and expressed as columns incorrect per minute.

3) Rate correct: the total number of correct columns attempted divided by the session length, five minutes, and expressed as columns correct per minute.

4) Algorithm preference: the algorithm of choice after contact with all three.

Recording and Scoring

Immediately following each session the experimenter checked each subject's work for accuracy. Percent accuracy, rate correct, and rate incorrect was recorded on the face of each worksheet. The subjects were never informed of their scores except during the preference phase.
Reliability

Reliability data were taken on the columns completed by the students. Since the calculations performed by each student on the daily worksheets produced a permanent product, it was possible to give their work to an independent grader. Twenty-five percent of the daily worksheets were randomly selected for reliability checks at the conclusion of the study. Each algorithm was equally represented in the sample to be checked. A comparison was made of the independent grader's scores to the experimenter's scores and a reliability coefficient was calculated. Columns that both graders scored the same were counted as agreements. Columns that were scored differently were scored as disagreements. The reliability coefficient was derived by dividing the number of agreements by the number of agreements plus disagreements.

Materials

A pre-test of 56 complex addition problems was administered to identify low performers (Appendix A). A test of 56 basic addition facts was given to ensure those identified as low performers knew the basic facts (Appendix B). Two practice sheets were given to each subject during algorithm training (Appendix C). Problem arrays other than 5x7 were included on the practice sheets. Each daily worksheet consisted of three addition problems in 5x7 arrays (Appendix D). The daily worksheets were typed on
8-1/2 x 11 inch paper with an IBM Selectric typewriter using the orator element. Triple spaces were used between columns and double spaces between rows. The stopwatch mode of a Casio F-81 Alarm Chronograph was used to time each session. Thus materials required for each daily session consisted of the stopwatch and the daily worksheets.

Procedure

Pretesting

Each subject for this study was required to score 90% or better on a pre-test of 56 basic addition facts (Appendix B). The common prerequisite for all three algorithms studies was competency with the basic facts. This test ensured that competency.

In order to identify low performers, a pre-test of 56 complex addition facts was given to twenty-four third grade students (Appendix A). A score less than 65% qualified that student as a subject for this study. There were no time constraints for either test. Each student was given as much time as necessary to complete each test.
Algorithm Training Procedures

Five subjects received instruction on the Hutchings "Low-stress" algorithm first, Fulkerson second, and a review of the conventional algorithm last. One subject received instruction on the Fulkerson algorithm first, Hutchings' "Low-stress" second, and the conventional algorithm last. All subjects received similar instruction in each algorithm. Training sessions followed the format used by VanHevel (1981) study incorporated a written record of the ones portion of each binary operation. Thus, instruction for the Fulkerson algorithm (Appendix G) used in the study departed from the VanHevel study by telling the subjects to remember the ones portion.

Training sessions consisted of approximately twenty minutes of instruction followed by ten minutes of practice on the practice worksheets. The next session was a review session of approximately ten minutes and twenty minutes of practice on the practice worksheets. During training the experimenter answered questions of students and gave feedback on the accuracy of computations. During the experimental session no questions were answered regarding the mechanics of any of the algorithms and no feedback was given other than general statements such as "You are all doing very well."
Daily Sessions

Each daily session consisted of approximately thirty minutes. The subjects sat facing the blackboard and spread out in the room to prevent copying. The day's algorithm sequence was shown on the board through three 2x3 addition problems (Appendix H). As each worksheet was given to each subject the experimenter also stated the type of algorithm to use. The subjects were instructed to wait until the experimenter said "One, two, three, go." When the experimenter said stop they were instructed to lay their pencils down. At the end of each session the subjects were allowed to play Hangman, a spelling game, for working hard.

Algorithm Preference

During the last four sessions of the study, all conditions were the same except each subject was required to choose which algorithm to use for each of the two five minute periods. Each subject was required to use only the algorithm chosen for a five minute period, but could change his choice for the second five minute period.

The same scoring criteria were used to determine accuracy. During this phase the experimenter told each student his percent accuracy score immediately following each five minute period.

Data were collected on the algorithm used, percent accuracy, rate correct, and the rate incorrect.
RESULTS

Reliability

Inter-scorer reliability data on columns correct and columns incorrect were collected on twenty-five percent of the daily worksheets. An overall agreement index of 96.2 percent was obtained. The agreement index ranged from 83.3 percent to 100 percent.

General Findings

In general, the results of this study indicate that the Hutchings' "Low-stress" algorithm was superior to the Fulkerson algorithm which in turn was superior to the conventional algorithm in producing efficient and accurate calculations. The order of introduction did not appear to make a difference in the rank order of superiority. In all graphic displays in this paper, students one (1) through five (5) were taught Hutchings' "Low-stress" first, Fulkerson second, and conventional third. Student six (6) was taught Fulkerson first, Hutchings' "Low-stress" second, and conventional third.

Table 1 (top) displays individual mean percent accuracy, group mean percent accuracy, median, and range for each algorithm. The results obtained show the highest accuracy scores were obtained under the Hutchings'
Table 1

Individual Means, Group Means, Range and Median for Percent Accuracy, Columns Incorrect per Minute, and Columns Correct Per Minute

<table>
<thead>
<tr>
<th>STUDENT (N)</th>
<th>LOW STRESS</th>
<th>FULKERSON</th>
<th>CONVENTIONAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (19)</td>
<td>66.85</td>
<td>23.12</td>
<td>10.69</td>
</tr>
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<td>10.25</td>
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<td>73.54</td>
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<td>69.03</td>
<td>33.94</td>
<td>21.93</td>
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<tr>
<td>RANGE</td>
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<td>20.00-71.35</td>
<td>9.58-61.57</td>
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<th>CONVENTIONAL</th>
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</thead>
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<td>1.46</td>
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<td>1.84</td>
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<td>4 (16)</td>
<td>.11</td>
<td>.10</td>
<td>.40</td>
</tr>
<tr>
<td>5 (18)</td>
<td>.93</td>
<td>1.06</td>
<td>2.65</td>
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<tr>
<td>6 (16)</td>
<td>.76</td>
<td>.31</td>
<td>.56</td>
</tr>
<tr>
<td>GROUP</td>
<td>.58</td>
<td>1.00</td>
<td>1.53</td>
</tr>
<tr>
<td>RANGE</td>
<td>.11-.93</td>
<td>.10-1.84</td>
<td>.40-2.85</td>
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<tr>
<td>MEDIAN</td>
<td>.70</td>
<td>1.07</td>
<td>1.50</td>
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<table>
<thead>
<tr>
<th>STUDENT (N)</th>
<th>LOW STRESS</th>
<th>FULKERSON</th>
<th>CONVENTIONAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (19)</td>
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<td>.16</td>
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<tr>
<td>2 (19)</td>
<td>.98</td>
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<td>.54</td>
</tr>
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<td>3 (18)</td>
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<td>.31</td>
</tr>
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<td>.32</td>
<td>.07</td>
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<td>2.02</td>
<td>.32</td>
<td>.07</td>
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<td>RANGE</td>
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<td>.28-.60</td>
<td>.07-.92</td>
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<tr>
<td>MEDIAN</td>
<td>1.0</td>
<td>.37</td>
<td>.30</td>
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"Low-stress" condition across all students. The next highest accuracy scores were obtained under the Fulkerson condition for all students except student number 2 who showed essentially no difference in accuracy between the Fulkerson and conventional algorithms. The lowest accuracy scores were obtained under the conventional condition. The mean percent accuracy across all students for the Hutchings' "Low-stress" algorithm ranged from 49.67% to 86.67% with a standard deviation of 14.62, for the Fulkerson algorithm the range was from 20% to 71.35% with a standard deviation of 20.31, and for the conventional algorithm the range was from 9.58% to 61.57% with a standard deviation of 20.02. The mean percent correct for all students under the "Low-stress" condition was 69.03%. The mean percent correct for all students under the Fulkerson condition was 33.94%. The mean percent correct for all students under the conventional condition was 21.93%. The median percent correct for the Hutchings' "Low-stress" algorithm was 70.19%. For the Fulkerson algorithm the median percent correct was 23.77%. For the conventional algorithm the median percent correct was 14.12%.

Table 1 (middle) presents individual means, group means, range, and median for columns incorrect per minute. The results obtained indicate a rank order of superiority similar to that obtained for percent accuracy. In general, error rates under the "Low-stress" condition were the lowest. Higher error rates were obtained under the Fulkerson condition and the highest were under the conventional condition. However, student number 3 showed a higher error rate with the Fulkerson algorithm than the conventional algorithm. The highest error rate for student number 6 was
with the Hutchings' "Low-stress" algorithm. This is the result of student number 6 completing many more columns with the "Low-stress" or either the Fulkerson or conventional algorithms. In view of the high rate correct and high accuracy scores for student number 6, the superiority of Hutchings "Low-stress" algorithm is maintained.

The mean number of columns incorrect per minute ranged from .11 to .93 with a standard deviation of .31 for "Low-stress", .10 to 1.84 with a standard deviation of .69 for the Fulkerson algorithm, and .40 to 2.85 with a standard deviation of .96 for the conventional algorithm. The mean number of columns incorrect per minute was .58 for the Hutchings' "Low-stress" algorithm, 1.00 for the Fulkerson algorithm, and 1.53 for the conventional algorithm. The median was .70 for the Hutchings' "Low-stress" algorithm, 1.07 for the Fulkerson algorithm, and 1.50 for the conventional algorithm.

Table 1 (bottom) presents individual means, group means, range, and median for columns correct per minute. The results obtained are consistent with the rank order of superiority indicated by percent accuracy and columns incorrect per minute. All students performed correct calculations at the fastest rate under the "low-stress" condition. The second fastest rate of correct calculations was obtained under the Fulkerson condition except for students 2 and 4. The slowest rate was obtained under the conventional condition. The mean correct columns per minute ranged from .54 to 3.17 with a standard deviation of .98 for the "Low-stress" algorithm, .28 to .60 with a standard deviation of .12 for the Fulkerson algorithm, and .07 to .92 with a standard deviation of .33 for the conventional algorithm. The mean
correct columns per minute for all students under the "Low-stress" condition was 1.43. The mean correct columns per minute under the Fulkerson condition was .40. The mean correct columns per minute for all students under the conventional algorithm condition was .38. The median correct columns per minute was 1.0 for "Low-stress", .38 for Fulkerton, and .31 for conventional algorithms. It should be noted that student number 2 computed addition facts by counting on his fingers instead of through rote memory.

Individual Performances

Figures 1,2,3,4,5, and 6 present percent accuracy scores for students 1,2,3,4,5, and 6 respectively. For all graphs, on day 12 the indication of no data collected represents three days that the students were on Thanksgiving vacation and all other indications represent one day of no data.

As can be seen from these figures, the accuracy scores for each student were generally highest during the "Low-stress" condition. Students 2,3,5, and 6 show superiority of accuracy under the "Low-stress" condition. Students 1 and 4 show more variability, but the superiority is maintained. Accuracy scores for both the Fulkerson and conventional algorithms tend to show more variability across all students.

An inspection of the same figures for columns added incorrectly per minute shows superiority under the "Low-stress" condition in producing the least amount of errors per minute. Students 2,3,4, and 5 show the greatest
Figure 1. Columns correct per minute, columns incorrect per minute, and percent accuracy on daily worksheets for student number 1 under the "low-stress", Fulkerson and conventional algorithms for addition problems using 5x7 arrays.
Figure 2. Columns correct per minute, columns incorrect per minute, and percent accuracy on daily worksheets for student number 2 under the "low-stress", Fulkerson and conventional algorithms for addition problems using 5x7 arrays.
Figure 3. Columns correct per minute, columns incorrect per minute, and percent accuracy on daily worksheets for student number 3 under the "low-stress", Fulkerson and conventional algorithms for addition problems using 5x7 arrays.
Figure 4. Columns correct per minute, columns incorrect per minute, and percent accuracy on daily worksheets for student number 4 under the "low-stress", Fulkerson and conventional algorithms for addition problems using 5x7 arrays.
Figure 5. Columns correct per minute, columns incorrect per minute, and percent accuracy on daily worksheets for student number 5 under the "low-stress", Fulkerson and conventional algorithms for addition problems using 5x7 arrays.
Figure 6. Columns correct per minute, columns incorrect per minute, and percent accuracy on daily worksheets for student number 6 under the "low-stress", Fulkerson and conventional algorithms for addition problems using 5x7 arrays.
difference in error rates between the Hutchings' "Low-stress" and the conventional algorithms. Error rates with the Fulkerson algorithm were generally greater than those obtained with the Hutchings' "Low-stress" algorithm appear to be fairly stable for all students. Error rates for both the Fulkerson and conventional algorithms show more variability except for student number 4 who counted on his fingers for all calculations.

Figures 1, 2, 3, 4, 5, and 6 also present the individual data on the number of columns added correctly per minute for each algorithm per session. Each student showed the greatest correct response rates under the Hutchings' "Low-stress" condition. Students 2 and 6 showed the greatest difference between Hutchings' "Low-stress" and the other two algorithms. The high correct rate for student 6 compensated for the high error rate and thus produced high accuracy scores. Overall, less variability is shown across all three algorithms for columns correct per minute than for percent accuracy or columns incorrect per minute.

Algorithm Preference

During the last three sessions of the study the students were given free choice of which algorithm to use to solve the problems on the daily worksheets. After completion of each five minute work session the students were told their respective accuracy scores before being required to do the next worksheet. All other conditions were maintained. Out of a total of 27 opportunities to choose an algorithm, 21 choices were for the Hutchings'
"Low-stress" algorithm and 6 were for the conventional algorithm. No student chose the Fulkerson algorithm. Each student made five choices except for students 1 and 5 who made 4 and 3 choices respectively. Only students 2 and 5 preferred the conventional algorithm. Student 2 chose the conventional algorithm 3 out of 5 times and student 5 chose the conventional algorithm 2 out of 3 times. Students 1, 3, and 6 chose Hutchings' "Low-stress" exclusively while student 4 chose Hutchings' "Low-stress" 4 out of 5 times.

The mean percent accuracy, mean incorrect rate, and mean correct rate were consistent with the findings previously reported. The mean percent accuracy for Hutchings' "Low-stress" during this phase was 78.72% and for the conventional algorithm it was 47.21%. Hutchings' "Low-stress" algorithm produced a mean error rate of .48 per minute. The conventional algorithm produced a mean error rate of .83 per minute. Hutchings' "Low-stress" algorithm produced a mean correct rate of 1.50 per minute. The conventional algorithm produced a mean correct rate of .50 per minute. The two students who chose the conventional algorithm did not perform better with the conventional algorithm than they did with the "Low-stress" algorithm.

Summary

The results of this study indicate the superiority of Hutchings' "Low-stress" algorithm in producing low error rates, high correct rates, and
high percent accuracy scores for 5x7 array addition problems. The Fulkerson method was superior to the conventional algorithm, but less effective than the Hutchings' "Low-stress" algorithm. The conventional algorithm was the least effective of all. Only one student showed no difference in percent accuracy between the Fulkerson method and the conventional algorithm. Only one student showed the highest error rate with the Hutchings' "Low-stress" algorithm. However, that rate was inflated due to the greater number of columns added by that student when using the "Low-stress" algorithm and was offset by the high correct rate and high accuracy scores. Only one student showed the highest error rate with the Fulkerson method. However, the lowest error rate for that student was obtained during the Hutchings' "Low-stress" conditions.

For algorithm preference, 21 of 27 choices made were for Hutchings' "Low-stress", 6 choices were for the conventional algorithm, and no choices were for the Fulkerson algorithm. During this phase, mean percent accuracy, mean error rate, and mean correct rate for the Hutchings' "Low-stress" were superior to the conventional algorithm.
DISCUSSION

The results of this study show that the Hutchings' "Low-stress" algorithm was superior to the Fulkerson algorithm, which in turn was superior to the conventional algorithm in producing efficient and accurate calculations. The superior performances were obtained quite quickly, especially when compared to the amount of time first semester third graders would have already used practicing the conventional algorithm.

In general the students enjoyed learning and practicing the new algorithms. During the first two weeks of the study the students were overheard to say repeatedly that they were using a secret way to do addition problems and even challenged their peers in the classroom to adding contests. It is interesting to note that the algorithm of choice for the adding contests was the Hutchings' "Low-stress" algorithm. This preference was in keeping with the results of the preference phase of this study. The students in the study wanted to know if they could use "Low-stress" in the classroom. The rest of the students in the regular classroom wanted to learn "Low-stress". Thus, the classroom teacher rearranged the daily schedule to teach "Low-stress" addition during the time the study was to run to avoid giving extra practice to the students in the study.

Even though each student came into contact with each algorithm each day complaints were heard when the required algorithm was not Hutchings' "Low-stress". Common statements were, "I hate slashes" (Fulkerson) or "I don't want to just add up" (conventional) and "I love numbers on the
corners," which was their nomenclature for Hutchings' "Low-stress".

**Instructional Procedures**

The procedures used to teach the new algorithms appear to have been effective as evidenced by the improved performance over the conventional algorithm. The students did not appear to have any difficulty in learning to follow the procedures required by each algorithm even though they were required to use all three each day.

**Data Analysis**

The results of this study are consistent with the findings of other researchers in the field, (Hutchings, 1972; Alessi, 1974; Dashiell, 1974; Boyle, 1975; Rudolph, 1976; Zoreff, 1976, Drew, 1981; and VanHevel, 1981) in showing superior performances in speed and accuracy of calculations with the Hutchings' "Low-stress" algorithm over the conventional algorithm. In addition, the results show the Fulkerson method to be less effective than the Hutchings' "Low-stress" algorithm and more effective than the conventional algorithm. Whereas the Fulkerson Full Record algorithm (VanHevel, 1981) was found to be very similar to the Hutchings' "Low-stress" algorithm. It would appear then, that the Fulkerson methods increased memory load over the Fulkerson Full Record and the Hutchings "Low-stress"
algorithms is a limiting factor in producing high rate calculations. This interpretation would seem to be consistent with the findings of McCallum, 1981 in a comparison of full record to partial record algorithms.

It was noted that student number 4 calculated the basic facts by counting on his fingers. Finger counting to obtain sums is a time consuming mechanism as reflected by the data on rate correct and incorrect for this student. However, accuracy scores for this student showed the same hierarchy of performance as the rest of the students in this study.

The grade level of the students was another factor that may have exerted some influence over the results. First semester third graders do not typically have an extensive history of doing addition with regrouping. Thus, the superiority of the Hutchings' "Low-stress" and Fulkerson algorithms may be inflated in this study. However, another way to look at the same factor is to say that relatively little practice with regrouping with the conventional algorithm would tend to equalize entry level performances among the three algorithms.

Another factor that may have affected the data is the use of only one student to compare the effects of the order the algorithms were taught. However, the results from this study parallel the results of other similar studies that used larger groups to compare order effects. In view of the similarity of results the reader can be fairly confident of the results of this study. A final factor that may have affected the mean percent accuracy data was the recording of 0% for any time a student did not attempt any columns for a particular algorithm. Student number 1 showed two no attempts
for the conventional algorithm. Student number 4 showed one no attempt for the Fulkerson method. Student number 6 showed four no attempts for the Fulkerson method. Thus, a total of only six no attempts were scored 0% for the conventional algorithm and five no attempts were scored 0% for the Fulkerson method. However, it is not clear that the percent accuracy data were greatly affected in view of the large number of scores used to obtain mean percent accuracy.

Student preferences for the Hutchings' "Low-stress" algorithm found in this study coincide with the findings of past researchers (Gillespie, 1976 and Buitendorp-Drew, 1980). Based on this study and past research, alternative algorithms, especially Hutchings' "Low-stress", appear to be realistic alternatives to the conventional algorithm in teaching and performing addition calculations. Informal assessment of the suitability of Hutchings' "Low-stress" algorithm indicates that students have no trouble using it on daily assignments in the regular classroom.

Future Research

A consideration of this and past research opens many doors to future research. In order to see if the demonstrated superiority of Hutchings' "Low-stress" algorithm and the Fulkerson method were inflated, a similar study could be run with students in higher grades.

It seems that students who compute basic addition facts on their fingers tend to equalize computation time irrespective of the algorithm
used. It might prove interesting to compare performances of students who compute on their fingers with those who solve basic facts through rote memory.

Other research may make a direct comparison of the Fulkerson method to the Fulkerson Full Record algorithm. Through this comparison more concrete statements concerning increased memory load and performance could be made.

Finally, due to the success of students using Hutchings' "Low-stress" in the regular classroom it may not be necessary to reprint math workbooks and exercise sheets to allow the use of half-step notation. A study could be run that compares materials that incorporate wide spacing with those that do not.
## APPENDIX A

### INVENTORY OF COMPLEX ADDITION FACTS

Inventory of Complex Addition Facts

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
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# Appendix B

## Inventory of Basic Addition Facts
*(Taken from VanHevel, 1981.)*

Inventory of Basic Addition Facts

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APPENDIX C

PRACTICE WORKSHEETS FOR TRAINING SESSIONS
(Taken from VanHevel, 1981.)

Practice Sheet #1

7
6
8
9
7
6
9
0
5
1
+874

6 8 7
4 8 3
6 9 5
6 9
+874

6 8 7 6 9 5
4 8 7 6 4 2
+876983

4 8 7 6 8 7 6
9 8 7 6 8 7 6
9 8 7 6 8 5 6
7 9 5 6 7 9 3
8 5 2 7 4 9 8
+6785678

45
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### APPENDIX D

#### SAMPLE DAILY WORKSHEET

| 2 2 1 7 6 |
| 1 9 3 6 2 |
| 1 6 7 7 2 |
| 7 8 4 3 7 |
| 3 2 8 3 8 |
| 9 3 2 2 5 |
| 7 8 7 6 5 |

\[ \mathbf{2 3 6 8 3} \]
\[ \mathbf{1 5 3 9 2} \]
\[ \mathbf{5 8 7 1 9} \]
\[ \mathbf{5 7 3 5 2} \]
\[ \mathbf{4 8 5 8 8} \]
\[ \mathbf{6 1 9 6 4} \]
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\[ \mathbf{8 8 5 6 5} \]
\[ \mathbf{9 7 2 8 5} \]
\[ \mathbf{1 2 9 6 5} \]
\[ \mathbf{8 5 9 4 5} \]
\[ \mathbf{3 8 6 4 4} \]
\[ \mathbf{5 3 4 4 9} \]
\[ \mathbf{4 7 6 6 6} \]

**NAME**

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I am going to show you a new way of writing number facts, but first I'll show you the old way.

This is the way you usually write your facts: \[
\begin{array}{c}
6 \\
+7 \\
\hline
13
\end{array}
\]

Using the new way the facts are written using two small numbers like this:

\[
\begin{array}{c}
6 \\
7 \\
13
\end{array}
\]

You still write thirteen, only it is written at the corners of the number instead of under it.

I'll show you another example. We can write eight plus six like this: \[
\begin{array}{c}
8 \\
+6 \\
\hline
14
\end{array}
\]

Do you see the fourteen? You always write the ones to the right and tens to the left.

Now I want you to try it. Using the paper I have given you, write down this problem: seven plus eight. Now I want you to write down the answer using the new way.

(Check to make sure they have all used the correct notation) \[
\begin{array}{c}
7 \\
+8 \\
\hline
15
\end{array}
\]

Very good. The little number on the left is said to be in the tens place and the little number on the right is said to be in the ones or units place.

Now I am going to show you a bigger problem. In this problem we are going to add three numbers; nine, seven and eight. Watch me:

Nine plus seven equals sixteen, so I'll write a little six to the right of the seven and a little one to the left. Do you see? Now using only the little number to the right I will add the little six to the eight. The answer is fourteen so I will write a little four to the right of the eight
and a little one to the left.

\[
\begin{array}{c}
9 \\
7 \\
16 \\
8 \\
14 \\
\end{array}
\]

Now I have the right half of the answer which is four. I just write the four at the bottom right like this:

\[
\begin{array}{c}
9 \\
7 \\
16 \\
8 \\
14 \\
\hline
4 \\
\end{array}
\]

To get the other half of the answer, I simply count the little ones at the left that I have forgotten about.

How many ones are there? Let's count one...two. So two is the other half of the answer.

\[
\begin{array}{c}
9 \\
7 \\
16 \\
8 \\
14 \\
\hline
24 \\
\end{array}
\]

The answer is 24. Do you see?

Now I want you to try an even bigger problem. Write this problem on your papers and don't forget to leave enough room between your big numbers to write your little numbers. Also, remember only add the little numbers at the right. Here is the problem: add six, plus seven, plus eight, plus five.

If you need help ask me.

Okay everybody is finished. The answer is 26. Let's see how you reached the answer.
Six plus seven equals thirteen, so you wrote the little three to the right of the seven and the little one to the left. That's right.

Next you added three plus eight. It equaled eleven, so you wrote a little one to the right and to the left of the eight. Remember we always forget about the little ones at the left until the very end.

Next you added one plus five. It equaled six, so you wrote a little six to the right of the five and nothing to the left because you had no tens.

Now you had half of the answer, so you wrote a six in the ones place under the line. Then you counted the ones on the left, one...two and wrote down the second half of your answer which was two.

Your answer was 26. Very good.

Did everybody get twenty six for their answer?

Good.
Now let me show you how to add when there is more than one column using the new way.

Look at problem:

\[
\begin{array}{cc}
6 & 8 \\
8 & 7 \\
4 & 15 \\
5 & 5 \\
1 & 0 \\
9 & \\
9 & \\
\end{array}
\]

Can we still write our left hand answer if there is more than one column? No, we can not.

When there is more than one column, each column can have only one number at the bottom, except for the last column. So the single number that we put at the bottom is always the right hand number.

What do we do with the left hand number? We carry it to the top of the next column and circle it.

Now when we begin to add the second column we always start with the number we carried. In this example we carried the two.
To finish this problem we add:

\[
\begin{array}{c}
\text{2 plus 6 = 8} \\
\hline
\text{8 plus 8 = 16} \\
\hline
\text{6 plus 4 = 10} \\
\hline
\text{0 plus 5 = 5} \\
\end{array}
\]

\[
\begin{array}{c}
\hline
6 \\
\hline
8 \\
\hline
16 \\
\hline
4 \\
\hline
5 \\
\hline
25 \\
\end{array}
\]

On the last column we write both the left and right side of the answer.

Watch me.

Five is the last right hand number so I'll write it at the bottom. There were two little ones at the left so I'll write two at the bottom. The answer is two hundred and fifty-nine.

\[
\begin{array}{c}
\text{2} \\
\hline
6 \\
\hline
8 \\
\hline
16 \\
\hline
4 \\
\hline
10 \\
\hline
5 \\
\hline
25 \\
\end{array}
\]

Remember, if there is a column to the left you always count your little ones and carry them to the top of the next column and circle the number.

Now I want you to do some practice problems.

(Hand out practice sheets)
Since you all know how to write your number facts, I thought I would spend a few minutes reviewing how to do column addition problems.

When you write number facts you add the two numbers and write down your answer. For instance, when you write six plus seven, all you do is write thirteen at the bottom, like this:

\[
\begin{array}{c}
6 \\
+7 \\
13
\end{array}
\]

The three is said to be in the ones place and the one in the tens place.

When you add more than two numbers, like in the problem nine plus seven plus eight, you add nine plus seven which equals sixteen. Then you add sixteen to eight to reach your answer of twenty four.

\[
\begin{array}{c}
9 \\
7 \\
+8 \\
24
\end{array}
\]

Does everybody know how to do this problem?

Good.

Okay, now I want you to write this problem down on the paper I gave you: six plus seven, plus eight, plus five. Let's see you do this problem.

Okay everybody is finished. The answer is twenty-six.

Let's see how you reached your answer. You added six plus seven and it equaled thirteen. Next you added thirteen plus eight. It equaled twenty one. You then added twenty-one plus five and it equaled twenty-six. That is the answer, you then wrote twenty-six at the bottom. Very good.
Now let me review how to add when there is more than one column.

Look at this problem:

\[
\begin{array}{c}
68 \\
87 \\
45 \\
+59 \\
\hline
9
\end{array}
\]

The sum of the first column is twenty-nine. Can we write the twenty-nine under that column? No we can not.

When there is more than one column, each column can have only one number at the bottom, except for the last column.

So the number we put at the bottom is always the right hand number or the units.

What do we do with the left hand or tens number? We carry it to the top of the next column and circle it. Watch me:

\[
\begin{array}{c}
68 \\
87 \\
45 \\
+59 \\
\hline
2
\end{array}
\]

Now when we begin to add the second column, we always begin with the number we carried. In this example we carried two.

To finish this problem we add:

\[
\begin{array}{c}
2+6=8 \\
8+8=16 \\
16+4=20 \\
20+5=25 \\
\hline 2
\end{array}
\]

On the last column we write both the left and right sides of the answer. Watch me.

\[
\begin{array}{c}
68 \\
87 \\
45 \\
+59 \\
\hline
259
\end{array}
\]

Your answer should be two hundred and fifty nine or 259.

Remember, if there is a column to the left, you always carry the tens or left hand answer to the top of the next column and circle it.
Now I want you to do some practice problems. (Hand out practice sheets)
I am going to show you a new way of writing number facts, but first I'll show you the old way.

This is the way you usually write your facts:

\[ \begin{align*}
6 + 7 &= 13 \\
\end{align*} \]

Using the new way the facts are written using a slash like this:

\[ \begin{align*}
6 \div 3 \\
\end{align*} \]

You still write thirteen, only it is written by making a slash through the seven in place of the one.

I'll show you another example. We can write eight plus six like this:

\[ \begin{align*}
8 + 6 &= 14 \\
\end{align*} \]

or using the new way like this:

\[ \begin{align*}
\div 4 \\
\end{align*} \]

Do you see the fourteen?

Now I want you to try it. Using the paper I have given you, write down this problem: seven plus eight. Now I want you to write down the answer using the new way.

(Check to make sure they have all used the correct notation)

\[ \begin{align*}
7 + 8 &= 15 \\
\end{align*} \]

Very good. The slash is said to be in the tens place and the number under the answer line is said to be in the ones place.

Now I am going to show you a bigger problem. In this problem we are going to add three numbers; nine, seven, and eight. Watch me:

Nine plus seven equals sixteen, so I'll make a slash through the seven and I'll remember the six.

\[ \begin{align*}
9 \div 7 + 8 \\
\end{align*} \]
Now using the six I remembered I will add it to the eight. Six plus eight is fourteen so I will make a slash through the eight and write the four under the answer line.

Now I have the right half of the answer which is four.

To get the other half of the answer, I simply count the slashes.

Let's see how many slashes there are. Let's count one...two. So two is the other half of the answer.

The answer is twenty-four. Do you see?

Now I want you to try an even bigger problem. Write this problem on your paper. Don't forget, you only add the number you "remember" to the next number. Here is the problem: add six, plus seven, plus eight, plus five.

If you need help, ask me.

Okay, everybody is finished. The answer is twenty-six. Let's see how you reached the answer.

Six plus seven equals thirteen, so you made a slash through the seven and remembered the three. That's right.

Next you added the three you remembered to the eight. It equaled eleven so you made a slash through the eight and remembered the one.

Next you added one plus five. It equaled six so your wrote the six under the answer line but you did not make a slash because you didn't have any tens.

Now you had the right half of the answer. Then you counted the slashes, one...two, and wrote down the left half of your answer which was two. Your answer was twenty-six.

Very good

Did everybody get twenty-six for their answer? Good
Now let me show you how to add when there is more than one column using the new way.

Look at this problem:

\[
\begin{array}{c}
68 \\
87 \\
45 \\
\_59
\end{array}
\]

Can we still write our left hand answer if there is more than one column? No, we can not.

When there is more than one column, each column can have only one number at the bottom, except for the last column.

So the single number we put at the bottom is always the right hand number.

What do we do with the left hand number? We carry it to the top of the next column and circle it.

Now when we begin to add the second column we always start with the number we carried.

In this example we carried the two.

To finish this problem we add:

\[
\begin{array}{c}
2 \\
8 \\
6 \\
0
\end{array}
\]

and write the five under the answer line.

To get the other part of the answer we simply count the number of slash marks in the last column. Let's count, one, two. So two is the far left number of the answer. The answer is two hundred and fifty-nine.

Remember, if there is a column to the left, you always count the slashes and carry that number to the top of the next column and circle it.

Now I want you to do some practice problems. (Hand out practice sheets)
### APPENDIX H

**SAMPLE DISPLAY OF ALGORITHM ORDER FOR DAILY SESSIONS**

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1. 2. 3.


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