A Comparison of Speed and Accuracy in Addition Calculation for High and Low Achieving Math Students using the Calculator, Conventional Algorithm and the Hutchings' “Low Stress Algorithm”

Drew

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A COMPARISON OF SPEED AND ACCURACY IN ADDITION CALCULATION
FOR HIGH AND LOW ACHIEVING MATH STUDENTS
USING THE CALCULATOR, CONVENTIONAL ALGORITHM AND
THE HUTCHINGS' "LOW STRESS ALGORITHM"

by

Edward S. Drew

A Project Report
Submitted to the
Faculty of the Graduate College
in partial fulfillment of the
requirements for the
Degree of Specialist in Education
Department of Psychology

Western Michigan University
Kalamazoo, Michigan
April 1981
A COMPARISON OF SPEED AND ACCURACY IN ADDITION CALCULATION
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Edward S. Drew, Ed.S.
Western Michigan University, 1981

This study was an attempt to compare the differential
calculation power (speed plus accuracy) with addition for
the conventional algorithm as compared with the Hutchings' low stress algorithm as compared to hand held calculators
for high and low achievers with two levels of difficulty. Also, it is a systematic replication of Zoref's (1976)
previous research. The subjects were eight third grade
students, four male and four female, approximately nine
years old. A multielement baseline design was used varying
type of calculation method within sessions and level of diff-
iculty across sessions. The Hutchings' "low stress" algo-
rithm produced markedly better results in correct rate,
error rate, and percent accuracy than the conventional algo-
rithm across both high and low achieving students with two
different sized problem arrays. The calculator also pro-
duced markedly better results than the conventional algo-
rithm regardless of the type of student or problem diff-
iculty. The Hutchings' algorithm could be adopted in
elementary mathematics curricula as an improved means of
addition instruction.
ACKNOWLEDGEMENTS

I would like to express my appreciation to the Faculty of Western Michigan University. A special thanks to the Department of Psychology and my advisor Galen Alessi, Ph.D. I would also like to thank the cooperative staff and students of the Andrew Jackson Elementary School.

I have appreciated the patience of my children, Malea, Juliann, and Brandon throughout this project. Furthermore without the assistance of an exceptional colleague, Pamela Drew, this would not have been possible.

Edward Drew
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WESTERN MICHIGAN UNIVERSITY, ED.S., 1981

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INTRODUCTION

The Problem

This study was an attempt to compare the differential calculation power (speed plus accuracy) with addition for the conventional algorithm as compared with the Hutchings' low stress algorithm as compared to hand held calculators for high achievers and low achievers. It is hoped that the results will further enhance others in developing specific teaching techniques and curriculum modifications to aid children's acquisition of knowledge. Also, it is a systematic replication of previous research by Zoref (1976).

Its Significance

One of a child's finest attainments in his learning experience is the concept of number ideas of quantity, weight, time, operation, numerical classification and problem solving. These principles begin early and develop as the individual grows. Mathematical "literacy" is becoming an increasingly important part of life. The role of computational skills in mathematics must be seen in the light of the contributions they make to one's ability to use mathematics in everyday living. In isolation, computational skills contribute little to
one's ability to participate in mainstream society.

Part of this surge in needing quantitative knowledge has been facilitated by the hand held calculator and the economical prices which they have come down to in the 1970's. During the 1970's many voices have been heard debating the virtues and dangers of calculator usage. A few have cried out in fear that the use of calculators will result in pupils who cannot remember basic facts or do traditional paper-and-pencil computation. Teachers, in particular, are concerned about how calculators will affect students' computational skills (Palmer, 1978; Reys, et. al. 1980). The "rot-the-mind-theory" has not been supported by research (Suydam, 1979). Although long-term effects of sustained calculator usage are not yet known, there is ample evidence that frequent use of calculators in elementary schools has no detrimental effect on achievement in mathematics. Some have proclaimed that the rapid growth and sales of the inexpensive calculators and their consequent widespread availability to pupils demand that the mathematics curriculum be reexamined and that teachers use calculators as an instructional tool. Meyer (1980), Beardslee (1978), and Zakariya et. al. (1980) emphasize that when you use a calculator you have to think and that students can comprehend more complex problems using a calculator. The official policy state-
ment by the National Council of Teachers of Mathematics has encouraged schools to formulate calculator policies and to make these tools available to pupils to be used in creative and innovative ways of learning mathematics.


Zakariya (1980) concludes that calculator usage added motivation and excitement to math lessons. Students with little prior success in math attained success with the use of the calculators. The use of the calculator can help motivate pupils to work with mathematics and provide a source of encouragement along the way so that success in learning mathematics skills can be reached. The most valuable benefit to most students is learning how to reason logically through a problem to reach a solution. Calculators can serve as a quick key and be used to reinforce math facts and concepts, in addition to being fun to use. (Hawthorne, 1973; Stultz, 1975; Meyer, 1980; Zakariya, 1980.)

More advantages are listed by the NCTM Instruction-
al Affairs Committee (1976) for using calculators in school. That committee lists the following: (a) promotes creative experimentation with mathematical problems, (b) helps one to become a well advised consumer, (c) can be used to demonstrate the concept of repeated operations, and (d) encourages independent problem solving. However, there are also certain obvious cautions when using a calculator, for example: there is no written record to help the student find the source of error, calculators do not illustrate mathematical operations, students can get answers without knowing the operations necessary to derive the answers, (Hawthorne, 1973), and traditional textbooks do not contain exercises appropriate for calculator exercises.

Paper-and-pencil work will always be necessary to some extent in a math curriculum, for the very reasons stated above. It is therefore important to impart calculations skills in as efficient a manner as possible. Certain practices in teaching computation need thoughtful reexamination because researchers say that most Americans appear to experience some degree of low-level "math avoidance". Gordon (1972) and Hutchings (1976) have the concern that poor computational skills often develop feelings of anxiety and failure about mathematics. Alessi (1974) expresses the opinion that tedious drill and concentration of math curricula on
calculation is responsible for negative feelings and that it is harmful to his/her self-esteem. They would agree with Skinner (1968) "that math is not a reinforcing event, for many people who have met with failure but rather a reaction of anxiety, guilt, or fear." (Skinner, 1968, pg. 18). Parents magazine (Fiske, 1980) recently ran an article on math anxiety and a recently published book deals with the topic Overcoming Math Anxiety by Sheila Tobias (1979).

Not only has math had negative press in regards to anxiety it can create, but also the sensational headline that students are declining in their math skills. Modern mathematics programs have received much of the blame and now the educational trend is "back to the basics". Minimum competency testing is also seen by some to be a solution. Some of these simplistic solutions are regrettable since one has only to look at the trends in all of technology and see that technology is rapidly expanding. Human knowledge is expanding rapidly in all fields but certainly quantitative knowledge is more important to more professions today. In a math curriculum each kind of learning, conceptual and computational, demands more time and it is hard to fit all this into the previous time allotted to math instruction.

"Back to the basics" is not a solution to such a complex dilemma. Hutchings (1976) suggests that "what
is needed are drastic changes in the structure and teaching of computational skills." Hutchings in 1972 developed a set of experimental algorithms for the four basic math operations at Syracuse University. Hutchings (1976) states: "they appear to permit easy mastery after brief training, to provide greater computational power than conventional algorithms, to operate with much less stress on the user than conventional algorithms, and to enjoy certain other advantages." (p. 219) Zoref (1976, p. 2) lists the following advantages of the Hutchings' algorithm: "(a) easy identification of errors, (b) facility with locating error patterns and prescribing appropriate remediation, (c) effective drill in basic math facts, (d) a full written record that allows for specific feedback on accuracy, and (e) useful as a teaching tool to demonstrate carrying (regrouping)."

Group design studies have been conducted by several individuals (Alessi, 1974; Boyle, 1975; Dasiell, 1974; Gordon, 1972; Hutchings, 1972) which examined the computational power (speed and accuracy) of the Hutchings' algorithms. Single subject design studies have investigated whether the Hutchings' algorithm is preferred over the conventional algorithm as well as the computational power of the Hutchings' algorithm (Gillespie, 1976; Rudolph, 1976; Zoref, 1976; P. Drew, 1980).
Hutchings (1976) defines the addition algorithm as follows:

"the low-stress addition algorithm uses a new notation, called half-space notation, to record individual steps. Half-space notation uses numerals of no more than a half-space in height to record the sum of two digits. With half-space notation, the units portion of the sum of two digits is written at the lower right of the bottom digit and the tens portion is written at the lower left of the bottom digit" (p. 220)

Given a single column of figures to add, one would

"start at the top, add the first two digits, and record the sum in the new notation... The complete sum of each two digit addition is recorded in half-space notation, but only the ones portion of each sum is used in the next addition. The ones portion of the column sum is always the same as the ones portion of the last two digits sum. The tens portion of the column sum is always the same as the number of tens recorded at the left of the column... For a column in some multicolumn addition exercise, the last step, that is, counting the tens at the left of the columns - would be slightly changed...the total number of tens is no longer written in the tens place of the first column's sum but instead at the top of the next column at the left...Note, however, that the column sum for the last column in a multicolumn example is recorded in exactly the same way as the sum of a single-column exercise" (Hutchings, pp. 221 - 223)

Example:

```
   3  6  4
  3  8  2
  1  6  4
   8  7  2
____1  7  9
  1  9  6  1
```
Growing out of the U.S. Public Law, 94-142, mandating special education services to all handicapped students, is a growing concern for the needs of the individual student. The advantages of the Hutchings' "low-stress" algorithm to "exceptional" children and mathematics are many. Frequently, these pupils have difficulty in math calculations as well as concepts. By using this full record notation the teacher can locate specific errors. Lankford (1974) found that "poor computers often made errors in whole number operations when their counting ... became too involved for their short memory spans". Bannatyne (1968, 1971, 1974) indicates that many learning disabled pupils have difficulty with sequential memory skills which consist of short-term memory storage of sequences of auditory and visual stimuli. Hutchings' algorithm would make it easier to analyze the pupil's work procedures which was suggested to help learning disabled students in math by Backman (1978), Inskeep (1978), Flinter (1979), and Moyer (1978). Hutchings (1976) is concerned about the math curriculum and feels the new procedures should be used for pupils with severe remedial needs. Hutchings (1976) further states:

"These algorithms, together with hand-held electronic calculators, might form the skills core. Indeed, low stress algorithms and hand-held calculators compliment each other and are a logical team. The calculator offers speed with complex operation; the algorithms offer independence from the machine, the power to
check the machine easily or even to exceed the machine's limitations, and a permanent, complete record of work. Facility in both and the option to use either as needed probably constitute the ideal skills package, necessitating only a fraction of the time and stress required by conventional algorithms."

(p. 236)

Hutchings does caution, however, about making any curriculum change too rapidly and encourages the use of the new algorithm with students in special education and remedial classes.

Reviews of the literature by Hutchings (1972), Gordon (1972), and Dasiell (1974) did not reveal much literature that related to new algorithms. Although a few algorithms are suggested, there does not appear to be research on them.

Sanders (1971) devises an aid to assist students who tend to forget where they are in computation. He suggests a system of holding up fingers to keep track of the tens. For example, when adding 8 plus 7, the pupil says "five" and holds up one finger to keep track of the tens portion of the sum. This may help eliminate a concern that Lankford (1974) had that students find computation difficult because the counting becomes too difficult and there is too much reliance on memory skills. The major advantage of this algorithm, like the Hutchings' low stress algorithm is that it requires only basic addition facts with no complex addition facts.

O'Malley (1969) has a similar solution only he has the child write down the one in the tens column, thus
adding the tens to that column. This algorithm decreases the memory load for complex facts and partial sums but it does not have all the advantages of the Hutchings' full written record.

Fulkerson (1963) has a similar algorithm whereby the pupil draws a line through the last digit used in adding a sum of ten or greater. He states some of the advantages, "to give pupils a better understanding of our number system, to motivate pupils to carry on needed practice in addition, and to make our pupils conscious of the fact that column addition can be performed by the use of algorithms other than the one to which all of us have been ordinarily accustomed." (Fulkerson, 1963, p. 140) This algorithm also doesn't require any complex addition facts.

Batarseh (1974) comes up with an alternative algorithm which he suggests would be useful to teach the concept of carrying to slow learners and educable retarded youngsters. It does, however, require some complex addition facts. It would not be convenient to use for large multicolumn addition problems and is suggested only as a method to illustrate the concept of carrying to slow youngsters.

Zoref (1976) studied computational speed and accuracy using a multielement baseline design. Type of calculation method was varied within sessions and level of difficulty was varied across sessions. In the multielement design (Ulman & Sulzer-Azaroff, 1975, p.377) one treatment is
applied across one baseline, which is broken into two parts. The treatment is applied to one part of the baseline, but withdrawn during the other part. The no treatment baseline is acting as if it is a kind of control. Alessi (1976) indicated the following advantages of using the multielement baseline design: That several treatments can be compared at the same time without the sequence or order affecting problems associated with the reversal design. Long periods are not needed to stabilize each baseline. The effects of the various treatments can be seen early in the study. These advantages make it useful for studies that are brief.

This study will replicate the study by Zoref (1976) which was designed to investigate the differential calculation power (speed plus accuracy) with addition for the conventional algorithm as compared with the Hutchings' low stress algorithm as compared to hand held calculators. The three methods are compared for two levels of difficulty of problems and for both high and low math achievers. In Zoref's (1976) study the Hutchings' low stress algorithm was generally the most stable method (i.e., had the lowest standard deviations) across all measures (error rate, correct rate, and the percent accuracy) regardless of the type of student or problem difficulty. This method also had the lowest error rate over the other two procedures, across the type of student as well as the level of difficulty. The Hutchings' low stress algorithm was also generally the most ac-
curate method used

Experiment one will examine only the low achievers and Experiment two will examine the high achievers. The present study attempts to examine the following questions:

1. Are calculation correct and error rates and accuracy percentages for the three methods different between high and low achievers in math?

2. Are calculation correct and error rates and percent of accuracy for the three methods affected by problem difficulty (number of binary additions per problem).
Experiment I

Method

Subjects

The subjects for this study were four third grade elementary students. There were two boys and two girls ranging in age from eight years and nine months to nine years and six months. These students were identified as low achievers in math. The identification was documented by their placement in low ability math sections (this school groups math students in low, average, and high sections). The results of the Iowa Tests of Basic Skills further confirmed the placement of these students, with math composite scores on national norms of 6th, 6th, and 33rd, and 42nd percentiles. The students had been placed in the math sections as a recommendation of the second grade teacher they had the previous year.

Setting

The study took place at an elementary school of 450 students in a medium sized city in Iowa which is located on the Mississippi River. Sessions were held in a vacated kindergarten classroom in the afternoon of the students' regular school day. The session times differed from 1:00 to 1:50, or 2:05 to 2:25 p.m. depending on the day. The time changes were due to teachers' academic schedules. A
total of 33 sessions were held. The sessions ranged from four to five days a week depending on schedule conflicts and vacations.

Independent Variables

This study is an attempt to replicate the study of Zoref, (1976) and so the independent variables as well as the dependent variables are similar. The two independent variables were: (a) the three calculation procedures: conventional algorithm, Hutchings' algorithm (refer to pages 5 and 6), and hand held calculators; and (b) two problem array sizes: five columns by seven rows of digits and two columns by seven rows of digits (see Appendix D).

Dependent Variables

Three dependent variables were considered: (a) rate of columns correct, (b) rate of columns incorrect, and (c) percent accuracy on attempted columns. The rate is determined by having a five minute time limit and calculating the number of columns correct and number of columns incorrect. Accuracy was calculated by dividing the number of columns correct by the number of columns attempted and multiplying the quotient by 100.

Reliability

Reliability data were taken on correcting the students' papers for the number of columns correct and incorrect. A
sample of 15 papers was taken from each set of the students' work sheets collected on Tuesdays and Fridays during the entire study. Reliability data were collected on scoring worksheet responses by two independent scorers using an opaque projector. Reliability was computed by dividing the total number of agreements by agreements plus disagreements multiplied by 100.

**Experimental Design**

A multielement baseline design was used (Ulman and Sulzer-Azaroff, 1975). The type of calculation procedure was varied across sessions. The results can be analyzed by examining the interaction between, as well as the main effects of, the variables manipulated.

**Materials**

A five minute basic addition math facts test (taken from Alessi, 1974) consisting of 56 basic addition facts was administered to the eight pupils as a pretest. Two identical worksheets were used for the three training sessions to ensure equal practice effects for each calculation procedure. The worksheets were made up of addition problems of varying array formats and sizes (Appendix D).

The following materials were used for each daily session: (a) on stopwatch (Winder Quartz); (b) one red pencil; (c) 21 dittoed sheets (seven sheets for each algorithm),
each with addition problems totaling 30 columns, printed
with an IBM Selectric Orator typing element, with three
spaces between the rows for each problem; (d) three answ-
er sheets, one for each type of calculation; and (e) three
hand held calculators.

Procedure

In order to ensure a satisfactory degree of competence
with basic addition, each student was pretested on a 56
basic addition math facts test. A criterion of 91% correct
or 51 correct within five minutes, was set as necessary for
participation in this study. Another requirement for selecting students was that both the principal and classroom
teacher felt that the low achieving students would profit
from the additional practice on column addition.

After the subjects were selected, a letter was sent
home to the students' parents explaining the purpose of the
study and permission slips to be filled out granting per-
mission of their child to participate in the study.

The instruction and review procedures were those adapt-
ed from Hutchings' (1972) and previously used by Alessi
(1974). See Appendix D for the instructional and review
sessions. The first session consisted of teaching the
Hutchings' algorithm. The second session consisted of demon-
strating how to perform addition on the calculator. The
third session was a review of the conventional algorithm.
During these sessions the same worksheets were completed, however, the order of the problems were rearranged.

Throughout the remainder of the study, sessions were run according the the following format. The students from Experiment one and two met at the same time and were divided into three groups (one group of two and two groups of three). The students were brought down from their rooms to the empty kindergarten room. There were nine stations in the room (each station was comprised of one chair and table with the students paper on the table. The conventional algorithm was worked on at stations one, two, and three; the Hutchings' algorithm at stations four, five, and six; and the calculator at stations seven eight, and nine.

When all the students were appropriately seated they were given the command to go. They were timed for five minutes with a stop watch. At the end of the five minutes, the students were told to "stop, put your pencils down, take your paper to the next station," where it was collected by the experimenter. Each session consisted of three trials, each for five minutes, with one trial per calculation procedure. The same three students rotated stations between one, four, and seven; another three rotated between two, five, and eight; and two students rotated between three, six, and nine. Each time the session began the order of the trial within sessions were on a random basis, using a random number table. The order of problem arrays.
across sessions was also random using a coin toss. However, no array was given more than two consecutive sessions.

Recording and Scoring

This experimenter would collect and correct the papers from the first trial as the students began their second trial. The papers of the second trial were corrected during the time students worked on the third trial. The papers were scored for the number of columns correct in each problem. Alessi (1974) explains the rational for scoring the columns correct:

"In order to correct for errors that might "snowball" in causing errors to accumulate across columns in the larger profiles, these were scored to prevent this effect. All children were asked to and did write the carrying numbers at the top of each column throughout the test. If it was clear that a column error had caused an incorrect number to be carried to the next higher column, and this column was correct except for the incorrect carried number, then this column was scored as correct. This procedure was not meant to correct for carrying errors per se, but only for column errors in preceding columns that caused incorrect carrying of tens to the adjacent column." (p. 42).

The experimenter would go over the errors made by the students at the end of the session. All papers were available for the students to check their performance. Time limits meant that the last set of papers could not be corrected for the students that session. It is felt, however, that feedback was given equally for each procedure over all experimental sessions since the order of procedures.
used by each student was randomized.
EXPERIMENT 1

Results

Reliability

Data for reliability yielded a range of 100% to 94% for a 98% mean agreement for scoring papers for number of columns correct and incorrect. Reliability was taken on 26% of the total student worksheets.

Organization of Dependent Data Presented

Results of this study are presented for the low math achievers in three topical sections: (a) comparison of performances for 2 x 7 problem arrays, (b) comparison of performances for 5 x 7 problem arrays, and (c) comparison of performances on both size problem arrays. Each of these comparisons for the three calculation procedures will be made in terms of three study measures: (a) correct calculation rate, (b) incorrect calculation rate, and (c) percent accuracy for calculations. Summary data for these comparisons are in two tables and four figures.

Comparison of Performances on 2 x 7 Problem Arrays

Correct rate data on 2 x 7 problem arrays. Table 1 presents the means and the standard deviations for
TABLE 1
MEANS AND STANDARD DEVIATIONS OF THE DAILY AVERAGE SESSION SCORES ACROSS THE STUDY USING EACH OF THREE CALCULATION METHODS WITH TWO DIFFERENT PROBLEM ARRAYS

<table>
<thead>
<tr>
<th></th>
<th>2x7 problem arrays¹</th>
<th>5x7 problem arrays ²</th>
<th>Mean Totals:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(13 binary operations)</td>
<td>(34 binary operations)</td>
<td>Correct ¹ Error ² Percent ³</td>
</tr>
<tr>
<td>Correct Rate</td>
<td>Mean ± sd</td>
<td>Correct Rate</td>
<td>Mean ± sd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Error Rate</td>
<td>Rate</td>
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<td></td>
<td></td>
<td>Percent Rate</td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>x sd</td>
<td>x sd</td>
<td>x sd</td>
</tr>
<tr>
<td></td>
<td>3.3 ± 2.3</td>
<td>3.0 ± 1.7</td>
<td>46.8 ± 28.5</td>
</tr>
<tr>
<td>HA</td>
<td>6.0 ± 2.9</td>
<td>1.5 ± 1.3</td>
<td>77.1 ± 20.8</td>
</tr>
<tr>
<td>EC</td>
<td>5.0 ± 2.2</td>
<td>1.2 ± 1.3</td>
<td>77.9 ± 24.9</td>
</tr>
<tr>
<td></td>
<td>4.5 ± 2.5</td>
<td>1.7 ± 1.9</td>
<td>67.5 ± 34.3</td>
</tr>
<tr>
<td></td>
<td>4.7 ± 2.4</td>
<td>1.5 ± 1.6</td>
<td>72.7 ± 29.6</td>
</tr>
</tbody>
</table>

Note: As all four students in each group were not present for all daily sessions nor each trial within daily sessions, some daily session scores reflect the average of less than three individual scores.

¹ each score represents averages from 16 sessions
² each score represents averages from 17 sessions
³ number of columns correct/5 minutes
⁴ number of columns incorrect/5 minutes
⁵ number of columns correct/number attempted

HA Hutchings Algorithm
CA conventional algorithm
EC electronic calculator
the average session scores for the three calculation procedures. As shown in the left column which is for the 2 x 7 problem arrays, the mean correct rates of the low achievers for the conventional algorithm (CA), Hutchings' (HA) and the electronic calculator (EC) were 3.3, 6.0, and 5.0, respectively. The students completed almost twice as many columns correct with HA than with the CA. The subjects completed one less column correct with the EC than HA.

Table 2 presents the means and standard deviations of the average daily scores for the three calculation types. Individual data for one of the four students was not representative of the group data. Student 3 had higher correct rates with the EC than with the HA.

Figure 1 presents the average daily scores of correct and error rates for the three calculation procedures with 2 x 7 problem arrays. It shows that HA yielded more correct columns than either EC or CA. The CA was the lowest but there was some overlapping of data for all three procedures. However, the trends tended to be fairly stable.

In the first one-half of the sessions for low achievers there is a frequent overlapping of data between HA and EC. In the last half of the study there is virtually no overlapping with the HA having a higher mean per sessions correct than the EC.

**Error rate data on the 2 x 7 problem arrays.** Table 1
### TABLE 2

MEANS AND STANDARD DEVIATIONS OF THE INDIVIDUAL DAILY TRIAL SCORES ACROSS THE STUDY USING THE THREE CALCULATION METHODS WITH TWO DIFFERENT PROBLEM ARRAYS

<table>
<thead>
<tr>
<th>Student Method</th>
<th>2 x 7 problem arrays</th>
<th>5 x 7 problem arrays</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>sd</td>
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<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>CA</td>
<td>16</td>
<td>2.06</td>
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<tr>
<td>HA</td>
<td>16</td>
<td>8.12</td>
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<tr>
<td>EC</td>
<td>16</td>
<td>7.12</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CA</td>
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<tr>
<td>HA</td>
<td>15</td>
<td>7.53</td>
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<tr>
<td>EC</td>
<td>15</td>
<td>2.86</td>
</tr>
<tr>
<td>3</td>
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<td></td>
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<tr>
<td>CA</td>
<td>13</td>
<td>1.46</td>
</tr>
<tr>
<td>HA</td>
<td>13</td>
<td>3.53</td>
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<tr>
<td>EC</td>
<td>13</td>
<td>5.76</td>
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<tr>
<td>CA</td>
<td>14</td>
<td>3.64</td>
</tr>
<tr>
<td>HA</td>
<td>14</td>
<td>4.78</td>
</tr>
<tr>
<td>EC</td>
<td>14</td>
<td>4.14</td>
</tr>
</tbody>
</table>

**NOTE:** Since all students were not present for all daily sessions nor all trials within each daily session, scores for each method reflect averages of different numbers of trials (and sessions) per method per student.

\( N^a = \) number of sessions attended
\( \frac{1}{2} \) number of columns correct/5 minutes
\( \frac{2}{3} \) number of columns incorrect/5 minutes
\( \frac{3}{3} \) number of columns correct/number attempted

CA = Conventional Algorithm
HA = Hutchings Algorithm
EC = Electronic Calculator
Figure 1
Mean rate of columns correct and incorrect per session using the three calculation methods with 2x7 problem arrays

- Hutchings'
- Conventional
- Calculator
presents the means and standard deviations for the average sessions scores for the three calculation procedures. As shown by the middle column of the left part of Table 1, the mean error rate for the low achievers was 3.0 for CA, 1.5 for HA, and 1.2 for EC. The CA produced the highest error rate with HA being lower and the EC the lowest. The CA's error rate was more than double that of both the HA and EC's error rate.

Table 2 shows that all low achievers had the highest error rates with the CA. Students 3 and 4 differed in the order of the error rate in that their HA rates were lower than the EC. This differs from the group data.

Figure 1 indicates that the mean error rate per session for low achievers was lowest with the HA at the end of the study. The EC was the lowest until session 12, then the HA replaced it. There was virtually no other variability. The CA yielded the highest error rate and had little variability.

Accuracy data on 2 x 2 problem arrays. Table 1 presents the means and the standard deviations for the average session scores for the three calculation procedures. As shown by the third column of the left part of Table 1, the mean percent accuracy is the lowest with the CA (46.8), while the HA (77.1) and the EC (77.9) are virtually the same. A significant difference arises with the CA being over thirty percent less accurate. Individual student data (Table 2)
are not consistent with the group results. Only two students had the same order of percentage accuracy as the group data. Two students performed better with HA than EC. All the students made more errors with the CA.

Figure 2 shows the average daily scores of percent correct for the three calculation methods with 2 x 7 problem arrays. Figure 2 shows that both the EC and HA were most accurate for the first two sessions. Then for the next five sessions the students were less accurate with HA and more accurate with EC. After session eleven the HA was the most accurate calculation method. While the CA was the least accurate with virtually no overlap in data with the other methods. This method also produced much more variability than the other two methods.

**Comparison Of Performance On 5 x 7 Arrays**

Correct rate data on 5 x 7 problem arrays. In the left column of the right half of Table 1 are the means and standard deviations for the correct rates (CA 3.4, HA 6.2, EC 4.5). Table 2 depicts the means and standard deviations of the average daily scores for individual data. This data are not in complete agreement with the group data. One subject had the same rank order as the group results. Student 2 had HA with the most columns correct the CA next and EC the lowest. Student 3 had EC with the most correct columns followed by the HA and CA the lowest. Finally Student 4 had the
Figure 2
Mean percent of columns correct per session using the three calculation methods with 2X7 problem arrays
CA with the most correct columns with HA next and the EC the lowest.

Figure 3 reflects the average daily scores of correct and error rates for the three calculation methods with the 5 x 7 problem arrays. It illustrates that the HA starts off with the highest correct rate and with minimal overlapping finishes the study the highest. The EC overlaps with CA but the trends still reflect the group results in that EC yields a higher correct rate than CA. The slopes for EC and CA both show little or no increase in correct rates. However, with the HA the last three sessions show a marked increase and it is reasonable to conclude that an increase of mastery began.

Error rate data on 5 x 7 problem arrays. As indicated by the data in the middle column of the right half of Table 1, the mean error rate is lowest with HA (1.4), EC (1.7), and CA (3.2) the highest. With these students the error rates with CA are over two times larger than using HA and almost double those for EC.

In Table 2 the individual error rate is shown to be consistent with the group data. Three students have the same rank order as the group data. One student was different and the change in order was between EC and HA. In no instance with the CA is any individual error rate lower than the mean error rate of HA.

Figure 3 illustrates that at the end of the study HA
Figure 3
Mean rate of columns correct and incorrect per session using the three calculation methods with 5x7 problem arrays.

- O Hutchings
- O Conventional
- O Calculator
has the lower error rate with the EC higher and CA highest. There is some overlapping but the slope for both HA and CA is fairly stable. The HA moves downward and CA stays the same. The EC's slope is much more variable. Some daily means for error rate are the highest and other are the lowest. The HA is clearly lower than CA with no overlapping but less clear when compared with EC.

Accuracy data on 5 x 7 problem arrays. The right column of the right half of Table 1 presents data showing that the HA was the most accurate (78.0%). The EC method was lower (67.5%) and the lowest was the CA (46.7%), with which students got less than one half of the number of columns correct.

There is little agreement on an individual basis. Three students had the same rank order as the group results. Three students exchanged places between HA and EC being the highest accuracy rate. Another student had EC the lowest with the CA next and HA the highest and lastly followed by EC and HA lowest.

Figure 4 illustrates the average daily scores of percent accuracy for the three calculation methods with the 5 x 7 problem arrays. Figure 4 shows the HA slope increasing, EC's slope decreasing and CA staying the same with some variability. When the study begins EC has a higher accuracy rate than HA. But by the end this reverses itself with HA being more accurate.
Figure 4
Mean percent of columns correct per session using three calculation methods with 5X7 problem arrays.

PERCENT OF COLUMNS CORRECT (MEAN)

SEQUENCES
Comparison of Performances

Correct rate data on 2 x 7 and 5 x 7 problem arrays.

The upper half of Table 1 presents the means and standard deviations for correct rates, error rates, and percent accuracy for low achievers on 2 x 7 and 5 x 7 problem arrays. The left-hand column of both quadrants shows that correct rates for all three methods remain clearly unchanged for the two different arrays. However, there is a slight decrease from 5.0 to 4.5 for the low achievers for the EC when calculating the 2 x 7 and 5 x 7 arrays, respectively. Both with the CA and HA the scores increased 0.1 and 0.2 when calculating the 5 x 7 as compared to the 2 x 7 arrays. The CA and HA increased slightly when using 5 x 7 than 2 x 7 (0.1 and 0.2). For the total correct rate regardless of problem difficulty, HA was highest (6.1), EC was lower (4.7), and the CA was the lowest (3.3). When comparing Figure 1 with Figure 3, it is seen that the correct rate for the HA are higher for both 2 x 7 and 5 x 7 problem arrays. EC is next and CA the lowest.

Error rate data on 2 x 7 and 5 x 7 problem arrays. Analyzing the student data in Table 1, it can be seen that there are similar error rates for HA and EC methods. However, the CA produced about double the error rate as either of the other two methods on both array sizes. There is no increase in error rate with HA, but a slight increase 0.2 with CA and a higher error rate with EC 0.5.
For total error rate regardless of problem difficulty, HA and EC were very similar. However the CA error rate was at least double that for the other two methods. Individual subject data described on Table 2 rank CA error rate the highest with all four students on both size problem arrays. There is an overlap of data between the HA and EC. On 2 x 7 arrays, two students have the lowest rate and on 5 x 7 three students have the lowest rate. Therefore, there is overall agreement with group data. Figures 1 and 3 show that CA had the highest error rate and there was overlapping with both HA and EC but HA's slope was lower than EC.

Accuracy data on 2 x 7 and 5 x 7 arrays. Table 1 presents data indicating that the accuracy levels are almost identical for the CA and HA across problem size. However, there is a drop in accuracy of EC when problem size increases (77.9 and 67.5). For total accuracy data regardless of problem difficulty, HA was highest (77.5), EC was somewhat lower (72.7) and the CA was markedly lower (46.8). Table 2 presents individual student data. There is some agreement with group data. In the 2 x 7 problem size the CA yielded the lowest accuracy rate for all four students while EC and HA produced two students with the most accurate method. On the 5 x 7 problem arrays there is less agreement with group data. The CA produced three students with the lowest accuracy while EC produced one. As for the highest accuracy HA produced three and EC produced one. Student 1 and 2's accuracy

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rate remained similar regardless of the problem size. Student 3 increased 7% on the CA with larger problems but dropped 11% on the EC. Student 4 increased 5% on the HA but dropped 30% on the EC. Thus two students had a significant drop in accuracy when 5 x 7 problems were calculated with the EC.

Figures 2 and 4 both illustrate much variability within the three calculation methods. The graphs illustrate an interesting trend after about one third of the study, the rank order of most accurate changes on both size problems, from EC, HA, CA to HA, EC, CA. The HA appears more accurate at the last half of the study. Could it be that the HA method improves with practice (more attempts)? When comparing Figure 2 with Figure 4, it is clear that EC has more variability for both problem array sizes.
EXPERIMENT 2

Method

Subjects

The subjects for the study were four third grade elementary students. There were two boys and two girls ranging in age from eight years and eleven months to nine years and six months. These students were identified as high achievers in math. The identification was documented by their placement in high ability math sections (this school groups math students in low, average, and high sections). The results of the Iowa Test of Basic Skills further confirmed the placement of these students, with math composite scores on national norms of 77th and three students at the 99th. The students had been placed in the math section as a recommendation of the second grade teacher they had the previous year.

Setting

The setting is the same as experiment one.

Independent Variables

The independent variables and the criteria for the independent variables for Experiment 2 are the same as in Experiment 1.
Dependent Variables

The independent variables were the same as in Experiment one.

Reliability

The procedures were the same as in Experiment 1.

Materials

The same materials were used as in Experiment 1.

Procedure

The same procedure was used as in Experiment 1.
EXPERIMENT 2

Results

Reliability

The reliability ranged from a high of 100% to a low of 91% for a mean of 95% agreement for scoring papers for the number of columns correct and incorrect. Reliability was taken on 29% of the total student worksheets.

Organization of Dependent Data Presented

Results of this study are presented for the high math achievers in three topical sections: (a) comparison of performances for 2 x 7 problem arrays, and (b) comparison of performances for 5 x 7 problem arrays, and (c) comparison of performances for high math achievers on both problem sizes. Each of these comparisons for the three calculation procedures will be made in terms of three study measures: (a) correct calculation rate, (b) incorrect calculation rate, and (c) percent accuracy for calculations. Summary data for these comparisons are in two tables and four figures.

Comparison Of Performances on 2 x 7 Problem Arrays

Correct rate data on 2 x 7 problem arrays. Table 3 gives the means and standard deviations for the average session scores for the three calculation procedures. As shown in the left column of the left half of Table 3, the
### TABLE 3
MEANS AND STANDARD DEVIATIONS OF THE DAILY AVERAGE SESSION SCORES ACROSS THE STUDY USING EACH OF THREE CALCULATION METHODS WITH TWO DIFFERENT PROBLEM ARRAYS

<table>
<thead>
<tr>
<th></th>
<th>2x7 problem arrays&lt;sup&gt;a&lt;/sup&gt; (13 binary operations)</th>
<th>5x7 problem arrays&lt;sup&gt;b&lt;/sup&gt; (34 binary operations)</th>
<th>Mean Totals:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8&lt;sup&gt;th&lt;/sup&gt; Grade</td>
<td>9&lt;sup&gt;th&lt;/sup&gt; Grade</td>
<td>8&lt;sup&gt;th&lt;/sup&gt; Grade</td>
</tr>
<tr>
<td></td>
<td>Correct&lt;sup&gt;1&lt;/sup&gt;</td>
<td>Error&lt;sup&gt;2&lt;/sup&gt;</td>
<td>Percent&lt;sup&gt;3&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>Rate</td>
<td>Rate</td>
<td>Rate</td>
</tr>
<tr>
<td>CA</td>
<td>7.3</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>HA</td>
<td>11.7</td>
<td>3.5</td>
<td>2.0</td>
</tr>
<tr>
<td>EC</td>
<td>10.6</td>
<td>3.7</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Note: As all four students in each group were not present for all daily sessions nor each trial within daily sessions, some daily session scores reflect the average of less than three individual scores.

---

<sup>a</sup>Each score represents averages from 16 sessions

<sup>b</sup>Each score represents averages from 17 sessions

1Number of columns correct/5 minutes

2Number of columns incorrect/5 minutes

3Number of columns correct/number attempted

CA = Conventional algorithm

HA = Hutchings algorithm

EC = Electronic calculator
mean correct rates for the conventional (CA) and Hutchings' (HA) algorithm and the electronic calculator (EC) are 7.3, 11.7, and 10.6. For the total correct rate data HA was highest (11.9), EC was lower (10.4), and CA was lowest (7.4).

Table 4 presents the means and standard deviations of the average daily scores for individual high achievers for the three calculations types. Individual data was not in general agreement for two of four students. Student 5 and 8 had higher correct rates with the EC than with HA.

Figure 5 presents the average daily scores of correct and error rates for the three calculation methods with 2 x 7 problem arrays. In the first half of the study there is frequent overlapping between HA and EC. At session 18, the HA clearly produced the most columns correct with EC next and CA the lowest. The group data remain stable with minimal fluctuations for the last half of the study.

Error rate data on the 2 x 2 problem arrays. Table 3 presents the means and standard deviations for the average sessions scores for the three calculation procedures. As shown by the middle column of the left half of Table 1, the mean error rate for the high achievers is 2.3 on the CA. However, both with HA and EC the high achievers had similar error rates, 2.0 and 1.9.

Table 4 shows only student 5's error rate was representative of group data. Student 6 had the highest error rate with EC, and lowest with HA. Student 7 had the highest
TABLE 4
MEANS AND STANDARD DEVIATIONS OF THE INDIVIDUAL DAILY TRIALS SCORES ACROSS THE STUDY
USING THE THREE CALCULATION METHODS WITH TWO DIFFERENT PROBLEM ARRAYS

<table>
<thead>
<tr>
<th>Student Method</th>
<th>2 X 7 problem arrays</th>
<th>5 X 7 problem arrays</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N (^2) Correct R. (^1) Error R. (^2) % accuracy (^3)</td>
<td>N (^2) Correct R. (^1) Error R. (^2) % accuracy (^3)</td>
</tr>
<tr>
<td></td>
<td>x sd</td>
<td>x sd</td>
</tr>
<tr>
<td>CA 5</td>
<td>16 7.56 1.96</td>
<td>3.18 2.32</td>
</tr>
<tr>
<td>HA 5</td>
<td>16 11.56 2.94</td>
<td>2.25 1.18</td>
</tr>
<tr>
<td>EC 5</td>
<td>16 14.50 3.38</td>
<td>1.56 1.50</td>
</tr>
<tr>
<td>CA 6</td>
<td>12 5.17 2.62</td>
<td>2.41 3.08</td>
</tr>
<tr>
<td>HA 6</td>
<td>12 6.56 2.67</td>
<td>1.08 0.66</td>
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<tr>
<td>EC 6</td>
<td>12 6.33 4.22</td>
<td>3.41 3.87</td>
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<td>14 6.71 1.89</td>
<td>2.28 2.40</td>
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<td>14 18.07 3.07</td>
<td>0.83 1.02</td>
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<tr>
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<td>14 10.50 4.25</td>
<td>1.78 1.62</td>
</tr>
<tr>
<td>CA 8</td>
<td>15 9.93 2.73</td>
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<td>HA 8</td>
<td>15 10.73 5.29</td>
<td>3.86 2.89</td>
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<tr>
<td>EC 8</td>
<td>15 11.20 2.78</td>
<td>0.73 0.88</td>
</tr>
</tbody>
</table>

NOTE: Since all students were not present for all daily sessions nor all trials within each daily session, scores for each method reflect averages of different numbers of trials (and sessions) per method per student.

N \(^2\) = Number of session attended

1Number of columns correct/5 minutes
2Number of columns incorrect/5 minutes
3Number of columns correct/number attempted

CA = Conventional Algorithm
HA = Hutchings Algorithm
EC = Electronic Calculator
Figure 5
Mean rate of columns correct and incorrect per session using the three calculation methods with 2X7 problem arrays.

![Graph showing mean rate of columns correct and incorrect per session using three calculation methods with 2X7 problem arrays.]
error rate with CA and lowest error rate with HA. Student 8 had the highest error rate with HA, and lowest with EC.

Figure 6 shows much overlapping in all three calculation performances. There is one trend with the high achievers. After session 15, HA has a lower error rate and is more stable than either the EC or CA.

Accuracy data on 2 x 7 problem arrays. Table 3 presents the means and the standard deviations for the average session scores for the three calculation procedures. As shown by the third column of the left half of Table 3, the mean percent accuracy is the lowest with CA (77.1), HA is higher (82.8), and EC (84.2) is the highest. Individual student data in Table 4 are not consistent with the group results. One student obtained the same order of percentage accuracy as the group data. One student performed better with HA than EC. Another student was most accurate with HA, CA was next and EC was last. One subject was most accurate with EC, CA next and the least accurate with HA.

Figure 6 shows the average daily scores of percent correct for the three calculation methods with 2 x 7 problem arrays. With three students there is a good deal of overlapping among the data from the three calculation procedures. At session 10 the group data remain stable with minimal fluctuations. The HA is clearly the most accurate. The EC is next and the CA is the least accurate.
Figure 6
Mean percent of columns correct per session using the three methods of calculation with 2X7 problem arrays.
Comparison Of Performance of 5 x 7 Arrays

Correct rate data on 5 x 7 problem arrays. The left column of the right half of Table 3 presents the means and the standard deviations for the correct rates for the high achievers. The CA produced the fewest number of correct problems (7.4). The EC was higher (10.1), while the HA had the most correct (12.0).

Table 4 depicts the means and standard deviations of the average daily scores for individual students for the three calculation procedures. Individual data are not in complete agreement with the group data.

Table 4 shows one student had the same rank order of calculation success with correct rate the same as the group data. Two students had the highest correct rate with EC followed by HA and CA respectively. The fourth student had similar success with all methods.

Figure 7 gives the average daily scores of correct and error rates for the three calculation methods with 5 x 7 problem arrays. The data for the higher achievers overlap during the first half of the study, as shown in Figure 7. During the last half of the study there is no overlapping and the rank order of the correct rate is the same as the group data. The trends for both HA and EC show an increase in correct rate. However, with CA there is virtually no rate increase.

Error rate data on 5 x 7 problem arrays. As indicated
Figure 7
Mean rate of columns correct and incorrect per session using three calculation methods with 5x7 problem arrays.

![Graph showing mean rate of columns correct and incorrect per session using three calculation methods with 5x7 problem arrays.](image)
in Table 3 the mean error rate is virtually the same for all calculation types: HA (2.0), EC (1.9), and CA (2.0).

Table 4 shows that individual students did not follow the group data. The CA produced one student with the highest error rate with HA next and the EC the lowest. The EC yielded two students with the highest error rate with the CA next and HA the lowest with an error rate less than one. The HA produced one student with the highest error rate with both EC and CA lower with the same error score. It is interesting to note that this student's error rate dropped from six to less than two at the end of the study.

Figure 7 depicts significant overlapping of data for all three calculation methods for error rates. Both with the CA and the EC the trend tends to stay the same or increase slightly. The HA's slope moves slightly downward. Looking only at the data presented after session 9 on the top part of Figure 7, a trend becomes clear: HA yielded a higher correct rate than did the EC, which in turn yielded a higher correct rate than the CA.

Accuracy data on 5 x 7 problem arrays. Table 3 presents the means and standard deviations for the average session scores for the calculation procedures. As shown by the third column of the right half of Table 3, the data shows that the HA is the most accurate (84.3). The EC is less accurate with the high achieving students attaining 83.7 accuracy rate. The CA is the least accurate with a rate of 77.5.
There is little agreement on an individual basis. Two students had the same rank order as the group results. Student 5 had EC as the most accurate with HA next and the CA the least accurate. Student 8 was most accurate with the CA with EC next and HA the least accurate.

Figure 8 illustrates the average daily scores of percent accuracy for the three calculation methods with the 5 x 7 problem arrays. In Figure 8 the HA appears to be more consistent and shows an increase in slope, while EC and CA appear to be lower than HA.

**Comparison Of Performances For High Achievers**

**Correct rate data on 2 x 7 and 5 x 7 problem arrays.**
The lower half of Table 3 shows the means and standard deviations for correct rate, error rate, and percent accuracy for the high achievers on 2 x 7 and 5 x 7 problem arrays. All three calculation procedures have similar results across the table form 2 x 7 to 5 x 7. The rank order from highest to lowest for correct rate is the same for both problem sizes: HA highest, EC lower, and CA lowest. There is a slight increase in correct rate for both CA and HA (.1 and .2) when the problem size increases. However, with EC there is a decrease in correct rate of 0.5 when the larger problem array are used.

For the total correct rate, regardless of problem difficulty, HA was highest (11.9), EC was next (10.4), and CA
Figure 8
Mean percent of columns correct per session using the three calculation methods with 5x7 problem arrays.
was lowest (7.4). Table 4 shows that individual student's data differed from group data. Students 5 and 3 differ on the 2 x 7 problem arrays by having a higher correct rate for EC than HA, but CA was the lowest for all four. On 5 x 7 only student 7 conforms to group data. Students 5 and 6 have EC higher than HA and student 8 has EC lowest and HA the highest. Three students dropped in correct rates with EC when problem size increased while only student 6 showed an increase (2.1).

Figure 5 shows some overlapping of data across methods in the first half of the study but none in the last half. The rank order remains constant with HA highest followed by EC and CA. Figure 7 illustrates more variability than with Figure 2 with EC but continues the same overall trends.

Error rate data on 2 x 7 and 5 x 7 problem arrays. Table 3 presents data showing similar error rates for all three calculation methods regardless of problem size. There is a slight decrease for both CA and HA from .3 and .1 when larger problem sizes are attempted while the EC error rate rises from 1.9 to 2.0 an increase of .1. Regardless of problem size EC yielded the lowest error rate for students 5 and 8. On the 2 x 7 problem array HA was next with CA highest for student 5. For student 8 on the 5 x 7 problem array CA and EC yielded the lowest error rate with HA next. The HA produced students 6 and 7 with the lowest error rate regardless of problem size. On the 5 x 7 problem array
EC produced the highest error rate. However, on the 2 x 7 problem array the CA yielded the highest error rate for student 6 and 7.

**Accuracy data on 2 x 7 and 5 x 7 problem arrays.** Table 3 shows that EC (84.2) is most accurate on 2 x 7 problem arrays while HA is less than 2% lower and CA is least accurate (77.1). However, on the 5 x 7 problem arrays the HA has the highest accuracy rate (85.9) with EC about 2% lower and CA lowest (77.9). Regardless of problem size the HA is the most accurate (84.3), with EC next 1% lower and the CA least accurate about 7% lower. With regards to the 2 x 7 and 5 x 7 arrays there was less than 1% change using CA and EC, but 3% increase in accuracy for the HA. Individual student data vary from group data. For student 5 the EC was the most accurate with HA next and CA the least accurate. However, for student 8 on the 2 x 7 problem arrays the EC was the most accurate with the CA next, and HA least accurate. This student was most accurate on the 5 x 7 problem arrays with the CA, followed by the EC and the HA the least accurate.

Figure 6 illustrates a great deal of overlapping of data for the three methods in the first half of the study but in the last half the HA is clearly the most accurate followed by EC and then CA. Figure 8 illustrates overlapping data throughout the study. The HA is less variable than either EC or CA. There is a slight overall trend for
the HA to be more accurate at the end of the study than at the beginning.
SUMMARY OF RESULTS BY CALCULATION METHOD

The Hutchings' "low stress" algorithm produced markedly better overall results in correct rate, error rate, and percent accuracy than the conventional algorithm across both low and high achieving students as well as level of problem difficulty. In other words, the Hutchings' algorithm was superior to the conventional algorithm in all aspects of the study and superior to the calculator for the low achievers.

The calculator produced clearly better results in correct rate, error rate, and percent accuracy than the conventional algorithm across both low and high achieving students as well as level of problem difficulty. The calculator results paralleled the Hutchings' algorithm except for the $5 \times 7$ problem arrays where it was less effective for the low achievers but was still more effective than the conventional algorithm.

The conventional algorithm yielded the lowest correct rate, accuracy rate, and the highest error rate regardless of the type of student or problem difficulty. In other words the conventional algorithm was inferior in performance in all variables in this study.
DISCUSSION

The results of these studies indicate that both the Hutchings' "low stress" algorithm and the electronic handheld calculator produced better results (higher correct rates, lower error rates, and higher accuracy) than the conventional algorithm regardless of problem size or high or low achieving math students. These results were expected and generally confirmed Zoref's previous findings. Furthermore, these results confirm findings from other studies with the Hutchings' "low stress" algorithm (Hutchings, 1972; Gordon, 1972; Alessi, 1974; Dashiell, 1974; Boyle, 1975; Rudolph, 1976; Gillespie, 1976; Zoref, 1976; and P. Drew, 1980.) These studies found that the Hutchings' algorithm is a more accurate calculation method than the conventional algorithm, especially in reducing calculation errors.

The high achievers had approximately twice the correct rates for all three calculation methods than the low achievers. This confirms the initial grouping of high and low achievers by mathematics class placement, teacher recommendation and standardized testing. It was also apparent that high achieving students were more receptive to the study than the low achievers, possibly because of success that they have had in the past in mathematics. Their pleasant faces were evident at the onset of the study and continued throughout. During the math facts tests, pre-training, and general explanation of procedure, the high achievers were
more attentive, followed the procedures more promptly and needed fewer reminders throughout the study.

The lack of motivation of the low achievers was obvious. They were looking around, a behavior which is incompatible with doing their calculations. They also had a variety of off task behaviors, daydreaming, fidgeting in their seats, making irrelevant marks on their papers, these behaviors were seen more often during the CA portion of the sessions.

It is reasonable to assume that the response effort required of the conventional algorithm was greater than with either of the other two calculation methods. The correct rate trend for the low achievers increased with Hutchings' algorithm. There was no increase noted for the other two methods. One low achieving student who knew the math facts could not do column addition with the conventional algorithm. This student did well with both the Hutchings' algorithm and the hand held calculator. The only way this student would do the conventional algorithm at the end of the study would be to discreetly write some of the partial sums in the margins. In other words he used his own modification of the Hutchings' algorithm.

Some high and low achieving students approached the new method (Hutchings') rather cautiously and somewhat reluctantly. This Experimenter felt that this accounted for a delay acquisition time for the mastery of the Hutchings' algorithm and influenced the mean results. Two main error patterns
were apparent with the Hutchings': (a) individual binary sums were incorrectly added; and (b) the number of tens written to the left of the number were incorrectly counted.

The calculator produced similar error patterns as with Zoref's (1976) study: "(a) failure to clear the calculator before starting a new problem, (b) repeating and/or omitting a row of numbers, and (c) punching in or recording the wrong number(s)." The low achievers worked as well as the high achievers but punched wrong buttons and had to start over which reduced the number of attempted problems. It was evident that the low achievers enjoyed using the calculator over the other two methods, but they had no better results with the calculator than with the Hutchings' algorithm.

This investigator did not confirm Zoref's large differences in percent accuracy between the 2 X 7 and 5 X 7 problem arrays. This may be due to the fact that students did not complete as many problems with the calculator. Also all students were approximately a year younger and did not have the additional year of educational experience.

Generalizations based on this study should be made with caution due to the limited population. Also generalizations should be limited to white middle class children from Iowa in a suburban setting in a public elementary school. It would appear that the results of this study might be generalized, with caution, to other students of similar socio-economic and cultural backgrounds.
APPENDIX A

HUTCHINGS' ADDITION ALGORITHM LESSON

(Adopted from Hutchings, 1972)
Hutchings' Addition Algorithm, Lesson

I am going to show you the usual way of writing number facts and then another way of writing them.

You have all seen number facts written like this:

\[
\begin{array}{c}
7 \\
+8 \\
\hline \\
15
\end{array}
\]

Well, they can also be written like this, using two small (half-space) numbers instead of the line and plus sign.

Do you still see the fifteen? (Point to both fifteens.)

I'll write the two examples next to one another.

Do you all see the fifteen? (Point )

Let's look at another one. I can write "9 plus 5 is 14" like this or like this

Both of these say "9 plus 5 is 14."

Tell me what these say:

\[
\begin{array}{cccccccc}
9 & 9 & 6 & 6 & 4 & 4 & 6 & 5 \\
+8 & +7 & +5 & 4 & +6 & 6 & +2 & 5 \\
\hline \\
17 & 13 & 9 & 12 & 7 & 7
\end{array}
\]

(Call on students, point to the full notation form when asking.)

The little number on the right* is understood to be in the one's place, as are 9 and 8.

The little number on the left* is understood to be in the ten's place.

In other words, this is the same as this (point from "big 7" to "little 7"). And this is the same as this (point from "big one" to "little one").
Now watch me write the following facts both ways.

\[
\begin{array}{cccccc}
9 & 9 & 8 & 8 & 4 & 4 \\
\pm 7 & \pm 5 & \pm 5 & \pm 5 & \pm 5 & 5 \\
\frac{16}{16} & \frac{12}{12} & \frac{13}{13} & \frac{9}{9} & \frac{9}{9} \\
\end{array}
\]

Look at the last pair. Are they different from the others? Note that there is no ten's place number and (do not draw \(\square\) until after saying this) there is no "little one" on the left.

Let's look at another.

a) \(4\) Is there any ten's number here? (Do not draw a box until after asking question.

c) So will there be any little number on the left?

d) \(4\) (Do not draw box until after asking question.)

\(\square\)

Again, \(4\)

\(\pm 3\)

\(\frac{7}{7}\)

\(\frac{3}{3}\)

If there is no ten's place number there is no "little number" on the left.

Now watch me write the rest of these.

Notice

\[
\begin{array}{ccc}
\frac{3}{4} & + 1 & 3 \\
\text{no ten's number here} & \text{so no "little number" here} \\
\end{array}
\]

but

\[
\begin{array}{ccc}
\frac{7}{15} & + 8 & 7 \\
\text{There is a ten's number here} & \text{so there is a "little number" here} \\
\end{array}
\]
Again, notice

\[
\begin{array}{c}
5 \\
\hline
6
\end{array}
\]

There is no ten's number here

So there is no "little number" here

but

\[
\begin{array}{c}
8 \\
\hline
13
\end{array}
\]

There is a ten's number here

So there is a "little number" here

\[
\begin{array}{cccccc}
5 & 5 & 6 & 6 & 1 & 1 \\
\hline
10 & & & & & 2
\end{array}
\]

\[
\begin{array}{cccccc}
3 & 3 & 5 & 5 & 5 & 4 \\
\hline
4 & 9 & 14 & 8 & 8
\end{array}
\]

Now I am going to show you a special way of adding that uses only those "little numbers" on the right.

I'll say that again (repeat previous statement).

This should make your addition very easy and accurate. It is a scientific method and many scientists do addition this way. Watch.

First, do you see that an example can be just number facts piled on atop the other? (Do not point with this question.)

OK! Here we go, starting at the top, writing facts as you learned and using only numbers at the right for addition.
8 a) Say, "The first fact we do may look a bit different because we do not have any little numbers yet." (Point)

b) Say, "This is the only time we will use two big numbers. In the rest of the example we use one little number and one big one."

c) Say, "Now, eight plus five is thirteen."

d) Write the thirteen, i.e., $5_3$ in the example.

8 a) Say, "we've written the thirteen but we'll use only the three."

b) Draw arrow $5_3$

c) Say, "Three plus seven is ten."

d) Write the 10, i.e., $7_0$ in the example.

8 a) Say, "We've written the ten but we'll use only the 0."

b) Draw arrow $9_0$

c) Say, "Zero plus nine is nine."

d) Write the 9, i.e., $9_0$ in the example.

8 a) Say, "We've written the nine and look that's all we have this time because zero and nine is just nine. But that's OK because we only use the right-hand number anyway."

b) Draw arrow $8_7$

c) Say, "Nine plus eight is seventeen."

d) Write the seventeen, i.e., $8_7$ in the example.
a) Say, "We've written the seventeen but we'll use only the seven."

b) Draw arrow $6 \frac{7}{2}$.

c) Say, "Seven plus six is thirteen."

d) Write the thirteen, i.e., 6 in the example.

a) Say, "We've written the thirteen but we'll use only the three."

b) Draw the arrow $8 \frac{3}{2}$.

c) Say, "Three plus eight is eleven."

d) Write the eleven, i.e., 8, in the example.

Now we're at the key part. All we've done is use number facts. We haven't done any "in your head" work.

Nevertheless, we already know the answer! Watch.

The last little number on the right is the right half of the answer.

To get the left half, we just count the little numbers on the left that we didn't use. One, two, three, four, five, there are five of them, so the first half of the answer is five. The answer is 58.
Now watch me do another. Remember we use only the right side "little numbers". We will not bother to write the arrows anymore, just say

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Now the last number on the right is a 2, so the right half of the answer is a 2! We get the left half of the answer by counting the little numbers on the left that we didn't use. One, two, three, four, five. There are five of them so the left half of the answer is 5. The answer is 52.

Now say the work for these with me as I do them at the board. (Children do not copy this.)

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Now copy these examples and do them by yourself. If you have any questions, ask me.

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After most have finished, say, "Check your work with mine as I do them at the board."

After doing the examples, say, "Now let's review."

I'll write the work for another one on the board. I want someone to raise his hand and tell me what the answer is.

6 6 plus 8 is 14
8 4 plus 9 is 13
3 3 plus 5 is 8
5 8 plus 7 is 15
5 5 plus 5 is 10
9 0 plus 9 is 9
3 9 plus 3 is 12

(Point to box.) Who will tell me what the right side of the answer is and how he got it.

(point to box.)

(Locate correct response.) Good! That's correct. The last little number on the right becomes the right side of the answer.

Who will tell me what the left side of the answer is and how he got it? (Locate correct response.) Good! That's correct, we count up the little numbers on the left for the left side of the answer.

Now, what do you suppose we do if there is more than one column? That is, if there is another column at the left of the column you're adding. Like this:

4 6
7 8
6 7
8 6
7 8

Can we still write our left-hand answer number at the
bottom if there is more than one column? No, we can't?

When there's more than one column, each column can have only one number at the bottom (except for the very last column which does have the usual two).

So the single number that we put at the bottom is always the right-hand number.

(Write and point)

What can we do with the left-hand number?

Would it make sense to throw it away? No, it's part of the problem. *So we will put it at the very top of the next column at the left. That way we don't lose it and it's still on the left side.

Watch! (Write on board.)

Count the little number on the left with me.

One, two, three, four.

There are four of them so we write a 4 at the top of the next column.

Now, when I start adding that column I will start with the four(4) first. Let's be sure you understand.

(Repeat twice from the *.)

This is called carrying, some of you already understand it. Good. Carrying is very easy.

But carrying is very important. You must never forget to carry.

Look at these examples and tell me what to write at the top of the left-hand column. (Write on board.)
(Do with volunteers from class at board.) Good, we write the left-hand answer number at the top of the next column. (Repeat three times.)

Remember though that for the last column only, the left-hand answer number is at the bottom as though it were a single column.

Now, copy these examples and do them with me.

Again, do you see that I always carry the number of tens to the top of the next column? (Point and illustrate example.) Except when there are no more columns. Then I write the number of tens on the bottom line as part of the answer. (Point and illustrate with each.)

Good! Are there any questions?

Now take these dittoed examples and do them by yourselves. If you have trouble, ask me for help.
Be sure to make and place your numbers neatly!

(Allow time needed for most to finish.)

Now, I will do them. Check your work against mine.

(Do examples on board. Answer questions. Emphasize the need to write neatly and the need to count the "carry number" correctly, demonstrate the latter while doing the work. State that the carry number is always written in at the top of the column to which it is carried.)
APPENDIX B

Review of Hutching's Algorithm
Lesson (Adapted From Hutching 1972)
We are going to review the new way of writing number facts which we practiced yesterday.

We are going to start at the top, writing number facts as you learned yesterday.

a) Say, "Remember that during the beginning of the example is the only time that we use two big numbers. In the rest of the example, we use one little number and one big number."

b) Say, "Five plus nine is fourteen."

c) Write the fourteen in the example as \(9\).

a) Say, "We've written the fourteen but we'll use only the four."

b) Say, "Four plus eight is twelve."

c) Write the twelve in the example as \(8\).

a) Say, "We've written the twelve but we'll use only the two."

b) Say, "Two plus six is eight."

c) Write the eight in the example as \(6\).
a) Say, "We've written the eight and we use just the eight."
b) Say, "Eight plus eight is sixteen."
c) Write the sixteen in the example as \(\frac{8}{8} \).

a) Say, "We've written the sixteen but we'll use only the six."
b) Say, "Six plus seven is thirteen."
c) Write the thirteen in the example as \(\frac{7}{3} \).

a) Say, "We've written the thirteen but we'll use only the three."
b) Say, "Three plus eight is eleven."
c) Write the eleven in the example as \(\frac{8}{8} \).

a) Say, "We've written the eleven but we'll use only the one."
b) Say, "One plus seven is eight."
c) Say, "The last little number on the right is the right half of the answer. To find the left half, we just count the little numbers on the left that we did not use. Who can tell me what the right half of the answer is? Eight! Right. Now, who can tell me what the left half of the answer is? Five! Right, the answer, then is 58."

a) Say, "Now let's try a bigger example. We are going to move faster this time because you have done so well."
b) Say, "Let's start with the right column (point to it). Seven plus five is twelve. (Write the twelve in the example as 5.) Two plus six is eight. (Write the eight in the example as 6.) Eight plus five is thirteen. (Write the thirteen in the example as 3.) Three plus nine is twelve. (Write the twelve in the example as 5.) We write the two below the right column and carry the three to the top of the next column." (Write the three above the second column.)

Say, "Now, when I start adding this column (point to second column), I will start with the three. Three plus seven is ten. (Write the ten in the example as 7.) Zero plus eight is eight. (Write the eight in the example as 8.) Eight plus seven is fifteen. (Write the eight in the example as 8.) Five plus nine is fourteen. (Write the fourteen in the example as 9.) Four plus three is seven. (Write the seven in the example as 7.) We write the seven below the column. Then we count the tens: One, two, three tens. We carry the three to the top of the next column." (Write the three above the last column.)

Say, "Now our example looks like this (pointing to example). Who can tell me the numbers we are going to add next? Right. We are going to add the three and the eight."

Say, "Three plus eight is eleven. (Write the eleven in the example as 1.) Who can tell me the numbers we are going to add next? Right. We are going to add the one and the six. One plus six is seven. (Write the seven in the example as 7.) Who can tell me the numbers we are going to add next? Right. We are going to add the seven and the four. Seven plus four is eleven. (Write the eleven in the example as 1.) Who can tell me the numbers we are going to add next? Right. We are going to add one and six. One plus six is seven. (Write the seven in the example as 7.) Who can tell me the numbers we are going to add next? Right. We are going to add the three and the eight."

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example as 67.) Seven plus eight is fifteen. (Write the fifteen in the example as 85.) Now we write the five below the column. (Write the five below the third column.) Then we count the tens: One, two, three tens. Because there are no more columns, we write the three to the left of the five." (Write the three to the left of the five in the example.)

Now copy these examples with me.

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Are there any questions? Good. Now take these dittoed examples and do them yourselves. If you have any trouble, ask me for help. Be sure to make and place your numbers neatly.

(Allow time for most to finish.)

Now I will do them. Check your work against mine.

(Do examples on the blackboard. Answer questions. Emphasize the need to write neatly and the need to count the "carry number" correctly, demonstrate the latter while doing the work. State that the "carry number" is always written at the top of the column to which it is carried.)
APPENDIX C

Conventional Algorithm, Lesson
(Adapted from Hutchings, 1972)
I am going to write some addition examples on the board. Begin to do them as soon as you can see them. After I finish writing all of them, I will go back and write in the answers. After you have finished working all of the examples, go back and check your answers against the answers I have written on the board. As soon as you have finished, turn your papers in.

Does everyone know what to do? (Pause momentarily.) Good. Begin...

\[
\begin{array}{cccccccccccc}
7 & 9 & 6 & 4 & 6 & 3 & 8 & 4 & 3 & 5 \\
+8 & +8 & +7 & +5 & +6 & +2 & +7 & +5 & +3 & +1 & +1 \\
\end{array}
\]

\[
\begin{array}{cccccccccccc}
5 & 6 & 1 & 3 & 5 & 4 & 3 & 2 & 8 & 5 \\
+5 & +8 & +7 & +1 & +4 & +9 & +8 & +3 & +2 & +8 & +7 \\
\end{array}
\]

\[
\begin{array}{cccccccccccc}
6 & 6 & 1 & 8 & 9 & 8 & 3 & 3 & 9 & 9 \\
+9 & +8 & +7 & +8 & +2 & +1 & +4 & +6 & +5 & +3 & +8 \\
\end{array}
\]

\[
\begin{array}{cccccccccccc}
8 & 8 & 9 & 4 & 6 & 4 & 6 & 8 & 6 & 5 \\
3 & 5 & 5 & 8 & 8 & 7 & 8 & 7 & 8 & 9 \\
7 & 6 & 3 & 3 & 9 & 6 & 7 & 6 & 7 & 9 \\
9 & 7 & 2 & 6 & 5 & 8 & 6 & 9 & 5 & 6 \\
8 & 9 & 6 & 1 & 7 & +7 & 8 & 9 & 4 & 6 \\
6 & 8 & 8 & 8 & 5 & 6 & 8 & 3 & 7 & 3 \\
8 & 3 & 7 & 7 & 9 & +7 & +9 & +6 & +3 & +8 & 2 \\
\end{array}
\]

\[
\begin{array}{cccccccccccc}
6 & 8 & 8 & 5 & 7 & 6 & 8 & 5 & 7 & 6 \\
5 & 9 & 7 & 6 & 7 & 7 & 1 & 8 & 5 & 9 \\
6 & 2 & 9 & 9 & 2 & 1 & 2 & 8 & 7 & 7 \\
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\end{array}
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APPENDIX D.

Practice Sheet and Sample Worksheets
APPENDIX E

Addition Math Facts Test

(From Alessi 1974)
Inventory of Basic Addition Facts

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SCORE:_________
APPENDIX F

Letter to Parents
Dear parent,

Your son/daughter has been selected to have the opportunity to participate in a research study I am doing for my Educational Specialist Degree. The study is to help decide which way of doing column addition is most accurate and preferred by students. Your child will not be identified in any way in the results and all material is destroyed after the study. Your child may stop participation at any time and the data will be destroyed at any point of his/her choosing.

This study will involve your child working in a small group doing extra practice on column addition. Three ways of deriving the answers will be used to see which is the most efficient: the hand held calculator, the conventional method and the Hutchings' "low stress" method. The conventional method has the student write down one answer for each column added and the Hutchings' method has the student note each two number sums, using small numbers. For example:

<table>
<thead>
<tr>
<th>Conventional</th>
<th>Hutchings'</th>
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</thead>
<tbody>
<tr>
<td>5</td>
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<tr>
<td>4</td>
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<td>24</td>
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</tbody>
</table>

Educationally yours,

Edward S Drew
School Psychologist

I give my permission for ______________________ to participate in this study, and I understand that he may withdraw at any time.

Signed ______________________________

Please send this form back with your child. Thank you for your cooperation.
### TABLE 5

Mean totals and standard deviations of the daily average session scores across the study for both low and high achievers using each of three calculation methods with two different problem arrays.

<table>
<thead>
<tr>
<th></th>
<th>2x7 problem arrays&lt;sup&gt;a&lt;/sup&gt; (13 binary operations)</th>
<th>5x7 problem arrays&lt;sup&gt;b&lt;/sup&gt; (34 binary operations)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correct Rate</td>
<td>Error Rate</td>
</tr>
<tr>
<td>CA</td>
<td>5.3  ± 2.3</td>
<td>2.6 ± 1.3</td>
</tr>
<tr>
<td>HA</td>
<td>8.9  ± 3.2</td>
<td>1.7 ± 1.4</td>
</tr>
<tr>
<td>EC</td>
<td>7.8  ± 2.9</td>
<td>1.5 ± 1.6</td>
</tr>
</tbody>
</table>

Note: As all four students in each group were not present for all daily sessions nor each trial within daily sessions, some daily sessions reflect the average of less than three individual scores.

<sup>a</sup> Each score represents averages from 16 sessions

<sup>b</sup> Each score represents averages from 17 sessions

1. Number of columns correct / 5 minutes
2. Number of columns incorrect / 5 minutes
3. Number of columns correct / number attempted

CA = Conventional algorithm
HA = Hutchings algorithm
EC = Electronic calculator


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Drew, P. Preferences of students for the Hutchings' "low stress" compared to the conventional algorithm under conditions of differentially increasing response effort with and without reinforcement. Unpublished Specialist in Education Project, Western Michigan University, 1980.


Fulkerson, E. Adding by tens. The Arithmetic Teacher, 1963. 10, 139-140.

Gillespie, C. L. Student preferences for the Hutchings' "Low Stress" versus the conventional addition algorithm under conditions of differentially increasing response effort with and without reinforcement. Unpublished Specialist in Education Project, Western Michigan University, 1976.


Lankford, F.G. What can a teacher learn about a pupil's thinking through oral interviews? The Arithmetic Teacher, 1974, 21, 26-32.


Meyer, P. When you use a calculator you have to think! The Arithmetic Teacher, 1980, 27, 18-21.


Sanders, W. J. Let's go one step farther in addition. The Arithmetic Teacher, 1971, 18, 413-415.


Weaver J. F. Big dividends from little interviews. The Arithmetic Teacher, 1955, 2, 46-47.


Zoref, I. A comparison of calculation speed and accuracy on two levels of problem difficulty using the conventional and Hutchings' "low-stress" addition algorithm and the pocket calculator with high and low achieving math students. Unpublished Specialist in Education Project, Western Michigan University, 1976.