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A Diagnostic Teaching Analysis of the Feasibility of Teaching Trainable Mentally Impaired Students Hutchings' Low-Stress Addition Algorithm

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A DIAGNOSTIC TEACHING ANALYSIS OF THE FEASIBILITY OF TEACHING TRAINABLE MENTALLY IMPAIRED STUDENTS HUTCHINGS' LOW-STRESS ADDITION ALGORITHM

by

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This study attempted to determine the feasibility of teaching trainable mentally impaired subjects Hutchings' low-stress addition algorithm. It was a non-experimental study of acquisition using diagnostic teaching techniques. Three trainable mentally impaired subjects participated in training. No subject knew basic addition facts, so use of a number line was substituted. Training was divided into four steps: a) use of the DOOF for a single binary; b) notation; c) adding two binaries in one column; and d) adding binaries in two columns. Tests were scored immediately, error patterns diagnosed and program modifications implemented. All subjects acquired skill for the first three training steps with between 80 and 100 percent accuracy. Two subjects were trained to add two columns. Time constraints prevented training completion, but correct calculation for all subjects showed an accelerating trend at the end of the study. Correct rates were very slow, averaging 2.1 binaries correct per minute.

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I would also like to thank my son, Linus, for his consideration and patience.

Marsha K. Markle

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CHAPTER I

INTRODUCTION

This project was designed to explore the feasibility of teaching Hutchings' low-stress addition algorithm to trainable mentally impaired students. Feasibility was defined as acquisition of program skills with 80% or better accuracy. It was an acquisition rather than a remedial or comparative study. The study was not experimental. It used the diagnostic teaching techniques of training, evaluation of progress and error patterns, and program modifications to correct errors.

Mathematics is an important part of the educational curriculum. Many critics have noted a decline in the competence of American students to master rudimentary calculation skills. The Conference Board Mathematical Science National Advisory Committee on Mathematics Education, in their _Overview and Analysis of School Mathematics Grades K-12_ (1975, pp. 106-107), point out that the mean score on the Scholastic Aptitude Test's quantitative section had declined each year from 1962 to 1975.

Concern with this problem has lead educators to investigate alternative approaches to mathematics instruction. One alternative is Hutchings' low-stress algorithm (Hutchings, 1972; 1976). Suggested advantages of Hutchings' method which apply to addition operations include: a) skills are taught in an overt (visible) format which allows for identification and correction of error patterns before they become habit-formed; b) components of the operation are discrete and may be taught separately and linked together later which allows for teaching "one step at a time"; c) there is a full record of every calculation which allows for planning efficient remedial practice to correct errors; and d) math facts required for accuracy are limited to basic facts (i.e., sums up to eighteen) which reduces by 90 percent the number of facts needed for column addition over that required with the standard algorithm (i.e., both basic and complex facts).

Considering the wealth of suggested advantages and the need for an effective, efficient method of computational training, there is a need to investigate the training effects of low-stress algorithms as initial instruction. Special student populations (e.g., trainable mentally impaired) have an even greater need for the very advantages alleged by the low-stress technique.

In recent years there have been several research studies comparing the relative effectiveness of Hutchings' low-stress algorithm to the standard method under varying conditions. The available literature supports the comparative superiority of the low-stress algorithm.

The earliest studies used group factorial designs (Alessi, 1974; Boyle, 1975; Dashiell, 1974; Hutchings, 1972). Alessi (1974) found that the low-stress algorithm produced higher accuracy scores for the number of columns correctly added and attempted in a 30-minute period. He compared the low-stress algorithm to the standard algorithm in situations of reinforcement versus no reinforcement and under varying degrees of problem difficulty. The relative superiority of the low-stress over the standard algorithm decreased as the problems increased in difficulty. Boyle (1975), in a systematic replication, found the low-stress procedure superior to the standard procedure under simulated test and non-test conditions.

The comparative superiority of the low-stress algorithm has also been supported by studies using within-subject designs. A description of those findings follows.

Rudolph (1976) investigated three research questions involving Hutchings' low-stress addition algorithm. First, is the algorithm effective in teaching "distractable" students computational skills? Second, what are the effects of a distracting versus a non-distracting environment on the subjects' performance? Third, how do performances of general education and emotionally impaired students compare when using the low-stress versus the standard algorithm within these two environments?

The subjects were four seventh grade male students, two from general education and two from an emotionally impaired classroom. All students were identified by their teachers as low performers with respect to math. All subjects passed a prerequisite test of basic addition facts with scores above a 95 percent criterion.

Results suggested (a) greater accuracy for student performance with Hutchings' low-stress algorithm compared with the standard algorithm, (b) no consistent trend for differences between distracting and non-distracting environments, and (c) no systematic difference between handicapped and non-handicapped student performance.

Zoref (1976) researched the differential calculation power of three procedures for addition: standard, Hutchings; low-stress, and the pocket calculator. In addition to the three procedures, other independent variables included two problem array sizes: 2 x 7 and 5 x 7; and two types of subjects with respect to math skills: high and low performers. Six fourth grade pupils served as subjects. They received a separate 20-minute training session or each addition procedure. Results indicated the performance for all subjects using the Hutchings' low-stress algorithm was the most stable and had the lowest error rates. The calculator yielded the most variable results and accuracy decreased with the larger problems. Zoref's study was directly replicated by Edward Drew (1981), yielding very similar results.

Gillespie (1976) studied student preferences for the Hutchings' low-stress versus the standard addition algorithm in two experiments: one without and one with response reinforcement. There were three conditions of response effort: equal number of problems for choosing both algorithms, a differential increase of 50 percent of the number of problems assigned for choosing the preferred algorithm, and a differential increase of 100 percent of the number of problems assigned for choosing the preferred algorithm.

There were twelve third grade subjects in the study. Six subjects were selected for each of two experiments. Each experiment contained high and low accuracy students with respect to math skills. Studetns were given three 20-minute instruction lessons for the low-stress training and one 20-minute lesson for the standard training.

Preference data were collected under separate response effort conditions. Results showed a preference for the low-stress algorithm in 20 out of 24 opportunities providing equal response effort conditions and 11 out of 14 opportunities where 50 percent more problems were assigned for the preferred method. Students tended to prefer the algorithm in which they were more competent. Gillespie's study was directly replicated by Pamela Drew (1980) with very similar results.

VanHevel (1981) compared the low-stress algorithm to a modified Fulkerson "full record" (Fulkerson, 1963) and standard algorithms. The Fulkerson procedure notates a ten's place digit of a binary operation with a slash through the column numeral. The slash stores the information for retrieval in carrying.

VanHevel's subjects were nine fourth grade regular education students. Results indicated that both Hutchings' low-stress and Fulkerson's "full record" algorithms were superior in producing accurate, efficient calculations. After training the subjects in each algorithm, They were given a choice of procedure to use when adding. Four students chose Fulkerson's "full record", three chose Hutchings' low-stress, and two chose the standard algorithm.

McCallum (1981) investigated the components of the low-stress algorithm to help determine which components might be critical to the superior performance found in previous studies. His study analyzed two components of the low-stress algorithm: binary addition and the full computational record. Fifteen fourth grade students who had mastered basic math facts served as subjects. All subjects were taught two new computation algorithms: (a) Hutchings' low-stress and (b) the standard algorithm with a written record. He also reviewed the standard algorithm with the subjects. The subjects used all three algorithms alternately. They were given worksheets with 5 x 7 size array problems and asked to solve as many as possible withing a five-minute time limit. Results showed superior performance in accuracy and speed with the full record algorithm versus no record. Hutchings' low-stress algorithm showed superior performance results for binary addition versus complex addition. All subjects had higher performances when using the low-stress algorithm compared with the standard algorithm.

Bracey et al. (1975) used the DISTAR Arithmetic I Program designed by Engelmann and Carnine (1969) to train four types of arithmetic skills. Their subjects were six institutionalized mentally retarded children. The subjects ranged in age from eleven to fourteen years and had I.Q. scores (Stanford-Binet) which ranged from 35 to 50. They were trained in object counting, making lines from numerals, the meaning of plus, and increment addition. "Plus one" was the only increment addition taught.

The subjects were trained in a group set in the classroom using one quarter hour lesson per day and individual worksheets. They received a total of thirty-two hours of group instruction. A token reinforcement program was used. Significant gains were recorded for each of the four skills trained. Gains were evaluated by a t-test on the difference between correlated means for pre- and post-test scores.

There have been no previous studies which attempted to train higher level increment addition for retarded subjects and no studies which train Hutchings' low-stress algorithm as an acquisition program. This study attempted to do both.

CHAPTER II

METHOD

Nature of the Study

This study investigates two main questions. First, is Hutchings' low-stress algorithm a feasible method for training acquisition of addition calculation skills for a trainable mentally impaired population? Second, what modifications of Hutchings' suggested training program will be required? The second question suggests that, because this is an acquisition rather than a remedial program, error patterns may result for which the original program has no corrective strategy.

This project was not experimental. It was a diagnostic teaching program covering training, analysis of progress and error patterns, and attempts to correct errors.

This study was formulated with the following constraints: a) as little disruption as possible of the ongoing school program be imposed; b) the study be terminated by the end of the subjects' school semester; and c) the study be run almost entirely by the trainer. In order to prevent possible difficulties, a letter was sent to each of the parents or guardians of the subjects, informing them of the study and asking permission for their child to participate. The study was approved by the Western Michigan University Human Subjects Committee.

Subjects

Three subjects participated in this project. All were selected on the basis of good preskills in math compared with others at the facility. Teachers were asked to select those students they believed to be most prepared to acquire new math skills. The age of each participant was: T.A., 21; T.J., 20; and S.B., 21½. Each student had been certified as trainable mentally impaired between the ages of six to eight.

Each student had been given a Wechsler Adult Intelligence Scale with a year of the study, scoring between 46 and 62 on the full scale. Their scores on the intelligence scale and grade equivalents for academic testing in math are listed below.

Subject	I.Q. Range	Math Grade Equivalent
T.A.	50-60	Kindergarten, fifth month (Brigance Diagnostic Test)
T.J.	51-62	Second Grade (Wide Range Achievement Test)
S.B.	46-58	First Grade (Wide Range Achievement Test)

Two students were male and one female. Due to delayed parent permission, the female student began participation in the study one month later than the two males.

Setting

The three subjects attended school at a center-based training facility serving only trainable mentally impaired students. They served students from ages 3 to 25 in full-time, self-contained classrooms. The center is located in a small town in southwest Michigan. The center mainly serves children from rural homes.

Training sessions were run during the subjects' regularly scheduled number skills tasks in the classroom, which was not daily. The subjects had swimming one day a week during that time and a variety of other specials another day each week. The classroom number skills program, during the time of the study, targetted telling time to the nearest half hour and reading the date from the calendar. Prior classroom training in basic math facts taught the strategy of drawing lines equal to the numerals and counting all the lines for the answer. T.J. was the only student who used this strategy on the placement test, but he did not use it accurately. The inconsistency with which he was accurate may contribute to the score he received on the Wide Range Achievement Test in math (Second Grade). The greater the numerals, the less accurate his addition skills.

Training took place in an isolated area of the classroom from October 15, 1980 until November 12, 1980. Then training was moved to a separate study room in the building.

Independent Variables

There are two independent variables in this study: the Hutchings' low-stress algorithm program (Hutchings, 1976) and a trainable mentally impaired population.

Dependent Variables

The three dependent variables for this study are: a) percent correct (the number of binary operations correctly completed divided by the number of binary operations in the test); b) correct rate (the number of binary operations correctly added divided by the length of time to complete the test); and c) incorrect rate (the number of binary operations incorrectly added divided by the length of time to complete the test).

Construction of Instruments

Worksheets were handwritten on unruled paper using a random order of numerals. Hutchings recommends that the identity element (zero) be avoided. However, zeros were included as addends because many binaries sum to ten, making zero appear in the subsequent binary operation at frequent intervals. Problems were set on the page with large size numerals and wide spacing. With the low-stress format, students need more space to write the notations.

The number of addition problems was held constant within each training step. There were ten problems per page for the first two training steps and five problems per page for the third and fourth steps. Time to complete each page was left variable, and elapsed times for each subject were recorded by the trainer.

Each student was provided with a copy of a DOOF (Discrete Overt Operations Format). This is a type of number line arranged in vertical format. The DOOF was constructed inside a manilla folder with numerals zero through 18. A half-inch box was drawn to the right of each numeral (see Appendix A).

Training Procedure

Training was divided into four major steps: a) use of the DOOF for a single binary (1 x 2 array); b) notation training (1 x 2 array); c) adding two binaries in one column (1 x 3 array); and d) adding binaries in two columns (2 x 2 array).

The following is a detailed description of tasks in each step of training:

Step 1: DOOF Usage:

a) Student places finger of the pencil-holding hand covering the first named addend and the box to the right of that numeral.

b) With a finger of the other hand, the student successively counts as many more boxes as named by the second addend in the column.

c) Student lifts the pencil-holding hand and writes the sume on the protocol. The sum is opposite the box to which the student is still pointing.

Step 2: Notation Usage:

a) For each binary operation, the student writes the answer directly below the final (second) addend in the column.

b) Answers are notated with the one's digit to the right of the column numeral and the ten's digit to the left.

$$\begin{array}{c} 6 \\ {}_{1}8_{4} \end{array} \quad \text{or} \quad \begin{array}{c} 6 \\ 3_{9} \end{array}$$

c) These digits are written smaller than those in the column (problem numerals).

<u>Step 3</u>: <u>Adding Two Binaries in One Column</u>:

a) Student performs the first binary operation as described in DOOF and notation usage.

b) Student uses the one's place digit of notated answer from first binary as the starting place for the next operation.

c) Student performs second binary operation with the one's place digit of notated answer and the next numeral in the column.

d) Student counts up any "1's" notated on the left side of the column and places that in the answer.

<u>Step 4</u>: <u>Adding Two Columns</u>:

a) Student completes the binary solution for the first column as described in DOOF and notation usage.

b) Student writes the one's place digit of the notated answer below the vinculum (bar).

c) If there is a ten's place notation for the first column, the student carries that to the top of the next column.

d) Student completes and writes the answer to the binary operation or operations in the second column.

e) The student counts any "1's" notated to the left of the second column and writes that number in the answer space. Two examples follow:

$$\begin{array}{rcc} & 3 & 2 \\ + & 6_9 & 5_7 \\ \hline & 9 & 7 \end{array} \qquad \begin{array}{rcc} & \overset{1}{4_5} & 9 \\ + & {}_1 7_2 & {}_1 3_2 \\ \hline 1 & 2 & 2 \end{array}$$

The lesson format for each training step was divided into three phases: a) a pre-test with a fixed number of problems; b) practice worksheets and instruction; and c) a post-test with the same fixed number of problems. The data collected from a pre-test and post-test for each lesson provided inspection of useful information. The difference between one lesson's pre-test and post-test helps evaluate the effect of each day's training and corrective adjustments. The change between one lessons's post-test and the next lesson's pre-test helps evaluate the amount of retention or forgetting between lessons. If the subjects did not learn the addition algorithm, there would be a need to inspect the tests for the acceptance or rejection of the presence of forgetting factors.

Tests were scored immediately so that praise could be given for accuracy or errors could be remediated quickly. The length of the practice session varied with the need of the student, based on test scores, but did not exceed 20 minutes.

Error Patterns and Program Modifications

One of the advantages of low-stress over the standard addition algorithm is that it supplies a full record of all binary operations used to calculate the sum. Inspection of the full record provides useful information for diagnosis of error patterns made (Ashlock, 1976) in the procedures used to obtain the sums. Alessi (1974, p. 72) and Boyle (1975, p. 54) identified some error patterns specifically related to procedures prescribed by Hutchings' addition algorithm. The patterns they identified were: a) misnotating the answer to the first binary, placing the digits between the first two numerals rather than below the second numeral; b) miscounting the "1's" in the ten's place of the first column, producing an error in the second column; and c) not writing the answer to the final binary in the column, thus missing the unrecorded last "tens" when adding the notation up to carry.

Test protocols were inspected to identify the error patterns found in this study. Tests were inspected daily and corrective adjustments were made to try to remediate the errors.

CHAPTER III

RESULTS

The results of this study will be discussed in the following sequence: a) placement test results; b) results of instruction in the program; and c) error patterns and program modifications.

Placement Test Results

A placement test was administered to each participant to determine math strengths and weaknesses. The results were also used to suggest program modifications. A list of the math skills tested and results of each student's answers follows. In order to pass a particular skill, the student had to score 90 percent or better accuracy for the sample items.

All students passed a separate test on the concept of addition. They correctly identified positive from negative examples of joining objects and performed object-joining operations.

Interobserver agreement data. Interobserver agreement data were calculated for the pre-test and post-test data and are presented in Table 2. The percent agreement was below 100 percent in only two instances and test protocols were rescored to obtain 100 percent agreement.

Results of Instruction in the Program

The results of this project are arranged in terms of three types

Table 1: Placement Test Skills and Results.

TABLE 1

Placement Test Skills and Results

Skill	Subject Response		
	T.A.	T.J.	S.B.
Match numerals to numerals	Pass	Pass	Pass
Match sets of objects to numerals and numerals to sets of objects	Fail	Fail	Fail
Arrange objects in numerical order from smallest to largest	Fail	Fail	Fail
Read numerals (random order)	Pass	Pass	Pass
Write numerals from sets of objects	Pass	Pass	Fail
Count by rote to 18	Pass	Pass	Pass
Count to 9 with one-to-one correspondence from sets of objects	Pass	Pass	Pass
Answer "How many?" from sets of objects	Pass	Fail	Pass
Count from a number to a number	Fail	Fail	Pass
Count from a number a given number of times	Fail	Fail	Fail
Identify addition symbol	Pass	Pass	Pass
Join numerical sets	Fail	Fail	Fail

Table 2: Interobserver Agreement Data.

TABLE 2

Interobserver Agreement Data

Date	Subject	Percent Agreement	
		Pre-Test	Post-Test
10-20-80	T.A.	100%	100%
	T.J.	100%	100%
10-30-80	T.A.	100%	100%
	T.J.	100%	100%
11-10-80	T.A.	100%	100%
	T.J.	100%	100%
11-20-80	T.A.	100%	100%
	T.J.	100%	100%
	S.B.	100%	100%
12-1-80	T.A.	100%	100%
	T.J.	100%	100%
12-4-80	S.B.	100%	100%
12-11-80	T.A.	93%	100%
	T.J.	100%	100%
	S.B.	90%	100%

of data: a) percent of binary calculations correct; b) percent of regrouping operations correct; and c) the rate of binaries calculated correctly per minute and the rate calculated incorrectly per minute.

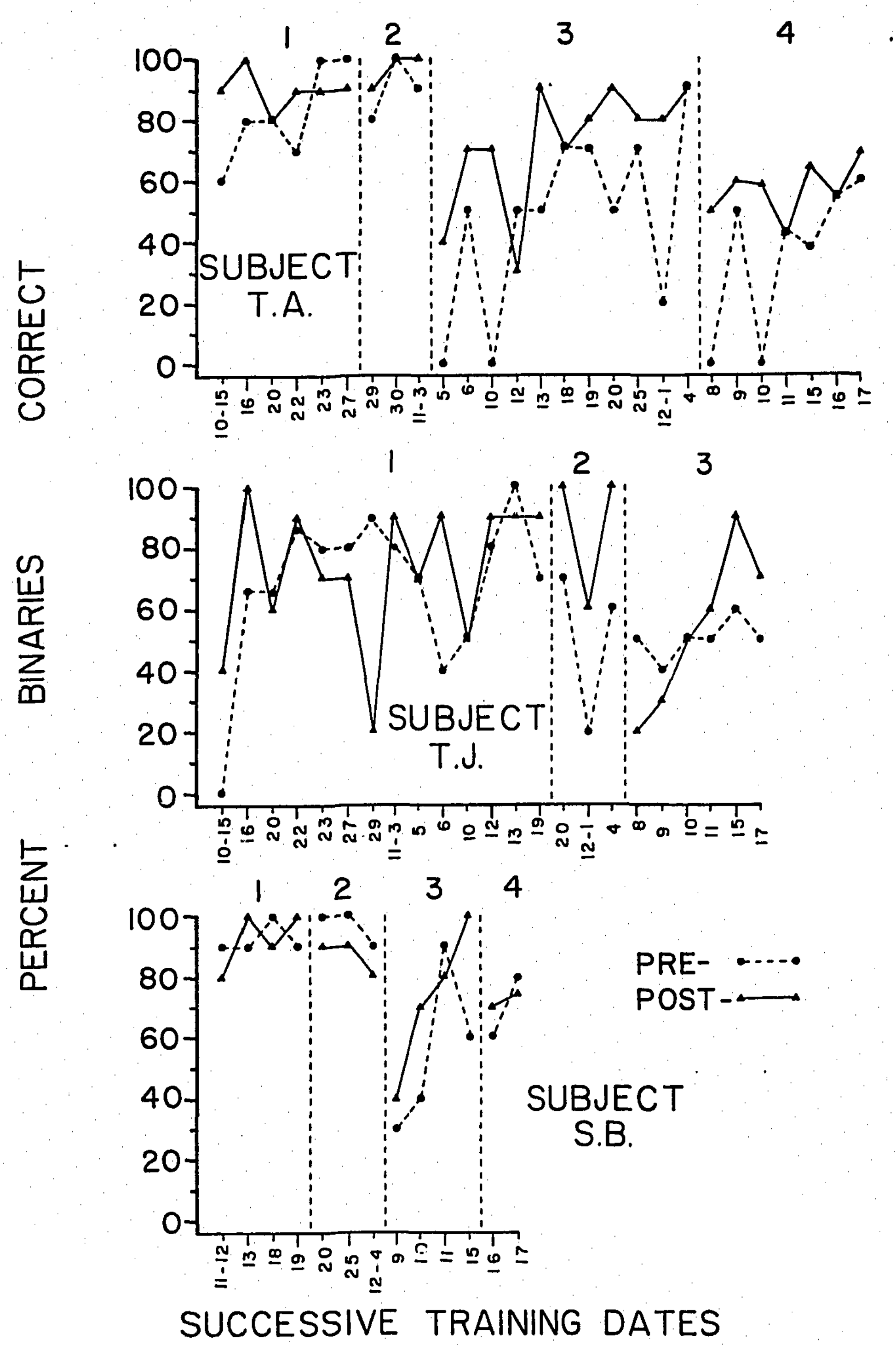
Binary calculation accuracy data. Figure 1 presents data for the percent of binaries correct for the pre-test and post-test of each student by training step. Training steps are numbered 1, 2, 3, and 4. Both T.A. and S.B. scored relatively high in accuracy at the start of Step 1 training (DOOF Usage). T.A. met consistency criterion of three consecutive post-test scores of 90 percent or better accuracy in six training sessions, and S.B. met criterion in four sessions for the first step of training.

T.J. began Step 1 with relatively low accuracy scores, proceeded through training with considerable variability, and met criterion for consistent high accuracy after fourteen training sessions. T.J. had difficulty mastering one-to-one correspondence counting. His counting errors included: a) lifting the counting finger very high off the DOOF instrument, consequently, either skipping one or more boxes or counting a box more than once; b) counting to a number other than that named in the second numeral; and c) counting up to a number just prior to the named numeral, stopping, and starting the count over from the wrong starting place.

Comparison of daily pre-test and post-test data reveals that binary operation accuracy improved in 50 percent of the sessions for each subject during Step 1. In the other 50 percent of the sessions, accuracy either remained the same or decreased from pre-test to

Figure 1: Percent Binaries Correct for the Pre-Test and Post-Test of Each Student by Training Step.

FIGURE 1


PERCENT CORRECT BINARIES
SUBJECT T.A.
SUBJECT T.J.
SUBJECT S.B.
PRE-
POST-
SUCCESSIVE TRAINING DATES

post-test. T.J.'s scores showed the most variability.

For Step 2, Notation Usage, the results show three data points for each subject but for different reasons. T.A. met consistency criterion for high accuracy in three training sessions. T. J. and S.B. were given only three training lessons before proceeding to Step 3 even though they did not meet criterion. The decision to waive the consistency criterion for these two subjects was based on conforming to the constraints of terminating the study by the end of the school semester while making every effort to complete the four steps of training. The second step of training was the least difficult compared to the previous step requirements.

The mean binary accuracy scores for Step 2 training were: T.A., 97%; T.J., 87%; and S.B., 93%. S.B.'s binary accuracy scores remained relatively high for Step 2 training, although her pre-test scores were always 10 percent higher than her post-test scores. T.J.'s binary accuracy scores for Step 2 were always higher for post-tests than pre-tests. T.A.'s post-test scores were always higher than or equal to pre-test scores.

Step 3 results, adding two binaries in one column, yielded considerable variability among accuracy scores for all subjects. This step required several new tasks to be performed. These tasks are enumerated in the Error Pattern section of the Results.

All subjects' binary accuracy scores decreased considerably at the beginning of Step 3 training, but improved across Step 3 training. T.A. met criterion for consistently high accuracy in 11 training sessions. Only one pre-test score was higher than that session's post-test. T.J., whose pre-test scores appear somewhat stable with a slight increase in trend across Step 3 training, steadily improved accuracy in post-tests across Step 3.

S.B.'s binary accuracy scores improved across Step 3 training, with only one post-test score below the corresponding pre-test. She did not meet criterion for consistently high accuracy to proceed to the next step in training. Her last three post-test scores averaged 83 percent. She began Step 4 training, however, becayse her scores were fairly high and because of time constraints coupled with an effort to expose her to all training steps.

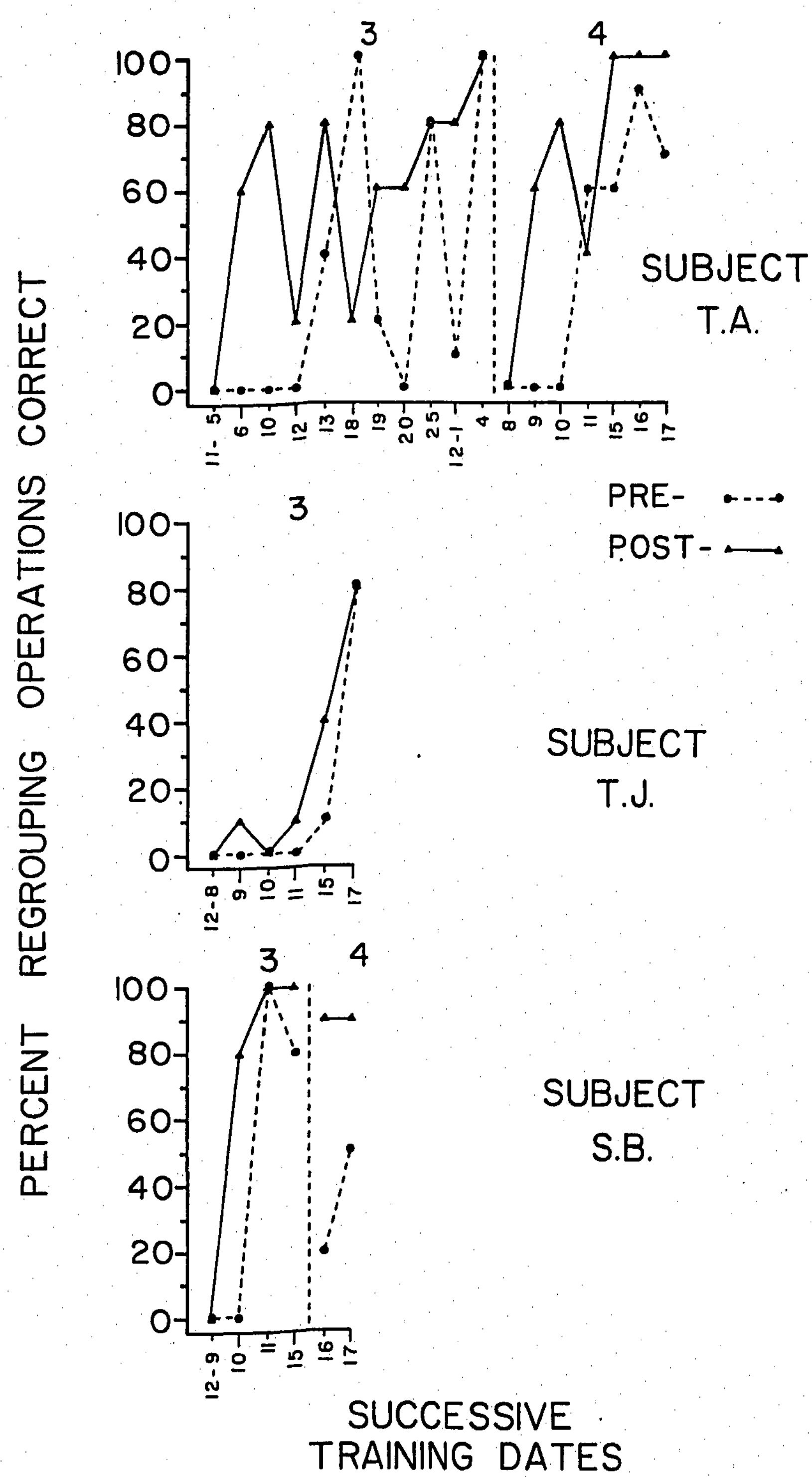
T.J. did not receive training in Step 4 because his accuracy scores did not improve until the study was terminated. T.A. received seven training lessons for Step 4 and S.B. received two.

T.A. and S.B.'s binary operation accuracy scores both decreased compared to Step 3 and did not recover to Step 3 levels of accuracy by the end of the study. T.A.'s post-test scores were always higher than or equal to pre-test schores. However, his pre-test scores were more variable than post-test scores. S.B., who received only two training sessions for Step 4, improved accuracy on the second day of training.

Regrouping data results. Figure 2 displays the results for regrouping operation accuracy. Steps 3 and 4 required regrouping operations as well as binary operations to be performed. None of the subjects had had prior classroom instruction for any regrouping algorithm. All subjects improved accuracy scores for regrouping across Step 3. Both subjects who received training in the fourth step (T.A.

Figure 2: Percent Regrouping Operations Correct for the Pre-Test and Post-Test of Each Student by Training Step.

FIGURE 2


PERCENT REGROUPING OPERATIONS CORRECT
3
4
100
80
60
40
20
0
11-5
6
10
12
13
18
19
20
25
12-1
4
8
9
10
11
15
16
17
SUBJECT
T.A.
PRE-
POST-
3
12-8
9
10
11
15
17
SUBJECT
T.J.
3
4
12-9
10
11
15
16
17
SUBJECT
S.B.
SUCCESSIVE
TRAINING DATES

and S.B.) improved regrouping skills across that training step. T.J. and S.B.'s post-test regrouping accuracy scores were consistently higher than or equal to pre-test scores, suggesting good within-session acquisition. T.A.'s post-test regrouping accuracy scores were higher than or equal to pre-test scores in all but one instance in Step 3 and one in Step 4. His accuracy scores show more variability for both pre-test and post-test scores than either T.J. or S.B.

Rate of calculation. Figure 3 displays the data on the rate of binary operations correct per minute and the rate incorrect per minute for each subject. In general, correct rates decreased for each subject across training steps, with rates decreasing at the point of introduction to a new training step. Incorrect rates, while generally lower than correct rates, tended to slightly increase across training steps. Both correct and incorrect rates for all subjects are low, indicating very slow work.

Mean post-test binary rates per minute. The following table lists the mean rate per minute of binary operations for the post-tests according to training step and by subject. The rates are listed separately for correct responses and incorrect responses. Also listed is the ratio of the mean correct rate to the mean incorrect rate.

In general, all subjects tended to have a declining rate of correct responding across training steps. Subject S.B. had an increased rate of correct responding in Step 4 over the Step 3 rate. The largest correct rate decline between training steps occurred in Step 3 for all subjects.

In general, all subjects tended to have an increasing rate of

Figure 3. The Rate of Binary Operations Correct Per Minute and the Rate of Binary Operations Incorrect Per Minute.

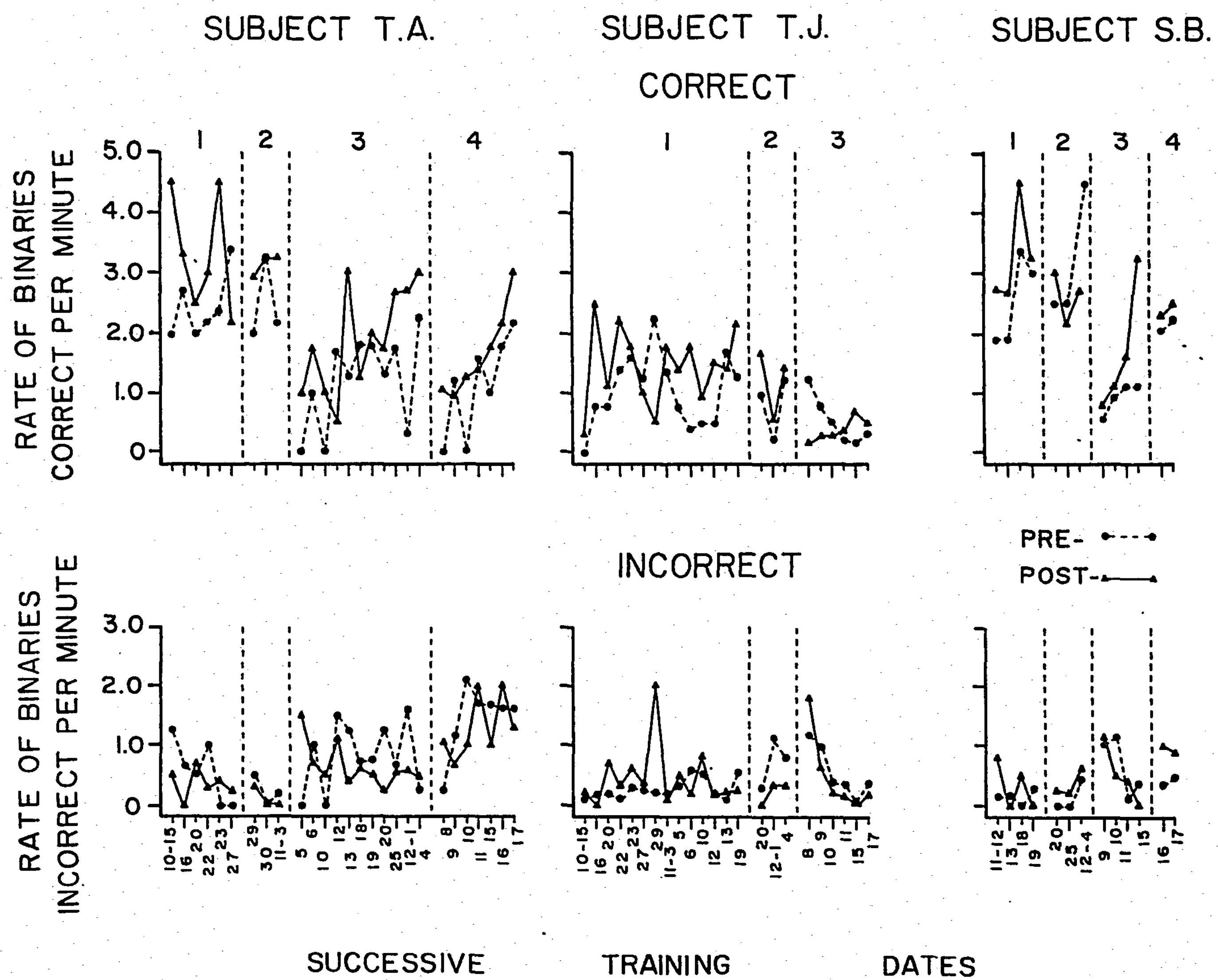

SUBJECT T.A.
SUBJECT T.J.
SUBJECT S.B.
CORRECT
RATE OF BINARIES CORRECT PER MINUTE
INCORRECT
RATE OF BINARIES INCORRECT PER MINUTE
PRE-
POST-
SUCCESSIVE TRAINING DATES

FIGURE 3

TABLE 3. Mean Post-Test Binary Rates Per Minute.

TABLE 3

Mean Post-Test Binary Rates Per Minute

Training Step	Subject	Mean Correct Rate	Mean Incorrect Rate	Ratio of Mean Correct to Mean Incorrect Rates
1	T.A.	3.37	.37	9.1
1	T.J.	1.47	.50	2.9
1	S.B.	3.25	.29	11.1
2	T.A.	3.22	.11	29.3
2	T.J.	1.25	.28	4.5
2	S.B.	2.64	.42	6.3
3	T.A.	1.89	.65	2.9
3	T.J.	.44	.60	.73
3	S.B.	1.72	.52	3.3
4	T.A.	1.72	1.29	1.33
4	S.B.	2.38	.87	2.71

incorrect responding across training steps except for the second step of training. T.A. and T.J. obtained their lowest incorrect rates for Step 2. The only procedural difference between Step 1 and Step 2 was the notation of the answer. S.B.'s incorrect rate consistently increased across training steps by small increments.

Rate Ratios

The ratios of the mean correct rate to the mean incorrect rate illustrates that, with one exception, subjects were responding correctly more often than incorrectly. The one exception occurred in Step 3 training for T.J. He responded quite slowly in general and proportionally produced more incorrect than correct answers. The mean ratio of correct rate to incorrect rate for all subjects was 6.7.

The proportion of correct to incorrect rates tended to decline across training steps, except for the second step. T.A. and T.J.'s correct to incorrect ratios were highest during the second step of training. This was due to reduced incorrect rates coupled with only mildly reduced correct rates. All the ratios declined sharply during Step 3 training compared with those of Step 3. The ratios reveal the trend in rates to decrease for correct responding across training steps while increasing for incorrect responding across steps.

Correct rates: Pre-test/post-test differences. During Step 1 training, the rate of binary operations correct per minute was always higher for post-tests than pre-tests for S.B., for T.A. except in one instance, and for T.J. in eleven out of fourteen training sessions.

During Step 2 training, the rate of binary operations correct per minute for post-tests was always higher than or equal to pre-test rates for T.A. and T.J., but was only higher than pre-test rates in the first lesson for S.B.

During the third step of training, the rate of binary operations correct per minute was always higher for post-tests than pre-tests for S.B., higher for T.A. in all but two sessions, but lower for T.J. in all but the last two training sessions.

During Step 4 training, post-test correct rates were higher in both sessions for S.B. and in five out of seven sessions for T.A.

Incorrect rates: Pre-test/post-test differences. During Step 1 training, the rates of binary operations incorrect per minute were lower for post-tests than pre-tests in three out of six sessions for T.A., in two out of four sessions for S.B., but only in four out of fourteen sessions for T.J. Since his incorrect rates were increasingly higher for post-tests than pre-tests, T.J. was offerred the option of a reduced practice session between tests contingent upon his accuracy improvement. The option was in effect for seven sessions (from November 3 to November 19, 1980) when T.J. met criterion for proceeding to the next training step.

During Step 2 training, the rates of binary operations incorrect per minute were lower for post-tests than pre-tests in all three sessions for T.A. and T.J. However, the rates were higher for post-tests than pre-tests for S.B. in all three sessions by a small amount.

During Step 3 training, incorrect rates were lower on post-tests than pre-tests in five out of six sessions for T.J., in nine out of eleven for T.A., and in two out of four for S.B.

During Step 4 training, incorrect rates were lower on post-tests than pre-tests in five out of seven sessions for T.A. They were higher on post-tests than pre-tests for S.B. in both sessions.

Error Patterns and Program Modifications

Test protocols from this study were inspected to identify error patterns in program procedures. Tests were inspected daily, error patterns diagnosed, and program modifications were made in an attempt to correct the errors. Some of the error patterns were unique for an individual subject and some were observed across subjects.

Error patterns: Step 1. The error patterns observed in Step 1 of training were unique to individual subjects. T.A. disregarded the second numeral in the binary operation, adding the first numeral to the first numeral. An example is:

$$\begin{array}{r} 3 \\ +\quad 5 \\ \hline 6 \end{array}$$

The following program modifications were made for each practice example: a) T.A. was asked to answer, "What is the first number in the problem?"; b) he was instructed to cover the named numeral on the DOOF; c) he was asked, "How many more?" and he named the second numeral; and d) he was instructed to count that many times more on the DOOF and write the number to which he was pointing in the answer space.

Program modifications (b) and (d) above were faded to "What do you do next?" and later eliminated altogether.

The error pattern observed in T.J.'s tests for Step 1 was that of not counting with one-to-one correspondence. He skipped or recounted numbers on the DOOF instrument. He lifted his counting finger very high off the DOOF and lost his place. He sometimes counted the top and the bottom of the box beside a numeral. The following program modifications were instituted: a) counting chips; b) physical guidance for DOOF counting; and c) a revised DOOF. The DOOF was changed by enlarging the space between numerals and replacing each box with a dash.

<u>Error pattern</u>: <u>Step 2</u>. The notation procedure for the low-stress algorithm is unique. The probability of errors in notation placement were predicted by logical analysis of the task and by the identification of this error pattern by Alessi (1974) and Boyle (1975). The training program was modified prior to instruction in Step 2 to prevent placement errors. Half-inch squares were drawn to the left and right of the second numeral in each problem to depict proper location for the notation. The squares were replaced by dashes (fading) during the second training lesson in Step 2. No other modification was made for notation in step 2. The subjects' error patterns in Step 2 training were not related to notation. T.A. and T.J. continued to make some errors similar to those they had made in Step 1, which were unrelated to notation. The program continued to be modified as it had in Step 1 for their particular error pattern.

<u>Error pattern</u>: <u>Step 3</u>: <u>Skipping digits</u>. Skill in Step 3, adding

two binaries in one column, required performance of several new tasks. These tasks are: a) accurate placement of notation for both binary operations; b) using the one's plase digit of notation as the starting place for the second binary; and c) regrouping and counting any ten's place notation digits for a complete answer. All subjects made a variety of common errors during Step 3 training.

One error pattern common to all subjects was skipping digits. The student either added the first and second numerals or, less often, the first and third numerals in the column for the complete answer. The program was modified in two ways to attempt to remediate this error pattern: a) a "build-up" format was used in practice sessions and, concurrently, b) students crossed out "used" numerals. The "build-up" format followed this sequence for each problem: a) a single binary was written on the page; b) students completed that binary operation as in Step 2; c) students crossed out the numerals used in the completed binary operation; d) the trainer wrote another numeral in the column, and e) students added the next binary pair. The trainer pointed to the proper numerals needed for the second binary operation. After two or three practice sessions of the "build-up" format, depending on the student's progress, another modification was presented.

Students were given isolated practice on performing the second binary operation. Subjects were presented with 1 x 3 array problems. The trainer completed the first binary operation in the students' presence and crossed out the numerals used in that operation after notating the answer. The students were asked, "What numbers do you add next?" The students were provided with the correct sum so that they did not use the DOOF. They wrote the sum in correct notation and location, with or without trainer help, depending on their performance. Any errors were corrected by the trainer and followed by repeating the question for another example.

Error pattern: Step 3: Miscounting notations. Another common error pattern identified in Step 3 tests was that of not counting or miscounting the ten's place digits of the notation for the final answer. Students usually brough down the ten's place digits which were located in the final binary. They did not, however, always count a ten's place notation digit if it were located after the first binary. The program was modified to include slashing out all numerals as they were used, including those in notation.

During practice lessons students were asked, "Are there any ones on the left to count?" Students responded "yes" or "no" and, without further prompting, counted any "1's" in the ten's place notation. This question was asked for every example, even if there were no "1's" to be counted. The rationale for this was that none of the students required prompting to count the "1's" after answering yes or no to the question referring them to check for this. The critical task appeared to be checking for numbers. Therefore, students were given directed practice to check for the digits. Students crossed out these digits as they counted them.

Error pattern: Step 3: Missing the starting point of the second binary. The students did not consistently cross out the numerals used on the tests even if they had done so during the practice session training. They were reminded to do so prior to taking a test, but this reminder was not repeated during testing. Failure to cross out the numerals used resulted in an error pattern.

The error pattern consisted of not using the one's place notation digit as the starting point for the second binary operation. If they had crossed out the numerals as they were used, it might have increased the probability of using the notated number for the second binary operation. Since they were not consistently crossing out numerals, the following modification was made to prompt using the notation digit for the second sum in a column.

Hutchings (1976) suggests drawing a curved arrow from the one's place of the notation to the next numeral in the column as a training prompt. The program was modified to include the curved arrows on the tests as well as for the lessons. An example is:

$$\begin{array}{r} 3 \\ 8 \\ +\quad 2 \\ \hline \end{array}$$

<u>Error pattern</u>: <u>Step 3</u>: <u>Misnotation</u>. Another common error pattern observed in Step 3 tests was that of misnotating the answer between the first and second numerals and/or between the second and third numerals. The notation should be placed at the lower corners of the last digit of the binary operation. The following examples illustrate the correct and incorrect notations described above.

<u>Correct</u>	<u>Incorrect</u>
3	3
/ 8 /	/ 8 / 9
+ 2 3	+ / 2 /

The reason the notation location is critical is that it defines the next binary operation pair of numerals. The error pattern this misnotation is likely to produce is ullustrated below:

```
    4
  / 9 3
  /   2
+   7 9
  -----
   29
```

As in the incorrect example above (3 + 8 + 2), the mislocation of the notation for the first binary could lead to the addition of the one's place digit of the notation with the second numeral in the column. That numeral had already been used to obtain the notated answer. Thus, the student produces an answer with an extra, erroneous operation in the answer.

Dashes were drawn at the lower corners of the appropriate numerals on the worksheets to prompt proper placement of the notation. Students were given a verbal rationale also. They were told to put the answer next to the place where they had stopped adding.

<u>Error pattern</u>: <u>Step 4</u>: <u>Carrying operation</u>. The error patterns observed in Step 4 tests were similar to those found in Step 3. The students misnotated answers, skipped numerals in the columns, and did not correctly count the ten's place notation digits for the answer. The program modifications for these errors were similar to those made

in Step 3.

The error pattern unique to Step 4 training occurred for the carrying operation from the first to the second column. Both students failed to carry any ten's place notation from column one. The topography was different for each student.

T.A. brought down the one's place digit of the answer to the first column into the answer space, ignored the ten's place notation, then performed the binary operation for the second column. An example of T.A.'s error pattern is:

$$\begin{array}{rcc} & 6 & 4 \\ + & 1 & 8 \\ & 7 & 12 \\ \hline & 7 & 2 \end{array}$$

The program was modified with the addition of a small circle at the ten's place digit of notation and a hooked arrow connecting the circle to the top of the second column. An example follows.

$$\begin{array}{rcc} & 2 & 3 \\ + & 4 & 9 \\ \hline & & \end{array}$$

He was directed to bring any number in the circle to the top of the second column, then add as he had been taught.

S.B. also did not carry when appropriate. Instead of ignoring the ten's place digit of notation, she brought the "1" down into the answer space. She added each of the two columns as if they were separate problems. An example of S.B.'s error pattern follows.

$$\begin{array}{rcc} & 2 & 3 \\ + & 4 & 9 \\ & 6 & 12 \\ \hline & 6 & 12 \end{array}$$

She, too, was given worksheets with added circles and hooked arrows as above.

Other Procedural Modifications

There were other modifications made during the study. All of the changes were made in an attempt to improve the chances of completing training within the time constraints.

The location of the training lessons was moved starting November 12, 1980, Session Number 13 for T.A. and T.J. and Session Number 1 for S.B. At the beginning of the study, the lessons took place in an isolated area of the classroom. The classroom was very large and had a workshop area as well as an area for desks. The lessons were conducted at tables near the workshop, away from students who were working at their desks. This was done to conform to a classroom lesson environment as closely as possible. Students remained on-task during most lessons, but some classroom activities (such as showing movies) proved disruptive. In addition, permission was granted for Subject S.B. to join the study. Tuere were then three subjects. They were all at different stages of training. Training was moved to a separate study room in the building in order to decrease distractions.

The reinforcement system used in the project changed after Lesson Number 20 on December 4, 1980. Up to that date, students were given praise for accuracy by the trainer and points for on-task behavior from the classroom teacher. The points were accumulated, at a rate equal to that in the classroom, to trade for class store items. Store items in the classroom included nail polish, aftershave lotion, small games and packaged snacks. In an effort to enhance the reinforcment value of training, a "lesson store" was created.

The "lesson store" items were dispensed in addition to the praise and points previously earned. The items were earned for on-task behavior and accuracy scores of 80 percent or higher on tests.

The students participated in choosing the items and activities to be included in the "lesson store". Student suggestions included candy, a game of tic-tac-toe, and a personal telephone. The candy and games were selected as feasible items. The candy chosen was small, solid milk chocolate "Santas" wrapped in brightly colored foil.

The trainer then displayed other options for the approval or disapproval of the students. Brightly colored, three-dimensional stickers of monsters, animals and space creatures were shown. The students said they would like to work to earn the stickers. They selected a favorite sticker to be placed on "lay away". The trainer demonstrated the use of three pocket electronic games. The games were: concentration, pinball, and a maze game. The games all had bright flashing lights and made noise. The students agreed they wished to work for time to play the games.

The candy, games and stickers were available after every session in which they were earned. No point accumulation was required. The subjects began earning the rewards on Lesson Number 21, December 8, 1980. The rewards were available during the last two weeks of training.

The lesson format was changed on Lesson 22, December 9, 1980, to include two practice sessions per training day. Up to that date.

sessions followed the pre-test, practice lesson, post-test sequence at one sitting. Permission was granted from the home economics teacher to take the subjects from her classroom for about twenty minutes for training. The format then changed to the following sequence: a) a morning session of pre-test and lesson; and b) an afternoon session of lesson and then a post-test. In short, the time available for practice was dounbled for the last two weeks of the project, six or seven sessions per student. The testing was not doubled, though, because of the possibility of fatigue of the subjects.

Forgetting

The potential for "forgetting" to influence acquisition was monitored throughout the study. As a guideline, "forgetting" was defined as a declining discrepancy of binary accuracy scores from one lesson's post-test to the next lesson's pre-test. The following table lists the mean post-test/pre-test discrepancy in percent accuracy by training step and by subject.

TABLE 4

Mean Percent Post-Test/Pre-Test Discrepancy

Step	Subject		
	T.A.	T.J.	S.B.
Step 1	-4%	+18%	+3.3%
Step 2	0% Change	-45%	+5.0%
Step 3	-18%	+2 %	0% Change
Step 4	-14%		+11.0%

CHAPTER IV

DISCUSSION

Preskills

None of the subjects in this study met the criterion for knowledge of basic addition facts. In previous experimental studies, all subjects had passed preskill requirements for accuracy with basic facts prior to training with Hutchings' low-stress algorithm. Furthermore, this was an acquisition program. The subjects had received minimal prior classroom instruction for increment addition. The subjects in past studies had between three and seven years of classroom instruction in the standard addition algorithm.

The use of the DOOF instrument substituted for knowledge of the basic facts. Compared with fact recall, locating the sums with the DOOF is a time-consuming procedure. This would predictably reduce the rate of responding for the subjects in this study compared with those of past studies. Rate comparisons across studies are enumerated later in the Discussion.

The preskills required for entry into DOOF instrument training were: a) read numerals; b) write numerals; c) match numerals; d) rote count in order; and e) count with one-to-one correspondence. As shown in Table 1, all subjects met criterion for each of these requirements except one subject. T.J. did not consistently count with one-to-one correspondence. He received additional program instructions to acquire that skill. All subjects were required to score

90 percent or bettwe accuracy on three consecutive post-tests for DOOF usage (Step 1) before they were eligible for further program instruction.

Acquisition of Skills

All subjects acquired skill in locating basic fact sums with the DOOF instrument (Step 1) with 90 (T.A. and T.J.) or 100 (S.B.) percent accuracy. All subjects acquired skill in notating sums with the low-stress format (Step 2) with 85 (S.B.) or 100 (T.A. and T.J.) percent accuracy. All subjects acquired skill in single column addition for two binaries (Step 3) with some degree of accuracy. According to the last two post-tests for Step 3, subjects obtained the following accuracy scores: 90 percent (S.B.); 85 percent (T.A.); and 80 percent (T.J.).

For the two subjects who received training in two column addition (Step 4), accuracy scores were lower than for other steps. According to the last two post-tests for Step 4, T.A. obtained a binary accuracy score of 62 percent. S.B.'s two post-tests averaged 73 percent. Both subjects were scoring with accelerating accuracy when the subjects' semester and training ended.

While subjects were acquiring skills with accelerating accuracy within each step of training, the trend across training steps was that of overall declining accuracy and decelerating correct rate.

Accuracy scores tended to decline most profoundly with training of the low-stress algorithm for solving two binaries in one column (Step 3). The subjects had received no prior classroom instruction for adding more than two numerals. The algorithm did not require an increased memory for complex facts, as would have been the case with the standard algorithm. However, the algorithm required an increase in memory for several sequenced procedural steps. The new procedural steps were: a) accurate placement of notation for two binary operations; b) using a notation unit as a starting place for the second binary operation; and c) regrouping. The most effective correction procedure for errors was isolated practice with one component. This suggests that separating and then chaining the sequence of new procedures might improve the program.

Forgetting

The most profound instance of "forgetting" occurred for T.J. in the second step of training. This decline occurred between the post-test prior to Thanksgiving vacation and the pre-test of the next lesson. T.J. was absent before the school vacation so that there were ten days between this particular post-test and the following pre-test.

T.A.'s discrepancy scores reveal some milder levels of "forgetting". He was the only subject for whom there was a mean decline or no change in post-test/pre-test scores throughout the study.

There are several instances of no mean change in accuracy or improved accuracy scores. In general, "forgetting" (as defined) was not a serious deterrent to acquisition for two subjects and was a mild deterrent for one subject.

Error Patterns

The test protocols in this study revealed error patterns similar to those identified by Alessi (1974) and Boyle (1975). The errors were found more frequently in this project and required more modifications for remediation.

Additional error patterns were identified in this study. Subjects skipped digits in columns, missed the starting point of the second and third binary operations, and did not carry from column to to column two. The subjects in this study had no prior training for adding more than one binary. The standard algorithm for increment addition was unfamiliar to them. They were previously untrained in the concepts and the mechanics of higher level addition.

Program Modifications

The program was modified for both antecedent and consequent events. Antecedent program alterations included worksheet prompts and isolated component skill practice. Consequences were modified to include immediate tangible and activity items designed as reinforcement.

Worksheet prompts, such as boxes or dashes and curved arrows, were designed to cue notation location, location of the starting point for the new binary operation, and carrying operations. Prompts were faded in several ways: a) from a different color than the written problem to the same color; b) from a bold print to a fine print; c) from a large size (box) to a small size (dash); and d) from a constant to an increasingly longer intermittent schedule of appearance.

The prompts reliably occasioned the desired response. Fading reduced dependency on the written prompts. However, time constraints contributed to presenting more than one prompt at a time if multiple errors were present in the subject's work. When this occurred, a sample worksheet problem appeared as follows:

```
    _2_  5
+   _9_  6_
-----------
```

In this case, even before hte student wrote a response, the worksheet was crowded with extra stimuli, The completed problem then appeared as follows:

```
     1
    _23  5
+   192  161
-----------
     1 2 1
```

The student was required to alternate DOOF usage and notation as well as navigate the written prompts. This provided several opportunities to lose one's place. If that happened, errors and a reduced rate of responding resulted. The advantage of using written prompts to reliably occasion accurate responses was countered by the disadvantage of the possible confusion of extraneous stimuli.

Another program modification was the use of isolating a component skill for practice. Sometimes this technique was combined with a written prompt which was faded within the lesson. The resulting advantage was a reliable improvement of accuracy for that component skill. The disadvantage was that the isolated skill later required time-consuming chaining exercises to build it into the sequence of procedures.

The modifications designed to increase the reinforcement value of the lessons were implemented during the last two weeks of training.

The effect of such changes is confounded by the simultaneous change of doubling the practice lessons. The students enjoyed the items form the "lesson store". They selected the use of the electronic games most frequently and often asked for more time to play with them. They smiled when displaying the stickers on books, notebooks and clothing. At the same time, they sometimes complained about coming to the morning and afternoon lessons. It is difficult to draw a conclusion about the effects of the "lesson store" on acquisition or tolerance for the doubled practice sessions. Both changes took place simultaneously and were in effect for a limited time.

Mean Percent Correct: Comparison Across Studies

Summary data have been compiled as a guideline for comparison of accuracy in this study with prior low-stress algorithm studies. The comparison data should be regarded with caution. Subjects from prior studies were regular education students or emotionally impaired special education students (Alessi, 1974; Boyle, 1975; Gillespie, 1976; Rudolph, 1976; and Zoref, 1976). Subjects from this study were trainable mentally retarded students. The data from other studies are calculated with scores for different size problems for the entire study. The scores from this study are calculated from the last three training post-tests from the third step of training. The reason that the third step was selected as the comparison was that: a) this step requires addition of more than one binary and b) only two subjects had training in the fourth step. Data on accuracy scores were limited to the last three post-tests because it was an acquisition program and scores from early lessons do not as accurately reflect terminal skill levels.

The summary data, then, compare accuracy of acquisition with that of performance after training. The summary compares different array sizes. The arrays from this study were 1 x 3; other studies had various, larger array sizes, such as 2 x 7, 3 x 7, and 5 x 7. All accuracy scores are reported as mean percent of correct answers. The data in the following table are presented as a very general guidleine of comparison across studies.

TABLE 5

Mean Percent Correct By Study

Study Source	Array Size	Low-Stress	Standard Algorithm
Markle (1981)	1 x 3	80%	
Alessi (1974)	2 x 7,		
	3 x 7,	82.5%	76.8%
	and 5 x 7		
Boyle (1975)	2 x 7	90%	78%
Rudolph (1976)	2 x 7	96%	82%
Zoref (1976)	2 x 7		
(low achievers)	and 5 x 7	88%	54%
Gillespie (1976)	4 x 5	90%	75%

Compared to the mean low-stress accuracy found in other studies, scores were slightly lower in this study. However, the mean percent was at a relatively high level (80%). Trainable mentally impaired subjects were performing the low-stress addition algorithm with accuracy close to that of subjects in regular education programs or emotionally impaired programs (Rudolph, 1976).

Compared to the mean standard algorithm accuracy found in other studies, scores were similar to compare favorably in this study. The subjects in this study had a higher mean accuracy score than did the regular education low achievers in the Zoref study.

Mean Binary Rates: Comparison Across Studies

Summary data have been compiled as a guideline for cdmparison of binary correct rates and incorrect rates in this study with prior low-stress studies (see Table 6). The same precautions apply as with the mean percent correct scores.

The data from other studies have been converted from rates per column to rates per binary operation for direct comparison with data units in the current study. Data from the current study used binary operations as the basic units. Data from other studies used column sums as the basic units. The number of binary operations completed in a column may be calculated by subtracting one from the number of numerals in a column (N-1). For example, a column of seven numerals contains six binary operations.

Finally, summary data are presented for the ratio of mean correct rates to mean incorrect rates (C/I). This table serves as a guideline representing the comparable ratio of accuracy to inaccuracy.

The mean binary correct rates are significantly lower in this study compared to other studies. Subjects from other studies

Table 6: Mean Binary Rates Per Minute By Study.

TABLE 6

Mean Binary Rates Per Minute By Study

Study Source	Mean Correct Rate	Mean Incorrect Rate	Mean Ratio (C/I)
Markle (1981)	2.1	.5	6.7X
Alessi (1974)			
Low-Stress Array Size:			
2 x 7	15.0	2.0	7.5X
3 x 7	11.4	2.6	4.4X
5 x 7	10.6	3.4	3.1X
Standard Algorithm Array Size:			
2 x 7	10.8	2.8	3.9X
3 x 7	9.8	2.8	3.5X
5 x 7	9.8	3.4	2.9X
Boyle (1975)			
Low-Stress	14.2	1.6	8.7X
Standard Algorithm	9.8	2.8	3.5X
Rudolph (1976)			
Low-Stress	15.0	1.5	10.0X
Standard Algorithm	15.0	4.4	3.3X

TABLE 6
(Continued)

Study Source	Mean Correct Rate	Mean Incorrect Rate	Mean Ratio (C/I)
Zoref (1976) - Low Achievers			
Low-Stress Array Size:			
2 x 7	10.2	1.4	7.1X
5 x 7	9.9	1.1	9.0X
Standard Algorithm Array Size:			
2 x 7	4.9	4.1	1.2X
5 x 7	5.3	4.1	1.3X

calculated binaries correctly from a range of two and one-half to seven times faster than did subjects from this study. The fact that the students in this project did not know basic facts and used the DOOF instrument to calculate sums may account for much of the discrepancy.

The mean incorrect rate found in this study is lower than rates found in other studies. It reflects a generally lower rate of responding of subjects in this study.

The correct rate is very low and the incorrect rate is lower in this study compared to other studies. The ratio data reveal that while the subjects in this study responded much more slowly in general, they were responding correctly almost seven times faster than incorrectly. This ratio compares favorably with those reported in other studies, using non-handicapped pupils. The lowest correct rate found in another study was for Zoref's low achievers using the standard algorithm. Those subjects were responding correctly approcimately two and one-half times faster than subjects from this study. However, the ratio of correct to incorrect rates also reveals that the low achievers were responding incorrectly almost as much as correctly. The subjects from this study were much slower but comparably more accurate than the low achiever regular education subjects from the Zoref study.

The most significant difference between this study and others is the very low rate of responding of the mentally retarded subjects. The substitution of the DOOF instrument for knowledge of basic facts did not significantly affect accuracy score differences. However, it appears to have contributed to a greatly reduced rate of responding. Further research might investigate the effect of a program to teach basic addition facts prior to increment addition training on the rate of responding.

One possible sequence to teach basic addition facts would consist of the following: a) the identity element (+0 facts); b) count to the next higher number (+1 facts); c) the commutative property of addition (2 + 4 = 4 + 2); and d) number "families" (all +2 facts, all +3 facts, all +4 facts, etc.) (Silbert, Carnine and Stein, 1981, pp. 224-234).

The advantages of training Hutchings' low-stress algorithm after mastery of rapid recall of basic facts would be twofold. Subjects use only basic facts rather than complex facts and there is a full record of all calculations available for diagnostic and remedial analysis purposes. Both these variables have been found to contribute to the superiority of the low-stress over the standard addition algorithm with regular education pupils (McCallum, 1981; VanHevel, 1981).

Attitudes of Subjects Toward Training

Anecdotal observations of the participants revealed a reluctant attitude toward training. Subjects T.A. and T.J. sometimes complained ("Oh no, not this again.") when the trainer entered the classroom to begin lessons. T.J. sometimes waved his hands over the test protocol, shook his head and said, "I don't want to do anymore." S.B. always came willingly and frequently made a positive comment about the lessons.

One possible reason for the observed reluctance is the nature of the training. The students were given lessons individually or in pairs. The close supervision resulted in a large increase in response effort and on-task behavior requirements when compared with the typical classroom situation. On the other hand, the training situation created an increased opportunity for praise and individual attention. The reinforcement value of praise and attention from the trainer seemed to vary among subjects. Tangible items and activities were dispensed during the last two weeks of training in an effort to increase the reinforcement value of the lessons. Although the subjects participated in selecting items and activities for the "lesson store", it did not noticably alter student attitudes within the two-week period. At the same time, practice lessons were doubled. This confounds interpretation of the effects of the "lesson store".

Academic Engaged Time

The subjects were engaged in the program three to four days weekly, averaging fifteen minutes per practice lesson. Tests were untimed, ranging in duration from two to didteen minutes. Students turned in test protocols to the examiner when they completed the math problems. Therefore, all test scores are based on the assumption that all problems were attempted. The number of binary operations per test was: a) 10 binaries for Steps 1 and 2 and b) 10 to 14 binaries for Steps 3 and 4. The number of binaries varied for Steps 3 and 4 due to a variety of regrouping operation requirements.

Feasibility

Fairly high levels of accurate acquisition existed in the results of the training program for all subjects. Time constraints interferred with training completion, but correct calculation for all subjects showed an accelerating trend at the end of the study. Accuracy scores were comparable with that of other studies with regular education (average I.Q.) pupils. The most significant difference between the results of this study and those using normal I.Q. pupils was the very low rate of responding of the mentally retarded subjects. But these subjects did not have prior training in the mechanics or the concepts of higher level addition.

Future research could investigate the effects of preskill mastery training on the ability of mentally retarded subjects to learn multi-place column addition. Preskill training could include both a concept program and a basic fact program. Training with DISTAR Arithmetic I program of preskills might improve the subjects' conceptual skills with higher level addition (Bracey, Maggs and Morath, 1975). Training rapid recall of basic facts could replace the continuous use of the DOOF, therefore increasing the rate of responding.

Future research could investigate comparable speed and accuracy of responding for retarded subjects with training in the standard and Hutchings' addition algorithms. Research on the comparable speed, accuracy, and preference with the pocket calculator versus standard and Hutchings' algorithm training could be useful, also.

APPENDICES

APPENDIX A

DOOF Instrument

0	☐
1	☐
2	☐
3	☐
4	☐
5	☐
6	☐
7	☐
8	☐
9	☐
10	☐
11	☐
12	☐
13	☐
14	☐
15	☐
16	☐
17	☐
18	☐

APPENDIX B

Protocol Example 1

6 4	9 3	6 7
8 5	3 9	8 4
7 8	9 7	4 6
8 9		

APPENDIX B
(Continued)

Protocol Example 2

6
4
9
—

3
6
7
—

8
5
3
—

9
8
4
—

7
8
9
—

APPENDIX B
(Continued)

Protocol Example 3

$$\begin{array}{r} 29 \\ +\,46 \\ \hline \end{array} \qquad \begin{array}{r} 73 \\ +\,58 \\ \hline \end{array}$$

$$\begin{array}{r} 31 \\ +\,45 \\ \hline \end{array} \qquad \begin{array}{r} 96 \\ +\,32 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ +\,74 \\ \hline \end{array}$$

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