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A COMPONENT ANALYSIS OF HUTCHINGS' LOW-STRESS ADDITION ALGORITHM

by

Daniel V. McCallum

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A COMPONENT ANALYSIS OF HUTCHINGS' LOW-STRESS ADDITION ALGORITHM

Daniel V. McCallum, M.A.

Western Michigan University, 1981

An A-B-A-C counterbalanced reversal design was used with two groups to analyze the two components of Hutching's Low-Stress addition algorithm. Fourth grade students achieving 96% accuracy on a pretest of basic math facts were subjects of this study. Subjects were taught two new methods of computation; Hutchings' Low-Stress and the Conventional algorithm with a written record, along with reviewing the Conventional algorithm. Students were given worksheets containing fixed size addition problems and asked to complete as many as possible with a five-minute timed session. Accuracy and speed were monitored across the three methods of computation. The results showed superior performance with the algorithm using a full record versus no record. A similar but less profound effect was seen with the algorithm that utilized only basic math facts versus complex facts. A most important finding was that all subjects had higher performances with the Low-Stress algorithm when compared with the Conventional algorithm.

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Daniel V. McCallum

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CHAPTER I

Introduction

A number of studies have been conducted with the Hutchings' algorithm since its inception (Hadden, 1981; Gillespie, 1976; Rudolf, 1976; Zoref, 1976; and Alessi, 1974). This initial research has concentrated on identifying the functional relationship between the algorithm and performance. Since this relationship has been reliably demonstrated, it is time for research in this area to move on to a component analysis of this algorithm. This was the undertaking of the present study.

There has been a growing concern in education over the past few years about hte competency of the students being graduated. The knowledge that there have been decreasing scores on the scholastic apptitude test given to high school seniors wanting to enter college has added pressure to parents, teachers and administators to correct the decreasing skills levels of their students. Re-evaluation of current teaching techniques has occurred. This has lead some institutions to adopt minimal competency exams that must be passed in order for a student to graduate from high school, and even between grades.

In mathematics there has been a developing trend over the years toward focusing on concepts and less emphasis on actual computational skills (Alessi, 1979). The National Council of Teachers of Mathematics (NCTM) has expressed its support for a balanced math curriculum that stresses basics in the context of total mathematics instruction (Arithmetic Teacher, 1976). They also point out the importance of simple computational skills in everyday situations. Cox (1975) pointed out that the development of basic computational skills should not be neglected. Yet many teachers continue to see older students who have not mastered these basic operations. This lack of mastery may be partially due to the students' negative reaction to mathematics in general. This reaction by the student often results in teachers spending as little time as possible on drill and more on material of greater inherent interest (as defined by the traditional education system).

Wheatley (1976) suggests that computational procedures that continually stress some degree of understanding may result in less computational facility. King (1972) seems to agree with this when he states that explanations of algorithms often confuse and frustrate students. It should be noted that King's statement is based upon personal experience and those of other teachers he has talked with rather than empirical data.

Remediation of computation skills is usually accomplished through drill and practice which has a number of disadvantages. One disadvantage is that practice often consumes valuable classroom time that could be used for other skills and in other areas. Teachers often have only a few techniques at their disposal for use in drill. Another disadvantage is the monotony of the tasks. Hutchings (1972) and Alessi (1974) share this viewpoint that monotony and boredom generated by repeated practice of math facts may result in a poor attitude toward the subject being practiced. Skinner states that "Figures and symbols of mathematics have become emotional stimuli... the glimpse of a column of figures, not to say an algerbraic symbol or an integral sign, is likely to set off, not mathematical behavior, but a reaction of anxiety, guilt, or fear" (Skinner, 1968, p. 18).

Alternative techniques for instruction and practice can be valuable to the teacher and student. Wheatley (1976) compared the methods of direct addition (summing each digit with the following digit) and the tens method (the student looks for combinations of digits that sum to ten). He found the direct method to be 15-18% faster and just as accurate as the tens method. He followed this up in 1977 (Wheatley and McHough) and found that this effect was seen across grade levels and student abilities. Wheatley and McHough suggest that the most efficient algorithm may be one that contains the least number of decisions. The Hutchings' algorithm seems to fit this description.

Nichol (1978) discussed the use of palindrones with students in the practice of basic skills. A palindrone is a word or number that can be read the same both forwards and backwards. Examples include noon, 212, 797, and 4004. The palindrones were found to be helpful in motivating students (by turning the task into a game) when the task involved practice of basic computational operations.

Other methods of addition computation have been suggested. One such method was suggested by Sanders in 1971. This addition procedure was developed for students who tended to get lost and forget where they were in addition problems. This method has the student silently compute the sum of two digits and then hold up fingers to represent the tens place. This method is continued down the column until all binary additions have been completed. This method does reduce the number of responses required by the student. It should be noted that Sanders pointed out that this method should only be used after the student is competent in addition, because there is no permanent product which could be used to identify error patterns.

O'Malley (1969) presented another computational method. Here, instead of holding up fingers for the tens place as done with the Sanders method, the student writes down the tens, hundreds, etc., portion of the sum of added pairs to the left of each column. This procedure thus provides the teacher with a partial written record of the student's work. An example looks like this:

$$\begin{array}{rr} & 268 \\ & \textcircled{1}\textcircled{1} \\ & 385 \\ & \textcircled{1} \\ \textcircled{1} & 462 \\ & \underline{581} \\ & 1696 \end{array}$$

Another partial record algorithm was developed by Fulkerson in 1963. This method is slightly different from O'Malley's (1969) method. Fulkerson has the student draw a line through the last digit used in obtaining a sum of ten or greater when adding two addends. When the student has finished adding a column, all that needs to be completed is adding the lines drawn through the digits to obtain the sum to be carried to the next column. This method does provide the teacher with a partial record of the student's work. An example looks like this:

$$\begin{array}{r} {}^{2}1{}^{2}46 \\ 23\cancel{5} \\ \cancel{6}\cancel{9}4 \\ \underline{3\cancel{9}\cancel{8}} \\ 1473 \end{array}$$

In 1972, Hutchings developed an alternate set of algorithms for the four basic mathematical operations. He states that they will "operate with much less stress on the user than Conventional algorithms" (Hutchings, 1976, p. 219). This method has some advantages over the algorithms mentioned above. It provides a full written record of all component operations, instead of the partial record provided by the O'Malley (1969) and Fulkerson (1963) methods. Sanders' (1971) algorithm provides for no permanent record of the student's work.

Since the introduction of the Hutchings; Low-Stress algorithm, recent research has been conducted to compare student performance using the Hutchings' Low-Stress addition algorithm to that with the Conventional algorithm (Hadden, 1981; Gillespie, 1976; Rudolf, 1976; Zoref, 1976; Alessi, 1974). The results of the above studies indicate better performance with the Low-Stress algorithm. The Gillespie (1976) study also indicated student preference for the Low-Stress algorithm. Since these studies indicate preferences for, and increased performance with the Hutchings' algorithm, a question arises concerning which components of the algorithm are critical to these results. Research with the algorithm has concentrated on the issues of preference, performance, acquisition rates, and replication of the findings with the addition algorithm. As of now, an empirical component analysis of the Hutchings' addition algorithm has not been udertaken. The purpose of this study was to conduct such an analysis.

There are two critical differences between Hutchings' Low-Stress algorithm and the Conventional algorithm. These are:

a) binary addition with the Low-Stress algorithm versus complex addition with the Conventional algorithm, and

b) the format of the Low-Stress algorithm requires that the student complete a full computational record. The Conventional algorithm requires no such record.

This study is concerned with the analysis of these two components.

Hutchings' Low-Stress full record algorithm has many advantages: a) it is easy to locate problem areas and specific errors made by the students, b) it requires less instructional time to meet a mastery level (Zoref, 1976), and c) only a knowledge of basic math facts vs. complex facts is needed for students to use the algorithm effectively. This last advantage reduces fact knowledge by 90% since there are only 100 basic facts and 900 complex facts. Note that complex facts involve the addition of a two-digit number with a single-digit number, e.g., 34 + 7, and that basic facts involve the addition of of two <u>single-digit</u> numbers, e.g., 4 + 7. One disadvantage of the full record algorithm may be the increased amount of space needed on a sheet of paper per problem. This occurs because Hutchings' Low-Stress algorithm uses half-step notation, which requires extra space between columns and rows for the student's pencil work. To provide this extra space, teachers would need to make up special worksheets for each lesson.

Could a full record Conventional algorithm requiring complex facts be as effective in acquisition and performance as the Hutchings' full record utilizing only basic facts? This question can only be answered by analyzing the components of the Hutchings' algorithm to determine the extent influence the full record alone has on performace.

Hutchings (1976) defines the algorithm as follows:

> The Low-Stress algorithm uses a new notation, called half-step notation, to record individual steps. Half-step notation uses numerals of no more than a half-space in height to record the sums of two digits. With half-space notation, the units portion of the sum of two digits is written at the lower right of the bottom...we add the first two digits...and record the sum in the new notation...The complete sum of each two-digit addition is recorded in half-space notation, but only the ones portion of the columns sum is always the same as the ones portion of the last two-digit sum...The tens portion of the column sum is always the same as the number of tens recorded at the left of the column. These are simply counted. For a column in some multi-column exercises then, the last step - that is, counting the tens at the left of the columns - would be slightly changed. The counting itself is not changed in any way, but the answer, the total number of tens, is no longer written in the tens place of the first column's sum but instead at the top of the next column at the left...Work continues in this manner until the exercise is completed. Note, however, that the column sum for the last column in a multi-column example is recorded in exactly the same way as the sum of a single-column exercise (pp. 220-223).

Examples:

$$\begin{array}{r}
7 \\
8 \\
15 \\
7 \\
12 \\
\hline
22
\end{array}
\qquad\qquad
\begin{array}{rrr}
2 & 1 & \\
1 & 9 & 4 \\
3 & 10 & \\
2 & 4 & 5 \\
5 & 4 & 9 \\
8 & 6 & 9 \\
13 & 10 & 18 \\
\hline
13 & 0 & 8
\end{array}$$

All addition operations are performed in each column, followed by all necessary regrouping. There is no need to alternate between addition and regrouping operations as with the Conventional algorithm.

CHAPTER II

Method

Subjects

The subjects of this study were fifteen fourth grade students attending the Galesburg-Augusta Elementary School. Galesburg and Augusta are two small rural communities located approximately thirteen miles from Kalamazoo, Michigan. The population of these two communities is primarily caucasian with the primary industry of farming.

All students of the fourth grade class were given a pretest to help select the subjects of this study, since a knowledge of basic math facts is considered a prerequisite for effective instruction in the Low-Stress algorithm. The pretest was comprised of 56 binary addition problems (used by Alessi, 1974). Fifteen students who completed the pretest in a five-minute period and achieved a 96% accuracy or better, became subjects of this study.

Informed consent forms, explaining the purpose, advantages and risks of the study, were signed and returned before the students could participate in this study. There were no parents who chose not to have their child participate in this study. Subjects were randomly assigned into two groups.

Setting

The setting of this study was a regular classroom not being used during the morning hours of each day. This room had no open space to other classrooms and was divided into three sections via a partition and a bookshelf. The room had a blackboard on each of two walls with chairs and tables throughout the room. Sessions were held once a day between 9:10 - 9:30 a.m. each school day. Sessions were also run in the afternoon for several subjects during the last week of the study. These additional sessions were held in the subjects' regular classroom between 2:30 - 3:00 p.m.

Instruments

Requirements for a measurement instrument for use in studies of computational speed and accuracy were suggested by Hutchings (1972). One of these is providing a range of examples that might occur in lessons but in a form which would not load for reading or eye movement skills. He also states "it is required that applications of the identity element (0) be avoided, as these are considered to load for a distinct peripheral concept while contributing very little to demands upon memory-retrieval functions" (p. 51).

To conform with one of these requirements, problems were set on standard 8½ by 11 inch paper, four per page, in two rows. Fixed size addition problems were used in this study. Following Alessi's suggestion (1974), an IBM Selectric typewriter utilizing an orator element was used to make up the worksheets. These problems were constructed by completing triple-spacing between columns and double-spacing between rows. There was also double-spacing between the last row of digits and the sum line. This construction allowed for sufficient space for a student's written responses when utilizing a full written record algorithm.

The problems were generated by a computer program. This program selected digits at random and placed them in the proper array format (5 x 7) used in this study. Following another requirement suggested by Hutchings (1972), zeros were not included in the problems. As mentioned above, the problem size array for this study was five columns by seven rows, (or 34 binary operations). This array size was chosen because parametric studies indicate a separation in accuracy between the Low-Stress and Conventional algorithm begins at levels greater than 15 binaries. Past research is extensive with 7 rows by 2 columns (13 binaries), 7 rows by 3 columns (20 binaries), and 7 rows by 5 columns (34 binaries). The greatest separation has occurred with the 7 x 5 array size. The number of binaries per problem is calculated by multiplying the number of rows by the number of columns, minus one. This assumes that all rows and columns are filled.

A Huer-Leonidas Trackmaster model stopwatch was used to time each session.

Scientific Research Association probes (M-1 and M-13) (Scientific Research Association, 1972) were used throughout the study to assess any changes in the subject's computational or place value skills. The numerical values on the probes were changed to help prevent any practice effects. However, the content of the material covered in the measurement instrument was consistent across presentations.

Independent variables

Independent variables involved the algorithm used by a particular student to complete the daily worksheets. These algorithms included: a) the Conventional algorithm, b) Hutchings' Low-Stress algorithm, and c) the Conventional algorithm with a full written record.

Dependent variables

There were three dependent variables involved in this study.

1. Percent correct: the number of colums computed correctly divided by the total number of columns attempted, and expressed as a percent.
2. Correct rate: calculated by adding the number of columns correctly added, divided by the session length and expressed as columns per minute.
3. Incorrect rate: calculated by adding the number of columns incorrectly added, divided by the session length and expressed as columns incorrect per minute.

Interscorer agreement

Agreement data were taken on correction of the students' papers for the number of columns correct and incorrect. Approximately 30% of the students' worksheets were collected in each phase of the study and scored by independent scorers. This agreement was calculated by dividing the total number of agreements by the total number of agreements plus disagreements, multiplied by 100 to yield a percent measure. In all instants where there were disagreements between the two scorers, the worksheets were again scored to determine the true measure. This in no way affected the reported agreement measures but did allow for the author to report the accurate measure of the dependent variables.

Design

A reversal design is commonly used to demonstrate reliable control of an important behavioral change (Baer, Wolf and Risley, 1968). Variations on this basic reversal design are often needed to overcome some of the problems of this design.

One problem that arises if more than one treatment condition is introduced is sequential effects. This effect can be detected if two groups of subjects are used with each group receiving both treatments but in different orders. Since this study was utilizing more than one treatment condition, thus producing the possibility of sequence effects, an A-B-A-C counterbalanced reversal design was used (Bailey and Bostow, 1979). The sequences used were A-B-A-C and A-C-A-B. This design is set up to detect and/or counterbalance any sequence effects that might occur due to the order of presentation of the algorithms.

Training procedures

All students received similar instructions on all three algorithms. A twenty-minute training session was conducted before the initial presentation of a new algorithm. This training was based on written lesson plans adopted from Hutchings (1972) and previously used by other investigators of the Hutchings' Low-Stress algorithm (Alessi, 1974; Boyle, 1975; Gillespie, 1976; Rudolph, 1976; Zoref, 1976).

This lesson plan was also adapted for use in training the Conentional algorithm with a full written record and the Conventional algorithm with no written record.

Before reinstituting a previously taught algorithm, a 10-minute review was conducted. This was done to help ensure the student utilized the correct computational algorithm. The review lesson plan wass adapted from Hutchings' (1972) review lesson plan.

Procedures

Before the actual study began, students were given the Scientific Research Association (SRA) probe to assess their current skills in addition utilizing the Conventional algorithm, and their knowledge of place value concepts. This probe was also given before the introduction of each new algorithm. These probes were then used to determine if and when any changes in skill levels occurred during this study.

Before the beginning of each computational session, the students were given a verbal prompt that specified the algorithm currently being used. Students were then given worksheets with addition problems and asked to write their name and the date on the space provided at the top of the paper. Once this was completed, they were asked to complete as many problems as they could in the allotted time. Each timed session began with the verbal prompt, "Ready, start", and ended with the prompt "Stop". At this point the students were instructed to bring their papers up to the investigator and line up to return to their regular classroom.

Each student remained on a given algorithm until a steady state of responding was achieved. The student was then switched to the next algorithm in the sequence.

The performance of two students (Subjects 3 and 7) was not consistent with that of their fellow students. Subject 3 had a very low correct rate with high accuracy, and Subject 7 had erratic performance. A simple reinforcement procedure was selected to achieve experimental control. Under this condition, the students were reinforced for either a higher correct rate (Subject 3) or for stable performance (stated as plus or minus 10 percentage points with Subject 7). The reinforcer was one that the subjects selected themselves (use of the stopwatch to time events, e.g. going to and from their regular classroom). These contingencies were in effect for the remainder of the study. This procedure was not used with any other subjects.

CHAPTER III

Results

Interscorer agreements

There were a total of fourteen agreement checks made approximately every third session throughout this study. These checks yielded scores ranging from 91.4% to 100% with a mean of 96.7%.

Organization of data

The results of this study are presented in three main sections: a) comparisons between the record vs. no record component. This involves students' performances on the Conventional algorithm (CA) and on the Conventional algorithm with a full written record; b) comparisons between the complex addition facts vs. basic facts component. This involves students' performances on the Hutchings' Low-Stress algorithm (HA) and on the Conventional algorithm with a full written record (WR); and finally, c) student performance on the standardized probes throughout this study. Comparisons will also be made between the Hutchings' algorithm (HA) and the Conventional algorithm (CA). The comparisons will first be made in terms of general effects across students and then by specific intra- and intersubject differences.

Performances in each of the first three sections will be made in terms of the three dependent variables: a) percent accuracy, b) correct rate, and c) incorrect rate. Daily scores for each subject are presented in Appendix D. Summary data for these comparisons are presented in three tables and four figures. The four figures are representative of the different general effects seen across all subjects.

Comparison of full record vs. no record

Percent of columns correct. Tables I and II present the means and standard deviations for the daily session scores across all computational methods. (Note that the methods are listed in order of their presentation.) As shown in the tables, 12 of 15 subjects had greater accuracy when utilizing the Conventional algorithm with the full written record, when compared to the same algorithm with no record. Subjects 3, 4 and 8 showed no observable differences between the two methods. Subject 4's data indicate that the Conventional algorithm without the record is more accurate when compared with the first condition (WR).

Figures 1 and 2 show the individual scores for Subject 4 throughour the study. These data are representative of those subjects who had no observable differences between these two algorithms. The exception to this is the low mean and high standard deviation for Subject 4 in the first condition (WR). A closer look at the daily raw scores and worksheets indicates that the subject made a number of systematic errors in the first condition (Sessions 1, 3 and 4) which resulted in a low mean of 63.5 and a high standard deviation of 36.6. There is a sharp upward trend in the beginning of the first condition (see Figure 1) because of these systematic errors

Table I. Individual student means and standard deviations for session performance across computational methods.

TABLE I

Individual Student Means and Standard Deviations for Session Performance Across Computational Methods

		%		RC		RI		#			%		RC		RI		#
		$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD				$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	
S_3	WR	81.2	14.6	1.72	.46	.38	.30	10	S_4	WR	63.7	36.6	1.5	.96	.53	.58	13
	CA	77.3	15.5	1.40	.28	.42	.29	8		CA	87.5	5.3	2.4	.23	.36	.16	5
	WR	84.4	8.7	2.17	.33	.40	.23	7		WR	91.8	6.0	2.3	.43	.20	.14	5
	HA	93.6	5.5	3.66	.66	.25	.22	12		HA	89.9	8.4	2.5	.50	.27	.23	18
	WR	92.1	11.9	3.08	.64	.24	.32	5		WR	83.7	7.7	2.2	.63	.4	.16	4
S_5	WR	92.3	7.7	1.9	.39	.16	.15	10	S_6	WR	93.3	5.3	1.7	.21	.13	.10	11
	CA	86.1	8.8	1.7	.10	.28	.19	7		CA	73.5	20.9	1.6	.32	.64	.51	5
	WR	94.8	3.5	2.6	.43	.15	.10	4		WR	89.7	7.5	1.0	.30	.22	.16	8
	HA	93.8	4.3	3.2	.49	.21	.14	14		HA	96.9	5.0	1.7	.61	.06	.09	10
	WR	96.4	7.1	2.8	.41	.10	.2	4		WR	96.8	6.3	1.8	.44	.05	.10	4
S_7	WR	49.6	23.2	.85	.35	.91	.47	12	S_8	WR	83.0	14.1	1.8	.43	.35	.27	8
	CA	30.8	19.8	.72	.46	1.56	.57	5		CA	79.9	12.2	3.0	.64	.73	.41	6
	WR	41.4	14.0	.88	.36	1.20	.24	5		WR	78.1	16.6	2.0	.33	.60	.52	6
	HA	63.9	19.0	1.5	.49	.79	.49	21		HA	83.6	11.2	3.1	.73	.45	.30	14
	WR	36.3	20.4	.65	.44	1.05	.25	4		WR	80.4	3.8	2.4	.34	.56	.08	6

TABLE I
(Continued)

		%		RC		RI		#
		$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	
S_9	WR	81.6	9.3	2.6	.40	.62	.37	10
	CA	62.2	23.0	2.2	.90	1.25	.56	7
	WR	91.5	5.3	3.7	.39	.33	.20	6
	HA	94.7	4.4	3.7	.80	.20	.18	10
	WR	96.4	2.3	3.9	.34	.15	.10	4
S_{11}	WR	82.9	14.7	1.8	.49	.35	.28	12
	CA	70.5	11.7	1.6	.41	.63	.23	6
	WR	84.5	5.8	2.1	.19	.40	.16	4
	HA	88.7	17.6	3.0	.94	.28	.28	16
	WR	78.3	13.1	2.1	.45	.60	.37	5

		%		RC		RI		#
		$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	
S_{10}	WR	70.3	14.2	1.7	.40	.70	.32	12
	CA	40	19.4	1.4	.70	1.73	.60	6
	WR	69.5	15.2	2.1	.49	.92	.46	11
	HA	81.4	10.1	2.9	.67	.64	.36	14
	WR	77.1	12.8	2.2	.60	.60	.24	5

Note: WR refers to Conventional algorithm with a written record
CA refers to Conventional algorithm
HA refers to Hutchings' algorithm

% indicates mean session percent accuracy
RC indicates mean session rate of columns correct/minute
RI indicates mean session rate of columns incorrect/minute
# indicates the number of sessions in each condition

Table II. Individual student means and standard deviations for session performance across computational methods.

TABLE II

Individual Student Means and Standard Deviations for Session Performance Across Computational Methods

		%		RC		RI		#			%		RC		RI		#
		$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD				$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	
S_1	WR	91.5	9.4	2.5	.31	.23	.26	11	S_2	WR	95.2	6.2	2.9	.80	.14	.19	10
	CA	76.5	13.0	2.4	.74	.70	.35	8		CA	90.3	6.8	3.3	.39	.34	.25	7
	WR	93.6	6.5	3.0	.31	.20	.20	5		WR	96.8	2.9	3.9	.52	.12	.11	5
	HA	94.6	5.6	3.7	.57	.20	.18	11		HA	96.7	6.0	4.2	.59	.10	.17	13
	CA	90.1	8.1	3.6	.28	.40	.28	4		CA	89.6	7.9	3.5	.54	.40	.28	5
S_{12}	WR	87.7	5.4	2.4	.37	.34	.16	10	S_{13}	WR	56.8	32.2	1.4	.82	1.1	.85	12
	CA	63.9	17.2	2.3	.62	1.3	.67	7		HA	72.9	17.5	1.9	.57	.66	.34	10
	WR	79.2	13.6	2.6	.44	.69	.45	11		WR	66.2	11.8	1.7	.26	.92	.36	5
	HA	92.8	7.8	3.6	.46	.28	.30	10		CA	29.8	17.2	1.1	.73	2.4	.54	5
	CA	69.3	8.0	2.9	.14	1.3	.42	2									
	WR	93.3	2.2	4.4	.11	.26	.11	3									

TABLE II
(Continued)

		%		RC		RI		#			%		RC		RI		#
		$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD				$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	
S_{14}	WR	86.3	7.4	2.1	.44	.33	.15	8	S_{15}	WR	84.9	10.7	2.0	.35	.35	.26	9
	HA	74.8	31.3	2.4	1.2	.51	.42	12		HA	81.5	7.4	2.2	.19	.50	.21	10
	WR	63.4	13.8	1.9	.57	1.0	.32	5		WR	67.7	11.0	1.5	.19	.70	.26	4
	CA	60.2	17.2	2.5	.75	1.6	.67	4		CA	59.2	15.6	2.2	.74	1.4	.53	5

WR refers to Conventional algorithm with a written record

CA refers to Conventional algorithm

HA refers to Hutchings' algorithm

% indicates mean session percent accuracy

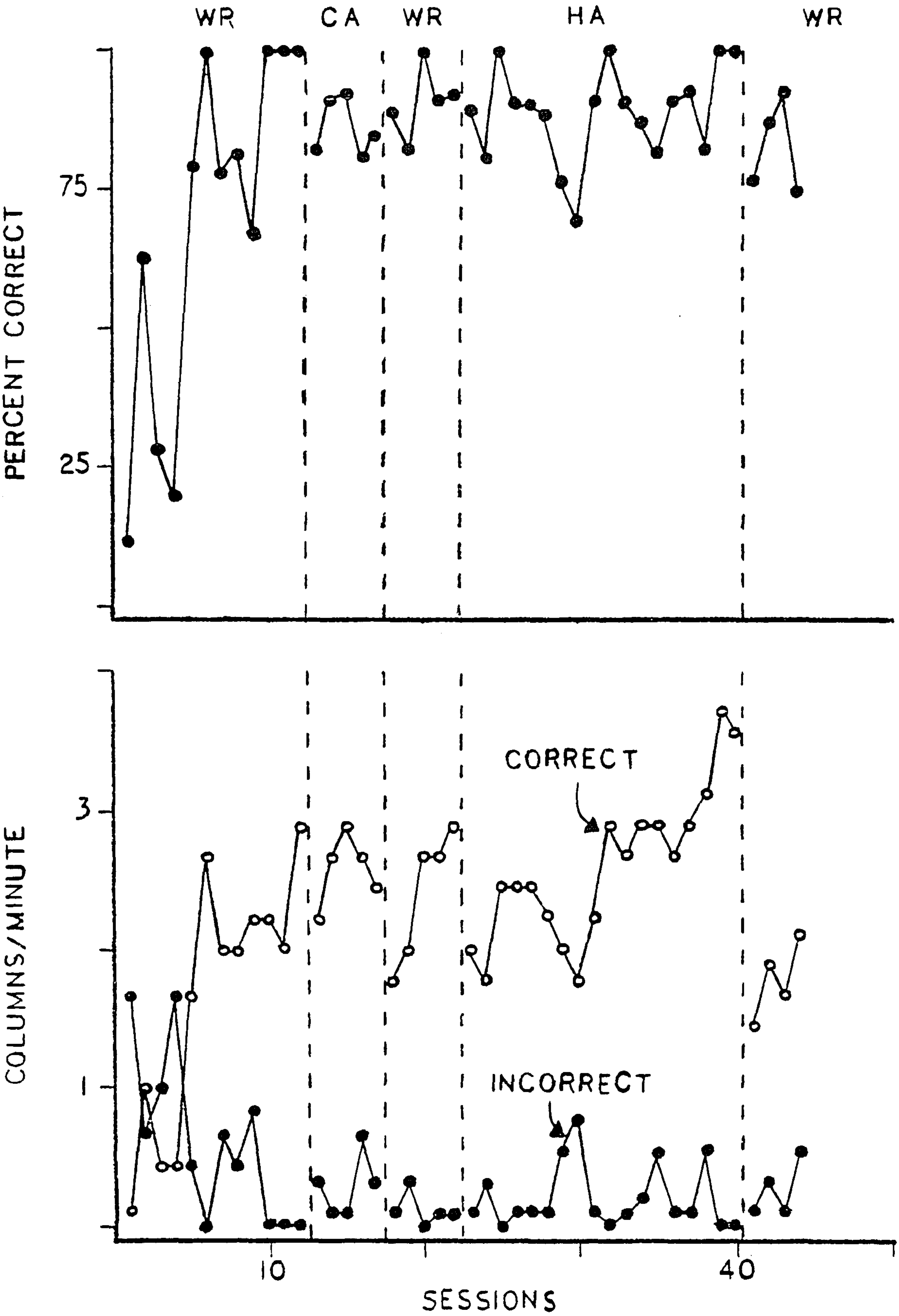
RC indicates mean session rate of columns correct/minute

RI indicates mean session rate of columns incorrect/minute

# indicates the number of sessions in each condition

Figure 1. Columns/minute correct and incorrect for all computational methods for Subject 4.

Figure 2. Percentage of columns added correctly for all computational methods for Subject 4.


WR
CA
WR
HA
WR
PERCENT CORRECT
75
25
COLUMNS/MINUTE
3
1
CORRECT
INCORRECT
10
40
SESSIONS

and hence a large standard deviation score. If these systematic errors are left out, the mean increases to 91.5 and the standard deviation drops to 6.9.

Figure 3 shows the individual scores for Subject 10 throughout the study. These data are representative of those subjects who had greater accuracy when utilizing the written record algorithm. There is some overlap among the raw scores between the written record (WR) and the Conventional algorithm (CA). But the range of correct responding is increased when the subject utilizes the algorithm with the full written record.

Figure 5 shows the individual scores for Subject 9. Again, higher accuracy is seen when a full written record is used with the Conventional algorithm. This subject also showed a decrease in variability when utilizing the written record algorithm. This same effect was seen with a number of other subjects who had greater greater accuracy with the written record vs. no record.

Rate of columns correct. The mean rates of columns correct with standard deviations for all subjects are presented in Tables I and II. Ten subjects had mean correct rates that were consistently higher with the written record algorithm (WR). Subjects 2, 4 and 15 had no observable difference between means with these two methods. Subjects 8 and 14 had slightly higher correct rates for the Conventional algorithm with no record (CA). Figure 2 shows no observable difference with Subject 4. There were upward trends for this subject in Conditions 3 and 5.

Figures 6 and 8 show the higher correct rates with the written

Figure 3. Columns/minute correct and incorrect for all computational methods for Subject 10.

Figure 4. Percentage of columns added correctly for all computational methods for Subject 10.

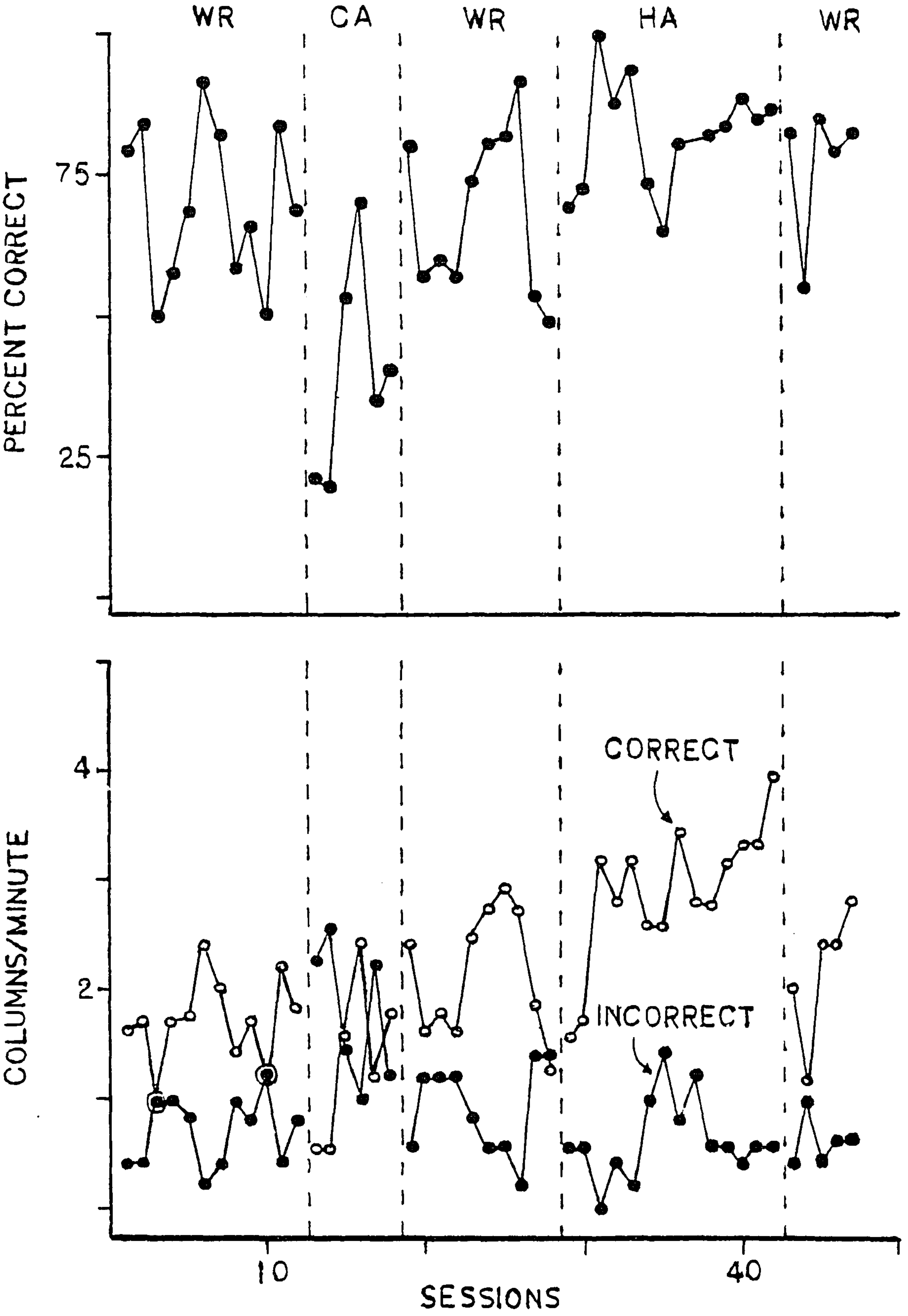

WR
CA
WR
HA
WR
PERCENT CORRECT
75
25
COLUMNS/MINUTE
4
2
CORRECT
INCORRECT
10
40
SESSIONS

Figure 5. Columns/minute correct and incorrect for all computational methods for Subject 3.

Figure 6. Percentage of columns added correctly for all computational methods for Subject 3.

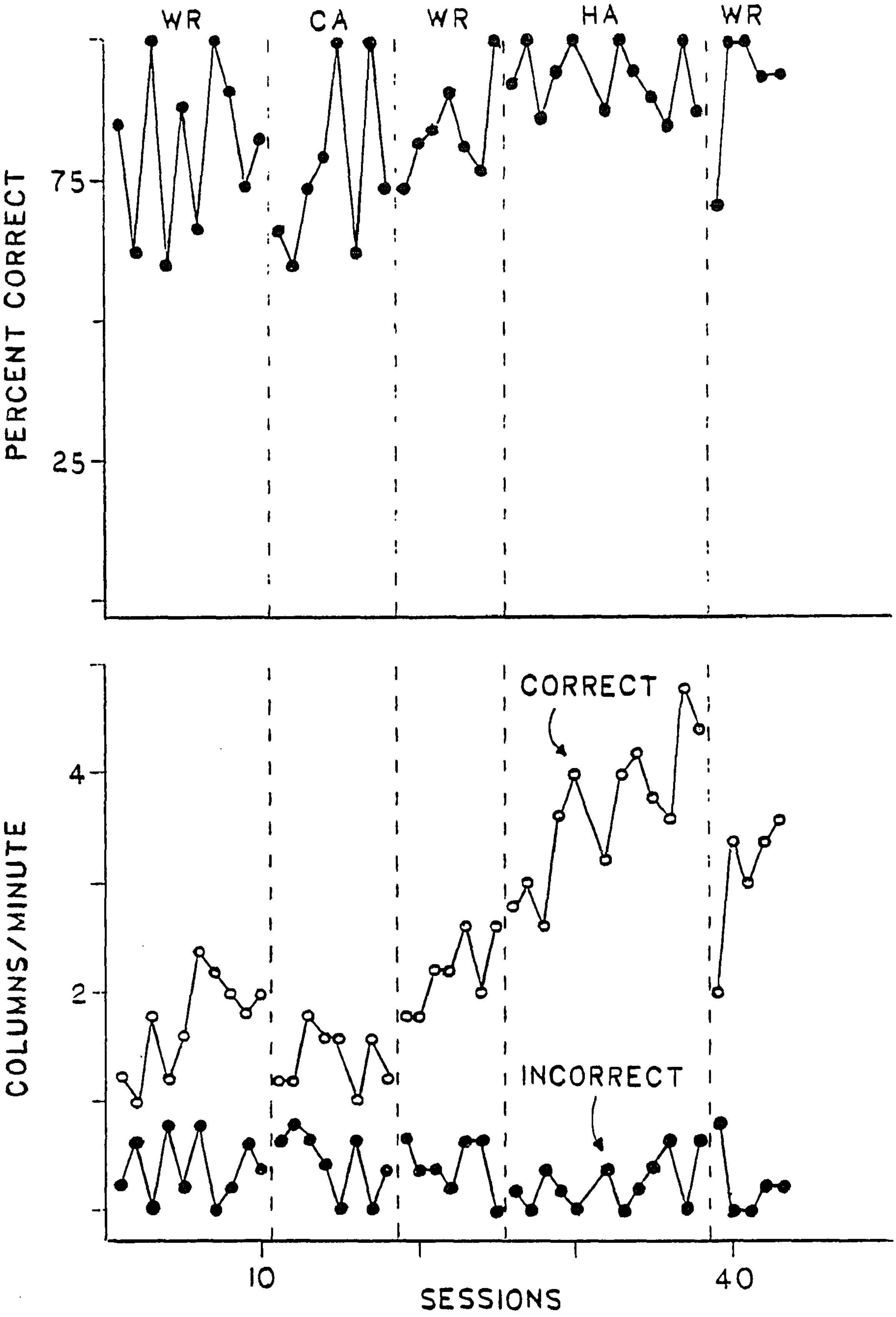
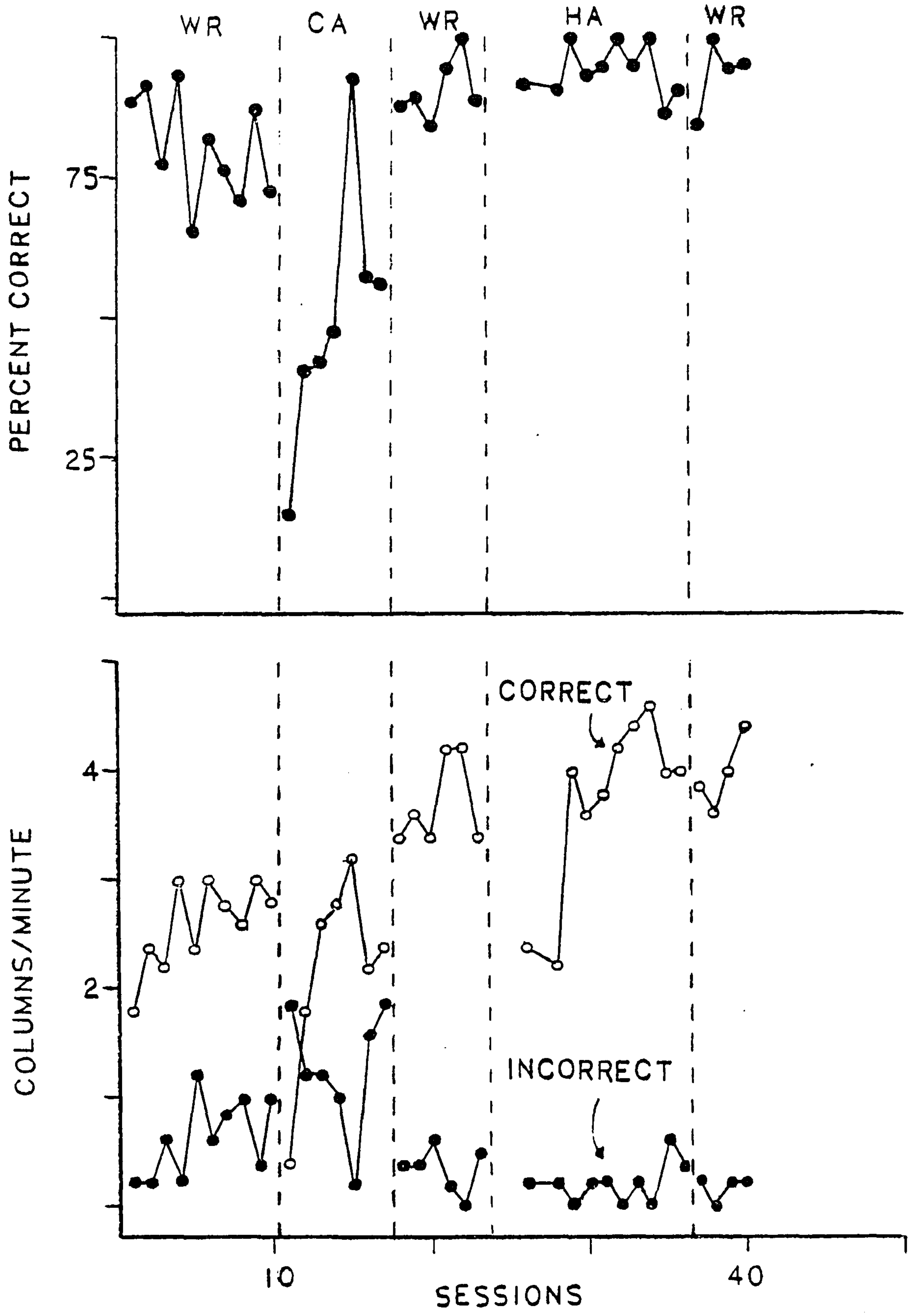

WR
CA
WR
HA
WR
PERCENT CORRECT
75
25
COLUMNS/MINUTE
4
2
CORRECT
INCORRECT
10
40
SESSIONS

Figure 7. Columns/minute correct and incorrect for all computational methods for Subject 9.

Figure 8. Percentage of columns added correctly for all computational methods for Subject 9.


WR
CA
WR
HA
WR
PERCENT CORRECT
75
25
COLUMNS/MINUTE
4
2
CORRECT
INCORRECT
10
40
SESSIONS

record (WR) versus no record (CA) for Subjects 3 and 9. There is an upward trend in the correct rate for Subject 9 in the last condition. There are also upward trends in Conditions 3 and 5 for Subject 3.

Figure 4 shows a slightly higher correct rate for Subject 10 when using the written record algorithm (WR) versus the Conventional algorithm (CA). Here again, there is an upward trend in the correct rate in the last condition for this subject.

Rate of columns incorrect. The mean rate of columns incorrect and the standard deviations for all subjects are presented in Tables I and II. In every case, except one, the mean rate of columns incorrect was lower when the subject utilized the written record (WR) versus no record (CA) algorithm. The one exception is in the first condition of Subject 4. Again, looking at the daily raw scores and worksheets, systematic errors are detected in Sessions 1, 3 and 4. Without these sessions, the error rate drops to .24 and is consistent with the data for this subject in subsequent conditions. The Conventional algorithm with the written record consistently shows lower error rates when compared with the Conventional algorithm with no record.

Comparisons of basic facts vs. complex facts

Percent of columns correct. The means and standard deviations for the percent accuracy across all computational methods are presented in Tables I and II. Eleven of the 15 subjects had greater mean accuracies when utilizing the Hutchings' Low-Stress algorithm

(HA) when compared with the Conventional algorithm with the full written record. No observable mean differences were seen with Subjects 2, 4, 5 and 9.

Not one subject had observable mean accuracies that were consistently higher with the Conventional algorithm with the written record (WR) when compared with the Hutchings' Low-Stress algorithm. However, these subjects did have one condition each where the Conventional algorithm with the record did have a slightly higher mean than the Low-Stress algorithm. Subjects 2 and 9 had positive mean differences for the Conventional algorithm of .1 and 1.1%, respectively. Subjects 4 and 5 had positive mean differences for the Conventional written record algorithm of 1.9 and 2.4%, respectively. Subjects 12 and 15 had only slight mean percentage differences in favor of the Low-Stress algorithm. Though there were no observable mean differences for Subject 9 looking at Figure 7 (which shows daily scores) there does appear to be a decrease in the variability of accuracy when the subject utilized the Low-Stress algorithm. This same effect was seen with a number of subjects, exemplified by Subject 3 (see Figure 5). Use of the Low-Stress algorithm either decreases the variability (seen with Subjects 3 and 9), increases the mean accuracy (Subject 10, see Figure 3), or shows no observable differences (Subject 4, Figure 1).

Rate of columns correct. All subjects except Subject 9 had consistently higher mean correct rates with the Low-Stress algorithm than the means obtained with the Conventional algorithm with the written record (see Tables I and II). Subjects 2, 4 and 5, who had no measurable differences in mean accuracies, did show increased mean rates when using the Low-Stress algorithm. Subject 9 showed no mean differences between these two algorithms (HA and WR) in the last two conditions (see Figure 8).

A number of subjects showed increasing trends in correct rates when using the Low-Stress algorithm. This is shown in Figures 2, 4, 6 and 8. There generally was a gradual increase in correct rates over all experimental conditions irrespective of the sequence in which the conditions were presented. When there was a decrease across experimental conditions, it generally occurred in the last condition (see Figures 2, 4 and 6).

Rate of columns incorrect. There are seven subjects who had lower mean error rates with the Low-Stress algorithm when compared with the written record algorithm (see Tables I and II). There were eight subjects for whom one comparison (across conditions) between the Conventional algorithm with a record and the Hutchings' algorithm showed the written record having a slightly lower mean error rate. In the other comparison the Low-Stress algorithm had the lower mean error rate. Figures 2, 4 and 6 show how close the actual error rates were between these two methods. Mean differences in error rates were usually in the tenths value with sometimes the only difference being seen in a hundredths of a point.

Comparisons of the Hutchings' Low-Stress algorithm (HA) and the Conventional algorithm without a full written record (CA)

Percent of columns correct. Tables I and II show that with every subject the Low-Stress algorithm yielded higher mean percent correct when compared with the Conventional algorithm (CA). The smallest difference is seen with Subject 4 (2.4%) to the largest difference with Subject 13 (43.6%). Subject 10 showed a difference between means of 41.4%. Figure 3 graphically shows that there is only one data point of overlap between these two methods (Session 14). Figure 7 shows the same effect with Subject 9: there is only one data point of overlap between the two methods (Session 15). Hutchings' Low-Stress algorithm often decreased the variability in accuracy which can be seen by looking at Figure 5 for Subject 3. Subject 4 has the smallest difference between the means. Figure 1 shows the daily scores for this subject. It is difficult to determine through visual inspection whether there is a difference in accuracy between the two methods with Subject 4. This graph is typical of those subjects who had mean differences of less than 10% (Subjects 1, 2, 4, 5 and 8).

Rate of columns correct. Fourteen out of the 15 subjects had higher mean correct rates when utilizing the Low-Stress algorithm as compared with the Conventional algorithm (CA). Subject 14 had a mean correct rate of 2.5 with the Conventional algorithm compared to a mean of 2.4 correct rate with the Low-Stress algorithm (Conditions 2 and 4 in Table I). This is a relatively small difference between the two means. Subjects 4, 10, 3 and 9 are typical of those subjects who had higher mean correct rates with the Low-Stress algorithm.

Figures 2, 4, 6 and 8 show that the daily scores were higher in almost all instances. It should be noted that there are increasing trends in the correct rate for all of these subjects when they used the Low-Stress algorithm. This effect was seen with 11 of the 15 subjects in this study. The remaining four had relatively stable correct rates when using this method of computation. Subject 9 (see Figure 8) showed an upward trend in the correct rate with the Conventional algorithm also. This same effect was seen with three other subjects.

Student performance on standardized probes

Addition computation probes. Table III presents the data for all subjects on all probes. Addition computation scores are indicated in the table by the letter "N". Ten subjects had no changes in their percent of items scored as correct across the entire study. Subjects 5, 6 and 15 had increases of 7.3%, 18.5% and 14.7%, respectively. This increase was not noted to occur with any particular condition of the study and was usually a gradual increase over the course of this investigation. Two subjects, 12 and 13, had decreases of 11.0% and 7.5%, respectively.

Place value probes. These data are presented in Table III. Place value scores are indicated in the table by the letter "P". The data here are somewhat different than the addition probes. Only four subjects showed no changes in percent of items scored correct (Subjects 3, 5, 8 and 12). Ten subjects showed some increase in scores. This ranged from 3.4% (one more item correctly answered) to 41.4% (12 more items correctly answered). Four subjects showed increases less than 10%, two had increases in the teens, and four

Table III. Individual student performance on place value and addition computation probes.

TABLE III

Individual Student Performance on Place Value and Addition Computation Probes

		Number of Probes			
Subject		1	2	3	4
S_1	N	92.6	92.6	92.6	92.6
	P	68.9	75.8	55.1	72.4
S_2	N	96.3	92.6	-	96.3
	P	37.9	65.5	-	79.3
S_3	N	81.5	88.8	81.5	81.5
	P	79.3	79.3	72.4	79.3
S_4	N	88.8	88.8	88.8	88.8
	P	65.5	65.5	72.4	79.3
S_5	N	81.5	92.6	88.8	88.8
	P	79.3	75.8	82.7	79.3
S_6	N	74.1	88.8	-	92.6
	P	51.7	51.7	-	58.6
S_7	N	66.6	59.2	62.9	66.6
	P	34.5	37.9	48.3	44.8
S_9	N	88.8	77.7	85.1	88.8
	P	31	41.4	48.3	51.7
S_{10}	N	70.4	70.4	70.4	70.4
	P	51.7	48.3	55.2	62.1
S_{11}	N	92.6	88.8	92.6	92.6
	P	62.1	44.8	44.8	58.6
S_{12}	N	85.1	77.7	81.5	74.1
	P	55.1	65.5	55.1	55.1
S_{13}	N	74.1	-	74.1	66.6
	P	51.7	-	55.1	55.1
S_{14}	N	81.5	85.1	-	81.5
	P	55.1	55.1	-	62.1
S_{15}	N	74.1	74.1	81.5	88.8
	P	17.2	17.2	48.3	44.8

TABLE III
(Continued)

Subject		Number of Probes 1	2	3	4
S_8	N	85.1	-	85.1	85.1
	P	37.9	-	20.6	37.9

Note: N indicates addition of whole numbers probe

P indicates place value probe

Probe 1 was administered before the study began

Probe 2 was administered after Condition 1

Probe 3 was administered after Condition 2

Probe 4 was administered after the last condition subjects had increases above 20%. Subject 11 had the only decrease in scores, from 62.1% to 58.6%. As mentioned above, this small change (3.4%) is only a change of one item correctly answered.

Sequence effects

The experimental design was such that if an order effect did occur, it would be detected. Both within subject and across subject analyses suggest that there were no sequence effects detected in this study. Responding in any one condition was not dependent upon the type of responding in a previous condition. It should be noted that this was determined through visual inspection of the subjects' graphs. Twelve who went through one sequence (A-B-A-C) and three who went through the other sequence (A-C-A-C). This interpretation is thus limited to the constraints of this study. This is consistent with other research that has been conducted with the Hutchings' Low-Stress algorithm (Gillespie, 1976; Rudolf, 1976; Zoref, 1976).

CHAPTER IV

Discussion

The results of this study indicate that student performances under a full record algorithm are generally superior to those under a recordless algorithm, with respect to both accuract and the rate of errors.

There is surprisingly a similar effect on the correct rate for the students of this study. The written record produced equal or higher correct rates for 13 of the 15 students even though this method required more overt behavior on the part of the student. Common sense might lead one to conclude that adding digits without writing would be faster than a method requiring pencil work. But, this was not the case. Only two students obtained higher mean correct rates with the Conventional algorithm and this difference was only slight. The obvious question to ask next is, why does the written record algorithm have these higher correct rates. One plausible reason could be that there is greater stimulus control over behavior chains than when no written record is used. Another reason could be that the students less often lose track of digits they are adding when a written record is used. Informal observations by the author during the sessions suggest that students who were using the recordless algorithm often added a column of digits more than once before writing a sum down. This seems to support the notion that the written record helps the student keep track of the digits being added. It is this author's opinion that both of these factors account for the superior student performance rather than one alone.

Results also indicate that the algorithm which required only basic math facts (Low-Stress) was superior to one requiring knowledge of both basic and complex facts. Ten of the 15 subjects had higher performances when using the Low-Stress algorithm. The other five subjects showed little or no increase in their performances. This variable was not as strong, apparently, as the written record variable. There are several possible reasons why this component did not show as great an effect as the written record component with the subjects in this study. First, all students in this study were high performers, as indicated by scores on the California Achievement Test (percentile ranks ranged from 49 to 98). These students probably already had mastered most of the 900 complex math facts. Another reason might be that these students were fourth graders who have had several years practice in recalling complex math facts.

The fact that the Low-Stress algorithm does not require a knowledge of complex math facts can be a disadvantage as well as an advantage. This algorithm does not attempt to teach these complex facts, since such are unnecessary when using the method. But, complex facts would be necessary were pupils to eventually add columns of digits without a written record, (although one might argue that such a task may be an irrelevant exercise). However, if the goal of teaching were to eventually lead to use of the Conventional algorithm, knowledge of complex facts would be needed.

A possible teaching sequence that might be efficient would be to start with the Hutchings' Low-Stress algorithm as an introductory teaching device. This affords a way of determining how well a student is acquiring addition operations and basic facts skills via the full written record of each binary sum. Also, it requires only a knowledge of basic math facts.

Once students are firm on this algorithm, a move can be made to the Conventional algorithm with a full written record. This teaches complex facts, but also allows the teacher to identify precise errors and complex facts in need of further drill or practice.

Finally, move to the Conventional algorithm without the record. This three stage sequence would involve a gradual shaping of student computational skills while fading along the components of the Hutchings' Low-Stress algorithm. This procedure would hopefully lead to the terminal behavior of adding digits covertly. Finally, were pupils to make many errors with the Conventional algorithm, the teacher could always have them return to the full record algorithm to precisely locate error patterns and fact knowledge deficiencies, and remediate these.

A common error pattern (Ashlock, 1976) made by the students on their worksheets was the multiplication of two single digits rather than addition of them. The students were currently mastering multiplication tables in their regular classroom, which could account for this type of error. As mentioned earlier, systematic errors were discussed with the student before the beginning of the next session, after the error pattern was discovered.

A most important finding was that all 15 subjects had higher performances with the Low-Stress algorithm when compared with the Conventional algorithm. The results of this study replicate findings by other researchers working with the Hutchings' Low-Stress algorithm (Hutchings, 1972; Alessi, 1974; Gillespie, 1976; Rudolf, 1976; Zoref, 1976). Though the level of this difference between the two algorithms is not as great as that reported in previous research with low performers, it does support findings of the comparative advantages of the Low-Stress over the Conventional algorithm, although this superiority may not be as profound with high as with low performers.

Ten out of 15 students had increases in their scores on the place value probes. As stated earlier this increase occurred gradually throughout the study and was not associated with any particular algorithm. The standardized probes of this study seem to support Brownell's (1939) suggestion that the type of notation which was used in computation might affect the acquisition of place value knowledge by students. It suggests that practice of addition skills with a full record can haave a positive effect on place value knowledge.

No effects were seen on the whole number addition probes. This could be due to a ceiling effect, as a number of students had a high number of items correct at the beginning of the study. There was little room for them to improve. Perhaps a larger number of items on this probe would have helped clarify any possible increase in addition skills.

Suggestions for future research include a systematic replication of this study with low-performing students. Fourth grade students may already have all of the skills necessary for the addition of large problems and may not need the advantages of the Hutchings' Low-Stress algorithm. A way to help determine this would be to administer a pretest similar to the one in this study but which would cover knowledge of complex, as well as basic, math facts. Students having this knowledge may not benefit as much from the advantages of the Low-Stress algorithm.

Improvements on this research include more time to run this investigation. Steady states of responding were obtained on only two of the dependent measures: accuracy and incorrect rate. This research was unable to determine just how high correct rates could have been because of generally rising correct rates in the Hutchings' Low-Stress algorithm condition when condition changes were made. This study was run for two months. Extention of all conditions would greatly help establish steady states of responding for all dependent measures, and thus help determine the upper limits for correct rate.

Future research might also involve a component analysis of Hutchings' Low-Stress subtraction algorithm. Again, with this algorithm there is no alternation between regrouping and subtraction. But, subtraction with decomposition algorithms uses only basic, and not complex, facts. Also, subtraction already uses a full (or almost full) record in the Conventional method. Still a component analysis of this algorithm would be very helpful.

APPENDICES

APPENDIX A

NAME ______________________________

1 8 3 5 2
5 4 7 9 8
6 2 3 5 8
2 7 4 3 7
6 1 1 3 9
6 2 9 7 7
6 9 6 4 5
____

9 4 6 5 7
5 2 5 1 2
7 2 7 4 3
4 9 2 5 1
3 5 9 6 6
9 1 4 1 6
1 7 2 1 6
____

2 3 3 6 5
4 9 9 8 8
6 4 3 8 6
2 1 4 6 6
4 1 3 8 4
1 9 8 5 5
7 3 7 5 6
____

9 2 9 3 6
7 9 8 9 9
8 5 7 1 6
4 8 8 2 2
8 5 6 9 7
7 9 9 2 4
1 7 4 3 9
____

APPENDIX B

I am going to show you the usual way of writing number facts and then another way of writing them.

You have all seen number facts written like this:

$$\begin{array}{r} 7 \\ +8 \\ \hline 15 \end{array}$$

Well, they can also be written like this, using two small (half-space) numbers instead of the line and plus sign.

$$\begin{array}{c} 7 \\ 1^{8}5 \end{array}$$

Do you still see the fifteen? (Point to both fifteens.)

I'll write the two examples next to one another.

$$\begin{array}{r} 7 \\ +8 \\ \hline 15 \end{array} \qquad \begin{array}{c} 7 \\ 1^{8}5 \end{array}$$

Do you all see the fifteen? (Point $\begin{array}{c} 7 \\ 1^{8}5 \end{array}$)

Let's look at another one. I can write "9 plus 5 is 14" like this $\begin{array}{r} 9 \\ +5 \\ \hline 14 \end{array}$ or like this $\begin{array}{c} 9 \\ 1^{5}4 \end{array}$.

Both of these say "9 plus 5 is 14."

Tell me what these say:

$$\begin{array}{r} 9 \\ +8 \\ \hline 17 \end{array} \quad \begin{array}{c} 9 \\ 1^{8}7 \end{array} \quad \begin{array}{r} 6 \\ +7 \\ \hline 13 \end{array} \quad \begin{array}{c} 6 \\ 1^{7}3 \end{array} \quad \begin{array}{r} 4 \\ +5 \\ \hline 9 \end{array} \quad \begin{array}{c} 4 \\ {}^{5}9 \end{array} \quad \begin{array}{r} 6 \\ +6 \\ \hline 12 \end{array} \quad \begin{array}{c} 6 \\ 1^{6}2 \end{array} \quad \begin{array}{r} 5 \\ +2 \\ \hline 7 \end{array} \quad \begin{array}{c} 5 \\ {}^{2}7 \end{array}$$

(Call on students, point to the full notation form $\begin{array}{c} 9 \\ 1^{8}7 \end{array}$ when asking.)

The little number on the right* is understood to be in the one's place, as are 9 and 8.

$$\begin{array}{c} 9 \\ 1^{8}7* \end{array}$$

The little number on the left* is understood to be in the ten's place.

$$\begin{array}{c} 9 \\ *1^{8}7 \end{array}$$

In other words, this is the same as this (point from "big 7" to "little 7"). And this is the same as this (point from "big one" to "little one").

$$\begin{array}{r} 9 \\ +8 \\ \hline 17 \end{array}$$

Now watch me write the following facts both ways.

$$\begin{array}{r} 9 \\ +7 \\ \hline 16 \end{array} \qquad \begin{array}{c} 9 \\ 1^{7}6 \end{array} \qquad \begin{array}{r} 8 \\ +5 \\ \hline 13 \end{array} \qquad \begin{array}{c} 8 \\ 1^{5}3 \end{array} \qquad \begin{array}{r} 4 \\ +5 \\ \hline 9 \end{array} \qquad \begin{array}{c} 4 \\ {}^{5}9 \end{array}$$

Look at the last pair. Are they different from the others? Note that there is no ten's place number <u>and</u> (do not draw until after saying this) there is no "little one" on the left.

Let's look at another.

a) $\begin{array}{r} 4 \\ +3 \\ \hline 7 \end{array}$ Is there any ten's number here? (Do not draw box until after asking question.

b) NO!! (repeat)

c) So will there be any little number on the left?

d) $\begin{array}{c} 4 \\ {}^{3}7 \end{array}$

(Do not draw box until after asking question.)

NO!! (repeat)

Again, $\begin{array}{r}4\\+3\\\hline 7\end{array}$ $\quad$ $\begin{array}{l}4\\3_7\end{array}$

If there is no ten's place number, there is no "little number" on the left.

Now watch me write the rest of these.

Notice $\begin{array}{r}3\\+1\\\hline 4\end{array}$ $\quad$ $\begin{array}{l}3\\1_4\end{array}$

no ten's number here so no "little number" here

but $\begin{array}{r}7\\+3\\\hline 10\end{array}$ $\quad$ $\begin{array}{l}7\\1^3 0\end{array}$

There <u>is</u> a ten's number here so there is a "little number" here

Again, notice $\begin{array}{r}5\\+1\\\hline 6\end{array}$ $\quad$ $\begin{array}{l}5\\1_6\end{array}$

There is <u>no</u> ten's number here so there is <u>no</u> "little number" here

but $\begin{array}{r}8\\+5\\\hline 13\end{array}$ $\quad$ $\begin{array}{l}8\\1^5 3\end{array}$

There is a ten's number here so there is a "little number" here

$$\begin{array}{r}5\\ \underline{+5}\\ 10\end{array}\qquad 1^{5}0\qquad \begin{array}{r}6\\ \underline{+8}\\ 14\end{array}\qquad 1^{8}4\qquad \begin{array}{r}1\\ \underline{+7}\\ 8\end{array}\qquad {}^{7}{}_{8}$$

$$\begin{array}{r}3\\ \underline{+1}\\ 4\end{array}\qquad 1_{4}\qquad \begin{array}{r}5\\ \underline{+4}\\ 9\end{array}\qquad 4_{9}\qquad \begin{array}{r}5\\ \underline{+9}\\ 14\end{array}\qquad 1^{9}4\qquad \begin{array}{r}4\\ \underline{+8}\\ 12\end{array}\qquad 1^{8}2$$

Now I am going to show you a special way of adding that uses only those "little numbers" on the right.

I'll say that again (repeat previous statement).

This should make your addition very easy and accurate. It is a scientific method and many scientists do addition this way. Watch.

$$\begin{array}{c}8\\ 5\\ 7\\ 9\\ 8\\ 6\\ 8\\ \underline{7}\end{array}$$

First, do you see that an example can be just number facts piled one atop the other? (Do not point with this question.)

OK! Here we go, start at the top, writing facts as you learned and using only numbers at the right for addition.

$$\begin{array}{c}8\\ 1^{5}_{7}3\\ 9\\ 8\\ 6\\ 8\\ \underline{7}\end{array}$$

a) Say, "The first fact we do may look a bit different because we do not have any little numbers yet." (Point)

b) Say, "This is the only time we will use two big numbers. In the rest of the example we use one little number and one big one."

c) Say, "Now, eight plus five is thirteen."

d) Write the thirteen, i.e., $1^{5}{}_{3}$ in the example.

$$\begin{array}{c}8\\ 1^{5}_{7}3\\ 1^{7}_{9}0\\ 8\\ 6\\ 8\\ \underline{7}\end{array}$$

a) Say, "We've written the thirteen but we'll use only the three."

b) Draw arrow 7^{3} .

c) Say, "Three plus seven is ten."

d) Write the 10, i.e., ${}_{1}7^{3}{}_{0}$ in the example.

$$\begin{array}{r} 8 \\ {}_{1}5_{3} \\ {}_{1}7^{3}_{0} \\ 9^{0}_{9} \\ 8^{9} \\ 6 \\ 8 \\ \underline{7} \end{array}$$

a) Say, "We've written the ten but we'll use only the 0."

b) Draw arrow 9^{0} .

c) Say, "Zero plus nine is nine."

d) Write 9, i.e., 9^{0}_{9} in the example.

$$\begin{array}{r} 8 \\ {}_{1}5_{3} \\ {}_{1}7^{3}_{0} \\ 9^{0}_{9} \\ {}_{1}8^{9}_{7} \\ 6^{7} \\ 8 \\ \underline{7} \end{array}$$

a) Say, "We've written the nine and look that's all we have this time because zero and nine is just nine. But that's OK because we only use the right-hand number anyway."

b) Draw arrow 3^{9} .

c) Say, "Nine plus eight is seventeen."

d) Write the seventeen, i.e., ${}_{1}8^{9}_{7}$ in the example.

$$\begin{array}{r} 8 \\ {}_{1}5_{3} \\ {}_{1}7^{3}_{0} \\ 9^{0}_{9} \\ {}_{1}8^{9}_{7} \\ {}_{1}6^{7}_{3} \\ 8^{3} \\ \underline{7} \end{array}$$

a) Say, "We've written the seventeen but we'll use only the seven."

b) Draw arrow 6^{9} .

c) Say, "Seven plus six is thirteen."

d) Write the thirteen, i.e., ${}_{1}6^{7}_{3}$ in the example.

$$\begin{array}{r} 8 \\ {}_{1}5_{3} \\ {}_{1}7^{3}_{0} \\ 9^{0}_{9} \\ {}_{1}8^{9}_{7} \\ {}_{1}6^{7}_{3} \\ {}_{1}8^{3}_{1} \\ \underline{7^{1}} \end{array}$$

a) Say, "We've written the thirteen but we'll use only the three."

b) Draw the arrow 8^{3} .

c) Say, "Three plus eight is eleven."

d) Write the eleven, i.e., ${}_{1}8^{3}_{1}$ in the example.

$$\begin{array}{ccc} & 8 & \\ & 5 & \\ 1 & & 3 \\ & 7 & \\ 1 & & 0 \\ & 9 & \\ & & 9 \\ & 8 & \\ 1 & & 7 \\ & 6 & \\ 1 & & 3 \\ & 8 & \\ 1 & & 1 \\ & 7 & \\ \hline & & 8 \end{array}$$

a) Say, "We've written the eleven but we'll use only the one.

b) Draw arrow 7^{1} .

c) Say, "One plus seven is eight."

d) Write the eight, i.e., 7^{1}_{8} in the example.

Now we're at the key part. All we've done is use number facts. We haven't done any "in your head" work.

Nevertheless, we already know the answer! Watch.

$$\begin{array}{ccc} & 8 & \\ & 5 & \\ 1 & & 3 \\ & 7 & \\ 1 & & 0 \\ & 9 & \\ & & 9 \\ & 8 & \\ 1 & & 7 \\ & 6 & \\ 1 & & 3 \\ & 8 & \\ 1 & & 1 \\ & 7 & \\ & & 8 \\ \hline 5 & 8 & \end{array}$$

The last little number on the right <u>is</u> the right half of the answer.

To get the left half, we just count the little numbers on the left that we didn't use. One, two, three, four, five, there are five of the, so the first half of the answer is five. The answer is 58.

Now watch me do another. Remember we use <u>only</u> the <u>right</u> side "little numbers." We will not bother to write the arrows anymore, just say

$$\begin{array}{ccc} & 6 & \\ & 8 & \\ 1 & & 4 \\ & 7 & \\ 1 & & 1 \\ & 6 & \\ & & 7 \\ & 9 & \\ 1 & & 6 \\ & 5 & \\ 1 & & 1 \\ & 8 & \\ & & 9 \\ & 3 & \\ 1 & & 2 \\ \hline 5 & 2 & \end{array}$$

6 plus 8 is 14
4 plus 7 is 11
1 plus 6 is 7
7 plus 9 is 16
6 plus 5 is 11
1 plus 8 is 9
9 plus 3 is 12

Now the last number on the right is a 2, so the right half of the answer is a 2! We get the left half of the answer by counting the little numbers on the left that we didn't use. One, two, three, four, five. There are five of them so the left half of the answer is 5. The answer is 52.

Now say the words for these with me as I do them at the board. (Children do <u>not</u> do this.)

```
 8         9         4
 5         5         8
1 3       1 4       1 2
 4         3         3
  7         7         5
 7         2         6
1 4         9       1 1
 9         6         1
1 3       1 5         2
 8         8         8
1 1       1 3       1 0
 3         7         7
  4       1 0         7
 6         9         6
1 0         9       1 3
___       ___       ___

52        49        43
```

Now copy these examples and do them by yourself. If you have any questions, ask me.

```
  6      8      5      9
  5      2      4      8
  9      7      9      3
  8      6      8      2
  5      9      7      7
  6      8      9      6
+ 9    + 5    + 8      8
___    ___    ___    + 9
                     ___
```

After most have finished, say. "Check your work with mine as I do them at the board."

After doing the examples, say, "Now let's review."

I'll write the work for another one on the board. I want someone to raise their hand and tell me what the answer is.

$$\begin{array}{ccc} & 6 & \\ & 8 & \\ 1 & & 4 \\ & 9 & \\ 1 & & 3 \\ & 5 & \\ & & 8 \\ & 7 & \\ 1 & & 5 \\ & 5 & \\ 1 & & 0 \\ & 9 & \\ & & 9 \\ & 3 & \\ 1 & & 2 \\ \hline \end{array}$$

6 plus 8 is 14
4 plus 9 is 13
3 plus 5 is 8
8 plus 7 is 15
5 plus 5 is 10
0 plus 9 is 9
9 plus 3 is 12

Say this part, do not write it, except as the half-space numerals.

(Point to box.) Who will tell me what the right side of the answer is and how he got it.

(Point to box.)

(Locate correct response.) Good! That's correct. The last little number on the right becomes the right side of the answer.

Who will tell me what the left side of the answer is and how he got it. (Locate correct response.) Good! That's correct, we count up the little numbers on the left for the left side of the answer.

Now, what do you suppose we do if there is more than one column? That is, if there is another column at the left of the column you're adding. Like this

$$\begin{array}{c} 4 \\ 7 \\ 6 \\ 8 \\ 7 \\ \hline \end{array} \qquad \begin{array}{ccc} & 6 & \\ & 8 & \\ 1 & & 4 \\ & 7 & \\ 1 & & 1 \\ & 6 & \\ & & 7 \\ & 8 & \\ \hline 1 & & 5 \end{array}$$

Can we still write our left-hand answer number at the bottom if there is more than one column? No, we can't?

When there's more than one column, each column can have only one number at the bottom (except for the very <u>last</u> column which does have the usual two).

So the single number that we put at the bottom is always the right-hand number.

(Write and point)

$$\begin{array}{ccc} & 8 & \\ & 7 & \\ 1 & & 5 \\ & 6 & \\ 1 & & 1 \\ & 9 & \\ 1 & & 0 \\ & 8 & \\ & & 8 \\ & 5 & \\ 1 & & 3 \\ \hline & 3 & \end{array}$$

What can we do with the left-hand number?

Would it make sense to throw it away? No, it's part of the problem. *So we will put it at the very top of the next column at the left. That way we don't lose it and it's still on the left side.

Watch! (Write on board.)
Count the little number on the left with me.
One, two, three, four.
There are four of them so we write a 4 at the top of the next column.

$$\begin{array}{c} 4 \\ 6 \\ 8 \\ 7 \\ 5 \\ 9 \\ 6 \\ 3 \\ 8 \\ \hline \\ \end{array} \qquad \begin{array}{ccc} & 5 & \\ & 7 & \\ 1 & & 2 \\ & 9 & \\ 1 & & 1 \\ & 6 & \\ & & 7 \\ & 4 & \\ 1 & & 1 \\ & 8 & \\ & & 9 \\ & 7 & \\ 1 & & 6 \\ & 2 & \\ & & 8 \\ \hline & 8 & \end{array}$$

Now, when I start adding that column I will start with the four (4) first. Let's be sure you understand. (Repeat twice from the *.)

This is called carrying, some of you already understand it. Good. Carrying is very easy.

<u>But</u> carrying is very important. You must never forget to carry.

Look at these examples and tell me what to write at the top of the left-hand column. (Write on board.)

$$\begin{array}{cccc} 6 & & 8 & \\ 5 & & 9 & \\ & 1 & & 7 \\ 6 & & 2 & \\ & & & 9 \\ 8 & & 5 & \\ & 1 & & 4 \\ 7 & & 6 & \\ & 1 & & 0 \\ 4 & & 3 & \\ & & & 3 \\ \hline \end{array} \qquad \begin{array}{cccc} 8 & & 5 & \\ 7 & & 6 & \\ & 1 & & 1 \\ 9 & & 9 & \\ & 1 & & 0 \\ 4 & & 5 & \\ & & & 5 \\ 3 & & 8 & \\ & 1 & & 3 \\ 4 & & 4 & \\ & & & 7 \\ \hline \end{array} \qquad \begin{array}{cccc} 7 & & 6 & \\ 6 & & 6 & \\ & 1 & & 2 \\ 9 & & 2 & \\ & & & 4 \\ 1 & & 8 & \\ & 1 & & 2 \\ 1 & & 3 & \\ & & & 5 \\ 4 & & 3 & \\ & & & 8 \\ \hline \end{array} \qquad \begin{array}{cccc} 8 & & 5 & \\ 1 & & 8 & \\ & 1 & & 3 \\ 1 & & 2 & \\ & & & 5 \\ 4 & & 7 & \\ & 1 & & 2 \\ 9 & & 5 & \\ & & & 7 \\ 6 & & 3 & \\ & 1 & & 0 \\ \hline \end{array}$$

(Do with volunteers from class at board.) Good, we write the left-hand answer number at the top of the next column. (Repeat three times.)

Remember though that for the <u>last</u> column only, the left-hand answer number is at the bottom as though it were a single column.

Now, copy these examples and do them with me.

$$\begin{array}{rr} 7 & 6 \\ 5 & 9 \\ 8 & 7 \\ 6 & 9 \\ 8 & 3 \\ 9 & 5 \\ \hline \end{array} \qquad \begin{array}{rrr} 7 & 9 & 8 \\ 8 & 7 & 7 \\ 6 & 8 & 5 \\ 4 & 7 & 6 \\ 6 & 9 & 5 \\ 8 & 3 & 9 \\ 4 & 2 & 2 \\ \hline \end{array} \qquad \begin{array}{rrrr} 4 & 7 & 8 & 5 \\ 5 & 7 & 6 & 9 \\ 8 & 7 & 6 & 2 \\ 8 & 5 & 7 & 6 \\ 8 & 3 & 9 & 5 \\ \hline \end{array}$$

Again, do you see that I always carry the number of tens to the top of the next column? (Point and illustrate example.) Except when there are no more columns. Then I write the number of tens on the bottom line as part of the answer. (Point and illustrate with each.)

Good! Are there any questions?

Now take these dittoed examples and do them by yourselves. If you have trouble, ask me for help.

$$\begin{array}{rrrr} & 6 & 8 & 7 \\ & 4 & 8 & 3 \\ & 6 & 9 & 5 \\ + & 8 & 7 & 4 \\ \hline \end{array} \qquad \begin{array}{rrr} & 6 & 9 \\ + & 5 & 8 \\ \hline \end{array} \qquad \begin{array}{rr} & 7 \\ & 6 \\ & 8 \\ & 4 \\ & 7 \\ & 6 \\ & 9 \\ & 0 \\ & 5 \\ & 1 \\ + & 3 \\ \hline \end{array} \qquad \begin{array}{rrr} & 6 & 5 \\ & 6 & 3 \\ & 3 & 9 \\ & 8 & 4 \\ + & 4 & 2 \\ \hline \end{array}$$

$$\begin{array}{rrrrrrr} & 6 & 8 & 7 & 6 & 9 & 5 \\ & 4 & 8 & 7 & 6 & 4 & 2 \\ + & 8 & 7 & 6 & 9 & 8 & 3 \\ \hline \end{array} \qquad \begin{array}{rrrrrrrr} & 4 & 8 & 7 & 6 & 8 & 7 & 6 \\ & 9 & 8 & 7 & 6 & 8 & 7 & 6 \\ & 9 & 8 & 7 & 6 & 8 & 5 & 6 \\ & 7 & 9 & 5 & 6 & 7 & 9 & 3 \\ & 8 & 5 & 2 & 7 & 4 & 9 & 8 \\ + & 6 & 7 & 8 & 5 & 6 & 7 & 8 \\ \hline \end{array}$$

Be sure to make and place your numbers neatly!

(Allow time needed for most to finish.)

Now, I will do them. Check your work against mine.

APPENDIX C

Review

We are going to review the new way of writing number facts which we practiced yesterday.

5
9
8
6
8
7
8
$\underline{7}$

We are going to start at the top, writing number facts as you learned yesterday.

5
${}_{1}9_{4}$
8
6
8
7
8
$\underline{7}$

a) Say, "Remember that during the beginning of the example is the only time that we use two big numbers. In the rest of the example, we use one little number and one big number."

b) Say, "Five plus nine is fourteen."

c) Write the fourteen in the example as ${}_{1}9_{4}$.

5
${}_{1}9_{4}$
${}_{1}8_{2}$
6
8
7
8
$\underline{7}$

a) Say, "We've written the fourteen but we'll use only the four."

b) Say, "Four plus eight is twelve."

c) Write the twelve in the example as ${}_{1}8_{2}$.

5
${}_{1}9_{4}$
${}_{1}8_{2}$
6_{8}
8
7
8
$\underline{7}$

a) Say, "We've written the twelve but we'll only use the two."

b) Say, "Two plus six is eight."

c) Write the eight in the example as 6_{8}.

$$\begin{array}{ccc} & 5 & \\ & 9 & \\ 1 & & 4 \\ & 8 & \\ 1 & & 2 \\ & 6 & \\ & & 8 \\ & 8 & \\ 1 & & 6 \\ & 7 & \\ & 8 & \\ & 7 & \\ \hline \end{array}$$

a) Say, "We've written the eight and we use just the eight."

b) Say, "Eight plus eight is sixteen."

c) Write the sixteen in the example as ${}_{1}{}^{8}{}_{6}$.

$$\begin{array}{ccc} & 5 & \\ & 9 & \\ 1 & & 4 \\ & 8 & \\ 1 & & 2 \\ & 6 & \\ & & 8 \\ & 8 & \\ 1 & & 6 \\ & 7 & \\ 1 & & 3 \\ & 8 & \\ & 7 & \\ \hline \end{array}$$

a) Say, "We've written the sixteen but we'll use only the six."

b) Say, "Six plus seven is thirteen."

c) Write the thirteen in the example as ${}_{1}{}^{7}{}_{3}$.

$$\begin{array}{ccc} & 5 & \\ & 9 & \\ 1 & & 4 \\ & 8 & \\ 1 & & 2 \\ & 6 & \\ & & 8 \\ & 8 & \\ 1 & & 6 \\ & 7 & \\ 1 & & 3 \\ & 8 & \\ 1 & & 1 \\ & 7 & \\ \hline \end{array}$$

a) Say, "We've written the thirteen but we'll use only the three."

b) Say, "Three plus eight is eleven."

c) Write the eleven in the example as ${}_{1}{}^{8}{}_{1}$.

$$\begin{array}{ccc} & 5 & \\ & 9 & \\ 1 & & 4 \\ & 8 & \\ 1 & & 2 \\ & 6 & \\ & & 8 \\ & 8 & \\ 1 & & 6 \\ & 7 & \\ 1 & & 3 \\ & 8 & \\ 1 & & 1 \\ & 7 & \\ & & 8 \\ \hline \end{array}$$

a) Say, "We've written the eleven but we'll use only the one."

b) Say, "One plus seven is eight."

c) Say, "The last little number on the right is the right half half of the answer. To find the left half, we just count the little numbers on the left that we did not use. Who can tell me what the right half of the answer is? Eight! Right. Now, who can tell me what the left half of the answer is? Five! Right, the answer, then is 58."

$$\begin{array}{ccc} 8 & 7 & 7 \\ 6 & 8 & 5 \\ 4 & 7 & 6 \\ 6 & 9 & 5 \\ 8 & 3 & 9 \\ \hline \end{array}$$

a) Say, "Now let's try a bigger example. We are going to move faster this time because you have done so well."

b) Say, "Let's start with the right column (point to it). Seven plus five is twelve. (Write the twelve in the example as ${}_1 5_2$.) Two plus six is eight. (Write the eight in the example as 6_2.) Eight plus five is thirteen. (Write the thirteen in the example as ${}_1 5_3$.) Three plus nine is twelve. (Write the twelve in the example as ${}_1 9_2$.) We write the two below the right column and carry the three to the top of the next column." (Write the three above the second column.

$$\begin{array}{ccc} 8 & 7 & 7 \\ 6 & 8 & 5 \\ 4 & 7 & 6 \\ 6 & 9 & 5 \\ 8 & 3 & 9 \\ \hline \end{array}$$

Say, "Now, when I start adding this column (point to second column), I will start with the three. Three plus seven is ten. (Write the ten in the example as ${}_1 7_0$.) Zero plus eight is eight. (Write the eight in the example as 8_0.) Eight plus seven is fifteen. (Write the fifteen in the example as ${}_1 8_5$.) Five plus nine is fourteen. (Write the fourteen in the example as ${}_1 9_4$.) Four plus three is seven. (Write the seven in the example as 3_7.) We write the seven below the column. Then we count the tens: One, two three tens. We carry the three to the top of the next column." (Write the three above the last column.)

$$\begin{array}{ccc} 3 & 3 & \\ 8 & {}_1 7_0 & 7 \\ 6 & 8_8 & {}_1 5_2 \\ 4 & {}_1 7_5 & 6_8 \\ 6 & {}_1 9_4 & {}_1 5_3 \\ 8 & 3_7 & {}_1 9_2 \\ \hline & 7 & 2 \end{array}$$

Say, "Now our example looks like this (pointing to example). Who can tell me the numbers we are going to add next? Right. We are going to add the three and the eight."

$$\begin{array}{ccc} 3 & 3 & \\ {}_1 8_1 & {}_1 7_0 & 7 \\ 6_7 & 8_8 & {}_1 5_2 \\ {}_1 4_1 & {}_1 7_5 & 6_8 \\ 6_7 & {}_1 9_4 & {}_1 5_3 \\ {}_1 8_5 & 3_7 & {}_1 9_2 \\ \hline \end{array}$$

Say, "Three plus eight is elevent. (Write the eleven in the example as ${}_1 8_1$.) Who can tell me the numbers we are going to add next? Right. We are going to add the one and the six. One plus six is seven. (Write the seven in the example as 6_7.) Who can tell me the numbers we are going to add next? Right. We are going to add the seven and the four. Seven plus four is eleven. (Write the eleven in the example as ${}_1 4_1$.) Who can tell me the numbers we are going to add next? Right. We are going to add one and six. One plus six is seven. (Write the seven in the example as 6_7.) Seven plus eight is fifteen. (Write the fifteen in the example as ${}_1 8_5$.)

(Continued from previous page)
Now we write the five below the column. (Write the five below the third column.) The we count the tens: One, two three tens. Because there are no more columns, we write the three to the left of the five." (Write the three to the left of the five in the example.)

Now, copy these examples and do them with me.

8	7
5	9
3	8
9	6
7	8
9	5

3	5	2
8	9	7
2	2	2
9	3	8
5	9	6
6	7	4
5	8	6

5	9	3	8
7	6	5	8
2	6	7	8
9	6	7	5
5	8	7	4

Are there any questions? Good. Now take these dittoed examples and do them by yourselves. If you have trouble, ask me for help. Be sure to make and place your numbers neatly.

6
8
7
3
3
5
8
4
9
1
7

6	7	8	6	7	8	4
6	7	8	6	7	8	9
6	5	8	6	7	8	9
3	9	7	6	5	9	7
8	9	4	7	2	5	8
8	7	6	5	8	7	6

(Allow time for most to finish.)

Now I will do them. Check your work against mine.

(Do examples on the blackboard. Answer questions. Emphasize the need to write neatly and the need to count the "carry number" correctly, demonstrate the latter while doing the work. State that the "carry number" is always <u>written</u> at the top of the column to which it is carried.

I am going to write some addition examples on the board. Begin to do them as soon as you can see them. After I finish writing all of them, I will go back and write in the answers. After you have finished working all of the examples, go back and check your answers against the answers I have written on the board. As soon as you have finished, turn your papers in.

Does everyone know what to do? (Pause momentarily.) Good. Begin...

$$\begin{array}[t]{r} 8\\5\\7\\9\\8\\6\\8\\+7\\\hline\end{array}\quad\begin{array}[t]{r} 8\\5\\6\\7\\9\\8\\3\\+6\\\hline\end{array}\quad\begin{array}[t]{r} 9\\5\\3\\2\\6\\8\\7\\+9\\\hline\end{array}\quad\begin{array}[t]{r} 4\\8\\3\\6\\1\\8\\7\\+6\\\hline\end{array}\quad\begin{array}[t]{r} 6\\8\\9\\5\\7\\5\\9\\+3\\\hline\end{array}\quad\begin{array}[t]{rr} 4&6\\7&8\\6&7\\8&6\\+7&8\\\hline\end{array}\quad\begin{array}[t]{r} 8\\7\\6\\+9\\\hline\end{array}\quad\begin{array}[t]{rr} 6&5\\8&7\\7&9\\5&6\\9&4\\6&8\\3&7\\+8&2\\\hline\end{array}\quad\begin{array}[t]{rr} 6&8\\5&9\\6&2\\8&5\\7&6\\+4&3\\\hline\end{array}$$

$$\begin{array}[t]{rrrr} 4&7&8&5\\5&7&6&9\\8&7&6&2\\8&5&7&6\\+8&3&9&5\\\hline\end{array}\quad\begin{array}[t]{rrr} 6&8&7\\4&8&3\\6&9&5\\+8&7&4\\\hline\end{array}\quad\begin{array}[t]{rr} 6&9\\+5&8\\\hline\end{array}\quad\begin{array}[t]{rr} 6&5\\6&3\\3&9\\8&1\\+4&2\\\hline\end{array}\quad\begin{array}[t]{rr} 6&9\\+8&5\\\hline\end{array}$$

$$\begin{array}[t]{rrrrr} 8&5&7&6&9\\4&8&3&6&9\\+7&9&7&2&4\\\hline\end{array}\quad\begin{array}[t]{rrrr} 5&9&7&6\\5&7&6&3\\7&4&6&6\\8&7&8&9\\+5&9&7&4\\\hline\end{array}$$

APPENDIX D

Raw Scores for All Subjects

S_1		S_2		S_3		S_4		S_5	
A	C	A	C	A	C	A	C	A	C
10	10	7	6	7	6	9	1	8	7
12	12	10	10	8	5	8	5	9	9
12	11	13	13	9	9	7	2	11	10
14	13	15	15	10	6	10	2	13	12
14	14	18	15	9	8	10	8	9	7
14	13	19	18	12	8	13	13	13	13
15	14	19	18	11	11	13	10	10	9
14	11	16	16	11	10	12	10	11	11
14	10	19	19	12	9	12	8	13	11
15	13	18	17	12	10	11	11	10	10
14	14	18	16	9	6	10	10	10	8
17	14	16	15	10	6	14	14	10	9
14	10	20	20	12	9	13	11	11	8
13	9	16	15	10	8	14	13	9	8
15	8	17	15	8	8	15	14	10	9
14	10	18	14	8	5	16	13	9	9
18	17	20	18	8	8	14	12	11	9
19	17	16	15	8	6	10	9	A	A
A	A	20	20	12	9	12	10	14	13
16	13	21	20	11	9	13	13	11	11
17	17	22	22	13	11	14	13	17	16
15	13	21	20	12	11	15	14	13	12
17	16	14	14	16	13	11	10	13	12
16	14	A	A	13	10	11	9	13	13
15	15	20	18	13	13	12	12	16	16
15	12	20	20	15	14	13	12	15	13
17	16	20	20	15	15	13	12	17	16
17	17	20	20	15	13	12	11	17	16
20	19	24	24	19	18	13	10	18	16
19	19	22	20	20	20	13	9	18	17
20	19	22	22	A	A	13	12	16	15
20	19	25	25	18	16	14	14	19	17
22	22	25	25	20	20	14	13	21	20
22	21	23	22	22	21	16	14	20	19
21	19	23	21	21	19	17	14	20	20
22	21	22	22	21	18	13	12	19	17
20	20	18	14	24	24	15	14	14	12
21	17	17	16	25	22	18	15	13	13
19	17	20	17	14	10	18	18	16	16
20	18	22	21	17	17	17	17	16	16
		20	19	15	15	9	7		
				18	17	16	14		
				19	18	14	13		
						13	10		

S_6		S_7		S_8		S_9		S_{10}	
A	C	A	C	A	C	A	C	A	C
6	6	6	5	8	7	10	9	10	8
9	9	7	6	10	8	13	12	11	9
10	9	7	4	11	9	14	11	10	5
9	9	8	2	10	5	16	15	12	7
10	9	9	6	11	10	18	12	13	9
10	10	10	5	12	11	18	15	13	12
10	9	11	5	11	10	18	14	12	10
9	8	9	1	12	11	19	13	12	7
10	9	7	4	16	14	17	15	12	8
9	8	11	6	20	15	19	14	12	6
9	8	10	2	18	14	11	2	13	11
13	9	10	4	17	10	18	12	13	9
8	8	7	1	20	18	19	13	14	3
11	6	11	7	21	19	19	14	15	3
A	A	12	2	12	10	17	16	15	8
11	10	12	4	13	12	19	11	17	12
13	7	15	4	14	11	21	12	17	6
10	10	8	2	13	10	19	17	15	6
9	9	10	5	15	7	20	18	15	12
12	11	11	6	11	10	20	17	14	8
13	11	12	6	12	7	22	21	15	9
11	9	11	3	13	9	21	21	14	8
11	10	8	8	14	13	19	17	16	12
A	A	11	9	16	15	A	A	16	13
10	8	9	7	19	17	A	A	17	14
9	8	9	7	18	16	13	12	14	13
8	7	13	10	16	15	A	A	17	9
9	9	11	6	20	19	12	11	15	7
11	10	11	8	19	19	20	20	10	7
9	9	12	8	21	16	19	18	11	8
11	10	10	3	20	18	20	19	16	16
9	9	10	5	19	16	21	21	16	14
11	11	12	7	21	19	23	22	17	16
10	10	11	3	21	18	23	23	18	13
13	13	10	6	12	9	23	20	20	13
10	10	15	13	14	11	22	20	21	17
8	8	13	10	15	12	20	19	20	14
8	7	15	6	17	14	18	18	17	14
10	10	12	6	15	13	21	20	19	16
12	12	12	9	15	12	23	22	19	17
		10	9					20	17
		16	11					22	19
		11	7					12	10
		8	1					11	6
		7	2					14	12
		10	6					15	12
		9	4					17	14

S_{11}		S_{12}		S_{13}		S_{14}		S_{15}	
A	C	A	C	A	C	A	C	A	C
8	7	10	9	10	3	10	9	10	8
9	8	12	10	12	1	10	9	11	8
8	5	13	11	15	1	13	12	10	9
11	7	13	11	12	3	10	7	11	8
12	12	15	13	13	8	14	13	12	11
11	10	14	13	A	A	12	10	12	12
12	10	15	15	A	A	14	12	14	12
12	10	16	13	15	7	15	13	13	12
10	5	16	14	13	10	A	A	13	9
11	10	15	13	12	11	A	A	9	7
13	11	14	11	12	9	A	A	12	8
12	12	18	12	13	11	7	0	12	11
13	11	18	15	13	12	8	5	13	11
12	9	18	8	12	10	8	2	13	9
12	7	20	16	8	3	14	11	A	A
10	8	19	9	11	11	15	15	15	11
10	7	13	12	13	11	A	A	14	11
11	10	16	15	15	12	18	15	14	13
13	10	16	16	13	9	17	16	13	11
14	12	18	15	15	11	15	14	14	11
13	11	17	13	16	13	17	16	A	A
8	2	17	14	12	8	20	18	A	A
12	10	18	14	13	7	20	19	14	11
12	11	18	13	11	9	17	14	10	8
14	13	16	9	13	8	14	7	11	7
16	15	17	10	14	7	14	8	11	8
16	14	14	11	14	10	13	7	20	13
16	14	A	A	15	10	16	13	11	6
18	16	A	A	11	1	16	12	19	16
16	16	16	16	18	6	20	13	17	8
18	17	19	19	20	4	20	8	14	7
18	17	18	16	20	11	20	11	20	10
20	19	17	16	19	6	21	17		
18	18	22	20						
19	18	21	19						
22	22	21	21						
22	20	19	14						
12	11	20	19						
14	13	21	20						
11	7	22	14						
17	12	20	15						
15	11	23	22						
		24	22						
		25	23						

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