A Comparison of Calculation Speed and Accuracy on Two Levels of Problem Difficulty using the Conventional and Hutchings' "Low Stress" Subtraction Algorithms and the Pocket Calculator

Margaret T. McGlinchey

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A COMPARISON OF CALCULATION SPEED AND ACCURACY ON TWO LEVELS OF PROBLEM DIFFICULTY USING THE CONVENTIONAL AND HUTCHINGS' "LOW STRESS" SUBTRACTION ALGORITHMS AND THE POCKET CALCULATOR

by

Margaret T. McGlinchey

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Submitted to the Faculty of the Graduate College
in partial fulfillment of the requirements for the Degree of Specialist in Education Department of Psychology

Western Michigan University
Kalamazoo, Michigan
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A COMPARISON OF CALCULATION SPEED AND ACCURACY ON TWO LEVELS OF PROBLEM DIFFICULTY USING THE CONVENTIONAL AND HUTCHINGS' "LOW STRESS" SUBTRACTION ALGORITHMS AND THE POCKET CALCULATOR

Margaret T. McGlinchey, Ed.S.
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This study involved a comparison of three different calculation procedures for solving two different sizes of subtraction problems. A multi-element baseline design was used with three subjects: 2 females and 1 male, whose ages were 9.6, 24, and 8.1, respectively. The calculation procedures were varied randomly within sessions, and the problem sizes varied randomly across sessions. Dependent measures included rate correct, rate incorrect, and percent accuracy. For all subjects, the conventional algorithm was the fastest calculation procedure. Accuracy data were not as consistent, since they varied across subjects and problem array sizes. Future research in this area might involve larger problems, and/or attempt to control for learning histories (i.e., use naive learners, or use a different base other than base 10).
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WESTERN MICHIGAN UNIVERSITY, ED.S., 1981
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CHAPTER I

Introduction

Mathematics in the public schools has been changing its emphasis in instruction from computation to more conceptual applications. This change in emphasis is what is commonly referred to as "New Math." In many ways, this extension in math appears to be advantageous for students. Most would agree that conceptual applications are important, but one consequence of the growing interest in "New Math" is that teachers are left with more to teach in the same amount of time. Despite this conceptual orientation, we are seeing declining measured achievement in mathematics among elementary school children (Alessi, 1979). Alessi suggests that part of the decline in measured calculation skills may be attributable to the reduced amount of time spent in algorithm drill.

Traditional education in mathematics focused on teaching these basic operations. Instruction, to a large extent, consisted of drills on basic facts and basic operations, which children often found tedious. Today, with our expanded mathematical knowledge, there is much more to teach children. However, it is still essential that they master the four basic operations which are so much a part of daily living.

Given that we have much more to teach children, but within the same time frame, it is important to teach children the basic operations as quickly and efficiently as possible. The conventional algorithm has not necessarily demonstrated this efficiency (Zoref, 1976).
One approach which has been offered as a viable alternative is the calculator. It has become more attractive as at least a partial solution due to its increased availability. The price of the pocket calculator has decreased dramatically over the last nine years (from $100.00 in spring of 1971 to less than $10.00 in 1980). This decrease in price makes it a reasonable tool for schools to purchase. Schultz (1975) recommends a minimum of one calculator for every two students to avoid chaos and frustration in the classroom.

Many uses for the calculator in the classroom have been suggested. It can be used to check answers which were completed with paper and pencil (Schultz, 1975), or to reduce tedium involved in large and more complicated problems (Hawthorne, 1973). Hawthorne also recommends calculators for practice with basic facts and states that children typically enjoy using them. The fact that children enjoy using calculators may minimize the tedium often associated with basic facts and operations.

There are also potential misuses of the calculator, such as using it in lieu of teaching basic facts or operations. Calculators do not demonstrate the steps involved in solving a particular problem, so therefore cannot teach the child how to perform a basic operation. Calculators should not eliminate the teaching of basic concepts (Harrington, 1976). They should be used only after the basic concepts are taught.

As a supplemental tool, the calculator could prove quite useful in the classroom. However, by itself, it does not solve the problem of teaching the basic operations quickly and efficiently. In 1972,
Dr. Lloyd B. Hutchings developed a set of experimental algorithms for the four basic math operations. According to Hutchings, the new algorithms "appear to permit easy mastery after brief training, to provide greater computational power than conventional algorithms, and to operate with much less stress on the user than conventional algorithms" (Hutchings, 1976, p. 219). All low stress algorithms have two distinct mechanical characteristics:

1. A concise, definable, easily read supplementary notation is used to record every step.

2. The learner can complete any intermediate step of a distinct kind rather than alternate between kinds of intermediate steps. (Hutchings, 1975, p. 227)

The algorithms facilitate identification of errors and error patterns, since all steps are overtized.

Several group design studies have been conducted to assess the computational power (speed and accuracy) of the "low stress" algorithms (Alessi, 1974; Boyle, 1975; Dasiell, 1974; Hutchings, 1972; Lester, 1978). Of these studies, only one (Lester) has assessed the effects of the "low stress" algorithm specifically in relation to subtraction. Lester assessed gains in subtraction basic facts and subtraction computation as a function of training in the "low stress" subtraction algorithm.

Students who were instructed in the "low stress" algorithm improved their subtraction basic facts and subtraction computation. The improvement was larger for this group when compared with the control group (Lester, 1978). The results were not statistically signification however. According to Lester, "subtraction computation
showed positive differential gains of magnitudes which the author considers important, but which do not sustain the preselected protection level" (Lester, 1978).

Single subject design studies have been conducted recently to investigate various components of Hutchings' algorithms as opposed to the conventional algorithm under varying response cost and reinforcement conditions. Gillespie (1976) found that given a free choice, most of the subjects preferred Hutchings'. They maintained this preference even when it required 50% more response effort. They were also more accurate with the Hutchings' algorithms.

Comparison of the Hutchings' and current addition algorithm for speed and accuracy in two school settings, and with regular and special education children, was investigated by Rudolph (1976). The data indicated greater accuracy using the "low stress" algorithm over the conventional algorithm. The error rates for each subject indicated similar results. This effect was not demonstrated as clearly in the correct rate for each student. With regard to the different school settings, Rudolph found the difference to be of a "relatively small magnitude (Rudolph, 1976, p. 49).

In a related study, calculation speed and accuracy on two levels of problem difficulty were compared, using the convention addition algorithm, Hutchings' "low stress" algorithm, and the pocket calculator (Zoref, 1976). Zoref used both high and low performing children and found that the Hutchings' algorithm was "generally the most stable method (i.e., had the lowest standard deviations) across all measures regardless of type of student or problem difficulty" (Zoref, 1976, p. 49).
Hutchings' algorithm had the lowest error rate and was generally the most accurate method used. These results are more dramatic when one considers the fact that all subjects were in the fourth grade and had quite a long history (two or three years) with the conventional algorithm.

Significant contributions have been made in the past with both group designs (Alessi, 1974; Boyle, 1975; Dasiell, 1974; Gordon, 1972; Hutchings, 1972; Lester, 1978) and single subject designs (Clark, 1976; Gilespie, 1976; Rudolph, 1976; Zoref, 1976). Most of this work, with the exception of Lester, has been conducted in the area of Hutchings' addition or multiplication algorithm.

Hutchings (1976) defines the subtraction algorithm as follows:

Low-stress subtraction has a special notation used to record the steps performed in solving a subtraction problem. The notation consists of two parts. The first is to record all upward regrouping of places by a half-space '1' placed at the upper left of numerals occupying such places. The second is to write the regrouped minued directly above its subtrahend, which helps some children in reading and organizing their work. The regrouped minued is written 'in the middle' rather than at the top, as shown in the following example:

\[
\begin{array}{cccc}
6 & 7 & 9 & 5 \\
\hline
6 & 6 & 8 & 5 \\
-5 & 2 & 9 & 6
\end{array}
\]

In subtraction, the entire minuend is regrouped and recorded before any subtraction occurs (Hutchings, 1976, p. 231).

The "low stress" subtraction algorithm appears to contain many of the features of the other low stress algorithms. These features are: (1) easy mastery after brief training; (2) greater computational power; (3) "less stress on the user" (Hutchings, 1976, p. 219); and (4) a complete record of component operations.
The purpose of the present study was to investigate the calculation power (speed plus accuracy) of the conventional subtraction algorithm, the Hutchings' "low stress" algorithm and the electronic pocket calculator. The three methods were compared for two levels of problem difficulty. The present study was a systematic replication of the work conducted by Zoref (1976) except the present one examined the subtraction "low stress" algorithm. Only three students participated in the present study. None of the students were identified as high or low performing. A multi-element baseline was used since the study focused on acquisition of academic behavior and changing baseline rates were expected (Ullman & Sulzer-Azaroff, 1975). A single-subject design was chosen rather than a group measure for the following reasons: (a) observations and analyses are afforded over time, (b) individual subject data are available, and (c) manipulation of variables is possible as well as the flexibility in manipulation needed to establish control (Zoref, 1976, p. 7).

The present study examined the following questions: (1) Are there differences in the calculation speed and accuracy among the three methods of calculation? (2) Are the methods of calculating subtraction problems affected by problem difficulty? In this case, difficulty was defined by the total numbers of the minuend and subtrahend.
CHAPTER II

Method

Subjects

Three subjects, two females and one male, participated in this study. Their ages were 9.6, 24, and 8.1, respectively. All of the subjects were attending a remedial education clinic for reading and/or mathematics. A requirement of selection for the study was written completion of a 56 basic subtraction facts worksheet at 90% accuracy and enrollment in the remedial education clinic.

Setting

The clinic was located on the campus of Western Michigan University in Kalamazoo, Michigan. The study took place in the University classrooms which were reserved for the clinic from 4:00 p.m. to 6:00 p.m. Monday through Thursday.

There were a total of five rooms reserved for the clinic. The subjects who participated in the study were in a total of three different rooms throughout the duration of the study. The daily sessions were run in the specific room the subject was in for tutoring. This varied throughout the study depending upon which reading group the subjects were in. A total of 32 sessions were run.
Independent Variables

This study involved two independent variables: (1) three types of calculation procedures—conventional, Hutchings' "low stress" (Hutchings, 1976), and the electronic pocket calculator, and (2) two problem array sizes (2 x 4 and 2 x 8).

Dependent Variables

Three dependent variables were investigated: (1) rate of columns correct, (2) rate of columns incorrect, and (3) percent accuracy on attempted columns. Rate of columns correct and incorrect were calculated using the number of columns completed per 5-minute sample. Percent accuracy was calculated by dividing the total amount correct by the number of columns attempted and multiplying by 100.

Reliability

Reliability data were obtained on correcting the students' papers for the number of columns correct and incorrect. A random sample of papers was selected from each set of student worksheets collected throughout the study. An independent observer then scored these worksheets. An agreement was scored if the two observers identified the same column as correct or incorrect. Reliability was computed by dividing the total number of scorer agreements by the total number of agreements plus disagreements and multiplying by 100.
Experimental Design

A multi-element baseline design was used (Ulman & Sulzer-Azaroff, 1975), varying the type of calculation procedure within sessions, and the type of problem array across sessions. This type of design allowed for examination of the main effects of the variables manipulated and the interaction effect (Zoref, 1976). This was accomplished by randomizing the procedures within sessions and problem arrays across sessions. This design was also chosen because changing baseline rates were expected given that academic behavior was being researched.

Materials

The pre-test consisted of one math worksheet of 56 basic subtraction facts (see Appendix A), two probe sheets from the SRA Diagnostic Mathematics Kit (probe L-2 and L-8, level A). One probe sheet tested subtraction with and without regrouping and the other tested place value.

A practice worksheet was used during training. The same worksheet was used across all three sessions to ensure an equal amount of practice for all three calculation procedures (see Appendix B). Additional worksheets were used during training for Hutchings' algorithm which only involved regrouping.

Materials for daily sessions consisted of the following: (1) one stopwatch (Sharp Elsimate EL-401A), (2) one lead pencil, (3) printed worksheets which contained subtraction problems totalling 40 columns per page (one to three for each calculation procedure); each page was
typed with an IBM Selectric Orator typing element (see Appendix C), (4) pocket calculators (Texas Instruments), (5) answer keys for worksheets, and (6) point cards. The point cards were part of the clinic's motivation program. Students earned points for working at the clinic. These points were exchanged daily at the Project store.

Worksheets were repeated at monthly intervals only. This was to avoid any possible practice effects.

Procedure

To ensure an adequate degree of competency with basic subtraction facts, a pre-test of 56 basic subtraction facts was administered to all potential subjects. A criterion of 90% accuracy was required to participate in this study.

After the subjects were selected, a letter describing the study was sent to their parents which requested permission for their child to participate. The parents were requested to sign a consent form and have their children return it. Written permission was obtained from the parents of all subjects who were chosen.

The first training session consisted of teaching the Hutchings' algorithm using an adaptation of Hutchings' training sequence (Appendix D) and a model-lead-test format (Englemann). The second session consisted of training the use of the pocket calculator to solve subtraction problems. The third session consisted of a review of the conventional algorithm. During each of these training sessions, a worksheet was provided for practice of the given calculation procedure. A fourth training session was conducted to review all three calculation procedures. Training sessions lasted for 15-20 minutes.
Daily sessions followed the same format throughout the study. Each session consisted of three trials, each five minutes long, with one trial for each calculation procedure. The order of presentation of calculation procedures within daily sessions was determined by using the random numbers table. Subjects were told which procedures to use prior to each trial. The order of problem array sizes across sessions was determined by a coin toss. Neither array was scheduled for more than two consecutive sessions.

The remedial clinic ran from 4:00 p.m. to 6:00 p.m. Monday through Thursday. The program was divided into two time blocks: 4:00 to 4:55 p.m. and 5:10 to 6:00 p.m. There was a 15 minute break between sessions. During this break time, students went to the project store and game room. All students who participated in this study were receiving reading instruction during both of the above time sessions. One session consisted of decoding and one session consisted of comprehension.

Efforts were made to minimize the amount of time students missed reading instruction. First, individual trials were split between sessions (i.e., one tutoring session included one 5-minute trial, and the second tutoring session included two 5-minute trials). Each day, the investigator went to the student's room and conducted the trials in the same room in which the student was receiving reading instruction. These rooms were large university classrooms. There were usually 2 small groups in each room. The investigator went to the room, set up desks, and placed the appropriate worksheets on the student's desk, then requested the student leave the group or tutor.
This minimized any "down time" due to transitions. When the subjects were seated with the investigator, they were given the cue "Ready... go". At the end of 5 minutes, they were told to stop and hand in their paper. When the trials were completed, the students were sent back to the tutor and/or group.

Students switched reading groups as their skills changed throughout the semester. This sometimes resulted in a change in their specific tutoring room. Therefore, the room in which sessions were conducted varied during the study. Due to these scheduling changes, it was also difficult to keep constant the time of day when the 5-minute trials were conducted. This was subject to change, as was the actual room in which trials were run.

As mentioned previously, the clinic utilized a point system with a "store" to help motivate clients to work hard. Points could be exchanged for items in the store, such as sugarless candy, toys, books, or posters. Two of the subjects in this study were on this point system. They were given points daily for working with the investigator. Points were administered for working hard, accuracy was not a part of the contingencies. Specific errors were brought to the attention of the subject by the investigator. This usually occurred prior to the first session of the following day.
CHAPTER III

Results

Reliability data yielded an index of 95% agreement for scoring papers for number of columns correct and incorrect.

The results of this study are presented in three topical sections: (a) a comparison of performance by calculation method, (b) a comparison of performance by problem difficulty, and (c) a summary of individual results. Each of these comparisons will be discussed with regard to the three study measures: correct rate, error rate, and percent accuracy. Included are 2 tables and 12 figures. Table 1 presents the means and standard deviations of individual student performance across the three calculation procedures and two types of problem arrays. Table 2 presents an analysis of columns completed with renaming, and types of errors across the conventional and Hutchings' calculation procedures.

Comparison of Performance by Calculation Method

The conventional algorithm was the fastest method of calculation across all three subjects and problem arrays (i.e., highest correct rate). The correct rate for each subject increased slightly when working 2 x 8 problem arrays as opposed to 2 x 4 problem arrays. Error rate data for the conventional algorithm are inconsistent across subjects and problem arrays. For two subjects (Lyn and Sara) overall accuracy (i.e., error rate and percent accuracy) on 2 x 4
problems is very similar across all three calculation procedures. They were highly accurate with all procedures (means of above 95%). Lou's error rate data are similar across the Hutchings' and conventional algorithms and percent accuracy for the conventional algorithm is 86.4. In 2 x 8 problem arrays, there is more variability across subjects with regard to accuracy. This fact makes it difficult to summarize findings. Specific error rates will be discussed for each subject when individual data are summarized (see Summary of Individual Results).

The Hutchings' "low stress" algorithm generally had the second fastest rate correct (with the exception of Sara on 2 x 4 problem arrays). The difference between the calculator and Hutchings' does not appear to be of large magnitude however (with the possible exception of Lou on 2 x 8 problem arrays). As mentioned previously, error rate data for Hutchings' algorithm and the convention algorithm on 2 x 4 problem arrays were very similar for all three subjects. Mean error rates with the Hutchings' algorithm on 2 x 8 problem arrays were at or below 1.1 for all three subjects, and mean percent ranged from 88.9 to 98.0.

The calculator was generally the slowest method of calculation across subjects and problem arrays. On both 2 x 4 and 2 x 8 problem arrays, error rate data were very low. With the exception of Lyn on 2 x 8 problems, all means were .7 or below. Means for percent accuracy were in the range of 84 to 88 (see Table 1).

In general, interpretation of the data is complicated by the fact that individual performance varied greatly across problem arrays and
Table 1: Means and standard deviations of each subject's scores across three calculation procedures and two problem arrays.
TABLE I

MEANS AND STANDARD DEVIATIONS OF INDIVIDUAL SCORES ACROSS THREE CALCULATION PROCEDURES AND TWO PROBLEM SIZES

<table>
<thead>
<tr>
<th>Student</th>
<th>Calculation</th>
<th>Correct Rate/5 min</th>
<th>Error Rate</th>
<th>Percent Accuracy</th>
<th>Correct Rate/5 min</th>
<th>Error Rate</th>
<th>Percent Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\bar{X}$</td>
<td>SD</td>
<td>$\bar{X}$</td>
<td>SD</td>
<td>$\bar{X}$</td>
<td>SD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N\textsuperscript{a}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lyn</td>
<td>Conventional</td>
<td>14</td>
<td>13.7</td>
<td>4.2</td>
<td>0.2</td>
<td>0.2</td>
<td>98.8</td>
</tr>
<tr>
<td></td>
<td>Hutchings' Calculator</td>
<td>14</td>
<td>9.2</td>
<td>2.8</td>
<td>0.2</td>
<td>0.2</td>
<td>98.2</td>
</tr>
<tr>
<td></td>
<td>Calculator</td>
<td>13</td>
<td>7.9</td>
<td>1.5</td>
<td>0.3</td>
<td>0.3</td>
<td>96.0</td>
</tr>
<tr>
<td>Sara</td>
<td>Conventional</td>
<td>15</td>
<td>6.6</td>
<td>1.7</td>
<td>0.2</td>
<td>0.4</td>
<td>95.3</td>
</tr>
<tr>
<td></td>
<td>Hutchings' Calculator</td>
<td>14</td>
<td>5.2</td>
<td>1.0</td>
<td>0.3</td>
<td>0.3</td>
<td>94.5</td>
</tr>
<tr>
<td></td>
<td>Calculator</td>
<td>15</td>
<td>5.5</td>
<td>0.9</td>
<td>0.1</td>
<td>0.3</td>
<td>98.1</td>
</tr>
<tr>
<td>Lou</td>
<td>Conventional</td>
<td>14</td>
<td>10.0</td>
<td>3.9</td>
<td>1.5</td>
<td>0.9</td>
<td>86.4</td>
</tr>
<tr>
<td></td>
<td>Hutchings' Calculator</td>
<td>14</td>
<td>6.2</td>
<td>2.6</td>
<td>1.2</td>
<td>0.9</td>
<td>82.9</td>
</tr>
<tr>
<td></td>
<td>Calculator</td>
<td>14</td>
<td>4.3</td>
<td>2.0</td>
<td>0.5</td>
<td>0.6</td>
<td>88.1</td>
</tr>
</tbody>
</table>

\textsuperscript{a} = Number of session attended
Correct Rate = Number of columns correct/5 minutes
Error Rate = Number of columns incorrect/5 minutes
Percent Accuracy = Number of columns correct / total number attempted \times 100
calculation methods. In an effort to determine possible causes for the different rates of performance for Hutchings' algorithm and the conventional algorithm, a behavior analysis was conducted of all problems completed and errors made.

The method of this study required students to work for 5 minutes with each calculation procedure. At the end of 5 minutes, students were told to stop. Columns completed during each 5-minute session were counted; therefore, partial credit was given for problems partially completed at the end of 5 minutes. This measurement procedure may have inflated the total scores of the conventional algorithm as compared to the Hutchings' algorithm. This is because Hutchings' algorithm requires renaming before subtracting, such that time was spent renaming near the end of an interval instead of subtracting. No partial credit was given for renaming correctly on the Hutchings' algorithm unless it was also subtracted. Of the total number of columns attempted throughout the study, a percentage of columns was renamed, but not subtracted because the subject did not have time. Those percentages were as follows: Lou, 4%; Lyn, 3%; Sara, 8% (see Table 2). Assuming it takes as much time to subtract as it does to rename, the total number of columns subtracted with the Hutchings' algorithm would be increased for each subjects as follows: Lou, 2%; Lyn, 1.5%; Sara, 4%.

For all three subjects, renaming was involved in 64-67% of the columns subtracted for both Hutchings' and the conventional algorithms. The data are summarized in Table 2. Of the renames attempted, two subjects (Lou and Sara) renamed more accurately with Hutchings' than
Table 2: An analysis of renaming operations for each subject.
### TABLE 2

**ANALYSIS OF RENAMING OPERATIONS**

<table>
<thead>
<tr>
<th>Subjects</th>
<th>Lou</th>
<th>Lyn</th>
<th>Sara</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Components</strong></td>
<td>HA</td>
<td>CA</td>
<td>HA</td>
</tr>
<tr>
<td>% columns renamed but not subtracted</td>
<td>4</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>% columns of correct renames</td>
<td>89</td>
<td>80</td>
<td>94</td>
</tr>
</tbody>
</table>

**ERROR ANALYSIS**

<table>
<thead>
<tr>
<th></th>
<th>Lou</th>
<th>Lyn</th>
<th>Sara</th>
</tr>
</thead>
<tbody>
<tr>
<td>% basic factor errors</td>
<td>64</td>
<td>71</td>
<td>64</td>
</tr>
<tr>
<td>% rename errors</td>
<td>58</td>
<td>71</td>
<td>69</td>
</tr>
</tbody>
</table>

HA = Hutchings' algorithm  
CA = Conventional algorithm
the conventional algorithm. This difference was of a very small magnitude for Sara. These results were not consistent with Lyn's who renamed more accurately with the conventional algorithm. An error analysis indicated that less renaming errors and basic fact errors were made with Hutchings' algorithm than with the conventional algorithm for two of the subjects (Lou and Sara). Basic fact errors and renaming errors are not mutually exclusive because one column could have had both types of errors. These data suggest that these two subjects made more errors with the conventional algorithm. These results are not consistent with Lyn's, who made more renaming errors with the Hutchings' algorithm.

Comparison of Performance by Problem Difficulty

A comparison of performance with each method of calculation across the two problem sizes shows inconsistent results. Each method of calculation will be addressed individually.

The correct rate and error rate increased with the conventional algorithm for all subjects on 2 x 8 problem arrays; the percent accuracy decreased. Although these changes were consistent for all subjects, they were so small for two subjects so that there were essentially no differences in performance (Sara and Lyn). There appeared to be a significant difference for Lou however.

Changes in performance with the Hutchings' algorithm were inconsistent across subjects. Two subjects (Sara and Lou) showed an increase in percent accuracy with 2 x 8 problem arrays; essentially, their correct rates and error rates remained the same. The third
subject (Lyn) also showed very little difference in correct rate (.8). Error rate increased and percent accuracy decreased, but these results appear to be due to one unusual data point. (See Figure 7.) If this data point was excluded, her error rate would be .3 as opposed to .8, and accuracy would be 97% as opposed to 94%.

Changes in performance with the electronic calculator across problem sizes were consistent for all three subjects. On 2 x 8 problem arrays, there was a decrease in rate correct, an increase in error rate, and a decrease in percent accuracy (see Table 1). This difference was very slight for two subjects (Lou and Sara). For the third subjects (Lyn), the difference was quite large.

Summary of Individual Results

Lyn's performance was consistently more efficient with the conventional algorithm. The most dramatic difference in the calculation procedures appeared to be in the rate correct (see Table 1). Less of a difference was obtained with error rates and percent accuracy. Lyn's overall performance was extremely accurate across all procedures and problem arrays (with the possible exception of the electronic calculator on 2 x 8 problem arrays). Figures 1 and 7 show a clean acquisition slope for the Hutchings' algorithm and the conventional algorithm. The curve for the conventional algorithm never overlaps that of the Hutchings' algorithm or the electronic calculation.

Correct rates for Sara were very similar. Overall accuracy was also extremely high across all calculation procedures and problem
arrays. Mean error rates ranged from .1 to .4 and mean percent accuracy ranged from 94.5 to 98.1. Correct rates were highest with the conventional algorithm across both problem arrays. This difference is not large however. Figures 2 and 8 show that the curve for the conventional algorithm was initially higher; but as the study progressed, more overlapping occurred across all three calculation procedures.

The conventional algorithm was the fastest method of calculation for Lou across both problem array sizes. Error rate data were also highest with the conventional algorithm. The difference between the Hutchings' algorithm and the conventional algorithm was very slight for 2 x 4 problem arrays however (see Table 1). On 2 x 8 problems, performance was most efficient with the Hutchings' algorithm (i.e., correct rate plus error rate plus percent accuracy). Performance with the electronic calculator was consistently slower. Although the means and percent accuracy across problem arrays are similar for the calculator, daily performance was quite variable (see Figures 5 and 12).
Figure 1: Rate of columns correct and incorrect per session for Lyn using the three calculation procedures with 2 x 4 problem arrays.
Figure 2: Rate of columns correct and incorrect per session for Sara using the three calculation procedures with 2 x 4 problem arrays.
Figure 3: Rate of columns correct and incorrect per session for Lou using the three calculation procedures with 2 x 4 problem arrays.
Figure 4: Percent of columns correct per session for Lyn using the three calculation procedures with 2 x 4 problem arrays.
Figure 5: Percent of columns correct per session for Sara using the three calculation procedures with 2 x 4 problem arrays.
Figure 6: Percent of columns correct per session for Lou using the three calculation procedures with 2 x 4 problem arrays.
Figure 7: Rate of columns correct and incorrect per session for Lyn using the three calculation procedures with 2 x 8 problem arrays.
Figure 8: Rate of columns correct and incorrect per session for Sara using the three calculation procedures with 2 x 8 problem arrays.
SARA - 2x8

CONVENTIONAL
BUTCHING'S
CALCULATOR

Rate of Columns Correct

Rate of Columns Incorrect

DAILY SESSIONS

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Figure 9: Rate of columns correct and incorrect per session for Lou using the three calculation procedures with 2 x 8 problem arrays.
Figure 10: Percent of columns correct per session for Lyn using the three calculation procedures with 2 x 8 problem arrays.
Lyn - 2x8

![Graph showing percent accuracy over daily sessions for Conventional, Hutchings, and Calculator methods.](image)

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Figure 11: Percent of columns correct per session for Sara using the three calculation procedures with 2 x 8 problem arrays.
Figure 12: Percent of columns correct per session for Lou using the three calculation procedures with 2 x 8 problem arrays.
Percent Accuracy

Conventional Hutchings' Calculator

Daily Sessions

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CHAPTER IV

Discussion

The results of this study indicate that for all three subjects, the conventional algorithm was the fastest calculation procedures. These results were consistent across the two problem sizes. For some, this difference in correct rate for the conventional algorithm and the Hutchings' algorithm does not appear to be large. Interpretation of this difference is further complicated by the variability in the data as evidenced by the large standard deviations.

On 2 x 8 problems, the Hutchings' algorithm was generally more accurate (i.e., higher percent accuracy and lower error rate) than the conventional algorithm, with the exception of Lyn. As mentioned previously, her error rate data was inflated due to one highly unusual data point (see Figure 7). If the apparently aberrant data were excluded, her mean error rate would be .3 as opposed to .8, and mean percent accuracy would be 97 as opposed to 94.3. These results appear across all three subjects; so that although they may not be statistically significant, they may have educational significance.

The set up of a problem using Hutchings' algorithm may cue the child to rename, since there is a rename box in each problem. In addition to this, the operations--renaming and subtracting--are separated. These two things may account for the increased accuracy on 2 x 8 problem arrays with the Hutchings' algorithm as opposed to the conventional algorithm.
Conclusive statements are difficult to make because the data are so variable across subjects. The requirements for participating in this study were: (1) completion of a basic subtraction facts worksheet at 90% accuracy, and (2) enrollment in the remedial education clinic. The subjects had different learning histories (i.e., two attended public schools in different grades and one attended a vocational rehabilitation center for retarded adults). These different histories may account for some of the variability across subjects.

In addition to this difference across subjects, all subjects had some history with respect to using the conventional algorithm. This fact may complicate the results of this study also. Of the three subjects involved, Lyn had the most successful history (successful defined by the amount of time to acquire the calculation skills and amount of practice to date). It is recommended that future research attempt to control for this history by using naive learners or teaching the algorithms using a different base (i.e., base 12 instead of base 10).

As mentioned previously, the recording method was an artifact in the study. This may account for some of the difference in the Hutchings' algorithm as opposed to the conventional algorithm. Future research should employ a recording method which is not time dependent. One alternative would be to measure the time it takes to complete x number of problems. When this artifact was discovered in the present study, changing to this type of recording method was considered, but rejected because the contingencies for the students (i.e., a behavior requirement vs. a time requirement) would have to be altered. If this
type of recording method were used in future research, it may be necessary to work individually with students.

All students were given feedback on their performance. If they were working well, the feedback was usually general in nature. If errors were made, specific feedback was given. Lou's data shows a sharp increase in incorrect rate with the conventional algorithm. This increase occurred over two days. Due to time constraints, he was not given feedback on the first day. On the second day, his data rose even higher. Feedback was given at this point and performance returned to its previous rate. The importance of providing feedback on an individual's performance should not be overlooked.

The correct rate results are also complicated by the fact that, all things being equal, Hutchings' will often require slightly more motor behavior since one rewrites all numbers regardless of whether it is renamed. If both conventional and Hutchings' problems require renaming of at least half of each problem, this will help to control for the increased motor behavior. That is, if renaming is required in at least half of the columns, it will involve similar amounts of motor behavior for both conventional and Hutchings' algorithms.

The issue of increased motor behavior has been raised in the past with regard to using a "crutch" in traditional subtraction (Brownell, 1939). More recently, the Hutchings' addition algorithm has received similar criticism. As problems get larger, it requires more and more motor behavior over the conventional algorithm. Yet, in spite of this fact, children perform faster and more accurately with Hutchings' algorithm as opposed to the conventional algorithm (Zoref, 1976).
This suggests that although Hutchings' algorithm involves more motor behavior, the separation of complex operations into binary operations outweighs the disadvantages of increased motor behavior.

The electronic calculator was generally the slowest procedures, with the exception of Sara on 2 x 4 problem arrays. Given that others have claimed that children enjoy using the calculator (Hawthorne, 1973), these results were surprising. The calculator was a novel instrument and takes time to become proficient at its use; therefore, it may have detracted attention from the task at hand, rather than stimulating interest. Anecdotally, this procedure received the most groaning and negative attention from all three subjects.

Lyn performed the slowest with the calculator across both problem array sizes. She was also the least accurate overall (i.e., both error rate and percent accuracy). Lyn's reinforcement history with regard to responding to calculation problems may be a factor in this slow rate. She has had much practice responding to calculation problems covertly and the novel instrument may have unnecessarily complicated the task. The fact that the scores were close for Sara across all three procedures may be due in part to less of a learning history with calculation procedures.

Generalization based on these findings should be made with caution. Although their ages and learning histories varied widely, subjects were all caucasian, lower middle class, and attending a tutorial program for remedial instruction.

Due to the variability in these findings, it is recommended that future research use a large group which is more homogenous (i.e., same
age, grade, similar standardized test scores). If possible, the subjects should have no differential history with respect to any particular algorithm. Using naive learners would meet this recommendation. Another alternative would be to eliminate this history by teaching the algorithms in a different base (i.e., base 12 instead of base 10). One would need to insure adequate teaching of the different base however. As mentioned previously, the recording method should not be time dependent. Future research might also focus on using larger problems when comparing the conventional algorithm with the Hutchings' algorithm. The use of the algorithm for initial teaching of subtraction vs. using it as a remedial approach could also be investigated.

With regard to the accuracy data, especially where larger problems are concerned, Hutchings' algorithm seems like a viable alternative to the conventional algorithm (Zoref, 1976). The algorithm is very quick to teach, it facilitates identification of errors, and provides drill in basic facts.
APPENDIX A

INVENTORY OF BASIC SUBTRACTION FACTS
<table>
<thead>
<tr>
<th>Name</th>
<th>18</th>
<th>16</th>
<th>15</th>
<th>10</th>
<th>3</th>
<th>4</th>
<th>14</th>
<th>8</th>
</tr>
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<td>11</td>
<td>10</td>
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<td></td>
</tr>
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<td>-3</td>
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<td>-8</td>
<td>-5</td>
<td></td>
</tr>
</tbody>
</table>

Score
APPENDIX C

SAMPLE DAILY WORKSHEETS
Worksheet - 2 x 4 1

\[
\begin{array}{ccc}
5964 & 9364 & 8712 \\
-5788 & -4992 & -7421 \\
8712 & 4979 & 4138 \\
-6843 & -3592 & -2528 \\
2941 & 3125 & 9522 \\
-1271 & -2941 & -3125 \\
\end{array}
\]

2972

-1294
Worksheet - 2 x 8 10

\[
\begin{array}{c}
29374134 \\
\hline \\
-15813285
\end{array}
\quad \begin{array}{c}
62861245 \\
\hline \\
-42934523
\end{array}
\]

\[
\begin{array}{c}
93731158 \\
\hline \\
-53347641
\end{array}
\quad \begin{array}{c}
69689428 \\
\hline \\
-42297587
\end{array}
\]

\[
\begin{array}{c}
48918829 \\
\hline \\
-19552963
\end{array}
\]

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APPENDIX D

HUTCHINGS' SUBTRACTION ALGORITHM LESSON

(Adapted from Hutchings, 1972)
First we're going to learn a special way of writing some of the subtraction work. This is called overload notation. Whenever we need a larger number in some place, we simply increase whatever number is already in the place by 10, and show this by writing a half-space one at the ordinal's upper left-hand corner. Like this:

If 0 is increased to 10, we write 0
If 1 is increased to 11, we write 1
If 2 is increased to 12, we write 2

When we regroup the old way, we usually write the numbers over the top number like this:

Example 1: \[ \begin{array}{c} 6 \ 7 \ 3 \ 8 \ 4 \ 8 \ \ \\
-1 \ 1 \ 8 \ 7 \ 9 \ 6 \end{array} \]
Example 2: \[ \begin{array}{c} 6 \ 7 \ 3 \ 8 \ 4 \ 8 \ \ \\
-1 \ 1 \ 8 \ 7 \ 9 \ 6 \end{array} \]

With the new subtraction, there is a box in the middle of the problem. This is called a rename box. What is this box called (see Example 2).

Now we're ready to do some problems.

First we look at each pair of numbers. If the top number is smaller than the bottom number, we have to rename. Say that statement with me: "If the top number is smaller than the bottom number, we have to rename." Again. All by yourself, say the statement. Let's try some problems.

My turn. First I look at the first pair of numbers and ask, "Is the 1 smaller than the 9?" Yes, so there will be renaming.

\[
\begin{array}{c}
4 \ 3 \ 1 \\
-4 \ 1 \ 9 \\
\hline \\
x \ x \ x
\end{array}
\]

I add 10 to the 1, so I have one less here (3).
4 3 1
\[ \begin{array}{c}
4 2 4 \\
- 4 1 9 \\
\hline
\text{xxxxx}
\end{array} \]

Then I bring down the last number.

Your turn. Starting at the right. Is the top number smaller than the bottom number? So you have to rename.

\[
\begin{array}{c}
5 7 3 4 \\
- 1 6 4 5
\end{array}
\]

Add a one to the four and make the three one less—-a 2. Now look at the next pair of numbers. Is the top number smaller than the bottom number? So what are you going to do? Right, go ahead and rename.

\[
\begin{array}{c}
5 7 3 4 \\
- 1 6 4 5
\end{array}
\]

Now, look at the next pair of numbers. Is the top number smaller than the bottom number? So do you have to rename? Good, write the number.

\[
\begin{array}{c}
5 7 3 4 \\
- 1 6 4 5
\end{array}
\]

Now, look at the next pair of numbers. Is the top number smaller than the bottom number? So do you have to rename?

\[
\begin{array}{c}
5 7 3 4 \\
- 1 6 4 5
\end{array}
\]

Now rename the following problems. Don't do the subtraction right now. Remember, if the top number is smaller than the bottom number, you have to rename.
Now we're going to do the subtraction problems too. With the new subtraction, the rule is: "First we rename, then we subtract."
What's the rule? Good, let's do these problems.
BIBLIOGRAPHY


Harrington, T. Those hand-held calculators could be a blinking useful tool for schools. The American School Board Journal, 1976, 163, 44-46.


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