4-1981

A Comparison of the Hutchings' “Low-Stress", Fulkerson "Full-Record" and Conventional Addition Algorithms for Speed, Accuracy and Preference with Regular Education Students

John Robert VanHevel
Western Michigan University

A COMPARISON OF THE HUTCHINGS' "LOW-STRESS", FULKERSON "FULL-RECORD" AND CONVENTIONAL ADDITION ALGORITHMS FOR SPEED, ACCURACY AND PREFERENCE WITH REGULAR EDUCATION STUDENTS

by

John Robert VanHevel

A Project Report
Submitted to the
Faculty of The Graduate College
in partial fulfillment of the
requirements for the
Degree of Specialist in Education
Department of Psychology

Western Michigan University
Kalamazoo, Michigan
April, 1981

A COMPARISON OF THE HUTCHINGS' "LOW-STRESS", FULKERSON "FULL-RECORD" AND CONVENTIONAL ADDITION ALGORITHMS FOR SPEED, ACCURACY AND PREFERENCE WITH REGULAR EDUCATION STUDENTS

John Robert VanHevel, Ed.S

Western Michigan University, 1981

The differential calculation power (speed and accuracy) of three different addition procedures were investigated using the Hutchings "Low-stress", Fulkerson "Full-record" and conventional algorithms. The subjects were nine fourth grade students, five female and four male, approximately ten years of age. Elements of both multiple baseline and reversal designs were employed, varying the type of calculation method across phases. After exposure to each algorithm, students were given a choice of methods to use in solving a problem. Results indicated that both the Hutchings "Low-stress" and Fulkerson "Full-record" algorithms were generally superior in producing stable, accurate and efficient calculations. When given a free choice four students chose the Fulkerson "Full-record", three chose the "Low-stress" and two chose the conventional algorithms. It was suggested that future research investigate the components of each algorithm.

ACKNOWLEDGEMENTS

I would like to thank Drs. Galen Alessi, Howard Farris and Joetta Long for serving as committee members on this project. A very special thanks is extended to Galen for his eternal sources of new references and manuscript corrections.

I would also like to thank Gary Williams, the principal of the two schools under study, without whom, this study would not have been possible. Special thanks go to Gerry Gados for his accurate reliability checks, and to Dr. Wayne Fuqua for his help with the experimental design (and very interesting classes).

Last, but not least, to my wife Judy, for her patient and unbiased typing and editing.

John Robert VanHevel

1316547

VANHEVEL, JOHN ROBERT
A COMPARISON OF THE HUTCHINGS' "LOW-STRESS", FULKERSON "FULL-RECORD" AND CONVENTIONAL ADDITION ALGORITHMS FOR SPEED, ACCURACY AND PREFERENCE WITH REGULAR EDUCATION STUDENTS.

WESTERN MICHIGAN UNIVERSITY, ED.S., 1981

University
Microfilms
International 300 N. Zeeb Road, Ann Arbor, MI 48106

TABLE OF CONTENTS

TABLE OF CONTENTS (CONT.)

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (CONT.)

INTRODUCTION

The Problem

In recent years the need for effective computational skills in the United States has increased dramatically. With the advent of new technologies, demands are being placed on our educational systems to graduate students who are proficient in computational skills. Ironically, even with this demand for higher mathematical skills, the fact is students are scoring lower on standardized achievement tests. The Conference Board Mathematical Science National Advisory Committee on Mathematics Education point out that the mean scores on the quantitative section of the Scholastic Aptitude Test (SAT) have declined from a high of 502 to a low of 472 from 1962 to 1975. The precentage of scores above 600 declined from 20.5 to 16.4. The committee report also pointed out that these scores show a general declining trend.

Some investigators have pointed out that recent emphasis on the use of more conceptual materials may have resulted in less time being spent practicing basic computational skills (Alessi, 1974). The results have been a failure of students to master these skills. Hutchings (1976) made a similar case:

> "The great increase in mathematical concepts and generalizations in the curriculum is fundamentally in conflict with the large amount of time and energy required for the mastery of conventional computational algorithms. Moreover, increased conceptual requirements in no way reduce the requirements for computational skill. . Understanding division is not the same as knowing how to divide quickly and accurately." (p. 219)

Hutchings also pointed out that with the addition of conceptual materials there is more mathematics to be learned but not more time to learn it.

In a paper presented at the Sixth National Conference of the Research Council for Diagnostic and Prescriptive Mathematics in 1979, Alessi cited some problems which have been associated with programs emphasizing conceptual materials in teaching mathematics. These problems ranged from teachers refusing to teach the conceptual material, to parent protests over the failure of their child to learn computational skills. In any event, according to Alessi (1979) there appears to be a "backlash" against the conceptual approach and a renewed interest in more traditional practices which stress drill in computational skills.

To many the obvious solution is a return to traditional practices (as mentioned above). According to Alessi (1974) however, there appears to be some evidence suggesting that failure to learn mathematics may be due to emotional as well as intellectual factors. This view holds that students may develop a conditioned emotional response towards mathematics. This negative attitude may be a result of extensive repetition and drill which is standard practice in most mathematic curricula. The resulting behavior may be that of a total avoidance of mathematics (Boyle, 1975; Skinner, 1968).

Thus far it has been speculated that the failure of students in the United States to master basic computational skills may in part be due to two factors. These factors were: a.) the learning of conceptual material has taken time away from practice in computational skills and b.) students may have developed negative attitudes toward mathematics as a result of current teaching practices of drill and repetition, which results in students avoiding the subject.

Recently, some investigators have suggested that what is needed to solve this current dilemma is a way of short-cutting the process of teaching the basic calculation processes or algorithms. In Japan, little time is spent on drill; instead children are taught to use an abacus at a very early age. The abacus functions in a similar manner to that of an electronic calculator and thus the need for drill is minimized. As a result, the amount of time devoted to drill and repetition of basic facts is decreased leaving teachers in Japan more time to deal with advanced concepts and generalizations (Alessi, 1974; Boyle, 1975).

Lately, a number of alternative methods have been developed which appeal to the aforementioned needs. The following section will address these methods in terms of their relevance to improving mathematics instruction.

Relevant Literature on Alternative Algorithms

According to Alessi (1974), an algorithm may be defined as, "Any of a number of defined sets of specific operations or procedures which, if carefully followed, will enable the user to correctly calculate or solve a problem of interest. Different algorithms are defined by their processes." (p. 4)

One of the best ways to analyze a problem is to observe it (Ashlock, 1976). For instance, if a child were to consistently get wrong answers, one way of approaching the problem would be to have him perform the problem orally. Using this method the child is required to state each rule and operation being used to solve a given problem. Errors in computation or reasoning are noted by the teacher. These errors should then be corrected. The problem with this method is that it would take a great deal of time with more than a few students. An alternative would be to have the students record all operations on their paper (a full record). If a mistake were made, the teacher would quickly be able to find and correct it.

Recently Lloyd B. Hutchings developed a system for solving addition problems which produces a full record of calculations. The system is called "Low-stress" (Hutchings, 1976). The "Low-stress" algorithm differs from the algorithm typically used in the United States in a number of ways. First, the "Low-stress" method uses a half space notation to express the sum of two digits. ie.

$$\begin{array}{c} 8 \\ +1 \\ 9 \end{array}$$

Second, if the sum is greater than nine, the tens portion is written to the lower left of the digit at the bottom. ie.

$$\begin{array}{cc} & 8 \\ & 7 \\ 1 & 5 \end{array}$$

In performing long columns, the ones portion of the column is always the same as the ones portion of the last two digits sum. The tens portion is always the same as the number of tens recorded at the left. In the following example there are no ones and three tens:

$$\text{add} \quad \begin{array}{c} 8 \\ 1^{7}5 \\ 1^{6}1 \\ 1^{9}0 \\ \hline 3\ 0 \end{array} \qquad \begin{array}{l} 8+7 = 15 \\ 5+6 = 11 \\ 1+9 = 10 \end{array}$$

In multi-column exercises, there is a need for extra wide spaces between columns to accommodate the half space notation. Also, the tens are summed and carried to the top of the next column at the left:

$$\begin{array}{cc} \overset{2}{1^{8}0} & 6 \\ {}^{2}{}_{2} & 1^{7}3 \\ {}^{4}{}_{6} & {}^{5}{}_{8} \\ {}^{1}{}_{7} & 1^{9}7 \\ \hline 1\ \ 7 & 7 \end{array}$$

Alessi (1979) pointed out that the Hutchings "Low-stress" (LS) method has several distinct advantages over the algorithm currently used to teach addition in the United States. Most notable of the advantages were that a full record of every calculation is available for easy identification of errors and subsequent practice. Traditionally in columnar addition the student adds each digit in a column relying on memory until the sum is written for each column. If a mistake is made in traditional columnar addition, a record of where the error occurred is difficult if not impossible to locate and thus more difficult to correct.

A second advantage of the "Low-stress" method according to Alessi (1979), is that the attention load (numbers to be remembered) in performing long sequences of calculations is substantially reduced. Take for example the following problem:

$$\begin{array}{r} 9 \\ 8 \\ 3 \\ 6 \\ \underline{7} \end{array}$$

Using the conventional method, a student would have to remember that 9+8=17, 17+3=20, 20+6=26 and 26+7=33. The "Low=stress" method on the other hand, would only require the following calculations without the need for remembering sums:

$$\begin{array}{r} 9 \\ 1\,{}^{8}7 \\ 1\,{}^{3}0 \\ {}^{6}6 \\ \underline{1\,{}^{7}3} \end{array}$$

9+8=17

7+3=10

0+6=6

6+7=13

1+1+1=3

Finally, calculations do not get more difficult as the size of the problem increases. Since the "Low-stress" method is made up of a combination of small operations, the student is never required to use complex addition facts. With the conventional method, complex addition facts are often used. According to Hutchings (1976, p.223), "The advantage offered by 'Low-stress' procedures is increased in proportion to the length of the column."

In an article entitled, "Adding by Tens", Elbert Fulkerson (1963) presented a method for performing columnar addition which eliminates the necessity for students to remember complex math facts. In the following example, a slash is used to indicate a ten, just as placing a one to the lower left of a digit in the "Low-stress" procedure indicated a ten. The student is then only required to remember the units portion as the columns are added. For example:

1	
8	9
5̸	7
6̸	5
20	1

The Fulkerson method is appealing for a couple of reasons. First, it does not require special spacing to accommodate the notations as does the "Low-stress", and second, it reduces the memory load required by eliminating complex facts from that load. However, this method provide a full record of calculations, making the identification of error patterns difficult to locate.

Using a similar approach to Hutchings and Fulkerson, O'Malley (1969) presented another method of calculation for columnar addition. Instead of making a slash or placing a one next to a digit to indicate tens, the digit is written within the column. This digit will be added as in the following example:

1st cycle	2nd cycle
6 8	6 8
①	①①
8 5	8 5
	①
6 2	6 2
	①
8 1	8 1
①	①
8 7	8 7
①	①①
6 8	6 8
1	4 5 1

1st cycle

a.) 8+5=13 (the digit 1 is written in the tens column)

b.) 3+2=5 (proceed to the next digit)

c.) 5+1=6 (proceed to the next digit)

d.) 6+7=13 (the digit 1 is written in the tens column)

e.) 3+8=11 (the digit 1 is written in the tens column and the sum of the units is 1)

2nd cycle

a.) 6+1=7 (proceed to the next digit)

b.) 7+8=15 (the digit 1 is written in the hundreds place) c.) 5+6=11 (the digit 1 is written in the hundreds place)

d.) 1+8=9 (proceed to the next digit)

e.) 9+1=10 (the digit one is written in the hundreds place)

f.) 8+1=9 (proceed to the next digit)

g.) 9+6=15 (the digit 1 is written in the hundreds place and the sum of the tens column is 5)

h.) The sum of the digits written to the left of the tens column is written at the bottom in the hundreds place.

Again, as with the Fulkerson method, only a partial record of the calculations are available. Also, special spacing would be required to accommodate the notation.

Sanders (1971) developed a system which is similar to both the O'Malley (1969) and Fulkerson's (1963) methods. Hutchings (1976), O'Malley (1969) and Fulkerson (1963) used the digit "1" or a slash to indicate a ten while just the unit digits were added. The Sanders (1971) method uses the same basic idea but instead of writing a digit or a slash, the student holds up a finger to indicate a ten and then vocalizes to himself the units. The following is an illustration of the Sanders (1971) method of addition:

8		
	8+9=17	(holds up one finger)
9		
	7+2=9	(holds up same finger)
2		
	9+6=15	(holds up second finger)
6		
	5+7=12	(holds up third finger)
7		
32		The number of fingers held up equals the number of tens.

The Sander's (1971) method appears to have at least two desirable qualities: first, it could be adopted with the current algorithm with no structural change in format, and second, it does reduce the memory load (eliminates complex facts) required over the conventional method. The one major disadvantage to this method is the lack of written calculations other than the answer, which makes locating and correcting errors more difficult.

Finally, Batarseh (1974) developed an algorithm which is designed to simplify the procedures involved in the carrying process. Using the Batarseh procedure, numbers are never carried to the top of a column, instead the digit is written underneath the column to which it will be carried and underlined. When the sum of the next column is obtained the underlined digit is added to that sum. This pattern is followed throughout the rest of the columns. However, this algorithm requires use of complex facts as well as basic facts. The following example illustrates this method:

$$\begin{array}{cccc} & 7 & 6 & 4 \\ + & 5 & 9 & 7 \\ & & \underline{1} & 1 \\ & \underline{1} & 6 & \\ 1 & 3 & 6 & 1 \end{array}$$

The algorithms described here should not be considered the complete set of methods for performing addition problems. They were presented because of particular relevance to the concerns of this study.

Research on Alternative Algorithms

Most of the literature concerned with the development of new algorithms has been of a descriptive nature. There exists a great paucity of publications presenting data from basic research. This lack of research was alluded to by Boyle (1975), Alessi (1974), and Rudolph (1976). Recently, however, a number of studies have been undertaken at the University of Maryland and Western Michigan University. The studies have compared the relative effects of the Hutchings "Low-stress" algorithm to the conventional method under varying conditions. The remainder of this section will review the results of these studies and implications.

The earliest studies, comparing the "Low-stress" to the conventional algorithm, used group factorial designs (Alessi, 1974; Boyle, 1975; Dashiell, 1974; Hutchings, 1972). The results of these studies show the "Low-stress" procedures to be more effective than the conventional method. Alessi (1974) found that the Hutchings "Low-fatigue" (stress) algorithm produced higher scores for the number of columns correctly added and attempted in a 30 minute period. It was also pointed out that as the problems increased in difficulty, the relative superiority of the "Low-stress" over the conventional procedures decreased respectively.

In his analysis and interpretation, Alessi (1974) noted that the majority of the children in the "Low-stress" group were amazed at their new found computational power and expressed enthusiasm over being able to add such large sums correctly.

In a similar study, Boyle (1975) investigated the effects of three conditions on the number of columns correctly added and attempted. The three conditions were: a.) antecedent algorithm instruction, b.) social reinforcement contigencies, and c.) simulated test and non-test conditions. Boyle concluded that, "The Hutchings 'Low-stress' algorithm produced significant increments in scores over the conventional algorithm under all conditions of reinforcement and testing."

In suggesting directions for future research, Alessi (1974) stated that:

> "..perhaps longer practice than allowed in this study is necessary to allow stable performances under varying environmental conditions. To test this idea perhaps a repeated measures design or series of single subject design studies could be conducted." (p. 98)

Recently a number of single subject design studies comparing the Hutchings "Low-stress" algorithm to other methods have been conducted at Western Michigan University (Rudolph, 1976; Gillespie, 1976; and Zoref, 1976). All of these studies found the "Low-stress" procedures generally superior to the conventional method in producing accurate calculations.

Rudolph (1976) compared the "Low-stress" algorithm to the conventional method under distracting and non-distracting conditions. The results indicated that the "Low-stress" method produced more consistent responding and a reduction in error rates over the conventional algorithm. The superior performance was not found to be dependent upon the level of distracting stimuli or the order in which the algorithms were presented. These findings do not support Boyle's (1975) speculation that students using the conventional algorithm would be more likely to lose their place when distracted and thus have to start over, while students using the "Low-stress" method would not encounter this difficulty.

In a study designed to assess student preferences for either "Low-stress" or the conventional algorithm, Gillespie (1976) found that given a free choice, students chose the "Low-stress" procedure. Once a preference was established, a penalty was imposed for that choice which required 50% and 100% more work to be completed. Most of the students chose the "Low-stress" method even at a 50% increase. The "Low-stress" method resulted in consistently higher accuracy and rates correct, and lower rates incorrect. Also, most students chose the method in which they were most competent.

Zoref (1976) presented data comparing both the "Low-stress" and conventional methods with the use of electronic pocket calculators. Her study incorporated 2 X 7 and 5 X 7 problem arrays with high and low achievers.

Overall, the results showed that the "Low-stress" algorithm was generally superior in terms of accuracy, rate correct and rate incorrect as compared to the conventional algorithm and the calculator. This was especially true with 5 X 7 arrays. Additionally, the "Low-stress" procedures resulted in the lowest incorrect rate in all phases of the study with high and low achievers regardless of problem size.

Rate correct was higher for the "Low-stress" algorithm than the calculator for high achievers with 2 X 7 and 5 X 7 arrays and for low achievers with 5 X 7 problem arrays. Greater variability was also noted for students using the calculators for 5 X 7 arrays. Generally, the use of the calculators seemed to hinder high achievers while helping low achievers.

While much of the past research supports the use of the Hutchings' "Low-stress" procedures, other methods must be examined if a most efficient model is to be found. The following section will address this question while presenting the purposes of this study.

The Purpose of This Study

It was stated previously that the Fulkerson (1963) method was advantageous because it reduced the memory load and did not require special spacing to accommodate the notation involved. The disadvantage of the Fulkerson method was that a full record of calculations was not produced. Thus a variation is proposed, whereby the slash would be retained to indicate tens, hundreds, etc., and half space notation would be used to indicate units. The following example illustrates the variation:

$$\begin{array}{l} 22\ 1 \\ 7_9\ 6_7\ 3 \\ \cancel{9}_8\ \cancel{4}_1\ \cancel{9}_2 \\ \cancel{6}_4\ \cancel{9}_0\ 7_9 \\ \hline 2\ 4\ \ 0\ \ 9 \end{array}$$

The same advantages presented by Alessi (1979) would appear to support this variation with the additional advantage of not requiring wide spaces between columns. This procedure will be referred to as the Fulkerson "Full-record" algorithm.

The purpose of this study was to compare the "Low-stress", Fulkerson "Full-record" and conventional algorithms for speed and accuracy in performing columnar addition. This study also attempted to determine which computational algorithm subjects preferred after exposure to each. In addition, an attempt was made to systematically replicate part of the earlier findings by Rudolph (1976) and Gillespie (1976).

METHOD

Special Considerations

This study was designed within the framework of the following limitations: a.) daily sessions would not exceed 15 minutes, b.) the sessions would be held each day at the same time, c.) the study would be run entirely by the experimenter, d.) all expenses would be the sole responsibility of the experimenter and e.) the experimenter would make the results available to the parents of the subjects and school personnel.

Subjects

The subjects of this study were nine fourth grade students, approximately ten years of age. Five of the subjects were males and four were females. All of the subjects were enrolled as full time students in regular education classrooms. Information regarding standardized test scores was unavailable to the experimenter.

Settings

The study took place at two different elementary school buildings located near Cadillac, Michigan. The schools will be referred to as school A and school B. Three students attended school A and the remaining six students attended school B. Daily sessions were held at 1:15 pm at school A and at 2:15 pm at school B, five days a week. At school A sessions were held in a hallway near the entrance of the school. Table and chairs were available. In school B, sessions were held in the boys' locker room located off the gymnasium. Both of these settings were considered distracting environments.

Experimental Task

During each daily session in all phases, except instructional days, students were given two work sheets. Each work sheet contained three 5 X 7 array addition problems. (Appendix F) The numbers used in constructing the arrays were obtained from a random numbers table. Also, as recommended by Hutchings (1972), the identity element zero was not used. The students were asked to do as many problems as they could within a five minute time period. The investigator used a stop watch to time sessions. No student ever completed all six problems.

Experimental Design

The experimental design used in this study incorporated elements of both reversal (Sidman, 1960) and multiple baseline designs (Bear, Wolf and Risley, 1968). Two reversal designs were used; and ABACA and ACABA in conjunction with a multiple baseline design. The use of the multiple baseline allowed each phase to be staggered over time. The total design allowed for a comparison of individual performances during each phase, while at the same time being sensitive to sequential as well as other time related effects.

Independent Variables

1.) Hutchings' "Low-stress" addition algorithm versus the Fulkerson "Full-record" addition algorithm versus the conventional addition algorithm.

2.) Order of algorithm taught: a.) Hutchings' "Low-stress" algorithm first or b.) Fulkerson "Full-record" first.

Dependent Variables

Four dependent variables were studied:

1.) Percent correct: the number of columns that are correctly computed divided by the total number attempted.

2.) Rate correct: the total number of columns correctly added, divided by the session length (five minutes) and expressed as columns correct/minute.

3.) Rate incorrect: the total number of columns incorrectly added, divided by the total session length (five minutes) expressed as columns incorrect/minute.

4.) Algorithm preference: the algorithm chosen after exposure to all three.

Recording and Scoring

Following each session the experimenter checked each students' papers. The number of columns correct and incorrect were divided by five minutes and the corresponding rates were recorded. In addition to the rate correct and incorrect, the percent accuracy was also recorded. Students were never given feedback on their scores. The experimenter told all the subjects that they were doing very well.

Reliability

Reliability checks were made on the problems completed by the students. Since the students' work (on daily assignments) left a permanent product, it was possible to make photo copies of their work. These photo copies were then given to an independent grader to be scored. The independent graders' scores were then compared to the experimenter's record and a reliability coefficient calculated. In calculating the reliability coefficient, the number of columns in which both graders' scored the same was counted as an agreement. Columns that were scored differently were counted as a disagreement. The reliability coefficient was calculated by dividing the number of agreements by the number of agreements plus disagreements. Reliability data were taken twice per phase.

Materials

A pretest of 56 addition facts was administered (Appendix A). During algorithm training, all subjects were given identical practice sheets consisting of a variety of problem arrays (Appendix B). Daily worksheets were composed of six addition problems in 5 X 7 arrays, three problems on each sheet (Appendix C). The daily work sheets were typed with an IBM Selectric typewriter using the orator element. Triple spaces were used between columns and double spaces between rows. Problem sheets were reused approximately every three to four weeks. Since the daily assignments consisted of two work sheets, it was possible not to pair the same two sheets together more than once. It was felt that the size of the problems and the fact that the same two never appeared twice, would minimize practice effects. A stop watch was also used during each session.

In summary the following materials were required each day: a.) stop watch and b.) 18 photo copies of the daily work sheets. Photo copies were used in place of dittos because it was felt that photo copies were easier to read.

Procedure

Pretesting

To participate in this study, all subjects were required to pass a pretest consisting of 56 basic addition problems (Appendix A). A passing score was 90% or above. This test was used to insure a satisfactory degree of competence with basic addition.

Informed Consent

After the subjects were selected, a letter of informed consent was sent home to their parents. This letter explained the functions of the study and asked for written permission for their child to participate. All subjects were granted written permission.

Algorithm Training Procedures

All subjects recieved similar instructions in the use of all three algorithms: conventional algorithm (Appendix C), Hutchings "Low-stress" algorithm (Appendix D), and the Fulkerson "Full-record" algorithm (Appendix E). Training sessions consisted of approximately ten minutes of instruction followed by a practice session. During the practice session, the students were required to complete two pages of problems (Appendix F) using the instructed method. During this time period, the investigator helped students who had questions or difficulties. Prior to reversal phases, students were given a ten minute review in which they were required to complete the same two practice sheets (described above) using the selected algorithm.

Daily Sessions

At the beginning of the study, the students were instructed to remain in their classrooms until the experimenter arrived. Once the students were in the work area, the daily work sheets were passed out. The subjects were instructed to leave the work sheets upside down on their desks until the experimenter said "begin". Once the students were told to begin, they were instructed to turn over their papers and do as many problems as they could. When the experimenter said "stop", the students were instructed to lay their pencils down.

Following the completion of their daily work sheets, the students were allowed to use the time remaining from the fifteen minute period, to play various games of their choice. This period was used as an incentive for working hard on their daily work sheets. Students were informed that if they worked hard on their problems, they would be allowed to use this remaining time to play games. If they did not work hard, they would have to return to their classroom. It was not necessary to impose this contigency during the study.

Experimental Conditions

During condition I, all subjects were given instructions using the conventional algorithm. Data were collected on daily work sheets. Following this first condition, the subjects were divided into two groups. Group A consisted of five students while Group B had four. The two groups were presented the algorithms in different orders. Within each group the timing in which the algorithms were presented was staggered in order that time related effects might be detected. These sub-groups were composed of two students each, with the exception of one group which had three.

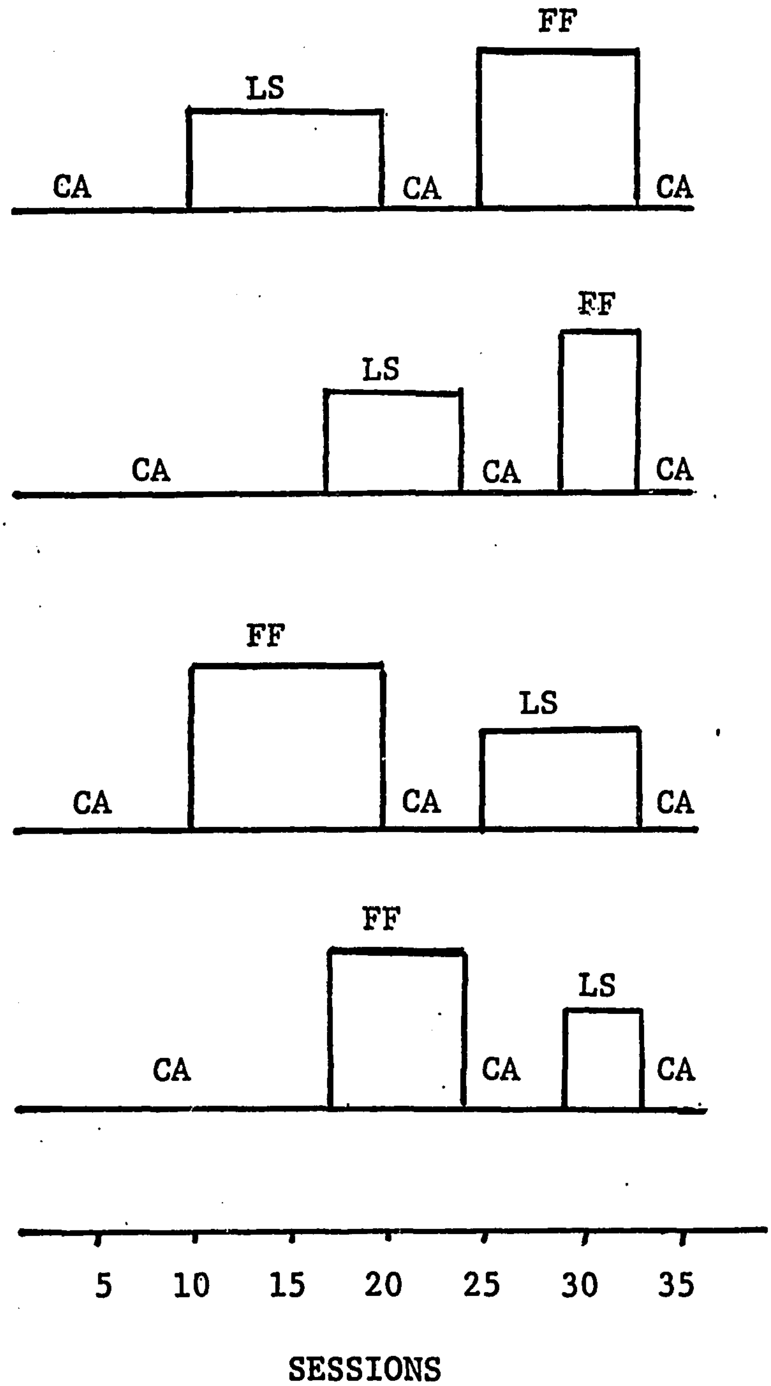
Students in Group A were taught the algorithms in the following sequence: a.) conventional, b.) "Low-stress", c.) conventional, d.) Fulkerson "Full-record", and e.) conventional. While Group B students were taught the algorithms in the following order: a.) conventional, b.) Fulkerson "Full-record", c.) conventional, d.) "Low-stress", and e.) conventional. Table I gives a visual display of the order and timing for each algorithm phase using the notation suggested by Johnston and Pennypacker (1980).

Algorithm Preference

On the last session day each student was required to

Table 1: A visual display of the order and timing in which each algorithm was taught for all subjects.

TABLE I



CA = Conventional
LS = "Low-stress"
FF = Fulkerson "Full-record"

complete a one page practice sheet containing three 5 X 7 array addition problems. In completing the three addition problems, the students were requested to use each of the algorithms they had been taught (one algorithm for each problem). This was done as a general review of each method. Following the completion of the practice problems, the students were given a super problem to complete (Appendix G). The super problem consisted of 17 columns and 12 rows. The students were instructed that they could use the method they liked best to complete the problem. Data were collected on the algorithm used and the number of columns correct out of 17.

RESULTS

Reliability

Data on reliability, collected over ten sessions (two each phase), yielded an overall agreement index of 96.6 for scoring columns correct and incorrect. The agreement index ranged from 85 to 100 percent.

General Findings

Overall the results of this study indicate that the Hutchings' "Low-stress" (LS) and the Fulkerson "Full-record" (FF) algorithms were generally superior to the conventional algorithm (CA) in producing efficient and accurate calculations. In addition, a comparison of the "Low-stress" and Fulkerson "Full-record" methods show that calculation patterns were similar under each condition. Data concerning individual performances across time will be examined later in this paper.

Table 2 presents individual and group means for percent accuracy for each algorithm phase. The results obtained across all students show higher accuracy scores using the "Low-stress" and Fulkerson "Full-record" as compared to the conventional algorithm. Also, the overall percent accuracy scores for both the "Low-stress" and Fulkerson "Full-record" algorithms are very similar. A com-

Table 2: Group and individual means for percent accuracy for groups A and B using 5 X 7 arrays.

TABLE 2

MEAN PERCENT ACCURACY FOR GROUPS A AND B

Group A Students	CA	LS	CA	FF	CA	Group B Students	CA	FF	CA	LS	CA
CR	74	85	60	92	52	BD	68	86	75	91	76
JS	88	95	91	93	76	TM	62	88	60	95	81
ER	81	97	75	96	77	DA	66	95	70	96	71
MR	92	96	86	99	89	MT	77	91	66	82	55
CS	79	98	83	96	74						
Total $\bar{x}$	82	94	79	95	73		68	90	67	91	70
$\bar{x}$ percent accuracy across all CA phases = 78						$\bar{x}$ percent accuracy across all CA phases = 69					

CA = Conventional
LS = Hutching's "Low-stress"
FF = Fulkerson "Full-record"

parison of each phase for all students indicates a clear superiority of the "Low-stress" and Fulkerson "Full-record" algorithms over the conventional algorithm in terms of percent accuracy. The average percent accuracy across each phase for all students ranged from 82% to 98% for the "Low-stress" method; from 86% to 99% for the Fulkerson "Full-record" procedures and; from 55% to 92% for the Conventional algorithm. The average percent correct for all students in each phase using the "Low-stress" method was 94% for group A and 91% for B. For all students using the Fulkerson "Full-record" algorithm, the average percent accuracy was 95% for group A and 90% for group B. The average percent accuracy for all students for all conventional algorithm phases was 78% for group A and 69% for B.

Table 3 presents individual and group means for columns incorrectly added per minute for each algorithm phase. An inspection of the overall error rates for all phases indicates results similar to the percent accuracy data. Both the Fulkerson "Full-record" and "Low-stress" methods were superior to the conventional algorithm in terms of producing lower error rates. Error rates averaged across each phase for all students ranged from .05 to .6 columns incorrect per minute for phases in which the "Low-stress" method was used. During sessions in which the Fulkerson "Full-record" algorithm was used, the error rates ranged from .04 to .3 columns incorrect per minute. In phases

Table 3: Group and individual means for columns incorrect per minute for groups A and B, using 5 X 7 arrays.

TABLE 3

MEAN COLUMNS INCORRECT/MINUTE FOR GROUPS A AND B

Group A Students	CA	LS	CA	FF	CA	Group B Students	CA	FF	CA	LS	CA
CR	.48	.32	1.15	.2	1.24	BD	1.0	.3	1.0	.3	1.0
JS	.24	.16	.2	.27	.72	TM	.9	.3	1.2	.2	.5
ER	.57	.08	.8	.12	.68	DA	1.1	.2	1.2	.2	1.1
MR	.26	.15	.6	.04	.53	MT	.4	.2	.6	.6	1.1
CS	.73	.05	.7	.08	1.0						
Total $\bar{\bar{x}}$	.45	.15	.69	.14	.83		.85	.25	1.0	.32	.92
$\bar{\bar{x}}$ columns incorrect/minute across all CA phases = .66						$\bar{\bar{x}}$ columns incorrect/minute across all CA phases = .93					

CA = Conventional
LS = Hutching's "Low-stress"
FF = Fulkerson "Full-record"

using the conventional algorithm, error rates ranged from .2 to 1.24 columns incorrect per minute. Across all phases the average error rate using the "Low-stress" algorithm was .15 for group A and .32 for group B. For all phases in which the Fulkerson "Full-record" procedures were in effect the average error rates were .14 for group A and .25 for B. Average error rates for phases in which the conventional algorithm was used were .66 for group A and .93 for group B. In phases using the "Low-stress" and Fulkerson "Full-record" algorithms similar computational patterns are shown in the form of decreased error rate levels.

Table 4 presents individual and group means for columns correctly added per minute for each algorithm phase. The results for correct rate per minute are not as uniform as those obtained for accuracy and incorrect rate. Correct rates averaged across each phase for all subjects ranged from 1.9 to 5.2 columns correct per minute for phases in which the "Low-stress" algorithm was used. During phases in which the Fulkerson "Full-record" was used, correct rates ranged from 2.6 to 4.2 columns correct per minute. In phases in which the conventional algorithm was used, correct rates ranged from 1.4 to 4.2 columns correct per minute. Across all phases, the average correct rate using the "Low-stress" algorithm was 3.1 for group A and 3.8 for group B. For phases in which the Fulkerson "Full-record" procedures were used the average correct rate was 3.3 for

Table 4: Group and individual means for columns correct per minute for groups A and B using 5 X 7 arrays.

TABLE 4

MEAN COLUMNS CORRECT/MINUTE FOR GROUPS A AND B

Group A Students	CA	LS	CA	FF	CA	Group B Students	CA	FF	CA	LS	CA
CR	1.4	1.9	1.8	2.7	1.5	BD	2.3	2.4	3.2	3.3	3.7
JS	2.0	3.2	2.3	3.8	2.2	TM	1.6	2.8	1.5	3.7	2.3
ER	2.5	3.4	2.5	3.3	2.4	DA	2.4	4.0	3.0	5.2	2.9
MR	3.3	4.3	4.0	4.2	4.2	MT	1.7	3.4	1.3	3.0	1.5
CS	2.9	2.9	3.4	2.6	3.0						
Total $\bar{x}$	2.4	3.1	2.8	3.3	2.6		2.0	3.1	2.2	3.8	2.6
$\bar{x}$ columns correct/minute across all CA phases = 2.6						$\bar{x}$ columns correct/minute across all CA phases = 2.3					

CA = Conventional
LS = Hutching's "Low-stress"
FF = Fulkerson "Full-record"

group A and 3.1 for group B. Average correct rates for phases in which the conventional algorithm was used were 2.6 for group A and 2.3 for group B.

While the total means across algorithm phases indicate that both the "Low-stress" and Fulkerson "Full-record" algorithms produced higher correct rate, an examination of individual means show that this effect was not indicative of each individual. Subject MR's correct rate increased during the "Low-stress" algorithm phase over the conventional algorithm levels, but failed to show a significant decline when phases were reversed. Virtually no correct rate changes were observed for subject MR for the Fulkerson "Full-record" method. For subject CS, both the "Low-stress" and Fulkerson "Full-record" algorithms resulted in lower correct rates. This may have been due to the fact that he used his fingers in tabulating column sums, and that writing the special notations for the "Low-stress" and Fulkerson "Full-record" algorithms required more time. For subject BD a general learning trend is noted across each algorithm phase.

Overall the data on means do not indicate that the order in which the algorithms were taught (LS or FF first) significantly affected student performances, with the exception of rate correct. Table 4 shows that for students in group A (LS taught first) the average correct rate was slightly higher during the Fulkerson "Full-record" phase

(LS = 3.1, FF = 3.3). For students in group B (FF taught first) the average correct rate was higher during the "Low-stress" phases (FF = 3.1, LS = 3.8). In both groups, the algorithm that was introduced last resulted in higher correct rates.

Individual Performances Across Algorithm Phases

Group A

Figures 1,2,3,4 and 5 present individual data for dependent measures on students in group A. Prior to examining individual performances, it should be noted that any systematic errors that were observed during the study were corrected by the investigator. Two subjects in group A presented systematic errors. As can be seen in Figure 1, data for Subject CR show a substantial decline in accuracy and correct rate, as well as an increase in the number of columns incorrectly added per minute, during the first day of the "Low-stress" phase. It was observed that subject CR consistently obtained the wrong sum when adding eight plus seven. This error was corrected prior to the next session. In Figure 5, it should be noted that subject CS failed to correctly calculate any column during the first session. He was asked to demonstrate his calculation method for the investigator. In the process of demonstrating his method, the subject discovered that he had been mis-

Figure 1: Correct and incorrect rate, and accuracy for subject CR on daily work sheets under different conditions of computation (Conventional, "Low-stress" and Fulkerson "Full-record") using 5 X 7 arrays.

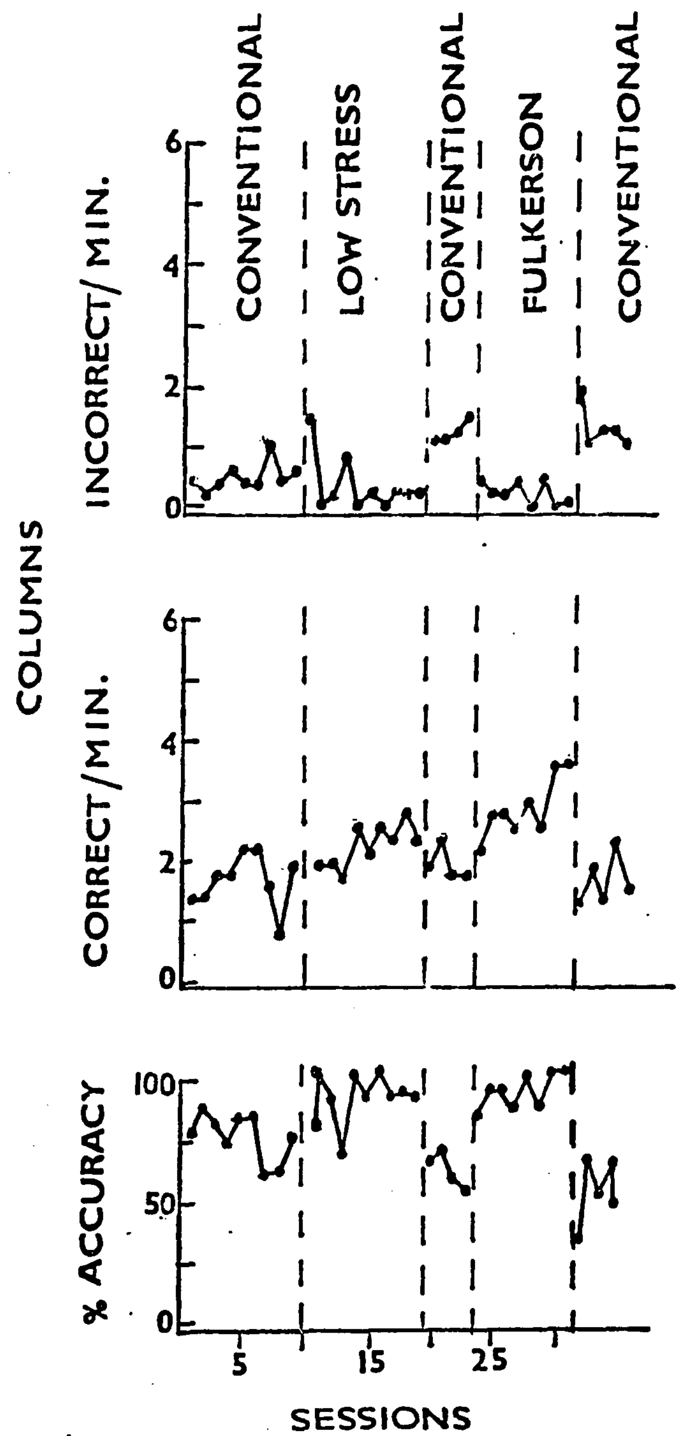

COLUMNS
INCORRECT/ MIN.
CORRECT/MIN.
% ACCURACY
CONVENTIONAL
LOW STRESS
CONVENTIONAL
FULKERSON
CONVENTIONAL
SESSIONS
5
15
25
0
2
4
6
50
100

Figure 2: Correct and incorrect rate, and accuracy for subject JS on daily work sheets under different conditions of computations (Conventional, "Low-stress" and Fulkerson "Full-record") using 5 X 7 arrays.

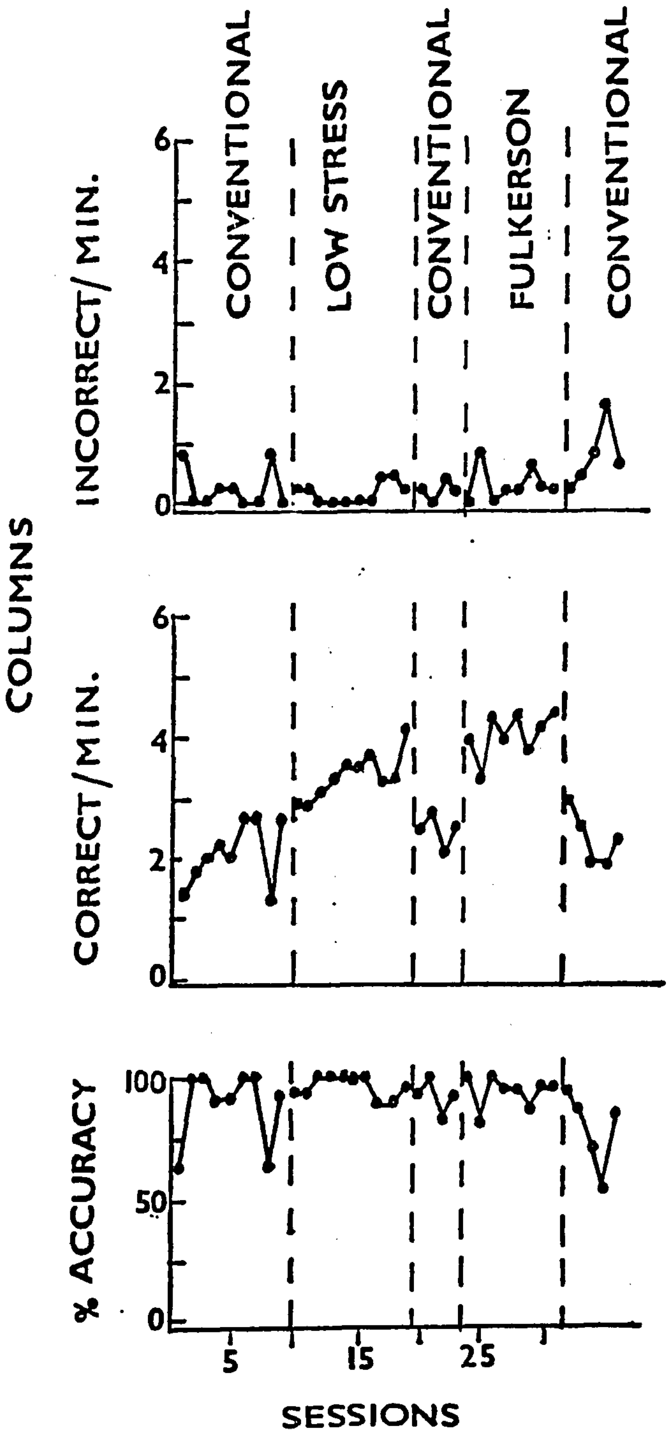

COLUMNS
INCORRECT/ MIN.
CORRECT/MIN.
% ACCURACY
CONVENTIONAL
LOW STRESS
CONVENTIONAL
FULKERSON
CONVENTIONAL
6
4
2
0
6
4
2
0
100
50
0
5
15
25
SESSIONS

Figure 3: Correct and incorrect rate, and accuracy for subject ER on daily worksheets under different conditions of computation (Conventional, "Low-stress" and Fulkerson "Full-record") using 5 X 7 arrays.

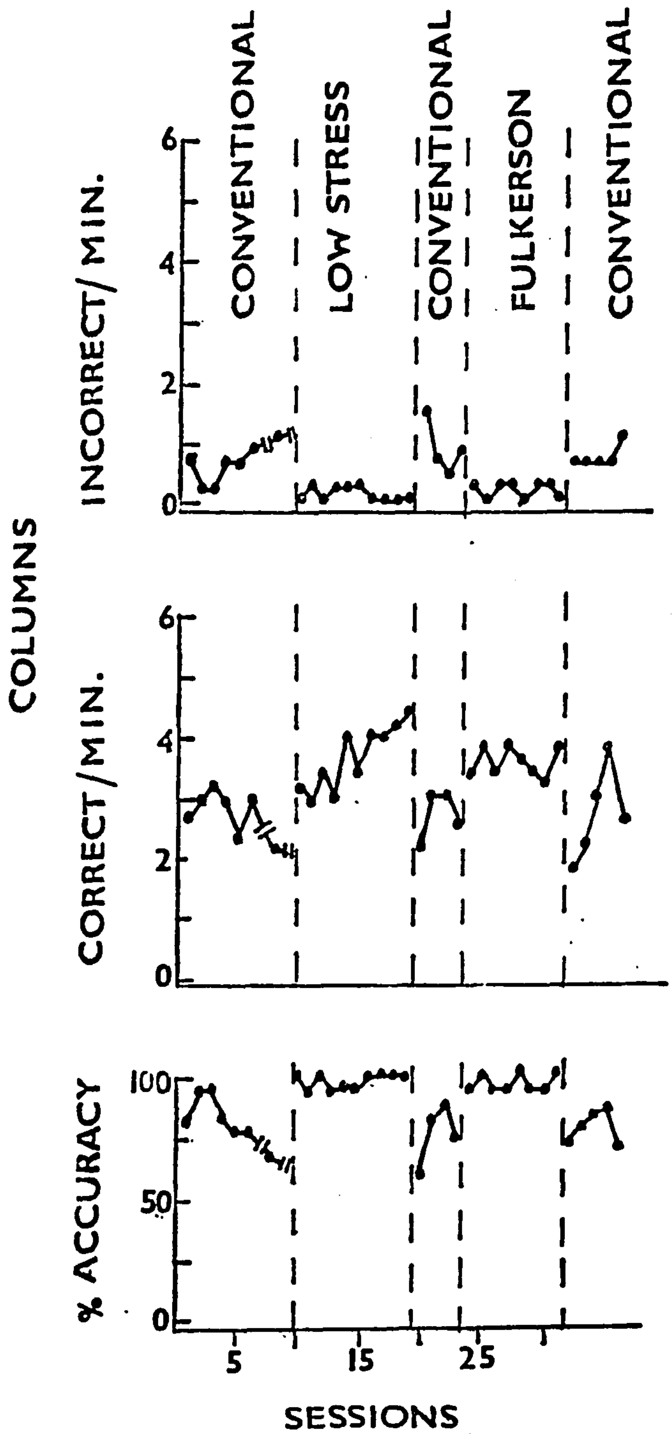

COLUMNS
INCORRECT/ MIN.
CORRECT/MIN.
% ACCURACY
CONVENTIONAL
LOW STRESS
CONVENTIONAL
FULKERSON
CONVENTIONAL
0
2
4
6
0
50
100
5
15
25
SESSIONS

Figure 4: Correct and incorrect rate, and accuracy for subject MR on daily work sheets under different conditions of computation (Conventional, "Low-stress" and Fulkerson "Full-record") using 5 X 7 arrays.

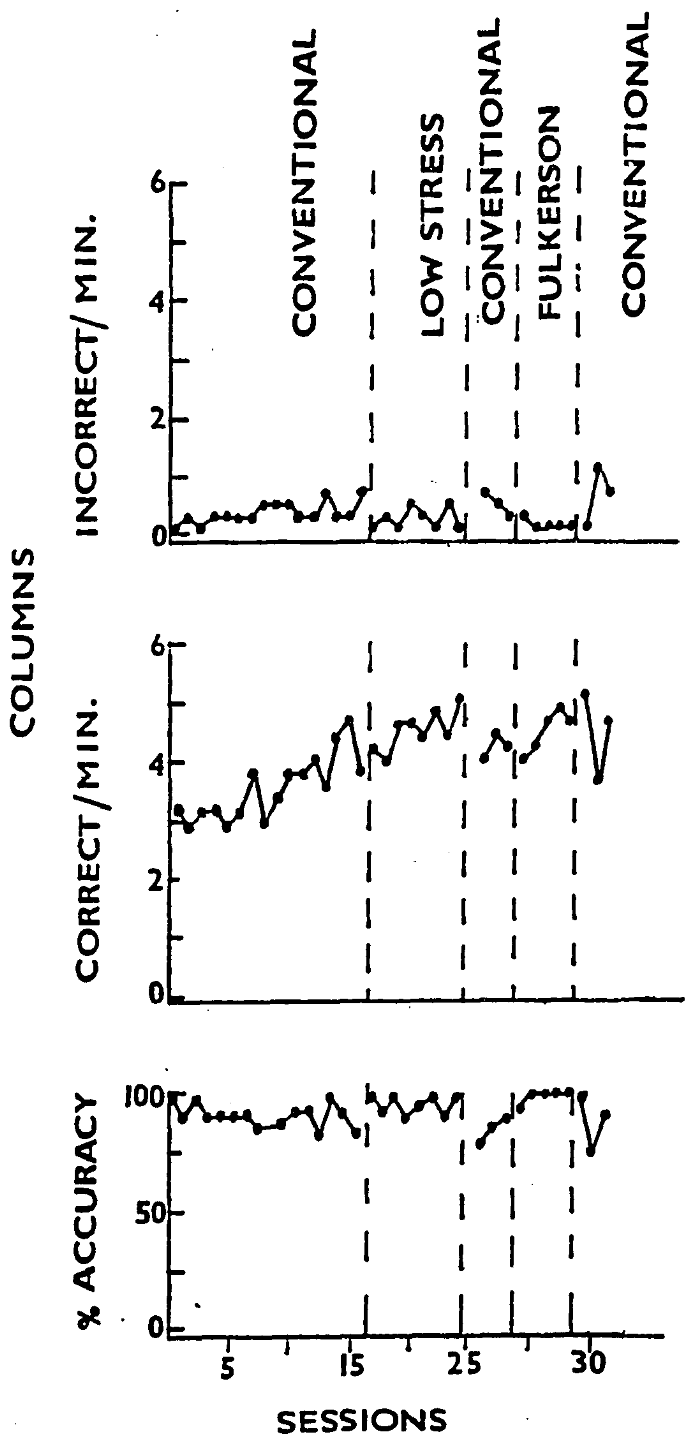
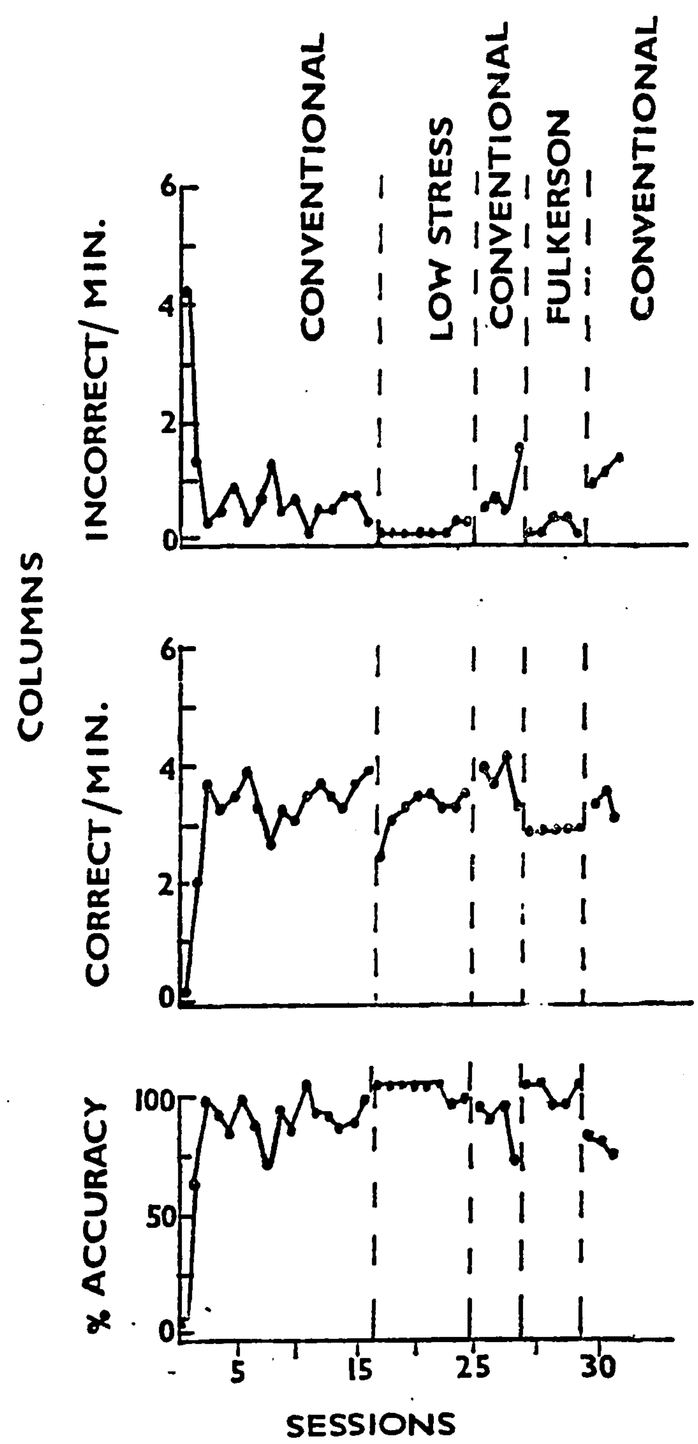

COLUMNS
INCORRECT/MIN.
CORRECT/MIN.
% ACCURACY
SESSIONS
CONVENTIONAL
LOW STRESS
CONVENTIONAL
FULKERSON
CONVENTIONAL
0
2
4
6
0
2
4
6
0
50
100
5
15
25
30

Figure 5: Correct and incorrect rate, and accuracy for subject CS on daily work sheets under different conditions of computation (Conventional, "Low-stress" and Fulkerson "Full-record") using 5 X 7 arrays.


COLUMNS
INCORRECT/ MIN.
CORRECT/MIN.
% ACCURACY
CONVENTIONAL
LOW STRESS
CONVENTIONAL
FULKERSON
CONVENTIONAL
SESSIONS

counting. For example, if he were adding six plus seven, he would start with the number six (instead of seven) and count seven numbers on his fingers arriving at 12 instead of 13 for his answer.

As can be seen from Figures 1,2,3,4, and 5, each student in group A showed greater accuracy scores and less variability during the "Low-stress" and Fulkerson "Full-record" phases as compared to the conventional algorithm. For subject MR (See Figure 4), accuracy scores were highest and most stable using the Fulkerson "Full-record" algorithm. Accuracy scores for subject CR (See figure 1), show similar results to those obtained for subject MR with the highest accuracy scores occurring during the Fulkerson "Full-record" phase. Comparable effects were obtained for accuracy, for both the Fulkerson "Full-record" and "Low-stress" algorithms with subjects JS, ER and CS.

In terms of columns incorrectly added per minute, figures 1,2,3,4,and 5 indicate that both the Fulkerson "Full-record" and "Low-stress" algorithms were overall more effective in producing lower error rates as compared to the conventional algorithm. Subject MR (See figure 4) displayed the least variability and also lowest error rate , during the Fulkerson "Full-record" phase. For subject JS, error rates were lowest and most stable using the "Low-stress" procedures. Data for subjects CR, ER and CS indicate that the "Low-stress" and Fulkerson "Full-record" algorithms were approximately equivalent in terms of reduced error rates over the conventional method.

For correct rate, the results are not as clear. Data for subject CS show a decrease in correct rate in phases using the "Low-stress" and Fulkerson "Full-record" algorithms as compared to the conventional algorithm. For subject MR, data on correct rate indicate minimal effects during the "Low-stress" and Fulkerson "Full-record" phases. Looking at the correct rate data for subject MR in figure 4, an ascending trend can be seen during the initial conventional algorithm phase. Since data in this trend overlap with data from subsequent phases, it is plausable to say that using the "Low-stress" and Fulkerson "Full-record" algorithms had little or no effect on correct rate. For subjects CR, JS and ER, data in figures 1,2, and 3 show that correct rate levels were higher using the "Low-stress" and Fulkerson "Full-record" methods compared with levels achieved using the conventional algorithm.

Group B

Figures 6,7,8 and 9 present individual data for all dependent measures on students in group B. As shown in these figures, each student in group B had higher accuracy scores using the "Low-stress" and Fulkerson "Full-record" algorithms then when using the conventional method. Variability was reduced during the "Low-stress" and Fulkerson

Figure 6: Correct and incorrect rate, and accuracy for subject BD on daily work sheets under different conditions of computation (Conventional, "Low-stress" and Fulkerson "Full-record") using 5 X 7 arrays.

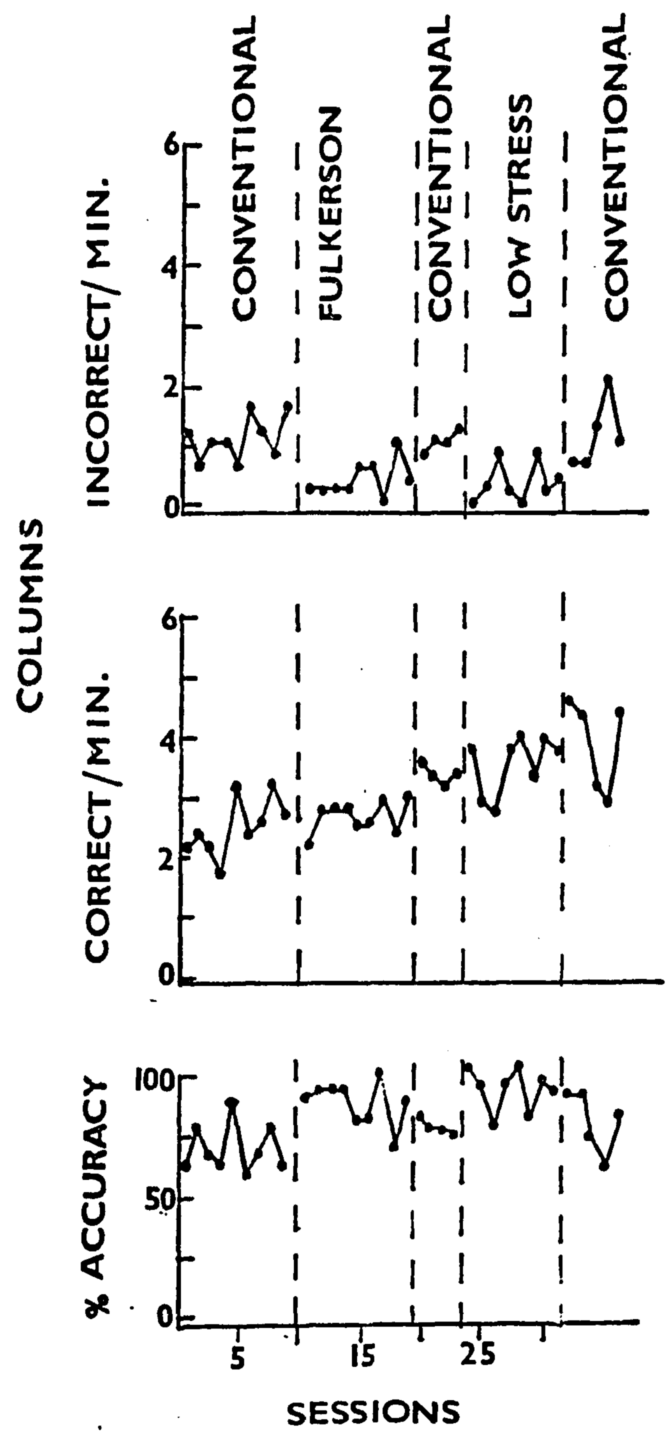

COLUMNS
INCORRECT/MIN.
CORRECT/MIN.
% ACCURACY
CONVENTIONAL
FULKERSON
CONVENTIONAL
LOW STRESS
CONVENTIONAL
0
2
4
6
0
2
4
6
0
50
100
5
15
25
SESSIONS

Figure 7: Correct and incorrect rate, and accuracy for subject TM on daily work sheets under different conditions of computation (Conventional, "Low-stress" and Fulkerson "Full-record") using 5 X 7 arrays.

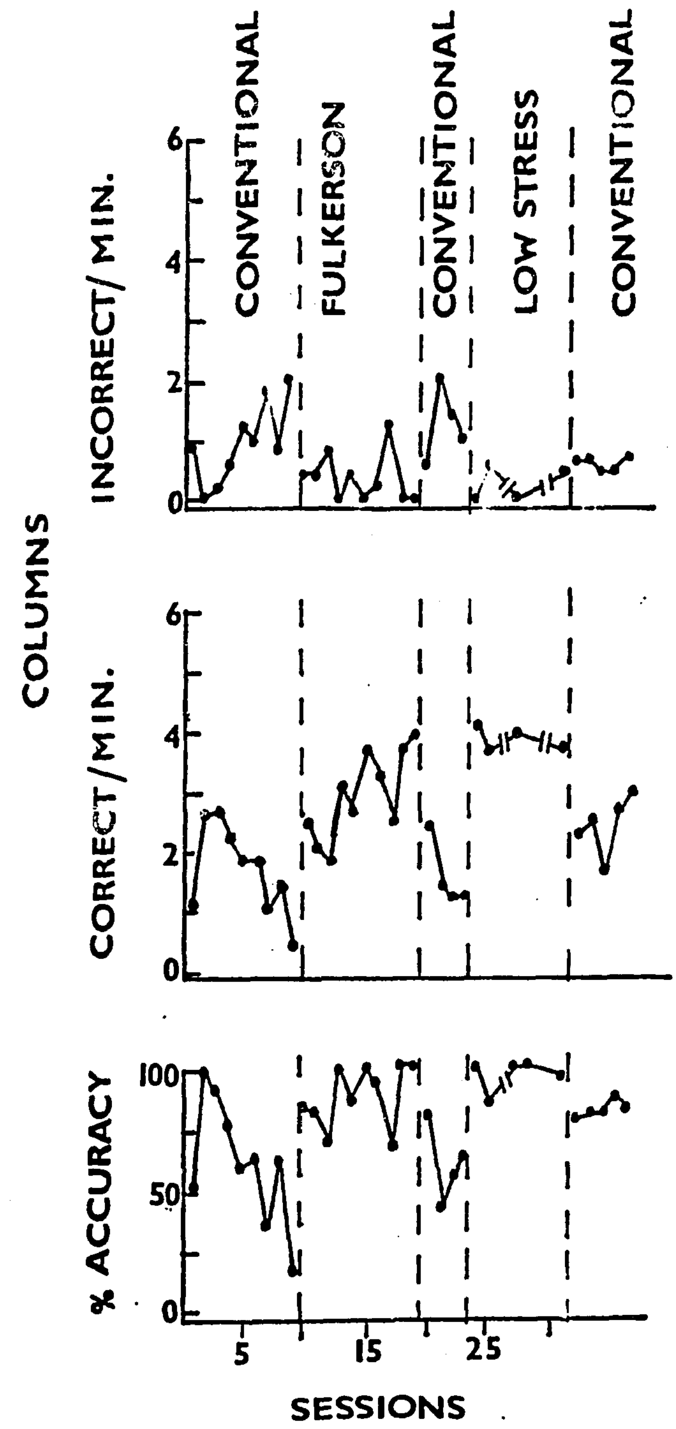

COLUMNS
INCORRECT/MIN.
CORRECT/MIN.
% ACCURACY
CONVENTIONAL
FULKERSON
CONVENTIONAL
LOW STRESS
CONVENTIONAL
0
2
4
6
50
100
5
15
25
SESSIONS

Figure 8: Correct and incorrect rate, and accuracy for subject DA on daily work sheets under different conditions of computation (Conventional, "Low-stress" and Fulkerson "Full-record") using 5 X 7 arrays.

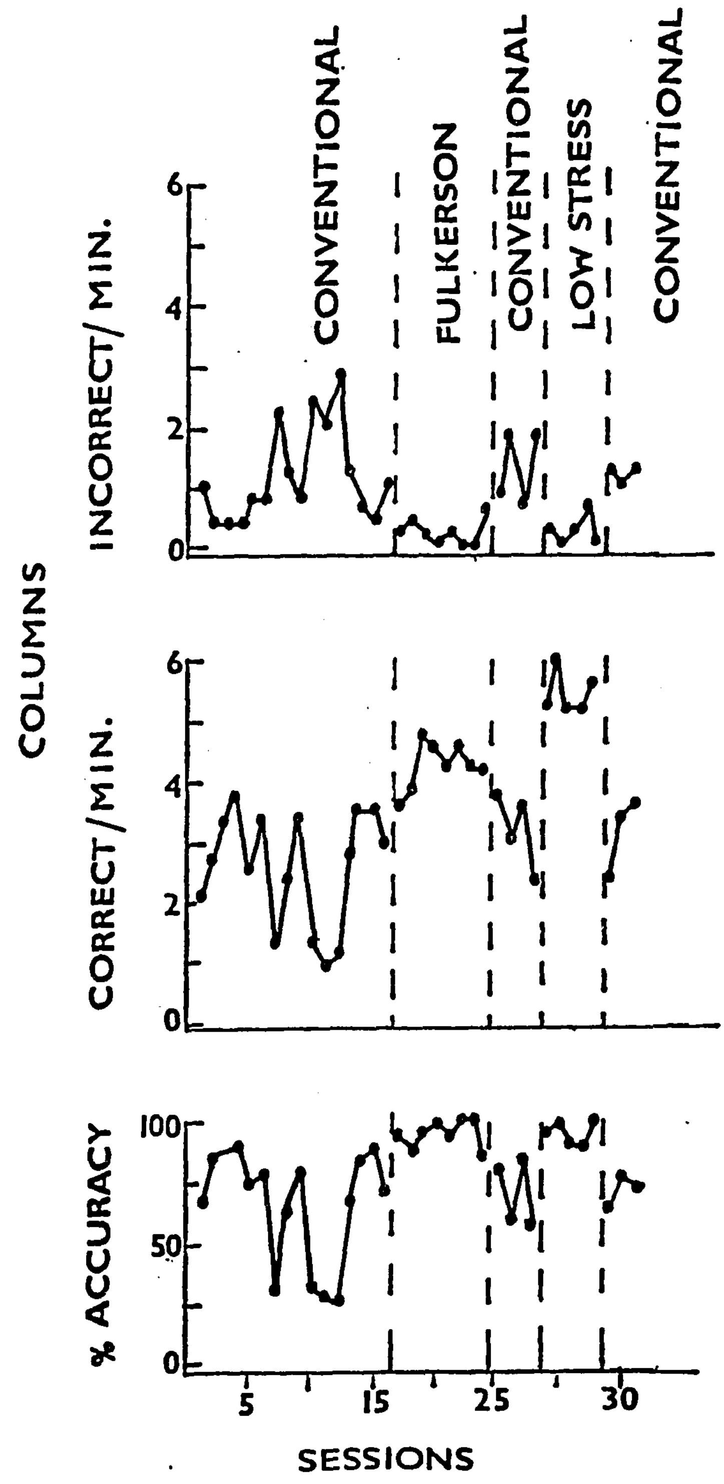
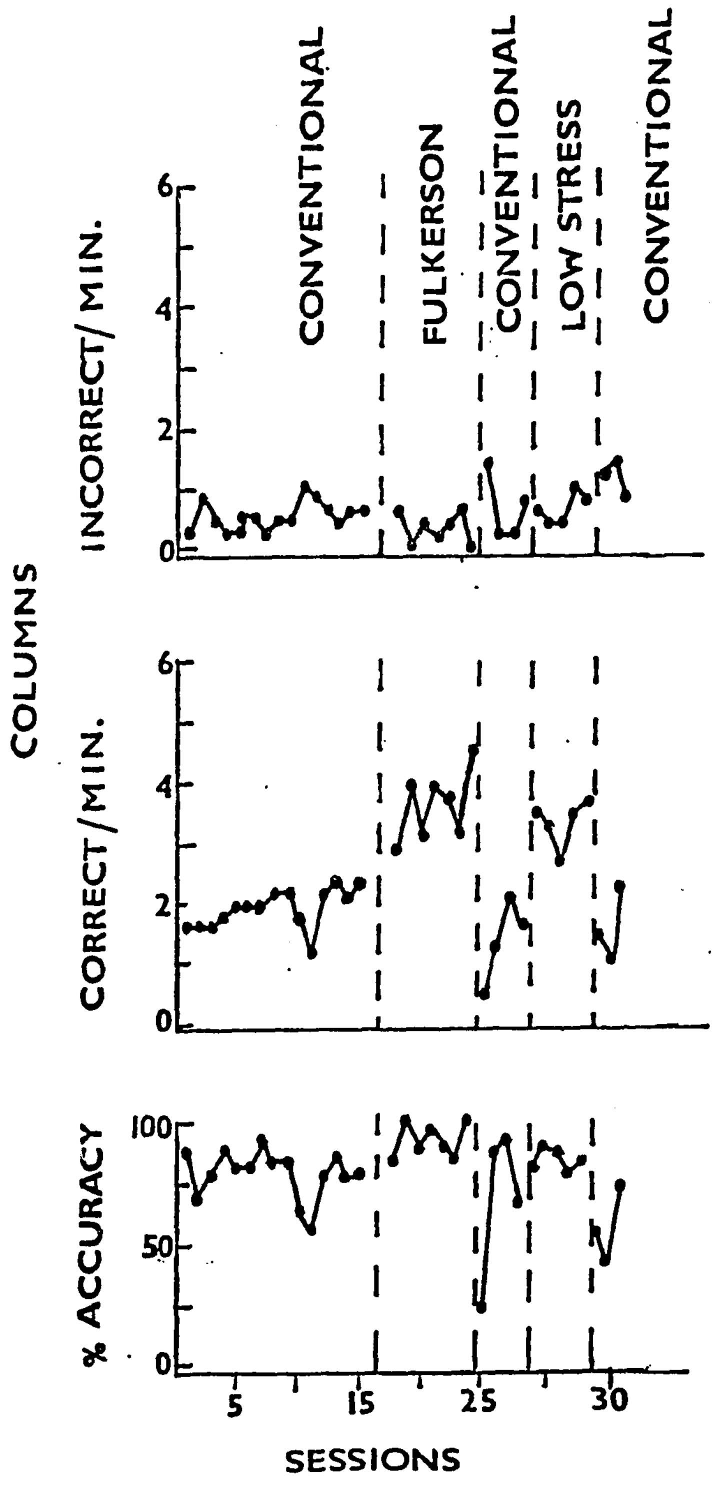

CONVENTIONAL
FULKERSON
CONVENTIONAL
LOW STRESS
CONVENTIONAL
COLUMNS
INCORRECT/MIN.
CORRECT/MIN.
% ACCURACY
SESSIONS
0
2
4
6
50
100
5
15
25
30

Figure 9: Correct and incorrect rate, and accuracy for subject MT on daily work sheets under different conditions of computational (Conventional, "Low-stress" and Fulkerson "Full-record") using 5 X 7 arrays.


COLUMNS
INCORRECT/ MIN.
CORRECT/MIN.
% ACCURACY
CONVENTIONAL
FULKERSON
CONVENTIONAL
LOW STRESS
CONVENTIONAL
0
2
4
6
0
2
4
6
0
50
100
5
15
25
30
SESSIONS

"Full-record" phases as compared to levels obtained for the conventional algorithm.

For rate incorrect, figures 6,7,8 and 9 show that when students were using the "Low-stress" and Fulkerson "Full-record" methods, they achieved lower error rates then when using the conventional algorithm. Variability was greatly reduced for subject DA (see figure 8) in both the ""Low-stress" and Fulkerson "Full-record" phases. Other subjects in group B showed only moderate declines in variability using the "Low-stress" and Fulkerson "Full-record" algorithms.

In terms of correct rate, figures 6,7,8 and 9 show,with the exception of subject BD, that the "Low-stress" and Fulkerson "Full-record" algorithms were superior to the convention algorithm. For subject BD (see figure 6), an ascending trend is noted which crosses all phases. Data on subject BD are not typical of others in group B.

The highest rate correct for both groups A and B was obtained by subject DA during the "Low-stress" condition (see figure 8). It can be seen from figure 8 that subject DA's rate correct scores, for the "Low-stress" phase, averaged over five columns per minute, with the highest being 5.8 columns per minute. Her highest rate correct for other phases was 4.6 using the Fulkerson "Full-record" algorithm and 3.6 using the conventional method. Also for subject DA, day to day variability was greatly reduced in both the "Low-stress" and Fulkerson "Full-record" algorithm phases.

Rate correct data for subject TM (see figure 7) show descending trend during the initial conventional algorithm phase followed by an ascending trend during the Fulkerson "Full-record" phase. Rate correct decreased when the conventional algorithm phase was reinstated followed by a subsequent increase during the "Low-stress" condition.

For subject MT, the data on correct rate (see figure 9)are similar to those of subject TM. It can be seen in figure 9, that correct rate scores were higher during the "Low-stress" and Fulkerson "Full-record" phases as compared to conventional phases. It should also be noted that correct rates are similar for both the "Low-stress" and Fulkerson "Full-record" algorithms. Correct rate scores, using the conventional algorithm do not overlap with scores obtained using either the "Low-stress" or Fulkerson "Full-record" algorithms.

Order Effects

Since the "Low-stress" and the Fulkerson "Full-record" algorithms are similar in terms of notation usage, it is plausable that having been taught one method would make learning the other easier. Looking at the data for subjects in groups A and B (see figures 1,2,3,4,5,6,7,8,and 9), order effects are only seen for rate correct, with subjects CR, JS, ER, and TM. Order effects were not noted for percent accuracy or rate incorrect.

For subject CR (see figure 1) an upward trend can be seen during the "Low-stress" phase followed by a decrease in rate correct for the conventional algorithm. When the Fulkerson "Full-record" algorithm was introduced, and upward trend was again noted. Order effects are assumed due to the fact that correct rates towards the end of the "low-stress" phase are similar to rates at the start of the Fulkerson "Full-record" condition, followed by a similar upward trend.

For subjects JS and ER (see figures 2 and 3), an upward trend is seen during the "Low-stress" phase for rate correct. This may indicate that higher rates were achieved due to practice. Using the conventional algorithm, rate correct scores decreased for both subjects. When the Fulkerson "Full-record" algorithm was introduced, rate correct levels immediately increased to levels similar to those obtained for the "Low-stress" phase with little or no practice effects noted.

The only individual to show possible order effects in group B was subject TM. Looking at figure 7, it can be seen that correct rates for subject TM are increasing during the Fulkerson "Full-record" phase, followed by a sharp decline in the conventional algorithm phase. Again, rate correct increased to levels obtained for the Fulkersone "Full-record" phase when the "Low-stress" algorithm was introduced but no practice effects were observed.

Algorithm Preference

Given a free choice of algorithms to use in solving a 17 X 12 array addition problem, two subjects, BD and CS, chose the conventional algorithm. Accuracy scores for both subjects BD and CS were 12% and 18% respectively. Both of these subjects chose the algorithm in which they achieved the highest rate correct scores during the study.

Three students chose the Hutchings' "Low-stress" method to solve their problem. Subjects DA, TM and CR obtained accuracy scores of 88%, 88% and 82%, respectively. These students also chose the algorithm in which they achieved the highest rate correct scores during the study.

Subjects ER, JS, MR and MT achieved accuracy scores of 88%, 88%, 94% and 71% respectively, using the Fulkerson "Full-record" algorithm. Only subject MT chose the algorithm in which he had the highest correct rate.

Students using the "Low-stress" and Fulkerson "Full-record" algorithms showed a definite superiority in terms of accuracy compared with students using the conventional algorithm. This finding supports Hutchings' (1976) view that the advantages offered by the "Low-stress" procedure increase in proportion to the column length. Since the Fulkerson "Full-record" algorithm is very similar to the "Low-stress" method, it is likely that these advantages would also apply to this method as well.

Summary

The instructional procedures used in this study were sufficient in teaching the students the two new algorithms (a 15 minute instruction followed by a 15 minute practice). All of the students learned the new addition procedures quickly and enthusiastically. Most students complained during reversal phases in which they were required to use the conventional algorithm. They commented that the conventional method was too hard and slow. Overall, most students had positive comments concerning the new addition procedures.

In terms of performance, the results of this study indicate that both the Hutchings' "Low-stress" and the Fulkerson "Full-record" algorithms were generally superior to the conventional algorithm, along all dependent measures. In addition, the data show that the effects achieved in phases using the "Low-stress" and the Fulkerson "Full-record" algorithms were highly similar. All students showed higher accuracy scores and lower error rates using the new algorithms as compared to the conventional method. Only two students failed to achieve higher correct rates for the two new methods. Possible order effects were noted for only correct rate. Data on four students show that having practiced with either of the new algorithms may have positively affected their performance using the other.

For algorithm preference, four students chose the Fulkerson "Full-record" algorithm; three chose the "Low-stress" method and two chose the conventional algorithm to solve the super problem (Appendix G). In addition, students choosing the new algorithms obtained accuracy scores which were distinctly higher to those who chose the conventional algorithm.

DISCUSSION

The students reacted positively towards most aspects of this study. This may have been due to the fact that they were allowed to leave their regular class to go with the investigator. However, changes that occurred during the study would suggest that learning the new algorithms was at least partially responsible. One subject often talked, disturbing others during the initial conventional algorithm phase. The investigator was considering some type of intervention plan to deal with his behavior. But once the new algorithms were taught his disruptive behavior decreased to almost non existent levels.

Some students wanted to know if they could use the new methods in their regular math class. Some students requested other methods for multiplication, division and subtraction. In addition to changes in calculation, speed and accuracy, future studies might examine behavior changes associated with the use of the Hutchings "Low-stress" and Fulkerson "Full-record" algorithms, such as attending.

Instructional Procedures

The instructional procedures used in this study were effective in teaching each of the new algorithms as evidenced by the changes in calculation accuracy and efficiency. Students readily understood the concepts involved and the use of the special notations in each new algorithm.

Error Patterns

Examination of daily work sheets indicated that three types of error patterns occurred in relation to the use of the new algorithms. The error patterns were: a.) the student would fail to write down the last binary sum, thus obtaining the correct units portion, but failing to carry the correct number to the next column, b.) the student would begin a problem by adding the first two binaries in the right column, only writing the sum between the first and second binaries instead of below, thus rendering the column sum incorrect, and c.) using the Fulkerson "Full-record" algorithm students would sometimes fail to correctly count the slashes, because the slashes were not made large enough to quickly identify. The first two of these patterns were observed by Alessi (1974). The pattern described in (c) was not observed for the "Low-stress" method. These error patterns (a,b,c) were not observed often and were easily corrected by the examiner. Boyle (1975) suggested that placing a box between the second and third binaries of the first column, might help eliminate the pattern described in (b) above.

Data Analysis

The results of the present study support the findings of past researchers, (Hutchings, 1972; Alessi, 1974; Dashiell, 1974; Boyle, 1975; Rudolph, 1976; Zoref, 1976; and Gillespie, 1976) in showing the Hutchings "Low-stress" algorithm generally superior to the conventional algorithm in producing accurate and efficient calculations. The results also indicated that the Fulkerson "Full-record" algorithm produced results similar to those obtained for the "Low-stress" procedures. Additionally, the results show that given a free choice of algorithms to use in solving a problem, more students chose the Fulkerson "Full-record" algorithm (four out of nine students) than the other two methods. Three students chose the"Low-stress" method and two students chose the conventional procedure.

Generally, students tended to pick the algorithm in which they were most competent. Students choosing the new algorithms had distinctly higher accuracy scores for the preference problem than students choosing the conventional method.

One factor that affected student performances, was whether or not they knew their basic addition facts. Although a pre-test was given in an attempt to eliminate students who had not mastered the basic math facts, the test did not eliminate those students who used their fingers in obtaining sums. It is felt that the "Low-stress" and Fulkerson "Full-record" algorithms would have shown even greater superiority had all the students known the basic addition facts by rote memory.

Another factor which might have had some effect on student performances, was the fact that they were excited about learning and using the new algorithms and might have tried harder (the same explanation is often used to explain failures of new programs). Even with this factor in mind, students using the new algorithms, with only brief instruction, generally increased their accuracy and speed of calculation over levels achieved using the conventional algorithm. This is surprising in the sense that most fourth graders would have had two years of instruction using the conventional procedures.

When students were using the Hutchings "Low-stress" and the Fulkerson "Full-record" algorithms, less day to day variability was noted in terms of calculation speed and accuracy, compared to the conventional phases. It would be expected that responding would be more stable during the conventional algorithm phases, since the students have had a longer history of it's use. On the other hand, perhaps the stability of performance was a reflection of the new algorithms' superiority.

In summary, it can be reported that both the Hutchings'

"Low-stress" and the Fulkerson "Full-record" algorithms produced more stable, accurate and efficient calculations in comparison to the conventional algorithm. In terms of preference, most students chose the Fulkerson "Full-record" and "Low-stress" algorithms when given a free choice.

Based on past research, the Hutchings "Low-stress" appears to be a viable alternative to the conventional algorithm in elementary education. It's adaptation into our current system would require extensive changes in problem formats, including extra wide spaces between rows and columns. The Fulkerson "Full-record" method on the other hand, would only require extra wide spaces between the rows. More research is needed on the Fulkerson "Full-record" method before adaptation is recommended.

Future Directions

In order to reduce the confounding aspects of order effects, perhaps future researchers might use a multi-element baseline design (Ulman and Sulzer-Azaroff, 1975). This type of design would allow each subject to use every algorithm under study during daily sessions. The use of such a design however, might present other problems such as confusion for the student and monitoring difficulties for the investigator.

Other research might focus on the various effects of each component of both the "Low-stress" and the Fulkerson "Full-record" methods. For instance, comparing the Fulkerson "adding by tens" method to the Fulkerson "Full-record" algorithm.

Finally, future reseach might investigate the feasibility of using the "Low-stress" or Fulkerson "Full-record" algorithms in teaching basic mathematics to mentally impaired individuals.

APPENDIX A
INVENTORY OF BASIC ADDITION FACTS
(Adapted from Otto, McMenemy, and Smith, 1973, p. 221;
taken from Alessi, 1974.)

Inventory of Basic Addition Facts

8 +2	6 +9	3 +6	9 +6	4 +7	6 +3	5 +4
5 +9	4 +5	8 +7	6 +4	5 +2	5 +7	3 +4
3 +2	6 +8	8 +5	2 +4	7 +6	8 +9	2 +9
3 +5	4 +2	2 +7	9 +3	7 +5	4 +9	6 +2
5 +3	9 +8	7 +4	6 +4	7 +8	3 +7	9 +2
2 +8	9 +4	4 +3	2 +6	2 +3	8 +3	3 +8
7 +9	8 +4	5 +6	6 +7	4 +8	8 +6	7 +2
7 +3	9 +5	6 +5	5 +8	3 +9	9 +7	2 +5

Score: ____________

APPENDIX B
PRACTICE WORKSHEETS FOR TRAINING SESSIONS

Practice Sheet #1

6 8 7
4 8 3
6 9 5
+ 8 7 4
______

6 9
+ 5 8
____

7
6
8
9
7
6
9
0
5
1
+ 3
__

6 5
6 3
3 9
8 1
+ 4 2
____

6 8 7 6 9 5
4 8 7 6 4 2
+8 7 6 9 8 3
____

4 8 7 6 8 7 6
9 8 7 6 8 7 6
9 8 7 6 8 5 6
7 9 5 6 7 9 3
8 5 2 7 4 9 8
+ 6 7 8 5 6 7 8
____

Practice Sheet # 2

	3 5 2	
8 7	8 9 7	
5 9	2 2 2	5 9 3 8
3 8	9 3 8	7 6 5 8
9 6	5 9 6	2 6 7 8
7 8	6 7 4	9 6 7 5
+ 9 5	+ 5 8 6	+ 5 8 7 4

6	
8	
7	
3	
3	
5	6 7 8 6 7 8 4
8	6 7 8 6 7 8 9
4	6 5 8 6 7 8 9
9	3 9 7 6 5 9 7
1	8 9 4 7 2 5 8
+ 7	+ 8 7 6 5 8 7 6

APPENDIX C
CONVENTIONAL ALGORITHM LESSON

Since you all know how to write your number facts, I thought I would spend a few minutes reviewing how to do column addition problems.

When you write number facts you add the two numbers and write down your answer. For instance, when you write six plus seven, all you do is write thirteen at the bottom, like this:

$$\begin{array}{r} 6 \\ +7 \\ \hline 13 \end{array}$$

The three is said to be in the ones place and the one in the tens place.

When you add more than two numbers, like in the problem nine plus seven plus eight, you add nine plus seven which equals sixteen. Then you add sixteen to eight to reach your answer of twenty four.

$$\begin{array}{r} 9 \\ 7 \\ +8 \\ \hline 24 \end{array}$$

Does everybody know how to do this problem?

Good.

Okay, now I want you to write this problem down on the paper I gave you: Six plus seven, plus eight, plus five. Lets see you do this problem.

Okay everybody is finished. The answer is twenty six.

Lets see how you reached your answer. You added six pous seven and it equaled thirteen. Next you added thirteen plus eight. It equaled twenty one. You then added twenty one plus five and it equaled twenty six. That is the answer. You then wrote twenty six at the bottom. Very good.

$$\begin{array}{r} 6 \\ 7 \\ 8 \\ +5 \\ \hline 26 \end{array}$$

Now let me review how to add when there is more than one column.

Look at this problem:

$$\begin{array}{r} 6\ 8 \\ 8\ 7 \\ 4\ 5 \\ +\ 5\ 9 \\ \hline 9 \end{array}$$

The sum of the first column is twenty nine. Can we write the twenty nine under that column? No we can not.

When there is more than one column, each column can have only one number at the bottom, except for the last column.

So the number we put at the bottom is always the right hand number or the units.

What do we do with the left hand or tens number? We carry it to the top of the next column and circle it. Watch me.

```
 (2)
  6 8
  8 7
  4 5
+ 5 9
-----
    9
```

Now when we begin to add the second column, we always begin with the number we carried. In this example we carried two.

To finish this problem we add:

```
                        2
2 plus 6 = 8   ------>  6 8
8 plus 8 = 16  ------>  8 7
16 plus 4 = 20 ------>  4 5
20 plus 5 = 25 ------>  5 9
                       -----
                          9
```

On the last column we write both the left and right sides of the answer. Watch me.

```
 (2)
  6 8
  8 7
  4 5
+ 5 9
-----
2 5 9
```

Your answer should be two hundred and fifty nine or 259.

Remember, if there is a column to the left, you always carry the tens or left hand answer to the top of the next column and circle it.

Now I want you to do some practice problems. (Hand out practice sheets)

APPENDIX D

HUTCHING' "LOW-STRESS" ADDITION ALGORITHM LESSON

I am going to show you a new way of writing number facts, but first I'll show you the old way.

This is the way you usually write your facts:

$$\begin{array}{r} 6 \\ +7 \\ \hline 13 \end{array}$$

Using the new way the facts are written using two small numbers like this:

$$\begin{array}{c} 6 \\ {}_{1}7_{3} \end{array}$$

You still write thirteen, only it is written at the corners of the number instead of under it.

I'll show you another example. We can write eight plus six like this $\begin{array}{r} 8 \\ +6 \\ \hline 14 \end{array}$ or using the new way like this $\begin{array}{c} 8 \\ {}_{1}6_{4} \end{array}$.

Do you see the fourteen? You always write the ones to the right and tens to the left.

Now I want you to try it. Using the paper I have given you, write down this problem: seven plus eight. Now I want you to write down the answer using the new way.

(Check to make sure they have all used the correct notation)

$$\begin{array}{c} 7 \\ {}_{1}8_{5} \end{array}$$

Very good. The little number on the left is said to be in the tens place and the little number on the right is said to be in the ones or units place.

Now I am going to show you a bigger problem. In this problem we are going to add three numbers; nine, seven and eight. Watch me:

Nine plus seven equals sixteen, so I'll write a little six to the right of the seven and a little one to the left. Do you see? Now using only the little number to the right I will add the little six to the eight.

$$\begin{array}{c} 9 \\ {}_{1}7_{6} \\ \underline{8} \end{array}$$

The answer is fourteen so I will write a little four to the right of the eight and a little one to the left.

$$\begin{array}{c} 9 \\ {}_{1}7_{6} \\ {}_{1}8_{4} \\ \hline \end{array}$$

Now I have the right half of the answer which is four. I just write the four at the bottom right like this:

```
  9
  7
1 8 6
1   4
-----
    4
```

To get the other half of the answer, I simply count the little ones at the left that I have forgotten about.

How many ones are there? Lets count one..two. So two is the other half of the answer.

```
  9
  7
1 8 6
1   4
-----
2   4
```

The answer is 24. Do you see?

Now I want you to try an even bigger problem. Write this problem on your papers and don't forget to leave enough room between your big numbers to write your little numbers. Also, remember only add the little numbers at the right. Here is the problem: add six, plus seven, plus eight, plus five.

If you need help ask me.

Okay everybody is finished. The answer is 26. Lets see how you reached the answer.

Six plus seven equals thirteen, so you wrote the little three to the right of the seven and the little one to the left. Thats right.

```
  6
  7
1 8 3
  5
  -
```

Next you added three plus eight. It equaled 11, so you wrote a little one to the right and to the left of the eight. Remember we always forget about the little ones at the left until the very end.

```
  6
  7
1 8 3
1 5 1
  -
```

Next you added one plus five. It equaled six, so you wrote a little six to the right of the five and nothing to the left because you had no tens.

```
  6
  7
1 8 3
1 5 1
    6
-----
```

Now you had half of the answer, so you wrote a six in the ones place under the line. Then you counted the ones on the left, one...two and wrote down the second half of your answer which was two.

```
  6
  7
 1 3
  8
 1 1
  5
   6
 ---
 2 6
```

Your answer was 26. Very good.

Did everybody get twenty six for their answer?

Good.

Now let me show you how to add when there are nore than one column using the new way.

Look at this problem:

```
6      8
8      7
4    1 5
5      5
     1 0
       9
       9
---------
```

Can we still write our left hand answer if ther is more than one column? No, we can not.

When there is more than one column, each column can have only one number at the bottom, except for the <u>last</u> column.

So the single number that we put at the bottom is always the right hand number.

What do we do with the left hand number? We carry it to the top of the next column and circle it.

```
2
6      8
8      7
4    1 5
5      5
     1 0
       9
       9
---------
       9
```

Now when we begin to add the second column we always start with the number we carried. In this example we carried the two.

```
2
6      8
8      7
4    1 5
5      5
     1 0
       9
       9
---------
       9
```

To finish this problem we add:

```
                                          2
2 plus 6 = 8   ------------------------>  6        8
                                            8
8 plus 8 = 16  ------------------------>  8        7
                                        1 6      1 5
6 plus 4 = 10  ------------------------>  4        5
                                        1 0      1 0
0 plus 5 = 5   ------------------------>  5        9
                                            5        9
                                        ____________

                                                     9
```

On the last column we write both the left and right side of the answer. Watch me.

Five is the last right hand number so I'll write it at the bottom. There were two little ones at the left so I'll write two at the bottom. The answer is thow hundred and fifty nine.

```
   2
   6           8
     8
 1 8          1 7
     6            5
   4           5
 1 5 0         9
     5           9
 ________________
   2 5           9
```

Remember, if there is a column to the left you always count your little ones and carry them to the top of the next column and circle the number.

Now I want you to do some practice problems.

(Hand out practice sheets)

APPENDIX E
FULKERSON "FULL-RECORD" ADDITIONAL ALGORITHM LESSON

I am going to show you a new way of writing number facts, but first I'll show you the old way.

This is the way you usually write your facts:

$$\begin{array}{r} 6 \\ +7 \\ \hline 13 \end{array}$$

Using the new way the facts are written using a slash and a small number like this:

$$\begin{array}{r} 6 \\ +\not{7}_{3} \end{array}$$

You still write thirteen, only it is written by making a slash through the seven in place of the one and writing a little three at the lower right corner of the seven instead of under it.

I'll show you another example. We can write eight plus six like this $\begin{array}{r} 8 \\ +6 \\ \hline 14 \end{array}$, or using the new way like this $\begin{array}{r} 8 \\ \not{6}_{4} \end{array}$.

Do you see the fourteen? You always write the units or ones to the right and make a slash to indicate the tens.

Now I want you to try it. Using the paper I have given you, write down this problem: seven plus eight. Now I want you to write down the answer using the new way.

(Check to make sure they have all used the correct notation $\begin{array}{r} 7 \\ \not{8}_{5} \end{array}$)

Very good. The slash is said to be in the tens place and the little number on the right is said to be in the ones or units place.

Now I am going to show you a bigger problem. In this problem we are going to add three numbers; nine, seven and eight. Watch me.

Nine plus seven equals sixteen, so I'll write a little six to the right of the seven and make a slash through the seven. Do you see?

$$\begin{array}{r} 9 \\ \not{7}_{6} \\ \underline{8} \end{array}$$

Now using only the little number to the right, I will add the little six to the eight. The answer is fourteen, so I'll write a little four to the right of the eight and make a slash through the eight.

9
7̸
6
8̸
4

Now I have the right half of the answer which is four. I just write the four at the bottom like this:

9
7̸
6
8̸
4

4

To get the other half of the answer, I simply count the slashes that I have forgotten about.

Lets see how many slashes there are? Lets count one..two. So two is the other half of the answer.

9
7̸
6
8̸
4

The answer is twenty four. Do you see? 24

Now I want you to try an even bigger problem. Write this problem on your papers and don't forget to leave enough room between your big numbers to write your little numbers. Also, remember that you only add the little numbers at the right. Here is the problem: add six, plus seven, plus eight, plus five.

If you nee help ask me.

Okay everybody is finished. The answer is twenty six. Lets see how you reached the answer.

Six plus seven equals thirteen, so you wrote a little three to the right of the seven and made a slash through the seven. Thats right.

6
7̸
3
8
5

Next you added three plus eight. It equaled eleven, so you wrote a little one to the right of the eight and made a slash. Remember we always forget about the slashes until the very end.

6
~~7~~
3
~~8~~
1
5

Next you added one plus five. It equaled six, so you wrote a little six to the right of the five and made no slash because you didn't have any tens.

6
~~7~~
3
~~8~~
1
5
6

Now you had the right half of the answer, so you wrote a six in the ones place under the line. Then you counted the slashes, one..two, and wrote down the left half of your answer which was two. Your answer was twenty six. Very good.

6
~~7~~
3
~~8~~
1
5
6

Did everybody get twenty six for their answer? Good.

26

Now let me show you how to add when there is more than one column using the new way.

Look at this problem:

6	8
8	~~7~~ 5
4	~~5~~ 0
5	9 9

Can we still write our left hand answer if there is more than one column? No, we can not.

When there is more than one column, each column can have only one number at the bottom, except for the last column.

So the single number that we put at the bottom is always the right hand number.

What do we do with the left hand number? We carry it to the top of the next column and circle it.

(2)
6 8
8 7
5
4 5
0
5 9
9
———
9

Now when we begin to add the second column we always start with the number we carried. In this example we carried the two.

(2)
6 8
8 7
5
4 5
0
5 9
9
———
9

To finish this problem we add:

2 plus 6 = 8 ⟶ 6
8
8 plus 8 = 16 ⟶ 8 7
6 5
6 plus 4 = 10 ⟶ 4 5
0 0
0 plus 5 = 5 ⟶ 5 9
5 9
———
9

On the last column we write both the left and right sides of the answer. Watch me.

Five is the last right hand answer, so I'll write it at the bottom. There were two slashes, so I'll write two at the bottom. The answer is two hundred and fifty nine.

(2)
6 8
8
8 7
6 5
4 5
0 0
5 9
5 9
———
2 5 9

Remember, if there is a column to the left, you always count the slashes and carry that number to the top of the next column and circle it.

Now I want you to do some practice problems. (Hand out practice sheets)

APPENDIX F
SAMPLE DAILY WORKSHEETS
FIVE-COLUMN, SEVEN ROW PROBLEM ARRAYS

9	4	8	1	3
7	4	4	5	7
4	8	5	4	5
7	5	1	2	1
9	2	9	4	1
6	9	9	2	6
9	4	9	7	2

1	9	1	5	2
3	6	2	4	8
9	4	4	5	8
5	4	1	5	8
7	5	5	1	9
4	2	2	8	6
9	4	9	3	8

6	2	2	8	6
9	4	2	9	1
6	8	2	2	7
1	9	8	2	8
7	7	1	1	4
7	8	1	3	8
2	7	4	8	2

Name____________________

1	9	6	4	9
8	5	2	4	4
3	1	5	2	1
9	4	2	3	7
8	2	3	7	1
8	7	4	8	1
1	9	3	2	5

____

3	9	6	6	3
2	1	8	1	6
8	8	4	4	8
1	8	6	2	2
8	6	2	2	5
4	9	7	6	7
5	8	6	9	7

____

4	3	8	5	2
6	8	9	7	1
1	8	4	7	7
8	3	7	4	5
1	6	9	3	3
2	3	6	8	7
4	1	9	6	3

____

Name__________________

APPENDIX G
SUPER PROBLEM
17 X 12 ARRAY ADDITION PROBLEM

SUPER PROBLEM

	3	8	8	5	8	5	5	5	8	8	8	4	7	8	4	8	8
	5	3	3	3	5	8	8	4	5	7	5	8	6	5	5	5	5
	8	4	4	7	7	7	3	8	3	5	7	7	5	7	7	6	6
	7	7	7	6	6	6	7	7	7	6	6	6	2	6	6	5	3
	6	6	6	5	8	8	9	6	6	3	8	8	9	9	8	8	5
	8	8	3	8	3	7	6	8	9	8	5	5	3	8	7	7	4
	4	5	5	3	5	5	8	7	8	9	7	4	6	7	9	6	8
	3	8	9	7	7	8	5	9	5	4	8	8	8	6	8	5	3
	2	5	7	6	6	7	4	8	3	8	3	3	5	8	5	8	9
	9	2	8	8	8	4	7	7	3	7	5	5	5	7	3	7	3
	6	5	3	5	9	6	8	6	2	7	6	9	9	3	6	7	6
+	3	3	6	9	3	3	7	7	9	6	5	3	6	8	3	6	5

BIBLIOGRAPHY

Alessi, G. J. Effects of Hutchings' "low fatigue" algorithm on childrens' addition scores compared under varying conditions of token economy reinforcement and problem difficulty. Unpublished Doctoral Dissertation, University of Maryland, 1974.

Alessi, G. J. Hutchings' Low Stress Algorithms: results from single subject exploratory intervention research studies. Paper presented at Sixth National Conference on Diagnostic and Prescriptive Mathematics, Tampa, April 1979.

Ashlock, R. B. Error Patterns in Computation: A Simi-programmed Approach. Columbus, Ohio: Charles E. Merrill Publishing Company, 1976.

Baer, D. M., Wolf, M. M. and Risley, T. R. Some current dimensions of applied behavior analysis. Journal of Applied Behavior Analysis, 1968, 1, 91-97.

Batarseh, G. J. Addition for the slow learner. The Arithmetic Teacher, 1974, 21, 714-715.

Boyle, M. Effects of Hutchings' "low stress" addition algorithm on childrens' addition scores under varying conditions of reinforcement and distraction. Unpublished Doctoral Dissertation, University of Maryland, 1975.

Dashiell, W. H. An analysis of changes in affect and changes in both computational power and computational stamina occurring in regular elementary school children after instruction in Hutchings' "low stress" addition algorithm, practice with unusually large examples, and exposure to one of two alternate performance options. Unpublished Doctoral Dissertation, University of Maryland, 1974.

Fulkerson, E. Adding by tens. The Arithmetic Teacher, 1963, 10, 139-140.

Gillespie, C. Student preferences for the Hutchings' low stress versus the conventional addition algorithm under conditions of differentially increasing response effort with and without reinforcement. Unpublished Specialist in Education Project, Western Michigan University, 1976.

Hutchings, L. B. An examination, across a wide range of socio-economic circumstance, of a format for field research of experimental numerical computation algorithms, an instrument for measuring computational power under any concise numerical addition algorithm, and the differential effects of short term instruction in two experimental numerical addition algorithms and equivalent practice with the conventional addition algorithm. Unpublished Doctoral Dissertation, Syracuse University, 1972.

Hutchings, L. B. Low-stress algorithm. In Measurement in School Mathematics, 1976 Yearbook, National Council of Teachers of Mathematics, Reston, Virginia, 1976.

Johnston, J. M. and Pennypacker, H. S. Strategies and Tactics of Human Behavioral Research. Book in Press, 1980.

National Advisory Committee on Mathematics Education. The Overview and Analysis of School Mathematics Grades K - 12. Washington D. C. : Conference Board of the Mathematical Sciences, 1975.

O'Malley, L. A sample method for addition. The Arithmetic Teacher, 1969, 16, 676.

Rudolph, L. A comparison of the Hutchings' Low stress" and the current addition algorithms for speed and accuracy in two school settings with regular and special education children. Unpublished Specialist in Education Project, Western Michigan University, 1976.

Sanders, W. J. Let's go one step farther in addition. The Arithmetic Teacher, 1971, 18, 413-415.

Sidman, M. Tactics of Scientific Research: evaluating experimental data in psychology. New York, N.Y. Basic Books, Inc. 1960.

Skinner, B. F. The Technology of Teaching. New York; Appleton Century-Crofts, 1968, 18.

Ulman, J. D. and Sulzer-Azaroff, B. Multielement baseline design in educational research. In E. Ramp and G. Semb (Eds.), Behavior Analysis: areas of research and application. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1975, p. 377-391.

Zoref, L. A comparison of calculation speed and accuracy on two levels of problem difficulty using the conventional and Hutchings' "low stress" addition algorithms and the pocket calculator with high and low achieving math students. Unpublished Specialist in Education Project, Western Michigan University, 1976.