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A Diagnostic-Teaching Investigation of the Feasibility of Using the Hutchings' "Low-Stress" Algorithm for Addition Skill Development in Trainable Mentally Impaired Pupils

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A DIAGNOSTIC-TEACHING INVESTIGATION OF THE FEASIBILITY OF USING THE HUTCHINGS' "LOW-STRESS" ALGORITHM FOR ADDITION SKILL DEVELOPMENT IN TRAINABLE MENTALLY IMPAIRED PUPILS

by

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The feasibility of using the Hutchings' "Low-Stress" algorithm for the development of addition skills in trainable mentally impaired pupils was investigated in this exploratory diagnostic study. The four subjects were identified as trainable mentally impaired pupils (IQ 44-63), three females and one male, between eighteen and twenty years of age. Results indicate increased power with counting and numeral recognition. Results from "Low-Stress" training phases indicate an increasing trend in binary accuracy and rate. Diagnosis of error patterns resulted in adjustments in teaching strategies. Results support the feasibility of "Low-Stress" algorithm for addition skill development in trainable mentally impaired pupils with counting and number recognition preskills. Training for all phases of the algorithm were not completed due to lack of time. It is suggested that future research investigate use of the Hutchings' "Low-Stress" algorithm with pupils having all the preskills.

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INTRODUCTION

The Problem

The present project was initiated to explore the effectiveness of the Hutchings' "Low-Stress" algorithm for the development of addition skills in students identified as trainable mentally impaired.

Research on teaching arithmetic to mentally impaired pupils has been sporadic. A review of published work revealed little agreement regarding an appropriate curriculum. Dunn (1956) compared twenty retarded and thirty normal children in a public school. He found that there was no significant difference between the normal and the retarded groups in arithmetic computations. He did, however, find a significant difference in arithmetic reasoning problems and concepts. Kirk (1972) in discussing development in arithmetic by the trainable students wrote:

"...they can learn some quantitative concepts ...and the vocabulary of quanitative thinking... The older children can learn to write numbers from 1 to 10 and some of them can learn time concepts... ." (p.231)

Bracy, Maggs and Morath (1974) used the Distar Arithmetic I Program designed by Engleman and Carnine (1969), to determine if moderately retarded children made significant gains in object counting, making sets of lines to match numbers, the meaning of plus, and increment addition (e.g. __ +1). Each child completed individual worksheets at the end of each session. A token reinforcement program along with social praise paired with tangible reinforcers were used in conjunction with the arithmetic program. Pre and post-test results showed significant gains in object counting, making sets of lines to match numbers, the meaning of plus, and increment addition. The study took place in Australia with six mentally retarded children (Stanford Binet IQ 35 to 50).

Generally, research on teaching methodology for the mentally retarded has not determined the effectiveness of one method over another. Further research is needed in this area. Technological aspects of teaching arithmetic to the mentally retarded will improve as more data is gathered to supplement earlier studies.

Relevant Literature - "Low-Stress" Algorithm

Many recent mathematics programs have emphasized conceptual meaning and application rather than computational skill. Facility with computational skill was included in the National Council of Teachers of Mathematics Position Statement (1976) as one of the basic skills. Hutchings (1976) states: "increased conceptual requirements in no way reduce the requirement for computational skills..." (p.219). Competence in computational skill may widen job opportunities for the retarded (Maggs et al, 1974) and may facilitate meeting mathematical competence requirements in applied professions and vocations (Hutchings, 1976).

Alessi (1979) cites some problems which have been associated with programs emphasizing conceptual material in teaching mathematics. These problems ranged from teachers refusing to teach conceptual material to parent protests over the failure of their children to learn computational skills. According to Alessi (1979) there appears to be a "backlash" against the conceptual approach and a renewed interest in more traditional practices which stress drill in computational skill.

Recent research has emphasized the development of quick, effective methods to teach calculations skills. Alternate algorithms are being re-examined, new algorithms are being developed. Research on the "Low-Stress" algorithm began at the Arithmetic Center at Syracuse University in 1967. A number of studies using this algorithm have been carried out in the last few years. Hutchings' (1976) wrote the following about these algorithms:

"They appear to permit easy mastery after brief training, to provide greater computational power than conventional algorithms, to operate with much less stress on the user than conventional algorithms." (p.219)

The "Low-Stress" algorithm differs from the one typically used in the United States. "Low-Stress" uses a half space notation to express the sum of each binary operation. i.e. $\begin{array}{r} 3 \\ +4 \\ {}^{7} \end{array}$ If the sum is greater than nine, the tens portion is written to the lower left of the digits. i.e. $\begin{array}{r} 6 \\ +7 \\ 1\;3 \end{array}$

When performing long column addition, the ones answer portion of the column is always the same as the ones portion of the sum of the last two digits. The tens portion is always the same as the number of tens recorded at the left of the column. The following is an example:

		4		4+3=7
		3	7	7+6=13
1+1=2	1	6	3	3+8=11
	1	8	1	
	2		1	

In multi-column problems the tens are summed and carried to the top of the next column at the left. There is a need for extra wide spaces between columns to accomodate the half space notations.

1+3=4		1					5+8=13
		3	4		5		
4+5=9	+	5	9	1	8	3	
			9			3	

Alessi (1979) points out that the Hutchings' "Low-Stress" method has several distinct advantages over the standard algorithm used to teach addition in the United States. The standard algorithm requires covert chains of calculation steps when computing columnar addition. If mistakes are made with the standard algorithm, a record of where the errors occurred is not available for error pattern analysis. In "Low-Stress", by contrast full and permanent record of every calculation performed makes identification of errors possible, and practical.

Alessi (1979) points out that another advantage is the substantial reduction in the number of addition facts needed to use the "Low-Stress" algorithm. Using the standard algorithm in the following problem:

$$\begin{array}{r} 6 \\ 5 \\ 9 \\ 4 \\ \underline{8} \end{array}$$

the student would have to covertly chain these steps:

Step 1 2 3 4

eg. 6+5=11; 11+9=20; 20+4=24; 24+8=32.

The first step requires basic fact knowledge while the last three steps require complex fact knowledge or covert regrouping operations.

By contrast, in the "Low-Stress" method only basic addition facts are needed (Steps 1 through 4) and all calculations are performed overtly, each leaving a permanent product record.

		6		6+5=1 Step 1
	1	5	1	1+9=10 Step 2
	1	9	0	
1+1+1=3 Step 5		4	4	0+4=4 Step 3
	+ 1	8	2	4+8=12 Step 4
	3		2	

The advantage of using only basic facts reduces the number of facts that must be mastered by the learner by 90%, or from 1000 to 100 (there are 100 basic facts, plus 900 complex facts, for a total of 1000 facts needed with the standard algorithm).

Alessi (1974) and Boyle (1975) investigated the effects of using the "Low-Stress" algorithm on computational rate and accuracy with regular education pupils. The subjects for this study were fourth grade students in a regular education program. Alessi (1974) concludes that the "low-fatigue" (stress) addition algorithm was superior to the conventional algorithm in accuracy and speed of calculation in a 30 minute test period. He also found that as the problems increased in difficulty the relative superiority of the "Low-Stress" over the conventional procedure decreased respectively.

Boyle (1975) carried out a systematic replication of the study by Alessi (1974). The subjects of this study were fifth grade students in regular education. He concluded that the "Low-Stress" algorithm was superior to the conventional algorithm in generating both improved quality and increased quantity of performance. Further, he stated that for children placed in special education "...the Hutchings' "Low-Stress" algorithm with its reduced demand on memory might offer them a chance to develop skills in computation which could enhance their self-concept and performance in other areas." (p.60)

Rudolph (1976) compared the Hutchings' "Low-Stress" and the "standard" algorithm with regular education students and students in a special education program for the emotionally impaired. The performance of these students was compared within "distracting" and "non-distracting" environments. Rudolph concluded that the "Low-Stress" method produced more consistent responding and a reduction in error rates over the current algorithm. Rudolph further states that the "comparability of the students' performance from the two populations (special education and general education) indicates the possible usefulness of the "Low-Stress" across mainstream or exceptional students". (p.53)

Zoref (1976) investigates differential calculation power (speed plus accuracy) using Hutchings' "Low-Stress" addition algorithm, the conventional algorithm and the pocket calculator. The results indicate that performance with the Hutchings' "Low-Stress" algorithm was the most stable and that error rates were lowest with this method. The subjects for this study were 6 fourth grade students. Half of the subjects were identified as high achievers in math and half as low achievers in math. Zoref (1976) found the Hutchings' algorithm to be an accurate, efficient method of instruction and suggests it be adopted in the elementary math curricula. Zoref's results were directly replicated by Edward Drew (1981) in investigating differential calculation power (speed plus accuracy) using Hutchings' "Low-Stress" addition algorithm, the conventional algorithm and the pocket calculator.

Gillespie (1976) investigates student preference for the Hutchings' "Low-Stress" verses the conventional addition algorithm under conditions of differentially increasing response effort with and without reinforcement. The subjects were high and low math fact accurate third grade students. A general preference for the use of the "Low-Stress" algorithm over the conventional method was found. Most of the pupils maintained the preference for "Low-Stress" even though doing it required completion of 50% more work. GIllespie's results were directly replicated by Pamela Drew (1980) on student preference for the Hutchings' "Low-Stress" verses the conventional addition algorithm under conditions of differentially increasing the number of problems required.

Van Hevel (1981) compares Hutchings' "Low-Stress", a modified Fulkerson's "Full Record" and conventional algorithms for speed, accuracy and preference. Results of this study indicate that both the Hutchings' "Low-Stress" and the modified Fulkerson's "Full Record" algorithms were generally superior in producing accurate and stable calculations. The subjects were 9 fourth grade regular education students.

General Error Patterns

An essential part of evaluating the feasibility of the Hutchings' "Low-Stress" addition algorithm as a procedure to use with trainable mentally retarded pupils involves a diagnosis of observed error patterns. This algorithm provides a full, permanent record of all binary operations involved in calculating the sum of the problem. Therefore these records could be studied for error patterns and analysis. (Ashlock 1976) After diagnosing error patterns, adjustments could be made in the teaching presentation in order to correct the erroneous procedures.

The following error patterns in using the "Low-Stress" algorithm have been identified in past research with regular education students, (Allessi, 1974; Boyle, 1975):

1. not writing down the last binary in each column;
2. not adding the 1's in the tens position of the first column

and carrying that sum up to the top of the second column;

3. not starting with the first pair of binaries in the right most column;
4. mis-noting the answer to the first binary operation under the first digit in the column rather than under the appropriate second digit.

Boyle (1975) recommended that "modifications in the construction of the dittoed practice sheets could preclude these mistakes during the students' very important initial contact with the procedure". (p.54)

Boyle (1975) further offered the following correction procedures:

error 1) "...two boxes could be placed under the last digit in each column as a visual cue for noting the last binary operation."

error 2) "...boxes could be placed above the top digit of each column after the first as a visual cue for correctly placing the 'tens' sum."

error 4) "...an additional half space could be inserted between the second and third digits as a visual cue for correctly placing the sum of the first binary." (p.54)

Purpose of This Study

Past research has consistently demonstrated that the "Low-Stress" algorithm is superior to the conventional algorithm for accuracy and speed of calculation. Past investigations have involved regular education and special education students from the program for the emotionally impaired. "Low-Stress" algorithm research has not been conducted with special education students in the programs for trainable mentally impaired pupils. Therefore, this study represents an exploratory, diagnostic teaching study of using the "Low-Stress" algorithm to teach trainable mentally impaired pupils.

METHOD

Special Considerations

Written informed consent was obtained from each of the parents of subjects. The research proposal was approved by the Human Subjects Committee at Western Michigan University. This study was formulated with the following constraints:

(a.) as little disruption as possible of the ongoing school program be imposed;

(b.) the study be run almost entirely by the investigator;

(c.) the study be terminated by the end of the subjects' school semester.

Subjects

The four subjects involved in this study were identifed as trainable mentally impaired according to the guidelines established by the State of Michigan Department of Education (Public Act 198, Rule 340.1704). The Rule states: "...the trainable mentally impaired shall be determined through manifestation of all of the following behavioral characteristics:

(a.) development at a rate approximately 3 to 4 standard deviations below the mean as determined through intellectual assessment.

(b.) lack of development primarily in the cognitive domain.

(c.) impairment of adaptive behavior.

2. A determination of impairment shall be based upon a comprehensive evaluation by a multidisciplinary evaluation team which shall include a psychologist.
3. A determination of impairment shall not be based solely on behavior relating to environmental, cultural, or economic differences...."

All four subjects had obtained full scale I.Q. score within the range of 44 - 56 as measured by a recent administration of the Wechsler Adult Intelligence Scale. They were placed full time in a center based school for the trainable mentally impaired. The program at this center involved teaching daily living skills to the students enrolled. The current annual goals for number skills for each subject were as follows:

Subject G.A.

(1.) identify which of 3 numerals is most/least (1-12);

(2.) make set to match numeral (0-19);

(3.) count numbers of a set (0-50);

Subject M.C.

(1.) order 4 lengths; from shortest to longest;

(2.) identify hour and minute hand;

Subject K.M.

(1.) simple addition and subtraction;

(2.) survival skills (money, time, measurement);

Subject M.B.

(1.) order picture object cards for numbers (1-9);

(2.) identify numbers on a line as after, before, between (0-12);

(3.) read time to the hour.

The subjects ranged in age from 18 years to 21 years.

Setting

The study took place at the subjects' school. Sessions were held during the subjects' scheduled math time and began at 9:30 a.m. Due to other events scheduled into the subjects' school day, sessions were held 2 to 3 times a week for each subject. Sessions took place in an unused classroom. The setting was considered nondistracting.

Independent Variables

(1.) Instruction in Hutchings' "Low-Stress" algorithm (Addition), using curriculum guide by Hutchings and McCuaig (1976) (p.26)

(2.) Handicap status of the student in the educational setting: trainable mentally impaired as defined by the Michigan Special Education Rules.

Dependent Variables

(1.) Percent correct - the number of binaries that the subject computed correctly divided by the total number attempted.

(2.) Rate correct - the number of binaries correctly added divided by the session length and expressed as binaries correct per minute.

(3.) Rate incorrect - the number of binaries incorrectly added divided by the session length and expressed as binaries incorrect per minute.

Reliability

Since the student's work in sessions provided a permanent product, independent graders were given these products to score on a random schedule. The investigator checked each student's paper. Papers were checked once at the end of the session and again at a later time. Reliability data using independent graders was taken two times during each phase (approximately 39% of the total sessions). When reliability were taken the investigator checked the students' papers first using a clear acetate sheet and a china marker. The independent grader scored directly on each student's sheet. The investigator's acetate sheet was then placed over the independent grader's scoring. In calculating the reliability coefficient all binary calculations which both graders scored the same way were counted as agreements. Binaries scored differently were counted as disagreements. The reliability coefficient was calculated by dividing the number of agreements by the number of agreements plus disagreements.

Materials

The instrument used in this study contained addition problems with the size fixed by the training phase. The problems were set on 8 1/2 by 11 inch paper. The five practice problems were placed 2 inches apart, 2 problems per row except the last row contained only one problem. Rows were placed 1 1/4 inches apart. Numerals contained in the problems were 1/4 inch in size. This size and spacing were used in order to reduce any possible interference from lack of clarity or lack of space to write answers. The problem array size varied with the teaching phase.

The following recommendations for the design of a measurement instrument for computational accuracy and speed were made by Hutchings' (1972):

> "It is required that the variations in example forms which load for reading or eye movement skills be avoided, e.g., interrupted rows, but that a range of profiles, as might occur in lessons or general experience, be presented." (p.51)

A Discrete Ordinality Operations Format (hereafter referred to as DOOF) was provided the investigator. A DOOF is a type of number line made up of numerals 0 through 18 written in a column with an empty box drawn next to each numeral. Pencils were also provided by the investigator. A wristwatch was used during each session. Instruction time was left variable and recorded for each session.

Placement Testing

All subjects were given a placement test (Appendix A) in October in order to determine math skills previously acquired and to determine possible modifications needed in the teaching format. These skills are considered prerequisite to beginning instruction with "Low-Stress" algorithm. This same test was again administered in early December.

The skills tested included: matching numerals; naming numerals; writing numerals; rote counting 1 through 18; counting from 1 to a given number; counting in sequence with one to one correspondence with objects; matching a numeral to a set (1-9); arranging sets (1-9) in sequential order; telling "how many" in a set; counting from a number to a number; counting from a number a given number of times; identifying the addition symbol.

Addition Concept Probe.

A probe for the concept of addition (Appendix B) was administered to all subjects in December. Since addition involves the union of disjoint sets, models were presented with items in sets. Various objects (pencils, paper clips, chips and squares of colored paper) were used. The following types of models were presented:

Part I: Objects were placed in two sets to represent each addend of a binary addition problem. The subject was asked "how many all together?" for each example presented.

Part II: A set representing either one addend or a sum was used. The investigator placed more objects (second addend) in the set (example of addition); removed objects from the set (non-example of addition); or simply moved objects around in the set (non-example of addition). The subject was instructed to "Watch what I do, Is this adding?"

Part III: The subjects were asked to show how a given fact problem would look using objects and sets. After arranging the sets the subjects were asked "How many altogether?"

Training Phases

Training phases were presented in the following order: DOOF training; notation training; notation with two binaries; notation with three binaries, and notation with four binaries. Modifications were made in the teaching format as error patterns were observed. These adjustments were used for each subject as empirically deemed necessary.

These four phases are described below:

I. Teaching the use of the DOOF for computing a single binary.

II. Teaching notation

The half space notation was used to write the answer to a single binary problem.

III. Teaching use of the notation with two binaries (1X3 array problem) as in the following example:

```
   5        5+7=12
 1 7 2
+  4        2+4=6
     6
 ------
```

IV. Teaching the use of notation with 2X2 array problems as in the following example:

```
              1
1+4=5         4 5      6       6+9=15
5+8=13      +1 8 3   1 9 5
            ----------------
                       5
```

Training Sessions

A pre-test was given before each training session followed by training and a post-test. The training session length was variable (between 25 to 30 minutes) and occurred during the subjects regularly scheduled math class. The number of training sessions for each phase was variable by subject. A criterion of three consecutive sessions at 90% or better accuracy were required before going on to the next phase of training.

Different colored chalk was used during training in order to clearly differentiate problem numbers from calculation work completed in the problems. This procedure was used only during training sessions. The subjects used lead pencils for pre and post-test responses.

The training sessions for the algorithm took place during the subjects' regularly scheduled math/vocational training period. While the subjects were involved in the algorithm training, "number activities" were not provided as usual in the classroom. Occasionally, however, practice sheets for the algorithm were provided. The subjects received daily clock reading instruction in the classroom. They were also involved in non-math activities in preparation for the upcoming holidays.

The classroom teacher awarded points to the students. These points could be used to purchase items from the "classroom store".

The subjects were escorted to and from the training sessions by the investigator.

Program Modifications

As a diagnostic teaching study, changes in the teaching methods were expected to be used to correct error patterns (Ashlock, 1976) as they occurred throughout the study.

RESULTS

Reliability

Data on reliability collected over 39% of sessions over all phases, yielded an overall agreement index of 100 for scoring binaries correct and incorrect.

Placement Test Results

A list of the skills tested on the placement test is presented in Table 1. The results are presented for each subject with "y" indicating the subject demonstrated the skills and "n" indicating the subject did not demonstrate the skill.

The results of the placement test administered before training was initiated indicate that none of the subjects had acquired all of the skills tested. As shown in Table 1, more of the test items were correctly answered by subjects K.M. and G.A. than subjects M.B. and M.C. Generally, all subjects demonstrated more counting skills than numeral recognition skills.

The results of the placement test also indicate that the subjects were not prepared for the "Low-Stress" algorithm instruction. The critical skill for this instruction is computing single binary (fact) problems. Only subject M.C. demonstrated this critical skill. However some of the subjects did have the preskills for DOOF instruction. These skills are counting from 1 to a given number; matching numeral to the same numeral; count in sequence with one to one correspondence; reading numerals. Subjects K.M., M.C. and G.A.

demonstrated these skills on the pre-test. The only DOOF preskill demonstrated by subject M.B. was counting from 1 to a given number. Therefore, training was begun on preskills through the use of a DOOF. DOOF training was initiated with all subjects after placement testing.

Table 1: Placement Test Results for Pre and Post Training

TABLE 1

Placement Test Results

		Pre-Test KM	Pre-Test MC	Pre-Test GA	Pre-Test MB	Post-Test KM	Post-Test MC	Post-Test GA	Post-Test MB
1.	Match a numeral to the same numeral	y	y	y	n	y	y	y	y
2.	Name numerals printed on a card and presented in a random order	y	y	y	n	y	y	y	y
3.	Write numerals	y	y	y	n	y	y	y	y
4.	Rote count 1 thru 18	y	n	y	y	y	y	y	y
5.	Count from 1 to a given number	y	y	y	y	y	y	y	y
6.	Count in sequence with one to one correspondence with objects	y	y	y	n	y	y	y	y
7.	Match numeral to a set (1-9)	y	n	y	n	y	y	y	y
8.	Arrange sets 1-9 in sequential order	y	n	n	y	y	y	y	y
9.	Tell "how many?", given a set (1-9)	n	y	y	y	y	y	y	y
10.	Count from a number to a number	y	n	n	n	y	n	y	n
11.	Count from a given number another number of times (i.e. start at 3 and count 4 more)	n	n	n	n	y	n	n	n
12.	Identify addition symbol	y	n	y	y	y	y	y	y
13.	Addition Probe					y	y	y	y
	Compute correct answer to addition						(DOOF used)		
	a. single binary (fact) problems	n	y	n	n	y	y	y	y
	b. 2X2 without regrouping	y	n	n	n	y	y	y	n
	c. regrouping required	n	n	n	n	y	n	y	n

Note: y = yes; n = no

The purpose of the DOOF is to substitute for a lack of knowledge of basic facts with a "tool" to locate these facts.

The following is a description of each subject's performance on the pre-test.

Subject K.M. also correctly answered items for: matching numerals; naming numerals; rote counting 1 to 18; counting in sequence with one to one correspondence; matching numerals to sets (1-9); arranging sets (1-9) in sequential order; counting from a number to a number; and identifying the addition symbol. Subject K.M. did not correctly answer items counting from a number a given times and items for "tell how many". It should be noted, however, that K.M. did respond correctly to "tell how many" items when prompted to "count not guess".

Subject G.A. correctly answered items for: matching numerals/ naming numerals; writing numerals; rote counting 1 to 18; counting in sequence with one to one correspondence; matching numerals to a set (1-9); telling "how many"; and identifying the addition symbol. Subject G.A. did not correctly answer items for arranging sets (1-9) in sequential order, counting from a number to a number, and counting from a number a given times.

Subject M.C. correctly answered items for: matching numerals; naming numerals; writing numerals; counting in sequence with one to one correspondence; and telling "how many" in a given set. Subject M.C. did not correctly answer items for rote counting 1 to 18 (the number 13 was not said); matching numerals to sets (1-9); arranging sets in sequential order (1-9); counting from a number to a number;

counting from a number a given times; and identifying the addition symbol.

Subject M.B. correctly answered items for: rote counting 1 to 18; arranging sets (1-9) in sequential order; telling "how many" for a given set; and identifying the addition symbol. Subject M.B., did not correctly answer items for: naming numerals; writing numerals; matching numerals to a set; counting from a number to a number; counting from a number a given times; counting in sequence with one to one correspondence; and matching numerals. Subject M.B. interchanged the numerals 6 and 9 in both reading and writing numerals.

Pre-Test Addition Computation Probe Results

Computation accuracy was also probed on the placement test. The probe contained single fact problems; 1X3 array size problems; 2X2 array size problems with and without regrouping required; and 3X2 array size problems. This probe was administered before and after the study.

Subject K.M. correctly answered one of the single fact problems on the pre-test of the probe. All 2X2 and 3X2 array size problems which did not require regrouping were correctly answered. When regrouping was required, subject K.M. either did not compute that binary or wrote both digits of the answer under the same column.

Subjects G.A. and M.B. wrote answers for all problems on the probe. However all answers were incorrect and no pattern for errors could be determined.

Subject M.C. correctly answered four single fact problems and one of the 1X3 array size problems. Tally marks representing each number in the binary were used in computing the answers. Subject M.C. did not attempt any of the other problems on the probe.

Post-Test Addition Computation Probe

All subjects correctly answered single binary (fact) problems. This skill is a major skill for "Low-Stress" algorithm instruction. The DOOF was used for these binary computations.

Subjects K.M., M.C. and G.A. correctly answered 1X3 array size problems. Subjects K.M. and G.A. also correctly answered 2X2 array problems which did and did not require regrouping.

Subject M.B. attempted the 1X3 array size problems. However answers were not correctly written.

Subject M.C. attempted 2X2 array problems. However, only problems which did not require regrouping were correctly answered.

The placement test was administered again after the study. All subjects correctly answered all items on the test except count from a number to a number and count from a number a given number of times.

Table 2: Addition Concept Probe Results

TABLE 2

ADDITION CONCEPT PROBE RESULTS

SUBJECTS

	M.C.	G.A.	M.B.	K.M.
Part I	Yes	Yes	Yes	Yes
Part II	Yes	Yes	Yes	Yes
Part III	Yes	Yes	Yes	Yes

Part I of the probe involved the union of disjoint sets in order to answer "how many all together?". Part II involved identifying whether the operation of addition was being performed using various objects. Part III involved writing an addition problem for two disjoint sets of objects and telling the sum of the disjoint sets.

The results of the concept probe (Table 2) administered in December indicate that all subjects had acquired the concept of addition.

Program Modifications

As a diagnostic teaching study, changes in the teaching method were used to correct error patterns. The following is a description of the error observed, and the teaching phase and the correction procedure used for the error.

Teaching: Error Patterns observed in present study and correctional procedures used:

Teaching Phase and Example of Errors	Description of Error	Correction Procedure Initiated
DOOF TRAINING	1. Subject did not count boxes on DOOF sequentially.	1. Enlarged size of DOOF so that boxes beside numerals were 3/4 inches by 3/4 inches. DOOF was drawn inside a 12X8 inch manilla folder.
	2. Subject lifted hand off DOOF in order to see numeral written on left side of box.	2. Numerals were written on both sides of the boxes on the DOOF in colored ink.
	3. Subject lifted hand off DOOF and therefore lost track of last box counted.	3. Modeled "sliding" movement of fingers on DOOF; physical prompting by moving

Teaching Phase and Example of Errors	Description of Error	Correction Procedure Initiated
DOOF TRAINING		the subject's hand in the "sliding" motion. These prompts were gradually faded.
	4. Subject did not start counting from first box under the box touched by pencil eraser	4. Same procedure as item 3 except subject touched pencil erasers with the tip of left hand and used "sliding" motion to get to the next box. The routine of touching the box with the pencil eraser in "writing hand", sliding from pencil eraser to next box with "counting hand" was prompted by the investigator. These prompts were gradually faded. The above terms were used rather than right and left

Teaching Phase and Example of Errors	Description of Error	Correction Procedure Initiated
		since all subjects responded correctly to the term "writing hand" without further teaching.
NOTATION New Notation: 5 1 7 2 Error: 5 +7 12	1. Subject drew line and plus sign as in conventional algorithm then wrote answer as in conventional method.	1a. Lines were drawn on either side of the second number in red ink; gradually faded to small lines in pencil; then to small dot in pencil.
		1b. Investigator wrote the answer using the conventional method, subject was then instructed to write the answer the "new way". The same binary was used.
1X3 ARRAY PROBLEMS 1 9 2 3 + 4	1. Misnoting the answer to the first binary operation under the first	1. Boyle's (1975) suggestion of an addition half space between the second

Teaching Phase and Example of Errors	Description of Error	Correction Procedure Initiated
	digit in the column rather than under the appropriate second digit.	and third digit used. An addition symbol was also placed between the first and second digit.
3 +7 4	2. Writing ones place answer on left side of problem rather than right side.	2. Similar procedure to #1 of Notation except line was drawn only on right side of problem in order to visually cue writing an answer on that side of problem.
9 + 3 12	3. Writing two place answer on left side of problem rather than placing one numeral on right side and one numeral on left side.	3. Box was drawn on either side of the problem, then faded to a line.
7 1 8 5 + 6 11	4. Writing answer to second binary under the answer bar	4. Same as #1 of notation except lines placed beside third digit.

Teaching Phase and Example of Errors	Description of Error	Correction Procedure Initiated
	rather than beside the third digit.	
7 1 8 5 +1 6 1	5. Not writing the answer below the answer bar.	5. Boxes were drawn below the bar. A small arrow was drawn from just above the answer bar pointing to the box. The box was gradually faded then the arrow was faded.
7 1 8 5 +1 6 4	6. Adding the second or third digits together rather than sum of the first binary and the third digit.	6a. As suggested by Hutchings' (1976) a small arrow was drawn beside the place the first binary sum would be written pointing to the third digit. 6b. A procedure of "crossing out" digits "already used" before computing the second binary. 6c. The investigator

Teaching Phase and Example of Errors	Description of Error	Correction Procedure Initiated
		computed the first binary and wrote the sum. The subject computed the second binary.
	7. Not counting the "1's" in the tens portion of the problem.	7. Circle "1's" in tens portion of problem while counting up.
2X2 ARRAY 1 6 0 3 + 4 4 2 5 1 4 5	1. Writing answer to first binary in tens portion beside first digit of binary rather than second number of binary and then adding as if there were 2 binaries.	1. A plus sign was placed between first and second digits. Crossing out digits "already used" before starting a new binary was also used.
	2. Not counting the "1's" for the tens portion and writing sum above tens column.	2. Similar to #7 of 1X3 array. A box was also drawn above the tens column as suggested by Boyle (1975).

The following procedures were used throughout all teaching phases:

1. Short, specific questions were asked by the investigator for each component of the operation to which the subject was required to respond (i.e., "What numbers are you adding?" "How many will you count?" etc.)
2. Requiring the subject to "think ahead" (verbalize each step of the operation) while the investigator listened for possible errors.

Reinforcement

Tangible reinforcers, in addition to the points from the classroom and social praise from both the classroom teacher and the investigator, were used during the last 8 sessions for all subjects. Subjects who obtained a post-test score higher than the pre-test score received a candy bar. Subjects who obtained a post-test score of 90% or better selected from any of the items available. The reinforcers available during these sessions were: computerized games; assorted candy bars; keys and key rings; time to talk with the investigator; an extra item from the classroom store. The computerized games were hand held and provided both a visual display of the sports characters and a sound to indicate the progress of the game. Subjects were able to play with the game for five minutes before returning to the classroom. Subjects who selected the candy could eat it immediately or take it with them to the classroom. Subjects who selected the keys were permitted to keep them. Subjects who selected time to talk with the investigator, selected the topic and the conversation lasted approximately five minutes. The classroom teacher provided the additional items from the "classroom store" and the subject selected the item upon return to the classroom.

The data for all subjects shows an improvement in binary accuracy between pre and post-test when the additional reinforcers were in effect. A similar variability is also seen in data for correct rate.

Session Occurrence

During the last month of the investigation sessions were held two times a day whenever possible. One session occured in the morning and one session in the afternoon. This was initiated in order to progress further along in the training.

Individual Performance Across Teaching Phases

A description of results for binary accuracy, binary correct rate and incorrect rate for each subject is presented below. Criterion for changing training phases was three consecutive sessions at 90% accuracy.

Figure 1 presents data on percent binaries correct for all subjects.

Subject M.B.

The data for the DOOF training phase (DF) shows an increasing trend for binary accuracy. Criterion of 3 consecutive sessions at 90 percent accuracy was met in 8 training sessions. The post-test range in scores for subject M.B. is 50 to 70 percent binary accuracy. This subject's post-test score range varied less than that of the pre-test score range (0-70) for this phase of training.

At the notation training phase (NT) criterion of 3 consecutive sessions at 90 percent was met in 4 sessions. The post-test range in scores for subject M.B. is 80 to 100 percent binary accuracy. A DOOF phase (DF) was reinstated after the notation phase in order to demonstrate maintenance of criterion at the DOOF phase. The post-test range in scores for the reinstated DOOF phase is 90 to 100 percent binary accuracy.

Criterion was not met at 1X3 array size phase. There were 12 training sessions at the 1X3 array size phase. The post-test range in scores is 20 to 80 percent binary accuracy. Pre-test score range (10 to 50) for the 1X3 array size phase varied slightly less than post-test score range. However sharper peaks are seen in the pre-test data.

<u>Subject M.C.</u> (Figure 1)

Criterion of 3 consecutive sessions at 90 percent binary accuracy was met in 5 sessions at the DOOF training phase (DF). The post-test range in scores for subject M.C. is 60 to 90 percent binary accuracy. The pre-test range in scores is 70 to 90 percent binary accuracy.

At the notation training phase there were 3 consecutive sessions at 80 percent accuracy before criterion of 3 consecutive sessions at 90 percent accuracy was met for this phase. There were 8 training sessions in all for this training phase. The post-test range in scores is 60 to 100 percent binary accuracy.

The DOOF probe phase was reinstated in order to demonstrate maintenance of criterion at the DOOF phase. The post-test range in

Figure 1: Percent Binaries Correct for Subjects M.B., M.C., G.A. and K.M.

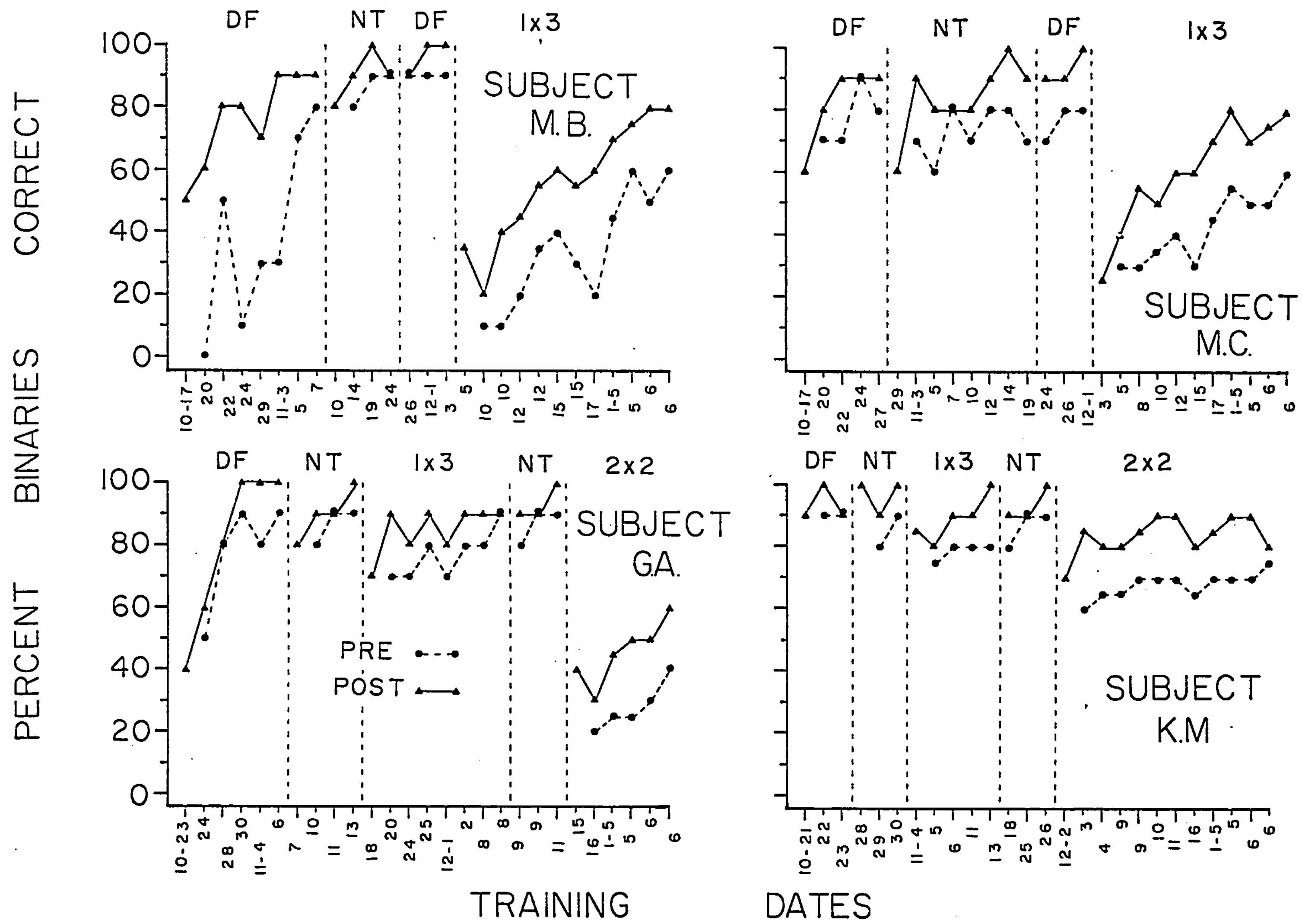


Figure I: Percent Binaries Correct for Subjects M.B., M.C., G.A. and K.M.

scores for the reinstated DOOF phase is 90 to 100 percent accuracy.

There is an increasing trend in both pre and post-test binary accuracy data for the 1X3 array size phase. However criterion was not met at this phase. There were 11 training sessions at the 1X3 array phase. The post-test range in scores is 25 to 80 percent binary accuracy. The pre-test range in scores is 30 to 60 percent binary accuracy.

Subject G.A. (Figure 1

There is an increasing trend in both pre and post-test accuracy data for the DOOF training phase. Criterion of 3 consecutive sessions at 90 percent accuracy was met in 6 sessions. The post-test range in scores for subject G.A. is 40 to 100 percent binary accuracy. The pre-test range in scores is 50 to 90 percent binary accuracy.

Criterion of 3 consecutive sessions was met in 6 training sessions at the notation training phase. The post-test range in scores is 80 to 100 percent binary accuracy. The pre-test range in scores is 80 to 90 percent binary accuracy.

Post-test accuracy scores fluctuated in the 80 to 90 percent range before criterion of 3 consecutive sessions at 90 percent accuracy was met. There were 8 training sessions at the 1X3 array size phase. The post-test range in scores is 70 to 90 percent binary accuracy.

The notation phase was reinstated to demonstrate maintenance of criterion at this phase. Post-test range in scores is 90 to 100 percent binary accuracy.

There is an increasing trend in both the pre and post-test accuracy data for the 2X2 array size phase. However, criterion was not met. There were 6 training sessions at this phase. The post-test range in scores is 30 to 60 percent binary accuracy. The pre-test range in scores is 20 to 40 percent binary accuracy.

Subject K.M. (Figure 1)

Criterion of 3 consecutive sessions at 90 percent accuracy was met within the first 3 sessions of each of the the DOOF (DF) and notation (NT) training phases. At the DOOF and notation training phases, the post-test range in scores for subject K.M. is 90 to 100.

Criterion of 3 consecutive sessions at 90 percent accuracy was met in 5 training sessions at the 1X3 array size phase. The post-test range in scores is 80 to 100 percent binary accuracy. The pre-test range in scores is 75 to 80 percent binary accuracy.

The notation training phase was reinstated in order to demonstrate maintenance of criterion at this phase. The post-test range in scores is 90 to 100 percent accuracy.

Criterion was not met at the 2X2 array size phase. There were 12 training sessions at the 2X2 array size phase. The post-test range in scores is 70 to 90 percent binary accuracy. The pre-test range in scores is 60 to 75 percent binary accuracy. The pre-test range in scores is 60 to 75 percent binary accuracy. There is a cyclical trend in both pre and post-test accuracy data.

Generally, across phases there is an increase in data for binary accuracy from pre-test to post-test.

Figures 2 and 3 present data for binary correct and incorrect per minute.

Subject M.B. (Figure 2)

At the DOOF phase, post-test data range for rate of binaries is .5 to 1.8 binaries correctly computed per minute. The post-test data range for rate of incorrect binaries is .6 to .2 binaries incorrect per minute. There is a trend of increasing rate in post-test correct rate data and a trend of decreasing rate in post-test incorrect rate data.

At the notation phase post-test range for rate of binaries is 1.6 to 2 binaries correctly computed per minute. The post-test range for rate of incorrect binaries is .4 to 0 binaries incorrect per minute.

At the 1X3 array size phase, the post-test range for rate of binaries is .4 to 1.6 binaries correctly computed per minute. The post-test range for rate of incorrect binaries is 1.2 to .4 binaries incorrect per minute. There is a trend of increasing rate in the post-test correct rate data and a trend of decrease rate in the incorrect rate data for this phase.

Subject M.C. (Figure 2)

At the DOOF phase, the post-test data range for rate of binaries is 1.4 to 1.8 binaries correctly computed per minute. The post-test data range for rate of incorrect binaries is 0.6 to 0.2 binaries incorrect per minute.

At the notation phase, the post-test data range for rate of binaries is 1.2 to 2.0 binaries correctly computed per minute. The post-test data range for rate of incorrect binaries is 0.8 to 0 binaries incorrect per minute.

Figure 2: Correct and Incorrect Rates for Subjects M.B. and M.C.

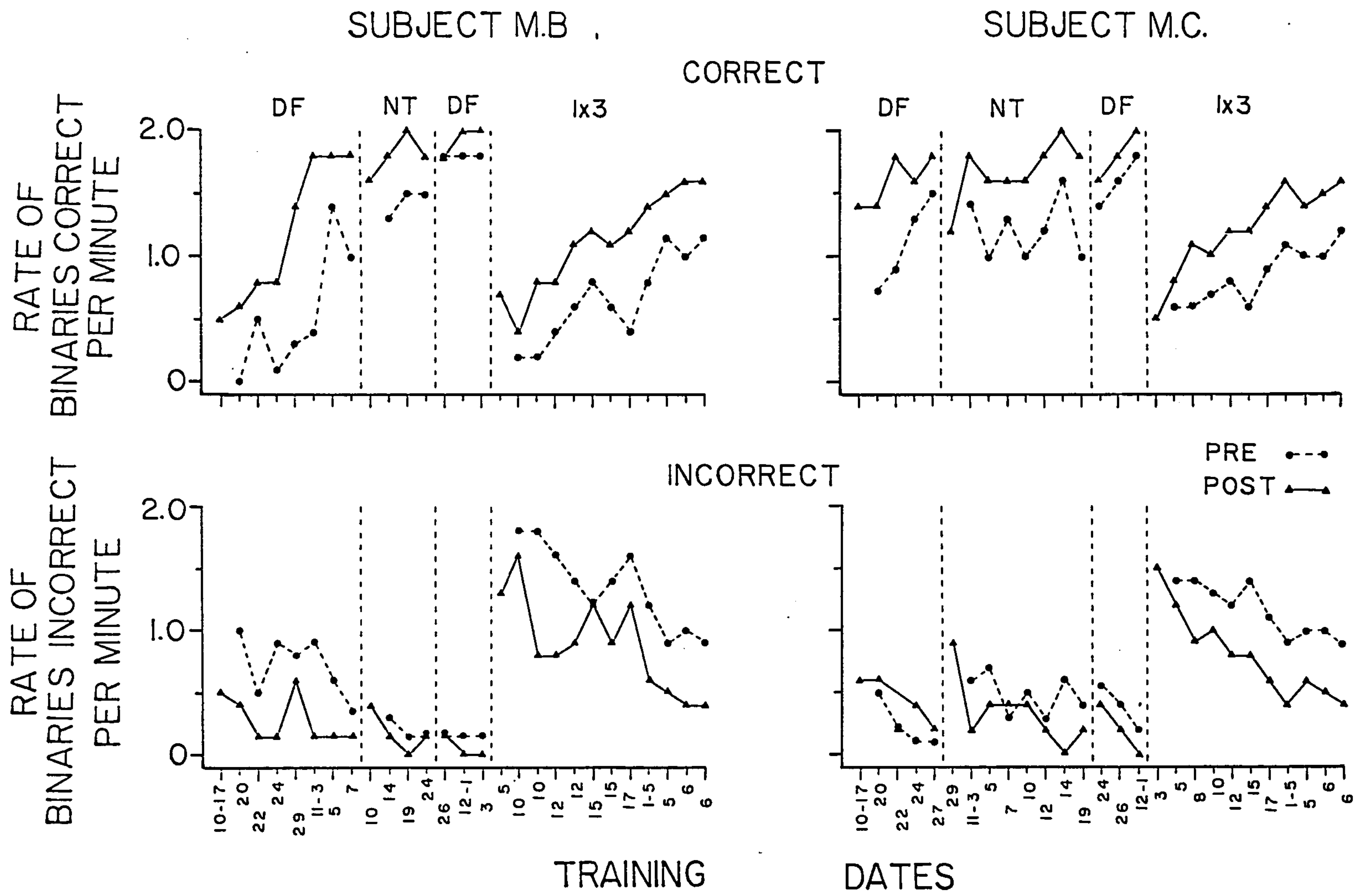


Figure 2: Correct and Incorrect Rates for Subjects M.B. and M.C.

At the 1X3 array size phase, the post-test data range for rate of binaries is 0.5 to 1.5 binaries correctly computed per minute. The post-test data range for rate of incorrect binaries is 1.5 to 0.4 binaries incorrect per minute. Data for correct rate shows a trend of increasing correct rate. Data for incorrect rate at this phase shows a trend for decreasing incorrect rate.

Subject K.M. (Figure 3)

At the DOOF phase, the post-test data range for rate of binaries is 1.8 to 3.0 binaries correctly computed per minute. The post-test data range for rate of incorrect binaries is 0.3 to 0 binaries incorrect per minute.

At the notation phase, the post-test data range for rate of binaries is 2.2 to 3.3 binaries correctly computed per minute. The post-test data range for rate of binaries incorrect is 0.2 to 0 binaries incorrect per minute.

At the 1X3 array size phase, the post-test data range for rate of binaries is 2.1 to 3.0 binaries correctly computed per minute. The post-test data range for rate of incorrect binaries is 0.6 to 0 binaries incorrect per minute.

At the 2X2 array size phase, the post-test data range for rate of binaries is 2.1 to 3.3 binaries correctly computed per minute. The post-test data range for rate of incorrect binaries is 0.9 to 0.3 binaries incorrect per minute.

Subject G.A. (Figure 3)

At the DOOF phase, the post-test data range for rate of binaries is 0.4 to 1.6 binaries correctly computed per minute. The post-test data range for rate of incorrect binaries is 0.6 to 0 binaries incorrect per minute.

At the notation phase, the post-test data range for rate of binaries is 1.1 to 1.4 binaries correctly computed per minute. The post-test data rate for rate of incorrect binaries is 0.3 to 0 binaries incorrectly computed per minute.

At the 2X2 array size phase, the post-test data range for rate of binaries is 0.9 to 1.8 binaries correctly computed per minute. The post-test data range for rate of incorrect binaries is 2.1 to 1.2 binaries incorrect per minute.

Figure 3: Correct and Incorrect Rates for Subjects K.M. and G.A.

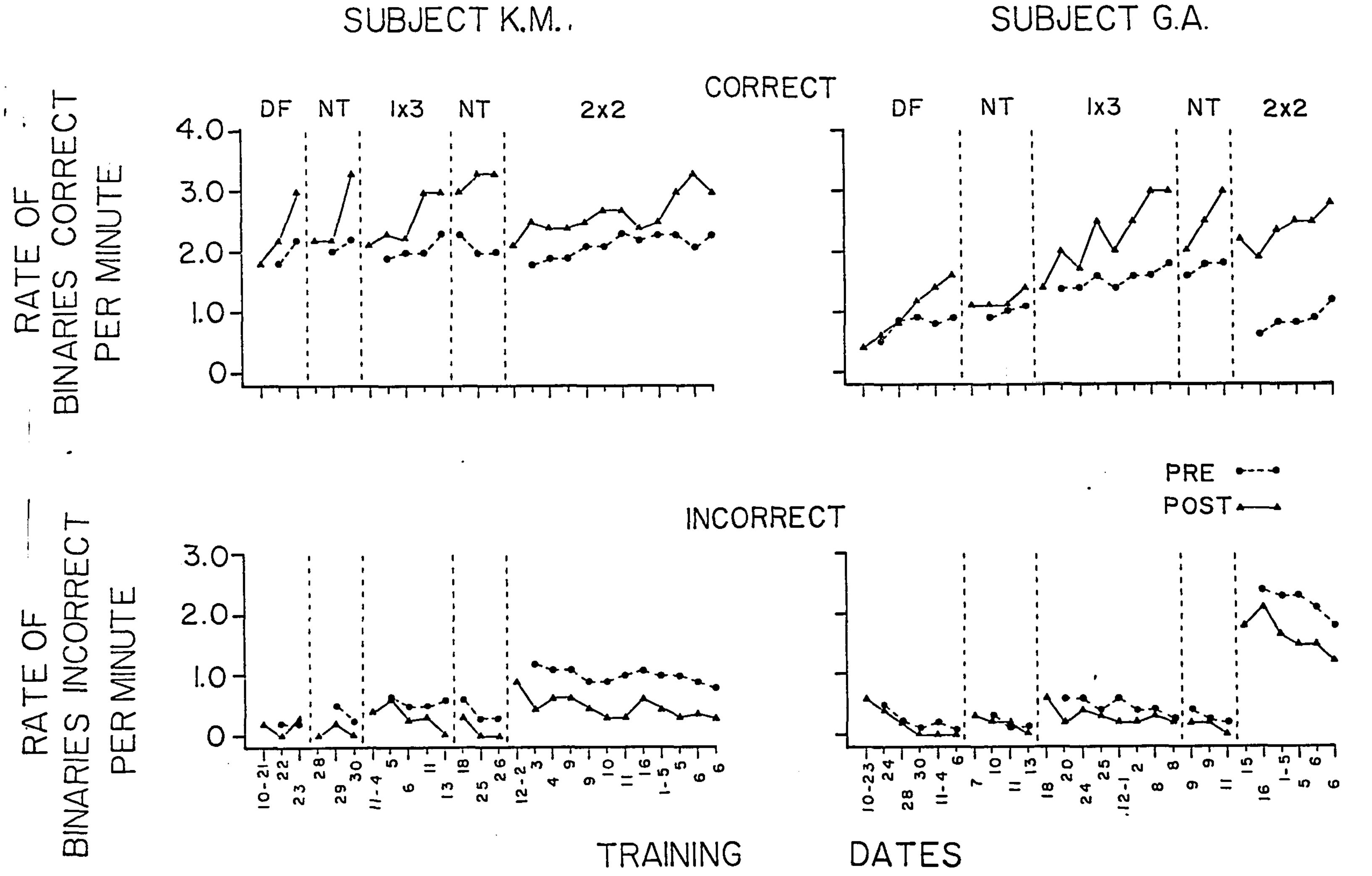


Figure 3: Correct and Incorrect Rates for Subjects K.M and G.A.

DISCUSSION

Limitations

It should be emphasized that the present study is an acquisition or diagnostic teaching investigation and not an experimental study. Correction procedures were used for any systematic errors observed during the study. The results can be used to identify future research for exploratory or experimental studies.

General Conclusions

Overall the results of this exploratory study indicate skill improvement for binary computation using the DOOF. Data for binary accuracy and correct rate for the DOOF and notation phases of training indicate more accuracy and higher rates than for data for the 1X3 array and the 2X2 array phases. In addition, the data from the post study administration of the placement test indicates an improvement in counting and numeral recognition skills.

Data Analysis

One of the limitations of the study was that it had to be completed by the end of the subjects' semester. Training days were lost due to holidays which occurred during the time of training. Therefore training was not completed for any of the subjects. Subjects K.M. and G.A. were in the 2X2 array phase when training ended. Subjects M.B. and M.C. were in the 1X3 array phase when training ended.

Subject M.B.

Figure 1 shows at the DOOF training phase, subject M.B.'s post-test accuracy score range varied less than that of the pre-test score range. Pre-test accuracy scores ranged from 0 to 70 and post-test accuracy scores ranged from 50 to 70 for this phase of training. An increasing trend in the data is seen for subject M.B. at the DOOF training phase.

Adjustments and correction procedures were used during training for the DOOF when error patterns were observed. The error pattern at this phase of training for subject M.B. was lifting the pencil high off the paper and thereby counting from an incorrect place on the DOOF. The "slide" procedure was used to correct this error. Subject M.B. also did not count the boxes on the DOOF sequentially. The DOOF was enlarged and numerals were written on both side of the boxes.

The upward trend in the data for the DOOF phase may reflect the effect of these adjustments and correction procedures on the percent of binaries correct. A binary percent range of 90 to 100 is seen when the DOOF phase is reinstated after the notation phase. This stable accuracy rate even after several sessions at a different phase may be further indication of the effectiveness of DOOF training on binary accuracy.

The range of pre-test accuracy scores at the notation phase is 80 to 90 and the post-test accuracy range is 80 to 100. The data for the end of the DOOF training phase through the notation training phase and the reinstated DOOF phase reflect a high stable accuracy of between 80 and 100 percent binary computation.

There was a slight decrease in percent accurate at the transition from the DOOF training phase to the notation training phase. However recovery to a higher percent accurate was fairly rapid.

Correction procedures and adjustments were also used during the notation training phase. The error pattern at this phase of training for subject M.B. was writing both numerals of the answer on the right side of the problem as a visual cue for writing the answer. These procedures may have produced the trend of increased percent binaries correct which is seen at the notation phase.

At the 1X3 array phase, subject M.B.'s pre-test accuracy scores ranged from 0 to 60 percent and post-test accuracy scores ranged from 20 to 80 percent. There is a large decrease in percent accuracy at the transition from the reinstated DOOF phase to the 1X3 array phase. Increasing trend is seen in the data for accuracy at the 1X3 array phase. However training at this phase was not completed.

Subject M.C.

Figure 1 shows that at the DOOF training phase, subject M.C. pre-test accuracy scores range from 70 to 90 and post-test accuracy scores range from 60 to 90. This relatively high accuracy at pre-test and relatively low increase in accuracy between pre and post-test scores may indicate a reduced effect in training and/or that the subject had acquired some of the skills necessary for using counting routines before training was initiated. It should be noted that this subject correctly computed addition fact problems on the placement test by counting vertical lines to represent numbers.

A lower accuracy score was obtained in the transition from the

DOOF training to the notation training. This is expected since another variable was added. Percent of binary accuracy for this phase ranged from 60 to 80 for pre-test and 60 to 100 for post-test scores.

Subject M.C. showed an error pattern of writing both numerals of a two place answer on the right side of the problem at the notation training phase. A box was drawn on both sides of the second digit of the binary as a visual cue for answer placement.

At the 1X3 array phase, subject M.C.'s accuracy scores ranged from 30 to 60 for pre-test scores and 25 to 80 for post-test scores. There is an upward trend in the data at this phase indicating some improvement in binary accuracy. However training at this phase was not completed.

Adjustments and correction procedures were used during the 1X3 array phase when error patterns were observed. The error pattern at this phase of training for subject M.C. was not writing the answer to the binary below the answer bar. Boxes were drawn below the bar and a small arrow was drawn from just above the answer bar pointing to the box. Subject M.C. also again showed the error of writing both numerals of a two place answer on the right side of the second digit of the binary. The correction procedure of a box drawn on both sides of the second digit of the binary was again used.

<u>Subject G.A.</u>

Figure 1 shows that at the DOOF training phase, subject G.A.'s pre-test accuracy scores ranged from 50 to 90 and post-test accuracy scores ranged from 40 to 100. The upward trend of both the pre-test and post-test scores may indicate the effects of training at this phase.

G.A. showed an error pattern of lifting the pencil high off the DOOF and thereby counting from the incorrect number. The "slide" correction procedure was used for this error pattern. This procedure may have contributed to the trend to increased binary accuracy seen at the DOOF training phase.

Pre and post-test scores in the notation phase ranged from 80 to 90 percent for pre-test and 80 to 100 percent for post-test. Correction procedures were not initiated for subject G.A. during this phase of training. The relative closeness of the ranges for pre and post-tests may reflect the cumulative effects of training for the DOOF and notation phases.

The range of the scores for the 1X3 array phase ranged from 70 to 90 for both the pre and post-tests. The closeness of the ranges and scores across the notation and the 1X3 array phases may indicate the effects of training on improvement binary accuracy.

A large decrease in binary accuracy is seen at the transition from the notation phase to the 2X2 array phase. The range of pre-test scores is 20 to 40 and post-test scores is 30 to 60. The trend in the data at this phase of training appeared to be upward. However training at this phase was not completed and criterion was not met.

Subject K.M.

Figure 1 shows that at the DOOF phase subject K.M. scored at 90 percent accuracy on the post-test on the first day of training (DOOF phase). Criterion at this training phase required fewer sessions for this subject than the other subjects. This subject also demonstrated acquisition of many of the pre-skills on the placement test. This may have affected the relative rapidness of meeting criterion. Correction procedures were not initiated for this subject during the DOOF training phase. Therefore the lack of trend in the data and the low variability between pre and post-test scores do not indicate an effect for DOOF training on accuracy in binary computation.

Criterion at the notation phase was also attained in fewer sessions by this subject than the other subjects. The high level of binary accuracy at transition from the DOOF phase to the notation phase may indicate a cumulative practice effect or that no effect was seen from the training. K.M.'s master of many of the preskills on the placement test may also have affected the high accuracy in binary computation.

Effects of Training

The results of the present investigation indicate that the subjects improved in counting and numeral recognition skills. These routines include matching numerals with sets; arranging numerals in sequential order; telling "how many" in a given set. The results also indicate a trend toward increasing binary accuracy and correct rate. Training was not completed in the present study. However, a trend of increasing power with the addition algorithm was reflected in the data. A higher binary accuracy and rate is reflected in the data for the DOOF and notation training phases than the 1X3 array and 2X2 array phases. The decrease in accuracy and rate observed at the 1X3 array may have resulted from the increase in procedural requirements. At the DOOF and notation phase the digits to use in the binary were clear and discernible (i.e., DOOF phase $\begin{array}{r}3\\ \underline{+\,5}\end{array}$: notation phase $\begin{array}{c}3\\5\end{array}$; only one counting routine was required to compute the answer; and placement of the written answer was not overly complex (i.e., one place answer on right of second digit of binary, two place answer on left and right of second digit of binary). However, error in answer placements was observed at these phases. These errors may have resulted from lack of skill and understanding in place value and multi-digit numerals.

In contrast, at the 1X3 array phase the digits to use in the binary were not clear and discernible for the "Low-Stress" naive pupil. i.e.,

$$\begin{array}{rccc} & & 3 & \\ & & 5 & 8 \\ + & 1 & 8 & 6 \\ \hline \end{array} \qquad \begin{array}{l} 3+5=8 \\ 8+8=16 \end{array}$$

At the 1X3 array phase two counting routines were required before arriving at an answer.

Another added procedural requirement involved answer placement for each binary. Answer placement errors similar to the description of the DOOF phase occurred at the 1X3 array phase. However, at this phase written answer placement was required three times before problem completion rather than the one written answer placement at the notation phase.

Correction procedures were initiated for any systematic error observed. The increasing trend for binary accuracy and rate may indicate the effectiveness of these procedures. However it should be noted that recovery of binary accuracy and rate required more training sessions at the 1X3 and 2X2 array phases than at the DOOF and notation phases.

Table 3 presents mean binary rates per minute and mean ratio of binary correct rate to binary incorrect rate which have been extrapolated from past research. The subjects in the studies reported in this table completed algorithm training. They were mostly regular education children. The problem array sizes were larger than that used in this study. Therefore the data is presented only as a reference point for interpreting the results of the present study. Table 4 presents mean binary rates per minute and mean ratio of binary correct rate to binary incorrect rate for each subject during the last three sessions of the 1X3 array size phase. The data for this phase is presented since all subjects of the present study received training at this phase.

Generally, the data in Table 3 indicates that the highest power for mean binary correct rate and mean binary incorrect rate was seen in the Rudolph (1976) study (mean binary correct per minute 15; mean binary incorrect per minute 1.5; mean ratio of correct rate to incorrect rate 10, these data represents the last few days of the study) and the Alessi (1974) study for the 2X7 array size (mean binaries correct per minute 15; mean binaries incorrect per minute 2; mean ratio of correct rate to incorrect rate 7.5).

The lowest power for mean binary correct rate and mean binary incorrect rate was seen at the Alessi (1974) study for 5X7 array size (mean binaries correct per minute 10.6; mean binaries incorrect 3.4; mean ratio of correct rate to incorrect rate 3.1).

Mean ratio of correct rate to incorrect rate for the subjects in the present study fall in the range of mean ratio of correct rate to incorrect rate from previous "Low-Stress" studies for two subjects (G.A. and K.M.) and below the range for two subjects (M.B. and M.C.)

Generally, the data indicate that the correct rates for binary computation for subjects in the present study is lower than correct rates for binary computation indicated in previous "Low-Stress" algorithm studies.

The rate of correct binaries per minute did not exceed 3 binaries for any subject in any phase of training. Factors which may have contributed to the slow rate are:

* the procedural aspects of the counting routines which required coordination of both hands; and
* the lack of adequate numeral recognition and counting routines.

Retention

Pre and post-test results indicates that the subjects were retaining skills practiced from session to session (including sessions separated by two or more days). The increasing trend of the data also indicates that the subjects were retaining previous skills and gaining power in binary accuracy. The investigator met with subjects for

Table 3: Mean Binary Rates Extrapolated from Past Research

TABLE 3

MEAN BINARY RATES PER MINUTE

Data Source		Mean Binary Correct Rate	Mean Binary Incorrect Rate	Mean Ratio CR/IR
by array size:				
Alessi (1974)	2X7	15	2	7.5
	3X7	11.4	2.6	4.4
Low-Stress	5X7	10.6	3.4	3.1
by array size:				
Alessi (1974)	2X7	10.8	2.8	3.86
	3X7	9.8	2.8	3.5
Standard Algo.	5X7	9.8	3.4	2.9
Boyle(1975)				
Low-Stress		14.2	1.64	8.7
Standard Algo.		9.8	2.76	3.6
Rudolph(1976) (Approx.-from graphs)				
(last few days of last phases only)				
Low-Stress		15	1.5	10
Standard Algo.		15	4.5	3.33
Zoref(1976)	2X7 array			
(low achievers)				
Low-Stress		10.2	1.44	7.1
Standard Algo.		4.9	4.08	1.2
Zoref(1976)	5X7 array			
(low achievers)				
Low-Stress		9.9	1.08	9.2
Standard Algo.		5.3	4.08	1.3

Table 4: Mean Binary Rate Per Minute for Present Study

TABLE 4

PRESENT STUDY RESULTS

MEAN BINARY RATES PER MINUTE

1X3 array size

Subject	Mean Binary Correct Rate	Mean Binary Incorrect Rate	Mean Ratio Correct Rate/ Incorrect Rate
M.B.	1.5	.62	2.4
M.C.	1.5	.5	3
G.A.	2.4	.24	10
K.M.	2.53	.32	7.9

a brief visit two months after the study's conclusion. These visits were requested by the subjects' classroom teacher. The subjects were observed using, accurately, skills presented during the DOOF training. Subjects K.M. and G.A. correctly computed 1X3 array problems using appropriate notation. These present results do not support any sizeable retention deficit. A factor which may have effected retention of the DOOF training was the cumulative practice effect of the DOOF training. The DOOF was used at each phase of training. Therefore, the subjects could have possibly "overlearned" these skills thereby enhancing retention.

Error Patterns

The error patterns observed by Alessi (1974) and Boyle (1975) were also observed in the present study. These studies were carried out with regular education pupils; the present study was carried out with trainable mentally impaired students. Since these error patterns (i.e., not writing down the last binary in each column; not adding the 1's in the tens position of the first column and carrying that sum up to the top of the second column; not starting with the first pair of binaries in the right most column; mis-noting the answer to the first binary operation under the first digit in the column rather than under the appropriate second digit) were common to both the regular and special education student, and may indicate inadequate teaching procedures used in the "Low-Stress" instruction. It may therefore be necessary to incorporate the proven effective correction procedures for these errors into the initial teaching procedure and thereby circumvent these error patterns before their occurrence.

Examination of worksheets from each session indicated errors which were not observed in the other studies of the "Low-Stress" algorithm. These errors are reported in the Program Modifications section of this study (see pg. 29). The most frequently observed errors were in carrying out the necessary procedural steps of the "Low-Stress" algorithm and random answering. Researchers of errors in computation (Roberts, 1968; Lankford, 1972; Schacht, 1967) have concluded that the largest number of errors across all ability levels result from incorrect algorithm techniques (errors other than number fact errors in performing an operation) Schacht (1967) concluded that:

> "differences in performance appear to be of degree and not of kind, with the less able making errors more frequently than the more able." (p.920)

Therefore, the supplementary error patterns identified in this study may not be specific to the trainable mentally impaired pupil but may instead result from the lack of prerequisite understanding and skill in counting, numeral recognition, and basic addition facts.

Teaching Strategy Recommendations

Probably one of the main reasons the subjects in the present study had difficulty completing training for the "Low-Stress" algorithm is that they did not have an adequate understanding of counting and numeral recognition for the DOOF training phase. However, the increasing trend in both accuracy and rate seen in the data for this phase across subjects may indicate improvement in counting and numeral recognition rather than proficiency with the DOOF. The procedure for using the DOOF provided the subjects with practice in: rote counting; matching numerals; numeral orientation (writing especially, reading to a lesser degree); counting in sequence with one to one correspondence with the boxes on DOOF; answer "how many"; count from 1 to a number (2-18); and writing numerals. These skills are elements of counting and numeral recognition routines. As can be seen from the results of the two administrations of the placement tests (Table 1) all subjects showed improvement in these skills.

Even though improvement in the elements of counting and numeral recognition were evidenced in the data, the DOOF may not be the most efficient and effective instructional aide for numeration instruction. When teaching numeration it is important that activities involving for sorting, trading, regrouping and counting in order to tell "how many" as well as constructing sets to show how much a numeral means be provided. The DOOF does not provide the materials for manipulations which is important to effective numeration instruction but the DOOF does provide a mechanical procedure for getting the answer.

Significant gains in elements of counting and numeral recognition skill (i.e., counting objects, making lines from numerals, the meaning of plus and increment addition) were found when using the Distar Arithmetic I Program (Engleman and Carnine, 1969) with moderately mentally retarded children (Bracy, et al, 1974). The Distar Arithmetic I Program may therefore be an appropriate prerequisite for using the Hutchings' "Low-Stress" addition algorithm with mentally impaired pupils. This program can be used for numeration instruction. The first eighty lessons of this program provide instruction for: rote counting; counting objects; symbol identification; symbol writing; counting from a number a given number of times; matching numerals and lines; and addition counting. These are many of the preskills outlined for the DOOF training phase. The Distar Arithmetic I Program could be followed by training for the "Low-Stress" algorithm.

The binary accuracy data for the DOOF training phase indicate the teaching strategy presented by Hutchings' along with the correction procedure is an effective strategy for binary accuracy. However a sharp decrease in accuracy is observed in the data for the 1X3 array phase. This may indicate that the operations involved in moving from the notation phase to the 1X3 phase are complex and should be separated into smaller component skills. One possibility for a teaching strategy at this phase is require the pupil only to compute the second binary. The first binary would be completed previous to presentation the pupil would be required to compute the second binary and write the sum and the answer below the answer bar. Crossing out digits "already used" is also recommended. The following is an example:

$$\begin{array}{r} 3 \\ 5\;{}_{8} \\ \underline{+\,4} \end{array}$$

Future Directions

Adjustments in the teaching format as presented by Hutchings' were used as error patterns were observed. Future research might investigate if it is necessary to incorporate these adjustments into the teaching format in order to improve the effectiveness of this algorithm when used with trainable mentally impaired pupils.

Other research might focus on using the "Low-Stress" addition algorithm in conjunction with the Distar Arithmetic I Program (Carnine and Engelmann, 1969) for increased counting, numeral recognition, binary accuracy and binary rate.

Finally, future researchers might use a changing criterion design (Hartman and Vance, 1976) in investigations of the "Low-Stress" algorithm with trainable mentally impaired pupils. This design requires initial baseline observations followed by the treatment phases. Each treatment phase is associated with a stepwise change in criterion and provides a baseline for the following phase. Experimental control is demonstrated when binary rate and accuracy change with each stepwise change in the criterion.

APPENDIX A

ADDITION PRESKILLS ASSESSMENT

ADDITION PRESKILLS ASSESSMENT

Before beginning this test you should be supplied with paper, pencil, poker or bingo chips, and a DOOF number line as well as the protocol and flash cards provided.

Student Name ______________________________

School ____________________ Teacher ____________________

Date ______________ Time Start ________ Time Stop ________

1. A. "Look at this number." "Now look at these numbers. Give me the one that is the same as that one."

2	(circle response)	3	5	2	7
6		8	6	9	5
4		4	7	1	9

1. B. (arrange cards in this order: 3, 8, 5, 1, 7)

"Look at these numbers." (Give student set in this order: (7, 8, 3, 1, 5).

"Find the same one for each number."

3 _____ 8 _____ 5 _____ 1 _____ 7 _____

2. "Look at this." (Use object cards.) "Give me the card with the same number of objects."

3	2	4	3	5
7	9	7	6	8
9	9	10	8	7

3. (Shuffle set of dot cards) "Put these in order starting from the smallest to the largest."

Response ____________________________________

4. (Arrange number cards in order 1-9). Hand student mixed dot cards.) "Put a dot card with the number that is the same."

1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ 7 _____

8 _____ 9 _____

5. "Look at this." (Show dot card.)

"Give me the number that is the same." (Display 1-9)

5 _____

8 _____

6 _____

6. (Show number cards in random order and ask for each: "What number?")

1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ 7 _____

8 _____ 9 _____ 10 _____ 11 _____ 12 _____ 13 _____ 14 _____

15 _____ 16 _____ 17 _____ 18 _____

7. (<u>Need paper and pencil</u>.) Randomly order dot cards 1-9. Present one at a time and ask: "Write this number."

1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ 7 _____

8 _____ 9 _____

8. "Count up to 18."

Response ______________________________

9. (<u>Need chips</u>. Give student stated amount of chips.)

"Count these." (Response)

4 chips ____________________

7 chips ____________________

9 chips ____________________

10. (<u>Need chips</u>.)

"Tell me how many."

6 chips ____________________

10. continued

8 chips ____________________

5 chips ____________________

11. "Count from 1 to ____."

1 to 7: ____________________

1 to 12: ____________________

1 to 15: ____________________

12. "Count starting from ____ to ____."

3 to 13: ____________________

6 to 16: ____________________

8 to 18: ____________________

13. A. "Start at ____ then count ____ more."

4	6 more	____________________
7	8 more	____________________

13. B. (Need DOOF) (Place finger on starting number.)

"Start at ____ then count ____ more."

12	3 more	____________________
9	5 more	____________________

---

14. (Place math symbol cards in front of student.) "Give me the card with the sign that tells you to add."

Response: ÷ X + —

15. (Give pencil to student and tell him/her to "Add these.")

See next page for problems.

16. (Place value assessment)

"There are 68 sticks. How many groups of 10 could you make?"

___

"There are 317 sticks. How many groups of 10 could you make?"

___

"6 tens plus 8 ones equals ______ ?"

"23 equals how many 10s?" ______"and how many ones?"_____

"3 hundreds plus 1 ten plus 7 ones equals ______ ?"

"526 equals how many hundreds?" ______ "how many tens?" ___
"how many ones? ______?"

"4321"

"The 1 is in the ______ place?"

"The 2 is in the ______ place?"

"The ______ is in the hundreds place?"

"The ______ is in the thousands place?"

17. Peer tutoring assessment. Use flash cards.

Model:	Test:
show problem side	______
look at answer side	______
"How many?"	______
if right, "good"	
or show next card	
if wrong, "no it's	______
So "how many?"	______

$$\begin{array}{r} 3 \\ +\ 5 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ +\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ +\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ +\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 2 \\ 4 \\ +\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ 5 \\ +\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 40 \\ +\ 50 \\ \hline \end{array} \qquad \begin{array}{r} 200 \\ +\ 500 \\ \hline \end{array}$$

25 + 43	47 + 30
624 + 201	213 + 714
56 + 29	25 + 65

839 + 867	465 + 846
1739 + 5632	7862 + 1338
2542 87 + 474	6391 807 134 + 7653

APPENDIX B
CONCEPT TEST

CONCEPT TEST

Binary operation of joining sets.

Part I

(use short sheets)

Place objects in each set; ask "how many all together"

a.) 5+3= b.) 1+6= c.) 4+2=

Part II

Examples -- Non examples -- "is this adding"

1+ 4-

2- 5-

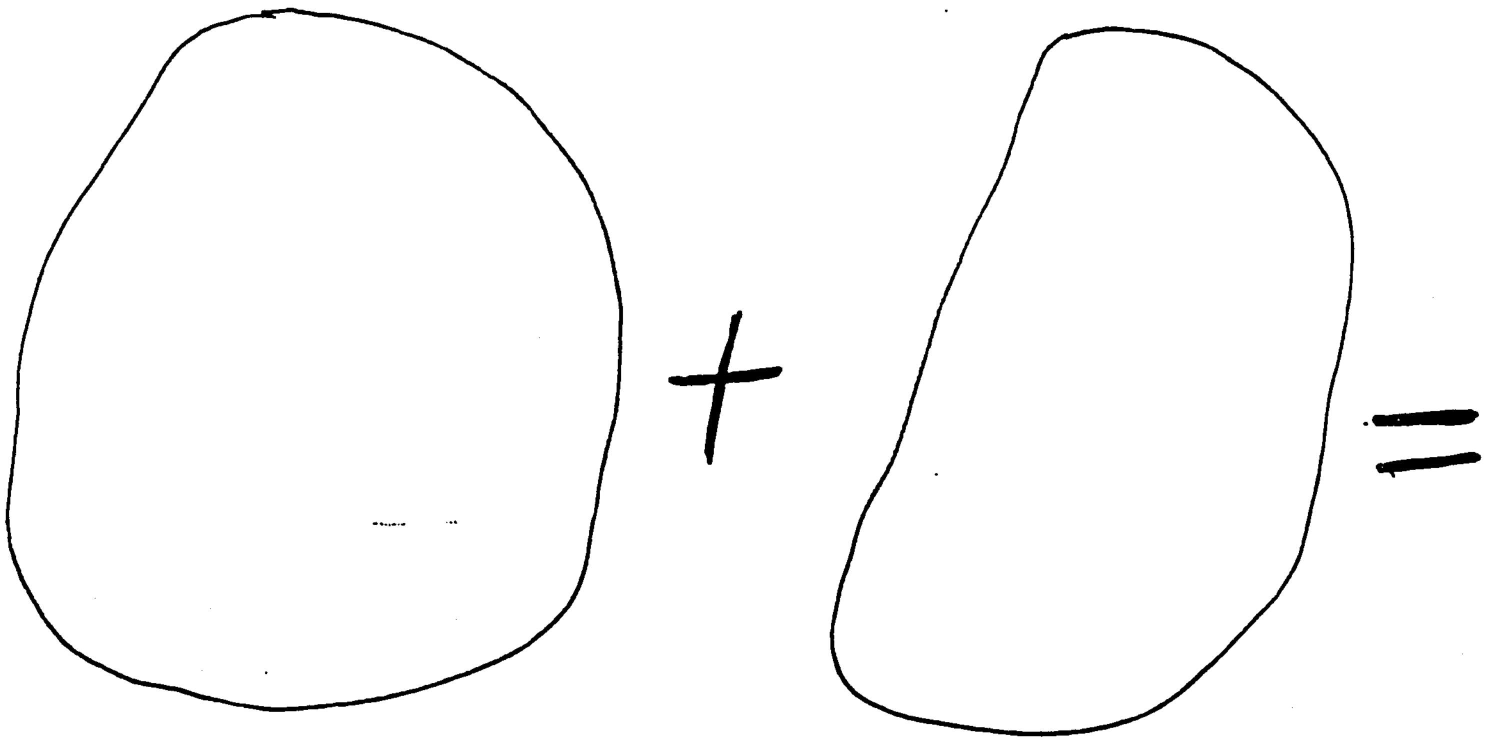
3+ 6+

Part III

Use sheet 0+0=

Present card with problem. Ask "S" to show you how this would look using the chips. Do it here - (point to sheet).

$$\begin{array}{r}3\\ \underline{+4}\end{array} \qquad \begin{array}{r}5\\ \underline{+1}\end{array} \qquad \begin{array}{r}2\\ \underline{+3}\end{array}$$



SETS SHEET

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