Preferences of Students for the Hutchings "Low Stress" Compared to the Conventional Algorithm under Conditions of Differentially Increasing the Number of Problems With and Without Reinforcement

Buitendorp Drew

Follow this and additional works at: https://scholarworks.wmich.edu/masters_theses

Part of the Psychology Commons, and the Science and Mathematics Education Commons

Recommended Citation
https://scholarworks.wmich.edu/masters_theses/1874

This Masters Thesis-Open Access is brought to you for free and open access by the Graduate College at ScholarWorks at WMU. It has been accepted for inclusion in Master's Theses by an authorized administrator of ScholarWorks at WMU. For more information, please contact wmu-scholarworks@wmich.edu.
PREFERENCES OF STUDENTS FOR THE HUTCHINGS' "LOW STRESS"
COMPARING TO THE CONVENTIONAL ALGORITHM UNDER CONDITIONS
OF DIFFERENTIALLY INCREASING THE NUMBER OF PROBLEMS
WITH AND WITHOUT REINFORCEMENT

by

Pamela G. Buitendorp Drew

A Project Report
Submitted to the
Faculty of the Graduate College
in partial fulfillment of the
requirements for the
Degree of Specialist in Education
Department of Psychology

Western Michigan University
Kalamazoo, Michigan
December 1980

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
This study was an attempt to determine which computation procedure students would select under varying classroom conditions. It is a systematic replication of previous research by Gillespie (1976). Algorithm preferences of 14 third grade students, 10 high accuracy and four low accuracy on a math facts test, were investigated under conditions of differentially increasing the number of problems with and without reinforcement. Reversal designs were used. The students completed 4x5 array problems and chose which algorithm to use, but after choosing the same algorithm for three consecutive sessions, they were required to use the alternative algorithm the next session. Preference was defined as using the same algorithm in six consecutive free choice sessions. Preference was determined during a baseline number of problems and also for 50% and 100% more problems. The Hutchings' algorithm was preferred 81% of the time in experiment 1 and 88% of the time in experiment 2. Algorithm preferences dissolved during 100% more problems and reinforcement did not alter the point where a shift from the preferred algorithm occurred.
ACKNOWLEDGEMENTS

I would like to thank the Department of Psychology at Western Michigan University, especially my advisor Galen Alessi, Ph.D. I would like to express my appreciation for the cooperation of the staff of the John Glenn Elementary School and to the students who participated in this study.

I would like to express my appreciation for the understanding and patience of my children (Juliann, Malea, and Brandon). And most of all I want to thank a fellow psychologist, and friend, Edward Drew, without his support it would not have been possible.

Pamela G. Buitendorp Drew
INFORMATION TO USERS

This was produced from a copy of a document sent to us for microfilming. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help you understand markings or notations which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure you of complete continuity.

2. When an image on the film is obliterated with a round black mark it is an indication that the film inspector noticed either blurred copy because of movement during exposure, or duplicate copy. Unless we meant to delete copyrighted materials that should not have been filmed, you will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., is part of the material being photographed the photographer has followed a definite method in "sectioning" the material. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.

4. For any illustrations that cannot be reproduced satisfactorily by xerography, photographic prints can be purchased at additional cost and tipped into your xerographic copy. Requests can be made to our Dissertations Customer Services Department.

5. Some pages in any document may have indistinct print. In all cases we have filmed the best available copy.
DREW, PAMELA G. BUITENDORP
PREFERENCES OF STUDENTS FOR THE HUTCHINGS' "LOW STRESS" COMPARED TO THE CONVENTIONAL ALGORITHM UNDER CONDITIONS OF DIFFERENTIALLY INCREASING THE NUMBER OF PROBLEMS WITH AND WITHOUT REINFORCEMENT.

WESTERN MICHIGAN UNIVERSITY, ED.S., 1980

University Microfilms International 300 N. Zeeb Road, Ann Arbor, MI 48106

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
PLEASE NOTE:

In all cases this material has been filmed in the best possible way from the available copy. Problems encountered with this document have been identified here with a check mark ☑.

1. Glossy photographs
2. Colored illustrations
3. Photographs with dark background
4. Illustrations are poor copy
5. Print shows through as there is text on both sides of page
6. Indistinct, broken or small print on several pages
7. Tightly bound copy with print lost in spine
8. Computer printout pages with indistinct print
9. Page(s) lacking when material received, and not available from school or author
10. Page(s) seem to be missing in numbering only as text follows
11. Poor carbon copy
12. Not original copy, several pages with blurred type
13. Appendix pages are poor copy
14. Original copy with light type
15. Curling and wrinkled pages
16. Other
TABLE OF CONTENTS

ACKNOWLEDGEMENTS............................................. ii
LIST OF TABLES.................................................. v
INTRODUCTION.................................................... 1
EXPERIMENT 1.................................................... 11
  Method......................................................... 11
  Subjects....................................................... 11
  Setting......................................................... 12
  Experimental Task........................................... 13
  Design......................................................... 14
  Independent Variables...................................... 15
  Dependent Variables........................................ 16
  Procedure..................................................... 16
  Recording and Scoring...................................... 20
  Reliability.................................................... 21
EXPERIMENT 1 Results......................................... 23
  Reliability.................................................... 23
  Algorithm Preference....................................... 23
  Algorithm Accuracy and Efficiency........................ 27
  Summary....................................................... 29
EXPERIMENT 2.................................................... 32
  Method......................................................... 32
  Subjects....................................................... 32

iii

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
LIST OF TABLES

Table 1: Individual and total student algorithm preferences within and across all experimental conditions ................... 24

Table 2: Individual and total student algorithm choices during free and forced choice sessions in each condition .......... 26

Table 3: Individual and total student mean session performance with the Hutchings' and the conventional algorithms during each experimental condition .................... 29

Table 4: Individual and total student algorithm preferences within and across all experimental conditions ..................... 39

Table 5: Individual and total student algorithm choices during free and forced choice sessions in each condition ............ 41

Table 6: Individual and total student mean session performance with the Hutchings' and the conventional algorithms during each experimental condition .................... 43
INTRODUCTION

This study was an attempt to determine which computation procedure students would select under varying classroom conditions. It is a systematic replication of previous research by Gillespie (1976).

Recently education has made national news with alleged reports of decline in computational skills and the requirements in some states for minimum competency testing. Modern math programs, which frustrated the parents because they could not help their child with her homework, have received part of the blame. Return to the basics now is the cycle that education finds itself in, and is seen as a solution.

The increasing sophistication of math information on a daily basis makes it more important to society than ever before to be a consumer of quantitative information. There is an increasing societal need for computational skills especially in view of the increase in quantitative occupations. The total curriculum in math is seeing an ever growing volume of material, especially with the emphasis in recent years on children understanding the conceptual basis of math. This leaves less time for the learning and maintaining of basic math computational skills. The conventional algorithms are often not mastered in the allotted time and are quickly reviewed the following year, leaving
the student in a repeated failure cycle which he can look forward to yearly.

Until recently most people have acquiesced to the assumption that some people "just can't handle figures" or "don't have a mathematical mind". Now however, with mathematical "literacy" becoming an increasingly important part of coping with modern life, educators, psychologists, and others are beginning to ask why numbers should be any more terrifying than letters and why students should not be expected to do just as well in math as they do in other subjects.

Hutchings (1972) and Gordon (1972) express the opinion that much of what is viewed as failure in computational skills is in reality due to feelings of anxiety and stress about mathematics. The tedious drill and concentration on calculation is felt to be responsible for emotional conditioning that is aversive to the child and harmful to his/her self-esteem and overall mental health. Hutchings (1972) and Alessi (1974) agree with Skinner (1968) that math is not a reinforcing event, for many people who have met with failure, but rather a "reaction of anxiety, guilt, or fear". (Skinner 1968, pg. 18). The emotional difficulties which people have experienced with math has generated enough concern that it is found in February, 1980 issue of Parents magazine and the following article: Found! A Cure for Math Anxiety. Sheila Tobias (1979) has recently written a book entitled Overcoming Math Anxiety which can be found
at the corner book store and hopes to appeal to educators and noneducators alike.

The needs of the individual student have received more concern since U.S. Public Law 94-142 has made it mandatory to provide the most appropriate education for every child in the least restrictive setting which of course is aimed at the exceptional child. Analyzing children's work procedures in the classroom (Backman, 1978) and diagnosing computational difficulty in the classroom (Inskeep, 1978) have received more attention. As a result of the trend toward mainstreaming, putting children with special needs into regular classrooms, the teaching of computation has become more challenging than ever. Of particular concern are learning disabled children, since they comprise the group most likely to be mainstreamed for a major part of their school day. Flinter (1979) and Moyer (1978) explore the area of computational difficulties experienced by children who may otherwise be doing average or above average work in other areas.

A set of experimental numerical computation algorithms was developed by Hutchings in 1972 at Syracuse University for his doctoral dissertation and later published in 1976. His research and subsequent studies by Gordon (1972), Alessi (1974), Dashiell (1974), Boyle (1975), Gillespie (1976), Rudolph (1976), Zoref (1976) and E. Drew (1981) have shown Hutchings' algorithms to be a useful alternative for calcu-
lation instruction. The above research support consistent calculation superiority with the Hutchings' "low stress" algorithm when compared with students using the conventional algorithm. Hutchings' states "that the new algorithms operate to permit easy mastery after brief training, to provide greater computational power than conventional algorithms, and to operate with much less stress on the user than conventional algorithms" (Hutchings, 1976, p. 219). Other advantages include: (a) it is easy to locate specific errors; (b) it requires only a knowledge of basic math facts; and (c) it has been designed to increase speed and accuracy (Alessi, 1974; Hutchings, 1976). Zoref deliniates the following advantages: "...(a) easy identification of errors, (b) facility with locating error patterns and prescribing appropriate remediation, (c) effective drill in basic math, (d) full written record that allows for specific feedback on accuracy, and (e) useful as a teaching tool to demonstrate carrying (regrouping)" (Zoref, 1976, p. 2). This new algorithm, it is pointed out by Alessi (1974), requires less in the way of sequential memory skills, less conceptual loading for conceptual regrouping skills, and more accuracy when computing under distracting conditions.

Hutchings (1976) felt that the most urgent need for the "low stress" algorithm was for students with extreme remedial needs. It is pointed out by (Moyer, 1978) that attention span and memory deficits are important limitations
in developing computation skills in learning disabled students, therefore, the "low stress" algorithms developed by Hutchings would appear to be useful. Lankford (1974) found that poor computers "often made errors in whole number operations when their counting...became too involved for their short memory spans" (p. 29). Many learning disabled students have difficulty with memory skills, according to Barnatyn (1968, 1971, 1974).

Hutchings' "low stress" algorithm appears to be a reasonable alternative to the conventional math algorithm presently taught in the school. With this algorithm the student records all the addition facts in a sequence. Only after this has been done, does the student do the necessary regroupings. With a full record there is less demand on covert chains of behavior. A major portion of the covert work is simplified in the Hutchings' algorithm compared to the conventional algorithm. Complex addition facts are not needed. Large sums can be obtained through a series restricted to basic addition facts. Hutchings' (1976) defines the addition algorithm as follows:

The low-stress addition algorithm uses a new notation, called half-space notation, to record individual steps. Half-space notation uses numerals of no more than a half-space in height to record the sum of two digits. With half-space notation, the units portion of the sum of two digits is written at the lower right of the bottom digit and the tens portion is written at the lower left of the bottom digit (p. 220).

Given a single column of figures to add, one would
at the top, add the first two digits, and record the sum in the new notation. The complete sum of each two digit addition is recorded in half-space notation, but only the ones portion of each sum is used in the next addition. The ones portion of the column sum is always the same as the ones portion of the last two digit sum. The tens portion of the column sum is always the same as the number of tens recorded at the left of the column. For a column in some multicolumn exercise, the last step—that is, counting the tens at the left of the columns—would be slightly changed—the total number of tens is no longer written in the tens place of the first column's sum but instead at the top of the next column at the left. Note, however, that the column sum for the last column in a multicolumn example is recorded in exactly the same way as the sum of a single column exercise (Hutchings, p. 221-223).

Example:

```
  3  2
  1  7  6
  6  8  8
  8  7  7
  19  9  3
  1  9  3  4
```

Alternative algorithms for faster and more efficient calculation skills are not abundant. Reviews of the literature by Hutchings (1972), Gordon (1972), and Dasiell (1974) have concluded that there is not much research literature on alternative algorithms.

In the literature there are suggestions for interviewing students to identify computational practices in children (for example Gray, 1966; Krutetskii, 1976, Lankford 1972, 1976; and Weaver 1955). Lankford reported some computational practices of poor computers through verbal interviews. Of significance is the finding that poor computer "often
made errors in whole number operations when their counting...became too involved for their short memory spans" (Lankford, 1974, p. 29).

Sanders (1971) recognizes that a source of error is that the "slow computer tends to forget where he was in a computation,..." (p. 413). He wants students to learn to get the right answer quickly and with a minimum of effort. He proposes a system of holding up fingers to keep track of the tens. For example when adding 8 plus 8, the child says "six" and extends one finger to keep track of the tens portion of the sum. In column addition then this would continue until the last pair of binaries are added and that become the one part of the column sum and the fingers are the tens portion to be added to the next column. While this may work for some students it can be seen that some students could be confused by it or have difficulty handling it conceptually. Hutchings' algorithm has the advantage of having a written record to analyze errors the student makes.

O'Malley (1969) suggests the student write the one in the next column, instead of holding up a finger. It does not have all the advantages of a full written record but may ensure proper carrying and lower the memory response effort.

Fulkerson (1963) suggests a similar algorithm but this time the student draws a line through the last digit used in adding a sum of ten or greater. It again does not
have all the advantages of the Hutchings' full written record. But he goes on to explain some advantages, "to give pupils a better understanding of our number system, to motivate pupils to carry on needed practice in addition, and to make our pupils conscious of the fact that column addition can be performed by the use of algorithms other than the one to which all of us have been ordinarily accustomed." (Fulkerson, 1963, p. 140).

From the above research evidence cited earlier it would appear that Hutchings' "low stress" algorithm may be a useful academic option. Educators have an obligation to use the most efficient method for teaching children basic operations as well as concepts. One area that also needs to be considered is the willingness of students to use a new algorithm.

Gillespie (1976) applied the Findley "minimax" preference procedure for the Hutchings' algorithm and the data supported the conclusion that it was preferred by most students, in that study. The purpose of this study was to replicate Gillespie's research. The "minimax" preference design (Findley, 1958; Lockart, Sexton, and Lea, 1973) permits one to study the preferences of individual subjects for various options. This design involves at least a two choice situation and the child is given a free-choice of either option, but within limits. For example in Gillespie's (1976) study the child had a free choice until one was chosen three
times in a row. At that point the student was "forced" to sample one trial with the other option. Preference then is operationally defined in terms of: (a) number of forced trials per opportunities available, (b) number of free choices per opportunities available, or (c) a sequence of several free choices for one to the exclusion of the other option.

One could ask the student for his/her verbal preferences but those verbal statements do not always reflect actual choice behavior. The area of choice-making behavior and its effects on school performance have only recently been looked at.

In this study two separate experiments will be run concurrently. This study will apply the Findley "minimax" preference to the academic area of mathematical addition computation. Will students prefer the Hutchings' "low stress" algorithm or the conventional algorithm when doing multiplace column addition?

The first experiment will attempt to answer the following questions:

1. In a free choice situation would third graders prefer to use the Hutchings' algorithm or the conventional algorithm?

2. After a preference for one calculation method has been established, how strong will that preference be? In other words, how much extra work are the students willing to do to maintain their preferences.

3. Is there a difference with students who know their addition math facts and those who do not in their math facts in algorithm preference?
4. Do students prefer to perform the algorithm that results in the greater accuracy and efficiency?

5. Are verbal preferences an accurate assessment of behavioral preferences for a specific algorithm?

The second experiment attempted to investigate the effects of reinforcement for accuracy on algorithm preference, under conditions similar to those of Experiment 1. The following question will attempt to be answered: What is the effect of reinforcement for accuracy on algorithm preferences under the conditions where performing the preferred compared to the nonpreferred algorithm, required doing more problems? How many additional problems does there need to be to cause a shift in preference from the preferred to the nonpreferred algorithm?
EXPERIMENT 1

Method

Subjects

Eight students were selected for this study in April of 1980 from the 32 students in the morning math class. This is the largest class in this rural modern public elementary school of 295 students. Scott County is in eastern Iowa and borders on the bend of the Mississippi where it flows east to west. It is a typical Iowa school district having a mixture of suburban and rural housing, and is a district that is still enjoying an increasing enrollment.

Alessi in 1974 developed a five minute basic addition math facts test, consisting of 56 basic addition facts, which was given to all 32 students in the class. The basis for selection was their percent accuracy on that test. For purposes of this study, and since it is a replication study, the same criteria were used as Gillespie, (1976), which was high accuracy was defined as 96% or better and low accuracy as 80% or less on the addition math facts test. A pool of possible students were selected on the basis of accuracy on the math facts test. Students were then not included if they had a history of poor attendance, would be going on vacation, and their health records were checked to make sure their were no known uncorrectable sensory...
acuity problems. The teacher then selected students who she thought would enjoy being in such a study. I then explained what we would be doing and told them that they could withdraw at any time from the study. Written parent permission was secured for the eight students, two low accuracy students and six high accuracy students. The high accuracy students all had 100% on the basic addition math facts test and the two low accuracy students (Students 1 and 2) had 80%.

Setting

The students in this school have their classes, grades three through six, in an open area surrounding a media center. The study took place down the hall in a conference room which is set aside for the school psychologist and speech clinician. This room is equipped with a large table surrounded by chairs and has a blackboard for instructional purposes. There are no windows and it is appropriately decorated with posters and a bulletin board.

Except during the instructional algorithm stage in the beginning of the study the experimenter sat out of the way of the students. During condition A it was noticed that one student (Student 1) tended to have a high rate of starting the problems on the wrong side (left to right). When necessary during condition A and B the experimenter sat at the side of the room that student was on and pointed where to start the problem (upper right). By condition C that was no longer necessary.
The answer keys were located in the hall right outside the door on a bulletin board. The students were allowed to mark on the papers in red marker since it would not be picked up by the machine when making a transparency to check the papers for the reliability checks.

To time the students, a stop watch was used that had the ability to record lap time. During the sessions where an observer was present for the reliability checks on the timing, he stood toward the door which was as out of sight as possible in a small conference room.

**Experimental Task**

Hutchings' (1972) made some specific recommendations for the design of a measurement instrument for the use of his algorithms as it relates to speed and accuracy:

It is required that variations in example forms which load for reading or eye movement skills be avoided, e.g., interrupted rows, but that a range of profiles, as might occur in lessons or general experience, be presented. It is required that applications of the identity element (0) be avoided, as these are considered to load for a distinct peripheral concept while contributing very little to demands upon memory-retrieval functions. It is required that a systematically balanced presentation of the universe of binary combinations be made (p. 51).

In conforming the above requirements of Hutchings' and also using the same experimental task as Gillespie (1976), students were given sheets of 4 x 5 (four columns, five rows) addition problems to complete on standard 8½ by 11 inch paper. The number arrays were obtained by using a random number table. In keeping with the above recommendation the
numeral 0 is not used. The problems were typed with an IBM Selectric typewriter using the Orator 10 element. Numbers in the rows were typed three spaces apart and with double spacing between the rows. The size of the type and spacing of the problems is considered a possible factor in the performance of the student because of the written responses necessary in the body of the problem when using the Hutchings' Algorithm. The number of $4 \times 5$ array problems on a page ranged from two to six, with no more than six problems on any page.

In Gillespie's (1976) study she stated: "It is felt that requiring different students to perform different number of problems during baseline conditions, may have resulted in certain interaction effects." She goes on to make the suggestion that in any replication of the study that all students should complete the same number of problems during baseline conditions. Therefore, that suggestion was implemented in this study. However, it should be noted that it was not possible to carry out her second suggestion which was to carry out the study in the regular classroom.

Design

A reversal design (Baer, Wolf, and Risley, 1968) of the pattern A-B-C-A was used for all students. As in Gillespie (1976) it was decided before the study to use condition C only for those students who did not shift preference from baseline during Condition B.
Independent Variables

1. The algorithm used by the student to compute daily exercises:  
   a) Hutchings' "low stress" algorithm (see p. 5 and 6) for definition and example,  
   b) Conventional algorithm: The only mark that is written down is the final answer. Example:  
      \[
      \begin{array}{c}
      5 \\
      6 \\
      9 \\
      8 \\
      28
      \end{array}
      \]

2. a) Baseline number of problems for the two forms of algorithms: The students are given the same number and size of problems no matter which algorithm they choose to complete them.
   b) Then the number of problems was differentially increased to 50% and 100% for the algorithm they established as the preferred one in the baseline condition. This is done by keeping the size of the problems the same but requiring the student to do 50% more problems (Condition B) and 100% more problems (Condition C) than during the baseline (Condition A). Students who change to their non-preferred algorithm are required to do only the same number of problems as in baseline.

3. High accuracy versus low accuracy students based on the addition math facts test and teacher recommendation. High accuracy is defined as 96% or better on the math facts
test and low accuracy is 80% or less on that addition facts test.

Dependent Variables

1. Algorithm preference. Which is defined as the student choosing to do the same algorithm for a minimum of six consecutive free choice sessions. That preference is determined during each condition.

2. Rate of Columns correct: \[ \frac{\text{number of columns correct}}{\text{number of minutes}} \]

3. Rate of columns incorrect: \[ \frac{\text{number of columns incorrect}}{\text{number of minutes}} \]

4. Percent accuracy: \[ \frac{\text{Number of columns correct}}{\text{number of columns attempted}} \]

Procedure

Pretraining. As previously stated a basic math addition facts test was administered which was taken from Alessi (1974), and students selected according to criteria stated previously. The instruction and review procedures used were those adapted from Hutchings' (1972) and previously used by Alessi (1974) and Boyle (1975). See Appendices 'A, B,' and C for the instructional and review sessions. Alessi (1974) states: "The instructional procedures functioned more than adequately, and at time it was felt by the experimenter that the sequence was too redundant for the level of children being worked with. It was felt that some of the drill and repetition could have been eliminated without markedly
affecting the acquisition of new algorithm procedures". (Pg. 50). Gillespie (1976) shortened review sessions to 20 minutes.

On the days of the algorithm instruction the experimenter got the students from their regular math class and accompanied them to the conference room. The instructional format for the Hutchings' algorithm instruction was followed from Alessi (1974), and consisted of one 30 minute session devoted to the procedures and practice of the Hutchings' algorithm. In all instruction and review sessions the students were provided with sheets of dittoed addition problems for practice. This was followed the next day with one 20 minute review and practice session in using the Hutchings' algorithm. The next day a 20 minute review session was held on the conventional algorithm.

Prebaseline Condition

The prebaseline condition started after the last review session. The following procedures are an attempt to replicate Gillespie (1976) and so therefore, are as close as possible to that study. Two ten minute sessions were held daily, one in the morning 10:10 and one in the afternoon 1:10.

During the first four sessions of the prebaseline condition the two types of algorithms were alternated to provide a forced exposure to each type of algorithm. The order for the students to start was counterbalanced across
students. During all subsequent sessions the student had a free choice of which algorithm to use. It was explained to the students that once they had chosen that algorithm that they were to do all their problems that session the way they had chosen. However, it was also explained that they would not be allowed to use any algorithm more than three times and would be forced (asked) to switch. They did not have to keep track of when that forced choice session would occur because they would be told. Also that after one forced choice session, the student would again be free to choose which ever algorithm he/she preferred.

Each time when the student had completed the problems he/she was to raise his/her hand and the experimenter recorded their time. Some of the students became very interested in the times and would come up and ask me so I would let them look in the book at their times. They were quite competitive as it related to others and in bettering their own time. The experimenter used a Armitron Quartz stop watch that could be stopped to record the lapsed time while it is being recorded. The answer keys were posted on the hall bulletin board and the students were instructed just to circle any numbers that were wrong. Their pencils were left in the room and a magic marker was there attached to a string to correct their work. They did not have to correct their work if they didn't want to. However, high accuracy students always choose to check their work and most of the time the other students did also. The students all
remained at the room until all were finished and they returned to the room as a group.

**Baseline: Condition A**

During condition A all students had a free choice as explained above as to what algorithm they would choose. The same number of problems were given to each student no matter what algorithm they choose. They all received four problems, as was decided upon in the prebaseline condition. This number could be reasonably handled by the low accuracy student in the time set aside for the task. The baseline condition ended when all students had established a preference for one algorithm. Condition A ended at one time but all students did not establish the trend toward that preference at one time. A preference for one algorithm was defined as choosing one algorithm six consecutive times during free choice sessions.

**Condition B**

The preferred algorithm, as established during baseline now required a 50% more problems than in baseline Condition A. The student was again allowed the same free choice situation as in Condition A, but now if they choose the algorithm that they had preferred they got six instead of four problems. If the student chose the nonpreferred algorithm from Condition A they would do the same number of problems as in Condition A, namely four. Condition B
ended for all students when each student had established a preference for either algorithm, or when it was clear that there was no algorithm preference.

**Condition C**

During this condition choosing the preferred algorithm resulted in a 100% increase in number of problems. If the preferred algorithm was chosen they got eight problems to do and four to do if they choose the nonpreferred algorithm. All students again had a free choice of the algorithm they wanted to use for that set of problems. Condition C ended for all students when each student had established a preference for either algorithm, or when it was clear that there was no algorithm preference.

**Return to Baseline Condition: Condition A**

The same procedure was used in this condition as in the initial Condition A. Condition A ended when a preference had been established for either algorithm or when it was clear that there was no algorithm preference.

**Recording and Scoring**

The students' papers were checked with the answer key as soon as possible after the session. The number of columns correct, number of columns incorrect, and the time taken to complete the problems were recorded. Alessi (1974) explains the rational for scoring a column correct.
"In order to correct for errors that might "snowball" in causing errors to accumulate across columns in the larger profiles, these were scored to prevent this effect. All children were asked to and did write the carrying numbers at the top of each column throughout the test. If it was clear that a column error had caused an incorrect number to be carried to the next higher column, and this column was correct except for the incorrect carried number, then this column was scored as correct. This procedure was not meant to correct for carrying errors per se, but only for column errors in preceding columns that caused incorrect carrying of tens to the adjacent column." (P. 42)

Reliability

For reliability checks on the timing both the observer and experimenter had separate stopwatches with lapped time. They were far enough out of each other's sight that they could not see what the other person was recording. The times after the session were then compared and each pair of times recorded plus or minus. A plus was recorded when the times came within (plus or minus) three seconds of each other, and a minus when it was outside of that range. Reliability coefficients were calculated by dividing agreements by disagreements plus disagreements (Type II reliability).

A permanent product is left by the problems the student completes. This provides an easy way for them to be corrected by another person other than the experimenter. This was done with a transparency. Reliability coefficients were calculated by dividing the number of agreements by the
number of agreements plus the disagreements and multiplying the quotient by 100 (Type II reliability).
EXPERIMENT 1

Results

Reliability

The reliability for the timing of the problems was taken for 10 of the 51 sessions and yielded a mean of 91% agreement. Papers were checked for 10 of the 51 sessions for each student and yielded a mean of 97% agreement.

Algorithm Preference

Table 1 indicates the algorithm preferences during all conditions for each student. In both baseline conditions all students preferred the Hutchings' algorithm. During Condition B, seven students continued that preference while one showed no preference for either algorithm. In Condition C, three students continued to prefer Hutchings' algorithm and the other five students showed no preference.

The bottom of Table 1 summarizes the results with percent of students preferring the Hutchings' algorithm, the conventional algorithm, or neither algorithm, during each condition and across all conditions. Across all conditions 81% of the students preferred the Hutchings' algorithm. No student ever showed a preference for the conventional algorithm and 19% had no preference demonstrated.
TABLE 1

Individual and Total Student Algorithm Preferences Within and Across All Experimental Conditions

<table>
<thead>
<tr>
<th>Experimental Conditions</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>* 1</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Hutchings</td>
</tr>
<tr>
<td>* 2</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>3</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Hutchings</td>
</tr>
<tr>
<td>4</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>5</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>6</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>7</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>8</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Hutchings</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Totals (Percent)</th>
<th>Totals Across Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutchings 100%</td>
<td>81%</td>
</tr>
<tr>
<td>Conventional 0%</td>
<td>0%</td>
</tr>
<tr>
<td>No Preference 0%</td>
<td>19%</td>
</tr>
</tbody>
</table>

Note * indicates students who did not know math facts consistently.
A detailed analysis of individual student preferences is presented in Table 2. In this table the individual and total student algorithm choices are presented during free choice and forced choice sessions in each condition. The number of sessions and the corresponding percentages are recorded. During all conditions where a preference for one algorithm was established, all high accuracy students choose to perform the Hutchings' (preferred) algorithm during 100% of the free choice sessions. The low accuracy students took longer to establish an algorithm preference.

The bottom of Table 2 shows that for all students the conventional algorithm was performed during no more than 38%, and the Hutchings' algorithm during no less than 62% of the free choice sessions, in any condition. No more than 21% of total algorithm trials were students forced to use the Hutchings' algorithm. However, during three conditions 100% of the students only used the conventional algorithm in forced choice sessions.

Verbal Preferences

The results of the questionnaire indicated that all students liked the Hutchings' algorithm better both at the beginning and the end of the study, which matched their behavioral preferences in the study. The students also stated that they got more problems correct and finished the problems quickly using the Hutchings' algorithm. This was an accurate assessment of the student's actual performance.
<table>
<thead>
<tr>
<th>Students</th>
<th>Condition A</th>
<th></th>
<th>Condition B</th>
<th></th>
<th>Condition C</th>
<th></th>
<th>Condition A</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forced</td>
<td>Free</td>
<td>#</td>
<td>%</td>
<td>Forced</td>
<td>Free</td>
<td>#</td>
<td>%</td>
</tr>
<tr>
<td>S1 H C</td>
<td>0/2</td>
<td>0</td>
<td>8/12</td>
<td>67</td>
<td>0/3</td>
<td>0</td>
<td>9/9</td>
<td>100</td>
</tr>
<tr>
<td>S2 H C</td>
<td>0/2</td>
<td>0</td>
<td>9/12</td>
<td>75</td>
<td>*6/12</td>
<td>50</td>
<td>0/0</td>
<td>0</td>
</tr>
<tr>
<td>S3 H C</td>
<td>0/3</td>
<td>0</td>
<td>11/11</td>
<td>100</td>
<td>0/3</td>
<td>0</td>
<td>9/9</td>
<td>100</td>
</tr>
<tr>
<td>S4 H C</td>
<td>0/2</td>
<td>0</td>
<td>7/7</td>
<td>100</td>
<td>0/2</td>
<td>0</td>
<td>9/9</td>
<td>100</td>
</tr>
<tr>
<td>S5 H C</td>
<td>0/2</td>
<td>0</td>
<td>8/8</td>
<td>100</td>
<td>0/2</td>
<td>0</td>
<td>8/8</td>
<td>100</td>
</tr>
<tr>
<td>S6 H C</td>
<td>0/2</td>
<td>0</td>
<td>9/9</td>
<td>100</td>
<td>0/3</td>
<td>0</td>
<td>9/9</td>
<td>100</td>
</tr>
<tr>
<td>S7 H C</td>
<td>0/2</td>
<td>0</td>
<td>6/6</td>
<td>100</td>
<td>0/2</td>
<td>0</td>
<td>7/7</td>
<td>100</td>
</tr>
<tr>
<td>S8 H C</td>
<td>0/3</td>
<td>0</td>
<td>9/9</td>
<td>100</td>
<td>0/2</td>
<td>0</td>
<td>8/8</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>3/3</td>
<td>100</td>
<td>0/9</td>
<td>0</td>
<td>2/2</td>
<td>0</td>
</tr>
</tbody>
</table>

*No preference for either algorithm established

H - Hutchings' algorithm
C - Conventional algorithm
This is interesting, especially since student 2 had no preference once the number of problems was increased at all. The results of the questionnaire were further reinforced by the students spontaneous remarks during the study e.g. when they had to do the conventional algorithm, statements such as "Oh no." or "Do I have to?". The students also became very anxious as exhibited by in seat rocking, counting out-loud, telling others to be quiet when they were doing the conventional algorithm. When they had to do the conventional algorithm many of them engaged in a variety of techniques to help them, the use of fingers, marks on paper, regrouping on the paper, and keeping track of where they were in the problem with their finger. One girl commented that her legs ached when she was done doing the math problems the conventional way, and it was a small wonder considering the effort that had gone into the task. They stated that the "new" way was so easy that they were concerned whether their next year's teacher would let them use it.

Algorithm Accuracy and Efficiency

Table 3 shows individual and total student mean session performance with the Hutchings' and the conventional algorithms during each experimental condition. The total student data is presented in Figures 1a and 1b which show that during all conditions, the Hutchings' algorithm had: (a) higher percentage of accuracy; (b) higher mean session rate of
TABLE 3: INDIVIDUAL AND TOTAL STUDENT MEAN SESSION PERFORMANCE WITH THE HUTCHINGS' AND THE CONVENTIONAL ALGORITHMS DURING EACH EXPERIMENTAL CONDITION

<table>
<thead>
<tr>
<th>Students</th>
<th>Condition A Baseline</th>
<th></th>
<th>Condition B 50% more problems</th>
<th></th>
<th>Condition C 100% more problems</th>
<th></th>
<th>Condition A Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>RC</td>
<td>RI</td>
<td>#</td>
<td>%</td>
<td>RC</td>
<td>RI</td>
</tr>
<tr>
<td>S1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>** H</td>
<td>86</td>
<td>2.84</td>
<td>.26</td>
<td>8</td>
<td>97</td>
<td>3.86</td>
<td>.11</td>
</tr>
<tr>
<td>C</td>
<td>47</td>
<td>1.42</td>
<td>1.52</td>
<td>6</td>
<td>78</td>
<td>1.89</td>
<td>.58</td>
</tr>
<tr>
<td>S2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>** H</td>
<td>88</td>
<td>2.08</td>
<td>.21</td>
<td>9</td>
<td>76</td>
<td>2.03</td>
<td>.83</td>
</tr>
<tr>
<td>C</td>
<td>63</td>
<td>1.68</td>
<td>1.04</td>
<td>5</td>
<td>54</td>
<td>1.50</td>
<td>1.27</td>
</tr>
<tr>
<td>S3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>99</td>
<td>6.02</td>
<td>.06</td>
<td>11</td>
<td>100</td>
<td>7.08</td>
<td>.03</td>
</tr>
<tr>
<td>C</td>
<td>96</td>
<td>4.66</td>
<td>.2</td>
<td>3</td>
<td>87</td>
<td>3.58</td>
<td>.61</td>
</tr>
<tr>
<td>S4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>98</td>
<td>5.19</td>
<td>.08</td>
<td>7</td>
<td>99</td>
<td>7.96</td>
<td>.07</td>
</tr>
<tr>
<td>C</td>
<td>97</td>
<td>4.60</td>
<td>.10</td>
<td>2</td>
<td>85</td>
<td>4.17</td>
<td>.78</td>
</tr>
<tr>
<td>S5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>98</td>
<td>5.59</td>
<td>.01</td>
<td>8</td>
<td>99</td>
<td>6.62</td>
<td>.10</td>
</tr>
<tr>
<td>C</td>
<td>91</td>
<td>4.39</td>
<td>.45</td>
<td>2</td>
<td>94</td>
<td>3.75</td>
<td>.25</td>
</tr>
<tr>
<td>S6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>99</td>
<td>5.75</td>
<td>.05</td>
<td>9</td>
<td>100</td>
<td>7.34</td>
<td>.03</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>4.19</td>
<td>0.0</td>
<td>2</td>
<td>91</td>
<td>4.11</td>
<td>.36</td>
</tr>
<tr>
<td>S7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>97</td>
<td>5.58</td>
<td>.18</td>
<td>6</td>
<td>98</td>
<td>6.15</td>
<td>.16</td>
</tr>
<tr>
<td>C</td>
<td>81</td>
<td>2.95</td>
<td>.68</td>
<td>2</td>
<td>90</td>
<td>3.94</td>
<td>.34</td>
</tr>
<tr>
<td>S8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>90</td>
<td>5.10</td>
<td>.5</td>
<td>9</td>
<td>96</td>
<td>6.68</td>
<td>.29</td>
</tr>
<tr>
<td>C</td>
<td>81</td>
<td>2.75</td>
<td>.48</td>
<td>3</td>
<td>78</td>
<td>2.68</td>
<td>.75</td>
</tr>
<tr>
<td>TOTAL (MEAN) H</td>
<td>94</td>
<td>5.10</td>
<td>.17</td>
<td>67</td>
<td>96.597</td>
<td>.20</td>
<td>19</td>
</tr>
<tr>
<td>C</td>
<td>82</td>
<td>2.75</td>
<td>.56</td>
<td>25</td>
<td>82</td>
<td>3.20</td>
<td>.61</td>
</tr>
</tbody>
</table>

H refers to Hutchings' algorithm  
C refers to conventional algorithm  
** students who didn't know math facts  
% indicates mean session percent accuracy  
RC refers to mean session rate of columns correct per minute  
RI indicates mean session rate of columns incorrect per minute  
# indicates the number of sessions in each condition during which the algorithm was performed
columns correct; and (c) lower mean session rate of columns incorrect, than the conventional algorithm. The overall trend is supported by individual student data, with the exception of the rate of columns incorrect for students 6 and 8 in Condition A, students 3, 6, and 7 in Condition C, and Student 5 in Condition A (return to baseline). Although Hutchings' algorithm was not always preferred by students when the number of problems was increased in Condition B and C, it remained the more accurate and resulted in a greater number of columns correct.

Table 3 at the bottom and Figure 1b show a general trend that the total mean rate of columns correct for the Hutchings' algorithm gradually increased. Overall student trends basically followed this pattern. Two exceptions were Student 4 whose rate peaked in Condition B and Student 8 whose rate peaked in Condition C.

Student 2 established no preference in Condition B or C, and Students 4, 5, and 6 established no preference in Condition C. In Condition C it appears that the 100% increase in number of problems was too great to continue to do the preferred algorithm.

Summary

Algorithm preference.

All students had a preference for the Hutchings' algorithm during the initial baseline condition. In the
Figure 1a: Total student mean session percent of columns correct in each experimental condition, for the Hutchings' algorithm and the conventional algorithm.

Figure 1b: Total student mean session rate of columns correct and incorrect, in each experimental condition, for the Hutchings' algorithm and the conventional algorithm.
final baseline condition, all students preferred the Hutchings' algorithm over the conventional algorithm. Seven students continued to show a preference for the Hutchings' algorithm over the conventional algorithm when it involved 50% more problems but four of those students had no preference when it required 100% more problems. Three students preferred the Hutchings' algorithm when it involved a 100% increase in the number of problems. One student always preferred the algorithm that required the lesser number of problems.

**Algorithm accuracy and efficiency**

During all conditions, with the minor exception of student 6 in Baseline A, each performed more accurately and efficiently with the Hutchings' algorithm than with the conventional algorithm. That is, the Hutchings' algorithm showed a higher accuracy and a greater mean session rate of columns correct. For most students that also meant a lower mean session rate of columns incorrect, except student 8 in Baseline A, student 2 in Condition C and student 5 in return to A. There was a gradually increasing trend of greater mean session rates of columns correct for the Hutchings' algorithm over the experimental conditions. All students, during all conditions chose as their preferred algorithm, the one that had the greater accuracy and efficiency.
EXPERIMENT 2

Method

Subjects

Six students were selected for this experiment from the afternoon third grade group which is made up of 31 students in this one section. This is a large class by the standards set up for class size in this modern public elementary school. Both the morning and afternoon sections were this large because of students moving into the district. This school's population and most of the district is white. This is a typical Iowa school district in that it has a mixture of suburban and rural housing. This study was run during the spring of the 1979 - 1980 school year.

Alessi (1974) developed a five minute basic addition math facts test, consisting of 56 basic addition facts, which was given to all 31 students in the class. Six students, three white males and three white females, were chosen for the study based on the same criteria as in Experiment 1. Four high accuracy (96% or better), and two low accuracy (80% or less) were chosen. The four high accuracy students had 100% and the two low accuracy students had 80% (Student 1) and 76% (Student 2). Written permission was secured for
the six students and it was explained to them that they could withdraw at any time.

Setting

The setting for this experiment was exactly the same as in experiment 1.

Experimental Task

The students in Experiment 2 received the same problems as in Experiment 1. No student was ever given the same problem twice. All students completed four problems during Condition A.

Design

A reversal design (Baer, Wolf, and Risley, 1968) of the pattern A-B-C-A was used for all students.

Independent Variables

The criteria and the independent variables were the same for Experiment 2 as in Experiment 1. One additional independent variable was introduced in Experiment 2 from Gillespie (1976).

Reinforcement versus nonreinforcement which was contingent upon the percent accuracy. This reinforcement was given during the conditions of increased response effort, Conditions B and C. The criteria were as follows: A self inking stamp was used that had a happy face. The happy
faces were worth one point. These points could in turn be exchanged for time in the regular classroom in preferred activities. The criteria for earning these happy faces were as follows:

<table>
<thead>
<tr>
<th>Points</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>100%</td>
</tr>
<tr>
<td>4</td>
<td>95-99%</td>
</tr>
<tr>
<td>3</td>
<td>90-94%</td>
</tr>
<tr>
<td>2</td>
<td>85-89%</td>
</tr>
<tr>
<td>1</td>
<td>80-84%</td>
</tr>
<tr>
<td>0</td>
<td>below 80%</td>
</tr>
</tbody>
</table>

Dependent Variables

The dependent variables and the criteria for the dependent variables for Experiment 2 are the same as in Experiment 1.

Procedure

Pretraining and Prebaseline Condition

These are both the same as in Experiment 1, Experiment 2 is an attempt to replicate Gillespie (1976) and therefore, all procedures are as close as possible to that study. Two 10 minute sessions were held daily following the last review session. They were at 11 - 11:10 a.m. and 2 - 2:10 p.m., daily. The prebaseline condition followed the same procedure and format that had been followed in Experiment 1.
**Condition A: Baseline**

The format for the baseline was the same for both experiments. This condition ended when all students had established a preference for one algorithm. All students ended Condition A, Baseline at the same time, but some students established a preference for one algorithm sooner than others.

**Condition B**

The preferred algorithm, as established during baseline now required a 50% greater number of problems than in Condition A. The students had to complete six problems now if they chose their preferred algorithm. As in Gillespie (1976) the token reinforcement program was introduced during this condition. The happy face stamp was explained and the number of stamps would depend on the accuracy of their answers, in other words how many columns they got correct. It was explained that each stamp was worth one point and the means of exchanging them for preferred activities in the regular classroom. The students had made up prior to this, a list of preferred activities in the classroom and points had been assigned to them by the experimenter.

Each student was given a sheet of paper of his/her favorite color to keep the happy face stamps on. The criteria for earning points and the procedure was gone over with the students so they all understood. During each session,
after they had checked their papers against the answer sheet, they could receive their happy face stamps. The experimenter would stamp the number of faces on the paper as set up in the criteria. If the students had made any errors in checking their answers, the number of points was adjusted in the next session.

This system of reinforcement was continued throughout Condition B. Condition B ended for all students when each student had established a preference for either algorithm, or when it was clear that there was no algorithm preference.

**Condition C**

The reinforcement system was continued during this condition. This condition was the same as in Experiment 1, namely that choosing the preferred algorithm resulted in 100% increase in number of problems. Students ended this condition during the same session when all students had established a preference for one algorithm or when there was a clear trend of no algorithm preference.

**Condition A**

The return to baseline was the same procedure as in Experiment 1. The questionnaire was filled out by all students after the last session. Condition A ended for all students at the same time but some students established a preference sooner than others. This condition ended when
all students had established a preference, for either algorithm.

**Recording and Scoring**

The procedures here were the same as in Experiment 1.

**Reliability**

The procedures here were the same as in Experiment 1. Reliability data was taken both on checking the time to complete the problems with the stopwatch and scoring of the written record of the problems.
EXPERIMENT 2

Results

Reliability

The reliability for the timing of the problems was taken for 10 of the 51 sessions and yielded a mean 95% agreement. Papers were checked for 10 of the 51 sessions for each student and yielded a mean of 99% agreement.

Algorithm Preference

Behavioral Preferences

Table 4 indicates student algorithm preferences during all conditions. In both baseline conditions all students preferred the Hutchings' algorithm. During Condition B, 50% more problems, five students continued that preference while one showed no preference for either algorithm. In Condition C, 100% more problems, two of the six continued to prefer the Hutchings' algorithm and the other four had no preference.

The bottom of Table 4 summarizes with percentages the preferences of the students, during each condition and across all conditions. Across all conditions, 87.5% of the students preferred the Hutchings' algorithm. No student ever showed a preference for the conventional algorithm and 12.5% had no preference demonstrated.

38
TABLE 4

Individual and Total Student Algorithm Preferences Within and Across All Experimental Conditions

<table>
<thead>
<tr>
<th>Experimental Conditions</th>
<th>Students</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>* 1</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Hutchings</td>
</tr>
<tr>
<td></td>
<td>* 2</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Hutchings</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Totals</th>
<th>Across Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutchings 100%</td>
<td>83%</td>
</tr>
<tr>
<td>Conventional 0%</td>
<td>0%</td>
</tr>
<tr>
<td>No Preference 0%</td>
<td>17%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hutchings, No Preference</th>
<th>33%</th>
<th>0%</th>
<th>67%</th>
<th>12.5%*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutchings, No Preference</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Hutchings, No Preference</td>
<td>17%</td>
<td>67%</td>
<td>0%</td>
<td>12.5%*</td>
</tr>
</tbody>
</table>
A detailed analysis of individual student preferences is presented in Table 5. In this table the individual and total student algorithm choices are presented during free choice and forced choice sessions in each experimental condition. The number of sessions and the corresponding percentages are recorded. For all students who had high accuracy with their math facts, they chose to perform the Hutchings' (preferred) algorithm during 100% of the free choice sessions, in the condition where a preference for one algorithm was established. The low accuracy students took longer to establish an algorithm preference.

At the bottom of Table 5 it shows that for all students the conventional algorithm was performed no more than 33.5% and the Hutchings' algorithm no less than 61.5% of the free choice sessions, in any condition. No more than 14% of total algorithm choices were students forced to use the Hutchings' algorithm. However, during three conditions 94% or more of the students were forced to use the conventional algorithm, in other words forced choice sessions.

**Verbal Preferences**

The results of the questionnaire indicated that all students liked the Hutchings' algorithm better both at the beginning and at the end of the study, which matched the behavioral preferences in this study. The students fur-
TABLE 5: INDIVIDUAL AND TOTAL STUDENT ALGORITHM CHOICES DURING FREE AND FORCED CHOICE SESSIONS IN EACH CONDITION. EXPERIMENT 2

<table>
<thead>
<tr>
<th>Students</th>
<th>Choice</th>
<th>Condition A</th>
<th>Condition B</th>
<th>Condition C</th>
<th>Condition A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Forced Free</td>
<td>Forced Free</td>
<td>Forced Free</td>
<td>Forced Free</td>
</tr>
<tr>
<td>S1</td>
<td>H</td>
<td>1/2 50 7/12 58 0/3 0 9/9 100 0/3 0 9/9 100 0/3 0 10/10 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1/2 50 5/12 42 3/3 100 0/9 0 3/3 100 0/9 0 3/3 100 0/10 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>H</td>
<td>0/3 0 10/11 91 1/2 50 5/10 50 *0/0 0 6/12 50 0/2 0 10/11 91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>3/3 100 1/11 9 1/2 50 5/10 50 0/0 0 6/12 50 2/2 100 1/11 9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>H</td>
<td>0/3 0 11/11 100 0/3 0 9/9 100 *0/0 0 6/12 50 0/2 0 9/9 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>3/3 100 0/11 0 3/3 100 0/9 0 0/1 0 8/11 73 3/3 100 0/10 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>H</td>
<td>0/3 0 11/11 100 0/3 0 9/9 100 *1/1 100 3/11 27 0/3 0 10/10 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>3/3 100 0/11 0 3/3 100 0/9 0 0/1 0 8/11 73 3/3 100 0/10 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>H</td>
<td>0/3 0 11/11 100 0/3 0 9/9 100 0/3 0 9/9 100 0/3 0 10/10 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>3/3 100 0/11 0 3/3 100 0/9 0 3/3 100 0/9 0 3/3 100 0/10 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>H</td>
<td>0/3 0 11/11 100 0/3 0 9/9 100 *0/0 0 7/12 58 0/2 0 7/7 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>3/3 100 0/11 0 3/3 100 0/9 0 0/0 0 5/12 42 2/2 100 0/7 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>H</td>
<td>1/17 6 61/67 91 1/17 6 50/55 91 1/7 14 40/65 61 0/15 0 56/57 98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>16/17 94 6/67 9 16/17 94 5/55 9 6/7 86 25/65 39 15 15 100 1/57 2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* No preference for either algorithm established
H refers to Hutchings' algorithm
C refers to conventional algorithm
they stated that they got more problems correct and finished the problems more quickly using the new (Hutchings') method. On behalf of the students this was an accurate assessment of their performance. They stated that they made more errors using the conventional (old) algorithm which the data supports.

The questionnaire results were further verified informally by the students various statements in regard to the "new" method of doing math problems. They did not mind doing the math problems the "new" way but they were not however, for the most part willing to do twice as many (Condition C). Student 6 quite strenuously objected to the conventional method and stated that she would not try to do the conventional way as fast as she could, and may not have. However, she is a very consistent and motivated student. Her data was not consistent with her verbal threat, that she wouldn't try. This student is also interesting in that she had developed some method of grouping and that when she was done with a sheet, doing it the conventional way, the margins of the paper are covered with numbers. A great deal of effort is going into the response.

Algorithm Accuracy and Efficiency

Table 6 shows individual and total mean session performance with the Hutchings' and the conventional algorithms during each experimental condition. The total student
<table>
<thead>
<tr>
<th>Students</th>
<th>Condition A Baseline</th>
<th>50% more problems</th>
<th>Condition B Baseline</th>
<th>100% more problems</th>
<th>Condition C Baseline</th>
<th>100% more problems</th>
<th>Condition A Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>RC</td>
<td>RI</td>
<td>#</td>
<td>%</td>
<td>RC</td>
<td>RI</td>
</tr>
<tr>
<td><strong>S_1</strong></td>
<td>H 96</td>
<td>2.18</td>
<td>.10</td>
<td>8</td>
<td>98</td>
<td>3.34</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td>C 71</td>
<td>1.96</td>
<td>.83</td>
<td>6</td>
<td>90</td>
<td>2.13</td>
<td>.20</td>
</tr>
<tr>
<td><strong>S_2</strong></td>
<td>H 91</td>
<td>2.20</td>
<td>.20</td>
<td>10</td>
<td>96</td>
<td>3.63</td>
<td>.24</td>
</tr>
<tr>
<td></td>
<td>C 68</td>
<td>1.84</td>
<td>.86</td>
<td>4</td>
<td>89</td>
<td>2.41</td>
<td>.31</td>
</tr>
<tr>
<td><strong>S_3</strong></td>
<td>H 98</td>
<td>3.80</td>
<td>.06</td>
<td>11</td>
<td>97</td>
<td>4.41</td>
<td>.17</td>
</tr>
<tr>
<td></td>
<td>C 100</td>
<td>3.67</td>
<td>0.3</td>
<td>3</td>
<td>94</td>
<td>3.62</td>
<td>.22</td>
</tr>
<tr>
<td><strong>S_4</strong></td>
<td>H 99</td>
<td>4.88</td>
<td>.03</td>
<td>11</td>
<td>100</td>
<td>6.60</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C 90</td>
<td>2.90</td>
<td>.32</td>
<td>3</td>
<td>90</td>
<td>3.91</td>
<td>.45</td>
</tr>
<tr>
<td><strong>S_5</strong></td>
<td>H 98</td>
<td>5.77</td>
<td>.09</td>
<td>11</td>
<td>99</td>
<td>6.56</td>
<td>.13</td>
</tr>
<tr>
<td></td>
<td>C 94</td>
<td>4.57</td>
<td>.28</td>
<td>3</td>
<td>92</td>
<td>3.44</td>
<td>.41</td>
</tr>
<tr>
<td><strong>S_6</strong></td>
<td>H 98</td>
<td>4.37</td>
<td>.09</td>
<td>11</td>
<td>100</td>
<td>7.12</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>C 94</td>
<td>3.88</td>
<td>.26</td>
<td>3</td>
<td>98</td>
<td>5.65</td>
<td>.11</td>
</tr>
<tr>
<td>Total (Mean)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H 97</td>
<td>3.87</td>
<td>.09</td>
<td>62</td>
<td>98</td>
<td>5.27</td>
<td>.11</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>C 86</td>
<td>3.14</td>
<td>.43</td>
<td>22</td>
<td>92</td>
<td>3.53</td>
<td>.28</td>
</tr>
</tbody>
</table>

H refers to Hutchings' algorithm  
C refers to Conventional algorithm  
** students who didn't know math facts  
% indicates mean session percent accuracy  
RI indicates mean session rate of columns  
# indicates the number of sessions in each condition during which the algorithm was performed.
data is presented in Figures 2a and 2b which show that during all conditions, the Hutchings' algorithm had: (a) higher percentage of accuracy; (b) higher mean session rate of columns correct; and (c) lower mean session rate of columns incorrect, than the conventional algorithm. The overall trend is supported by individual student data, with the exception of some students in the rate of columns incorrect. These exceptions were in Condition A (baseline) for Student 3 and Condition C for students 2, 4, and 5. Student 6 had 100% accuracy for both algorithms in the final baseline condition. The Hutchings algorithm was not always preferred when the number of problems was increased but it remained the more accurate algorithm and resulted in a higher rate of columns correct, in other words it was faster for the students to do per column.

During Conditions B and C (the reward system was used during those conditions) the percent accuracy did not change appreciably, with Hutchings algorithm and increased slightly with the conventional algorithm. However, the increase was maintained in the final Condition A, return to baseline.

Table 6 at the bottom and Figure 2b shows a general increasing trend in the rate of columns correct both with the conventional and the Hutchings' algorithm but the increase with the Hutchings' is greater. Overall the individual student trends support this with two exceptions. Student 3's rate of columns correct drops in Condition C.
Figure 2a: Total student mean session percent of columns correct, in each experimental condition, for the Hutchings' and conventional algorithm. Data from experiment 2.

Figure 2b: Total student mean session rate of columns correct and incorrect, in each experimental condition, for the Hutchings' and conventional algorithm. Data from experiment 2.
and Student 5 stays about the same in Condition B and C.

Table 6 indicates that during the initial baseline and Condition B (50% more problems) with one exception the students chose the algorithm that resulted in higher accuracy, higher rate correct (faster), and lower rate of columns incorrect. The one exception was in Condition B by Student 2. During return to baseline Condition A all students preferred their more accurate and efficient algorithm with the exception that Student 6 had 100% accuracy with both algorithms. During condition C (100% more problems), even under reinforcement conditions, the students switched to their less accurate and less efficient algorithm.
EXPERIMENTS 1 AND 2

Summary

Format

The format of Experiments 1 and 2 were a replication of Gillespie (1976) with the exception of one change that she suggested, that all students receive the same number of problems. Experiment 1 and 2 were the same in all aspects, except that in Experiment 2 the students received reinforcement for percent accuracy in Conditions B and C.

Behavioral Algorithm Preference

In reviewing the data there are essentially no differences in algorithm preferences between the reinforcement and nonreinforcement groups during the baseline conditions. During all baseline conditions the students preferred the Hutchings' algorithm. During Condition B in each experiment, one student from each experiment had no algorithm preference.

In Condition C all students performed more accurately and efficiently with the Hutchings' algorithm and that was even though it required a 100% increase in the number of problems over the conventional algorithm. Even though it was more accurate and efficient all students were not willing to put forth the effort for a 100% increase in number of problems. In Condition C 37.5% in Experiment 1 and 33% in
Experiment 2 had a preference for the Hutchings' algorithm. No student in either experiment established a preference for the least accurate and efficient algorithm during any condition. The reinforcement given for percent accuracy during Condition C of Experiment 2, did not appear to effect the students' preferences for the algorithm that was more accurate.

Students from both experiments who did not consistently know their basic addition math facts, had a lower rate correct (slower) and had a lower accuracy or percentage correct. But no essential difference was found between the students who had high accuracy and those who had low accuracy on the math facts test, in behavioral preference in regards to which algorithm was chosen.

**Verbal Algorithm Preference**

All of the 14 students in both experiments had accurate verbal assessments of their behavioral algorithm preferences. Therefore there was no difference from the two experiments on this behavior.

**Algorithm Accuracy and Efficiency**

When a preference for one algorithm was established the students chose to perform their more accurate and efficient algorithm. For student 6 in Experiment 2 she had 100% accuracy in Condition C with both algorithms but was
more efficient with Hutchings' which was her preference. The only exception was in Experiment 1, Student 6, in baseline Condition A who had 99% accuracy with Hutchings and 100% accuracy with the Conventional. Students overall chose to perform the Hutchings algorithm 81% of the time in Experiment 1 and 87.5% of the time in Experiment 2. For both experiments, the Hutchings' algorithm overall was more efficient and accurate than the conventional algorithm.

Figure 3b shows that in Experiment 1 the students had a higher mean session rate of columns correct with the Hutchings' algorithm in conditions baseline A and B. Students from Experiment 2 were higher in Conditions C and return to baseline A. Mean session rate of columns correct with the Hutchings' algorithm had a larger increase from baseline A to return to baseline A for Experiment 2 than for Experiment 1. The increase was as follows: 2.76 columns per minute for Experiment 2 and 1.21 columns per minute for Experiment 1. Figures 1b and 2b show for Experiment 2 a lower mean session rate of columns incorrect with the Hutchings' and conventional algorithm, in all conditions than in Experiment 1.

Figure 3b shows that the Hutchings' algorithm was faster (rate correct) in all conditions in both experiments. Figure 3a shows that accuracy was higher for the Hutchings in all conditions with one exception (Condition C, Experiment 1).
Figure 3a: Total student mean session percentage accuracy in Hutchings’ and the conventional algorithm, in each condition, with data from Experiment 1 and 2.

Figure 3b: Total student mean session rate correct, in each experimental condition, for the Hutchings’ and the conventional algorithm. Data from experiment 1 and 2.
DISCUSSION

The results of the present study indicate that most third graders when given a free choice choose to compute multiplace addition problems using the Hutchings' "low stress" algorithm, rather than the conventional algorithm. Not only do they prefer the Hutchings' algorithm but it is also the more accurate and efficient of the two algorithms. Algorithm preference has been addressed in previous research. Alessi (1970) and Hutchings' (1972) suggest: 1) there may be less stress involved in using the Hutchings' algorithm, for the student when confronted with large addition problems; and 2) that there is less demand on sequential memory skills in the Hutchings' algorithm than in the conventional algorithm. Gillespie (1976) further suggests 3) that students may prefer the Hutchings' algorithm because it was their more accurate and efficient method. The findings in this study further confirm previous research (Hutchings, 1972; Gordon, 1972; Alessi, 1974; Dashiell, 1974; Boyle, 1975; Rudolph, 1976; Gillespie, 1976; and E. Drew, 1981) that showed the Hutchings' algorithm is a more accurate addition calculation method that has superior efficiency.

Gillespie (1976) cautions that with any new procedure there may be novelty effects, in other words the students prefer a new method simply because it is new. Alessi (1974) states: "This possibility for distortion is accepted, but
felt to be minimal in view of research controlled for such novelty effects in relation to the conventional and Hutchings' algorithms" (p. 46). Keep in mind also that novelty is a two-edged sword: if the pupils do better, it's because of "novelty" (interested in it); and if the pupils do worse it is because of "novelty" (not used to it). Therefore, novelty may not be a viable explanation until research data demonstrate such an effect as well as its direction.

The data from this study shows that most students (93%) preferred the Hutchings' algorithm even when choosing that algorithm resulted in 50% more problems to complete. Why? It may be that the Hutchings' algorithm reduced the response effort for the students by lowering the sequential memory load and lowering the stress involved in large addition problems. It is also possible that the students preferred to do the addition problems with their more accurate and efficient method. Another possibility is that since the students received feedback by checking their papers this in turn may have affected their choices. The students overall were more accurate with Hutchings' algorithm and they may have preferred to be accurate. Therefore the feedback may have differentially reinforced students for selecting the low stress algorithm. Or it could be that the students did not perceive the 50% added problems as being much different from baseline and did not mind doing the additional problems and didn't mind the additional time.
that it took to complete them. It is this author's opinion that all those factors played a roll but that from the observation of the students' behavior, that lowering the amount of stress with the Hutchings' "low stress" algorithm was the most obvious factor. The students when they were required to do addition problems by the conventional algorithm showed overt observable behaviors associated with anxiety or stress.

When the number of problems was increased to 100% the students may have still preferred the Hutchings' algorithm but not enough to do twice as many problems. There were 36% of the students who still preferred to use the Hutchings' algorithm. Some students wanted to finish quickly and verbally expressed that they had finished before other students who chose to do the problems with the Hutchings' algorithm. However, in this condition the Hutchings' algorithm continued to be the more accurate and efficient method. The point in this study that caused a shift in performance from the preferred to the nonpreferred algorithm is the same in this study as in Gillespie (1976). A shift was observed in 7% of the students at the 50% increase in response effort and an additional 57% of the students shifted in the 100% more problems. It may be that the 100% increase in the number of problems outweighed, part of the time, their consistently choosing their more preferred algorithm.
It was pointed out previously by Gillespie (1976) that the requirements for defining preference in this instance are quite strenuous. But on reviewing individual student data it would not have made a significant difference if the way preference were defined were changed in this study. Gillespie (1976) felt that in hers it would have made a difference in what the preferences would be. In this study when the number of problems became too many the students appeared to randomly make choices and to be influenced by what a friend was doing. They became less concerned overtly with the accuracy of the task and more interested in the time the task took. For two of the low accuracy students the task was just too demanding for them because they did not know their math facts. Since there was no equal mastery criteria in training the "low stress" and the conventional algorithms, that may have effected the results.

This study showed that reinforcement for percent accuracy did not alter the point where a shift from the preferred algorithm occurred. Gillespie (1976) suggests the following reason: "the reinforcement for percent accuracy may not have been effective enough to over come the 100% increased response effort required for performing the preferred algorithm". This seems to be the case in this study as well.

Based on the findings of this study, students prefer to perform the algorithm that results in the greatest
accuracy and efficiency. It can be see that wanting to perform more accurately and efficiently may dissolve when greater response effort is required. which in this case was having to complete more problems. This could have educational significance if a student would choose to use a non-preferred method that was not accurate or efficient. Educators should note that students need enough reinforcement to perform high quality work which is essential to develop the most effective learning environment.

Educators could take note from these results and ask themselves why shouldn't students be permitted to choose the most efficient method for them. Some of the students were concerned that their teacher next year may not allow them to use their newly learned skill because it struck the students as so easy that their teacher would not want them to use it! One question should be noted that was posed by Lockhard et. al. (1973) which is: does a student prefer a condition because he/she does well in it, or must a student like a given condition to do well? The first alternative may be what has taken place in this study. More research on the relationships between preference and performance would be useful in educational settings.

Although verbal algorithm preferences were an accurate measure of behavioral preferences in this study, one
would caution that this does not imply that they are as good as behavioral preferences. Verbal preferences are certainly reinforcing to the experimenter and are another means of confirming behavioral preferences. Behavioral preferences themselves are a reinforcer for the choice response and would reflect continuing behavior (Lockhart et.al. 1973) than just a verbal statement in the conclusion of the study.

As any study this one should be interpreted with caution. The student data are variable and the study was conducted outside the regular classroom. The number of students involved is a small sample and that sample is of a white suburban-rural population in Iowa. The results of this study may be generalized to other third grade students of similar socioeconomic, cultural and achievement backgrounds, and taught in a small group setting. This author also had some concerns about the students who did not know their math facts and it is felt that in any future study that it might be better to have as a minimum criteria knowledge of math facts as in Zoref (1976). Equal mastery criteria in training both algorithms could be closer if all students had the minimal starting criteria of knowing their math facts. The students could then be divided into high accuracy and low accuracy groups on the basis of an instrument to test abilities in doing multiple-place column addition. It is felt that consistent accuracy
with basic addition facts and regrouping are essential for proper use of the Hutchings' or any other algorithm.

If success of a technique depends upon student preferences, then the Hutchings' algorithm can be recommended to educators. But any changes in the math curriculum should be done with caution and after careful research to explore all variables. Future research could also apply the Findley minimax procedure to other educational environments and curriculums.
APPENDIX A

HUTCHINGS' ADDITION ALGORITHM LESSON

(Adopted from Hutchings, 1972)
Hutchings' Addition Algorithm, Lesson

I am going to show you the usual way of writing number facts and then another way of writing them.

You have all seen number facts written like this:

\[
\begin{array}{c}
7 \\
+8 \\
\hline
15
\end{array}
\]

Well, they can also be written like this, using two small (half-space) numbers instead of the line and plus sign.

\[
\begin{array}{c}
7 \\
8 \_5
\end{array}
\]

Do you still see the fifteen? (Point to both fifteens.)

I'll write the two examples next to one another.

Do you all see the fifteen? (Point \(7 \_5\))

\[
\begin{array}{c}
7 \\
+8 \\
\hline
15
\end{array}
\]

Let's look at another one. I can write "9 plus 5 is 14" like this

\[
\begin{array}{c}
9 \\
+5 \\
\hline
14
\end{array}
\]

or like this

\[
\begin{array}{c}
9 \\
5 \_5
\end{array}
\]

Both of these say "9 plus 5 is 14."

Tell me what these say:

\[
\begin{array}{cccccccccc}
9 & 9 & 6 & 6 & 4 & 4 & 6 & 6 & 5 & 5 \\
+8 & 8 & 7 & 7 & 5 & 5 & 6 & 6 & 2 & 2 \\
\hline 17 & 13 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 \\
\end{array}
\]

(Call on students, point to the full notation form when asking.)

The little number on the right* is understood to be in the one's place, as are 9 and 8.

The little number on the left* is understood to be in the ten's place.

In other words, this is the same as this (point from "big 7" to "little 7"). And this is the same as this (point from "big one" to "little one").
Now watch me write the following facts both ways.

\[
\begin{array}{cccccc}
9 & 9 & 8 & 8 & 4 & 4 \\
+7 & +5 & +5 & +5 & 5 & 5 \\
\hline
16 & 13 & 13 & 9 & 9 & 9 \\
\end{array}
\]

Look at the last pair. Are they different from the others? Note that there is no ten's place number and (do not draw \(\square\) until after saying this) there is no "little one" on the left.

Let's look at another.

a) \(4\) Is there any ten's number here? (Do not draw a box until after asking question.)

\(\square\)

b) NO!! (repeat)

c) So will there be any little number on the left?

\(\square\)

d) \(4\)

\(\square\)

(Do not draw box until after asking question.)

NO!! (repeat)

Again,

\[
\begin{array}{cc}
4 & 4 \\
+3 & 3 \\
\hline
7 & 7 \\
\end{array}
\]

If there is no ten's place number there is no "little number" on the left.

Now watch me write the rest of these.

Notice

\[
\begin{array}{c}
3 \\
+1 \\
\hline
4 \\
\end{array}
\quad
\begin{array}{c}
3 \\
1 \\
\end{array}
\]

no ten's number here  so no "little number" here

but

\[
\begin{array}{c}
7 \\
+8 \\
\hline
15 \\
\end{array}
\quad
\begin{array}{c}
7 \\
8 \\
\end{array}
\]

There is a ten's number here  so there is a "little number" here
Again, notice

\[
\begin{array}{c}
5 \\
+ 1 \\
\hline
6
\end{array}
\]  \hspace{2cm} \begin{array}{c}
5 \\
+ 1 \\
\hline
6
\end{array}
\]

There is no ten's number here

so there is no "little number" here

but

\[
\begin{array}{c}
8 \\
+ 5 \\
\hline
13
\end{array}
\]  \hspace{2cm} \begin{array}{c}
8 \\
+ 5 \\
\hline
13
\end{array}
\]

There is a ten's number here

so there is a "little number" here

\[
\begin{array}{cccc}
5 & 5 & 6 & 6 \\
+ 5 & + 8 & 8 & + 7 \\
\hline
10 & 14 & 8 & \frac{23}{8}
\end{array}
\]

\[
\begin{array}{cccc}
\frac{3}{4} & \frac{5}{4} & \frac{5}{4} & \frac{4}{4} \\
+ \frac{1}{4} & + \frac{5}{9} & + \frac{5}{14} & + \frac{8}{12}
\hline
\frac{9}{4} & \frac{10}{4} & \frac{11}{4} & \frac{12}{4}
\end{array}
\]

Now I am going to show you a special way of adding that uses only those "little numbers" on the right.

I'll say that again (repeat previous statement).

This should make your addition very easy and accurate. It is a scientific method and many scientists do addition this way. Watch.

8 First, do you see that an example can be just number facts piled on atop the other? (Do not point with this question.)

8 OK! Here we go, starting at the top, writing facts as you learned and using only numbers at the right for addition.
a) Say, "The first fact we do may look a bit different because we do not have any little numbers yet." (Point)

b) Say, "This is the only time we will use two big numbers. In the rest of the example we use one little number and one big one."

c) Say, "Now, eight plus five is thirteen."

d) Write the thirteen, i.e., $5\times$ in the example.

a) Say, "we've written the thirteen but we'll use only the three."

b) Draw arrow $7\times$

c) Say, "Three plus seven is ten."

d) Write the 10, i.e., $7\times$ in the example.

a) Say, "We've written the ten but we'll use only the 0."

b) Draw arrow $9\times$

c) Say, "Zero plus nine is nine."

d) Write the 9, i.e., $9\times$ in the example.

a) Say, "We've written the nine and look that's all we have this time because zero and nine is just nine. But that's OK because we only use the right-hand number anyway."

b) Draw arrow $8\times$

c) Say, "Nine plus eight is seventeen."

d) Write the seventeen, i.e., $8\times$ in the example.
a) Say, "We've written the seventeen but we'll use only the seven."

b) Draw arrow \(6\).  

c) Say, "Seven plus six is thirteen."

d) Write the thirteen, i.e., 6 in the example.

---

a) Say, "We've written the thirteen but we'll use only the three."

b) Draw the arrow \(3\).  

c) Say, "Three plus eight is eleven."

d) Write the eleven, i.e., 8, in the example.

---

a) Say, "We've written the eleven but we'll use only the one."

b) Draw arrow \(1\).  

c) Say, "One plus seven is eight."

d) Write the eight, i.e., 8, in the example.

Now we're at the key part. All we've done is use number facts. We haven't done any "in your head" work.

Nevertheless, we already know the answer! Watch.

The last little number on the right is the right half of the answer.

To get the left half, we just count the little numbers on the left that we didn't use. One, two, three, four, five, there are five of them, so the first half of the answer is five. The answer is 58.
Now watch me do another. Remember we use only the right side "little numbers". We will not bother to write the arrows anymore, just say

\[
\begin{align*}
6 & \quad 6 \text{ plus } 8 \text{ is } 14 \\
7 & \quad 4 \text{ plus } 7 \text{ is } 11 \\
6 & \quad 1 \text{ plus } 6 \text{ is } 7 \\
9 & \quad 7 \text{ plus } 9 \text{ is } 16 \\
5 & \quad 6 \text{ plus } 5 \text{ is } 11 \\
8 & \quad 1 \text{ plus } 8 \text{ is } 9 \\
3 & \quad 9 \text{ plus } 3 \text{ is } 12 \\
52 & \quad \text{Now the last number on the right is a } 2, \text{ so the right half of the answer is a } 2! \text{ We get the left half of the answer by counting the little numbers on the left that we didn't use. One, two, three, four, five. There are five of them so the left half of the answer is } 5. \text{ The answer is } 52.
\end{align*}
\]

Now say the work for these with me as I do them at the board. (Children do not copy this.)

\[
\begin{align*}
8 & \quad 9 & \quad 4 \\
5 & \quad 5 & \quad 8 \\
6 & \quad 3 & \quad 3 \\
7 & \quad 2 & \quad 6 \\
2 & \quad 8 & \quad 8 \\
3 & \quad 9 & \quad 2 \\
52 & \quad 49 & \quad 43 \\
\end{align*}
\]

Now copy these examples and do them by yourself. If you have any questions, ask me.

\[
\begin{align*}
6 & \quad 8 & \quad 5 & \quad 9 \\
5 & \quad 2 & \quad 4 & \quad 8 \\
9 & \quad 7 & \quad 9 & \quad 3 \\
8 & \quad 6 & \quad 8 & \quad 2 \\
5 & \quad 9 & \quad 7 & \quad 7 \\
6 & \quad 8 & \quad 9 & \quad 6 \\
\pm 2 & \quad \pm 5 & \quad \pm 8 & \quad \pm 9 \\
\end{align*}
\]
After most have finished, say, "Check your work with mine as I do them at the board."

After doing the examples, say, "Now let's review."

I'll write the work for another one on the board. I want someone to raise his hand and tell me what the answer is.

6 plus 8 is 14
4 plus 9 is 13
3 plus 5 is 8
8 plus 7 is 15
5 plus 5 is 10
0 plus 9 is 9
9 plus 3 is 12

(Point to box.) Who will tell me what the right side of the answer is and how he got it.

(point to box.)

(Locate correct response.) Good! That's correct. The last little number on the right becomes the right side of the answer.

Who will tell me what the left side of the answer is and how he got it. (Locate correct response.) Good! That's correct, we count up the little numbers on the left for the left side of the answer.

Now, what do you suppose we do if there is more than one column? That is, if there is another column at the left of the column you're adding. Like this:

4 6
7 8
6 7
8 6
7 8

Can we still write our left-hand answer number at the
bottom if there is more than one column? No, we can't?

When there's more than one column, each column can have only one number at the bottom (except for the very last column which does have the usual two).

So the single number that we put at the bottom is always the right-hand number.

(Write and point)

What can we do with the left-hand number?

Would it make sense to throw it away? No, it's part of the problem. *So we will put it at the very top of the next column at the left. That way we don't lose it and it's still on the left side.

Watch! (Write on board.)

Count the little number on the left with me.
One, two, three, four. There are four of them so we write a 4 at the top of the next column.

Now, when I start adding that column I will start with the four(4) first. Let's be sure you understand. (Repeat twice from the *)

This is called carrying, some of you already understand it. Good. Carrying is very easy.

But carrying is very important. You must never forget to carry.

Look at these examples and tell me what to write at the top of the left-hand column. (Write on board.)
(Do with volunteers from class at board.) Good, we write the left-hand answer number at the top of the next column. (Repeat three times.)

Remember though that for the last column only, the left-hand answer number is at the bottom as though it were a single column.

Now, copy these examples and do them with me.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>8</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

Again, do you see that I always carry the number of tens to the top of the next column? (Point and illustrate example.) Except when there are no more columns. Then I write the number of tens on the bottom line as part of the answer. (Point and illustrate with each.)

Good! Are there any questions?

Now take these dittoed examples and do them by yourself. If you have trouble, ask me for help.

\[
\begin{array}{ccc}
7 & 6 & 9 \\
8 & 7 & 6 \\
9 & 3 & 5 \\
\end{array}
\]

\[
\begin{array}{ccc}
7 & 6 & 9 \\
8 & 7 & 6 \\
9 & 3 & 5 \\
\end{array}
\]

\[
\begin{array}{ccc}
6 & 8 & 7 \\
4 & 9 & 1 \\
+ 8 & 6 & 3 \\
\end{array}
\]

\[
\begin{array}{ccc}
6 & 8 & 7 \\
4 & 9 & 1 \\
+ 8 & 6 & 3 \\
\end{array}
\]

\[
\begin{array}{ccc}
7 & 6 & 9 \\
8 & 7 & 6 \\
9 & 3 & 5 \\
\end{array}
\]

\[
\begin{array}{ccc}
7 & 6 & 9 \\
8 & 7 & 6 \\
9 & 3 & 5 \\
\end{array}
\]

\[
\begin{array}{ccc}
6 & 8 & 7 \\
4 & 9 & 1 \\
+ 8 & 6 & 3 \\
\end{array}
\]

\[
\begin{array}{ccc}
6 & 8 & 7 \\
4 & 9 & 1 \\
+ 8 & 6 & 3 \\
\end{array}
\]

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Be sure to make and place your numbers neatly!

(Allow time needed for most to finish.)

Now, I will do them. Check your work against mine.

(Do examples on board. Answer questions. Emphasize the need to write neatly and the need to count the "carry number" correctly, demonstrate the latter while doing the work. State that the carry number is always written in at the top of the column to which it is carried.)
APPENDIX B

Review of Hutching's Algorithm
Lesson (Adapted From
Hutching 1972)
We are going to review the new way of writing number facts which we practiced yesterday.

We are going to start at the top, writing number facts as you learned yesterday.

a) Say, "Remember that during the beginning of the example is the only time that we use two big numbers. In the rest of the example, we use one little number and one big number."

b) Say, "Five plus nine is fourteen."

c) Write the fourteen in the example as ,9.

a) Say, "We've written the fourteen but we'll use only the four."

b) Say, "Four plus eight is twelve."

c) Write the twelve in the example as ,8.

a) Say, "We've written the twelve but we'll use only the two."

b) Say, "Two plus six is eight."

c) Write the eight in the example as ,8.
a) Say, "We've written the eight and we use just the eight."

b) Say, "Eight plus eight is sixteen."

c) Write the sixteen in the example as $\frac{8}{8}$. 

---

a) Say, "We've written the sixteen but we'll use only the six."

b) Say, "Six plus seven is thirteen."

c) Write the thirteen in the example as $\frac{7}{3}$.

---

a) Say, "We've written the thirteen but we'll use only the three."

b) Say, "Three plus eight is eleven."

c) Write the eleven in the example as $\frac{1}{8}$.

---

a) Say, "We've written the eleven but we'll use only the one."

b) Say, "One plus seven is eight."

c) Say, "The last little number on the right is the right half of the answer. To find the left half, we just count the little numbers on the left that we did not use. Who can tell me what the right half of the answer is? Eight! Right. Now, who can tell me what the left half of the answer is? Five! Right, the answer, then is 58."

---

a) Say, "Now let's try a bigger example. We are going to move faster this time because you have done so well."
b) Say, "Let's start with the right column (point to it). Seven plus five is twelve. (Write the twelve in the example as \(7_2\).)
   Two plus six is eight. (Write the eight in the example as \(6_2\).) Eight plus five is thirteen. (Write the thirteen in the example as \(\text{\underline{,5}}_2\).) Three plus nine is twelve. (Write the twelve in the example as \(\text{\underline{,5}}_2\).)
   We write the two below the right column and carry the three to the top of the next column." (Write the three above the second column.)

\[
\begin{array}{c}
3 & 3 \\
8 & 7_6 \\
6 & 5_2 \\
4 & 9_5 \\
6 & 9_3 \\
8 & 2_3 \\
7 & 2 \\
\end{array}
\]

Say, "Now, when I start adding this column (point to second column), I will start with the three. Three plus seven is ten. (Write the ten in the example as \(7_0\).)
   Zero plus eight is eight. (Write the eight in the example as \(8_6\).) Eight plus seven is fifteen. (Write the eight in the example as \(\text{\underline{,5}}_5\).)
   Five plus nine is fourteen. (Write the fourteen in the example as \(\text{\underline{,9}}_7\).) Four plus three is seven. (Write the seven in the example as \(3_7\).)
   We write the seven below the column. Then we count the tens: One, two, three tens. We carry the three to the top of the next column." (Write the three above the last column.)

\[
\begin{array}{c}
3 & 3 \\
8 & 7_6 \\
6 & 5_2 \\
4 & 9_5 \\
6 & 9_3 \\
8_5 & 2_7 \\
7 & 2 \\
\end{array}
\]

Say, "Now our example looks like this (pointing to example). Who can tell me the numbers we are going to add next? Right. We are going to add the three and the eight."

\[
\begin{array}{c}
3 & 3 \\
8 & 7_6 \\
6 & 5_2 \\
4 & 9_5 \\
6 & 9_3 \\
8_5 & 2_7 \\
7 & 2 \\
\end{array}
\]

Say, "Three plus eight is eleven. (Write the eleven in the example as \(\text{\underline{,8}}_6\).) Who can tell me the numbers we are going to add next? Right. We are going to add the one and the six. One plus six is seven. (Write the seven in the example as \(6_7\).) Who can tell me the numbers we are going to add next? Right. We are going to add the seven and the four. Seven plus four is eleven. (Write the eleven in the example as \(\text{\underline{,4}}_7\).) Who can tell me the numbers we are going to add next? Right. We are going to add one and six. One plus six is seven. (Write the seven in the example as \(7_7\).)"
example as 6 \_7 \_ ) Seven plus eight is fifteen. (Write the fifteen in the example as 8 \__. ) Now we write the five below the column. (Write the five below the third column.) Then we count the tens: One, two, three tens. Because there are no more columns, we write the three to the left of the five." (Write the three to the left of the five in the example.)

Now copy these examples with me.

\[
\begin{array}{cccccccc}
8 & 7 & 3 & 5 & 2 & 5 & 9 & 3 & 8 \\
5 & 9 & 8 & 9 & 7 & 7 & 6 & 5 & 8 \\
3 & 8 & 2 & 2 & 2 & 2 & 6 & 7 & 8 \\
9 & 6 & 9 & 3 & 8 & 9 & 6 & 7 & 5 \\
7 & 8 & 5 & 9 & 6 & 5 & 8 & 7 & 4 \\
\hline
9 & 5 & 6 & 7 & 4 \\
\hline
5 & 8 & 6
\end{array}
\]

Are there any questions? Good. Now take these dittoed examples and do them yourselves. If you have any trouble, ask me for help. Be sure to make and place your numbers neatly.

(Allow time for most to finish.)

Now I will do them. Check your work against mine.

(Do examples on the blackboard. Answer questions. Emphasize the need to write neatly and the need to count the "carry number" correctly, demonstrate the latter while doing the work. State that the "carry number" is always written at the top of the column to which it is carried.)
APPENDIX C

Conventional Algorithm, Lesson
(Adapted from Hutchings, 1972)
I am going to write some addition examples on the board. Begin to do them as soon as you can see them. After I finish writing all of them, I will go back and write in the answers. After you have finished working all of the examples, go back and check your answers against the answers I have written on the board. As soon as you have finished, turn your papers in.

Does everyone know what to do? (Pause momentarily.) Good. Begin...

\[
\begin{array}{cccccccccc}
7 & 9 & 6 & 4 & 6 & 3 & 9 & 8 & 4 & 3 \\
+8 & +8 & +7 & +5 & +6 & +2 & +7 & +5 & +3 & +1 \\
\hline
5 & 6 & 1 & 3 & 5 & 4 & 3 & 2 & 8 & 5 \\
+5 & +8 & +7 & +1 & +4 & +9 & +8 & +3 & +2 & +8 \\
\hline
6 & 6 & 1 & 8 & 9 & 8 & 3 & 9 & 9 & 9 \\
+9 & +8 & +7 & +8 & +2 & +1 & +4 & +6 & +5 & +3 \\
\hline
8 & 8 & 9 & 4 & 6 & 4 & 6 & 8 & 6 & 5 \\
3 & 5 & 5 & 8 & 8 & 7 & 8 & 7 & 8 & 7 \\
7 & 6 & 3 & 3 & 9 & 6 & 7 & 6 & 7 & 9 \\
9 & 2 & 6 & 5 & 8 & 6 & +9 & 5 & 6 & 4 \\
8 & 8 & 1 & 7 & +7 & 8 & 9 & 6 & 8 & 4 \\
6 & 3 & 7 & 7 & 9 & 3 & 7 & 6 & 8 & 5 \\
\hline
7 & 6 & 8 & 5 & 7 & 6 & 8 & 5 & 7 & 6 \\
5 & 9 & 7 & 6 & 7 & 7 & 1 & 8 & 5 & 9 \\
6 & 2 & 9 & 9 & 9 & 2 & 1 & 2 & 8 & 7 \\
8 & 5 & 4 & 5 & 1 & 8 & 4 & 7 & 6 & 9 \\
7 & 6 & 3 & 8 & 1 & 3 & 9 & 5 & 8 & 3 \\
+4 & 3 & +4 & 4 & +4 & 3 & +6 & 3 & +9 & 5 \\
\hline
7 & 9 & 8 & 4 & 7 & 8 & 5 & 6 & 8 & 7 & 6 & 9 \\
8 & 7 & 7 & 5 & 7 & 6 & 9 & 4 & 8 & 3 & 5 & 8 \\
6 & 8 & 5 & 8 & 7 & 6 & 2 & 6 & 9 & 5 \\
4 & 7 & 6 & 8 & 5 & 7 & 6 & +8 & 7 & 4 \\
6 & 9 & 5 & +8 & 3 & 9 & 5 \\
8 & 3 & 9 & +4 & 2 & 2 \\
\end{array}
\]
APPENDIX D.

Practice Sheet and Sample Worksheet
<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>5</td>
<td>3</td>
<td>9</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>2</td>
<td>7</td>
<td>4</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>NAME</td>
<td>DATE</td>
<td>AM</td>
<td>PM</td>
<td>NEW</td>
<td>OLD</td>
<td>FREE CHOICE</td>
</tr>
<tr>
<td>------</td>
<td>------</td>
<td>----</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
<td>-------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4593</td>
<td>2275</td>
<td>2268</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6584</td>
<td>6894</td>
<td>2392</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3586</td>
<td>2257</td>
<td>7775</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8272</td>
<td>8232</td>
<td>9963</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9573</td>
<td>7663</td>
<td>8973</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6389</td>
<td>7344</td>
<td>9935</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8253</td>
<td>7845</td>
<td>6398</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5343</td>
<td>7436</td>
<td>5244</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5377</td>
<td>5637</td>
<td>4666</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8546</td>
<td>9559</td>
<td>2474</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX E

Addition Math Facts Test

(From Alessi 1974)
<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>6</td>
<td>3</td>
<td>9</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>8</td>
<td>2</td>
<td>7</td>
<td>8</td>
<td>2</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>2</td>
<td>9</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>4</td>
<td>2</td>
<td>8</td>
<td>2</td>
<td>8</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>6</td>
<td>3</td>
<td>8</td>
<td>6</td>
<td>2</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

**SCORE:**
APPENDIX F

Questionaire

(From Gillespie, 1976)
QUESTIONNAIRE

1. Which way, old or new, was easier to do?

2. a) Which way did you like to do better in the beginning?

   b) Which way did you like to do better at the end?

3. Did you get more problems correct when you used the old way or when you used the new way?

4. Which way did you make more mistakes?

5. Using which way did you do the problems more quickly?

* In all the above questions, students were asked to either write old or new on their paper, in response to each question. Students used the term old to refer to the conventional method, and the term new to refer to the Hutchings' algorithm. For questions 6, 7, and 8, the choices for answers were placed on a blackboard. The questions were asked orally by the experimenter. Students were asked to write down on their papers the letters that corresponded to their answer choices. Students were told that they could write down only one letter, or more than one letter.

6. Remember when I told you that if you did the problem one way you would have to do more problems, but if you chose to do the problems another way, you could do fewer problems? One time I told some of you that you could do either two or three problems. I told some of you that you could do either four or six problems. And still others of you could do either six or nine problems. Now when you had a choice of doing these numbers of problems, what helped you decide whether to do more problems and your favorite way, or else fewer problems and your less favorite way? Did you pick:

   A. The way with the easiest problems.

   B. The way where you could get the most problems correct.

   C. The way where you could be done the quickest.
D. One way only because another friend chose that same way.

E. None of the above reasons.

7. Now remember when I told some of you that you had a choice of doing either four or eight problems, or six or twelve problems. Now when you had a choice of doing these numbers of problems, what helped you decide whether to do more problems and your favorite way, or else fewer problems and your less favorite way? Did you pick:

A. The way with the easiest problems.
B. The way where you could get the most problems correct.
C. The way where you could be done the quickest.
D. One way only because another friend chose the same way.
E. None of the above reasons.

8. Over the whole study, in doing the problems, were you more concerned about getting done quickly or more concerned about getting them all correct?

A. done correctly
B. done quickly
C. both
APPENDIX G

Letter to Parents
Dear Parent,

Your son/daughter has been selected to have the opportunity to participate in a research study I am doing for my Educational Specialist Degree. This study is to decide which way of doing column addition is most accurate and preferred by students. Your child will not be identified in any way in the results and all material is destroyed after the study. Your child may stop participation at any time and the data will be destroyed at any point of his/her choosing.

This study will involve your child working in a small group doing extra practice on column addition. Two ways of deriving the answers will be used to see which is the most efficient: the conventional method and the Hutchins' "low stress" method. The conventional method has the student write down one answer for each column added and the Hutchings' method had the student note each two number sums, using small numbers.

For example:  

Conventional  

| 5  | 4  | 7  | 8  | 24 |
--- | --- | --- | --- | ---|
 5  | 4  | 7  | 8  | ![](24)

Hutchings'  

| 5  | 4  | 7  | 8  | 24 |
--- | --- | --- | --- | ---|
 5  | 4  | 7  | 8  | ![](24)

Educationally yours,

Pamela G.B. Drew
School Psychologist

I give my permission for__________________________

to participate in this study, and I understand that he/she may withdraw at any time.

Signed__________________________________

Please send this form back with your child. Thank you for your cooperation.
BIBLIOGRAPHY


Boyle, M. Effects of Hutchings' "low stress" addition algorithm on childrens' addition scores under varying conditions of reinforcement and distraction. University of Maryland Arithmetic Center Monograph #7, 1975.

Dashiell, W. H. An analysis of changes in affect and changes in both computational power and computational stamina occurring in regular elementary school children after instruction in Hutchings' "low stress" addition algorithm, practice with unusually large examples, and exposure to one of two alternative performance options. University of Maryland Arithmetic Center Monograph #7, 1974.


Fulkerson, E. Adding by tens. *The Arithmetic Teacher,* 1963, 10, 139-140.

Gillespie, C. L. Student preferences for the Hutchings' "Low Stress" versus the conventional addition algorithm under conditions of differentially increasing response effort with and without reinforcement. Unpublished Specialist in Education Project, Western Michigan University, 1976.


Lankford, F. G. What can a teacher learn about a pupil's thinking through oral interviews? The Arithmetic Teacher, 1974, 21, 26-32.


Sanders, W. J. Let's go one step farther in addition. The Arithmetic Teacher, 1971, 18, 413-415.


Weaver, J. F. Big dividends from little interviews. Arithmetic Teacher, 1955, 2, 46-47.


Zoref, L. A comparison of calculation speed and accuracy on two levels of problem difficulty using the conventional and Hutchings' "low stress" addition algorithms and the pocket calculator with high and low achieving math students. Unpublished Specialist in Education Project, Western Michigan University, 1976.