A Comparison of Calculation Speed and Accuracy on Two Levels of Problem Difficulty Using the Handheld Calculator and Hutchings' "Low Stress" Algorithms and Traditional Subtraction

Allman A. Todd

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A COMPARISON OF CALCULATION SPEED AND ACCURACY ON TWO LEVELS OF PROBLEM DIFFICULTY USING THE HAND-HELD CALCULATOR AND HUTCHINGS' "LOW STRESS" ALGORITHMS AND TRADITIONAL SUBTRACTION

by

Allman A. Todd III

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Submitted to the
Faculty of The Graduate College
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requirements for the
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Finally, I must express my indebtedness to my family. To my parents whose generosity and caring made my college career possible, and to my wife, Vickie, whose love and support has made it all worthwhile.

Allman A. Todd III
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Western Michigan University

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INTRODUCTION

This study investigated the differential calculation power (speed plus accuracy) for subtraction using the traditional algorithm, as compared with the Hutchings' low-stress algorithm, Hutchings (1976), and hand-held calculator on problems having two levels of difficulty. During the past twenty to thirty years there have been numerous and drastic changes in elementary school mathematics. Two major issues have developed from these changes. One issue addresses itself to the goals of elementary school mathematics, and the other to methods of instruction. Among the most noteworthy changes in the goals has been a decreased emphasis on computational proficiency with an increased emphasis on the acquisition of mathematical concepts and principles (Ashlock, 1976).

These issues have become dichotomies for many people. In the late sixties and early seventies, reports began coming in of students' failing scores on tests of computational skills. Proponents of computational proficiency rallied efforts to shift the emphasis back to computational skill acquisition and drill. Ashlock (1976), suggests that it is a false dichotomy to separate mathematical concepts from computational skill and to separate teaching for meaning from the administration of drill.

In the past, traditional programs were usually based on computational drill and memorization of facts. Contemporary programs stress understanding of basic mathematical structure and concepts. Memorization and drill must be preceded by an understanding of the
concept and a comprehension of why basic facts are true and why computational procedures (algorithms) work. An effective contemporary mathematics program combines understanding with skill to provide the child with real power in mathematics (Heddens, 1974).

With the amount of essential mathematical knowledge increasing it becomes more critical that the acquisition and maintenance of basic computational skills be made quickly and accurately. The traditional algorithms often fail this challenge (Carpenter, Coburn, Reys, and Wilson, 1975). Alternatives to the traditional algorithms must be explored. Engelmann (1969) states that "If the child who is failing in school is to catch up in arithmetic performance, the teacher cannot adhere to the grade level achievement norms established for middle class children. The time normally devoted to fact learning should be used for teaching the slow learning child the structure of arithmetic" (p. 252). By "structure of arithmetic" it might be assumed that the author was referring to the concepts of arithmetic. By developing new, more efficient algorithms, more time may be afforded to this concept formation.

In 1972, Barton Hutchings developed a set of algorithms for the four basic operations of addition, subtraction, multiplication, and division. According to Hutchings (1976), the new algorithms "appear to permit easy mastery after brief training, to provide greater computational power than conventional algorithms, and to operate with much less stress on the user than conventional algorithms" (p. 219). Hutchings (1975) states that all low-stress algorithms have two
distinct mechanical characteristics: one, a concise, definable, easily read, supplementary notation is used to record every step, and two, the learner can complete an intermediate step of a distinct kind rather than alternate between different kinds of intermediate steps.

Heddens (1974) defines an algorithm as a standard form for writing a calculation of a particular kind. Hutchings (1975) defines his subtraction algorithm as follows:

Explicit uninterrupted expression of renamed minuends by coherent, easily perceived numerals is the essential feature of low stress subtraction. This is accomplished by the following:

1. Formally defining a half-space, left-side superscript as the way of expressing, at any place value, numbers greater than nine, and requiring the use of this notation. That is, 64, six tens and four ones, may be expressed as 5^4, five tens and fourteen ones. In fact, sixty-five can be called fifty-fourteen if necessary.

2. Requiring that all regrouping in a problem be completed before any subtraction is done.

3. Requiring that all digits in a minuend that contains any regrouping be recorded as part of the renamed minuend.

4. Requiring that a renamed minuend be written between the minuend and subtrahend, so that digits of the renamed minuend are immediately above their respective digits in the subtrahend. After all regrouping is completed, subtraction is performed (p. 228).

There have been many suggested advantages for the use of low-stress algorithms for addition. Alessi (1979) enumerates these suggested advantages:

1. Each skill in the entire sequence is clearly defined, discrete and logically linked to each other skill.
2. Each new skill introduced in the low-stress system extends previous learning by very efficiently teaching the minimum set of new component skills needed to yield a maximum range of mathematical competence.
3. Each new skill builds directly and cumulatively on
previously taught skills.
4. Each skill is taught in a totally visible (overt) format to allow the teacher to observe actual pupil learning and assure high quality learning by identifying and correcting error patterns before they become habit formed.
5. Each operation (addition/subtraction/multiplication/division) allows the teacher to separate (a) mechanical from (b) meaningful use of the computational procedures.
6. Because of the special properties of the low stress notation, meaningful use may automatically be taught to most pupils merely through the use of mechanical computational procedures.
7. Components of each operation (addition/subtraction/multiplication) are discrete in low stress procedures and may be taught separately at first, and then linked together later, allowing the teacher to teach 'one step at a time'. (e.g., in addition, 'regrouping' is separated from simple addition; in subtraction, 'borrowing' is separated from simple subtraction; in multiplication, 'regrouping' is separated from simple multiplication, as well as from addition of partial products.)
8. The full record of every pupil calculation is available to the teacher for identifying errors and planning practice.
9. The 'memory load' for math facts is substantially reduced over the conventional procedures.
10. The 'attention load' for actually performing long sequences of calculations is substantially reduced over conventional procedures.
11. Low stress procedures are not harder to use when problems get very large, as is the case with conventional procedures. Low stress is as easy to use with very large as with moderate size problems. (pp. 4-5)

Several studies have been conducted to assess the computational power (speed plus accuracy) of the Hutchings' algorithms (Alessi, 1974; Zoref, 1976; Boyle, 1975; Gordon, 1972; Dashiell, 1974; Hutchings, 1972). Gillespie (1976) investigated whether Hutchings' algorithm is preferred over the traditional algorithm in varying response cost and reinforcement conditions. Rudolph (1976) studied the computational power of the Hutchings' algorithms versus the traditional algorithm with normal and emotionally disturbed children.
in distracting and non-distracting environments.

Zoref (1976) investigated the differential calculation power (speed plus accuracy) for addition using the conventional (traditional) and Hutchings' low-stress algorithms, and the pocket (hand-held) calculator on problems having two levels of difficulty. The subjects were three male and three female fourth grade students. Half of the subjects were identified as low achievers and half as high achievers in mathematics. The study employed a multi-element baseline (Ulman and Sulzer-Azaroff, 1975) within a multi-element manipulation design (Sidman, 1960). The two baseline elements were comprised of the two types of problem arrays (2X7 and 5X7), and the three manipulation elements were the three methods of computation. The dependent measures were percent accuracy, rate correct, and rate incorrect.

Zoref (1976) found that the low-stress algorithm was generally superior in accuracy, rate correct and rate incorrect, compared with the conventional algorithm and calculator. The low-stress algorithm also yielded the lowest rate of incorrect adding across all phases of the study.

Although most elementary students have encountered subtraction problems involving regrouping, Carpenter et al. (1975), found these skills are clearly not mastered by the majority of these students. It seems that subtraction computation improves greatly from ages 13 to 17. In this study, Carpenter (1975), found two types of errors most frequently. One was "subtraction with some regrouping errors." Probably the most notable difference between low-stress and tradi-
tional subtraction algorithms is the method of regrouping. This, among other things, makes the low-stress algorithm worthy of study.

Cacha (1975) notes that students are often unsuccessful in subtracting because they are not able to regroup (borrow) successfully. She found that subtraction algorithms requiring regrouping can be solved more successfully and more quickly by a flexible method of regrouping.

Cleminson (1973) has also found that a major source of error in subtraction is improper regrouping. He suggested the development of an alternative algorithm to help remedy this problem.

In the mid 1940's there was some debate as to the relative advantages of the "equal addition" and of the "decomposition" methods of subtraction. Of the procedures, the "equal addition" method was favored in Great Britain and the "decomposition" method in the United States (Brownell, 1949). For illustration, take the following subtraction problem as an example:

\[
\begin{array}{c}
72 \\
-54 \\
\hline
18
\end{array}
\]

Brownell (1949) states:
According to EA (equal addition) the child is taught (at once or later on) to think merely: "Four from 12, 8; write 8; 6 from 7, 1; write 1."

According to D (decomposition), the child, sooner or later thinks: "Four from 12, 8; write 8; 5 from 6, 1; write 1."

The term "equal addition" is appropriate for the first procedure because a ten is added to both terms in the example, as 10 ones to the 2 of 72, and as 1 ten to the 5 of 54. Likewise, "decomposition" is appropriate for the second procedure, because the minuend, 72, is "decomposed" (made equivalent) to 60 + 12 (p. 5).

The comparison between the "equal addition" and "decomposition"
method of subtraction is an important one, because they are alternative ways of subtracting. Equal addition however, does not require the operation of borrowing.

Until 1947, almost without exception, investigators had found the "equal addition" better than the "decomposition" method of subtraction in terms of pupil computation accuracy. However, Brownell (1947) found that the "equal addition" was superior only when subtraction was taught mechanically. He found that "decomposition" was superior to the "equal addition" method when subtraction was taught, (a) rationally, and (b) when understanding and ability to transfer were regarded as important learning outcomes. Brownell (1947) also found that the "equal addition" procedure was difficult to teach to a level of functional understanding. He recommended that the decomposition method be taught over the equal addition method, for this reason. Low-stress algorithms, like the traditional one, employ the "decomposition" method of regrouping.

Another alternative for efficient mathematics instructions that does not involve the revision of algorithms is the hand-held calculator. At approximately one-tenth the price they were six years ago, hand-held calculators are a bargain. They have progressed from being considered a status symbol to the point where many consider them a necessity. Marketing figures in 1978 indicated that over 80 million calculators had been sold in this country (Suydam, 1978). The calculator has been readily accepted at the college level as a tool for mathematics, statistics, chemistry, physics, and other
courses, for all levels of students, from remedial to advanced. At the secondary school level there has also been a high degree of acceptance. From the junior high school downward, there seems to be more doubt among educators about the use of calculators.

Most educators believe that the use of calculators should not replace instruction on skills or concepts, but be an adjunct to facilitate learning. The goals of many studies have been to ascertain whether or not the use of calculators would harm students' mathematical achievement. As Suydam (1979) states, the answer continues to be "no". The calculator does not appear to affect achievement adversely.

There has been a belief that calculators should perhaps be used with higher achieving students but not with lower achieving students. This is due, at least in part, to the fact that teachers still hold firmly to the belief that students must master the basic facts before they use calculators. This in effect is depriving these lower performing student of the possible benefits afforded by the use of the calculator. Zoref (1976) found that low achievers benefitted more from the use of the calculator than did the high achievers in her study. In fact it helped narrow the performance gap between the low and high achievers. Kasnic (1977), also found that use of the calculator helped low ability students compete more successfully with high ability students.

One of the values found in using calculators is that they are motivating to students and help break up the monotony of daily
instruction. It has been said that they encourage curiosity, positive attitudes, and independence. Hutton (1976), found that students thought using the calculator was enjoyable and helpful. The teachers in her study reported that the calculators seemed to motivate students and that no instructional time was lost by including the calculator component.

A common criticism of calculators is that they could be used as substitutes for developing computational skills, and this could undermine students' motivation to master basic facts and algorithms. Rudnik and Krulik (1976), found that overall achievement, including the ability to perform paper and pencil algorithms, did not suffer from calculator availability and use.

As Suydam (1979) notes, the research on calculators differs from most other bodies of research on particular areas within mathematics education, in that the trend of the findings is toward a positive direction. "Almost all of the studies comparing achievement of groups using or not using calculators either favor the calculator group or (in about equal number) reflect no significant differences. This contrasts with the 'typical' case within mathematics education, in which a bell-shaped, 'normal' curve, reflecting a preponderance of findings of no significant differences, prevails" (p. 1).

The calculator is being recognized as an instructional tool which has certain capabilities. Moreover, it has certain limitations, and teachers must accept the responsibility for teaching children
how and when to use calculators. Students will have calculators! Further research is necessary to find their most efficient means of use.

The present study was designed to investigate the differential calculation power (speed plus accuracy) for subtraction using the traditional algorithm as compared with the Hutchings' low-stress algorithm, and hand-held calculator. The three methods are compared for two levels of problem difficulty, both of which require re-grouping (borrowing).

The study employed a multi-element baseline (Ulman and Sulzer-Azaroff, 1975) within a multi-element manipulation design (Sidman, 1960). This design combination allowed the study of the main treatment effects as well as possible interaction effects, for the three computation methods compared across both 4X2 and 8X2 problem arrays. The three manipulation elements were the three methods of computation, and the two baseline elements were comprised of the two types of problem arrays.
METHOD

Subjects

A pre-screening test was given to all of the sixth grade students at the school where the study took place. This pre-screening test consisted of all of the possible whole number subtraction combinations from 18 to 0 (Appendix A). This test was administered so subjects could be chosen who had a high subtraction fact recall. It was important for subjects to have this skill because the study was designed only to assess the performance rate of the three subtraction procedures, not subtraction fact acquisition. Therefore, by virtue of this pre-screening process, the subjects selected were average or above in mathematics calculation.

The subjects for this study were six sixth grade students, three male and three female, approximately 12 years of age. All students were identified as average or above in mathematics, as mentioned above. This was confirmed by each students' California Achievement Test scores in mathematics. The percentiles based on national norms were 52nd, 57th, 75th, 86th, 87th, and 93rd for Total Math, and the 44th, 68th, 83rd, 84th, 75th, and 78th for Computation respectively.

Setting

The study took place at the elementary school the students regularly attended in Allegan, Michigan. Sessions were held along
a side wall in the Library each school day directly after announcements, from 9:15 to 9:45 a.m. There were three station pairs corresponding with each of the three calculation procedures. The station pairs consisted of study carrels that faced each other. This arrangement helped to guard against sharing of answers. The calculator station pair was first, or far left, the Hutchings low-stress algorithm station pair was second or middle, and the traditional algorithm station pair was third or far right. Facing the station pairs, they were arranged in alphabetical order: calculator, Hutchings, and traditional (C, H, T). Three training sessions and 26 data gathering sessions were held. At least four sessions were run each week. Training sessions lasted 20 minutes each. Data collecting sessions lasted five minutes for each procedure. Data were collected on all three procedures during each daily session.

**Independent Variables**

This study involved two independent variables: (a) three types of calculation procedures: hand-held calculator, Hutchings low-stress algorithm (Hutchings, 1975), and traditional algorithm; and (b) two problem array sizes: eight columns (vertical) by two rows (horizontal) and 4 columns by two rows of digits. Approximately 50% of the columns required regrouping (borrowing).

**Dependent Variables**

Three dependent variables were investigated: (a) rate of
columns correct, (b) rate of columns incorrect, and (c) percent accuracy on attempted columns. Rate of columns correct was obtained by counting the number of columns completed correctly for each five minute sample. Rate of columns incorrect was obtained by counting the number of columns completed incorrectly in each five minute sample. Percent accuracy was calculated by dividing the number of columns correct by the number of columns attempted, and multiplying the resulting quotient by 100.

Reliability

Reliability data were taken on correct scoring of the students' papers for number of columns correct and incorrect. Reliability checks were taken an average of every third session day, with a total of eight reliability checks taken, or 31% of total sessions run. Reliability data were collected by two independent scorers, having the first one record scoring responses on a clear overhead transparency and the second one actually score columns correct and incorrect on the worksheet itself. Agreements were checked by placing the first scorer's transparency over the scored worksheet. Interobserver agreement percentages were computed by dividing the total number of scored agreements by the total number of agreements plus disagreements, multiplied by 100.

Experimental Design

This study employed a multi-element baseline (Ulman and Sulzer-Azaroff, 1975) within a multi-element manipulation design (Sidman,
1960). This design combination allowed the study of the main treatment effects as well as possible interaction effects, for the three computation methods compared across both 4X2 and 8X2 problem arrays. The three manipulation elements were the three methods of computation, and the two baseline elements were comprised of the two types of problem arrays.

**Materials**

For the pre-screening test, eighty-five identical ditto sheets of the 100 basic subtraction facts were used, consisting of all the possible whole number subtraction combinations from 18 to 0 (Appendix A). For the first training session the students were given what was referred to as the "Handbook for Hutchings Low-Stress Subtraction" (Appendix B). To ensure equal practice effects for each procedure, two worksheets for each of the three training sessions, corresponding with each of the three calculation procedures, were used. The worksheets consisted of varying problem sizes, but all in the low-stress format (triple space between columns and double space between rows). For the second training session six identical calculators (Texas Instruments 1000) were used for the worksheets, one for each student.

Daily sessions required the following materials: (a) one stop watch (Casio F-100 quartz digital); (b) 54 Xeroxed worksheets for 4X2 problem arrays, and 36 Xeroxed worksheets for 8X2 problem arrays (Appendix C). The worksheets were printed with an IBM Selectric Orator typing element, with triple spacing between columns and double spacing between rows; and (c) two hand-held calculators (TexasInstru-
ments 1000). (Note: When necessary, Xeroxed worksheets were recycled at bi-weekly intervals. It was felt that two weeks allowed sufficient time for possible memory effects to be minimal.)

Procedure

An initial letter was sent out to all parents of sixth grade students in attendance at the school where the study took place. This letter was designed to inform the parents that their children were going to be given a pre-screening test. The test (Appendix A) was described to the parents and the purpose for the research was briefly explained.

The pre-screening test was handed out to each of the three sixth grade classes. The tests were handed out face down and the students were asked to fill in their name, teacher's name, address, phone number, and parent(s') name(s). The test procedures were then explained and followed by a timed 2 minutes and 30 seconds test. After the test, they were told to turn their papers over and hand them in.

The papers were then graded, and the fastest, most accurate students' parents were contacted to seek approval for participation in the study. These students were selected, since good fact recall is a prerequisite for algorithm work. The six subjects were then chosen and parent permission forms were signed. None of the parents refused their permission. Each selected student was then told that he/she would receive a piece of chewing gum each Friday if they participated in the activities, and as a final reward, they would be
bought a bottle of soda from the teachers' lounge on the last day of the study. These rewards were not contingent on performance, but simply on participation in the study.

The first training session was devoted to getting acquainted and teaching the Hutchings low-stress procedures. The training for the low-stress algorithm was conducted according to a modified version of a training manual devised by Hutchings and McCuaig (1976). At the beginning of the session, each of the students were given their own copy of the "Handbook for Hutchings Low-Stress Subtraction" (Appendix B). The second training session consisted of demonstrating how to perform subtraction with the hand-held calculator. The third session was devoted to the review of the traditional algorithm. During each of the training sessions, worksheets were handed out and completed using whichever procedure was being taught that day. Questions were answered as they arose during the completion of worksheets. Each training session lasted approximately 20 minutes and was timed with a stop watch.

For the remainder of the study, sessions were run with a consistent format. The students were randomly assigned daily to one of three treatment conditions, and directed to whatever calculation procedure they were to do. Members of each station pair were seated in study carrells that faced each other. This arrangement guarded against sharing of answers. Corrected worksheets were placed facedown on the desks, with the student's name on the page. This indicated where they were to be seated at the start of their session day. They were then instructed to review their corrected papers and ask questions
as needed.

The corrected papers were then gathered, and the worksheets for that day and that calculation procedure were distributed face-down. The students were then instructed to put their name and calculation procedure on the back of the worksheets which were stapled together. All students would do the same worksheets for that station, that session day, with different problems for each of the three calculation methods.

When all of the students had written their names and calculation procedure on their papers, they were given the cue, "Ready... begin". They were then timed for five minutes with a stop watch. When the time was up they were told to stop, flip their papers over, and advance two seats (one station). All the papers were then gathered and placed in their appropriate folders for correction. New worksheets were again handed out and the students were again instructed as above to write their names and calculation station on the backs of their worksheets. They were then given the cue to begin. Each session consisted of three trials, each five minutes in duration, with one trial per calculation procedure. The order of problem arrays (8X2; 4X2) was alternated every other session.

The investigator would correct the worksheets that evening and return the worksheets the next day as described above. The process was then repeated, exactly as the previous day.
RESULTS

Reliability

Reliability data yielded an index of 100% agreement for scoring papers for number of columns correct and incorrect for each of the eight reliability checks that were run.

Organization of Dependent Data Presented

The results of this study are presented in two main sections: (a) Comparison of performances for 4X2 problem arrays, and (b) Comparison of performances for 8X2 problem arrays. Each of the performance comparisons for the three calculation procedures will be made with respect to three measures: (a) rate of columns correct, (b) rate of columns incorrect, and (c) percent accuracy of columns. Summary data for these comparisons are presented in one table and four figures. Table 1 presents the means and standard deviations for each of the three dependent measures for the two main sections listed above. All tables and figures are located together for convenience of visual examination.

Comparison of performances on 4X2 problem arrays

Correct rate data for 4X2 problem arrays. Table 1 displays the means and standard deviations for the three calculation procedures for both 4X2 and 8X2 problem arrays. As shown on the table, the mean column correct rate (125.41) for traditional (T) was higher
Table 1

Means and Standard Deviations of the Daily Average Session Scores Across the Study Using Each of Three Calculation Methods With Two Different Problem Arrays (Column Data)

<table>
<thead>
<tr>
<th>Problem Array</th>
<th>Method of Calculation</th>
<th>Number of Sessions</th>
<th>Correct Rate</th>
<th>Incorrect Rate</th>
<th>Percent Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \overline{x} )</td>
<td>SD</td>
<td>( \overline{x} )</td>
</tr>
<tr>
<td>4 X 2</td>
<td>Calculator</td>
<td>13</td>
<td>72.35</td>
<td>6.97</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td>Hutchings</td>
<td>13</td>
<td>96.92</td>
<td>9.22</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>Traditional</td>
<td>13</td>
<td>125.41</td>
<td>9.12</td>
<td>1.85</td>
</tr>
<tr>
<td>8 X 2</td>
<td>Calculator</td>
<td>13</td>
<td>61.05</td>
<td>4.73</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td>Hutchings</td>
<td>13</td>
<td>96.87</td>
<td>8.88</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>Traditional</td>
<td>13</td>
<td>126.88</td>
<td>8.47</td>
<td>2.57</td>
</tr>
</tbody>
</table>

\( \overline{x} \) = Mean  
SD = Standard Deviation
Table 2 (A)

<table>
<thead>
<tr>
<th>WEEK</th>
<th>CALCULATOR</th>
<th>HUTCHINGS</th>
<th>TRADITIONAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55.0</td>
<td>64.5</td>
<td>107.0</td>
</tr>
<tr>
<td>2</td>
<td>65.0</td>
<td>72.7</td>
<td>119.0</td>
</tr>
<tr>
<td>3</td>
<td>78.2</td>
<td>74.8</td>
<td>122.0</td>
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<td>72.8</td>
<td>73.2</td>
<td>127.0</td>
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<td>5</td>
<td>76.3</td>
<td>78.2</td>
<td>131.3</td>
</tr>
<tr>
<td>6</td>
<td>80.2</td>
<td>105.0</td>
<td>131.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DAY</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
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<tbody>
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Mean Raw Data for Daily Session Scores Across the Study for Each of Three Calculation Methods for 4X2 Problem Arrays

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Table 2 (B)

Mean Raw Data for Daily Session Scores Across the Study for Each of Three Calculation Methods for 8X2 Problem Arrays

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<td>123.2</td>
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|      |            |           |             |
| 1    | 3.50       | 3.50      | 1.17        |
| 2    | 3.33       | 2.33      | 1.67        |
| 3    | 1.17       | 1.67      | 3.00        |
| 4    | 1.83       | 0.67      | 1.67        |
| 5    | 0.67       | 0.67      | 2.67        |
| 6    | 3.17       | 2.33      | 4.00        |

|      |            |           |             |
| 1    | 93.8       | 97.2      | 98.8        |
| 2    | 94.8       | 97.5      | 98.5        |
| 3    | 98.0       | 98.1      | 97.7        |
| 4    | 97.2       | 99.4      | 98.7        |
| 5    | 98.9       | 99.0      | 97.9        |
| 6    | 95.3       | 97.7      | 96.9        |

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Figure 1: Mean rate of columns correct per session using the three calculation methods with 4X2 problem arrays.
Figure 2: Mean rate of columns incorrect and percent accuracy per session using the three calculation methods with 4X2 problem arrays.
FIGURE 2

4x2 PROBLEM ARRAYS
Figure 3: Mean rate of columns correct per session using the three calculation methods with 8X2 problem arrays.
8X2 PROBLEM ARRAYS

Traditional ($\bar{X}=126.88$)

Hutchings ($\bar{X}=96.87$)

Calculator ($\bar{X}=61.05$)

Mean Rate of Columns Correct (Number of Columns/5 Minutes)

SESSIONS

FIGURE 3
Figure 4: Mean rate of columns incorrect and percent accuracy per session using the three calculation methods with 8X2 problem arrays.
FIGURE 4

Percent Columns Correct

Mean Rate Columns Incorrect (Number of Columns/5 Minutes)

Calculator (X̄=96.75)

Traditional (X̄=98.05)

Hitchins (X̄=98.49)

Hutch (X̄=98.55)

Calculator (X̄=2.4)

Traditional (X̄=2.57)

8X2 PROBLEM ARRAYS
than that (96.92) for the Hutchings low-stress (H). This in turn was higher than that (72.35) for the calculator (C). Figure 1 presents the graphic display of the data. There were distinct acquisition slopes for each of the three calculation procedures. Because there is no overlap among the data, it can be concluded that T was higher than H, which was in turn higher than C, for correct rate on 4X2 problem arrays.

**Error rate data for 4X2 problem arrays.** As shown on Table 1, the mean column incorrect rate for T was highest (1.85), followed by C (1.60), which was just slightly higher than H (1.58). Looking at the top graph in Figure 2, it can be seen that there is a great deal of overlap among the data displayed for the three calculation procedures. This makes it difficult to make any firm statements about comparative error rate.

**Percent accuracy data for 4X2 problem arrays.** Table 1 shows that T was most accurate (98.54%), followed closely by H (98.35%), and the least accurate was C (97.90%). Once again there is a great deal of overlap among the data displayed, as seen in the bottom graph in Figure 2. This makes it difficult to make any firm conclusions about comparative accuracy.

**Comparison of performances on 8X2 problem arrays**

**Correct rate data for 8X2 problem arrays.** Table 1 shows the mean column correct rate of T to be highest (126.88), followed by
H (96.87), with C yielding the lowest column correct rate (61.05).

Figure 3 presents the graphic display of the data. There were substantial acquisition slopes for T and H, and a slight acquisition slope for C. Because there was no overlap among the data, it can be concluded that T was higher than H, which was higher than C, for correct rate on 8X2 problem arrays.

**Error rate data for 8X2 problem arrays.** As shown on Table 1, the mean column error rate for T was highest (2.57), followed by C (1.02), with H yielding the lowest (0.91) column error rate. Looking at the top graph in Figure 4, it can be seen that there is substantial overlap among the data, with the greatest amount of overlap among H and C. This makes it difficult to make any firm statements about comparative error rate.

**Percent accuracy data for 8X2 problem arrays.** Table 1 shows that H was most accurate (98.49%), followed by T (98.05%), with the least accurate being C (96.75%). There is again a substantial amount of overlap among the data, with the greatest amount of overlap among C and T, as seen in the bottom graph of Figure 4. This again makes it difficult to make any firm statements regarding comparative accuracy.

**Summary of Results by Calculation Method**

The calculator was found to yield the lowest column correct rate for both 4X2 and 8X2 problem arrays. Unlike the Hutchings
and traditional method, it was also found to be substantially slower for the 8X2 problem arrays than it was for the 4X2 problem arrays. This method produced the lowest accuracy scores regardless of problem difficulty. The calculator also had the second highest error rate.

The Hutchings "low-stress" algorithm was found to yield the second highest correct rate. It did not fluctuate in correct rate because of problem difficulty. This method yielded the lowest error rate, regardless of problem difficulty. It was also more accurate for the 8X2 problem arrays, than the 4X2 arrays. This is in contrast to both the traditional algorithm and calculator whose accuracies dropped with the more difficult problem arrays.

The traditional algorithm was found to yield the highest correct rate for both the 4X2 and 8X2 problem arrays. But this method also produced the highest error rate for both problem arrays. It yielded a much higher error rate for the 8X2 problem arrays than it did for the 4X2 problem arrays. This method was the most accurate for 4X2 problem arrays, but deteriorated to fall behind the low-stress algorithm's accuracy with 8X2 problem arrays.
DISCUSSION

**Traditional Algorithm**

Results of the present study indicate that the traditional algorithm yielded the highest column correct rate, but also highest incorrect rate, for the three methods compared. While most accurate for 4X2 problem arrays, it was second most accurate for the more difficult 8X2 problem arrays. These results were expected, especially in view of the fact that these subjects were average or above in mathematics achievement, which itself implies a history of success with traditional algorithms. Another explanation would be that as sixth graders, these subjects had a four to five year history of practice with the traditional algorithm for subtraction. The calculator and Hutchings low-stress processes were novel except for the single training session devoted to each of them at the start of the study. It is interesting to note that the traditional algorithm incorrect rate increased and the accuracy decreased as problems became larger (i.e., 4X2 versus 8X2 arrays).

**Hand-Held Calculator**

The calculator was found to yield the lowest column correct rate. As the problems became larger computation speed decreased. This decreasing rate can have a number of possible explanations. First, eye contact must be broken with the problem when the student makes an entry on the calculator. Depending on that student's skill in correctly pushing long sequences of number keys after brief exposure to...
the sample problem, this could require as many as five or six back and forth scans between the problem on the worksheet and the calculator. Each of these consume time and increase the possibility of error by breaking the pupil's eye contact with the worksheet. This can also help explain why the correct rate went down dramatically as the problems became larger.

A second possibility, related to the one mentioned above, could be that due to the students' naivety with the key placement on the face of the calculator, they may have spent excessive time searching for the proper keys to push. Third, if a student makes a mistake on any part of the entry process, he/she generally clears the whole entry and starts the entire problem over again from the beginning. Although the calculators were equipped with a "clear entry" button, students would generally clear an entire problem when they made a mistake. This loses much time.

This has been the long standing argument between the abacus and written algorithms. If you make a mistake with an abacus, you must start all over from the beginning. If you make a mistake with written algorithms, you need only trace backwards until you find the error. Once you find the error, you can then continue on to solve the problem, thereby saving all previously done correct work. This advantage is more important as longer chains of calculations are performed. Newer, more expensive calculators with printout records will allow this advantage to calculators over the abacus. But calculators without printout options are at the same disadvantage as the abacus.

Another possible consideration that should be made is that since
the student has not had the calculator as part of his/her mathematics instruction, it may be associated more with toys and play than with on-task or study behavior. More technically, it may not have been established as a discriminative stimulus for on-task behavior. Finally, there may be some excitement elicited by the novelty of using the calculator. This may either detract from, or facilitate the concentration necessary for maximal study behavior.

Novelty is an area which requires much thought, for novelty almost always is brought up to explain both positive and negative results. Often times if a pupil does worse with a novel method, teachers say that it was the novelty undermining his/her performance. If, on the other hand, the pupil does better with a novel method, teachers say it was novelty facilitating his/her performance by perking up the pupil's interest and breaking up the routine.

The calculator was also found to yield the second highest incorrect rate. An informal error analysis found that approximately 90% of the errors were made as a result of pushing the wrong number or operation keys. The remaining 10% of the errors were committed when the student failed to correctly depress the equal sign key. This resulted in his/her recording the subtrahend (i.e., last term entered) as the answer, rather than the difference.

The calculator was also found to be the least accurate of the three methods, regardless of problem difficulty. Similar results were found for addition with high achievers, by Zoref (1976). Zoref (1976) found that low achieving students benefitted more from the use of the calculator than did the high achievers and Kasnic (1977)
found that use of the calculator helped low ability students compete more successfully with high ability students.

These results have serious implications regarding the calculator's use in the classroom. The belief that calculators should perhaps be used with higher achieving students but not with lower achieving students needs to be scrutinized. Teachers who still hold firmly to the belief that weaker students must master the basic facts before using calculators need to consider that they may be depriving the subject who might benefit the most by use of the calculator. Many students who have not mastered the basic facts, need much more practice with basic fact combinations. The calculator allows them this practice within a more functional and meaningful context: that of solving actual problems. Like previous research, the present study indicates that high achieving mathematics students are not the most appropriate group, and surely not the exclusive groups appropriate, for use of the calculator.

**Hutchings' Low-Stress Algorithm**

The Hutchings low-stress algorithm yielded the second highest correct rate. It is important to note that the correct rate did not fluctuate with problem size. This suggests that this procedure does not become more difficult when problems get very large. There are possible explanations for the slower speed of the low-stress versus the traditional algorithm. First, the low-stress algorithm was a new process for all of the subjects (none of the students had even heard of it before). They had only one training prior to the commence-
ment of the study. Second, there are more written steps involved with this method. This is one of the benefits of this algorithm, allowing the easy location and correction of errors.

The low-stress algorithm yielded the lowest incorrect rate, regardless of problem size. It is important to note that this method's incorrect rate did not fluctuate with problem difficulty, whereas the traditional algorithm and calculator incorrect rates increased markedly with problem difficulty. Because of its consistently low incorrect rate, the low-stress algorithm had the highest percent accuracy for the larger 8x2 problems. It was even more accurate for 8x2 than 4x2 problem arrays. Similar results were found for addition by Zoref (1976).

Problem data results are presented in Appendices D and E for the interested reader. This study analyzed column data because it was thought to be more sensitive. For example, if a student completed three of four columns correctly on a 4x2 problem, he/she would receive a 75% accuracy score for columns and a 0% accuracy score for problems. Also, by using column data, partial credit is allowed for incompleted problems which again may be more sensitive. If problem data were used, the investigator would have expected a lowered accuracy and an increased incorrect rate for larger problems, since the mathematical probability of making an error increases as the size of the problem increases.

Problem data are not sensitive to some differences between the calculator and algorithms, whereas column data are. For example, most mistakes made with the algorithm problems involved only a one digit
(column) error, whereas with the calculator, most of the mistakes involved the entire answer. Therefore, if problem data were used, an inflation of results would have been evidenced for the calculator, and a deflation of results would have occurred for the algorithms.

Caution should be taken when generalizing these results to other students. All students were white, middle class sixth grade students. All were average or above in mathematics achievement. All attended a rural elementary school. The students had positive attitudes and remained on-task without drastic intervention. They were also very competitive with one another, which was further incentive to do well. These results might be generalized to other pupils in similar cultural, geo- and socio-economic circumstances.

It is this investigator's suggestion that further study be done using both high and low achieving students as subjects. More time should be spent in training the two novel methods to students, as it seemed the traditional algorithm had an unfair practice advantage. Also, it would be interesting to study and compare the effects of the three procedures used as curricula taught to children as their first exposure to subtraction. Future research might also compare these three procedures for the operations of multiplication and division.
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APPENDIX A

INVENTORY TEST- SUBTRACTION COMBINATIONS

(From Directed Practice in Arithmetic Whole Numbers, p. 55; Educational Services, Inc. Benton Harbor, Michigan, 1964)
INVENTORY TEST—SUBTRACTION COMBINATIONS

Subtract and write the differences quickly.

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Perfect score is 100.

My score is.............
APPENDIX B
Hutchings' "Low-Stress" Handbook
**How let's look at a larger problem.**

\[
\begin{array}{ccccccc}
4 & 2 & 7 & 1 & 6 & 3 & 8.5 \\
-2 & 1 & 6 & 4 & 8 & 7 & 4.5 \\
\end{array}
\]

Bring down the 5. Can I take 5 from 5? Yes.

\[
\begin{array}{ccccccc}
4 & 2 & 7 & 1 & 6 & 3 & 8.5 \\
-2 & 1 & 6 & 4 & 8 & 7 & 4.5 \\
\end{array}
\]

Bring down the 8. Can I take 4 from 8? Yes.
Bring down the 3. Can I take 7 from 3? No.

Add 10 to the 3, that's 13, thirteen. Take 1 from the 6, that's 5.

Can I take 8 from 5? No.

Add 10 to the 5, that's 15, fifteen. Take 1 from 1, that's 0.

Can I take 4 from 0? No.
Add 10 to the 0, that's 10, ten. Take 1 from the 7, that's 6.

Add 10 to the 0, that's 10, ten. Take 1 from the 7, that's 6.

Can I take 6 from 6? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

Bring down the 2. Can I take 1 from 2? Yes.

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APPENDIX C
SAMPLE WORKSHEETS
Worksheet #2

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</table>

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<tr>
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<td>4 1 8 2</td>
<td>4 2 6 7</td>
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<tr>
<td>- 1 4 5 5</td>
<td>- 3 8 1 7</td>
<td>- 2 3 2 9</td>
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<tr>
<td>9 7 1 4</td>
<td>4 3 3 1</td>
<td>3 0 4 7</td>
</tr>
<tr>
<td>- 3 8 3 3</td>
<td>- 1 1 8 9</td>
<td>- 2 6 8 7</td>
</tr>
</tbody>
</table>

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Worksheet #35

Name ____________________________

50 26955524

2 6 9 5 5 5 2 4

5 6 6 9 4 0 8 5

3 2 6 1 7 5 2 9

- 2 8 4 8 1 4 8 6

- 2 7 3 6 4 8 6 6

1 6 4 7 3 5 3 0

6 9 7 9 2 1 5 1

- 1 0 6 6 4 9 2 7

- 2 4 5 1 2 8 8 6

2 5 8 9 2 8 4 5

5 5 6 3 0 3 9 0

- 1 4 1 9 3 6 4 6

- 5 1 1 5 0 4 8 5

8 9 8 3 3 3 1 1

7 6 7 6 4 8 9 3

- 3 6 6 7 0 6 8 4

- 2 9 7 8 0 1 3 7

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APPENDIX D

Means and Standard Deviations of the Daily Average Session Scores Across the Study Using Each of Three Calculation Methods with Two Different Problem Arrays (Problem Data)

<table>
<thead>
<tr>
<th>Problem Array</th>
<th>Method of Calculation</th>
<th>Number of Sessions</th>
<th>Correct Rate</th>
<th>Incorrect Rate</th>
<th>Percent Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\bar{x}$</td>
<td>SD</td>
<td>$\bar{x}$</td>
</tr>
<tr>
<td>4 X 2</td>
<td>Calculator</td>
<td>11</td>
<td>18.24</td>
<td>1.09</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Hutchings</td>
<td>11</td>
<td>23.55</td>
<td>1.07</td>
<td>1.49</td>
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<td></td>
<td>Traditional</td>
<td>11</td>
<td>30.01</td>
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<td>1.97</td>
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<td>8 X 2</td>
<td>Calculator</td>
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<td>0.70</td>
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<td>Hutchings</td>
<td>11</td>
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<td>1.05</td>
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<tr>
<td></td>
<td>Traditional</td>
<td>11</td>
<td>13.80</td>
<td>1.07</td>
<td>2.31</td>
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</table>

$\bar{x}$ = Mean  
SD = Standard Deviation
APPENDIX E(1)

Mean Raw Data for Daily Session Scores Across the Study for Each of Three Calculation Methods for 4x2 Problem Arrays

<table>
<thead>
<tr>
<th>WEEK</th>
<th>CALCULATOR</th>
<th>HUTCHINGS</th>
<th>TRADITIONAL</th>
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</thead>
<tbody>
<tr>
<td>Mean Rate Correct</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1</td>
<td>15.5</td>
<td>17.7</td>
<td>18.3</td>
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<tr>
<td>2</td>
<td>19.0</td>
<td>8.0</td>
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<tr>
<td>3</td>
<td>17.8</td>
<td>18.2</td>
<td></td>
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<tr>
<td>4</td>
<td>18.7</td>
<td>18.8</td>
<td>18.8</td>
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<tr>
<td>5</td>
<td>19.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Rate Incorrect</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.00</td>
<td>0.83</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>0.83</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>1.33</td>
<td>0.50</td>
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<tr>
<td>5</td>
<td>0.40</td>
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<td></td>
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<tr>
<td>Column Percent Accuracy</td>
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</tr>
<tr>
<td>1</td>
<td>93.9</td>
<td>95.5</td>
<td>99.1</td>
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<td>2</td>
<td>95.8</td>
<td>94.7</td>
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<td>3</td>
<td>94.7</td>
<td>99.1</td>
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APPENDIX E(2)

Mean Raw Data for Daily Session Scores Across the Study for Each of Three Calculation Methods for 8X2 Problem Arrays

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<thead>
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<th>CALCULATOR</th>
<th>HUTCHINGS</th>
<th>TRADITIONAL</th>
</tr>
</thead>
<tbody>
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<td>13.2</td>
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<td>13.5</td>
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<td>7.7</td>
<td>11.5</td>
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<td>10.3</td>
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<td>2</td>
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<td>85.7</td>
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<td>85.7</td>
<td>81.6</td>
<td>80.0</td>
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