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Wobbling Gear Drivetrain for Cordless Screwdriver

David K. Kedrowski
Western Michigan University, davidkedrowski@yahoo.com

Scott P. Slimak
Western Michigan University

Western Michigan University

Department of Mechanical and Aeronautical Engineering

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by
David K. Kedrowski
and
Scott P. Slimak

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David Kedrowski and Scott Slimak, having been admitted to the Carl and Winifred Lee Honors College in 1988, have satisfactorily completed the senior oral examination for the Lee Honors College on April 13, 1993.

The title of the paper is:

"Wobbling Gear Drivetrain for Cordless Screwdriver"

Dr. Philip Guichelaar
Mechanical Engineering

Dr. Jerry Hamelink
Mechanical Engineering

Lowell E Wilcox

Dr. Lowell Wilcox
Gleason Works

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Faculty Mentor

Dr. Philip Guichelaar
Associate Professor
Department of Mechanical and Aeronautical Engineering
Western Michigan University
Kalamazoo, Michigan

Industrial Mentor

Mr. Robert Moores
Director, Cordless Product Development
Black & Decker Corporation
Towson, Maryland

Others

Mr. Mike Buckner
Science Reference Librarian
Western Michigan University
Kalamazoo, Michigan

Mr. Max Mumford
General Manager
Edwards Industrial Sales, Inc.
Kalamazoo, Michigan

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Abstract

Manufacturers of cordless screwdrivers use two planetary gear trains in series to reduce speed and multiply torque from a small electric motor. Alternately, a nutating gear drivetrain has the potential to create comparable speed reductions and torque multiplications. To apply nutating gear theory to a cordless screwdriver drivetrain, a study of spur gears, planetary gear reducers, bevel gears, and the theory of nutation was undertaken. Finally, a nutating design was proposed. The advantages of a nutating gear drivetrain over a conventional planetary gear train include: a decreased number of drivetrain parts, an ability to use lower-strength gear materials, a higher overall reliability, and a decreased manufacturing cost. An analysis of bending and surface fatigue stresses showed that nylon gears would have an infinite life.

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Introduction

Cordless screwdrivers are a common household commodity. Today, base models can be purchased for less than \$20. Homeowners and do-it-yourselfers are discovering that these labor saving devices, with their wall-mounted recharging units, are excellent investments.

The small electric motors used as the power source in cordless screwdrivers have high rotational speeds and low torque ratings. The typical electric motor operates at speeds near 10,000 revolutions per minute (rpm); these speeds are impractical for the common user. In addition, most around-the-house jobs require more than the standard 0.25 $in \cdot lb_f$ torque supplied by these electric motors. Therefore, it is necessary to use a gear reduction system which reduces the motor's initial speed to a practical level. The same gear system also multiplies the motor's torque to a useful level.

Cordless screwdrivers currently use planetary (epicyclic) gear trains for speed reduction and torque multiplication. Planetary gear trains use spur gears, provide high speed reduction ratios, and have sufficient strength and rigidity to handle the resulting production of high torques. Common characteristics of planetary gear trains include a high number of parts and the use of gears which must be made from high strength materials.

Nutating, or "wobbling" gears, explained later in this report, are also capable of high speed reduction ratios. Coupling the mathematical concept of nutation with straight bevel gears, a class of gears has been developed which provides high speed reductions and torque multiplication factors in a high shaft angle configuration. This type of drivetrain requires fewer parts than

conventional planetary drivetrains. It also allows for a high number of teeth to be in contact at any instant (contact ratio), which diminishes the need for high strength gear materials.

Benchmarks

The first step in determining the feasibility of adapting nutating gears to cordless screwdrivers involved an investigation of the cordless screwdrivers currently in production. This study determined the type of drivetrains major companies use for speed reductions and torque multiplications, as well as output specifications. The major distributors of small inexpensive cordless screwdrivers are Black & Decker Corporation, S & B Power Tools (formerly Skil Corporation), and Craftsman (a division of Sears-Roebuck & Co.). The specifications of several cordless screwdrivers are shown in Table I.

Literature Search

A literature search of Western Michigan University's Waldo Library and a computer search of the COMPENDEX database (through Waldo Library's OARS program) yielded little information concerning nutating gears. A computer search of the INSPEC database (also through OARS) found two patent abstracts dating back to 1969 and 1970. Within the time frame of the project, it was not possible to obtain the complete patents for review. The majority of the technical information specific to nutating gears was obtained through Dr. Lowell Wilcox at Gleason Works (see *References*).

Table I

Benchmarking Summary

Brand Name	Model No.	Consumer Price ($)	Torque Output ($in \cdot lb_f$)	Output (rpm)
Black/Decker	9018	18	20	130
Black/Decker	9072-1	18	20	130
Black/Decker	9074-1	30	40	180
Black/Decker	9076-1	35	40	180
Master Mechanic (S & B Power Tools)	MM8520	26	N/A	180
Master Mechanic	MM8521	20	N/A	130
Master Mechanic	MM8523	26	N/A	180
Master Mechanic	MM8524	32	N/A	180
Craftsman	911120	28	N/A	140
Craftsman	911124	40	N/A	140
Craftsman	911141	20	N/A	140

Alec Stokes' *Gear Handbook: Design and Calculations*, contains bevel gear formulas that proved useful for bending fatigue analysis. In addition, *Fundamentals of Machine Component Design* by R. Juvinall and K. Marshek and *Kinematics and Dynamics of Machines* by G. Martin proved useful for traditional gear theory, gear nomenclature, bending fatigue analysis, and surface fatigue analysis.

Traditional Gear Theory

Spur Gears

Spur gears (see Figure 1) are the most common type of gear in use today. Spur gears are used to transmit motion between parallel axes. Ratios of tooth numbers (gear ratios) allow for speed reduction or speed multiplication. Gear nomenclature is derived primarily from spur gears.

To fully understand how spur gears work, first imagine two smooth cylinders parallel to and in contact with each other along their sides. If there is a high level of friction (a no-slip condition) between the two cylinders and one (the first cylinder) is turned, the second cylinder will also turn, but in the opposite direction. That is, if the first cylinder is turned in a clockwise direction, the second cylinder will turn in a counterclockwise direction (Figure 2a). If both cylinders have the same circumference and there is no slip between them, they both turn at the same speed. However, if the first cylinder is smaller than the second cylinder, the second cylinder will turn at a rate slower than the first cylinder. Finally, if the first cylinder is larger than the second cylinder, the second cylinder will turn at a rate faster than the first cylinder. This is due to the difference in circumferences. When the first cylinder is rotated once, a point on its outer edge moves a distance equal to its circumference. Since there is no-slip contact, a point on the second cylinder must move the same distance. If the circumference of the second cylinder is the same as the circumference of the first cylinder, then each rotates one time. If the circumference of the second cylinder is larger than the circumference of the first cylinder and the first cylinder is rotated once, a point on the

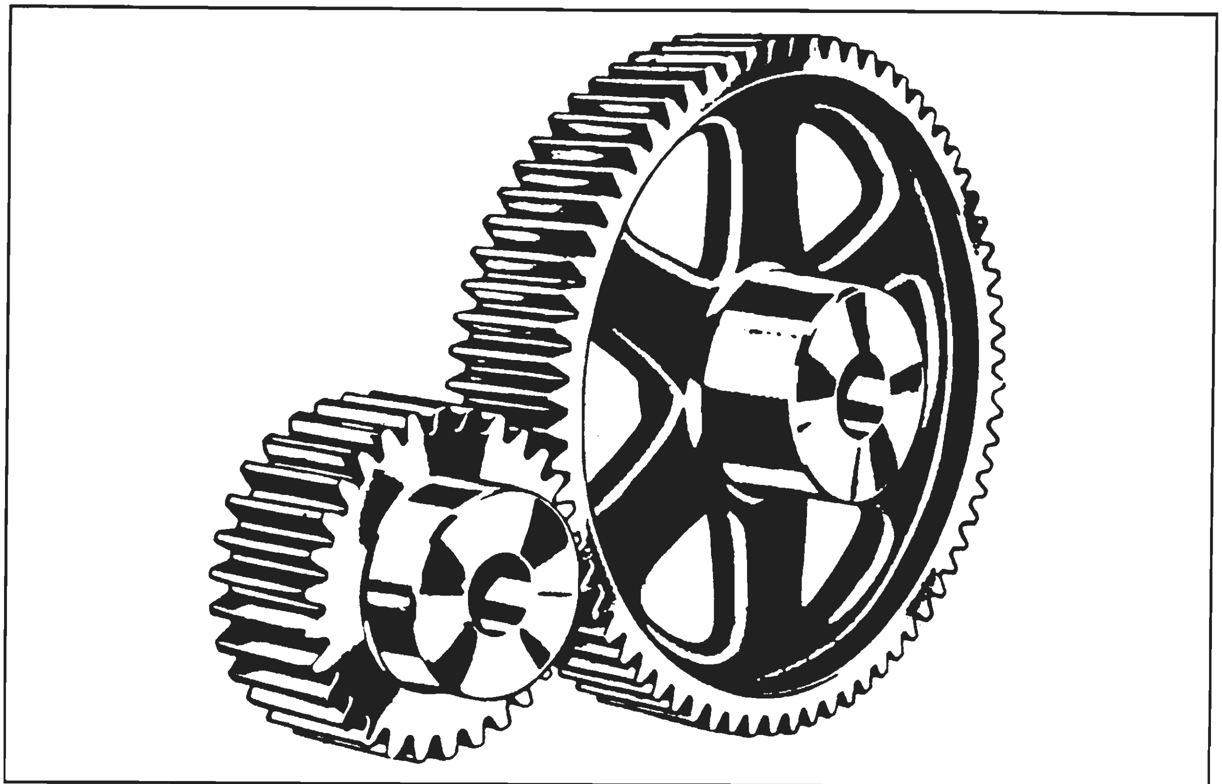

Figure 1 Typical Spur Gear Set (R.C. Juvinall & K.M. Marshek, 1991)

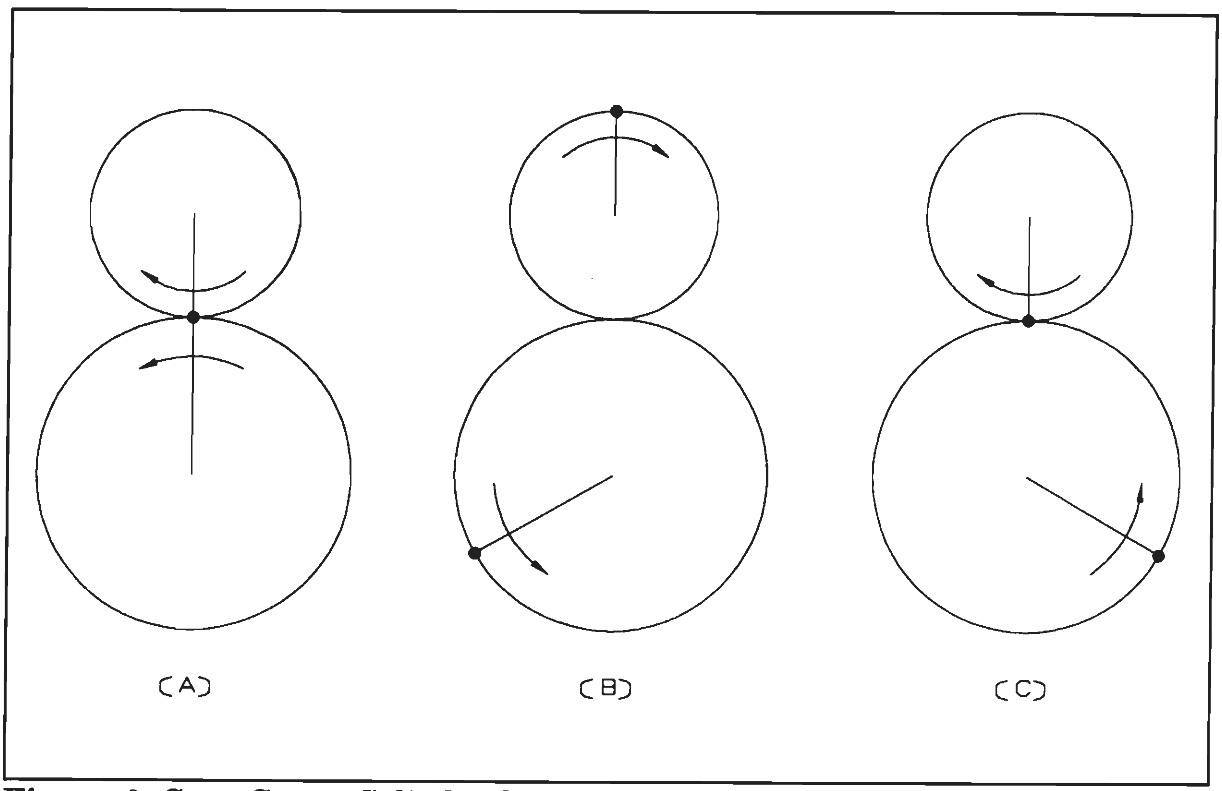

Figure 2 Spur Gear: Cylinder Speed Reduction

circumference of the second cylinder moves a distance equal to the circumference of the first cylinder (which is less than the circumference of the second cylinder as seen in Figure 2). The second cylinder has undergone less than one revolution for a full revolution of the first cylinder (Figure 2c). The second cylinder is now rotating at a speed less than the first cylinder. Conversely, if the second cylinder is turned one revolution in this case, it follows that the first cylinder will turn more than one revolution (a speed increase).

A no-slip condition, which is a high friction condition with no energy losses, is difficult to create between two cylindrical surfaces. However, noting that the no-slip condition is created by friction, and realizing that friction is caused by surface imperfections such as microscopic burrs and holes, it seems logical to increase the size of the imperfections to increase the friction. If these imperfections are increased to a visible size, gear teeth result. Gear teeth must be made such that when in contact, a line perpendicular to the tooth surfaces at the contact point passes through a point P (the pitch point) which is defined by the intersection of the pitch circles (see Figure 3) of the two mating gears. This criterion must be met in order to assure uniform rotational motion of the mating gears. To satisfy this criterion, gear teeth must be carefully cut into what is termed the involute shape. According to Juvinall and Marshek (1991), "an involute curve is the curve generated by any point on a taut thread as it unwinds from a circle."

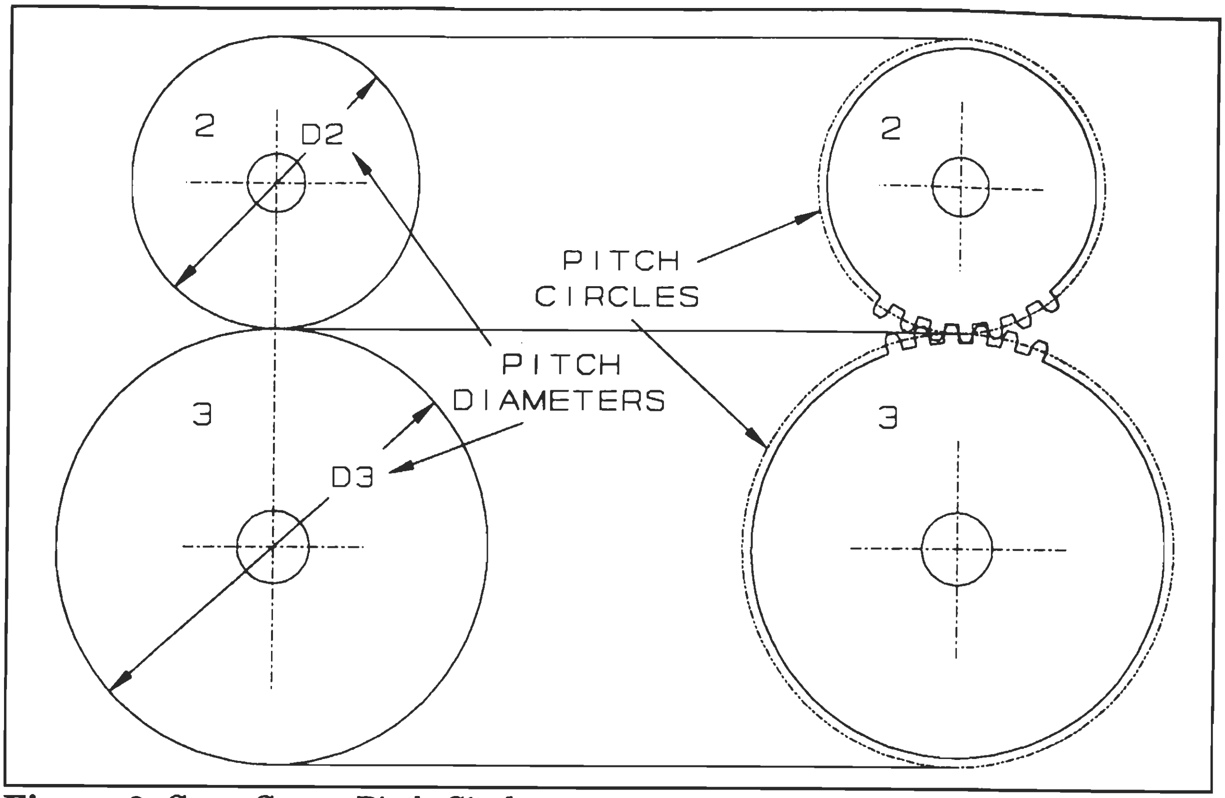

Figure 3 Spur Gear: Pitch Circles

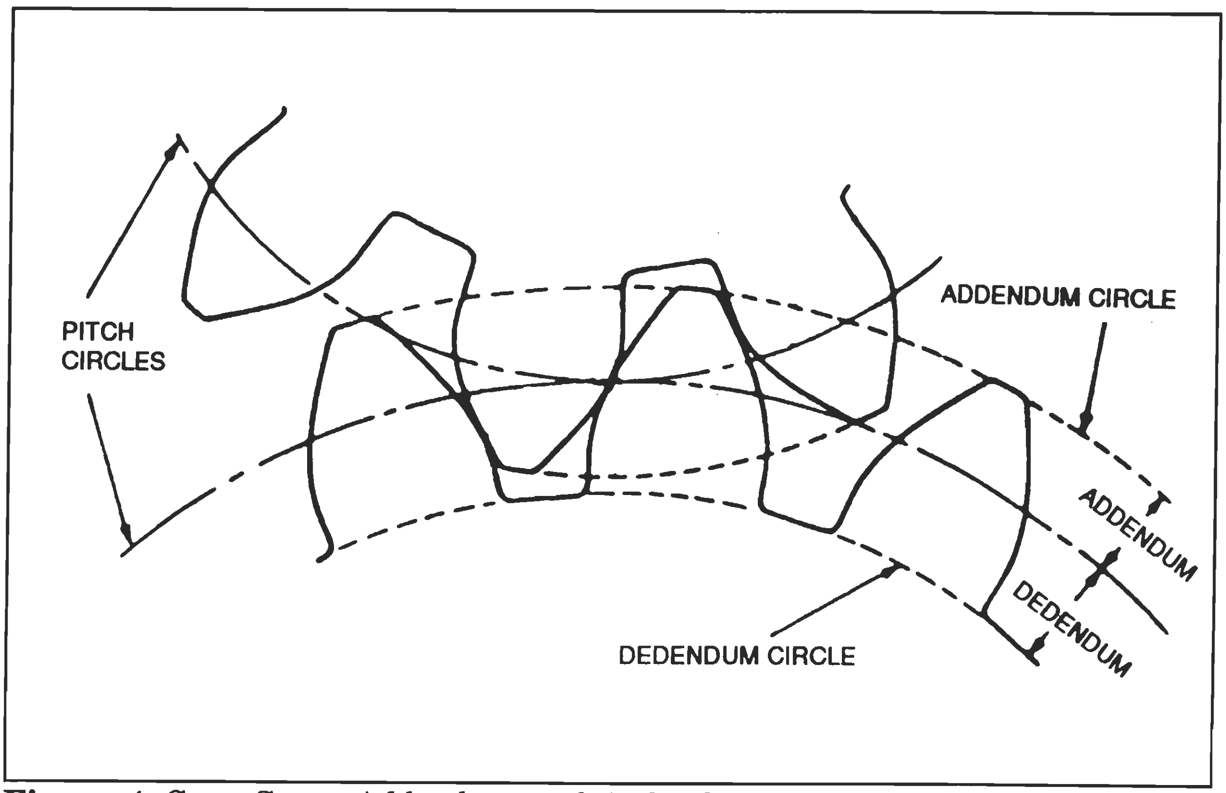

Figure 4 Spur Gear: Addendum and Dedendum

Spur gears are compared on the basis of their diametral pitch (P). In a

$$Diametral\ Pitch = \frac{Number\ of\ Teeth}{Pitch\ Diameter}$$
$$P = \frac{N}{d} \qquad (1)$$

set of mating spur gears, both gears must have the same diametral pitch. Note that since $P_1 = P_2$ in any gear set, the ratio of the numbers of teeth on the gears is equal to the ratio of the diameters of the gears. Since the diameter of a cylinder (gear) is related to the circumference of the cylinder (gear) by the constant pi ($c=\pi \cdot d$), the ratio between diameters and circumferences is identical. Therefore,

$$\frac{c_1}{c_2} = \frac{d_1}{d_2} = \frac{N_1}{N_2} \qquad (2)$$

which shows that, in effect, the circumferences of the gears are compared just as for the cylinder.

If the original cylinders are superimposed over the gears, the diameter of each cylinder becomes the pitch diameter (d) of the corresponding gear (see Figure 3). The circle defined by the pitch diameter (the pitch circle) represents the average point of contact of all the teeth on the gear. The gear teeth themselves extend both above and below the pitch circle by the addendum and the dedendum respectively (see Figure 4). The addendum and dedendum must be chosen such that the teeth on opposing gears come in contact and go out of contact without interference which would cause the gears to bind or lock together. Choice of addendums and dedendums for gears is usually based on diametral pitch.

Planetary Gear Trains

Planetary gear trains use two different types of spur gears--external spur gears and internal spur gears. Internal spur gears are "hollow" gears with gear teeth on their inner (internal) surface instead of their outer (external) surface. The gears in a planetary gear train, as shown in Figure 5, are the sun gear (external), the planet gears (external), and the ring gear (internal).

The first gear in a typical cordless screwdriver's planetary gear train is the sun gear. The sun gear is driven by the electric motor. This sun gear drives three planet gears, each of which is equally spaced about the sun. Because the planet gears are larger than the sun gear and have more teeth, they rotate at a slower speed than the sun gear.

Each of the planet gears meshes with the ring gear, a gear with internal instead of external teeth. The ring gear in the Model 9018 screwdriver is the inside of the drivetrain housing which is fixed against rotation. This causes each of the planet gears to revolve around the sun, in addition to rotating about their own axes. The planet gears revolve around the sun at a rotational speed slower than the speed at which they rotate, providing another speed reduction and an increase in torque.

Each of the planet gears is mounted on a three-armed planet carrier, an arm. The revolution of the three planet gears around the sun is thus translated to a rotating motion of the arm.

A sun gear is attached to the center of the rotating arm. It is identical to the input sun gear. This second sun gear drives a gear train which is identical to the first one. This second drivetrain further reduces speed and

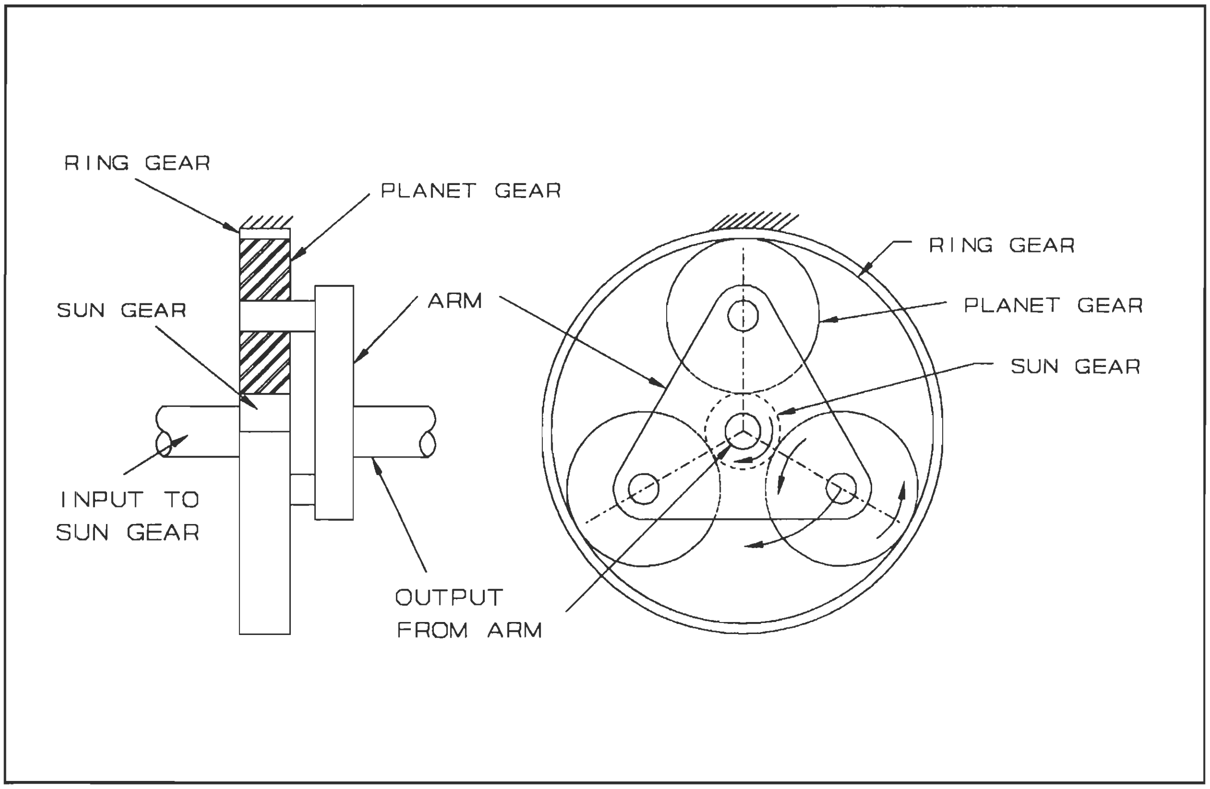

Figure 5 Planetary Gear Train: Schematic

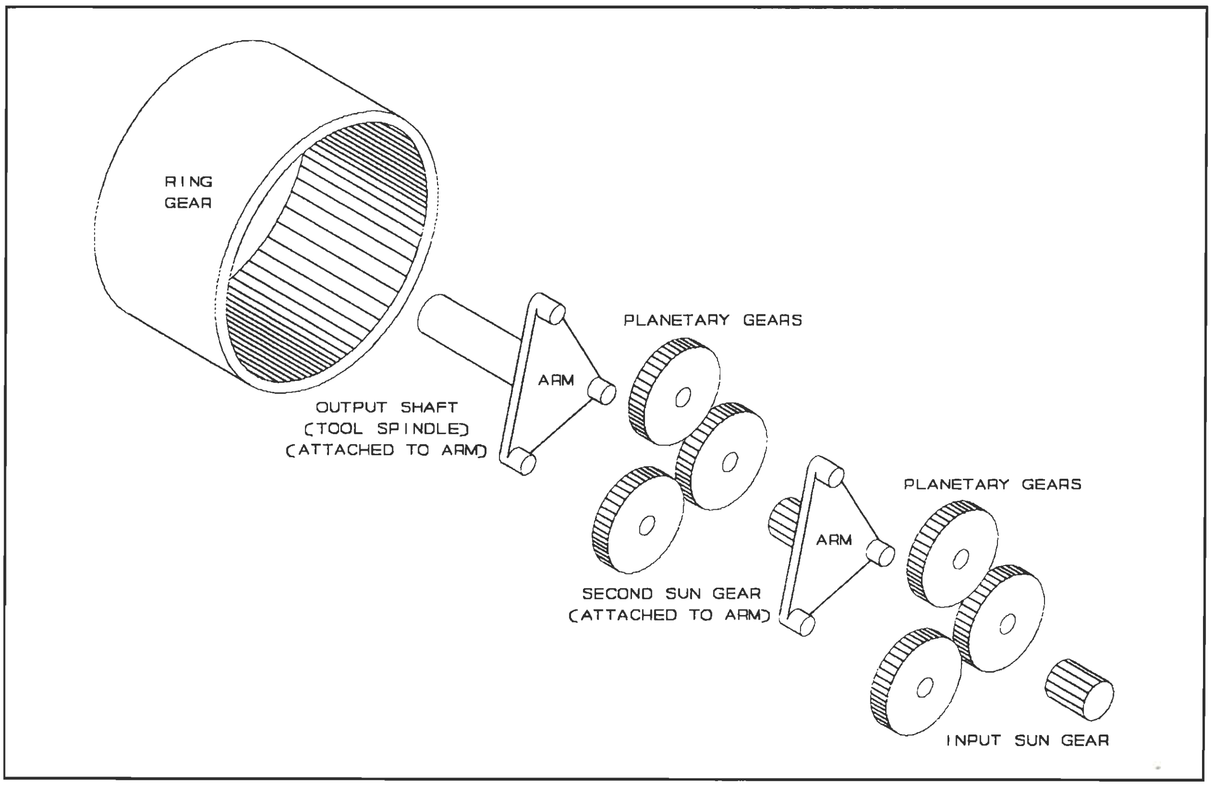

Figure 6 Planetary Gear Train: Isometric

increases torque. Finally, the arm of the second planetary gear train is the output arm which is attached to the tool spindle. Figure 6 shows an exploded view of a double planetary system, as described above.

In the Black & Decker 9018 Cordless Screwdriver, the gear ratio (speed reduction ratio) for each of the two planetary gear sets is 9:1. Since these two gear sets are in series, the first reduction of 9:1 is then reduced by the same factor of 9:1. This results in a total speed reduction of 81:1. See Appendix A for calculations.

The Model 9018 has an output speed of 130 rpm and an output torque of 20 $in \cdot lb_f$. Applying the 81:1 gear ratio to both specifications results in an initial electric motor speed of 10,530 rpm with a torque of 0.25 $in \cdot lb_f$. See Appendix B for calculations. This torque results in transmitted forces of 6.80 lb_f at the point where the second set of planetary gear teeth contact the ring gear teeth. See Appendix C for calculations. Since the contact ratio, which is the average number of teeth in contact at any time, is approximately one, each tooth must bear this load. If the contact ratio was two, then the transmitted force would decrease to 3.4 lb_f. Spur gears are usually limited to contact ratios of approximately one.

Bevel Gears

Bevel gears, while less common than spur gears, are an important class of gears. Bevel gears allow the transfer of motion between non-parallel intersecting axes. Bevel gears are commonly used in automobile applications.

Where spur gears are used to transmit motion between parallel axes, bevel gears are used to transmit motion between intersecting axes. Because the axes intersect, cylinders cannot be used to model bevel gears. Instead, bevel gears are based on cones, as shown in Figure 7. To apply the analogy given for spur gears to bevel gears, consider two identical cones with sides at a 45° angle from the base. The cross section of each cone is an isosceles triangle. If the sides of the two cones are placed in contact such that the tips of the cones are in contact and the bases are in contact, a right triangle results. Assuming no slip contact, rotate one of the cones. The other cone will rotate in the opposite direction, and because the cylinders are identical in size, at the same speed as the first cone. Speed reductions are achieved in the same manner as spur gears. A large bevel gear will always turn at a slower speed than a smaller bevel gear when they are in contact.

Nutating Gear Theory

Nutation

The concept of nutation was discovered by Robert Davison almost by accident. To pass time during a convalescence, Davison did what many have done at some time--spun coins on a table. He noticed that as a spinning coin begins to decrease in speed, a peculiar wobble takes place. This wobble is nutation. It is this innovative concept that can be adapted for high gear speed reductions.

As a spinning coin slows, it "topples" on its edge. The motion of the coin can now be described as a "wobbling," or nutating, motion. As the coin

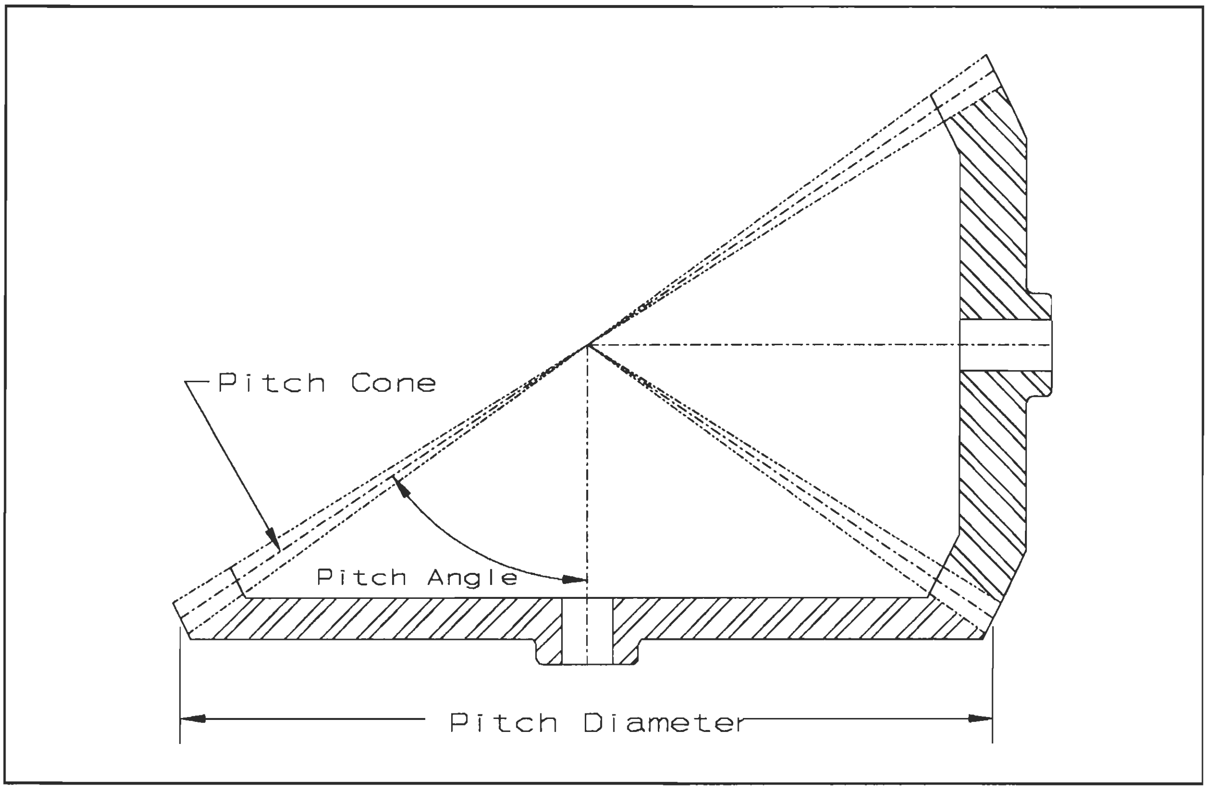

Figure 7 Bevel Gear: Pitch Cones

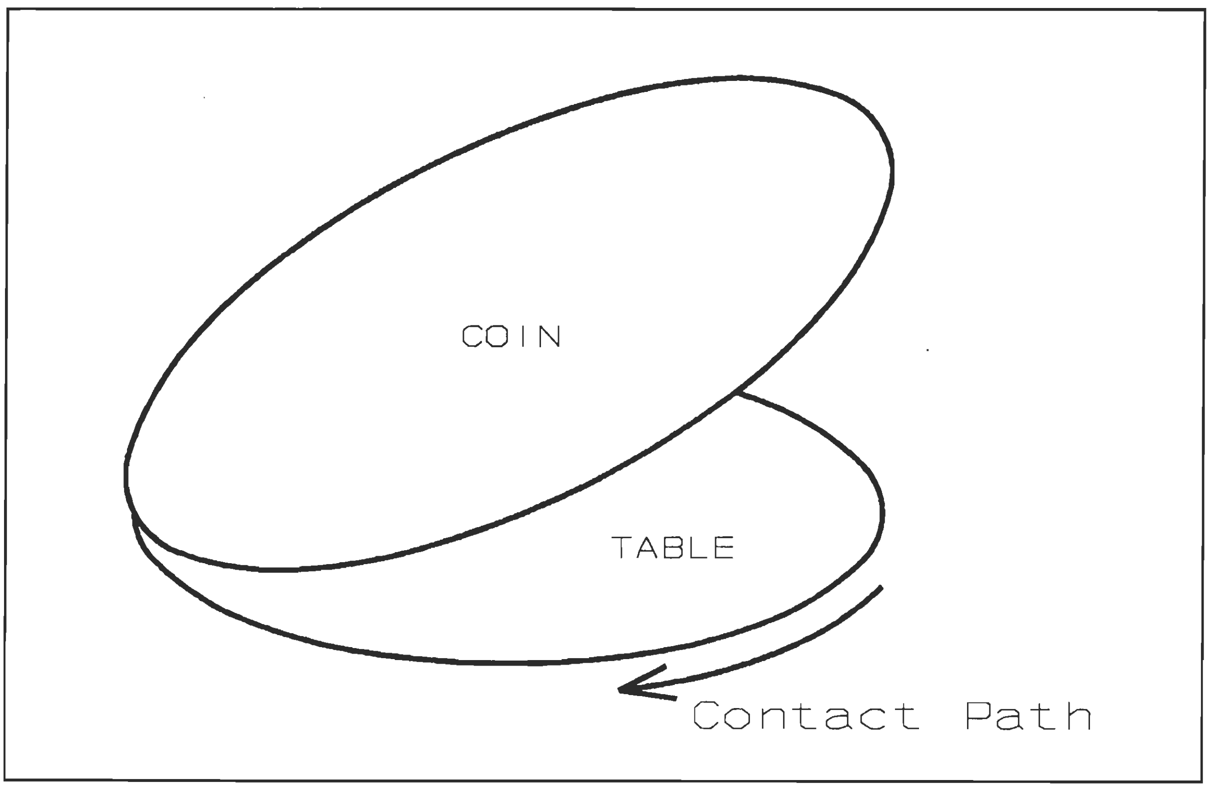

Figure 8 Wobbling Coin: Contact Path

wobbles, it traces a circle on the top of a table, as shown in Figure 8. Until the coin comes to rest on the table, the diameter of the circle traced on the table top is always smaller than the diameter of the coin. Suppose R is the radius of the spinning coin and θ is the angle between the table-top axis and the coin axis. The axis of the coin is perpendicular to the coin's face and passes through the center of the coin. The table-top axis is any convenient axis perpendicular to the table-top. As seen in Figure 9, the radius of the table-top circle is $R \cdot \cos(\theta)$--a value smaller than R.

To understand how nutation works, focus on one point, "A", on the edge of the coin. Referring to Figure 10a, as the coin wobbles, "A" comes in contact with the table at point "B". As shown in Figure 10b, the coin continues to wobble. When "A" contacts the table a second time, the edge of the coin has traced out a path on the table equal to the circumference of the coin ($2\pi R$). However, the circle on which the path has been traced is only $2\pi R \cdot \cos(\theta)$ in circumference. Therefore, "A" does not contact "B" at its second contact; it contacts a point "C" further along the table-top circle (see Figure 10c). Equation 3 shows how far "C" is from "B" along the table-top circle (see

$$\textit{Linear Advance of Point } \boldsymbol{A} = 2\pi R \cdot (1-\cos\boldsymbol{\theta}) \qquad \textbf{(3)}$$

Appendix D). Since the point of contact for "A" has changed, the orientation of the coin about its axis must have changed. In other words, the coin has rotated a small amount. This small rotation, which is dependent on the angle θ, is given in Equation 4 (refer again to Appendix D). Every time "A" contacts

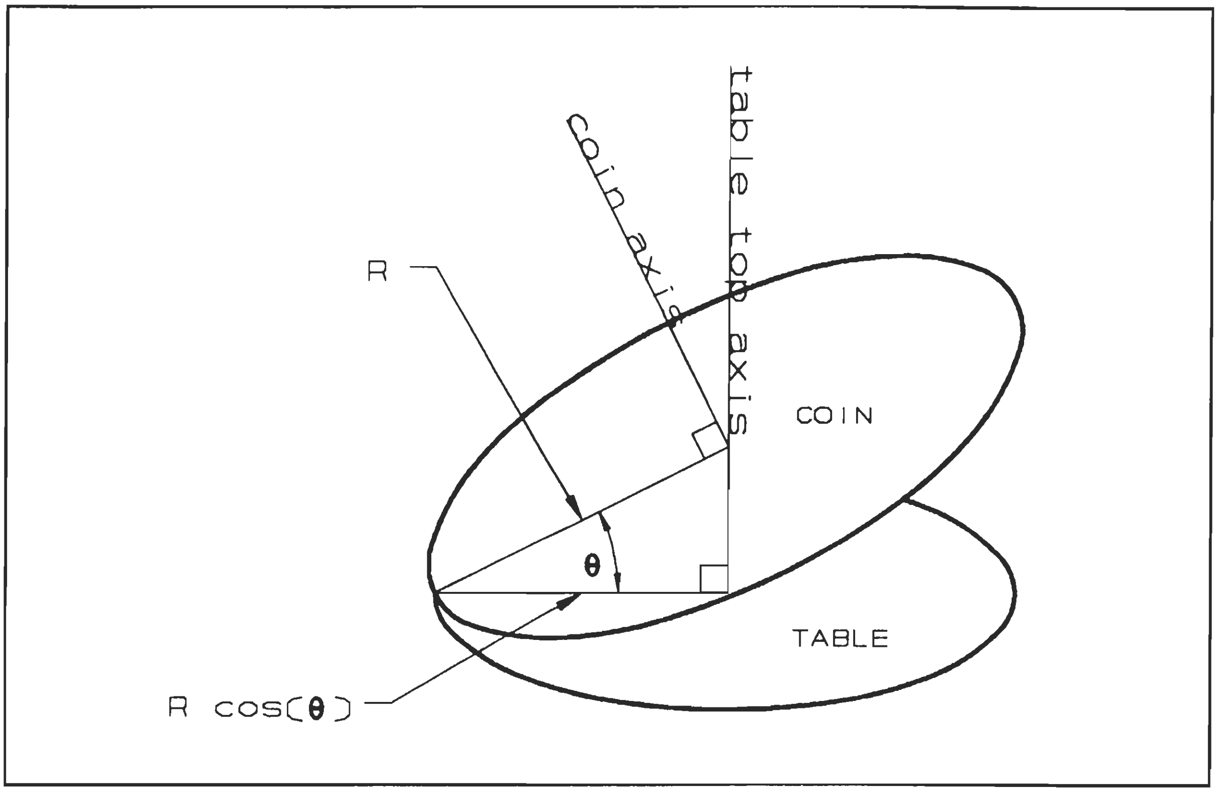

Figure 9 Wobbling Coin: Geometry

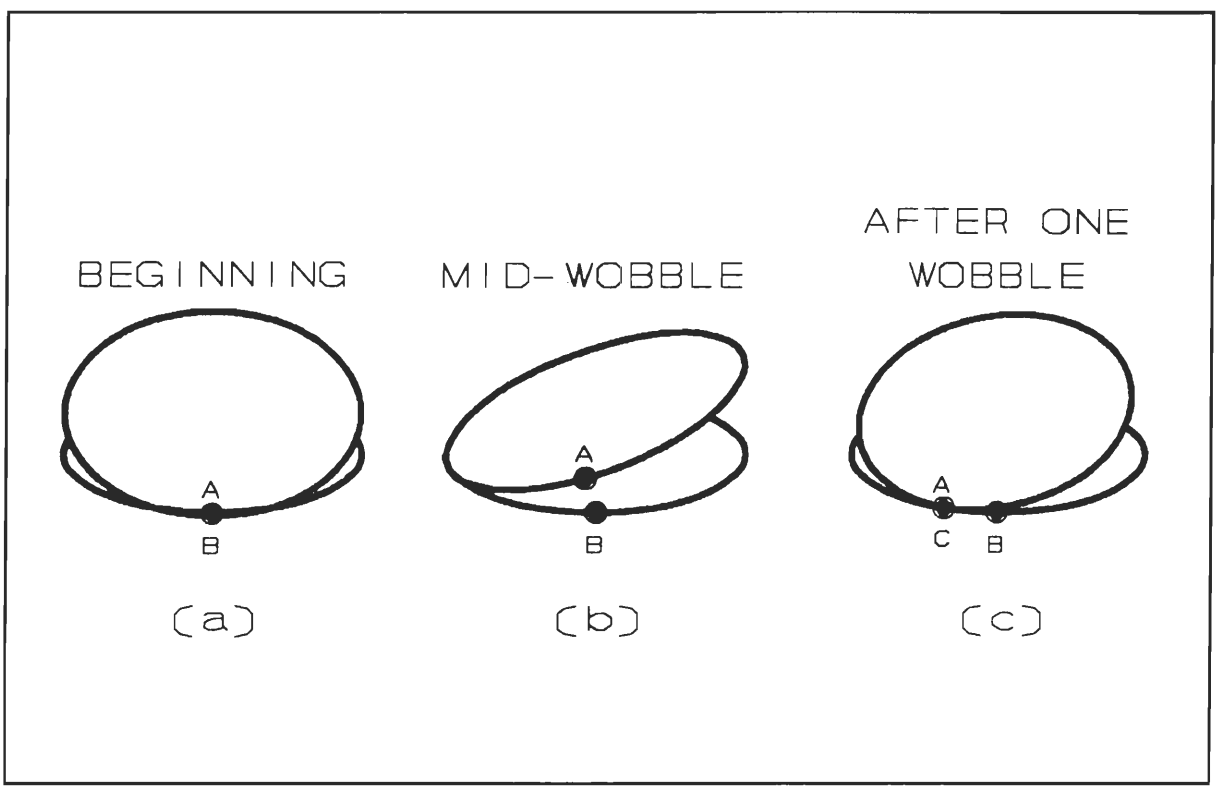

Figure 10 Wobbling Coin: Contact Path Advance

$$\mathit{Angle\ of\ Advance} = (1-\cos\theta)\cdot 360° \quad \textbf{(4)}$$

the table, the coin has rotated this small amount. This creates a slow rotation of the coin as it wobbles on the table. Taken together, the wobble of the coin and the slow rotation of the coin describe what is known as nutation ("Nutating Gear Device," 1980).

A simple boundary condition check verifies Equation 3. If the coin is spinning and has not yet begun to topple, the resulting angle θ is 90°, and the traced path from Equation 3 is 0, or a point. Conversely, if the coin is flat on the table, the resulting angle θ is 0°, and the traced path is 2πR, or the same as the coin circumference.

Simply put, for every "nutation" (one wobble) of the coin there is a corresponding partial rotation of the coin. If the wobble rate and angle θ can be chosen and controlled, nutation can be utilized for gear reductions.

Nutating Gears

Incorporating nutating theory into gear design is a relatively undocumented area of gear design. From Equation 3, the smaller the angle θ, the higher the speed reduction per nutation. This is due to a decreasing difference in path lengths which results in less rotation per nutation. In the simple nutating gear train design shown in Figure 11, the input shaft lies on the same axis as the output shaft and the input gear is nutated by a crank or motor.

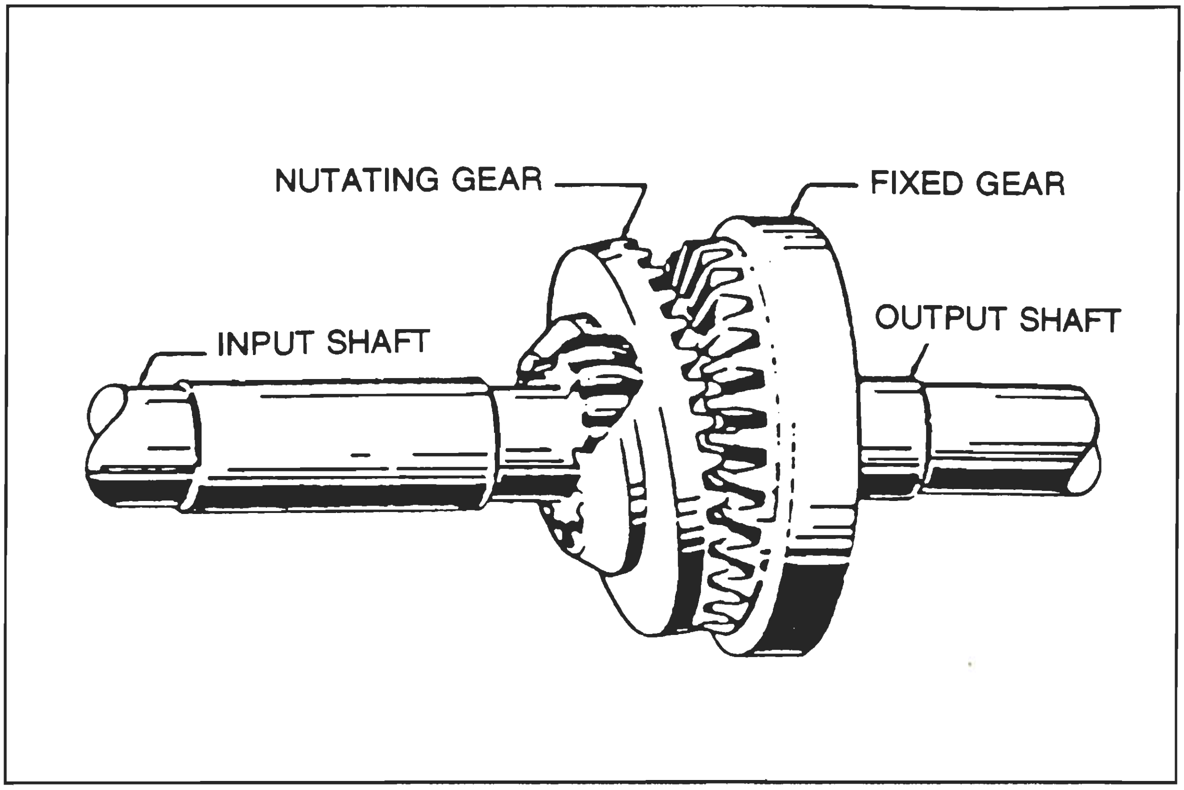

Figure 11 Simple Nutating Gear Train

Basically, nutating gears are high shaft angle bevel gear arrangements where one bevel gear is nutating instead of rotating. The rotation of one gear nutating against a fixed gear is now based upon a difference in tooth numbers instead of a difference in circumferences as illustrated above with the coin. For instance, consider a fixed gear with 20 teeth and a nutating gear, or pinion, with 19 teeth. Assume each gear has its teeth numbered and as the pinion begins nutating relative to the fixed gear, tooth #1 on the pinion is in contact with tooth #1 on the fixed gear. As the pinion continues to nutate, the #2 teeth will come into contact, followed by the #3 teeth, until eventually the #19 teeth come into contact. Since the pinion has 19 teeth and the gear has 20 teeth, the next pair of teeth to come into contact will be the #1 tooth on the pinion and the #20 tooth on the fixed gear. If the pinion continues to nutate, tooth #1 will

contact tooth #19 on the fixed gear after the next nutation. After 20 nutations, the #1 teeth will again be in contact, and the pinion will have rotated once (while nutating 20 times).

When only two of the mating gears have a different number of teeth, the resulting system is called a simple nutating gear arrangement. However, in order to restrain the nutating gear from rotating relative to the output axis, a second set of gears must be added. This second set of gears not only constrains the motion of the input gear, but also provides support and rigidity to the entire drivetrain. If the tooth numbers of the added set are identical to each other, the added gears have no effect on the overall reduction ratio of the gear train.

However, in a major departure from conventional gear theory, the diametral pitches of nutating gears must be slightly different to avoid interference problems.

The reduction ratio (R) is the ratio of input speed to output speed for a gear system. From the previous discussion, the reduction ratio for a simple nutating gear design can be expressed as:

$$R = \frac{N_o}{N_o - N_i} \tag{5}$$

where: N_o = number of teeth of the output gear member

N_i = number of teeth of the input gear member

The reduction ratio for a compound (four gear) design (see Figure 12) is defined as:

$$R = \frac{N_1 \cdot N_4}{N_1 \cdot N_4 - N_2 \cdot N_3} \tag{6}$$

where: N_1 = number of teeth of member fixed to member N_3

N_2 = number of teeth of member mating with member N_1. This member must be fixed from rotation.

N_3 = number of teeth of the input gear member

N_4 = number of teeth of the output member (mates with N_3)

Dividing the numerator and denominator of Equation 6 by N_1 yields:

$$R = \frac{N_4}{N_4 - \left(\frac{N_2}{N_1}\right) \cdot N_3} \tag{7}$$

If N_2 is set equal to N_1, Equation 7 reduces to Equation 5, or the simple gear design (taking $N_4 = N_o$ and $N_3 = N_i$).

Finally, if the sign of the reduction ratio is positive (+), the direction of rotation of the output shaft is the same as the input shaft. If the sign is negative (-), the output shaft rotates in a direction opposite to the input shaft.

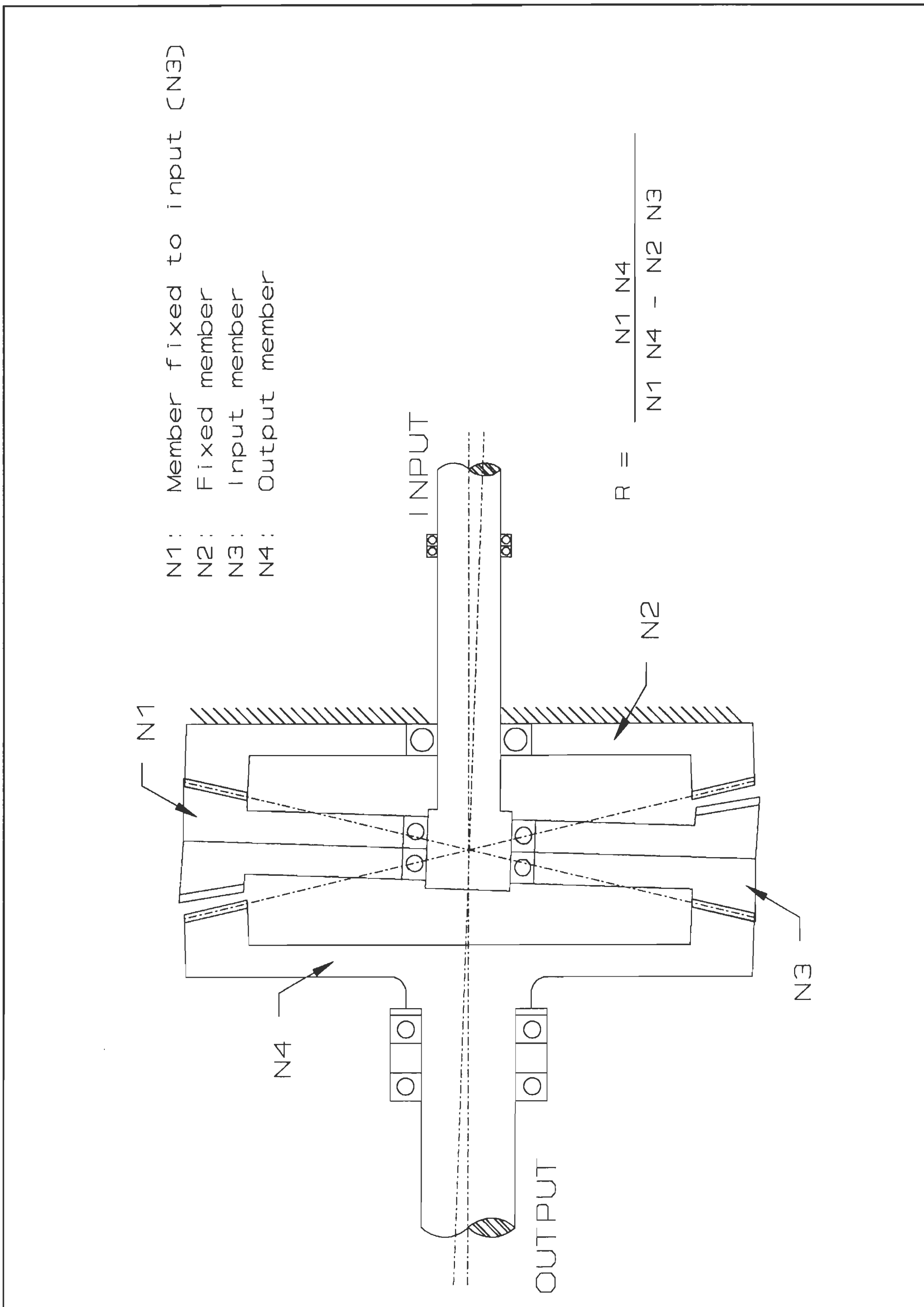

Figure 12 General Compound Nutating Gear Train

Design Requirements

The following requirements were determined for the design of the nutating gear drivetrain and its effect on the screwdriver:

1. The new drivetrain shall allow for an increase in output torque.
2. The new drivetrain shall provide a speed reduction identical to the current drivetrain.
3. The new drivetrain shall have increased reliability/life compared to the current drivetrain.
4. The new drivetrain shall be such that any change in the external dimensions of the housing are minimized.
5. The new drivetrain shall use fewer parts than the current drivetrain.
6. The new drivetrain shall require less time for assembly than the current drivetrain.
7. The new drivetrain shall demonstrate a cost savings with respect to the current drivetrain.
8. The new drivetrain shall adhere to all applicable standards.
9. The new drivetrain shall use recyclable plastic parts where possible.
10. The output torque from the new drivetrain shall be adequate for normal household loads.
11. The output speed and output torque of the new drivetrain shall provide for reasonable handling by the average consumer.

Initial Design Specifications

Based on the above design requirements, the following numerical specifications were determined. The values are based on the Black & Decker Corporation Model 9018 Cordless Screwdriver.

Output torque from spindle	$\geq$ 20 in·lb$_f$
Output speed at spindle	130 rpm
Input motor speed	10,530 rpm
Input motor torque	$\geq$ 0.25 in·lb$_f$
Motor power	$\approx$ .041 hp
Gear reduction	81
Gear tooth friction	power loss in kW·hr
Gears	
Pitch	$\approx$ 67
Diameter	$\leq$ 1.25 in
Pitch cone angles	$\approx$ 70° and $\approx$ 108°
Teeth	3 @ 80, 1 @ 81
Outer housing diameter	$\leq$ 1.6"
Lubrication	as required (similar to current)
Reliability	$\geq 10^6$ cycles, $\geq$ 99.9%
Materials & fabrication costs	$\leq$ \$1.25
Assembly costs	$\leq$ \$0.25
Assembly expertise	low
Total manufacturing cost	$\leq$ \$1.50/gears & mounts

Design Proposal

Proposed Nutating Gear Drivetrain

Due to the extraordinary advantages of nutating gears, the theory of nutation was combined with bevel gear theory to design a workable nutating gear drivetrain. The dimensions of the proposed design were loosely based upon Black & Decker's Model 9018 cordless screwdriver housing. With further investigation, this design could be adapted to other models of cordless screwdrivers currently produced.

The proposed design is similar to the compound nutating gear drivetrain shown in Figure 12. Based on Equation 7, the teeth on gears N_1 and N_2 are set equal. The actual sizes of the gears are based on the housing dimensions of the 9018. The inner diameter of this housing is nominally 30.188 mm or 1.189 inches.

Equation 5 was used to determine the number of teeth necessary for a simple nutating gear design to achieve the existing 81:1 speed reduction. Therefore, N_4 was chosen to have 81 teeth while N_3 was chosen to have 80 teeth. Note that since the output gear has more teeth than the input gear, the 81:1 gear ratio is positive, indicating that both input and output shafts rotate in the same direction.

According to a graph supplied by Gleason Works titled "Maximum Shaft Angle Versus Ratio," a gear tooth ratio of $N_4/N_3 = 81/80 = 1.0125$ can be used with a maximum shaft angle of 178°. The graph was created to allow a designer to choose shaft angles for nutating gears which will not create tooth interference, a common problem in bevel gear pairs with one internal member.

There are three basic reasons for selecting the highest shaft angle possible in a nutating gear set. First, a higher shaft angle results in a higher contact ratio. A higher contact ratio means more of the pinion teeth and driven gear teeth are in contact at any one time. Since a relatively large number of teeth are in contact and therefore are sharing dynamic loads, tooth stresses are substantially lowered. This is shown in detail in *Gear Tooth Stress Analysis*. Second, because a high shaft angle requires the angle θ between the gears to be quite small, the diameters of the gears will be almost

equal. This allows the diametral pitches to be close in value, minimizing the chance for interference problems. Finally, the slightly eccentric position of the input gear creates dynamic forces associated with vibration along axes other than the axis of rotation during operation. As the angle of eccentricity decreases, these vibrational forces decrease. A shaft angle of 178° indicates an angle of eccentricity of only 2°. Since 178° is the maximum possible shaft angle that can be readily produced, it was chosen as the proposed design shaft angle.

To achieve the required shaft angle, the end of the input shaft inside the housing is bent 2° from the traditional 180° before it enters the input gear, N_1 and N_3, in Figure 12. This creates the 178° shaft angle determined above. Observe the schematic Figure 12 considering the input shaft as "right," the output shaft as "left," above the shafts as the "top," and below the shafts as the "bottom." Assume that the left end of the input shaft is positioned so the bent end is facing "top"; therefore, the teeth located at the top of the housing for gears N_1 and N_2 are in contact at this instant. Because gear N_2 is fixed from rotation and gear N_1 is in direct contact with gear N_2, gear N_1 cannot rotate with the input shaft. Since N_1 is restricted from rotating, a bearing must be inserted in gear N_1 allowing the shaft to turn within the input gear (N_1 & N_3). As the input shaft rotates, the orientation of the bent end of the shaft changes, forcing gear N_1's orientation to change. This change of orientation takes place such that when the input shaft has rotated through 180°, the bottom teeth of gear N_1 come in contact with the bottom teeth of gear N_2. Realizing that the continual change of orientation is a gradual wobbling motion, it can be seen that the input gear is actually nutating. For every rotation of the input shaft,

N_1 nutates once. Setting $N_1 = N_2$, no speed reduction takes place in this first set of nutating gears. Since N_3 is rigidly fixed to N_1, N_3's rotational speed is identical to the speed of N_1. The output gear N_4 is free to rotate and rotates once for every 81 nutations of the input gear N_1. The speed reduction of 81:1 is thus created between N_3 and N_4 (refer to Equation 7).

The inner housing diameter of the Model 9018 is 1.189 inches. The pitch diameter for N_3 was based on this value and was chosen to be $d_3 = 1.189$ inches. The diametral pitch for P_3 is 67.28. From trigonometry, $d_4 = d_3 \cdot \cos 2°$, or $d_4 = 1.188$ inches. The corresponding diametral pitch for P_4 is 68.17. Because common manufacturing tolerances are ± .005 inches and the difference in calculated pitch diameters for d_3 and d_4 is .001 inch, both gears were designed with a pitch diameter of 1.189 inches. This common pitch diameter change results in a diametral pitch for P_4 of 68.12.

Next, using equations given in Appendix E of Stokes' *Gear Handbook: Design and Calculations*, the pitch angles of the bevel gears were chosen. The values, based on the above pitch diameters, are 69.4° and 108.6°. These two values add to 178°, the given shaft angle. The pitch angle 69.4° represents an external bevel gear and the pitch angle 108.6° represents an internal bevel gear. A 0° pitch angle (γ) represents an external spur gear. When $0° < \gamma < 90°$ an external bevel gear is indicated. When $90° < \gamma < 180°$ an internal bevel gear is indicated. A 180° pitch angle represents an internal spur gear.

Design Recommendations

Adapting a nutating gear system to the current drivetrain housing would involve minimal redesign.

- A new shaft with the desired 178° shaft angle can be manufactured and inserted over the motor output shaft, replacing the current initial sun gear.
- As shown later, due to the lower stresses, in part due to the high contact ratio on the gear teeth, the gears can be manufactured from an injection molded polymer such as nylon (6,6). Nylon (6,6) is a plastic which is much cheaper to produce and mold into parts than the die cast zinc gears currently used in epicyclic drivetrains. Where torque is low enough, some manufactures are using polymer gears in planetary systems to reduce costs.
- Referring to Equation 7, the number of teeth for N_2 is set equal to N_1. Both N_3 and N_1 are opposite sides of the same part. Setting the number of teeth on N_3 and N_1 (and therefore N_2) equal to 80 allows this part to be injection molded with identical die halves. In addition, this symmetry lessens the chance of assembly error. Because of the symmetry of the gear, either side can be inserted onto the input shaft first. This symmetric gear will also lower the assembly time usually needed for an operator to properly orient the part before assembling. Attempting to minimize assembly time by part symmetry is one manufacturing process stressed in Design for Manufacturing Assembly (DFMA).

- Since the ring teeth currently cast inside the housing would no longer be needed, they could be eliminated.
- The following two options are proposed for the manufacturing of N_2. First, the dies currently used for the injection molded plastic battery housing could be redesigned to add teeth where N_2 would be located. The second option would be to produce N_2 as a separate gear and "snap-fit" the gear into the proper location in the battery housing. Two openings near the output shaft of the electric motor could be used to secure a snap-fit part.
- The output shaft must be redesigned with N_4 as a fixed member. This could be done with a plastic gear snap-fit onto the current output shaft assembly. If both N_2 and N_4 were designed to be snap-fit parts, the two gears could be injection molded, again out of nylon (6,6), further reducing material costs.

Gear Tooth Stress Analysis

A gear tooth stress analysis was performed. Due to a lack of design information for plastics and bevel gears, several assumptions had to be made to perform a bending stress and fatigue analysis on the system. The following equations calculate:

1) The largest tooth force acting on the bevel gear system,

2) The bevel gear tooth bending stress,

3) The bevel gear tooth endurance limit (infinite life),

4) The bevel gear tooth surface fatigue stress, and

5) The bevel gear tooth surface fatigue endurance limit.

This stress analysis follows the procedures outlined in *Fundamentals of Machine Component Design* by Robert C. Juvinall.

1) Bevel Gear Tooth Force

To calculate the tooth bending force acting in a bevel gear arrangement, the average pitch diameter is first found.

$$d_{av} = d - b \cdot \sin\gamma \qquad \textbf{(8)}$$

where: b (tooth face width) = 10/(P·d) = 10/(68.12·1.189) = 0.123 inches

γ = pitch angle

(sin 108.6° ≈ sin 69.4°)

Substituting:

$$d_{av} = 1.189 - 0.123 \cdot (\sin 108.6°)$$

$$d_{av} = 1.072 \text{ inches}$$

The average velocity of the gears is then calculated using this average diameter.

$$V_{av} = \pi \cdot d_{av} \cdot n \qquad \textbf{(9)}$$

where: V_{av} is in feet/min and n is in rpm

$$V_{av} = \pi \cdot (1.072) \cdot (130)$$

$V_{av} = 437.8 / 12$ inches

$V_{av} = 36.48$ ft/min

Finally, the tangential (maximum) force acting on the gear teeth is calculated.

$$F_t = 33{,}000 \cdot W / V_{av} \qquad \textbf{(10)}$$

where: W is in horsepower

$F_t = 33{,}000 \cdot (0.0413) / 36.48$

$F_t = 37.31 \text{ lb}_f$

Geometry based equations are not available to calculate the contact ratio for high shaft angle bevel gears. However, a literature reference which discusses a nutating gear drivetrain with a 42:1 gear ratio states that the contact ratio for the drivetrain is about 40%. The proposed 81:1 gear ratio has a larger shaft angle than the 42:1 gear ratio; therefore, the proposed design is presumed to have a larger contact ratio. Accordingly, a contact ratio of 60% is assumed for these calculations. Because the nutating motion involves more sliding of teeth than actual tooth impact, this ratio also takes into account a less conservative estimate of the impact of the gear teeth. For stress analysis, this 60% contact ratio means that the calculated force is distributed over 60% of the teeth (48 teeth) rather than the single tooth assumption used in conventional calculations. Since the stress is directly proportional to the forces acting on the teeth, dividing by 48 yields an actual force per tooth of:

$F_t = 0.7774$ psi

2) Bevel Gear Tooth Bending Stress

The bevel gear tooth bending stress is calculated using the tangential force found in Equation 10.

$$\sigma = \quad (F_t \cdot P \,/\, (b \cdot J)) \cdot K_v \cdot K_o \cdot K_m \qquad \textbf{(11)}$$

where: $J =$ Bevel gear geometry factor, based on the *Lewis form factor* and a stress concentration factor. $J = 0.2$.

$K_v =$ Velocity (dynamic) factor. Indicates the severity of impact as teeth engage and is dependent upon the pitch line velocity and manufacturing accuracy.

$K_o =$ Overload factor. Indicates the degree of non-uniformity of driving and loading torques.

$K_m =$ Mounting factor. Indicates the accuracy of the mating gear alignments.

The velocity factor is calculated through Equation 12 below which assumes that the gears are manufactured to an accuracy equivalent to shaved and ground gears.

$$K_v = \quad (50 + \sqrt{V_{av}}) \,/\, 50 \qquad \textbf{(12)}$$

Solving: $K_v = \quad 1.12$

An overload factor (K_o) of 1.25 is chosen assuming that the gear system would experience moderate shock.

Finally, a mounting factor (K_m) of 1.3 is chosen assuming that the design mountings would be "accurate mountings," with "small bearing clearances, minimum deflection," and "precision gears" (Juvinall, 1991). Substituting the

calculated K values into Equation 11 results in a bevel gear tooth bending stress of:

$\sigma =$ 3906 psi.

3) Bevel Gear Tooth Endurance Limit (Infinite Life)

The endurance limit is estimated by:

$$S_n = S_n' \cdot C_L \cdot C_G \cdot C_S \cdot K_r \cdot K_t \cdot K_{ms} \quad \textbf{(13)}$$

where: S_n' = Standard endurance limit (the material ultimate strength, S_u, divided by two, as with steels).

C_L = Load factor = 1.0 for bending loads (although nutation is not a typical bending load, a conservative 1.0 is assumed).

C_G = Gradient factor = 1.0 for diametral pitches > 5.

C_S = Surface factor. Because the figures available are completed for metals only, a conservative estimate is made, $C_s = 0.9$.

K_r = Reliability factor = .753. Assumed for a desired 99.9% reliability.

K_t = Temperature factor = 1.0. It is assumed the screwdriver will be used intermittently, keeping temperatures to a minimum.

K_{ms} = Mean stress factor = 1.0. Since the current 9018 screwdriver allows the operator to turn a screw in or out, this constitutes bending in two different directions.

Since S_u = 12,000 psi for nylon (6,6), then:

$S_n =$ 4066 psi.

To survive an infinite life cycle, S_n must always be greater than the maximum calculated bevel gear tooth bending stress on the system. Since the maximum calculated stress acting on the gear teeth is lower than the endurance limit S_n, the nylon (6,6) theoretically would survive for an infinite product life.

4) Bevel Gear Tooth Surface Fatigue Stress

A bevel gear can also fail from surface fatigue. The equation for the bevel gear surface fatigue stress is:

$$\sigma_H = C_p \sqrt{\frac{F_t}{b \cdot d_p \cdot I} \cdot K_v \cdot K_o \cdot K_m} \tag{14}$$

where: C_p = Elastic coefficient. The tables available have limited material elastic coefficients and therefore a conservative assumption must be made. A conservative value for C_p can be found assuming Poisson's ratio, ν, to be 0.5, and the tensile modulus, E, to be 475,000 psi.

$$C_p = 0.694 \sqrt{\frac{1}{2 \cdot \frac{1 - \nu^2}{E}}} \tag{15}$$

Therefore, $C_p \approx 400 \sqrt{psi}$.

I = Geometry factor ≈ 0.80. This value is assumed for straight bevel gears with a pressure angle of 20° and a shaft angle of 90°.

Taking the remaining variables previously assigned, F_t = 0.7774 psi, b = 0.123 inches, d_p = 1.189, K_v = 1.12, K_o = 1.25, and K_m = 1.3, and solving:

σ_H = 1389 psi.

5) Bevel Gear Tooth Surface Fatigue Endurance Limit

Surface endurance strength is estimated by:

$$S_H = S_{fe} \cdot C_{Li} \cdot C_{Rn} \tag{16}$$

where: S_{fe} = Surface fatigue strength. S_{fe} values for various steels, nodular iron, aluminum bronze, and tin bronze were compared to the materials' respective ultimate tensile strength. On average, S_{fe} was 72% of S_u. Therefore, S_{fe} for nylon (6,6) is taken to be 8640 psi.

C_{Li} = Life factor ≈ 1.1. Assumed from a figure for steel gears.

C_R = Reliability factor = 0.80 for an assumed 99.9% reliability.

Inserting these values and solving yields:

S_H = 7603 psi.

The value for σ is less than this value. The surface stresses will not cause failure.

Recommendations

To improve the nutating gear drivetrain design, we recommend:

1. The assumed maximum possible 178° shaft angle was derived from machine capabilities dating from the early 1970's. The introduction of more sophisticated machinery may make it possible to cut gears for a shaft angle even closer to 180°, thus increasing the speed reduction, if desired.
2. The assumptions made for specific stress factors were taken from empirical, or experimentally proven, tables. If nutation were to begin to be adapted for screwdrivers, finite element stress analysis (FEA) should be performed on the drivetrain. This should be followed with life cycle testing.
3. There are currently no formulas to derive the contact ratio for nutating gears. Although face contact formulas exist for spiral bevel gears, the high shaft angle bevel gear design did not yield a realistic contact ratio when inserted into the standard bevel gear face contact ratio formula. Further study should be done to develop the proper contact ratio.
4. Material selection can be varied. Nylon (6,6) is widely available and attractively priced. Other possible materials are shown in Table II with their associated tooth bending endurance limits. The tooth calculated bending stress is $\sigma = 3906$ psi.

Table II
Additional Gear Materials

Material	Max. Ultimate Tensile Strength (ksi)	Calculated Tooth Bending Endurance Limit (ksi)
25% Glass Coupled Acetal	16	5422
Cast, Rigid Epoxy Novolacs	12	4066
Nylon 6	12.8	4337
Glass Reinforced PET	min. 16.1	5455
PEEK	16	5422
30% Glass Fiber Reinforced PS	14	4744

Conclusion

The gear train design of current cordless screwdrivers was investigated. Current drivetrains were found to consist of two planetary gear trains connected in series to reduce motor speed and to multiply torque from a small electric motor. A nutating gear drivetrain was found to have the potential to create speed reductions and torque multiplications similar to that of current planetary designs, yet with many advantages. The advantages of a nutating gear drivetrain over a conventional planetary gear train include:

- a decreased number of drivetrain parts,
- low tooth bending and surface fatigue stresses,
- an ability to use inexpensive, low-strength gear materials, and
- a high product reliability.

For cordless screwdrivers, nutating gear drivetrains are an excellent improvement over the current epicyclic drivetrains.

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Appendices

Appendix A

Planetary Gear Train Speed Reduction

The method of superposition was used to calculate the speed reduction for the Model 9018 cordless screwdriver drivetrain. This method calculates the number of revolutions for any gear in the system by adding the number of turns each gear makes with the arm to the number of turns each gear makes relative to the arm (Martin, 1982). For the Model 9018, the number of teeth for each type of gear was found to be:

Sun: $N_s = 6$
Planets: $N_p = 19$
Ring: $N_r = 48$

For analysis purposes, a positive (+) sign denotes gear movement in the counterclockwise direction, while a (-) sign denotes gear movement in the clockwise direction. The following table shows how the speed reduction for one planetary gear system was derived.

Member	Sun (Input)	Planets	Ring	Arm (Output)
Train locked, arm given one positive turn	+1	+1	+1	+1
Arm fixed, ring given one negative turn	$+(48/19)\cdot(19/6)$	-48/19	-1	0
Resultant turns	+9	-1.53	0	+1

In other words, for every nine input turns of the sun in one direction, the first arm turns one time in the same direction (due to the same sign). A 9:1 reduction has taken place (refer to Figures 5 and 6). If this analysis was repeated for the second planetary gear system, another 9:1 reduction would be achieved. Since the second set output arm now moves nine times slower with respect to the first output arm, which is itself moving nine times slower than the first input sun, the total reduction is $9\cdot9$, or 81. An 81:1 reduction has taken place.

Appendix B

Electric Motor: Power, Torque, and Speed

Equation 1 can be used to calculate the power at the output if the output torque and output shaft speed are known. Assuming negligible friction losses through the drivetrain, the power at the input will be equal to the power at the output. Since the power is constant, the reduction ratio for the gear train can then be used to calculate the input torque and the input shaft speed.

$$\dot{W} = \frac{T_o \cdot n_o}{63024}$$

where:
- W = power (hp)
- T_o = output torque (in·lb$_f$)
- n_o = output shaft speed (rpm)

Note that $n_o = n_i/R_{gt}$ and $T_o = T_i \cdot R_G$, where:

- n_i = input shaft speed (rpm)
- R_{gt} = speed reduction factor for the gear train
- T_i = input torque (in·lb$_f$)
- R_G = torque multiplication factor for the gear train ($R_G < R_{gt}$ when friction losses are present)

Substituting into Equation 1 gives:

$$\dot{W} = \frac{(T_i \cdot R_G) \cdot \frac{n_i}{R_{gt}}}{63024}$$

which must reduce to:

$$\dot{W} = \frac{T_i \cdot n_i}{63024}$$

This only happens if $R_G = R_{gt}$, which is only true under the assumption of negligible power (friction) losses through the drivetrain.

For the Black & Decker Corporation Model 9018:

T_o = 20 in·lb$_f$, n_o = 130 rpm, and R = 81.

From the above analysis,

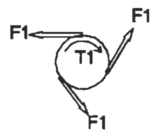
W = .0413 hp, T_i = 0.25 in·lb$_f$, and n_i = 10,530 rpm.

Appendix C

Planetary Gear Train Force Analysis

$T_i = 0.225$ in·lb$_f$

Sun (from rotor)



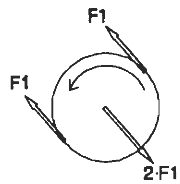
$F_1 = 1/3 \cdot (T_i/r_s)$

$r_s = 0.104$ in

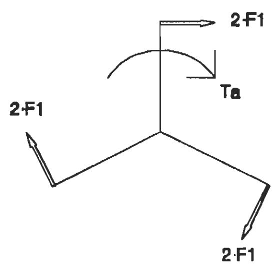
$F_1 = 0.722$ lb$_f$

$2 \cdot F_1 = 1.44$ lb$_f$

Planet



Arm

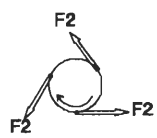


$T_a = 3 \cdot (2 \cdot F_1) \cdot r_a$

$r_a = 0.490$ in

$T_a = 2.12$ in·lb$_f$

Sun

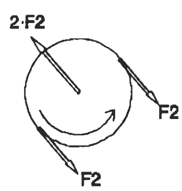


$F_2 = 1/3 \cdot (T_a/r_s)$

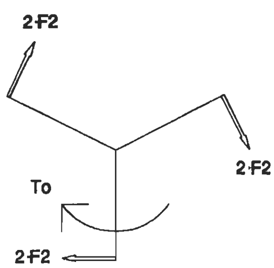
$F_2 = 6.80$ lb$_f$

$2 \cdot F_2 = 13.6$ lb$_f$

Planet



Arm (to spindle)



$T_o = 3 \cdot (2 \cdot F_2) \cdot r_a$

$T_o = 20$ in·lb$_f$

Appendix D

Linear and Angular Advance of a Nutating Coin

Linear advance of the point **A** = s

$$s = C_{coin} - C_{table\text{-}top}$$

where: C = circumference

Substituting for the circumferences,

$$s = 2\pi R - 2\pi R \cdot \cos\theta$$

Simplifying gives:

$$s = 2\pi R \cdot (1 - \cos\theta)$$

Angle of Advance = ϕ

From the geometry identity, $\phi = \frac{s}{R}$

Substituting for s gives:

$$\phi = \frac{2\pi R \cdot (1 - \cos\theta)}{R}$$

Finally, converting to degrees and simplifying yields:

$$\phi = 2\pi \cdot (1 - \cos\theta) \cdot \left(\frac{360°}{2\pi}\right)$$

$$\phi = (1 - \cos\theta) \cdot 360°$$

Appendix E

Bevel Gear Pitch Cone Angles

The following equations were taken from Stokes' *Gear Handbook: Design and Calculations* to calculate the pitch cone angles for the pinion and fixed gears.

1 Pitch cone angle$_{pinion}$:

$$= \tan^{-1} \frac{\sin(180° - \textit{shaft angle})}{\dfrac{\textit{No. of teeth}_{\textit{fixed gear}}}{\textit{No. of teeth}_{\textit{pinion}}} - \cos(180° - \textit{shaft angle})}$$

2 Pitch cone angle$_{fixed\ gear}$:

$$= \tan^{-1} \frac{\sin(180° - \textit{shaft angle})}{\dfrac{\textit{No. of teeth}_{\textit{pinion}}}{\textit{No. of teeth}_{\textit{fixed gear}}} - \cos(180° - \textit{shaft angle})}$$

where: Shaft angle = 178°
No. teeth$_{fixed\ gear}$ = 81
No. teeth$_{pinion}$ = 80

Solving for **1** and **2** yields:

Pitch cone angle$_{pinion}$ = 69.4°
Pitch cone angle$_{fixed\ gear}$ = 108.6°

(The pitch cone angle$_{fixed\ gear}$ result was actually a negative number,

but 180° was added to reflect the same angle with a positive notation.)