Acquisition of Problem-Solving Behavior via Precurrent Training

Diane E. Nelles
ACKNOWLEDGEMENTS

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Diane E. Nelles
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>METHOD</td>
<td>6</td>
</tr>
<tr>
<td>RESULTS</td>
<td>12</td>
</tr>
<tr>
<td>DISCUSSION</td>
<td>13</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>17</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td>22</td>
</tr>
<tr>
<td>TABLE 1</td>
<td>23</td>
</tr>
<tr>
<td>TABLE 2</td>
<td>24</td>
</tr>
<tr>
<td>FIGURE 1</td>
<td>25</td>
</tr>
<tr>
<td>FIGURE 2</td>
<td>26</td>
</tr>
<tr>
<td>FIGURE 3</td>
<td>27</td>
</tr>
</tbody>
</table>
Teaching students to apply computational skills in practical situations, such as in solving written or oral problems, is the goal of most if not all mathematics curricula. However, few currently available programs make any effort to teach students how to specifically solve word problems. Typically, one or two examples are provided and the student is expected to be able to complete the remainder of the problems with little difficulty (Addison-Wesley Publishing Company, 1971; American Book Company, 1963; General Learning Corporation, 1973; Holt, Rinehart and Winston, Incorporated, 1974; Houghton-Mifflin Company, 1974; Science Research Associates Mathematics, 1974; Science Research Associates Greater Cleveland Mathematics Program, 1961; Scott, Foresman and Company, 1975; The Economy Company, 1970). Often it is felt that the ability to solve numerical equations is a sufficient pre-requisite to solving word problems successfully. Thus, specific problem solving steps are rarely presented and, as a result, it is not surprising that teachers are frequently concerned over students' ability to solve word problems (Lerch and Hamilton, 1970; Richardson, 1975; Spitzer and Flor­noy, 1956).

Word problems require the integration of reading and mathematical skills. Separately, both areas have received considerable attention as research topics. It has been demonstrated that math computation skills have improved as a result of manipulating a number of variables. Accuracy improved when contingent reinforcement in the form of tokens (Fer­r­itor, Buckholdt, Hamblin and Smith, 1972; Harris and Sherman, 1973; Johnson and Bailey, 1974), teacher praise and feedback (Kirby and Shields,
1972), and principal praise (Copeland, Brown and Hall, 1974) was presented. Other strategies which resulted in high rates of completion and accuracy have included reinforcement of homework assignments (Harris and Sherman, 1974), dividing work periods into small intervals of time (Van Houten and Thompson, 1976), gradually reducing the amount of time in which to work (Allyon, Garber and Pisor, 1976) and using an increasing multiple ratio of reinforcement (Lovitt and Esveldt, 1970).

Other studies have dealt with the effect a number of variables have on reading skills. Several studies examined the effect of contingent reinforcement on the oral reading of words and sentences (Gray, Baker and Stancyk, 1969; Staats and Butterfield, 1965; Staats, Finley, Minke and Wolf, 1964a; Staats, Staats, Schutz and Wolf, 1962; Whitlock and Bushell, 1967). The use of college tutors and contingency contracts with seventh graders (Schwartz, 1977) and a DRH procedure with hard of hearing subjects (Wilson and McReynolds, 1973) increased oral reading rate. Word recognition skills have been strengthened by interspersing known with unknown items (Neef, Iwata and Page, 1977) and as the result of fading out the picture stimulus (Corey and Shamow, 1972). Reading comprehension skills have been studied with conflicting evidence of success. Correct responses to comprehension questions increased as the result of token reinforcement in the studies conducted by Knapcyk and Livingston (1973) and Lahey, McNees and Brown (1973). Staats, et al. (1965) and Staats, et al. (1967) observed little change when they reinforced correct responses.

Thus, research has shown that both math computation and reading skills can improve as a result of manipulating contingencies of reinforce-
ment and antecedent conditions. It would appear likely, then, that
the combined abilities needed to solve word problems could be strength­
ened through the implementation of behavioral techniques.

Much traditional research has focused on word problems. Several
studies had the subjects engage in discussions and interviews where
they talked about math concepts and steps they took when solving word
problems (Irish, 1964; Corle, 1958; Pace, 1961). Burch (1953) conducted
a study where specific questions were asked of the subject such as "What
does the problem tell you to do?" and "What must you find?". Stern
(1976) found little effect when the subjects were required to verbalize
problem solving steps. Lovitt and Curtiss (1968) had an 11 year old
verbalize the problem prior to writing his answer to simple equations
with marked improvement. In another study, insignificant improvement
in problem solving ability was noted after the subjects studied quanti­
tative vocabulary (Vanderlinde, 1965). Several authors have presented
suggestions for improving problem solving skills. These include break­
ing the skills into a series of steps, using concrete objects, initially
presenting problems without numbers, using diagrams and pictures, locat­
ing relevant and irrelevant data, using a "discovery" approach, orally
presented problems, writing mathematical sentences, having the students
formulate their own problems and estimating answers (Dutton, Petrie and
Richardson, 1975; Reidesel, 1964; Sims, 1969; Spitzer, 1967; Trueblood,
1969). Other suggestions included what not to do: teaching the student
to rely on word cues, using concrete aids and specific steps (Brownell,
As can be seen there is much inconclusive and conflicting evidence as to which techniques are most successful for improving problem solving skills and little actual research has systematically evaluated methods for teaching problem solving. The studies cited above frequently involved subjective analyses and the suggestions offered by many authors had little or no data to support them.

The purpose of the present study was to evaluate problem solving from a behavioral analysis approach. Skinner (1965) defined problem solving as "any behavior which through the manipulation of variables, makes the appearance of a solution more probable." He called these behaviors precurrents to the solution response. Such precurrent behaviors increase the probability that a solution response will occur and therefore be reinforced. This problem solving sequence is characteristic of a chain of behaviors. A chain consists of a sequence of responses in which each response serves as a discriminative stimulus for the following response. Each discriminative stimulus, in turn, reinforces the preceding response. The last response in the chain produces reinforcement which maintains the whole chain.

According to Skinner, precurrent behaviors may be emitted either overtly or covertly. Many problem solving techniques involve covert responses, and this fact is probably the major source of difficulty in gaining access to controlling variables. Behaviors emitted by the organism between the presentation of the problem and the solution response are not always at a level where they can be seen. These behaviors are often termed as "thinking". They may involve emitting verbalizations at a non-vocal level or re-calling circumstances of past similar prob-
lems. Regardless of whether covert problem solving skills are acquired through natural contingencies or from instruction by others, they are acquired in an overt form and can be carried out at an overt level (Skinner, 1976). Parsons (1976) demonstrated this by training preschool subjects to emit precurrent vocal counting responses before responding to a simple matching exercise. When compared to simply reinforcing correct solutions, the use of precurrents was more successful. It was concluded that reinforcing precurrent behaviors can lead to rapid conditioning of problem solving skills. Lovitt and Curtiss (1968) demonstrated a similar technique when they trained an 11 year old to verbalize simple addition and subtraction problems before writing his response. Rate of responding and accuracy increased as a result of the verbalizations. Word problems are more complex than computational problems and may require the manipulation of more antecedent variables than verbalization. In a study of a different nature, Briscoe, Hoffman and Bailey (1975) trained and reinforced a community board for verbalizing a set of identified steps necessary for problem solving. These steps could be called precurrent behaviors.

Becker, Englemann and Thomas (1975) identify problems as "tasks that are taught by identifying sets of tasks embodying essential characteristics of the concepts and operations to be taught." In other words, the task must be broken down into smaller steps. Problem solving behavior becomes a chain of concepts and operations. Concepts can be identified as a signal or discriminative stimulus which controls the subsequent operation or response. This is the same chaining concept presented by Skinner. Problem solving involves the use of operations in new combina-
tions to solve problems that have not been seen before. In the Distar
Arithmetic Programs (1975), series of concepts and operations or pre­
current behaviors are taught to facilitate the solving of word problems.
The word problem is presented and lead questions are asked such as,
"How many did he have?", "How many did he plus?" and "How many did he
end up with?". The students are taught to place the numbers in an
equation form. These precurrent behaviors are very similar to those
taught in the present study. Unlike the Distar presentations, the pre­
sent study attempted to train subjects to solve word problems when the
unknown fell in different positions in the equations and when the compo­
nents were not arranged sequentially in the word problem.

The current study utilized the concept of reinforcing precurrent
behaviors to strengthen the terminal response of providing the solution
to word problems. Identification of the various components in simple
word problems were the precurrent responses reinforced. The word prob­
lems were broken down into five components: the initial, change and
resulting sets, the operation and the solution response. The subject
was reinforced for identifying the components by underlining or point­
ing and placing the responses in an equation form before the solution
was determined. It was assumed that the identification of each succes­
sive component would function to strengthen the probability that the
correct solution response was emitted.

Method

Subjects and Setting

Two male students at the Kalamazoo Valley Multihandicap Center
(KVMC), a program for the physically and mentally impaired served as subjects. Their ages were 19 and 23, and their IQ scores were 46 and 72 respectively. The second subject was deaf and communicated via sign language.

Several pre-requisite skills were required for inclusion in the study. These included the ability to compute addition and subtraction equations where the sums were less than or equal to 10. Furthermore, the subjects had to be able to compute simple algebraic equations when the unknown was in any position. The required reading skills included the ability to comprehend the meaning of the sample of words used in the word problems (about second grade level). These skills were assessed informally.

Training was conducted in the classroom at KVMC. Other students were engaged in one-to-one or small group academic activities in the same room while training was conducted.

**Stimulus Materials**

**Word problems and format.** The subjects were presented with worksheets on which there were five word problems. Below each word problem, answer blanks in the following format were provided: □ ○ □ = □. See Appendix A for an example.

The word problems involved addition and subtraction where the sums were equal to or less than 10. The sample of words used in the problems included 10 different proper names, 11 different verbs and 20 different nouns. These words were used in different combinations to formulate the pool of problems used during training and on the probes.

Each word problem indicated whether an individual received or gave
away objects, and could be reduced to the general equation of $A + B = C$. One of the variables, $A$, $B$ or $C$ was unknown and the subject solved for this unknown. Depending upon which variable was unknown there were six specific equations to which the problem could be reduced. Within each word problem the variables could be presented in different sequences. For each of the six possible equations, there were two sequences in which the variables were presented in the word problem. These appear in Table 1.

Response components. The equations were divided into five components: the initial set, the change set, the operation, the resulting set and the solution set. The initial set was determined by words indicating the number of objects the person started out with in the beginning. Verbs stating that something was added or subtracted specified the change set. Those same verbs gave indication of the operation. The phrase denoting the number of objects the person had in the end was the resulting set. Given the following word problem as an example, "If Mary began with seven balls and ended up with five balls, how many did she give away?", the various components were as follows:

- **initial set** - "...Mary began with seven balls..."
- **change set** - "...how many did she give away?"
- **operation** - "...give away?"
- **resulting set** - "...ended up with five balls, ..."
- **solution set** - "...how many did she give away?"

Either the initial, change or resulting sets were unknown and provided the question of the problem. The answer to that question formed the
solution set.

**Procedure**

**Training sequence.** Training involved teaching the subject to identify the five components of the equation in the following order: the initial set, the change set, the operation, the resulting set and the solution set. During the training of the initial, change and resulting sets, there were three conditions. First, the subject was presented with worksheets where the component to be trained was always a known number. Responses were preceded by a specific prompt (see Table 2). Secondly, the subject was presented with problems where the component being trained was randomly unknown. Again, responses were preceded by a specific prompt. In the third condition, the subject was presented with problems where the component was randomly unknown and the specific prompt was not given. During training of the operation and solution, there were only two conditions. In the first condition, the specific prompt was presented and in the second condition the specific prompt was not presented. All components were randomly unknown throughout both conditions.

The procedure for Subject 1 differed slightly as he was the pilot subject. The randomly unknown and unprompted condition for the initial set, change set and operation began on the same day. The resulting set was trained totally under the randomly unknown and unprompted condition. Subject 1 responded before the trainer could provide the prompt.
on the results during the last two probes, it was determined that the solution set did not need to be trained. Training for Subject 2 also varied slightly in that he did not need training on the resulting and solution sets. Again, this decision was based on the results of the previous probes.

Immediately prior to each condition, the trainer provided a demonstration of the correct responses. The trainer presented the specific prompt, if during a prompted condition, and modelled the correct response. The demonstration consisted of the presentation of five word problems.

Training was cumulative in that the subject responded to the previously trained components along with the one currently being trained. Once mastery criteria were met, the procedures remained the same as during the last training condition.

Training sessions. A trial was defined by the responses for the previous and currently trained components for one word problem. Each training session consisted of 10 trials or problems not including remedial trials. Criteria for advancement to subsequent conditions were as follows: one session of 100% accuracy across all previous and currently trained components during both prompted conditions and two consecutive sessions of 100% accuracy across all previous and currently trained components during the unprompted conditions. Whenever a subject reached criteria for an unprompted condition, a probe session was conducted.

Consequences. Correct responses throughout the various stages of training are defined in Table 2 and were immediately followed by social reinforcement. In the event of an incorrect response, the trainer mod-
elled the correct response on the worksheet with a different colored pen. A remedial trial using a different word problem of the same format was presented. During all conditions, the trainer presented the specific prompt for each previous and currently trained component as the subject responded. Correct responses on the remedial trial were socially reinforced. If the error had occurred on a previously trained component, training continued using the remedial problem. If the error had occurred on the currently trained component, the next training trial was initiated. Following an incorrect response on the remedial trial, the trainer again modelled the correct response and presented another remedial trial in the same manner. Correct responses had to be obtained on all previous and currently trained components before the next training trial was presented.

Probe sessions. A probe session consisted of the presentation of 10 word problems. The subject was requested to "Read each problem out loud. Find the answer putting your work here (as the trainer pointed to the boxes and circles)."

Correct responses were defined as: the correct numbers in the appropriate boxes for the known quantities, an X above the appropriate box for the unknown quantity, the correct operation symbol in the circle and the correct solution in the box of the unknown quantity. Incorrect responses were defined as any other responses or no response at all. Intermittent social reinforcement was provided for effort to eliminate extinction of responding. The subjects were allowed 20 minutes to complete each probe.

Reliability. The experimenter and observer independently recorded
the subjects' written responses after training and probe sessions. Following data collection, experimenter and observer records were compared. Interobserver reliabilities were calculated by dividing the number of agreements for each part of a problem by the agreements plus disagreements and multiplying by 100. Reliability checks were taken at least once during each training stage.

Reliability checks on probe sessions yielded a mean score of 99.5%. Observations taken on training sessions yielded a mean score of 99.9%.

Experimental design. This study utilized an intrasubject multiple baseline design across behaviors (Baer, Wolf and Risley, 1968). Probe data recorded on the five component skills, identification of the initial set, change set, operation, resulting set and the solution set, were the dependent variables. Training was begun on the first component and proceeded sequentially through the other four components.

Results

Figure 1 shows the results of the probe data for both subjects across baseline and training conditions. The training method resulted in an increase in correct responding for both subjects. In terms of mean performance before and after training, Subject 1 increased his level of accuracy from 4.0 to 9.6 out of a possible 10 on the initial set, from 2.2 to 8.5 on the change set, from 2.0 to 8.7 on the operation and from 0.2 to 9.0 on the resulting set. The solution set did not require training since the subject had previously demonstrated the ability to compute an unknown given a simple equation. Once the other components had been trained mean accuracy rose from 1.2 to 7.5 on the
solution set.

Subject 2 demonstrated more generalization across components than did Subject 1 as training progressed. His level of accuracy increased from a mean of 6.7 to 9.3 on the initial set, from 3.9 to 9.5 on the change set and from 5.8 to 10.0 on the operation. Once the first three components had been trained, mean accuracy rose from 0.4 to 9.5 on the resulting set and from 2.6 to 8.5 on the solution set.

Individual training data are presented in Figures 2 and 3. For both subjects, fewer sessions were required to reach criterion as training progressed across components. Subject 1 reached mastery criterion in 43 sessions for the initial set, 36 sessions for the change set, 17 sessions for the operation and two sessions for the resulting set. Subject 2 met criterion in 28 sessions for the initial set, 16 sessions for the change set and seven sessions for the operation.

Discussion

These data suggest that the training procedure was effective in teaching mentally impaired students to use overt precurrent behaviors to solve simple word problems. There was marked improvement in their performance on the first four components for Subject 1 and the first
three components for Subject 2 after training. The use of these pre­
current behaviors generalized to the remaining components without the
benefit of training for Subject 2. It is not felt that this was a de­
monstration of a lack of experimental control. On the contrary, it
would be desirable to have the use of these precurrent behaviors gen­
eralize to other situations. However, a factor one must consider is
the simple nature of the problems used during this study. Subject 2
could have merely responded to the resulting set through a process of
elimination. It would be worthwhile to investigate whether the use of
precurrent behaviors would generalize to other types of problems.

Previous research has emphasized the effects that computation and
reading skills per se have on mathematical problem solving ability.
While these skills are definite pre-requisites for problem solving,
they do not insure the development of problem solving behavior. Knifong
and Holton (1977) found that even though students could answer compre­
hension questions about word problems, they still had difficulties solv­
ing them. The ability to verbalize problem solving steps (Stern, 1971)
did little to increase the number of correct solution responses. The
current sub­j­ects had mastered math computation skills, could read the
word problems and could not reliably solve simple word problems. This
supports the premise that in order to solve word problems students must
be taught all behaviors precurrent to the final solution response. Par­
sons (1976) also demonstrated this when pre-schoolers were only suc­
cessful on a matching exercise after engaging in precurrent counting
behaviors. A community board became more effective after being trained
to use problem solving steps (Briscoe, et al., 1975).
As pointed out by Skinner (1965), problem solving involves the emission of precurrents which make the solution response more probable. As the subjects correctly responded to each component of the word problem, the probability of a correct solution increased. Each subsequent response reinforced the previous response until the goal of the solution was attained. At the same time, each response sets the occasion for the subject to make the next response. These formed the chain of behaviors mentioned by Skinner (1966). This chain of responses were established by a set of rules which the trainer reinforced. Upon continued presentation it is hoped that they would come under the control of the natural contingency of a successful solution (Skinner, 1966).

Many previous studies attempted to investigate variables which occurred at a covert level (Corle, 1958; Irish, 1964; Pace, 1961). Instead of trying to define these variables in observable terms and manipulating them, these researchers attempted to evaluate thinking processes through verbal interviews. This method of data collection is highly unreliable and therefore, the validity of their results is almost impossible to evaluate. As Skinner (1976) suggests, covert behaviors are trained at an overt level and can be maintained as such. Researchers need to define the covert behaviors which lead to problem solutions (Becker, et al., 1975) and train them at an overt level. Only in this way can results be useful and valid.

Once overt behaviors are strengthened, they frequently recede to a covert level because there is a greater efficiency at that level (Skinner, 1976). Therefore, it would be desirable for precurrent behaviors to become covert. If the subjects in the present study were
to become highly proficient with the trained problem solving behaviors, it is assumed that these behaviors would eventually recede to a covert level.

The skills taught in this study were of a simple nature. However, the concept has implications for many areas. Break a problem solving task into small units so as to form a chain of behaviors leading to the solution response. Reinforce and strengthen those behaviors in a sequential manner. Gradually, the terminal solution response should maintain the chain of precurrents. As the subject becomes more adept, the overt precurrents may be reduced to a covert level.

Techniques used in this study could be applied to additional types of word problems; those using multiplication and division, combinations of two or more operations, extraneous information and different wording. It would be interesting to measure the amount of generalization to other types of word problems that would occur after training on several specific types. Adapting these techniques for a group instruction format would not be difficult and would have implications for curriculum design in the public schools. Populations with age appropriate skills would undoubtably acquire skills in using precurrent behaviors at a much faster rate than the current subjects. This would be a more efficient method of teaching problem solving skills than those currently in use. There is much further investigation that could be done with precurrent behaviors. They are applicable to many areas in addition to education.
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Appendix A: Sample Worksheet

Name______________________

1. How many hot dogs did Nan start out with if she ate 3 hot dogs and had 5 left?

\[
\begin{array}{ccc}
\text{\_\_\_\_\_} & \text{\_\_\_\_\_} & = \\
\end{array}
\]

2. If Bob began with 2 dogs and bought 7 more, how many did he have in the end?

\[
\begin{array}{ccc}
\text{\_\_\_\_\_} & \text{\_\_\_\_\_} & = \\
\end{array}
\]

3. If Sam had 10 books in the beginning and lost 8 books, how many did he have left?

\[
\begin{array}{ccc}
\text{\_\_\_\_\_} & \text{\_\_\_\_\_} & = \\
\end{array}
\]

4. If Ann started out with 6 dogs and had 2 left, how many dogs did she lose?

\[
\begin{array}{ccc}
\text{\_\_\_\_\_} & \text{\_\_\_\_\_} & = \\
\end{array}
\]

5. How many hot dogs did Fred make if he began with 2 hot dogs and ended up with 10?

\[
\begin{array}{ccc}
\text{\_\_\_\_\_} & \text{\_\_\_\_\_} & = \\
\end{array}
\]
Table 1
General Equations and Sequences of Word Problems

<table>
<thead>
<tr>
<th>Equation</th>
<th>Word Sequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A + B = ?$</td>
<td>1. If (name) started out with $A$ objects and was given $B$ objects, how many did he end up with?</td>
</tr>
<tr>
<td></td>
<td>2. How many objects did (name) end up with if he started out with $A$ objects and was given $B$ objects?</td>
</tr>
<tr>
<td>$A - B = ?$</td>
<td>1. If (name) started out with $A$ objects and gave away $B$ objects, how many did he end up with?</td>
</tr>
<tr>
<td></td>
<td>2. How many objects did (name) end up with if he started out with $A$ objects and gave away $B$ objects?</td>
</tr>
<tr>
<td>$A + ? = C$</td>
<td>1. If (name) started out with $A$ objects and ended up with $C$ objects, how many was he given?</td>
</tr>
<tr>
<td></td>
<td>2. How many objects was (name) given if he started out with $A$ objects and ended up with $C$ objects?</td>
</tr>
<tr>
<td>$A - ? = C$</td>
<td>1. If (name) started out with $A$ objects and ended up with $C$ objects, how many did he give away?</td>
</tr>
<tr>
<td></td>
<td>2. How many objects did (name) give away if he started out with $A$ objects and ended up with $C$ objects?</td>
</tr>
<tr>
<td>$? + B = C$</td>
<td>1. If (name) was given $B$ objects and ended up with $C$ objects, how many did he start out with?</td>
</tr>
<tr>
<td></td>
<td>2. How many objects did (name) start out with if he was given $B$ objects and ended up with $C$ objects?</td>
</tr>
<tr>
<td>$? - B = C$</td>
<td>1. If (name) gave away $B$ objects and ended up with $C$ objects, how many did he start out with?</td>
</tr>
<tr>
<td></td>
<td>2. How many objects did (name) start out with if he gave away $B$ objects and ended up with $C$ objects?</td>
</tr>
<tr>
<td>Component</td>
<td>Specific Prompt</td>
</tr>
<tr>
<td>----------------</td>
<td>--------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Initial Set</td>
<td>How many objects did (name) start out with?</td>
</tr>
<tr>
<td>Change Set</td>
<td>What happened next?</td>
</tr>
<tr>
<td>Operation</td>
<td>Was that number added or subtracted from the first number?</td>
</tr>
<tr>
<td>Resulting Set</td>
<td>How many objects did (name) end up with?</td>
</tr>
<tr>
<td>Solution Set</td>
<td>The phrase which denotes the unknown set. For example, &quot;How many pigs did Sam sell?&quot;</td>
</tr>
</tbody>
</table>
Training Sessions

Fig. 1. Number of correct responses across components as a function of probe sessions for Subjects 1 and 2.
Fig. 2. Number of correct responses across components as a function of the known (K), random prompted (RP) and random unprompted (RU) training sessions for Subject 1.
Fig. 3. Number of correct responses across components as a function of the known (K), random prompted (RP) and random unprompted (RU) training sessions for Subject 2.