"Integration of Math and Music in the Secondary Classroom"

Brian O'Neill
Western Michigan University, brian.m.oneill@wmich.edu

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Integration of Mathematics and Music in the Secondary Classroom

Brian O’Neill
Western Michigan University
Lee Honors College Undergraduate Thesis
Introduction

It is disheartening to hear the motivation behind why a math student chooses to study mathematics and a music student prefers to study music. Rarely does one find a pupil that audibly expresses enjoyment of both professions simultaneously at face value. Too many times students say that they choose music because they were either never good at math, never want to take another math class as long as they live, or are genetically inept for mathematical study. On the other end of the spectrum, there are mathematics students who would not dare pick up an instrument, or even want to learn to sing in a choir. No wonder kids in high school are saying the same thing with this looming sentiment magnified by our future mathematics and music teachers of primary and secondary education! What if one had an appreciation for both math and music? What if we had teachers who taught each subject through the other? Does true learning stem from teaching one subject in a classroom? Hopefully teachers are catering to well-rounded ideals through the cultivation of a liberal arts education.

When one thinks of mathematics, the first word that may come to mind is structure. Math is full of theorems, formulas, and graphs. All of these things have a certain way that they are computed or created. When one thinks of music, expression might come to mind initially. Music does have a lot of self-revelation; one would not be so easily moved to tears or joy if there was a lack of expression in music. If one chooses to look deeper into each subject, one may find that math can be very expressive and music can be very structured. Within math, there are many different ways to solve the same problem. Expression is shown in the venue one chooses to approach a mathematical problem. One may choose a particular way over another for a given reason. The language of math is so diverse that it is hard to express math in a common language despite the popular thought of math being a universal language. Expression is definitely a factor in math. On the other hand, there is so much structure in music. Songs composed by classical artists contain reoccurring themes. There is also structure in jazz; certain forms are followed in jazz music such as 12 bar blues. Thus, there are numerous crossovers between math versus music and structure versus expression.

Music is not foreign to a high school student. Walking down the hallways of a high school, one can expect to observe numerous students listening to their headphones; they are lost in a sea of tunes essentially disconnected from events happening immediately around them. Why not use this obsession to aid in teaching the subject that most students dread to learn, mathematics.

“Integration of Mathematics and Music in the Secondary Classroom “is an innovative approach to the way that mathematics is traditionally taught in high school. This project is a culmination of rote procedural mathematics and inquiry through musical concepts. There are three main lessons of math and music inquiry. In these lessons, questioning is the essential task and tool given to students to enable them to construct mathematical ideals. The theory of constructive learning is supported by routine mathematics sprinkled throughout the unit. A healthy mixture of inquiry and procedural mathematics is essential to developing Bloom’s taxonomic notion of higher order thinking.
In addition to the quadratics algebra unit, I have further exemplified my desire to connect mathematics to music through self-formulating a high school A Capella ensemble on the campus of Parchment High School. This group is called Bassically Treble. In this choir, I composed select songs that were purposefully written with mathematics concepts in mind. The inspiration of forming such a group only furthers the connection between music and mathematics.
Write each definition with your own words or examples

Distributive Property –
Factors –
Multiples –
Like Terms –

Example 1
Distribute $8(2 + x)$

Example 2
Distribute $(x + 2)(x + 5)$

Use the distributive property
1. $3(x + 4) = \underline{\underline{\underline{\underline{\underline{\underline{}}}}}}$
2. $(4 + 3y)5 = \underline{\underline{\underline{\underline{\underline{\underline{}}}}}}$
3. $-2x(x - 8) = \underline{\underline{\underline{\underline{\underline{\underline{}}}}}}$

4. $(y + 3)(y - 2) = \underline{\underline{\underline{\underline{\underline{\underline{}}}}}}$
5. $(x + 6)(x - 7) = \underline{\underline{\underline{\underline{\underline{\underline{}}}}}}$
6. $(3 - y)(4 - 8) = \underline{\underline{\underline{\underline{\underline{\underline{}}}}}}$

7. $(y + 9)(y + 5) = \underline{\underline{\underline{\underline{\underline{\underline{}}}}}}$
8. $(x + 2)(x - 2) = \underline{\underline{\underline{\underline{\underline{\underline{}}}}}}$
9. $(3 - y)(4 - y) = \underline{\underline{\underline{\underline{\underline{\underline{}}}}}}$
10. \((y + 6)(y + 6) = \) ______________
11. \((x + 4)(x - 0.5) = \) ______________
12. \((1.5 - y)(y - 10) = \) ______________


List all Positive Factors

13. \(10\) _____ _____ _____ _____ _____
14. \(15\) _____ _____ _____ _____ _____
15. \(18\) _____ _____ _____ _____ _____
16. \(25\) _____ _____ _____ _____ _____

List Some Multiples

17. \(3\) _____ _____ _____ _____ _____
18. \(7\) _____ _____ _____ _____ _____
19. \(9\) _____ _____ _____ _____ _____
20. \(15\) _____ _____ _____ _____ _____

Simplify

21. \(3x^2 + 7x + 4 + 4x + 2x\)
22. \(5x - 4x^2 + 8x + 2x^2 - 2x^3\)
23. \(3 + 9x - 5x^2 - x^2 + 3x^3\)
24. \(6x^3 + 8x - 4 + 8 - 4x^2\)
25. \(2x + 1 - 6 + 7x^2 - x^2\)
26. \(3x - 2x + 7x^2 + 4x^3\)
Algebra: Warm Up

1) Distributive Property (use the box method to help you out):
   a. \((x + 2) (x - 5)\)
   b. \((7 - y) (y + 3)\)

2) Write an equation for the following lines:
   Line A =
   Line B =
   Line C =
It is good to Question!

Write down questions you have that are related to the mathematics being studied in this inquiry lesson. It is a good habit to ask questions. Use the bubbles to ask general questions and the branches off the bubbles to ask questions that may arise from the original inquiry.
Inquiry Lesson 1: Problems without Polyrhythm

In previous study, you explored how linear equations are important to describing numerous situations. You also identified key components such as slope and intercept from the general form $y = mx + b$.

Ever Wonder, What if . . .

What if everything in the world was composed of straight lines?

What if there were no curves, slants, circles, or spirals?

What if rules were straightforward; no one ever ‘bent the rules’?

Rhythm and beat are two essential parts to a composition of music. How many times have you said to friends that you enjoy a particular song because of the beat?! Beats occur in units called measures. Examine the following common monorhythm:

![4 beats pattern]

Polyrhythm is when two totally different rhythms are played simultaneously. Polyrhythm can be found in African music, jazz music, and even popular music.

![Polyrhythm example]

Think About This

1) How might the beats above be written linearly (ie: $y = mx + b$)
   a) 4 beats per measure
   b) 3 beats per measure

2) What are some popular songs today that have a 4 beat feel?

3) What are some other beats, besides 4 and 3, you might find in familiar songs?

In this lesson you will
- Explore how the equations of lines are the important building blocks to creating higher order functions
- Learn about parabolas
- Graph and produce charts for parabolic functions
- Learn about polyrhythm
Exploration 1: Rectangles and Circles

1) The rectangle at the right is growing at a constant rate. The length (L) is increasing by 1 ft per second while the width (W) increases at 2 ft per second.

   a) Let x represent the number of seconds. Write a linear equation that represents the changing length of the rectangle
      \[ L(x) = \]

   b) Write a linear equation that represents the changing width of the rectangle \( W(x) \)
      \[ W(x) = \]

   c) Represent each linear equation in a table and graph your points

      \[ \begin{array}{c|c|c|c}
          \text{x} & \text{L(x)} & \text{W(x)} & \text{L(x) \cdot W(x)} \\
          \hline
          -4 & \_ & \_ & \_ \\
          -3 & \_ & \_ & \_ \\
          -2 & \_ & \_ & \_ \\
          -1 & \_ & \_ & \_ \\
          0 & \_ & \_ & \_ \\
          1 & \_ & \_ & \_ \\
          2 & \_ & \_ & \_ \\
          3 & \_ & \_ & \_ \\
          4 & \_ & \_ & \_ \\
        \end{array} \]

   d) Describe the shape of the \( L(x) \cdot W(x) \) graph.

   e) Think About This: This graph is called a parabola. When length and width of a rectangle are multiplied, the area is found. If the line \( L(x) \) is multiplied by the line \( W(x) \) what do you think the parabola produced represents in terms of the rectangle?

   f) Think About This: What is the relationship between the equation of the linear functions \( L(x) \) and \( W(x) \) and the x-intercepts of the parabola above?
2) Use the following graph to fill out the chart and answer the questions:

<table>
<thead>
<tr>
<th>x</th>
<th>f1(x)</th>
<th>f2(x)</th>
<th>f1(x) · f2(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
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<td>2</td>
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<td></td>
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<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) What is the relationship between the equation of the linear functions L(x) and W(x) and the x-intercepts of the parabola above?

b) Look at the third column of both of your charts above. What similarities do you notice about the equations of each parabola?

**Exploration 2: Polyrhythm**

1) Let’s revisit our polyrhythm from above. Let \( x \) represent the number of measures and \( y \) the number of beats.

<table>
<thead>
<tr>
<th>Measure 1</th>
<th>Measure 2</th>
<th>Measure 3</th>
<th>Measure 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 beats</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
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<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>3 beats</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
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<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

a) Write a linear equation that represents 4 beats per measure.

\[ \text{Poly}1(x) = \]

b) Write a linear equation that represents 3 beats per measure.

\[ \text{Poly}2(x) = \]
2) What if a drummer of a jazz band wanted to combine two rhythms to form a new polyrhythm? The first rhythm is 3 beats per measure and starts at the beginning of the song. The second rhythm is 8 beats per measure and does not start until measure 3. The situation is depicted below:

<table>
<thead>
<tr>
<th>Measure 1</th>
<th>Measure 2</th>
<th>Measure 3</th>
<th>Measure 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 beats</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Measure 1</th>
<th>Measure 2</th>
<th>Measure 3</th>
<th>Measure 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 beats</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>

a) Write a linear equation that represents 3 beats per measure.

Poly1(x) =

b) Write a linear equation that represents 8 beats per measure starting at measure 3.

Poly2(x) =

c) Multiply the two linear equations from part (a) and part (b) to see what the parabola of this polyrhythm is.

d) Graph the parabola on your NSpire and sketch a graph of the parabola.

You Should Know:

1) When you multiply two linear equations by each other, what kind of graph is produced?

2) Describe the shape of a parabola.

3) What is a characteristic of every parabolic equation?
Algebra: x-intercepts of a Parabola

Part A

For each of the following:

a. Graph the linear functions $f_1(x)$ and $f_2(x)$ on your calculator.
b. Sketch $f_1(x)$ and $f_2(x)$, indicating the x-intercept of each.
c. Graph the product of the two linear functions $f_3(x) = f_1(x) \cdot f_2(x)$. Sketch $f_3(x)$.

1. $f_1(x) = x + 1$
   $f_2(x) = 2x + 4$

2. $f_1(x) = -x + 5$
   $f_2(x) = -0.5x + 1$

3. $f_1(x) = -2x + 3$
   $f_2(x) = 4x + 2$

4. $f_1(x) = -x - 3$
   $f_2(x) = 3x$

5. What do you notice about the x-intercepts of the graphs?
Example 1: Factor $x^2 - x - 42$

**STEP 1:** Write the $x^2$ (first) term and the constant (last) term in the box (in the lower left and upper right)

\[
\begin{array}{c|c}
  x^2 & -42 \\
  x & \hline
\end{array}
\]

**STEP 2:** Multiply these two numbers on the diagonal and place that value in the top of the diamond

\[
\begin{array}{c|c}
  x^2 & -42x^2 \\
  x & -x \\
\end{array}
\]

Also in this step, place the middle term into the bottom of the diamond

**STEP 3:** Finish the diamond (finding two values that ADD to make the bottom value and MULTIPLY to make the top value)

\[
\begin{array}{c|c|c|c}
  -7x & -42 & \hline
  x^2 & 6x \\
\end{array}
\]

Then put the answers to the diamond into the empty boxes in the square.
**STEP 4:** Perform the box method backwards

The numbers on the outside become the factors

<table>
<thead>
<tr>
<th>-7x</th>
<th>-42</th>
</tr>
</thead>
<tbody>
<tr>
<td>x^2</td>
<td>6x</td>
</tr>
</tbody>
</table>

x 6

So, the factored form looks like this: $x^2 - x - 42 = (x - 7)(x + 6)$

Example 2: Factor $5x^2 - 13x + 6$

$$5x^2 - 13x + 6 = (x - 2)(5x - 3)$$

Practice Problems

1. $2x^2 + 9x + 9 = \underline{\hspace{2cm}}$

2. $x^2 - 28x + 196 = \underline{\hspace{2cm}}$

3. $6x^2 - x - 15 = \underline{\hspace{2cm}}$

4. $4x^2 + 18 = \underline{\hspace{2cm}}$
5. \(x^2 + 18x + 81 = \) ________________

6. \(x^2 + 4x - 21 = \) ________________

7. \(3x^2 - 20x - 32 = \) ________________

8. \(3x^2 - 21 = \) ________________ (the middle term is 0x)

9. \(4x^2 + 20x + 25 = \) ________________

10. \(x^2 - 5x + 6 = \) ________________

11. \(x^2 - 12x + 36 = \) ________________

12. \(x^2 - 3x - 54 = \) ________________

13. \(2x^2 + 15x + 18 = \) ________________

14. \(16x^2 - 1 = \) ________________

15. \(x^2 - 14x + 49 = \) ________________

16. \(x^2 + 8x + 15 = \) ________________
1) Write an equation that represents the table:

The following table represents the quadratic equation \( x^2 + 2x + 5 = y \)

2) Inquire: Write down a question that you have about these tables and equations.
Algebra 1
Skill-Drill LIOF, Factoring, and Graphing

Standard form of a parabola (quadratic function):

\[ Y = ax^2 + bx + c \]

Place the following in standard form and factor:

1. \( 6x = -5 - x^2 \)
2. \( x^2 = -8x - 15 \)

Standard form: ________________

Factored Form: ________________

What is the relationship between the zeros of the graph and the factored form of a parabola?

Why is the standard form equal to 0?
3. $x^2 - 10x = -21$

Standard form: ____________________

Factored Form: ____________________

4. $3x^2 = -4x - 1$

Standard form: ____________________

Factored Form: ____________________

What is the relationship between the zeros of the graph and the factored form of a parabola?

5. $x^2 = 4$

Standard form: ____________________

Factored Form: ____________________

6. $-10x = -2x^2 + 72$

Standard form: ____________________

Factored Form: ____________________
7. $12x^2 - x = 6$

Standard form: ________________

Factored Form: ________________

8. $x^2 - 6x = -9$

Standard form: ________________

Factored Form: ________________

What is the relationship between the zeros of the graph and the factored form of a parabola?

9. $x^2 - 5x = 36$

Standard form: ________________

Factored Form: ________________

10. $2x^2 = 50$

Standard form: ________________

Factored Form: ________________
1) Think about the y values of a table

   a) If the **first difference** of the y values of a table are the same, what kind of equation would result?
      
      A. Quadratic Function  
      B. Linear Function  
      C. Absolute Value Function

   b) What if the **second difference** were the same?
      
      A. Quadratic Function  
      B. Linear Function  
      C. Absolute Value Function

2) Write the factored form of the following quadratic equation: \(x^2 + 4x + 4\)

3) **Review:** Solve \(|x + 2| = 4\)
**Relationships of Quadratic Functions**

**Graph**
- Why is the graph curved?
- What is a parabola?
- Why does multiplying two lines “look like” a V?
- Why isn’t f1(x) x f2(x) an absolute value “V” shape?

**Tables**
- Why are there two differences in the chart for a quadratic equation?
- What is ‘finding the difference’ in the y’s of a chart called?
- Why is the first difference in the chart for a quadratic equation not equal?
- Why do we not look at the x’s?
- How can you tell whether the table is a parabola or not without knowing the equation?
- How do you know it is a parabola?
- How do you get an equation out of the table?

**Factored form and x-intercepts**
- Why does the factored form use opposite numbers?
- Why does the graph cross at those numbers used in factored form?
- What is the point of having factored form and standard form?
- Why is this relationship important?
- What does the zero do?

**Linear Equations**
Algebra: Rediscovering Inquiry 1

Group Members

______________
______________
______________
______________

Standard Form

Factored Form

Graph:

x-Intercepts:

x = ______  x = ______

y-intercepts:

y = ______

Table

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Relationships

Between x-intercepts and table
1) ___________________________________________________________
2) ___________________________________________________________

Between table and graph
1) ___________________________________________________________
2) ___________________________________________________________
Between factored form and graph
1)_
2)_

Between factored form and table
1)_
2)_

Between Linear functions and factored form
1)_
2)_

Between Linear functions and parabolas
1)_
2)_

Question
_________________________________________________________
_________________________________________________________
_________________________________________________________
Algebra: Quiz Review

*(2 percentage pts on quiz if completed)*

**Distribute (Use the box method)**

1. \(3(x + 4) = \) ____________  
2. \((4 + 3y)5 = \) ____________  
3. \(-2x(x - 8) = \) ____________

4. \((y + 3)(y - 2) = \) ____________  
5. \((x + 6)(x - 7) = \) ____________  
6. \((3 - y)(4 - 8) = \) ____________

**Place into factored form (use the box method)**

1. \(2x^2 + 9x + 9 = \) ____________  
2. \(x^2 - 28x + 196 = \) ____________

3. \(6x^2 - x - 15 = \) ____________  
4. \(4x^2 + 18 = \) ____________

5. \(x^2 + 18x + 81 = \) ____________  
6. \(x^2 + 4x - 21 = \) ____________

7. \(3x^2 - 20x - 32 = \) ____________  
8. \(3x^2 - 21 = \) ____________ (the middle term is 0x)
Place the following quadratics in standard form \((y = ax^2 + bx + c)\) and Graph

1. \(-x - 6 = y - x^2\)
   Standard Form ____________________

2. \(x^2 = 25 + y\)
   Standard Form ____________________

3. \(y + 10x - 16 = x^2\)
   Standard Form ____________________

4. \(x^2 + 10x + 9 = y\)
   Standard Form ____________________
Questions: Answer in complete sentences

1) When you multiply two linear equations together, what results? Give an example of two linear equations that do this.

2) Describe, in complete sentences, what the shape of a parabola looks like.

3) What is the relationship between factored form and the x-intercepts of a graph?

4) Name some characteristics of every quadratic equation.

5) What is the relationship between two linear equations and factored form?

6) What is true about the differences in the y values of the table of a quadratic equation?
Algebra: Warm Up

Name ______________________

Solve the following system:
\[
\begin{align*}
x &= 4y + 2 \\
y &= 5x - 1
\end{align*}
\]

Tell whether the following are functions, and then give the domain and range

Function?     Y     N  Function?     Y      N  Function?     Y      N
Domain________________  Domain_______________  Domain______________
Range _________________  Range________________  Range________________

Algebra: Warm Up

Name ______________________

Solve the following system:
\[
\begin{align*}
x &= 4y + 2 \\
y &= 5x - 1
\end{align*}
\]

Tell whether the following are functions, and then give the domain and range

Function?     Y     N  Function?     Y      N  Function?     Y      N
Domain________________  Domain_______________  Domain______________
Range _________________  Range________________  Range________________
Algebra: Warm up (pre-assessment)

1. Draw a line of symmetry through the following parabolas:

2. Draw a graph of the following situation:
   - You throw a water balloon from a window
   - You jump in a long jump event for track
Algebra: Quiz 1
Quadratics

Indicate which family from the given chart, table, or graph: Quadratic, Linear, or Absolute Value (1pt each)

1. Family: ______________

2. Family: ______________

3. Family: ______________

4. Family: ______________

5. Family: ______________

6. Family: ______________

7. Family: ______________

8. Family: ______________

* (Hint for the tables think about the first and second differences)

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>

9. Family: ______________

<table>
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<tr>
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<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
</tr>
</tbody>
</table>

10. Family: ______________
Distribute the following and place in standard form (1 pt distribute, 1pt standard form)

11. \((x + 5) (x - 5) = y\)

12. \((3 - x) (x - 7) = y\)

Factor the following and indicate the x-intercepts (1 pt factored, 2 pts x-intercepts)

13. \(x^2 + 4x - 21 = \) ________________
   \(X = _____ \quad X = _____\)

14. \(x^2 - 64 = \) ________________
   \(X = _____ \quad X = _____\)

15. \(x^2 + 5x + 6 = \) ________________
   \(X = _____ \quad X = _____\)

Use your NSpire to graph the following (5 pts)

16. Standard Form: Graph \(x^2 - x - 2 = y\)

17. Factored Form: ________________

18. X-intercepts:
   \(X = _____ \quad X = _____\)

19. Make a table:

<table>
<thead>
<tr>
<th></th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Algebra: Warm Up

1) Find the x-intercepts by placing the following into factored form: \( x^2 + 4x - 21 = y \)

Factored Form __________________

\( X = ________ \)

\( X = _______ \)

2) Given the following graphs:

   ![Graph 1](image1.png)

   \( \text{Domain} \)___________________
   \( \text{Range} \)___________________

   ![Graph 2](image2.png)

   \( \text{Domain} \)___________________
   \( \text{Range} \)___________________

3) Write a quadratic equation in factored form knowing that the x-intercepts were 4 and -2?

Algebra: Warm Up

1) Find the x-intercepts by placing the following into factored form: \( x^2 + 4x - 21 = y \)

Factored Form __________________

\( X = ________ \)

\( X = _______ \)

2) Given the following graphs:

   ![Graph 1](image1.png)

   \( \text{Domain} \)___________________
   \( \text{Range} \)___________________

   ![Graph 2](image2.png)

   \( \text{Domain} \)___________________
   \( \text{Range} \)___________________

3) Write a quadratic equation in factored form knowing that the x-intercepts were 4 and -2?
Suzie Q. and her twin sister Suzie X. are going on a totally radical vacation for spring break. They decided to take a hop, skip, and a jump down to the luscious land of Jamaica maaaaan. They absolutely love to take pictures when they go on trips so that their memories will forever be caught on film. They say that a picture is worth 1000 words; Suzie squared has about a googolplex worth of words in this case!!!

Anyways, when Suzie squared came back from the trip, they reminisced about their travels by flipping through their photos. As they thumbed through, a strange occurrence kept showing up; there were parabolic happenings caught on film!! Oh, how the amazing world of math likes to sneak its way into our everyday lives, including vacations! The reason that they made the connection between parabolas and the events that were happening in their photos was because their amazing math teacher opened up a whole new world of quadratic functions before they went on spring break (kind of like the time Aladdin opened a whole new world for Jasmine....Look what that led to....unconditional love!! Get the correlation I am making....whole new world....math....loving math) Their teacher told them not to forget what they had learned about quadratics while they were on vacation. HINT HINT, WINK, WINK, NUDGE NUDGE. Sure enough, those parabolas were everywhere to be found! Here are some of the pictures that Suzie squared took while they were on vacation. They all represent parabolas!

Graph 1 - Reggae music with the locals:

Graph 2 - Swimming with Shamu and Flipper:

Graph 3 - A coconut fell and almost hit Suzie X on the head!
Write a story about your spring break adventure. Think of the parabolas that could occur on your trip.

Qualitative Graphing

Include the following:
1) 5 graphs from your story that represent a parabola
2) Labeled axis
3) Graph title
4) Explanation of the mathematics and real life application (any maximums, minimums, zeros, y-intercepts).

What do these critical points represent in real life?
1. Tell whether the following are linear, quadratic, or absolute value. Then **justify**.

\[ Y = (x + 2)(x + 5) \]

Family ______________
Justify:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
</tr>
</tbody>
</table>

Family ______________
Justify:

2. Find the solutions (aka x-intercepts) of the following quadratic equation by factoring: \( x^2 - x - 2 = y \)
Write down questions you have that are related to the mathematics being studied in this inquiry lesson. It is a good habit to ask questions. Use the bubbles to ask general questions and the branches off the bubbles to ask questions that may arise from the original inquiry.
Inquiry Lesson 2: MA-THEMATICS

In the previous inquiry lesson, you explored how linear equations compose the building blocks of a quadratic function. You also found that all parabolas contain a squared component to their equations. You discovered that there were important relationships between quadratic equations, x-intercepts, and tables.

Ever Wonder, What If . . .

What if you there was only one version of ‘Have Yourself a Merry Little Christmas’ that played during the holidays?
What if our bodies never grew and transformed?
What if you could never alter the ‘original’ of something?

Thematic transformation is a very long term that describes a type of variation on music. If you break the phrase down, it becomes easier to understand. “Thematic” comes from the idea of a theme. Think how weird it would be to watch your favorite TV show without the theme song to introduce it. Transformation is a change in the formal appearance of something. For example, if you were to change your hair style by dying it blue and creating long spikes, your hair is said to have gone through a transformation; it remains your hair, but it is changed.

Themes exist in music scores of films too! Often times, movies will present a couple of reoccurring themes throughout the plot. These familiar tunes develop and change by using a number of transformative techniques to alter the rhythm or character of the sound. A movie score composer can essentially drag the audience through a slew of emotions by using the original theme, but tweaking it a little.

The following are popular movie scores that use thematic transformation:

Pirates of the Caribbean  Star Wars  Disney’s UP  Shrek

Think About This

1) How might you change a particular theme song to impose the following ‘feel’:
   a) Happy
   b) Sad
   c) Racing
   d) Slow
2) Can you name some other popular movie scores that use thematic transformation?
3) How might thematic transformation compare to transformations of a graph?

In this lesson you will
- Learn what the vertex of a parabola is and why it is important
- Explore the axis of symmetry on a parabola
- Learn what happens when you changing the coefficients of a, b, and c of \( y = ax^2 + bx + c \)
- Learn about transformations of the Parent quadratic function
Exploration 1: ABC EASY AS 123

1. Vocabulary – Write each definition in your own words
   
   a. Parabola

   b. Parent Function for Quadratic

   c. Vertex

   d. Axis of symmetry

   e. Standard Form for Quadratic

2. Graph the Parent graph \( y = x^2 \)

<table>
<thead>
<tr>
<th>a</th>
<th>( f(x) = ax^2 )</th>
<th>Graph</th>
</tr>
</thead>
</table>
| 3  | \( f(x) = 3x^2 \)                 | a. What is the vertex? ___________  
|    |                                   | b. Does the graph appear wider (outside) or narrower (inside) the graph of \( Y = x^2 \) ___________  
|    |                                   | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________  |
| 2  | \( f(x) = 2x^2 \)                 | a. What is the vertex? ___________  
|    |                                   | b. Does the graph appear wider (outside) or narrower (inside) the graph of \( Y = x^2 \) ___________  
|    |                                   | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________  |
| 1  | \( f(x) = x^2 \)                  | a. What is the vertex? ___________  
|    |                                   | b. Does the graph appear wider (outside) or narrower (inside) the graph of \( Y = x^2 \) ___________  
|    |                                   | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________  |
| -1 | \( f(x) = -x^2 \)                 | a. What is the vertex? ___________  
|    |                                   | b. Does the graph appear wider (outside) or narrower (inside) the graph of \( Y = x^2 \) ___________  
|    |                                   | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________  |
| -2 | \( f(x) = -2x^2 \)                | a. What is the vertex? ___________  
|    |                                   | b. Does the graph appear wider (outside) or narrower (inside) the graph of \( Y = x^2 \) ___________  
|    |                                   | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________  |
| -3 | \( f(x) = -3x^2 \)                | a. What is the vertex? ___________  
|    |                                   | b. Does the graph appear wider (outside) or narrower (inside) the graph of \( Y = x^2 \) ___________  
|    |                                   | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________  |
3. What relationship do you notice between the vertex of each graph and the axis of symmetry?

4. Using the sliders on your NSpire, discover what happens to the parent function as the values of \( a \) change.

   a) Indicate this change in motion by placing arrows on the following graph depicting the direction the graph moves from the parent function:
   
   i) Negative number for \( a \)  
   
   ii) Positive number for \( a \)

   b) Write a sentence describing the effects changing the value of \( a \) has on the graph.

Again, graph the Parent graph \( y = x^2 \)

<table>
<thead>
<tr>
<th>( b )</th>
<th>( f(x) = x^2 + bx )</th>
<th>Graph</th>
</tr>
</thead>
</table>
| 3 | \( f(x) = x^2 + 3x \) | a. What is the vertex? ____________  
 b. How does the graph change from the parent function \( Y = x^2 \) ____________
 c. Does the axis of symmetry change? ____________ Write an equation for the axis ____________ |
| 2 | \( f(x) = x^2 + 2x \) | a. What is the vertex? ____________  
 b. How does the graph change from the parent function \( Y = x^2 \) ____________
 c. Does the axis of symmetry change? ____________ Write an equation for the axis ____________ |
| 1 | \( f(x) = x^2 + x \) | a. What is the vertex? ____________  
 b. How does the graph change from the parent function \( Y = x^2 \) ____________
 c. Does the axis of symmetry change? ____________ Write an equation for the axis ____________ |
| -1 | \( f(x) = x^2 - x \) | a. What is the vertex? ____________  
 b. How does the graph change from the parent function \( Y = x^2 \) ____________
 c. Does the axis of symmetry change? ____________ Write an equation for the axis ____________ |
| -2 | \( f(x) = x^2 - 2x \) | a. What is the vertex? ____________  
 b. How does the graph change from the parent function \( Y = x^2 \) ____________
 c. Does the axis of symmetry change? ____________ Write an equation for the axis ____________ |
| -3 | \( f(x) = x^2 - 3x \) | a. What is the vertex? ____________  
 b. How does the graph change from the parent function \( Y = x^2 \) ____________
 c. Does the axis of symmetry change? ____________ Write an equation for the axis ____________ |
5. What relationship do you notice between the vertex of each graph and the axis of symmetry?

6. Using the sliders on your Nspire, discover what happens to the parent function as the values of $b$ change.

   a) Indicate this change in motion by placing arrows on the following graph depicting the direction the graph moves from the parent function:

   i) Negative number for $b$ → → →

   ii) Positive number for $b$ → → →

b) Write a sentence describing the effects changing the value of $b$ has on the graph.

Again, graph the Parent graph $y = x^2$

<table>
<thead>
<tr>
<th>$c$</th>
<th>$f(x) = x^2 + c$</th>
<th>Graph</th>
</tr>
</thead>
</table>
| 3   | $f(x) = x^2 + 3$ | a. What is the vertex? ___________
      |                 | b. How does the graph change from the parent function $Y = x^2$ ___________
      |                 | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________ |
| 2   | $f(x) = x^2 + 2$ | a. What is the vertex? ___________
      |                 | b. How does the graph change from the parent function $Y = x^2$ ___________
      |                 | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________ |
| 1   | $f(x) = x^2 + 1$ | a. What is the vertex? ___________
      |                 | b. How does the graph change from the parent function $Y = x^2$ ___________
      |                 | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________ |
| -1  | $f(x) = x^2 - 1$ | a. What is the vertex? ___________
      |                 | b. How does the graph change from the parent function $Y = x^2$ ___________
      |                 | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________ |
| -2  | $f(x) = x^2 - 2$ | a. What is the vertex? ___________
      |                 | b. How does the graph change from the parent function $Y = x^2$ ___________
      |                 | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________ |
| -3  | $f(x) = x^2 - 3$ | a. What is the vertex? ___________
      |                 | b. How does the graph change from the parent function $Y = x^2$ ___________
      |                 | c. Does the axis of symmetry change? ___________ Write an equation for the axis ___________ |
7. What relationship do you notice between the vertex of each graph and the axis of symmetry?

8. Using the sliders on your NSpire, discover what happens to the parent function as the values of \( c \) change.

a) Indicate this change in motion by placing arrows on the following graph depicting the direction the graph moves from the parent function:

i) Negative number for \( c \)  

ii) Positive number for \( c \)

b) Write a sentence describing the effects changing the value of \( c \) has on the graph.

**Exploration 2: Thematic Transformations in Music Scores**

Let’s revisit the idea of a movie score theme and the transformation of that theme. Figure 1 below shows the music for the Pirates of the Caribbean theme from the movie.

**Figure 1: Theme**

1. How might the Pirates of the Caribbean theme song be similar to a parent function?
Now, take a look at figure 2. This figure shows a different variation on the Pirates of the Caribbean theme. Notice that the notes are written differently. For example, the time in which the music is played is at a slower pace; Figure 2 depicts 4/4 time (four beats per measure) while figure 1 depicts ¾ time (three beats per measure).

**Figure 2: Variation on Theme**

2. The following chart names some differences and similarities between the theme (figure 1) and the variation (figure 2). How do you think these differences or similarities affect the way the music sounds?

<table>
<thead>
<tr>
<th>Changes</th>
<th>Sound of the music</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theme (figure 1)</td>
<td>Variation (Figure 2)</td>
</tr>
<tr>
<td>Written in ¾ faster time</td>
<td>Written in 4/4 slower time</td>
</tr>
<tr>
<td>Expression is a ‘march’ feel</td>
<td>Expression is flowing</td>
</tr>
<tr>
<td>More movement with notes because note lengths are smaller</td>
<td>Less movement with notes because note lengths are larger</td>
</tr>
</tbody>
</table>

**Similarities**

Same note structure

---

**You Should Know:**

1) Explain in words what happens to the parent graph of a quadratic function $y = ax^2 + bx + c$ if you:

   a. Change $a$

   b. Change $b$

   c. Change $c$

2) How does the axis of symmetry relate to the vertex of a parabola?

3) What is a parent function and what does it mean to transform that parent function?
In words, describe what happens to a quadratic function in standard form $y = ax^2 + bx + c$ when you change $a$, $b$, and $c$? (6 pts: 2 pts for describing each transformation in detail)

__________________________________________________________________________________________________
__________________________________________________________________________________________________
__________________________________________________________________________________________________
__________________________________________________________________________________________________
__________________________________________________________________________________________________
__________________________________________________________________________________________________
__________________________________________________________________________________________________

What form are the following quadratic equations in? **standard form, factored form, vertex form**

(x + 9)(x – 7) = y

Form: _________________

$y = 6x^2 + 4x + 3$

Form: _________________

$y = 3(x – 5)^2 + 2$

Form: _________________
There are many themes in movie music scores. Choose from the movie soundtracks below (or another soundtrack):

- Star Wars
- Lord of the Rings
- Shrek
- Indiana Jones
- Harry Potter
- Batman

Soundtrack ___________________________

1. Listen to the main Theme
   Record the time of the sound clip
   Describe the theme music in words
   \[(\text{Time } \underline{___} \underline{___} - \underline{___} ___)\]

2. Listen to a Variation I on the Theme
   Record the time of the sound clip
   Describe in words how the theme music changed
   \[(\text{Time } \underline{___} \underline{___} - \underline{___} ___)\]

3. Graph the following on the same axis:
   \[f(x) = ax^2\]
   - \[f(x) = 2x^2\]
   - \[f(x) = 4x^2\]
   - \[f(x) = -5x^2\]

The theme is like the Parent function; everything stems from it.
5. Listen to a *Variation II* on the Theme
   Record the time of the sound clip
   Describe in words how the theme music changed

6. Graph the following on the same axis:

   \[ f(x) = x^2 + bx \]

   \[ f(x) = x^2 + 4x \]
   \[ f(x) = x^2 - 10x \]
   \[ f(x) = x^2 - 2x \]

7. Transformation II
   Describe in words the effect of changing the value of \( b \)

8. Listen to a *Variation III* on the Theme
   Record the time of the sound clip
   Describe in words how the theme music changed

9. Graph the following on the same axis:

   \[ f(x) = x^2 + c \]

   \[ f(x) = x^2 + 5 \]
   \[ f(x) = x^2 - 3 \]
   \[ f(x) = x^2 + 2 \]

10. Transformation III
    Describe in words the effect of changing the value of \( c \)
Algebra 1
Skill-Drill Radicals and More

Solve the following. Place into simplest form

1. $6^2 + \sqrt{25}$
2. $\frac{3+5}{\sqrt{4}}$
3. $3 \pm \sqrt{4^2 + 9}$
4. $\sqrt{100} + 7$
5. $-5 \pm \sqrt{5^2 - 2 \times 0}$
6. $\sqrt{10 + 10 \times 9}$
7. $\frac{\sqrt{16}}{\sqrt{25}}$
8. $\frac{1+2\sqrt{3}}{4}$
9. $\sqrt{64}$
10. $\pm \frac{\sqrt{36}}{2}$
11. $\sqrt{4 \times 5 - 4}$
12. $\frac{3\pm\sqrt{3^2 - 5 \times 2 \times 2}}{2 \times 1}$

Evaluate $\sqrt{b^2 - 4ac}$ when $a = 1$, $b = -2$, and $c = -3$

Evaluate $\sqrt{6a + b}$ when $a = 5$ and $b = 6$
Evaluate \( \frac{10 \pm 2\sqrt{b}}{a} \) when \( a = 2 \) and \( b = 4 \)

Evaluate \( \sqrt{b^2 - 4ac} \) when \( a = 4 \), \( b = 5 \), and \( c = 1 \)

### Solve

1. \( 3x^2 - 48 = 0 \)  
2. \( x^2 - 25 = 0 \)  
3. \( 2x^2 - 8 = 0 \)
4. \( 6x^2 - 54 = 0 \)  
5. \( 4x^2 - 3 = 57 \)  
6. \( 5x^2 + 10 = 20 \)

### Review

Factor the following and tell what the \( x \)-intercepts are

1. \( 2x^2 + 9x + 9 = \) ________________  
2. \( x^2 - 28x + 196 = \) ________________
3. \( 6x^2 - x - 15 = \) ________________  
4. \( 4x^2 + 18 = \) ________________
Algebra: Vertex Form

Name___________________________

Vertex form of a parabola (quadratic equation)

\[ f(x) = a(x - h)^2 + k \]

1. Graph to the right \( f(x) = (x - 3)^2 + 3 \)
   - a. Vertex? _________________
   - b. Relationship between vertex and formula
     _____________________________________________
   - c. Line of symmetry equation?_________________
   - d. Graph open up or down?___________________

2. Graph to the right \( f(x) = (x + 2)^2 - 1 \)
   - a. Vertex? _________________
   - b. Relationship between vertex and formula
     _____________________________________________
   - c. Line of symmetry equation?_________________
   - d. Graph open up or down?___________________

3. Relationships between vertex form of a parabola and vertex on the graph.
   a_________________________________________________________________________________________________
   b_____________________________________________________________________
   ______________________________

4. Graph to the right \( f(x) = -5 (x - 3)^2 + 3 \)
   - a. Vertex? _________________
   - b. Line of symmetry equation?_________________
   - c. Graph open up or down?___________________
   - d. How is this equation similar to #1?_________________
     How is it different? _________________________

5. Graph to the right \( f(x) = -2 (x + 2)^2 - 1 \)
   - a. Vertex? _________________
   - b. Line of symmetry equation?_________________
   - c. Graph open up or down?___________________
   - d. How is this equation similar to #2?_________________
     How is it different? _________________________
6. Relationships between vertex form of a parabola and whether the parabola opens up or down.
   a.____________________________________________________________________________________________
   b.____________________________________________________________________________________________

7. Graph to the right $f(x) = -\frac{1}{2}(x - 3)^2 + 3$
   a. Vertex? _________________
   b. Line of symmetry equation?_________________
   c. Graph open up or down?_________________
   d. How is this equation similar to
      #4?_________________
      How is it different? _________________

8. Graph to the right $f(x) = -\frac{1}{4}(x + 2)^2 - 1$
   a. Vertex? _________________
   b. Line of symmetry equation?_________________
   c. Graph open up or down?_________________
   d. How is this equation similar to
      #5?_________________
      How is it different? _________________

9. Relationships between vertex form of a parabola and how wide the graph is.
   a.___________________________________________________________________________________________
   b.___________________________________________________________________________________________

10. Give an example of a function in vertex form that has the following qualities:
    a. Graph opens up
    b. Vertex at (3, 8)
    c. Graph 6 units wide
    Vertex Form: ________________________

11. Give an example of a function in vertex form that has the following qualities:
    a. Graph opens down
    b. Vertex at (-9, 6)
    c. Graph 0.75 units narrow
    Vertex Form: ________________________

12. Give an example of a function in vertex form that has the following qualities:
    a. Graph opens down
    b. Vertex at (-1, -8)
    c. Graph 1 unit wide
    Vertex Form: ________________________
Complete the table. What happens to the graph given the following:

<table>
<thead>
<tr>
<th>Parabola Form</th>
<th>$y = a(x - h)^2 + k$ (vertex)</th>
<th>$Y = ax^2 + bx + c$ (standard)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative h or b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive h or b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative k or c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive K or c</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Find the x-intercepts of the following quadratic: \( y = x^2 - 2x - 3 \)

Give two characteristics of the graph that has the equation \( y = -2(x - 7)^2 + 6 \) (hint: think vertex form)

How many solutions does \( x = 0 \) have for the following graphs (zero, one, or two)?

Solutions ________________

Solutions ________________

Solutions ________________
It is good to Question!

What did you learn?

Write down questions you have that are related to the mathematics being studied in this inquiry lesson. It is a good habit to ask questions. Use the bubbles to ask general questions and the branches off the bubbles to ask questions that may arise from the original inquiry.
In the previous inquiry, you saw the different transformations a parabola can undergo by changing a, b, and c in the standard form for a quadratic equation $y = ax^2 + bx + c$. You also explored what the vertex and axis of symmetry was on a parabola.

Ever Wonder, What If...?
What if you were not able to create a model or diagram of something (ie: a map, or a science concept)?
What if there were no role models in the world?
What if everything was a perfect representation?

What makes music so enjoyable? We can attribute the notion that music is pleasing to the ears on the fact that a single artist is simply good at singing that high note, or an all-star saxophonist can kill a solo. What about the back up singers and instrumentalists?! A solo is nothing without those sweet harmonies mixed in with melodies! We can measure harmony mathematically through frequencies. Each note on a scale has a frequency.

When we hear a note, most people would say that they only hear that one note. The fact is that there are other notes built on top of that one note when it is sung or played alone; these notes are called overtones. The overtone sounds are harmonically related to that original note.

If you were to graph the overtones according to rank, they would form a close representation of a partial parabolic function. The following graph is a model of the points that make up the overtones.
Think About This
1) What is the significance of creating a model from chaotic or random data?
2) What other real-world situations might be modeled by a parabolic shape and quadratic function?
3) How might you formulate an equation that can accommodate most of the points on a scatter plot?

In this Lesson you will
- Model real-world situations to a quadratic function pattern
- Make inferences about modeled data
- Use your knowledge of vertex form, standard form, and factored form
- Use your knowledge of transformations
- Learn about overtones and undertones

Exploration 1: Models in the real world

Recall the three different forms for writing quadratic equations:

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Factored Form</th>
<th>Vertex Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = ax² + bx + c</td>
<td>y = (x + a)(x - b)</td>
<td>y = a(x - h)² + k</td>
</tr>
</tbody>
</table>

1. The St. Louis arch is a popular structure that takes the shape of a parabola. Its highest point reaches to 630 feet. The structure spans a width of 630 feet. The arch has a tram that acts as an elevator. The tram moves inside the parabola shape to transport people around the structure. What if the tram got stuck and repairmen needed to get to the tram? They would need to know the exact place on the parabolic structure to fix the problem. Let's model this parabola.

Given The graph on the right has three points that this situation gives us:

1) The highest point (315 ft, 630 ft)
2) The starting point (0 ft, 0 ft)
3) The ending point (630 ft, 0 ft)
**Form** What form of the quadratic equation would most help us model this situation from this information? Why?

**Model** Use your NSpire and knowledge of transformations to model the St. Louis Arch.

**Use your model**

Equation______________________

X represents? ____________________

Y represent?_____________________

1) Use your model to find where the truck team is if the tram is stuck at a height of 300 feet in the air?

2) Use your model to find how high a repairman would have to reach if the repair truck was on the ground at the 410 foot mark?

2. The stunts that a kite boarder performs can be described by a parabolic shape. A professional kite boarder wants to always know what their record height is, so they can try and beat it next time. Let’s model the parabola.

**Given** The table shows certain heights and times of the kite boarder

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Height (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.05</td>
<td>0.71</td>
</tr>
<tr>
<td>0.15</td>
<td>1.89</td>
</tr>
<tr>
<td>0.3</td>
<td>3.06</td>
</tr>
<tr>
<td>0.469</td>
<td>3.52</td>
</tr>
<tr>
<td>0.5</td>
<td>2.66</td>
</tr>
<tr>
<td>0.8</td>
<td>1.76</td>
</tr>
<tr>
<td>0.85</td>
<td>1.19</td>
</tr>
<tr>
<td>0.9</td>
<td>0.54</td>
</tr>
<tr>
<td>0.938</td>
<td>0</td>
</tr>
</tbody>
</table>

**Form** What form of the quadratic equation would most help us model this situation from this information? Why?

**Model** Use your Nspire and knowledge of transformations to move the parent function onto the path of the kite boarder’s height.

**Use your model**

Equation______________________

X represents? ____________________

Y represent?_____________________

1) Use your model and your chart to approximate the highest point at which the kite boarder reaches?

2) Use your model to find how high the kite boarder will be after 0.5 seconds?
3) At what time(s) will the kite boarder be touching the water?

**Exploration 2: Overtones**

Let’s revisit our overtone graphs. Sound can be measured by numbers. For example, we can measure how loud or soft something sounds to the ear via the amount of decibels a sound gives off. Sound can also be measured according to pitch or tone. We measure pitch with frequencies. The following charts give the frequencies of the first overtones from a given starting note C:

**Given** The table shows 1\textsuperscript{st} overtone frequencies of select notes.

<table>
<thead>
<tr>
<th>Notes away from CO</th>
<th>Overtone 1 Frequency (hertz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32.7</td>
</tr>
<tr>
<td>13</td>
<td>65.4</td>
</tr>
<tr>
<td>25</td>
<td>130.82</td>
</tr>
<tr>
<td>37</td>
<td>261.62</td>
</tr>
<tr>
<td>49</td>
<td>523.26</td>
</tr>
<tr>
<td>61</td>
<td>1046.5</td>
</tr>
</tbody>
</table>

**Form** What form of the quadratic equation would most help us model this situation from this information? Why?

**Model** Use your Nspire and knowledge of transformations to move the parent function onto the path of the kite boarder’s height.

**Use your model**

Equation ________________________

X represents? ________________________

Y represent?________________________

1) What would be the 1\textsuperscript{st} frequency of the note Ab if it is 8 notes away from CO.

2) How many notes away is an Eb if the 1\textsuperscript{st} overtone frequency is 329.62?

**Given** The table shows 2\textsuperscript{nd} overtone frequencies of select notes.

<table>
<thead>
<tr>
<th>Notes away from CO</th>
<th>Overtone 2 Frequency (hertz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>49.05</td>
</tr>
<tr>
<td>13</td>
<td>98.1</td>
</tr>
<tr>
<td>25</td>
<td>196.23</td>
</tr>
<tr>
<td>37</td>
<td>392.43</td>
</tr>
<tr>
<td>49</td>
<td>784.89</td>
</tr>
<tr>
<td>61</td>
<td>1569.8</td>
</tr>
</tbody>
</table>

**Form** What form of the quadratic equation would most help us model this situation from this information? Why?
**Model** Use your Nspire and knowledge of transformations to move the parent function onto the path of the kite boarder’s height.

**Use your model**

Equation______________________
X represents? ______________________
Y represent?_______________________

1) What would be the 2nd frequency of the note F if it is 32 notes away from CO.

2) How many notes away is an Bb if the 2nd overtone frequency is 330?

**You Should Know:**

1) How do you determine which form (factored, standard, or vertex) is best to model data?
   a) Factored Form
   b) Standard Form
   c) Vertex Form

2) How do transformations help us to model a situation?

3) What is the relationship between modeling and analyzing information?
Algebra: Quiz Review

*(2 percentage pts on quiz if completed)*

Distribute (Use the box method)

1. \((y + 3)(y - 2) = \) ______________
2. \((x + 6)(x - 7) = \) ______________
3. \((3 - y)(4 - 8) = \) ______________

Give the zeros of the graph

1. \(2x^2 + 9x + 9 = \) ______________
2. \(x^2 - 28x + 196 = \) ______________
3. \(x^2 + x - 2 = \) ______________
4. \(x^2 - 15x + 56 = \) ______________

Place the following quadratics in standard form \((y = ax^2 + bx + c)\) and Graph

1. \(-x - 6 = y - x^2\)
   Standard Form ______________

2. \(x^2 = 25 + y\)
   Standard Form ______________
Write an equation that has the following criteria (use vertex form, standard form, or factored form)

a. Vertex at (6,7)  
  a. Vertex at (3,-5)  
  b. Parabola opens down  
  b. Parabola opens up  
  c. Stretch of 0.5  
  c. Narrow of 7

Equation__________________  
Equation__________________

a. zeros at x = 5 and x = -2  
  a. Zeros at x = 4 and x = 9  
  b. Parabola opens up  
  b. Parabola opens down

Equation__________________  
Equation__________________

Predict how each graph in standard form will change if we start with the parent function \( y = x^2 \)

\[
\begin{align*}
    f(x) &= x^2 - 2x \\
    f(x) &= x^2 - 8 \\
    f(x) &= 2x^2 \\
    f(x) &= x^2 + 4 \\
    f(x) &= 5x^2 \\
    f(x) &= x^2 + 4x
\end{align*}
\]

<table>
<thead>
<tr>
<th>Parabola Form</th>
<th>( y = a(x - h)^2 + k ) (vertex)</th>
<th>( Y = ax^2 + bx + c ) (standard)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative h or b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive h or b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative k or c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive K or c</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solve the following Radicals

1. \( \pm \sqrt{36} \) 
2. \( \sqrt{4 \times 5 - 4} \) 
3. \( \frac{3 \pm \sqrt{3^2 - 5 \times 2 \times 2}}{2 \times 1} \)

4. \( 3x^2 - 48 = 0 \) 
5. \( x^2 - 25 = 0 \)

Questions: Answer in complete sentences

1) What is true about the differences in the y values of the table of a quadratic equation?

2) How does the axis of symmetry relate to the vertex of a parabola?

3) When you multiply two linear equations together, what results? Give an example of two linear equations that do this.

4) What is the relationship between factored form and the x-intercepts of a graph?

5) Name three situations in which you might use standard form, vertex form, or factored form to model a situation.
   - Standard Form:
   - Vertex Form:
   - Factored Form:
Algebra: Warm Up

1. Evaluate \( \frac{10 \pm 2\sqrt{b}}{a} \) when \( a = 2 \) and \( b = 4 \)  
2. Evaluate \( \sqrt{b^2 - 4ac} \) when \( a = 4 \), \( b = 5 \), and \( c = 1 \)

Domain ______________________
Range ______________________

Clifton Bridge Height

Domain ______________________
Range ______________________

1. Evaluate \( \frac{10 \pm 2\sqrt{b}}{a} \) when \( a = 2 \) and \( b = 4 \)  
2. Evaluate \( \sqrt{b^2 - 4ac} \) when \( a = 4 \), \( b = 5 \), and \( c = 1 \)

Domain ______________________
Range ______________________

Clifton Bridge Height

Domain ______________________
Range ______________________
Quadratics

Describe what the parabolic will look like given the following form of a quadratic equation (1pt per description)

1. \( y = -0.5 (x - 4)^2 + 5 \)  
   Form________________  
   3 Descriptions of parabola  
   •  
   •  
   •  

2. \( y = 6x^2 + 2x - 3 \)  
   Form________________  
   4 Descriptions of parabola  
   •  
   •  
   •  
   •  

3. \( y = 7 (x + 3)^2 - 6 \)  
   Form________________  
   3 Descriptions of parabola  
   •  
   •  
   •  

4. \( y = (x - 8) (x + 3) \)  
   Form __________________  
   2 Descriptions of parabola  
   •  
   •  

What are the \( x \)-intercepts of the following quadratic equations, find algebraically (2 pts each)

5. \( y = x^2 - 6x - 7 \)  
   \[ x = _____ \]  
   \[ x = _____ \]

6. \( y = x^2 + x - 12 \)  
   \[ x = _____ \]  
   \[ x = _____ \]

Given the following form, tell what the equation for the line of symmetry will be (2 pts each)

7. \( y = 3 (x -2)^2 + 6 \)  
   Vertex ____________  
   Axis of symmetry equation ____________

8. \( Y = -2 (x + 7)^2 - 4 \)  
   Vertex ____________  
   Axis of symmetry equation ____________
Justify which form of a quadratic you would use in the following situation. Give an equation that might model the situation (3 pts)

9. A basketball player wants to analyze their free throw shot and so they videotape themselves shooting a ball. The ball travels in the shape of a parabola shown at the right. The shooter knows that the ball’s highest point is at (10ft, 6ft).

Form ________________

Justify

Equation ________________

Indicate which family from the given chart, table, or graph: Quadratic, Linear, or Absolute Value (1pt each)

Y = 4x + 2

11. Form ________________

12. Form ________________
**Quadratic Formula**

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Where \(a\), \(b\), and \(c\) come from the coefficients of the standard form \(y = ax^2 + bx + c\)

**Example**

\(Y = 2x^2 + 5x + 3\)

Step 1: \(a = \) \\
\(b = \) \\
\(c = \)

Step 2: Plug these values into the formula

Step 3: write the zeros (x-intercepts) of the function

\(\text{Zeros} = ( , ) ( , )\)

**Example**

\(Y = x^2 - 2x + 3\)

Step 1: \(a = \) \\
\(b = \) \\
\(c = \)

Step 2: Plug these values into the formula

Step 3: write the zeros (x-intercepts) of the function

\(\text{Zeros} = ( , ) ( , )\)

**Practice Problems**

1. \(y = 5x^2 + 12x - 2\) 
2. \(y = 9x^2 + 12x + 4\)

3. \(y = x^2 - 6x + 7\) 
4. \(y = 2x^2 + 5x + 3\)
5. \( y = 5x^2 + 12x - 2 \)  
6. \( y = 4x^2 + 7x - 15 \)  

7. \( y = 2x^2 - 3x + 1 \)  
8. \( y = 3x^2 + 6x + 3 \)  

9. \( y = x^2 - 6x + 7 \)  
10. \( y = x^2 + 4x - 5 \)  

Solve the following problems using the quadratic formula. Show work. Make sure the equation is \( = 0 \) before starting.

11. \( 0 = 4x^2 + 7x - 15 \)  
12. \( 5x^2 - 47x = 156 \)
Factor the following and then use the quadratic formula to check your answers

15. \( x^2 - 5x - 14 = y \)

Factored Form ________________

x- intercepts at \( x = \) ______ \( x = \) ______

Using quadratic Formula:

16. \( y = x^2 - x - 2 \)

Factored Form ________________

x- intercepts at \( x = \) ______ \( x = \) ______

Using quadratic Formula:

15. \( x^2 + 7x + 12 = y \)

Factored Form ________________

x- intercepts at \( x = \) ______ \( x = \) ______

Using quadratic Formula:

16. \( y = 2x^2 + 5x - 2 \)

Factored Form ________________

x- intercepts at \( x = \) ______ \( x = \) ______

Using quadratic Formula:

What do you notice about using the quadratic formula and the factored form?

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
Skill Drill: Quadratic Formula

7 Steps to Solving quadratic equations

\[ Y = 2x^2 - 5x + 1 \]

Step 1: Quadratic formula

Step 2: \[ a = \]
\[ b = \]
\[ c = \]

Step 3: Substitute

Step 4: Discriminant

Step 5: Simplify

Step 6: Solve

Step 7: Check graph
Practice Problems on a separate piece of paper, solve the following quadratic equations using the quadratic formula and the 7 steps listed above.

1. \( Y = x^2 + 7x \ - \ 3 \)  
   Solutions:

2. \( Y = x^2 \ - \ 6x \ + \ 2 \)  
   Solutions:

3. \( Y = 3x^2 \ + \ 8x \ - \ 1 \)  
   Solutions:

4. \( Y = x^2 \ - \ 10x \ + \ 3 \)  
   Solutions:

5. \( Y = 2x^2 \ + \ 4x \ - \ 2 \)  
   Solutions:

6. \( Y = -x^2 \ - \ 3x \ + \ 1 \)  
   Solutions:

7. \( y = -3x^2 \ - \ 12x \ + \ 3 \)  
   Solutions:

8. \( y = x^2 \ + \ 4x \ - \ 1 \)  
   Solutions:

9. \( y = -4x^2 \ - \ 9x \ + \ 2 \)  
   Solutions:

10. \( y = x^2 \ - \ 3x \ + \ 1 \)  
    Solutions:
Algebra: Warm up

Use the 7 step method to solve the following

1. \( y = x^2 + 4x - 2 \)  
2. \( y = -x^2 - 10x + 3 \)
Algebra
Skill-Drill Triple Threat (graphs, tables, equation)

Graph
Step 0: Graph points

Step 0: Pick points on the graph

Step 1: Create table
Step 2: Follow steps 1-5 (table to Equation)

Step 3: Graph point on axis
Step 2: Solve for y
Step 1: Plug in values for x

Table

Step 0: Pick points on the graph

Step 1: c is where x = 0
Step 2: Find Second Difference
Step 3: a is Half of Second Difference
Step 4: Plug in a point for x and y
Step 5: Plug in b when solved

Equation

Y = ax^2 + bx + c

Work:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>10</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>
Complete the following triple threat triangles given the following information:

Equation __________________

Equation ____________

x  y
-4  6
-3  0
-2  -4
-1  -6
0  -6
1  -4
2  0
3  6

Equation __________________

Equation ______ y = -2x² + 2x + 1

Equation ______________

Equation ______________

x  y
-3  18
-2  3
-1  -6
0  -9
1  -6
2  3
3  18
Distribute (Hint: Use the box method)

1. \((y + 5)(3y - 2)\) = ______________
2. \((x + 2)(x + 9)\) = ______________
3. \((3 - x)(x - 8)\) = ______________

Factor and find the x-intercepts (Hint: Use the box method)

1. \(x^2 + 18x + 81\) = ______________
   \(x = ______\) \(x = ______\)
2. \(x^2 + 4x - 21\) = ______________
   \(x = ______\) \(x = ______\)
3. \(2x^2 + 9x - 35\) = ______________
   \(x = ______\) \(x = ______\)
4. \(3x^2 - 21\) = ______________ (the middle term is 0x)
   \(x = ______\) \(x = ______\)

Place the following quadratics in standard form \(y = ax^2 + bx + c\) and Graph

1. \(y + 10x - 16 = x^2\)
   Standard Form ______________
2. \(x^2 + 10x + 9 = y\)
   Standard Form ______________

\[\text{Graphs:} \]

\[\text{Graph 1:} \]

\[\text{Graph 2:} \]

\[\text{Graph 3:} \]

\[\text{Graph 4:} \]
Write an equation that has the following criteria (use vertex form, standard form, or factored form)

a. Vertex at (3,7)  
   b. Parabola opens down  
   c. Stretch of 0.5

Equation__________________

a. Vertex at (7,-2)  
   b. Parabola opens up  
   c. Narrow of 7

Equation__________________

a. zeros at x = -7 and x = 6  
   b. Parabola opens up  
   c. 5 units to the right  
   d. 9 units down

Equation __________________

Predict how each graph in standard form will change if we start with the parent function \( y = x^2 \)

<table>
<thead>
<tr>
<th>Parabola Form</th>
<th>( y = a(x - h)^2 + k ) (vertex)</th>
<th>( Y = ax^2 + bx + c ) (standard)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative h or b</td>
<td></td>
<td></td>
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<tr>
<td>Positive h or b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative k or c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive K or c</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Use the quadratic formula to solve the following:

1. $y = 5x^2 + 12x - 2$
2. $y = 9x^2 + 12x + 4$

3. $y = x^2 - 6x + 7$
4. $y = 2x^2 + 5x + 3$

5. $y = 5x^2 + 12x - 2$

Complete the following triple threat triangles given the following information:

Equation _________________

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>26</td>
</tr>
<tr>
<td>-2</td>
<td>17</td>
</tr>
<tr>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Equation $y = -2x^2 + 3x + 1$
Questions: Answer in complete sentences

1) How does the axis of symmetry relate to the vertex of a parabola?

Give the equation for the axis of symmetry of the given parabolas:

a. \( f(x) = 2x^2 + 4x - 5 \)  
   Equation __________
b. \( f(x) = -5(x - 4)^2 + 7 \)  
   Equation__________

2) When you multiply two linear equations together, what results? Give an example of two linear equations that do this.

3) Name three situations in which you might use standard form, vertex form, or factored form to model a situation.
   
   - Standard Form:
   - Vertex Form:
   - Factored Form:

4) Describe, in complete sentences, what the shape of a parabola looks like.

5) What is the relationship between factored form and the x-intercepts of a graph?
6) What is the relationship between two linear equations and factored form?

7) What is true about the differences in the y values of the table of a quadratic equation?

8) How can you tell whether a quadratic equation has two solutions, one solution, or no solutions?

9) Why is it important to be able to produce a graph, table, and equation for a given situation?
The chart above describes the parabolic path of distance and height water from a fountain travels (2 pts each)

1. Using the chart, create an equation in standard form: Equation ________________________________

2. Create an equation in factored form if the graph has zeros at (0,0) and (5,0) Equation ________________________________

3. Create an equation in vertex form if the vertex is at (2.5, 6.25) and the stretch is 3 Equation ________________________________

4. Give an equation for the axis of symmetry. Equation ________________________________

Solve the following quadratics using any method; indicate your method (4 pts each)

5. \[ y = x^2 + 4x - 21 \] Method __________________

Solution(s) ________ ________
6.  \( y = x^2 + 5x + 6 \)

Method __________________

Solution(s) ____________ __________

7.  \( y = 2x^2 + 6x + 1 \)

Method __________________

Solution(s) ____________ __________

8.  \( y = 4x^2 - 7x + 2 \)

Method __________________

Solution(s) ____________ __________
Indicate which family from the given chart, table, or graph: Quadratic, Linear, or Absolute Value (1pt each)

9. Family: ________________

10. Family: ________________

11. Family: ________________

12. Family: ________________

13. Family: ________________

14. Family: ________________

15. Family: ________________

16. Family: ________________

17. Family: ________________

18. Family: ________________
Describe what the parabolic will look like given the following form of a quadratic equation

(1pt per description)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>19. $y = -0.2(x + 7)^2 + 5$</td>
<td>20. $y = -3x^2 + 2x - 8$</td>
</tr>
<tr>
<td>Form________________</td>
<td>Form________________</td>
</tr>
<tr>
<td>3 Descriptions of parabola</td>
<td>4 Descriptions of parabola</td>
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</tr>
</thead>
<tbody>
<tr>
<td>21. $y = 9(x - 5)^2 - 8$</td>
<td>22. $y = (x - 6)(x + 2)$</td>
</tr>
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<td>Form________________</td>
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<td>3 Descriptions of parabola</td>
<td>2 Descriptions of parabola</td>
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Concluding Thoughts

Despite the immense research and creativity that went into writing this project, “Integration of Mathematics and Music in the Secondary Classroom” is in the beginning stages of development. I am a firm advocate in the use of inquiry and the liberal arts to teach mathematics to high school students. This approach to teaching mathematics is in line with the newly developed Common Core standards for mathematics. It teaches students to take ownership of their learning. Most importantly it begins to answer the question that most students ask and all teachers dread, “When are we ever going to use this?” I believe that a mixture of teaching mathematics through a more applicable medium alongside traditional ways of teaching is a recipe to initiate answers to why mathematics should be important to every student. My hope is to use the lessons supplied through this thesis in my future math classroom to help students of music appreciate mathematics and students of mathematics to value music.
Presentation schedule

Brian O’Neill
Lee Honors College Thesis: “Integration of Mathematics and Music in the Secondary Classroom”
Defense Date: April 22, 2013
Location: Parchment High School Auditorium

I. Introduction
   a. Bassically Treble (arranged by Brian O’Neill)
      i. Names/Part/Class
      ii. Love for music to music in math
   b. “Integration of Mathematics and Music in the Secondary Classroom”
      i. Research from pedagogical knowledge and high school mathematics teachers
      ii. Inquiry and the Common Core to construct understanding
      iii. Rote mathematics
      iv. Quadratics unit composition

II. Inquiry One: Problems Without Polyrhythm
   a. Polyrhythm
   b. Relation to mathematics
   c. Inquiry poster highlights
   d. Some Nights (arranged by Brian O’Neill)

III. Inquiry Two: Ma-Thematics
   a. Thematic Transformations
   b. Relation to math
   c. Inquiry poster highlights
   d. Pirates of the Caribbean Theme (arranged by Brian O’Neill)
IV. Inquiry Three: The Undertones of Overtones
   a. Overtones
   b. Relation to math
   c. Inquiry poster highlights
   d. Past Life Melodies

V. Closing Remarks
   a. Hope for project
   b. Future plans
   c. Animal