A Study of Forward vs. Backward Chaining, using Hutching's "Low Stress" Algorithm in Simple and Complex Addition Problems with Third Graders

Linda Sue Clark
Western Michigan University

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Linda Sue Clark

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A STUDY OF FORWARD VS. BACKWARD CHAINING,
USING HUTCHING’S "LOW STRESS" ALGORITHM
IN SIMPLE AND COMPLEX ADDITION PROBLEMS
WITH THIRD GRADERS.

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INTRODUCTION

Stimuli that accompany reinforcers may themselves acquire conditioned reinforcing properties. Much of our behavior is believed to be controlled by conditioned reinforcement. We have learned that stimuli that have served as discriminative stimuli (S^D's) for reinforced responding acquire the status of conditioned reinforcers. As such, the S^D's can control the rate and pattern of responding that produces them. The strength of a conditioned reinforcer is a function of the length of time between its occurrence and the subsequent occurrence of the unconditioned reinforcer.

By properly utilizing conditioned reinforcers, we can construct elaborate chains of behavior that culminate in only a single primary reinforcement. For example, bananas are a powerful reinforcer for a chimpanzee. If a chimp is given poker chips that can be traded in for bananas, the poker chips become conditioned reinforcers (Wolfe, 1936). The chimp will learn to operate a vending machine which dispenses only poker chips. What we have here is a chain of behavior: the chimp operates the machine, obtains a poker chip, exchanges the chip for a banana, and eats the banana. Thus, the poker chip is both a conditioned reinforcer for responding to obtain it and a discriminative stimulus for responding to a banana.

Far more complex chains are commonplace in human behavior. Indeed, much of our behavior is controlled by conditioned reinforcers.
Consider what the seemingly straightforward activity of going to the movies on a cold winter's day entails: selection of the movie, determination of the show times, dressing in warm clothes, walking to the car, warming the car's engine, and so forth. All of which, may or may not, be reinforced by the particular movie. Much of human behavior seems to comprise such bits and pieces of complex response chains that are performed for reinforcers far more remote than often realized.

In a chain of behavior, the conditioned reinforcer that is closest to the unconditioned reinforcer is the strongest. The response it follows is the first to be learned and the last to be extinguished. Behavioral chains are, in fact, taught backwards. For example, Barnabus, the Rat of Brown University, was trained to climb a spiral staircase when a light (the first of several $S^D$s) was illuminated (Pierrel & Sherman, 1963). He would then push down a raised drawbridge and cross it to another platform from which he would ascend a ladder, pull a chain to summon an attached car, pedal the car over a bridge, mount another flight of stairs, run through a tube, enter an elevator at the end, and raise a Brown banner. When the flag was raised, the elevator descended to the ground floor. Here when a buzzer sounded, Barnabus ran over to a lever, pressed it, and was at long last reinforced with a food pellet. When the buzzer stopped, Barnabus turned back to the spiral staircase and waited for the light, signaling the beginning of a new performance. This rather unusual performance can be described as a stimulus-response chain:
This stimulus-response chain was conditioned by the backward chaining technique. Barnabus first learned to press the level at the sound of the buzzer in order to receive a food pellet (the primary reinforcer). He was then required to ride an elevator in order to reach the box containing the buzzer and the lever, which had both become $S^D$s for the pressing response and conditioned reinforcers for riding. Once the elevator ride had become an $S^D$, it could become a conditioned reinforcer for raising the flag, which, in turn, reinforced running through the tube and entering the elevator, etc.

For humans, self-help behavior such as dressing and feeding one's self can be taught to retarded individuals by using a backward chaining technique with food or praise as a reinforcer.
Backward chaining was used to teach 40 profoundly retarded males, ranging in age from seven to 20, to feed themselves properly with utensils (Colwell, 1965). A spoon was filled with a highly reinforcing food, placed in the person's hand, and the hand raised by the teacher nearly to the mouth. Then the teacher removed his hand and the individual completed the rest of the chain by placing the food in his mouth. When this response was conditioned, the individual's hand was released farther away from the mouth. After a brief time it was necessary only to assist the individual in filling the spoon; this final component is easily mastered. Many stimulus-response chains of this sort, which retarded individuals customarily lack, can be conditioned using a backward chaining procedure and a little patience. Efforts of this sort by the teacher allow a retarded individual to go through life with more dignity than would otherwise be possible.

These techniques might be of value in teaching normal children also. We rarely spend much time thinking about such everyday tasks as how we get dressed unless we are in a situation where someone has not learned the appropriate responses. Suppose you had to teach a child how to tie his shoes. What normally seems so simple and matter-of-fact now seems difficult and complex. The act of shoe-tying becomes a major undertaking which is best analyzed in terms of a stimulus-response chain. A child learning to tie bows in his shoelace or, later in life, learning to tie a necktie goes through a series of separate movements, with one movement preparing the
stage and supplying the stimuli for the next. Such tasks will be reinforced only at the end of the activity, when the sequence of steps is completed. Under such conditions the reinforcer will be closest in time to the very last movements made. These last movements than will be best learned. The early stages in the chain will be relatively longer removed from the reinforcement and will be learned less well. Thus, it seems logical for the teacher to examine each skill being taught to determine whether it is of any importance that all parts of the task be equally well learned. If so, the earlier phases will require more practice or perhaps, separate reinforcement.

We can derive some practical advice from Hull's concept of "backward learning", i.e., "determine the relative need for practice at each step in a sequential act and schedule such differential practice, including the separate reinforcement of each step, until all parts of the task are equally well learned (1943)."

Differential learning occurs when a student learns a poem or speech. The last part is learned best; the middle, next best; and the beginning, only poorly. Often, however, if the performance of a chain is interrupted in the middle for some reason, it may be difficult to continue from the point of interruption. It may even be necessary to start the chain over from the beginning (Anderson & Faust, 1973). If the student does not recite the entire "piece" through on each occasion, but stops as soon as he makes an error and returns to the beginning on every such occasion, the beginning will be overlearned, while the end will suffer. In either event,
it is clear that unless particular effort is taken to insure equal practice and equal reinforcement at each step along the way, the learning will be uneven, with the end of the operation relatively better learned. It may be more effective with some tasks, for instance, tying a shoe, to begin with the end or final action and teach the whole task backward, taking advantage of this "goal-gradient" effect.

Whaley and Malott (1971) discuss the relative merits of forward and backward chaining. They indicate that the use of backward chaining is not always necessary with human beings. They discuss a standard stimulus-response chain experiment which is commonly done in introductory lab courses. The stimulus-response chain consists of:

- **D** (light off)
- **R** (pull chain from top of cage)
- **S** (light on)
- **R** (push response lever)
- **S** (click of water dipper)
- **R** (approach water dipper)
- **S** (water)
- **R** (drink water)
- **S** (consume water)

How is this taught using backward chaining? The response of approaching the water dipper is conditioned in the presence of the water, in a short amount of time. Then the response of approaching the water immediately following the sound of the dipper click is conditioned. This phase is always preliminary to the conditioning of the bar-pressing response. Imagine the difficulty of conditioning a bar-press response when the rat has not been trained to go to the water dipper at the sound of the dipper click. Without such
training, if the rat pressed the bar and the dipper clicked, the rat might not approach the water dipper. It could be several minutes before he would actually consume the water reinforcer. Such delays in reinforcement might make conditioning of the bar-press response nearly impossible. Thus, the procedure of backward chaining is essential in conditioning components of the chain leading to the consumption of the reinforcer. Otherwise, conditioning of subsequent components of the chain might never occur.

Now consider the necessity of conditioning the bar-press response before the chain-pulling response. According to Whaley and Malott, it might not be essential, but it certainly seems convenient. Suppose the response of pulling the chain is conditioned, which is immediately reinforced by the dipper click, and the rat approaches the water dipper. Usually the response of pulling the chain itself acts as an S\textsuperscript{D} for approaching the water dipper, regardless of whether the click appears or not. Thus, it would be difficult if we now tried to incorporate the bar-press response in the chain. The rat pulls the chain which turns on the light, and now he immediately goes to the water dipper, even though the dipper does not click. His going to the water would probably be a fairly well-conditioned response that would interfere with the conditioning of intermediate links of the chain such as bar pressing. This happens when the consuming responses have been associated with earlier links of the S-R chain, in this instance, chain pulling.

Whaley and Malott content that if the reinforcer's presentation
did not require this interfering sequence of consuming responses, backward chaining might not be necessary. Suppose we were dealing with a normal human being and that the reinforcer was the conditioned reinforcer praise. We will say "Good" every time the human makes a correct response. The human hearing us say "Good" constitutes a consuming response analogous to the rat's approaching the water dipper and drinking the water. However, the human being's consuming response of hearing "Good" does not interfere with his other responses. We could condition the response of pulling a chain when the light is off. Every time the human pulled the chain we would say, "Good." During the next phase of the experiment, every time the human pulled the chain we would turn on the light and wait until he pressed the bar before we said "Good." We could get through this sequence with little difficulty of one response interfering unduly with another.

Conventional instruction is typically organized in a "logical" sequence, commencing at the beginning and ending with the completed behavior: for example, if a student is learning to write his name, "John", he writes "J," and the teacher says, "Good." As he writes each succeeding letter, he is praised by the teacher. Contrast this with the backward chaining approach: Joh is supplied. John must first fill in the last letter, then the last two letters, then the last three, and finally, all letters. There are several questions that might be asked regarding the advisability or necessity for teaching a behavior by means of a backward chaining procedure:

1) Is there sufficient evidence to support using this approach in
any applied situations? Can people learn just as well with behavioral components added from the beginning? 2) Does the effectiveness of backward chaining and/or forward chaining vary as a function of task complexity? The answers to these questions are presently unclear. Research conducted with human subjects on this topic is minimal. A review of the literature revealed few experimental studies which generalize to an applied situation prior to this study. Where the procedure has been used, especially in teaching a behavior that has been difficult for an individual to learn, it has been successful.

Cox and Boren (1965) trained 30 men to perform a 72-action procedure on Nike Hercules equipment. Three different training procedures were used, ten men being trained with each technique. First, the actions were organized into seven operant spans (short groups of steps) and taught in reverse chronological order (using backward chaining). The second group was taught the same operant spans in chronological order (using forward chaining). The third group was taught the complete procedure without grouping actions into operant spans. Each subject was required to learn the procedure to one perfect performance. Two propositions were tested. The first was that training men to perform a fixed procedure taught by chaining is more efficient than is training men when the material is presented in normal chronological order. The second proposition was that training men to perform a fixed procedure when the material is presented in operant spans is more efficient than is training men to perform on the unbroken procedure. Comparisons were made
between the mean training times for the three techniques. Insignificant differences were found. In addition, no differences in training time were found to be attributable to instructors or effects of interaction (treatments X instructors).

The effectiveness of forward and backward chaining were compared as techniques for establishing vocational skills in six mentally retarded subjects (Patterson, Panyan, Wyatt, & Morales, 1974). A manual chain (assembling metal pieces) of six separate component tasks was taught via backward and forward chaining in two separate experiments. Comparisons between the two chaining procedures were made on: length of time to learn the tasks, number of errors in training, and durability of the response chain measured post treatment. Neither procedure was found to be superior for the tasks employed. The authors argue against prescribing one technique for teaching, and discuss the possibility that for other types of tasks, one procedure might be more appropriate.

This study attempted to empirically compare the effectiveness of forward vs. backward chaining across tasks of varying complexity. The task to be learned was an algorithm used for solving additional problems. The dependent variables were: 1) percentage of columns correct based on final column answers, 2) percentage of component errors (errors within a problem), and 3) the amount of time needed to complete a set of problems. The independent variables were: 1) the chaining procedure used, backward or forward, and 2) the complexity of the task.
METHOD

Subjects

Subjects for the study were 24 third-grade children of similar social and cultural background. They attended an elementary school in close proximity to Kalamazoo, Michigan. Subjects were divided into four groups: two experimental and two control. All students were experimentally naive, having no prior knowledge of the task or procedure for performing the task.

Setting

The subjects were taught on an individual basis outside their regular classroom. The experimenter attempted to maximize contact with each student by: 1) sitting at a table, beside the student, 2) arranging to meet in a setting with little or no interruptions, e.g., school lunch room, conference room, library, etc., and 3) assuring that the setting was relatively self-contained and free from distracting stimuli to reduce the possibility of the student looking out the window, attending to articles present in the room, etc. Necessary materials consisted of colorful marking pens and paper, so as to make the activity attractive. Examples were presented on 1/4 or 1/2 sheet of ditto paper; numbers were 1/4 inch in size and spaced 1/4 inch apart.
Independent Variables

I. Chaining Procedure Used (Forward or Backward)

II. Complexity of the Task (as determined by number of components within and homogeneity of the problem).

Dependent Variables

I. Percentage of Columns Correct = \( \frac{\text{Number of Columns Correct}^*}{\text{Total Number of Columns Presented}} \)

* Number of Columns Correct is based on Column Outcome (Final Answer).

II. Percentage of Components Correct = \( \frac{\text{Number of Component Steps Correct}}{\text{Total Number of Components Necessary to Complete Problem}} \)

III. Amount of Time Spent on Daily Tasks

Arithmetic Materials and Measures

A method of column addition was taught to the subjects and used as a measure for comparison. This method has been described as a "low stress algorithm" on the basis of the operations it requires (Hutchings, 1976). This method of calculation involves the addition of only two digits at a time, with each digit of no greater value than nine. Contrast this with the conventional algorithm which requires the cumulative addition of two or more digits. The algorithm can be described as follows:

Half-space notation uses numerals of no more than a half-space in height to record the sum of two digits. With half-space notation, the units portion of the sum of two digits is written at the lower right of the bottom digit.
and the tens portion is written at the lower left of the bottom digit....The ones portion of the column sum is always the same as the ones portion of the last two-digit sum....The tens portion of the column sum is always the same as the number of tens recorded at the left of the column. These are simply counted [Example A]....For a column in some multi-column exercise....the total number of tens, is no longer written in the tens place of the first column's sum but instead at the top of the next column at the left [Example B] (Hutchings, 1976).

Example A

<table>
<thead>
<tr>
<th>Column</th>
<th></th>
</tr>
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<tbody>
<tr>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

9 + 6 = 15  
5 + 5 = 10  
0 + 7 = 7   
7 + 3 = 10  
0 + 8 = 8

Example B

<table>
<thead>
<tr>
<th>Column</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

9 + 6 = 15  
5 + 5 = 10  
0 + 7 = 7   
7 + 3 = 10  
0 + 8 = 8

The performance can be thought of as the following stimulus-response chain:

$S^D$ Presentation of the problem.

R Add first two numbers in ones column.

$S^D$ First two numbers added in ones column.

R Write answer in small numbers below second number and above third number with units to the right and the tens notation (if present) to the left.

4 7 1 3 3 4
3 2 2 2 4 7
7 9 6 7 5 6
SD First binary step complete.

R Ones notation of previous binary (disregarding tens notation if present) is added to the next number in the column, with units written to the right and tens notation to the left.

```
 4 7 1 3 3 4
 3 2 2 2 4 73
 7 9 6 7 5 6
 1 4 2 9 1 8
```

SD One of the previous binary is added to next number

R Continue same process until all column numbers have been added.

SD Final binary sum is entered below the last number and above answer line.

R Units answer to last binary is written directly below the column, beneath answer line.

```
 4 7 1 3 3 4
 3 2 2 2 4 73
 7 9 6 7 5 6
 1 4 2 9 1 8
```

SD Units answer written beneath answer line.
R Count tens notation and write tally above top number in the next column to the left (the tens column).

\[
\begin{array}{cccc}
4 & 7 & 1 \\
3 & 2 & 2 & 3 \\
7 & 9 & 6 & 9 \\
1 & 4 & 2 & 9 \\
\hline
1 & & & \\
\end{array}
\quad
\begin{array}{cccc}
3 & 3 & 4 \\
2 & 4 & 7 & 1 \\
7 & 5 & 6 & 7 \\
9 & 1 & 8 & 5 \\
\hline
5 & & & \\
\end{array}
\]

SD Tens written above top number in tens column.

R Add written tens to top number in tens column and write sum below first number in the column (units to the right and tens to the left).

\[
\begin{array}{cccc}
4 & 7 & 1 \\
3 & 2 & 2 & 3 \\
7 & 9 & 6 & 9 \\
1 & 4 & 2 & 9 \\
\hline
8 & & & \\
\end{array}
\quad
\begin{array}{cccc}
3 & 3 & 4 \\
2 & 4 & 7 & 1 \\
7 & 5 & 6 & 7 \\
9 & 1 & 8 & 5 \\
\hline
5 & & & \\
\end{array}
\]

SD Written tens added to top number and answer written directly below, following same format.

R Continue entire process until last column is completed.

SD Last column complete

R Final tally of tens notation(s) is entered in the position to the left of the ones tally for final binary sum in problem.

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Prerequisite Behaviors (Entering Skills)

The prerequisite skill needed to perform the task, the algorithm, was the ability to add any two one-digit numbers (commonly referred to as the basic 100 addition facts). A prerequisite check was given to determine if each of the 24 students in the classroom had this skill. This check consisted of a worksheet unit of problems involving the addition of single-digit numbers (e.g., $2 + 5 + \_\_\_$, $3 + 7 = \_\_\_\_$, etc.).

Pretest

Following administration of the prerequisite check, a pretest was given to each student, which contained either simple or complex problems, depending on to which group he had been assigned. The students were pretested in groups of five to six students. A percentage correct was determined based upon final column answers. Standard instructions were presented for the pretest, post test,
and retention test as follows: "Take your time and work each problem carefully. Don't spend too much time on any one problem. You can skip a problem if it is too difficult for you, but do as many as you can one by one. Show all of your work. Hand in your paper to me as soon as you finish. Begin now." A stopwatch was used to record the time, which was started with the words, "Begin now," and each student's time recorded when he handed his paper to the experimenter. When each student finished, he was asked if he knew of any other way to use in solving the problems. All students responded that they did not.

Sample Selection and Assignment

The results of the prerequisite check revealed which students in the classroom had the prerequisite skill needed to perform the task—i.e., adding any two single-digit numbers. Criterion for mastery was established at 85% of the 50 problems correct. All students in the classroom met this criterion. A table of random numbers was used to assign the 24 students to one of four groups. The students were assigned a number, i.e., from 1 through 24. A row in the table of numbers was entered, three consecutive numbers were added, and the first six different numbers between 1 and 24 inclusive as they occurred were assigned to the forward simple group, the next six to the forward complex group, the next six to the backward simple group, and the remaining students to the backward complex group. Thus, six students were randomly assigned to each of four teaching methods.
Instruction

Twenty-four subjects were given individual instruction of the algorithm. In the instructional setting, the experimenter gave one example of the problem, teaching only certain steps on any particular session. The student performed four problems of identical complexity and received immediate feedback on each step. Instruction at any particular step continued on additional sessions, until the student could perform the complete sequence without error. Behavioral objectives were written for each group, specifying specific steps of the algorithm (see Appendix A). This 100% criterion was determined on the basis of the percentage of component errors made on the set of four task problems done by the subject independently in that session. Refer to Appendix B for examples of the format used in presenting the task problems. The instructor (experimenter) practiced a prepared set of techniques for each instructional method prior to the study so that individuals within groups would receive identical instruction. The order that the groups received instruction was randomized for each session.

Treatments

One of the following treatments was given to each student according to his random assignment:

1) Backward Simple. A simple addition problem was taught using the algorithm with the backward chaining procedure. Initially the most simple problems were presented and difficulty increased until all students could perform the least simple problem. Each
student was given a demonstration of the last component step in the problem (with all preceding steps previously done). The whole problem was then demonstrated to the student. The student was told to follow a second demonstration of the problem until the instructor stopped (she stopped at the last step). The student was required to perform the last step. This procedure was continued with the student receiving feedback on the correctness of his responses on four additional problems. Then the student was given a task, consisting of four problems, requiring him to complete the last step only. Demonstration and task problems consisted of different numbers, but were identical in complexity. When the student performed with 100% accuracy on one task, he was given a demonstration of the last two component steps, which he was eventually required to perform himself. This process was continued, dropping the next component from the experimenter's demonstration and adding it to the student's performance, until the student performed the least simple problem without error. Each time the student made an error, the experimenter demonstrated the relevant task sequence through the error, and the student completed the sequence.

2) Backward Complex. A complex addition problem was taught using backward chaining. The procedure for teaching is described in Treatment 1 above; however, number of steps from least to most complex ranged from 13 to 42, respectively.

3) Forward Simple. A simple addition problem was taught using forward chaining. Each student was given a demonstration of
the first component and required to attempt to perform it himself. If the student made an error, he was corrected, and given the three additional practice problems. A task of four problems was then presented, in which the student was required to perform the first step. When the student acquired 100% accuracy on this task, he was taught the second component using the same procedure, only demonstrations were provided with both components presented in proper sequence. Other components were taught in the same way, in chronological order.

4) Forward Complex. A complex addition problem was taught using forward chaining. The procedure for teaching is described in Treatment 3 above; however, number of steps from least to most complex ranged from 13 to 42, respectively.

Data Collection

Data were recorded and charted daily for each subject. Each student was provided with one demonstration using the procedure called for during that session; the student then worked four problems of identical complexity to the stimulus problem and was given feedback on the correctness of his responses. This implies that the amount of feedback given during any one session was determined by the number of component steps the student was asked to perform. Then the instructions were given and the four-problem task was presented. Reliability checks on scoring for all students were taken at random intervals on the average of one out of five sessions. Each student was presented four problems, either simple
or complex (depending on his group assignment), and asked to show his work. These were scored on the basis of component errors made at each step in doing the problem and on column outcome, the final answer. The percentage of components correct was calculated as follows: Number of components correct/Total number of component steps possible X 100. The length of time required to complete the task was recorded daily. Refer to Appendix C for examples of task problems.

Design and Data Analysis

A two-way analysis of variance was used to assess the difference between groups based on column outcome, using the percentage of columns correct from the post test scores. In this analysis, sources of variance were the two treatments (forward and backward), the two levels of complexity (simple and complex), and the treatments X complexity interaction. A two-way analysis of variance was also performed on the mean scores for each student on: 1) average amount of time needed to complete the tasks (i.e., total number of minutes on all sessions/total number of sessions), and 2) percentage of components correct. A table of the results may be presented as follows:

<table>
<thead>
<tr>
<th>Complexity</th>
<th>Treatment</th>
<th>Forward</th>
<th>Backward</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td></td>
<td>n = 6</td>
<td>n = 6</td>
</tr>
<tr>
<td>Complex</td>
<td></td>
<td>n = 6</td>
<td>n = 6</td>
</tr>
</tbody>
</table>

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RESULTS

A one-way analysis of variance was performed on the percentage scores obtained on the prerequisite check. This analysis served the purpose of assuring complete randomization of the groups. Group means for the four groups are presented in Table 1. There were no mean differences between groups as randomly assigned larger than chance would predict: F(3, 20) = .45, p > .723. Pre-test results are also presented in Table 1. These results revealed no significant differences between the groups: F(1, 20) = .52, p > .477. Mean scores were high for simple groups and relatively low for complex groups. A simple explanation for these results is that the subjects had a great deal of practice solving simple addition problems using the conventional method, but not with solving complex problems.

Data recorded on completion of all daily instructional tasks were analyzed using two measures for each subject: 1) the average number of minutes needed to complete the tasks, and 2) the average percentage of components correct. Group means for the four groups are presented in Table 2. There were no interaction effects between groups on the percentage of components correct larger than chance would predict: F(1, 20) = .09, p > .767. However, mean differences between backward and forward groups on both simple and complex levels approached significance: F(1, 20) = 3.83, p > .064; F(1, 20) = 3.27, p > .086, respectively. Mean differences on average number of minutes needed to complete the daily tasks were highly

22
TABLE 1. Mean Scores on Prerequisite Check and Pretest.
### TABLE 1

MEAN SCORES ON PREREQUISITE CHECK AND PRETEST

<table>
<thead>
<tr>
<th>Prerequisite Check</th>
<th>Group (n=6)</th>
<th>Pretest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>SD</td>
<td></td>
</tr>
<tr>
<td>98.0</td>
<td>3.10</td>
<td>Backward complex</td>
</tr>
<tr>
<td>96.6</td>
<td>3.50</td>
<td>Backward simple</td>
</tr>
<tr>
<td>96.0</td>
<td>4.56</td>
<td>Forward simple</td>
</tr>
<tr>
<td>98.0</td>
<td>3.35</td>
<td>Forward complex</td>
</tr>
</tbody>
</table>
TABLE 2. Scores Derived from Daily Instructional Tasks: Average Number of Minutes & Percentage of Components Correct.
TABLE 2
SCORES DERIVED FROM DAILY INSTRUCTIONAL TASKS

<table>
<thead>
<tr>
<th>Average Number of Minutes</th>
<th>Group (n=6)</th>
<th>Percentage of Components Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>SD</td>
<td>Backward complex</td>
</tr>
<tr>
<td>162.33</td>
<td>55.83</td>
<td>93.67</td>
</tr>
<tr>
<td>33.67</td>
<td>14.9</td>
<td>Backward simple</td>
</tr>
<tr>
<td>39.0</td>
<td>4.86</td>
<td>Forward simple</td>
</tr>
<tr>
<td>215.83</td>
<td>74.62</td>
<td>Forward complex</td>
</tr>
</tbody>
</table>

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significant on the simple level between forward and backward groups: $F(1,20) = 62.71, p>.001$. No interaction effects were evidenced on this measure: $F(1,20) = 1.56, p>.226$.

A two-way analysis of variance was also performed on the post test scores for both the percentage of columns and percentage of components correct. Group means for the four groups are presented in Table 3. Mean differences on percentage of columns and percentage of components correct were not significant as indicated, respectively: $F(1,20) = .02, p>.883; F(1,20) = .05, p>.834$. However, differences between the forward simple and forward complex groups on the percentage of components correct again approached significance $F(1,20) = 4.06, p>.058$.

Checks on reliability were done to assess the accuracy of experimenter recordings. Reliability on number of component errors was checked at least once every five sessions by an independent observer who scored the papers and recorded the total number of errors on a separate score sheet, making no marks on the original paper. Following this, the experimenter scored the papers and a comparison was made of the two sets of scores. Reliability was calculated by dividing the total number of agreements by the total number of disagreements plus agreements and multiplying by 100. An agreement was scored if both observers recorded the same answer provided by the student as an error. A disagreement was scored if one observer recorded the answer and the other did not. Reliability data were obtained for four experimental sessions, yielding an overall mean of 96% and percentage scores of 93, 93, 100, and 100.
TABLE 3. Post Test Scores: Percentage of Columns Correct & Percentage of Components Correct.
### TABLE 3

**POST TEST SCORES**

<table>
<thead>
<tr>
<th>Percentage of Columns Correct</th>
<th>Group (n-6)</th>
<th>Percentage of Components Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td>94.00</td>
<td>8.41</td>
<td>95.67</td>
</tr>
<tr>
<td>95.33</td>
<td>5.72</td>
<td>98.50</td>
</tr>
<tr>
<td>97.67</td>
<td>3.61</td>
<td>98.00</td>
</tr>
<tr>
<td>95.67</td>
<td>1.97</td>
<td>94.50</td>
</tr>
</tbody>
</table>
A retention test obtained three weeks after the post test was given indicated a significant difference in mean number of components correct for the two backward groups (simple and complex): \( F(1,19) = 4.31, p > .052 \). Group means for the four groups are presented in Table 4. Interaction effects were not significant: \( F(1,19) = .65, p > .432 \). An analysis was also performed on the percentage of columns correct. Interaction effects were in the direction of significance \( F(1,19) = 4.11, p > .057 \). There was a highly significant difference between forward simple and complex groups: \( F(1.19) = 9.91, p > .005 \). Tables 3 and 4 allow a comparison of group means for the post test and retention test. The retention test analysis of the percentage of columns correct showed that the backward and forward simple groups retained the chaining process to a greater extent than the complex groups, with means either the same as or slightly higher than those obtained on the post test. Regarding the percentage of components correct, three out of four groups (backward complex, backward simple, and forward simple) obtained lower mean scores on the retention test as compared with post test results. Thus, the backward complex group performed significantly lower on the retention test as compared with post test results.

The mean length of time it took each group to attain criterion on all tasks is shown in Figure 1. The mean time to reach criterion on the five steps in the algorithm chain for the forward simple group was 233 minutes, while the backward group took an average of 216 minutes. The mean time to reach criterion on the nine steps for the backward group was 3059 minutes as compared with
TABLE 4. Retention Test Scores: Percentage of Columns Correct & Percentage of Components Correct.
<table>
<thead>
<tr>
<th>Percentage of Columns Correct</th>
<th>Group (n=6)</th>
<th>Percentage of Components Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>SD</td>
<td></td>
</tr>
<tr>
<td>85.80</td>
<td>10.03</td>
<td>Backward Complex</td>
</tr>
<tr>
<td>98.83</td>
<td>2.86</td>
<td>Backward Simple</td>
</tr>
<tr>
<td>97.67</td>
<td>5.72</td>
<td>Forward Simple</td>
</tr>
<tr>
<td>94.67</td>
<td>3.39</td>
<td>Forward Complex</td>
</tr>
</tbody>
</table>
FIGURE 1. Total number of minutes required to meet criterion for the backward and forward chaining groups on simple and complex tasks.
3919 minutes for the forward complex group.

The total number of tasks completed for each group at each of the nine steps is shown in Figure 2. Variability between groups appears to be greater when subjects were learning the middle steps of the algorithm chain for each group. For example, Step 3 took the backward complex group a total of 17 tasks and the forward group, 14; Step 4: 16 for the backward complex group and 12 for the forward complex group; Step 5: 9 for the backward complex group and 17 for the forward complex group. The backward simple group took a total of 9 tasks to reach criterion on Step 3, while the forward simple group took 6; on Step 4, the backward simple group required 8 tasks, while the forward simple group took 12. Total number of tasks for each group required at the initial and final stages is not significantly different.
FIGURE 2. Total number of tasks required by the four groups to reach criterion on each of the steps in the chain.
DISCUSSION

Generalization from results reported in experimental literature has led us to believe that chaining should make human learning more efficient. Because of the great number of variables involved in teaching a chain, it is difficult to formulate a conclusion with regard to promoting either a backward or forward technique as the best way of teaching skills or information to human subjects. This study was designed to investigate the effectiveness of the two major chain sequences, forward and backward, as basic learning procedures. Data collected allowed the comparison of:

a) time required, and b) accuracy of responses, using backward and forward chaining techniques to perform simple and complex mathematical procedures. The comparison showed that neither procedure was significantly superior for this task and for the subjects employed in the study.

Prior to instruction, subjects were randomly assigned to one of four groups. Results from the prerequisite check produced mean scores which were remarkably similar, indicating no difference. Scores obtained for the simple group had a mean of 96 and both complex groups revealed a mean score of 98. Individual subject scores ranged from 88 to 100, indicating that each subject had the prerequisite skill of adding any two single-digit numbers needed to perform the mathematical task used in the study. The task appeared to be relatively simple for all subjects, and
competition among subjects for completion in the shortest amount of time was evident. However, this did not generally appear to affect the accuracy of their responses. The pretest also revealed no significant differences between backward and forward groups. All students used the traditional method of addition for solving the problems. None of the subjects were familiar with another way to solve the problems prior to instruction.

Data gathered from all daily instructional tasks provided two measures of comparison. The mean percentage of components correct and the mean of the individual student average number of minutes for each group was computed. An analysis of the percentage of components correct showed that both forward groups performed with greater accuracy during instruction than the two backward groups. On the other hand, the forward group spent a significantly greater amount of time than the backward group on the daily tasks consisting of simple problems. Differences between the two complex groups on the amount of time required were not significant, but mean scores were again higher for the forward group. Thus, although the two forward groups achieved a greater degree of accuracy on daily instructional tasks, they required more time to complete the tasks as compared with the backward groups.

An analysis of the post test results revealed that the backward and forward simple groups performed better than their respective complex groups on two measures: 1) the percentage of components correct, and 2) the percentage of columns correct. However, subjects performing complex tasks may have experienced fatigue...
because the complex problems required a great deal more effort and concentration than the simple ones. A close examination of individual post test results suggest that more errors were made on problems placed on the second half of the paper than those presented on the first half. The fact that the problems increased in complexity as the subject worked down the page is irrelevant in that both simple and complex groups were subjected to this arrangement. An analysis of the percentage of components correct revealed slightly different information than the analysis of columns correct. With regard to the second measure, group means were not significantly different and groups performed equally well. With regard to the percentage of components correct, which is a more detailed analysis, the forward simple group performed significantly better than the forward complex group. The mean percentage scores of components correct obtained by the backward groups were not significantly different. In summary, the simple groups performed with a higher degree of accuracy on the post test, but backward vs. forward differences were not significant.

Retention test results were analyzed in terms of the identical two measures obtained on the post test, i.e., percentage of columns and percentage of components correct. Mean scores were significantly higher for the backward complex group as compared with the backward simple group on the percentage of components correct. The reverse was true of the two forward groups; however, the results were not significant. A comparison of mean scores on the percentage of columns correct showed that the forward simple group performed
significantly higher than the forward complex group. On this measure, the forward group performed significantly higher than the backward group on complex problems only. In addition, the forward complex group was the only one of the four groups which obtained higher mean scores on the percentage of components correct, compared with post test results. In general, forward groups obtained significantly higher mean scores on the percentage of components correct than the backward groups when considering retention. To summarize, both forward groups retained the process and mean scores for these two groups approximated those obtained on the post test. Both backward groups performed significantly lower scores than the forward groups on the retention test. The difference could be related to the fact that overall teaching time was longer for both forward groups.

Figure 1 shows the mean length of time for the four groups to reach criterion on the complete task. Results are similar to those obtained from the data representing individual subject averages during instructional sessions. These results show a significant difference between forward and backward complex groups, with the subjects in the forward group taking an average of about 900 minutes more than subjects in the backward group. Differences were evident for the two simple groups, with the forward group averaging 17 minutes more to learn the complete task than subjects in the backward group.

Figure 2 shows the total number of tasks completed for each group at each of the nine steps. Although differences in the totals
for each step are greater, both within and between groups in the middle steps, totals for each group for this set of steps are quite similar. Specifically, the backward simple and forward simple groups required 23 and 24 total tasks, respectively, for Steps 2 through 4; the backward and forward complex groups required a total of 54 and 56 tasks, respectively, for Steps 3 through 6. Similarly, the total number of tasks required to reach criterion on the complete task for the backward simple, forward simple, backward complex, and forward complex groups are 38, 36, 110, and 110, respectively, with no significant differences.

The data from this study support the results obtained for retarded subjects learning a complex manual assembly task in the Patterson, Panyan, Wyatt, and Morales (1974) study, Experiment I. Those authors found that subjects who learned a forward chain took a total of 137 minutes longer than the backward chain subjects. In that study, group comparisons on mean teaching time to criterion on each of the six task components of the chain revealed no significant differences. In contrast, results of the Patterson, et al. study (1974) showed that the backward group took longer than the forward group on a simple task; however, differences were not significant. Those authors, together with Cox and Boren (1965) concluded that there was no significant difference between the effectiveness of the forward and backward chaining procedures. It is interesting to note that although differences in the Cox and Boren study (1965) were not significant, comparisons in mean teaching times revealed that the two backward groups took slightly
longer to learn the task. They taught 30 men a 72-action procedure on Nike-Hercules equipment, a complex task, using two instructors. Differences in this study were not significant, but the comparison between the two complex groups showed that average number of minutes spent was higher for the forward group than it was for the backward group.

Patterson, et al. (1974) also measured durability of the response chain post treatment. Those authors indicated that subjects taught a complex task performed the task with fewer errors than those taught a simple task on a retention test. However, no significant differences were reported between the forward and backward subjects on both simple and complex levels. These results do not agree with results obtained from the retention test of the present study, which showed that the two forward groups performed better than the two backward groups.

Patterson, et al. (1974) reported similar patterns of responding for complex groups in terms of the chaining errors (i.e., beginning the chain with the wrong task or failure to properly sequence the tasks within the chain) and the within-task errors. Both simple groups showed an increase in the number of correctly sequenced tasks and a decrease in both within-task errors and chaining errors, as compared with the complex groups. This study supports these results in that the simple groups performed with a higher degree of accuracy on the post test, and backward vs. forward differences were not significant.

The two studies previously discussed with respect to the present
study are the only two applied studies relevant to the question of forward vs. backward chaining. Laboratory data show that chaining can be broken down into forward and backward sequences, but laboratory experiments have not compared the two procedures. Thus, generalization from the laboratory to the applied setting has been a major problem for psychologists. Data from the present study indicate that the question of forward and backward chaining is complex. Certainly, there appears to be a multitude of factors involved in teaching a chain (e.g., number of steps involved in the chain, similarity of steps in the chain, type of task taught, number of subjects involved, etc.). Also, a great number of dependent variables need to be studied. Obviously, the need for investigation is considerable, particularly in the applied setting.

Three main observations led the author to conclude that student acceptance of the task and instructor was positive. This conclusion was based on the observation that everyday, when the instructor entered the classroom to select the subject with whom she was to work, many other students began to plead for permission to work with the instructor. In addition, the instructor received many requests from individual subjects to work on another task after they had already completed one within that session. This occurred consistently if the subject had met the 100% criterion on that task. The third observation was that the instructor was often asked about the next time she would return to give the subject another task.

However, negative aspects did exist. Remarks and questions such as, "Oh, I hate this!" and "How much longer do I have to be
here?" were made. Also, subjects occasionally asked for aid from the instructor on the tasks when uncertain as to the correctness of their responses. A few subjects commented that they had forgotten the procedure while solving a certain set of problems.

It is felt by the author that these behaviors, both positive and negative, were observed in members of all four groups, with one group not exceeding another in number of approach and avoidance responses. It appears that positive responses were greater than avoidance responses because of three factors: 1) the subjects responded well to the individual attention given by the instructor (particularly, removal from the routine of the classroom, individual feedback on quality of performance, etc.), 2) the chaining sequence maximized the opportunity for success on any given task and, 3) the subjects displayed a competitive spirit and were anxious to convey their results to peers.

The procedure worked best for subjects who had been previously successful in an individualized program planned and implemented by the classroom teacher. For example, the teacher had contracted with one student to minimize the work he needed to have "redone", because the student was continually guessing on answers. In this study, this subject resorted to guessing in adding two larger single-digit numbers (e.g, \(8 + 9 = \_\_\)). This occurred mainly because, seeing the stopwatch, he assumed timing was an important factor, contingent upon praise. When he was finally convinced by the instructor that timing was irrelevant, guessing was minimized. This was largely accomplished by removing the stopwatch and
surreptitiously timing him on a wall clock.

In a few sessions, superstitious behavior was observed in two subjects who told the instructor that they knew they would not reach criterion on the task, even before attempting it. When asked for their rationale, they explained that in the past, this remark had usually resulted in completion of a perfect set of problems. The instructor responded with encouragement to do their best work, and a reminder to take their time and work carefully.

Group comparisons on three different measures provided much more information than that afforded by studies done prior to this study. Measures taken were those typically concerning classroom teachers, i.e., student accuracy of responding and amount of time spent. All subjects were naive, having had no previous exposure to the algorithm that was used as the task in this study. In addition, none of the subjects were experienced in using a backward chaining procedure to solve the addition problems. An immense amount of data was collected and certainly more conclusions could be drawn, particularly in the area of individual subject response patterns. Certain advantages to the design of this study are also realized. First, the author did not need to rely on information obtained by another instructor, who could possibly have confounded the results and related inaccurate or incomplete impressions. In addition, the author did not depend on a co-instructor, which eliminated the possibility of treatment X instructor interaction. Individualized instruction provided a great deal of control over individual subjects, allowed provisions for ample
feedback and observations, and maximized positive learning conditions for the subject (i.e., sitting next to the subject, one-to-one interaction). Acquaintance with the subjects and the classroom teacher facilitated implementation of the study in many ways, e.g., finding the subject and removing him from the classroom, knowledge of the subjects' schedule for extra-curricular activities and social functions at school, accompaniment with the instructor was not aversive, etc.

Suggestions for alteration in the design include: 1) allowing plenty of space for working the algorithm, 2) lessening the amount of feedback provided, 3) setting a regular schedule on a daily basis, and 4) lessening the number of problems done within one session. Although the author attempted to allow adequate working space, some subjects had difficulty writing within the space provided. Often, subjects did not print the numerals small enough and were observed to be writing one numeral over another. The result was the possibility of an incorrect sum merely because they could not read the number they had written, and did not recheck the sum of the two previous components. Also, the subjects used colored markers which could not be erased. These markers served the purpose of motivating the subjects to perform the instructional task. In the case of an error, subjects crossed out the original response and squeezed in the alternative. Again, if the subject was distracted for some reason, this number was difficult to read when he returned to the problem. Subjects also expressed some uncertainty in reading the ditto. This was not due to inadequate
ditto preparation, but to unfamiliarity with typewritten numerals. For example, they occasionally confused "ones" with "sevens". Feedback on correctness of responses was originally provided for every component step the subject performed on the set of problems presented prior to the task. This practice was discontinued early in instruction due to the possibility of the subjects becoming satiated on receiving a considerable amount of reinforcement. The alternative approach consisted of reinforcing correct completion of each column correctly, which appeared to be effective. A routine schedule during the same hours daily is suggested for elimination of irregularity of instructional time as a confounding variable. This is not often completely possible due to subject illnesses and school social functions; however, the time period and number of days per week should have been regularly scheduled each week in this study. The number of problems performed by each subject appeared to be rather cumbersome, particularly for the complex groups, when solving problems consisting of $4 \times 5$ and $5 \times 6$ arrays. It may be beneficial to reduce this number at a certain step in the chain for each complex group to decrease the chances of the subject becoming fatigued before presentation of the task.
REFERENCES


APPENDICES
APPENDIX A

FORWARD COMPLEX

**Step 1**

Given a problem in 4 x 3 array, the student will add the first two numbers in the one's column, and write the sum in small notation to the right of and slightly below the second number.

**Step 2**

Given a problem in 4 x 3 array, the student will write the sum of the first three numbers in the one's column, using the following rules: with half-space notation, the units portion of the sum of two digits is written at the lower right of the bottom digit and the tens portion is written at the lower left of the bottom digit.

**Step 3**

Given a problem in 4 x 3 array, the student will add the binary sums for the whole one's column, and check for numbers written in half-space notation to the left of the one's column, add them and carry this sum above the first digit in the ten's column.

**Step 4**

Given a problem in 4 x 3 array, the student will complete the problem up to and including adding the number carried above
the ten's column to the first number in that column, and writing
the sum of this binary.

**Step 5**

Given a problem in 4 x 3 array, the student will complete the problem up to and including writing the sum of the half-space numbers to the left of the ten's column at the top of the hundredth's column.

**Step 6**

Given a 4 x 3 problem, the student must complete the addition of all columns with 100% accuracy.

**Step 7**

Given a problem in 4 x 4 array, the student must complete the problem with 100% accuracy.

**Step 8**

Given a problem in 5 x 5 array, the student must complete the problem with 100% accuracy.

**Step 9**

Given a problem in 6 x 6 array, the student must complete the problem with 100% accuracy.
BACKWARD SIMPLE

Step 1

Given a problem in 2 x 1 array and the sum written in half-space notation to the right of and above the horizontal line, the student will complete the problem by writing the sum of the two numbers directly below the horizontal line with 100% accuracy.

Step 2

Given a problem consisting of a single-digit and a two-digit number and with the one's column complete, the student will write the number in the ten's column in half-space notation directly under the column but above the horizontal line, and again beneath the horizontal line aligned with the ten's column with 100% accuracy.

Step 3

Given a problem consisting of a single-digit and a two-digit number with the sum of the first two numbers in the ones column written in small notation to the right of the second number, the student will carry that sum below the horizontal line and complete the problem with 100% accuracy.

Step 4

Given a problem in 3 x 2 array with the first two digits in the one's column completed, the student will complete the problem
Step 5

Given a problem in 3 x 2 array, the student must complete all component steps with 100% accuracy.

BACKWARD COMPLEX

Step 1

Given a problem in 4 x 3 array, the student will write the sum of the half-space numbers to the left of the hundredths column directly under the horizontal line.

Step 2

Given a problem in 4 x 3 array partially completed, the student will write the half-space number written to the right of the last number in the hundredths column directly under the horizontal line and bring the sum of the half-space numbers written to the left of the hundredths column down directly under the horizontal line.

Step 3

Given a problem in 4 x 3 array completed up to the last two digits in the hundredths column, the student will complete the problem with 100% accuracy.
Step 4

Given a problem in 4 x 3 array, completed up to the half-space notation written above the hundredths column, the student must complete the problem with 100% accuracy.

Step 5

Given a problem in 4 x 3 array, completed up to and including adding the sum of the half-space numbers written to the left of the ten's column, the student will write this sum above the hundredths column and complete the problem with 100% accuracy.

Step 6

Given a problem in 4 x 3 array completed up to and including writing the sum of the half-space numbers written to the left of the one's column above the first number in the ten's column, the student will complete the problem with 100% accuracy.

Step 7

Given a problem in 4 x 3 array, the student must complete the problem with 100% accuracy.

Step 8

Given a problem in 5 x 5 array, the student must complete the problem with 100% accuracy.
Step 9

Given a problem in 6 x 6 array, the student must complete the problem with 100% accuracy.

FORWARD SIMPLE

Step 1

Given two single-digit numbers arranged vertically, the student must place the sum of these in half-space notation to the right of the second number and slightly above the horizontal line.

Step 2

Given two numbers (one-digit and two-digit), the student will perform the following with 100% accuracy: 1) add two numbers in ones column and place sum in small notation to the right of second number, 2) carry this sum and write it beneath horizontal line directly below the one's column, 3) carry number in ten's column below that column and write it in small notation above horizontal line.

Step 3

Given a problem in 2 x 2 array, the student completed the problem with 100% accuracy, adding two digits in each column, writing the sum in half-space notation to the right of the second number in each column, and then below their respective columns.
beneath the horizontal line.

Step 4

Given a problem consisting of one one-digit and two two-digit addends, the student will complete the problem with 100% accuracy.

Step 5

Given a problem in 3 x 2 array, the student will complete the problem with 100% component accuracy.
APPENDIX B

A set of task problems as presented to the forward simple group (Step 4):

\[
\begin{align*}
3 & 0 & 5 & 1 & 6 & 3 & 1 & 5 \\
2 & 7 & 2 & 4 & 2 & 0 & 4 & 1 \\
+ & 1 & + & 2 & + & 2 & + & 3 \\
\end{align*}
\]

A set of task problems as presented to the backward simple group (Step 2):

\[
\begin{align*}
3 & 6 & 4 & 2 & 2 & 5 & 8 & 3 \\
+ & \frac{17}{7} & + & \frac{7}{4} & + & \frac{1}{2} & + & \frac{2}{8} \\
\end{align*}
\]

A set of task problems as presented to the forward complex group (Step 3):

\[
\begin{align*}
3 & 6 & 1 & 3 & 0 & 2 & 5 & 3 & 7 & 7 & 9 & 0 \\
5 & 2 & 0 & 6 & 7 & 3 & 1 & 5 & 4 & 4 & 3 & 2 \\
9 & 7 & 3 & 1 & 4 & 5 & 9 & 8 & 2 & 5 & 8 & 1 \\
+ & 2 & 4 & 8 & + & 7 & 2 & 1 & + & 6 & 0 & 7 & + & 1 & 6 & 3 \\
\end{align*}
\]

A set of task problems as presented to the backward complex group (Step 3):

\[
\begin{align*}
\frac{2}{3} & 6 & 7 & 1 & 3 & 4 & 0 & 2 & 5 & \frac{2}{3} & 7 & \frac{2}{5} & 9 & 0 \\
5 & 6 & 2 & 9 & 0 & 1 & 6 & \frac{7}{8} & 3 & 5 & 1 & \frac{6}{5} & 4 & 1 & \frac{4}{3} & \frac{3}{2} & \frac{2}{2} \\
9 & 1 & 7 & 3 & 4 & 1 & \frac{4}{5} & 5 & 6 & \frac{9}{8} & 2 & 3 & 5 & \frac{8}{1} & 1 & 2 \\
+ & 2 & \frac{4}{2} & \frac{8}{2} & + & \frac{7}{2} & \frac{2}{1} & + & \frac{6}{0} & \frac{7}{0} & + & \frac{1}{6} & \frac{3}{6} \\
\end{align*}
\]

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APPENDIX C

Examples of Task Problems:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>374</td>
<td>27</td>
<td>2673</td>
</tr>
<tr>
<td>1</td>
<td>929</td>
<td>11</td>
<td>4538</td>
</tr>
<tr>
<td>+3</td>
<td>+456</td>
<td>+61</td>
<td>9125</td>
</tr>
</tbody>
</table>

(Simple) (Complex) (Simple) (Complex)
APPENDIX D

Definitions

**Backward Chaining Procedure** - Effecting the development of performing the algorithm stimulus-response chain, by training the last component in the chain first; the next to the last, next; and so on until the entire chain is emitted as a single complex behavior.

**Chain** - Two or more performances combined into the more complex algorithm sequence, occurring in a determinate order, whereby each component serves a dual purpose, i.e., as both discriminative stimuli ($S^D$s), which occasion the subsequent component response, and as reinforcing stimuli, which reinforces the response that occurred immediately prior to it.

**Complex** - Refers to a problem in which it is necessary to place the tens notation to the left of and slightly below the second number when two numbers were added. Also, the "tens" were counted, carried, and added to the next column of numbers. Three to six numbers (addends) consisting of three to six digits.

**Component** - Any single writing response contained within the stimulus-response chain.

**Error** - An incorrect written response or failure to write a required response.

**Forward Chaining Procedure** - Effecting the development of performing the algorithm stimulus-response chain, by training the first component in the chain first, the second component next, etc, until the entire chain is emitted as a single complex behavior.
**Problem Complexity** - As determined by: 1) number of numerals to be added (addends), 2) number of digits contained in each addend, and 3) which steps of the algorithm are required, within any problem.

**Reinforcement** - Presentation of a social stimulus (e.g., approval, descriptive praise, smiles, etc.), contingent upon a component response, which results in an increase or maintenance of that response. For example, saying, "That's fine. You brought this number down and under the line," as opposed to saying, "That's fine."

**Simple** - Refers to a problem composed of digits such that when the binary operation was performed, it was unnecessary to place numbers in the tens place (to the left and below the second number being added). In addition, there were no "tens" carried to the next column, any number being added consisted of at most two digits, and not more than three numbers were added within one problem. The total number of steps used of the algorithm for simple problems ranged from two to six.

**Session** - Consists of four consecutive trials following training.

**Trial** - The completion of one or more components of behavior, i.e., one addition problem.