Student Preferences for the Hutchings' "Low Stress" Versus the Conventional Addition Algorithm under Conditions of Differentially Increasing Response Effort With and Without Reinforcement

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STUDENT PREFERENCES FOR THE HUTCHINGS' "LOW STRESS" VERSUS THE CONVENTIONAL ADDITION ALGORITHM UNDER CONDITIONS OF DIFFERENTIALLY INCREASING RESPONSE EFFORT WITH AND WITHOUT REINFORCEMENT

by

Carolyn Louise Gillespie

A Project Report
Submitted to the Faculty of The Graduate College in partial fulfillment of the Specialist in Education Degree

Western Michigan University Kalamazoo, Michigan August 1976
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Carolyn Louise Gillespie
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INTRODUCTION

A number of educators in recent years have focused their efforts on two major concerns: What are the most efficient methods for teaching children basic operations and concepts? What are the optimal conditions under which this learning should take place? It is of the utmost importance that children are taught with the most effective methods, in the shortest time possible, and under conditions that insure they will continue to approach learning situations. Of what benefit is education if children are not willing to use in one setting, what they have learned in another?

Becker, Engelmann and Thomas (1975) have dealt extensively with the technology of teaching for effective education. Johnston and Penny-packer (1971) took a behavioral approach to college teaching and looked at elements in their teaching technology that make for efficient learning and student motivation. Skinner (1968), using a scientific analysis of behavior, investigated student motivation and the elements that are essential for it. Keller (1968) in his statement that "The student is always right" (p. 88), leads us to believe that educators, by focusing on these two major concerns, are headed in the right direction. These two concerns are not mutually exclusive; they must be dealt with together.

In an attempt to look at conditions for optimal student learning, the area of choice-making behavior and its effects on school performance has only recently received attention. The importance of choice-making behavior to education is evident. Information on student choices and
preferences is invaluable to educators if they are to develop learning situations which students will approach and perform well in. Choice-making behavior is particularly significant in light of the fact that it seems to be related to high academic performance. For example, Lovitt and Curtiss (1969) found that higher academic rates occurred when the pupil arranged the contingency requirements rather than when the teacher specified them. Also rates of responding were greater during choice periods than during no choice periods (Lovitt and Curtiss, 1968a).

Lockhart, Sexton, and Lea (1973) were among the first to assess student preferences in an educational setting. They investigated whether students preferred and performed better with fill-in-the-blank or multiple-choice items. Student preference for forced excellence or grade choice testing situations (Lea and Lockhart, 1974) were also examined. Both these studies used the Findley "minimax" procedure to assess student preferences.

Findley (1958) first used concurrent schedules of reinforcement to assess preferences. "If a sample of the behavior shows the organism to be emitting one operant to the partial exclusion of the other, then a relative preference is defined for that operant" (Findley, p. 123). However, as Lockhart et al. (1973) stated:

"Although these methods typically generate substantial switching behavior with a higher frequency of responding under one condition than the other, the schedules themselves provide for no control over the switching responses or order of presentation of conditions. Indeed, they do not require that the organism even experience both (or all) conditions. If there is not exposure to all conditions, one cannot directly infer that the organism prefers the condition in which he worked to the other condition." (p. 2)
Findley (1962) developed his previous procedure into a minimax design. In this situation the subject was forced to experience both conditions. The subject was exposed to one condition for a maximum number of reinforcements. However, when the maximum number of reinforcements was reached, the subject was forced exposure to the other condition for a minimum number of reinforcements. Following this condition, free choice was once again instituted. It is assumed that a subject who consistently stayed in Condition A for the maximum number of reinforcements, and switched voluntarily back to Condition A, after a minimum number of reinforcements in Condition B, established a preference for Condition A. "Thus, the direction and the timing of the switching response in this procedure assumes significance" (Lockhart et al., 1973, p. 3).

In Findley's minimax procedure, the subject remained in one condition until receiving reinforcement. This Findley referred to as a nonreversible option. Lockhart et al. (1973) further stated that this procedure allows for a simplified data analysis since there is no confounding of stimulus conditions within a given component. Each subject serves as his own control.

Assessing behavioral preferences reaches significance particularly in view of the findings that verbal preferences do not always reflect ongoing behavior. Morgan and Lindsley (1966) found differences in subjects' behavioral and stated preferences for stereophonic or monophonic music. Whitehurst and Whitehurst (1973) found little difference between student evaluations of forced excellence grading and grade choice, while
Lea and Lockhart (1974) in a replication of this study, found a behavioral preference for the grade choice condition in 13 out of 16 subjects. Thus, it is highly questionable whether it is appropriate or beneficial to rely on students' verbal preferences. Future studies on student academic preferences will give educators feedback on the effectiveness of their teaching methods and learning environments.

Educators in mathematics, as educators from other disciplines, are searching for more efficient methods of teaching basic operations. Learning basic math operations poses a problem for many children. As Skinner (1968) stated:

"Few pupils ever reach the stage at which automatic reinforcements follow as the natural consequence of mathematical behavior. On the contrary, the figures and the symbols of mathematics have become standard emotional stimuli. The glimpse of a column of figures, not to say an algebraic symbol or an integral sign, is likely to set off, not mathematical behavior, but a reaction of anxiety, guilt, or fear" (p. 18).

Hutchings (1972) and Gordon (1972) are in accord with Skinner that failure in mathematics is likely to result in emotional difficulties. In addition, as Alessi (1974) stated, in recent years there has been a deemphasis on the amount of time that can be devoted to mastery of the basic computational skills. Conceptual materials are taking over more and more of the classroom mathematics period. It is therefore essential that more efficient methods of teaching the basic operations be developed.

Lovitt and Curtiss (1968b) tried one such method. They instructed one student to verbalize the math problem before writing it down. Under this condition the students' error rate dropped and rate of correct
responding increased.

In an effort to teach addition calculations efficiently, different algorithms have been offered as an alternative to the traditional addition algorithm. Batarseh (1974) developed a procedure for addition whereby the concept of carrying is simplified. Instead of carrying the digit, it is written underneath the appropriate column (under the column to which it would have been carried), and underlined. After the sum of the next column has been obtained, the underlined number is then added. The same procedure continues with successive columns.

Example:

\[
\begin{array}{c}
5 \\
+ \\
8 \\
\hline
13
\end{array}
\]
\[
\begin{array}{c}
1 \\
2 \\
\hline
1423
\end{array}
\]

Lankford (1974) found some specific computational practices of poor computers through verbal interviews. Of particular importance is the finding that poor computers "often made errors in whole number operations when their counting...became too involved for their short memory spans" (Lankford, 1974, p. 29). In addition, they tended to rely heavily on a few retained facts and relied on mechanical aids to memory.

Sanders (1971) went one step further in addition and developed a procedure for learners who tend to forget where they are in an addition problem. His method is of particular significance because it involves a faster method for performing columnar addition. It would help eliminate
some of the major problems that Lankford found with poor computers. In this addition procedure, the student says the units place to himself and holds up fingers to represent the tens place. For instance, when presented with a problem, the student would add it in the following way:

\[
\begin{align*}
7 & \quad 7 + 8 = 15 \quad \text{(holds up one finger)} \\
8 & \\
4 & \quad 5 + 4 = 9 \\
7 & \quad 9 + 7 = 16 \quad \text{(holds up additional finger)} \\
5 & \quad 6 + 5 = 11 \quad \text{(holds up additional finger)} \\
\hline
31 & \quad \text{The units place equals 1 (from 11) and} \\
& \quad \text{the tens place equals 3 because 3 fingers} \\
& \quad \text{have been held up.}
\end{align*}
\]

However, Sanders emphasized that this method should only be used after students are fairly competent in addition.

O'Malley (1969) presented a simple method of dealing with addition in all bases that is slightly different from Sanders'. Instead of holding up a finger, the digit is written under the tens, hundreds, etc. (whichever is appropriate) when any sum of numbers exceeds ten. Only units are added. When the end of a column is reached, the ones written under the next column are added to the succeeding column. (See the example on the following page.)
Example:

3 5
\[\begin{array}{c}
\text{7} \\
8 6 \\
9 4 \\
3 5 \\
8 2 \\
9 9 \\
\end{array}\]

\[
\underline{4 3 1}
\]

(O'Malley, 1969, p. 676)

In this example, the student would proceed as follows:

1. Five plus six equals 11. (Writes down the digit 1 in the tens column).

2. One plus four equals five.

3. Five plus five equals 10. (Writes down the digit 1 in the tens column.)

4. Zero plus two equals two.

5. Two plus nine equals 11. (Writes down the digit 1 in the tens column.)

The student writes the digit 1 at the bottom of the ones column. He then counts up the number of digits (the digit 1) he has written in the tens column, and carries them to the tens column. He then begins adding again. The tens column is added in the same manner as the ones column. Whenever a sum exceeds ten, the digit 1 is written outside and beside the tens column. Thus, this procedure provides a partial written record of the tens, hundreds, etc. units.

Fulkerson's (1963) procedure also provides for a partial written
record of the component operations, but is slightly different from
that of O'Malley's (1969). Instead of recording the tens, hundreds,
etc. units by writing the digit 1, his method involves drawing a line
through the last digit used in obtaining tens, hundreds, etc.

Example:  2 0 4
          6 5 6
          7 6 8
          3 2 5  

1 9 5 6

The columns can be added downward or upward. Beginning at the top
right in the example shown, 4 plus 9 equals 13, which can be renamed
as 1 ten and 3 ones. A line is drawn through the 9 to show that it was
the last digit used in obtaining ten. The ten does not need to be
held in mind because the line represents it. Now with the 3 which was
left over, add until another ten is obtained. In this example, 3 plus
8 equals 11, which can be renamed as 1 ten plus 1. Another line is
drawn through the 8 to indicate another ten. The sum of the 1 left
over and 5 is 6, which is recorded as the unit's digit at the bottom
of the column.

The two lines drawn in the units column represent 2 tens. These
are carried to the tens column and added to zero. Adding continues in
the tens columns until there is a sum greater than 10 tens; 2 tens plus
5 tens plus 6 tens equals 13 tens. A line is drawn through the 6 to
represent 10 tens, and 3 tens remain. Continuing, 3 tens plus 2 tens
equals 5 tens, which is recorded as the tens digit at the bottom.
The one line drawn in the tens columns represents 10 tens or 1 hundred, and is added to the 2 hundreds in the next column. Adding continues, similarly, until the problem is finished.

Thus, Fulkerson's method also provides for a partial written record of the component operations.

The Hutchings' "low stress" algorithm has been offered as a reasonable alternative to the conventional math algorithm taught in schools (Alessi, 1974; Hutchings, 1972). It has been found to have some distinctive advantages over the algorithms previously mentioned. Instead of providing a partial written record (Fulkerson, 1963; O'Malley, 1969) of the component operations, it provides a complete record of all component operations. With a full record there is less demand on memory or covert chains of behavior. The student records all the addition facts in a sequence, and only after this has been done, does the student perform the necessary regroupings. Hutchings (1976) defined the algorithm by its process:

"The low-stress addition algorithm uses a new notation, called half-space notation, to record individual steps. Half-space notation uses numerals of no more than a half-space in height to record the sum of two digits. With half-space notation, the units portion of the sum of two digits is written at the lower right of the bottom digit, and the tens portion is written at the lower left of the bottom digit...[When presented with single column addition problems], we add the first two digits...and record the sum in the new notation...The complete sum of each two-digit addition is recorded in half-space notation, but only the ones portion of each sum is used in the next addition...The ones portion of the column sum is always the same as the ones portion of the last two-digit sum...The tens portion of the column sum is always the same as the number of tens recorded at the left of the column. These are simply counted... For a column in some multicololumn exercise, then, the last
step - that is, counting the tens at the left of the column - would be slightly changed. The counting itself is not changed in any way, but the answer, the total number of tens, is no longer written in the tens place of the first column's sum but instead at the top of the next column at the left... Work continues in this manner until the exercise is completed. Note, however, that the column sum for the last column in a multicolumn example is recorded in exactly the same way as the sum of a single-column exercise. (pp. 220-221).

Example:

\[
\begin{array}{c}
\ 6 \\
\ 9 \\
\ 1 \\
\ 7 \\
\ 7 \\
\end{array}
\]

2 7

In this example, the student would proceed as follows:

1. Five plus six equals 11.
2. One (from the unit's place in 11) plus nine equals 10.
3. Zero (from the unit's place in 10) plus seven equals seven.
4. The student brings down the digit seven and adds the ten's digit, to complete the column sum.

It is Hutchings' opinion that there is less stress involved for the student using this algorithm when he is presented with challenging problems. It is further his opinion that this may be due to the fact that all the addition facts are performed at once, followed by all necessary regroupings. The student does not need to alternate between addition and regrouping. As Hutchings stated, "The advantage offered by the low stress procedures is increased in proportion to the length
of the column" (Hutchings, 1976, p. 223).

There are two major advantages in being able to perform large problems correctly (Hutchings, 1976). The first is psychological. The student will not be intimidated but rather challenged by large problems. Second, being able to perform large problems correctly and efficiently has a practical advantage.

The Hutchings' "low stress" algorithm reduces the amount of instructional time necessary for mastery. Other advantages include: (a) it is easy to locate specific errors; (b) it requires only a knowledge of basic math facts; and (c) it has been designed to increase speed and accuracy (Alessi, 1974; Hutchings, 1976). Several studies using group factorial experimental designs, found the "low stress" algorithm to be more efficient in calculation than the conventional procedures (Alessi, 1974; Boyle, 1975; Dashiell, 1974; Gordon, 1972; Hutchings, 1972). Furthermore, several single subject design studies found similar computational superiority for the "low stress" algorithm over the conventional one (Rudolph, 1976; Zoref, 1976). Further, the Hutchings' algorithm "complements well the current emphasis in mathematics curricula on the set theory and place value concepts" (Alessi, 1974, p. 14). For the reasons listed above and the advantages it demonstrates over other alternative algorithms, the Hutchings' "low stress" algorithm seems a very reasonable alternative to the conventional addition algorithm.

In recent research with the Hutchings' "low stress" algorithm, Alessi (1974) found that the Hutchings' algorithm, as opposed to the
conventional algorithm, produced higher scores for the number of columns correct and columns attempted in a 30 minute time period. As the problems presented increased in difficulty (more binary operations to perform), the Hutchings' algorithm showed a decrease in superiority. Also, when children were reinforced for either number of columns correct or number of whole problems completed correctly, those reinforced for columns correct attempted fewer columns, and added fewer columns correctly, as compared to the group reinforced for whole problems correct.

Alessi (1974) stated that the children using the Hutchings' algorithm were extremely impressed by their ability to correctly perform large addition problems (2 X 7, 3 X 7, 5 X 7). They worked hard with the new algorithm, asked for feedback, and verbally expressed happiness and self assurance when told that they had performed the problem correctly. Parents, in addition, stated that both they and their children were pleased with the Hutchings' algorithm. Thus, from this evidence, it would seem that children may prefer the Hutchings' addition algorithm over the traditional algorithm.

However, as previously stated, verbal preferences are not always an accurate assessment of behavioral preferences (Lea and Lockhart, 1974; Morgan and Lindsley, 1966). From the evidence provided so far on the Hutchings' "low stress" algorithm, it would seem that in terms of both needed instructional time and efficiency, it is a reasonable alternative to the conventional algorithm. However, the success of the Hutchings' "low stress" algorithm will ultimately depend upon
how willing children are to use it.

This study is composed of two separate experiments. Run concurrently, they attempted to apply the Findley "minimax" preference procedure to another academic area, that of mathematical addition computations. Do children prefer to use the conventional algorithm or the Hutchings' "low stress" algorithm in addition computations?

The first experiment attempted to answer the following questions:

1. When given a free choice, would third grade students prefer to do addition problems using the conventional algorithm or the Hutchings' "low stress" algorithm?

2. What effect does increased response effort have on algorithm preference? That is, to what extent will children continue to perform the preferred algorithm under conditions of increased response effort over the nonpreferred algorithm?

3. Is there a difference in algorithm preference between students who consistently know their basic addition math facts and those who do not?

4. Do children prefer to perform the algorithm that results in the greater accuracy and efficiency?

5. Are verbal preferences for a certain algorithm an accurate assessment of behavioral preferences for that algorithm?

The second experiment attempted to investigate the effects of reinforcement for accuracy on algorithm preference, under conditions similar to those of Experiment 1. The following question was asked:

1. What is the effect of reinforcement for accuracy on algorithm
preference under the condition where performing the preferred as opposed to the nonpreferred algorithm, required increased response effort? That is, how large must the differential response effort be to cause a shift in preference from the preferred to nonpreferred algorithm?
A five minute basic addition math facts test (taken from Alcan, 1976) consisting of 36 basic addition facts, was administered to 23 third grade students at an elementary school near Kalamazoo, Michigan. Six students, three white males and three white females, were chosen for the study, based on their percent accuracy on the math facts test. Four high accuracy students and two low accuracy students were chosen. High accuracy was defined as 96% or better, and low accuracy as 80% or less on the addition math facts test. The basis for selection was solely on students' accuracy on the addition math facts test. The six students for the study were selected by the homeroom teacher from the possible pool of students who attained 96% or better, or 80% or less on the math facts test. Students who the teacher thought would enjoy being in such a study, were chosen. However, students who planned to take any vacations during the time period of the study were excluded as possible participants. The four high accuracy students chosen, two females (Students 1 and 2) and two males (Students 5 and 6), all had 100% accuracy on the math facts test. The two low accuracy students chosen, one female (Student 3) and one male (Student 4), had 80% and 70% accuracy, respectively.
Setting

The study took place in four different rooms in the elementary school. No one room in the school building was consistently available for the purposes of this study. However, during the majority of the sessions one room was consistently used.

For four sessions during the algorithm instructional stage, the six students met in a large classroom. The students sat in chairs, arranged in one row facing and approximately five feet from the blackboard. The experimenter stood between the row of chairs and the blackboard, facing the students.

The majority of the sessions took place in a smaller room. This room was partitioned in half by bookshelves and a portable blackboard. The students sat in chairs at two long rectangular tables. The tables were situated perpendicular to one another and separated by approximately seven feet. During most of the sessions three students sat at each table. However, during the final week of the study, the bookshelves were removed from the room. This allowed the students to spread out. At this point, students could either choose to sit at one of the two tables or to take two chairs into another area of the room. Students sat on one chair, placed their papers and worked on the other chair. At his point most of the students chose not to sit at the table, and to work in an area that was at least six feet from any other student.

Two additional rooms, located off of the main part of the library, were also used. The first, used for approximately one week, contained two long rectangular tables that were placed perpendicular to one another.
During all sessions three students sat at each table. There were no windows or other visual distractions in this room. The second room was used for only three sessions. It was enclosed with glass and was smaller than the others. One long rectangular table was in the middle of the room. All students sat in chairs at this one table.

During all sessions, except those during the instructional algorithm stage, the experimenter sat in a chair or on the floor behind the students and as far out of their view as possible. During the instructional stage, the experimenter was either located between the students and the blackboard, or walked around giving individual help to the students.

During sessions where an observer was present, the observer and the experimenter sat as far out of the students' view as possible.

Answer keys were located in corners of the room as far removed from the students as possible. These were placed either on the floor or on chairs.

**Experimental Task**

During each session in all conditions, with the exception of the instructional algorithm stage, students were given sheets of 4 X 5 (four columns, five rows) addition problems to complete. The addition problems were obtained from a computer printout of 4 X 5 random number arrays. In accord with specific recommendations from Hutchings (1972), the numeral 0 was never used in any of the problems. As Hutchings (1972) stated, "It is required that applications of the
identity element be avoided, as those are considered to load for a distinct peripheral concept while contributing very little to demands upon memory - retrieval functions" (p. 51).

All numerals in the addition problems were typed with the IBM Orator element, with the uppercase printstyle. Digits in all rows were typed three spaces apart, with double-spacing between rows. Further, the area between the bottom row of digits in a problem and the bottom line was double-spaced. Spacing of digits in the problems is considered critical for experimental tasks using so-called "full record algorithms" (Alessi, 1974). The number of 4 X 5 array problems on a page ranged from two to six, with no more than six problems on any page. No student ever received the same problem twice.

The number of problems given a student during any session depended upon (a) the individual student's speed of completing addition problems, established during the prebaseline phase of the study, and (b) the response effort required during the condition in which the session occurred. During the baseline condition, three students (Students 1, 5, and 6), received six problems per session. Students 2 and 4 were given four problems, and Student 3 received 2 problems. The number of problems assigned were chosen so that each student spent approximately the same amount of time on the calculation task (5-6 minutes).

**Design**

In Experiment 1 two different reversal designs (Baer, Wolf, and
Risley, 1968) were employed. It was decided before the study to implement Condition C only for those students who did not shift preference from baseline during Condition B. Students 1, 2, 3, and 4 received a reversal design of the pattern A-B-C-A. Students 5 and 6 received an A-B-A-B-A reversal design.

Independent Variables

1. Hutchings' "low stress" addition algorithm versus the conventional addition algorithm.

2. Equal response effort for both algorithms versus differentially increased response effort of 50% and 100% for the preferred algorithm.

3. High accuracy versus low accuracy students based on the addition basic math facts test and teacher recommendation.

Criteria for Independent Variables

1. a) Hutchings' "low stress" full record algorithm (see pages 9 and 10 for definition and example).

   b) Conventional algorithm: The only mark written down is the final answer; all other computations are done covertly.

Example:

\[
\begin{array}{c}
5 \\
6 \\
9 \\
7 \\
27
\end{array}
\]
2. a) Equal response effort for the two types of algorithms:

The students are given the same number of problems and size of problems regardless of which algorithm they choose to use in completing the problems.

b) Differentially increased response effort for the preferred algorithm: Increased response effort is defined by the requirement to complete more problems. The size of the problems remains constant. Students choosing to perform their preferred algorithm (defined below) are required to complete 50% more problems (Condition B) or 100% more problems (Condition C) than during baseline. Students switching to their non-preferred algorithm are required to do only the same number of problems as in baseline conditions.

3. a) High accuracy: On the basic math facts test, 96% or better.

b) Low accuracy: On the basic math facts test, 80% or less.

Dependent Variables

1. Algorithm preference
2. Rate of columns correct
3. Rate of columns incorrect
4. Percent accuracy

Criteria for Dependent Variables

1. Preference for one algorithm defined by students' choosing to do the same algorithm for at least six consecutive free choice sessions. Preference is determined during each condition.
2. Rate of columns correct: \[
\frac{\text{number of columns correct}}{\text{number of minutes}}
\]

3. Rate of columns incorrect: \[
\frac{\text{number of columns incorrect}}{\text{number of minutes}}
\]

4. Percent accuracy: \[
\frac{\text{number of columns correct}}{\text{number of columns attempted}}
\]

Procedure

Pretraining. The basic math facts test was administered and students selected according to criteria stated previously. Students were then taught the Hutchings' "low stress" algorithm during two 20 minute instruction sessions held on two consecutive days. One 20 minute session of review was then held on the Hutchings' "low stress" algorithm (students' spring vacation intervened between direct instruction and review session). A 20 minute review session was also held on the conventional addition algorithm. In all instruction and review sessions, students were given dittoed sheets of addition problems for practice. The instruction and review procedures used were those adapted from Hutchings (1972) and previously used by Alessi (1974) and Boyle (1975).

In the last instruction session each student was asked to complete two problems using the conventional addition algorithm and two problems using the Hutchings' algorithm. The time to complete these sets of two problems was recorded by the experimenter. This was done in an effort to estimate how many problems each student should be given during each baseline session, so that all students would finish in approximately six minutes. This time elapse was established so that
future sessions could be planned to fit within the 10 minute time
limit needed, so that session length would not interfere with school
classes.

**Prebaseline condition.** Following the last review session, the
prebaseline condition was started. Two 10 minute sessions were held
daily. One session was held in the morning between approximately
9:05 and 9:15. One session was held in the afternoon between approx­
imately 12:50 and 1:00. Following Session 19, and until the end of
the study (Session 56), the times of the sessions were changed, due
to scheduling problems. The first session was held between 11:25
and 11:35. The second session was held between 12:50 and 1:00. The
experimenter went to the classroom and got the students before each
session.

During the first four sessions of the prebaseline condition,
students were forced to alternate using the two types of algorithms,
in order to provide forced exposure to both types. The starting order
was counterbalanced across students.

The prebaseline condition ended for each student, with the excep­
tion of Student 5, when he/she had established stable performances during
exposure to the four forced choice sessions. During the prebaseline
condition, it became evident that for Student 5 the number of problems
given in each session needed to be increased to get a stable perform­
ance elapsed time of approximately six minutes. The prebaseline con­
dition ended for Student 5 when the number of problems given per ses­
session was increased and a stable performance noted.
With the exception of the four forced choice sessions during the prebaseline condition, during all subsequent conditions, each student was asked before each session which algorithm he/she preferred to use in doing addition problems. The student was then instructed to use that algorithm in doing the addition problems during that session. However, students were not allowed to use the same algorithm on any more than three consecutive sessions. After three consecutive sessions of using the same algorithm, the students were forced to use the alternate algorithm for one session. After this one forced choice session, the students were again allowed free choice sessions.

When each student had completed the required number of problems, he/she was instructed to raise his/her hand or call his/her name. The experimenter recorded the time taken to complete the problems for each student. The experimenter used a stopwatch to measure the time taken. Answer keys were provided so that students could check their answers when finished. Students were instructed to circle any errors, but not to change any answers. Different colored magic markers were provided for checking papers. Students could then either remain in the room until all were finished or return to their regular classroom.

**Baseline condition (Condition A).** During baseline condition all students had a choice of what type of algorithm they wished to perform during each session. Response effort was equal across both types of algorithms. The students were given the same number of problems to complete no matter which algorithm they chose to use. Students 1, 5, and 6 received six problems. Students 2 and 4 were given four
problems, and Student 3 received two problems. The baseline condition ended for each student when a preference for one algorithm had been established. All students did not establish preference during the same session.

Condition B. Students again had a free choice of which algorithm they wished to use during each session. However, the preferred algorithm, as established during baseline for each student, required a 50% increase in response effort than in Condition A. Students were instructed that they could again choose which algorithm to use, but if they chose the one they preferred in Condition A, they would have to do three, six, or nine problems (depending on whether they had previously been completing two, four, or six problems, respectively). If they chose the nonpreferred algorithm from Condition A, they would have to do the same number of problems as in Condition A. Students 1, 5, and 6 would be required to do nine problems of their preferred algorithm. If they chose their preferred algorithms, Students 2 and 4 would be required to do six, and Student 3, three problems. Condition B ended for each student after a preference for either algorithm had been established.

Condition C. Students again had a free choice of the algorithm they wished to use during each session. During this condition choosing the preferred algorithm (as established in Condition B), required a 100% increase in response effort from Condition A, and over that required if the nonpreferred algorithm was chosen. The same response effort was required for the nonpreferred algorithm as during Condition
A. In other words, Student 1 now had to complete 12 problems, Students 2 and 4, eight problems, and Student 3, four problems, if they chose to use their preferred algorithm. Students 5 and 6 were not exposed to Condition C. Condition C ended when each student had established a preference for either algorithm, or when it was clear that there was no algorithm preference. Again, all students did not complete Condition C during the same session.

**Return to Baseline Condition (Condition A).** This condition, the fourth condition, was a return to baseline. The exact same procedure was used as during the initial baseline of Condition A. Condition A ended when a preference had been established for either algorithm, or when school ended for the summer. During the last session of this condition, students were asked to answer certain questions. These questions were presented both orally and visually on the blackboard by the experimenter. The questions pertained to algorithm preference, accuracy level, and rate of completion.

**Condition B.** Students 5 and 6 were exposed once again to this condition. The procedure used during this condition was identical to the one used during the initial B condition.

**Condition A.** Students 5 and 6 experienced a second return to baseline condition. The exact same procedure was used as during the initial and second baseline. Students 5 and 6 were administered the questionnaire during the last session of this condition.
Recording and Scoring

Following each session the experimenter checked the students' papers against the answer key. The number of columns incorrect and the time taken to complete all problems were recorded. The experimenter recorded which algorithm the student had used for that session. For each session the rate of columns correct, rate of columns incorrect, and percent accuracy were calculated.

Reliability

Reliability checks on the students' elapsed times for completing problems were taken by an independent observer. The observer, like the experimenter, recorded when the students had completed their required problems. The observer and the experimenter each had separate stopwatches. They sat across the room from one another and could not see the recorded times of each other. The times recorded by the observer and experimenter for each student were compared. Each pair of recorded times was scored either plus or minus. A plus resulted when recorded times were in agreement within three seconds, high or low. A minus resulted when times were in disagreement more than three seconds, high or low. Reliability coefficients were calculated by dividing agreements by agreements plus disagreements (Type II reliability).

The problems completed by the students leave a permanent product. One independent observer corrected the answers periodically, after they had been corrected by the experimenter, to act as a reliability check. Reliability coefficients were calculated by dividing the
number of agreements by the number of agreements plus disagreements (Type II reliability).
EXPERIMENT 1

Results

Reliability

Reliability for the students' times to complete the problems was taken for seven of the 56 sessions, and yielded a mean of 95% agreement. Papers from 10 of the 56 sessions for each student were also checked. A mean of 93% agreement was obtained.

Algorithm Preference

Students 5 and 6 did not receive Condition C since they showed a shift in algorithm preference from baseline during Condition B. For clarity, the results for Students 5 and 6 will be presented separately from the rest of the students.

Behavioral Preference (Students 1 through 4). Table 1 (top) shows the algorithm preferences for each student during all conditions. All four students showed a preference for the Hutchings' algorithm during both baseline conditions. During Condition B, three students continued to prefer the same algorithm, while one student showed no preference for either algorithm. During Condition C, three students showed no preference for either algorithm. Student 4 had a preference for the Hutchings' algorithm during this condition.

Table 1 (bottom) shows the percentage of students preferring
Table 1: Individual and total student algorithm preferences within and across all experimental conditions.
<table>
<thead>
<tr>
<th>Experimental Conditions</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>Student 2</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>* Student 3</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>* Student 4</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
<td>Hutchings</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Totals (Percent)</th>
<th>Totals Across Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutchings</td>
<td>100% 75% 25% 100% 75%</td>
</tr>
<tr>
<td>Conventional</td>
<td>0%   0%   0%   0%   0%</td>
</tr>
<tr>
<td>No Preference</td>
<td>0%   25%  75%   0%   25%</td>
</tr>
</tbody>
</table>

NOTE: * indicates students who did not know math facts consistently.
the Hutchings' algorithm, the conventional algorithm, or neither algorithm, during each condition and across all conditions. Across all conditions, 73% of the students preferred the Hutchings' algorithm, and 23% had no preference. No student ever showed a preference for the conventional algorithm.

Table 2 presents a more detailed analysis of individual student preferences. The number of sessions (and corresponding percentages) during which each student chose or was forced to perform either algorithm, during each condition, for both free and forced choice sessions, are presented. During all conditions where a preference for one algorithm was established, each student, with one exception, chose to perform the preferred algorithm during 100% of the free choice sessions. During the initial baseline condition, Student 4 chose to perform the preferred algorithm during 75% of the free choice sessions. Table 2 shows that during two of the four conditions when no preference for either algorithm was established, the Hutchings' algorithm was performed for 67% and 77% of the free choice sessions. The conventional algorithm was performed for 62% of the free choice sessions during the third no preference condition. In the final no preference condition, the Hutchings' and the conventional algorithm each were performed for 50% of the free choice sessions. Across all students the conventional algorithm was performed during no more than 37.5%, and the Hutchings' algorithm during no less than 62.5% of the free choice sessions, in any condition. Total algorithm choices show that in no more than 33% of the forced choice sessions, were students forced to perform the
Table 2: Individual and total student algorithm choices during free and forced choice sessions in each condition.
<table>
<thead>
<tr>
<th>Students</th>
<th>Condition A Baseline</th>
<th>Condition B 50% Response Effort</th>
<th>Condition C 100% Response Effort</th>
<th>Condition A Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forced Choice</td>
<td>Free Choice</td>
<td>Forced Choice</td>
<td>Free Choice</td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>S1</td>
<td>Hutchings</td>
<td>0/3 0.0</td>
<td>9/9 1.0</td>
<td>0/3 0.0</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>3/3 1.0</td>
<td>0/9 0.0</td>
<td>3/3 1.0</td>
</tr>
<tr>
<td>S2</td>
<td>Hutchings</td>
<td>0/3 0.0</td>
<td>9/9 1.0</td>
<td>0/3 0.0</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>3/3 1.0</td>
<td>0/9 0.0</td>
<td>3/3 1.0</td>
</tr>
<tr>
<td>S3</td>
<td>Hutchings</td>
<td>0/2 0.0</td>
<td>6/6 1.0</td>
<td>0/2 0.0</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>2/2 1.0</td>
<td>0/6 0.0</td>
<td>2/2 1.0</td>
</tr>
<tr>
<td>S4</td>
<td>Hutchings</td>
<td>0/3 0.0</td>
<td>9/12 .75</td>
<td>* 0/2 0.0</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>3/3 1.0</td>
<td>3/12 .25</td>
<td>2/2 1.0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>Hutchings</td>
<td>0/11 0.0</td>
<td>33/36 .92</td>
<td>0/10 0.0</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>11/11 1.0</td>
<td>3/36 .08</td>
<td>10/10 1.0</td>
</tr>
</tbody>
</table>

* No preference for either algorithm established.
Hutchings' algorithm. During two conditions students were forced to perform the conventional algorithm during 100% of the forced choice sessions.

There was no difference in algorithm preference between groups of students who consistently knew their basic addition math facts (Students 1 and 2) and those who did not (Students 3 and 4). The preferences for Student 3 were identical to those of Students 1 and 2. Student 4 showed the same preference for the Hutchings' algorithm during baseline conditions as did the other three students. Under conditions of increased response effort for the Hutchings' algorithm, Student 4 showed preferences that were reversed from those of the other three students; that is, Student 4 had no preference in Condition B (50% increase in response effort), and preferred the Hutchings' algorithm for Condition C (100% increase in response effort).

Verbal preferences. The results of the questionnaire showed that all four students' behavioral preferences matched their verbal preferences. All students said that the Hutchings' algorithm was the one they liked better both at the beginning and at the end of the study. In addition, the students said they got more problems correct and finished the problems quicker using the Hutchings' algorithm. All indicated that they made more mistakes using the conventional algorithm. This is an accurate assessment of each individual student's performance during each condition, with one exception. During Condition B, Student 4 was more accurate and more efficient with the conventional algorithm (to be presented later in Table 3).
The above results from the questionnaire are further substantiated by the fact that, when in a forced choice session, Student 3 refused on several occasions to use the conventional algorithm. During the study, several students asked the experimenter to make up huge problems so that they could practice the new way. The experimenter heard several times the students say that they liked the new way better. Furthermore, the homeroom teacher mentioned to the experimenter at the end of the study that she was pleased to see several of the students using the Hutchings' algorithm in their classroom math assignments.

**Algorithm Accuracy and Efficiency**

Table 3 shows individual and total student mean session accuracy, mean session rates of columns correct and incorrect for the Hutchings' and conventional algorithms during all conditions. Figures la and lb present total student data. Figures la and lb show that during all conditions, the Hutchings' algorithm had: (a) higher accuracy; (b) greater mean session rate of columns correct; and (c) lower mean session rate of columns incorrect, than the conventional algorithm. This overall trend is supported by all individual student data during each condition, with one exception. During Condition B, Student 4 was more accurate and more efficient with the conventional algorithm. Thus, during Conditions B and C, where choosing the Hutchings' algorithm required increased response effort (50% and 100% increases, respectively), the Hutchings' algorithm was still a more efficient and
Table 3: Individual and total student mean session performance with the Hutchings' and the conventional algorithms during each experimental condition.
### TABLE 3

**INDIVIDUAL AND TOTAL STUDENT MEAN SESSION PERFORMANCE WITH THE HUTCHINGS' AND THE CONVENTIONAL ALGORITHMS DURING EACH EXPERIMENTAL CONDITION**

<table>
<thead>
<tr>
<th>Students</th>
<th>Condition A</th>
<th>Condition B 50% Response Effort</th>
<th>Condition C 100% Response Effort</th>
<th>Condition A Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>X</strong> RC RI <strong>#</strong></td>
<td><strong>#</strong> RC RI <strong>#</strong></td>
<td><strong>#</strong> RC RI <strong>#</strong></td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>.977 8.44 .18 9</td>
<td>.973 9.53 .24 9</td>
<td>.97 10.11 .33 6</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>.945 5.59 .29 3</td>
<td>.945 6.83 .41 3</td>
<td>.94 7.05 .44 8</td>
</tr>
<tr>
<td>S1</td>
<td>H</td>
<td>.945 5.59 .29 3</td>
<td>.945 6.83 .41 3</td>
<td>.94 7.05 .44 8</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>.945 5.59 .29 3</td>
<td>.945 6.83 .41 3</td>
<td>.94 7.05 .44 8</td>
</tr>
<tr>
<td>S2</td>
<td>H</td>
<td>1.0 5.88 0.0 3</td>
<td>1.0 5.88 0.0 3</td>
<td>1.0 5.88 0.0 3</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>.916 2.97 .31 3</td>
<td>.916 2.97 .31 3</td>
<td>.916 2.97 .31 3</td>
</tr>
<tr>
<td>S4</td>
<td>H</td>
<td>.862 3.68 .61 9 **</td>
<td>.862 3.68 .61 9 **</td>
<td>.862 3.68 .61 9 **</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>.668 2.92 1.31 6</td>
<td>.668 2.92 1.31 6</td>
<td>.668 2.92 1.31 6</td>
</tr>
</tbody>
</table>

**TOTAL (MEAN)**

| H | .929 5.34 .286 33 | .89 5.31 .568 35 | .94 5.87 .338 27 | .89 6.27 .60 35 |

**NOTE:**
- H refers to Hutchings' algorithm
- C refers to conventional algorithm
- * indicates preferred algorithm in each condition
- ** no algorithm preference established
- * indicates mean session percent accuracy
- RC refers to mean session rate of columns correct/minute
- RI indicates mean session rate of columns incorrect/minute
- # indicates the number of sessions in each condition during which the algorithm was performed
Figure 1a: Total student mean session percent of columns correct, in each experimental condition, for the Hutchings' algorithm and the conventional algorithm. Data from the group of four students (Students 1, 2, 3, and 4) in Experiment 1 are included.

Figure 1b: Total student mean session rate of columns correct and incorrect, in each experimental condition, for the Hutchings' algorithm and the conventional algorithm. Data from the group of four students (Students 1, 2, 3, and 4) in Experiment 1 are included.
FIGURE 1a

PERCENT OF COLUMNS CORRECT (MEAN)

CONDITIONS

FIGURE 1b

MEAN RATE OF COLUMNS CORRECT AND INCORRECT (NUMBER OF COLUMNS/MINUTE)

CONDITIONS

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The bottom of Table 3 and Figure 1b show that, over all conditions, the total mean session rate of columns correct for the Hutchings' algorithm gradually increased. Generally, individual student data support this finding. However, the mean session rate of columns correct decreased for Student 3 during the C condition, and decreased slightly for Students 2 and 4 during the B condition.

During all conditions, each student chose as his/her preferred algorithm, the one that resulted in (a) higher accuracy, (b) lower mean session rates of columns incorrect, and (c) higher mean session rates of columns correct. For one student (#4) under Condition B, and for three students under Condition C (#1, #2, and #3), no preference for either algorithm was established. In Condition C it appears that the 100% increase in response effort for choosing the preferred algorithm (Hutchings' algorithm), affected students' choices for consistently performing the more accurate and efficient algorithm. In Condition B Student 4 showed no preference for the conventional algorithm, although that algorithm was associated with (a) the lower response effort, (b) the higher accuracy, (c) greater mean session rates of columns correct, and (d) the lower mean session rates of columns incorrect. However, in Condition C Student 4 established a preference for the Hutchings' algorithm which involved greater response effort. In Condition C, Student 4 performed with higher accuracy, greater mean session rates of columns correct, and lower mean session rates of columns incorrect with the preferred (Hutchings')
Algorithm Preference (Students 5 and 6)

Behavioral preferences. Students 5 and 6 were similar in their algorithm preferences (Table 4 top) during all conditions. Both preferred the conventional addition algorithm during the initial baseline and switched preference to the Hutchings' algorithm during the second baseline. During both B conditions, both students preferred the algorithm that involved the lesser response effort. Due to time limitations only four sessions were conducted in the third baseline condition. Thus, no algorithm preference could be established. The individual choices of each student during each of these four sessions are presented in the table. Both students chose the Hutchings' algorithm for three of these four sessions.

Total student percentages for choosing the Hutchings' algorithm, the conventional algorithm, or no preference (neither algorithm), within each condition and across all study conditions, are presented at the bottom of Table 4. Students showed preferences for each algorithm during 50% of the conditions. Each showed a preference for the Hutchings' algorithm during one baseline and one increased response effort condition. Each student also showed a preference for the conventional algorithm during one baseline and one B condition.

Table 5 presents individual and total algorithm choices for Students 5 and 6, in each condition, during free and forced choice sessions. In each experimental condition, Student 5 performed the
Table 4: Individual and total algorithm preferences for Students 5 and 6 within and across all experimental conditions.
### TABLE 4

**INDIVIDUAL AND TOTAL ALGORITHM PREFERENCES FOR STUDENTS 5 AND 6 WITHIN AND ACROSS ALL EXPERIMENTAL CONDITIONS**

<table>
<thead>
<tr>
<th>Experimental Conditions</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
<th>a</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 5</td>
<td>C</td>
<td>H</td>
<td>H</td>
<td>C</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Student 6</td>
<td>C</td>
<td>H</td>
<td>H</td>
<td>C</td>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Totals (Percent)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutchings</td>
<td>0%</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>Conventional</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>No Preference</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td></td>
<td>0%</td>
</tr>
</tbody>
</table>

**Totals Across Conditions**

**NOTE:**
- a indicates algorithm choices, H = Hutchings, C = Conventional
- b indicates forced choice session
- c totals exclude final A condition
Table 5: Individual and total algorithm choices for Students 5 and 6 during free and forced choice sessions in each condition.
<table>
<thead>
<tr>
<th>Students</th>
<th>Condition A Baseline</th>
<th>Condition B SOX Response Effort</th>
<th>Condition A Baseline</th>
<th>Condition B 50% Response Effort</th>
<th>Condition A Baseline</th>
<th>Condition B SOX Response Effort</th>
</tr>
</thead>
<tbody>
<tr>
<td>S5</td>
<td>H</td>
<td>2/2 1.0 0/6 0.0</td>
<td>0/3 0.0 10/10 1.0</td>
<td>0/2 0.0 8/8 1.0</td>
<td>2/2 1.0 1/10 .10</td>
<td>0/0 0.0 3/4 .75</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0/2 0.0 6/6 1.0</td>
<td>3/3 1.0 0/10 0.0</td>
<td>2/2 1.0 0/8 0.0</td>
<td>0/2 0.0 9/10 .90</td>
<td>0/0 0.0 1/4 .25</td>
</tr>
<tr>
<td>S6</td>
<td>H</td>
<td>2/2 1.0 2/10 .20</td>
<td>0/2 0.0 8/9 .89</td>
<td>0/2 0.0 6/6 1.0</td>
<td>3/3 1.0 0/9 0.0</td>
<td>0/1 0.0 3/3 1.0</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0/2 0.0 8/10 .80</td>
<td>2/2 1.0 1/9 .11</td>
<td>2/2 1.0 0/6 0.0</td>
<td>0/3 0.0 9/9 1.0</td>
<td>1/1 1.0 0/3 0.0</td>
</tr>
<tr>
<td>TOTALS</td>
<td>H</td>
<td>4/4 1.0 2/16 .125</td>
<td>0/5 0.0 18/19 .95</td>
<td>0/4 0.0 14/14 1.0</td>
<td>5/5 1.0 1/19 .05</td>
<td>0/1 0.0 6/7 .86</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0/4 0.0 14/16 .875</td>
<td>5/5 1.0 1/19 .05</td>
<td>4/4 1.0 0/14 0.0</td>
<td>0/5 0.0 18/19 .95</td>
<td>1/1 1.0 1/7 .14</td>
</tr>
</tbody>
</table>

NOTE: H refers to Hutchings' algorithm
C refers to the conventional algorithm
preferred algorithm during no less than 90% of the total free choice sessions. Student 6 performed the preferred algorithm during no less than 80% of the total free choice sessions.

Verbal preferences. On the questionnaire, verbal preferences matched behavioral preferences for one of the two students. Student 5 indicated that he liked the conventional algorithm better in the beginning and the Hutchings' algorithm better at the end of the study. This corresponds exactly with his behavioral preferences. Student 6 said that he liked the Hutchings' algorithm better both at the beginning and at the end of the study. This contradicts his behavioral preference, which was identical to Student 5. Thus, his verbal preferences were not an accurate assessment of his behavioral preferences.

Furthermore, Student 5's verbal assessment of how he performed over the whole study with the two different algorithms, was also accurate. He felt that the Hutchings' algorithm was easier and that using it, he got more problems correct. Student 5 indicated that when using the conventional algorithm he made more mistakes but finished the problems more quickly. These statements match Student 5's behavioral performance. However, Student 6's verbal statements of how he performed over the whole study with the two different algorithms, did not match his behavioral performance. Over all conditions, Student 6 performed, both in speed and accuracy, approximately equal with both algorithms. Yet Student 6 indicated that he made more mistakes with the Hutchings' algorithm and got more problems correct with the conventional algorithm. He further indicated that neither algorithm
was easier than the other, not could he finish more quickly using one algorithm over another. It should be noted that Student 6 said that his main concern was finishing quickly.

**Algorithm Accuracy and Efficiency**

Table 6 presents individual and total student mean session accuracy, mean session rates of columns correct, and mean session rates of columns incorrect for the Hutchings' and conventional algorithms during all conditions. The total mean session data is presented in Figures 2a and 2b. Figure 2 reveals that in the initial baseline condition, the conventional algorithm yielded (a) higher accuracy, (b) a greater mean session rate of columns correct, and (c) a slightly lower mean session rate of columns incorrect than did the Hutchings' algorithm. Individual data for both students support this finding. In the following four conditions, the total mean session results (Table 6, Figures 2a and 2b) show that the Hutchings' algorithm had higher accuracy, greater mean session rates of columns correct, and lower mean session rates of columns incorrect, with two exceptions. In the final B condition, (Figure 2b), the conventional algorithm had a greater mean session rate of columns correct than the Hutchings' algorithm. Accuracy was equal for both algorithms during the second baseline condition (see Figure 2a). Individual data substantiates this finding with three exceptions:

1. During the initial B condition, Student 5 had a slightly lower mean session rate of columns correct with the
Table 6: Individual and total student mean session performance with the Hutchings' and the conventional algorithms during each experimental condition.
## TABLE 6

**INDIVIDUAL AND TOTAL STUDENT MEAN SESSION PERFORMANCE WITH THE HUTCHINGS' AND THE CONVENTIONAL ALGORITHMS DURING EACH EXPERIMENTAL CONDITION**

<table>
<thead>
<tr>
<th>Students</th>
<th>Condition A Baseline</th>
<th>Condition B 50% Response Effort</th>
<th>Condition A Baseline</th>
<th>Condition B 50% Response Effort</th>
<th>** Condition A Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X RC RI #</td>
<td>X RC RI #</td>
<td>X RC RI #</td>
<td>X RC RI #</td>
<td>X RC RI #</td>
</tr>
<tr>
<td>H</td>
<td>81 3.33 .81 2</td>
<td>90 4.40 .52 10</td>
<td>89 4.67 .60 8</td>
<td>90 4.58 .53 3</td>
<td>86 4.9 .79 3</td>
</tr>
<tr>
<td>S5</td>
<td>*86 4.66 .75 6</td>
<td>86 4.56 .72 3</td>
<td>83 3.45 1.92 2</td>
<td>71 4.19 1.69 9</td>
<td>75 4.2 1.4 1</td>
</tr>
<tr>
<td>H</td>
<td>93 6.95 .54 4</td>
<td>*97 7.61 .22 8</td>
<td>97 7.76 .29 6</td>
<td>93 7.53 .52 3</td>
<td>90 7.4 .78 3</td>
</tr>
<tr>
<td>S6</td>
<td>*94 7.67 .49 8</td>
<td>95 6.23 .29 3</td>
<td>96 7.17 .29 2</td>
<td>*95 8.82 .35 9</td>
<td>92 6.61 .60 1</td>
</tr>
<tr>
<td>TOTAL (MEAN)</td>
<td>89 5.74 .63 6</td>
<td>93 5.83 .39 18</td>
<td>92 6.0 .46 14</td>
<td>92 6.1 .52 6</td>
<td>88 6.17 .79 6</td>
</tr>
<tr>
<td>C</td>
<td>90 6.42 .60 14</td>
<td>91 5.40 .51 6</td>
<td>92 5.31 1.1 4</td>
<td>83 6.5 1.0 18</td>
<td>84 5.41 1.0 2</td>
</tr>
</tbody>
</table>

**Note:**
- H refers to Hutchings' algorithm
- C refers to conventional algorithm
- * Indicates preferred algorithm in each condition
- ** Indicates no algorithm preference established
- X Indicates mean session percent accuracy
- RC indicates mean session rate of columns correct/minute
- RI indicates mean session rate of columns incorrect/minute
- # Indicates the number of sessions in each condition during which the algorithm was preferred
Figure 2a: Total student mean session percent of columns correct, in each experimental condition, for the Hutchings' algorithm and the conventional algorithm. Data from the group of two students (Students 5 and 6) in Experiment 1 are included.

Figure 2b: Total student mean session rate of columns correct and incorrect, in each experimental condition, for the Hutchings' algorithm and the conventional algorithm. Data from the group of two students (Students 5 and 6) in Experiment 1 are included.
Hutchings' than with the conventional algorithm.

2. During the final B condition, Student 6 performed more accurately and more efficiently with the conventional algorithm than with the Hutchings' algorithm.

3. For Student 6, during the final baseline condition, using the Hutchings' algorithm, he was less accurate and had a slightly greater mean session rate of columns incorrect.

When the total mean accuracy, the total mean session rate of columns correct, and the total mean session rate of columns incorrect for the Hutchings' and conventional algorithms are averaged across the three baseline conditions (equal response efforts), both algorithms were found to be equal in percent accuracy (90%). Across the three baseline conditions, the conventional algorithm had a slightly higher mean session rate of columns correct (Hutchings' - 5.98 columns/minute; conventional - 6.10 columns/minute). The Hutchings' algorithm had a lower mean session rate of columns incorrect (Hutchings' - .58 columns/minute; conventional - .74 columns/minute).

When the three dependent variables are averaged for each student across all conditions (regardless of differences in response effort required for each algorithm), the results show that for Student 5, the Hutchings' algorithm was more accurate and efficient than the conventional algorithm (Hutchings': 88% accuracy, 4.48 columns correct/minute, .597 columns incorrect/minute; Conventional: 79% accuracy, 4.34 columns correct/minute, 1.29 columns incorrect/minute). Student 6,
across all conditions, performed equally well with either algorithm
(Hutchings': 95% accuracy, 7.51 columns correct/minute, .398
columns incorrect/minute; Conventional: 95% accuracy, 7.84 columns
correct/minute, .395 columns incorrect/minute).

Table 6 and Figures 2a and 2b show a gradually increasing mean
session rate of columns correct for the Hutchings' algorithm across
all conditions (total). Individual student data substantiates this
finding, although both students decreased their mean session rate of
columns correct for the Hutchings' algorithm during the final B con-
dition. The mean session rate of columns correct for Student 6 also
decreased during the final baseline condition.

Individual student data from Table 6 illustrate that each student,
with one exception, chose as their preferred algorithm, the one that
yielded the (a) higher accuracy, (b) the greater mean session rate
of columns correct, and (c) the lower mean session rate of columns
incorrect. The one exception was for Student 5, who during the final
B condition, chose as his preferred algorithm, the less accurate
and efficient one (conventional algorithm). Both students chose as
their preferred algorithm during both B conditions the one that involved
the lesser response effort. Thus, for Student 5, during the final B
condition, the increased response effort required for performing the
preferred algorithm, may have interfered with his preference for
the more accurate and more efficient algorithm.

Summary

Algorithm preference. Four of six students had a preference for
the Hutchings' algorithm during the initial baseline condition.

In the final baseline condition, all students preferred the Hutchings' algorithm over the conventional algorithm. Three students continued to show a preference for the Hutchings' algorithm when it involved 50%, but had no preference when it required 100% increase in response effort. One student preferred the Hutchings' algorithm when it involved 100% increase in response effort but had no preference under 50% increase in response effort. Two students under Condition B always preferred the algorithm that required the lesser response effort.

**Algorithm accuracy and efficiency.** During all conditions, with one exception, for the group of four students, each performed more accurately and efficiently with the Hutchings' algorithm than with the conventional algorithm. That is, the Hutchings' algorithm showed a higher accuracy, a greater mean session rate of columns correct, and a lower mean session rate of columns incorrect. Total mean session data from the group of two students showed that the Hutchings' algorithm was superior in four experimental conditions, while the conventional algorithm was superior in one condition. For Student 5, across all conditions, the Hutchings' algorithm was more accurate and efficient than the conventional one. The performance of Student 6, across all conditions, was equal in speed and accuracy for both algorithms. Both groups of students showed a gradually increasing trend of greater mean session rates of columns correct for the Hutchings' algorithm over the experimental conditions.

All students, during all conditions, except Student 5, chose as
their preferred algorithm, the one that had the greater accuracy and efficiency.
EXPERIMENT 2

Method

Subjects

A five minute basic addition math facts test (taken from Alessi, 1974) consisting of 56 basic addition facts, was administered to 27 third graders at an elementary school near Kalamazoo, Michigan. Six students, three white males and three white females, were chosen for the study based on their percent accuracy on the math facts test. Four high accuracy students and two low accuracy students were chosen. High accuracy was defined as 96% or better, and low accuracy as 80% or less on the addition math facts test. Of the four high accuracy students, three, one female (Student 1) and two males (Students 3 and 4), had 100% accuracy. One male (Student 2) had 96% accuracy. The two low accuracy students, two females (Students 5 and 6), had 70% and 73% accuracy, respectively. The homeroom teacher chose these students from the possible student pool of those who attained 96% or better, or 80% or less on the addition math facts test. Student selection was based on the same criterion as in Experiment 1.

Setting

The setting for Experiment 2 was exactly the same as in Experiment 1. However, all students in Experiment 2 preferred to sit at the tables rather than to use two chairs (sitting on one chair, working
Experimenat Task

Students received the exact same problems as students from Experiment 1. No student was ever given the same problem twice. Students 1, 7, and 8 received six problems, and Students 4, 5, and 6 received four problems during each session of the baseline condition. As in Experiment 1, the number of problems assigned was tailored to the individual rates of calculation for the students. This was to ensure approximately five to six minutes of on task calculation per baseline session.

Design

A reversal design (Baer, Wolf, and Risley, 1968) of the pattern A-B-C-A was used for all students.

Independent Variables

The independent variables and the criteria for the independent variables were the same as in Experiment 1. One additional independent variable was introduced in Experiment 2.

4. Reinforcement versus nonreinforcement for percent accuracy. This independent variable was added during conditions of increased response effort, identical to those of Conditions B and C of Experiment 1.

Criteria for Independent Variables

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4. The reinforcement used was in the form of "happy face" stamps. Each "happy face" was worth one point. Points could be exchanged for time in preferred activities in the regular classroom. "Happy faces" were awarded for accuracy on columns correct according to the following criteria:

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>5</td>
</tr>
<tr>
<td>95% - 99%</td>
<td>4</td>
</tr>
<tr>
<td>90% - 94%</td>
<td>3</td>
</tr>
<tr>
<td>85% - 89%</td>
<td>2</td>
</tr>
<tr>
<td>80% - 84%</td>
<td>1</td>
</tr>
<tr>
<td>below 80%</td>
<td>0</td>
</tr>
</tbody>
</table>

**Dependent Variables**

The dependent variables and the criteria for the dependent variables were identical to those of Experiment 1.

**Procedure**

**Pretraining.** This procedure and format was identical to that of Experiment 1.

**Prebaseline condition.** This condition was started following the last review session. Two 10 minute sessions were held daily. One session was held in the morning from 8:40 to 8:50. An afternoon session was held from 1:05 to 1:15. Following Session 19, and until the end of the study (Session 56), the times of the sessions were changed due to scheduling problems. The first session was held between 11:45 and
11:55. The second session took place between 1:05 and 1:15. The experimenter went to the classroom and got the students before each session.

The prebaseline condition followed the same format as in Experiment 1. During the prebaseline condition it became evident that, for all students, the number of problems given in each session would have to be increased, in order to get a stable performance of approximately five to six minutes. The prebaseline condition ended for each student when the number of problems given per session had been increased so that each student worked for approximately five to six minutes.

The session format for Experiment 2 was identical to that of Experiment 1.

Prebaseline condition (Condition A). Baseline condition was the same for both experiments. Students 1, 2, and 3 received six problems during each session of baseline; Students 4, 5, and 6 received four problems. The condition ended when all students had established a preference for one algorithm. All students ended baseline at the same time, although some students established a preference earlier than others.

Condition B. As in Experiment 1, students had a 50% increase in response effort if they chose the preferred algorithm, as established during baseline. In other words, Students 1, 2, and 3 had to complete nine problems, and Students 4, 5, and 6, six problems, when using their preferred algorithm.
During Condition B, a token reinforcement program was introduced. Students were informed that after they had completed their problems in a session, they might receive "happy face" stamps. The number of "happy face" stamps they received would depend on how accurate their answers were. Their accuracy was to be based on how many columns they added correctly out of the total number of columns they had been given. Students were informed that each "happy face" equalled one point. Each student was given an index card with his or her name on it. The "happy faces" were put on these cards.

Previously, the six students had made up a list of preferred activities with their homeroom teacher. The list of these activities, with assigned point values (determined by the experimenter), was shown to the students. Another identical list was placed on the bulletin board in the homeroom. The criteria for the number of points the students could receive in any one session and the procedure of exchanging "happy faces" (points) for the activities was explained to the students. Any questions were answered at this point.

Students were told that during each session, after they had checked their answers, they were to come to the experimenter, so that they could receive their "happy faces". The experimenter or the students stamped the appropriate number of "happy faces" on the index cards. The experimenter checked the students' answers after each session was over. If the students had made any errors in checking their answers, the number of points was adjusted, accordingly, during the next session.

The token reinforcement program was used throughout Condition B.
Condition B ended after all students had established a preference for one algorithm. All students did not establish preference during the same session.

**Condition C.** The token reinforcement program was used throughout this condition. Other than reinforcement, this condition was identical to Condition C of Experiment 1. Students 1, 2, and 3 were required to complete 12 problems, and Students 4, 5, and 6, eight problems with their preferred algorithm (preferred algorithm established during Condition B). Students ended this condition during the same session when all students had either established a preference for one algorithm or when it was clear that they had no preference for either algorithm.

**Condition A.** The return to baseline condition was identical to that of Experiment 1. The questionnaire was administered to all students after the last session. The condition ended when school let out for the summer.

**Recording and Scoring**

Recording and scoring were identical to that of Experiment 1.

**Reliability**

Reliability data collected was identical to that of Experiment 1. Reliability was taken on both elapsed times to complete the problems and scoring of permanent products of the sessions.
EXPERIMENT 2

Results

Reliability

The reliability for students' times to complete the problems was taken for seven of the 30 sessions, and yielded a mean of 100% agreement. Papers from 10 of the 30 sessions for each student were also checked. Reliability on number of errors from each paper resulted in a mean of 95% agreement.

Algorithm Preference

Behavioral preferences. Table 7 (top) presents algorithm preferences for each student under all four experimental conditions. During initial baseline all students had a preference for the Hutchings' algorithm. Student 6 switched preference to the conventional algorithm during Condition B. During Condition C two additional students switched preference to the conventional algorithm. Two other students established no preference for either algorithm in this condition. The sixth student established an early preference for the Hutchings' algorithm in Condition C, and then switched to no preference for either algorithm in the same condition. During the return to baseline condition, four students returned to a preference for the Hutchings', one student showed no preference for either algorithm, and one student preferred the conventional algorithm.
Table 7: Individual and total student algorithm preferences within and across all experimental conditions.
TABLE 7

INDIVIDUAL AND TOTAL STUDENT ALGORITHM PREFERENCES WITHIN AND ACROSS ALL EXPERIMENTAL CONDITIONS

<table>
<thead>
<tr>
<th>Experimental Conditions</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 1</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>Student 2</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>Student 3</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>No Preference</td>
<td>Hutchings</td>
</tr>
<tr>
<td>Student 4</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Conventional</td>
<td>Conventional</td>
</tr>
<tr>
<td>Student 5</td>
<td>Hutchings</td>
<td>Hutchings</td>
<td>Conventional</td>
<td>No Preference</td>
</tr>
<tr>
<td>Student 6</td>
<td>Hutchings</td>
<td>Conventional</td>
<td>Conventional</td>
<td>No Preference</td>
</tr>
</tbody>
</table>

Student Totals (Percent)  

<table>
<thead>
<tr>
<th></th>
<th>Totals Across Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutchings</td>
<td>100% 66.7% 62.5%</td>
</tr>
<tr>
<td>Conventional</td>
<td>0% 50% 16.7% 20.8%</td>
</tr>
<tr>
<td>No Preference</td>
<td>0% 50% 16.7% 16.7%</td>
</tr>
</tbody>
</table>

NOTE:  
a indicates the student switched preference from Hutchings in middle of condition  
b indicates students who did not know math facts consistently
Table 7 (bottom) presents, both within and across conditions, the total student percentages of preferences for the Hutchings' algorithm, the conventional algorithm, or no preference for either algorithm. Across conditions the Hutchings' algorithm was preferred for more than 60% of the conditions. Within each condition, with the exception of Condition C, the Hutchings' algorithm was the most preferred method.

Students 5 and 6, who did not consistently know their basic addition math facts, showed a slightly different algorithm preference than the students who consistently knew their math facts. Student 6 was quicker than the other students to change preference during Condition B. Both Students 5 and 6 preferred the conventional algorithm during Condition C, while three out of the other four students showed no preference for either algorithm.

Table 8 presents a more detailed analysis of individual student preferences. The number of sessions (and the corresponding percentages of total sessions) during which each student either chose or was forced to perform the Hutchings' or the conventional algorithm during each condition, are presented. For each student, during each condition where a preference for the Hutchings' algorithm was established, the Hutchings' algorithm was preferred in no less than 90% of the free choice sessions. In conditions where a preference for the conventional algorithm was established, the conventional algorithm was performed in no less than 67% of the free choice sessions. During Condition C, where three students showed no preference for either algorithm,
Table 8: Individual and total student algorithm choices during free and forced choice sessions in each condition.
<table>
<thead>
<tr>
<th>Students</th>
<th>Condition A Baseline</th>
<th>Condition B 50% Response Effort</th>
<th>Condition C 100% Response Effort</th>
<th>Condition A Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forced Choice</td>
<td>Free Choice</td>
<td>Forced Choice</td>
<td>Free Choice</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0/2 0.0 7/7 1.0</td>
<td>1/3 .33 *9/12 .75</td>
<td>0/3 0.0 9/9 1.0</td>
</tr>
<tr>
<td>S1</td>
<td>Hutchings 3/3 1.0 0/9 0.0</td>
<td>2/2 1.0 0/7 0.0</td>
<td>2/3 .66 3/12 .75</td>
<td>3/3 1.0 0/9 0.0</td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hutchings 0/3 0.0 10/11 .91</td>
<td>0/2 0.0 7/7 1.0</td>
<td>1/3 .33 *9/12 .75</td>
<td>0/3 0.0 9/9 1.0</td>
</tr>
<tr>
<td>S2</td>
<td>Conventional 3/3 1.0 1/11 .09</td>
<td>2/2 1.0 0/7 0.0</td>
<td>2/3 .66 3/12 .75</td>
<td>3/3 1.0 0/9 0.0</td>
</tr>
<tr>
<td></td>
<td>Hutchings 0/3 0.0 10/11 .91</td>
<td>0/2 0.0 7/7 1.0</td>
<td>0/2 0.0 *9/13 .62</td>
<td>0/2 0.0 9/10 .90</td>
</tr>
<tr>
<td>S3</td>
<td>Conventional 3/3 1.0 1/11 .09</td>
<td>2/2 1.0 0/7 0.0</td>
<td>2/2 1.0 3/13 .365</td>
<td>2/2 1.0 1/10 .10</td>
</tr>
<tr>
<td></td>
<td>Hutchings 0/4 0.0 12/12 1.0</td>
<td>0/2 0.0 7/7 1.0</td>
<td>2/2 1.0 4/12 .33</td>
<td>3/3 1.0 0/9 0.0</td>
</tr>
<tr>
<td>S4</td>
<td>Conventional 4/4 1.0 0/12 0.0</td>
<td>2/2 1.0 0/7 0.0</td>
<td>0/2 0.0 8/12 .67</td>
<td>0/3 0.0 9/9 1.0</td>
</tr>
<tr>
<td></td>
<td>Hutchings 0/3 0.0 9/9 1.0</td>
<td>0/2 0.0 7/7 1.0</td>
<td>3/3 1.0 0/10 0.0</td>
<td>0/3 0.0 9/9 1.0</td>
</tr>
<tr>
<td>S5</td>
<td>Conventional 3/3 1.0 0/9 0.0</td>
<td>2/2 1.0 0/7 0.0</td>
<td>0/3 0.0 10/10 1.0</td>
<td>3/3 1.0 0/9 0.0</td>
</tr>
<tr>
<td></td>
<td>Hutchings 0/3 0.0 9/9 1.0</td>
<td>1/1 1.0 1/8 .125</td>
<td>3/3 1.0 0/12 0.0</td>
<td>1/1 .50 *9/10 .90</td>
</tr>
<tr>
<td>S6</td>
<td>Conventional 3/3 1.0 0/9 0.0</td>
<td>0/1 0.0 7/8 .875</td>
<td>0/3 0.0 12/12 1.0</td>
<td>1/1 .50 5/10 .90</td>
</tr>
<tr>
<td></td>
<td>Hutchings 0/19 0.0 59/61 .97</td>
<td>1/11 .09 36/43 .84</td>
<td>10/16 .625 28/71 .39</td>
<td>4/16 .25 41/56 .73</td>
</tr>
<tr>
<td></td>
<td>Conventional 19/19 1.0 2/61 .03</td>
<td>10/11 .91 7/43 .16</td>
<td>6/16 .30 43/71 .61</td>
<td>12/16 .75 15/56 .27</td>
</tr>
</tbody>
</table>

* No preference for either algorithm established.
the Hutchings' algorithm was performed in no less than 58% of the free choice sessions. In the no preference condition during the return to baseline condition, both the algorithms were chosen for 50% of the free choice sessions.

**Verbal preferences.** Five out of six students' verbal preference statements matched their behavioral algorithm preferences. Students 1, 2, 3, and 5 indicated that they liked the Hutchings' algorithm better both at the beginning and at the end of the study. Student 4 indicated that he liked the Hutchings' algorithm better in the beginning and the conventional way better at the end. All these are accurate assessments of their behavioral preferences for the algorithms. Student 6 indicated that she liked the conventional method better throughout the study. This does not match her behavioral algorithm preference at the beginning of the study, although it does match her preference behavior after the initial baseline condition.

Four of six students accurately assessed their overall performances with both algorithms. Students 1, 2, 3, and 5 said that using the Hutchings' algorithm they (a) got done more quickly, (b) had more problems correct, and (c) had fewer mistakes. Their verbal statements are supported by individual behavioral performances during all conditions. Students 4 and 6 said that they had more problems correct with the conventional method and made more mistakes with the Hutchings' algorithm. These statements are not in accord with their individual accuracy and efficiency data.
Algorithm Accuracy and Efficiency

Table 9 presents individual student and total student mean session accuracy, mean session rates of columns correct, and mean session rates of columns incorrect for the Hutchings' and the conventional algorithm within all experimental conditions. Total student data from Table 9 are presented in Figures 3a and 3b. Figure 3a shows that the Hutchings' algorithm was the more accurate algorithm within each experimental session. Figure 3b shows that the Hutchings' algorithm was more efficient than the conventional algorithm within each experimental condition. That is, the Hutchings' algorithm had consistently higher accuracy, greater mean session rates of columns correct, and lower mean session rates of columns incorrect, than the conventional algorithm. Individual student results (Table 9), with several exceptions, support this general finding. The individual exceptions include: 1) Student 6 performed with higher accuracy, with greater mean session rates of columns correct, and with lower mean session rates of columns incorrect, when using the conventional algorithm during Condition B and during the final baseline condition. Also, Student 6 had approximately the same accuracy for both algorithms during the initial baseline condition, although the Hutchings' algorithm yielded a greater mean session rate of columns correct, and a lower mean session rate of columns incorrect, than the conventional algorithm. 2) Student 3 had equal accuracy for both algorithms during the final baseline condition. The mean session rate of columns correct was greater with the Hutchings' algorithm and the mean session rate of columns incorrect was lower with
Table 9: Individual and total student mean session performance with the Hutchings' and the conventional algorithms during each experimental condition.
TABLE 9

INDIVIDUAL AND TOTAL STUDENT MEAN SESSION PERFORMANCE WITH THE HUTCHINGS' AND THE CONVENTIONAL ALGORITHMS DURING EACH EXPERIMENTAL CONDITION

<table>
<thead>
<tr>
<th>Students</th>
<th>Condition A Baseline</th>
<th>Condition B 50% Response Effort</th>
<th>Condition C 100% Response Effort</th>
<th>Condition A Baseline</th>
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<tr>
<td></td>
<td>X</td>
<td>RC</td>
<td>RI</td>
<td>#</td>
</tr>
<tr>
<td>S1</td>
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<tr>
<td>H</td>
<td>.97</td>
<td>5.98</td>
<td>.17</td>
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<tr>
<td>C</td>
<td>.91</td>
<td>3.25</td>
<td>.346</td>
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<td>S2</td>
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<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>H</td>
<td>* .93</td>
<td>6.13</td>
<td>.46</td>
<td>10</td>
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<tr>
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<td>10</td>
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<tr>
<td>H</td>
<td>* .98</td>
<td>5.13</td>
<td>.10</td>
<td>10</td>
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<td>H</td>
<td>* .96</td>
<td>3.31</td>
<td>.13</td>
<td>12</td>
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<tr>
<td>C</td>
<td>.75</td>
<td>2.17</td>
<td>.63</td>
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<td></td>
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<td>H</td>
<td>* .90</td>
<td>2.49</td>
<td>.26</td>
<td>9</td>
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<tr>
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<td>9</td>
</tr>
<tr>
<td>H</td>
<td>* .88</td>
<td>2.70</td>
<td>.40</td>
<td>9</td>
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<tr>
<td>C</td>
<td>.88</td>
<td>2.48</td>
<td>.35</td>
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<tr>
<td>TOTAL (MEAN)</td>
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<td>C</td>
<td>.86</td>
<td>3.06</td>
<td>.424</td>
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</table>

NOTE: H refers to Hutchings' algorithm
C refers to conventional algorithm
* indicates preferred algorithm in each condition
** indicates no algorithm preference established in each condition
X indicates mean session percent accuracy
RC indicates mean session rate of columns correct/minute
RI indicates mean session rate of columns incorrect/minute
# indicates the number of session in each condition during which the algorithm was performed.
Figure 3a: Total student mean session percent of columns correct, in each experimental condition, for the Hutchings' algorithm and the conventional algorithm. Data from Experiment 2 are included.

Figure 3b: Total student mean session rate of columns correct and incorrect, in each experimental condition, for the Hutchings' algorithm and the conventional algorithm. Data from Experiment 2 are included.
the conventional algorithm. 3) Student 5 during the B condition was
more accurate and had a lower mean session rate of columns incorrect
with the Hutchings' algorithm, but had a slightly higher mean session
rate of columns correct with the conventional algorithm.

During Conditions B and C (when percent accuracy was rewarded with
points), the total percent accuracy for both algorithms was higher
than during either of the baseline conditions (Table 3, Figure 3a).
Individual student data support this finding generally for the Hutch­
ings' algorithm, with two exceptions. Student 4 had the same mean
session percent accuracy during Conditions B and C, as during both
baseline conditions. Student 1 slightly increased her mean session
accuracy across all experimental conditions. Individual student data
do not support this finding with the conventional algorithm.

The total mean session results for the Hutchings' algorithm show
that the mean session rate of columns incorrect was lower during the
reinforcement conditions then during both baselines (Table 9, Figure 3b).
Generally, individual student data support this finding, with two
individual exceptions. Student 4 had approximately the same mean
session rate of columns incorrect during baseline conditions as during
Conditions B and C. In addition, Student 1 increased her mean session
rates of columns incorrect during the reinforcement conditions.

For the total mean session results for the Hutchings' algorithm,
the mean session rates of columns correct increased during the last
two experimental conditions. This is substantiated by individual student
data, with two exceptions. Student 4 decreased slightly during Condition
C and Student 6 decreased slightly during the final baseline condition. Table 9 shows that during initial baseline and the B condition, all students, with two exceptions, chose as their preferred algorithm, the one that had the higher accuracy, the greater mean session rate of columns correct, and the lower mean session rate of columns incorrect. During initial baseline, Student 6 had the same mean session accuracy for both algorithms, (i.e., percent correct) although the Hutchings' algorithm was more efficient, (i.e., faster). During Condition B, the preferred algorithm for Student 5 had a lower mean session rate of columns correct than the nonpreferred algorithm. During the return to baseline condition, three out of five students had a preference for their more accurate and efficient algorithm. Student 4 had a preference for the lesser accurate and efficient algorithm. Student 3 showed equal accuracy for both algorithms, with a higher mean session rate of columns correct for the preferred algorithm, and a lower mean session rate of columns incorrect for the nonpreferred algorithm. During Condition C, all three students who established a preference for one algorithm, did so for the lesser accurate and efficient one. For both Students 5 and 6 the nonpreferred algorithm had a slightly lower mean session rate of columns correct than the preferred algorithm.

Thus, during initial baseline and with 50% increased response effort, with minor exceptions, students preferred to perform the algorithm that had the higher accuracy, the greater mean session rate of columns correct, and the lower mean session rate of columns incorrect. This holds true also for the return to baseline condition, with the
exception of Student 4. However, during 100% increased response effort, even under reinforcement conditions, the students switched to their less accurate and efficient algorithm.
EXPERIMENTS 1 AND 2

Summary

Format

Experiments 1 and 2 were identical in all respects, except that students in Experiment 2, during Conditions B and C, received reinforcement for percent accuracy.

Behavioral Algorithm Preference

There were essentially no differences in algorithm preferences between the reinforcement and nonreinforcement groups during the baseline conditions. With two exceptions from both experiments, students preferred the Hutchings' algorithm during all baseline conditions. During B conditions, one student from Experiment 2 and two students from Experiment 1 chose to perform the algorithm that involved the lesser response effort.

All students exposed to Condition C performed more accurately and efficiently during this condition with the Hutchings' algorithm, even though it required a 100% increase in response effort over the conventional algorithm. Half of the students from Experiment 2, exposed to the C condition, showed a tendency toward preferring the Hutchings' algorithm, although no preference was established for this algorithm. Twenty-five percent of the students from Experiment 1 had a preference for the Hutchings' algorithm during this condition. Twenty-five
percent of the students from Experiment 1 showed a tendency toward establishing a preference for the Hutchings' algorithm by choosing to perform the algorithm during 77% of the total free choice sessions. However, there is an essential difference in algorithm preference, during Condition C, between students of Experiments 1 and 2. Fifty percent of the students from Experiment 2 established a preference for their lesser accurate and efficient algorithm (conventional). No student from Experiment 1 established a preference for the lesser accurate and efficient method during this condition. Thus, the reinforcement given for percent accuracy during Condition C of Experiment 2, did not maintain the students' preferences for the algorithm that was more accurate.

Both students from Experiment 2 who did not consistently know their basic addition math facts, (Students 5 and 6), showed an algorithm preference for the conventional algorithm (which was less accurate and efficient) during Condition C. This did not occur for the two students from Experiment 1 who did not consistently know their basic addition math facts. No other essential differences were found between students who had high accuracy and those who had low accuracy on the math facts test.

**Verbal Algorithm Preference**

Ten of the 12 students (83%) in both Experiments 1 and 2 had accurate verbal assessments of their behavioral algorithm preferences. One student from each experiment was inaccurate in matching verbal
assessment to behavioral preference. Thus, there was no essential difference between students from the two experiments on this dimension.

**Algorithm Accuracy and Efficiency**

Students chose to perform their more accurate and efficient algorithm during Experiment 1 in 95%, and in Experiment 2, during 80% of the total experimental conditions where a preference for one algorithm was established.

For both experiments, the Hutchings' algorithm overall was more efficient and accurate than the conventional algorithm. In Experiment 1, only during 15% of the total number of experimental conditions, did three students perform consistently better with the conventional algorithm. In Experiment 2, only during 8% of the total number of experimental conditions did one student clearly perform more accurately and efficiently with the conventional algorithm.

Figures 1a, 2a, and 3a show that students from Experiment 2 were more accurate in all conditions with the Hutchings' algorithm than were students in Experiment 1. The group of two students from Experiment 1 were more accurate in all conditions (with one exception) with the conventional algorithm than the students from Experiment 2. During the initial B condition students from Experiment 2 and the group of two students from Experiment 1, performed with equal accuracy on the conventional algorithm. The group of four students from Experiment 1 had a lower accuracy with the conventional algorithm in all conditions than the students in Experiment 2.
Figures 1b, 2b, and 3b show that students from Experiment 1 had a higher mean session rate of columns correct with the Hutchings' algorithm over all conditions, with the exception of the final baseline condition, than students from Experiment 2. The mean session rate of columns correct during final baseline condition was equal for the group of four students from Experiment 1 and all students from Experiment 2. Mean session rate of columns correct for the Hutchings' algorithm had a greater increase from initial to final baseline for Experiment 2 than for Experiment 1. (Increase: Experiment 2 - 1.97 columns/minute; Experiment 1 - .93 columns/minute, and .425 columns/minute for the group of four and two students, respectively.) The results from Experiment 2 show a lower mean session rate of columns incorrect with the Hutchings' algorithm in all conditions than in Experiment 1 (Figures 1b, 2b, and 3b). The mean session rate of columns correct with the conventional algorithm was higher for Experiment 1 than for Experiment 2. The mean session rate of columns incorrect for the conventional algorithm was lower in all conditions for Experiment 2.
DISCUSSION

This study demonstrated that when in a free choice situation, most third grade students in this study preferred to compute addition problems using the Hutchings' "low stress" addition algorithm, rather than the conventional addition algorithm. The most logical question to ask next is, why was the Hutchings' algorithm more preferred than the conventional one? There are several possible reasons. As stated in previous research with the Hutchings' algorithm (Alessi, 1974; Hutchings, 1972), two possible reasons could be that: 1) there is less stress involved for the student using this algorithm, when he is presented with challenging problems; and 2) the Hutchings' algorithm requires less demand on memory or covert chains of behavior than the conventional algorithm. Further, students may have preferred the Hutchings' algorithm over the conventional algorithm because it was their more accurate and efficient method. Thus, the present study supports the findings from previous research (Alessi, 1974; Boyle, 1975; Dashiell, 1974; Gordon, 1972; Hutchings, 1972; Rudolph, 1976; Zoref, 1976) that showed the Hutchings' algorithm to be superior to the conventional one in accuracy and efficiency. It is the author's opinion that all three factors together, and none alone account for the students' preferences for the Hutchings' algorithm.

However, a fourth factor must not be overlooked. In introducing any new procedure, there is always the possibility of novelty effects. Students could have preferred the Hutchings' algorithm only because
it was something different, something new and exciting. To investigate this possibility would require a full length preference study conducted throughout the first few years of students' schooling.

Why did several students show no preference for the Hutchings' algorithm during free choice conditions? This question, a corollary to the one asked above, needs to be answered. It is possible that during the initial baseline condition, some students feared using a new method, especially when the conventional algorithm had already been perfected to a high degree. Or, on the other hand, during the final baseline condition, a previous preference for the Hutchings' algorithm, due only to novelty effects, may have worn off. Both reasons seem equally plausible.

This study demonstrated that most students showed a preference for the Hutchings' algorithm, as opposed to the conventional algorithm, even when this choice required them to complete 50% more problems. Why? It is possible that students saw this condition as no different from baseline. That is, students did not mind doing more problems, and had little concern about the time needed to complete the problems. After all, students were being excused from class during the study sessions. However, it is also possible that students wanted to perform with their more accurate and efficient algorithm especially since all students were receiving verbal feedback on their percent accuracy. In the author's opinion, the first alternative seems less reasonable than the second. Students were not required to return to the classroom until the whole group had completed their assigned problems.
Why did several students fail to establish a preference for the Hutchings' algorithm when this choice required completing 50% more problems? One possible answer is obvious. Students' preferences for the Hutchings' algorithm may have broken down when they were required to complete more problems. After all, why should they choose to do more problems when they could just as easily choose to do fewer problems? Students may have preferred the Hutchings' algorithm but may not have preferred it enough to compensate for completing more problems. Students may have been concerned with the time it would take them to finish. It must not be overlooked that two students performed more accurately and efficiently with the conventional algorithm, when choosing this algorithm required completing 50% less problems than if choosing the Hutchings' algorithm.

Why was the conventional algorithm preferred in more sessions, during the 100% increased response effort condition, than the Hutchings' algorithm? In the author's opinion there is only one plausible reason; performing the conventional algorithm involved completing a lot fewer problems. The preference for the Hutchings' algorithm was not strong enough to outweigh the increased response effort required for choosing it. Wishing to finish quickly may have been an additional influencing factor on the students. A greater percentage of students who did not consistently know their basic addition math facts, as compared to those who did, preferred to complete fewer problems. It is suggested that the increase in response effort may have differentially affected this group, since their math facts were less consistent. During the 100% increased response effort condition, no student performed more accurately or
efficiently with the conventional algorithm (consistently across all
students, no student established a preference for their more accurate
and efficient algorithm). This finding which is in direct conflict
with the results from the other two conditions, may well be explained
by the 100% increased response effort required for performing the more
accurate and efficient algorithm.

What is the point of differential response effort that will
cause a shift in performance from the preferred to the nonpreferred
algorithm? For 25% of the students this point occurred at 50% in­
creased response effort, while for an additional 17% of the students
it occurred at 100% increased response effort. But what is more in­
teresting is that for 58% of the students, there was never a shift
from the preferred to the nonpreferred algorithm. Why? Students may not
have shifted preference because during some of the sessions they still
wished to perform the more accurate and efficient algorithm. Or, in
other words, the 100% increased response effort may not have been great
enough to outweigh performing, at least some of the time, with the
algorithm they liked better. Both reasons have equal merit.

However, during the 100% increased response effort condition, the
requirement for "algorithm preference", defined in this study, presents
problems. It is a more stringent definition of preference than those
described in other studies. One previous study (Lockhart et al., 1973)
defined preference in terms of percent of free sessions chosen for one
alternative over another. This definition of preference would allow
a finer analysis of student algorithm preferences during "no preference
for either algorithm" conditions. At the same time, it is this author's opinion that the number of forced switches may be a more accurate assessment of consistent preference for one condition, rather than the number of voluntary switches between both alternatives. However, future research in this area will allow educators to develop a more definite and accurate criteria of what constitutes a preference.

The findings of this study showed that reinforcement for percent accuracy, only slightly, if at all, altered the point of increased response effort that caused a shift from the preferred to the nonpreferred algorithm. Why? Possibly the reinforcement for percent accuracy may not have been effective enough to overcome the 100% increased response effort required for performing the preferred algorithm. This study showed that the reinforcement procedure was not effective enough to increase the individual student's accuracy, or to decrease error rates with the conventional algorithm.

Under a free choice situation do children prefer to perform the algorithm that results in the greatest accuracy and efficiency? Based on the findings of this study, the answer, for most students, is yes. Is there a point of differential response effort where wanting to perform with the more accurate and efficient algorithm dissolves? The answer is also, yes. This point appeared in this study as the response effort associated with preference changed from equal to 50% to 100% increase over the nonpreferred choice. Either a switch to the less accurate and efficient algorithm, or a no preference for the more accurate and efficient algorithm has great educational significance. It
is unlikely, that students in an educational setting, who are required to work under conditions of increased response effort, will chose to use an initially preferred or more efficient algorithm over another. However, the more essential point is that during conditions when students do not perform high quality work, one of two factors may be operating. Either the reinforcement for performing high quality work is not great or effective enough, or there are other factors present, (e.g., differential response effort), that offset the effectiveness of the reinforcers. It is the responsibility of educators to single out these factors if they are to develop the most effective reinforcing learning environment for students.

It is the responsibility of educators to use the technology available to assess student preferences and to assess the effects of preference on performance. Why shouldn't students be given a choice if they are able to perform a task equally well using either of two alternatives? And what happens in the case where a student prefers the least accurate and efficient method? Should students still be given a free choice? Before these questions can be answered, one essential question, posed by Lockhart et al. (1973), needs to be asked. Does a student come to prefer a condition simply because he does well in it, or must a student like a given condition to do well? The results of this study tend to substantiate the first alternative. However, the results of this study cannot even attempt to answer the second alternative. More research on the relationships between preference and performance is needed.
Verbal algorithm preferences were a fairly accurate measure of behavioral preferences, thus substantiating the findings of Whitehurst and Whitehurst (1973). However, this does not imply that, in looking at choice making behavior, verbal preferences are a better tool than behavioral preferences. The author concurs with Lea and Lockhart (1974) that, verbal preferences may have no reinforcing power in themselves or may only be reinforcing to the experimenter. Behavioral preferences themselves are a reinforcer for the choice response and would better reflect ongoing behavior (Lockhart et al., 1973) than just a verbal statement at the conclusion of the study.

This study, although it established an overall preference for the Hutchings' algorithm, should be interpreted with caution. This study has certain limitations. First, student data are somewhat variable. Second, this study was conducted in a setting outside of rather than in the regular classroom. Third, it is felt that requiring different students to perform different number of problems during baseline conditions (keeping elapsed time on task constant), may have resulted in certain interaction effects. Several students wished to perform as many problems as the rest of the students, while several students resisted performing more problems than the rest of the students. A replication of this study, with all students completing the same number of problems during baseline conditions (with variable elapsed time on task) in the regular classroom, is suggested.

Future research might be geared toward applying the Findley minimax procedure to a variety of other educational settings. The success of
the Hutchings' "low stress" algorithm, if its success depends ultimately on student preferences and quality of performance, seems to be assured. However, before any major changes in the curriculum are attempted, a full length preference and performance study from the time of students' entrance into the math curriculum, should be conducted. Any changes in math curriculum should be done with extreme caution and only after much careful research.


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Fulkerson, E. Adding by tens. The Arithmetic Teacher, 1963, 10, 139-140.


Lankford, F. G. What can a teacher learn about a pupil's thinking through oral interviews? The Arithmetic Teacher, 1974, 21, 26-32.


Zoref, L. A comparison of calculation speed and accuracy on two levels of problem difficulty using the conventional and Hutchings' "low stress" addition algorithms and the pocket calculator with high and low achieving math students. Unpublished Specialist in Education Project, Western Michigan University, 1976.
APPENDIX A
INVENTORY OF BASIC ADDITION FACTS
(Adapted from Otto, McMenemy, and Smith, 1973, p. 291; Taken from Alessi, 1974)
<table>
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APPENDIX B

HUTCHINGS' ADDITION ALGORITHM, LESSON
(Adapted from Hutchings, 1972)
I am going to show you the usual way of writing number facts and then another way of writing them.

You have all seen number facts written like this:

Well, they can also be written like this, using two small (half-space) numbers instead of the line and plus sign.

Do you still see the fifteen? (Point to both fifteens.)

I'll write the two examples next to one another.

Do you all see the fifteen? (Point)

Let's look at another one. I can write "9 plus 5 is 14" like this or like this.

Both of these say "9 plus 5 is 14."

Tell me what these say:

(Call on students, point to the full notation form when asking.)
The little number on the right* is understood to be in the one's place, as are 9 and 8.

The little number on the left* is understood to be in the ten's place.

In other words, this is the same as this (point from "big 7" to "little 7"). And this is the same as this (point from "big one" to "little one").

Now watch me write the following facts both ways.

\[
\begin{array}{cccc}
9 & 9 & 8 & 4 \\
+7 & 17 & 13 & 9 \\
\hline
16 & 7 & 3 & 9
\end{array}
\]

Look at the last pair. Are they different from the others? Note that there is no ten's place number and (do not draw box until after saying this) there is no "little one" on the left.

Let's look at another.

a) 4 Is there any ten's number here? (Do not draw box until after asking question.)

b) NO!! (repeat)

c) So will there be any little number on the left?

d) 4 (Do not draw box until after asking question.)

NO!! (repeat)
Again, \[ \frac{4 + 3}{7} = 1. \] If there is no ten's place number there is no "little number" on the left.

Now watch me write the rest of these.

Notice

\[ \frac{3}{4} + \frac{1}{4} = \frac{4}{4} \] so no "little number" here

\[ \frac{3}{4} \]

but

\[ \frac{7}{15} + \frac{8}{15} = \frac{15}{15} \] so there is a "little number" here

\[ \frac{7}{15} \]

Again, notice

\[ \frac{5}{6} + \frac{1}{6} = \frac{6}{6} \] so there is no "little number" here

\[ \frac{5}{6} \]

but

\[ \frac{8}{13} + \frac{5}{13} = \frac{13}{13} \] so there is a "little number" here

\[ \frac{8}{13} \]
Now I am going to show you a special way of adding that uses only those "little numbers" on the right.

I'll say that again (repeat previous statement).

This should make your addition very easy and accurate. It is a scientific method and many scientists do addition this way. Watch.

First, do you see that an example can be just number facts piled one atop the other? (Do not point with this question.)

OK! Here we go, starting at the top, writing facts as you learned and using only numbers at the right for addition.

a) Say, "The first fact we do may look a bit different because we do not have any little numbers yet." (Point)

b) Say, "This is the only time we will use two big numbers. In the rest of the example we use one little number and one big one."

c) Say, "Now, eight plus five is thirteen."

d) Write the thirteen, i.e., 13 in the example.

a) Say, "We've written the thirteen but we'll use only the three."

b) Draw arrow 73.

c) Say, "Three plus seven is ten."

d) Write the 10, i.e., 10 in the example.
a) Say, "We've written the ten but we'll use only the 0."

b) Draw arrow \(9\).

c) Say, "Zero plus nine is nine."

d) Write the 9, i.e., \(9_9\) in the example.

---

a) Say, "We've written the nine and look that's all we have this time because zero and nine is just nine. But that's OK because we only use the right-hand number anyway."

b) Draw arrow \(9\).

c) Say, "Nine plus eight is seventeen."

d) Write the seventeen, i.e., \(8_7\) in the example.

---

a) Say, "We've written the seventeen but we'll use only the seven."

b) Draw arrow \(6\).

c) Say, "Seven plus six is thirteen."

d) Write the thirteen, i.e., \(6_7\) in the example.

---

a) Say, "We've written the thirteen but we'll use only the three."

b) Draw the arrow \(8\).

c) Say, "Three plus eight is eleven."

d) Write the eleven, i.e., \(8_3\) in the example.
a) Say, "We've written the eleven but we'll use only the one."

b) Draw arrow 7² .

c) Say, "One plus seven is eight."

d) Write the eight, i.e., 7₈ in the example.

Now we're at the key part. All we've done is use number facts. We haven't done any "in your head" work.

Nevertheless, we already know the answer! Watch.

The last little number on the right is the right half of the answer.

To get the left half, we just count the little numbers on the left that we didn't use. One, two, three, four, five, there are five of them, so the first half of the answer is five. The answer is 58.

Now watch me do another. Remember we use only the right side "little numbers." We will not bother to write the arrows anymore, just say

Now the last number on the right is a 2, so the right half of the answer is a 2! We get the left half of the answer by counting the little numbers on the left that we didn't use. One, two, three, four, five. There are five of them so the left half of the answer is 5. The answer is 52.
Now say the work for these with me as I do them at the board. (Children do not copy this.)

<table>
<thead>
<tr>
<th>8</th>
<th>9</th>
<th>4</th>
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<tbody>
<tr>
<td>5</td>
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<td>6</td>
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<td>6</td>
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<td>52</td>
<td>49</td>
<td>43</td>
</tr>
</tbody>
</table>

Now copy these examples and do them by yourself. If you have any questions, ask me.

<table>
<thead>
<tr>
<th>6</th>
<th>8</th>
<th>5</th>
<th>9</th>
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</thead>
<tbody>
<tr>
<td>5</td>
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<td>9</td>
<td>6</td>
</tr>
<tr>
<td>+9</td>
<td>+5</td>
<td>+8</td>
<td>+9</td>
</tr>
</tbody>
</table>

After most have finished, say, "Check your work with mine as I do them at the board."

After doing the examples, say, "Now let's review."
I'll write the work for another one on the board. I want someone to raise his hand and tell me what the answer is.

6
8
4
9
3
5
7
6
8
7
5
9
0
3
9
plus 8 is 14
4 plus 9 is 13
3 plus 5 is 8
8 plus 7 is 15
5 plus 5 is 10
0 plus 9 is 9
9 plus 3 is 12

Who will tell me what the right side of the answer is and how he got it.

(Point to box.)

(Locate correct response.) Good! That's correct. The last little number on the right becomes the right side of the answer.

Who will tell me what the left side of the answer is and how he got it.

(Locate correct response.) Good! That's correct, we count up the little numbers on the left for the left side of the answer.

Now, what do you suppose we do if there is more than one column? That is, if there is another column at the left of the column you're adding. Like this

4
7
6
8
7
6
8
7
8

Can we still write our left-hand answer number at the bottom if there is more than one column? No, we can't?

When there's more than one column, each column can have only one number at the bottom (except for the very last column which does have the usual two).
So the single number that we put at the bottom is always the right-hand number.

(Write on board.)

What can we do with the left-hand number?

Would it make sense to throw it away? No; it's part of the problem. So we will put it at the very top of the next column at the left. That way we don't lose it and it's still on the left side.

Watch! (Write on board.)

Count the little number on the left with me.

Now, when I start adding that column I will start with the four (4) first. Let's be sure you understand. (Repeat twice from the 4.)

This is called carrying, some of you already understand it. Good. Carrying is very easy.

But carrying is very important. You must never forget to carry.

Look at these examples and tell me what to write at the top of the left-hand column. (Write on board.)

(Do with volunteers from class at board.) Good, we write the left-hand answer number at the top of the next column. (Repeat three times.)

Remember though that for the last column only, the left-hand answer number is at the bottom as though it were a single column.
Now, copy these examples and do them with me.

```
7 6 7 9 8
5 9 6 8 5
8 7 4 7 6
6 9 6 9 5
8 3 8 3 9
9 5 6 2 2
```

Again, do you see that I always carry the number of tens to the top of the next column? (Point and illustrate example.) Except when there are no more columns. Then I write the number of tens on the bottom line as part of the answer. (Point and illustrate with each.)

Good Are there are any questions?

Now take these dittosed examples and do them by yourselves. If you have trouble, see me for help.

```
7 6 7 9 8
5 9 6 8 5
8 7 4 7 6
6 9 6 9 5
8 3 8 3 9
9 5 6 2 2
```

Be sure to make and place your numbers neatly!

(Allow time needed for most to finish.)

Now, I will do them. Check your work against mine.
APPENDIX C
HUTCHINGS' ADDITION ALGORITHM, REVIEW
(Adapted from Hutchings, 1972)
REVIEW

We are going to review the new way of writing number facts which we practiced yesterday.

We are going to start at the top, writing number facts as you learned yesterday.

a) Say, "Remember that during the beginning of the example is the only time that we use two big numbers. In the rest of the example, we use one little number and one big number."

b) Say, "Five plus nine is fourteen."

c) Write the fourteen in the example as \( 9y \).

a) Say, "We’ve written the fourteen but we’ll use only the four."

b) Say, "Four plus eight is twelve."

c) Write the twelve in the example as \( 8z \).

a) Say, "We’ve written the twelve but we’ll use only the two."

b) Say, "Two plus six is eight."

c) Write the eight in the example as \( 6z \).

a) Say, "We’ve written the eight and we use just the eight."

b) Say, "Eight plus eight is sixteen."

c) Write the sixteen in the example as \( 6z \).
a) Say, "We've written the sixteen but we'll use only the six."

b) Say, "Six plus seven is thirteen."

c) Write the thirteen in the example as $7_3$.

---

a) Say, "We've written the thirteen but we'll use only the three."

b) Say, "Three plus eight is eleven."

c) Write the eleven in the example as $8_3$.

---

a) Say, "We've written the eleven but we'll use only the one."

b) Say, "One plus seven is eight."

c) Say, "The last little number on the right is the right half of the answer. To find the left half, we just count the little numbers on the left that we did not use. Who can tell me what the right half of the answer is? Eight! Right. Now, who can tell me what the left half of the answer is? Five! Right, the answer, then is 58."

---

a) Say, "Now let's try a bigger example. We are going to move faster this time because you have done so well."

b) Say, "Let's start with the right column (point to it). Seven plus five is twelve. (Write the twelve in the example as $5_2$.) Two plus six is eight. (Write the eight in the example as $6_1$.) Eight plus five is thirteen. (Write the thirteen in the example as $7_3$.) Three plus nine is twelve. (Write the twelve in the example as $9_2$.) We write the two below the right column and carry the three to the top of the next column." (Write the three above the second column.)
Say, "Now, when I start adding this column (pointing to second column), I will start with the three. Three plus seven is ten. (Write the ten in the example as \(7\).) Zero plus eight is eight. (Write the eight in the example as \(8\).) Right plus seven is fifteen. (Write the fifteen in the example as \(15\).) Five plus nine is fourteen. (Write the fourteen in the example as \(14\).) Four plus three is seven. (Write the seven in the example as \(7\).) We write the seven below the column. Then we count the tens: One, two, three tens. We carry the three to the top of the next column." (Write the three above the last column.)

Say, "Now our example looks like this (pointing to example). Who can tell me the numbers we are going to add next? Right. We are going to add the three and the eight."

Say, "Three plus eight is eleven. (Write the eleven in the example as \(11\).) Who can tell me the numbers we are going to add next? Right. We are going to add the one and the six. One plus six is seven. (Write the seven in the example as \(7\).) Who can tell me the numbers we are going to add next? Right. We are going to add the seven and the four. Seven plus four is eleven. (Write the eleven in the example as \(11\).) Who can tell me the numbers we are going to add next? Right. We are going to add one and six. One plus six is seven. (Write the seven in the example as \(7\).) Seven plus eight is fifteen. (Write the fifteen in the example as \(15\).) Now we write the five below the column. (Write the five below the third column.) Then we count the tens: One, two, three tens. Because there are no more columns, we write the three to the left of the five." (Write the three to the left of the five in the example.)

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How copy these examples and do them with me.

\[
\begin{array}{ccc}
0 & 1 & 2 \\
3 & 4 & 5 \\
6 & 7 & 8 \\
9 & 0 & 1 \\
\end{array}
\quad
\begin{array}{ccc}
3 & 3 & 2 \\
6 & 6 & 7 \\
9 & 9 & 8 \\
2 & 2 & 4 \\
1 & 1 & 6 \\
\end{array}
\quad
\begin{array}{ccc}
5 & 9 & 3 \\
8 & 6 & 5 \\
3 & 6 & 7 \\
2 & 6 & 7 \\
3 & 6 & 7 \\
\end{array}
\quad
\begin{array}{ccc}
7 & 8 & 4 \\
9 & 8 & 6 \\
5 & 8 & 7 \\
6 & 7 & 6 \\
7 & 6 & 5 \\
\end{array}
\]

Are there any questions? Good. Now take these dittoed examples and do them by yourselves. If you have trouble, ask me for help. Be sure to make and place your numbers neatly.

\[
\begin{array}{ccc}
6 & 6 & 6 \\
7 & 7 & 8 \\
3 & 8 & 6 \\
3 & 9 & 7 \\
3 & 9 & 7 \\
\end{array}
\quad
\begin{array}{ccc}
6 & 7 & 8 \\
6 & 7 & 8 \\
6 & 5 & 8 \\
3 & 9 & 7 \\
8 & 9 & 4 \\
8 & 7 & 6 \\
\end{array}
\quad
\begin{array}{ccc}
6 & 8 & 4 \\
7 & 8 & 9 \\
7 & 8 & 9 \\
7 & 8 & 9 \\
7 & 5 & 8 \\
7 & 6 & 5 \\
\end{array}
\]

(Allow time for most to finish.)

Now I will do them. Check your work against mine.

(Do examples on the blackboard. Answer questions. Emphasize the need to write neatly and the need to count the "carry number" correctly. Demonstrate the latter while doing the work. State that the "carry number" is always written at the top of the column to which it is carried.)
APPENDIX D

CONVENTIONAL ALGORITHM, LESSON

(Adapted from Hutchings, 1972)
I am going to write some addition examples on the board. Begin to do them as soon as you can see them. After I finish writing all of them, I will go back and write in the answers. After you have finished working all of the examples, go back and check your answers against the answers I have written on the board. As soon as you have finished, turn your papers in.

Does everyone know what to do? (Pause momentarily.) Good. Begin...

```
7 9 6 4 6 5 9 8 4 3 5 5
+8 +8 +7 +5 +6 +2 +7 +5 +3 +1 +1 +5

6 1 3 5 5 4 3 2 8 5 6 6
+8 +7 +1 +4 +9 +8 +3 +2 +8 +7 +9 +8

1 8 9 9 8 3 3 9 9
+7 +8 +2 +1 +4 +6 +5 +3 +8

8 8 9 4 6 4 6 8 5 5 6 8
3 5 5 8 8 7 8 7 8 7 5 9
7 6 3 3 9 6 7 6 7 9 6 2
9 7 2 6 5 8 6 +2 5 6 8 5
8 9 6 1 7 +7 +8 9 4 7 6
6 8 8 8 5 6 8 +4 3
8 3 7 7 9 3 7
+7 +6 +9 +6 +3 +8 2

8 5 7 6 8 5 7 6 7 9 8
7 6 7 7 1 8 5 9 8 7 7
9 9 9 2 1 2 8 7 6 8 5
4 5 1 8 4 7 6 9 4 7 6
3 8 1 3 9 5 8 3 6 9 5
+6 +4 3 +6 3 +9 5 8 3 9
+4 2 2
```
APPENDIX E

Sample Sheets of Addition Problems
Given to the Students During Each Study Session
| 2 5 8 3 |
| 6 9 6 5 |
| 1 7 9 1 |
| 1 4 4 1 |
| 8 3 4 2 |

| 2 5 3 6 |
| 7 7 8 9 |
| 2 9 6 7 |
| 8 8 6 9 |
| 8 5 3 7 |

**NAME:**
**DATE:**
**NEW**  **OLD**
**FREE CHOICE**
**AM**     **PM**
| 7 | 6 | 6 | 8 | 4 | 8 | 6 | 2 |
| 4 | 3 | 6 | 6 | 5 | 4 | 8 | 5 |
| 9 | 8 | 4 | 9 | 4 | 9 | 3 | 1 |
| 2 | 5 | 9 | 9 | 7 | 8 | 5 | 4 |
| 1 | 8 | 9 | 2 | 7 | 8 | 5 | 2 |

| 7 | 5 | 5 | 9 | 9 | 3 | 4 | 2 |
| 4 | 7 | 2 | 7 | 2 | 1 | 6 | 1 |
| 9 | 3 | 1 | 1 | 5 | 4 | 7 | 3 |
| 6 | 5 | 8 | 1 | 5 | 9 | 8 | 7 |
| 9 | 5 | 8 | 6 | 8 | 7 | 1 | 2 |

**Name:**
**Date:**
**AM**   **PM**
**NEW**  **OLD**
**Free Choice**
APPENDIX F
QUESTIONNAIRE GIVEN TO ALL STUDENTS
DURING THE LAST EXPERIMENTAL SESSION
QUESTIONNAIRE

1. Which way, old or new, was easier to do?

2. a) Which way did you like to do better in the beginning?
   
   b) Which way did you like to do better at the end?

3. Did you get more problems correct when you used the old way or when you used the new way?

4. Which way did you make more mistakes?

5. Using which way did you do the problems more quickly?

* In all the above questions, students were asked to either write old or new on their paper, in response to each question. Students used the term old to refer to the conventional method, and the term new to refer to the Hutchings' algorithm. For questions 6, 7, and 8, the choices for answers were placed on a blackboard. The questions were asked orally by the experimenter. Students were asked to write down on their papers the letters that corresponded to their answer choices. Students were told that they could write down only one letter, or more than one letter.

6. Remember when I told you that if you did the problem one way you would have to do more problems, but if you chose to do the problems another way, you could do fewer problems? One time I told some of you that you could do either two or three problems. I told some of you that you could do either four or six problems. And still others of you could do either six or nine problems. Now when you had a choice of doing these numbers of problems, what
helped you decide whether to do more problems and your favorite way, or else fewer problems and your less favorite way? Did you pick:

A. the way with the easiest problems
B. the way where you could get the most problems correct
C. the way where you could be done the quickest
D. one way only because another friend chose that same way
E. None of the above reasons.

7. Now remember when I told some of you that you had a choice of doing either two or four problems. Others had a choice of doing either four or eight problems, or six or 12 problems. Now when you had a choice of doing these numbers of problems, what helped you decide whether to do more problems and your favorite way, or else fewer problems and your less favorite way? Did you pick:

(Choices the same as in question six.)

8. Over the whole study, in doing the problems, were you more concerned about getting done quickly or more concerned about getting them all correct?

A. done correctly
B. done quickly
C. both