Ohmic Magnetoresistance in Nonparabolic Semiconductors: Application to n-InSb

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OHMIC MAGNETORESISTANCE IN NONPARABOLIC SEMICONDUCTORS: APPLICATION TO n-InSb

by

Mahmoud Jaafarian

A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment
of the
Degree of Master of Arts

Western Michigan University
Kalamazoo, Michigan
August 1976
ACKNOWLEDGEMENTS

I am indebted to Professor V. K. Arora for his valuable assistance and encouragement, without which the realization of this thesis would not have been possible.

Special thanks are due Professor M. Soga for reviewing this thesis. His teaching of quantum mechanics has been especially helpful in completing some of the calculations in this thesis.

Professor K. K. Rao deserves a special word of appreciation for his comments from an experimental point of view.

The financial support of the Department of Physics during my study at Western Michigan University is highly appreciated. Thanks are due to the Computer Center for making computer time available for numerical calculations in this work.

Mahmoud Jaafarian
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To my brother,

Mohamad Reza Jaafarian
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CHAPTER I

INTRODUCTION

Magnetotransport studies are quite effective in unscrambling various interactions in semiconductors, and hence provide a deeper insight into the response of the electronic system to external stimuli. Some of these semiconductors have complicated band structure, and are thus difficult to treat theoretically. In the present investigation, relatively simple nonparabolic band structure of the n-InSb type is treated to study the effect of magnetic field on the electric conductivity.

Review of Earlier Works

In a classical model of a semiconductor, the drift velocity \( v_x \) of an electron in the presence of an electric field \( E_x \) (with magnetic field zero) is given by

\[
v_x \sim \frac{eE_x \tau}{m^*}.
\]

(1.1)

The scalar conductivity \( \sigma \) defined by

\[
J_x = \sigma E_x,
\]

(1.2)

where \( J_x = -N_e e v_x \) is the electronic current density with electronic concentration \( N_e \), can then be written as

\[
\sigma \sim \frac{N_e e^2 \tau}{m^*}.
\]

(1.3)

1
Improvements in this simple model have been obtained by using Block electron waves, where these waves are scattered either by lattice vibrations (phonons) or lattice imperfections [1]. \( \tau \) can then be interpreted as electron-phonon or electron-impurity relaxation time. The transport studies [1, 2, 3] with these improvements in the absence of magnetic field have used theoretical techniques based on the solution of the Boltzmann transport equation, in which the statistical behavior of the myriad of electrons was described by nonequilibrium Boltzmann transport function. The works based on these guidelines were fairly successful in interpreting the experimental data.

The above approach cannot be used in a satisfactory way when, in addition to electric field, a magnetic field is also imposed on the crystal. Without going into further detail, let us examine how the simple Equation (1.3) is affected by the magnetic field. In the presence of a magnetic field, the binding of the electrons to the host atoms will increase. This decreases the probability that electrons will be found in a free state, and hence the number of conduction electrons will decrease, especially at low temperatures. This "freeze-out effect" [4] could be neglected for nondegenerate semiconductors for not too low temperatures. The number of conduction electrons in the magnetic field, \( N(B) \), then could be approximated by its zero field value:

\[
N(B) \approx N(0) . \tag{1.4}
\]
The effective mass problem, which is strongly related to the band structure of solids, can be conveniently handled by using the pseudopotential approach [5]. In this approach, part of the kinetic energy is replaced by a repulsive potential which cancels most of the strong attractive electron-lattice ions interaction (cancellation theorem) [5], making the use of perturbation theory effective. How is this problem affected by a magnetic field? It has been shown by Zak and Zawadzki [6] that when

\[ \lambda = \left( \frac{\hbar c}{eB} \right)^{1/2} \gg a, \]

where \( \lambda \) is the radius of the cyclotron orbit and \( a \) the inter-atomic spacing, the fluctuations in the wave-function over the lattice cell will be small and hence the effective mass approximation holds good for magnetic fields up to 200 KG (\( \lambda \sim 10^{-6}\text{cm} \)). For high magnetic fields, where \( \hbar \omega_c \) is comparable to the band gap, the nonparabolic character of the energy band plays an active role. This makes the effective mass parallel to the magnetic field energy-dependent and hence field-dependent.

Any change in conductivity then should be through the effect of the magnetic field on \( \tau \), the relaxation time. That this is indeed the case can be easily seen by examining the curvature in the free path of an electron. In the longitudinal configuration, where electric and magnetic fields are parallel to each other, the components of mean free path in
the direction of electric field are not affected, making conductivity field-independent in the classical model. The effect of the quantized motion of the electron in a magnetic field makes \( \tau \) field-dependent \([7, 8]\). But the absence of curvature still allows us to treat the problem by the Boltzmann transport equation \([9]\) with results in agreement with the experimental data. For anistropic energy surfaces, even for the longitudinal case, there are anistropic effects \([10]\) which cannot be averaged by the Boltzmann distribution function \([11]\).

For the transverse configuration, the component of the free path in the direction of the external electric field is strongly affected by the curvature in the free path of the electron. This curvature effect makes the velocity operator nondiagonal in any representation with a magnetic field \([10]\). For small magnetic fields, this curvature effect could be considered small in the Boltzmann transport equation with field-independent relaxation time. Theories based on this ansatz \([12]\) predicted a saturation in the magnetoresistance, which we very well know now is not present in the experimental results; rather, magnetoresistance varies almost linearly with the magnetic field. This casts serious doubts about the universal applicability of the Boltzmann transport equation.

The semiclassical Boltzmann transport equation has been fairly successful for problems involving no, or at most a
low, magnetic field because the de Broglie wavelength $\lambda_D$ of the electron has always been smaller than the mean free path $\lambda_M$ and the radius of the cyclotron orbit $\lambda$, i.e., $\lambda_D < \lambda_M < \lambda$. For such fields, an electron can be treated like a classical particle and the effect of a magnetic field on its motion can be treated as a perturbation. But for strong magnetic fields, this semiclassical situation may change when $\lambda \sim \lambda_M$ (or equivalently $\omega_c \tau \sim 1$, where $\omega_c$ is the cyclotron frequency of the electron in a magnetic field, and $\tau$ the relaxation time). In this case, the effect of a magnetic field cannot be treated as a perturbation. In strong magnetic field transverse to electric field, the component of the free path of the electron in the direction of the electric field has a curvature in it. This curvature introduces nondiagonal matrix elements in any quantum-mechanical representation, and gives zero expectation value for the current when its averaging is attempted by using the Boltzmann transport function, in conflict with the experimental observation on transverse currents. It is at this point that the quantum-mechanical technique has a distinct advantage over Boltzmann type techniques. This technique has a further advantage when $\lambda \sim \lambda_M$ or $\hbar \omega_c \sim \xi$, the Fermi energy ($k_B T$, for nondegenerate electrons); then the quantization of energy levels of an electron also plays a prominent role. A review paper by Dresden [13] does an excellent job in explaining why the semiclassical Boltzmann transport equation cannot be used for magnetic
fields of arbitrary strength. Implicit in this equation are many assumptions, including the existence of a relaxation time, which are not always satisfied. This reference [13] also emphasizes the need of a quantum-mechanical approach such as using the density matrix.

Starting with the earlier work of Adams and Holstein [12], there have been numerous quantum-mechanical treatments to study the transport properties in the simultaneous presence of electric and magnetic fields. A review of these works has been given by Roth and Argyres [8]. In these calculations, a perturbation expansion was made under the assumption \( \omega_c \gg \frac{1}{\tau} \), where \( \omega_c = \frac{eB}{m^*c} \), is the cyclotron frequency of the electron with effective mass \( m^* \) and charge \(-e\) in the presence of a magnetic field of magnitude \( B \), and \( \tau \) is the relaxation time of the electron. These theories suffered from an unpleasant drawback of divergent results. Various cutoff mechanisms have been suggested to offset this divergence difficulty [8]. This divergence comes from the infinite density of states at the bottom of the Landau subband in the conduction band of a semiconductor. For those electrons making transition to the bottom of Landau level, \( \tau^{-1} \) diverges, invalidating the condition \( \omega_c \gg \frac{1}{\tau} \). This is the cause of divergence in those works where an expansion in terms of \( (\omega_c \tau)^{-1} \) is attempted. A method to avoid the perturbation expansion in terms of \( (\omega_c \tau)^{-1} \) was suggested by Arora and Miller [10]. They used the quantum-mechanical approach based
on the solution of Liouville's equation for the density matrix. This theory was further elaborated by Arora and Peterson [7] and was applied to study the magnetophonon structure in a simplified parabolic model of n-type InSb. They showed that the divergence difficulty could be eliminated by extending the scattering dynamics beyond the strict Born approximation, and as such no artificial cutoff mechanism was necessary. Other details, like phonon-drag, inelasticity, nonparabolicity, etc., could be incorporated when deemed important.

It has been shown recently that the nonparabolicity of the conduction band may considerably affect the transport properties of semiconductors. For example, Wu and Spector [14] have shown that the nonparabolicity will introduce magnetic field dependence of ultrasound propagation in a longitudinal magnetic field. The effect of nonparabolicity is to introduce an energy and hence a magnetic field-dependent effective mass of the conduction electrons. The effective mass increases with the magnetic field, therefore decreasing the conductivity or increasing the magnetoresistance. Sharma and Phadke [15] used the Boltzmann transport equation to show that this alone could lead to non-zero longitudinal magnetoresistance, even if the effect of the magnetic field on the relaxation time is neglected. In a later work [16], they included the magnetic field dependence on the relaxation time and found that the effect of nonparabolicity is to give rise
to a higher longitudinal magnetoresistance in the extreme quantum limit. Pal and Sharma [17] found that in the extreme quantum limit this nonparabolicity may give rise to a stronger dumping of helicon waves in the transverse configuration. It is therefore thought to be a worthwhile effort to investigate the effect of nonparabolicity of the conduction band on transport properties in semiconductors.

An Outline of the Thesis

Use is made here of the Arora-Miller magnetotransport theory as elaborated by Arora and Peterson [7] to calculate the magnetoresistivity components for nonparabolic semiconductors of the n-InSb type. Chapter II describes the density matrix appropriate to the nonparabolic band structure. Liouville's equation is solved in a convenient representation to obtain a density matrix when the electron system is subjected to simultaneous electric and magnetic fields. This density matrix is then used in Chapter III to find the expectation value of the ohmic electric current density. The numerical evaluations of the magnetoconductivity components so obtained are presented and discussed in Chapter IV. A comparison is then made with other works, and the findings of this investigation are summarized.
CHAPTER II

DENSITY MATRIX

In this chapter we present the solution of Liouville's equation for the density matrix with the Landau gauge. The electric field interaction and the electron-phonon interaction are treated strictly as perturbation.

Hamiltonian Formulation of the Problem

The Hamiltonian for an electron-phonon system with electric field $E = (E_x', E_y', E_z')$, and magnetic field $B$ parallel to $Z$-direction with magnetic potential $A = (0, B_x', 0)$, Landau gauge, in a nonparabolic model of a semiconductor has the form

$$\mathcal{H}_T = \mathcal{H}_0 + \mathcal{H}_e + \mathcal{H}_L$$

where $\mathcal{H}_0$ is unperturbed Hamiltonian including electron and lattice Hamiltonian $\mathcal{H}_e$ and $\mathcal{H}_L$ given by

$$\mathcal{H}_e = -\frac{E_g}{2} + \frac{Lq}{2} \left[ 1 + \frac{4}{E_g} \frac{\pi^2}{2m^*} \right]^{1/2}, \quad (2.2)$$

$$\mathcal{H}_L = \sum_q (N_q + \frac{1}{2})\omega_q, \quad (2.3)$$

where

$$\vec{p} = \vec{p} + \frac{e}{c} \vec{A}. \quad (2.4)$$

Summation in Equation (2.3) is over all phonon wave vectors $q$ in the acoustic branch of the lattice. $N_q$ and $\omega_q$ are,
respectively, the phonon occupation number operator and phonon angular frequency. The perturbation part \( \mathcal{H} \) is

\[
\mathcal{H} = V + e\mathbf{E} \cdot \mathbf{r},
\]

(2.5)

where \( V, \) the electron lattice deformation potential interaction for the phonon acoustic branch, is given by

\[
V = \sum_{q} C(q) b_{q} e^{i\mathbf{q} \cdot \mathbf{r}} + \text{Hermitian Conjugate}.
\]

(2.6)

\( C(q) \) for the acoustic branch is

\[
|C(q)|^2 = \frac{E_{1}^{2} k_{\omega_{q}}}{2 \rho_{d} U_{q}^2},
\]

(2.7)

where \( r \) is the electron position operator; \( b_{q} \), the phonon destruction operator; \( U_{q} \), the average longitudinal sound velocity; \( \rho_{d} \), the crystal density; \( \Omega \), the crystal volume; and \( E_{1} \), the deformation potential energy for acoustic phonons.

The term \( F = e\mathbf{E} \cdot \mathbf{r} \) is interaction of the electron with external electric field \( \mathbf{E} \). As only the linear effect of \( \mathbf{E} \) is investigated in the present work, we are treating this field as a perturbation. This provides the convenience of uniform distribution of electrons when perturbation is absent.

The eigenfunctions of the electronic Hamiltonian of Equation (2.2), \( \mathcal{H}_{e} \) (normalized in a unit volume), are given by

\[
|\psi_{nk}\rangle = \psi_{nk} e^{i(k_{y}y + k_{z}z)} \phi_{n}(\xi),
\]

(2.8)

where

\[
x_{k} = -\lambda^{2}k_{y} \text{ and } \lambda^{2} = \frac{\hbar c}{eB}.
\]

(2.9)

The quantum number \( k \) in the above equation stands for \( k_{y} \) and
\( k_z \). \( \Phi_n(\xi) \) is the harmonic oscillator wavefunction, which in terms of Hermite polynomials is given by

\[
\Phi_n(\xi) = \left( \frac{1}{\sqrt{\pi n!}} \right)^{1/2} e^{-\xi^2/2} H_n(\xi),
\]

(2.10)

The eigenvalues of \( \mathcal{H}_e \) of Equation (2.2) are given by

\[
E_{nk} = - \frac{e^2}{4\varepsilon_0} \left\{ 1 - \left[ 1 + \frac{4}{\varepsilon_0} \left( (n+\frac{1}{2})\hbar \omega_e + \frac{\hbar^2 k_z^2}{2m^*_e} \right) \right]^{1/2} \right\},
\]

(2.11)

where \( \omega_e = \frac{eB}{m^*_e} \) is the cyclotron frequency of the electron.

When

\[
(n + \frac{1}{2})\hbar \omega_c + \frac{\hbar^2 k_z^2}{2m^*_e} \ll E_e,
\]

the eigenvalues reduce to those obtained using the parabolic model for the band structure.

The Landau levels for the nonparabolic band model of Indium Antimonide are shown in Appendix A. To make the comparison, we have also exhibited in the same figure the energy levels for the parabolic model. It is clear from the comparison that energy levels are greatly modified at high magnetic fields and higher values of the momentum parallel to the magnetic field. In transport problems, usually the electrons with low momentum,

\[
k_z \sim 0 \text{ to } k_z \text{ max } \sim \left( \frac{2m^*_e k_B T}{\hbar^2} \right)^{1/2},
\]

are important. As such, the modification of the energy levels at high magnetic fields may play an active role.
Density Matrix

As discussed in the introduction, serious doubts exist in the universal applicability of the Boltzmann transport equation. It is necessary, therefore, to start from more basic notions of statistical mechanics. In treating quantum-mechanical systems, it is necessary to deal with two types of uncertainties. The first type is due to the probabilistic nature of the wavefunction, as is illustrated by the "uncertainty principle." The second uncertainty occurs when one does not have sufficient information to find the quantum-mechanical state of the system, and can be handled by the density matrix in the absence of perturbation ($\mathcal{H}' = 0$). The state of the electronic system is well described by the wavefunctions given by Equation (2.10). When perturbation is present, we do not know exactly in which state the system is, necessitating an expansion of wavefunction $\psi^i$ of the i-th electron in terms of orthonormal set of Equation (2.10):

$$\psi^i(t) = \sum_\alpha a^i_\alpha(t) |\alpha\rangle .$$

(2.12)

The expectation value of the physically observable property $\Lambda$ of such an electron is then given by

$$\Lambda^i = \langle \psi^i | \Lambda | \psi^i \rangle = \sum_{\alpha,\alpha'} a^i_\alpha a^\dagger_\alpha \langle \alpha | \Lambda | \alpha' \rangle .$$

(2.13)

An ensemble average for $N$ electrons in the system then can be described by

$$\langle \Lambda \rangle = \frac{1}{N} \sum_i \Lambda^i = \sum_{\alpha,\alpha'} \langle \alpha | \rho | \alpha' \rangle \langle \alpha' | \Lambda | \alpha \rangle = \text{Tr}(\rho \Lambda) ,$$

(2.14)
where
\[ \langle \alpha' \mid \rho \mid \alpha \rangle \equiv \frac{1}{N} \sum_{i,j} a_{\alpha}^{i}(t) \alpha_{\alpha}^{i}(t). \] (2.15)

The time dependence of coefficients \( a_{\alpha}^{i} \rho \) can be obtained from the time-dependent Schrödinger equation for wavefunction \( \psi_{i} \) of Equation (2.12):
\[ i\hbar \frac{d a_{\alpha}^{i}(t)}{dt} = \sum_{\alpha'} \langle \alpha \mid \mathcal{H} \mid \alpha' \rangle a_{\alpha'}^{i}(t). \] (2.16)

Using this time dependence in Equation (2.15), we obtain Liouville's equation for the density matrix
\[ i\hbar \frac{d \rho}{dt} = [\mathcal{H}, \rho]. \] (2.17)

To solve Equation (2.17), we need to describe the time development of the system. We assume that at \( t = -\infty \), the system was in equilibrium with perturbation absent (\( \mathcal{H}' = 0 \)). We then turn on the perturbation slowly, so that
\[ \mathcal{H}(t) = \mathcal{H}_{0} + \mathcal{H} e^{St}, \] (2.18)
where \( S \) is a small positive number. The density matrix of the system will follow a similar time dependence:
\[ \rho(t) = \rho_{0} + \rho' e^{St}, \] (2.19)
where \( \rho_{0} \) is a uniform density matrix independent of external electric field which is diagonal in the Landau representation of Equation (2.10):
\[ \langle \alpha' \mid \rho_{0} \mid \alpha \rangle = f_{\alpha} \delta_{\alpha' \alpha} = \frac{1}{e^{(E_{n}k' - 3)/k_{B}T} + 1} \delta_{n' n} \delta_{k' k}. \] (2.20)
\( \zeta \) is the Fermi energy of the system which can be evaluated from the normalization condition

\[
\text{Tr}(\rho) = \sum_{\alpha} \langle \alpha | \rho_0 | \alpha \rangle = N ,
\]

(2.21)

with the result for nondegenerate electrons (see Appendix B)

\[
e^{\zeta/k_B T} \approx \frac{N_e (2\pi \hbar^2/m^* k_B T)^{3/2}}{2\hbar \omega_c \sum_{n=0}^{\infty} e^{-E_g(a_n-1)/2k_B T}\sqrt{a_n}} ,
\]

(2.22)

where

\[
a_n = \left[ 1 + \frac{4}{E_g(n+\frac{1}{2})\hbar \omega_c} \right]^{1/2}.
\]

(2.23)

Our purpose is to find the steady state \( \rho \) at \( t = 0 \), when \( \mathcal{H} = \mathcal{H}_0 + \mathcal{H}' \) as given by Equation (2.1). This is easily done by taking the limit \( S \to 0 \). Substitution of Equations (2.12) and (2.19) in Equation (2.17) yields

\[
(E_{\alpha'\alpha} - i\hbar S)\langle \alpha' | \rho' | \alpha \rangle = \langle \alpha' | [\rho_0, \mathcal{F}] | \alpha \rangle +
\]

\[
+ f_{\alpha'\alpha} \langle \alpha' | \mathcal{V} | \alpha \rangle + \langle \alpha' | [\rho', \mathcal{H}'] | \alpha \rangle e^{St} \, ,
\]

(2.24)

with

\[
E_{\alpha'\alpha} = E_{\alpha'} - E_{\alpha} ,
\]

(2.25)

\[
f_{\alpha'\alpha} = f_{\alpha'} - f_{\alpha} .
\]

(2.26)

Equation (2.24) is a coupled equation in matrix elements of \( \rho' \). To decouple this equation, usually a linearization procedure is adopted [12]. According to this procedure, the last term involving \([\rho', \mathcal{H}']\) is neglected on the ground that this involves higher order terms. This allows us to solve \( \langle \alpha' | \rho' | \alpha \rangle \) from Equation (2.24) in terms of first order terms in \( \mathcal{H}' \). This first order expression of \( \langle \alpha' | \rho' | \alpha \rangle \) is then used
to generate second and higher order terms. This procedure is equivalent to generation of a series in terms of \((\omega_c t)^{-1}\), of which only the first two terms are kept under the assumption of a strong magnetic field \((\omega_c \gg \frac{1}{t})\). But higher order terms in the series may also become important for those electrons making transition to the bottom of conduction band \((k_z \approx 0)\), thereby invalidating the expansion. This is avoided by solving Equation (2.24) formally:

\[
\langle \alpha' | \rho' | \alpha \rangle = \frac{\langle \alpha' | \{ \rho_0, F \} | \alpha \rangle + \int \alpha' \alpha \langle \alpha' | V | \alpha \rangle + \langle \alpha' | \{ \rho', F' \} | \alpha \rangle e^{St}}{E_{\alpha'\alpha} - i\hbar \omega}. \tag{2.27}
\]

This formal expression is then used in the last term of Equation (2.24) to get

\[
\langle E_{\alpha'\alpha} - i\hbar \omega \rangle \langle \alpha' | \rho' | \alpha \rangle = \langle \alpha' | \{ \rho_0, F \} | \alpha \rangle + \int \alpha' \alpha \langle \alpha' | V | \alpha \rangle + \sum_{\alpha''} \langle \alpha'' | \{ \rho_0, F \} | \alpha'' \rangle + \int \alpha' \alpha'' \langle \alpha'' | V | \alpha'' \rangle + \langle \alpha' | \{ \rho', F' \} | \alpha'' \rangle e^{St} \frac{E_{\alpha'\alpha''} - i\hbar \omega}{E_{\alpha''\alpha''}} \langle \alpha'' | \rho'' | \alpha'' \rangle e^{St} - \sum_{\alpha''} \langle \alpha'' | \rho'' | \alpha'' \rangle e^{St}.
\]

At this stage, we make approximations to decouple the exact equation (2.28). Since we are interested only in the ohmic cases, we will neglect all terms involving \([\rho', F]\), as these will give rise to non-ohmic currents. We now assume that the scattering of electrons is elastic \((\hbar \omega \ll E_{nk})\). Use is then made of certain sum rules for the scatterers in
non-polar semiconductors [10, 18, 19]:

\[ \sum_{k'} \langle nk | \mathbf{v} | mk', v | n'k \rangle = \sum_{k'} \langle nk | \mathbf{v} | mk', v | n'k \rangle 2 \delta_{n', n} \delta_{m', m}, \]  \hspace{1cm} (2.29)

\[ \sum_{k'} \langle nk | \mathbf{v} | mk', v | n'k \rangle = 0. \]  \hspace{1cm} (2.30)

We now take an ensemble average over the scatterers (dropping all first order terms in \( V \)), take the limit \( S \to 0 \), and use the identity

\[
\lim_{S \to 0} \frac{1}{x - iS} = P \left( \frac{1}{x} \right) + i \pi \delta(x),
\]  \hspace{1cm} (2.31)

where \( P \) stands for the principal part. In the irreversible behavior of the current, the principal part term will not contribute to the current when the elastic scattering assumption is made [19] for non-polar semiconductors, although this may contribute to line shift in cyclotron resonance experiments involving polar semiconductors [19]. Consistent with our assumption of elastic scattering in non-polar semiconductors, we will drop this principal part term. Finally, we make use of a property of density matrix due to space invariance, described by Fano [20], according to which all the non-diagonal in \( k \)-space (\( k'_y \neq k_y, k'_z \neq k_z \)), elements of \( \rho \) are zero. This simplifies Equation (2.28) to the extent given by

\[ E_{n'k', nk} \langle n'k' | \rho | nk \rangle = \langle n'k' | [\rho_0, F] | nk \rangle + \]

\[ + \langle n'k' | \rho | nk \rangle \frac{i \hbar}{\tau_{n'k', nk}}, \]  \hspace{1cm} (2.32)

which can be solved to give for \( \langle n'k' | \rho | nk \rangle \), the expression
\[ \langle n', k' | \rho | nk \rangle = \frac{\langle n', k' | [\rho_0, F] | nk \rangle}{E_{n', k', nk} - \frac{i\hbar}{\tau_{n', k', nk}}} , \tag{2.33} \]

with
\[
\frac{1}{\tau_{n', k', nk}} = \frac{1}{2} \cdot \frac{1}{\tau_{n', k'}} + \frac{1}{2} \cdot \frac{1}{\tau_{nk}} , \tag{2.34} \]
\[
\frac{1}{\tau_{nk}} = \frac{2\pi}{\hbar} \sum_{n''} |\langle nk | V | n'' k'' \rangle|^2 \delta(E_{nk}, E_{n'' k''}) . \tag{2.35} \]

Equation (2.33) contains a Breit-Wigner [21] type of collision broadening. The presence of the last term in Equation (2.32) is thus of great importance. The neglect of such a term would be a violation of the Heisenberg uncertainty principle, leading to divergence difficulty encountered by others [8, 22]. This built-in broadening thus has an important effect in magnetotransport work [7]. Not only will this avoid divergence difficulty, but it will also give the zero field results obtained from the Boltzmann transport equation when the limit \( B \to 0 \) is made. Moreover, due to the curvature effect in the transverse configuration, the relaxation time \( \tau \) now behaves like a pseudotensor, which is an arithmetic average of the relaxation of two states of the density matrix. Some authors [23, 24] have included this kind of collision broadening by introducing a collision term in the density matrix changing Equation (2.17) to
\[
\frac{i\hbar}{\partial t} = [\mathcal{H}, \rho] + i\hbar \frac{\partial \rho}{\partial t}_{\text{collision}} . \tag{2.36} \]

This method of treating the density matrix does avoid divergence, but is artificial and leaves out the details of
the collision relaxation time. Since \( \mathcal{H} \) includes the inter-
action responsible for collision, it is not natural to
include the collision term from outside. The collision term
introduced by Lifshitz [25] and others [12, 23, 24] does not
take into account the effect of applied fields, for example.

Liouville's equation, Equation (2.17), is thus solved
in the ohmic limit to give for the matrix elements of

\[
\langle n'k'|\rho|nk \rangle = f_{nk} \delta_{n'n} \delta_{k'k} + \frac{\langle n'k'| [\rho_0, F]|nk \rangle}{E_{n'k',nk} - \frac{i\hbar}{\tau_{n'k',nk}}} , \tag{2.37}
\]

with

\[
\langle n'k'| [\rho_0, F]|nk \rangle = \frac{f_{nk}'}{E_{n'k',nk}} \cdot \frac{e|k|}{1} \cdot \frac{1}{2m^*} \left[ \left( 1 + \frac{2E_{nk'}}{E_g} \right)^{-1} + \right.
\]

\[
+ \left. \left( 1 + \frac{2E_{nk'}}{E_g} \right)^{-1} \right] \cdot \left[ E_{x}^{*} \sqrt{\frac{k_{x}c}{2m^*}} \left( \sqrt{n+1} \delta_{n',n} - \sqrt{n} \delta_{n',n+1} \right) + \right.
\]

\[
E_{y}^{*} \sqrt{\frac{k_{y}c}{2m^*}} \left( \sqrt{n+1} \delta_{n',n+1} + \sqrt{n} \delta_{n',n-1} \right) + E_{z}^{*} k_{z} \delta_{n',n} \right] \delta_{k'k} . \tag{3.38}
\]

The relaxation time \( \tau_{nk} \) of Equation (2.35) for the
electron-acoustic phonon scattering of Equation (2.6) is
given by

\[
\frac{1}{\tau_{nk}} = A_{ac} \left( 1 + \frac{4E_{nk}}{E_g} \right)^{1/2} \sum_{n'} \left[ E_{nk} - \left( n' + \frac{1}{2} \right) k_{x}c \right]^{1/2} , \tag{2.39}
\]

with

\[
A_{ac} = \frac{E_{1}^{2}k_{B}T(2m^*)^{1/2}}{2m^2 \rho_{d}^{2} \lambda^2} . \tag{2.40}
\]
The density matrix of Equation (2.37) will be used in the next chapter to find the expectation values of the current density.
CHAPTER III

MAGNETORESISTIVITY

In this chapter we calculate the matrix elements of the electronic current density, which are multiplied with the density matrix of the last chapter to calculate the expectation value of the current from Equation (2.14).

Magnetoconductivity Components

The electronic current operator \( \mathbf{J}_{\text{op}} \) can be obtained from the Heisenberg equation of motion:

\[
\mathbf{J}_{\text{op}} = -e \mathbf{V}_{\text{op}} = -ie \left[ \mathbf{\pi}, \mathbf{\mathcal{H}} \right].
\]

(3.1)

The matrix elements of Equation (3.1), with the Landau representation of Equation (2.10) as the basis, are given by

\[
\langle n'k'| J_x | nk \rangle = -\frac{ie}{2} \sqrt{\frac{\hbar \omega_c}{2m^*}} \left[ \left( 1 + \frac{2E_{n'k'}}{E_g} \right)^{-1} + \left( 1 + \frac{2E_{n'k'}}{E_g} \right)^{-1} \right] (n',n+1 - \sqrt{n} \delta_{n',n-1}) \delta_{kk'}.
\]

(3.2)

\[
\langle n'k'| J_y | nk \rangle = -e \sqrt{\frac{\hbar \omega_c}{2m^*}} \left[ \left( 1 + \frac{2E_{n'k'}}{E_g} \right)^{-1} + \left( 1 + \frac{2E_{n'k'}}{E_g} \right)^{-1} \right] (n',n+1 + \sqrt{n} \delta_{n',n-1}) \delta_{kk'}.
\]

(3.3)

\[
\langle n'k'| J_z | nk \rangle = -e \frac{\hbar k_z}{m^*} \left[ 1 + \frac{2E_{nk}}{E_g} \right]^{-1} \delta_{n',n} \delta_{k',k}.
\]

(3.4)

The structure of these matrix elements clearly suggests the need of the density matrix. The expectation value of
transverse currents $J_x$ and $J_y$ whose matrix elements are non-diagonal in the eigenfunction scheme of Equation (2.10) can be obtained only through nondiagonal elements of the density matrix. On the other hand, the longitudinal current operator is diagonal in the representation of Equation (2.10), justifying the use of the Boltzmann transport function in finding the expectation value of $J_z$. This generalization of the zero field method is reasonable since the wavefunction $e^{ikz}z$ associated with the electronic motion parallel to the magnetic field remains unchanged with or without a magnetic field.

Using the matrix elements of the density matrix from Equation (2.37) and those of current operators from Equations (3.2) through (3.4), the expectation values of the current operator and hence the magnetoconductivity tensor $\sigma$ defined by

$$\mathbf{J} = \mathbf{\Sigma} \cdot \mathbf{E}$$

is obtained as

$$\mathbf{\sigma} = \begin{bmatrix} \sigma_1 & -\sigma_2 & 0 \\ \sigma_2 & \sigma_1 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix},$$

with

$$\sigma_1 = -e^2 \left( \frac{\hbar^3 \omega}{2m^*} \right) \sum_{nk} \left[ \left( 1 + \frac{2E_{n'k'}}{Eg} \right)^{-1} + \left( 1 + \frac{2Enk}{Eg} \right)^{-1} \right]^2 \left( \frac{f_{nk,n'k'}}{E_{nk,n'k'}} \right),$$

$$\sigma_2 = -e^2 \left( \frac{\hbar^3 \omega}{2m^*} \right) \sum_{nk} \left[ \left( 1 + \frac{2E_{n'k'}}{Eg} \right)^{-1} + \left( 1 + \frac{2Enk}{Eg} \right)^{-1} \right]^2 \left( \frac{1}{E_{nk,n'k'}} \right),$$

$$\sigma_3 = -e^2 \left( \frac{\hbar^3 \omega}{2m^*} \right) \sum_{nk} \left[ \left( 1 + \frac{2E_{n'k'}}{Eg} \right)^{-1} + \left( 1 + \frac{2Enk}{Eg} \right)^{-1} \right]^2 \left( \frac{1}{E_{nk,n'k'}} + \frac{\hbar^2}{\mathcal{L}^2} \right).$$
\[ \sigma_2 = -e^2 \left( \frac{k^2 \omega_c}{2m^*} \right) \sum_{n,k} \left[ \left( 1 + \frac{2E_{n,k'}}{E_g} \right)^{-1} - \left( 1 + \frac{2E_{n,k}}{E_g} \right)^{-1} \right]^2 \]

\[ f_{n,k,n',k'}(n+1) \frac{1}{E_{n,k,n',k'}} + \frac{\hbar^2}{\gamma^2} \]

\[ \sigma_3 = \frac{e^2}{k^2 B^2} \sum_{n,k} f_{n,k} \left( 1 + \frac{2E_{n,k}}{E_g} \right)^{-2} \left( \frac{h k^2}{m^*} \right)^2 \gamma_{n,k}. \]

From these equations it is clear as \( 1/\gamma_{n,k} \to \infty \) for slowly moving electrons in the direction of the magnetic field at the bottom of the Landau level \( (k_z = 0) \), conductivity elements are nondivergent.

For large forbidden band gap \( (E_g \to \infty) \), the quantized energy of Equation \( 2.11 \) reduces to the results obtained for parabolic model of a semiconductor when \( E_{n,k} \approx \epsilon_{n,k} \). Then, the above results for conductivity reduce to those obtained earlier by Arora \[26\] for parabolic band model:

\[ \sigma_1' \approx \frac{2e^2}{m^*} \sum_{n,k} f_{n,k,n',k'}(n+1) \frac{1}{\omega_c^2} + \frac{1}{\gamma_{n,k,n',k'}}. \]

\[ \sigma_2' \approx \frac{2e^2}{m^*} \sum_{n,k} f_{n,k,n',k'}(n+1) \frac{\omega_c}{\omega_c^2} + \frac{1}{\gamma_{n,k,n',k'}}. \]

\[ \sigma_3' \approx -2e^2 \sum_{n,k} \frac{d f_{n,k}}{d E_{n,k}} \left( \frac{h k^2}{m^*} \right)^2 \gamma_{n,k}. \]

These results are much more simplified for the case of
metals, where $\mathcal{F}$ is assumed to be constant, independent of quantum numbers $n$ and $k$. In this case, using the following relations

$$\sum_{nk,p} f_{nk,n'k'}(n+1) = N_e,$$  \hspace{1cm} (3.13)

$$\frac{df_{nk}}{d\epsilon_{nk}} \propto -\delta(\epsilon_{nk} - \mathcal{F}).$$ \hspace{1cm} (3.14)

we find the elements of the conductivity tensor as

$$\sigma_1 = \frac{ne^2}{m^*} \cdot \frac{\mathcal{F}}{(\omega_c \mathcal{F})^2 + 1},$$ \hspace{1cm} (3.15)

$$\sigma_2 = \frac{ne^2}{m^*} \cdot \frac{\omega_c \mathcal{F}^2}{(\omega_c \mathcal{F})^2 + 1},$$ \hspace{1cm} (3.16)

$$\sigma_3 = \frac{ne^2}{m^*}. \hspace{1cm} (3.17)$$

For the zero magnetic field ($B \to 0$), the conductivity tensor becomes diagonal with all components equal to

$$\sigma_1 \sim \sigma_3 = \frac{ne^2}{m^*}, \text{ and } \sigma_2 \sim 0.$$ \hspace{1cm} (3.18)

The conductivity tensor for the nonparabolic semiconductor given by Equation (3.6), in the absence of a magnetic field ($B \to 0$), is diagonal with all components equal to

$$\sigma(0) = \frac{2ne^2}{3m^*kT^2} \cdot \frac{\int_0^{\mathcal{F}_1/2} \mathcal{F}_1^{1/2} \mathcal{F}_2(0) e^{-E_k/kT} dE_k}{\int_0^{\mathcal{F}_1/2} e^{-E_k/kT} dE_k},$$ \hspace{1cm} (3.19)
where

\[ \gamma = \frac{E_k}{2} \left(1 + \frac{E_k}{E_g}\right) \]  
(3.20)

\[ \gamma' = \left(1 + \frac{2E_k}{E_g}\right) \]  
(3.21)

\[ E_k = \frac{E_g}{2} + \frac{E_g}{2} \left[1 + \frac{4}{E_g} \cdot \frac{\hbar^2 k^2}{2m^*}\right]^{1/2} \]  
(3.22)

\[ \frac{1}{\mathcal{T}(0)} = \frac{\sqrt{2}\pi k_B T_m^{*3/2}}{\pi k^4 \rho \mu^2} \gamma' \gamma^{1/2} \]  
(3.23)

Experimentally Measurable Parameters

For a theoretician, it is easy to talk in terms of conductivity, whereas an experimentalist finds delight in measuring the resistivity of a sample. The components of the resistivity tensor are obtained by inverting the conductivity tensor:

\[ \mathbb{R} = \begin{bmatrix}
\sigma_1/(\sigma_1^2 + \sigma_2^2) & -\sigma_2/(\sigma_1^2 + \sigma_2^2) & 0 \\
-\sigma_2/(\sigma_1^2 + \sigma_2^2) & \sigma_1/(\sigma_1^2 + \sigma_2^2) & 0 \\
0 & 0 & 1/\sigma_3
\end{bmatrix} \]  
(3.24)

The experimentally measurable parameters are relative change in resistivities \( \Delta R_{xx}/R_0 \) (the transverse magnetoresistance), \( \Delta R_{zz}/R_0 \) (the longitudinal magnetoresistance), and the normalized Hall coefficient \( R_H/R_0 \) where \( R_0 = -1/n_ee^2 \) is the high-field Hall coefficient for parabolic semiconductors and \( R_0 = \sigma^{-1}_0 \) is the zero-field resistivity. In terms of \( \sigma_1, \sigma_2, \)
and $\sigma_3$, these can be written as

$$\Delta R_{xx}/R_0 = \sigma_1/(\sigma_1^2 + \sigma_2^2)R_0 ,$$  \hspace{1cm} (3.25)$$

$$\Delta R_{zz}/R_0 = 1/\sigma_3 R_0 ,$$  \hspace{1cm} (3.26)$$

$$R_H/R_H^0 = -\sigma_2 N_e e c/(\sigma_1^2 + \sigma_2^2)B .$$  \hspace{1cm} (3.27)$$

The numerical results for these components are presented and discussed in the next chapter.
CHAPTER IV  

NUMERICAL RESULTS AND DISCUSSION 

In this chapter, we present the method to compute the experimentally observable parameters and the results obtained from computer calculations.

Numerical Evaluation of Magnetoresistivity Tensor 

Numerical evaluation of the magnetoconductivity components is greatly facilitated by using the transformation and resummation technique introduced earlier by Arora [26] and Peterson [27]. According to this technique, \( \frac{\hbar^2 k_z^2}{2m^*} \) can be expressed in terms of \( \hbar \omega_c \):

\[
\frac{\hbar^2 k_z^2}{2m^*} = (m+y)\hbar \omega_c ,
\]

where \( m \) is an integer and \( y \) is a fraction whose value is between zero and one. The energy function and distribution function with the new variables are

\[
E_{nk} = -\frac{E_g}{2} + \frac{E_g}{2} \left[ 1 + \frac{4}{E_g} (n+y+\frac{1}{2})\hbar \omega_c \right]^{1/2} ,
\]

\[
f_{nk} = e^{-\frac{E_g}{k_B T} - \left[ 1 + \frac{4}{E_g} (n+y+\frac{1}{2})\hbar \omega_c \right]^{1/2} \frac{E_g}{2k_B T}} ,
\]

where

\[
N \equiv n + m .
\]
The summation over \( k \) changes to integral by using the results obtained from the free particle in a box:

\[
\sum_{k_y k_z} \rightarrow \frac{2}{(2\pi\lambda)^2} \sum_{m=0}^{N} \int_{0}^{1} \frac{2m^2 \omega_c}{\hbar} \left( m+y \right)^{-1/2} \, dy \quad .
\]

(4.4)

Using this scheme, conductivity components will have the following forms suitable for numerical evaluation on computer:

\[
\sigma_1 = C_1 \int_{0}^{1} \sqrt{y} \, dy \sum_{N=0}^{\infty} \frac{F_{N2} \cdot F_{N1}}{F_{N}} \cdot \left[ \frac{N+1 + \sqrt{y} \sum_{m=1}^{N} \frac{(N-m+1)}{(m+y)^{1/2}}} {y \frac{E^2 g_{F}^2}{4} + \frac{Aa_c^2}{4\hbar \omega_c} T_{2N}^2 (y)} \right],
\]

(4.5)

\[
\sigma_2 = C_2 \int_{0}^{1} \sqrt{y} \, dy \sum_{N=0}^{\infty} \frac{F_{N2} \cdot F_{N1}}{F_{N}} \cdot \left[ \frac{N+1 + \sqrt{y} \sum_{m=1}^{N} \frac{(N-m+1)}{(m+y)^{1/2}}} {y \frac{E^2 g_{F}^2}{4} + \frac{Aa_c^2}{4\hbar \omega_c} T_{2N}^2 (y)} \right],
\]

(4.6)

\[
\sigma_3 = C_3 \int_{0}^{1} \sqrt{y} \, dy \sum_{N=0}^{\infty} \frac{N}{(CN)^{3/2} e^{-CN/2} \varepsilon g^a/2} \quad .
\]

(4.7)

where

\[
\varepsilon g = \frac{E g}{\hbar \omega_c} \quad ,
\]

(4.8)

\[
a = \frac{\hbar \omega_c}{k_{B} T} \quad ,
\]

(4.9)
\[ C_N = 1 + \frac{4}{\varepsilon_g} (n+y+\frac{1}{2}) \]  
(4.10)

\[ F_{N_2} = \frac{1}{4} \left[ C_{N+1}^{-1} + C_N^{-1} \right] \]  
(4.11)

\[ F_{N_1} = e^{-\frac{1}{2} \varepsilon_g a/2} - e^{-\frac{1}{2} \varepsilon_g a/2} \]  
(4.12)

\[ F_N = C_{N+1}^{1/2} C_N^{1/2} \]  
(4.13)

\[ T_N'(y) = \left[ C_{N+1}^{1/2} C_N^{1/2} \right] \left[ 1 + \sqrt{y} \sum_{N'=1}^{N} (N'+y)^{-1/2} \right] + \]  
(4.14)

\[ C_1 = \frac{2 \sqrt{2}}{\hbar m^{1/2}} \left( \frac{e}{2\pi \hbar \omega_c} \right)^2 \frac{A_{ac}}{\varepsilon_g} \cdot \frac{\sqrt{k_B T}}{\epsilon} \frac{E_g/2k_BT}{e} \]  
(4.15)

\[ C_2 = \frac{2 \sqrt{2} \omega_c}{\hbar m^{1/2}} \left( \frac{e}{2\pi \hbar \omega_c} \right)^2 \left( \frac{\hbar \omega_c}{\sqrt{2m\hbar \omega_c}} \right)^{1/2} \frac{\sqrt{k_B T}}{\epsilon} \frac{E_g/2k_BT}{e} \]  
(4.16)

\[ C_3 = \frac{4e^2}{m^* k_B T} \left( \frac{1}{2m} \right)^2 \frac{A_{ac}}{\hbar \omega_c} \left( \frac{2m^*}{\hbar \omega_c} \right)^{1/2} \frac{\sqrt{k_B T}}{\epsilon} \frac{E_g/2k_BT}{e} \]  
(4.17)

The physical constants used to perform numerical computations are \( E_1 = 30 \) eV; \( u = 3.7 \times 10^5 \) cm/sec.; \( \rho = 5.77 \) gr/an\(^3\); \( N = 10^{14} / \text{cm}^3 \); \( T = 77 \) K; and \( E_g = 0.265 \) eV or \( \infty \) (parabolic band).

The longitudinal magnetoresistance is independent of \( E_1 \), while the transverse magnetoresistance is sensitive to the value of \( E_1 \). Unfortunately, this value is not very well established in the published literature. Tsidilkovskii and
Demchuk [28] conclude very strongly that $E_1 = 30$ eV. We will therefore use this value in present calculations.

**Discussion of Numerical Results**

The expressions for longitudinal magnetoresistance when reduced to the extreme quantum limit are quite similar to those obtained by Sharma and Phadke [15] by using the Boltzmann transport equation. A detailed comparison with these works was not possible, as Sharma and Phadke analyze their results in the extreme quantum limit only. But we arrive at the same qualitative conclusion that nonparabolicity enhances the longitudinal magnetoresistance. In the extreme quantum limit, approximate dependence of the effective mass in a direction parallel to the magnetic field is given by [15, 16]

$$m^*(B) = m^*(0) \left[ 1 + \frac{2 \nu \omega_c}{E_g} \right]^{1/2}.$$  \hspace{1cm} (4.18)

This increase in effective mass with the magnetic field reduces the conductivity and hence increases the magnetoresistance.

The expressions for transverse magnetoresistance were not expected to agree with those obtained earlier [16]. As stated previously, these works are based on Kubo's formalism [29] which gives divergent results, the divergence difficulty becoming more apparent when electrons tend to move slowly in the direction of strong magnetic field. The reasons for this divergence difficulty and disagreements with older
theories were carefully explained by Arora and Peterson [7], where the results obtained were shown to reduce those obtained from the Boltzmann transport equation in the low field limit. The numerical results for the transverse magnetoresistance also show an enhancement due to nonparabolicity, the enhancement increasing with increasing values of the applied magnetic field. The results for the Hall coefficient are quite interesting. Earlier theoretical works show that the normalized Hall coefficient \( \frac{R_H}{R_H} \) is close to unity independent of scattering. But our results indicate that nonparabolicity will decrease this Hall coefficient. This is in agreement with the low field work of Nag and Dutta [30], where Hall effect is found to decrease with magnetic field (see Appendix C).

For low magnetic fields \( \hbar \omega_c \ll E_g \), the effect of nonparabolicity is quite small. When \( \hbar \omega_c \sim E_g \), there is a marked increase in magnetoresistance, both transverse and longitudinal. In conclusion, we have shown for a very simple case of elastic acoustic phonon scattering that nonparabolicity may have a pronounced effect on magnetotransport properties, the effect increasing with the increasing values of magnetic field.

Conclusion

In the above work, we have applied the density matrix formalism of Arora and co-workers [7, 10] to obtain all the experimentally measurable magnetoresistance coefficients for
nonparabolic semiconductors. A detailed comparison with experiments could not be made due to lack of experimental data and lack of knowledge on which scattering mechanisms are dominant, but the values presented are representative of what are found experimentally [31, 32].

In his review article, Dresden [13] repeatedly emphasizes the need of a more general approach such as using the density matrix. The semiclassical Boltzmann equation has been fairly successful for problems involving no, or at most a low, magnetic field of the order of a kilogauss because the de Broglie wavelength of the electron has always been smaller than the mean free path and the radius of the cyclotron orbit (it is infinity for zero magnetic field). But for strong magnetic fields, when the de Broglie wavelength is comparable to mean free path or the radius of cyclotron orbit, the semiclassical picture breaks down. In this case, the effect of the magnetic field cannot be treated as a perturbation. It is at this point that quantum theory offers a special advantage.

With the advent of an era of strong magnetic fields available with superconducting magnets, it is hoped that this work will allow others to interpret a practical situation in terms of more meaningful results.
Fig. 1. The energy schematic of the non-parabolic model of n-InSb, for small forbidden band gap (solid curves) and for large forbidden band gap (dashed curves). The dashed curve is in agreement with energy of parabolic model of n-type Indium Antimonide.
APPENDIX B

In this appendix, we calculate the Fermi energy expression of Equation (2.22) from the normalization condition

$$\sum_{n, k_y k_z} f_{nk} = N_e,$$  \hspace{1cm} (A.1) \n
where $f_{nk}$ is given by Equation (2.20).

The electron energy $E_{nk}$ of Equation (2.11) can be approximated \[28\] by the expression

$$E_{nk} \sim -\frac{1}{2}E_g + \frac{1}{2}E_g a_n + \frac{\mathcal{K}_x^2 k_z^2}{2m^* a_n},$$  \hspace{1cm} (A.2) \n
since

$$\left(\frac{\mathcal{K}_z \text{Max}}{2m^*}\right)^2 \simeq k_B T \ll E_g$$  \hspace{1cm} (A.3) \n
for InSb at temperature $T = 77$ K. [In Equation (A.2), $a_n$ is given by Equation (2.23).]

The summation over spin states gives a factor of 2, and the summations over $k_y$ and $k_z$ can be replaced by integrations

$$\sum_{k_y k_z} \rightarrow \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} dk_z \int_{-\frac{1}{2}\lambda^2}^{+\frac{1}{2}\lambda^2} dk_y. $$  \hspace{1cm} (A.4) \n
The limits over $k_y$ result from the fact that the center of the cyclotron orbit, $x_k = -\lambda^2 k_y$, must reside within the
crystal \((-\frac{1}{2} \leq \lambda^2 k_y \leq +\frac{1}{2}\)), assumed to be a unit cube. Since 
\(f_{nk}\) is independent of \(k_y\) and is an even function of \(k_z\), we can write Equation (A.1) as

\[
\frac{\mathcal{S}}{k^* g T} \sum_n -E g (a_n -1)/2 k^* g T \frac{4}{(2\pi \alpha)^2} \int_0^\infty -\frac{\hbar^2 k_z^2}{(2m^* a_n k g T)} = N_e .
\]

(E.5)

Evaluating the integral over \(k_z\) and writing \(\lambda = (\hbar/m^* \omega_c)^{1/2}\) leads us to Equation (2.22).
Fig. 2. Magnetoresistance ratio and normalized Hall coefficient versus $a = \frac{\hbar \omega_c}{k_B T}$ for the nonparabolic band model (solid curves) and parabolic band model (dashed curves) of n-type InSb at temperature $T = 77$ K, assuming electron-acoustic phonon scattering to be the dominant mechanism of scattering.
REFERENCES


[21] See, for example, S. Debenedetti, in: Nuclear Interactions, John Wiley and Sons, New York 1964 (pp. 331-335).


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