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Self-Reinforcement in the Classroom: Assessing Efficacy, Durability, Prior History and Unmerited Self-Reward

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SELF-REINFORCEMENT IN THE CLASSROOM: ASSESSING EFFICACY, DURABILITY, PRIOR HISTORY AND UNMERITED SELF-REWARD

by

Mark Henry Lewis

A Thesis
Submitted to the
Faculty of The Graduate College
in partial fulfillment
of the
Degree of Master of Arts

Western Michigan University
Kalamazoo, Michigan
December 1976

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The author wishes to gratefully acknowledge the assistance and cooperation of Professors Neil D. Kent, Howard E. Farris, and Kathleen A. Lockhart. I am especially grateful to Dr. Lockhart for a great deal of valuable academic, professional, and personal guidance. Additionally, I would like to acknowledge the assistance and cooperation of Ms. Nadine Bawkey, Principal, Schoolcraft Elementary School, Ms. Jean Elzey, classroom teacher, and Messrs. Frank Barefield, Robert Leopold and Brian Shumway, who served so willingly as experimenters.

Mark Henry Lewis
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CHAPTER I
REVIEW OF THE LITERATURE

The major social institutions in this country have long devoted much attention to the subject of self-control. Its importance is stressed both in our religious and in our educational systems as well as in our child-rearing practices. The reason for this should be obvious. Those who cannot manage their own behavior are in need of external, societal supports which often postpone the acquisition of effective behavior perpetuating the need for continued reliance on external agents (Skinner, 1976). It would seem desirable, then, to teach people early in life to guide and direct their own actions competently and successfully. In order to accomplish this, various components of the self-control process must be examined experimentally so as to provide teachers and other mediators of social norms with some understanding of the principles of self-management and some guidelines as to how they may be efficiently and effectively acquired.

Applications of an operant technology for use in such experimental investigations have become increasingly frequent (Mahoney & Thoresen, 1974; Thoresen & Mahoney, 1974; Watson & Tharp, 1972). This model assumes

"...that self-control is a set of learned skills, that a technology of teaching and learning those skills and applying them in one's own life is possible, and that the effectiveness of such a technology can be demonstrated." (Graziano, 1975, p. 530)
Much of the research generated from this model has addressed itself specifically to the issue of self-reward or self-reinforcement. Although there appear to be some discrepancies as to what is characterized as self-reinforcement, this typically involves the self-administration of reward contingent on meeting some performance standard, either self- or externally prescribed. The assumption here is that self-reinforcement should have the same effect on the acquisition, maintenance and extinction of responding as does external reinforcement. A number of studies, both laboratory (Kanfer & Marston, 1963; Kanfer & Duerfeldt, 1967; Bandura & Perloff, 1967) and applied (Lovitt & Curtiss, 1969; Glynn, 1970; Bolstad & Johnson, 1972), have demonstrated that contingent self-reward is as effective as external reward in the acquisition and maintenance of behavior. This appears to hold true for self-administration of rewards and/or self-determination of performance standards. Further, data indicate that self-reward rates generally tend to parallel prior external reward rates (Kanfer & Duerfeldt, 1967; Bolstad & Johnson, 1972; Johnson & Martin, 1973); that is, when subjects begin to dispense their own rewards only slight average increases in frequency or magnitude of reinforcement occur across sessions.

Discrepancies arise, however, when one looks at the non-reinforcement of responses previously self-reinforced versus those externally reinforced. For example, Johnson (1970) hypothesized greater generalization and greater resistance to extinction for
self-reinforcement via (a) the enhanced discrimination of the reinforced behavior and (b) the conditioning of self-evaluative responses as secondary reinforcers. Using a matching-to-sample discrimination task, measures were taken of the effects of external versus self-reward on the attending behavior of first- and second-grade boys. Results indicated greater initial resistance to extinction for the self-management group with no significant differences on a measure of generalization.

Additionally, by teaching self-evaluative responses, Johnson and Martin (1973) attempted a modified replication of the above study in an effort to clearly demonstrate greater resistance to extinction for behaviors maintained under self-imposed contingencies. The results paralleled the earlier finding and demonstrated only initially greater durability and only in the presence of one of the schedules used in the study (FR-3). Further, the return to baseline condition consisted of only two sessions.

Unfortunately, the two laboratory studies cited above contain serious methodological deficits that render their findings questionable. These problems include verbally announcing withdrawal of reinforcement in one group but not in the others, varied presence of the experimenters in different conditions, and the external reward of self-evaluative responses within a self-reward condition (Thoresen & Mahoney, 1974).

A later study, also done in the laboratory (Weiner & Dubanoski, 1975), further attempted to examine the issue of
relative extinction rates. The authors used 172 children (grades 2-4) to examine the effect of self-selection versus external selection of performance standards on relative rates of extinction. When measuring time to extinction criterion, performances for subjects allowed to select one of three reinforcement schedules showed greater resistance to extinction than the performances of their yoked partners. This proved true only under one schedule (FR-4), however. A multiple comparison test was not reported for the second dependent measure, defined as number of responses emitted to extinction criterion.

Examinations of the effects of self-reinforcement on the acquisition, maintenance, and extinction of responses would seem particularly appropriate in a classroom situation. First, it is here that self-management skills might best be acquired. Secondly, an increased emphasis on individualizing curricula for students in public schools has brought a concomitant increase in the amount of time and effort that teachers must devote to classroom management. Although many teachers grant that reinforcement systems may be quite effective in maintaining and accelerating students' academic performance, it is questionable whether they have the time to administer such a system properly for a classroom of twenty-five or more children. If teachers are to be persuaded to use consistent reinforcement with their students, a method must be devised that accomplishes the goal of maintenance and acceleration of academic behaviors yet does not add significantly to teaching workloads. It
would seem that self-reinforcement procedures would be one such method. Thirdly and perhaps most importantly, it would provide children with the opportunity to successfully manage their own behavior, perhaps for the first time.

In just such an attempt at classroom application, Bolstad and Johnson (1972) found self-reinforcement procedures as successful as external reinforcement in reducing the disruptive behavior of school children. This study failed to demonstrate the hypothesized result of greater resistance to extinction for self-management, however, and failed to control for differences in magnitude of reinforcement via a yoking procedure.

Using on-task behavior as the dependent measure, Glynn, Thomas, and Shee (1973) replicated an earlier finding (Glynn, 1970) suggesting the equal effectiveness of self-reward and external reward. The authors additionally reported less variability in on-task behavior under the self-control conditions. Goodlet and Goodlet (unpublished, 1969) further supported findings that both external and self-reinforcement conditions were equally effective in a study done with 10 year-old disruptive children. The above three studies introduced the self-management condition only following prior history with external reward. To assess the effects of instituting self-reinforcement procedures without previously imposing external contingencies, Glynn and Thomas (1974) used on-task behavior and found self-control techniques ineffective until paired with a cueing procedure.
In examining self-reinforcement in 14 year-old residents of a psychiatric hospital school, Kaufman and O'Leary (1972) found that under the self-reward procedure (following externally imposed contingencies) low rates of disruptive behavior were maintained, but that the residents generally awarded themselves the highest rating possible. There was no significant correlation between the pupils' evaluations and the teacher's ratings. It should be noted that the self-control condition remained in effect only seven days.

Using the same population, Santogrossi, O'Leary, Romanczyk, and Kaufman (1973) failed to replicate the findings of the above-cited study. Instead, they found that a self-reward phase alone failed to decrease the rate of inappropriate behaviors. Within four days of the onset of self-control, which followed a teacher imposed condition, children exhibited high rates of disruptive behavior while awarding themselves the highest number of tokens possible. It might be noted that the lack of a return-to-baseline condition failed to demonstrate the effect of reinforcement and there was no adequate control for differences in magnitude of reward (e.g., yoked conditions or yoked partners).

In the preceding two studies, the source of evaluations and subsequent reinforcement was abruptly transferred. The effects of gradually transferring this locus of control were studied by Drabman, Spitalnik, and O'Leary (1973), utilizing disruptive third-grade boys and assessing generalization via a focus on teaching self-evaluative behavior and by training matching-to-
teacher evaluations. Under these stimulus conditions, significant generalization was obtained with low rates of disruptive behavior and a few instances of unmerited self-reward.

The control of disruptive and on-task behaviors was further examined in a study by Frederiksen and Frederiksen (1975). Using 14 sixth and seventh grade special education students, the authors demonstrated the long term (11 weeks) effectiveness of self-determined token reinforcement following a teacher-determined reinforcement condition. Additionally, self-assessments were reportedly highly correlated with teacher assessments. The authors suggest a functional relationship between the effectiveness of the self-control procedures and an extended (13 weeks) history of teacher-determined reinforcement. Further, they cite lack of assessment of teacher behavior as a possible source of invalidity.

In terms of evaluating the efficacy of self-control as a classroom management procedure, it would appear critical to examine its effects on academic performance. Only five such published studies have been noted by this reviewer. Lovitt and Curtiss (1969) analyzed the effect of child-specified versus teacher-specified contingencies on the academic response rate of a 12 year-old student. Results indicated an increased rate of correct responses in several academic areas under the self-reward condition and support the authors' earlier finding (unpublished, 1968) that students' rates of responding were greater during choice periods than no-choice periods. These results have not, as
yet, been replicated and subsequent statistical analyses suggested no significant differences in rate for the two conditions (Glass, Willson, & Gottman, 1975). In addition, the superiority of the child-specified condition was confounded by a concomitant increase in magnitude of reward. The validity of a post-facto control used by Lovitt and Curtiss to offset this confound may be questioned on the basis of multiple treatment interference (Campbell, 1969).

Also using academic rate (number of correct items/time) as the dependent variable, Glynn(1970) compared self-determined, experimenter-determined, and chance-determined treatments in a class of ninth-grade girls. Experimenter- and self-determined conditions were equally efficacious and, further, previous experience with token reinforcement was found to influence the subsequent rate of self-determined reward. However, the lack of random assignment to conditions, unequal mean performances in baseline plus the limited range of self-selection of performance standards (i.e., students could choose only a limited number of performance criteria) are methodological drawbacks to the above-cited study. Perhaps even more serious is the inclusion of self-correcting and self-administration of tokens in the experimenter-determined condition. The author failed to report the accuracy of grading or the presence or absence of appropriate token administration in this condition. Finally, token exchange was carried out at the end of each token phase, demonstrating the effects of promise of reward rather than contingent reinforcement.
Felixbrod and O'Leary (1973) examined changes in self-selected performance standards over time and the effects of contingent reward under self- and externally imposed performance standards. Rate of correct answers to test items served as the major dependent variable. Results indicated that contingent reward was equally effective whether performance standards were self- or experimenter-imposed. Also, there was a consistent trend toward more lenient self-imposed standards across sessions. This study appears to suffer from the lack of a chance-determined or non-contingent reward group, from the reactive arrangement of individual testing outside the classroom, and from its short duration of six sessions (a total of 120 minutes).

In a series of seven self-management projects with academically or socially handicapped elementary school children, Lovitt (1973) reported successful demonstrations of pupil management of "teacher" behaviors. Using responses per minute as the dependent variable, subjects performed such behaviors as self-selection of academic tasks, pupil-scheduling, self-recording and pupil-specified contingencies. Engaging in such behaviors appeared to accelerate academic performance and decelerate such behaviors as talk-outs and hitting. These demonstration projects most often involved single subject AB designs, however, and results must be interpreted with a good deal of caution.

Finally, Ballard and Glynn (1975), extended the applications of academic self-management to include creative story writing with
elementary school children. Fourteen third grade students served as subjects in a multiple baseline across behaviors design (Baer, Wolf, & Risley, 1968) that included number of sentences, number of different action words and number of describing words. Results indicated that self-assessment plus self-recording yielded no increases over baseline. When a self-assessment plus self-recording plus self-reinforcement package was used, there were significant increases in each of the three behaviors targeted. This otherwise well designed study might include sequence effects (Campbell, 1969) and the lack of systematic manipulations of the total treatment package as possible drawbacks.

A possible criticism of several of the studies examining self-reward (Drabman, Spitalnik, & O'Leary, 1973; Glynn, 1974; Glynn, Thomas, & Shee, 1973; Santogrossi, O'Leary, Romanczyk, & Kaufman, 1973) includes the amount and time of reward being specified to a greater or lesser degree by the experimenter, and, therefore, not qualifying as self-reward in any meaningful way. A proper study of self-reward would involve no control by the experimenter (or anyone other than the subject). However, this viewpoint implicitly assumes that self- and external control compose a dichotomy, and, in fact, are two separate phenomena which are a function of separate variables. Skinner (1953) and others have contended that all self-control (self-reward or otherwise) is a function of external variable, i.e., the environment, past history, etc. This view allows one to examine the control issue as a continuum, and where a
behavior is to be placed on the continuum is a function, in turn, of the degree to which the individual is able to directly manipulate the variables controlling his behavior. With respect to self-reward it is true that most studies have placed limits on the amount and time of reward, yet the subject is responsible for determining if he should receive a reward, and within limits, how much. If one argues that this is not self-reward on the grounds that the subject is under certain external constraints necessitated by practicality during the experimental situation, one must likewise demand that prior history, another external variable, have no role in the subject's actions, and this is clearly not the case. Controlling for history would be desirable, certainly, yet this is nearly impossible in the absolute sense when using human subjects. History may be manipulated as a part of the experiment, yet nothing can be done about the years of past training that have occurred prior to the experiment. Likewise, self-reward should at some future point also be examined systematically as a function of its location on the continuum.

The above argument notwithstanding, it becomes apparent from synthesizing the above-cited research that additional work is needed to establish fully the controlling variables in self-reward behavior. It appears that self-reinforcement procedures can be at least as effective as externally imposed contingencies in maintaining performance, if preceded by a period of external control. Few other variables have been systematically investigated in a classroom setting, however. The question of
relative rates of extinction generated by both self- and external reward is still an open one. The effect of prior history with external reinforcement appears to be a critical variable and this has not, as yet, been systematically examined. The important areas of generalization and maintenance (key ones in determining the cost efficiency of teaching self-control) have only been preliminarily explored with animals (Bandura & Mahoney, 1974; Mahoney & Bandura, 1972; Mahoney, Bandura, Dirks, & Wright, 1974) and the applied research leaves much to be desired methodologically (Johnson, 1970; Johnson & Martin, 1973). Finally, discrepant data exist as to the effects of unmerited self-reward on performance (Kaufman & O'Leary, 1972; Mahoney & Bandura, 1972).

The present study, then, attempted to examine some of these variables more closely using a functional analysis of behavior. The variables under examination included the relative efficacy of self- versus external reinforcement, relative extinction rates generated by both conditions, the effects of prior history with external reinforcement on subsequent ability to reliably self-reward, and the effects of unmerited self-reward on performance.
CHAPTER II

METHOD

Subjects

Twenty-three children (13 boys, 10 girls) from a conventionally organized fourth grade classroom served as subjects in the present experiment. The elementary school involved in the study was located in a predominantly white, rural, lower-middle class area of western Michigan and was chosen due to its participation in a performance contracting project that used faculty and students of the Psychology Department at Western Michigan University as resource personnel.

Subjects were randomly assigned to one of three experimental groups resulting in group sizes of 8, 8 and 7. Data analyses included only 22 subjects, however, with one child excluded as a result of extended absence from school. This resulted in group sizes of 8, 7 and 7.

Materials

Academic responses per minute served as the primary dependent measure and were obtained via performance scores on Hutchings' "low fatigue" addition algorithm (Alessi, 1974). Eighteen addition problems, each set up as a five column by seven row matrix of computer generated random numbers, comprised the daily material...
given to subjects. Zero was excluded as it would have introduced the conceptual notion of the identity element (Alessi, 1974). This format insured equivalent materials yet provided non-identical content, thereby controlling for level of difficulty across time (Campbell & Stanley, 1963). The experimental algorithm essentially consisted of continuously generated binary responses yielding permanent product rate data (see Figure 1). These provided a direct and reliable measure of the effects of experimental interventions while providing face validity for both students and teacher (Alessi, 1974). It should be noted, however, that the present study is one of the first uses of this algorithm as a dependent measure and further work is necessary to establish its sensitivity to the manipulation of independent variables.

Additionally, a second set of materials was provided subjects in order to test for generalization effects. A series of fifteen reading exercises referred to as "rate builders" was taken from the Scientific Research Associates (SRA) Reading Laboratory 11 A. All fifteen exercises appropriate for each child's reading level were used.

Preliminary Training

All students were exposed to the experimental algorithm via the author presenting Hutchings' addition algorithm lesson and Hutchings' addition algorithm review (Alessi, 1974). (See Appendix A). These consisted of two thirty-minute standardized presentations
Figure 1. Computed example of Hutchings' "low-fatigue" addition algorithm.
taught on the first two consecutive days of the study.

Prior to this the respective reading levels of the subjects were determined via administration by the author of the Starting Level Guide (SLG) which accompanied the SRA exercises.

Following preliminary training a multiple-intervention reversal design was initiated to examine the effects of experimental manipulations. This design allowed for both between and within subject comparisons.

Baseline

All subjects in all groups were exposed to an initial baseline condition in which they were asked to work on the addition algorithm problems as a daily ten-minute exercise. Subjects were instructed to work as quickly and accurately as they could. Following completion of this session relatively immediate feedback (within approximately ten minutes) was provided in the form of experimenters correcting each student's completed problems. SRA exercises were then distributed and an additional ten minutes was provided students. These exercises were then simply collected at the end of the allotted time. A decision was made later in this condition, however, to discontinue the SRA exercises.

The experimental sessions were held at the same time each day and the classroom teacher was instructed to avoid commenting on the study to the children and to respond to their comments as neutrally as possible. She was further asked to have the children
direct any questions concerning the study to the author.

Experimenters were instructed while in this phase to respond to correct solutions by saying "right" or "correct," and to reserve comment on incorrect solutions. This was done in order to control for the absence of external social reinforcement in later self-reward conditions. Although some argue for the existence of covert conditioned self-evaluative statements (Johnson, 1973), this control was used in the absence of the direct measurement of such phenomena.

Finally, all groups rotated through two experimental settings (classroom and cafeteria) as both settings would later be needed to separate self- and external reward groups. This separation was included to control for possible modeling confounds (i.e., the observation of subjects assigned to a different sequence of conditions). Experimenters were randomly assigned to the experimental settings.

Following baseline, generalized conditioned reinforcers in the form of points were made contingent on addition algorithm performance. Records of students' point totals were kept at the classroom teacher's desk. Individual students were allowed access to these point totals on request. Students were given the opportunity to exchange these points for back-up reinforcers following completion of each Tuesday and Thursday session. Back-up
rewards were selected from a reinforcement "menu" which consisted of items previously indicated by the children as being desirable. This "menu" included such items as marbles, colored pencils, notebooks, hair ribbons, etc.

Group I

Subjects assigned to Group 1 were given no experience with external reward but were placed directly into a self-reward (SR) condition, here defined as self-evaluation in the form of self-correcting plus self-administration of token reward. Here, students were asked to correct their own math problems and to be responsible for self-administration of points. The experimenter verbally presented the correct answer, with subjects being informed during the initial session of the two-points-per-correct-problem performance standard. This combination of self-correction plus self-administration of reward, while not encompassing all possible components of self-reward, was chosen in order to isolate and examine the effects of self-administration independent of self-determined rates of reinforcement or self-imposed performance standards.

In order to control for subjects' expectancies concerning possible punishment for unwarranted self-reward, the children were provided with slips of paper on which to record their point totals and these were collected separately from the math exercise sheets. Experimenters were informed not to attend to recorded point totals.
in the presence of subjects.

Finally, Group 1 subjects were physically separated from Groups 2 and 3 in order to control for modeling confounds.

**Group 2**

Group 2 subjects experienced an externally imposed condition (ER) in which the experimenter provided feedback as to correct answers just as in baseline. The experimenter then awarded points to subjects at the rate of two per correct problem. Students had the opportunity to exchange their points for back-up reinforcers following completion of each Tuesday and Thursday session.

**Group 3**

Group 3 subjects experienced exactly the same contingencies as those described for Group 2 students.

**Reversal**

In order to demonstrate experimental control, provide evidence for the efficacy of contingent token reinforcement, and test for relative extinction rates, Token I was followed by a withdrawal-of-token reinforcement phase. Conditions were identical to the initial baseline period described earlier.

**Token II**

Following a recovery of baseline performance rates, a Token II
condition was instituted for all groups. Group 1, previously functioning under self-reward, was now placed in an external reward condition. Here, as whenever an external control condition followed a self-control condition, the phases were yoked to control for differences in magnitude of reinforcement occurring as a result of unwarranted self-reward.

Group 2, previously functioning under an external reward condition, was now exposed to self-reward as previously described for Group 1.

Group 3 was again placed in an external reinforcement condition for the purpose of further manipulating prior history with externally imposed contingencies. This manipulation was designed to provide a measure of the effects of differential experience with external reward on the ability to reliably self-reward.

Token III

The final condition in the study allowed for a comparison of rates of responding under self- versus external reward conditions for each student in the counter-balanced groups (Group 1 and 2). Group 1 subjects returned to a self-reward condition while Group 2 students were placed back into an external reward condition. Group 3 children, after an extended period of external reinforcement, were given the opportunity to self-reward for the first time. The sequence of conditions is summarized in Table 1.
Table 1

The Sequence of Conditions for All Experimental Groups

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</tr>
<tr>
<td>2</td>
<td>B ER B SR ER</td>
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<tr>
<td>3</td>
<td>B ER B ER SR</td>
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CHAPTER III

RESULTS

Data were primarily analyzed as frequency of correct binaries per minute. Figure 2 indicates rates of responding for all three groups over all fifty-eight sessions of the study. Figures 3, 4, and 5 indicate rates of responding for Groups 1, 2, and 3 respectively.

Baseline

Data were recorded for twenty baseline sessions in order to attain the maximum rate of correct responding possibly attributable to practice effects. A one-way analysis of variance (ANOVA) for unequal sample sizes was performed on these raw scores. Results indicated no significant differences among groups for baseline performances ($F = .82, \text{df} = 2, 19$) suggesting the absence of any pre-experimental group inequalities. Additionally, an analysis of standardized achievement test scores in arithmetic (available before the study) also indicated no significant difference among groups ($F = .21, \text{df} = 2, 16$) further suggesting the absence of pre-experimental dissimilarities.

An attempt was made in this condition to use the SRA rate builder exercises as a secondary dependent measure to assess the relative generalization effects of self-reinforcement versus external reinforcement. These data proved highly variable, however, and the SRA exercises were discontinued after nineteen sessions.
Figure 2. Number of correct binaries per minute for all experimental groups.
Figure 3. Number of correct binaries per minute for Group 1.
Figure 4. Number of correct binaries per minute for Group II.
Figure 5. Number of correct binaries per minute for Group III.
Immediate feedback on performance was not feasible under the present experimental conditions and delayed feedback failed to generate more stable rates of responding. It appeared that children were attending largely to speed and not accuracy. This was further suggested by the observation that some children were finishing all fifteen exercises in approximately three to four minutes and appeared to be in competition as to whom could finish most quickly.

Token I

Figure 2 indicates changes in rate of responding as a result of the introduction of contingent token reinforcement. All groups exhibited some increase from baseline rates and a one-way ANOVA performed on raw scores resulted in the lack of significant differences among groups (F = 1.23, df = 2, 19). This result suggests the equal efficacy of self-reinforcement versus external reinforcement.

Further investigation of these data point to reduced variability or more stable rates of responding generated by self-reward subjects. A similar finding was suggested by Glynn (1970).

Reversal

As indicated in Figure 2 when children were informed that they would no longer be receiving tokens for correctly solved problems, decreased rates of responding resulted for all groups. A one-way ANOVA resulted in the lack of significant differences among groups.
(F = .24, df = 2, 19) for this condition. A closer inspection of these data, however, suggests that subjects previously functioning under externally imposed reward (Groups 2 and 3) exhibited a precipitous drop in performance level after two to three sessions of non-reinforced responding. This is in contrast to those subjects having experienced self-imposed reward (Group 1) whose performance was maintained over five non-reinforced sessions. Within the present experiment, then, self-reward subjects exhibited slightly greater resistance to extinction than did their external reinforcement counterparts.

Token II

The re-introduction of token reinforcement resulted in the recovery of previous performance levels for Groups 2 and 3, as indicated in Figure 2. Group 1, however, or those subjects previously functioning under self-reward, showed only a temporary recovery (two sessions) before demonstrating a rapid deceleration to below initial baseline rates of responding. This occurred in spite of the fact that, for Group 1 subjects, the Token I and Token II conditions were yoked to control for any differences in magnitude of reward. A comparison of overall differences among groups for this condition, using a one-way ANOVA, barely failed to yield significant results (F = 3.46, df = 2, 19).

Further inspection of these data (see Figure 2) suggests a replication of two earlier findings: 1) That responding generated
by subjects functioning under self-reward appeared less variable than for external reinforcement subjects and 2) that self-reinforcement and external reinforcement appeared to be of equal efficacy in maintaining academic behavior. This latter replication can be seen via a comparison of Group 2 and 3 subjects, all of whom entered this condition with similar histories of externally imposed reward.

**Token III**

As can be seen in Figure 2 the academic response rates of Group 1 did not recover, indeed continued to decline, despite placing these children back in a self-reward condition. Groups 2 and 3, however, continued to maintain previous rates of responding. Statistical analyses of these data resulted in the finding of a significant difference among groups (F = 5.19, df = 2, 19, p<.05).

An analysis of between group differences using an extended Tukey procedure revealed no significant difference between Groups 2 and 3 (q = .57, df = 3, 18). A comparison of Group 1 with Group 2 using the same procedure did prove significant, however, (q = 4.14, df = 3, 18, p<.05) while a Group 1 with Group 3 comparison barely failed to yield significant results (q = 3.57, df = 3, 18).

**Rate of Self-Reward**

The self-reward behaviors of self-correcting plus self-administration of token reward were examined using the number of
awarded tokens per correct problem as the dependent measure. It will be recalled that the children had been previously informed that they should award themselves two points for every correctly solved problem. As indicated in Figure 6, all groups awarded themselves points at a rate exceeding the established standard of two per correct problem. The two per correct problem standard is designated by the heavy black line in the figure. This unmerited self-reward (USR) typically took the form of 1) changing existing answers to correspond to correct answers or 2) students simply awarding themselves more points than were merited by performance. The former type of USR was the more frequent while the latter type usually occurred later in the self-reward condition.

As depicted in Figure 6, Group 1 subjects, or those children who had no prior experimental history with external reward, self-awarded at the highest rate (\( \bar{x} = 3.32 \) points/problem). Group 3 demonstrated the lowest rates of USR while having the most extended history with external control (\( \bar{x} = 2.38 \) points/problem).

It is also interesting to note that for all groups there were no trends toward increased rates of unmerited self-reward over time. This was in spite of the fact that no aversive consequences were directly manipulated contingent on evidence of USR or "cheating".

Unmerited Self-Reward

The finding of USR and its relationship to prior history with external reward was further examined via an analysis of the
Figure 6. Number of tokens awarded per correct problem for all experimental groups.
frequency of unmerited self-reward per available opportunity. This was done in order to control for artificially low rates of USR as a function of accuracy of performance.

As indicated by Figure 7, Group 1 had the highest frequency of USR, taking advantage of approximately 59% of all available opportunities. Group 2, who, unlike Group 1, had prior experience with external reinforcement, exhibited a lower frequency of USR than did Group 1, taking advantage of approximately 38% of all available opportunities. In the final condition, those subjects having the most extended experimental history with external reward (Group 3) engaged in USR approximately 49% of the time, while Group 1 remained fairly consistent taking advantage of approximately 54% of the available opportunities. Again, the lack of experimental history with external reward appeared to be functionally related to a greater probability of engaging in USR.

Both of the above analyses further suggest that unmerited self-reward apparently had no immediate deleterious effect on academic performance. It will be recalled that the equal efficacy of self-reward versus external reward was demonstrated in two different conditions despite the occurrence of USR.

Preference

Following termination of the study, the children were asked to indicate their preference for self-reinforcement versus external reinforcement. Nineteen of twenty-two students or 86% of the class
Figure 7. Frequency of unmerited self-reward per opportunity for all experimental groups.
stated that they would prefer to function under an externally imposed reinforcement system. Verbal reports generally indicated that the lack of preference for self-reinforcement was related to the opportunity to engage in unmerited self-reward.
CHAPTER IV
DISCUSSION

The present investigation yielded several findings. Supportive of previous research (e.g., Glynn, 1970) was the demonstration that self-reinforcement proved to be of equal efficacy in the maintenance of academic behavior when compared with external reinforcement. Additionally, it appeared that self-reinforcement was related to the emittance of more stable rates of responding, a finding noted in an earlier classroom study also using academic response rate as the dependent measure (Glynn, et al., 1973).

The relationship between self-reinforcement and reduced variability is an interesting one from two perspectives. In terms of the experimental analysis of behavior, the emittance of steady state behavior is considered most desirable (Sidman, 1960) and self-reinforcement, if its stability continues to be demonstrated, may prove to be a useful baseline from which to carry out experimental interventions. From an applied perspective, self-reinforcement may be potentially useful in reducing variability in the academic behavior of such target populations as children labelled hyperkinetic, behaviorally disordered or emotionally disturbed.

It may well be the case that self-reinforcing behavior requires greater attending behavior not necessary when an external agent is charged with evaluating and reinforcing responses. This increase in attending behavior may result in a more consistent
Another finding of this investigation was the somewhat greater resistance to extinction evidenced by the self-reward group. A number of studies (Bolstad & Johnson, 1972; Johnson, 1970; Johnson & Martin, 1973; Weiner & Dubanoski, 1975) have attempted to demonstrate this phenomenon and, like the present study, offer some support for increased maintenance. No conclusive statements can be made on the basis of these data, however, and further research is necessary to empirically demonstrate the potential of self-control procedures as an effective behavioral maintenance system.

It is not felt that increased relative rates of extinction were due to the emittance of conditioned positive self-evaluative statements or covert reinforcement, as suggested by some (e.g., Johnson & Martin, 1973). These behaviors may occur under external reinforcement as well as self-reinforcement. More probably, it may be functionally related to increases in attending behavior as described above.

Unmerited self-reward (USR) was found to occur for all groups in the study, although self-reward rates tended to parallel prior external reward rates for those groups having such experimental history. The lack of such history with external reward was found to be related to increases in rate of self-reward and frequency of USR per opportunity.

Contrary to what one might expect there was no evidence of increased rates of USR over time. This is somewhat discrepant with
Felixbrod and O'Leary's (1973) finding of a consistent trend toward more lenient self-imposed standards across sessions.

It appeared that the occurrence of USR did not result in decrements in academic response rate. Previous research has yielded inconsistent data with respect to this parameter. For example, Kaufman and O'Leary (1972) reported the maintenance of low rates of disruptive behavior in spite of subjects awarding themselves the highest rating possible. Santogrossi et al. (1973) failed to replicate this finding, however, reporting high rates of disruptive behavior following USR. In an infrahuman study, Mahoney and Bandura (1972) reported a similar finding (i.e., significant decrements in performance following the occurrence of USR). Within the present experiment, the maintenance of academic responding might well be attributed to the relatively small increases in the magnitude of self-awarded token reinforcement.

Perhaps the most obvious difference among the experimental groups involves the large discrepancy in rates of responding following the reversal condition. Here, self-reward subjects failed to recover previous performance levels when placed in an external reward condition; in fact, performance continued to decline even throughout the last condition (self-reward). The external reward groups, however, successfully recovered prior rates of responding and these were maintained throughout the final two conditions.

It may have been the case that the yoking procedure, in which each subject was reinforced at his or her mean self-reward rate,
was responsible for the decrement. The children were well aware that they were receiving tokens at a rate exceeding the expected two per correct problem. In fact, they were well aware that they were receiving more tokens per correct problem than were their fellow external reward, albeit non-yoked, counterparts. The realization that the magnitude of reinforcement was unrelated to their actual performance score may have had far more serious consequences than failing to control for differences in magnitude of reward via yoking (Bolstad & Johnson, 1972).

Verbal preference data suggested that 86% of the children serving as subjects in the study indicated a preference for external reinforcement conditions. Students' verbal reports generally suggested that their preference was based on avoiding the possibility of engaging in "cheating" or USR. These results are consistent with the findings of Rachlin and Green (1972) and Atkins and Lockhart (1975) whose data suggest that subjects will avoid a choice situation (delayed but larger reward VS immediate but smaller; self-paced VS instructor-paced quizzing) if one of the choices is immediately reinforcing but may have delayed aversive consequences. Opting for externally administered reward is seen as just such a "pre-commitment device" (Ainslie, 1974). Behavioral preference data are considered necessary, however, to adequately demonstrate this phenomenon.

It is felt that the present study suffers from several drawbacks. Included among these is the lack of a larger increment in rate of
responding following the presentation of contingent token reinforcement. This may well have been due to the limited use of potential reinforcers. Prizes were exclusively used and such potential accelerating consequences as free time, preferred activities, edibles, and money were considered either not feasible or inappropriate by the classroom teacher. Additionally, the lack of positive social reinforcement for performance, the use of experimenters in lieu of the classroom teacher, and the reinforcement of problems rather than binaries might also be included. A "ceiling effect" suggests itself as a final possibility. It may well have been the case that the demand characteristics present resulted in such high baserates. For example, asking the children to work as quickly and accurately as possible is much like the instructions used in standardized testing situations.

Another drawback to the investigation includes the failure to obtain a stable baserate for Group 3 before terminating baseline. Initiation of Token I was considered necessary, however, due to temporal constraints and the urging of the classroom teacher.

The findings of the present experiment suggest a number of areas for future investigation. Certainly, reduced variability in responding and slower extinction rates are two key parameters requiring a good deal more research. An analysis of attending behavior under both self- and external reinforcement might prove fruitful and may suggest some functional relationships with respect
to variability in responding and relative extinction rates.

Further analyses of USR or "cheating", particularly under self-determined rates of reward and self-imposed performance standards are warranted. It is felt that the experimental algorithm used shows considerable promise with respect to the precise measurement of cheating behavior as a function of a variety of independent variables. It should also provide a reliable assessment of the effect of USR on performance.

The use of an additional experimental algorithm such as Hutching's multiplication algorithm may provide a sensitive measure of the relative generalization effects of self-reinforcement versus external reinforcement. Clearly, the experimental algorithm may prove to be a most useful, reliable independent and dependent variable.
References


Farris, H. E. Personal communication, July, 1975.


Rachlin, H., & Green, L. Delay of gratification as simple choice. Unpublished manuscript, State University of New York at Stony Brook, 1972.


Watson, D. L., & Tharp, R. G. *Self-directed behavior: Self-
modification for personal adjustment.* Monterey, Calif.: 

Weiner, H. R., & Dubanoski, R. A. Resistance to extinction as a 
function of self- or externally determined schedules of 
reinforcement. *Journal of Personality and Social Psychology,* 
APPENDIX A

HUTCHINGS' ADDITION ALGORITHM LESSON
(Adapted from Hutchings, 1972)
I am going to show you the usual way of writing number facts and then another way of writing them.

You have all seen number facts written like this: \[ 7 + 8 = 15 \]

Well, they can also be written like this, using two small (half-space) numbers instead of the line and plus sign. \[ 7 \quad 185 \]

Do you still see the fifteen? (Point to both fifteens.)

I'll write the two examples next to one another.

Do you all see the fifteen? (Point.) \[ 7 \quad 185 \]

Let's look at another one. I can write "9 plus 5 is 14" like this \[ 9 + 5 = 14 \] or like this \[ 154 \]

Both of these say "9 plus 5 is 14".

Tell me what these say.

\[
\begin{array}{cccccccccc}
9 & 9 & 6 & 6 & 4 & 4 & 6 & 6 & 5 & 5 \\
+8 & 187 & +7 & 173 & +5 & 59 & +6 & 162 & +2 & 27 \\
\hline
17 & 13 & 9 & 12 & 7
\end{array}
\]

(Cal: on students; point to the full notation form when asking.)
The little number on the right* is understood to be in the one's place, as are 9 and 8.

The little number on the left* is understood to be in the ten's place.

In other words, this is the same as this (point from "big 7" to "little 7"). And this is the same as this (point from "big 1" to "little 1").

Now watch me write the following facts both ways.

\[
\begin{array}{cccc}
9 & 9 & 8 & 4 \\
+7 & 16 & +5 & +5 \\
13 & 13 & 53 & 59 \\
187 & 187 & 187 & 187 \\
\end{array}
\]

Look at the last pair. Are they different from the others? Note that there is no ten's place number and (do not draw until after saying this) there is no "little 1" on the left.

Let's look at another.

a) 4 Is there any ten's number here? b) NO!! (repeat)
   (Do not draw box until after asking question.)

   c) So will there be any little number on the left?

   d) 4 (Do not draw box until after asking question.)
      NO!! (repeat)
Again, \[4 + 3 \]
\[
\begin{array}{c}
7
\end{array}
\]

If there is no ten's place number, there is no "little number" on the left.

Now watch me write the rest of these.

Notice \[3 + 1\]
\[
\begin{array}{c}
4
\end{array}
\]

no ten's number here, so no "little number" here

But \[7 + 8\]
\[
\begin{array}{c}
15
\end{array}
\]

there is a ten's number here, so there is a "little number" here

Again, notice \[5 + 1\]
\[
\begin{array}{c}
6
\end{array}
\]

there is no ten's number here, so there is no "little number" here

But \[8 + 5\]
\[
\begin{array}{c}
13
\end{array}
\]

there is a ten's number here, so there is a "little number" here

\[
\begin{array}{cccccc}
5 & 5 & 6 & 6 & 1 & 1 \\
+5 & 10 & +8 & 18 & +7 & 18 \\
10 & 14 & 18 & 18 & 8 & 8 \\
\end{array}
\]
Now I am going to show you a special way of adding that uses only those "little numbers" on the right.

I'll say that again. (Repeat previous statement.)

This should make your addition very easy and accurate. It is a scientific method and many scientists do addition this way. Watch.

8 First, do you see that an example can be just number facts piled one atop the other? (Do not point with this question.)

8 OK! Here we go, starting at the top, writing facts as you learned and using only numbers at the right for addition.

8 a) Say, "The first fact we do may look a bit different because we do not have any little numbers yet." (Point)

8 b) Say, "This is the only time we will use two big numbers. In the rest of the example we use one little number and one big one."

8 c) Say, "Now, eight plus five is thirteen."

8 d) Write the thirteen, i.e., 753 in the example.

8 a) Say, "We've written the thirteen but we'll use only the three."

8 b) Draw arrow 7.

8 c) Say, "Three plus seven is ten."

8 d) Write the 10, i.e., 170 in the example.

8 a) Say, "We've written the ten but we'll use only 0."

8 b) Draw arrow 9.

8 c) Say, "Zero plus nine is nine."

8 d) Write the 9, i.e., 90 in the example.

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a) Say, "We've written the nine and look that's all we have this time because zero and nine is just nine. But that's OK because we only use the right-hand number anyway."

b) Draw arrow 8.

c) Say, "Nine plus eight is seventeen."

d) Write the seventeen, i.e., 187 in the example.

---

a) Say, "We've written the seventeen but we'll use only the seven."

b) Draw arrow 6.

c) Say, "Seven plus six is thirteen."

d) Write the thirteen, i.e., 163 in the example.

---

a) Say, "We've written the thirteen but we'll use only the three."

b) Draw the arrow 3.

c) Say, "Three plus eight is eleven."

d) Write the eleven, i.e., 183 in the example.

---

a) Say, "We've written the eleven but we'll use only the one."

b) Draw arrow 1.

c) Say, "One plus seven is eight."

d) Write the eight, i.e., 18 in the example.

---

Now we're at the key part. All we've done is use number facts. We haven't done any "in your head" work.

Nevertheless, we already know the answer! Watch.
The last little number on the right is the right half of the answer.

To get the left half, we just count the little numbers on the left that we didn't use. One, two, three, four, five; there are five of them, so the first half of the answer is five. The answer is 58.

Now watch me do another. Remember we use only the right side "little numbers". We will not bother to write the arrows anymore, just say

Now the last number on the right is a 2, so the right half of the answer is a 2! We get the left half of the answer by counting the little numbers on the left that we didn't use. One, two, three, four, five. There are five of them so the left half of the answer is 5. The answer is 52.

Now say the work for these with me as I do them at the board.
(Children do not copy this.)
Now copy these examples and do them by yourself. If you have any questions, ask me.

<p>| | | | |</p>
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<td>-9</td>
<td>+5</td>
<td>+8</td>
<td>+9</td>
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</tbody>
</table>

After most have finished, say, "Check your work with mine as I do them at the board."

After doing the examples, say, "Now, let's review."
I'll write the work for another one on the board. I want someone to raise his hand and tell me what the answer is.

6  
8  
8  
6  
6 plus 8 is 14
4 plus 9 is 13
3 plus 5 is 8  Say this part, do not write it, except as the half-space numerals.
8 plus 7 is 15
5 plus 5 is 10
0 plus 9 is 9
9 plus 3 is 12

(Point to box.) Who will tell me what the right side of the answer is and how he got it.

(Point to box.)

(Locate correct response.) Good! That's correct. The last little number on the right becomes the right side of the answer.

Who will tell me what the left side of the answer is and how he got it. (Locate correct response.) Good! That's correct, we count up the little numbers on the left for the left side of the answer.

Now what do you suppose we do if there is more than one column? That is, if there is another column at the left of the column you're adding. Like this

4  
7  
6  
8  
7  
184  
17  
67  
18

Can we still write our left-hand answer number at the bottom if there is more than one column? No, we can't.

When there's more than one column, each column can have only one number at the bottom (except for the very last column which does have the usual two).
So the single number that we put at the bottom is always the right-hand number.

(Write and point.)

8
175
151
163

What can we do with the left-hand number?

Would it make sense to throw it away? No, it's part of the problem. *So we will put it at the very top of the next column at the left. That way we don't lose it and it's still on the left side.*

Watch! (Write on board.)

\[
\begin{array}{c}
\sqrt{137} \\
6 \\
5
\end{array}
\]

Count the little number on the left with me. One, two, three, four. There are four of them so we write a 4 at the top of the next column.

Now when I start adding that column I will start with the four (4) first. Let's be sure you understand. (Repeat twice from the *.)

This is called carrying; some of you already understand it. Good. Carrying is very easy.

But carrying is very important. You must never forget to carry.

Look at these examples and tell me what to write at the top of the left-hand column. (Write on board.)

\[
\begin{array}{ccccccccc}
6 & 8 & 8 & 5 & 7 & 6 & 8 & 5 \\
5 & 7 & 7 & 1 & 6 & 62 & 1 & 83 \\
6 & 9 & 9 & 24 & 1 & 24 \\
8 & 154 & 4 & 5 & 1 & 82 & 4 & 82 \\
7 & 150 & 3 & 1 & 3 & 9 & 12 \\
4 & 33 & 4 & 47 & 4 & 38 & 6 & 30 \\
\end{array}
\]

(Do with volunteers from class at board.) Good, we write the left-hand answer at the top of the next column. (Repeat three times.)
Remember though that for the last column only, the left-hand answer number is at the bottom as though it were a single column.

Now, copy these examples and do them with me.

\[
\begin{array}{c}
7 & 6 & 7 & 9 & 8 \\
5 & 9 & 8 & 7 & 7 \\
8 & 7 & 4 & 7 & 6 \\
6 & 9 & 6 & 9 & 5 \\
8 & 3 & 8 & 3 & 9 \\
9 & 5 & 4 & 2 & 2 \\
\end{array}
\]

\[
\begin{array}{c}
4 & 7 & 8 & 5 \\
5 & 7 & 6 & 9 \\
8 & 7 & 6 & 2 \\
8 & 5 & 7 & 6 \\
8 & 3 & 9 & 5 \\
\end{array}
\]

Again, do you see that I always carry the number of tens to the top of the next column? (Point and illustrate example.) Except when there are no more columns. Then I write the number of tens on the bottom line as part of the answer. (Point and illustrate with each.)

Good! Are there any questions?

Now take these dittoed examples and do them by yourselves. If you have trouble, ask me for help.

\[
\begin{array}{c}
7 \\
6 \\
8 \\
4 \\
7 \\
6 \\
9 \\
6 \\
5 \\
1 \\
+3 \\
\end{array}
\]

\[
\begin{array}{c}
6 & 8 & 7 \\
4 & 8 & 3 \\
6 & 9 & 5 \\
+8 & 7 & 4 \\
\end{array}
\]

\[
\begin{array}{c}
0 & 6 & 3 \\
5 & 3 & 9 \\
1 & 8 & 1 \\
+5 & 8 \\
+4 & 2 \\
\end{array}
\]

\[
\begin{array}{c}
4 & 8 & 7 & 6 & 8 & 7 & 6 \\
9 & 8 & 7 & 6 & 8 & 7 & 6 \\
9 & 8 & 7 & 6 & 8 & 5 & 6 \\
6 & 8 & 7 & 6 & 9 & 5 \\
4 & 8 & 7 & 6 & 4 & 2 \\
+8 & 7 & 6 & 9 & 8 & 3 \\
+6 & 7 & 8 & 5 & 6 & 7 & 8 \\
\end{array}
\]

Be sure to make and place your numbers neatly!

(Allow time needed for most to finish.)

Now, I will do them. Check your work against mine.
(Do examples on board. Answer questions. Emphasize the need to write neatly and the need to count the "carry number" correctly, demonstrate the latter while doing the work. State that the carry number is always written in at the top of the column to which it is carried.)
HUTCHINGS' ADDITION ALGORITHM REVIEW

(Adapted from Hutchings, 1972)
REVIEW

We are going to review the new way of writing number facts which we practiced yesterday.

a) Say, "Remember that during the beginning of the example is the only time that we use two big numbers. In the rest of the example, we use one little number and one big number."

b) Say, "Five plus nine is fourteen."

c) Write the fourteen in the example as 14.

a) Say, "We've written the fourteen but we'll use only the four."

b) Say, "Four plus eight is twelve."

c) Write the twelve in the example as 12.

a) Say, "We've written the twelve but we'll use only the two."

b) Say, "Two plus six is eight."

c) Write the eight in the example as 8.
a) Say, "We've written the eight and we use just the eight."

b) Say, "Eight plus eight is sixteen."

c) Write the sixteen in the example as \( 186 \).

a) Say, "We've written the sixteen but we'll use only the six."

b) Say, "Six plus seven is thirteen."

c) Write the thirteen in the example as \( 173 \).

a) Say, "We've written the thirteen but we'll use only the three."

b) Say, "Three plus eight is eleven."

c) Write the eleven in the example as \( 181 \).

a) Say, "We've written the eleven but we'll use only the one."

b) Say, "One plus seven is eight."

c) Say, "The last little number on the right is the right half of the answer. To find the left half, we just count the little numbers on the left that we did not use. Who can tell me what the right half of the answer is? Eight! Right. Now, who can tell me what the left half of the answer is? Five! Right, the answer is 58."
a) Say, "Now let's try a bigger example. We are going to move faster this time because you have done so well."

b) Say, "Let's start with the right column (point to it). Seven plus five is twelve. (Write the twelve in the example as $7_2$.) Two plus six is eight. (Write the eight in the example as $6_8$.) Eight plus five is thirteen. (Write the thirteen in the example as $5_3$.) Three plus nine is twelve. (Write the twelve in the example as $9_2$.) We write the two below the right column and carry the three to the top of the next column." (Write the three above the second column.)

Say, "Now, when I start adding this column (point to second column), I will start with the three. Three plus seven is ten. (Write the ten in the example as $7_0$.) Zero plus eight is eight. (Write the eight in the example as $8_8$.) Eight plus seven is fifteen. (Write the fifteen in the example as $5_5$.) Five plus nine is fourteen. (Write the fourteen in the example as $4_4$.) Four plus three is seven. (Write the seven in the example as $7_7$.) We write the seven below the column. Then we count the tens: One, two, three tens. We carry the three to the top of the next column." (Write the three above the last column.)

Say, "Now our example looks like this (pointing to example). Who can tell me the numbers we are going to add next? Right. We are going to add the three and the eight."

Say, "Three plus eight is eleven. (Write the eleven in the example as $18_1$.) Who can tell me the numbers we are going to add now? Right. We are going to add the one and the six. One plus six is seven. (Write the seven in the example as $7_7$.) Who can tell me the numbers we are going to add now? Right. We are going to add the seven and the four. Seven plus four is eleven. (Write the eleven in the example as $14_4$.) Who can tell me the numbers we are going to add next? Right. We are going to add one and six. One plus six is seven. (Write the seven in the example as $67_7$.)

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Seven plus eight is fifteen. (Write the fifteen in the example as 15.) Now we write the five below the column. (Write the five below the third column.) Then we count the tens: One, two, three tens. Because there are no more columns, we write the three to the left of the five. (Write the three to the left of the five in the example.)

Now copy these examples and do them with me.

```
  8  7  3  5  2
  5  9  8  9  7
  3  8  2  2  8
  9  6  9  3  8
  7  8  5  9  6
  9  5  6  7  4
       5  8  6
```

Are there any questions? Good. Now take these dittoed examples and do them by yourselves. If you have trouble, ask me for help. Be sure to make and place your numbers neatly.

```
  6
  8
  7
  3
  3
  5  6  7  8  6  7  8  4
  8  6  7  8  6  7  8  9
  4  6  5  8  6  7  8  9
  9  3  9  7  6  5  9  7
  1  8  9  4  7  2  5  8
  7  8  7  6  5  8  7  6
```

(Allow time for most to finish.)

Now I will do them. Check your work against mine.

(Do examples on the blackboard. Answer questions. Emphasize the need to write neatly and the need to count the "carry number" correctly, demonstrate the latter while doing the work. State that the "carry number" is always written at the top of the column to which it is carried.)