A Comparison of the Hutchings' “Low-Stress” and Current Addition Algorithms for Speed and Accuracy in Two School Settings with Regular and Special Education Children

Laurence E. Rudolph
Western Michigan University

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A COMPARISON OF THE HUTCHINGS' "LOW-STRESS" AND CURRENT ADDITION ALGORITHMS FOR SPEED AND ACCURACY IN TWO SCHOOL SETTINGS WITH REGULAR AND SPECIAL EDUCATION CHILDREN

by

Laurence E. Rudolph

A Project Report
Submitted to the
Faculty of The Graduate College
in partial fulfillment
of the
Specialist in Education Degree

Western Michigan University
Kalamazoo, Michigan
August 1976
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There are so many people to thank, that I am not sure how to go about it. The many friends that have helped me to feel good about being me: Mickey Lutz, Alan Zamosky, Carl Robinson, and of course, Judye, Robin and Sidra; Jane Howard, Wendy Leys, Dave Fossum, Dan and Marsha Kane, Leslie Zoref, Karen Lascurettes, Chloe, Paul Coyne and Alice (Mary Pat) Dybsky; Bill Swartz, Cary Reisman, Steve Popuch, and a great many people from the south side of Chicago; Kenneth McClelland who was the first best teacher I ever had (there have been a few); Mr. Scott; Louis (Doc) Popuch; Sylvia Popuch; Mike and Millie O'Clock; Nick and Nina O'Clock; the Southeast Little League; Diane Neuhauser; Debbie Reisman; Rick Brown, Dave Fridling, Jay Morohan, Doug Brenner, Dave Goodfriend, Pete Taylor, Arnie (Steak) Sirlin, and the rest of the gang; Patsy Bulmash who I spent what seemed to be an entire lifetime with and helped me more than she will ever know; Henry Wolfe and Thelma Wolfe; Harry Griner who has showed me the simpler things in life; Sandy Hitzing; Julia and Hays, who have shown me what is is really all about; Kalamazoo, Michigan, the best place in the world to be (right now); Rae Perls, a friend and teacher; Dusty, the best friend I ever had; Marsha Morgan and Trisha Bentley, who will hate being in the same breath; Jane Lull and Candy White for being more than efficient, more than nice helping put up with it all; the entire Department of Psychology at Western Michigan University for showing that it does not always have to be aversive and a waste of time to go to
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Most of all there is my family; Dave, Millie, Steve and Maria. I could not have made it without them. I could not have come close.

Laurence E. Rudolph
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A COMPARISON OF THE HUTCHINGS' "LOW-STRESS" AND CURRENT ADDITION ALGORITHMS FOR SPEED AND ACCURACY IN TWO SCHOOL SETTINGS WITH REGULAR AND SPECIAL EDUCATION CHILDREN.

Western Michigan University, Ed.S., 1976
Psychology, general

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<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
</tr>
<tr>
<td>INTRODUCTION                                  1</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td></td>
</tr>
<tr>
<td>METHODS                                        10</td>
<td></td>
</tr>
<tr>
<td>Nature of the Study                           10</td>
<td></td>
</tr>
<tr>
<td>Instruments                                    11</td>
<td></td>
</tr>
<tr>
<td>Construction of Instruments                   11</td>
<td></td>
</tr>
<tr>
<td>Independent Variables                         12</td>
<td></td>
</tr>
<tr>
<td>Dependent Variables                            15</td>
<td></td>
</tr>
<tr>
<td>Subjects and Setting                          16</td>
<td></td>
</tr>
<tr>
<td>Training Procedure                            18</td>
<td></td>
</tr>
<tr>
<td>Experimental Environments                     19</td>
<td></td>
</tr>
<tr>
<td>Reinforcement for Attendance and Performance   20</td>
<td></td>
</tr>
<tr>
<td>Procedure                                      21</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td></td>
</tr>
<tr>
<td>RESULTS                                         23</td>
<td></td>
</tr>
<tr>
<td>Comparisons of the &quot;Low-Stress&quot; and Current Algorithm 23</td>
<td></td>
</tr>
<tr>
<td>Comparisons of Performances of Students With Different Handicap Status 49</td>
<td></td>
</tr>
<tr>
<td>Comparisons of the Effects of the Order in Which the Algorithms were Presented 49</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td></td>
</tr>
<tr>
<td>DISCUSSION                                     51</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td></td>
</tr>
<tr>
<td>REFERENCES                                     55</td>
<td></td>
</tr>
</tbody>
</table>
INTRODUCTION

This study was designed to investigate an alternative approach to elementary school mathematics instruction. An alternative is of timely importance for two reasons. First, the increasing amount of mathematic computation called for in the day-to-day routine of the average American citizen requires emphasis on such skills in the school curriculum. Second, several critics have noticed a decline in the level of measured competence of American students in mathematics skills. This combination of increased mathematical complexity of today's society and the failure of American students to acquire the rudimentary calculation skills necessary to deal with this complexity, creates a situation that is of major concern to the field of education, and to society as a whole.

The notion that American students are somehow actually decreasing in their abilities to deal with mathematics seems at first hard to accept in light of the recent emphasis on "New Math" skills. However, the Conference Board Mathematical Science National Advisory Committee on Mathematics Education, in their Overview and Analysis of School Mathematics Grades K-12 (1975, pp. 106-107) cite the following data on performance of the American students:

"The pattern of results for the Scholastic Aptitude Test used for college admissions decision making, is unmistakable and widely known. From 1962 to 1975, the mean score on the quantitative section of the SAT had declined each year. The total has been from a high of 502 to the present 472...Perhaps more significantly, the percentage of scores above 600 in mathematics declined 20.5 to 16.4."
A number of local studies have also verified the decline in achievement of basic skills during the late 1960's and early 1970's. For example, a study of the Stanford Achievement Test scores of fourth graders in a modern mathematics program in New Jersey schools showed a decline in mathematics subtest scores..."

A second indicator of the failure of American students to acquire mathematics skills can be seen by comparing performances of these students with students from other nations. In one survey, when ranked by level of mathematics skills, United States students finished second to last (Hutchings, 1972, p. 35). Japanese students ranked first and West German students last. Countries that ranked at the top of the list (specifically Oriental countries) may differ in certain cultural parameters that affect the performance of students of those countries.

However, a more specific and perhaps parsimonious interpretation of this performance difference may be found by examining the practices used to teach mathematics in different countries. Hutchings (1972) reports comparisons of the educational systems of Japan and the United States, with the conclusion that a likely cause of the apparent superiority of the Japanese students may in part be the early use of the abacus in the Japanese system. While the American approach to computation has been historically to drill on basic facts and the different algorithms used for computation, the Japanese approach has been to teach students to use the abacus, thus obviating such tedious drilling procedures.

An analysis of the differences in the skills needed for using the algorithm and abacus approaches to computation suggested several variables of possible importance to educators. First, it has been argued
that the use of the abacus decreases the demand for "Short Term Memory" (covert response chains) on the part of the student (Alessi, 1974, p. 14) thus lowering the possibility of any confusion during instruction and practice as well as the lowering of possibilities for careless errors in calculations. Second, it is speculated by Alessi (1974) that the performance of the American students may be due in part to conditioned negative emotional responses resulting from emphasis on the tedious drill required to perfect any precision skill. Boyle (1975) states that "The current practice of drilling or repetition in computational skills (e.g., multiplication tables, basic addition and subtraction facts) in contemporary mathematics curricula may well be conditioning students to develop negative emotional responses early in their academic endeavors." This situation is of concern because as Poffenberger and Norton (1959) state "...some attitudes involving mathematics once formed are unlikely to change" (p. 19).

It also is speculated that the decrease in practice time required for computation proficiency using the abacus allows the Japanese educators to spend relatively more time on advanced mathematical skills involving applications, basic concepts, generalization and theory (Alessi, 1974).

If the above analysis was accurate, one approach to correcting the situation for American students would be to teach the use of the abacus, thus copying the model of the Japanese. This possibility was supported by Hutchings (1972). Alessi (1974) also offers the possibility of the use of electronic calculators in place of the abacus.
However, instead of adopting an abacus or calculator approach, both authors prefer using an alternative algorithm. The use of alternative algorithms is more in line with American cultural expectations, but is also advantageous for at least three other reasons: a) algorithms can be accommodated more readily within the current mathematics instructional system used in the United States; b) algorithms do not involve the use of instruments more complicated than the traditional paper and pencil; and c) algorithms have historically retained their usefulness through the fact that their operation leaves a permanent record of the calculations performed. This last fact has particular advantages for the correctional aspects of the instructional systems. The use of the abacus or electronic calculator leaves no permanent product of the calculations of the student. In the event that an error is made, it is difficult or impossible to isolate the exact place that the error was made. Therefore, the entire chain of calculations must be rerun from the start. It should also be noted that the above comparison of the two instructional approaches (algorithm and abacus) suggests that the advantage (or difference) would be the amount of time and effort that it takes for the student to acquire the computational skills, and not the lack of devices to assist the computational skills once they are acquired, (i.e., availability of abacus, calculator or pencil and paper).

Therefore, it would seem that solutions to the problem would ideally incorporate the following advantages: a) ease of assimilation into the American educational system; b) relatively short amount of time
needed to teach computational skills; c) possibilities for teachers
to analyze errors made during the acquisition of computational skills;
and d) allowance for the student to compute the answers quickly and
accurately, with little response effort.

A system which appears to meet the above characteristics has
been developed by Lloyd B. Hutchings at Syracuse University. His
research (1972), and subsequent studies by Gordon (1972), Alessi (1974),
Dashiell (1974) and Boyle (1975) have shown this system to be a promising
alternative for calculation instruction. Students using this new pro­
cedure, called the Hutchings' "low-stress" algorithm (Hutchings, 1976),
have shown consistent superiority when compared to students using the
conventional procedure.

The Hutchings' "low-stress" algorithm is defined in terms of its
operation:

"Half-space notation uses numerals of no more than a half­
space in height to record the sum of two digits. With half­
space notation, the units portion of the sum of two digits
is written at the lower right of the bottom digit and the
tens portion is written at the lower left of the bottom digit...
The ones portion of the column sum is always the same as the
ones portion of the last two-digit sum...The tens portion of
the column sum is always the same as the number of tens re­
corded at the left of the column. These are simply counted
(Example A)...For a column in some multi-column exercise...
the total number of tens is no longer written in the tens place
of the first column's sum but instead at the tope of the next
column at the left (Example B).

(See the example on the following page.)
### Example A

<table>
<thead>
<tr>
<th>Column</th>
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<tr>
<td>6</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>5 9 + 6 = 15</td>
<td>1 0 5 + 5 = 10</td>
<td>1 5 9 + 6 = 15</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>7 0 + 7 = 7</td>
<td>1</td>
<td>7 0 + 7 = 7</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>1 7 + 3 = 10</td>
<td>1 3 7 + 3 = 10</td>
<td>1 0 7 + 3 = 10</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>8 0 + 8 = 8</td>
<td>1 0 8 0 + 8 = 8</td>
<td>8</td>
</tr>
<tr>
<td>3 8</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

(Hutchings, 1976, p. 221)
The Hutchings' "low-stress" algorithm meets three of the four above listed requirements for improved calculation procedures. It is similar in all basic didactic requirements to the present algorithm used in American schools. It is a "Full Record System" providing a permanent self-generated record by the student of each operation performed within the entire calculation chain. This allows for precise error pattern analysis by teachers and others (Ashlock, 1972). Finally, from the current available research cited above, it appears that the use of the "low-stress" algorithm results in many fewer errors, while requiring less computational time than the conventional algorithm.

Relevant Literature on Alternate Algorithms

Research in alternative algorithms for faster and more efficient calculation have not been reported often in recent literature. Three reviews of the literature by Hutchings (1972), Gordon (1972), and Dasiell (1974) have similarly concluded that, "relatively little of the existing literature has an immediate relevance" (Hutchings, 1972). Gordon reports, "References reflecting new developments or new ideas regarding algorithms for unassisted numerical computation are very scarce indeed" (1972, p. 14). Gray (1965) states:

"There is little data indeed from research studies upon which to base an evaluation of professional judgment on the effectiveness of current mathematics programs. There is a need, then, for carefully controlled experimental investigations of the effectiveness of programs based on mathematical principles to promote pupil growth as measured by achievement and understanding."

From the available research on development of new computational
procedures, it should be noted that several different algorithms have recently been proposed. Sanders (1971) presented an addition algorithm which partially meets the requirements listed above. The procedure developed by Sanders resembles that of the "low-stress" algorithm, with the exception that it lacks the "Full Record System" of notation.

The available literature on the "low-stress" algorithm conclusively supports its comparative superiority. The "low-stress" algorithm has been associated with highly significant main effects in all of the inferential studies reported up to 1975, including Hutchings (1972), Gordon (1972), Alessi (1974), Dasiell (1974), and Boyle (1975). The Hutchings' "low-stress" algorithm has been compared to the Current Algorithm (CA) in situations of reinforcement versus no reinforcement (Alessi, 1974); test versus no test situation (Boyle, 1975); under varying degrees of problem difficulty (Alessi, 1974); as well as the interactions of these variables. The consistency and levels of significance of the results of these studies suggest the "low-stress" algorithm to be a valuable subject for further investigation.

Alessi (1974) states that the Hutchings' "low-stress" algorithm has "...particular relevance in special education when teaching highly 'distractable' or disorganized students with inadequate attending behaviors...[this is] the author's opinion based on the logical approach of task analysis, and further research will support or reject support for these speculations" (p. 14). This suggestion forms the basis for the present study. This study will attempt to investigate the relative effects of the Hutchings' "low-stress" algorithm and the Current Algorithm (CA); when used by subjects selected from mainstream and non-mainstream...
populations; while working in distracting and non-distracting environments.

All previous experimental investigations comparing the "low-stress and the CA employed large group factorial designs with single observation sessions. This study is a first attempt to compare the two algorithms using a single subject design with repeated daily observations. This later factor is expected to allow a comparison of the differential acquisition effects as well as the final performance effects for the two algorithms.
METHODS

Nature of the Study

This study investigates three main questions. First, is the Hutchings' "low-stress" algorithm an effective procedure for teaching "distractible" students computational skills? Second, what are the effects of "distracting" and "non-distracting" environments on the performance of these students? Third, how comparable are the performances of these and other mainstream students when using either the "low-stress" or current algorithm within these different situations? A secondary question investigated by this study concerns the use of an individual subject research design in studying acquisition effects for the primary questions.

The design of this study was formulated with the following constraints: a) as little disruption as possible of the ongoing school program be imposed; b) the study be run before the beginning of the formal school day; c) the study be run almost entirely be the experimenter; and d) the financial burden be assumed solely by the experimenter. In order to prevent possible difficulties, a letter was sent to each of the parents of the subjects, informing them of the study, and asking permission for their child to participate.

With the above factors in mind, the study was designed to optimally discern answers to the experimental questions, while allowing for as little disruption of the regular school instructional activities as possible.
Because questions of interactions among variables in addition to main variable effects were being asked, and experimental design was chosen using a reversal procedure (ABABA) imposed over multiple elements. The conditions of distraction were alternated daily using the multielement baseline design (Ulman and Sulzer-Azaroff, 1975), while the algorithms used ("low-stress" or current) were reversed (ABABA) over the course of the study (Baer, Wolf, and Risley, 1968). Subjects were selected from populations of mainstream and non-mainstream students to form the two groups for this study.

Instruments

Requirements: Hutchings (1972) made some specific recommendations for the design of a measurement instrument for use in studies of computational speed and accuracy:

"It is required that variations in example forms which load for reading or eye movement skills be avoided, e.g., interrupted rows, but that a range of profiles, as might occur in lessons or general experience, be presented. It is required that applications of the identity element (0) be avoided, as these are considered to load for a distinct peripheral concept while contributing very little to demands upon memory-retrieval functions. It is required that a systematically balanced presentation of the universe of binary combinations be made" (p. 51).

Also the instrument should have face validity.

Construction of Instruments

In conforming to the above requirements, the instruments used in this study were fixed size addition problems. The problems were set on 8 1/2 by 11 inch paper, five per page, in two rows, with four
problems on the first row, and one problem on the second row. The problems were typed with an IBM Selectric typewriter using the Orator 10 element (for overhead images). There were three spaces between each integer in the addends, two lines between the addends in a problem, five spaces between problems in the same row, and seven lines between rows of problems (see Table 1). The size of the type and spacing of the problems is considered a possible factor in student performance, due to the use of student written responses in the body of the problem when using the "low-stress" algorithm. With the "low-stress" algorithm, the students need more space to write.

In compliance with the Hutchings' recommendations, the problems contained no zeros in the addends. The addends themselves were generated by a computer program, which was designed to construct addition problems by selecting addends at random. The problem array format selected for this study included three columns, each containing seven rows of digits, yielding a total of twenty binary computations per problem (the total number of binary operations equals the number of rows multiplied by the number of columns, minus one, given that all the rows and columns are filled).

**Independent Variables**

1. The algorithm used by the student to compute the daily exercises:
   a) Current Algorithm (CA), or b) "low-stress" algorithm, as defined by their respective operations.

2. The environment in which the student computed the daily exercises:
Table 1: Example of Daily Worksheet.
TABLE 1

EXAMPLE OF DAILY WORKSHEET

<p>| | | | | |</p>
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<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

---

3 3 6
4 8 5
6 9 3
1 7 4
3 6 4
2 5 7
6 8 4

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a) distracting setting, or b) non-distracting setting as defined by
the school situation in which the exercises were performed (see page
19).

3. Handicap status of the student in the educational system:
a) mainstream, or b) Emotionally Impaired (hyperactive/distractable)
as defined by the Michigan Special Education Guidelines (see page
17).

4. Response costs: a) no cost for errors, or b) cost for errors
as defined in the Procedure Section (see page 21).

Dependent Variables

1. Percent correct: the number of columns that the subject
computed correctly, divided by the total number of columns attempted
(always fifteen), and expressed as a percent.

2. Correct rate: the number of columns correctly added, divided
by the session length, and expressed as a ratio of columns correct
per minute.

3. Incorrect rate: the number of incorrect columns divided by
the session length, and expressed as a ratio of columns incorrect per
minute.

It should be noted that in scoring the papers of the students an
error carried over from one columns to the next was considered to be
one error even though that error might affect two or more column sums.
For example, if a column correctly added summed to fifteen, but in
error was summed to twenty-one, this single error would affect the total
in this and the next adjacent column. Such "chain reaction" errors are
easily discriminated, and were counted only once in scoring the results
of the study.

**Subjects and Setting**

The four students for this study attended seventh grade at the
Comstock Northeast Middle School. Comstock is an incorporated town-
ship of some 15,000 individuals, located approximately six miles from
Kalamazoo, Michigan (in the southwest corner of the state). The popu-
lation of Comstock is mostly caucasian, the standard of living for the
majority of the population is in the lower socioeconomic strata.

All four children were selected on the basis of poor math skills.
Teachers were asked to select those students they believed to be most
in need of supportive help in math. The files of the selected students
were searched for any objective evidence to support the selection by
the teacher. Scores on the math subtest of the Metropolitan Achieve-
ment Test and the Peabody Individual Achievement Test, and the Key Math
Test scores (when available) were used to support the teachers' reports.
The grade equivalents for these seventh grade students were between
the second and third grade levels in math related areas.

Since a knowledge of basic addition facts is considered a pre-
requisite to effective instruction in the "low-stress" algorithm, a pre-
test on basic addition facts was administered to the students for this
study. The pretest was comprised of fifty-two binary addition problems
(two single digit positive whole numbers). An arbitrary cut off of
95% accuracy was used for selection. Also, to insure that the students would be able to perform the necessary calculation within the time limit restraint of the daily sessions, an average of no more than five seconds per problem was used for a cut off selection figure. All of the four identified students were able to meet these criteria. Their scores are listed below.

<table>
<thead>
<tr>
<th>SUBJECT</th>
<th>% CORRECT</th>
<th>MEAN SECONDS PER PROBLEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stan</td>
<td>96</td>
<td>3.75</td>
</tr>
<tr>
<td>Todd</td>
<td>100</td>
<td>4.10</td>
</tr>
<tr>
<td>Dan</td>
<td>100</td>
<td>4.23</td>
</tr>
<tr>
<td>Kim</td>
<td>100</td>
<td>4.51</td>
</tr>
</tbody>
</table>

The students were selected from two populations within the school. Todd and Stan were from General Education classes (GED) and showed no school problems other than their poor math skills. Dan and Kim were selected from the School Adjustment Program, a Special Education, self-contained classroom for students who have been labeled emotionally impaired (EI), according to the guidelines established by the State of Michigan Department of Education (Public Act 198, Rule 340.1713). These guidelines are not explicit in their description of a student who qualifies as EI. In general, subjective evaluations by the classroom teacher, a school psychologist, a school social worker, and other professionals may be sufficient for labeling a child EI. Students who are labeled EI are usually behavior problem students with a high level of distractability, hyperactivity, and/or disruption of the regular classroom activities. The files of both Special Education children (Kim and
Dan) indicated observations of both "distractibility" and "hyperactivity" by classroom teachers and other school personnel.

**Training Procedure**

All students received similar instructional lessons in the "low-stress" algorithm and similar review lessons for the current algorithm. Two students (one from Special Education and one from General Education) started baseline with the current algorithm and later reversed to the "low-stress" algorithm, while two other students started with the "low-stress" algorithm and later reversed to the current algorithm. This counterbalancing was devised to check possible order effects due to a cumulative sequence of learning through the different algorithms.

The students were randomly assigned (by the use of assigned numbers and reference to a table of random numbers) to the two groups (with blocking across Special and General Education characteristics). While one group was taught the "low-stress" algorithm, the other group was given a review lesson on the current algorithm.

On the first day of training, each of the groups was given thirty minutes of instruction. On the second day each group was given a fifteen minute follow-up session to check and/or firm up their skills for the respective algorithm. These training procedures used written lesson plans that were essentially identical to those used by several other investigators using the "low-stress" algorithm for addition (Alessi, 1974; Boyle, 1975; Hutchings, 1972; and Dashiell, 1974).
For both groups, colored chalk was used to aid discrimination of student-written responses from the digits in any given sample problem. Discrimination of written responses is considered to be a more critical factor in the "low-stress" algorithm training sessions due to the increased use of such written responses with that algorithm. The colored chalk was also anticipated to be a possible factor in the maintenance of student attention (Alessi, 1974), and therefore was used to instruct in both the "low-stress" and the current algorithm.

During the course of the study, those students who were initially taught the "low-stress" algorithm were reversed to performance using the current algorithm, while those students who initially received review on the current algorithm were reversed to the "low-stress" algorithm. The procedures for training at the reversal point were identical to the above procedure.

**Experimental Environments**

The students were required to perform the addition tasks in two different environmental settings. For the sake of brevity, these environments will be called distracting and non-distracting. However, it is understood that distraction is a subjective term. What is distracting to one student may not be distracting to another. Distracting and non-distracting environments, as defined in this study, refer to certain characteristics of the school settings at the time of the day that the study was run. The "distracting setting" was designated as a section of the media center (library). This area was an open area and
was in constant use with various sorts of activities ongoing. Since the school was arranged according to an open space plan, there were no walls to separate this area from the other ongoing classes. At the time of day that the study was run, other students were on a free period in this area, and a great deal of student activity accompanied this free time period.

The "non-distracting setting" was designated as the "Cafetorium", a multipurpose room that was physically isolated from the rest of the school building. The room was not in use during the time of day that the study was run, and thus was relatively quiet in comparison with the media center.

**Reinforcement for Attendance and Performance**

It was decided that in order to maintain student interest for the extended length of this study, some artificial (non-intrinsic) means of motivation would be appropriate. Therefore, a token system was implemented. Marks were made on a 3 X 5 inch card with each mark equalling one cent.

Three of the students had previous experience with token reinforcement, and therefore needed no priming. The other student was quite willing to work under these conditions, so pre-sensitization was limited to verbal instruction. From the data, this appeared to be sufficiently effective.

Except where noted below, the following conditions were in effect for all the students throughout the study:
arriving on time 5 credits
arriving on time for all five sessions in a week 25 credits
each correct column added 1 credit *
completing the study 100 credits

(Each credit was worth one cent)

*This contingency was modified for Dan (see the Procedure Section for more specific information).

Procedure

The students were asked to meet the experimenter each school morning during the free activity period preceding "home room" (i.e., the first class of the day where attendance was taken). At that time the students were escorted to one of the two experimental environments described above. They were then asked to record on the back of the assignment sheet their name, the setting, and the date. They were instructed to turn over their worksheet and begin the task of computing the problems for the session on a signal from the experimenter. They were also instructed to raise their hands when finished with the assignment. Each performance was timed on a wrist watch by the experimenter, rounded to the nearest five seconds. After all students had completed the assignment, they were allowed to take their sheets to the desk where they could check their answers against an answer key.

During baseline and subsequent phases of the study, when the performance of a student had stabilized, the algorithm which the student was
using was reversed as described above.

The performance of one student (Dan) was so erratic, that an alternative procedure was implemented for this student only. A response cost procedure was selected to achieve experimental control in this instance. Under this additional condition, Dan still would be reinforced with one cent for each correct column added, as with the other three students. However, a penalty of one cent would also be levied for each column error that was made, unlike the other students. In this condition, the following contingencies were in effect:

<table>
<thead>
<tr>
<th>Columns Correct</th>
<th>Reward</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>14</td>
<td>13</td>
</tr>
<tr>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>7 or less</td>
<td>0</td>
</tr>
</tbody>
</table>

This set of contingencies remained in effect for the remainder of the study for this student only. Response cost procedures were not used with the other students.
RESULTS

The results of this study will be discussed in the following sequences: a) comparisons of the students' performances with the two different algorithms (Hutchings' "low-stress" algorithm for addition and the Current Algorithm); b) comparisons of the students' performances within the different experimental settings (distracting and non-distracting); c) comparisons of the performances of the students from the two different populations (Special Education and General Education); and finally d) any possible effects of the order in which the two groups were sequenced in the different algorithms ("low-stress" or Current Algorithm first). The comparisons will be made first in terms of general effects across students, and second by comparisons of specific intra- or intersubject differences.

Comparisons of the "Low-Stress" and Current Algorithm

Table 2 presents data on individual phases using either algorithm. This is presented separately for all students. For calculation accuracy, the results obtained across all students indicates greater accuracy using the "low-stress" algorithm as opposed to the Current Algorithm (CA). A consistent trend can be seen indicating an increasing quality of computation over time in favor of the "low-stress" algorithm.

The calculation accuracy (percent correct) is measurably higher in those phases where the "low-stress" algorithm was used as compared to the phases where the Current Algorithm (CA) was used. One exception
Table 2: Student Performances.
<table>
<thead>
<tr>
<th>Subject</th>
<th>&quot;Low-Stress&quot;</th>
<th>Current Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Phases</td>
<td>Phases</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Stan:</td>
<td>% Correct</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>Correct Rate</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>Incorrect Rate</td>
<td>0.07</td>
</tr>
<tr>
<td>Todd:</td>
<td>% Correct</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>Correct Rate</td>
<td>2.72</td>
</tr>
<tr>
<td></td>
<td>Incorrect Rate</td>
<td>0.12</td>
</tr>
<tr>
<td>Kim:</td>
<td>% Correct</td>
<td>94.2</td>
</tr>
<tr>
<td></td>
<td>Correct Rate</td>
<td>2.19</td>
</tr>
<tr>
<td></td>
<td>Incorrect Rate</td>
<td>0.11</td>
</tr>
<tr>
<td>Dan:</td>
<td>% Correct</td>
<td>72.2</td>
</tr>
<tr>
<td></td>
<td>Correct Rate</td>
<td>1.69</td>
</tr>
<tr>
<td></td>
<td>Incorrect Rate</td>
<td>0.51</td>
</tr>
</tbody>
</table>
is noted in phase one for Dan. As stated in the Procedure Section, Dan's performance was erratic during the early stages of the study (in conditions of both algorithms). The implementation of a response cost procedure appears to have affected his performance in such a way that the results obtained are similar to those obtained on the other students. Excepting Dan's performance in phase one, a comparison of all remaining phases for all students indicates a clear superiority of the "low-stress" algorithm. The average percent correct for all phases of all students ranged from 91.8% to 99.0% for the "low-stress" algorithm. The average percent correct for all phases of all students ranged from 69.1% to 89.0% for the CA (see Table 2). The average percent correct for all phases for all students combined yielded 92.7% for the "low-stress" algorithm and 78.6% for the CA (this last set of figures includes the first phase of Dan's performance).

Performances using the two algorithms can be compared by examining the frequency with which daily accuracy scores were obtained within the 15 possible accuracy scores obtainable (6.66% for each column correctly added). Figure 1 presents the frequency with which daily accuracy scores are distributed by algorithm and student. The distribution shows that the sessions for the "low-stress" algorithm have a higher frequency in the 90 to 100 percent range than those obtained on the CA. A higher frequency of scores below 90 were obtained for the CA than with the "low-stress" algorithm. Using 90 percent as a level of mastery, the percent of daily scores obtained over all sessions is 82 with the "low-stress" algorithm. The percent of daily scores above 90 percent for...
Figure 1: Frequency distribution of daily percent correct scores.
the CA is 19.

An inspection of the error rates for the four students (see Table 2) indicates results similar to the percent correct data. Except for phase one of Dan's performance (before response cost), the error rates for all students averaged for each phase, ranged from .19 to .02 columns incorrect per minute for sessions in which the "low-stress algorithm was sued. In sessions using the CA, the error rates ranged from .69 to .32 columns incorrect per minute. The average error rate for all students across phases using the "low-stress" algorithm was .10 columns incorrect per minute. Across phases using the CA, the average error rate was .43 columns incorrect per minute, or 4.3 times as high an error rate as observed with the "low-stress" algorithm.

There is no overlap of scores by algorithm type for the averages drawn across phases for percent correct or error rates (excepting the first phase of Dan's performance). The lowest percent correct for any phase of performance using the "low-stress" algorithm (91.8), is higher than the highest percent correct score for any phase using the Current Algorithm (89.0). The highest error rate for any phase with the "low-stress" algorithm (.19 columns incorrect per minute) is lower than the lowest error rate for any phase using the Current Algorithm (.32 columns incorrect per minute).

The results for the correct rate are not as clear cut. Referring to the figures of the individual performances (see Figures 1b, 2b, 3b, and 4b), the data for two of the students (Todd and Kim) resemble the results obtained for the percent correct and error rate, although this
Figure 1b: Correct and Incorrect rate for Subject 1 on daily assignments under different conditions of computation (current versus "low-stress" algorithms) and environment (distracting versus non-distracting).
Columns Incorrect Per Minute
Non-Distracting
Distracting

Columns Correct Per Minute

Sessions

Current
"Low-Stress"
Current
"Low-Stress"
Current
Figure 2b: Correct and incorrect rate for Subject 2 on daily assignments under different conditions of computation (current versus "low-stress" algorithms) and environment (distracting versus non-distracting).
Figure 3b: Correct and incorrect rate for Subject 3 on daily assignments under different conditions of computation (current versus "low-stress" algorithms) and environment (distracting versus non-distracting).
Figure 4b: Correct and incorrect rate for Subject 4 on daily assignments under different conditions of computation (current versus "low-stress" algorithms) and environment (distract-versus non-distracting).
is not as pronounced. The data for the two remaining students (Stan and Dan) indicates a trend of increase in correct rate over the course of the study, regardless of the algorithm used. When all phases for all students are averaged, the obtained results are 2.3 columns correct per minute for the "low-stress" algorithm, and 2.1 columns correct per minute for the Current Algorithm, a difference of ten percent.

Table 3 gives the variability for daily percent accuracy scores by algorithm phases of the study. The standard deviations of scores for all the "low-stress" algorithm phases was 4.5 and for all the CA phases was 12.6.

Comparison of Settings

Table 2 shows a) comparisons of the average scores by setting for each phase of each student; b) comparisons of average scores by setting across all students and phases; and c) comparisons of average scores by setting across students, phases, and algorithms. No consistent trend is seen in these data. In the first phase of Stan's performance (see Figure 1a), a great deal of variability is seen in the scores. Much of this seems to be accounted for by the difference in settings. The magnitude of this difference moderates over the course of the phase. Another such difference occurs in the final phase of the study for Todd. The accuracy of his performance for those sessions in the distracting setting was 21 percentage points lower than those sessions in the non-distracting setting (see Figure 2a). In the last phase of Kim's performance using the Current Algorithm (see Figure 3a), her accuracy was 9 percentage points higher in the distracting setting than in the
Table 3: Comparisons of student performances by settings.
### TABLE 3

**COMPARISONS OF STUDENT PERFORMANCES BY SETTINGS**

<table>
<thead>
<tr>
<th>Name</th>
<th>CA</th>
<th>LS</th>
<th>CA</th>
<th>LS</th>
<th>CA</th>
<th>LS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D</td>
<td>N</td>
<td>D</td>
<td>N</td>
<td>D</td>
<td>N</td>
</tr>
<tr>
<td>Stan</td>
<td>60</td>
<td>82</td>
<td>95</td>
<td>98</td>
<td>80</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Todd</td>
<td>83</td>
<td>75</td>
<td>96</td>
<td>93</td>
<td>70</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kim</td>
<td>79</td>
<td>76</td>
<td>93</td>
<td>95</td>
<td>76</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dan</td>
<td>76</td>
<td>80</td>
<td>75</td>
<td>80</td>
<td>79</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Dist.</th>
<th>Non-Dist.</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Low-Stress&quot;</td>
<td>94</td>
<td>93</td>
</tr>
<tr>
<td>Current Alg.</td>
<td>76</td>
<td>82</td>
</tr>
<tr>
<td>Both Alg. Together</td>
<td>85</td>
<td>88</td>
</tr>
</tbody>
</table>

*ALL SCORES ARE FOR PERCENT CORRECT*
Figure 1a: Percent of columns correct for Subject 1 on daily assignments under different conditions of computation (current versus "low-stress" algorithms) and environments (distracting versus non-distracting).
Figure 2a: Percent of columns correct for Subject 2 on daily assignments under different conditions of computation (current versus "low-stress" algorithms) and environments (distracting versus non-distracting).
Figure 3a: Percent of columns correct for Subject 3 on daily assignments under different conditions of computation (current versus "low-stress" algorithms) and environments (distracting versus non-distracting).
Figure 4a: Percent of columns correct for Subject 4 on daily assignments under different conditions of computation (current versus "low-stress" algorithms) and environments (distracting versus non-distracting).
Students using the "low-stress" algorithm showed less daily variability in their performances than when using the Current Algorithm, regardless of the setting in which they were performing. However, a difference is noted between distracting and non-distracting settings in the first phase of Dan's performance when using the "low-stress" algorithm. His accuracy performance was 8 percentage points higher in the non-distracting setting when compared to the distracting setting (82 and 74 percent). All other comparisons between settings within phases for all students using the "low-stress" algorithm yielded a maximum different of 2.7 percent or less in the average scores for those phases.

A comparison of all phases for all students using the Current Algorithm yielded the following averages in the percent correct scores: for the distracting setting, 78.6; for the non-distracting setting, 82.0 (a difference of 3.4 percent). The results for all phases of all students using the "low-stress" algorithm yielded these accuracy figures: for distracting setting, 92.7; for the non-distracting setting, 93.7 (a difference of 1.0 percent). For all phases of all students across algorithms, the results were 85.6 percent correct in the distracting setting, and 87.9 percent correct in the non-distracting setting (a difference of 2.3 percent).

From these data it appears that there was an impact on the performances of the students that coincided with the changes in environmental settings. This difference was observed to be of a relatively
small magnitude when compared to the impact of the different algorithms on the performances of the students. It would appear also that the impact of the settings may have been more pronounced with the Current Algorithm than with the "low-stress" algorithm, but no consistent trend was noted across the different students or within any one of the students.

The above data on environmental settings is given for accuracy performance only. The data for the rate performances (correct and incorrect columns per minute) are similar to that of the accuracy data, and thus need not be presented.

Comparisons of Performances of Students with Different Handicap Status

The levels of performance and magnitude of changes in performance vary for each of the individual students. However, these changes are not systematically associated with the presence of the handicap status of the student. As mentioned above, one student (Dan) did require an alteration in procedure (the addition of response cost). Dan was one of the students that was selected from the Special Education population. The other student from the Special Education population, (Kim) performed similarly to the other students who were from the General Education population.

Comparisons of the Effects of the Order in Which the Algorithms were Presented

The order in which the students performed the different algorithms (either Current Algorithm or "low-stress" algorithm first) did not
appear to affect the results obtained on their performances. The "low-stress" algorithm was consistently more accurate, and lower in error rate, regardless of the order in which the two algorithms were introduced. The magnitude of these differences does not appear to change with the different orders of presentation.
DISCUSSION

Present results indicate that student performance, when using the "low-stress" algorithm, are superior to those student performances using the Current Algorithm (CA). This superiority in performance does not appear dependent upon the handicap status of the student (whether Special Education or General Education); the level of distracting stimuli in the environment; nor the order in which the students were asked to perform the two algorithms (either "low-stress" or Current first). The degree and consistency of these differences is seen in both the accuracy of the student performances, and in the error rates of their performances. There is a similar but less substantial difference seen in the correct performance rate of these students. This superiority of the "low-stress" algorithm becomes apparent very early in the acquisition phase and maintains across some 40 daily sessions.

Aside from these objective measures, anecdotal observations of these students over the course of the study indicated a preference for the "low-stress" algorithm. A one point in the study, the experimenter was not able to monitor the sessions for two days. During these two days, both of the students that were using the Current Algorithm switched to using the "low-stress" algorithm. These sessions are in parentheses on the figures of the individual performances of these students (Kim and Todd). The student performances for these three sessions where they switched to the "low-stress" algorithm were all of 100 percent accuracy.
At those points in the study where the students were instructed to switch from the "low-stress" algorithm to the Current Algorithm, complaints were noted. No such complaints were noted when they were instructed to switch from the Current Algorithm to the "low-stress" algorithm.

Discussions with the teachers indicated that they observed a general uptrend in the performance and attitude toward math among these students. The teacher of the two Special Education students indicated a general overall increase in performance in all academic subjects. The teacher indicated that he believed this to be a factor related to the students' ability to perform competently in a situation with General Education students. This is an important point and worthy of further investigation.

The lack of noticeable change in student performances in the different settings indicates the possibility of the "low-stress" algorithm being useful in various types of school environments regardless of the level of distraction. This is of special interest to educators in "open space" schools where the noise levels are likely to be high. The distracting setting had a limited effect for either algorithm or type of student. The students appeared to adapt to the distracting environment over a period of several sessions. After this period their performances were similar to those recorded in the non-distracting setting. This point may have implications for special education practices which restrict the classroom environment for the so-called "hyperactive" or distractable student. Such procedures may by iatrogenic rather than
therapeutic, pre-empting the student's opportunity to learn to adjust to the more complex classroom environment.

The comparability of the students' performances from the two populations (Special Education and General Education) indicates the possible usefulness of the "low-stress" algorithm across mainstream or exceptional students. It would appear that any student capable of performing simple addition facts, could (with help) be taught to perform longer and more complex addition problems. This also indicates the feasibility of including in the regular classroom students once thought to be functioning at too low a level in math curricula. This would be an area of interest to models that emphasize mainstreaming of special education students.

Aside from offering evidence in support of the possible use of the "low-stress" algorithm in special education, this study also demonstrated the use of a single student research design. The ability to monitor the performance of an individual student over time, and to note trends in performance is seen by this author to be a useful tool when studying the effects of acquisition of skills. The comparability of the results obtained in this study to those obtained in other studies which used group-factorial statistical designs is an indication of the comparability of this type of design in answering similar questions in a different way (this is a useful way of replicating factorial design results to insure a greater degree of reliability and generality across educational settings).

A final note of importance to this author is that this study
investigated an alternative to a procedure to which no alternatives have been sought for several centuries. It would appear that the field of education viewed the computational algorithm as a given or constant. This study indicates that there is a very reasonable alternative to this procedure. Perhaps many other such variables in the field of education that we have assumed to be constant could benefit from a similarly refreshing analysis.
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